Foundations of an empirical input-output SFC model

An application to the Danish economy

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Abstract

In recent years, the need for more integrated macroeconomic models has increased, as financial imbalances and sector-specific shocks can have significant macroeconomic effects. While Denmark has a suite of well-established macroeconomic models, current frameworks often lack a sufficiently disaggregated representation of real-financial interactions and the financial balances across sectors and industries. Furthermore, existing stock-flow consistent (SFC) models tend to rely on highly aggregated structures, limiting their ability to capture sectoral dynamics and transmission channels.

In this paper, we propose a methodology to address these gaps by developing the foundations for a quarterly empirical input–output stock-flow consistent (E-IO-SFC) model for Denmark. The model integrates national accounts, financial accounts, and input–output tables, producing a quarterly, sector- and industry-disaggregated database aligned with accounting principles. We set up the core accounting structure through a transaction flow matrix and balance sheet matrix, enabling an internally consistent representation of real-financial linkages.

We describe the necessary data transformations and estimation strategy for key behavioral equations, focusing on the alignment between accounting identities and dynamic behavioral mechanisms. The framework is designed to allow endogenous determination of financial balances and their feedback effects on the real economy. Finally, we reflect on the practical challenges encountered in building such a model and outline the next steps toward developing a fully operational E-IO-SFC model for the Danish economy.

Contents

1 Introduction

Over the past decades, the need for more integrated macroeconomic models has grown, reflecting an increasing recognition that financial imbalances and sector-specific shocks can have systemic effects. Standard macroeconomic models used to assess economic dynamics and guide policy decisions have generally failed to capture the complex interactions between the real economy and financial systems, as noted by Wren-Lewis (2016). According to Svartzman et al. (2023), these limitations can be addressed through the use of stock-flow consistent (SFC) models, which offer a more comprehensive representation of the interdependencies between the real and financial sectors. However, as Valdecantos (2024a) points out, conventional SFC models tend to be too aggregated, frequently portraying the economy as a single-sector system.

This project puts forward a proposal to address these challenges by building on SFC literature, which offers an accounting-consistent framework for analyzing sectoral interactions and financial balances. Specifically, we lay the foundations for an empirical input-output stock-flow consistent (E-IO-SFC) model using Denmark as a case study, combining the integrated economic accounts with input-output data and sector and industry-specific financial accounts on a quarterly basis. Besides developing the methodological foundations for building such a model (including the challenges that this entails, mainly data-related), this project contributes to the existing literature by developing a quarterly dataset with a proper level of disaggregation. The next step, not covered in this project due to time and space limitations, will be to run, validate, test and use the model. We expect the foundations developed in this project, alongside the reflections made about the challenges we encountered, to be useful for researchers aiming to build a model of a similar nature.

Our contribution is twofold. First, we construct a quarterly database integrating national accounts, financial accounts, and input-output tables across six institutional sectors and five industries. Second, we disaggregate the traditionally aggregated non-financial corporations (NFC) sector, showing how this can be done from a data perspective and how it enhances the analysis of transmission mechanisms both from a real and financial viewpoint.

As mentioned, the aim of this project is not to provide a fully operational policy tool, but to establish a detailed and internally consistent model framework that can support future analytical work. According to Valdecantos (2023b), the development of an empirical SFC model involves several distinct steps. This project follows this guideline. The first step is the formulation of a research question. The second step, defining the model's structure and scope, including the construction of a transaction flow matrix and balance sheets, is carried out in section 4. The third step, developing a consistent database, is documented in sections 5 and 6. The fourth step, constructing the exogenous model[1] through a system of equations, and the fifth step, estimating the behavioral equations to be integrated into the model, are presented in section 7. In section 8, we highlight the current challenges, while section 9 discusses the methodology applied in this project as well as the overall process. Finally, section 10 concludes the project.

1.1 Problem statement

What are the necessary steps to lay the foundations for the construction of a quarterly E-IO-SFC model, using Denmark as a case study, and what are the main challenges that such an endeavor entails?

[1] By exogenous, a model that runs solely on in-sample data, where no endogenous behavioral responses are activated is meant.

2 Problem identification

In this section, the economic problem motivating this thesis is identified and discussed. Through this, we highlight two key gaps in the current literature that remain unaddressed and justify the need of a modeling approach like the one proposed in this paper. The identification begins at a general level and is then gradually narrowed down to focus on the current macroeconomic modeling landscape in Denmark, including an overview of existing models that may contribute to addressing these gaps. In section 2.2, we present our perspective on how these gaps can be addressed and propose a way forward.

2.1 Identification of the macroeconomic problem

The adequacy of the current macroeconomic modeling landscape[2] to address pressing questions and to support policymakers in responding to early warning signs and taking preventive measures has been, and continues to be, subject to debate. These pressing questions are often raised by policymakers and stakeholders, typically at a general or global level. For instance, one might ask: "What are the direct and indirect effects of geopolitical tensions across industries and labor markets? And would the development of firms balance sheets remain sustainable under such conditions?" Lari (2024) is one of many advocates of this long-standing critique, emphasizing not only its historical roots but also its continued relevance today. In his paper 'The Problems of Macroeconomics as Institutional Problems', Lari highlights how many of the foundational issues raised in the past remain unresolved and are still highly relevant in contemporary policy discussions. One of the early advocates highlighting these issues from a policy-making perspective was Jean-Claude Trichet, President of the European Central Bank at the time. In a speech delivered in November 2010, he stated:

> "When the crisis came, the serious limitations of existing economic and financial models immediately became apparent. Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. As a policymaker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools".

From a more specific viewpoint, Wren-Lewis (2016) highlights the lack of concern. He argues:

> "The DSGE approach involves detrending techniques that pre-filter the data in a way that the models can set aside the interdependency of credit availability and consumption, better understanding of which would have been crucial for policymakers during and after the crisis."

The viewpoint expressed by Wren-Lewis (2016) is further supported by Nobel laureate in economics, Joseph E. Stiglitz, who argues that the DSGE approach, which has been central to macroeconomic modeling over the past decades, fundamentally fails due to its flawed microfoundations. According to Stiglitz (2018), these models rely on assumptions of perfectly rational agents and complete markets, thereby neglecting key economic behaviors identified in both information economics and behavioral economics. Stiglitz's critique aligns with that of Trichet, particularly in the argument that current macro models inadequately represent the financial sector, rendering them incapable of anticipating or responding effectively to financial crises.

Oliver Blanchard's critique is also in line with this perspective. In his paper "On the Future of Macroeconomic Models", Blanchard (2018), former Chief Economist at the IMF, addresses similar shortcomings of DSGE models. He raises several serious objections and openly acknowledges their flaws: (i) a reliance on implausible assumptions that contradict empirical evidence, (ii) limited usefulness in deriving credible conclusions, and (iii) estimation procedures that often rely on a mix of calibration and Bayesian techniques.

[2] The macro modeling landscape in Denmark encompasses the various models used to analyze macroeconomic dynamics and inform economic policy. It includes both established models, such as DREAM, MAKRO, and ADAM, and ongoing academic efforts to develop alternative frameworks. These models play a central role in shaping fiscal, climate, and structural policy decisions.

These observations can potentially highlight a gap within the macroeconomic framework namely, the passive role of the financial sector and, consequently, its limited influence in shaping macroeconomic dynamics. According to Byrialsen and Raza (2020), it is crucial to fully incorporate the linkages between the financial and real spheres of the economy under analysis, as such an integrated understanding is essential for designing models capable of effectively supporting macroprudential policy decisions. The analysis between the real and financial sectors is inseparable, in the sense that one cannot adequately examine real economic outcomes without considering the constraints and dynamics embedded in the financial balances of each sector or industry[3]. A solid understanding of the interaction between the real and financial spheres can therefore enable policymakers to implement preventive measures that mitigate the adverse effects of economic shocks.

Narrowing the focus to a national perspective, several efforts have been made to develop empirical macroeconomic models tailored to the Danish economy in order to address economic policy implications. However, one may ask whether these models are sufficiently adequate when considering the concerns raised by Trichet, Wren-Lewis, Blanchard and Stiglitz regarding the limitations of conventional macroeconomic modeling frameworks. A range of macroeconomic models is currently in use across various institutions in Denmark, many of which can broadly be classified as general equilibrium models. One of the leading institutions for macroeconomic model development in Denmark is the Danish Research Institute for Economic Analysis and Modeling group (DREAM), which has developed models such as GreenREFORM and MAKRO. The main purpose of GreenREFORM is to assess the climate and environmental effects of policy initiatives, such as greenhouse gas taxes (Kirk et al., 2024). The latter is used to analyze structural and cyclical interventions, and are employed by, among others, the Danish Ministry of Finance[4] (Bonde et al., 2023). In order to clearly describe the limited representation of the financial side in the models developed by the DREAM group, it is relevant to consider the models' underlying financial structure. For firms, this structure is primarily represented by EBITDA[5]. This reflects the use of an aggregate measure for the financial position of each business industry, rather than a detailed or decomposed financial structure. For the household sector, financial balances is likewise reported in a broad, net financial variable[6], rather than through a disaggregated representation of financial instruments.

On the other hand, a noteworthy model is the Annual Danish Aggregate Model (ADAM), developed by Statistics Denmark (DST). According to DST (2012) ADAM represents the traditional synthesis between Keynesian and neoclassical theory. In the short run, production and employment are determined by aggregate demand, whereas in the long run, they are driven by supply-side factors. The model is empirically grounded, with the majority of its behavioral equations estimated using national accounts data. According to its developers, ADAM is a large and relatively disaggregated macroeconomic model. It can be understood as a compromise between empirically oriented time series models and theoretically oriented equilibrium models. Over time, the development of ADAM has moved it closer to the structure of equilibrium models, although it cannot strictly be classified as one. Unlike dynamic general equilibrium models, ADAM does not rely on rational expectations, nor does it incorporate policy reaction functions. Expectations in the model are either adaptive or constant. One of ADAM's key features is its fully specified input-output system, which provides detailed accounts of values, quantities, and prices related to supply and use. In terms of long-run properties, there is little difference between ADAM and standard equilibrium models such as DREAM.

In relation to financial instruments and ADAM's financial sub-model wealth is allocated across five financial instruments. This means that the sectoral savings surplus must either be invested in or financed through these five instruments, as defined in the model[7]. The allocation of the savings surplus is treated as a financial transaction,

[3]In particular, household consumption decisions are strongly influenced by the evolution of credit constraints. Changes in credit availability directly affect households' ability to smooth consumption over time, making it essential to model financial conditions alongside real economic variables in order to capture these behavioral responses accurately.

[4]The model is build upon a new technical framework, which features a system of sub-models. These sub-models can be decoupled to operate independently when necessary (Kirk et al., 2024).

[5]A financial metric that captures a company's operating profit before interest, taxes, depreciation, and amortization.

[6]This consists of the sum of domestic equities, cash and bank deposits, foreign equities, and bonds (Bonde et al., 2023).

[7]See DST (2012, p. 168) for a detailed specification of the financial instruments.

whereby assets are acquired or disposed of by mutual agreement. Changes in the value of financial wealth occur not only through transactions, but also through revaluations, such as shifts in bond or equity prices. The most important feedback mechanism from ADAM's financial sub-model to the rest of the economy operates through private consumption. Consumption is influenced by income from financial assets, which depends on both the level and composition of financial wealth across instruments. Portfolio allocation in ADAM follows either fixed shares, constrained to sum to one, or a residual approach, where one asset class absorbs the remaining balance.

To summarize on the national perspective, the main critique identified in this problem identification has centered on the insufficient attention paid to the financial balances in the highlighted models. By "insufficient attention," we refer to the absence of a disaggregated and endogenously determined representation of the financial balances reported in the national accounts, covering both assets and liabilities across sectors and industries across all financial instruments on a quarterly basis. This concern applies to both the models of the DREAM group and ADAM. This omission of a detailed description of the financial side and its links with the real spheres obey to the fact that the purpose of these models may be different, and that since no model can include and explain everything, it is "reasonable" that they had to be simplistic in some aspects. Lastly, from a more theoretical perspective, Byrialsen (2018) argues that the modeling tradition in Denmark has gradually shifted toward general equilibrium frameworks. While the use of DSGE models has certain advantages, it is important to emphasize that these models have also been subject to criticism for various reasons by prominent academics, including Blanchard (2018), and Stiglitz (2018). In light of these concerns, Blanchard (2018) argues that the macroeconomic modeling landscape should become more open to alternative approaches and less 'imperialistic', highlighting the need for greater methodological pluralism and the development of new types of macroeconomic models. Furthermore, Stiglitz (2018) advocates for the development of alternative modeling approaches that incorporate more realistic economic behavior. On a final note, Svartzman et al. (2023) argues that existing macroeconomic models tend to underestimate important relationships due to their reliance on overly simplified mechanisms and optimization assumptions. To address these limitations, researchers have proposed several promising alternative modeling approaches. Among these are stock-flow consistent models, which offer a more comprehensive framework for capturing financial and real sector interactions.

The above shortcomings in current models applied in Denmark can, to a certain extent, be addressed through the use of the empirical SFC modeling approach, as argued by Byrialsen and Raza (2020) and further supported by Svartzman et al. (2023). Their grounding on the system of national accounts and emphasis on the fulfillment of budget constraints (and, most importantly, how these constraints are satisfied and which dynamic implications this entails) make the SFC approach a powerful tool to detect potential instabilities in balance sheet structures and their subsequent adverse effects on the economy. Within this framework, as Byrialsen and Raza (2020) explain, the real and financial sectors are linked through standard accounting principles, while the dynamics is governed by behavioral equations. This approach allows for an integrated understanding of the economy as a single system, where the real and the financial spheres have a relative degree of autonomy (the agents making decisions in them are different and have different goals) but are inextricably connected.

However, as noted by Valdecantos (2024a), standard SFC models are too aggregated because they represent the economy as a single sector system, thus limiting their ability to capture inter-industry dynamics and industry-specific sources of macroeconomic instability. The importance of these dynamics is further underscored in Platitas and Ocampo (2025), which analyzes the inflationary effects of supply chain disruptions following the COVID-19 pandemic. Although the study does not explicitly employ an input-output structure, it clearly illustrates how shocks in specific sectors or industries, such as commodity-producing industries, can propagate through the economy and lead to broad-based inflation. The findings suggest that global supply chain shocks can induce significant, sizeable, and persistent effects on inflation, particularly in food and tradable goods. Thus, the study emphasizes the need to monitor both global supply chain pressures and commodity prices when analyzing the sources of supply-side inflationary pressures[8].

[8] This cascade effect is captured in the model via input prices from other industries, cf. section 7.2.2.

Taking the above into consideration, this paper aims to address two key gaps in the current macroeconomic modeling landscape in Denmark:

1. **The limited role of the financial sector and the missing link between the real and financial sides of the economy:** Existing macro models often assign a passive role to the financial sector and the financial balances within a model, despite its importance in shaping macroeconomic dynamics. While Denmark has a well-established suite of macroeconomic models that are largely fit for purpose, many of these frameworks lack an integrated treatment of real and financial interactions at the sectoral or institutional level.

2. **Lack of disaggregation in existing SFC models:** Current SFC models generally lack IO structures and therefore do not account for interactions between multiple industries and the sectors. As a result, they are often limited to highly aggregated representations of the economy, which restricts their ability to analyze sector- and industry-specific dynamics and it's policy impacts.

2.2 How to tackle the identified economic problem

SFC models have emerged as a crucial tool for understanding the detailed relationships between financial flows and stocks within an economy. SFC models are currently used at the Bank of England, as developed by Burgess et al. (2016), at the Ministry of Finance in Italy (Hermitte et al., 2023), and in France by Le groupe Agence française de développement (AFD), which has developed an SFC model for the Colombian economy to assess a global low-carbon transition (Godin et al., 2023).

The pressing questions currently facing the economics discipline, and those likely to arise in the coming decades, require more detailed representations of the financial side of the economy. Despite the notable strengths of the SFC approach, further development is necessary. In this section, we therefore outline how the identified gaps (Gap 1 and Gap 2) can be addressed using an SFC modeling framework. The aim is to ensure a more detailed and disaggregated representation of the connections between the real and financial sides of the economy on a quarterly basis[9].

Addressing the aforementioned gaps in the modeling landscape requires two tasks. First, to break down production and its related flows (intermediate and final consumption, labor demand, etc.) into different industries. Second, to build a detailed representation of the financial side of the economy. To address the first task and disaggregate the production of the non-financial corporations (NFC) sector as reported in the national accounts, we make use of input-output tables and accounting statistics, also provided by DST. Svartzman et al. (2023) highlights that combining the SFC approach with input-output structures holds significant promise, as input-output tables capture interactions across multiple industries and regions. Such an integration would ideally result in a more flexible and dynamic modeling framework, capable of reflecting complex real-world inter-dependencies. To address the second task, we use financial accounts from DST, as described in detail in section 5.2.

Although few IO-SFC models currently exist, there is growing academic interest in this approach and increasing efforts to further develop it. Among the existing models, there is an empirical IO-SFC model developed by Thomsen et al. (2024) for the Danish economy, as well as a partially empirical model for the Italian economy developed by Passarella (2023a). Furthermore, ongoing work is being carried out at the World Bank and at the AFD on empirical IO-SFC models for Brazil.

The richness of the integration of IO and SFC methodologies can further be strengthened if the financial sphere of the economy was described in more detail, including the construction of balance sheets at the industry level. Among the key advantages of such an integrated approach would be their ability to track how inter-sectoral transactions affect financial stocks and flows. It would also provide a more realistic and detailed representation of the economy by accounting for heterogeneity across industries and sectors, and how the exposure of some specific sectors or industries to different kinds of risks could lead to macroeconomic instabilities that are hard to detect when working with more aggregated models. Furthermore, IO-SFC models along the lines proposed in this thesis could simulate the effects of sector-specific policies on the broader economy and analyze how these policies propagate across industries (Passarella, 2023b).

2.2.1 Dividing the different industries within the NFCs

The redevelopment of the SFC modeling approach proposed in this paper shifts the focus from an aggregated to a more disaggregated structure, specifically through the breakdown of the NFC sector into five industries: energy (E), agriculture (A), transport (T), manufacturing and construction (MC), and other manufacturing and services (OMS). This selection of industries is primarily based on their contributions to gross value added (GVA), employment distribution, and export shares in Denmark, as seen in figure 1 and 2. While it may appear economically insignificant to include the energy and agricultural industries, particularly given their relatively small

[9] Having a macroeconomic model with quarterly real- and financial balance data allows for more accurate estimation of behavioral equations. This higher frequency improves the model's empirical fit and helps capture short-term dynamics between the real and financial sectors and industries.

contributions to the overall GVA, employment, and exports, this inclusion is justified by their current political attention (e.g. the Green Tax Reform (Institut for Fødevare- og Ressourceøkonomi, 2024)) and their central role in recent global and national crises. Notably, the energy and food crises triggered by the war in Ukraine have had significant macroeconomic consequences, such as rising inflation, which underline the importance of these industries in the broader economic context (Branner and Ingholt, 2023). Thus, although this paper does not emphasize environmental considerations at the moment, the inclusion of these industries remains relevant for addressing today's pressing policy questions and their potential impact on the broader economy. The election of the U.S. President Donald Trump, in 2024, marked a shift in the global agenda, raising new pressing questions such as tariffs and protectionism, while leaving the green transition increasingly sidelined in both policy and political discourse (Merkadeau, 2023). This shift highlights the adaptability of the SFC approach, as it allows for reorientation towards emerging economic and policy challenges without the need for structural redesign. On the contrary, starting from the benchmark structure the model could potentially be extended, or some of its building blocks developed more thoroughly, as new questions emerge and the model is used to address them.

Taken together, this paper takes initial steps toward two main contributions. First, it explores the construction of an E-IO-SFC model and a corresponding quarterly database for the Danish economy, incorporating financial balances at the sectoral level. Second, it proposes a preliminary disaggregation of the commonly defined NFC sector at industry level within the SFC framework, with an emphasis on the real side of the economy and it's financial balances.

