Study nr.
20242377

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Mathematical assignment

Question 1.1

We have a three asset portfolio, where the goal is to chose the weights that minimize the overall variance of the portfolio.

At the same time we have to bear in mind that we have the constraint of $W_A + W_B + W_C = 1$

The lagrangian formula forms to minimize the variance whilst adhering to the constraints:

$$\mathcal{L}(W_A, W_B, W_C, \lambda) = \sigma_P^2 + \lambda(W_A + W_B + W_C - 1)$$

First order conditions (setting derivatives to zero)

$$\frac{\vartheta\mathcal{L}}{\vartheta W_A} = 2W_A\sigma_A^2 + 2W_B\sigma_{AB} + 2W_C\sigma_{AC} + \lambda$$

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$$\frac{\vartheta \mathcal{L}}{\vartheta W_B} = 2W_B\sigma_B^2 + 2W_A\sigma_{AB} + 2W_C\sigma_{BC} + \lambda = 0$$

$$\frac{\vartheta \mathcal{L}}{\vartheta W_C} = 2W_C\sigma_C^2 + 2W_A\sigma_{AC} + 2W_B\sigma_{BC} + \lambda = 0$$

$$\frac{\vartheta \mathcal{L}}{\vartheta \lambda} = W_A + W_B + W_C - 1 = 0$$

The gradient vector of the function G is perpendicular to the constraint curve on all points along it.

Question 1.2

The purpose of the restraint being the sum of all weights equals to one, is the mimicking of an investment scenario. You want all of your money to be fully invested, because without it you can make any return. If this restraint was not introduced, then all weights would've been 0, because that's where variance would be the lowest. At the same time any leverage wasn't being introduced, so the weights wouldn't be able to be over 1 either.

Question 1.3

Now, suppose that the following quantities have been estimated: $\sigma_A^2 = 9$, $\sigma_B^2 = 16$, $\sigma_C^2 = 16$, $\sigma_{AB} = 3$, $\sigma_{AC} = 3$, and $\sigma_{BC} = -4$.

Consider the following constrained optimization problem:

$$\min_{w_A, w_B, w_C} \sigma_P^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + \\ +2w_Aw_B\sigma_{AB} + 2w_Aw_C\sigma_{AC} + 2w_Bw_C\sigma_{BC} \quad (3)$$

subject to

$$w_A + w_B + w_C = 1. \quad (4)$$

Solve this problem by finding the weights w_A, w_B, and w_C.

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The portfolio variance would thus look like

$$\sigma_P^2 = 9W_A^2 + 16W_B^2 + 16W_C^2 + 2 \cdot 3W_A \cdot W_B + 2 \cdot 3W_A \cdot W_C + 2(-4) \cdot W_B \cdot W_C$$

Simplified to

$$\sigma_P^2 = 9W_A^2 + 16W_A^2 + 16W_C^2 + 6W_A \cdot W_B + 6W_A \cdot W_C - 8W_B \cdot W_C$$

With a constraint of $W_A + W_B + W_C = 1$

Like in 1.1 we apply the lagrangian

$$\mathcal{L}(W_A, W_B, W_C, \lambda) = \sigma_P^2 + \lambda(W_A + W_B + W_C - 1)$$

Which gives the equation

$$9W_A^2 + 16W_A^2 + 16W_C^2 + 6W_A \cdot W_B + 6W_A \cdot W_C - 8W_B \cdot W_C + \lambda(W_A + W_B + W_C - 1)$$

Now we take the derivatives with regards to W_A, W_B, W_C $and\ Lamda\ respectively$

$$\frac{\vartheta\mathcal{L}}{\vartheta W_A} = 18W_A + 6W_B + 6W_C + \lambda$$

$$\frac{\vartheta\mathcal{L}}{\vartheta W_B} = 6W_A + 32W_B - 8W_C + \lambda = 0$$

$$\frac{\vartheta\mathcal{L}}{\vartheta W_C} = 6W_A - 8W_B + 32W_C + \lambda = 0$$

$$\frac{\vartheta\mathcal{L}}{\vartheta\lambda} = W_A + W_B + W_C - 1 = 0$$

To simplify the system, we eliminate $\lambda$by subtracting the W_C first order condition from the W_C first order condition. This works because λ appears identically in both equations, so it cancels, revealing the relationship between $w_B$and w_C. The reason why we do it with B and C rather than A and B/C is because B & C FOC's have $6W_A$ in common

$$(6W_A + 32W_B - 8W_C + \lambda) - (6W_A - 8W_B + 32W_C + \lambda) = 0$$

Then we distribute

$$= 6W_A + 32W_B - 8W_C + \lambda - 6W_A + 8W_B - 32W_C - \lambda$$

We can now cancel out $6W_A \; and \; 6W_A$

And $\lambda - \lambda$

Leaving us with

$$32W_B + 8W_B + (-8W_C - 32W_C) = 0$$

We combine the terms to get

$$40W_B - 40W_C = 0$$

Divide 40 and get:

$$W_B - W_C = 0 = W_B = W_C$$

We can now treat B & C as equals and we now only need to know what the weight of A is in order to be able to solve all weights.

We know that $W_A + W_B + W_C = 1$

Thus we can use this information to set up the equation

$W_A + 2x = 1$

To isolate the value of W_A we minus by 2x on both sides

$$W_A = 1 - 2x$$

We can now plug this into the first order condition and find the weights of all three variables

$$\frac{\vartheta \mathcal{L}}{\vartheta W_A} = 18W_A + 6W_B + 6W_C + \lambda$$

Remember that $W_B \; and \; W_C \; are \; treated \; as \; equals \; and \; as \; X$

$$18(1 - 2x) + 6x + 6x + \lambda$$

Simplifying = $18 - 36x + 12x + \lambda$ becomes $18 - 24x + \lambda$

We can now do the substitution in one of the other first order conditions, as we need them to cancel out lambda

$$\frac{\vartheta \mathcal{L}}{\vartheta W_B} = 6W_A + 32W_B + 8W_C + \lambda = 0$$

$$6(1 - 2x) + 32x - 8X + \lambda = 0$$

$$6 - 12x + 32x - 8x + \lambda) = 0$$

$$6 - 12x + \lambda = 0$$

We now subtract the two to eliminate lambda and solve for x

$$(18 - 24x + \lambda) - (6 + 12x + \lambda) = 0$$

$$12 - 36x = 0$$

$$x = \frac{1}{3}$$

$$W_A = 1 - 2 \cdot \frac{1}{3} = \frac{1}{3}$$

Meaning $W_A = \frac{1}{3} + W_B = \frac{1}{3} + W_C = \frac{1}{3} = 1$

Question 1.4

The difference between 1.3 and 1.4 is that 1.4 introduces another constraint, which requires another lambda term.

Variances are the same as in 1.3

Now, suppose that the following quantities have been estimated: $\sigma_A^2 = 9$, $\sigma_B^2 = 16$, $\sigma_C^2 = 16$, $\sigma_{AB} = 3$, $\sigma_{AC} = 3$, and $\sigma_{BC} = -4$.

