MASTER THESIS

The Role of Cryptocurrencies in Portfolio Diversification and Optimization

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Abstract

This thesis investigates the role of cryptocurrencies as diversification instruments within traditional equity portfolios consisting of Danish and U.S. stocks. Using a comprehensive dataset spanning February 2019 to January 2025, the study applies mean-variance optimization, risk-adjusted performance metrics (Sharpe, Sortino, and Calmar ratios), downside risk measures (VaR, CVaR, and maximum drawdown), factor models (Fama-French 3- and 5-factor models), and a robust battery of statistical tests, including Wilcoxon Signed-Rank, non-parametric benchmark, bootstrap procedures, and regime-dependent bootstrapping. The findings demonstrate that including cryptocurrencies generally enhances portfolio efficiency by increasing risk-adjusted returns across multiple portfolio constructions, both in-sample and out-of-sample. Crypto-inclusive portfolios consistently show superior Sharpe and Sortino ratios relative to traditional portfolios, although not all improvements reach formal statistical significance under bootstrap testing. Factor model analyses reveal significant positive alphas for portfolios with cryptocurrency exposure, suggesting that their returns are not fully attributable to conventional systematic risk factors. However, the diversification benefit of cryptocurrencies exhibits regime dependency. While they maintain low correlations with equities during stable periods, these correlations rise during market crises, reducing their protective function during financial stress. Crypto-inclusive portfolios also exhibit heightened downside risk, particularly in unconstrained and leveraged strategies, emphasizing the importance of applying realistic portfolio constraints. Overall, the study concludes that cryptocurrencies can serve as effective diversification tools that improve portfolio efficiency when integrated into thoughtfully constructed portfolios with appropriate risk management, while their regime-dependent characteristics necessitate caution during periods of market turmoil.

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1. Introduction

1.1 Introduction to the Cryptocurrency Market

The cryptocurrency market has emerged as a significant financial innovation over the past decade, reshaping traditional notions of investment and asset management. Cryptocurrencies are digital or virtual currencies that leverage cryptographic techniques to secure transactions and control the creation of new units. Unlike fiat currencies, which are issued and regulated by central banks, cryptocurrencies operate on decentralized networks, often utilizing blockchain technology to ensure transparency and security (The Investopedia Team et al., 2024).

The origins of the cryptocurrency market can be traced back to 2008 when an anonymous entity known as Satoshi Nakamoto introduced Bitcoin in a white paper titled "Bitcoin: A Peer-to-Peer Electronic Cash System." Bitcoin, launched in 2009, was designed as an alternative to traditional financial systems, offering a decentralized means of transferring value without the need for intermediaries. Over time, Bitcoin's success inspired the creation of thousands of alternative cryptocurrencies (altcoins), including Ethereum, Ripple (XRP), and Litecoin, each with unique functionalities and use cases (Lee et al., 2018).

Since its inception, the cryptocurrency market has experienced exponential growth, driven by increasing adoption, institutional interest, and advancements in blockchain technology. In 2017, the total market capitalization of cryptocurrencies surged to an unprecedented $800 billion, fueled by speculative trading and initial coin offerings (ICOs). Despite periodic downturns, the market has continued to expand, with a market capitalization surpassing $2 trillion in 2021 and hitting all time high around $3.8 trillion in December 2024 (Statista, 2025). This rapid evolution highlights both the potential and the volatility inherent in cryptocurrency investments.

One of the defining characteristics of cryptocurrencies is their high volatility. Unlike traditional asset classes such as equities and bonds, cryptocurrencies are prone to extreme price fluctuations, often influenced by market sentiment, regulatory developments, and technological advancements. While this volatility poses risks, it also creates opportunities for portfolio diversification, particularly when cryptocurrencies exhibit low or negative correlations with traditional financial assets (Lee et al., 2018). This aspect makes them an intriguing subject of study in the context of portfolio optimization.

The decentralized and borderless nature of cryptocurrencies has also introduced regulatory challenges. Governments and financial institutions worldwide continue to grapple with the implications of digital assets, leading to diverse regulatory frameworks. Some nations have embraced cryptocurrencies, integrating them into their financial ecosystems, while others have imposed strict regulations or outright bans. The regulatory uncertainty adds another layer of risk for investors considering cryptocurrencies as part of their portfolios (The Investopedia Team et al., 2024).

Despite these challenges, cryptocurrencies have gained recognition as an emerging asset class with unique characteristics. Their potential for high returns and their role in financial innovation make them an attractive consideration for investors looking to diversify their portfolios. Given their distinct risk-return profile, it is important to explore how cryptocurrencies interact with traditional assets and whether they can serve as an effective tool for portfolio diversification. The following sections will introduce these aspects, setting the stage for a deeper discussion on the role of cryptocurrencies in modern investment strategies.

1.2 Portfolio Diversification and cryptocurrency as a potential tool

Portfolio diversification is a fundamental strategy in investment management, aiming to optimize returns while mitigating risks. The principle of diversification is based on the idea that a well-balanced mix of assets can achieve better risk-adjusted returns than individual investments. By spreading investments across multiple asset classes, investors reduce their exposure to the idiosyncratic risks associated with any single asset or market sector (Troy Segal, 2023). The importance of diversification has been widely recognized, as it helps investors build resilient portfolios capable of withstanding economic fluctuations and financial downturns.

Diversification addresses two types of risk: systematic and unsystematic. Systematic risk, also known as market risk, is inherent to the overall financial system and cannot be eliminated through diversification. Factors such as economic downturns, interest rate changes, and geopolitical events contribute to systematic risk. On the other hand, unsystematic risk, also referred to as diversifiable risk, is specific to a particular company or industry and can be mitigated through a well-diversified portfolio

(James Chen, 2024). By incorporating assets that react differently to macroeconomic variables, investors can significantly reduce their exposure to unpredictable market shocks.

One of the most widely recognized frameworks for portfolio diversification is Markowitz's Mean-Variance Optimization (MVO), introduced in modern portfolio theory. MVO is a portfolio optimization framework that seeks to minimize portfolio variance for a given target level of expected return, based on the correlations and variances of the included assets. The theory suggests that an efficient portfolio is one that minimizes risk for a given level of expected return or, equivalently, maximizes expected return for a given level of risk. This framework remains a cornerstone of modern portfolio theory, shaping investment strategies across both institutional and individual investors (Markowitz, 1952).

Beyond its theoretical foundations, diversification has practical applications that benefit investors across different risk tolerances. A well-diversified portfolio provides stability during periods of economic uncertainty, as negatively correlated assets tend to offset losses incurred in other investments. Additionally, diversification enhances long-term wealth accumulation by smoothing out returns over time, reducing exposure to extreme market fluctuations. Investors who effectively diversify can achieve a more resilient portfolio, allowing them to participate in market growth while mitigating downside risk. The empirical success of MVO-based investment strategies has led to its widespread adoption in portfolio construction by financial institutions and fund managers (Markowitz, 1952).

While traditional diversification strategies have relied on asset classes such as equities, bonds, real estate, and commodities, the increasing digitization of financial markets has introduced new potential diversifiers, including cryptocurrencies. The growing interest in digital assets presents an opportunity to examine whether their inclusion can enhance portfolio performance or introduce additional risk. Unlike traditional assets, cryptocurrencies are highly volatile and speculative, yet their potential for outsized returns and their evolving correlation with other assets make them a compelling subject of study.

1.3 Research Purpose and Questions

Given the growing significance of cryptocurrencies in financial markets, this thesis aims to investigate their role in portfolio diversification, with a particular focus on their ability to improve returns while maintaining risk at a level that increases the risk adjusted return. While traditional asset classes have been extensively studied in the context of portfolio optimization, the inclusion of cryptocurrencies remains a debated topic due to their high volatility and speculative nature.

This study seeks to address the following research questions:

To what extent does adding cryptocurrencies to a traditional investment portfolio improve risk-adjusted returns?

And the supporting questions:

- *How do cryptocurrencies correlate with Danish and American equities under different market conditions?*
- *How do extreme market conditions (e.g., financial crises, periods of high volatility) impact the diversification benefits of cryptocurrencies?*

To answer these questions, the study applies a range of quantitative techniques, including Markowitz mean-variance optimization, risk-return metrics (Sharpe, Sortino, and Calmar ratios), and downside risk measures such as Value-at-Risk and Conditional VaR. Additionally, the analysis incorporates market regime segmentation to assess how performance varies during periods of financial stress, and factor models (Fama–French 3- and 5-factor frameworks) to evaluate whether excess returns can be explained by exposure to systematic risk factors.

The empirical analysis is based on a dataset comprising U.S. and Danish equities, chosen for their sectoral diversity and reliable data coverage, alongside major cryptocurrencies. This design allows for a broad yet consistent comparison of diversification effects across different market structures.

By bridging theoretical models with empirical testing, the study aims to evaluate the role of cryptocurrencies in modern portfolio strategies and assess whether their inclusion offers sustainable improvements in portfolio efficiency across market conditions.

The remainder of the thesis is structured as follows: Section 2 reviews relevant literature on portfolio diversification and the evolving role of cryptocurrencies. Section 3 outlines the data, methodological approach, and portfolio construction techniques, including risk metrics and statistical tests. Section 4 presents the empirical results, followed by Section 5, which discusses the findings, addresses limitations, and outlines directions for future research. Finally, Section 6 provides a concluding summary of the main insights.

2. Literature Review

This section reviews the academic foundations relevant to this thesis. It focuses on modern portfolio theory, empirical studies on cryptocurrency as a diversification tool, the dynamics of correlation during financial crises, the importance of robust statistical testing, and factor-based return attribution. Each part concludes by highlighting how these prior contributions inform the design of this thesis or reveal gaps that it seeks to address.

2.1 Modern Portfolio Theory and Diversification

The foundation for portfolio optimization is found in the work of Markowitz (1952), who introduced the concept of mean-variance optimization (MVO). His model proposes that an efficient portfolio is one that either maximizes expected return for a given level of risk or minimizes risk for a given level of return. This is formalized through the concept of the efficient frontier, which represents the set of optimal portfolios under assumptions of rational investor behavior, normally distributed returns, and constant correlations between assets.

MVO marked a paradigm shift by quantifying the benefits of diversification. However, its assumptions have since been widely challenged. In particular, the assumption of normality often fails in real markets where return distributions are skewed or exhibit excess kurtosis. Moreover, correlations between asset classes are rarely stable, particularly during periods of market stress.

For this thesis, MVO serves as the starting point for portfolio construction. However, by extending the analysis to assets like cryptocurrencies, known for their volatility and non-normal return profiles, this study addresses some of the key limitations of traditional MVO applications and investigates whether theoretical diversification translates into empirical performance under modern asset structures.

2.2 Cryptocurrencies in Portfolio Construction

The emergence of cryptocurrencies has prompted growing interest in their potential role as diversifiers. Lee, Guo, and Wang (2018) conducted one of the more comprehensive empirical studies in this area, analyzing the correlation between the CRIX index and traditional assets using a DCC-GARCH model. Their findings suggest that cryptocurrencies maintain low correlations with equities and fixed-income securities during calm market periods, which supports the hypothesis that they can reduce portfolio risk through diversification.

At the same time, the authors highlight limitations. Correlation structures are not stable and may rise during periods of financial turmoil. Moreover, cryptocurrencies are influenced by investor sentiment, speculative dynamics, and lack of intrinsic valuation anchors, all of which complicate their role as reliable risk mitigators. Despite these concerns, Lee, Guo, and Wang conclude that moderate allocations to crypto assets can still enhance a portfolio's risk-return profile under dynamic allocation strategies.

In the context of this thesis, these findings justify the empirical testing of cryptocurrencies alongside traditional assets. However, Lee, Guo, and Wang's study is limited to global index-level analysis and does not investigate performance under regime changes or across smaller equity markets such as Denmark. This study fills that gap by analyzing cryptocurrencies diversification potential under different market conditions and across distinct national portfolios.

2.3 Correlation During Financial Crises

A critical concern in portfolio management is that diversification benefits may diminish in crises. Longin and Solnik (2001) explore this by examining how correlations among international equity markets evolve under extreme conditions. Using extreme value theory, they find that correlations increase significantly in the tails of return distributions. This implies that during market stress, asset classes that appear uncorrelated in stable periods begin to move together, effectively collapsing diversification benefits when they are needed most.

This dynamic, often referred to as correlation breakdown, suggests that static correlation estimates are insufficient for stress testing portfolio resilience. Longin and Solnik's findings have become a cornerstone in the literature on regime-sensitive investment strategies, emphasizing the need to analyze correlation patterns under different market conditions.

For this thesis, their work provides the rationale for dividing the sample period into market regimes (e.g., crisis vs. stable periods). It supports the use of regime-based testing to examine whether cryptocurrencies maintain their low-correlation behavior under stress, or whether their diversification benefits are conditional and time-varying.

2.4 Statistical Inference and the Importance of Robust Testing

Validating portfolio performance requires more than observing improvements in Sharpe or Sortino ratios. A central concern in the financial econometrics' literature is that standard tests often assume return normality, homoscedasticity, and independence - assumptions that are rarely satisfied in real-world data, particularly in cryptocurrency markets.

Ledoit and Wolf (2008a) argue that many widely used Sharpe ratio tests are statistically invalid under realistic return dynamics. They emphasize the importance of robust hypothesis testing and caution against relying on a single test statistic. While their work proposes a heteroskedasticity-consistent analytical alternative, the broader implication supports a multi-test framework: different statistical methods can reveal different facets of performance significance, especially when data characteristics are complex.

In line with this reasoning, this thesis adopts a multi-method approach, employing three statistical tests: the Wilcoxon Signed-Rank Test, a Sharpe ratio-specific test, and a bootstrap procedure. Among these, the bootstrap plays a vital role due to its flexibility and minimal assumptions. By resampling returns to construct empirical confidence intervals, the bootstrap method is especially suited to non-Gaussian and volatile data environments, such as those involving cryptocurrencies.

Together, these tests offer complementary perspectives, enhancing the robustness of performance evaluation. This approach ensures that the conclusions drawn are not dependent on a single statistical lens, but rather hold across multiple methodological frameworks - addressing the limitations identified by Ledoit and Wolf and increasing the reliability of the empirical findings.

2.5 Factor Models and Performance Attribution

Even when crypto-inclusive portfolios outperform on a raw basis, such returns could reflect exposure to systematic risk factors rather than true alpha. To assess this, researchers often turn to multi-factor asset pricing models. The Fama–French 3-factor model by Fama & French (1993) introduces size and value factors alongside market risk, while the 5-factor extension Fama & French (2015) adds profitability and investment factors. These models have become central tools in evaluating whether excess returns are justified by risk exposures.

In this thesis, both models are applied to test whether the observed outperformance of cryptocurrency-enhanced portfolios can be explained through known factor sensitivities. If statistically significant alpha remains after controlling for these exposures, this would suggest that cryptocurrencies offer non-replicable diversification benefits.

Factor-based analysis in existing cryptocurrency literature is still limited. Many studies stop at correlation or Sharpe comparisons, without determining whether excess returns result from systematic factors. This thesis contributes by incorporating rigorous multi-factor regression tests across weekly and monthly data frequencies, thereby strengthening the evidence base for (or against) the claim that cryptocurrencies adds unique value.

2.6 Summary and Relevance of This Study

The reviewed literature highlights several important points:

- MVO offers a robust but assumption-sensitive framework for portfolio optimization.
- Cryptocurrencies exhibit low correlations in normal markets, but their behavior under stress remains less certain.
- Correlation structures are regime-dependent, and diversification benefits may vanish during crises.
- Performance improvements require rigorous, assumption-agnostic testing for validation.
- Factor attribution is necessary to separate genuine alpha from risk-based returns.

Despite these advances, gaps remain, especially regarding:

- The interaction of cryptocurrencies with smaller or non-U.S. markets,
- The behavior of crypto assets under different market regimes, and
- The lack of multi-test, statistically validated, factor-attributed studies.

This thesis addresses these gaps by combining Danish and U.S. equities with cryptocurrencies, applying regime-based segmentation, and validating performance through robust statistical methods and factor models. The goal is to provide a more comprehensive and credible evaluation of the diversification role of cryptocurrencies in modern portfolio construction.

3. Methodology and theoretical framework

3.1 Data Collection and Sample Design

The dataset employed in this thesis spans six years of weekly observations, covering the period from February 1, 2019 to January 31, 2025. It comprises 27 of the largest U.S. stocks, 15 of the largest Danish stocks, and 3 major cryptocurrencies, resulting in a total of 45 assets. Equity data was sourced from FactSet, while cryptocurrency data was obtained from Investing.com (Investing.com)(Factset.com).

The equity selection was based on market capitalization, using FactSet's screening tools to identify the largest listed companies in the United States and Denmark. Similarly, the cryptocurrencies were selected by market capitalization via Investing.com, excluding stablecoins and any tokens explicitly pegged to fiat currencies. The three included cryptocurrencies are those with the largest market capitalization that exhibit independent price behavior.

The composition of the dataset reflects structural characteristics of the respective markets. U.S. equities are skewed toward technology and consumer discretionary sectors, while Danish equities are heavily weighted toward pharmaceuticals, healthcare, and biotechnology. Additional sectors represented across the dataset include banking, energy (renewables and oil & gas), retail, industrials, and consumer goods, supporting the creation of diversified portfolios. The inclusion of cryptocurrencies facilitates the investigation of their potential contribution to portfolio efficiency and directly addresses the thesis' central research question.

The dataset is split into two distinct periods:

- Estimation Period: February 1, 2019 – January 26, 2024 (5 years).
  This segment is used for all portfolio construction and statistical testing.
- Performance Period: January 26, 2024 – January 31, 2025 (1 year).
  This final year serves as an out-of-sample window to evaluate portfolio robustness and real-world applicability.

The data is split into an estimation period and a performance period to distinguish between model development and real-world evaluation. The estimation period (2019–2024) is used for portfolio construction, optimization, and all statistical testing. The performance period (2024–2025) is reserved for assessing out-of-sample performance, simulating how the portfolios would behave under new market conditions. This separation helps prevent overfitting and supports a more robust evaluation of strategy effectiveness, as recommended in empirical finance literature (DeMiguel et al., 2009a).

A key challenge encountered during data preparation was the mismatch in weekly data intervals between equities and cryptocurrencies. Crypto markets operate continuously, with weekly data typically recorded from Sunday to Sunday, whereas equity markets follow a Friday-to-Friday schedule. To ensure temporal consistency across assets, daily cryptocurrency data was collected and manually filtered to extract Friday closing prices. Aligning observation intervals across all assets is essential to preserve the validity of return comparisons, correlation analysis, and portfolio optimization outcomes (Tsay, 2005).

3.2 Correlation Matrix

To assess the diversification potential of cryptocurrencies within traditional equity portfolios, this thesis conducts a detailed analysis of the correlations between asset classes. Correlation is a critical factor in modern portfolio theory, as assets with low or negative correlations can significantly reduce portfolio variance and enhance the risk-return tradeoff (Markowitz, 1952).

3.2.1 Estimation Period Correlation Matrix

The primary correlation analysis is conducted over the five-year estimation period (February 1, 2019 - January 26, 2024). Pearson correlation coefficients are calculated between each of the three cryptocurrencies (Bitcoin, Ethereum, Binance Coin) and all individual equities within the Danish and U.S. stock selections.

The Pearson correlation coefficient $\rho_{X,Y}$ is calculated as:

$$\rho_{X,Y} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$$

where:

- $Cov(X, Y)$ = covariance between asset returns X and Y,
- $\sigma_X \sigma_Y$ = standard deviations of asset returns X and Y.

Weekly returns are used throughout, and Friday-to-Friday closing prices ensure synchronization between equities and cryptocurrencies.

This analysis aims to establish the average degree of linear relationship between traditional stocks and crypto assets over a relatively stable long-term horizon. Low average correlations would suggest the potential for meaningful diversification benefits when cryptocurrencies are introduced into a traditional portfolio (Tsay, 2005)(Markowitz, 1952).

3.2.2 Regime-Based Correlation Analysis

Recognizing that asset correlations are not static, the study further segments the estimation period into five market regimes, classified based on observed volatility in the S&P 500 index:

- Pre-Crisis
- Crisis 1
- Between Crises
- Crisis 2
- Post-Crisis

These periods are defined using a rolling 12-week volatility filter, with specific volatility thresholds applied to distinguish periods of high and low market stress (Ang & Bekaert, 2002a).

Within each volatility regime, separate correlation matrices are computed to capture the dynamic evolution of cross-asset relationships. This approach addresses the well-documented phenomenon that correlations tend to rise during periods of financial market stress, reducing the effectiveness of diversification precisely when it is most needed (Longin & Solnik, 2001).

3.2.3 Data Handling, Adjustments, and Presentation of Results

As discussed in Section 3.1, all data was harmonized to a weekly frequency using Friday closing prices. Since cryptocurrencies trade continuously while equity markets are closed on weekends, daily crypto data was collected and manually adjusted to match the equities' weekly timing.

The correlation results are presented in two forms:

- A full estimation-period matrix summarizing the average correlations between cryptocurrencies and traditional stocks.
- Regime-specific matrices highlight how correlations evolve across periods of differing market volatility.

These correlation patterns provide an essential foundation for evaluating how cryptocurrencies may enhance portfolio efficiency and directly feed into the subsequent portfolio optimization procedures.

3.3 Portfolio Optimization and the Minimum Variance Frontier

To evaluate the impact of including cryptocurrencies in a traditional stock portfolio, this thesis applies the Markowitz Mean-Variance Optimization (MVO) framework. The objective is to construct efficient portfolios that minimize portfolio variance for a given expected return, with and without exposure to cryptocurrencies.

The analysis is conducted across two optimization settings:

- Unconstrained portfolios, which allow for short-selling (i.e., asset weights may be negative).
- Constrained portfolios, where short-selling is not permitted, and weights are restricted to be non-negative with a full investment constraint $\sum W_i = 1$.

This dual approach allows for a comparison between theoretical efficiency (unconstrained) and realistic investment scenarios (constrained), where regulatory and practical limits apply (Jagannathan & Ma, 2003).

3.3.1 Optimization Model

This thesis applies the standard Mean-Variance Optimization (MVO) framework, originally developed by Markowitz and formalized in modern portfolio theory by Elton et al. (2014). MVO provides a mathematical foundation for constructing portfolios that balance risk and return, where risk is measured as portfolio variance. Two optimization objectives are applied: minimizing portfolio variance across a range of target returns, and maximizing the Sharpe ratio under varying portfolio constraints.

In both optimization strategies, portfolio return, and variance are calculated as:

$$\mu_p = w^\top \mu, \;\; \sigma_p^2 = w^\top \Sigma w$$

where:

- $\boldsymbol{w}$ is the vector of portfolio weights,
- $\boldsymbol{\mu}$ is the vector of expected asset returns,
- $\boldsymbol{\Sigma}$ is the covariance matrix of asset returns.

The minimum variance portfolio (MVP) identifies the combination of assets that yields the lowest possible risk for a given target level of expected return. This is obtained by solving the following optimization problem:

$$\min_{w} w^\top \Sigma w$$

In the frontier portfolios constructed in this thesis, the optimization is subject to the following constraints:

$$w^\top \mu = \mu_p \; (target\ return)$$

$$w^\top 1 = 1 \; (fully\ invested)$$

$$w \geq 0 \; (optional\ in\ a\ frontier\ without\ short\ selling)$$

By solving this problem repeatedly across a range of target returns μ_p, the minimum variance frontier is constructed. Each point on the frontier represents the most efficient portfolio for a specific level of expected return, offering the lowest possible risk under the given constraints. In this thesis, the optimization is implemented in Excel using the Solver tool. Portfolio variance is minimized subject to constraints on the target return and, in the constrained case, a no short-selling condition.