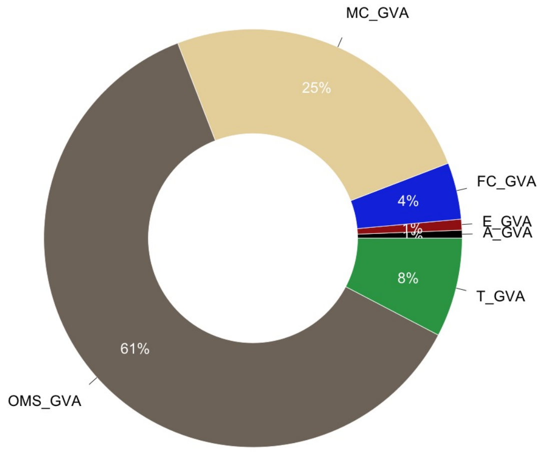
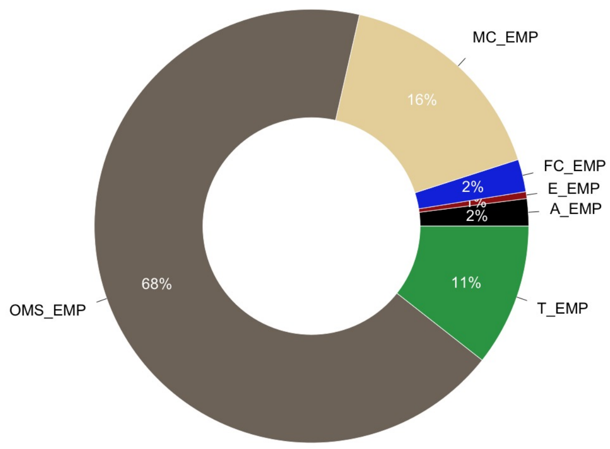
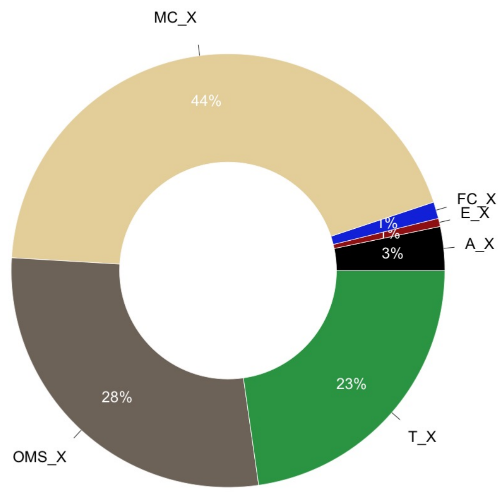


Figure 1: Contribution to GVA on industry level in 2024Q4 (DST, 2024f)



(a) Distribution of employment on industry level in 2024Q4



(b) Share of export on industry level in 2024Q4

Figure 2: Employment and share of export on industry level (DST, 2024e) (DST, 2024j)

3 Literature review

In section 3.1, we present a literature review of SFC models from a general perspective, focusing on their historical development and its theoretical foundations. Subsequently, section 3.2 provides a review of existing SFC models in both the Danish and international contexts.

3.1 SFC models from a historical viewpoint and emerging trends

SFC models[10], rooted in post-Keynesian economics, have emerged as a crucial tool for understanding the detailed relationships between financial flows and stocks within an economy. The foundations for SFC models were laid in the late 1940s by Morris Copeland, who aimed to provide comprehensive measurements of money flows to answer central questions about the sources and uses of money in national economies. His pioneering work culminated in the development of the flow-of-funds accounts, as detailed in his 1949 publication. Following Copeland's groundwork, Richard Stone introduced national accounting systems in USA. However, Stone's system faced criticism for not adequately capturing the flows of financial assets and liabilities, leading to a disconnection between the real and financial sides of the economy. This criticism highlighted the necessity for models that integrated these different types of flows comprehensively. A direct predecessor of what is nowadays known as SFC models gained substantial attention in the 1970s and 1980s through the contributions of James Tobin and Wynne Godley. Working within the neoclassical tradition, Tobin focused on portfolio allocation theories, while Godley, in collaboration with the Cambridge Economic Policy Group, utilized the principles of contemporary SFC models to analyze the British economy's sustainability trends, particularly addressing the balance of payments issues. Their efforts were instrumental in demonstrating the robustness of the spirit of SFC models in capturing the dynamic interactions between different sectors of the economy (Byrialsen, 2018).

A significant development in the history of SFC models occurred in 1989 when Godley and his colleague Gennaro Zezza were involved in a research project in Denmark. They developed a simple SFC model to discuss the real economic costs associated with high foreign debt. This model evolved into the first empirical SFC model for the Danish economy by 1992, aiming to simulate economic dynamics over short to medium terms (Godley and Zezza, 1992). A few years later, Godley collaborated with the Levy Institute which led to development of an empirical SFC model. A salient contribution of Godley at the Levy Institute came in 1999, when he identified seven unstable trends in the US economy (Godley, 1999). Among these trends were a rise in real money supply growth and an increase in the current account deficit. Based on these scenarios, Godley predicted with significant accuracy that the US economy would likely face a recession by 2010. This achievement, the prediction of the 2008 financial crisis, is remarkable and noteworthy as it highlights the capabilities and strengths of SFC models (Byrialsen, 2018).

According to Valdecantos (2024a) standard SFC models are too aggregated because they describe a single sector economy. This follows from the fact that the cornerstone of SFC models is the system of national accounts, where all production is aggregated in a single variable (output). However, the pressing questions the economics discipline is facing (and will most likely face in the coming decades) call for more detailed descriptions of the real and the financial side of the economy. Despite the mentioned strengths, SFC models still need upgrading.

An IO-SFC model integrates traditional input-output analysis within the SFC framework. Although there are not many IO-SFC models developed yet, there is a growing interest in this area and an increasing number of researchers are actively working on expanding this modeling approach.

Furthermore, ecological SFC modeling has also gained traction in recent years for analyzing climate policies from macroeconomic, financial, and ecological perspectives. Building on standard SFC modeling methods, ECO-SFC models are distinguished by their focus on the dynamic interactions between macroeconomic and financial variables and their implications (Dafermos and Nikolaidi, 2022).

[10] This review builds upon the same as presented in Nortvig et al. (2024).

3.2 Empirical literature and on-going projects

The first empirical SFC model in a Danish context was developed by Godley and Zezza (1992) at Aalborg University. Since then, SFC modeling has gained increasing attention within macroeconomic research, particularly through the development of empirical models that remain largely aggregated at the sectoral level. Most of these contributions have either advanced theoretical frameworks or adapted the empirical methodologies initiated by Godley and Zezza (1992).

This empirical literature review aims to provide an overview of existing empirical SFC models, both internationally and in Denmark, by comparing them within a broader historical and modeling context. It draws primarily on the classification presented in Pierros (2024), in which empirical SFC models are categorized based on the complexity of their structure, the number of institutional sectors, and the range of financial and physical assets included. This classification further enables the division of existing empirical SFC models into three main types: the New Cambridge (NC) type, the Godley–Lavoie (GL) type[11], and the High Complexity (HC) type. In order to emphasize overarching trends and facilitate comparative insights, the review focuses on classifications of models rather than examining each model individually. While this is not always the ideal approach, it allows for a clearer overview of the main characteristics across model types. However, since only a few IO-SFC models have been developed so far, we will briefly review those existing IO-SFC models, while maintaining a broader focus on the general landscape of already developed models, particularly in terms of their sectoral level, financial detail, and methodological structure.

The focus is placed on HC-SFC models, which are distinguished by their detailed institutional structure and their ability to capture complex transmission mechanisms within the economy. It is within this category that IO-SFC models would ideally be placed, given their potential to incorporate inter-industry linkages into the SFC framework. HC-models aim to reflect the real-world interactions between sectors more accurately, particularly in terms of financial flows and balance sheets.

From a broader perspective the existing SFC models tend to fit the real side of the economy relatively well, but often fail to adequately capture financial dynamics. This shortcoming can be attributed to several factors: (i) the use of heterogeneous data sources that are not fully consistent with national accounts, (ii) model closures and residual assumptions typically associated with the behavior of the financial sector, and (iii) simplified behavioral frameworks, such as Tobinsque rules, among others.

Type	Number of models	Median year of publication	Average financial assets	Average capital assets	Average sectors
NC	7	1999	4	1	3
GL	8	2019	6	2	5
HC	13	2022	11	4	6

Table 1: Empirical SFC models by type and main characteristics (Pierros, 2024).
Note: NC, New Cambridge; GL, Godley and Lavoie; HC, High Complexity.

Table 1 above presents key characteristics of the existing empirical SFC models as reported in Pierros (2024). It indicates that the vast majority of existing models belong to the HC-type and have been the focus of considerable research activity in recent years, which also underscores the relevance of the foundations of the E-IO-SFC model that we are currently developing the foundations of. Existing SFC-models typically incorporate a large amount of financial assets but the ones developed so far only include the common six institutional sectors, suggesting

[11] The NC type is characterized by the aggregation of households, firms, and banks into a single private sector entity. The GL type, named after the collaborative work of Godley and Lavoie, treats institutional sectors separately.

that they generally operate at a commonly aggregated sectoral level[12].

Authors	Country	Fit (Real)	Fit (Financial)	Notable closure
Cripps/Izurieta (2014)	Global	N/A	N/A	N/A
Burgess et al. (2016)	UK	N/A	Poor	Household pension wealth
Miess/Schmelzer (2016)	Austria	N/A	N/A	N/A
Jackson/Victor (2020)	Canada	N/A	N/A	Loan provision / Equity issuance
Valdecantos (2021)	Argentina	N/A	N/A	Financial sector bond holdings
Mazier/Reyes (2022)	France	Adequate	Adequate	Accommodative banking sector
Meijers/Muysken (2022)	Netherlands	Good	Poor	Central bank reserves
Zezza/Zezza (2022)	Italy	Good	Poor	'Other financial assets'
Godin et al. (2023)	Colombia	N/A	N/A	Households' equity holdings
Hermitte et al. (2023)	Italy	Good	Very poor	Households' equity holdings
Espagne et al. (2023)	Vietnam	N/A	N/A	Financial sector bond holdings
Yilmaz et al. (2023)	Tunisia	N/A	N/A	Financial sector bond holdings
Byrialsen et al. (2024)	Denmark	N/A	N/A	Accommodative banking sector

Table 2: Overview of empirical HC models (Pierros, 2024).

The structures of the HC-models presented in table 2 are highly heterogeneous and strongly dependent on the specific research questions addressed. Increased model complexity is typically accompanied by significant data requirements, as key variables are often either unobserved or not publicly available. The UK-model developed by Burgess et al. (2016) explicitly models the housing market and features a substantial role for insurance companies and pension funds within its financial balances. Similarly, the model by Godin et al. (2023) for Colombia, as well as the models for Italy by Hermitte et al. (2023), incorporate a broader set of financial instruments. The model by Miess and Schmelzer (2016) offers a comprehensive dataset sourced from Eurostat and presents a highly detailed financial sector.

Models by Zezza and Zezza (2022) for the Italian economy and Byrialsen et al. (2024) for Denmark include a large number of financial and physical asset categories. Meanwhile, the works of Mazier and Reyes (2022) and Meijers and Muysken (2022) delve deeply into the processes of financialization, incorporating an extended set of financial assets and a more complex representation of the financial sector. According to Pierros (2024), recent efforts have been made to enhance the representation of the productive structure in empirical SFC models, such as the case of Argentina presented by Valdecantos (2021). However, this remains a challenging task due to data limitations, particularly concerning the NFC sector. Pierros emphasizes that while input-output tables allow for a high level of sectoral disaggregation, they generally lack corresponding financial data. Pierros (2024) concludes that significant room for improvement remains in the current generation of SFC models, especially concerning the financial and NFC sectors.

A Benchmark Ecological IO-SFC Model for Denmark

This working paper, presented by Thomsen et al. (2025), presents an empirical annual ecological SFC input–output model for the Danish economy. The model integrates ecological and financial dimensions within a unified macroeconomic framework by combining post-Keynesian SFC modeling with national IO accounting. Its primary purpose is to offer a consistent analytical tool for evaluating green investment strategies and climate-related policy interventions, while simultaneously capturing distributional and financial stability effects.

[12] The sectors commonly included are Households, Non-Financial Corporations (NFC), Financial Corporations (FC), Government, Central Bank (CB), and Rest of the World (RoW).

The model includes five institutional sectors: households, NFCs, FCs, government, and the RoW. The NFC sector is disaggregated into nine industries using IO data from DST, allowing for detailed tracking of production and emissions flows. In contrast, the financial dimension is treated at the aggregated sector level. Technical coefficients from the IO table are used to distribute production, intermediate inputs, and ecological burdens across industries in a coherent and empirically grounded manner. While still in working paper form, the model represents a significant step toward integrated macro-ecological modeling, with potential applications in policy evaluation, sustainability assessment, and long-term scenario analysis. Several key behavioral equations are empirically estimated, including household consumption, investment, the share of equity in the financial wealth of households, and wage formation. These estimations ensure that the model exhibits realistic macroeconomic dynamics and can respond meaningfully to policy shocks and structural changes.

The model envisaged with the foundations laid out in this project will share many similarities with the work of Thomsen et al. (2025). The main contributions we make are, first, having balance sheets for each industry instead of aggregated financial accounts for the NFC sector and, second, more reliable econometric estimations based on the higher number of observations that the quarterly frequency allows for.

A Prototype IO-SFC Model for a Small Open Peripheral Economy

The model presented by, Valdecantos (2024b), is an empirical IO-SFC model for the Colombian economy[13]. The model includes multiple sectors in the Colombian economy, households, non-financial corporations, banks, government, Central Bank, and the Rest of the World. The IO technical aspect lies in the division of the NFC sector, which is divided into three industries: primary sector, manufacturing, and services. The model for the Colombian economy has been a guideline to the model represented in this paper in relation to building the system of equations for an E-IO-SFC model for Denmark. The model has drawn inspiration from the construction of the equation system, especially regarding the intermediate transactions between sectors. Among this, the inspiration to incorporate technical coefficients, which determine the amount of intermediate input required from each sector per unit of output in the receiving (buying) sector. Since the model for the Colombian economy is an ongoing project, no conclusions can be drawn yet.

Circular economy innovations in a 2-area input-output stock-flow consistent dynamic model

The first draft of the paper by, Passarella (2023a), is twofold[14]. First it develops a theoretical ecological 2-area, input-output SFC dynamic model. Second the model is used to test impacts of ‘circular economy’ (CE) innovations on the economy, society and ecosystem. The ‘so-called’ economic area is made up of five macroeconomic sectors, households (which are split into wage-earners and rentiers), private production firms, government, banks, and the Central Bank. Instead of the common ‘home’ country and Rest of the World, this model includes two areas where both trade and financial flows takes place across.

The CE denotes a set of policies which targets reusing, repairing, sharing, and recycling products and resources to create a closed-loop system, thus minimizing waste, pollution, and (CO_2) emissions. The paper introduces CE in the model by simple method. This is done by considering a 5-industry economy, in which the first four industries produce goods and services and manage waste. The fifth industry deals with waste recycling. The idea with the CE innovation is a reduction in the quantities of products and services used as inputs in the same industries. This occurs because recycled waste now enters their production processes. The CE innovation can thereby be seen as a technique to incorporate some degree of input sustainability.

[13]This review builds upon the same as presented in Nortvig et al. (2024).
[14]This review builds upon the same as presented in Nortvig et al. (2024).

4 Defining the structure and scope of the E-IO-SFC model

According to Valdecantos (2023b), the second step in building an empirical SFC model concerns defining the scope and structure of the model, which necessitates the development of a Balance Sheet (BS) and a Transaction Flow Matrix (TFM). Therefore, the central matrices defining the skeleton of the E-IO-SFC model are presented in this section, namely the BS and the TFM. In their description, emphasis will be placed on the model's assumptions that deviate from the national accounts approach, cf. United Nations (2009a).

4.1 SFC models: A general perspective

As in most macroeconomic models, when dealing with SFC modeling, the equations pertaining to the dynamic system can be categorized into two types. The first are behavioral equations, which typically refer to behavioral relationships between economic agents and relationships that can be estimated using econometric methods. These are the equations that drive the model's dynamics and, as a result of it, explain why different models representing the same economy for the same period could generate different outcomes. The second type of equations are the accounting identities, which ensure internal consistency within the system of national accounts by stating relationships that must always hold by definition. In the context of SFC models, this internal consistency is further disaggregated into horizontal and vertical consistency, which will be discussed later in this section[15].

Following the international conventions, all sources of funds in a sectoral account take a plus sign, while the uses of these funds take a minus sign. Any transaction involving an incoming flow, the proceeds of a sale, or the receipts of some monetary flow, thus takes a positive sign. Conversely, a transaction involving an outgoing flow must take a negative sign. A use of funds can be a purchase of consumption goods, for instance. The sign convention used in the Flow of Funds (FoF) section of the TFM, which precedes the 'Net Worth' row, cf. table 4, can be somewhat counterintuitive, since the acquisition of a financial asset, that would add to the existing stock of asset, say deposits, by households, is recorded with a positive sign e.g., the households accumulate more deposits). Conversely, a negative sign is used when the transaction represents a source of funds (e.g., the households acquire more loans)[16](Godley and Lavoie, 2012, p. 40).

The consistency relationships in the TFM of SFC models are also closely related with the national accounting terminology related to income and expenditure. This is based on the so-called product balance method, which reconciles income and expenditure at a detailed level. A product balance must, by definition, find balance between supply and use (demand), which can be linked to row and column consistency as in the TFM (Hjulsager, 2022, p. 12). Furthermore, it closely aligns with the concepts of 'source' and 'use' of funds.

The proposed model is a representation of a complete inter-dependent dynamic system, which fully satisfies the national accounts for the Danish economy. At first, the focus is on the essential principles of SFC modeling, which are described by Zezza and Zezza (2019). When using an SFC model to represent a national economy, it is crucial to ensure complete accounting consistency. The following five principles must be met:

1. Horizontal consistency
2. Vertical consistency
3. Flows-to-stocks consistency
4. Balance-sheet consistency
5. Stock-to-flows feedback.

The first principle means that every transaction noted in the accounts should be recorded as an outflow for one sector and as an inflow for the corresponding counter-sector. By doing so it can be registered who pays whom. If

[15]This section builds upon the same ideas presented in Nortvig et al. (2024).

[16]The sign convention in the FoF section follows a financing perspective. For example, if a household purchases bonds worth 100 DKK, it uses 100 DKK to acquire a financial asset, this would be recorded as +100, representing a use of funds. If the household finances this by taking out a loan, it receives 100 DKK, this is recorded as −100, representing a source of funds. In this framework, an increase in assets is positive (a use), while an increase in liabilities is negative (a source).

this were not the case, the involved flow of funds would either come from or go nowhere. Like Godley and Lavoie (2012) put it: “Everything comes from somewhere and everything goes somewhere”. To ensure the horizontal consistency the sum of the elements in the row must add up to zero.

Regarding the second principle, it states that each current payment/transaction should be recorded in the current account for the individual agents presented in the model (this could for instance be households consumption), while at the same time it should also occur at least once more in their respective financial accounts (this could be the funds used for the households consumption). In sum, this would fulfill the vertical consistency. To clarify, the first two principles should apply to the sectors representing households, each of the different industries in the NFC sector, Financial Corporations (FC), the government (G), the Central Bank (CB), and the Rest of the World (RoW) in the present model.

The third principle refers to the fact that any stock of assets at current prices, either real or financial, at the end of an accounting period, is given by the relevant flows during that period and net capital gains arising from the fluctuations in asset prices. This could for instance be the change in a specific equity where the capital gain reflects a source of funds for the NFC sector and a use of funds for the household sector. In the TFM, if a financial instrument is considered a net asset for a given sector or industry, its transactions will be registered with a positive sign in the flow of funds section, as its represents a use of funds. Conversely, a liability is recorded with a negative sign because it represents a source of funds. The third principle will thereby guarantee the flow-to-stock consistency.

The fourth principle states that each financial instrument must appear as an asset for at least one agent while also appearing as a liability for at least one other agent. To clarify the balance sheet consistency, consider the following example. When households take out a loan, it will appear as a liability for them, while for the financial corporations, it will appear as an asset at the same time.

