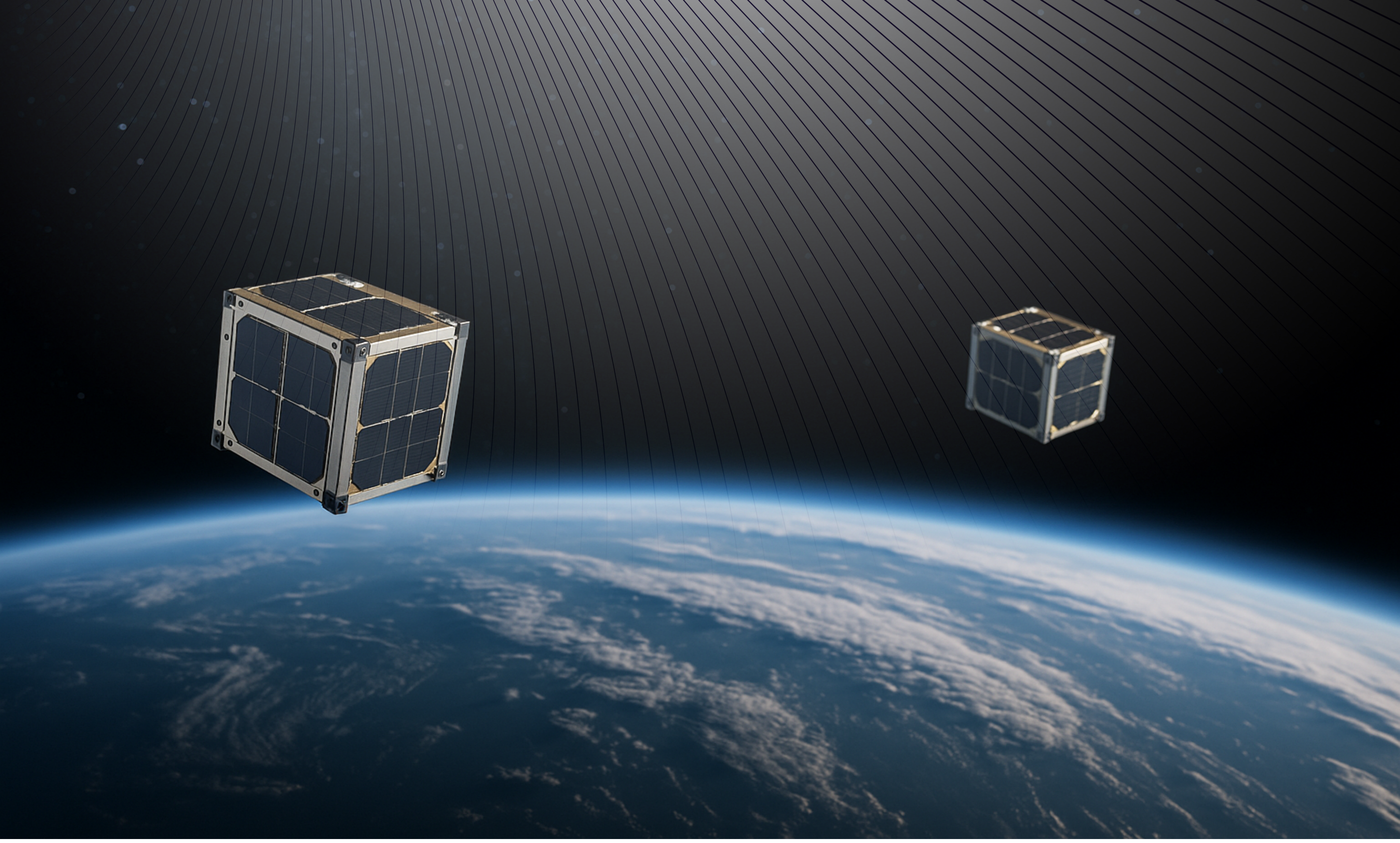

Control of Satellite Swarms

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Electronic Systems, ES10

June 4, 2025

STUDENT REPORT

AALBORG UNIVERSITY

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Abstract:

This project aims to test the feasbility of controlling two cube satellites with magnetorquers for swarming capabilities. This novel concept for swarm control based on magnetic actuation is investigated. The success criterion is to make the controller generate enough electromagnetic force to have two satellites attract and settle at some desired distance in between.
A linear state-space control strategy is developed for a highly non-linear system by a means of translation and scheduled gains. An analysis of this non-linear is conducted and an approximation of the model is done. This is followed by linearization at the desired operating point and also at different points in order to schedule gains for the linear controller. A fictitious force is calculated that acts on the satellites in different orbits around Earth, which acts as the minimum requirement for the feasibility of this project. This is followed by the controller design.
The strategy, namely, observer based control with full state-feedback is finally implemented in MATLAB Simulink platform and tested for the derived requirements.

Preface

This report is written by group 1029b, as a thesis report on the fourth semester of the Master in Electronic Systems at Aalborg University (AAU). The project is written during the period February 2025 to June 2025.

This thesis is addressed to the academic reader with technical knowledge in science and electrical engineering. The reader is expected to have an understanding of electronic systems, calculus, control theory and electromagnetism.

The project was suggested by Professor Jakob Stoustrup, who deserves a special thanks for his constant supervision and support during the development of this project.

Aalborg University, June 4, 2025

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Contents

Chapter 1

Introduction

CubeSats are a category of small satellites that have a standard size and form factor. They have become very popular nowadays due to their compactness, cheap cost of development, testing and deployement. A standard CubeSat uses 'one unit' or '1U' which measures 10x10x10 centimetres [6]; which can be extended to larger units by stacking up the 'units', depending on the mission requirements.

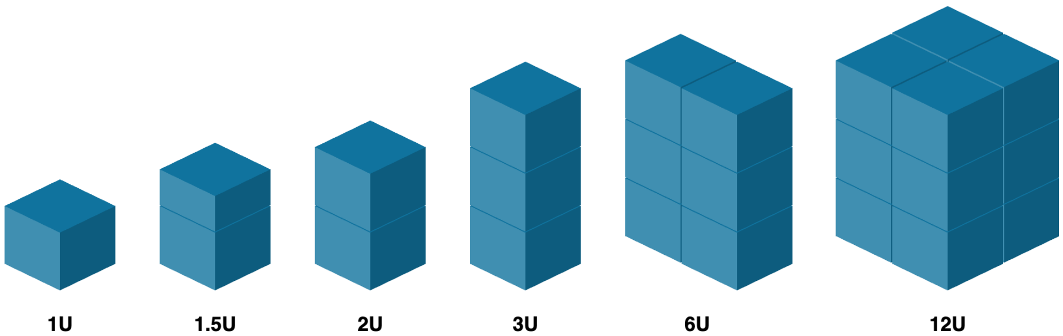

Figure 1.1: Different configurations of CubeSats

Thanks to the concept of CubeSats, it has become much more feasible to conduct scientific investigations and technology demonstrations. One such effort is the concept of satellite swarms. So far, the CubeSats have majorly operated alone as independent units, but combining these multiple mini satellites could prove useful in undergoing larger scale missions. Some of the applications for satellite swarms are discussed as following:

- **Imagery** - Due to the small size of CubeSats, the on-board camera sensors are small as well, which cannot output very detailed imagery. So multiple CubeSats could be combined for imaging earth, resulting in a bigger effective camera sensor, improving the resolution and depth. Similarly, it can also be used to photograph space.

- **Communication** - The small sized antennas on CubeSats could be combined on demand to effectively make a bigger antenna. This could be used as a make-shift antenna for telecommunication, broadcasting, in emergency situations for instaneous coverage, or generally, for remote sensing and navigation.
- **Space observations** - Satellite swarms could be useful to conduct experiments and observations in low-Earth orbits. Many different CubeSats from different launches could come together and act as a single test lab unit.
- **Docking** - The method for satellite swarming could be tested for spatial docking of spacecrafts, which requires high precision and accuracy.

Chapter 2

Project Formulation

It is desired to test the feasibiilty and control of magnetorquer based cube satellites, for swarming applications. This work investigates a control strategy for electromagnetic force based actuation to keep two satellites in a swarm formation. As illustrated in Figure 2.1, a current controller is to be designed which can make two satellites get close for swarming and maintain a desired distance in-between.

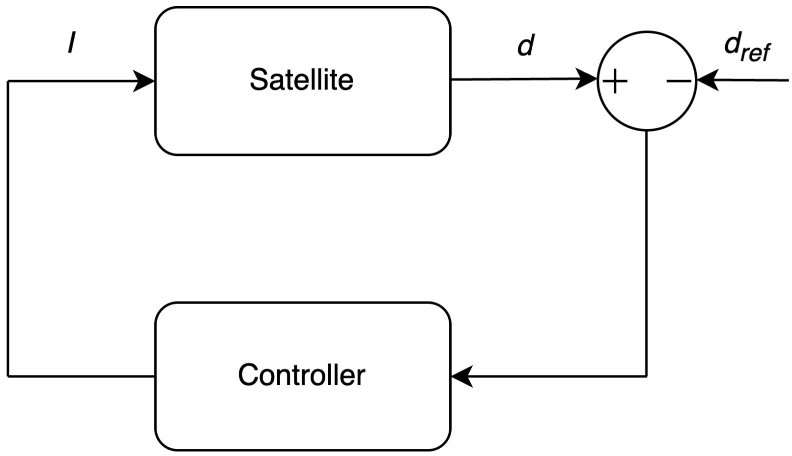

Figure 2.1: Desired closed loop of the satellite

Chapter 3

Requirements

From the project formulation in chapter 2, it is possible to derive a series of requirements for this project. The requirements are presented in the following Table 3.1.