Weight constraint is still $W_A + W_B + W_C = 1$

Target expected return = 18

$$10W_A + 20W_B + 20W_C = 18$$

Portfolio variance =

$$\sigma_P^2 = 9W_A^2 + 16W_C^2 + 16W_C^2 + 6W_A \cdot W_B + 6W_A \cdot W_C - 8W_B \cdot W_C$$

Because we have two equality constraints, two lagrangian multipliers will be used

$$\mathcal{L} = \sigma_P^2 + \lambda_1(W_A + W_B + W_C - 1) + \lambda_2(10W_A + 20W_B + 20W_C - 18)$$

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First order conditions

$$\frac{\vartheta \mathcal{L}}{\vartheta W_A} = 18W_A + 6W_B + 6W_C + \lambda + 10\lambda_2 = 0$$

$$\frac{\vartheta \mathcal{L}}{\vartheta W_B} = 6W_A + 32W_B - 8W_C + \lambda + 20\lambda_2 = 0$$

$$\frac{\vartheta \mathcal{L}}{\vartheta W_C} = 6W_A - 8W_B + 32W_C + \lambda_1 + 20\lambda_2 = 0$$

$$\frac{\vartheta \mathcal{L}}{\vartheta \lambda_1} = W_A + W_B + W_C - 1 = 0$$

$$\frac{\vartheta \mathcal{L}}{\vartheta \lambda_2} = 10W_A + 20W_B + 20W_C - 18 = 0$$

```
> cat("Sum of weights:", sum(w), "\n")
Sum of weights: 1
> cat("Expected return:", as.numeric(t(mu) %*% w), "\n")
Expected return: 18
> cat("Portfolio variance:", as.numeric(t(w) %*% Sigma %*% w), "\n")
Portfolio variance: 5.16
>
> cat("\nLagrange multipliers:\n")

Lagrange multipliers:
> cat("lambda1 =", lambda1, "\n")
lambda1 = -6
> cat("lambda2 =", lambda2, "\n")
lambda2 = -0.24
> cat("Sum of weights:", sum(w), "\n")
Sum of weights: 1
> w <- sol[1:3]
> w

0.2 0.4 0.4
> |
```

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Question 2.1

```
> print(format(round(X, 4), nsmall = 4))
      Intercept x1        x2
 [1,] " 1.0000" " 0.9148" " 1.8563"
 [2,] " 1.0000" "-0.7939" " 0.0292"
 [3,] " 1.0000" "-0.9954" "-1.5624"
 [4,] " 1.0000" "-1.5661" "-0.5250"
 [5,] " 1.0000" " 0.0245" " 0.2905"
 [6,] " 1.0000" "-0.4453" "-0.1761"
 [7,] " 1.0000" "-0.6943" "-0.4767"
 [8,] " 1.0000" " 0.8374" "-0.8831"
 [9,] " 1.0000" " 0.1891" "-0.3481"
[10,] " 1.0000" " 1.2689" "-0.0865"
>
```

Question 2.2

```
> dim(y)
[1] 10  1
> y
            [,1]
 [1,] 12.3155970
 [2,]  0.2111123
 [3,] -6.4599263
 [4,] -5.0539623
 [5,]  4.0159367
 [6,]  0.4382559
 [7,] -1.5706465
 [8,]  1.0984719
 [9,]  1.7673143
[10,]  5.7966909
>
```

Question 2.3

```
> xtx_inv_inv

[1,] 0.10524375  0.00604910  0.02381274
[2,] 0.00604910  0.16072153 -0.07549729
[3,] 0.02381274 -0.07549729  0.17709840
>
```

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Question 2.4

```
> dim(Xty)
[1] 3 1
> print(format(round(Xty, 4), nsmall = 4))
          [,1]
Intercept "12.5588"
x1        "35.0467"
x2        "35.3655"
>
```

Question 2.5

```
> dim(beta_hat)
[1] 3 1
> rownames(beta_hat) <- c("beta0 (Intercept)", "beta1 (x1)", "beta2 (x2)")
> print(format(round(beta_hat, 4), nsmall = 4))
                  [,1]
beta0 (Intercept) "2.3759"
beta1 (x1)        "3.0387"
beta2 (x2)        "3.9163"
>
```

Question 2.6

They are the same

```
> print(format(round(beta_hat, 4), nsmall = 4))
                  [,1]
beta0 (Intercept) "2.3759"
beta1 (x1)        "3.0387"
beta2 (x2)        "3.9163"
> lm_fit <- lm(y ~ x1 + x2, data = dat)
> coef(lm_fit)
(Intercept)          x1          x2
   2.375891    3.038726    3.916311
```

Factor Model - REITS

Introduction

The U.S Real Estate Investment Trust (REIT) market bears great significance in the American real estate market as well as the American economy. REITS are financial vehicles founded with the sole purpose of investing in real estate and its subsectors and distributing profits as dividends to the investors. REITS as equities allows for investors to get a direct financial exposure to the broad real estate market, which is otherwise a market for experienced and cash-heavy participants. The public offered REIT market consists of an equity market capitalization of 1.483 trillion as of November 2024 (REIT Industry Financial Snapshot, 2024) excluding mortgage REITS; an alternative form of REIT, that only engages in investments in mortgages. Public REITS owns 580.000 properties, which carries a gross asset value of 4 trillion dollars including listed and non-listed REITS. 170 million Americans reside in homes owned by REITS, which is approximately half of the population of the United States. (REIT by the Numbers, 2023)

REITS were introduced in the 1960s to make real estate investing more accessible to smaller investors. Created by legislation and accompanied by requirements to govern various aspects of a REIT's operations, income distribution, asset composition and shareholder structure. This includes amongst others the requirement to keep 75% of assets in real estate, cash or U.S treasuries, derive 75% of gross income from rent, interest on mortgages or real estate sales and to distribute 90% of taxable income to shareholders through dividends.

The inherent characteristics of REITS resembles those of fixed income investments, due to its predictable, cash heavy returns which form the basis for regular dividend payments. In addition to fixed-income-like structure, REITS are seen as a hedge to inflation concerns, as leases and market rents tend to follow the development of the consumer pricing index. Conversely investors are exposed to the change of underlying market conditions, which might lead to properties in the portfolio depreciating in value, rent levels decreasing or increasing vacancies.

Prior studies claim that REIT returns are driven by macroeconomic factors, such as GDP growth, employment and interest rates (Essa & M.S 2023). These factors amongst others influence real estate fundamentals, such as property demand, rental levels and financing costs, making REITS sensitive to broader economic trends. For instance, the literature posits that GDP growth drives demand across office, retail and industrial REITS, while employment levels directly impact residential and office REITS.

Another study suggests that this effect has been strengthened by the increased level of professionalism and institutional ownership (Essa & M.S 2023), which has been a trend in REITS for the last 25 years, as a result of the introduction of the REIT modernization act of 1999, making the investment of REITS more enticing to institutional investors.