The last and fifth principle addresses the accumulation of financial assets and liabilities. It turns out that these have dynamic implications for the current accounts, as an interest-bearing asset for one agent will generate a future inflow of income payments, while for the counterpart it will generate a future outflow of payments. This illustrates the interaction between credit and debt.

The five essential principles of SFC modeling have now been presented, but notably, it is also worth mentioning that taking the third and fifth principles together describes an important characteristic of SFC models (Valdecantos, 2023a). These two principles ensure a path-dependent system, where choices and decisions have a crucial impact on the future development of the system and the economy being analyzed (Jespersen, 2007).

In the remainder of this section we present the two matrices that make up the cornerstone of every SFC model (and the one we are building).

4.2 Balance Sheet

As mentioned in section 2, and in line with the second identified gap, the model distinguishes six sectors, with the disaggregated NFC sector at industry level. These sectors and industries are incorporated into the balance sheet framework presented here[17]. Typically, SFC models comprise various components, including accounting matrices and dynamic equations. This section presents the first accounting matrix, which is aligned with the national accounts as reported by DST. Every sector and industry are listed in the columns, and in each of the eight rows, every financial instrument is represented as a financial asset or liability for the specific sector and industry in this economy. This representation helps to understand the financial structure of the E-IO-SFC model.

The general understanding of this BS is that when an entry appears with a plus sign, it indicates that the given sector or industry holds the corresponding instrument as an asset, as it adds to its net worth. Net worth, in this context, refers to the difference between assets and liabilities, representing the sector's or industry's net wealth. For example, if households have more assets than liabilities, they will have a positive net worth or wealth. Conversely, when an entry appears with a minus sign, it indicates that the given sector or industry holds liabilities in that element, signifying that the stock of the element is a liability for the specific sector or industry. These principles imply that wealth is represented with a minus sign, as it is located on the liability side, following the standard accounting identity stating that Assets = Liabilities + Net Worth, cf. Godley and Lavoie (2012). These conventions ensure that all the rows and columns of the BS sum to zero, thus providing consistency and coherence in the accounting of the financial stock variables contained in the E-IO-SFC model, in line with the principles of SFC modeling presented in section 4.1 above.

	Households	Energy	Transport	Agriculture	Manufacturing and Construction	Other manufacturing and services
Monetary gold and SDR (F.1)						
Currency and deposits (F.2)	$F.2^H$	$F.2^E$	$F.2^T$	$F.2^A$	$F.2^{MC}$	$F.2^{OMS}$
Securities (F.3)	$F.3^H$	$-F.3^E$	$-F.3^T$	$-F.3^A$	$-F.3^{MC}$	$-F.3^{OMS}$
Loans (F.4)	$-F.4^H$	$-F.4^E$	$-F.4^T$	$-F.4^A$	$-F.4^{MC}$	$-F.4^{OMS}$
Equity (F.5)	$F.5^H$	$-F.5^E$	$-F.5^T$	$-F.5^A$	$-F.5^{MC}$	$-F.5^{OMS}$
Insurance tech. reserves (F.6)	$F.6^H$	$-F.6^E$	$-F.6^T$	$-F.6^A$	$-F.6^{MC}$	$-F.6^{OMS}$
Financial derivatives (F.7)	$-F.7^H$	$-F.7^E$	$-F.7^T$	$-F.7^A$	$-F.7^{MC}$	$-F.7^{OMS}$
Trade credits (F.8)	$-F.8^H$	$F.8^E$	$F.8^T$	$F.8^A$	$F.8^{MC}$	$F.8^{OMS}$
Net Worth	NW^H	NW^E	NW^T	NW^A	NW^{MC}	NW^{OMS}

	Financial corporations	Government	Central Bank	Rest of the World	$\sum$
Monetary gold and SDR (F.1)			$F.1^{CB}$	$F.1^{RoW}$	+
Currency and deposits (F.2)	$-F.2^{FC}$	$F.2^G$	$F.2^{CB}$	$F.2^{RoW}$	0
Securities (F.3)	$F.3^{FC}$	$-F.3^G$	$-F.3^{CB}$	$F.3^{RoW}$	0
Loans (F.4)	$F.4^{FC}$	$F.4^G$	$F.4^{CB}$	$-F.4^{RoW}$	0
Equity (F.5)	$F.5^{FC}$	$F.5^G$	$-F.5^{CB}$	$-F.5^{RoW}$	0
Insurance tech. reserves (F.6)	$-F.6^{FC}$	$F.6^G$		$F.6^{RoW}$	0
Financial derivatives (F.7)	$F.7^{FC}$	$F.7^G$	$F.7^{CB}$	$-F.7^{RoW}$	0
Trade credits (F.8)	$-F.8^{FC}$	$-F.8^G$	$-F.8^{CB}$	$-F.8^{RoW}$	0
Net Worth	NW^{FC}	NW^G	NW^{CB}	NW^{RoW}	0

Table 3: Balance sheet for the E-IO-SFC model.

[17]This section builds upon the ideas presented in Nortvig et al. (2024).

The BS is constructed in alignment with the structure of the national accounts and based on the financial variables reported by DST. However, since DST does not provide complete balance sheet data at the industry level, a method has been developed to approximate these positions. This approach constitutes a key contribution of the E-IO-SFC model, enabling the construction of sector- and industry-specific balance sheets. For each period/quarter, the stock of assets is subtracted from the stock of liabilities for the specific element, i.e., the financial instruments are treated in net terms[18]. For specific content of each variable included in the BS, the reader is referred to section 5.2. In the first row of the BS, monetary gold and SDRs are presented as assets for the Central Bank in the domestic economy and for the RoW. It is essential that all the rows and columns add up to zero for row consistency. However, this specific column should be understood as tangible capital, contrasting with other financial assets and liabilities, which represent claims between parties. In the case of monetary gold and SDRs, they do not specifically have a counterpart. For the other rows in the BS, one row is described because the same procedure applies to the rest. In row 4 of the BS, cf. table 3, loans F.4 are represented. This indicates that households, NFCs, and the RoW hold loans as liabilities, their counterparts being financial corporations and the government, who are the providers/suppliers of credit. The signs presented in the BS align with the net position of each agent with respect to each financial instrument in the real world. For instance, households hold loans as liabilities to finance their purchases (say mortgages to purchase their homes), the outstanding balance of the loan being listed as a liability[19]. For detailed values from 2019Q4, please refer to the appendix, specifically table 24.

[18]DST reports, for each sector, the holdings of various financial instruments as both assets and liabilities. While incorporating this level of detail could enhance the model, a simplified approach is chosen for pragmatic reasons.

[19]According to McLeay et al. (2014), commercial banks create money in the form of bank deposits by issuing new loans. When a bank issues a loan, it credits the borrower's bank account with a deposit equal to the loan amount. This deposit increases the household's assets while the loan itself increases their liabilities. Thus, new money is created. It is important to note that bank deposits are merely records of how much the bank owes its customers, meaning deposits are a liability for the bank rather than an asset that can be lent out. As result, while loans appear as a liability (F.4), they are functioning as deposits (F.2) which can be used to buy final consumption by the households. Similarly, for commercial banks, loans represent an asset, but in practice, they are functioning as a liability through currency and deposits (F.2) which is transferred to any of the sectors.

4.3 Transaction Flow Matrix

The structure of the TFM for the E-IO-SFC model is represented with the institutional agents in the columns and transactions in the rows. The model consists of six different sectors as introduced in section 4.2: Households (incl. NPISH), NFC's (disaggregated into five different industries), FC's, G, CB, and RoW. One of the key contributions of this paper is the disaggregation of the NFC sector, which is clearly visible in both the TFM, cf. table 4, BS, cf. table 3 and described in section 5.3.2. For each institutional agents a 'current' and 'accumulation' column is represented. These columns record the current and accumulation transactions, respectively, the latter including the capital and financial accounts. To compare these two columns with the structure of national accounts, the 'current'-column consists of the following: The production account, the generation of income account and the allocation of primary income account. On the other side, the 'accumulation'-column comprises the capital and the financial accounts. Having presented the structure of the columns of the TFM, this section now offers a detailed description of the individual entries, along with the assumptions made to tailor the framework to the specific requirements of the E-IO-SFC model. The description follows a row-by-row structure, beginning with the definition of aggregate demand. Then, it proceeds with the generation of income account, followed by the allocation of primary and secondary income. Subsequently, the use of income account, the capital account, and the financial account are described. This construction and approach follow the methodology outlined in Valdecantos (2023a). In table 4 below, the representation of the TFM for the E-IO-SFC model is shown with variable names. For detailed values for 2019Q4, please refer to the **attached document** called "TFM2019Q4". To simplify the structure of the E-IO-SFC model, several assumptions are introduced, for instance, treating only NFC's and FC's as producers, or redistribute certain institutional flows. While these assumptions are necessary for analytical tractability and data feasibility, it is important to emphasize that they do not result in accounting inconsistencies. This is because any production or financial flows initially excluded from a sector or industry due to such simplifications are subsequently reallocated or "returned" through carefully designed adjustments in the E-IO-SFC model. In this way, sectoral balances, and the integrity of the full accounting framework, are preserved.

	Households		Energy		Transport		Agriculture	
	Current	Accumulation	Current	Accumulation	Current	Accumulation	Current	Accumulation
Private Consumption (P.3)	$-C^H$		C^E		C^T		C^A	
Government Consumption (P.3)			G^E		G^T		G^A	
Investment (P.51g P.53)		$-I_a^H$	I_c^E	$-I_a^E$	I_c^T	$-I_a^T$	I_c^A	$-I_a^A$
Changes in Inventories (P.52)			ΔINV^E	$-\Delta INV^E$	ΔINV^T	$-\Delta INV^T$	ΔINV^A	$-\Delta INV^A$
Exports (P.6)			X^E		X^T		X^A	
Imports (P.7)			$-IM^E$		$-IM^T$		$-IM^A$	
Intermediate purchases			$-IG_p^E$		$-IG_p^T$		$-IG_p^A$	
Intermediate sales			IG_s^E		IG_s^T		IG_s^A	
Gross Domestic Product [Y] (B.1*g)			$[Y^E]$		$[Y^T]$		$[Y^A]$	
Net indirect taxes less subsidies on production (D.21-D.31 and D.29-D.39)			$-NIT^E$		$-NIT^T$		$-NIT^A$	
[Gross value added at factor cost] (B1.GF)			$[GVA_F^E]$		$[GVA_F^T]$		$[GVA_F^A]$	
Wages (D.1)	WB^H		$-WB^E$		$-WB^T$		$-WB^A$	
Gross operating surplus and mixed income redistributed	GOS^H		$-GOS_{H,G}^E$		$-GOS_{H,G}^T$		$-GOS_{H,G}^A$	
Net taxes on imports (D.2-D.3)								
Net Interest (D.41 + D.44 + D.45)	NI^H		NI^E		NI^T		NI^A	
Distributed income from corporations (D.42)	DI^H		DI^E		DI^T		DI^A	
Reinvested earnings on FDI (D.43)			F_{FDI}^E		F_{FDI}^T		F_{FDI}^A	
Current taxes on income and wealth (D.5)	$-T^H$		$-T^E$		$-T^T$		$-T^A$	
Social contributions (D.61)	$-SC^H$							
Social benefits (D.62)	SB^H							
Other current transfers (D.7)	OC^H		OC^E		OC^T		OC^A	
Adj. change in pension entitl. (D.8)	PE^H							
[Saving (B.8g) / Current external balance (B.12)]	$[S^H]$		$[S^E]$		$[S^T]$		$[S^A]$	
Capital and non-financial and non-produced assets transfers (D.9 + NP)		$-NPL^H$		NPL^E		NPL^T		NPL^A
Adjustment variable		Adj^H		Adj^E		Adj^T		Adj^A
[Net lending (B.9)]		$[NL^H]$		$[NL^E]$		$[NL^T]$		$[NL^A]$
Monetary gold and SDR (F.1) - transactions								
Currency and deposits (F.2) - transactions		$F2^H$		$F2^E$		$-F2^T$		$F2^A$
Securities (F.3) - transactions		$-F3^H$		$-F3^E$		$F3^T$		$-F3^A$
Loans (F.4) - transactions		$-F4^H$		$-F4^E$		$-F4^T$		$-F4^A$
Equity (F.5) - transactions		$-F5^H$		$F5^E$		$F5^T$		$F5^A$
Insurance tech. reserves (F.6) - transactions		$F6^H$		$-F6^E$		$-F6^T$		$-F6^A$
Financial derivatives (F.7) - transactions		$F7^H$		$F7^E$		$F7^T$		$F7^A$
Trade credits (F.8) - transactions		$-F8^H$		$F8^E$		$F8^T$		$F8^A$
Monetary gold and SDR (F.1) - roc								
Currency and deposits (F.2) - roc		$-F2_{roc}^H$		$-F2_{roc}^E$		$-F2_{roc}^T$		$-F2_{roc}^A$
Securities (F.3) - roc		$-F3_{roc}^H$		$F3_{roc}^E$		$F3_{roc}^T$		$F3_{roc}^A$
Loans (F.4) - roc		$F4_{roc}^H$		$-F4_{roc}^E$		$-F4_{roc}^T$		$-F4_{roc}^A$
Equity (F.5) - roc		$F5_{roc}^H$		$-F5_{roc}^E$		$-F5_{roc}^T$		$-F5_{roc}^A$
Insurance tech. reserves (F.6) - roc		$-F6_{roc}^H$		$-F6_{roc}^E$		$-F6_{roc}^T$		$-F6_{roc}^A$
Financial derivatives (F.7) - roc		$F7_{roc}^H$		$F7_{roc}^E$		$F7_{roc}^T$		$F7_{roc}^A$
Trade credits (F.8) - roc		$F8_{roc}^H$		$-F8_{roc}^E$		$-F8_{roc}^T$		$-F8_{roc}^A$
Change in Net Worth		NW^H		NW^E		NW^T		NW^A

	Manufacturing & Construction		Other Manufacturing & Services		Financial Corporations	
	Current	Accumulation	Current	Accumulation	Current	Accumulation
Private Consumption (P.3)	C^{MC}		C^{OMS}		C^{FC}	
Government Consumption (P.3)	G^{MC}		G^{OMS}		G^{FC}	
Investment (P.51g P.53)	I_c^{MC}	$-I_a^{MC}$	I_c^{OMS}	$-I_a^{OMS}$		$-I_a^{FC}$
Changes in Inventories (P.52)	ΔINV^{MC}	$-\Delta INV^{MC}$	ΔINV^{OMS}	$-\Delta INV^{OMS}$		
Exports (P.6)	X^{MC}		X^{OMS}		X^{FC}	
Imports (P.7)	$-IM^{MC}$		$-IM^{OMS}$		$-IM^{FC}$	
Intermediate purchases	$-IG_p^{MC}$		$-IG_p^{OMS}$		$-IG_p^{FC}$	
Intermediate sales	IG_s^{MC}		IG_s^{OMS}		IG_s^{FC}	
Gross Domestic Product [Y] (B.1*g)	$[Y^{MC}]$		$[Y^{OMS}]$		$[Y^{FC}]$	
Net indirect taxes less subsidies on production (D.21-D.31 and D.29-D.39)	$-NIT^{MC}$		$-NIT^{OMS}$		$-NIT^{FC}$	
[Gross value added at factor cost] (B1.GF)	$[GVA_F^{MC}]$		$[GVA_F^{OMS}]$		$[GVA_F^{FC}]$	
Wages (D.1)	$-WB^{MC}$		$-WB^{OMS}$		$-WB^{FC}$	
Gross operating surplus and mixed income redistribution	$-GOS_{H,G}^{MC}$		$-GOS_{H,G}^{OMS}$		$-GOS_{H,G}^{FC}$	
Net taxes on imports (D.2-D.3)						
Net Interest (D.41 + D.44 + D.45)	NI^{MC}		NI^{OMS}		NI^{FC}	
Distributed income from corporations (D.42)	DI^{MC}		DI^{OMS}		DI^{FC}	
Reinvested earnings on FDI (D.43)	F_{FDI}^{MC}		F_{FDI}^{OMS}		F_{FDI}^{FC}	
Current taxes on income and wealth (D.5)	$-T^{MC}$		$-T^{OMS}$		$-T^{FC}$	
Social contributions (D.61)					SC^{FC}	
Social benefits (D.62)					$-SB^{FC}$	
Other current transfers (D.7)	OC^{MC}		OC^{OMS}		OC^{FC}	
Adj. change in pension entitl. (D.8)					$-PE^{FC}$	
[Saving (B.8g) / Current external balance (B.12)]	$[S^{MC}]$		$[S^{OMS}]$		$[S^{FC}]$	
Capital and non-financial and non-produced assets transfers (D.9 + NP)		NPL^{MC}		NPL^{OMS}		NPL^{FC}
Adjustment variable		Adj^{MC}		Adj^{OMS}		Adj^{FC}
[Net lending (B.9)]		$[NL^{MC}]$		$[NL^{OMS}]$		$[NL^{FC}]$
Monetary gold and SDR (F.1) - transactions						
Currency and deposits (F.2) - transactions		$F2^{MC}$		$F2^{OMS}$		$-F2^{FC}$
Securities (F.3) - transactions		$-F3^{MC}$		$-F3^{OMS}$		$F3^{FC}$
Loans (F.4) - transactions		$-F4^{MC}$		$-F4^{OMS}$		$F4^{FC}$
Equity (F.5) - transactions		$F5^{MC}$		$F5^{OMS}$		$F5^{FC}$
Insurance tech. reserves (F.6) - transactions		$-F6^{MC}$		$-F6^{OMS}$		$-F6^{FC}$
Financial derivatives (F.7) - transactions		$F7^{MC}$		$F7^{OMS}$		$-F7^{FC}$
Trade credits (F.8) - transactions		$-F8^{MC}$		$F8^{OMS}$		$-F8^{FC}$
Monetary gold and SDR (F.1) - roc						
Currency and deposits (F.2) - roc		$-F2_{roc}^{MC}$		$-F2_{roc}^{OMS}$		$-F2_{roc}^{FC}$
Securities (F.3) - roc		$F3_{roc}^{MC}$		$F3_{roc}^{OMS}$		$-F3_{roc}^{FC}$
Loans (F.4) - roc		$-F4_{roc}^{MC}$		$-F4_{roc}^{OMS}$		$-F4_{roc}^{FC}$
Equity (F.5) - roc		$-F5_{roc}^{MC}$		$-F5_{roc}^{OMS}$		$F5_{roc}^{FC}$
Insurance tech. reserves (F.6) - roc		$-F6_{roc}^{MC}$		$-F6_{roc}^{OMS}$		$F6_{roc}^{FC}$
Financial derivatives (F.7) - roc		$F7_{roc}^{MC}$		$F7_{roc}^{OMS}$		$-F7_{roc}^{FC}$
Trade credits (F.8) - roc		$-F8_{roc}^{MC}$		$-F8_{roc}^{OMS}$		$F8_{roc}^{FC}$
Change in Net Worth		NW^{MC}		NW^{OMS}		NW^{FC}

	Government		Central Bank		Rest of the world		Σ
	Current	Accumulation	Current	Accumulation	Current	Accumulation	
Private Consumption (P.3)							0
Government Consumption (P.3)	-G						0
Investment (P.51g P.53)		$-I_a^G$					0
Changes in Inventories (P.52)							0
Exports (P.6)					$-X$		0
Imports (P.7)					IM		0
Intermediate purchases							-
Intermediate sales							+
Gross Domestic Product [Y] (B.1*g)							[+]
Net indirect taxes less subsidies on production (D.21-D.31 and D.29-D.39)	NIT^G						0
[Gross value added at factor cost] (B1.GF)							[+]
Wages (D.1)			$-WB^{CB}$		WB^{RoW}		0
Gross operating surplus and mixed income redistribution	GOS^G						0
Net taxes on imports (D.2-D.3)	NTI^G				$-NTI^{RoW}$		0
Net Interest (D.41 + D.44 + D.45)	NI^G		NI^{CB}		NI^{RoW}		0
Distributed income from corporations (D.42)	DI^G		DI^{CB}		DI^{RoW}		0
Reinvested earnings on FDI (D.43)					$-FDI^{RoW}$		0
Current taxes on income and wealth (D.5)	T^G				$-T^{RoW}$		0
Social contributions (D.61)	SC^G				SC^{RoW}		0
Social benefits (D.62)	$-SB^G$				SB^{RoW}		0
Other current transfers (D.7)	OC^G		OC^{CB}		OC^{RoW}		0
Adj. change in pension entitl. (D.8)							0
[Saving (B.8g) / Current external balance (B.12)]	$[S^G]$		$[-S^{CB}]$		$[-S^{RoW}]$		
Capital transfers and net acquisition of non-financial non-produced assets (D.9 + NP)		NPL^G				$-NPL^{RoW}$	0
Adjustment variable		Adj^G		Adj^{CB}		Adj^{RoW}	Adj^{Σ}
[Net lending (B.9)]		$[NL^G]$		$[NL^{CB}]$		$[NL^{RoW}]$	[0]
Monetary gold and SDR (F.1) - transactions				$F1^{CB}$		$-F1^{RoW}$	0
Currency and deposits (F.2) - transactions		$-F2^G$		$-F2^{CB}$		$-F2^{RoW}$	0
Securities (F.3) - transactions		$F3^G$		$-F3^{CB}$		$-F3^{RoW}$	0
Loans (F.4) - transactions		$-F4^G$		$F4^{CB}$		$-F4^{RoW}$	0
Equity (F.5) - transactions		$F5^G$		$F5^{CB}$		$-F5^{RoW}$	0
Insurance tech. reserves (F.6) - transactions		$F6^G$				$F6^{RoW}$	0
Financial derivatives (F.7) - transactions		$F7^G$		$F7^{CB}$		$F7^{RoW}$	0
Trade credits (F.8) - transactions		$F8^G$		$-F8^{CB}$		$-F8^{RoW}$	0
Monetary gold and SDR (F.1) - roc				$-F1_{roc}^{CB}$		$-F1_{roc}^{RoW}$	+
Currency and deposits (F.2) - roc		$-F2_{roc}^G$		$-F2_{roc}^{CB}$		$F2_{roc}^{RoW}$	0
Securities (F.3) - roc		$F3_{roc}^G$		$-F3_{roc}^{CB}$		$F3_{roc}^{RoW}$	0
Loans (F.4) - roc		$F4_{roc}^G$		$F4_{roc}^{CB}$		$F4_{roc}^{RoW}$	0
Equity (F.5) - roc		$F5_{roc}^G$		$F5_{roc}^{CB}$		$F5_{roc}^{RoW}$	0
Insurance tech. reserves (F.6) - roc		$F6_{roc}^G$				$-F6_{roc}^{RoW}$	0
Financial derivatives (F.7) - roc		$-F7_{roc}^G$		$F7_{roc}^{CB}$		$-F7_{roc}^{RoW}$	0
Trade credits (F.8) - roc		$F8_{roc}^G$		$F8_{roc}^{CB}$		$-F8_{roc}^{RoW}$	0
Change in Net Worth		NW^G		NW^{CB}		NW^{RoW}	0

Table 4: Structure of the Transaction Flow Matrix represented symbolically.