ID	Specification	Note
R.1	The controller must be able to create a force of attraction between the two satellites at some distance and keep them $10cm$ apart, without any oscillations.	Work here is defined as being able to create an attractive force and maintain it.
R.2	The controller must be able to overcome the fictitious force acting on the satellite due to orbital velocity, within the maximum available current limit of $1A$.	Work here is defined whether the controller is able to generate enough actuation to overcome this force and does not let the two satellites drift apart.

Table 3.1: Requirements

Chapter 4

Methodology

In the following sections, a general overview of the theory involved will be presented, along with a more in-depth study of the magnetorquer based control system.

4.1 Magnetorquers

A magnetorquer or magnet torquer is an electromagnetic coil which is used for attitude control of cube satellites. Situated on the six sides of the cube, it creates a magnetic dipole which when interacted with the ambient magnetic field of Earth, produces useful torque which can detumble and stabilize the satellite [10]. The torque τ with dipole moment $\vec{m}$ and Earth's magnetic field $\vec{B}$ is related as:

$$\tau = \vec{m} \times \vec{B} \tag{4.1}$$

The magnetic dipole generated by the magnetorquer is given by:

$$m = nIA \tag{4.2}$$

where n is the number of loops of wire in the coil, I the current supplied and A the area enclosed by the coil. It can be seen from the relation that the dipole moment is directly proportional to the number of wire turns and the supplied current. The area of the coil is constrained to the maximum length of a side of a CubeSat, which is 10 centimetres.

4.2 Magnetic Forces

From the last section about *Magnetorquers*, it is evident that magnetic forces are involved. Magnetic forces arise whenever electrical current flow through conductors. This magnetic force is given by the Lorentz force law [1] in the following Equation 4.3:

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \tag{4.3}$$

Which says that a charged particle q moves with a velocity v through an electric field E and magnetic field B, it experiences a total electromagnetic force given by the above Equation 4.3. Although the Lorentz law originally concerns point charges, it extends to continuous currents as well. For a straight wire of length L oriented along some unit vector $\hat{l}$, the force becomes:

$$F = IL \times B \tag{4.4}$$

If the wire is curved, then the cross product is integrated along the wire as follows:

$$F = I \int (dl \times B) \tag{4.5}$$

To find the magnetic field B at a point in space produced by a steady current I, the Biot-Savart law [8] gives a relation for that. For an infinitesimal current element dl at a position r', the magnetic field at the field point r is given as:

$$dB(r) = \frac{\mu_0}{4\pi} \frac{Idl \times (r - r')}{|r - r'|^3} \tag{4.6}$$

Where μ_o i the vaccuum permeability. To find the total B, integration is done over the entire current path:

$$B(r) = \frac{\mu_0}{4\pi} \int_{\text{wire}} \frac{I\,dl \times (r - r')}{|r - r'|^3} \tag{4.7}$$

For a circular loop of radius R, at a point on the its axis at a distance x from center, the total magnetic field is given as [4]:

$$B(x) = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}} \tag{4.8}$$

4.3 Coaxial Force between Two Coils

As discussed above, the magnetorquers are basically coils of current carrying wire. In practice, these coils are square shaped, that follow the perimeter of the CubeSat's square sides. So, the analysis will be done using a simple setup of two square coaxial current carrying coils, which represent one out of six sides of the CubeSat, facing another CubeSat side with some distance in between, as depicted in Figure 4.1.

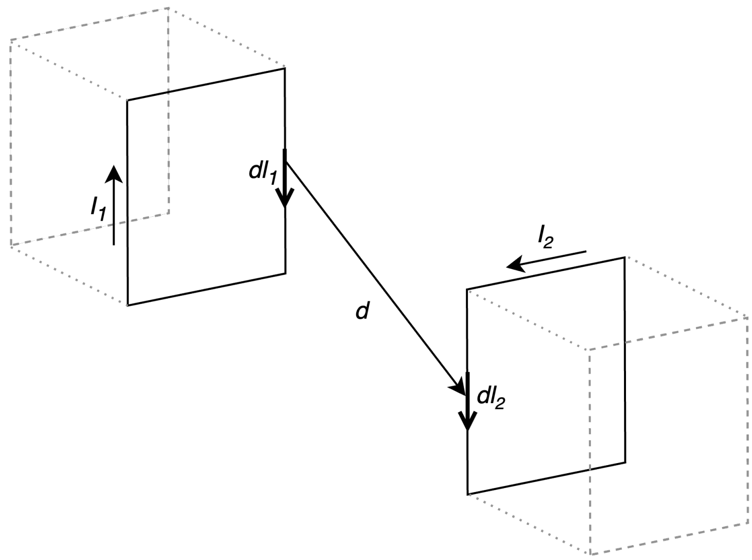

Figure 4.1: Two coaxial magnetorquer square coils

To get the dynamics of the system, it is necessary to take into consideration the forces acting on these magnetorquer coils. Hence, a model is required which describes the magnetic force acting between two current carrying coaxial coils. One such model is presented in [3] using the Lorentz force law and the Biot-Savart law, as follows:

$$\vec{\mathbf{F}} = -\frac{\mu_0}{4\pi} I_1 I_2 \oint \oint \frac{\hat{\mathbf{r}}}{r^2} d\vec{\mathbf{l}}_1 \cdot d\vec{\mathbf{l}}_2 \tag{4.9}$$

Where:

$\vec{F}$	Force acting on coil 2 due to coil 1	$[N]$
μ_0	Permeability of free space	$[N.A^{-}2]$
$\hat{r}$	Unit vector pointing along the line connecting two segments	[1]
r	Distance between two coils	$[m]$
$d\vec{\mathbf{l}}_1$	Wire segment of coil 1	$[m]$
$d\vec{\mathbf{l}}_2$	Wire segment of coil 1	$[m]$

Now a resultant force is needed from the Equation 4.9 that gives a force value due to all the segments on first coil onto the second coil's segments. Hence, the following section will discuss a vector analysis to solve for the double surface integrals.

4.3.1 Vector Analysis

The double closed surface integral is needed to be solved in order to find the resultant force on coil 2 due to coil 1. Which means that resultant force will be the sum of forces from all the infinitesimal wire segments of coil 1 on all the infinitesimal wire segments of coil 2. To do so, a parametrization of the coil segments will be done. As shown in the following Figure 4.2, center of coil 1 is assumed to be at origin of a local three dimensional cartesian coordinate system. The x-axis and y-axes are in the plane of the coil, while the z-axis is perpendicular to the plane and pointing towards the center of coil, which is placed at a distance d from coil 1.

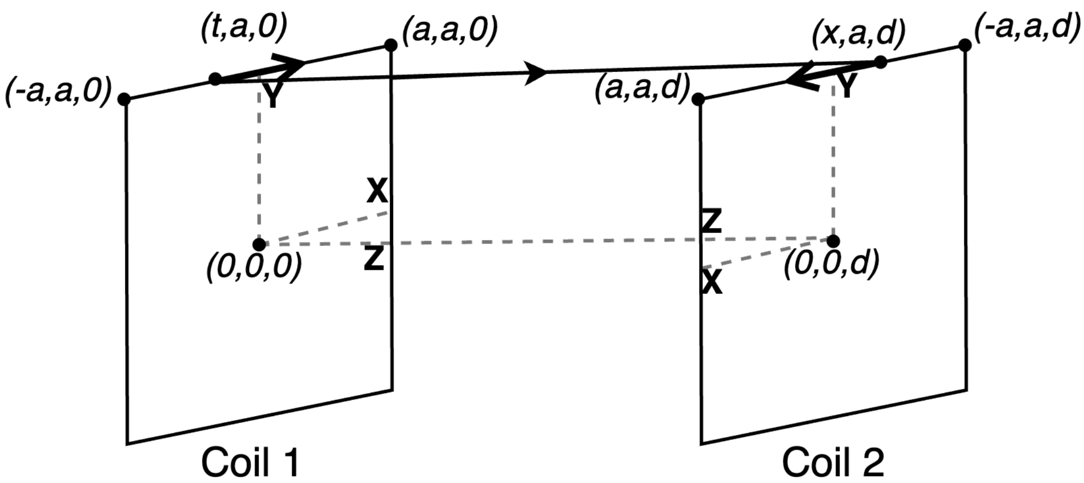

Figure 4.2: Electromagnetic force due to coil 1 top wire on coil 2 top wire (top-top)

As shown in the Figure 4.2, it is the case when the top wire of coil 1 is exerting electromagnetic force on the top wire segment of coil 2. Assumptions for all the different cases will be made from this top-top arrangement. For parametrization, it is assumed that the length of the side of the square is $2a$, and the infinitesimal point on the side of wire segment on coil 1 is situated at some point $(t, a, 0)$, where $t \in [-a, a]$. Similarly for coil 2, the point lies at (x, a, d), where $x \in [-a, a]$. With this established, the vectors of Equation 4.9 could be derived.