The second approach maximizes the Sharpe ratio, which measures excess return per unit of risk:

$$\max_{w} \frac{w^{\top}(\mu - r_f 1)}{\sqrt{w^{\top}\Sigma w}}$$

where rf is the risk-free rate. This portfolio, also known as the tangency portfolio, lies at the point of tangency between the capital market line and the efficient frontier. It represents the optimal portfolio in the presence of a risk-free asset, delivering the highest return per unit of total volatility.

In the thesis, the Sharpe ratio is calculated using annualized weekly returns, the risk-free rate, and portfolio standard deviation. Solver is used to maximize this objective by adjusting portfolio weights, while ensuring the portfolio remains fully invested and adheres to any specified short-selling bounds. This method is applied in six of the eight portfolios evaluated in the empirical analysis.

By applying both minimum variance and maximum Sharpe optimization techniques under different constraint configurations, the thesis evaluates how the inclusion of cryptocurrencies affects optimal portfolio structure and risk-adjusted performance across a spectrum of investment preferences (Elton et al., 2009, chap. 4).

3.3.2 Frontier Construction

For both the unconstrained and constrained cases, portfolios are generated across a range of target returns. For each return level, the corresponding portfolio with the minimum variance is computed, resulting in a Minimum Variance Frontier (MVF). This frontier illustrates the set of optimal portfolios offering the best possible tradeoff between risk and return (Elton et al., 2009, chap. 4).

The MVFs were constructed using Excel's Solver tool, which was used to minimize portfolio standard deviation (volatility) for a series of increasing target returns. At each target return level, Solver iteratively adjusts the asset weights, subject to the specified constraints, to find the portfolio with the lowest variance. Separate Solver models were used for both constrained and unconstrained scenarios, and the process was repeated for portfolios with and without cryptocurrencies.

The MVF is constructed for two asset sets:

- Traditional portfolios consisting only of U.S. and Danish equities.
- Crypto-inclusive portfolios incorporating Bitcoin, Ethereum, and Binance Coin alongside traditional stocks.

This setup enables a visual and analytical comparison of risk-return profiles with and without cryptocurrency exposure. A frontier that lies above and/or to the left of another indicates superior efficiency.

3.3.3 Practical Considerations

While the unconstrained optimization offers insights into the theoretical upper bound of diversification benefits, it often results in extreme portfolio weights that may be impractical or unrealistic for actual investors. In contrast, the constrained version reflects real-world investment limitations, such as regulatory restrictions on short-selling and institutional risk controls (Jagannathan & Ma, 2003).

Therefore, the constrained MVF is used as the primary basis for interpreting whether the inclusion of cryptocurrencies can enhance portfolio efficiency in a feasible investment setting.

3.4 Portfolio Performance Evaluation

This section evaluates the performance of eight constructed portfolios over the five-year estimation period. The assessment is based on both traditional and advanced performance metrics, emphasizing risk-adjusted returns and downside risk. Portfolios are analyzed in matched pairs to isolate the impact of cryptocurrencies. All performance metrics are computed using weekly return data and annualized where relevant.

3.4.1 Portfolio pairs

Eight portfolios are constructed using a mix of equal weighting and Markowitz Mean-Variance Optimization under various constraints. These are grouped into four matched pairs, each consisting of a portfolio without cryptocurrencies and one with:

- A vs B: Equal-weighted portfolios, without vs. with crypto
- C vs D: Maximum Sharpe portfolios, unconstrained

- E vs F: Constrained maximum Sharpe portfolios (max weight ≤ 10%, min 20 assets)
- G vs H: Constrained portfolios allowing shorting (weights: -20% to 100%)

Portfolio A, C, E and G are without cryptocurrency and B, D, F and H is with cryptocurrency included. All portfolios are fully invested with weight sums constrained to one. This setup allows for consistent comparison of performance across different levels of flexibility and exposure to cryptocurrencies.

3.4.2 Risk-Adjusted Performance Metrics: Sharpe, Sortino, and Calmar Ratios

The Sharpe ratio and Sortino ratio are central to evaluating the risk-adjusted performance of the constructed portfolios. These measures quantify how much excess return is earned per unit of risk and are widely used in both academic research and professional portfolio management.

The Sharpe ratio is defined as:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

where:

R_p is the annualized portfolio return

R_f is the annualized risk-free rate (set to 4.3%),

σ_p is the annualized standard deviation of portfolio returns.

This ratio evaluates return relative to total volatility and is particularly useful for comparing portfolios with different risk levels under the assumption of normally distributed returns (Sharpe, 1966).

The Sortino ratio improves upon the Sharpe ratio by focusing only on downside volatility, thus penalizing negative deviations while ignoring upside risk:

$$Sortino\ Ratio = \frac{R_p - R_f}{\sigma_d}$$

where σ_d is the annualized downside deviation, calculated using the full sample denominator to reflect typical investor practice (Sortino & Price, 1994).

Both ratios are calculated as point estimates for each portfolio across the full five-year estimation period, used for descriptive analysis and performance comparison. They are also used in statistical testing, where they are calculated as respectively rolling ratios and bootstrapped ratios, with will be elaborated in the section about the test.

In addition to these, the Calmar ratio is also included as a complementary descriptive measure of performance. It is defined as:

$$Calmar\ Ratio = \frac{R_p}{Max\ Drawdown}$$

The Calmar ratio assesses how much return a portfolio delivers relative to its worst historical drawdown - the largest peak-to-trough decline over the period. This ratio is particularly relevant for portfolios with high volatility or exposure to assets such as cryptocurrencies, where short-term losses may be severe. While the Sharpe and Sortino ratios reflect average risk-adjusted returns, the Calmar ratio captures the risk of extreme loss (Young, 1991).

No formal statistical testing is applied to the Calmar ratio in this thesis. It is instead used to provide additional contextual insight into the tradeoff between return and downside exposure.

3.4.3 Value-at-risk (VaR), Expected Shortfall (CVaR) and Maximum drawdown

To capture the tail risk of the constructed portfolios, this thesis includes two downside risk measures: Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR), also known as Expected Shortfall. Both are calculated at a 95% confidence level, using historical simulation methods based on weekly return data. Furthermore, the maximum drawdown will be examined in extension as well.

To measure downside tail risk, the study includes:

Value-at-Risk (VaR) estimates the maximum expected loss over a one-week horizon, with 95% confidence:

$$VaR_{95\%} = Percentile_{5\%}(R)$$

Conditional Value-at-Risk (CVaR 95%) (also known as Expected Shortfall) quantifies the expected loss given that the loss exceeds the VaR threshold:

$$CVaR_{95\%} = E[R \mid R \leq VaR_{95\%}]$$

Both are calculated using historical weekly return data. These measures are reported weekly (not annualized) to avoid compounding errors and better reflect risk from weekly fluctuations. The mathematical definitions and theoretical basis for these measures are drawn from McNeil, Frey, & Embrechts (McNeil et al., 2015, chap. 4).

Unlike volatility-based measures such as VaR and CVaR, maximum drawdowns capture actual realized loss extremes, providing a more intuitive measure of the potential severity of losses under adverse market conditions. This makes them particularly relevant when analyzing cryptocurrencies, which are characterized by high tail risk and extreme price fluctuations.

Maximum drawdown (MDD) as an extension of tail risk analysis, is included to capture the largest peak-to-trough loss experienced by a portfolio over the estimation period. It is defined as:

$$MDD = \underset{t \in [0,T]}{max} \left(\frac{Peak_t - Though_t}{Peak_t} \right)$$

Unlike VaR and CVaR, which reflect statistical risk based on return distributions, MDD is a realized historical loss metric. It provides valuable insight into worst-case capital erosion, making it highly relevant for risk-averse investors and particularly important for portfolios exposed to volatile assets such as cryptocurrencies (Chekhlov et al., 2005).

In this thesis, VaR, CVaR, and MDD are computed for each of the eight portfolios during the five-year estimation period and used for descriptive analysis only. Together, these measures help assess whether the inclusion of cryptocurrencies leads to increased downside exposure. Because all metrics are calculated at the weekly frequency, they offer a detailed and granular view of tail risk and drawdown behavior, especially crucial in understanding the trade-offs introduced by cryptocurrency allocation.

3.4.4 Jensen's Alpha

Jensen's Alpha is used to evaluate whether a portfolio generates returns above what is expected based on its exposure to market risk. Unlike Sharpe and Sortino ratios, which assess performance in isolation, Alpha measures relative performance against a benchmark - the S&P 500 in this thesis.

It is calculated as:

$$\alpha = R_p - [R_f + \beta(R_m - R_f)]$$

Where

$R_p = portfolio\ return,$

$R_m = market\ return\ (proxied\ by\ the\ S\&P\ 500),$

$\beta = \ portfolio\ beta\ estimated\ via\ linear\ regression\ of\ portfolio\ returns\ on\ market\ returns.$

A positive alpha indicates outperformance relative to the market, after adjusting for systematic risk. Alongside Alpha, Beta is also reported to assess how sensitive each portfolio is to market movements.

In this thesis, Jensen's Alpha is used to compare each crypto-inclusive portfolio to its non-crypto counterpart, helping determine whether any observed performance improvements stem from alpha generation or merely higher market exposure. The measure is used descriptively and complements the Sharpe and Sortino ratios by offering a market-relative perspective on risk-adjusted performance (Jensen, 1968a).

3.5 Statistical tests

While descriptive performance metrics such as Sharpe and Sortino ratios provide initial insights into portfolio efficiency, they do not in themselves establish whether observed differences are statistically meaningful. To formally test whether differences between crypto-inclusive and traditional portfolios are significant, this section applies four complementary statistical methods.

The first three tests aim to determine whether there is general statistical evidence that the inclusion of cryptocurrencies leads to higher or lower risk-adjusted returns compared to traditional portfolios:

- Wilcoxon Signed-Rank Test (paired rolling performance),
- Non-Parametric Benchmark Test (comparison to fixed targets),
- Bootstrap Test (full-sample difference distribution).

The fourth test focuses specifically on the consistency of performance across market regimes, comparing portfolio behavior during crisis and non-crisis periods.

Together, these methods form a robust framework for evaluating whether the inclusion of cryptocurrencies results in systematic performance differences, and whether those differences vary under different market conditions. This multi-method approach improves the reliability of statistical inference, addressing potential issues like non-normality, sample-specific bias, and model sensitivity (White, 2000)(Ledoit & Wolf, 2008b).

3.5.1 Wilcoxon Signed-Rank Test: Rolling Sharpe and Sortino Ratios

The Wilcoxon Signed-Rank Test is a non-parametric test used to compare paired samples. In this thesis, it is applied to rolling Sharpe and Sortino ratios calculated over 52-week windows for each portfolio pair. The test evaluates whether the median difference between crypto and non-crypto portfolios is significantly different from zero - without assuming normality of the underlying data.

This test is particularly well-suited to financial time series, where return distributions often violate assumptions of normality and constant variance. These features - such as skewness, fat tails, and volatility clustering - are well-documented empirical properties across asset classes and time periods (Cont, 2001). Prior normality tests confirmed that the distributional assumptions required for parametric tests are not met in this case.

Let X_t and y_t denote the rolling performance ratios (Sharpe or Sortino) for a matched pair of portfolios at time *t*. The test is performed on the differences $D_t = X_t - Y_t$, assessing the null hypothesis:

$$H_0: Median(D_t) = 0 \ vs \ H_1: Median(D_t) \neq 0$$

The test is applied to the following portfolio pairs:

- A vs B (Equal-weighted portfolios)
- C vs D (Max Sharpe, unconstrained)
- E vs F (Constrained Max Sharpe with weight limits)
- G vs H (Constrained with shorting)

The resulting p-values determine whether the crypto-enhanced portfolios significantly outperform their traditional benchmarks in terms of rolling Sharpe and Sortino ratios.

This test provides an initial assessment of significance under minimal distributional assumptions, but since financial return series may violate the assumption of independent and identically distributed (i.i.d.) samples, additional tests are introduced in the following sections to confirm the robustness of the results (Wilcoxon, 1992).

3.5.2 Non-Parametric Benchmark Test: Rolling Sharpe and Sortino vs Target

This test evaluates whether the rolling Sharpe and Sortino ratios of each crypto-enhanced portfolio tend to lie above or below a fixed benchmark value, defined as the full-sample Sharpe or Sortino ratio of the corresponding non-crypto portfolio in each pair.

For each 52-week rolling window, the crypto portfolio's ratio is compared to the benchmark. The proportion of rolling values falling below the benchmark is recorded. This proportion acts as a directional indicator of relative performance, without requiring distributional assumptions. The method is inspired by an applied walkthrough from NEDL (2022), who demonstrates non-parametric directional testing in a rolling performance context, and aligns with the broader academic rationale for distribution-free comparison methods (Ledoit & Wolf, 2008a).

The null hypothesis assumes no consistent performance difference between the portfolios, implying the rolling values are equally likely to fall above or below the benchmark. A low proportion (e.g., <5%)

suggests that the crypto-enhanced portfolio outperforms consistently, while values close to 50% suggest no directional advantage.

Formally:

$$Proportion_{below} = \frac{1}{T}\sum_{t=1}^{T} II\left(R_{crypto}^{(t)} < R_{benchmark}\right)$$

Where $R_{crypto}^{(t)}$ is the rolling Sharpe or Sortino ratio at time t, and $R_{benchmark}$ is the benchmark value

This approach is simple and intuitive but comes with key limitations: it does not produce a p-value, and the overlapping nature of rolling windows violates independence, complicating interpretation (Lo, 2002). Therefore, while useful for descriptive and directional assessment, it does not provide formal statistical inference.

To address these limitations and assess significance under more flexible assumptions, the next section introduces a bootstrap-based test, which evaluates full-sample Sharpe and Sortino differences using resampling techniques.

3.5.3 Bootstrap Test: Full-Sample Sharpe and Sortino Differences

To formally test whether the risk-adjusted performance differences between crypto and non-crypto portfolios are statistically significant, this thesis applies a non-parametric bootstrap test based on full-sample Sharpe and Sortino ratios.

The bootstrap procedure estimates the distribution of the difference in performance metrics under the null hypothesis that any observed advantage is due to chance. Unlike the Wilcoxon or benchmark-based tests, the bootstrap method accounts for heteroskedasticity, non-normality, and serial correlation in financial return data, making it particularly suitable for portfolios with volatile assets such as cryptocurrencies (Ledoit & Wolf, 2008a).

For each matched portfolio pair, the test is performed as follows:

1. The weekly return series of each portfolio is resampled with replacement based on original return series.

2. For each resample (1,000 iterations), Sharpe and Sortino ratios are calculated for both portfolios.

3. The difference between crypto and non-crypto portfolios is recorded in each iteration.

4. The empirical distribution of these differences is used to form a confidence interval.

Let $\Delta^{(i)} = SR_{crypto}^{(i)} - SR_{benchmark}^{(i)}$ be the difference in the ith bootstrap sample. After B iterations ($B = 1000$), the 95% confidence interval for Δ is estimated from the empirical distribution. If this interval does not include zero, the difference is considered statistically significant (Efron & Tibshirani, 1994, chap. 5).

Application

This test is applied to the full estimation period for each pair of portfolios (A vs B, C vs D, etc.) and for both Sharpe and Sortino ratios. Since the test uses full-sample returns, it captures overall performance differences, complementing the rolling and directional tests in previous sections.

By using bootstrapping, the test avoids strong parametric assumptions and provides a more robust inference framework suitable for real-world financial data (Ledoit & Wolf, 2008a).

3.5.4 Performance Across Market Regimes

To investigate whether the performance of crypto-enhanced portfolios is sensitive to market conditions, this thesis conducts a separate performance evaluation across distinct volatility regimes. This analysis tests whether the risk-adjusted benefits or drawbacks of cryptocurrencies persist during periods of market stress.

Market Regime Definition

The estimation period is segmented into five regimes based on volatility in the S&P 500 index, using a 12-week rolling standard deviation to classify conditions as either high or low volatility. The regimes include:

- Pre-Crisis
- Crisis 1

- Between Crises
- Crisis 2
- Post-Crisis

Each regime contains non-overlapping return data and reflects differing macroeconomic conditions and market sentiment (Ang & Bekaert, 2002b).

Testing Procedure

To evaluate whether portfolio performance changes systematically across market conditions, a bootstrap test is applied within each portfolio across different market regimes. Specifically, Sharpe and Sortino ratios are calculated for each regime segment (e.g., a low-volatility period and its adjacent crisis period), and then compared using a bootstrapped difference test.

This involves:

1. Resampling return data separately within each regime (e.g., Low1 and Crisis1) for a given portfolio.
2. Computing the Sharpe and Sortino ratios for each resample.
3. Recording the difference between the two regime-specific metrics.
4. Constructing a 95% confidence interval for the distribution of these differences.

If the interval excludes zero, the difference is interpreted as statistically significant. This process is repeated for key regime pairs (e.g., Crisis1 vs Low1, Crisis2 vs post-Crisis) and across all portfolios.

Purpose

This test serves two purposes:

- To assess whether the inclusion of cryptocurrencies leads to consistent outperformance or underperformance under different market conditions.
- To determine whether any observed effects are regime-dependent, particularly during crisis periods when diversification benefits are most critical.

This regime-based bootstrap analysis strengthens the robustness of the overall performance evaluation by incorporating time-varying risk dynamics often seen in financial markets (Longin & Solnik, 2001).

3.5.5 Summery of Testing Framework

In summary, this thesis applies four complementary statistical tests to evaluate whether the inclusion of cryptocurrencies leads to statistically significant improvements in risk-adjusted performance. The Wilcoxon Signed-Rank Test assesses paired rolling performance without assuming normality. The Non-Parametric Benchmark Test provides directional evidence relative to fixed portfolio targets. The Bootstrap Test estimates confidence intervals for full-sample Sharpe and Sortino differences under minimal distributional assumptions. Finally, the Crisis Regime Test examines how performance varies across market volatility states. Together, this multi-method framework allows for robust inference, accounting for the non-normality, heteroskedasticity, and autocorrelation often present in financial return series.

3.6 Factor models

To assess whether the performance of the constructed portfolios - particularly those including cryptocurrencies - can be explained by traditional risk exposures, this thesis applies two widely used multi-factor models from asset pricing literature: the Fama–French 3-factor model and an extended Fama–French 5-factor model including a momentum factor. These models decompose portfolio returns into components attributable to systematic effects such as market, size, value, profitability, investment, and momentum characteristics (Fama & French, 1993) (Fama & French, 2015) (Carhart, 1997).

3.6.1 Model Specifications

The Fama–French 3-factor model is specified as:

$$R_p - R_f = \alpha + \beta_m(R_m - R_f) + \beta_s \cdot SMB + \beta_v \cdot HML + \varepsilon$$

The Fama–French 5-factor + Momentum model adds three additional factors:

$$R_p - R_f = \alpha + \beta_m(R_m - R_f) + \beta_s \cdot SMB + \beta_v \cdot HML + \beta_r \cdot RMW + \beta_c \cdot CMA + \beta_{mom} \cdot UMD + \varepsilon$$

where:

$R_p = portfolio\ return,$

$R_f = risk\ free\ rate,$

$R_m = market\ return,$

$SMB = size\ premium\ (Small\ Minus\ Big),$

$HML = value\ premium\ (High\ Minus\ Low),$

$RMW = profitability\ (Robust\ Minus\ Weak),$

$CMA = investment\ (Conservative\ Minus\ Aggressive),$

$UMD = momentum\ (Up\ Minus\ Down),$

$\alpha = regression\ intercept\ representing\ abnormal\ return,$

$\varepsilon = regression\ residual.$

The 3-factor model originates from Fama & French (1993), the 5-factor model is based on their later work Fama & French (2015), and the momentum extension follows Carhart (1997).

3.6.2 Frequency and Data Application

In this thesis:

- The Fama–French 3-factor model is estimated using weekly return data, as weekly factor returns are available for market, size, and value factors.
- The extended 5-factor + momentum model is applied to monthly return data, reflecting the monthly publication frequency of those factors.

This approach balances temporal resolution and factor availability: the 3-factor model provides a high-frequency view of market-based exposures, while the 5-factor + Momentum model captures broader sources of systematic variation - albeit at lower temporal resolution.

To address the frequency mismatch and improve comparability, a robustness check is conducted using the 3-factor model estimated on monthly data. This enables a direct comparison with the 5-factor results under a consistent time frame.

3.6.3 Purpose and Interpretation

The primary purpose of this analysis is to determine whether the observed portfolio returns - especially for crypto-inclusive portfolios can be explained by exposure to conventional risk factors, or whether they exhibit abnormal returns not accounted for by these models.

The intercept term α serves as a measure of abnormal return generation. A statistically significant positive alpha implies that the portfolio produces returns unexplained by its factor loadings, potentially due to cryptocurrency exposure or other latent return drivers (Jensen, 1968b).

This factor model analysis complements the earlier performance metrics (Sharpe ratio, Sortino ratio, and Jensen's Alpha) by offering a multi-factor attribution framework grounded in asset pricing theory. It allows us to distinguish between return patterns driven by systematic risk exposures and those that represent genuine alpha.

It is important to recognize that both the Fama-French 3-factor and 5-factor models were originally developed to explain the cross-section of equity returns. While they provide a well-established framework for assessing systematic risk exposures in traditional assets, their applicability to cryptocurrencies may be limited. Cryptocurrencies are driven by different risk factors, such as adoption dynamics, technological developments, regulatory shifts, and speculative market behavior, which are not directly captured by these equity-based factor models. Consequently, while these models serve as a valuable benchmark to control for known systematic risks, any remaining alpha observed in crypto-inclusive portfolios may reflect return components unrelated to the conventional factor structure (Liu & Tsyvinski, 2021).

3.7 Out-of-Sample Validation

To evaluate the real-world applicability and robustness of the constructed portfolio strategies, this thesis conducts an out-of-sample performance test. This validation examines how portfolios perform when implemented in a forward-looking context using data that was not available during model construction or optimization. The aim is to assess whether the risk-adjusted performance observed in-sample carries over to new, unseen market conditions or whether it reflects overfitting to the estimation window (Bailey & López de Prado, 2014).

3.7.1 Period and Setup

The out-of-sample period spans from January 26, 2024 to January 31, 2025, immediately following the five-year estimation window. Each portfolio is implemented using the fixed weights determined at the end of the estimation period. These weights remain constant throughout the out-of-sample year - no re-optimization or parameter adjustment is performed.

To maintain these weights despite market fluctuations, portfolios are mechanically rebalanced on a weekly basis using price data. This simulates a realistic passive investment strategy in which an investor commits to a fixed-weight allocation and rebalances periodically to maintain the intended structure. This structure reflects common practices in strategy validation literature, where fixed-weight portfolios are used to test out-of-sample robustness under practical constraints (DeMiguel et al., 2009b).

3.7.2 Evaluation Metrics

The out-of-sample evaluation uses the same performance measures as the in-sample analysis, allowing for consistent comparison. The following metrics are computed based on weekly return data:

- Sharpe ratio

- Sortino ratio
- Calmar ratio
- Jensen's Alpha (relative to the S&P 500)

All metrics are calculated using fixed portfolio weights without any re-estimation of parameters during the evaluation period, preserving the integrity of the out-of-sample framework (Bailey & López de Prado, 2014).

3.7.3 Purpose and Interpretation

The purpose of this validation is not to forecast returns, but to test whether the portfolios' in-sample performance reflects a true investment advantage or merely a favorable historical window. By applying the strategies to new data, this analysis tests whether they can maintain superior performance in a realistic investment context.

The out-of-sample period is limited to one year, which constrains statistical power. As such, results are interpreted as a robustness check, not a conclusive forecasting evaluation. Nevertheless, consistency between in-sample and out-of-sample results, particularly in risk-adjusted metrics, provides evidence in favor of the generalizability and practical relevance of crypto-inclusive portfolio strategies (Bailey & López de Prado, 2014).