The aggregate demand components of the E-IO-SFC model

The first two rows of the TFM represent final consumption by households, C^H, and by the government, G. Final consumption refers to expenditure on goods and services used for the direct satisfaction of individual needs or wants. Only households and the government incur final consumption expenditures, which are recorded as outflows in their respective entries. Correspondingly, these appear as inflows for the industries and the FC's, as they represent income generated through the production of final consumption goods[20]. A production-related assumption has been made in the context of the E-IO-SFC model, namely that only NFC's and FC's are engaged in production activities. Accordingly, references to the NFC sector should henceforth be understood to include FC's, unless otherwise stated. This treatment of FC's as part of the producing industries is based solely on this modeling assumption[21].

The third row in the TFM is investment and consists of gross fixed capital formation and acquisition less disposals of valuables, represented by I_a^i and I_c^i for accumulated and current investment respectively[22]. Here a minus sign is seen for the industries in their accumulation columns. All the accumulation is allocated to the industries' current accounts, indicating that the firms are the entities selling and producing these investments. On the other hand, it is shown as a negative sign for households, FC's and the government because it represents an outflow, which is recorded in their accumulation account. It is relevant to note that the possibility for FC's being able to have an inflow of investment is assumed away even though it is reported in the national accounts. It is decided to remove the sign of the FC's in their current column, because of the minor share of total investment (3,8%) OECD (2021). This assumption simplifies the E-IO-SFC model structure and the definition of the system of equations. The fundamental assumption made here is that the industries are the sole entities accounting for investments as inflows in their current accounts. However, examining the national accounts of any country reveals that all agents contribute to production and, consequently, to value added. By attributing investment activities exclusively to the industries, it is assumed that households, FC's, and the government are not receiving inflows resulting from investment decisions, which would otherwise be reported in their respective current accounts.

To account for the fact that the producing industries cannot always meet demand instantaneously, changes in inventories, ΔINV^i, are introduced in the fourth row of the TFM, consistent with their representation in the national accounts. In this context, it is acknowledged that households may, in principle, produce and store consumption goods. However, for the sake of simplicity, this possibility is abstracted from, and all household inventories are instead attributed to the producing industries. The sign appearing in the TFM depends on which account it is reported on. The same variable has been taken for each account and divided between current and accumulation since it is already measured as inventory, and therefore already includes goods that have not been sold. Hence, the variable will be as it is in the current account and treated with a minus sign in the accumulation account.

Row five and six represent exports and imports, X^i and IM^i for Denmark[23]. It is assumed that only the NFC sector, are involved in all import and export activities. Imports into Denmark are assumed to occur through this sector. This is the reason why imports are represented with a negative sign in the TFM for the producing sectors and a positive sign for the RoW and the opposite for exports.

Since the E-IO-SFC model is specified at the industry level, it is essential to account for intermediate purchases, IG_p^i, and sales, IG_s^i, between industries to ensure consistency with the national accounts. In this framework, intermediate transactions are reported as both intermediate purchases[24] and sales. These flows

[20] The series used to construct final consumption does not include transfer income from the government.

[21] According to the IO table from DST, FC's account for 0.02% of total production related to government final consumption and 8% for household final consumption.

[22] These are non-financial assets that are not consumed or used in production, do not deteriorate physically under normal conditions, and are primarily acquired and held as stores of values.

[23] Imports includes intermediate imported goods from RoW.

[24] Intermediate purchases include only domestic transactions.

are distributed within the NFC sector to match aggregate values from the input-output tables. Given this modeling choice, appropriate adjustments must be made when defining the system of equations to ensure that intermediate purchases and sales are balanced at the industry level. This issue will be discussed further in section 8. Intermediate flows are allocated across the producing sectors, including FC's. The inclusion of FC's here is motivated by the need to attribute a share of gross operating surplus (GOS) to them, as they also generate income through their role in the production process. This point will be elaborated in a later section. To illustrate how households purchases of products positively impact foreign economies, consider the following example involving a household intending to purchase a vehicle, specifically a Tesla[25]. The household buys the car from the MC industry. To manufacture this vehicle, the MC industry must import goods from abroad, such as steel for the doors or rubber for the tires. This necessitates a transaction from MC companies to the RoW through the 'Import' row, which explains the negative entry for imports in the NFC's and a positive entry for imports in RoW. This occurs due to the project's assumption that all intermediate imports are categorized in the import row. In reality, the household might directly purchase a Tesla from the RoW; however, for simplicity, it is assumed that only final goods can be bought from the NFC sector within the domestic market.

The rows described so far represent the aggregate demand in the Danish economy, which corresponds to gross value added (GVA) in market prices[26]. GVA reflects the total monetary value of final purchases of goods and services from the sectors included in the E-IO-SFC model. In the TFM, this is summarized in a 'fictive' row labeled 'Gross Domestic Product' (GDP). The term 'fictive' refers to the fact that this row does not represent an actual transaction, but rather aggregates all previous transactions listed above in the TFM. The GDP components identified here will serve as the basis for defining the income of each sector and industry in the system of equations. As shown in table 4, the notation for GDP is presented as a current inflow to the NFC sector.

To complete this section of the TFM, it is important to note that the NFC sector collects taxes, which are subsequently transferred to the government, recorded as an outflow from the NFC sector and an inflow to the government. For simplicity, taxes less subsidies on products and other taxes have been combined into a single term, denoted as NIT^i. Although, all value added is assumed to be generated by the NFC sector, and taxes are therefore imputed to them, this does not imply that households do not pay VAT. Like other indirect taxes, VAT is embedded in the market prices paid by consumers and is thus indirectly passed through to the NFC sector.

The generation of income in the E-IO-SFC model

Now that we have outlined the definition of GDP or GVA at factor prices[27] in terms of the supply and demand of the aggregate economy, our focus will shift to the macroeconomic identity known as income. This is found by looking at how income is generated. GDP created in Denmark goes to compensation of employees, resident producer surplus, mixed income, and payments of productions taxes. The next two rows include the wage bill, WB^i, and GOS^i. In the E-IO-SFC model the compensation of employees are paid by the NFC sector and are transferred to the households in the domestic economy and to the Danish workers in RoW. It should be noted: In reality households receive wages from the government. Within this framework this is entirely removed. Consequently, the industry-specific values for the wage bill are taken from the data figures (which will be reported in section 5), where government wages are already incorporated into the industries. This approach ensures that the government wage bill is accurately allocated across industries based on their respective production activities.

[25] We are aware that Tesla isn't produced in Denmark. The example is just for illustrative purposes and to demonstrate how the IO structure works within the E-IO-SFC model.

[26] Market prices refer to the prices at which goods and services are traded, i.e., the prices paid by consumers (Hjulsager, 2022, p. 26).

[27] Factor prices are the prices that pay or compensate the production factors labor and capital. They are calculated as basic prices minus other production taxes, net. Basic prices are the prices that a company ultimately receives for its goods, since indirect taxes accrue to the public sector (Hjulsager, 2022, p. 26-27).

The row that typically represents gross profits is not reported in the conventional way due to the modeling assumption regarding the disaggregation of the producing sector in the E-IO-SFC model. Ordinarily, this account reflects the profits earned by each sector based on their capital contribution to the production process. However, given the assumption that production is carried out exclusively by the NFC sector, it becomes necessary to "return" the portion of surplus generated by households and the government to the producing industries. This adjustment is essential to ensure accurate net lending figures across all sectors and industries. If the effect of this assumption on production is not accounted for prior to the computation of net lending, discrepancies will emerge between the current and financial accounts of the sectors. Therefore, in the TFM, as shown in table 4, the GOS generated by the household and government sectors is redistributed as an outflow (indicated by a minus sign) from the different industries comprised in the NFC sector to the households and government. This redistribution is proportional to households or governments contribution to GVA for each industry. For instance, if households account for 5% of GVA in the agriculture sector, they receive 5% of the agriculture's total GOS. This adjustment ensures that the implications of the production assumption are fully incorporated (Valdecantos, 2023a, p. 6). In this context, it is essential to identify a value that captures the surplus before production costs are deducted. GVA at factor costs serves this purpose. Since the NFC sector is assumed to be the sole producer, introducing GVA enables the NFC sector to repay a larger portion of GOS^i, thereby preserving accounting consistency.

Allocation of primary income in the E-IO-SFC model

The allocation of the primary income account aims to illustrate how different sectors and institutional groups in the economy receive and pay various types of income (Hjulsager, 2022). Primary income refers to the returns derived from the ownership of assets, whether real or financial. The interaction between the Rest of the World (RoW) and the domestic economy appears as the first entry in this account, namely the net taxes on production and imports levied by the government on firms, denoted by NTI^i. Net interest income, NI^i, is introduced in the E-IO-SFC model as a merged variable[28]. In the national accounts, these components are typically reported as interest receivable and payable. However, for the sake of simplicity, they are consolidated here as net interest received i.e., interest income minus interest expenses. This aggregation is motivated by two key considerations. First, the components 'other investment income' and 'rent' capture residual property income not covered by interest-bearing instruments, but in practice, they cannot be allocated to specific financial assets reported in the balance sheets due to data limitations from DST. Second, separating these components would not enhance the model's empirical performance. By constructing a net interest variable, the model captures whether a given sector or industry acts as a net payer or receiver of interest, thereby preserving analytical clarity without sacrificing relevant economic information.

The next items to be addressed are dividends, referred to in the national accounts as 'distributed income of corporations' and 'reinvested earnings on foreign direct investment' (FDI). A portion of the distributed corporate income is paid to RoW, while reinvested earnings reflect profits generated by foreign-owned companies operating in Denmark that are not distributed but instead retained and reinvested. In economic terms, the latter corresponds to cases where foreign firms hold equity stakes in Danish corporations. Both variables are expressed in net terms and denoted as DI^i (net dividends) and F^i_{FDI} (net reinvested earnings on FDI). A positive entry in the TFM implies that Denmark, or the specific industry in question, is a net receiver of the respective income flow i.e., in the case of F^i_{FDI}, this could indicate foreign ownership of Danish firms resulting in retained profits accruing to foreign investors.

Secondary income distribution in the E-IO-SFC model

A current transfer occurs when an institutional unit provides goods, services, or financial resources to another institutional unit without receiving anything of equivalent economic value in return. Such transfers do not have

[28] Comprised of interest-bearing assets, 'other investment income', and 'rent'.

a direct counterpart transaction. When a transfer is made by an institutional unit, it is recorded as an outflow and indicated with a minus sign. Conversely, transfers received are recorded as inflows and marked with a plus sign.

The first variable involved in the above description is the government's income and wealth tax revenue, denoted by T^i in the TFM. This revenue is collected from households and the NFC sector, which are required to pay taxes on income, profits, and wealth. As shown in the TFM, the government also receives tax revenue from RoW. This source should be understood as Danes residing abroad, but subject to some taxes in Denmark, such as property taxes. The government's tax revenue is the primary source of financing the public sector.

Secondly, the financing of the social insurance system, called social contributions and denoted as SC^i, is described. The financing occurs through actual or imputed payments imposed on all participants in the labor market, ensuring that the institutions providing this form of social insurance have sufficient funds. The contribution is paid by employers on behalf of employees, making it part of the employee's wage bill. It appears with a minus sign for households, thereby reducing their disposable income. The interpretation of social contributions in the TFM should be understood as follows: these contributions, along with payments made by households, are recorded with a minus sign as they represent a use of funds, while for FC and the government, they are recorded as a source of funds. Note that payments to the RoW are expected to be significantly smaller compared to domestic payments.

Third comes social benefits, denoted by SB^i, which is an inflow for households and RoW and paid by the government and FC's. This includes cash assistance, unemployment benefits, sickness benefits, education, early retirement pensions, and old-age pensions. Consequently, in the case of households, the interpretation of social benefits will be registered as a source of funds (indicated by a plus sign), and as a use of funds for the mentioned institutions (indicated by a minus sign). The contributions are paid by FC's and the government.

The current transactions, which are not included in the aforementioned, are recorded in the last item called 'other current transfers', and denoted by OC^i.

Use of disposable income in the E-IO-SFC model

The next entries in the TFM relate to how disposable national income is used for consumption and savings. It represents the income that Danish residents have at their disposal. Broadly speaking, it shows how the components of aggregate demand are used by households and the government. One of these components is denoted by PE^i, which refers to adjustments in pension entitlements. Note here the description in the 'secondary income distribution account' regarding households' contributions to the social security system. This implies that the funds households contribute to the social insurance system are only accessible once pensions are paid out. To ensure that actual disposable income is accurately represented, the adjustment in PE^i must be included. This adjustment credits households with a portion of the social contributions from FC before reporting each industry's saving position, S^i.

Capital and financial accounts in the E-IO-SFC model

Capital transfers and acquisitions less disposals of non-produced non-financial assets are reported next in the TFM as a merged item. The reason for combining these is that they both relate to changes in ownership: one involves capital goods, while the other pertains to non-produced non-financial assets. The capital-related item describes changes in ownership that occur when a physical asset is transferred from one sector to another. The second item, which relates to non-produced non-financial assets, registers changes in physical assets or their sale, as well as changes in the ownership of intangible assets such as patents. For households, this has the following interpretation: if households receive a donation of a capital good, it would be reflected in two ways. First, there will be an increase in their gross fixed capital formation, which appears with a minus sign because it is a use of funds. However, that capital accumulation does not stem from an actual expenditure by the household but from a donation. Therefore, it is not truly a use of funds. To prevent the registered increase in gross fixed capital formation from distorting the net lending position, the capital transfer is written with a plus sign, thereby offsetting the values.

The next item recorded in this account is ‘net lending (+) / net borrowing (-)’ which appears in the TFM as a ‘fictive row’. This item is crucial as it serves as a balancing entry, reflecting each sector’s or industry’s final financial position once all transactions have been recorded. It is instrumental in indicating whether a sector is in a surplus or deficit. This implies that a surplus in one sector necessitates a corresponding deficit in another. When the net lending balances of all sectors and industries sum to zero, it confirms the achievement of both vertical and horizontal consistency, as stated in the key principles outlined in section 4.1[29]. The last item in this account is ’Consumption of fixed capital’, D^i, which measures the depreciation of physical assets. Note that, even though this item does not determine net lending it still affects each sector and industry’s net worth. It should also be noted, that D^i is not a transaction between sectors and industries.

The financial accounts, which are all the variables reported in the bottom of the TFM also referred to as FoF, is followed by an adjustment variable[30]. In simpler terms, the financial account details the flow of funds between sectors, recording transactions involving financial assets and liabilities that occur between resident units and the RoW. These transactions can be either pure financial operations or the monetary counterpart (payment) of a real transaction (for instance, a households consumption from firms). This level of disaggregation of financial variables is one of the contributions of this paper. Additionally, it tracks changes in the value of the sectors and industries asset and liability holdings. These aspects are reported as transactions and as revaluations[31] and other changes[32] (ROC), respectively, for each financial item introduced in the TFM.

[29] According to attached table ”TFM2019Q4”, the horizontal consistency is not achieved in the case of summing to zero due to the way data are reported by DST. The horizontal sum of net lending is not too large in terms of aggregate income. This discrepancy is 0,00000832 in terms of nominal GDP in 2019Q4. This means that the inconsistencies in the data should not necessarily be a cause for concern. The discrepancies arises for the difference between intermediate purchases and sales (-1), compensation of employees (-1), net interest (-1), reinvested earnings on FDI (-1) and current taxes on income and wealth (-1). This is also the case for the flow of funds.

[30] This variable does not appear in either the national accounts or DST’s reports. It is incorporated to maintain consistency between the current and capital accounts, ensuring alignment with the data reported in the financial accounts. This inclusion is crucial for ensuring that the model and all related identities are accurately met, thereby eliminating any discrepancies. This is additionally important because databases representing the current and the accumulation accounts are not naturally consistent. The method by which this is calculated is based on the sum of all transactions in the financial accounts F.1 to F.8 minus each sector’s net lending position. This method ensures internal accounting consistency within the model.

[31] Revaluations occur because of a change in the monetary value of a financial asset or liability due to changes in the level and structure of its price (United Nations, 2009b).

[32] Other changes in the volume of financial assets and liabilities are any changes in the value of these assets that are due neither to transactions nor to revaluation. These changes include those due to cancellation and write offs, economic appearance and disappearance of assets, reclassification, and the changes in financial assets arising from entities changing their economy of residence (United Nations, 2009b).

5 Statistical inputs for the E-IO-SFC model database

According to Valdecantos (2023b) the third step in building an empirical SFC models is to build the data bank. Therefore, this section introduces the data foundations and methodological considerations underlying the construction of the E-IO-SFC model on a quarterly time frequency ranging from 2001Q1 until 2024Q4 in millions DKK. The time span is based on the data availability, when taking all the variables entering the E-IO-SFC model into account. The structure outlined in this section follows the TFM row by row to ensure transparency and traceability. The data sources used to map the Danish national accounts and construct the database are drawn from DST. Section 5.1 outlines the data used to construct the upper and middle part of the TFM, capturing the real side of the economy. Section 5.2 focuses on the financial side, describing the flow of funds between sectors and the compilation of financial transactions in the lower part of the TFM. Additionally, this section presents the data and methodology used to build the balance sheet at the industry level, thereby addressing the **first gap** identified in section 2.1. Lastly, section 5.3 details the model's main contributions: (i) why an input-output (IO) structure is essential, (ii) how we propose to incorporate the structure, addressing the **second gap**, and (iii) the current use of the IO-table provided by DST. Each subsection concludes with a description of the empirical methods applied to close the relevant data gaps and ensure consistency across the modeling framework. The data bank consists of non-seasonally adjusted data in current prices, which is necessary for the system of accounting identities to hold. For instance, when the variables are in current prices, they describe the sums of money that actually change hands each period. The use of seasonal adjustment is only necessary for the variables entering estimation of behavioral equations. We describe this later in section 7.2.