This project will extrapolate on this prior literature, introducing a purpose of understanding the relationship between macroeconomic factors and REIT returns in the US for different subsectors of REITS.

Research question : To what extent do selected macroeconomic variables explain monthly returns of residential, office, and retail REIT portfolios in the pre-COVID period (1988–2019)

Methodology

In this study, AI tools (including ChatGPT) were used as a supportive aid for idea generation and inspiration for phrasing during the writing process. The AI outputs were treated as suggestions only and were critically reviewed and adapted where relevant. All analysis, argumentation, structure, and final wording of the thesis including interpretation of sources and conclusions were produced by me, and I take full responsibility for the content.

In order to investigate the relationship between REIT returns and the macroeconomic, a factor model will be employed. The purpose of factor models is to create a model, that explains the relationship between the factors and the dependent variable to an extensive degree in an unequivocal manner, that doesn’t impose a high degree of variance in the model. This allows for each factor to be interpreted in an isolated manner. The real-life application of such study could be utilized by portfolio managers, choosing their next investment. If the study proves to be statistical significant, it could help portfolio managers navigate which investment has the best risk profile, that matches their investment strategy. In this context, the risk profile is based on macroeconomic factors. This study will focus on three subsectors of REITS, specifically residential REITS, Office REITS and retail REITS. A portfolio manager might find, that office REITS bear the lowest sensitivity to a set of variables, thus making it an adequate investment for their portfolio.

The project is characterized by a confirmatory approach, as the purpose is to test predefined relationships between REITS and macroeconomic factors based on existing financial literature. The variables and existing literature will be accounted for later in this section. The project will be hypothesis-driven, meaning a hypothesis for the relationship between the direction of the returns and the macroeconomic variable will be outlined. Econometric methods such as multiple regression will be utilized to asses the fit of the hypothesized model to the data and diagnostic test will ensure the model fits the predefined assumptions.

The analytic framework behind this factor analysis is the Arbitrage Pricing Theory (APT), a theory that claims that asset returns can be explained by a set of factors, yet only regarding its systematic risk (Chen et al., 1986). The theory is closely related to the widely known asset pricing model of CAPM, which also claims that asset pricing can only be predicted by its broad exposure to the markets, which is given by its beta. These theories are based on the notions of Modern portfolio theory (Markowitz, 1952), which argues that investors can eliminate all unsystematic risk or firm-specific risk, by holding a well-diversified portfolio. This leaves investors with only a risk of being invested in the markets and the magnitude of this risk is characterized by its sensitivity to the markets given by beta. Individually curated factors models like one of this kind differs from the traditional Capital Asset Pricing Model, as it delves deeper into the underlying factors that governs the market movements, and attempts to isolate these factors in order to be able to preemptively predict expected asset returns. The arbitrage pricing theory rests on the principle of no-arbitrage, meaning they assume that markets are efficient (Fama, 1991). Any difference in valuations for the same cash-flow profile must mean that investors deem the two assets to have a differential in their underlying risk of these expected cash flows.

The Arbitrage Pricing Theory is well suited for identifying systematic risk factors within REITS, as real estate markets are heavily influenced by systematic risk factors of macroeconomic conditions.(Chen et al., 1986)

The factor model is expressed as:

$$R_i = \alpha_i + \sum_{j=1}^{k} \beta_{ij} F_j + \varepsilon_i$$

Where

$$R_i = The\ return\ on\ asset\ i$$

$$a_i = The\ intercept, which\ represents\ the\ asset's\ return\ independent\ of\ the\ factors.$$

Under this specification, the intercept α_i captures the average component of sector returns that is not explained by the included macro factors. A statistically different intercept does not imply mispricing by itself. It may reflect omitted risk factors (for example broader equity/REIT market exposure), sector-specific fundamentals, measurement or remaining misspecification. For this reason, $\alpha_i$is interpreted as unexplained average return rather than as evidence of arbitrage opportunities.

$$\beta_{ij} = The\ sensitivity\ of\ asset\ i\ to\ factor\ j$$

$$F_j = The\ value\ of\ factor\ j\ which\ represents\ a\ systematic\ economic\ varialbe$$

$$\varepsilon_i = The\ idiosyncratic\ error\ term\ representing\ the\ asset - specific\ risk$$

In a diversified portfolio the error term of the individual assets should be negligible

This model provides a flexible framework for capturing systematic risks, with coefficients quantifying the sensitivity of REIT returns to each factor. The inclusion of log transformations for variables such as GDP and M2 money supply aligns with findings by Chen, Roll, and Ross (1986), who demonstrated the significance of logarithmic changes for capturing proportional relationships in macroeconomic data.

Factor Selection

Criteria for Selection

The initial factors for the model have been chosen on its theoretical relevance, empirical evidence, and availability of high-quality data. A confirmatory approach ensures that selected variables align with established economic theories and prior findings in the literature.

Selected Factors and Rationales

Interest Rates

Interest rates is a key component In capital markets and a driving factor in the economy. The fed-funds effective rate will be used as a proxy for interest rates, as it represents an approximation of the borrowing costs for REITS and its comparable opportunity cost for alternative investments such as investments in treasury bills. If the returns of the REITS are in proximity to of the risk-free rate, investors could be more likely to invest in the risk-free treasuries. The underlying conditions of the operations and financing of the properties could also be affected by these conditions, as the income is being consumed by the costs of financing. Another influencing factor of interest rates could be its effect on the consumer behavior in the geographical area, where the REITS operate. This could lead to a change in the spending power of consumers, meaning the available funds for housing, office and retail spaces are expanded or constrained. Interest rates and the risk-free rate are direct inputs, in the calculation of capitalization rate, which is the discount rate used for valuations. An increase in capitalization rates for the same amount of income will lead to a decrease in property values.

Based on prior empirical studies a negative relationship is expected, meaning that an increase in interest rates will lead to a decrease in REIT returns and vice-versa (Ling & Naranjo, 1997)

Source: Federal Reserve Economic Data (FRED).

Inflation

Inflations impacts REITS performance, due to its inherent properties of increasing prices, which in sustained periods will lead to increased interest rates. This theory is the key argument of the fisher effect. (Fisher, 1932).

As argued earlier real estate investments and REITS are claimed to be inflation proof, as landlords can index their market rents with the consumer pricing index, meaning that the income is not fixed.(Chen et al. 1986) This carries with it the risk of higher vacancies in the properties, as tenants are not able to pay rent, which in sustained periods will force landlords to create leases with lower rents levels, thus effectively eliminating the claim of a inflation hedge.

The year-over-year (YoY) Consumer Price Index (CPI) is used as a measure of inflation.

Inflation is expected to have a negative relationship with REIT returns, but might have a high degree of multicollinearity due to correlation with interest rates.

Data Source: Bureau of Labor Statistics (BLS).