$$\vec{dl}_1 = (dt, 0, 0) \tag{4.10}$$

$$\vec{dl}_2 = (-dx, 0, 0) \tag{4.11}$$

$$\vec{r} = (x, a, d) - (t, a, 0) = (x - t)\hat{i} + d\hat{k} \tag{4.12}$$

$$|r| = \sqrt{(x-t)^2 + d^2} \tag{4.13}$$

Now, the equation of force for this case becomes:

$$\vec{F_{TT}} = -\frac{\mu_0}{4\pi} I_1 I_2 \int_{-a}^{a} \int_{-a}^{a} -\frac{(x-t)\hat{i} + d\hat{k}}{\sqrt{(x-t)^2 + d^2}^3} dt dx \tag{4.14}$$

Solving this integral will give the force vector on coil 2's top wire due to the top wire of coil 1. Similarly, all sixteen possible cases, including the top-top case will be derived. For the cases where the wire segments are perpendicular to each other, the dot product of $\vec{dl}_1$ and $\vec{dl}_2$ becomes zero, resulting in zero force. Hence, including all the parallel cases, the resultant force vector is found by summing all the non-zero integrals. This operation is done in MATLAB with varying the distance between the coils from $0.01m$ to $1m$. The force versus distance graph is plotted in Figure 4.3. The values of the constants are as follows:

1. Length of square side: $a/2 = 0.05m$
2. Permibiliity of free space: $\mu_0 = 4 * \pi * 10^-7$
3. The choice of current is dependent on the expected real world value of maximum current allowed. Equal currents in both coils: $I1 = I2 = 1A$
4. Number of coil turns: $N = 250$ (multiplied as N^2 due to the presence of two coils)

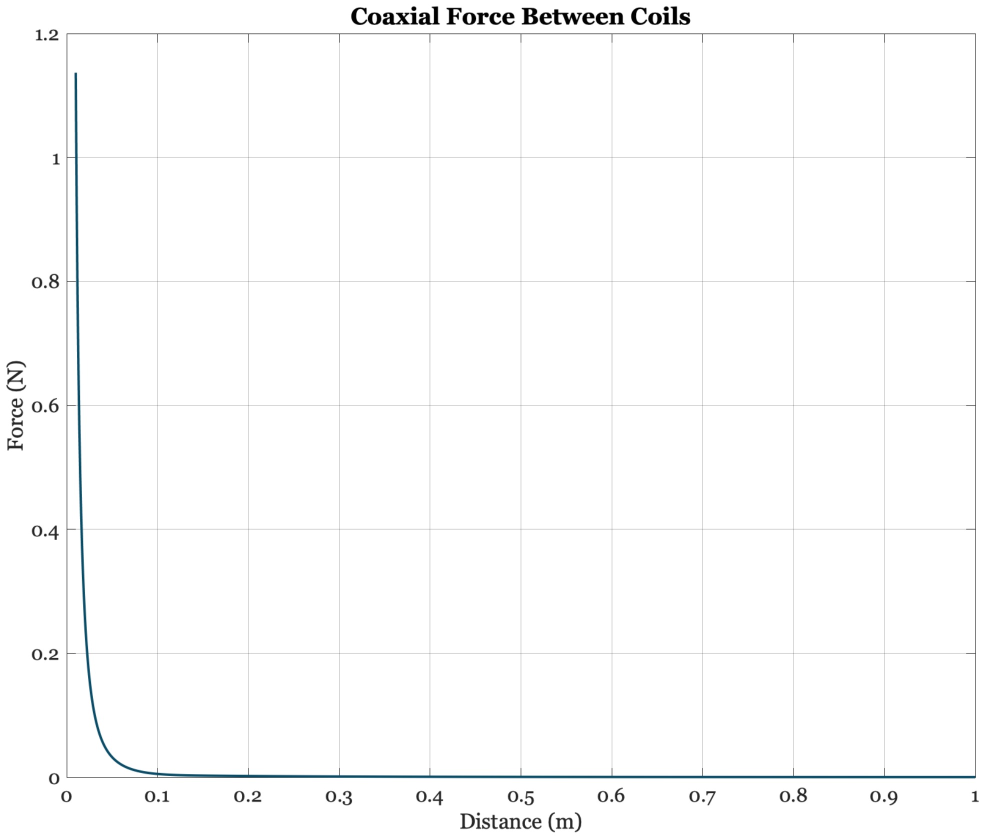

Figure 4.3: Electromagnetic force between two coaxial coils

As it can be seen from the Figure 4.3 that the model is highly non-linear. To save computing power on the double closed surface integral, a function approximation will be done. This approximation will be useful for the controller, as it will speed up the calculations rather than computing the integrals at every time step.

4.3.2 Model Approximation

In this section, an approximation of the highly non-linear model will be done. The choice of approximation method is log-log regression, which involves transforming both the dependent and independent variables to their logarithms and then fitting a polynomial to the transformed data. The reason for using this method is due to the fact that the model is exponential in nature, so log-log transformations can linearize the relationship for precise curve fitting. The following are the steps taken for this approximation:

- Transforming the relationship by taking the logarithm of both the independent variable (r) and the dependent variable (F).

$$log_{10}F = a_n(log_{10}r)^n + a_{n-1}(log_{10}r)^{n-1} + a_{n-2}(log_{10}r)^{n-2} \dots a_0 \tag{4.15}$$

- Fitting a polynomial with a suitable degree using least-squares regression to the log-transformed data. It is favorable to have the mean error of less than five percent. After testing with different degree of polynomials, the polynomial of degree 5 is chosen as it has the mean error of 3.6% as illustrated in Figure 4.5, which fulfills the requirement of less than 5%. The fitted function of degree 5 along with its constants is as follows:

$$\begin{aligned} log_{10}F = 0.5161(log_{10}r)^5 + 1.7131(log_{10}r)^4 + 1.1754(log_{10}r)^3 \\ - 0.5115(log_{10}r)^2 - 2.1781(log_{10}r) - 7.8197 \end{aligned} \tag{4.16}$$

The plot of the log transformed original and fitted polynomial is shown in Figure 4.4.

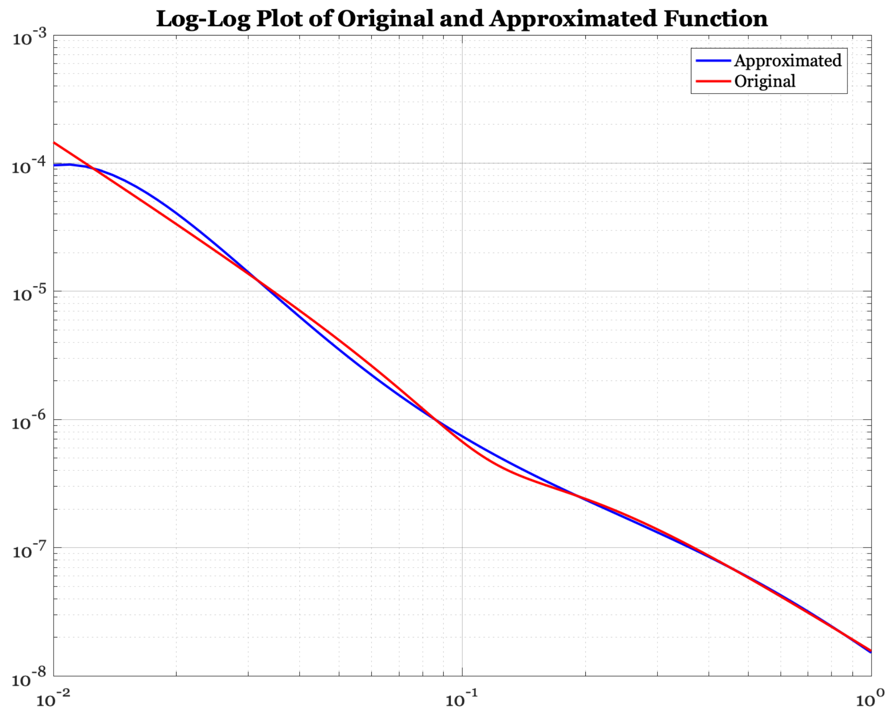

Figure 4.4: Transformed original function to log-scale and fitted polynomial

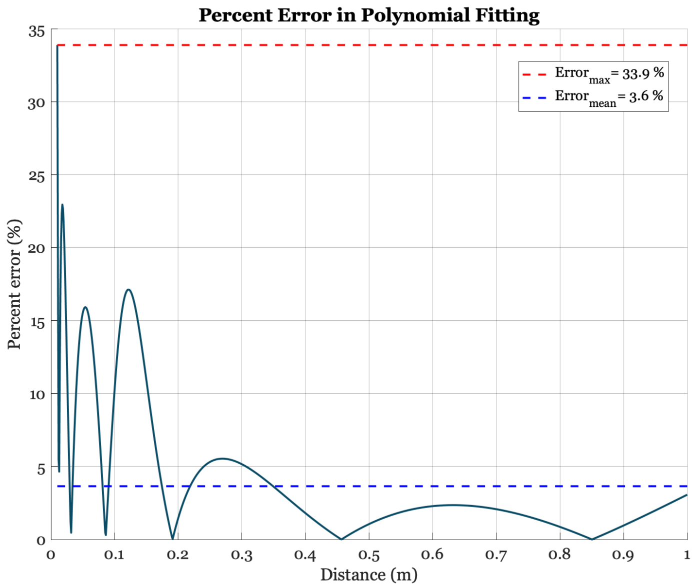

Figure 4.5: Percent error in polynomial fitting in antilog scale

- Taking the antilog of the fitted polynomial to convert it to linear scale. The original non-linear model with the approximated function can be seen in the Figure 4.6