5.1 DST tables for the real side

Tables 5 and 6 provide an overview of the DST tables referenced in this subsection, including sectoral classifications, transaction types, and data frequency. All relevant figures used in constructing the TFM for the underlying DST tables that reports transactions in both receivable and payable terms are in net values.

Upper Part of the TFM

As outlined in Valdecantos (2023a), the structure of the TFM diverges slightly from the standard national accounting framework. While national accounts begin with the production account and proceed sequentially through the income, capital, and financial accounts, the TFM used for the E-IO-SFC starts with a partial representation of the 'use of disposable income' (i.e., household and government consumption), the 'capital account' (i.e., investment), and the 'goods and services account' (i.e., exports and imports). Thus, the TFM combines elements from various accounts of the System of National Accounts (SNA), providing a representation of the goods and services market equilibrium. This ordering offers a more intuitive introduction to the structure of the SNA, as it reflects the fundamental identity that all output must equal the sum of aggregate demand components (including the change in inventories), which in turn corresponds to the gross income generated in the economy. Accordingly, the upper part of the TFM includes the following transactions: household and government consumption (P.3), gross fixed capital formation and acquisitions less disposals of valuables (P.51g + P.53), and changes in inventories (P.52), followed by production-related flows like exports (P.6), imports (P.7), intermediate consumption (P.2), and net taxes on production and products (D.21–D.31 and D.29–D.39). The data underpinning these entries are sourced from NKSO2 (DST, 2024k), NKSO1 (DST, 2024j), NKHO2 (DST, 2024g), and NKBP10 (DST, 2024f), where NKHO2 provides information at the level of the total economy and NKBP10 offers a breakdown by industry. Methodological procedures for allocating and completing these figures in the upper part of the TFM are detailed in section 5.1.1.

Middle Part of the TFM

The middle part of the TFM contains data from the generation and distribution of income accounts. For compensation of employees (D.1), the total wages received by households is taken from NKSO2, while sectoral breakdowns are obtained from NKBP10 (for the industries and sectors) and NKSO1 (for FC). The allocation of these wage payments across sectors and industries is described in section 5.1.1. Additionally, wages paid to Danish workers living abroad are computed as the difference between the domestic and total values in NKSO1 and NKSO2, respectively, yielding the RoW component. Regarding gross operating surplus and mixed income (B.2g+B.3g), values for households and government are retrieved from NKSO2. As only NFC and FC are assumed to engage in production activities, the portion of surplus attributed to non-producing sectors is redistributed back to producing sectors to ensure a correct net lending figure at industry level. The adjustment procedure is discussed in section 5.1.1.

The remaining components of the current and capital accounts are primarily sourced from NKSO2 (DST, 2024k). This includes primary income, such as interest (D.41), dividends (D.42), reinvested earnings (D.43), other investment income (D.44), and rent (D.45), as well as secondary income transfers, including current taxes on income and wealth (D.5), social contributions (D.61), social benefits (D.62), and other current transfers (D.7). The same source also provides data for adjustment for the change in pension entitlements (D.8), gross saving (B.8g), and components of the capital account, including capital transfers (D.9), acquisitions less disposals of non-produced non-financial assets (NP), and the balancing item for net lending/borrowing (B.9). Taxes on production and imports less subsidies from the RoW (D.2–D.3) are drawn from NKN2 (DST, 2024h). For the Central Bank, D.41 and D.42 are retrieved from NASD21 (DST, 2024c), while D.7 transfers are based on NASD22 (DST, 2024d).

DST table	Information	Frequency
NKBP10	Industry divided SNA (production and generation of income)	Quarterly up until 2024Q4
NKSO1	SNA by sector for production and generation of income	Quarterly up until 2024Q4
NKSO2	SNA by sector for allocation and use of income	Quarterly up until 2024Q4
NKN1	Demand and supply by transaction for the whole economy (used for the components of aggregate demand)	Quarterly up until 2024Q4
NKN2	Real gross national disposable income for the whole economy	Quarterly up until 2024Q4
NKHO2	Production, GDP and generation of income for the whole economy	Quarterly up until 2024Q4
NAN1	Demand and supply by transaction and price unit	Annual up until 2024
NASD12	Generation of income by transaction and sector	Annual up until 2023
NASD21	Allocation of primary income by transaction and sector	Annual up until 2023
NASD22	Secondary distribution of income by transaction and sector	Annual up until 2023
NASD23	Use of disposable income by transaction and sector	Annual up until 2023
NASD24	Capital by transaction and sector	Annual up until 2023

Table 5: Information contained in each of the DST tables.

DST tables	Sector, Type of Supply, Transactions
Sectoral names	S.1 Total economy, S.11 Non-financial corporations (NFC), S.12 Financial corporations (FC), S.121 The central bank (CB), S.13 General government (G), S.14+S.15 Households and non-profit institutions serving households (H), S.2 Rest of the world (RoW)

DST tables	Sector, Type of Supply, Transactions
NKBP10	P.1 Output, P.2 Intermediate consumption, B.1g Gross value added, D.29-D.39 Other taxes less subsidies on production, D.1 Compensation of employees, B.2g+B.3g Gross operating surplus and mixed income
NKSO1	P.6 exports, P.7 imports, D.1 Compensation of employees
NKSO2	P.3 Final consumption expenditure, P.51g Gross fixed capital formation, P.53 Acquisitions less disposals of valuables, P.52 Changes in inventories, D.1 Compensation of employees, B.2g+B.3g Gross operating surplus and mixed income, D.41 Interest, D.42 Distributed income of corporations, D.43 Reinvested earnings on direct foreign investments, D. 44 Other investment income, D.45 Rent, D.5 Current taxes on income, wealth, etc., D.61 Net social contributions, D.62 Social benefits other than social transfers in kind, D.7 Other current transfers, D.8 Adjustment for the change in pension entitlements, B.8g Saving, gross, B.12 Current external balance, D.9 Capital transfers, NP Acquisitions less disposals of non-produced non-financial assets, B.9 Net lending/net borrowing
NKN2	D.2-D.3 taxes on production and imports less subsidies from RoW
NKHO2	D.21-D.31 Taxes less subsidies on products, D.29-D.39 Taxes less subsidies on production
NASD12	D.1 Compensation of employees
NASD21	D.41 Interest, D.42 Distributed income of corporations
NASD22	D.7 Other current transfers
NASD23	B.8g Saving, gross
NASD24	B.12 Current external balance

Table 6: Information contained in each of the DST tables used for the real side of the economy. Note, several variables (D.41 - D.9) in the NKSO2 table consists of receivable (r) and payable (p) transactions.

5.1.1 Empirical method for filling the gaps on the real side

Completing the data for the upper part

The decision to disaggregate the NFC sector into specific industries requires that the demand components, retrieved on a aggregated level from DST, such as NKSO1 and NKSO2, has to be distributed across the selected industries included in the model framework. To enable this allocation, the NAIO1F table (DST, 2025b) is used, as it provides disaggregated information on household and NPISH consumption, government consumption, gross fixed capital formation, inventories, and exports among other things, all broken down by supplying and demanding industries. The shares obtained from NAIO1F, are used to allocate macro-level demand components across industries such as Energy, Transport, and Agriculture, as illustrated in figure 4. Although the full IO-structure is presented later in section 5.3, the NAIO1F shares are already essential for ensuring consistent industry-level allocation in the upper part of the TFM.

In the case of intermediate consumption (IP), the value can be retrieved directly from NKBP10. However, to isolate the values for sectors E and T, both of which are otherwise included within broader aggregates in NKBP10, it is necessary to apply proportional shares. Specifically, output and IP for these industries are grouped under D_E ('Utility services') and G_I ('Trade and transport etc.'), respectively. To extract disaggregated values, industry-specific shares are calculated using the NAIO1F table. More accurate, the share of output of D (energy) represented by D_E and the share of H (transport) represented by G_I are derived from NAIO1F and applied to the aggregated NKBP10 figures[33]. Since the NAIO1F data is only available annually up to 2023, it has been necessary to expand and split the series to a quarterly frequency. For this purpose, it is assumed that

[33] For more details, please refer to lines 1386-1400 of the attached R-script "IO-TABLE WORK".

the shares for D and H remain constant within each year and the recent annual value is kept constant for the remaining periods. These shares are used consistently whenever NKBP10 data requires disaggregation, allowing for a coherent and internally consistent allocation of IP, as well as other figures, across the selected industries in the TFM.

While the value of IP can be obtained directly from DST sources in quarterly frequency, the corresponding figure for intermediate sales is not explicitly reported (it is only available in annual basis in the IO tables). To construct this missing component in the quarterly series, we apply an indirect calculation method using the identity stating that the gross value added at basic prices is equal to total output (in turn, equal to total sales) minus intermediate purchases. Rearranging this identity we get that intermediate sales for each industry are computed as the sum of gross value added and taxes on production and imports less subsidies from RoW, plus intermediate consumption, minus all components of aggregate demand attributed to that industry. This ensures full accounting consistency in the production account of the industries in quarterly frequency. For example, om a general level, the calculation is given by:

$$IS^i = GVA^i + D21D31^i - P3.C^i - P3.G^i - P51gP53^i - P52^i - P6^i + P7^i + IP^i.$$

Ensuring consistency between intermediate purchases and sales is particularly important when distributing values among the disaggregated NFC industries to properly account for inter-industry interactions. However, this empirical method is being revised in section 8.

During the construction of the model, a comparison between the industry classification 'Financial and insurance (K)' from NKBP10 and the institutional sector 'Financial Corporations (S.13)' from NKSO1 revealed a discrepancy in key variables such as GVA, GVA_F, and D29–D39. This mismatch appears to result from a small portion of financial corporations being classified outside of 'K' in NKBP10. To resolve this, the affected FC variables are now consistently sourced from NKSO1 when NKBP10 is used. Consequently, when using NKBP10, the difference between "K" and the "full" FC[34] sector is absorbed into the residual OMS industry, being this industry that most likely is associated with financial activities after Financial Corporations. This correction also necessitated an adjustment in how net indirect taxes less subsidies on production are calculated. D29–D39 is now sourced directly from NKBP10 and allocated across industries as intended. For D21–D31, output shares from NKBP10 are used to distribute the total, after which the net tax variable is obtained by summing the two components.

Completing the data for the middle part

Regarding the figure for compensation of employees (D.1) the same procedure as described above is applied. Specifically, the value is distributed across industries using NKBP10, with the amount for FC sourced from NKSO1, and the value for the OMS industry calculated as a residual. Additionally, the share of D.1 attributable to CB is derived from the total figure of FC by applying a proportion calculated from NASD12. This share is subtracted from the FC component, yielding the final values for both FC and CB as recorded in the TFM.

In a similar manner, the redistribution of GOS has been addressed using supplementary data from DST. A key source here is a report that outlines how GVA generated by each industry is attributed to different institutional sectors (Jørgensen et al., 2021, p. 24–25). Table 7 provides insight of this data for the agricultural industries, along with an overview of the total distribution of GVA across all industries and sectors.

[34]"Full" refers to the fully aggregated financial corporations sector as reported by DST in tables such as NKSO2 (DST, 2024k), which present data at the sectoral level, as opposed to the industry-level breakdown used in NKBP10 DST (2024f).

Industries	Total	S.11	S.12	S.13	S.14	S.15
01 Crop and animal production, hunting and related service activities	0.9	0.3	0	0	0.6	0
02 Forestry and logging	0.1	0	0	0	0.1	0
03 Fishing and aquaculture	0.1	0.1	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Total (Share in %)	100	60.4	6.1	21.1	11.4	1.4

Table 7: Distribution & GVA by industries and institutional sectors in percent (Jørgensen et al., 2021, p. 24-25)

A complete overview of this table can be found in the **attached document** titled "Distribution & GVA by industries and institutional sectors 2016". It should be noted that GOS generated by the Central Bank is assumed to be assigned to Financial Corporations.

In order to do this reallocation of industries' GOS by sector, first the industries from the table above is distributed according to how this project disaggregated the NFC sector. Second, it is necessary to examine how much of the GVA across all industries is generated by the government and households, respectively, in order to calculate how much the respective industries within this framework must redistribute (by deducting it from its own primary account) to ensure accounting consistency across all relevant sectors [35]. For clarification, an example is provided below showing how the amount of redistributed GOS by the Agriculture industry to the relevant sectors is computed. As shown in table 7, the government does not contribute to GVA within the Agriculture industry, meaning that the latter does not redistribute anything to the former. The household sector, on the other hand, contributes 0.7% to the GVA. Hence, this percentage must be deducted from the Agriculture industry's primary income account by redistributing to the households. The amount that the household and the government must "receive" from the Agriculture industry out of its total GVA is calculated as:

$$B2g3g^{A,H}_{\text{share}} = \frac{(0.6 + 0.1)}{(11.4 + 1.4)} = 0.055$$
$$B2g3g^{A,G}_{\text{share}} = 0$$

According to the **attached document** titled **"TFM2019Q4"**, the household sector receives DKK 44.783 million (GOS^H) and the government DKK 15.185 million (GOS^G) across all producing industries in redistributed GOS. Hence, the amount that the Agriculture industry must deduct from its primary income account is calculated as $GOS^A_H = GOS^H \cdot B2g3g^{A,H}_{\text{share}} = 2449.07$ million DKK. This approach is applied across all producing industries[36]. Thereafter, the θ-values needed in equations 227-236 can be obtained implicitly by taking the figures for the amount the specific industry distributes to the households (or the government) and dividing by the total GOS of same industry. Following the example of the agriculture industry the θ-value is calculated as:

$$\theta^A_H = \frac{GOS^A_H}{GOS^A},$$

[35] Recall that we assume, as is standard in SFC models, that all production is carried out by NFC and FC. Hence, in order for our figures to match the net lending position of each sector reported by DST it is necessary to redistribute the gross operating surplus to each sector according to its real contribution to the production process

[36] For clarification, the household and government sectors do not contribute to either the energy industry or the FC, meaning they do not "receive" anything. In contrast, the transport industry "gives back" 1.42% to the government and 3.91% to households, while MC "gives back" 0.95% to the government and 4.69% to households, and OMS 97.63% and 85.94%, respectively.

where GOS^A is calculated as gross value added at factor costs subtracted with compensation of employees. For reference, please refer to the following equation shown on the general level:

$$GOS^i = GVA^i_F - WB^i.$$

By using this information, the accuracy of the database is improved, as the actual shares of each sector's ownership of the different industries are applied. However, the information provided by DST is only available for 2016. Therefore, it has been decided to assume that the GVA generated by each industry remains constant across the entire input data.

Some additional data manipulation has been carried out to simplify the presentation of the TFM, specifically within the primary income account. This has involved merging variables with either zero or limited relevance, as well as combining variables with overlapping purposes. In particular, the variables D.41, D.44, and D.45 have been aggregated into a single item referred to as 'Net interest'[37]. However, it remains necessary to distribute the merged net interest variable across the industries within the NFC sector. As will be discussed in the upcoming section 5.2.1, a proxy for net interest allocation has been constructed based on the average contribution of selected interest bearing financial variables. These shares are derived using accounting statistics data, which is also introduced in section 5.2.1. The proxy for each industry's share in the merged net interest item is calculated as the simple average of four interest bearing indicators. For example, on a general level, the proxy is defined as:

$$i.\text{Net}.i.\text{proxy} = \frac{i.F2^{AS}_{share} + i.F3^{AS}_{share} + i.F4^{LI}_{share} + i.F8^{Net}_{share}}{4}.$$

Turning to distributed income of corporations (D.42) and reinvested earnings on direct foreign investments (D.43), these figures have been distributed across the industries within the NFC sector using the share of GVA as a benchmark. For D.42 specifically, the net value attributed to the CB, retrieved from NASD21, has been subtracted from the total amount recorded under FC, yielding the final values for both FC and CB as reported in the TFM.

We also use the share of GVA as a benchmark to distribute the figures of the secondary income account - current transfers (D5) and direct taxes on income and wealth (D7). The choice regarding the distribution of D.5 and D.7 assumes that GVA somehow reflects the contribution of each sector or industry to the total value added in the economy. To retrieve the CB contribution from D7, we use the same method previously applied to D.42, but via NASD22.

Again, some additional data manipulation has been carried out to simplify the presentation of the TFM, specifically within the capital account. This has involved merging D9 and NP which is referred to as 'Capital and non-financial and non-produced asset transfer'. To distribute this figure the respective GVA share of the industries have been used.

Taking all of the above corrections into account, the net lending position of the various sectors and industries is calculated as the sum of all income and expenditure transactions mentioned above. Compared with the DST figures[38], a discrepancy is observed between DST and the net lending calculation in our database. However, this discrepancy is offset when the respective adjustment variables are included in the sector/industry-specific net lending calculation. Hence, the values reported in the TFM regarding net lending include the adjustment variables. It is important to note that DST also exhibits a discrepancy between the sum of all income and expenditure transactions and sum of all financial transactions (i.e. assets minus liabilities). However, DST does not explicitly report an adjustment variable to account for this gap.

[37]Note that D.41 is further divided into payable and receivable components based on the relationships defined in the system of equations; see the appendix for further details.

[38]The figure "B9 Net lending (+)/net borrowing (-)" within NKSO2.

5.2 DST tables for the financial side

Tables 8 and 9 provide an overview of the DST tables referenced in this subsection, including sectoral classifications, transaction types, and data frequency. All figures used in constructing the TFM and BS are expressed in net terms, as the underlying DST tables report transactions and stocks in both asset and liability positions.

Lower part of the TFM

The final component of the TFM is the financial account, which constitutes the lower part of the matrix. In essence, the financial account captures the flow of funds between sectors, recording transactions in financial assets and liabilities that take place both among resident institutional units and between residents and the RoW.

As described in section 4.3, the financial account also reflects changes in the value of sectors' holdings of assets and liabilities. To retrieve the necessary data at the sectoral level, the NKSFK dataset (DST, 2024i) is used, covering the period from 2001Q4 to 2024Q4. The financial instruments included from this source are: 'Monetary gold and special drawing rights (SDRs)' (F.1), 'Currency and deposits' (F.2), 'Debt securities' (F.3), 'Loans' (F.4), 'Equity and investment fund shares or units' (F.5), 'Insurance, pension and standardized guarantee schemes' (F.6), 'Financial derivatives and employee stock options' (F.7), and 'Other accounts receivable/payable' (F.8). For each instrument, three components are retrieved: 'Transactions, net', 'Revaluation', and 'Other changes in volume'. The counterpart sector for all transactions is recorded as 'Total', meaning that flows to both the domestic economy and the RoW are included in the aggregate.

Balance Sheet

For the construction of the BS, the same NKSFK dataset is used at the sectoral level. However, in this context, the relevant component is the 'Closing balance sheet', which captures end-of-period stock positions for each financial instrument and sector. These values serve as the initial and final stocks for financial assets and liabilities in the model and are essential for ensuring stock-flow consistency in the overall framework. Specifically, the stock value recorded for a given quarter (e.g., 2019Q4) represents the end-of-period position, that is, the stock as of the last day of the quarter. These values are used directly as initial stocks in the subsequent period and final stocks in the current period, thereby ensuring temporal consistency in the E-IO-SFC model's flow-of-funds framework.

DST table	Information	Frequency
NKSFK	SNA for balance sheets, account entry, financial instrument and counterpart sector, disaggregated by sector	Quarterly from 1999Q1 up until 2024Q4
REGN2	Accounts statistics by industry, items and size	Yearly from 2000 up until 2021
REGNLA4	Accounts statistics by industry (DB07) and items	Yearly from 2008 up until 2022

Table 8: Information contained in each of the DST tables.