GDP Growth

Economic expansion drives demand for real estates, which should have a positive effect on REIT returns (Chaudhry et al., 2022). As the gross domestic product increase, this should derive from the underlying businesses in the economy thriving. This leads to a higher spending level for both businesses and consumers, which in turn is expected to drive the demand for real estate in terms of increased capacity for businesses and increased available funds for consumers to spend on housing. This latter claim will affect rental levels, but also capitalization rates, as housing demand increases, the supply is supposed to be constrained on the short-term resulting in a yield compression.

GDP growth is adjusted from quarterly to monthly data using a cubic spline method to maintain temporal consistency.

Data Source: Federal reserve of economic data (FRED)

Unemployment Rate

Unemployment rates affect disposable income and consumer spending, impacting residential and retail segments as well as an underlying indicator of the health of the office sector.

Unemployment is expected to have an inverse relationship with REIT returns. (Chaudhry et al., 2022)

Data Source: U.S. Bureau of Labor statistics : Wheaton (1999) emphasizes the adverse effects of unemployment on property demand.

Government Bond Spreads

The yield spread between 10-year and 3-month Treasury bonds captures economic growth expectations and is widely known as the term spread. It reflects investors perceptions of long- and short term economic conditions.

The yield curve is a visual representation of the relationship between long- and short-term treasury securities. The usual shape of the yield curve is an upward-sloping one, reflecting its typical concavity. Maturity being on the x-axis and yield being on the y-axis, this typical relationship means that investors require a higher yield on securities with a longer maturity - also called the term premium. In times of economic turmoil yield curves tend to be inverse, meaning that investors require higher yield on short-term securities as opposed to long-term ones.

As the bond markets are highly efficient and controlled by real-time trades, they can offer crucial insights to REIT returns and the underlying economic factors that affect it.

Empirical studies confirms a positive relationship between the term spread and REIT returns. (Chen et al., 1986)

Data Source: FRED.

Market liquidity

The M2 money supply, a measure of liquidity that includes cash, checking deposits, and easily convertible near money, is a relevant factor for a REIT model as it reflects the availability of capital in the economy. Liquidity conditions captured by M2 significantly influence borrowing costs, consumer spending, and investment flows, which are central to REIT performance.

When liquidity in the economy is abundant (high M2 growth), borrowing costs tend to be lower, and capital availability increases, enabling REITs to finance acquisitions and development projects more efficiently. Additionally, higher liquidity often boosts economic activity, driving demand for commercial and residential real estate, increasing rental income, and improving occupancy rates. Conversely, when liquidity tightens, borrowing costs rise, reducing REITs' profitability and dampening property demand.

Empirical studies support the inclusion of liquidity measures like M2 in REIT models. For instance, Chen, Roll, and Ross (1986) identified the money supply as a significant macroeconomic factor influencing asset returns. Similarly, Ling and Naranjo (1997) found that changes in monetary conditions, including liquidity measures, substantially impact real estate securities, highlighting the role of M2 in explaining REIT return dynamics.

Data Source: Federal Reserve.

Hypothesized relationship between factors and returns

Factor	Relationship	Rationale
GDP Growth	**+**	**Positive economic growth drives demand/cash-flows**
Employment	**+**	**More employment - more economic**

		prosperity, which means higher demand and purchasing power
Fed funds rate	**-**	**Higher cost of capital, means higher cap rates, essentially devaluing assets on balance sheet and making refinancing more expensive**
Inflation	**+/-**	**Multifaceted - higher inflation creates higher cap rates, but also higher indexation of rents possible**
Term spread	**+**	**Growth expectation**
M2 growth	**+**	**More liquidity in market - more transactions, increasing liquidity and driving prices on assets. Lower borrowing costs. Drives demand**

Data Transformation and Validation

To capture as many market dynamics as possible, whilst keeping the validity of the findings a monthly frequency was chosen. The study engages with well documented data such as macroeconomic factors, as well as public security returns, which means that there are no big trade-offs in terms of the frequency of the observations and the precision of the data.

Albeit some changes were necessary.

Choosing between inflation measures

Year-over-year figures for monthly inflation is preferred as it offers several advantages in the scope of this project. Firstly it is season-adjusted meaning it compares to periods in the same yearly cyclical pattern, which mitigates potential influence of seasonal patterns and eliminates any kind of bias that might be inferred, if one was unaware of these conditions. The year-over-year measure represents long-term trends, reducing the risk of potential short-term inflation shocks, that might have been apparent with month-over-month figures. This does not mean that unsustained Year-over-year price shocks in prices do not happen. If there is a substantial price shock in the base year, this might still appear as an anomaly on the Year-over-year figures, but generally because the prices development is aggregated over 12-months, the effect should be smoothened out. This means that the variance of year-over-year figures should be smaller than corresponding figures in month-over-month. This mitigation of variance should decrease the risk of heteroscedasticity in the econometric diagnostics.

GDP adjustment from quarterly to monthly data

GDP is reported on a quarterly basis, which leads to the need to convert these figures into monthly. This will be done through a cupic spline interpolation. This process generates a smooth estimate of the GDP values in the months in between the quarters.

The cupic spline interpolation method was chosen, as it mimics the natural development of the gross domestic product, as opposed to linear interpolation.

Log Transformations

To account for the exponential nature of GDP and M2 money supply, these variables will be converted into log-growth transformations. The two variables are exponential by nature, due to their inherent connection to inflationary pressure. Throughout history, inflationary periods has been substantially prevalent compared to those of deflationary periods. It’s a natural mechanic of economies.

Log-growth aligns with the underlying exponential nature of these variables, making growth figures over longer time periods more comparable and avoids overstatement of growth over long periods.

REIT prices are converted into monthly log returns for three reasons. First, log returns are time-additive, meaning multi-period returns can be aggregated by summing log returns. This is convenient when aligning financial returns with macroeconomic variables that are observed at monthly frequency and when comparing returns across different horizons. Second, log returns treat proportional price changes symmetrically and provide a more stable scale for regression modelling; for typical monthly return magnitudes, log returns are also very close to simple percentage returns, so the economic interpretation remains intuitive. Third, working at the monthly frequency reduces microstructure noise and day-to-day volatility that is less relevant for macroeconomic transmission, and it aligns the dependent variable with the frequency of the explanatory variables (GDP growth, inflation, monetary policy measures, etc.). We therefore compute monthly log returns from daily prices by taking the log of the ratio between the last and first observed price within each month.

Transforming the data into log-growth figures will be an advantage, if the growth rates span over a long time, thus allowing for compounding effects to take effect. Another scenario would be if changes are large, from year to year.

Transforming the data into log-figures increases it stationarity, a critical factor in Ordinary least squares for the regression model, as well as stabilize variance increasing the reliability of the regression model.

This method is consistent with other empirical studies in similar studies. (Chen et. al,. 1986),

To confirm that the GDP and M2 data is exponential the raw data will be plotted onto a curve. If the it is exponential developing, the log transformation will be plotted to confirm linearity

Scope and Sources

Data spans 30 years (from 1988-2019), covering multiple economic cycles. Whilst avoding the regime change of covid, which due to it's extremities might introduce dynamics into the inferences, that might not be aligned with how markets act in under normal conditions.