4. Analysis and interpretation of results

4.1 Introduction to analysis and interpretation of results

This thesis seeks to examine whether the inclusion of cryptocurrencies can improve the risk-adjusted return of a stock portfolio consisting of Danish and U.S. equities. To investigate this, the analysis begins by exploring the potential diversification benefits, focusing on the Markowitz Mean-Variance Optimization model, the role of correlations between assets, and the structure of the minimum variance frontier.

Subsequently, the performance of eight portfolios is evaluated in pairs - four without cryptocurrencies and four with. To determine whether the inclusion of cryptocurrencies leads to superior risk-adjusted returns, the analysis compares the Sharpe, Sortino, and Calmar ratios of each portfolio. This comparison is supported by three statistical methods: the Wilcoxon Signed-Rank Test, a bootstrap-based test, and a non-parametric benchmark test. While all three ratios are used for descriptive comparison, formal statistical testing is applied to the Sharpe and Sortino ratios due to their widespread use and well-established distributional properties.

Furthermore, the thesis investigates how these performance metrics behave during periods of elevated market volatility, with particular emphasis on financial crises. Downside risk measures such as Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR), and Maximum Drawdown are also examined. The analysis concludes with a multi-factor regression using the Fama–French 3-factor and 5-factor models, followed by an out-of-sample performance evaluation. Overall, this section aims to provide insights into the primary research question:

To what extent does adding cryptocurrencies to a traditional investment portfolio improve risk-adjusted returns (e.g., Sharpe ratio)?

And the supporting questions:

- *How do cryptocurrencies correlate with Danish and American equities under different market conditions?*
- *How do extreme market conditions (e.g., financial crises, periods of high volatility) impact the diversification benefits of cryptocurrencies?*

4.2 Correlations matrix

One of the most important components in portfolio diversification is correlation, because if its effect on risk reduction. To assess the potential diversification benefits of cryptocurrencies, it is essential to examine their correlation with traditional equity assets. Figure 1 presents the average correlation coefficients between three major cryptocurrencies (Bitcoin, Ethereum, and Binance Coin) and both US and Danish equities over the full sample period. Across all cryptocurrencies, the correlations with equities remain relatively low and positive, ranging from approximately 0.11 to 0.20. Bitcoin, for example, shows a correlation of 0.1233 with US stocks and 0.1603 with Danish stocks, while Ethereum exhibits slightly higher correlations.

These findings suggest that cryptocurrencies tend to move relatively independently of traditional stock markets, which supports the hypothesis that they can serve as diversification tools within a portfolio of US and Danish equities. The modest correlations imply that including cryptocurrencies in a traditional portfolio may help reduce overall portfolio risk and improve its risk-return profile, particularly in non-crisis periods.

Figure 1 - Correlations of cryptocurrencies VS respectively US and Danish equities

	US stocks	Danish Stocks
Bitcoin	0.1233	0.1603
Etherum	0.1907	0.1973
Binance coin	0.1072	0.1483

Source: Own construction

To further understand how these relationships evolve under different market conditions, figure 2 reports time-divided correlations between cryptocurrencies and equities, split into five periods: Pre-Crisis, Crisis 1, Between Crises, Crisis 2, and Post-Crisis. During the pre-crisis, between-crisis and post-crisis periods, correlations remain low, typically below 0.20. This reaffirms the potential for cryptocurrencies to contribute to diversification under stable market conditions. However, a notable pattern emerges during periods of high volatility - specifically in Crisis 1 and Crisis 2. In these periods, correlations between cryptocurrencies and equities increase significantly, particularly with Danish stocks. For example, during Crisis 1, Ethereum and Binance Coin have correlations of 0.6404 and 0.6501 with

Danish equities, respectively. Bitcoin's correlation with Danish stocks also rises to 0.5728. Though less pronounced, correlations with US equities also increase during crises.

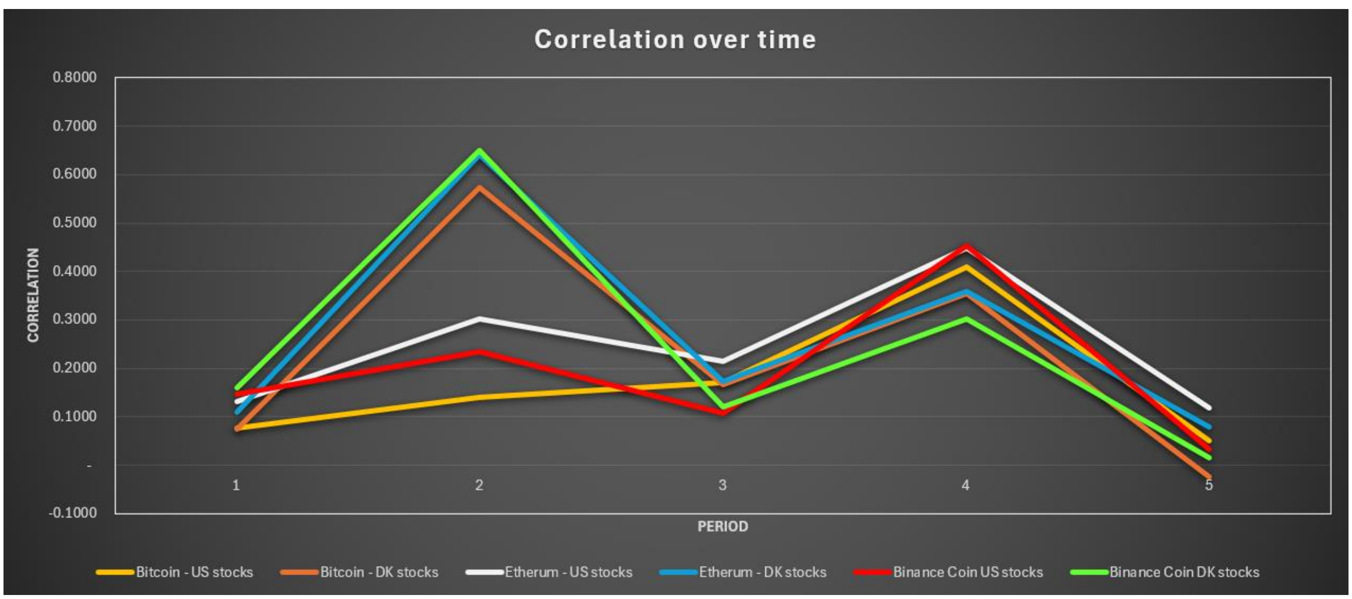
Figure 2 - Correlations of cryptocurrencies VS respectively US and Danish equities - time divided

	Pre Crisis	Crisis 1	Between	Crisis 2	Post
Bitcoin - US stocks	0.0768	0.1408	0.1702	0.4087	0.0507
Bitcoin - DK stocks	0.0740	0.5728	0.1672	0.3555	- 0.0246
Etherum - US stocks	0.1311	0.3024	0.2135	0.4487	0.1189
Etherum - DK stocks	0.1089	0.6404	0.1737	0.3584	0.0783
Binance Coin US stocks	0.1474	0.2331	0.1069	0.4536	0.0336
Binance Coin DK stocks	0.1603	0.6501	0.1194	0.3031	0.0163

Source: Own construction

This dynamic behavior reflects a well-documented phenomenon in finance: correlation breakdown during market stress. Assets that are typically uncorrelated or weakly correlated tend to become more synchronized in times of crisis, which reduces the benefits of diversification when they are needed most. The development of the correlation can be seen in figure 3 below.

Figure 3 – Correlation over time



Source: Own construction

In summary, the correlation analysis indicates that while cryptocurrencies offer meaningful diversification potential in normal market conditions, their effectiveness in mitigating portfolio risk declines in periods of financial stress. These findings provide important context for evaluating the overall contribution of cryptocurrencies to portfolio efficiency and align with one of the supporting research questions regarding how correlation patterns change under varying market conditions. The two tables can be found below, and further correlations between individual equities from thesis data and cryptocurrencies can be found in appendixes.

4.3 Min Variance Frontier

It is now established that it would be relevant to investigate cryptocurrencies as a diversification tool in a portfolio of Danish and US equities, primarily due to their relatively low correlation with traditional assets. Using the Markowitz Mean-Variance Optimization model, the thesis constructs minimum variance frontiers (MVF) for two setups: one including cryptocurrencies (Bitcoin, Ethereum, Binance Coin), and one excluding them. This enables a visual and analytical comparison of the risk-return tradeoff with and without cryptocurrencies.

The MVF illustrates the set of portfolios that deliver the lowest possible variance for a given level of expected return. When assets with low correlation are introduced, the efficient frontier typically shifts upward or leftward - offering either higher return for the same level of risk or lower risk for the same level of return. This is the theoretical foundation for testing whether cryptocurrencies can meaningfully enhance portfolio efficiency.

To evaluate this, two sets of frontiers have been constructed:

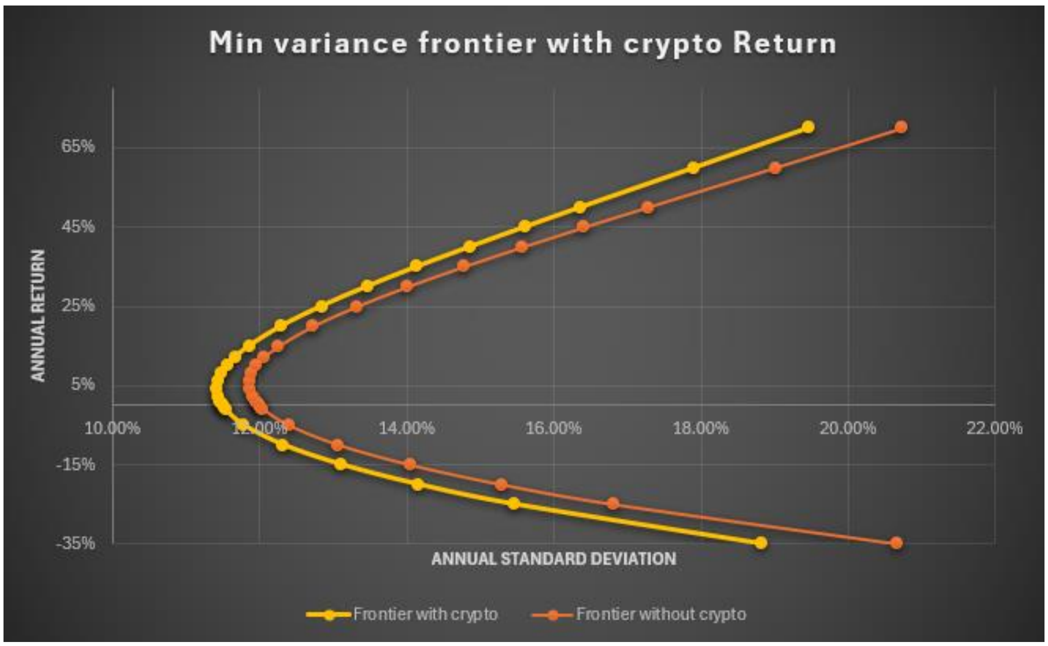
- **Unconstrained frontier**: where short selling is allowed (weights can be negative).
- **Constrained frontier**: where short selling is restricted (weights must be ≥ 0), reflecting a more realistic investment scenario.

In both frontiers, the total weight is constrained to sum to 1 ($\sum w = 1$), representing fully invested portfolios without leverage. Both the unconstrained and constrained frontiers can be found in Figure 4 and Figure 5 below.

Results – Unconstrained Frontier

In the unconstrained setup, the frontier including cryptocurrencies significantly outperforms the one excluding them, particularly at higher levels of expected return. This is primarily driven by the historically high returns and low correlation of cryptocurrencies. However, this comes at the cost of extreme portfolio allocations (e.g., highly negative or positive weights), which are unlikely to be feasible in practice. While the unconstrained frontier demonstrates theoretical potential, it should be viewed as an idealized benchmark rather than a practical portfolio choice.

Figure 4 – Unconstrained Min Variance Frontiers



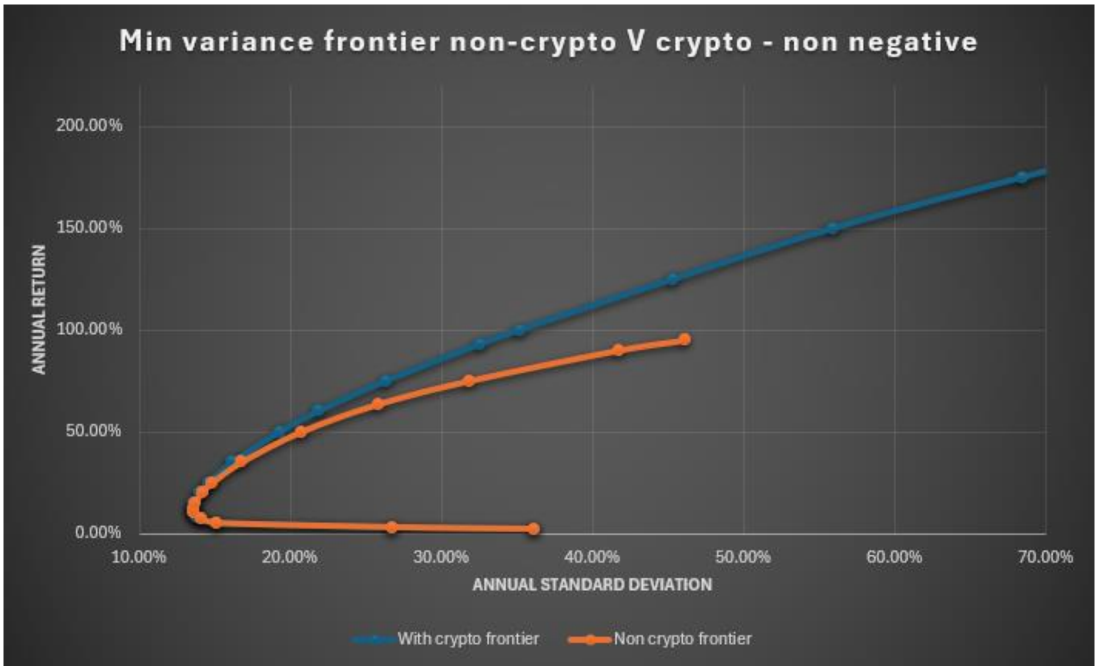
Source: Own construction

Results – Constrained Frontier

When restricting the weights to non-negative values, the analysis becomes more applicable to real-world investors. Even with this constraint, the frontier that includes cryptocurrencies consistently lies above the non-crypto frontier across almost the entire range. This suggests that even modest crypto allocations can improve the risk-adjusted performance of a traditional equity portfolio. The difference becomes particularly clear at medium to high risk levels, where the optimizer leverages the return potential of crypto assets.

For example, with a target return of 50%, the crypto-inclusive portfolio reaches that level with lower volatility than the non-crypto version. Conversely, for a given level of volatility, the inclusion of crypto assets allows for higher expected returns, indicating a superior tradeoff.

Figure 5 – Unconstrained Min Variance Frontiers



Source: Own construction

Implications

The frontier analysis strongly supports the hypothesis that adding cryptocurrencies can enhance the efficiency of a traditional equity portfolio. Even under non-leveraged and short-sale-constrained conditions, the inclusion of Bitcoin, Ethereum, and Binance Coin improves the frontier, supporting their role as potential diversification enhancers.

These results serve as a theoretical baseline. In the next sections, this will be followed by an empirical performance evaluation using Sharpe and Sortino ratios, tested statistically, to determine whether the improved efficiency shown here is also reflected in realized returns.

4.4 Portfolio Performance

As described in the methodology section, the dataset is divided into a 5-year estimation period and a 1-year performance period. First, the analysis will focus on the estimation period, evaluating the risk-adjusted performance of the portfolios including three different tests of Sharpe and Sortino ratios. Afterwards, the performance data will be used to assess how the portfolios actually performed following the estimation period.

4.4.1 Distribution Statistics – Estimation Period

Figure 6 – Distribution statistics + Maximum drawdown for each portfolio

Distribution statistics	Mean return annual	Standard deviation annual	Kurtosis	Skewness	Drawdown	VaR at 0.05	VaR at 0.025	CVaR at 0.05	CVaR at 0.025
Market (S&P 500)	14.82%	19.86%	6.0446	-0.5507	31.81%	-3.35%	-4.92%	-6.43%	-8.20%
A - Equally weigthed without crypto	23.44%	17.94%	3.7839	-0.6411	27.11%	-3.03%	-4.80%	-5.75%	-7.24%
B - Equally weigthed with crypto	30.08%	18.99%	4.1393	-0.8426	28.34%	-3.54%	-4.85%	-6.18%	-7.81%
C - Max Sharpe Ratio without crypto	63.54%	25.85%	1.2270	-0.0443	26.16%	-4.70%	-6.43%	-7.15%	-8.40%
D - Max Sharpe Ratio with crypto	85.86%	30.01%	2.0321	-0.0008	33.63%	-5.65%	-7.78%	-8.49%	-9.77%
E - Max Sharpe Ratio without crypto, Weights ≤ 10%, Number of assets ≥ 20	43.53%	20.38%	2.2036	-0.3998	23.59%	-4.06%	-4.61%	-5.81%	-7.08%
F - Max Sharpe Ratio with crypto, Weights ≤ 10%, Number of assets ≥ 20	59.74%	23.44%	2.7784	-0.4306	27.46%	-4.20%	-5.72%	-6.84%	-8.45%
G - Max Sharpe Ratio without crypto, Negative values allowed, Weigts ≤ 100% and ≥ -20%	988.92%	127.22%	0.3478	-0.0826	77.36%	-24.78%	-29.42%	-33.41%	-38.93%
H - Max Sharpe Ratio with crypto, Negative values allowed, Weigts ≤ 100% and ≥ -20%	6745.45%	257.42%	3.5448	0.0696	100.00%	-45.34%	-66.07%	-74.72%	-93.19%

Source: Own construction

Before proceeding to formal statistical tests, this section presents an overview of the distribution statistics for the constructed portfolios during the five-year estimation period. These descriptive metrics provide valuable insight into the return characteristics, risk exposure, and distributional properties of each strategy. Note that return-related metrics such as mean return, standard deviation, Sharpe and Sortino ratios are annualized for comparability, the Value at Risk (VaR) and Conditional Value at Risk (CVaR) are presented on a weekly basis, reflecting the data frequency and avoiding overestimation of long-term tail risk.

The eight portfolios analyzed in this thesis are constructed using a combination of equal weighting and Markowitz Mean-Variance Optimization, under varying constraints. This enables a structured and consistent analysis of risk-adjusted returns across different portfolio construction methods and asset compositions.

The portfolios are organized into four matched pairs, where each pair consists of a version with and without cryptocurrencies. The design of these pairs mirrors the structure used in the upcoming hypothesis tests and is as follows:

- A vs B: Equal-weighted portfolios with and without cryptocurrencies
- C vs D: Maximum Sharpe ratio portfolios without vs with crypto (no constraints on weights)
- E vs F: Constrained maximum Sharpe ratio portfolios with weight limits (≤ 10%) and minimum number of assets (≥ 20)
- G vs H: Constrained maximum Sharpe ratio portfolios allowing short positions within a band (weights between -20% and 100%)

Each portfolio is fully invested (weights sum to 1), but the presence or absence of shorting and asset caps introduces a range of risk profiles. The aim is to understand how the inclusion of cryptocurrencies affects portfolio distribution characteristics across multiple strategies.

Return, Risk, and Distributional Characteristics

When comparing the performance of the four portfolio pairs, a consistent pattern emerges: portfolios that include cryptocurrencies tend to deliver higher average returns than their non-crypto counterparts. This finding holds across all configurations - whether equal-weighted or optimized for Sharpe ratio under various constraints.

For instance, the equal-weighted crypto portfolio (Portfolio B) achieves an annualized return of 30.08%, outperforming its non-crypto equivalent, Portfolio A, which returns 23.44%. The same trend is observed in the unconstrained maximum Sharpe portfolios, where Portfolio D (with crypto) reaches 85.86%, compared to 63.54% for Portfolio C. Even in constrained setups, the crypto-inclusive portfolios maintain their outperformance: Portfolio F exceeds E, and Portfolio H - allowing both long and short positions - achieves a return of 6745.45%, largely due to concentrated crypto exposure and the effects of leverage.

However, this increased return potential comes with substantially elevated risk. Across all portfolio pairs, the inclusion of cryptocurrencies results in higher volatility, more severe tail risk, and deeper realized losses. These effects are seen consistently across key risk metrics: standard deviation, Value-at-Risk (VaR), Conditional Value-at-Risk (CVaR), and Maximum Drawdown (MDD).

For example, in the constrained portfolios, Portfolio F (with crypto) shows a higher standard deviation than its non-crypto counterpart E (23.44% vs. 20.38%), a more negative VaR at 5% (-4.20% vs. -4.06%), and a lower CVaR at 5% (-6.84% vs. -5.81%). It also experiences a greater maximum drawdown (27.46% vs. 23.59%), indicating larger historical losses despite tight portfolio constraints. The divergence becomes more pronounced in the leveraged long-short portfolios. Portfolio H reaches a standard deviation of 257.42%, compared to 127.22% for Portfolio G. Its VaR at 5% is -45.34% vs. -24.78%, CVaR at 5% is -74.72% vs. -33.41%, and maximum drawdown hits 100%, compared to 77.36% for its non-crypto counterpart.

This same risk amplification pattern is observed in the other portfolio pairs. While VaR and CVaR reflect distribution-based downside risk, Maximum Drawdown captures the worst observed loss, serving as a tangible measure of capital erosion. Together, these metrics confirm that crypto-enhanced portfolios, while often superior in return, carry significantly greater risk across both statistical and historical dimensions.

Beyond traditional measures of risk, the distributional characteristics of returns further emphasize the differences. Many of the crypto-inclusive portfolios exhibit non-normal return distributions, characterized by negative skewness and excess kurtosis. This indicates fatter tails and a higher likelihood of extreme losses - key elements of downside risk that standard deviation alone cannot capture. These effects are particularly strong in portfolios that allow high allocation concentration or short positions, such as Portfolios D and H. Their high kurtosis values signal an increased probability of both extreme gains and severe losses, making them more sensitive to market shocks. Conversely, constrained portfolios like E and F manage to maintain more balanced distributional profiles. While still benefiting from crypto exposure, these portfolios exhibit more moderate skewness and lower kurtosis, suggesting that effective constraint design can harness the upside potential of cryptocurrencies without fully absorbing their tail risk.

In summary, the inclusion of cryptocurrencies improves return potential but significantly alters the return distribution - introducing greater volatility and tail risk. The results underline the importance of risk-aware portfolio construction. Constraints on asset weights, especially in portfolios involving volatile assets like cryptocurrencies, can be crucial in mitigating downside exposure while preserving the upside.

4.4.2 Risk-Adjusted Performance and Market Exposure

While portfolios that include cryptocurrencies display higher volatility and greater downside risk, their risk-adjusted performance improves notably across all portfolio pairs. This is clearly reflected in both Sharpe and Sortino ratios, which measure return per unit of total and downside risk, respectively. In each of the four matched portfolio pairs, the crypto-inclusive portfolios consistently outperform their traditional counterparts:

Sharpe Ratios

- Portfolio B: 1.35 vs A: 1.06
- Portfolio D: 2.72 vs C: 2.29
- Portfolio F: 2.36 vs E: 1.92
- Portfolio H: 26.19 vs G: 7.74

Sortino Ratios

- Portfolio B: 2.01 vs A: 1.58
- Portfolio D: 4.79 vs C: 3.07
- Portfolio F: 3.91 vs E: 3.09
- Portfolio H: 44.42 vs G: 13.47

These improvements suggest that even after adjusting for increased volatility and downside risk, the return per unit of risk is consistently higher when cryptocurrencies are included. Particularly in the constrained portfolios (e.g., D vs C and F vs E), the results underscore the value of controlled crypto exposure as a diversification tool that enhances efficiency without necessitating excessive risk.