DST tables	Financial Instruments and Accounts statistics
Sectoral names	S.1 Total economy, S.11 Non-financial corporations (NFC), S.12 Financial corporations (FC), S.121 The central bank (CB), S.13 General government (G), S.14+S.15 Households and non-profit institutions serving households (H), S.2 Rest of the world (RoW)

DST tables	Financial Instruments and Accounts statistics
NKSFK	F.1 Monetary gold and special drawing rights (SDRs), F.2 Currency and deposits, F.3 Debt securities, F.4 Loans, F.5 Equity and investment fund shares or units, F.6 Insurance, pension and standardized guarantee schemes, F.7 Financial derivatives and employee stock options, F.8 Other accounts receivable/payable
REGN2 & REGNLA4	Financial fixed assets, Debts receivable from customers, Long-term Debts, Other Short-term Debts, Capital and reserves, Short term debts to suppliers, Provisions for liabilities and charges

Table 9: Information contained in each of the DST tables regarding the financial account.

5.2.1 Empirical method for filling the gaps in the lower part of the TFM and BS

Completing the data for the lower part and BS

As highlighted in the first identified gap in section 2, existing macroeconomic models often assign a passive role to the financial sector and lack an integrated treatment of real and financial interactions at the sectoral or industrial level. This subsection addresses how we propose to disaggregate both financial stocks and flows at the industry level, thereby enabling the model to capture financial interactions between sectors and industries more accurately. By linking the financial account (lower part of the TFM) with balance sheet positions across disaggregated industries, the framework allows for a more comprehensive representation of financial dynamics and their interplay with real economic activity. The following subsections describe the specific methods used to allocate financial instruments across industries and how this information is used to construct industry and sector-specific balance sheets that are consistent with the flow data recorded in the TFM[39].

Because DST (as all statistical institutes around the world) does not publish the financial account by industry, it has been necessary to calculate shares when allocating all the aforementioned financial variables among the industry-divided NFC sector. These shares are needed to distribute the aggregate figures for the NFC sector into the different industries present in the E-IO-SFC model. To do this for the financial variables, the REGN2 table from DST (2024l) has been used. Although it does not provide a classification of financial assets and liabilities with the level of detail done in the financial account of the system of national accounts, it still provides balance sheet statistics by industry, items, and size that we can use to build proxies. The REGN2 table represents 17 industries, as shown in table 10. In this table, it can also be seen how the 17 industries are distributed among 4 of the 5 NFC industries applicable in our framework. It is noteworthy that data from this source provided by DST excludes agriculture, fishing, harbor etc. To get data for the agricultural sector the REGNLA4 table from DST (2024m) has been used. As indicated in table 5 REGN2 and REGNLA4 provide data on a yearly basis for different periods. It is important to communicate this information in order to understand the manipulation that will be described later in this section.

The selected accounting items include 'Intangible fixed assets', 'Land and buildings', 'Production machinery and equipment', 'Other tangibles', 'Financial fixed assets', 'Debts receivable from customers', 'Total Assets = Total Liabilities', 'Capital and reserves', 'Provisions for liabilities and charges', 'Long-term debts', 'Short-term debts to suppliers', 'and Other short-term debts'. All figures are stated in millions of DKK. It should be noted that due to missing data, only accounting items linked to F.2, F.3, F.4, F.5, and F.8 have been identified. These aforementioned accounting items are allocated among the aggregates of financial variables presented in the SNA (from F.1 to F.8) by adding their values. The table below illustrates how these different accounting items are distributed. The chosen scale among the industries is the total among all enterprises.

In order to match the time frequency used in the model some data manipulations is necessary. To gain a

[39]This subsection builds upon the ideas presented in Nortvig et al. (2024).

Industry	Division of the REGN2 table
Energy	Electricity, gas, steam, and air conditioning supply (D)
Transport	Transportation (H)
Manufacturing and Construction	Mining and quarrying (B), Manufacturing (C), Construction (F)
Other manufacturing and services	Water supply, sewerage and waste management (E), Materials recovery (383), Wholesale and retail trade and repair of motor vehicles etc. (45), Wholesale trade, except of motor vehicles etc. (46), Retail trade, except of motor vehicles etc. (47), Accommodation and food service activities (I), Information and communication (J), Real estate activities (L), Knowledge-based services (M), Travel agent, cleaning, and other operational services (N), Repair of household goods (95)

Note: Agriculture is excluded from the REGN2 division used in this model.

Table 10: Division of the REGN2 table into the different industries.

Financial Variable	Accounting Item
$F.2^{AS}$	50% of Financial fixed assets
$F.3^{AS}$	50% of Financial fixed assets
$F.4^{LI}$	Long-term Debts and Other Short-term Debts
$F.5^{LI}$	Capital and reserves
$F.8^{AS}$	Debts Receivable from Customers
$F.8^{LI}$	Short-term debts to suppliers and Provisions for Liabilities and Charges

Note: The subscripts ^{AS} and ^{LI} refer to assets and liabilities, respectively. The two F.8 variables are subtracted from each other to form $F.8^{Net}$.

Table 11: Distribution of accounting items to F.2, F.3, F.4, F.5, and F.8.

better understanding of the method used to obtain quarterly shares of financial stocks, they will be described sequentially. Note that the focus will be on the energy and agriculture industries, as only these two lack information in several time periods. Common to all variables the shares have been calculated on an annual basis, subsequently distributed quarterly, thus achieving the desired time frequency. The method used to obtain the quarterly distribution involves expanding and keeping the respective yearly value constant for four periods, as explained in detail below.

The first two variables are $F.2^{AS}$ and $F.3^{AS}$ respectively. Data for the period 2014Q1 to 2021Q4 is complete for every industry, with the exception of agriculture. Hence, it is needed to find a proxy for how much the sum of the two instruments is augmented when including agriculture. This assumption was indeed necessary due to the fact that the total value represented by all industries in the accounting statistics excludes the agriculture industry in REGN2 and is completely left out of REGNLA4. Including their share of GVA might be a good solution as it provides a representation of agriculture's contribution to the economic activity and could thereby offer a good approximation of financial transactions. Therefore, it is important to make an adjustment by extending the total sum of these two variables in each time period where there is no data for agriculture's financial stocks. Here, the share of GVA of agriculture is added to the total value. For clarification, consider the following formula, where the share of $F.2^{AS}$ is calculated for the energy industry:

$$E.F2^{AS}_{share} = \frac{E.F2^{AS}}{(SUM.F2^{AS} * (1 + GVA^{A}_{share}))}.$$

The share of agriculture will be calculated as the residual. However, in periods from 1999Q2 to 2013Q4 data for the energy industry is also missing which is why the total sum of $F.2^{AS}$ and $F3^{AS}$ respectively needs to be further extended. The value used in this case is the last known value from 2014Q4, which is held constant for this period. Due to the aforementioned manipulation, the share of agriculture is now complete as well as the other industries except for energy. Thereby, the energy industry in the period from 1999Q2 to 2013Q4 will be the residual and by definition ensuring all the shares are adding up to one.

The next variables are $F.4^{LI}$ and $F5^{LI}$ respectively. Data for the period 2014Q1 to 2021Q4 are complete for every industry, eliminating the need for manipulation during this time frame. However, due to different periods with missing data for both the energy and agriculture industries from 1999Q2 to 2013Q4, additional steps were required. Firstly, the dataset is divided into the period 2008Q1 to 2013Q4, during which only the energy industry lacks data. Therefore, the last known value (2014Q1) for the representative financial variable of this industry is used to extend the total sum of $F.4^{LI}$ and $F.5^{LI}$. Hence, the share of the energy industry during this period is calculated as the residual. Secondly, the data is divided into the period 1999Q2 to 2007Q4, where both the energy and agriculture industries lack data. In this case, the sum must be extended by two values since both industries are affected. The values used to extend the sum are the last known values for the representative financial variables and industries. Additionally, the values used are those calculated in the first data split. It was decided to keep the share of agriculture constant, resulting in the calculation of the energy industry as the residual.

The required information to distribute F.6 and F.7 are not contained in the REGN2 and REGNLA4. Therefore, it is decided to allocate these variables across industries proportionally to each industry's share of GVA. While GVA may not capture financial exposure directly, it serves as a reasonable proxy for the relative economic importance of each industry, ensuring a transparent and consistent allocation method.

Lastly, the next variables that require manipulation are $F.8^{AS}$ and $F.8^{LI}$. To begin with, it was decided to treat $F.8^{AS}$ and $F.8^{LI}$ separately. While the composition of $F.8^{AS}$ and $F.8^{LI}$ was available for all periods from 2014Q1 to 2021Q4 for all industries, except agriculture, the composition from REGNLA4 is only accessible for $F.8^{AS}$ from 2008Q1 to 2011Q4 and for $F.8^{LI}$ from 2011Q1 to 2021Q4, resulting in an uneven distribution. Therefore, it was chosen to subtract $F8^{AS}$ from $F8^{LI}$, creating a net variable for F.8, now denoted as $F.8^{Net}$. This decision aligns with the registration of financial instruments in this framework, as described in section 5.2. The total sum of $F.8^{Net}$ is extended by the GVA share of agriculture as in the manipulation concerning $F.2^{AS}$ and $F.3^{AS}$ respectively. However, in the period from 1999Q2 to 2013Q4 both the energy and agriculture industries lack data. Therefore, the total sum of $F.8^{Net}$ is extended with the two last known values for the industries. It was decided to maintain the share of agriculture constant, resulting in the calculation of the energy industry as the residual (Nortvig et al., 2024).

Interpolation, extrapolation and ARIMA-based forecasting

In order to introduce fluctuations into the dataset and simultaneously complete the time series from 2001Q1 to 2024Q4, we rely on a combination of interpolation, extrapolation, and ARIMA-based forecasting. When source data is only available on an annual basis, it is necessary to adopt methods that can disaggregate and extend these values into quarterly frequency. To address this, linear interpolation is applied to generate seasonally fluctuating quarterly time series. The transformation from annual to quarterly shares is performed by assigning the annual share to the first quarter of each year and linearly interpolating between these annual points to avoid constant values across quarters[40]. This procedure is applied from 1999Q1 to the last point for which annual data are available. To complete the dataset from the last known value to 2024Q4, an ARIMA model is used to forecast each share i quarters ahead[41]. The choice of ARIMA is motivated by its ability to handle univariate time series with non-linear, non-stationary patterns[42]. For an illustrative example of the interpolation, extrapolation and

[40]The interpolation is implemented using the 'approx' function in R.

[41]According to Enders (2015, pp. 79–80), ARIMA models are particularly suitable for forecasting as they capture key time series characteristics such as trends, cycles, seasonality, and autoregressive components. Their flexibility in modeling non-linear dynamics makes them well suited for this purpose.

[42]In R, the forecasting process is carried out using the 'auto.arima' function, which selects the best-fitting model based on information criteria such as AIC or BIC. The selected model is then passed to the 'forecast' function to project values out of sample. See Hyndman and Athanasopoulos (2018).

ARIMA-based forecast please refer to figure 3

It is worth noting that most of the fluctuations introduced through the figures of REGN2 and REGNLA4 occur during the period from 2014Q1 to 2021Q4, where shares are not held constant. Finally, for each financial variable, the share attributed to the agriculture industry is calculated residually after disaggregating the other sectors.

In summary, the computed financial shares based on REGN2 and REGNLA4 have been used to allocate the relevant financial stocks and flows across the industry-disaggregated NFC sector. These shares form the empirical foundation for distributing financial instruments such as F.2, F.3, F.4, F.5, and F.8 in both the lower part of the TFM and the balance sheet. For those instruments or periods where industry-specific balance sheet data are unavailable or incomplete, the allocation has instead been based on each industry's relative contribution to GVA, such as F.6 and F.7. This approach ensures internal consistency and continuity with the methodology applied in earlier parts of the model construction.

5.2.2 Error and trial

Initially, it was decided to allocate the financial variables across industries based on each industry's share of GVA. However, this approach was not ideal, as the objective was to establish a comprehensive data foundation for the E-IO-SFC model under development. Subsequently, data was sought from the balance sheets of 5000 of the largest Danish companies with over 34 employees. This data was to be sourced from Proff.dk, but it was ultimately not utilized due to insufficiency in the provided information. This prompted us to consider DST, specifically the REGN2 table, which includes financial positions for all industries except agriculture. We then contacted DST to find out whether data for agriculture was available. They informed us about the REGNLA4 table, which provides information on agriculture, though in a limited amount, but covering the same accounting items as in REGN2. Since this data does not provide the same in-depth as the financial data sources (e.g. NKSFK) a sent scheme from DST is used to distribute the different accounting items. This scheme contains non-aggregated variables for every item listed for the two sources. Hence, it could be decided how the accounting items should be distributed among the financial instruments. However, due to the incompleteness of REGN2 and REGNLA4 only a small proportion of the accounting items could be distributed among the financial instruments. Hence, it can be discussed whether this composition of the accounting items contains the same information as the financial instruments in e.g. NKSFK. Therefore, it should be noted that this composition of the accounting items in the calculations of shares for financial data is only temporary until more detailed sources are available. Another problem that occurred regarding REGN2 and REGNLA4 is the incompleteness of the time series and the missing out of an accounting item in the periods available. For instance, Financial fixed assets is completely left out of REGNLA4, why data for this is made upon assumptions, as described in previous section 5.2.1. Furthermore, the composition of $F8^{AS}$ we've made is only available in the period from 2008 to 2011, and $F8^{LI}$ is available from 2011 to 2022. Due to this incompleteness, it is chosen to make a simpler representation, where the two instruments for the other sectors (collected via REGN2) are subtracted from each other and thereby making a net variable in which a share can be calculated. In relation to REGN2, the only problem was directed towards the compositions of all the accounting items for the energy industry, as information between 2000Q1 and 2013Q4 was not available. Due to these issues across the two datasets, further manipulation was needed. This includes extending the total sum of the representative financial instrument by either one or two values if there is missing data for one or two financial instruments (Nortvig et al., 2024).

5.3 The input-output structure and used DST tables

This subsection aims to address (i) why an IO-structure is essential for understanding the structural dynamics of an economy, (ii) how we propose to distribute the aggregated NFC sector, and thus addressing the identified gap 2, and (iii) the current use of the IO-table (NAIO1F) from DST.

5.3.1 Input-output tables

An IO-table is a central component of the national accounts, as it describes the relationships between the supply and use of goods and services across an economy. It provides a comprehensive account of how the value generated through production is allocated, including intermediate consumption (e.g., raw materials), final consumption in the domestic economy, and external trade through imports and exports. IO tables capture the structure of production by detailing the value of output, value added, and inputs used across industries, as well as identifying which industries supply which goods and services (Hjulsager, 2022).

By definition, IO tables are particularly well suited for disaggregating national accounts data, as they provide the necessary level of industry-level detail to trace economic flows across sectors. According to Jensen (2024), their primary function is "by means of an organized tabulation of detailed economic statistics to inform about the interactions between production, imports and uses in the economy for a given period." This formal purpose underscores their analytical potential in applied modeling frameworks.

In the E-IO-SFC model, disaggregating the NFC sector enables the incorporation of an IO structure, which unlocks several analytical capabilities as described above. This is essential for evaluating the industry-specific effects of policy measures, trade interventions, or supply shocks. The resulting disaggregated structure thus offers a robust foundation for examining transmission mechanisms and sectoral responses within the broader economy.

5.3.2 How to technically distribute the NFC sector into the chosen industries

As outlined in section 2.2.1, the NFC sector is disaggregated into five industry groups: Energy (E), Transport (T), Agriculture (A), Manufacturing and Construction (MC), and Other Manufacturing and Services (OMS). Financial Corporations (FC) are treated separately, as they are reported distinctly in both the SNA and the IO-tables. To implement this disaggregation, the annual NAIO1F table DST is used, ranging from 1999 to 2023. This table provides detailed data on supply and use by product, industry, and transaction type, and forms the empirical foundation for mapping, among other things, final demand to the disaggregated industries in the model. The OMS category serves as a residual, capturing all industries in the IO system that are not explicitly assigned to the four NFC subcategories and FC. A full overview of how specific industries from the NAIO1F table are assigned to each category can be found in table 12.

Industry	**Division of the NAIOF1 table**
Agriculture	Agriculture, forestry and fishing (A)
Energy	Electricity, gas, steam, and air conditioning supply (D)
Transport	Transportation (H)
Manufacturing and Construction	Mining and quarrying (B), Manufacturing (C), Construction (F)
Other manufacturing and services	Water supply, sewerage and waste management (E), Wholesale and retail trade (G) Accommodation and food service activities (I), Information and communication (J), Real estate activities and renting of non-residential buildings (LA), Dwellings (LB), Other business services (M_N), Public administration, education and health (O_Q), Arts, entertainment and other services (R_S)
Financial Corporations	Financial and insurance services (K)

Table 12: Shows the division of the NAIO1F table into the different industries.

This classification structure is applied consistently whenever IO data is used. In particular, it is essential to allocate the upper part of the TFM, where demand components such as private consumption, government consumption, gross fixed capital formation, inventories, and exports must be assigned to the supplying industries.

5.3.3 Empirical method for filling the gaps in the IO-tables

For each component, the total supply is calculated in all industries, after which the shares corresponding to each disaggregated industry are derived. Table 13 illustrates this allocation for the year 2019.

Supplying industry / Final demand	$\mathbf{C}^H$	$\mathbf{C}^G$	**I**	**ChINV**	**X**
E	23.239,92	0,00	1.470,46	410,40	4.349,27
T	23.773,12	5.079,95	1.145,58	0,00	286.875,93
A	4.711,21	0,01	198,48	-108,93	22.931,49
MC	52.525,02	7.431,81	233.297,61	5.290,39	506.609,75
OMS	637.151,86	539.740,53	113.386,44	2.571,80	247.995,65
FC	69.288,90	0,00	(5.749,97)	1,12	12.714,73
$\sum$	**810.690,03**	**552.252,30**	**355.248,55**	**8.164,78**	**1.081.476,81**

Table 13: The supplying industries of final demand in 2019 in million DKK (DST, 2025b)

When calculating the shares of investment demand across the remaining industries, the value of FC must still be accounted for to ensure the full distribution of total demand. In practice, the value attributed to the OMS industry is computed as the residual, thereby preserving consistency in the total allocation.

To go from annual to quarterly shares, we take the computed share for the year as the share for the first quarter and then interpolate between them to avoid constant proportions within each year[43] ensuring the data from NAIO1F to align with the quarterly frequency of the E-IO-SFC model. The method used for this transformation is interpolation and ARIMA-based extrapolation as shown in the previous section 5.2.1. As an example, the share of private consumption supplied by the energy industry is calculated as:

$$C_H^E \text{ share} = \frac{23.239,92}{810.690,03} = 0,287.$$

Figure 3 illustrates the resulting quarterly time series for this share before interpolation. This series based on the original annual values are shown in black, while the interpolated series appear in blue and extrapolated values

[43]The function used for interpolation is called 'approx'.

appear in red. This process is applied to all calculated shares in the dataset to ensure the IO-derived components of the database reflect realistic quarterly fluctuations.