The independent variable data has been sourced from U.S Bureau of Labaor Statistics and Federal Reserve of Economic Data.

Data and Sample Construction

This study combines daily REIT price data with monthly macroeconomic variables to estimate sector-level factor models. Daily REIT prices are converted into monthly log returns by computing the log of the ratio between the last and first observed price within each calendar month. This study focuses on three subsectors of REITS, which are residential, office and retail, where each sector return series is constructed as the equal-weighted average of the individual REITs' monthly log returns. All macroeconomic series are converted to a monthly date index (first day of each month) and merged by month. Because macro series do not all start/end at the same date, the analysis uses the common available range and then applies explicit sample windows for estimation.

Linear Factor Model Specification

For each sector (residential, office & retail) the baseline empirical model is a linear regression of monthly sector returns on macro factors:

Here the dependent variable is the sector's monthly log return, and the regressors represent economic activity (GDP growth), labor market conditions (employment rate), monetary policy stance (Fed Funds level, inflation, term spread, and liquidity (M2 growth). The models are estimated separately for each sector using OLS.

Data Integrity and Alignment Checks

Before estimation, the merged monthly dataset is checked for date consistency and alignment across series. Each input series is sorted by date, converted to a month index, and merged using a full join on the monthly date. The final analysis dataset is ordered chronologically and checked for irregular gaps and duplicates.

A series of exploratory data methods will be employed to check the dependent and independent variables for outliers, anomalies and trends.

Diagnostics

Following estimation of the baseline sector regressions, the model residuals are evaluated using a sequence of diagnostic tests. This diagnostic workflow determines whether classical OLS assumptions are violated and whether specification adjustments or robust inference are required.

Diagnostic tests are evaluated at the 5% significance level. For the Breusch–Pagan test, rejection of the null of homoskedasticity indicates heteroskedasticity in the residuals. For the Breusch–Godfrey test (order 12), rejection of the null indicates residual serial correlation. These outcomes are used as decision rules for inference: when either heteroskedasticity or autocorrelation is detected,

Diagnostic Testing Strategy and Econometric Validity

Following estimation of the baseline linear factor regressions by ordinary least squares (OLS), a structured diagnostic procedure is applied to assess whether the assumptions underlying conventional OLS inference are credible in a monthly macro–finance setting. Because the dependent variables are asset returns and the explanatory variables are macroeconomic time series, violations such as heteroskedasticity, serial correlation, and functional-form misspecification are plausible. If present, these violations can lead to biased standard errors, unreliable hypothesis tests, and unstable coefficient interpretations creating potential false inferences about the relationship between macro-factors and REIT returns. The diagnostic stage therefore has two roles - one is to figure out the right way to handle standard errors and OLS inference, another is to catch useless variables early on, allowing for adjustments of these variables, such as introducing lags or other kinds of manipulations.

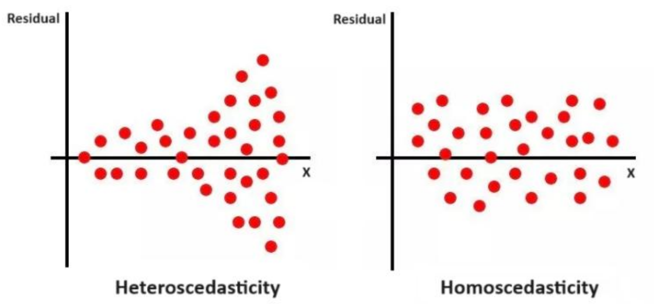
Baseline OLS Estimation as a Reference Specification

Each sector model is first estimated using OLS to obtain coefficient estimates and residuals. Under exogeneity, OLS coefficient estimates can remain unbiased and consistent; however, the usual t- and F-tests rely on additional assumptions about the error structure, including homoskedasticity and the absence of serial correlation. These assumptions are frequently violated in return regressions, particularly at monthly frequency, making residual-based diagnostics essential before interpreting statistical significance.

Heteroskedasticity: Breusch–Pagan Test

Residual heteroskedasticity is evaluated using the studentized Breusch–Pagan (BP) test. The null hypothesis is homoskedasticity, i.e., constant conditional variance of the disturbance term

$$H0: Var(\varepsilon t \mid Xt) = \sigma 2.$$



(figure 1)

Rejection of this null indicates that the variance of the residuals varies systematically with the regressors or across time. In the context of asset returns, heteroskedasticity is economically plausible due to volatility clustering and changing market conditions. While heteroskedasticity does not, by itself, imply biased coefficient estimates under exogeneity, it invalidates conventional OLS standard errors, motivating robust inference methods.

In this setting, exogeneity means that the part of REIT returns the model does not explain (the error term ε_t) is not related to the macro variables $X_t$used in the regression. In other words, there should not be a systematic link between the regression residuals and the included factors. If this holds, the estimated coefficients can be read as the sector's return sensitivity to each factor, rather than being driven by reverse causality or missing variables.

Autocorrelation: Breusch–Godfrey Test

Residual serial correlation is tested using the Breusch–Godfrey (BG) test. The test is run with 12 lags, which is a natural choice for monthly data because it allows correlation over roughly one year. The null hypothesis is that the residuals are not autocorrelated up to lag 12:

$$H0:\ \rho 1 = \rho 2 = \cdots = \rho 12 = 0.$$

If the null is rejected, it indicates that the residuals are correlated over time rather than behaving like random noise. In macro–return regressions, this can happen because macro effects take time to feed into asset prices, returns can be persistent, or relevant variables/dynamics are missing from the model. When serial correlation is present, standard OLS standard errors and hypothesis tests are not reliable, which supports using HAC/Newey–West standard errors for inference.

Functional Form and Misspecification: Ramsey RESET Test

Model specification is checked using Ramsey's RESET test. RESET is a general test for whether the chosen linear model leaves out important structure. The idea is to add extra terms based on the model's fitted values (typically squared and cubed fitted values) and test whether these added terms help explain returns. The null hypothesis is that the additional terms are not jointly significant, meaning the baseline model is adequate. If the null is rejected, it suggests that the model may be misspecified—for example because important variables are missing, the timing/dynamics are not captured (lags), or the chosen transformations are not appropriate. RESET does not identify the exact reason for misspecification, so a rejection is treated as a signal to consider a small set of theory-based linear refinements (such as adding lags or using changes in interest rates).

Multicollinearity: Variance Inflation Factors (VIF)

Multicollinearity is assessed using the Variance Inflation Factor (VIF). VIF measures how strongly each regressor is correlated with the other regressors in the model. High multicollinearity does not bias the coefficient estimates, but it increases standard errors and makes it harder to detect statistically significant effects. It can also lead to unstable coefficient estimates, where signs or magnitudes change across similar specifications. This is especially relevant when several variables capture related channels (for example, monetary policy rates, inflation, and yield spreads). When VIF indicates problematic overlap, the specification is adjusted in a theory-consistent way—for instance by replacing the level of the policy rate with its monthly change—to reduce redundancy and improve estimation stability.