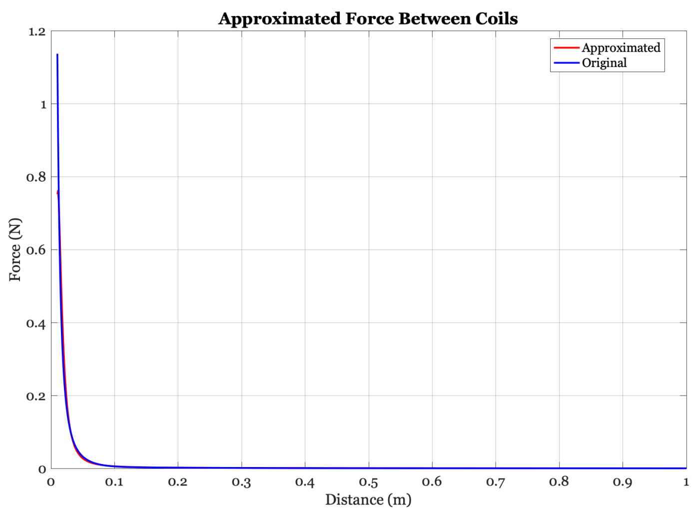

Figure 4.6: Approximated function and original function

As can be seen from the Figure 4.6 that the approximated function is a very good approximation of the highly non-linear model. Only at very close distances of less than $5cm$ that the approximation starts behaving erratic, as shown in Figure 4.7

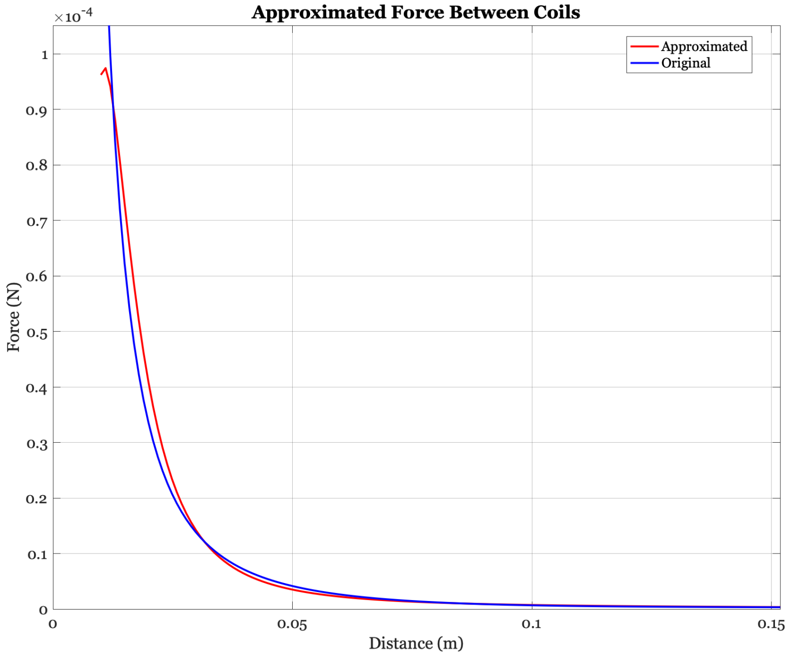

Figure 4.7: Approximated function and original function (zoomed)

But that is not a problem since the operating point of the system will be at least at $10cm$, as the length of each side of CubeSat is $10cm$. Hence, this approximated model will now represent the highly non-linear model of the satellite system.

4.3.3 Model Linearization

As the model of the system is highly non-linear, a linearization is required around the operating point to design and apply linear control techniques. As mentioned before, the operating point of the satellite system will be around $10cm$, so linearization will be done at the same point.

From Newton's law,

$$F = m.a = m.\dot{v} = m.\ddot{d} \tag{4.17}$$

The attractive force on coil 2 due to coil 1 will make it accelerate towards coil 1. But, due to the absence of friction, coil 1 will also move towards coil 2, thus making it a non-inertial frame. So, Newton's law will become,

$$2F = m(a_1 - a_2) \tag{4.18}$$

$$F = \frac{ma}{2} \tag{4.19}$$

Now, linearizing at the operating point x_0:

$$F(d, I) = I.f(d) = m.\ddot{d} \tag{4.20}$$

$$\simeq \left.\frac{\partial F}{\partial d}\right|_{x_0} d + \left.\frac{\partial F}{\partial \dot{d}}\right|_{x_0} \dot{d} + \left.\frac{\partial F}{\partial I}\right|_{x_0} I \tag{4.21}$$

$$\left.\frac{\partial F}{\partial d}\right|_{x_0} = I.\left.\frac{\partial f(d)}{\partial d}\right|_{x_0} = I_0 m \tag{4.22}$$

Where I_0 is the current at linearization and m is the slope of the line.

$$\left.\frac{\partial F}{\partial \dot{d}}\right|_{x_0} = 0 \tag{4.23}$$

$$\left.\frac{\partial F}{\partial I}\right|_{x_0} = f(d)\Big|_{x_0} = mx_0 + c \tag{4.24}$$

$$\therefore \frac{m.\ddot{d}}{2} = I_0.m.d + (m.x_0 + c).I \tag{4.25}$$

$$\implies \ddot{d} = \dot{v} = 2\frac{I_0 m}{M} d + 2\frac{mx_0 + c}{M} I \tag{4.26}$$

Where M is the mass of the coil and c is the y-intercept of the linear function.

The perturbations in the linearization will be considered negligible as the linearization will be done for the exact point and not for the small region around it. The reason for doing so is because there will be multiple linearizations that will be done along the non-linear function. This will result in multiple controllers for different points on the curve, which will be taken care by scheduled gains of the slopes and y-intercepts.

$$\therefore \dot{v} = 2\frac{I_0 md}{M} + 2\frac{mx_0 + c}{M} I \tag{4.27}$$

In state-space form, with $I_0 = 1A$ and $x_0 = 0.1m$ the linearization becomes:

$$\begin{bmatrix} \dot{d} \\ \ddot{d} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2\frac{m}{M} & 0 \end{bmatrix} \begin{bmatrix} d \\ \dot{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 2\frac{0.1m+c}{M} \end{bmatrix} I \tag{4.28}$$

$$d = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} d \\ \dot{d} \end{bmatrix} \tag{4.29}$$

As mentioned earlier, the values of m and c will be found out for different points on the curve, with a fixed current I_0. So m and c are dependent on the distance d, making this state-space non-linear. This behaviour will be treated in next section by a way of translation. Now that the model is developed, in the following sections, the controller design will be discussed.

4.4 Control Analysis

It is chosen to design a state-space controller for the SISO system, where the output will be distance d. From the state-space representation in Equation 4.28, the model is non-linear due to $m(d)$ and $c(d)$. So an analysis will be done on a linear second order integrator with a translated input. Since the control variable is current I and the input to the system will be force F, the input translation will be done like so:

$$\begin{bmatrix} \dot{d} \\ \ddot{d} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d \\ \dot{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{F}{M} \tag{4.30}$$

$$d = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} d \\ \dot{d} \end{bmatrix} \tag{4.31}$$

Where the control input h will be given as:

$$h = \frac{1}{M}.F = \ddot{d} \tag{4.32}$$

Taking the linearized function F from Equation 4.26:

$$h = \frac{2}{M}(I_0.m.d + (m.x_0 + c)I) \tag{4.33}$$

To make the current as input, isolating I:

$$I = \frac{Mh - 2I_0 m(d)d}{2(m(d)x_0 + c(d))} \tag{4.34}$$

Now, the idea is to design a controller for the linear system in Equation 4.30, that can be used for the original non-linear system by using scheduled gains to translate the inputs for non-linearities. It is better illustrated in following Figure 4.8.

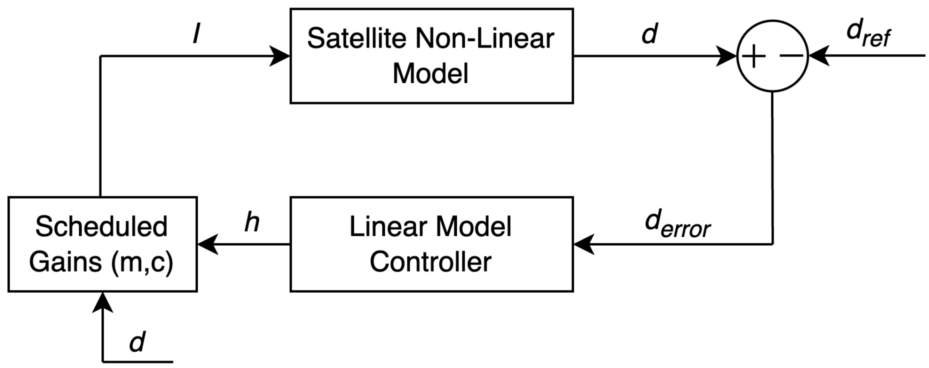

Figure 4.8: General control scheme

Where I is the control current that goes in as input into the satellite's non-linear model which gives out the distance between the two satellites. This distance is compared with the desired separation distance d_{ref}, which gives out the error distance d_{error}. This error in the distance is the input to the linear controller which gives out the signal h, which will be translated to the required current I by the means of scheduled gains, as derived in Equation 4.34. The controlled current I will be the input to the non-linear function $f(d)$ which will give out the input F for the double integrator system in Equation 4.30, since $F = I.f(d)$. Hence, making a linear controller work for a non-linear model. A more detailed illustration of the scheme can be seen in the following Figure 4.9.