Calmar ratios, which capture risk-adjusted returns relative to maximum drawdown, reinforce this conclusion. Across most configurations, crypto-enhanced portfolios exhibit higher Calmar ratios than their non-crypto counterparts, despite larger drawdowns. For example, Portfolio F achieves a significantly better balance between return and drawdown than Portfolio E. However, extreme setups like Portfolio H exhibit unstable and excessively high Calmar values due to leveraged exposure and sharp swings, including instances of weekly returns below -100%. This highlights the importance of practical constraints in managing tail risk.

To further contextualize performance, Jensen's Alpha and Beta are also reported. Crypto-inclusive portfolios generally deliver higher Alpha values, reflecting returns in excess of what would be expected given their market exposure (S&P 500):

- Portfolio B: Alpha of 16.69% vs A: 10.23%
- Portfolio D: Alpha of 71.36% vs C: 49.82%
- Portfolio F: Alpha of 45.85% vs E: 30.18%
- Portfolio H: Alpha of 6705.64% vs G: 959.24%

That said, this outperformance comes with greater systematic risk in more flexible portfolios. Portfolio G has a Beta of 2.43, and Portfolio H climbs to 3.40, indicating strong sensitivity to broader market movements, primarily driven by leverage and high crypto concentrations. Figure 7 below displays the Jensen's Alpha together with the Sharpe, Sortino, and Calmar ratios across all portfolios.

Figure 7 – Sharpe ratios, Sortino ratios, Calmar ratios and Jensen's alpha

Sharpe, Sortino & Calmar Ratio	Return	Std Deviation	Downside Risk	Drawdown	Sharpe ratio	Sortino Ratio	Calmar ratio
Market (S&P 500)	14.82%	19.86%	14.67%	31.81%	0.524	0.710	0.466
A - Equally weigthed without crypto	23.44%	17.94%	12.06%	27.11%	1.061	1.578	0.865
B - Equally weigthed with crypto	30.08%	18.99%	12.78%	28.34%	1.352	2.009	1.061
C - Max Sharpe Ratio without crypto	63.54%	25.85%	15.06%	26.16%	2.288	3.928	2.429
D - Max Sharpe Ratio with crypto	85.86%	30.01%	17.01%	33.63%	2.715	4.788	2.553
E - Max Sharpe Ratio without crypto, Weights ≤ 10%,	43.53%	20.38%	12.65%	23.59%	1.920	3.092	1.845
F - Max Sharpe Ratio with crypto, Weights ≤ 10%, Number of assets ≥ 20	59.74%	23.44%	14.17%	27.46%	2.361	3.906	2.176
G - Max Sharpe Ratio without crypto, Negative values allowed, Weigts ≤ 100% and ≥ -20%	988.92%	127.22%	73.09%	77.36%	7.739	13.471	12.783
H - Max Sharpe Ratio with crypto, Negative values allowed, Weigts ≤ 100% and ≥ -20%	6745.45%	257.42%	151.75%	100.00%	26.187	44.423	67.455
Risk free rate	4.40%						

Jensen's alpha	Return	Beta	Jensen's alpha
Market (S&P 500)	14.82%	1.00	0.00%
A - Equally weigthed without crypto	23.44%	0.846	10.23%
B - Equally weigthed with crypto	30.08%	0.863	16.69%
C - Max Sharpe Ratio without crypto	63.54%	0.894	49.82%
D - Max Sharpe Ratio with crypto	85.86%	0.970	71.36%
E - Max Sharpe Ratio without crypto, Weights ≤ 10%,	43.53%	0.859	30.18%
F - Max Sharpe Ratio with crypto, Weights ≤ 10%, Number of assets ≥ 20	59.74%	0.911	45.85%
G - Max Sharpe Ratio without crypto, Negative values allowed, Weigts ≤ 100% and ≥ -20%	988.92%	2.427	959.24%
H - Max Sharpe Ratio with crypto, Negative values allowed, Weigts ≤ 100% and ≥ -20%	6745.45%	3.400	6705.64%
Risk free rate	4.40%		

Source: Own construction

In summary, the inclusion of cryptocurrencies enhances both total and downside-adjusted performance across all strategies. Yet, these results also highlight the importance of portfolio design and constraint management: while unconstrained portfolios may offer extreme returns, they also introduce considerable risk, whereas constrained portfolios like D and F achieve a more efficient balance between return and risk.

4.5 Statistical tests

To assess whether the observed risk-adjusted performance differences are statistically significant, the empirical results from the four statistical tests described in Section 3.5 are presented below. Detailed test results can be excessed in the appendixes.

4.5.1 Wilcoxon Signed-Rank Test: Rolling Sharpe and Sortino Ratios

To assess whether the improvement in risk-adjusted returns observed in the cryptocurrency-enhanced portfolios is statistically significant, a Wilcoxon Signed-Rank Test is applied. This non-parametric test compares paired samples of rolling Sharpe and Sortino ratios across time for each portfolio pair, allowing for a robust comparison that does not assume normality of the underlying data - a key advantage, as Shapiro-Wilk tests confirmed non-normal distributions in nearly all return series. Shapiro-Wilk tests can be found in appendix 8.6.

The results are summarized as follows:

- **A vs B (Equal-weighted portfolios)**
    - Sharpe: *p-value = 1.94e-15*
    - Sortino: *p-value = 1.55e-14*
      → The crypto-inclusive portfolio B significantly outperforms portfolio A.
- **C vs D (Unconstrained max Sharpe portfolios)**
    - Sharpe: *p-value = 0.5313*
    - Sortino: *p-value = 0.1936*
      → No statistically significant difference between the crypto (D) and non-crypto (C) portfolios.
- **E vs F (Constrained max Sharpe portfolios with weight and asset limits)**
    - Sharpe: *p-value = 0.0037*
    - Sortino: *p-value = 0.0024*
      → Portfolio F, which includes crypto, significantly outperforms portfolio E.

- **G vs H (Constrained portfolios with shorting allowed)**
  - Sharpe: *p-value = 0.0004*
  - Sortino: *p-value = 0.0031*
    → Crypto-inclusive portfolio H significantly outperforms portfolio G.

These results provide statistical support for the thesis' core hypothesis: adding cryptocurrencies to traditional portfolios improves risk-adjusted returns in most tested configurations. The only exception is the unconstrained Sharpe-maximized portfolios (C vs D), where the difference is not statistically significant. This could potentially be explained by the extreme values and volatility driven by heavy crypto weights in those portfolios, which may have affected the consistency of rolling risk-adjusted performance.

Despite the statistical strength of the Wilcoxon results, it is important to acknowledge certain limitations of the test. Although the Wilcoxon Signed-Rank Test is non-parametric and does not rely on normality assumptions, it still requires that the data are independent and identically distributed (i.i.d.). This assumption is often violated in financial time series, as confirmed by previous tests in this thesis that identified autocorrelation and heteroskedasticity in the rolling Sharpe and Sortino ratios.

These issues suggest that while the Wilcoxon test provides useful insights, complementary testing approaches are necessary to validate the robustness of the findings. For that reason, the thesis now turns to a non-parametric benchmark test, which compares rolling Sharpe and Sortino ratios against a fixed benchmark value, providing a different perspective on relative outperformance. After this, a bootstrap analysis will be employed, offering stronger robustness to autocorrelation and volatility clustering, and serving as a more distributionally flexible method.

4.5.2 Non-Parametric Benchmark Test: Rolling Sharpe and Sortino vs Target

To complement the pairwise Wilcoxon tests, this section applies a non-parametric benchmark test, which compares rolling Sharpe and Sortino ratios of the crypto portfolios against the benchmark ratios of their traditional counterparts. This test is distribution-free and well-suited for financial time series that exhibit non-normality and time-dependent structures, as is the case here.

For each portfolio pair, the rolling Sharpe and Sortino ratios (calculated over 52-week windows) of the crypto-enhanced portfolio are compared to the full-period (non-rolling) Sharpe and Sortino ratios of the corresponding non-crypto portfolio. The test simply counts how often the rolling values fall below the benchmark. While this method does not yield a formal p-value, it offers a practical and time-aware indication of relative performance across periods.

The results are summarized as follows:

- Sharpe Ratio – % of Rolling Values Below Benchmark
  - B vs A: 44.5%
  - D vs C: 44.0%
  - F vs E: 43.1%
  - H vs G: 90.4%

- Sortino Ratio – % of Rolling Values Below Benchmark
  - B vs A: 44.5%
  - D vs C: 47.9%
  - F vs E: 42.6%
  - H vs G: 89.0%

In three of the four portfolio pairs (B vs A, D vs C, F vs E), fewer than 50% of the rolling Sharpe and Sortino ratios fall below their respective benchmarks. This suggests that in most rolling periods, the crypto-enhanced portfolios outperform their traditional counterparts in terms of risk-adjusted returns, reinforcing earlier findings from the descriptive analysis.

However, Portfolio H vs G stands out as an exception: over 89% of H's rolling Sharpe and Sortino values fall below the benchmark. This discrepancy, despite H's extremely high average Sharpe and

Sortino ratios, likely stems from large negative outliers early in the sample. Due to the use of overlapping 52-week windows, these outliers impact a significant portion of the rolling calculations, suppressing performance metrics for much of the sample. This highlights a key limitation of rolling analysis when returns are volatile and heavily skewed, especially in leveraged portfolios.

While the benchmark test provides valuable directional insight, it does not offer statistical significance in the formal sense. It also assumes independence between rolling observations, which is violated due to overlapping windows. Additionally, it focuses solely on Sharpe and Sortino ratios, without incorporating sampling variability or distributional uncertainty from the full return series.

To address these limitations, as well as those of the Wilcoxon test, the next section introduces a bootstrap-based test. This approach resamples full return series to generate empirical distributions of Sharpe and Sortino ratio differences, offering a more robust, assumption-light framework for statistical inference.

4.5.3 Bootstrap Test: Full-Sample Sharpe and Sortino Differences

To formally test the statistical significance of the observed improvements in risk-adjusted returns, a non-parametric bootstrap method is applied. Unlike the previous two tests, which relied on rolling metrics, this approach draws from the full return distributions of each portfolio, offering a robust way to capture the randomness and variability in performance.

The procedure resamples weekly returns 1,000 times with replacement and recalculates annualized Sharpe and Sortino ratios for each draw. In the second stage, the test computes Sharpe and Sortino differences between each benchmark and its paired crypto-enhanced portfolio. Confidence intervals and summary statistics are then derived from the empirical distribution of these differences.

Results Summary

Figure 8 - Sharpe Ratio Differences (Benchmark - Crypto Portfolio)

Portfolio Pair	**Mean Difference**	**95% CI**	**% Diffs < 0**	**Statistically Significant?**
A - B	-0.3032	[-1.97, 1.26]	61.9%	❌
C - D	-0.4001	[-2.59, 1.73]	63.7%	❌
E - F	-0.4433	[-2.38, 1.35]	67.1%	❌
G - H	-34.0955	[-175.46, 9.99]	84.3%	❌

Source: Own construction

Figure 9 - Sortino Ratio Differences (Benchmark - Crypto Portfolio)

Portfolio Pair	**Mean Difference**	**95% CI**	**% Diffs < 0**	**Statistically Significant?**
A - B	-0.4266	[-3.52, 2.47]	62.4%	❌
C - D	-1.0854	[-6.60, 3.70]	64.7%	❌
E - F	-0.8934	[-5.37, 2.99]	67.2%	❌
G - H	-62.6715	[-346.14, 19.89]	81.3%	❌

Source: Own construction

Interpretation

The mean differences in both Sharpe and Sortino ratios are negative across all portfolio pairs, suggesting that crypto-enhanced portfolios consistently outperform their traditional counterparts. Furthermore, in every case, more than 60% of the resampled differences are below zero, further supporting the directional outperformance of crypto portfolios.

However, the confidence intervals for all comparisons include zero, and the two-sided p-values are large, indicating that these differences are not statistically significant under a 5% threshold.

As with earlier tests, the H vs G pair displays the most extreme values, which are driven by Portfolio H's heavy leverage and exposure to volatile crypto assets. These factors create a wide spread in the bootstrap distribution, inflating the confidence interval and diluting significance despite the large average outperformance.

The bootstrap method excels in capturing the true variability of returns without relying on distributional assumptions. It is also robust to autocorrelation and heteroskedasticity, unlike the Wilcoxon Signed-Rank or t-tests.

That said, bootstrapping assumes that resampled observations are representative of the underlying process, and in heavily skewed, volatile, or outlier-dominated portfolios (e.g. Portfolio H), this assumption can be problematic. Moreover, it focuses on the mean of the distribution, which may be influenced by extreme outcomes.

Despite the lack of formal significance, the bootstrap test strengthens the thesis' case by reinforcing earlier findings: risk-adjusted performance improves when cryptocurrencies are included - particularly when proper portfolio constraints are applied.

The analysis now turns to performance under changing market conditions, specifically whether crypto portfolios continue to outperform in periods of market stress compared to periods of stability.

4.5.4 Crisis Period Performance Test: Directional Analysis of Bootstrapped Risk-Adjusted Ratios

To investigate how portfolios behave during periods of market distress compared to more stable times, this section applies a bootstrap-based Sharpe and Sortino ratio difference test across identified crisis and non-crisis intervals. Unlike prior sections focused on overall performance significance, the goal here is to assess whether risk-adjusted performance systematically deteriorates during crisis periods - particularly for portfolios containing cryptocurrencies.

The test compares bootstrapped Sharpe and Sortino ratios between two time segments (e.g., Crisis1 vs Low1) for each portfolio, using 1,000 resamples per comparison. For each pair, it calculates the mean and median differences, confidence intervals, and most importantly, the proportion of differences below zero, which directly reflects how frequently the portfolio performed better during non-crisis periods.

Result Summary and Interpretation

The 3 figures below shows a snapshot of some of the results from the test, full results can be found in appendixes.

Figure 10 – Sharpe ratio comparison between crisis period and normal period.

Sharpe Ratio Comparison

Portfolio	Comparison	Mean_Diff	Median_Dif	CI_Lower	CI_Upper	Below_0_%	Above_0_%	Significance
A	Crisis1 vs Low1	-1.7901	-2.5674	-6.4907	7.8922	80.3	19.7	Not Significant
B	Crisis1 vs Low1	-2.398	-3.2368	-7.2729	8.1409	84.2	15.8	Not Significant
C	Crisis1 vs Low1	-0.6833	-1.7164	-6.984	11.9084	66.6	33.4	Not Significant
D	Crisis1 vs Low1	-1.4142	-2.8048	-8.3713	14.7049	75.1	24.9	Not Significant
E	Crisis1 vs Low1	-2.2028	-3.288	-8.2382	11.2494	80.4	19.6	Not Significant
F	Crisis1 vs Low1	-2.6139	-3.6301	-9.2008	10.5033	81.1	18.9	Not Significant
G	Crisis1 vs Low1	633.637	8.1145	-68.3088	4086.333	42.1	57.9	Not Significant
H	Crisis1 vs Low1	3014.394	-24.9841	-2443.653	17908.5	64.5	35.5	Not Significant
A	Crisis1 vs Mid	0.2209	-0.7133	-3.5274	9.4593	61.8	38.2	Not Significant
B	Crisis1 vs Mid	-0.3692	-1.419	-4.3366	9.9544	70.5	29.5	Not Significant

Source: Own construction

Figure 11 – Sortino ratio comparison between crisis period and normal period

Sortino Ratio Comparison

Portfolio	Period_Comparison	Mean_Diff	Median_Diff	CI_Lower	CI_Upper	% Below	% Above	Significance
A	Crisis1 vs Low1	inf	-5.2454	-17.8672	36.136	0.84	0.16	Not Significant
B	Crisis1 vs Low1	inf	-7.164	-20.7047	20.4751	0.883	0.117	Not Significant
C	Crisis1 vs Low1	-0.8193	-4.9763	-20.6175	41.8763	0.745	0.255	Not Significant
D	Crisis1 vs Low1	inf	-7.1838	-26.2613	52.9187	0.809	0.191	Not Significant
E	Crisis1 vs Low1	inf	-8.3676	-25.3474	56.6037	0.827	0.173	Not Significant
F	Crisis1 vs Low1	inf	-8.6117	-27.2363	59.4904	0.861	0.139	Not Significant
G	Crisis1 vs Low1	inf	1.4812	-202.1198	23257.29	0.483	0.517	Not Significant
H	Crisis1 vs Low1	inf	-46.6686	-5834.238	212808.6	0.649	0.351	Not Significant
A	Crisis1 vs Mid	inf	-1.092	-5.9772	42.0327	0.641	0.359	Not Significant
B	Crisis1 vs Mid	inf	-2.0843	-7.6715	27.4647	0.71	0.29	Not Significant

Source: Own construction

Figure 12 – Sharpe & Sortino ratio mean and median duing different times

Sharpe ratio mean & median duing different times

Portfolio	Period	Mean Sharpe	Median Sharpe
A	Crisis1	1.2882	0.1597
B	Crisis1	1.5411	0.2626
C	Crisis1	3.1941	1.6217
D	Crisis1	3.0395	1.2907
E	Crisis1	2.6788	1.1389
F	Crisis1	2.2311	0.904
G	Crisis1	751.3041	24.1974
H	Crisis1	3121.588	9.5845
A	Crisis2	2.0577	0.5099
B	Crisis2	2.4209	0.6987
C	Crisis2	2.962	0.8557
D	Crisis2	3.4806	0.8643
E	Crisis2	3.2261	1.7769
F	Crisis2	4.5316	1.5697
G	Crisis2	1977.708	22.4556
H	Crisis2	1545358	11.1456
A	Low1	3.0684	2.9966
B	Low1	3.7285	3.6243

Sortino ratio mean & median duing different times

Portfolio	Period	Mean Sortino	Median Sortino
A	Crisis1	inf	0.3093
B	Crisis1	inf	0.311
C	Crisis1	8.3576	2.4028
D	Crisis1	inf	1.6345
E	Crisis1	inf	1.5086
F	Crisis1	inf	1.4254
G	Crisis1	inf	34.5033
H	Crisis1	inf	11.8504
A	Crisis2	inf	1.1871
B	Crisis2	inf	0.8099
C	Crisis2	inf	1.4712
D	Crisis2	inf	1.1444
E	Crisis2	inf	3.5747
F	Crisis2	inf	3.3213
G	Crisis2	inf	24.646
H	Crisis2	inf	11.777
A	Low1	7.013	6.3099
B	Low1	9.148	8.1575

Source: Own construction

The crisis period analysis reveals a compelling pattern in the behavior of risk-adjusted performance across different market regimes. Specifically, when comparing bootstrapped Sharpe and Sortino ratios between crisis and non-crisis periods, a majority of the differences fall below zero. This outcome suggests that portfolios generally deliver stronger risk-adjusted returns during more stable market environments. While this effect is observable across both traditional and crypto-enhanced portfolios, it is especially pronounced in those that include cryptocurrency exposure.

One clear example is the comparison between Crisis1 and Low1. Portfolio A, constructed without crypto assets, shows that 80.3 percent of the bootstrapped Sharpe ratio differences fall below zero, indicating significantly better performance in the normal period. This pattern is even more evident in Portfolio B, which includes cryptocurrencies, where 84.2 percent of Sharpe differences fall below zero. Portfolio H, which allows for shorting and leverage and also includes crypto, shows a similar directional tendency: 64.5 percent of Sharpe differences and 64.9 percent of Sortino differences are below zero. Across nearly all crisis-to-normal comparisons, portfolios that include crypto assets exhibit a more consistent tendency to underperform during periods of financial stress.

This directional outcome is further supported by the bootstrapped distribution medians. For instance, Portfolio B's median Sharpe ratio falls sharply from 3.62 during Low1 to just 0.26 in Crisis1. Its Sortino ratio follows a similar decline, from 8.16 to 0.31. Portfolio H experiences an even steeper drop:

its median Sharpe decreases from 93.1 in Low1 to only 9.58 in Crisis1, and its Sortino median drops from 150.0 to 11.85. Other crypto-inclusive portfolios, such as F and D, display the same behavior, with reductions in both median Sharpe and Sortino ratios from normal to crisis periods. These patterns reinforce the observation that portfolios with crypto allocations are especially sensitive to systemic risk.

Traditional portfolios also show performance declines during crises, though generally less extreme. For example, Portfolio A's median Sharpe ratio drops from 2.99 to 0.16, while Portfolio C's Sortino ratio decreases from 8.16 to 2.40 between Low1 and Crisis1. These changes underscore the impact of broader market stress, but the effect is less consistent and severe compared to portfolios that include crypto assets.

Collectively, these results support the interpretation that the performance of crypto-inclusive portfolios is highly regime-dependent. While they improve overall performance metrics across full-sample periods, their behavior in crisis regimes reveals a clear vulnerability. Their higher volatility, sensitivity to market sentiment, and lower liquidity appear to amplify downside risk when market conditions deteriorate. In effect, the characteristics that drive crypto outperformance during bull markets become liabilities in more turbulent times. However, while the tests consistently point to a directional trend of weaker performance during crises, this pattern is not statistically significant at the 95% confidence level. The bootstrapped confidence intervals for the Sharpe and Sortino ratio differences remain wide and often cross zero, indicating that the observed differences, although consistent in direction, do not meet conventional thresholds for statistical significance.

Test Limitation and Justification

While the bootstrap test offers an intuitive, distribution-free way to compare risk-adjusted returns across regimes, it does carry certain limitations. Notably, it assumes that returns within each subsample (e.g., Crisis1, Mid, Low2) are independently and identically distributed (i.i.d.), and that the dynamics of asset returns - particularly for cryptocurrencies - remain relatively stationary within those windows. In reality, financial returns often exhibit autocorrelation, volatility clustering, and other time-dependent structures, especially during turbulent periods.

More advanced techniques like block bootstrapping could address such dependencies, but they are impractical in this setting due to the relatively short duration of the crisis periods, in particular Crisis1.

Block bootstrapping would limit the number of usable resamples and reduce statistical power. Additionally, each crisis has unique features, and lumping them into a generalized regime model could obscure meaningful differences.

To mitigate this, the analysis treats Crisis1 and Crisis2 as distinct events and compares them separately to surrounding non-crisis periods. This approach respects the path-dependent nature of financial crises without overcomplicating the test design, ensuring that regime-specific effects are still captured in a robust and interpretable way.

4.5.5 Summery

The analysis consistently finds that adding cryptocurrencies to a traditional equity portfolio tends to improve average risk-adjusted returns. This directional pattern is observed across full-sample performance metrics and supported by multiple statistical tests, including the Wilcoxon signed-rank test, non-parametric benchmark comparisons, and bootstrap-based approaches. While not all results are statistically significant at conventional levels, the tests consistently point in the same direction: crypto-enhanced portfolios generally exhibit higher Sharpe and Sortino ratios than their traditional counterparts, particularly in unconstrained or lightly constrained portfolio settings.

However, the improvement in performance is not uniform across all market conditions. The regime-based analysis reveals that while crypto-inclusive portfolios outperform in calm markets, they tend to underperform during crisis periods. This is evident both from directional results (a majority of bootstrapped differences falling below zero) and sharp drops in median Sharpe and Sortino ratios during stress regimes. Although these crisis-period differences do not meet the 95% confidence threshold for statistical significance, the consistency in direction highlights a clear vulnerability under financial stress.

Overall, the inclusion of cryptocurrencies enhances long-term portfolio efficiency, but their impact is highly regime-dependent. They contribute positively to return potential and risk-adjusted performance in stable markets, while also introducing meaningful downside risk during periods of volatility and uncertainty. These findings suggest that investors considering crypto allocations must balance the potential for performance enhancement with the conditional risks that arise under adverse market conditions.

4.6 Factor models

The purpose of this multi-factor analysis is to assess whether the excess returns observed in the portfolios - particularly those including cryptocurrencies - can be attributed to exposure to systematic risk factors, or whether they exhibit abnormal return generation (alpha). This is essential in deepening the understanding of whether the strong risk-adjusted returns observed in previous sections (Sharpe, Sortino, Jensen's alpha) reflect true outperformance or are simply compensation for known factor risks.