Decision Rules for Remedies and Final Inference

The diagnostic tests guide how inference and specification are handled. If heteroskedasticity or serial correlation is detected, conventional OLS standard errors are not used. Instead, inference is based on Newey–West HAC standard errors, which remain valid when residuals are both heteroskedastic and autocorrelated. Though if only Heteroskedasticity is detected, one can use HC robust SE. If RESET indicates misspecification, a limited number of adjustments are considered that remain within linear regression, such as including lagged macro variables to reflect delayed transmission or using first differences (Δ) for interest rates to capture shocks rather than levels. If VIF indicates multicollinearity, overlapping variables are transformed or prioritized to improve coefficient stability. After any adjustment, the diagnostic tests are repeated to confirm whether model adequacy and stability improve, and the final reported results are based on the specification that is most defensible both economically and econometrically.

Results

The sampled data in the baseline dataset ran from 1988-04-01 till 2019-12-01 and included 381 data points.

The final model looks as follows

$$rs,t = \alpha_s + \beta_{1_s} GDP_{growth_{t-6}} + \beta_2, sEmployment_t + \beta_{3_s} \Delta Fed_t + \beta_{4_s} Inflation_t + \beta_{5_s} YieldSpreadt_t + \beta_{6_s} M2\_growth_t + \varepsilon_{s_t}$$

Baseline results - coeffecients and magnitudes

Appendix 2-4 shows the standard OLS models of the baseline data before any diagnostics have been performed and any further manipulations have taken place.

All three models has the expected relationship dynamics with GDP_log_growth, inflation rate, & yield spread

For both residential and retail employment rate has a controversial relationship in regards to what the theory links to, yet office aligns.

The opposite applies to m2 growth, here residential and retail has a relationship consistent with theory and office has the opposite.

The magnitudes of the coefficients looks realistic with fed_funds_rate, inflation and yield_spread having the largest contributors due to their non-log-conversions, as the figures are raw.

Diagnostics

Breusch-Pagan test

All three models (appendix 5-7) has p-values under 0,05, meaning that we can reject the null hypothesis homoscedasticity being apparent and assume that heteroscedasticity is the case instead.

Breusch-Godfrey Test

All three models (appendix 8-10) has p-values under 0,05, meaning that we can reject the null hypothesis of the regression residuals not being serially correlated and assume autocorrelation is apparent.

Ramsey's Reset Test

All three models (appendix 11-13) has p-values under the significance level meaning we can reject the null hypothesis of the model being adequately fitted and thus the model needs to be manipulated with additional factors or tweaked factors to have a significant explanatory power in REIT returns.

Solutions to Ramsey's test

First solution would be to introduce a lagged gdp log growth instead of one consistent with the actual timing of the return. Using a lagged GDP growth term is economically reasonable in a real estate context because the sector tends to adjust slowly. Changes in GDP reflect broader improvements in economic activity, but those improvements usually take time to show up in the channels that matter for property companies tenant demand, occupancy, rental growth, and leasing outcomes (Gallin J) For example, higher GDP growth often comes from stronger household and business spending, but that spending typically happens first and only later shows up in measured GDP, and it can take additional time before firms expand office space or households upgrade housing choices. At the same time, many real estate cash flows are set through leases that adjust gradually rather than instantly. For these reasons, a lagged GDP variable can better reflect the timing of how macroeconomic conditions translate into real estate fundamentals and, in turn, REIT returns.

Ramsey's Reset Test - with adjustments

After the models have been fitted with a lagged GDP growth term with a lag of 6 months, the models improve drastically (appendix 14-16). The inclusions remove evidence of misfitting for residential and retail models, the office model continues to reject RESET, suggesting remaining omitted structure or sector-specific dynamics. Office estimates are therefore interpreted more cautiously.

Breusch-Pagan/Godfrey test on adjusted model

Appendix 17–18 report the Breusch–Pagan and Breusch–Godfrey test results. The tests indicate heteroskedasticity and residual serial correlation in the sector regressions. Because these conditions violate the assumptions behind standard OLS standard errors, inference is based on Newey–West HAC standard errors in the reported results.

VIF

Across the three sector regressions, the Fed Funds rate is the only regressor with a VIF above the commonly used threshold of 5 (Appendix 19), indicating comparatively strong multicollinearity with the other factors. This level of collinearity can inflate standard errors and make coefficient estimates less stable. Replacing the policy rate level with its monthly change, could reduce multicollinearity with other slow-moving macro variables and better captures policy shocks that are more relevant for asset returns, leading to more stable estimation and inference.

The introduction of a delta fed funds rate instead of a standard rate, proves to have solved the variance inflated factor problem. Appendix 19 shows that all factors are now below 5 VIF.

Inference

Statistical significance is assessed using two-sided tests at the 5% level ($\alpha = 0.05$). Results significant at the 10%, 5%, and 1% levels are flagged for reference. Because heteroskedasticity and serial correlation are present, p-values and confidence intervals are based on Newey–West HAC standard errors.

Significance of final model

Appendix 20-22 shows the model statistics for the three adjusted models. After applying Newey–West HAC standard errors, none of the macro factors are statistically significant at the 5% threshold in any of the three sector regressions. This suggests limited evidence that the selected macro variables have stable, separately identifiable marginal effects on monthly REIT sector returns in the sample period.

Explanatory power of final model

The final linear macro-factor regressions exhibit low explanatory power across sectors. The residential model has $R^2 = 0.015$(Adj. $R^2 = -0.001$), the retail model $R^2 = 0.018$(Adj. $R^2 = 0.002$), and the office model $R^2 = 0.026$(Adj. $R^2 = 0.010$), indicating that the included macro variables account for only a small share of monthly return variation. Under conventional OLS standard errors, the change in the Fed Funds rate is significant in the office and retail regressions; however, this result does not hold under Newey–West HAC inference, consistent with heteroskedasticity and serial correlation in residuals. Overall, the evidence suggests that, at monthly frequency, the selected macroeconomic indicators provide limited standalone explanatory power for REIT sector returns in the pre-COVID sample

Overview of results

Coeffecients, HAC SE and P-Value for each sector

```
term               Residential                Office                     Retail
<chr>              <chr>                      <chr>                      <chr>
(Intercept)        0.0652 (0.0862) [0.450]    -0.0920 (0.1124) [0.414]   0.0701 (0.1170) [0.550]
gdp_log_growth_l6  -0.0143 (0.0151) [0.344]   -0.0354 (0.0243) [0.146]   -0.0223 (0.0234) [0.341]
employment_rate    -0.0809 (0.1252) [0.519]   0.1633 (0.1637) [0.319]    -0.0864 (0.1749) [0.622]
d_fed              2.0353 (2.2933) [0.375]    6.5796 (4.7731) [0.169]    4.5480 (5.2883) [0.390]
inflation_rate     0.0099 (0.3305) [0.976]    -0.2996 (0.4133) [0.469]   0.0420 (0.4964) [0.933]
yield_spread       0.2007 (0.2333) [0.390]    0.3280 (0.3573) [0.359]    0.1815 (0.3320) [0.585]
m2_log_growth      0.0030 (0.0042) [0.480]    -0.0019 (0.0057) [0.746]   0.0014 (0.0061) [0.815]
```

```
> fit_stats
        Model   N       Adj_R2
1 Residential 375 -0.001381502
2      Office 375  0.010170201
3      Retail 375  0.001628779
>
```

Conclusion

This paper examined whether a set of macroeconomic variables can explain monthly returns of U.S. REIT sector portfolios (residential, office, and retail) over the pre-COVID period. A linear factor model was estimated for each sector using GDP growth (lagged), labour market conditions, changes in the Federal Funds rate, inflation, the yield spread, and money supply growth. Diagnostic testing indicated heteroskedasticity and serial correlation in the regression residuals, implying that conventional OLS standard errors are not appropriate. For this reason, inference throughout is based on Newey–West heteroskedasticity-and-autocorrelation-consistent (HAC) standard errors.