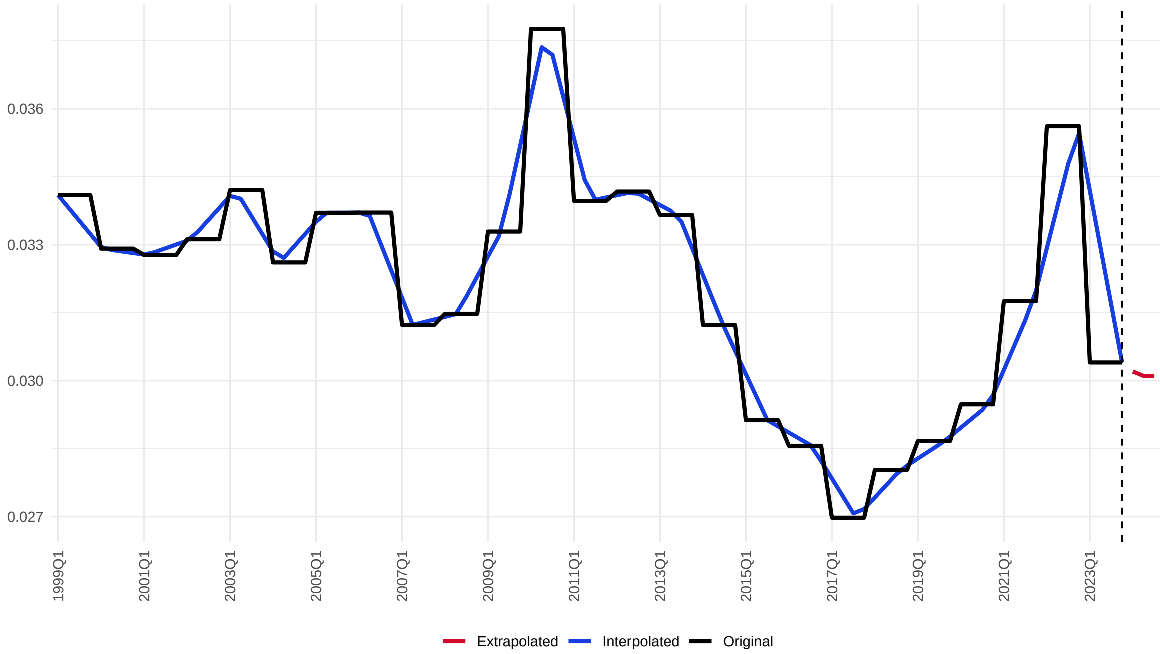


Figure 3: Consumption received by households from the energy industry.

5.4 DST tables for data satisfying the system of equations

In constructing the system of equations, certain variables arise that are not directly represented in either the TFM or the BS, but which nevertheless play a crucial role in simulating both the baseline and endogenous model. Additionally, several components used within the system of equations are not explicitly reported in any single figure from DST, but are instead constructed based on combinations of variables, proxies, or assumptions drawn from different sources.

Taking this into account, the data collection process includes variables related to labor market statistics, prices, depreciation rates, interest rates, expectations of real sales, the desired level of inventories, and the desired level of investment in inventories for the exogenous version of the model. For the endogenous model, the focus shifts towards informing behavioral equations. This involves data on actual, structural, and cyclical unemployment, targeted wage rates and productivity, real export determinants such as foreign income, and the real interest rate.

To inform these parts of the model, the relevant figures have been retrieved from the sources listed in table 14 and table 15. These tables provide the necessary information for calculating deflators, unobserved variables, proxies, and nominal-to-real conversions across industries and sectors. In conclusion, the following section 6 discusses how each specific source from DST and external datasets is used to operationalize the system of equations for both the baseline and endogenous versions of the model.

DST table	Information	Frequency
NKBB10	Employment and hours by socioeconomic status, industry, and seasonal adjustment	Quarterly from 1990Q1 up until 2025Q1
AKU110K	Labor market statistics in total	Quarterly from 2008Q1 up until 2024Q4
AKU100	Labor market statistics in total	Quarterly from 1996Q1 up until 2019Q4
PRIS111	Net price index (2015=100) by commodity group and unit	Monthly from 2001M1 up until 2025M03
NKBP10	Industry divided SNA (production and generation of income by transaction and price unit)	Quarterly from 1999Q1 up until 2024Q4
NAN1	Demand and supply by transaction and price unit	Yearly up until 2024
NAHK	...	...
NASK	Accumulation account and balance sheets	Yearly from 1995 up until 2023
DNRUURI	Mortage rates for households	Monthly from 2003M1 up until 2025M3
MPK18	Lending and deposits rate	Quarterly from 2002Q1 up until 2025Q1
MPK3	Security rates	Monthly from 1985M1 up until 2025Q4
DNRENTM	Lending and deposits rate (Nationalbanken)	Yearly from 1985 up until 2024
FORV1	Confidence indicator for the overall economy	Monthly up until 2025M05
ETILLID	Sentiment indicator for businesses by indicator	Monthly up until 2025M05
UHM	External trade by items, imports and exports, country, unit and seasonal adjustment	Monthly up until 2025M03

Table 14: Information contained in each of the DST tables.

Data sources	Information	Frequency
FRED	Federal Reserve Bank of ST. Louis	
GDPC1	Real Gross Domestic Product, Billions of Chained 2017 Dollars, Quarterly, Seasonally Adjusted Annual Rate	Quarterly up until 2024Q4
CLVMNACSCAB1GQSE	Real Gross Domestic Product for Sweden, Millions of Chained 2010 Swedish Krona, Quarterly, Seasonally Adjusted	Quarterly up until 2024Q4
CLVMNACSCAB1GQDE	Real Gross Domestic Product for Germany, Millions of Chained 2010 Euros, Quarterly, Seasonally Adjusted	Quarterly up until 2024Q4
GDP	Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate	Quarterly up until 2024Q4
CPMNACSCAB1GQSE	Gross Domestic Product for Sweden, Millions of Swedish Krona, Quarterly, Seasonally Adjusted	Quarterly up until 2024Q4
CPMNACSCAB1GQDE	Gross Domestic Product for Germany, Millions of Euros, Quarterly, Seasonally Adjusted	Quarterly up until 2024Q4
WITS	United States AHS Weighted Average in percentage for all products Denmark	Annual up until 2022
CPB World Trade Monitor	CPB Netherlands Bureau for Economic Policy Analysis - Global trade indicator	Monthly up until 2025

Table 15: Information contained in each of the external sources.

6 Data adjustments

In this section, we proceed with the third step outlined by Valdecantos (2023b). It provides a detailed guide for constructing a consistent database that aligns with the underlying system of equations. This database serves as the foundation for the E-IO-SFC model, ensuring that the fully exogenous version of the model adheres to all accounting identities. This database must meet the requirements for running the exogenous baseline model. By exogenous, a purely data-driven version of the model is implied, one that is used to assess whether the underlying data structure is coherent and capable of replicating official statistics from DST at both the industry and sectoral levels. If the model structure fails to reproduce observed figures, the database is considered inconsistent, indicating that it may yield unreliable or implausible results. This step is therefore essential to ensure internal consistency between modeled and observed macroeconomic aggregates; refer to section 4.1 for the five consistency checks.

6.1 Methodological considerations for the baseline model

In addition to the observable variables presented in section 5, the model requires the generation of several non-observable variables, such as prices, interest rates, and expectations about real sales. These are referred to as non-observable because they are explicitly defined through calibrated model mechanisms to ensure consistency across industries and sectors. For example, specific price indices are computed to ensure that the transformation from real to nominal terms fulfills the consistency checks. All variables and operations required for this process are introduced below, including both directly observed data and those derived implicitly through accounting identities, model assumptions, or balancing constraints. Data adjustments that are presented here is relevant for the exogenous E-IO-SFC model.

6.1.1 Allocation keys at the general level

Across the E-IO-SFC models system of equations, numerous macroeconomic allocation keys (α's), taxes (τ's) and financial allocation keys (β's) can be implicitly calculated. To gain an overall understanding of these for the following subsections, a brief description is provided. For instance, by referring to the equations for received distributed income of corporations (D.42r) for the industries within the NFC sector, equation 242-246, the respective expression can be rearranged, allowing the specific α-coefficient to be isolated, as α_r^i is the only unknown factor:

$$\alpha_r^i = \frac{DI_r^i}{(DI_p^E + DI_p^T + DI_p^A + DI_p^{MC} + DI_p^{OMS} + DI_p^{FC} + DI_p^{RoW})}.$$

A similar method is applied in cases where an equation contains a single unknown factor. This approach is particularly useful when analyzing the internal distribution mechanisms of the model, as it may give insights into how income, taxes or capital flows are allocated between sectors and institutional agents. Moreover, this method is especially valuable as it guarantees that the restructured expressions preserve the E-IO-SFC models internal consistency, ensuring that both sides of the equation exactly matches.

6.1.2 Labor market statistics

According to the structure of the E-IO-SFC model, an empirical description of the Danish labor market is required. This includes data on employment, unemployment (both structural and actual), labor force participation, and productivity. To construct this, several data sources and methodological steps have been employed. Employment data is essential for all sectors with employed workers, including the five industries, the financial sector, and the Central Bank. To obtain detailed employment figures across sectors and industries, data has been sourced from a DST table named NKBB10, which contains key employment statistics at the industry level. As NKBB10 exclusively covers employment data without information on unemployment or labor force participation it is used solely to determine the share of employed persons within each industry relative to total employment. This allows for precise figures of the sectoral distribution of employment, as it is based on raw, disaggregated data. Ensuring consistency between employment, unemployment, and labor force data is crucial for the internal consistency of the E-IO-SFC model. Using data sources with differing methodologies would risk introducing discrepancies and result in inconsistent model outcomes. Therefore, maintaining methodological alignment across datasets has been a central concern in the construction of the labor market component of the model. In table 16 below the sectoral/industry shares and levels on employment (in 10.000 persons) are reported in 2019Q4. These shares have been used on the aggregated employment measure shown in figure 4 below[44].

	Energy	**Transport**	**Agriculture**	**Manufacturing and construction**	**Other manufacturing and services**	**Financial corporations**
EMP^{share}	0.0039	0.1133	0.0228	0.1628	0.6709	0.0260
EMP^{level}	1.2	32.8	6.6	47.2	194.0	7.5

Table 16: Sectoral employment shares and employment levels (10,000 persons) in 2019Q4.

Note: The exact employment share for financial corporations is 0.07553618.

In the system of equations for the Central Bank[45], it is evident that an employment level specific to the Central Bank is required. However, according to the tables provided by DST, employment data for the Central Bank is already aggregated within the broader category of financial corporations. Given this limitation, it is necessary to make an assumption and perform a data adjustment to isolate the level of employment of the Central Bank. This step ensures consistency in the model and enables the central bank's labor-related variables to be separately from the rest of the financial sector. To find this employment level, it is decided to calculate the total wage bill for both the Central Bank and the broader category of financial corporations which assumes a equal wage rate between CB and FC. Based on this, the share of the total wage bill attributable to the Central Bank is determined. This share is then applied to the reported employment level for financial corporations as a whole seen in table 16. In doing so, the employment level for the Central Bank is approximated, ensuring a consistent and proportionate allocation of labor within the financial sector. In table 17, updated figures on the employment levels for the Central Bank and financial corporations are reported in 10.000 persons. The values appear reasonable, showing that 508 persons were employed in the Central Bank and 75.027 in financial corporations in 2019Q4.

	Financial corporations	**Central Bank**
EMP^{level}	7.502719	0.050898

Table 17: Updated employment numbers for financial corporations and the Central Bank.

Following the industry-level employment shares, the AKU[46] tables are used to supplement the labor market representation. These tables contain data on employment, unemployment (as defined by the AKU[47]), and the number of individuals outside the labor force. They are employed here due to their methodological consistency across employment and unemployment figures, offering a reliable foundation for modeling purposes.

The AKU tables are based on the Labour Force Survey (Arbejdskraftundersøgelsen, AKU), which is one of Denmark's largest continuous interview-based surveys. It is conducted quarterly using a representative sample,

[44] For presentation purposes, the employment level has been rounded and re-scaled to units of 10.000 persons rather than in millions, which are used within the model.

[45] $EMP^{CB} = \alpha^{CB}_{EMP} \cdot EMP^{FC}$.

[46] AKU100 and AKU110K, cf. table 5.

[47] The documentation on the AKU definition can be found here: DST (2024).

with approximately 72.000 individuals aged 15–74 participating annually. AKU represents Denmark's contribution to Eurostat's Labour Force Survey and has been carried out continuously since 1994. The primary objective of the AKU is to map the population's relationship with the labor market. Individuals are classified as either employed, unemployed (according to the AKU definition), or outside the labor force. The survey is particularly suitable for international comparisons, for analyzing unemployment among individuals not eligible for benefits, youth unemployment (15–24 years), and patterns in working hours (DST, 2025).

A notable limitation of the AKU tables is that they are reported in two separate time series, as shown in table 5 in section 5.1. To address this issue, a backdating procedure has been applied to extend the most recent table ending in 2008Q1. To maintain consistency in data usage and adhere to the accounting methodology used by DST, the AKU100 table is used exclusively to calculate quarterly growth rates. To backdate the AKU110K table, growth rates for employment and the labor force (defined as LF = Employed + Unemployed) have been calculated and applied. For the sake of clarity, the formula used for backdating the employment level is represented below, where g denotes the quarterly growth rate between 2007Q4 and 2008Q1. In the example, we use total employment figures. The same method is applied for backdating the series on the labor force.

$$EMP_{2007Q4} = \frac{EMP_{2008Q1}}{1+g}.$$

The total backdated series of total employment and the labor force is plotted in millions below in figure 4. The series for unemployment is generated as the difference between the labor force and employment.

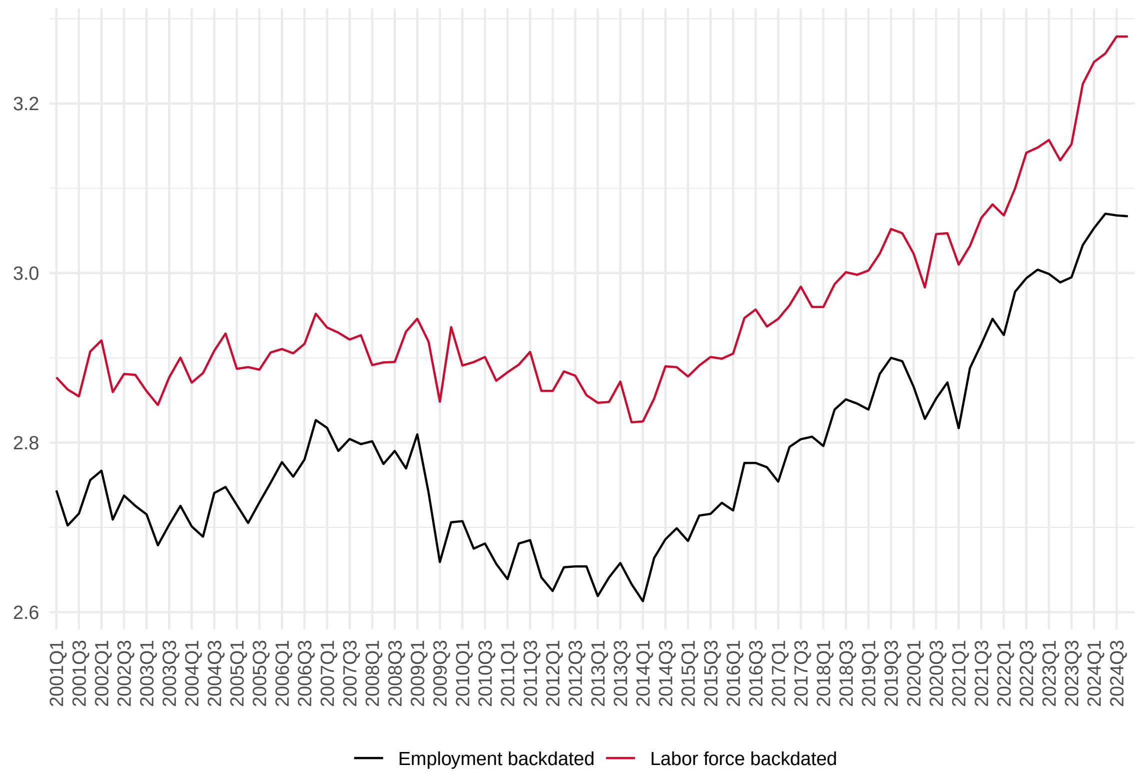


Figure 4: Backted series for employment and labor force.

In addition to being relevant for employment, the labor force and unemployment, it has also been essential to collect data on actual unemployment rate and to construct a timeseries on structural and the cyclical deviation of unemployment. As these indicators are central to the model, they will be further elaborated in section 6.2.

6.1.3 Prices and indirect taxes (τ)

Prices play a crucial role in any SFC model, as they connect nominal flows with real quantities and shape key dynamic mechanisms such as inflation and wage formation. Without an explicit price structure, it would be impossible to consistently translate nominal variables, such as consumption or government expenditure, into real economic activity. Moreover, nominal growth in variables could easily be misinterpreted as real growth. For instance, an increase in nominal flows may simply reflect rising prices rather than a genuine expansion in output or consumption. By introducing price indices, the model can distinguish between movements driven by inflation and those reflecting real economic changes. This distinction is essential for ensuring that the model's dynamics reflect actual economic behavior rather than nominal measures.

In the E-IO-SFC model, price variables serve several operational purposes: (i) they capture inflation dynamics and allow for a proper decomposition of nominal and real developments, (ii) prices influence labor market outcomes, as wages respond, either fully or partially, to price changes and (iii) all nominal flows are deflated using sector-specific or aggregate price indices, ensuring internal consistency and a faithful representation of the real side of the economy.

As mentioned in section 5.4, price data is initially obtained from the Danish statistical source NKBP10 provided by DST. NKBP10 is reported in basic prices, which can be interpreted as producer prices. The base year for this table is 2020, in that sense it matches the rest of the data entering the E-IO-SFC model. As stated in section 5.1, NKBP10 is distributed across six industries defined in the model. For each industry, the nominal and real values are summed, and the ratio between them is used to compute the corresponding producer price.
Figure 5 below displays the resulting producer prices over time at the applied industry level.

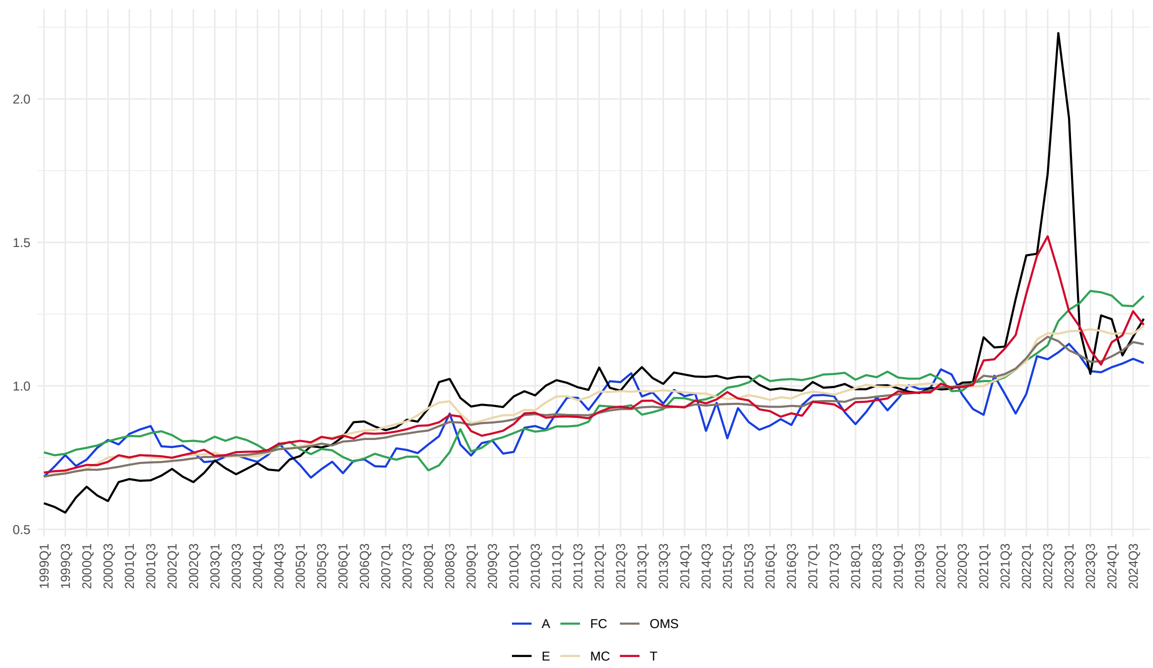


Figure 5: Producer prices in the model.