To maintain consistency with the portfolio design throughout this thesis, interpretation centers on the four portfolio pairs.

Across all three model specifications - the Fama–French 3-factor (weekly and monthly) and the 5-factor model with momentum (monthly) - a consistent pattern emerges: portfolios with crypto exposure (B, D, F, H) exhibit higher alpha estimates than their traditional counterparts, often significantly so. This suggests that their superior risk-adjusted performance, already evidenced in previous sections, cannot be fully explained by traditional equity risk factors.

For example, in the 3-factor weekly model:

- Portfolio A has an alpha of 0.0016, while Portfolio B's alpha is higher at 0.0026.
- This gap widens in the C–D pair (0.0068 vs 0.0092) and in E–F (0.0044 vs 0.0064).
- The largest difference appears in G–H, where Portfolio H (crypto) shows an alpha nearly double that of G.

These relationships persist in the 5-factor + momentum model, where alpha estimates increase further for the crypto portfolios. In the monthly regressions:

- Portfolio B's alpha nearly triples that of A (0.0125 vs 0.0050),
- D outpaces C (0.0495 vs 0.0290),
- and F again exceeds E (0.0323 vs 0.0177).
- Portfolio H stands out dramatically with an alpha of 0.494, far surpassing G's 0.182, albeit with reduced explanatory power ($R^2 = 0.17$).

Figure 13 - Main results from the Factor Models

Portfolio	Alpha (3-Factor)	R_squared (3-Factor)	Alpha (5-Factor)	R_squared (5-Factor)
A	0.0016	0.8989	0.005	0.9569
B	0.0026	0.844	0.0125	0.8309
C	0.0068	0.6222	0.029	0.6229
D	0.0092	0.5057	0.0495	0.2649
E	0.0044	0.7398	0.0177	0.8326
F	0.0032	0.6012	0.0323	0.5277
G	0.0182	0.5654	0.1824	0.4272
H	0.0494	0.17	0.4945	0.1718

Source: Own construction

While high alphas suggest outperformance, they must be interpreted alongside model fit. Crypto-inclusive portfolios tend to exhibit lower R^2 values, indicating that a smaller portion of their returns is explained by traditional equity risk factors. This may imply exposure to novel return drivers - such as speculative demand, behavioral momentum, or market segmentation - but may also reflect model misspecification. The Fama–French frameworks were developed for traditional equities and may omit factors relevant to digital assets, such as liquidity shocks or regulatory risks. In this light, the observed alpha could stem from unmodeled risk exposures rather than genuine outperformance. Future research incorporating crypto-specific factor models may help clarify this ambiguity and improve explanatory power.

These findings strengthen the conclusions drawn from the bootstrapped Sharpe and Sortino ratio tests. There, we observed consistent directional evidence that portfolios B, D, F, and H outperformed their crypto-free counterparts in normal markets, albeit with increased downside risk during crises. The factor regressions help to contextualize this behavior: while crypto-enhanced portfolios outperform on average, that outperformance is not attributable to conventional risk premia. Instead, they appear to generate unexplained alpha, which may be appealing from a diversification perspective, but also introduces regime sensitivity and model uncertainty.

In summary, the multi-factor analysis affirms the core empirical finding of this thesis: portfolios enhanced with cryptocurrency have the potential to improve long-term risk-adjusted returns. However, they do so in a way that deviates from conventional risk-based explanations, confirming their distinct role within asset allocation. This makes their inclusion in diversified portfolios both attractive and conditional - requiring careful consideration of their behavior across different market regimes, as evidenced by both the bootstrapped ratio tests and the crisis-period analysis. Detailed results from the factor models are available in the appendices.

4.7 Out-of-Sample Validation

To evaluate the robustness of the portfolio strategies beyond the in-sample estimation window, this section presents the actual out-of-sample performance following the inception date of each portfolio. Figure X below summarizes the realized returns and risk-adjusted metrics across all portfolios for this evaluation period.

Figure 14 - Performance data results

Actrual performance after inception date									
	Portfolio A	Portfolio B	Portfolio C	Portfolio D	Portfolio E	Portfolio F	Portfolio G	Portfolio H	S&P 500
Weekly Return	0.41%	0.49%	0.71%	0.96%	0.68%	0.77%	5.01%	9.08%	0.40%
Annual Return	23.89%	29.15%	44.78%	64.13%	41.99%	49.39%	1169.34%	9084.86%	22.78%
Weekly Std.	1.50%	1.58%	4.45%	4.31%	2.84%	2.93%	25.32%	31.22%	1.76%
Annual Std.	10.83%	11.40%	32.06%	31.09%	20.46%	21.11%	182.55%	225.13%	12.70%
Beta	77.53%	79.91%	180.36%	172.45%	130.21%	132.22%	903.65%	989.04%	100.00%
Downside risk	7.69%	8.15%	23.42%	22.05%	7.69%	14.63%	14.63%	14.63%	15.15%
Sharpe ratio	1.81	2.18	1.26	1.92	1.84	2.14	6.38	40.33	1.45
Sortino ratio	2.55	3.05	1.73	2.71	4.90	3.08	79.65	620.85	1.22
Alpha	5.27%	10.08%	7.15%	27.96%	13.63%	20.66%	998.04%	8897.78%	0.00%

Source: Own construction

The results indicate a consistent pattern: portfolios that include cryptocurrencies outperform their traditional counterparts across nearly all risk-adjusted performance measures. Portfolio B, which equally weighs traditional assets alongside cryptocurrencies, outperforms Portfolio A - its crypto-free counterpart - in Sharpe ratio (2.18 vs. 1.81), Sortino ratio (3.05 vs. 2.55), and annualized return (29.15% vs. 23.89%). This trend holds across all portfolio pairs: Portfolio D (max Sharpe with crypto) achieves a higher Sharpe (1.92 vs. 1.26) and a much higher alpha (27.96%) compared to Portfolio C (max Sharpe without crypto). Portfolio F, which applies a more constrained optimization with crypto exposure, still outperforms Portfolio E (its constrained traditional pair) in both Sharpe and Sortino ratios. Portfolio H, the most unconstrained and leveraged portfolio, delivers exceptionally high absolute and risk-adjusted returns (Sharpe: 40.33; Sortino: 620.85; Alpha: 8997.78%), far surpassing Portfolio G.

These patterns closely mirror the findings from the in-sample estimation period (see Figures X and X), where crypto-enhanced portfolios consistently outperformed across Sharpe ratio, Sortino ratio, and Jensen's Alpha metrics. For instance, Portfolio B again exceeded A in every performance dimension, and the unconstrained Portfolio H maintained the highest return and efficiency measures of all portfolios.

The consistency between estimation and actual out-of-sample results strengthens the thesis findings: incorporating crypto assets into a diversified portfolio meaningfully improves long-term performance. The benefits are most pronounced in setups that allow greater flexibility in allocation and risk exposure, confirming that cryptocurrencies contribute positively not only to total return but also to portfolio efficiency on a risk-adjusted basis.

However, this conclusion must be contextualized by the earlier crisis period analysis. While crypto allocations enhance average performance, their contributions are highly regime-dependent. As previously established, performance deteriorates during financial stress, a factor which the out-of-sample data also alludes to, particularly through the elevated standard deviations and downside risks in crypto-enhanced portfolios.

In summary, this out-of-sample analysis validates the performance gains observed during the estimation phase. Crypto assets demonstrably improve portfolio returns and efficiency across a range of optimization frameworks, though these gains are coupled with increased volatility. The risk-adjusted performance premium from crypto inclusion appears consistent, making it a compelling, though conditional, addition to modern portfolio strategies.

5. Discussion

5.1 General Evaluation of Results

The analysis demonstrates that the inclusion of cryptocurrencies in traditional equity portfolios generally improves risk-adjusted returns, as measured by Sharpe, Sortino and Calmar ratios. This trend is consistent across several portfolio strategies, including equal-weighted, maximum Sharpe ratio (constrained and unconstrained), and portfolios that allow for short selling. Across the estimation period, portfolios containing crypto assets consistently show higher annualized Sharpe, Sortino and Calmar ratios than their non-crypto counterparts.

Moreover, the improvement in performance is not isolated to a specific construction method, which suggests a systematic effect rather than random variation. However, this conclusion is tempered by the results of formal statistical testing on Sharpe and Sortino ratios. The Wilcoxon signed-rank test shows significant improvements in several crypto vs. non-crypto pairings (e.g., Portfolios A vs. B, E vs. F), while other cases do not reach statistical significance. The bootstrap tests, which resample the entire return distribution, further reveal that although the average difference in Sharpe and Sortino ratios favors crypto portfolios, confidence intervals often include zero, suggesting that these differences may not be statistically robust.

Additionally, the simple benchmark test - comparing rolling ratios to static targets - consistently indicates that crypto portfolios outperform their benchmarks in the majority of rolling windows. While this provides supportive directional evidence, it lacks the statistical rigor to serve as definitive proof. Taken together, the body of evidence supports the notion that cryptocurrencies can improve portfolio efficiency, but the strength of this conclusion depends on the statistical lens applied.

These results highlight an important tension between statistical robustness and practical investment relevance. While the p-values do not uniformly meet conventional thresholds, the consistent directional advantage raises the question of whether cryptocurrencies offers latent risk premia or merely reflects sample-specific anomalies. This mirrors ongoing academic discussions about the challenges of measuring performance persistence in alternative assets - particularly when traditional risk metrics may not fully capture structural shifts or evolving return dynamics (Corbet et al., 2019a)(Liu et al., 2022).

Further supporting this interpretation, the factor model analysis adds a critical dimension. The Fama–French 3-factor and 5-factor regressions reveal that crypto-inclusive portfolios tend to generate higher alphas, often statistically significant and even after controlling for standard risk factors such as market (Mkt-RF), size (SMB), value (HML), profitability (RMW), investment (CMA), and momentum. For instance, Portfolios D and H displayed especially elevated alpha values in both weekly and monthly model specifications. This implies that the performance advantage of crypto portfolios is not fully explained by conventional sources of systematic risk. Instead, it may reflect either the capture of novel risk dimensions - possibly related to technological adoption, regulatory shifts, or behavioral drivers, or genuine alpha generation through structural inefficiencies in the crypto market.

The regime analysis adds further nuance: while crypto-enhanced portfolios perform well on average, their benefits appear conditional on market environments. This points toward an essential insight: the effectiveness of cryptocurrencies in portfolio optimization may not be constant over time, and its performance must be interpreted in a broader market context. This also complements the factor model findings, suggesting that cryptocurrencies contribution may not be stable across different volatility regimes, further emphasizing the importance of dynamic risk management frameworks.

5.2 Sustainability of Historical Returns: Hype or Fundamentals?

A central consideration is whether the historically high Sharpe and Sortino ratios observed in crypto-enhanced portfolios can be sustained going forward. The cryptocurrency market has been driven by considerable hype, media attention, and speculative behavior over recent years (Shiller, 2020). Unlike equities, which can be valued through fundamental metrics like earnings and discounted cash flows, cryptocurrencies lack a standardized or widely accepted valuation framework (Yermack, 2024).

This absence of intrinsic value anchors makes cryptocurrencies particularly sensitive to investor sentiment and speculative cycles (Corbet et al., 2019b). For example, portfolios like H and F, which feature substantial crypto exposure, exhibit extreme returns that may reflect a euphoric market phase rather than enduring outperformance. If the exceptional historical returns are driven primarily by hype and early-stage adoption dynamics, then they may not be repeatable under more mature market conditions.

This leads to broader reflections on their role in portfolio theory. In contrast to equities, whose prices are tethered to tangible business activity, crypto assets are not fundamentally linked to cash-generating activities (Yermack, 2024). Therefore, their inclusion in portfolio models should be accompanied by caution and consideration of their speculative characteristics.

The observed performance of crypto assets may also be partially driven by investor sentiment, which plays an outsized role in pricing cryptocurrencies due to their lack of intrinsic valuation anchors. Market narratives, social media influence, and speculative enthusiasm can amplify both momentum and reversals, leading to return patterns that appear as alpha in traditional models. This sentiment-driven behavior contributes to volatility clustering and fat tails, which in turn complicates portfolio optimization and performance attribution. Incorporating sentiment-based indicators in future research could improve the understanding of cryptocurrencies return dynamics (Lee et al., 2018).

5.3 Risk, Correlation, and Regime Dependence

Cryptocurrencies exhibit relatively low correlation with traditional equities under normal market conditions, which suggests diversification potential. However, this correlation increases sharply during periods of market stress, as shown in the regime-based correlation analysis. This phenomenon, where diversification benefits break down during crises, diminishes the stabilizing effect of cryptocurrencies precisely when it is most needed (Longin & Solnik, 2001).

Moreover, Sharpe and Sortino ratios drop significantly in high-volatility regimes, particularly for portfolios that include crypto assets. This reflects the asymmetric risk profile of crypto: high upside during bull markets, but disproportionately large downside in crisis periods. The regime-dependent nature of crypto performance suggests that static allocations may be suboptimal. A more dynamic approach, such as volatility-sensitive rebalancing or regime-switching models, may be better suited to harness cryptocurrencies benefits while mitigating its risks (Ang & Bekaert, 2002c).

Compared to traditional diversifiers such as gold, which tends to maintain or even strengthen its low correlation with equities during crises, cryptocurrencies appear less reliable in turbulent markets. While gold’s diversification benefit has been well-documented over decades, the role of cryptocurrency as a hedging asset remains less consistent. This underscores the need for careful allocation frameworks when considering cryptocurrencies in defensive strategies. Future studies could compare crypto assets directly to alternatives like gold or inflation-protected securities to assess their relative effectiveness across market regimes (Carolyn Barnette, 2025).

5.4 Portfolio Construction and Practical Constraints

The downside risk profile of these portfolios further illustrates the importance of careful construction. Crypto-enhanced portfolios - particularly those with fewer constraints - exhibit significantly higher maximum drawdowns and downside deviations. Portfolio H, which applies leverage and allows short positions, demonstrates this risk starkly: its weekly return distribution includes losses exceeding negative 100%, which implies total capital wipeout and portfolio bankruptcy risk under extreme market movements. This highlights how the return potential of crypto assets comes at the cost of exposure to severe tail risk, especially when not properly bounded by allocation controls.

This contrast suggests that cryptocurrencies may serve best as a tactical component within a diversified portfolio. Allocation constraints such as maximum weights, minimum number of assets, and prohibition of short-selling can help balance the potential rewards of crypto exposure against the risks of excessive volatility and drawdowns. In practical terms, these constraints are essential for maintaining portfolio robustness (Jagannathan & Ma, 2003).

Beyond allocation constraints, another practical challenge lies in the inherent sensitivity of mean-variance optimization itself. The MVO framework is known to produce unstable weight allocations, especially when based on volatile or highly correlated inputs. Small changes in expected returns or covariance estimates can lead to disproportionately large shifts in optimal weights, often resulting in extreme allocations that are impractical or difficult to implement. As Michaud (1989) argues, optimized portfolios may appear mathematically optimal but are often empirically suboptimal due to estimation error and weight instability. These issues are amplified in portfolios that include assets like cryptocurrencies, where return distributions are non-normal and historical performance may not be reliable for forward-looking optimization. These limitations highlight the importance of incorporating practical constraints and validating portfolio strategies through out-of-sample testing, as done in this study (Michaud, 1989).

5.5 Implications, Limitations, and Directions for Future Research

While the analysis indicates that cryptocurrencies can enhance portfolio efficiency, it is important to maintain a realistic perspective. The extreme returns observed in this study are unlikely to be sustained indefinitely. As the market matures and regulatory scrutiny increases, structural shifts in investor behavior, liquidity, and volatility may reshape the investment profile of crypto assets (OECD, 2022).

From a methodological standpoint, this study is limited by a relatively short out-of-sample period and potential sample selection bias. Macroeconomic uncertainty, geopolitical risks, and market-specific events have also been excluded from the analysis but could significantly impact crypto behavior (Corbet et al., 2020).

Future research could explore more granular segments of the crypto market - such as DeFi tokens, stablecoins, and NFTs - and their distinct roles in portfolio construction. Dynamic allocation models, potentially powered by machine learning or regime-detection techniques, may offer more resilient approaches to managing crypto exposure (Fang et al., 2022). Additionally, extending the out-of-sample testing period would provide stronger evidence for the robustness and generalizability of the observed effects.

5.6 Final Reflections

Cryptocurrencies present an intriguing and controversial addition to modern portfolios. While they clearly offer diversification benefits under normal conditions, their utility diminishes, and risk increases under financial stress. Their behavior challenges traditional assumptions in portfolio theory, especially regarding valuation, correlation stability, and risk asymmetry.

In conclusion, cryptocurrencies should neither be dismissed outright nor embraced uncritically. Its role in portfolio management requires a nuanced approach that balances innovation and performance potential with skepticism and careful risk control. As financial markets evolve, the strategic inclusion of crypto assets may prove valuable, but only within frameworks that account for their unique and volatile nature.

6. Conclusion

This thesis set out to investigate to what extent the inclusion of cryptocurrencies in traditional equity portfolios improves risk-adjusted returns. Supporting questions explored how cryptocurrencies correlate with Danish and American equities across different market regimes, and how extreme market conditions affect their diversification benefits. Using a combination of portfolio optimization, performance evaluation, statistical testing, factor modeling, and out-of-sample validation, the analysis offers a comprehensive and data-driven response to these objectives.

The results indicate that portfolios including cryptocurrencies generally deliver higher risk-adjusted returns compared to traditional portfolios. This trend holds across multiple optimization strategies - such as equal-weighted, maximum Sharpe ratio, and constrained portfolios - and is evident in all key performance metrics, including the Sharpe, Sortino, and Calmar ratios. These improvements persist in both estimation and out-of-sample periods, suggesting they are not merely the result of model overfitting. Although the magnitude of outperformance varies, and some bootstrap confidence intervals include zero, the consistency of directional results across methods underscores the practical relevance of cryptocurrency inclusion.

Factor model regressions further reinforce this conclusion. Portfolios with cryptocurrency exposure often generate statistically significant alphas, meaning their returns are not fully explained by conventional risk factors such as market, size, value, profitability, or investment behavior. This points toward structural return characteristics or inefficiencies unique to crypto assets.

The correlation analysis supports the idea that cryptocurrencies can act as diversifiers in stable markets, as they exhibit low correlation with both Danish and American equities during normal periods. However, these relationships are not static. During times of financial stress, correlations increase - particularly with Danish equities - reducing the diversification benefit precisely when it is most needed. This finding aligns with broader literature on regime-dependent asset behavior.

Extreme market conditions also highlight the downside risks associated with crypto assets. Tail risk metrics such as conditional value-at-risk and maximum drawdown show that portfolios with higher crypto exposure are more vulnerable to severe losses. One leveraged portfolio (H) even demonstrated potential for total capital loss, emphasizing the need for caution. Although more constrained portfolios mitigate this risk, the pattern remains: while cryptocurrencies may improve return potential, they also introduce heightened volatility and asymmetric risk.

This pattern is further reflected in the regime-based bootstrap tests. While statistical significance at the 95% level is not consistently achieved, the majority of tests show underperformance of crypto portfolios during crises. This directional consistency reinforces the conclusion that crypto assets are sensitive to market regimes and require careful implementation within a risk-aware framework.

In summary, this thesis concludes that cryptocurrencies can enhance portfolio efficiency under many conditions, particularly when used within well-structured and risk-controlled strategies. However, the associated risks - especially during periods of high volatility - necessitate thoughtful portfolio design. Dynamic allocation, regime awareness, and robust risk management are essential to capturing the upside potential of cryptocurrencies while avoiding unacceptable downside exposure. With such measures in place, cryptocurrencies can serve as valuable tools for diversification and performance enhancement in modern asset allocation strategies.

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8. Appendixes

8.1 Appendix 1 - Wilcoxon Signed-Rank Test

9.
```
### Wilcoxon Signed-Rank Test ###
portfolio_pairs <- list(
  c("A", "B"),
  c("C", "D"),
  c("E", "F"),
  c("G", "H")
)

for (pair in portfolio_pairs) {
  portfolio_1 <- pair[1]
  portfolio_2 <- pair[2]

  cat("\nComparing Rolling Annualized Sharpe Ratios for:", portfolio_1, "vs.",
portfolio_2, "\n")
  print(wilcox.test(rolling_annual_ratios_df[[paste0(portfolio_1, ".Sharpe")]],
                    rolling_annual_ratios_df[[paste0(portfolio_2, ".Sharpe")]],
paired = TRUE))

  cat("\nComparing Rolling Annualized Sortino Ratios for:", portfolio_1, "vs.",
portfolio_2, "\n")
  print(wilcox.test(rolling_annual_ratios_df[[paste0(portfolio_1, ".Sortino")]]
,
                    rolling_annual_ratios_df[[paste0(portfolio_2, ".Sortino")]]
, paired = TRUE))
}
```

10.
```
##
## Comparing Rolling Annualized Sharpe Ratios for: A vs. B
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  rolling_annual_ratios_df[[paste0(portfolio_1, ".Sharpe")]] and rollin
g_annual_ratios_df[[paste0(portfolio_2, ".Sharpe")]]
## V = 4017, p-value = 1.936e-15
## alternative hypothesis: true location shift is not equal to 0
##
##
## Comparing Rolling Annualized Sortino Ratios for: A vs. B
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  rolling_annual_ratios_df[[paste0(portfolio_1, ".Sortino")]] and rolli
ng_annual_ratios_df[[paste0(portfolio_2, ".Sortino")]]
## V = 4246, p-value = 1.545e-14
```

```
## alternative hypothesis: true location shift is not equal to 0
##
##
## Comparing Rolling Annualized Sharpe Ratios for: C vs. D
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  rolling_annual_ratios_df[[paste0(portfolio_1, ".Sharpe")]] and rollin
g_annual_ratios_df[[paste0(portfolio_2, ".Sharpe")]]
## V = 10424, p-value = 0.5313
## alternative hypothesis: true location shift is not equal to 0
##
##
## Comparing Rolling Annualized Sortino Ratios for: C vs. D
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  rolling_annual_ratios_df[[paste0(portfolio_1, ".Sortino")]] and rolli
ng_annual_ratios_df[[paste0(portfolio_2, ".Sortino")]]
## V = 9834, p-value = 0.1936
## alternative hypothesis: true location shift is not equal to 0
##
##
## Comparing Rolling Annualized Sharpe Ratios for: E vs. F
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  rolling_annual_ratios_df[[paste0(portfolio_1, ".Sharpe")]] and rollin
g_annual_ratios_df[[paste0(portfolio_2, ".Sharpe")]]
## V = 8428, p-value = 0.003658
## alternative hypothesis: true location shift is not equal to 0
##
##
## Comparing Rolling Annualized Sortino Ratios for: E vs. F
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  rolling_annual_ratios_df[[paste0(portfolio_1, ".Sortino")]] and rolli
ng_annual_ratios_df[[paste0(portfolio_2, ".Sortino")]]
## V = 8320, p-value = 0.002449
## alternative hypothesis: true location shift is not equal to 0
##
##
## Comparing Rolling Annualized Sharpe Ratios for: G vs. H
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  rolling_annual_ratios_df[[paste0(portfolio_1, ".Sharpe")]] and rollin
```

```
g_annual_ratios_df[[paste0(portfolio_2, ".Sharpe")]]
## V = 7868, p-value = 0.0003911
## alternative hypothesis: true location shift is not equal to 0
##
##
## Comparing Rolling Annualized Sortino Ratios for: G vs. H
##
##  Wilcoxon signed rank test with continuity correction
##
## data:  rolling_annual_ratios_df[[paste0(portfolio_1, ".Sortino")]] and rolli
ng_annual_ratios_df[[paste0(portfolio_2, ".Sortino")]]
## V = 8382, p-value = 0.003088
## alternative hypothesis: true location shift is not equal to 0
```