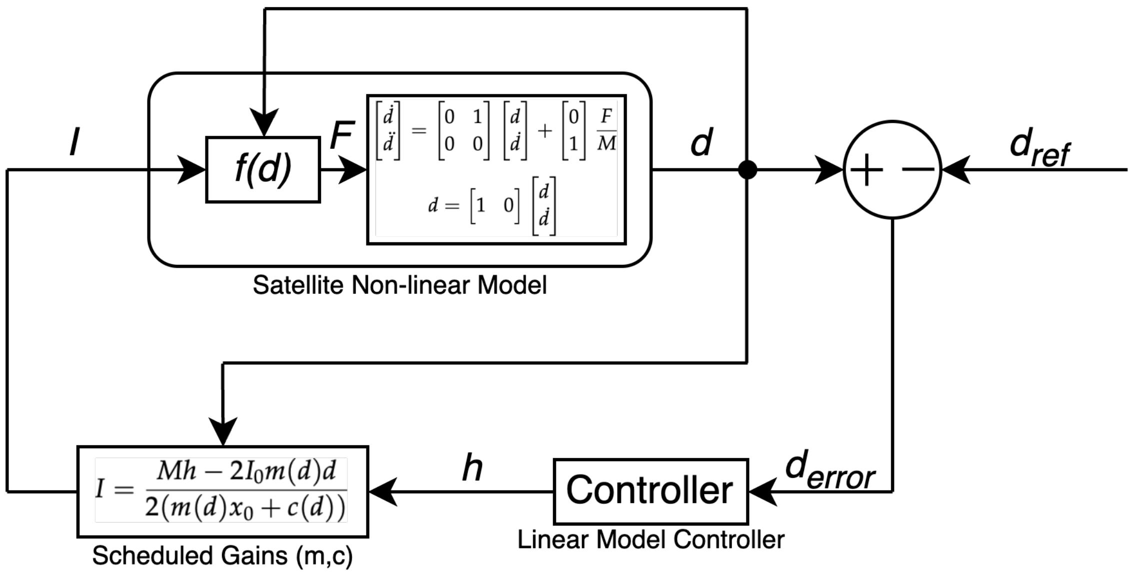

Figure 4.9: Detailed control scheme

4.5 Forces Acting on the Satellite

When the CubeSats are launched in space, they might acquire different orbits. Or, it might be needed to swarm different satellites in different orbits generally. The orbital velocities differ with the satellite's distance from the Earth. In order for two satellites to have a swarm formation, this difference in velocities have to be overcome.

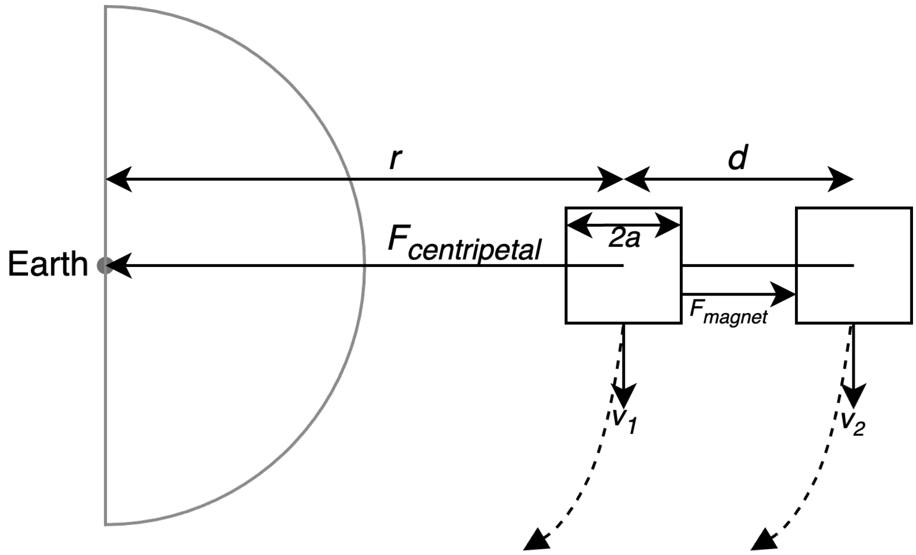

Figure 4.10: Forces acting on two satellites

The figure above illustrates the forces acting on the satellites in orbit, which consists of the following parameters:

r	Radius of orbit for the satellite	$[m]$
d	Distance between two satellites	$[m]$
$2a$	Length of a side of CubeSat	$[m]$
$F_{centripetal}$	Centripetal force acting on satellite towards center of Earth	$[N]$
F_{magnet}	Electromagnetic force acting between the satellites	$[N]$
v_1	Orbital velocity of satellite 1	$[m/s]$
v_2	Orbital velocity of satellite 2	$[m/s]$

As can be seen from the Figure 4.10, that there are two forces acting on a satellite: centripetal force towards the center of Earth and tangential force which provides the orbital velocity to the satellite. The orbital velocity [7] is given by following Equation 4.35:

$$v = \sqrt{\frac{G * M_{earth}}{r}} \tag{4.35}$$

Where:	G	Gravitational constant	$[6.67430 * 10^{-11} \frac{N.m^2}{kg^2}]$
	M_{earth}	Mass of Earth	$[5.97210^{24} kg]$
	r	Radius of orbit for the satellite	$[m]$

It can be deduced from the orbital velocity Equation 4.35 that a satellite at lower orbit will have greater velocity than the satellite at higher orbit. Due to this fact, if the satellite 2 has to electromagnetically attract satellite 1, the actuation force should be greater than at least this fictitious force that acts on the satellite 1, due to difference in orbits. This fact will be crucial for controller validation. Now, it is required to find this minimum force that needs to be overcome by the controller. Since the CubeSats are often operated in low Earth orbits, the orbital radii is around 2000 *km* [9]. Taking this radii as r, further analysis will be done.

Since $v_1 > v_2$, it is needed to find the force needed to pull satellite 1 by satellite 2. In other words, force required to make $v_1 = v_2$. So, initial velocity is v_1 and final velocity is v_2. Therefore:

$$v_1 = \sqrt{\frac{G.M_{earth}}{r}} \tag{4.36}$$

$$v_2 = \sqrt{\frac{G.M_{earth}}{r + d}} \tag{4.37}$$

From Newton's laws of motion:

$$F = m.a \tag{4.38}$$

$$v^2 = u^2 + 2ad \tag{4.39}$$

Where v is the final velocity and u is the initial velocity.

$$\because a = \frac{v^2 - u^2}{2d} \tag{4.40}$$

$$\therefore F = m.a = m.\frac{v^2 - u^2}{2d} = m.\frac{v_2^2 - v_1^2}{2d} \tag{4.41}$$

Where m is the mass of the satellite $m = 1kg$.

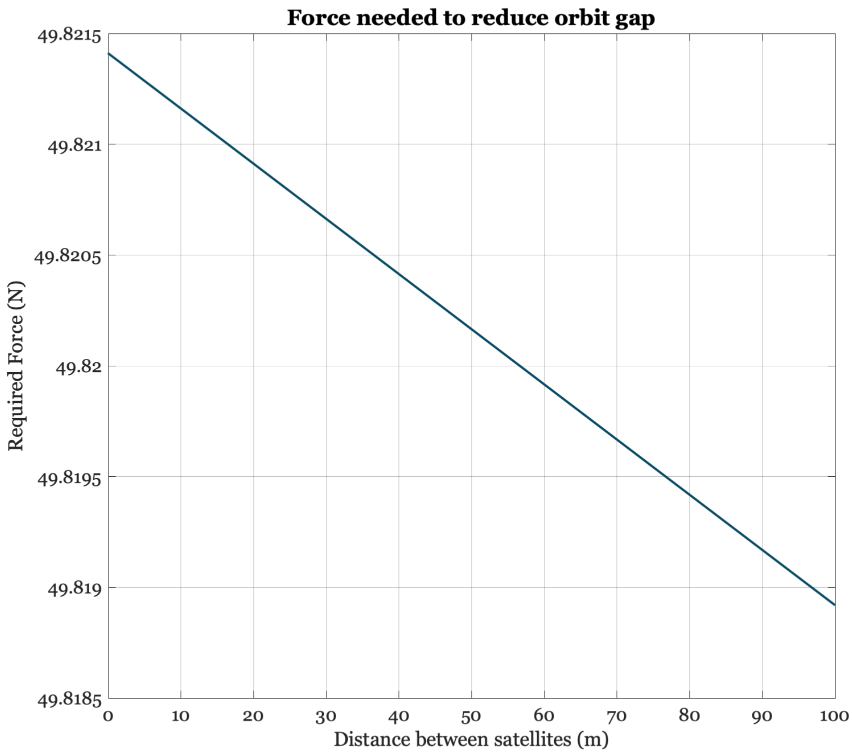

Figure 4.11: Force needed to reduce orbit gap

As per the project requirements, the desired distance between the satellites is $d = 0.1m$. For this distance, the value of this fictitious force after calculating is $F = 49.8214N$. For different distances $(0.01m - 100m)$ between the satellites, the following Figure 4.11 shows the change. The force versus distance almost seems linear without much change in the force over this separation range.

Chapter 5

System Development

The following sections will present the theory on the chosen state-space control strategy: Observer based control with full state-feedback, followed by its implementation and testing. The reason for choosing this control strategy is because of the requirement R1 which demands that there should be no oscillations in between the two satellites. So, with full state-feedback, the system can be made critically damped to achieve this goal.