Across all three sectors, the final HAC results provide limited evidence that the selected macro factors have stable, separately identifiable marginal effects on monthly REIT sector returns. While coefficient signs can be discussed in relation to expected cash-flow and discount-rate channels, none of the estimated factor loadings are statistically significant at conventional levels once inference is corrected for heteroskedasticity and autocorrelation. Model fit is also low, with R^2 values in the range of approximately 1–3%, suggesting that the included macro variables explain only a small share of monthly return variation. This outcome is consistent with the fact that REIT returns at a monthly horizon are highly noisy and are likely driven by market-wide and sector-specific shocks that are not fully captured by a small set of macroeconomic indicators.

Specification refinements motivated by diagnostic results improved model adequacy in parts of the analysis. Introducing lagged GDP growth is economically consistent with delayed transmission from macroeconomic activity to real estate fundamentals and reduced evidence of misspecification for the residential and retail specifications, although the office model continued to show remaining specification concerns. Replacing the level of the policy rate with its monthly change reduced multicollinearity relative to other macro variables and better reflects monetary policy shocks, but did not materially strengthen statistical evidence for factor effects under HAC inference.

In conclusion the results and inference suggest that, within a linear regression framework and at monthly return frequencies, the examined macroeconomic factors have limited explanatory power for REIT sector returns in the pre-COVID sample in their current form. A key limitation is that the model does not include broader market or real-estate-specific risk factors, which may account for a substantial portion of return variation and could clarify the incremental role of macroeconomic variables. Future work could extend the specification by incorporating extended factors, exploring sector-specific lag structures more formally, and assessing parameter stability across subperiods.

Appendix

1.

```
> df_pre_covid %>%
+   summarise(
+     start = min(date),
+     end   = max(date),
+     N     = n()
+   )
# A tibble: 1 × 3
  start      end            N
  <date>     <date>     <int>
1 1988-04-01 2019-12-01   381
>
```

2.

```
> print(m_res_pre)

Call:
lm(formula = avg_log_return_residential ~ gdp_log_growth + employment_rate + 
    fed_funds_rate + inflation_rate + yield_spread + m2_log_growth, 
    data = df_pre_covid)

Coefficients:
    (Intercept)   gdp_log_growth  employment_rate   fed_funds_rate   inflation_rate     yield_spread    m2_log_growth  
        0.17033          0.02997         -0.25135          0.07950         -0.13783          0.20627          0.00277  
```

3.

```
> print(m_ret_pre)

Call:
lm(formula = avg_log_return_retail ~ gdp_log_growth + employment_rate + 
    fed_funds_rate + inflation_rate + yield_spread + m2_log_growth, 
    data = df_pre_covid)

Coefficients:
    (Intercept)   gdp_log_growth  employment_rate   fed_funds_rate   inflation_rate     yield_spread    m2_log_growth  
       0.227246         0.061113        -0.350278         0.065376        -0.131116         0.183371         0.001761  
```

4.
```
> print(m_off_pre)

Call:
lm(formula = avg_log_return_office ~ gdp_log_growth + employment_rate +
    fed_funds_rate + inflation_rate + yield_spread + m2_log_growth,
    data = df_pre_covid)

Coefficients:
    (Intercept)  gdp_log_growth  employment_rate   fed_funds_rate   inflation_rate     yield_spread    m2_log_growth
      -0.019461        0.068092         0.018141        -0.295612        -0.186547         0.065285        -0.002403

> view(df_16)
```

```
> bptest(m_res_pre)

        studentized Breusch-Pagan test

data:  m_res_pre
BP = 47.01, df = 6, p-value = 1.863e-08
```

5.
```
> bptest(m_off_pre)

        studentized Breusch-Pagan test

data:  m_off_pre
BP = 40.714, df = 6, p-value = 3.297e-07
```

6.
```
> bptest(m_ret_pre)

        studentized Breusch-Pagan test

data:  m_ret_pre
BP = 56.993, df = 6, p-value = 1.833e-10
```

7.
```
> bgtest(m_res_pre, order = 12)

        Breusch-Godfrey test for serial correlation of order up to 12

data:  m_res_pre
LM test = 25.884, df = 12, p-value = 0.01115
```

8.
```
> bgtest(m_off_pre, order = 12)

        Breusch-Godfrey test for serial correlation of order up to 12

data:  m_off_pre
LM test = 29.383, df = 12, p-value = 0.003455
```

9.
```
> bgtest(m_ret_pre, order = 12)

        Breusch-Godfrey test for serial correlation of order up to 12

data:  m_ret_pre
LM test = 51.53, df = 12, p-value = 7.51e-07
```

10.

11.
```
> resettest(m_res_pre)

	RESET test

data:  m_res_pre
RESET = 5.515, df1 = 2, df2 = 372, p-value = 0.004362
```

12.
```
> resettest(m_off_pre)

	RESET test

data:  m_off_pre
RESET = 3.8799, df1 = 2, df2 = 372, p-value = 0.02149
```

13.
```
> resettest(m_ret_pre)

	RESET test

data:  m_ret_pre
RESET = 7.3121, df1 = 2, df2 = 372, p-value = 0.0007677
```

14.
```
> resettest(m_res_gdplag6)

	RESET test

data:  m_res_gdplag6
RESET = 2.7352, df1 = 2, df2 = 366, p-value = 0.06621
```

15.
```
> resettest(m_office_gdplag6)

	RESET test

data:  m_office_gdplag6
RESET = 5.3968, df1 = 2, df2 = 366, p-value = 0.004899
```

16.
```
> resettest(m_retail_gdplag6)

	RESET test

data:  m_retail_gdplag6
RESET = 1.0951, df1 = 2, df2 = 366, p-value = 0.3356
```

```
> bptest(m_res_gdplag6)

        studentized Breusch-Pagan test

data:  m_res_gdplag6
BP = 34.063, df = 6, p-value = 6.541e-06

> #office
> bptest(m_office_gdplag6)

        studentized Breusch-Pagan test

data:  m_office_gdplag6
BP = 34.752, df = 6, p-value = 4.813e-06

> #retail
> bptest(m_retail_gdplag6)

        studentized Breusch-Pagan test

data:  m_retail_gdplag6
BP = 27.83, df = 6, p-value = 0.0001012

> |
```