8.2 Appendix 2 - Non-Parametric Benchmark Test: Rolling Sharpe and Sortino vs Target

```
# Print results
for (name in names(final_results)) {
  cat("\n---", name, "---\n")
  res <- final_results[[name]]
  cat("Target Sharpe Ratio:", round(res$Target_Sharpe, 4), "\n")
  cat("Annual Sharpe Ratio:", round(res$Annual_Sharpe, 4), "\n")
  cat("Bootstrap samples:", res$n_boot, "\n")
  cat("Standard Error:", round(res$SE_Sharpe, 4), "\n")
  cat("t-statistic:", round(res$t_stat, 4), "\n")
  cat("Parametric p-value:", round(res$p_value, 4), "\n")
  cat("Non-parametric % below target:", round(res$prop_below * 100, 2), "%\n")
}

##
## --- B_vs_A ---
## Target Sharpe Ratio: 1.0667
## Annual Sharpe Ratio: 1.3573
## Bootstrap samples: 209
## Standard Error: 0.1085
## t-statistic: 2.678
## Parametric p-value: 0.008
## Non-parametric % below target: 44.5 %
##
## --- D_vs_C ---
## Target Sharpe Ratio: 2.2922
## Annual Sharpe Ratio: 2.7183
## Bootstrap samples: 209
## Standard Error: 0.1432
## t-statistic: 2.9756
## Parametric p-value: 0.0033
## Non-parametric % below target: 44.02 %
##
## --- F_vs_E ---
## Target Sharpe Ratio: 1.9254
## Annual Sharpe Ratio: 2.3664
## Bootstrap samples: 209
## Standard Error: 0.1196
## t-statistic: 3.687
## Parametric p-value: 3e-04
## Non-parametric % below target: 43.06 %
##
## --- H_vs_G ---
## Target Sharpe Ratio: 7.7384
## Annual Sharpe Ratio: 26.1889
## Bootstrap samples: 209
## Standard Error: 6.5346
```

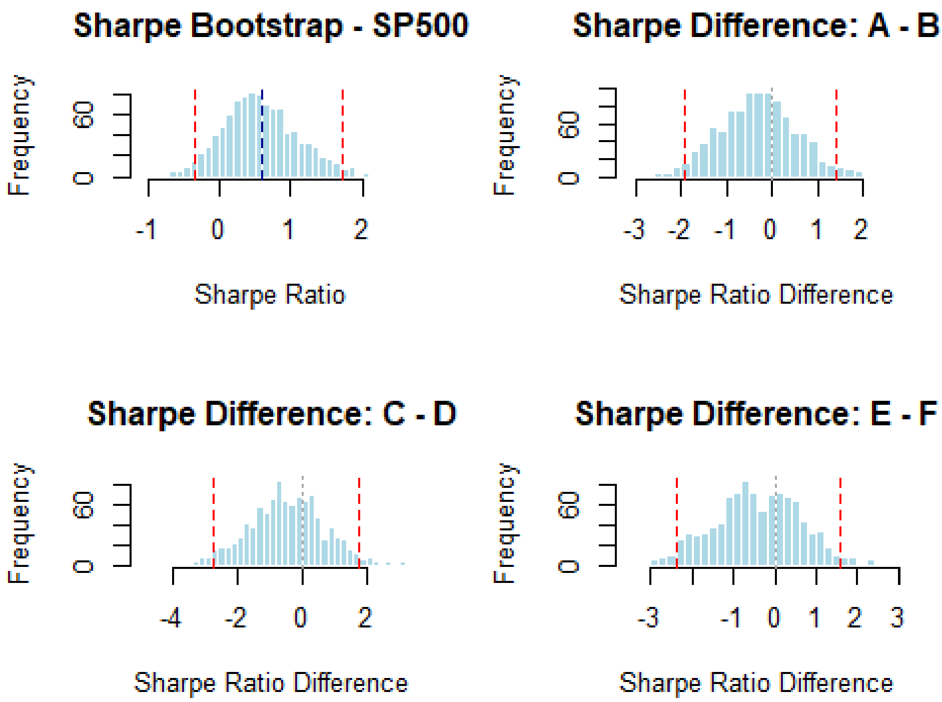
```
## t-statistic: 2.8235
## Parametric p-value: 0.0052
## Non-parametric % below target: 90.43 %

# Print results
for (name in names(final_sortino_results)) {
  cat("\n---", name, "---\n")
  res <- final_sortino_results[[name]]
  cat("Target Sortino Ratio:", round(res$Target_Sortino, 4), "\n")
  cat("Annual Sortino Ratio:", round(res$Annual_Sortino, 4), "\n")
  cat("Bootstrap samples:", res$n_boot, "\n")
  cat("Standard Error:", round(res$SE_Sortino, 4), "\n")
  cat("t-statistic:", round(res$t_stat, 4), "\n")
  cat("Parametric p-value:", round(res$p_value, 4), "\n")
  cat("Non-parametric % below target:", round(res$prop_below * 100, 2), "%\n")
}

##
## --- B_vs_A ---
## Target Sortino Ratio: 1.5892
## Annual Sortino Ratio: 2.0203
## Bootstrap samples: 209
## Standard Error: 0.216
## t-statistic: 1.9961
## Parametric p-value: 0.0472
## Non-parametric % below target: 44.5 %
##
## --- D_vs_C ---
## Target Sortino Ratio: 3.9357
## Annual Sortino Ratio: 4.8043
## Bootstrap samples: 209
## Standard Error: 0.359
## t-statistic: 2.4193
## Parametric p-value: 0.0164
## Non-parametric % below target: 47.85 %
##
## --- F_vs_E ---
## Target Sortino Ratio: 3.1076
## Annual Sortino Ratio: 3.9222
## Bootstrap samples: 209
## Standard Error: 0.2681
## t-statistic: 3.0388
## Parametric p-value: 0.0027
## Non-parametric % below target: 42.58 %
##

##
## --- H_vs_G ---
```

```
## Target Sortino Ratio: 13.4961
## Annual Sortino Ratio: 44.5129
## Bootstrap samples: 209
## Standard Error: 13.8419
## t-statistic: 2.2408
## Parametric p-value: 0.0261
## Non-parametric % below target: 89 %
```

8.3 Appendix 3 - Bootstrap Test: Full-Sample Sharpe and Sortino Differences

```
  # Print results
  cat("\n--- Bootstrapped Sharpe Difference:", benchmark, "-", crypto, "---\n")
  cat("Mean Sharpe Difference:", round(mean_diff, 4), "\n")
  cat("95% Confidence Interval:", round(ci_diff[1], 4), "to", round(ci_diff[2], 4)
, "\n")

  # Plot
  hist(sharpe_diff,
       breaks = 40,
       main = paste("Sharpe Difference:", benchmark, "-", crypto),
       xlab = "Sharpe Ratio Difference",
       col = "lightblue",
       border = "white")
  abline(v = ci_diff, col = "red", lty = 2)
  abline(v = 0, col = "darkgray", lty = 3)
}

##
## --- Bootstrapped Sharpe Difference: A - B ---
## Mean Sharpe Difference: -0.3109
## 95% Confidence Interval: -1.9344 to 1.3901

##
## --- Bootstrapped Sharpe Difference: C - D ---
## Mean Sharpe Difference: -0.4615
## 95% Confidence Interval: -2.7656 to 1.799

##
## --- Bootstrapped Sharpe Difference: E - F ---
## Mean Sharpe Difference: -0.4266
## 95% Confidence Interval: -2.3915 to 1.5716
```

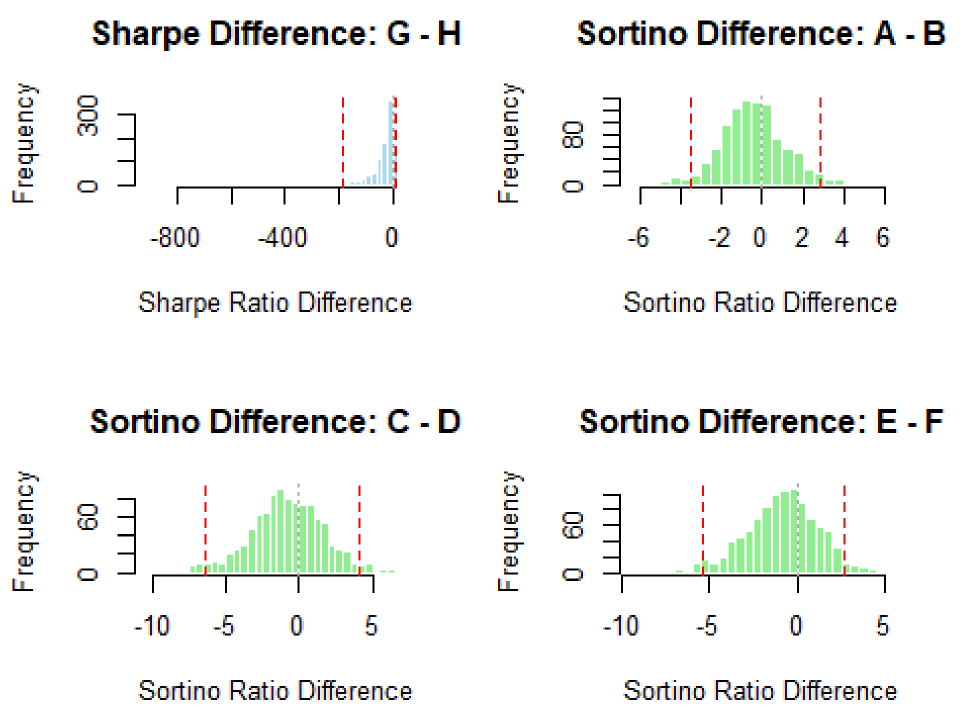
```
##
## --- Bootstrapped Sharpe Difference: G - H ---
## Mean Sharpe Difference: -35.9528
## 95% Confidence Interval: -187.6872 to 9.5136
```




```
### Expanded Bootstrap Stats (Extra Analysis) - Sharpe ratios###
# Re-run the bootstrap loop to include additional stats
for (pair in bootstrap_pairs) {
  benchmark <- pair[1]
  crypto <- pair[2]

  # Run bootstrap difference again (or reuse if stored earlier)
  sharpe_diff <- bootstrap_sharpe_diff(
    returns1 = returns_ts[, benchmark],
    returns2 = returns_ts[, crypto],
    risk_free = 0.043,
    n_boot = 1000
  )

  # Calculate summary stats
  mean_diff <- mean(sharpe_diff)
  ci_diff <- quantile(sharpe_diff, c(0.025, 0.975))
  prop_below_zero <- mean(sharpe_diff < 0)
  one_sided_p <- mean(sharpe_diff > 0)  # If testing if crypto > benchmark
  two_tailed_p <- 2 * min(
```

```r
    mean(sharpe_diff > mean_diff),
    mean(sharpe_diff < mean_diff)
  )

  # Output
  cat("\n+++ Additional Bootstrap Analysis:", benchmark, "-", crypto, "+++\n")
  cat("% Sharpe diffs < 0:", round(prop_below_zero * 100, 2), "%\n")
  cat("One-sided p-value (crypto > benchmark):", round(one_sided_p, 4), "\n")
  cat("Two-tailed p-value:", round(two_tailed_p, 4), "\n")
}

##
## +++ Additional Bootstrap Analysis: A - B +++
## % Sharpe diffs < 0: 62.8 %
## One-sided p-value (crypto > benchmark): 0.372
## Two-tailed p-value: 1
##
## +++ Additional Bootstrap Analysis: C - D +++
## % Sharpe diffs < 0: 65.1 %
## One-sided p-value (crypto > benchmark): 0.349
## Two-tailed p-value: 0.978
##
## +++ Additional Bootstrap Analysis: E - F +++
## % Sharpe diffs < 0: 66 %
## One-sided p-value (crypto > benchmark): 0.34
## Two-tailed p-value: 0.958
##
## +++ Additional Bootstrap Analysis: G - H +++
## % Sharpe diffs < 0: 85 %
## One-sided p-value (crypto > benchmark): 0.15
## Two-tailed p-value: 0.632

cat("\n--- Bootstrapped Sortino Difference:", benchmark, "-", crypto, "---\n")
  cat("Mean Sortino Difference:", round(mean_diff, 4), "\n")
  cat("95% Confidence Interval:", round(ci_diff[1], 4), "to", round(ci_diff[2], 4)
, "\n")

  # Plot
  hist(sortino_diff,
       breaks = 40,
       main = paste("Sortino Difference:", benchmark, "-", crypto),
       xlab = "Sortino Ratio Difference",
       col = "lightgreen",
       border = "white")
  abline(v = ci_diff, col = "red", lty = 2)
  abline(v = 0, col = "darkgray", lty = 3)
}
```

```
##
## --- Bootstrapped Sortino Difference: A - B ---
## Mean Sortino Difference: -0.4136
## 95% Confidence Interval: -3.5222 to 2.8781

##
## --- Bootstrapped Sortino Difference: C - D ---
## Mean Sortino Difference: -0.8524
## 95% Confidence Interval: -6.3887 to 4.1332

##
## --- Bootstrapped Sortino Difference: E - F ---
## Mean Sortino Difference: -0.9341
## 95% Confidence Interval: -5.416 to 2.6949

##
## --- Bootstrapped Sortino Difference: G - H ---
## Mean Sortino Difference: -64.7822
## 95% Confidence Interval: -358.3377 to 23.0736
```




```
### Expanded Bootstrap Stats (Extra Analysis) - Sortino ratios###
for (pair in bootstrap_pairs) {
  benchmark <- pair[1]
  crypto <- pair[2]

  sortino_diff <- bootstrap_sortino_diff(
```

```
    returns1 = returns_ts[, benchmark],
    returns2 = returns_ts[, crypto],
    risk_free = 0.043,
    n_boot = 1000
  )

  mean_diff <- mean(sortino_diff)
  ci_diff <- quantile(sortino_diff, c(0.025, 0.975))
  prop_below_zero <- mean(sortino_diff < 0)
  one_sided_p <- mean(sortino_diff > 0)
  two_tailed_p <- 2 * min(
    mean(sortino_diff > mean_diff),
    mean(sortino_diff < mean_diff)
  )

  cat("\n+++ Additional Bootstrap Sortino Stats:", benchmark, "-", crypto, "+++\n"
)
  cat("% Sortino diffs < 0:", round(prop_below_zero * 100, 2), "%\n")
  cat("One-sided p-value (crypto > benchmark):", round(one_sided_p, 4), "\n")
  cat("Two-tailed p-value:", round(two_tailed_p, 4), "\n")
}

##
## +++ Additional Bootstrap Sortino Stats: A - B +++
## % Sortino diffs < 0: 62.6 %
## One-sided p-value (crypto > benchmark): 0.374
## Two-tailed p-value: 0.994
##
## +++ Additional Bootstrap Sortino Stats: C - D +++
## % Sortino diffs < 0: 67.2 %
## One-sided p-value (crypto > benchmark): 0.328
## Two-tailed p-value: 0.994
##
## +++ Additional Bootstrap Sortino Stats: E - F +++
## % Sortino diffs < 0: 67.4 %
## One-sided p-value (crypto > benchmark): 0.326
## Two-tailed p-value: 0.952
##
## +++ Additional Bootstrap Sortino Stats: G - H +++
## % Sortino diffs < 0: 80.8 %
## One-sided p-value (crypto > benchmark): 0.192
## Two-tailed p-value: 0.584
```