5.1 State-Feedback Control

From the last chapter *Methodology*, the linear state-space system was established as follows, with mass $M = 1Kg$:

$$\begin{bmatrix} \dot{d} \\ \ddot{d} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d \\ \dot{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} F \tag{5.1}$$

$$d = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} d \\ \dot{d} \end{bmatrix} \tag{5.2}$$

From the general state-space format, the matrices A, B, C and D are as follows:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{5.3}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{5.4}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{5.5}$$

$$D = 0 \tag{5.6}$$

Before proceeding further, it is crucial to check for open-loop stability and to find out whether the above system is controllable, so the closed loop poles can be placed via state feedback, and observable, so an observer can be made to estimate the states.

5.1.1 Open-Loop System

Firstly, an analysis will be done on the open-loop system, which is without any control, to check for stability via its poles and step response. The eigenvalues of the system matrix A determine stability, since they are the poles of the transfer function [5]. The eigenvalues of the system matrix is given as the values of s that are the solutions of:

$$det(sI - A) = 0 \tag{5.7}$$

Solving the above Equation 5.7 gives two poles at 0, which is expected from a double integrator. The significance of having both poles at origin means there is no inherent decay in the system. The following Figure 5.1 illustrates the same, where the step response is a quadratic ramp. This behaviour comes from the fact that in the first integration, the step input (constant) is integrated, producing a ramp proportional to time t. And in the second integration, this ramp is integrated, which is proportional to t^2, producing a quadratic ramp, which grows unbounded over time.

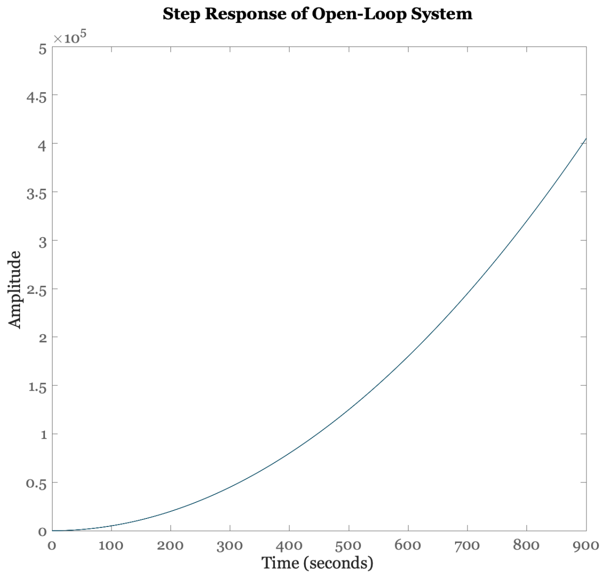

Figure 5.1: Open-loop step response

Due to this open-loop instability, there is need for appropriate feedback gains to stabilize in the closed-loop.

5.1.2 Controllability and Observability

As mentioned earlier, it is important to know the system's controllability and observability before proceeding with the control design. A system is is said to be controllable if there exists a suitable sequence of inputs that when applied over a finite time interval, can steer a state from any initial value to a desired final value. In other words, it is the ability of controlling a system's behaviour through its inputs. To check for controllability, an LTI system is controllable only if its controllability matrix $\mathcal{C}$ has full rank, or rank($\mathcal{C}$) = number of states.

$$\mathcal{C} = \begin{bmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{bmatrix} \tag{5.8}$$

After solving the Equation 5.8, the rank of the controllability matrix $\mathcal{C}$ is found to be 2, which is in fact equal to the number of states. Hence, the system is controllable. Next, the system is checked for observability. A system is observable if the initial states can be uniquely inferred from the knowledge of its measured outputs and inputs over a finite time interval. For a system to be observable, its observability matrix $\mathcal{O}$ must have full rank, or rank($\mathcal{O}$) = number of states.

$$\mathcal{O} = \begin{bmatrix} C \\ C\,A \\ C\,A^2 \\ \vdots \\ C\,A^{n-1} \end{bmatrix} \tag{5.9}$$

By solving the above Equation 5.9, the rank of the observability matrix $\mathcal{O}$ is found to be 2, which is equal to the number of states. Hence, the system is observable. Since both controllability and observability are dual concepts, they cannot exist without the other being true [5].

5.1.3 Closed-Loop System with Full-State Feedback

In this section, the step response of the system will be analyzed by closing the loop with pole placement approach, also known as full-state feedback. By full-state feedback, it means that all the states of the systems are known at all times, which would not be required for the desired controller, as only the first state d is of interest. Also, the other state is simply the derivative of the first state, so it can be easily calculated. In this section, for the proof of concept, all the states are being considered.

Since it is desirable to have a critically damped system, the poles need to be real, negative and equal. The pole placement scheme is shown in the following Figure 5.2.

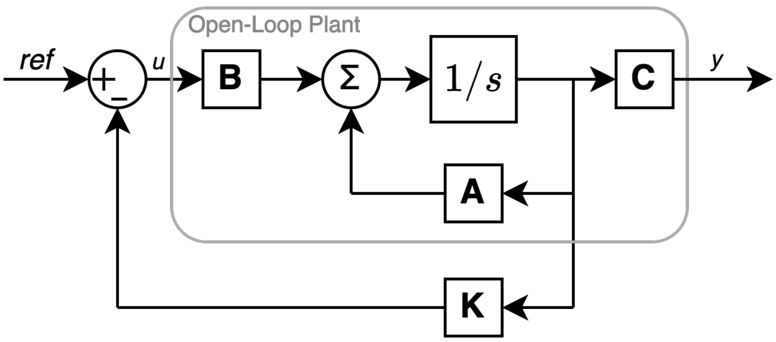

Figure 5.2: Closed loop with full-state feedback

Where K is the state-feedback gain matrix. It is assumed that the reference is zero. Then the input, with the state vector $\mathbf{x}$, becomes:

$$u = -K\mathbf{x} \tag{5.10}$$

The state-space equations for this closed-loop feedback system are as follows:

$$\dot{\mathbf{x}} = (A - BK)\mathbf{x} \tag{5.11}$$

$$y = C\mathbf{x} \tag{5.12}$$

Now with the state-feedback gain matrix K, the poles of the system can be adjusted based on the requirements. Assuming the poles are needed to be placed at $[-1, -1]$. The closed loop state matrix is now:

$$A_{cl} = A - BK \tag{5.13}$$

With:

$$K = [k_1 \; k_2] \tag{5.14}$$

Finding its characteristic polynomial and matching coefficients with the desired polynomial:

$$s^2 + k_2 s + k_1 = (s+1)^2 \tag{5.15}$$

Hence:

$$K = [k_1 \; k_2] = [1 \; 2] \tag{5.16}$$

Now, the closed loop state matrix becomes:

$$A_{cl} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \tag{5.17}$$

Whose eigen values are indeed both at -1. As can be seen from the step response in Figure 5.3, the system behaves critically damped. Now, poles can be adjusted based on how fast or slow the response is needed and how much actuation power is available in the real system.

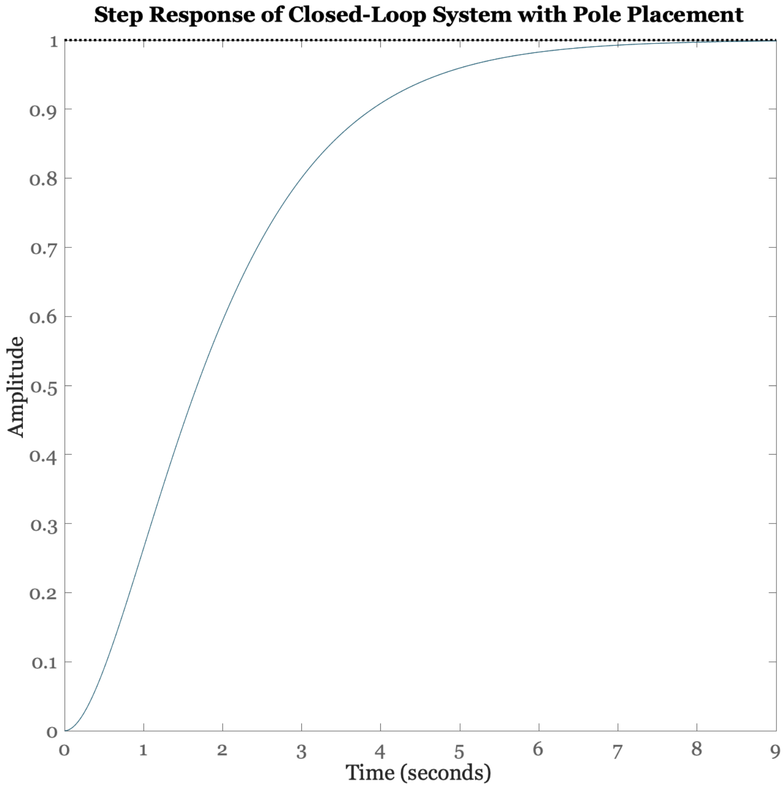

Figure 5.3: Closed-loop step response with poles at -1

5.1.4 Observer Design

As it is chosen to have the state-feedback controller, it is desirable to design an observer. The reason being that it is often difficult to measure all the states of the system, so an observer is needed to estimate them. It is basically a copy of the original system, with the same output. The measured output of the original plant is compared with the estimated output of the observer, which helps to correct the estimates of the states. As mentioned before, that only the first state d is of interest, so now onward the term full-state feedback would denote the same. The required observer design is illustrated in the following Figure 5.4, as described by [2].