17.

```
> bgtest(m_res_gdplag6, order = 12)

        Breusch-Godfrey test for serial correlation of order up to 12

data:  m_res_gdplag6
LM test = 25.67, df = 12, p-value = 0.01195

> bgtest(m_office_gdplag6, order = 12)

        Breusch-Godfrey test for serial correlation of order up to 12

data:  m_office_gdplag6
LM test = 27.631, df = 12, p-value = 0.006263

> bgtest(m_retail_gdplag6, order = 12)

        Breusch-Godfrey test for serial correlation of order up to 12

data:  m_retail_gdplag6
LM test = 47.191, df = 12, p-value = 4.319e-06
```

18.

```
> vif(m_res_l6_dfed)
gdp_log_growth_l6   employment_rate           d_fed    inflation_rate      yield_spread     m2_log_growth 
         1.213261          1.951488        1.137414          1.445576          1.438048          1.061239 
> vif(m_office_l6_dfed)
gdp_log_growth_l6   employment_rate           d_fed    inflation_rate      yield_spread     m2_log_growth 
         1.213261          1.951488        1.137414          1.445576          1.438048          1.061239 
> vif(m_retail_l6_dfed)
gdp_log_growth_l6   employment_rate           d_fed    inflation_rate      yield_spread     m2_log_growth 
         1.213261          1.951488        1.137414          1.445576          1.438048          1.061239 
```

19.

```
> coeftest(m_office_l6_dfed, vcov = NeweyWest(m_office_l6_dfed, lag = 12, prewhite = TRUE))

t test of coefficients:

                    Estimate Std. Error t value Pr(>|t|)
(Intercept)       -0.0920211  0.1124308 -0.8185   0.4136
gdp_log_growth_l6 -0.0353698  0.0242621 -1.4578   0.1457
employment_rate    0.1632793  0.1636777  0.9976   0.3191
d_fed              6.5795858  4.7731426  1.3785   0.1689
inflation_rate    -0.2995674  0.4133209 -0.7248   0.4690
yield_spread       0.3279508  0.3572626  0.9180   0.3592
m2_log_growth     -0.0018576  0.0057209 -0.3247   0.7456
```

20.

```
> coeftest(m_retail_l6_dfed, vcov = NeweyWest(m_retail_l6_dfed, lag = 12, prewhite = TRUE))

t test of coefficients:

                    Estimate Std. Error t value Pr(>|t|)
(Intercept)        0.0700983  0.1170268  0.5990   0.5495
gdp_log_growth_l6 -0.0222860  0.0233692 -0.9536   0.3409
employment_rate   -0.0863560  0.1748910 -0.4938   0.6218
d_fed              4.5479963  5.2882939  0.8600   0.3903
inflation_rate     0.0420331  0.4964101  0.0847   0.9326
yield_spread       0.1815324  0.3319836  0.5468   0.5848
m2_log_growth      0.0014142  0.0060518  0.2337   0.8154
```

21.

```
> coeftest(m_res_l6_dfed, vcov = NeweyWest(m_res_l6_dfed, lag=12, prewhite=TRUE))

t test of coefficients:

                    Estimate Std. Error t value Pr(>|t|)
(Intercept)        0.0651851  0.0862227  0.7560   0.4501
gdp_log_growth_l6 -0.0142933  0.0150865 -0.9474   0.3440
employment_rate   -0.0808682  0.1252007 -0.6459   0.5187
d_fed              2.0353490  2.2933020  0.8875   0.3754
inflation_rate     0.0098709  0.3305254  0.0299   0.9762
yield_spread       0.2007494  0.2333019  0.8605   0.3901
m2_log_growth      0.0029746  0.0042104  0.7065   0.4803
```

22.

```
> summary(m_res_l6_dfed)

Call:
lm(formula = avg_log_return_residential ~ gdp_log_growth_l6 + 
    employment_rate + d_fed + inflation_rate + yield_spread + 
    m2_log_growth, data = df_l6)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.281740 -0.025072 -0.000843  0.025580  0.161976 

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)        0.065185   0.120479   0.541    0.589
gdp_log_growth_l6 -0.014293   0.013669  -1.046    0.296
employment_rate   -0.080868   0.173947  -0.465    0.642
d_fed              2.035349   1.658626   1.227    0.221
inflation_rate     0.009871   0.329562   0.030    0.976
yield_spread       0.200749   0.280018   0.717    0.474
m2_log_growth      0.002975   0.003792   0.784    0.433

Residual standard error: 0.05251 on 368 degrees of freedom
Multiple R-squared:  0.01468,	Adjusted R-squared:  -0.001382 
F-statistic: 0.914 on 6 and 368 DF,  p-value: 0.4847
```

23.

```
> summary(m_retail_l6_dfed)

Call:
lm(formula = avg_log_return_retail ~ gdp_log_growth_l6 + employment_rate + 
    d_fed + inflation_rate + yield_spread + m2_log_growth, data = df_l6)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.43497 -0.02928  0.00014  0.03247  0.47402 

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)  
(Intercept)        0.070098   0.167489   0.419   0.6758  
gdp_log_growth_l6 -0.022286   0.019002  -1.173   0.2416  
employment_rate   -0.086356   0.241819  -0.357   0.7212  
d_fed              4.547996   2.305802   1.972   0.0493 *
inflation_rate     0.042033   0.458153   0.092   0.9270  
yield_spread       0.181532   0.389277   0.466   0.6413  
m2_log_growth      0.001414   0.005271   0.268   0.7886  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.07299 on 368 degrees of freedom
Multiple R-squared:  0.01765,	Adjusted R-squared:  0.001629 
F-statistic: 1.102 on 6 and 368 DF,  p-value: 0.3607
```

24.

25.
```
> summary(m_office_l6_dfed)

Call:
lm(formula = avg_log_return_office ~ gdp_log_growth_l6 + employment_rate +
    d_fed + inflation_rate + yield_spread + m2_log_growth, data = df_l6)

Residuals:
     Min       1Q   Median       3Q      Max
-0.38429 -0.03919  0.00180  0.03539  0.45205

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)       -0.092021   0.199611  -0.461   0.6451
gdp_log_growth_l6 -0.035370   0.022647  -1.562   0.1192
employment_rate    0.163279   0.288196   0.567   0.5714
d_fed              6.579586   2.748025   2.394   0.0172 *
inflation_rate    -0.299567   0.546021  -0.549   0.5836
yield_spread       0.327951   0.463936   0.707   0.4801
m2_log_growth     -0.001858   0.006282  -0.296   0.7676
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08699 on 368 degrees of freedom
Multiple R-squared:  0.02605,	Adjusted R-squared:  0.01017
F-statistic:  1.64 on 6 and 368 DF,  p-value: 0.1349
```

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