8.4 Appendix 4 - Crisis Period Performance Test: Directional Analysis of Bootstrapped Risk-Adjusted Ratios

```
### Sharpe Ratio ###

# Store with summary
    port_results[[period]] <- list(
      values = boot,
      mean = mean(boot),
      median = median(boot),
      CI = quantile(boot, c(0.025, 0.975))
    )

    cat("\n---", port, "-", period, "---\n")
    cat("Mean Sharpe:", round(mean(boot), 4), "\n")
    cat("Median:", round(median(boot), 4), "\n")
    cat("95% CI:", round(quantile(boot, c(0.025, 0.975)), 4), "\n")
  }
  all_results[[port]] <- port_results
}

##
## --- A - Low1 ---
## Mean Sharpe: 3.1037
## Median: 3.0477
## 95% CI: 0.6462 5.8695
##
## --- A - Crisis1 ---
## Mean Sharpe: 1.2949
## Median: 0.2426
## 95% CI: -1.7334 10.2974
##
## --- A - Mid ---
## Mean Sharpe: 1.0167
## Median: 0.99
## 95% CI: -0.5301 2.8961
##
## --- A - Crisis2 ---
## Mean Sharpe: 2.0085
## Median: 0.6431
## 95% CI: -3.2418 15.4697
##
## --- A - Low2 ---
## Mean Sharpe: 1.3132
## Median: 1.1768
## 95% CI: -0.4724 3.5514
##
## --- B - Low1 ---
```

```
## Mean Sharpe: 3.7823
## Median: 3.6847
## 95% CI: 1.3228 6.7373
##
## --- B - Crisis1 ---
## Mean Sharpe: 1.5053
## Median: 0.2252
## 95% CI: -1.7312 13.4581
##
## --- B - Mid ---
## Mean Sharpe: 1.5691
## Median: 1.5184
## 95% CI: -0.1436 3.5846
##
## --- B - Crisis2 ---
## Mean Sharpe: 2.0457
## Median: 0.7664
## 95% CI: -2.7135 15.8079
##
## --- B - Low2 ---
## Mean Sharpe: 1.3703
## Median: 1.2815
## 95% CI: -0.456 3.8347
##
## --- C - Low1 ---
## Mean Sharpe: 3.786
## Median: 3.6183
## 95% CI: 1.0023 7.4321
##
## --- C - Crisis1 ---
## Mean Sharpe: 3.2199
## Median: 1.691
## 95% CI: -1.5626 17.1316
##
## --- C - Mid ---
## Mean Sharpe: 1.4543
## Median: 1.3751
## 95% CI: -0.2676 3.796
##
## --- C - Crisis2 ---
## Mean Sharpe: 2.6367
## Median: 0.8543
## 95% CI: -2.796 18.1815
##
## --- C - Low2 ---
## Mean Sharpe: 4.3561
## Median: 4.1556
## 95% CI: 1.3187 8.4717
```

```
##
## --- D - Low1 ---
## Mean Sharpe: 4.5309
## Median: 4.3301
## 95% CI: 1.3341 8.711
##
## --- D - Crisis1 ---
## Mean Sharpe: 2.918
## Median: 1.1428
## 95% CI: -1.4062 18.1825
##
## --- D - Mid ---
## Mean Sharpe: 2.7494
## Median: 2.6176
## 95% CI: 0.6147 5.6498
##
## --- D - Crisis2 ---
## Mean Sharpe: 3.9428
## Median: 1.2204
## 95% CI: -2.0866 27.4465
##
## --- D - Low2 ---
## Mean Sharpe: 3.8055
## Median: 3.5891
## 95% CI: 1.2177 7.3243
##
## --- E - Low1 ---
## Mean Sharpe: 4.422
## Median: 4.3901
## 95% CI: 1.7494 7.6271
##
## --- E - Crisis1 ---
## Mean Sharpe: 2.3002
## Median: 0.9686
## 95% CI: -1.5608 14.6448
##
## --- E - Mid ---
## Mean Sharpe: 1.2203
## Median: 1.1433
## 95% CI: -0.4367 3.169
##
## --- E - Crisis2 ---
## Mean Sharpe: 3.4155
## Median: 1.4639
## 95% CI: -3.2378 22.2101
##
## --- E - Low2 ---
## Mean Sharpe: 3.0539
```

```
## Median: 2.8927
## 95% CI: 0.6403 6.3583
##
## --- F - Low1 ---
## Mean Sharpe: 5.1697
## Median: 5.0069
## 95% CI: 2.0367 9.3231
##
## --- F - Crisis1 ---
## Mean Sharpe: 2.5487
## Median: 0.7433
## 95% CI: -1.4758 16.7509
##
## --- F - Mid ---
## Mean Sharpe: 2.2444
## Median: 2.1864
## 95% CI: 0.35 4.6096
##
## --- F - Crisis2 ---
## Mean Sharpe: 4.0499
## Median: 1.741
## 95% CI: -2.3508 22.2063
##
## --- F - Low2 ---
## Mean Sharpe: 2.9282
## Median: 2.7976
## 95% CI: 0.6033 5.9649
##
## --- G - Low1 ---
## Mean Sharpe: 24.2417
## Median: 13.857
## 95% CI: 1.0916 120.3355
##
## --- G - Crisis1 ---
## Mean Sharpe: 568.4953
## Median: 21.4535
## 95% CI: -0.4674 5365.359
##
## --- G - Mid ---
## Mean Sharpe: 4.1173
## Median: 2.6977
## 95% CI: -0.2236 17.433
##
## --- G - Crisis2 ---
## Mean Sharpe: 1289.857
## Median: 18.0673
## 95% CI: -0.799 7859.007
##
```

```
## --- G - Low2 ---
## Mean Sharpe: 23.54
## Median: 15.6263
## 95% CI: 1.4691 96.3676
##
## --- H - Low1 ---
## Mean Sharpe: 465.8394
## Median: 96.1376
## 95% CI: 1.4424 3790.496
##
## --- H - Crisis1 ---
## Mean Sharpe: 7088.805
## Median: 10.303
## 95% CI: -0.2673 29975.65
##
## --- H - Mid ---
## Mean Sharpe: 254.9283
## Median: 55.5471
## 95% CI: 0.7886 1590.759
##
## --- H - Crisis2 ---
## Mean Sharpe: 770530.9
## Median: 9.1539
## 95% CI: -0.4017 1682734
##
## --- H - Low2 ---
## Mean Sharpe: 23.9999
## Median: 8.9957
## 95% CI: -0.0911 127.1578
##
## --- SP500 - Low1 ---
## Mean Sharpe: 1.7805
## Median: 1.6806
## 95% CI: -0.3444 4.4904
##
## --- SP500 - Crisis1 ---
## Mean Sharpe: 0.6956
## Median: -0.2527
## 95% CI: -1.8627 8.8955
##
## --- SP500 - Mid ---
## Mean Sharpe: 0.8438
## Median: 0.8175
## 95% CI: -0.6191 2.6358
##
## --- SP500 - Crisis2 ---
## Mean Sharpe: 1.1766
## Median: 0.1172
```

```
## 95% CI: -3.1839 11.614
##
## --- SP500 - Low2 ---
## Mean Sharpe: 0.6083
## Median: 0.5606
## 95% CI: -1.0698 2.5811

      # Output
      cat("\n---", p1, "vs", p2, "---\n")
      cat("Mean Difference:", round(mean_diff, 4), "\n")
      cat("Median Difference:", round(median_diff, 4), "\n")
      cat("95% CI:", round(ci_diff[1], 4), "to", round(ci_diff[2], 4), "\n")
      cat("% Below 0:", round(prop_below_zero * 100, 2), "% | % Above 0:", round(p
rop_above_zero * 100, 2), "%\n")
      cat("Significance:", sig_flag, "\n")
    }
  }
}
## ============================
## Portfolio: A
##
## --- Crisis1 vs Low1 ---
## Mean Difference: -1.8088
## Median Difference: -2.5713
## 95% CI: -6.4681 to 7.8775
## % Below 0: 80.5 % | % Above 0: 19.5 %
## Significance:  ❌ Not Significant
##
## --- Crisis1 vs Mid ---
## Mean Difference: 0.2782
## Median Difference: -0.6223
## 95% CI: -3.4854 to 9.4604
## % Below 0: 60.1 % | % Above 0: 39.9 %
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Mid ---
## Mean Difference: 0.9917
## Median Difference: -0.2915
## 95% CI: -4.5238 to 14.7914
## % Below 0: 54 % | % Above 0: 46 %
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Low2 ---
## Mean Difference: 0.6953
## Median Difference: -0.5106
## 95% CI: -4.9124 to 14.0704
## % Below 0: 57.3 % | % Above 0: 42.7 %
```

```
## Significance:  ❌  Not Significant
##
## ============================
## Portfolio: B
##
## --- Crisis1 vs Low1 ---
## Mean Difference: -2.2771
## Median Difference: -3.2384
## 95% CI: -7.1959 to 9.3017
## % Below 0: 81.9 % | % Above 0: 18.1 %
## Significance:  ❌  Not Significant
##
## --- Crisis1 vs Mid ---
## Mean Difference: -0.0638
## Median Difference: -1.1406
## 95% CI: -4.2562 to 10.8825
## % Below 0: 66 % | % Above 0: 34 %
## Significance:  ❌  Not Significant
##
## --- Crisis2 vs Mid ---
## Mean Difference: 0.4766
## Median Difference: -0.73
## 95% CI: -4.986 to 14.6098
## % Below 0: 60.1 % | % Above 0: 39.9 %
## Significance:  ❌  Not Significant
##
## --- Crisis2 vs Low2 ---
## Mean Difference: 0.6754
## Median Difference: -0.4177
## 95% CI: -5.3531 to 14.8159
## % Below 0: 55.7 % | % Above 0: 44.3 %
## Significance:  ❌  Not Significant
##
## ============================
## Portfolio: C
##
## --- Crisis1 vs Low1 ---
## Mean Difference: -0.5662
## Median Difference: -1.814
## 95% CI: -7.0332 to 14.5114
## % Below 0: 67.9 % | % Above 0: 32.1 %
## Significance:  ❌  Not Significant
##
## --- Crisis1 vs Mid ---
## Mean Difference: 1.7656
## Median Difference: 0.1836
## 95% CI: -3.6673 to 16.406
```

```
## % Below 0: 48.1 % | % Above 0: 51.9 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid ---
## Mean Difference: 1.1825
## Median Difference: -0.518
## 95% CI: -5.258 to 17.1128
## % Below 0: 54.3 % | % Above 0: 45.7 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 ---
## Mean Difference: -1.7194
## Median Difference: -3.0482
## 95% CI: -9.2745 to 14.8298
## % Below 0: 75.5 % | % Above 0: 24.5 %
## Significance: ❌ Not Significant
##
## ============================
## Portfolio: D
##
## --- Crisis1 vs Low1 ---
## Mean Difference: -1.6129
## Median Difference: -3.1369
## 95% CI: -8.3798 to 14.1462
## % Below 0: 74.6 % | % Above 0: 25.4 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid ---
## Mean Difference: 0.1687
## Median Difference: -1.4765
## 95% CI: -5.5923 to 16.2403
## % Below 0: 63.8 % | % Above 0: 36.2 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid ---
## Mean Difference: 1.1934
## Median Difference: -1.4426
## 95% CI: -6.1727 to 25.1501
## % Below 0: 63.1 % | % Above 0: 36.9 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 ---
## Mean Difference: 0.1372
## Median Difference: -2.3269
## 95% CI: -7.8114 to 25.5991
## % Below 0: 69.6 % | % Above 0: 30.4 %
## Significance: ❌ Not Significant
```

```
##
## ============================
## Portfolio: E
##
## --- Crisis1 vs Low1 ---
## Mean Difference: -2.1219
## Median Difference: -3.2083
## 95% CI: -7.6851 to 10.0239
## % Below 0: 78.6 % | % Above 0: 21.4 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid ---
## Mean Difference: 1.0799
## Median Difference: -0.1237
## 95% CI: -3.5849 to 14.3024
## % Below 0: 52.1 % | % Above 0: 47.9 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid ---
## Mean Difference: 2.1952
## Median Difference: 0.4037
## 95% CI: -5.1632 to 20.8005
## % Below 0: 45.6 % | % Above 0: 54.4 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 ---
## Mean Difference: 0.3616
## Median Difference: -1.3468
## 95% CI: -7.2245 to 19.1516
## % Below 0: 63 % | % Above 0: 37 %
## Significance: ❌ Not Significant
##
## ============================
## Portfolio: F
##
## --- Crisis1 vs Low1 ---
## Mean Difference: -2.6211
## Median Difference: -3.7598
## 95% CI: -9.4561 to 11.8832
## % Below 0: 81.2 % | % Above 0: 18.8 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid ---
## Mean Difference: 0.3043
## Median Difference: -1.3636
## 95% CI: -4.883 to 14.9729
## % Below 0: 63.3 % | % Above 0: 36.7 %
```

```
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Mid ---
## Mean Difference: 1.8055
## Median Difference: -0.2989
## 95% CI: -5.4284 to 21.6471
## % Below 0: 53.1 % | % Above 0: 46.9 %
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Low2 ---
## Mean Difference: 1.1216
## Median Difference: -1.0574
## 95% CI: -6.3288 to 21.3338
## % Below 0: 60 % | % Above 0: 40 %
## Significance:  ❌ Not Significant
##
## ===============================
## Portfolio: G
##
## --- Crisis1 vs Low1 ---
## Mean Difference: 544.2536
## Median Difference: 4.7318
## 95% CI: -104.1543 to 5359.746
## % Below 0: 45.7 % | % Above 0: 54.3 %
## Significance:  ❌ Not Significant
##
## --- Crisis1 vs Mid ---
## Mean Difference: 564.378
## Median Difference: 17.9032
## 95% CI: -10.6247 to 5365.092
## % Below 0: 28.4 % | % Above 0: 71.6 %
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Mid ---
## Mean Difference: 1285.74
## Median Difference: 14.1121
## 95% CI: -10.6022 to 7856.099
## % Below 0: 32.6 % | % Above 0: 67.4 %
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Low2 ---
## Mean Difference: 1266.317
## Median Difference: 2.0156
## 95% CI: -73.3672 to 7850.273
## % Below 0: 47.4 % | % Above 0: 52.6 %
## Significance:  ❌ Not Significant
##
```

```
## ============================
## Portfolio: H
##
## --- Crisis1 vs Low1 ---
## Mean Difference: 6622.966
## Median Difference: -27.9503
## 95% CI: -3555.612 to 28412.24
## % Below 0: 64.7 % | % Above 0: 35.3 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid ---
## Mean Difference: 6833.877
## Median Difference: -11.3949
## 95% CI: -1329.95 to 29751.6
## % Below 0: 61.5 % | % Above 0: 38.5 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid ---
## Mean Difference: 770276
## Median Difference: -6.8933
## 95% CI: -1294.839 to 1682555
## % Below 0: 58.2 % | % Above 0: 41.8 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 ---
## Mean Difference: 770506.9
## Median Difference: 1.3467
## 95% CI: -82.1377 to 1682680
## % Below 0: 47.9 % | % Above 0: 52.1 %
## Significance: ❌ Not Significant
##
## ============================
## Portfolio: SP500
##
## --- Crisis1 vs Low1 ---
## Mean Difference: -1.0849
## Median Difference: -1.7421
## 95% CI: -5.1988 to 6.7225
## % Below 0: 75.2 % | % Above 0: 24.8 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid ---
## Mean Difference: -0.1481
## Median Difference: -0.8669
## 95% CI: -3.3962 to 7.8174
## % Below 0: 66.3 % | % Above 0: 33.7 %
## Significance: ❌ Not Significant
```

```
##
## --- Crisis2 vs Mid ---
## Mean Difference: 0.3328
## Median Difference: -0.641
## 95% CI: -4.5645 to 10.7583
## % Below 0: 59.4 % | % Above 0: 40.6 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 ---
## Mean Difference: 0.5683
## Median Difference: -0.3252
## 95% CI: -4.4171 to 10.6848
## % Below 0: 55.4 % | % Above 0: 44.6 %
## Significance: ❌ Not Significant
```

```
### Sortino Ratio ###

cat("\n---", port, "-", period, "(Sortino) ---\n")
    cat("Mean Sortino:", round(mean(boot), 4), "\n")
    cat("Median:", round(median(boot), 4), "\n")
    cat("95% CI:", round(quantile(boot, c(0.025, 0.975)), 4), "\n")
  }
  sortino_results[[port]] <- port_results
}
## --- A - Low1 (Sortino) ---
## Mean Sortino: 7.1062
## Median: 6.3907
## 95% CI: 1.0425 17.4897
##
## --- A - Crisis1 (Sortino) ---
## Mean Sortino: 4.2646
## Median: 0.3131
## 95% CI: -1.7885 24.6347
##
## --- A - Mid (Sortino) ---
## Mean Sortino: 1.8226
## Median: 1.6365
## 95% CI: -0.8097 5.5925
##
## --- A - Crisis2 (Sortino) ---
## Mean Sortino: Inf
## Median: 1.264
## 95% CI: -2.9825 52.902
##
## --- A - Low2 (Sortino) ---
## Mean Sortino: 2.1277
## Median: 1.8678
## 95% CI: -0.7517 6.5119
```

```
##
## --- B - Low1 (Sortino) ---
## Mean Sortino: 8.5835
## Median: 7.8077
## 95% CI: 1.8768 19.7007
##
## --- B - Crisis1 (Sortino) ---
## Mean Sortino: 5.209
## Median: 0.0893
## 95% CI: -1.6766 46.0933
##
## --- B - Mid (Sortino) ---
## Mean Sortino: 3.0105
## Median: 2.6063
## 95% CI: -0.0518 8.0373
##
## --- B - Crisis2 (Sortino) ---
## Mean Sortino: Inf
## Median: 0.9366
## 95% CI: -2.6812 134.8308
##
## --- B - Low2 (Sortino) ---
## Mean Sortino: 2.3535
## Median: 2.0791
## 95% CI: -0.4966 6.7134
##
## --- C - Low1 (Sortino) ---
## Mean Sortino: 8.9958
## Median: 7.8277
## 95% CI: 1.9641 22.3491
##
## --- C - Crisis1 (Sortino) ---
## Mean Sortino: Inf
## Median: 2.4288
## 95% CI: -1.6959 89.6913
##
## --- C - Mid (Sortino) ---
## Mean Sortino: 2.6394
## Median: 2.2751
## 95% CI: -0.2876 7.9032
##
## --- C - Crisis2 (Sortino) ---
## Mean Sortino: Inf
## Median: 1.2963
## 95% CI: -2.7175 48.6519
##
## --- C - Low2 (Sortino) ---
## Mean Sortino: 8.5727
```

```
## Median: 7.2826
## 95% CI: 1.8367 22.551
##
## --- D - Low1 (Sortino) ---
## Mean Sortino: 11.2975
## Median: 9.5845
## 95% CI: 2.008 28.1764
##
## --- D - Crisis1 (Sortino) ---
## Mean Sortino: Inf
## Median: 1.9026
## 95% CI: -1.473 92.285
##
## --- D - Mid (Sortino) ---
## Mean Sortino: 5.786
## Median: 5.1484
## 95% CI: 0.9852 14.6169
##
## --- D - Crisis2 (Sortino) ---
## Mean Sortino: Inf
## Median: 0.9627
## 95% CI: -2.267 61.5719
##
## --- D - Low2 (Sortino) ---
## Mean Sortino: 7.8726
## Median: 6.8006
## 95% CI: 1.4866 19.0079
##
## --- E - Low1 (Sortino) ---
## Mean Sortino: 11.7869
## Median: 10.6966
## 95% CI: 3.4021 26.2132
##
## --- E - Crisis1 (Sortino) ---
## Mean Sortino: Inf
## Median: 1.4448
## 95% CI: -1.668 50.8311
##
## --- E - Mid (Sortino) ---
## Mean Sortino: 2.2214
## Median: 1.8674
## 95% CI: -0.6092 6.8246
##
## --- E - Crisis2 (Sortino) ---
## Mean Sortino: Inf
## Median: 2.8721
## 95% CI: -3.0169 64.5554
##
```

```
## --- E - Low2 (Sortino) ---
## Mean Sortino: 5.5261
## Median: 4.9575
## 95% CI: 0.9919 13.1673
##
## --- F - Low1 (Sortino) ---
## Mean Sortino: 12.3883
## Median: 11.0531
## 95% CI: 3.3497 30.1427
##
## --- F - Crisis1 (Sortino) ---
## Mean Sortino: Inf
## Median: 1.0955
## 95% CI: -1.4797 90.0787
##
## --- F - Mid (Sortino) ---
## Mean Sortino: 4.6416
## Median: 4.063
## 95% CI: 0.5576 12.3511
##
## --- F - Crisis2 (Sortino) ---
## Mean Sortino: Inf
## Median: 3.7704
## 95% CI: -2.3613 102.9327
##
## --- F - Low2 (Sortino) ---
## Mean Sortino: 5.4152
## Median: 4.6617
## 95% CI: 0.9473 13.6817
##
## --- G - Low1 (Sortino) ---
## Mean Sortino: 57.1413
## Median: 29.3421
## 95% CI: 1.339 305.703
##
## --- G - Crisis1 (Sortino) ---
## Mean Sortino: Inf
## Median: 51.2492
## 95% CI: -0.5401 31087.39
##
## --- G - Mid (Sortino) ---
## Mean Sortino: 7.3505
## Median: 4.1101
## 95% CI: -0.3528 34.6094
##
## --- G - Crisis2 (Sortino) ---
## Mean Sortino: Inf
## Median: 28.0542
```

```
## 95% CI: -0.8691 414177.9
##
## --- G - Low2 (Sortino) ---
## Mean Sortino: 55.0644
## Median: 30.9161
## 95% CI: 2.1567 263.7888
##
## --- H - Low1 (Sortino) ---
## Mean Sortino: 1071.931
## Median: 167.4534
## 95% CI: 2.9228 6847.399
##
## --- H - Crisis1 (Sortino) ---
## Mean Sortino: Inf
## Median: 7.5433
## 95% CI: -0.2969 340176.1
##
## --- H - Mid (Sortino) ---
## Mean Sortino: 545.4894
## Median: 104.0926
## 95% CI: 1.5913 4449.099
##
## --- H - Crisis2 (Sortino) ---
## Mean Sortino: Inf
## Median: 23.1766
## 95% CI: -0.4356 52275463
##
## --- H - Low2 (Sortino) ---
## Mean Sortino: 49.9914
## Median: 13.862
## 95% CI: 0.015 310.5208
##
## --- SP500 - Low1 (Sortino) ---
## Mean Sortino: 3.4237
## Median: 3.0279
## 95% CI: -0.569 9.9842
##
## --- SP500 - Crisis1 (Sortino) ---
## Mean Sortino: 1.9363
## Median: -0.4217
## 95% CI: -1.8446 19.4827
##
## --- SP500 - Mid (Sortino) ---
## Mean Sortino: 1.4399
## Median: 1.2152
## 95% CI: -1.003 4.9784
##
## --- SP500 - Crisis2 (Sortino) ---
```

```
## Mean Sortino: Inf
## Median: 0.6281
## 95% CI: -2.7687 49.0937
##
## --- SP500 - Low2 (Sortino) ---
## Mean Sortino: 1.034
## Median: 0.7523
## 95% CI: -1.1944 4.6035
```

```
      cat("\n---", p1, "vs", p2, "(Sortino) ---\n")
      cat("Mean Difference:", round(mean_diff, 4), "\n")
      cat("Median Difference:", round(median_diff, 4), "\n")
      cat("95% CI:", round(ci_diff[1], 4), "to", round(ci_diff[2], 4), "\n")
      cat("% Below 0:", round(prop_below_zero * 100, 2), "% | % Above 0:", round(p
rop_above_zero * 100, 2), "%\n")
      cat("Significance:", sig_flag, "\n")
    }
  }
}
```

```
##
## ============================
## Portfolio (Sortino): A
##
## --- Crisis1 vs Low1 (Sortino) ---
## Mean Difference: -2.8416
## Median Difference: -5.3729
## 95% CI: -17.3052 to 15.9937
## % Below 0: 85.5 % | % Above 0: 14.5 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid (Sortino) ---
## Mean Difference: 2.4421
## Median Difference: -1.1732
## 95% CI: -5.568 to 23.7065
## % Below 0: 64 % | % Above 0: 36 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -0.3517
## 95% CI: -6.4898 to 51.7564
## % Below 0: 53.8 % | % Above 0: 46.2 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -0.4521
```

```
## 95% CI: -7.0813 to 49.6743
## % Below 0: 53.2 % | % Above 0: 46.8 %
## Significance: ❌ Not Significant
##
## ============================
## Portfolio (Sortino): B
##
## --- Crisis1 vs Low1 (Sortino) ---
## Mean Difference: -3.3744
## Median Difference: -6.9587
## 95% CI: -19.1645 to 33.0025
## % Below 0: 87.2 % | % Above 0: 12.8 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid (Sortino) ---
## Mean Difference: 2.1985
## Median Difference: -2.0705
## 95% CI: -8.0566 to 44.378
## % Below 0: 75.3 % | % Above 0: 24.7 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -1.6171
## 95% CI: -8.0457 to 134.147
## % Below 0: 61.5 % | % Above 0: 38.5 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -0.8749
## 95% CI: -7.474 to 132.5455
## % Below 0: 57.4 % | % Above 0: 42.6 %
## Significance: ❌ Not Significant
##
## ============================
## Portfolio (Sortino): C
##
## --- Crisis1 vs Low1 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -4.7632
## 95% CI: -20.1469 to 80.9719
## % Below 0: 74.4 % | % Above 0: 25.6 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid (Sortino) ---
## Mean Difference: Inf
```

```
## Median Difference: 0.2571
## 95% CI: -7.1349 to 89.648
## % Below 0: 47.7 % | % Above 0: 52.3 %
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -1.0212
## 95% CI: -7.8408 to 46.5045
## % Below 0: 55.5 % | % Above 0: 44.5 %
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Low2 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -5.3178
## 95% CI: -21.9684 to 42.5293
## % Below 0: 74.1 % | % Above 0: 25.9 %
## Significance:  ❌ Not Significant
##
## ============================
## Portfolio (Sortino): D
##
## --- Crisis1 vs Low1 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -6.6528
## 95% CI: -26.0255 to 77.8111
## % Below 0: 78.5 % | % Above 0: 21.5 %
## Significance:  ❌ Not Significant
##
## --- Crisis1 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -2.6924
## 95% CI: -12.7821 to 85.3094
## % Below 0: 66.3 % | % Above 0: 33.7 %
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -3.4933
## 95% CI: -13.5197 to 56.7402
## % Below 0: 70.1 % | % Above 0: 29.9 %
## Significance:  ❌ Not Significant
##
## --- Crisis2 vs Low2 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -5.1202
## 95% CI: -18.4186 to 54.6014
```

```
## % Below 0: 73.8 % | % Above 0: 26.2 %
## Significance: ❌ Not Significant
##
## ============================
## Portfolio (Sortino): E
##
## --- Crisis1 vs Low1 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -8.2447
## 95% CI: -24.3314 to 39.4452
## % Below 0: 85.3 % | % Above 0: 14.7 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -0.377
## 95% CI: -6.3868 to 49.8732
## % Below 0: 52.9 % | % Above 0: 47.1 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: 0.8902
## 95% CI: -7.0024 to 61.6188
## % Below 0: 44 % | % Above 0: 56 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -2.0027
## 95% CI: -12.7045 to 55.4575
## % Below 0: 60.4 % | % Above 0: 39.6 %
## Significance: ❌ Not Significant
##
## ============================
## Portfolio (Sortino): F
##
## --- Crisis1 vs Low1 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -8.8138
## 95% CI: -28.0409 to 69.7648
## % Below 0: 86.2 % | % Above 0: 13.8 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -2.6381
```

```
## 95% CI: -10.9978 to 82.5468
## % Below 0: 69.5 % | % Above 0: 30.5 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -0.3123
## 95% CI: -10.5138 to 99.8504
## % Below 0: 51.5 % | % Above 0: 48.5 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -1.1821
## 95% CI: -12.5847 to 100.7464
## % Below 0: 54.8 % | % Above 0: 45.2 %
## Significance: ❌ Not Significant
##
## ============================
## Portfolio (Sortino): G
##
## --- Crisis1 vs Low1 (Sortino) ---
## Mean Difference: Inf
## Median Difference: 13.5072
## 95% CI: -229.1276 to 30892.22
## % Below 0: 44.7 % | % Above 0: 55.3 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: 44.4643
## 95% CI: -20.1126 to 31085.6
## % Below 0: 25.6 % | % Above 0: 74.4 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: 21.3682
## 95% CI: -22.5001 to 414170
## % Below 0: 32.6 % | % Above 0: 67.4 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 (Sortino) ---
## Mean Difference: Inf
## Median Difference: 0.0158
## 95% CI: -203.5531 to 414129.2
## % Below 0: 50 % | % Above 0: 50 %
```

```
## Significance: ❌ Not Significant
##
## ============================
## Portfolio (Sortino): H
##
## --- Crisis1 vs Low1 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -54.3527
## 95% CI: -5652.745 to 339202.7
## % Below 0: 67.3 % | % Above 0: 32.7 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -29.5601
## 95% CI: -3477.177 to 340099.3
## % Below 0: 63.2 % | % Above 0: 36.8 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -14.9502
## 95% CI: -3742.967 to 52275154
## % Below 0: 57.8 % | % Above 0: 42.2 %
## Significance: ❌ Not Significant
##
## --- Crisis2 vs Low2 (Sortino) ---
## Mean Difference: Inf
## Median Difference: 1.5033
## 95% CI: -228.941 to 52275222
## % Below 0: 48.4 % | % Above 0: 51.6 %
## Significance: ❌ Not Significant
##
## ============================
## Portfolio (Sortino): SP500
##
## --- Crisis1 vs Low1 (Sortino) ---
## Mean Difference: -1.4874
## Median Difference: -2.9021
## 95% CI: -10.1391 to 16.46
## % Below 0: 79.5 % | % Above 0: 20.5 %
## Significance: ❌ Not Significant
##
## --- Crisis1 vs Mid (Sortino) ---
## Mean Difference: 0.4964
## Median Difference: -1.3292
## 95% CI: -5.6735 to 16.8921
```

```
## % Below 0: 68.7 % | % Above 0: 31.3 %
## Significance: ✗ Not Significant
##
## --- Crisis2 vs Mid (Sortino) ---
## Mean Difference: Inf
## Median Difference: -0.7453
## 95% CI: -5.9124 to 48.5677
## % Below 0: 55.6 % | % Above 0: 44.4 %
## Significance: ✗ Not Significant
##
## --- Crisis2 vs Low2 (Sortino) ---
## Mean Difference: Inf
## Median Difference: -0.1912
## 95% CI: -5.6129 to 49.5495
## % Below 0: 52.5 % | % Above 0: 47.5 %
## Significance: ✗ Not Significant
```