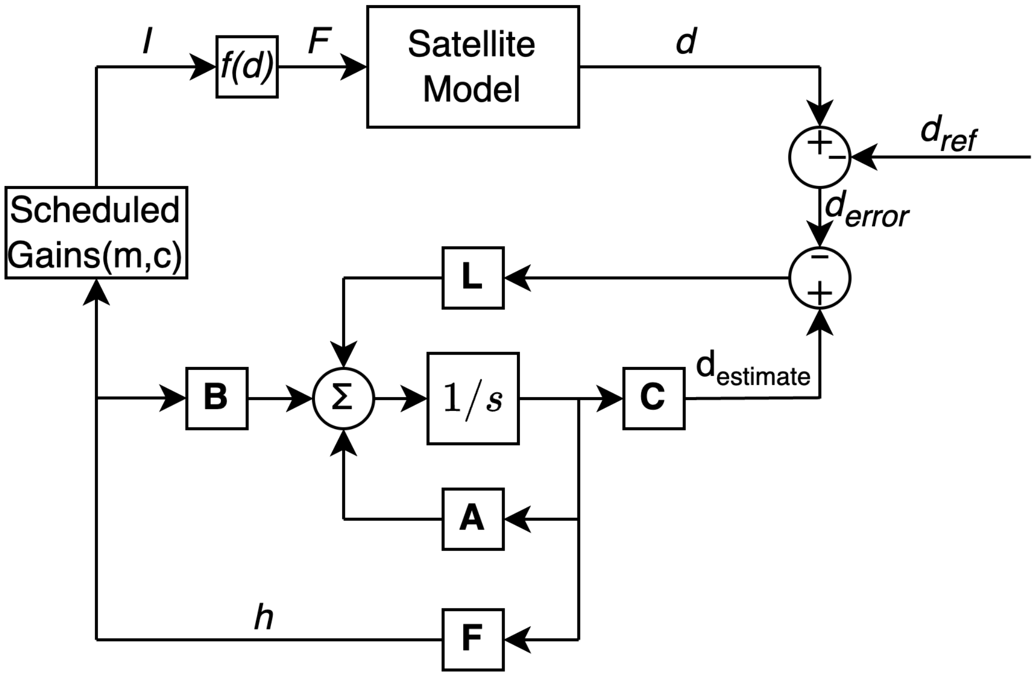

Figure 5.4: Full state-feedback with observer

The state-space equations for the observer are as follows:

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \tag{5.18}$$

$$\hat{y} = C\hat{x} \tag{5.19}$$

In full-state observer, the **F** matrix behaves like the state-feedback gain matrix K. The observer gain matrix **L** decides the poles for the observer, which can make the estimation faster or slower. It is wanted to make the dynamics of the observer faster than the system, so the poles must be placed farther to the left than the dominant poles of the system. The farthest they can be put is limited by the noise in the system, because the observer gain amplifies the noise as well.

5.2 Controller Implementation

Now that the controller design is derived in Figure 5.4, it will be implemented in MATLAB Simulink platform.

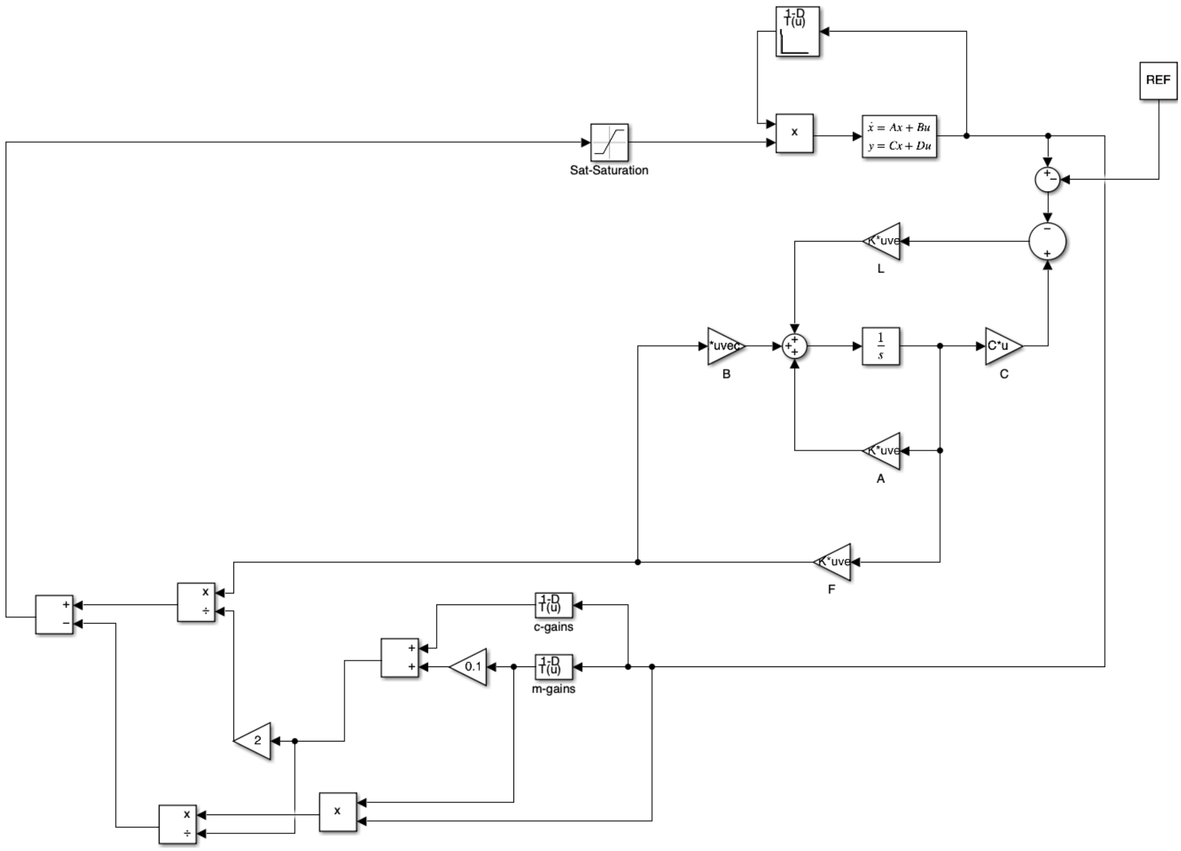

Figure 5.5: Controller implementation in MATLAB Simulink

In the above Simulink model, the m and c gains are derived by linearizing at twenty points on the non-linear model between $0.01m$ and $0.2m$. The reason being the model is highly non-linear in this range, whereas is becomes almost linear beyond $0.2m$. Also, the operating point of the system is chosen to be $0.1m$. To simulate the actual non-linear model $f(d)$ of the satellite setup, the approximated model derived in Equation 4.16 is used here.

Now, this controller will be tested for the requirements in the next chapter *Acceptance Testing*.

Chapter 6

Acceptance Testing

As prescribed in the chapter *Requirements*, it is desired to test the derived observer based control system and check for the requirements R1 and R2.

6.1 Test of R.1

R.1 criteria:

- The controller must be able to create a force of attraction between the two satellites at some distance and keep them $10cm$ apart, without any oscillations.

Procedure R.1

1. Initialize all the constants and variables to MATLAB workspace.
2. Set the initial conditions of the system and observer at distance $0.2m$.
3. Set the reference signal to be $0.1m$
4. Run the simulation for 100 seconds.
5. Check the response of the system in Simulink scope.

Success criterion R.1:

- If the controller is able to create an attractive force and maintains the distance of $0.1m$ between them.

Results of R.1:

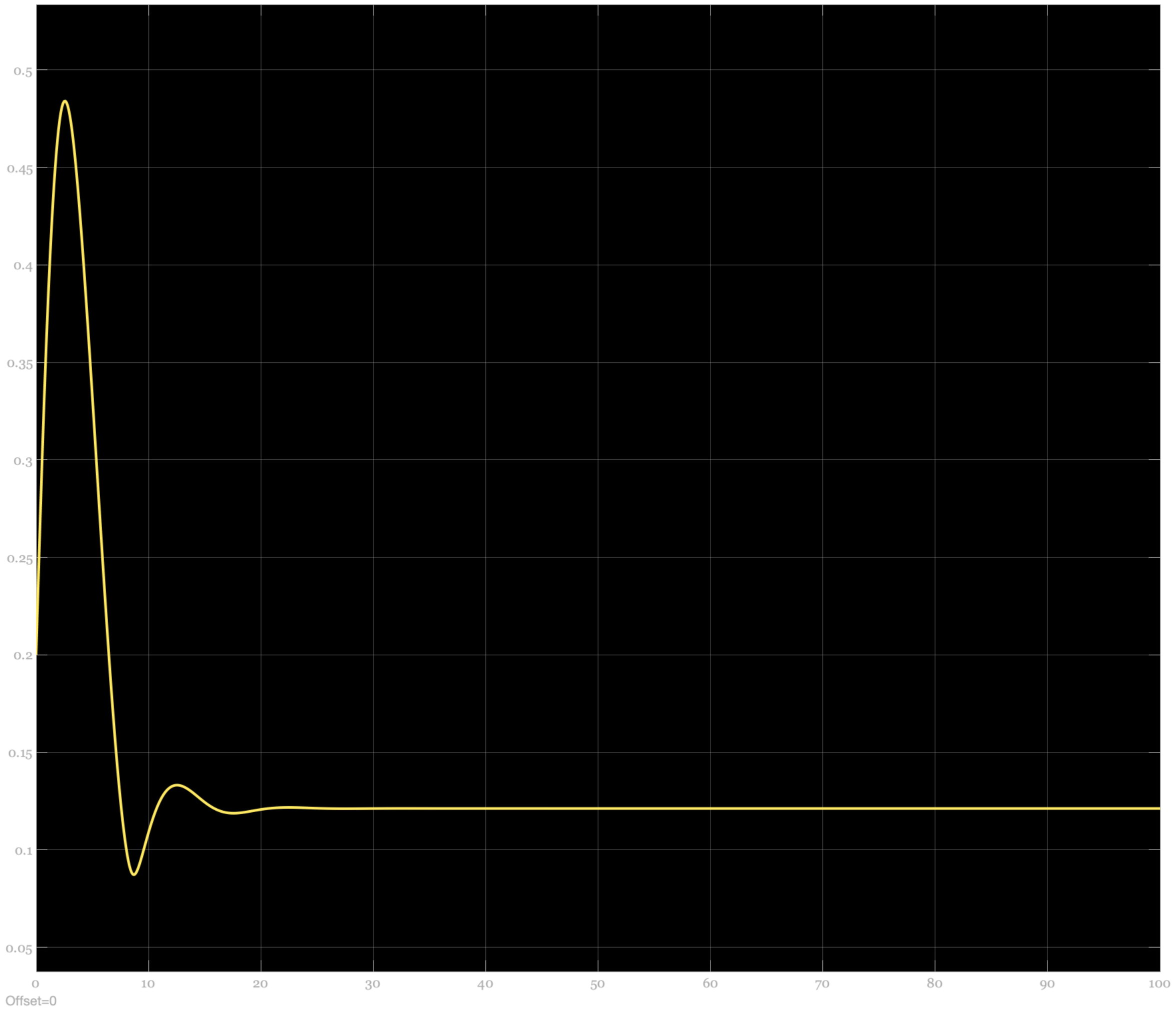

Figure 6.1: Response of the controlled system (x-axis: Time(s), y-axis: Distance(m))

As can be seen from the response of the controlled system in Figure 6.1, the system is behaving as expected. There are oscillations in the system and steady state error, as it does not settle on the reference value $0.1m$. Thus, requirement R.1 is not met.

6.2 Test of R.2

R.2 criteria:

- The controller must be able to overcome the fictitious force acting on the satellite due to orbital velocity, within the maximum available current limit of $1A$.

Procedure R.2

1. Initialize all the constants and variables to MATLAB workspace.
2. Set the initial conditions of the system and observer at distance $0.2m$.
3. Set the reference signal to be $0.1m$.
4. Set the saturation block for current limit constraint.
5. Run the simulation for 100 seconds.
6. Check the input F generated by the controller and current I.

Success criterion R.2:

- The controller must be able to generate enough actuation within current limit to overcome this force and does not let the two satellites drift apart.

Results of R.2:

The generated input F and I is shown in the following Figure 6.2 and Figure 6.3.

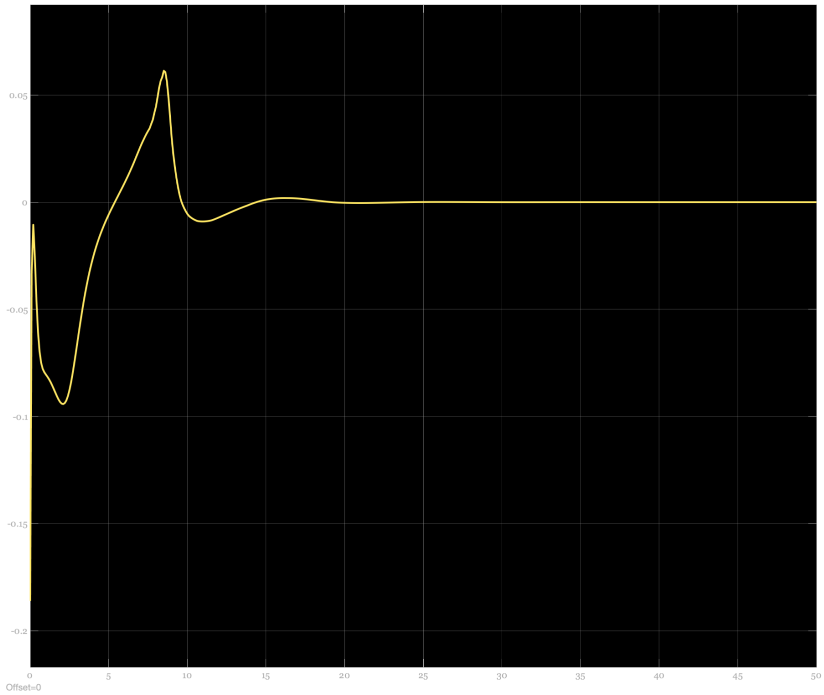

Figure 6.2: Force generated (x-axis: Time(s), y-axis: Force(m))

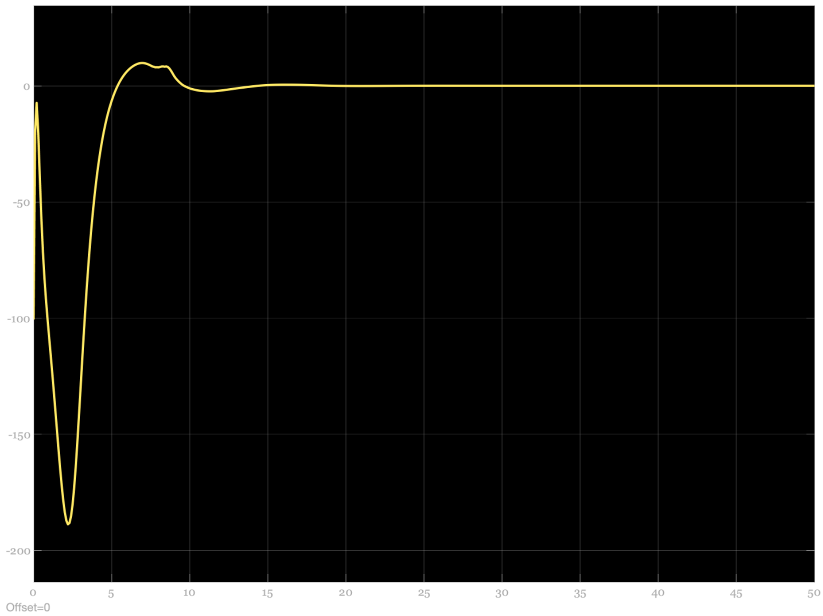

Figure 6.3: Current generated (x-axis: Time(s), y-axis: Current(A))

As can be seen from responses, they are irregular and erratic. The force value never reaches the required value but there seems to be some error in the simulation, so it might not be the true response. Similarly with current, the value reaches unrealistic levels, much greater than the limit. Hence, the requirement R.2 is not met.

In summary, both the requirements are not fulfilled by the implemented controller.

Requirement	**Fulfilled**
R.1	No
R.2	No

Table 6.1: Summary of tested requirements.

Chapter 7

Discussion

During the development of this project, multiple questions were encountered. While most of them were highlighted in the report, there are some that need to be discussed. This chapter will be an attempt to address them here.

7.1 Acceptance Testing

As presented in the chapter *Acceptance Testing*, the tests for two requirement R.1 and R.2 were not successful. The reason for that is most likely due to an implementation mistake in MATLAB Simulink, because the theory behind checks out. This shall be thoroughly investigated before the evaluation and attempt will be made to get concrete result out of it.

7.2 Future Work

Since this work deals with electromagnetic forces in one-dimension due to simplified interaction between two magnetorquer coil, it will be important to extend it to higher dimensions to consider real world scenarios. The CubeSats have at least three magnetorquers in its three sides, but also can be extended to all six faces as well. This availability of extra coils should be included to find the equivalent force of attraction for swarming applications. So the model presented in this work should be extended to incorporate the other magnetorquers available in the system.

Since the magnetorquers are already used to de-tumble and stabilize CubeSats, this attitude control should be used in conjunction with the swarming system so that different satellites in various orbits can be used for swarming.

The fictitious forces acting on satellites due to their orbits should be investigated further. In this work, the force derivation only holds true for the moment when both magnetorquers are coaxial. This should be extended to the cases when the actuation is initiated by both satellites with a means of tracking each other's attitude, so the resulting elctromagnetic force vector can be adjusted accordingly.

Chapter 8

Conclusion

In the chapter *Project Formulation*, it was specified to test the feasibiilty and control of magnetorquer based cube satellites, for swarming applications.

To be able to conclude whether it is possible for the satellites to use their magnetorquers for swarm formations, two major requirements were established in the chapter *Requirements*. In the following chapter *Methodology*, the theory involved was presented and an analysis was conducted for the highly non-linear system. From this chapter, the choice of controller design was concluded: Observer based full state-feedback, which was developed in the following chapter *System Development*. From initial tests, the linear controller looked promising and it was extended to the full setup, with the non-linear translations.

In the final phase of development, the controller was checked was the requirements prescribed in the chapter *Requirements*. The tests were not successful, as mentioned in the the chapter *Discussion*, likely due to faulty implementation in MATLAB Simulink. Again, this will be thoroughly investigated until the evaluation so more concrete conclusion can be made for this project's goals.

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