8.5 Appendix 5 - Factor models

```
### ------------------------------------------- ###
### 3 Factor Model with weekly data             ###
### ------------------------------------------- ###
```

```
### View regression results ###
print("=== Regression Results 3-Factor Model ===")

## [1] "=== Regression Results 3-Factor Model ==="

print(regression_results_3factor)

##   Portfolio       Alpha Beta_Mkt_RF    Beta_SMB   Beta_HML R_squared
## 1         A 0.001592706   0.8275312 -0.01244820 -0.1071154 0.8988788
## 2         B 0.002612180   0.8371157  0.06366382 -0.1104248 0.8439917
## 3         C 0.006756185   0.9036950 -0.09932449 -0.5067448 0.6222197
## 4         D 0.009192916   0.9565856  0.07775717 -0.4402097 0.5056643
## 5         E 0.004353456   0.8612828 -0.11038482 -0.3248989 0.8043721
## 6         F 0.006356587   0.9046235 -0.02372628 -0.3229614 0.6790190
## 7         G 0.040207240   2.4797485  0.41424925 -2.6180952 0.3094403
## 8         H 0.077944468   3.0758841  4.07724211 -2.4581202 0.1443324
##     Alpha_pval
## 1 2.251269e-03
## 2 1.568121e-04
## 3 2.568685e-06
## 4 1.351089e-06
## 5 3.110314e-07
## 6 2.577088e-07
## 7 2.296621e-05
## 8 2.512420e-04

### View diagnostics
print("=== Model Diagnostics 3-Factor Model ===")

## [1] "=== Model Diagnostics 3-Factor Model ==="

print(diagnostics_results_3factor)

##     Portfolio  DW_stat    DW_pval    BP_stat       BP_pval
## DW          A 2.486076 0.99996052 16.8112107 7.728177e-04
## DW1         B 2.311738 0.99438964 17.3416203 6.011288e-04
## DW2         C 1.927811 0.28321984  2.3921864 4.950908e-01
## DW3         D 1.831419 0.08788604  1.3110114 7.265200e-01
## DW4         E 2.271146 0.98634408 24.4677652 1.994805e-05
## DW5         F 1.998012 0.49808016  4.8299799 1.846787e-01
## DW6         G 2.000283 0.50541466  5.4721289 1.403152e-01
## DW7         H 2.074479 0.73052878  0.7617949 8.585816e-01
```

```r
### ------------------------------------------- ###
### 5 Factor Model with Monthly data            ###
### ------------------------------------------- ###

### View regression results ###
print("=== Regression Results 5-Factor + Momentum ===")

## [1] "=== Regression Results 5-Factor + Momentum ==="

print(regression_results_5factor)

##   Portfolio       Alpha Beta_Mkt_RF    Beta_SMB    Beta_HML    Beta_RMW
## 1         A 0.004981349   0.9175742 -0.07156446 -0.09374222  0.15148449
## 2         B 0.012511379   0.7950280 -0.04550294 -0.06728308  0.04473871
## 3         C 0.029022478   0.9661280 -0.24630122 -0.54911061  0.12455398
## 4         D 0.049519560   0.6084657 -0.05430439 -0.36551851 -0.12620768
## 5         E 0.017680469   0.9678732 -0.22722500 -0.32703641  0.03781002
## 6         F 0.032347170   0.8013633 -0.14332054 -0.16493538 -0.05720987
## 7         G 0.182387225   3.4287842 -0.41045733 -2.70130845 -1.26937246
## 8         H 0.494480460  -2.1269071  0.31480167 -0.85647530 -7.36824438
##       Beta_CMA    Beta_Mom R_squared   Alpha_pval
## 1 -0.002201888  0.03135124 0.9568512 6.335620e-04
## 2 -0.092636290 -0.04658986 0.8309843 3.425755e-04
## 3 -0.172564194  0.12462355 0.6229627 5.297382e-05
## 4 -0.448046461 -0.11433918 0.2648623 5.291050e-04
## 5 -0.047459542  0.02031152 0.8499603 1.862349e-06
## 6 -0.405344892 -0.11452602 0.5277459 3.037661e-04
## 7 -2.372988712  0.46035872 0.4272240 1.662806e-04
## 8 -6.403568346 -3.61646570 0.1718178 4.858522e-04

### View diagnostics ###
print("=== Diagnostics 5-Factor + Momentum ===")

## [1] "=== Diagnostics 5-Factor + Momentum ==="

print(diagnostics_results_5factor)

##     Portfolio  DW_stat   DW_pval  BP_stat   BP_pval
## DW          A 2.194162 0.7663694 9.012980 0.1728494
## DW1         B 1.828861 0.2416660 6.556830 0.3637798
## DW2         C 1.871258 0.2963190 5.630478 0.4658290
## DW3         D 1.774249 0.1800012 5.786128 0.4475697
## DW4         E 2.208893 0.7837514 3.492977 0.7449039
## DW5         F 2.018986 0.5165398 6.546992 0.3647770
## DW6         G 2.137286 0.6927636 6.278550 0.3927211
## DW7         H 1.808901 0.2179248 6.549932 0.3644787
```

```
### ------------------------------------------  ###
### 3 Factor Model with Monthly data            ###
### ------------------------------------------  ###
```

```
### View regression results
print("=== Regression Results: 3-Factor Model (Monthly) ===")

## [1] "=== Regression Results: 3-Factor Model (Monthly) ==="

print(regression_results_3factor_monthly)

##   Portfolio      Alpha Beta_Mkt_RF    Beta_SMB    Beta_HML  R_squared
## 1         A 0.00571286   0.9264729 -0.17062564 -0.09820695 0.95377059
## 2         B 0.01226141   0.8269827 -0.05195128 -0.10199582 0.82743597
## 3         C 0.02972405   0.9493289 -0.30430354 -0.68358221 0.61464330
## 4         D 0.04736737   0.6803187  0.09097349 -0.56000262 0.24803736
## 5         E 0.01794428   0.9619308 -0.22090617 -0.39349460 0.84643834
## 6         F 0.03064711   0.8741885 -0.03012026 -0.35150666 0.50560835
## 7         G 0.17466961   3.2717943  0.93054055 -4.04556232 0.41216711
## 8         H 0.43542645  -1.2448434  6.72358533 -3.66405135 0.08689089
##     Alpha_pval
## 1 2.791327e-04
## 2 6.432061e-05
## 3 1.595306e-05
## 4 8.187392e-05
## 5 3.447664e-07
## 6 3.936527e-05
## 7 1.422934e-04
## 8 4.641856e-04

### View diagnostics
print("=== Diagnostics: 3-Factor Model (Monthly) ===")

## [1] "=== Diagnostics: 3-Factor Model (Monthly) ==="

print(diagnostics_results_3factor_monthly)

##     Portfolio  DW_stat   DW_pval   BP_stat   BP_pval
## DW          A 2.343026 0.9061939 1.1699041 0.7602312
## DW1         B 1.821104 0.2357570 2.5808837 0.4608507
## DW2         C 1.900580 0.3408275 0.7525111 0.8607887
## DW3         D 1.815824 0.2294515 1.8214401 0.6102805
## DW4         E 2.222185 0.7998740 0.2366977 0.9714583
## DW5         F 2.057204 0.5785179 1.7908692 0.6169239
## DW6         G 2.149231 0.7109259 1.7350761 0.6291639
## DW7         H 1.900188 0.3402681 2.2539888 0.5213927
```

8.6 Appendix - Diagnostics of return data and rolling Sharpe and Sortino ratios

```
### Check for autocorrelation and heteroskedasticity returns data ###
```

```
  # Print short summary
  cat("Ljung-Box p-value:", round(lb_test$p.value, 4), "\n")
  cat("ARCH Test p-value:", round(arch_test$p.value, 4), "\n")
}

##
## ------------------------------
## Diagnostics for Portfolio: A
## Ljung-Box p-value: 0.3486
## ARCH Test p-value: 0
##
## ------------------------------
## Diagnostics for Portfolio: B
## Ljung-Box p-value: 0.4478
## ARCH Test p-value: 0
##
## ------------------------------
## Diagnostics for Portfolio: C
## Ljung-Box p-value: 0.2715
## ARCH Test p-value: 0.0665
##
## ------------------------------
## Diagnostics for Portfolio: D
## Ljung-Box p-value: 0.3673
## ARCH Test p-value: 0
##
## ------------------------------
## Diagnostics for Portfolio: E
## Ljung-Box p-value: 0.0669
## ARCH Test p-value: 0
##
## ------------------------------
## Diagnostics for Portfolio: F
## Ljung-Box p-value: 0.2077
## ARCH Test p-value: 0
##
## ------------------------------
## Diagnostics for Portfolio: G
## Ljung-Box p-value: 0.486
## ARCH Test p-value: 0.3028
##
## ------------------------------
## Diagnostics for Portfolio: H
## Ljung-Box p-value: 0.8314
## ARCH Test p-value: 0
```

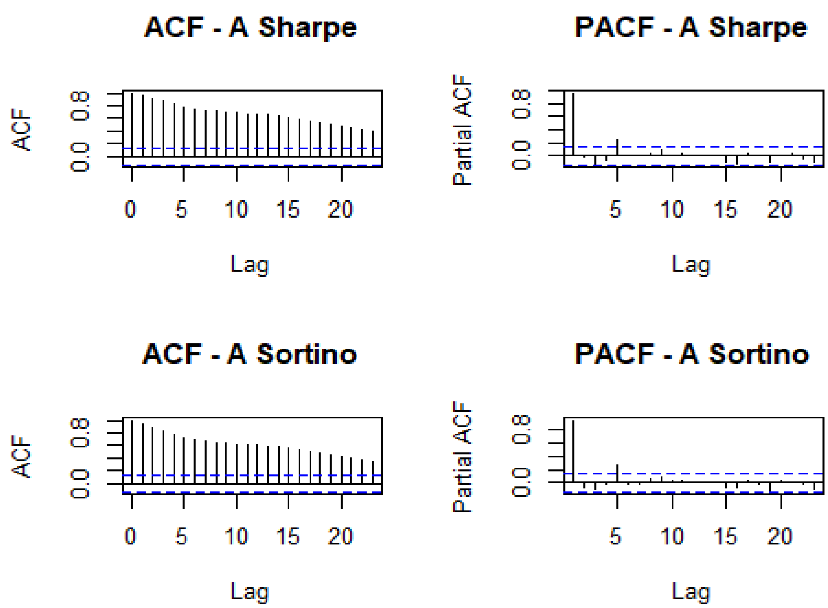
```
##
## ------------------------------
## Diagnostics for Portfolio: SP500
## Ljung-Box p-value: 0.6465
## ARCH Test p-value: 0
```

```
### Normality Test (Shapiro-Wilk) return data ###
for (col in colnames(returns_data)) {
  cat("\nChecking normality for portfolio:", col, "\n")
  normality_test <- shapiro.test(returns_ts[, col])  # Perform Shapiro-Wilk test
  print(normality_test)
}

##
## Checking normality for portfolio: A
##
##  Shapiro-Wilk normality test
##
## data:  returns_ts[, col]
## W = 0.94526, p-value = 2.802e-08
##
##
## Checking normality for portfolio: B
##
##  Shapiro-Wilk normality test
##
## data:  returns_ts[, col]
## W = 0.94526, p-value = 2.799e-08
##
##
## Checking normality for portfolio: C
##
##  Shapiro-Wilk normality test
##
## data:  returns_ts[, col]
## W = 0.98568, p-value = 0.01059
##
##
## Checking normality for portfolio: D
##
##  Shapiro-Wilk normality test
##
## data:  returns_ts[, col]
## W = 0.97467, p-value = 0.000139
##
##
```

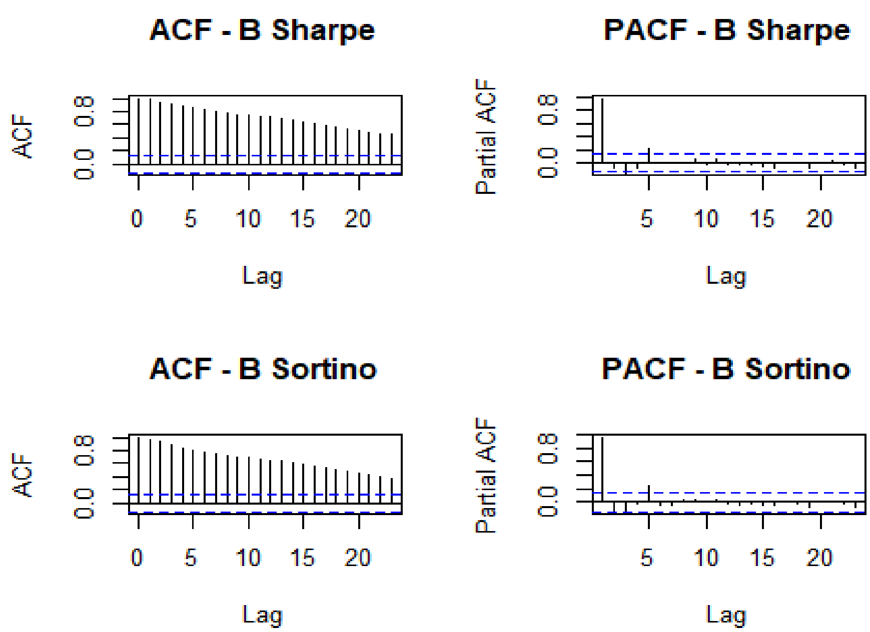
```
## Checking normality for portfolio: E
##
##  Shapiro-Wilk normality test
##
## data:  returns_ts[, col]
## W = 0.97321, p-value = 8.315e-05
##
##
## Checking normality for portfolio: F
##
##  Shapiro-Wilk normality test
##
## data:  returns_ts[, col]
## W = 0.96616, p-value = 8.161e-06
##
##
## Checking normality for portfolio: G
##
##  Shapiro-Wilk normality test
##
## data:  returns_ts[, col]
## W = 0.99568, p-value = 0.6878
##
##
## Checking normality for portfolio: H
##
##  Shapiro-Wilk normality test
##
## data:  returns_ts[, col]
## W = 0.95227, p-value = 1.598e-07
##
##
## Checking normality for portfolio: SP500
##
##  Shapiro-Wilk normality test
##
## data:  returns_ts[, col]
## W = 0.92234, p-value = 2.1e-10
```

```r
### ------------------------------------------  ###
### ✅ Diagnostic Check: Autocorrelation & ARCH ###
### ------------------------------------------  ###
  cat("Sharpe - Ljung-Box p:", round(lb_sharpe$p.value, 4), " | ARCH p:", round(ar
ch_sharpe$p.value, 4), "\n")
  cat("Sortino - Ljung-Box p:", round(lb_sortino$p.value, 4), " | ARCH p:", round(
arch_sortino$p.value, 4), "\n")
}

##
## ==== Autocorrelation & Heteroskedasticity Checks for A ====
## Sharpe - Ljung-Box p: 0  | ARCH p: 0
## Sortino - Ljung-Box p: 0  | ARCH p: 0
##
## ==== Autocorrelation & Heteroskedasticity Checks for B ====
## Sharpe - Ljung-Box p: 0  | ARCH p: 0
## Sortino - Ljung-Box p: 0  | ARCH p: 0
##
## ==== Autocorrelation & Heteroskedasticity Checks for D ====
## Sharpe - Ljung-Box p: 0  | ARCH p: 0
## Sortino - Ljung-Box p: 0  | ARCH p: 0
##
## ==== Autocorrelation & Heteroskedasticity Checks for E ====
## Sharpe - Ljung-Box p: 0  | ARCH p: 0
## Sortino - Ljung-Box p: 0  | ARCH p: 0
##
## ==== Autocorrelation & Heteroskedasticity Checks for F ====
## Sharpe - Ljung-Box p: 0  | ARCH p: 0
## Sortino - Ljung-Box p: 0  | ARCH p: 0
##
## ==== Autocorrelation & Heteroskedasticity Checks for G ====
## Sharpe - Ljung-Box p: 0  | ARCH p: 0
## Sortino - Ljung-Box p: 0  | ARCH p: 0
##
## ==== Autocorrelation & Heteroskedasticity Checks for H ====
## Sharpe - Ljung-Box p: 0  | ARCH p: 0
## Sortino - Ljung-Box p: 0  | ARCH p: 0
```

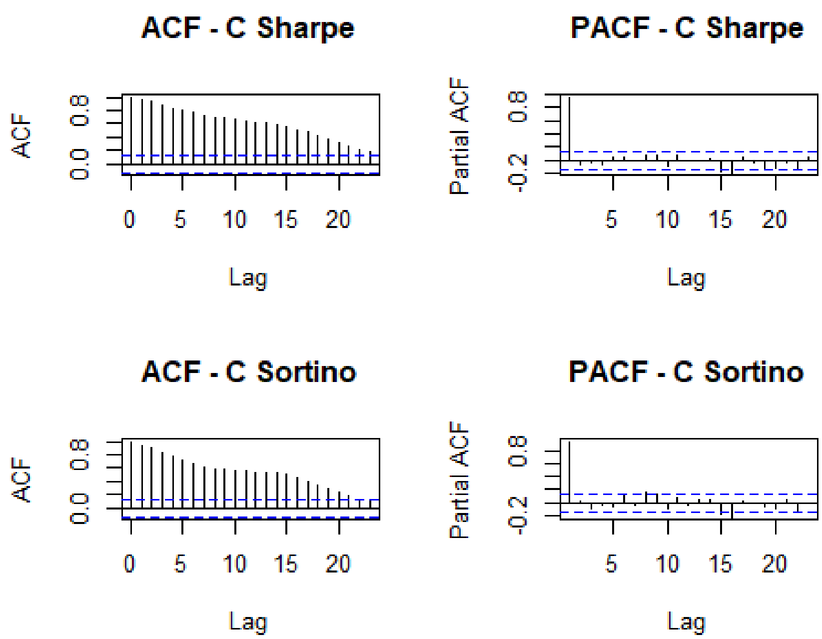
```
# ----------------------------------------------------------
# 📊 ACF & PACF of Rolling Sharpe and Sortino Ratios
# ----------------------------------------------------------
##
## === Plotting ACF & PACF for A ===
```



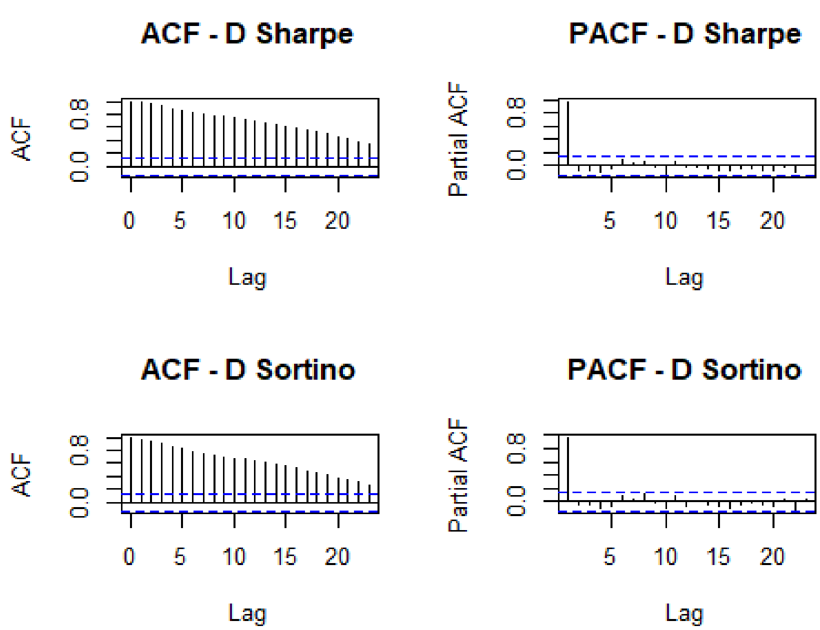

```
##
## === Plotting ACF & PACF for B ===
```



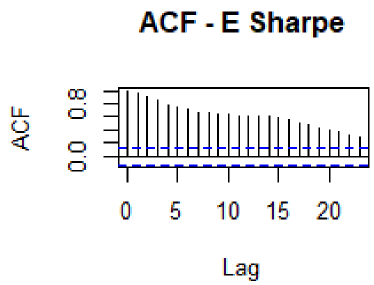
```
##
## === Plotting ACF & PACF for C ===
```

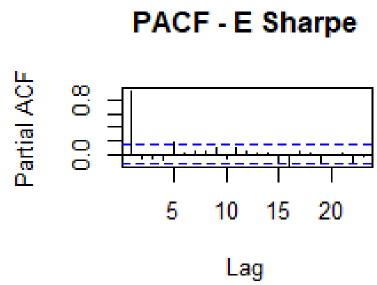


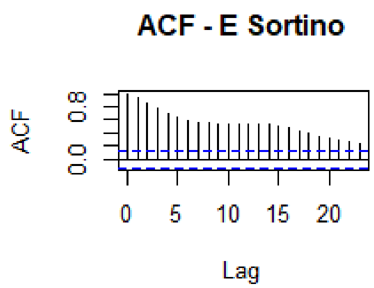

```
##
## === Plotting ACF & PACF for D ===
```

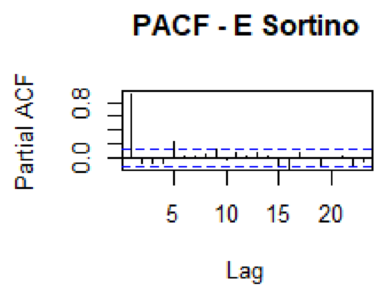


```
##
## === Plotting ACF & PACF for E ===
```

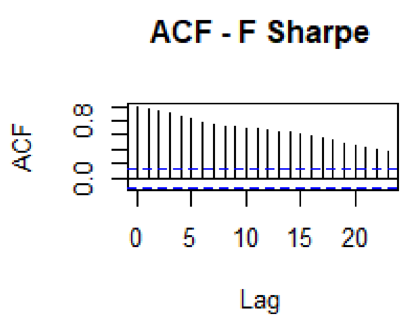


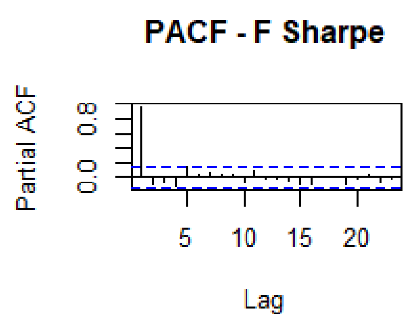


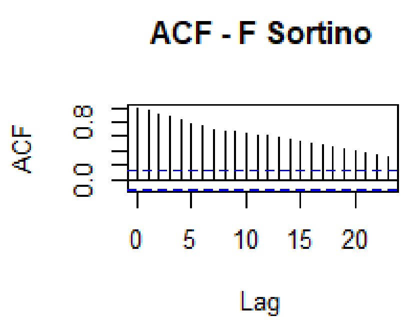


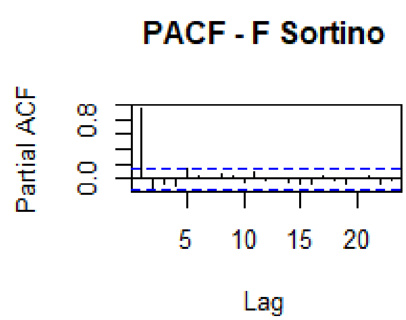



```
##
## === Plotting ACF & PACF for F ===
```

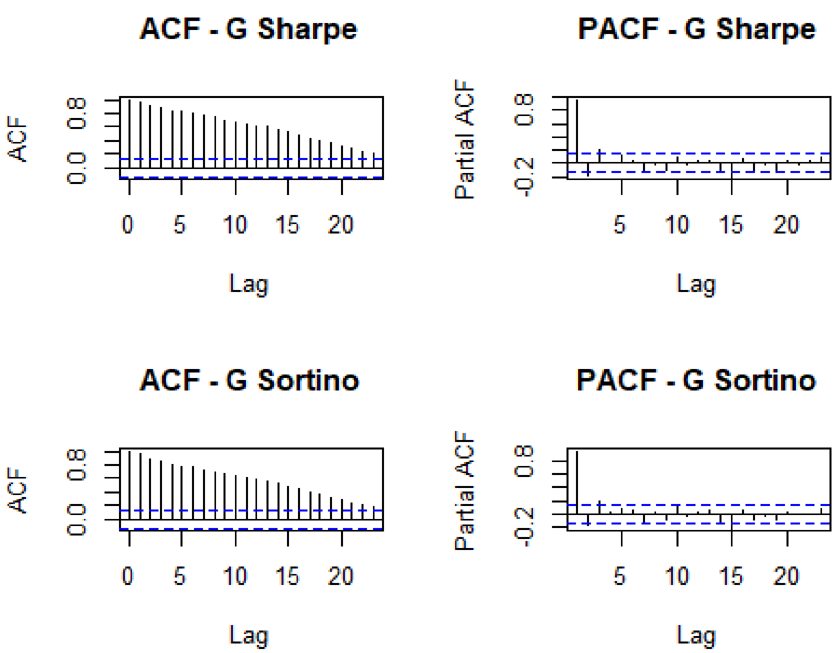








```
##
## === Plotting ACF & PACF for G ===
```




```
##
## === Plotting ACF & PACF for H ===
```