Having established producer prices by industry, the next step is to calculate the corresponding consumer prices[48]. The objective is to distinguish between basic prices (producer prices) and market prices (consumer prices), as

[48]In previous versions of the model, consumer prices were constructed using the PRIS111 dataset from DST, based on COICOP-classified product groups and their corresponding expenditure weights. Each subgroup was matched to industries using visual classification, and aggregate indices were computed by applying DST's published weights. While this method allowed for detailed price construction, it did not guarantee consistency with producer prices or with the Input–Output structure of the model. The current approach, based on τ-values and NKH1 data, ensures full coherence between producer and consumer prices across all sectors and components of aggregate demand.

both are needed within the model. Conceptually, the difference between the two is captured by their respective τ-values, which reflect the ratio of net indirect taxes to gross output. Using data from the NKH1 table, it is possible to extract the components of aggregate demand, namely private consumption, government consumption, investment, change in inventories, imports and exports. By dividing the nominal values by their corresponding real quantities, one can derive consumer price indices for each demand component. These indices, in turn, allow for the calculation of τ-values using the following relation:

$$\tau_C = 1 - \frac{\sum \alpha_C^i \cdot pp^i}{p_C},$$

where p_C denotes the consumer price index for consumption, pp^i the producer price in industry i, and α_C^i the consumption share of industry i. An identical procedure is applied to compute the τ-values for the remaining demand components.

To compute the τ-values empirically, net indirect taxes for each industry are divided by the total production of each industry. To ensure that the difference between producer prices and consumer prices is fully and consistently captured by the τ-values, it is necessary that the total value of net indirect taxes exactly matches the implied gap between the two price levels across all sectors. In practice, however, small inconsistencies may arise due to rounding errors, data limitations, or structural imbalances in the input-output tables. To preserve the accounting consistency of the model, any residual amount of net indirect taxes not allocated through the standard procedure is assigned to the OMS industry. This creates a residual τ value for OMS, ensuring that the overall system of prices and taxes remains internally balanced. Below the τ-values can be seen for all six industries.

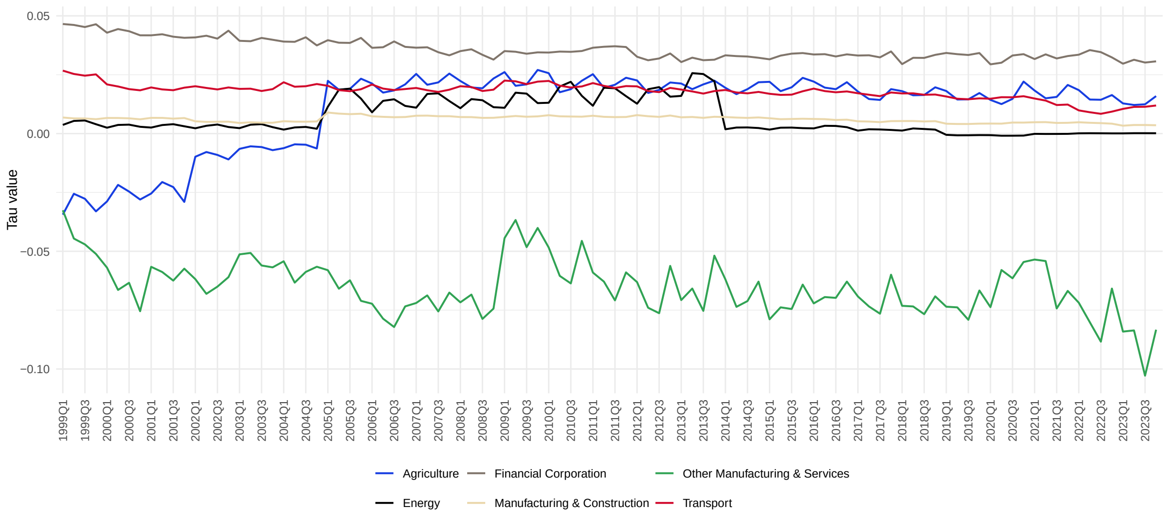


Figure 6: Estimated τ-values used within the E-IO-SFC model.

As the attentive reader may notice, the τ value for the OMS industry is negative throughout most of the sample period. This outcome reflects the fact that the industry, on aggregate, receives more in subsidies than it pays in indirect taxes such as VAT. Since τ captures net indirect taxation as a share of gross output, a negative value implies that the government provides a net fiscal transfer to this industry.

Negative τ-values can arise in sectors where public support is significant, such as education, health care, or certain cultural and social services, which are often classified in the OMS industry. Additionally, since OMS is used as the residual industry for balancing the overall τ-values, any unallocated remainder from the computation of net indirect taxes is absorbed. Therefore, the negative value observed for τ_{OMS} should be interpreted as both a reflection of the industry's fiscal treatment and the E-IO-SFC models internal accounting mechanism.

6.1.4 Imported investments

The total investments in the model should be considered from both a domestic and foreign perspective. Total imports in the model include intermediate goods, investments, and both public and private imports presented below:

$$IM = IG^{E}_{IM} + IG^{T}_{IM} + IG^{A}_{IM} + IG^{MC}_{IM} + IG^{OMS}_{IM} + IG^{FC}_{IM} + I^{E}_{IM} + I^{T}_{IM} + I^{A}_{IM} + I^{MC}_{IM} + I^{OMS}_{IM} + C_{IM}$$

Imported investments are defined as an α-share of the total accumulated investments across the six producing industries. According to the data breakdown from the DST(NAIO1F) as described in section 5.3, the imported share of investments is assumed to be a time-varying yet fixed parameter relative to the total accumulated investments within each sector and industry. The α-parameter is derived from the input-output table by dividing imported gross fixed capital formation by the sum of imported and domestic gross fixed capital formation, representing total investments. Data on domestic and foreign gross fixed capital formation is only available at an annual level up to 2021. Therefore, it was necessary to first convert the data from annual to quarterly frequency before performing an extrapolation to extend the series until 2024Q3.

6.1.4.1 Depreciation rate

In order to relax the assumption of a fixed exogenous depreciation rate, δ^i, which is presented below:

$$D^i = \delta^i \cdot (K^i_{t-1}),$$

it has been necessary to incorporate industry- and sector-specific capital stock data sourced from DST NAHK database. However, as these data are available only in the form of annual opening and closing balances, a procedure is required to interpolate quarterly capital stock figures in a consistent and economically meaningful way. Rather than applying linear interpolation, where the annual change in capital stock is evenly distributed across the four quarters, a more refined method has been utilized. Specifically, the annual change in capital stock is apportioned across quarters in proportion to the observed quarterly distribution of gross fixed capital formation (investment) from NASD24. This approach assumes that intra-annual variation in investment serves as a proxy for the timing of capital accumulation, thereby capturing seasonal or cyclical dynamics in capital formation more accurately.
By construction, the level of capital stock in the fourth quarter of each year is anchored to the reported closing balance from DST. This ensures consistency with the official annual data while allowing for quarterly data.

Practical Example
To illustrate the method, consider the following case. Since the annual closing balances of the capital stock are known, it is only necessary to construct the intermediate quarterly values for Q1, Q2, and Q3.

Annual gross fixed capital formation (2023): 10,000
Quarterly distribution of investment:

- Q1: 3,000 (30% of annual investment)
- Q2: 2,500 (25%)
- Q3: 2,000 (20%)
- Q4: 2,500 (25%)

Capital stock:

- Closing balance 2022 (i.e., opening balance 2023): 30,000

- Closing balance 2023: 40,000

The annual change in capital stock is thus:

$$\Delta K_{2023} = 40{,}000 - 30{,}000 = 10{,}000$$

Using the quarterly investment shares as weights, the capital stock is estimated to evolve as follows:

$$K_{Q1} = 30{,}000 + 10{,}000 \cdot 0.30 = 33{,}000$$

$$K_{Q2} = 33{,}000 + 10{,}000 \cdot 0.25 = 35{,}500$$

$$K_{Q3} = 35{,}500 + 10{,}000 \cdot 0.20 = 37{,}500$$

$$K_{Q4} = 40{,}000 \quad \text{(by construction)}$$

This interpolation technique ensures that the quarterly evolution of the capital stock reflects the temporal distribution of investment, thereby improving the alignment between capital accumulation and economic activity within the year. Returning to the depreciation rates, these are computed by dividing the estimated values of the capital stock by the corresponding depreciation levels and other changes in non-financial assets in each period, obtained from NASD24 and NABK10, respectively. This approach ensures internally consistent, sector-specific depreciation rates that reflect both the capital intensity and the actual rate of capital consumption over time.

	H	**E**	**T**	**A**	**MC**	**OMS**	**FC**	**G**
Depreciation rates	0.0072	0.0055	0.0355	0.0085	0.0402	0.0150	0.0380	0.0138

Table 18: Depreciation rates for the different sectors/industries in 2019Q4.

6.1.5 Interest rates

In the E-IO-SFC model, financial assets and liabilities play a central role. Since these may generate interest flows over time, incorporating interest rates is essential to capture the financial dynamics and inter-sectoral relationships in the economy. Interest rates form the transmission between the level of financial stocks and the flows of interest income and expenses. They influence income distribution, spending behavior, and changes in sectoral balance sheets. Without explicitly modeling interest rates, it would be impossible to consistently explain how changes in wealth and debt affect economic activity over time.

In the model, sector-specific interest rate series are constructed to capture relevant financial flows. These include official central bank rates, market interest rates on securities, and deposit and lending rates for households, non-financial corporations, financial corporations, and the government. The data used to construct these series are sourced from various datasets provided by DST. Central bank rates are obtained from the DNRENTM table, which provides end-of-month observations for both lending and deposit facilities at the Danish central bank. Monthly values were converted into quarterly frequency by averaging across the three months of each quarter, following the same procedure used for prices (see section 6.1.3). In the E-IO-SFC model, these rates serve as policy instruments used by the Central Bank to influence macroeconomic conditions. The lending rate reflects the cost at which commercial banks can borrow from the central bank, affecting the broader structure of interest rates in the economy. The deposit rate determines the return on excess reserves held by banks, influencing their willingness to supply credit. Both rates play a key role in shaping consumption, investment, and portfolio decisions across sectors, and are therefore critical in the transmission of monetary policy in the model.

Deposit and lending rates for the various sectors and industries are based on the MPK18 table, which reports average interest rates in Danish financial institutions. This includes both deposits and loans for households, corporations, and the government, and these monthly series were likewise converted to quarterly averages. These rates are key to determining the interest income and expenses of each sector, and thereby influence sectoral saving, borrowing, and overall net lending positions. In particular, they affect household consumption and saving decisions, firms' investment behavior, and the government's interest payments on debt. By linking these rates to sector-specific balance sheets, the model captures how changes in financial conditions propagate through the economy.

However, for households specifically, the MPK18 dataset is not considered suitable to represent borrowing costs, since it only covers bank interest rates. As approximately 80% of the household debt in Denmark consists consistently of mortgage loans (Danmarks Nationalbank, 2024), relying solely on MPK18 would risk significantly overestimating actual borrowing costs. Instead, the DNRUURI table from Danmarks Nationalbank is used, as it provides average interest rates on mortgage loans and is therefore considered a more accurate reflection of household borrowing conditions.

Finally, interest rates on securities were drawn from the MPK3 table, which includes rates on bonds. The average yield on all bonds was used as a representative interest rate for securities held by each sector. In the E-IO-SFC model, this rate determines the return on bond holdings across sectors and thus directly affects their income from financial assets. Changes in bond yields also influence portfolio allocation decisions between money, deposits, and securities, and serve as a transmission channel for monetary policy through asset markets. This dynamic is supported empirically in the estimation of the financial beta for securities reported in section 7.2.3.

As the MPK18 and DNRUURI tables starts in 2002 and 2003, respectively, the earliest observed values are assumed to hold for all prior periods with missing data. By systematically linking these interest rate series with sector and industry-specific stocks, the model computes gross interest flows for each sector and industry. This ensures full internal consistency between monetary flows and balance sheet dynamics throughout the entire economy.

All interest rates used in the model is converted to effective quarterly rates using the standard compounding transformation:

$$r_q = (1 + r_{\text{annual}})^{1/4} - 1.$$

This adjustment reflects two key considerations. First, the model operates with quarterly time steps, and interest payments are computed on the outstanding debt from the previous period. Since this implicitly results in compounding over the year, using nominal rates divided by four would underestimate the actual interest flows. The compounding formula ensures consistency between the timing of interest payments and the effective annual rate reported in the data.

Second, the data are expressed in percentage points. That is, a value of 4.0 in the dataset corresponds to 4% per year, or 0.04 in decimal form. Before applying the compounding transformation, the values must therefore be scaled by dividing by 100. Without these adjustments, interest flows would be misaligned with the reported rates and would either overstate or understate the true financial burden associated with debt.
In addition, an implicit interest rate for F.6. (insurance technical reserves) was constructed, as this category is assumed to correspond to the series D44 in DST's financial accounts. The rate is derived using the following formula:

$$\iota^H_{F.6} = \frac{D.44^H_r}{F.6^H_{S(-1)}},$$

where $\iota^H_{F.6}$ is the interest rate on F.6 for the household sector, $D44^H_r$ denotes the reported interest income for households under D.44, and $F.6^H_{S(-1)}$ is the lagged stock of F.6 held by households. This approach ensures that the imputed interest rate reflects actual income flows and the underlying asset position.

For each financial instrument in the model, except monetary gold and SDR (F.1), financial derivatives (F.7) and trade credits (F.8), interest flows are calculated as the product of the relevant interest rate and the lagged stock of the asset or liability. This approach ensures internal consistency with the timing convention of the E-IO-SFC model, where interest accrues based on the stock position at the beginning of the period. The computed flows are as follows:

$$\text{Interest}_{F,i} = \iota_{F,i} \cdot F_i^{S(-1)},$$

where $F \in \{F.2, F.3, F.4\}$ and i indexes sectors such as households, the five industries, financial corporations, government, rest of world. For F.6 (insurance technical reserves), which corresponds to the D.44 category in the national accounts, the interest rate is not directly observed and is therefore calculated implicitly. The imputed interest rate is defined as:

$$\iota_{F.6}^i = \frac{D.44_r^i}{F.6_{S(-1)}^i}.$$

This ensures that interest flows from F.6 match the recorded income in the national accounts. In the national accounts, the total interest income for each sector and industry (D.41D.45) includes components that cannot be directly attributed to any observed financial instrument in the model. After calculating the interest flows for F.2, F.3, and F.4, a residual interest flow remains. This residual was initially attempted to be attributed to F.7 and F.8 by constructing implicit interest rates using the same method as for F.6. However, this approach resulted in unrealistically large values, suggesting that such a method was not appropriate for these instruments.
Given the limited importance and poor data quality of F.7 and F.8, a pragmatic approach was chosen: the residual interest income is evenly split between the two instruments. This ensures that the model preserves consistency with the reported total interest income, without introducing distortions from unreliable imputations. Thus, the interest income from F.7 and F.8 is given by:

$$\text{Residual}^i = D.41D.45_r^i - \text{Interest}_{F.2}^i - \text{Interest}_{F.3}^i - \text{Interest}_{F.4}^i.$$

$$\text{Interest}_{F.7}^i = \text{Interest}_{F.8}^i = \frac{1}{2} \cdot \text{Residual}^i.$$

This method allows the model to preserve the full structure of financial flows even when some rates or flows are not directly observed in the data. Figure 7 illustrates the effective deposit and lending interest rates for both households and NFCs over the full sample period. The figure captures key monetary policy events, including the global financial crisis (2008–09), the low interest rate regime of the 2010s, and the rapid tightening of policy starting in 2022. The spread between F.4 and .F2 for each sector represents the interest margin earned by financial corporations, which reflects their profitability from traditional inter-industry activities.

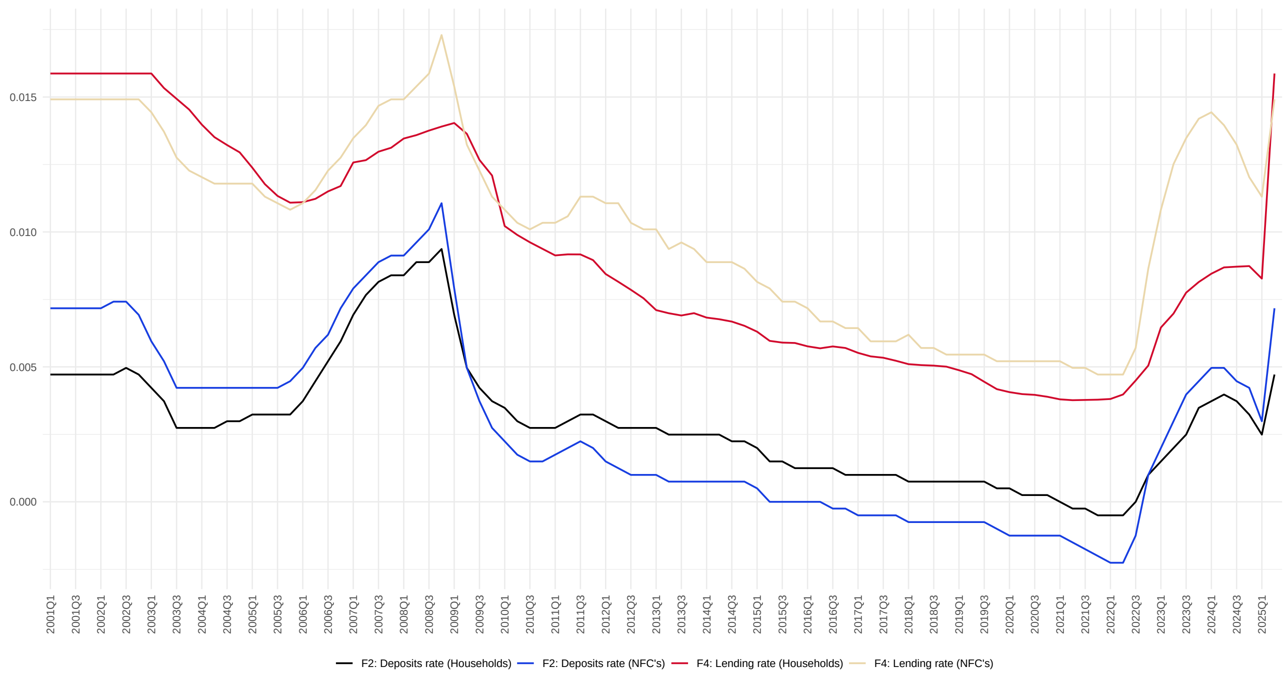


Figure 7: Deposit and lending rates for households and NFCs.

Note: The Y-axis is expressed in decimals. Annual interest rates are converted to effective quarterly rates.

6.1.6 Expectations of real sales

Expected real sales[49], as presented in the equation below, are used at the industry level to determine actual real production for each industry, as shown below. The definition builds on the assumptions in Godley and Lavoie (2012, p. 319), where actual real sales are multiplied by an indicator, represented in their case as a random variable.

$$y_P^i = y_e^i + i_{inv,d}^i.$$

This requires a measure of expectations regarding sales. In DST, a series called 'ETILLID', reflected as a sentiment indicator, is available as a monthly series[50]. The indicator can be used to obtain a quick snapshot of the assessment of the overall business sector and the underlying industries compared to the immediate past. According to the methodological documentation for 'ETILLID', cf. DST (2024a), it is a time series generated based on a qualitative survey assessing the past three months' developments and expectations for the next three months. 'ETILLID' is constructed so that the value of 100 reflects the historical average, and values above 100 therefore indicate positive assessments. Each deviation of 10 reflects that assessments deviate by one standard deviation from the variation in the aggregated confidence indicator. According to DST (2025d), a typical situation would be between 90 and 110. 'ETILLID' further captures historical shocks in the Danish economy over a period of more than 25 years, as well as periods of economic growth. The indicator is based on monthly data, which in this case has been converted into quarterly observations using a simple arithmetic average. Based on these facts regarding 'ETILLID', it is considered a valid measure for creating expected real sales, a variable that will play a

[49] Note here: Expected sales are calculated based on sales at basic prices to ensure consistency with real production, which is always measured at basic prices. This is because: (i) sales measured at market prices will create a level discrepancy compared to production measured at basic prices; and (ii) sales at market prices include imported final and intermediate goods, which overstate actual domestic sales.

[50] The indicators included in the confidence indicator are standardized and weighted in the aggregation, with industry accounting for 40%, construction for 5%, services for 30%, retail trade for 5%, and consumer expectations for 20%. In the aggregation of the business confidence indicator, consumer expectations are not included, and the 20% weight is distributed proportionally among the four business sectors (DST, 2024a).

crucial role in the model as it determines the production decision.
The first step in the transformation is to divide the series by 100, which constitutes a linear transformation. The result is shown by the black line in figure 8.

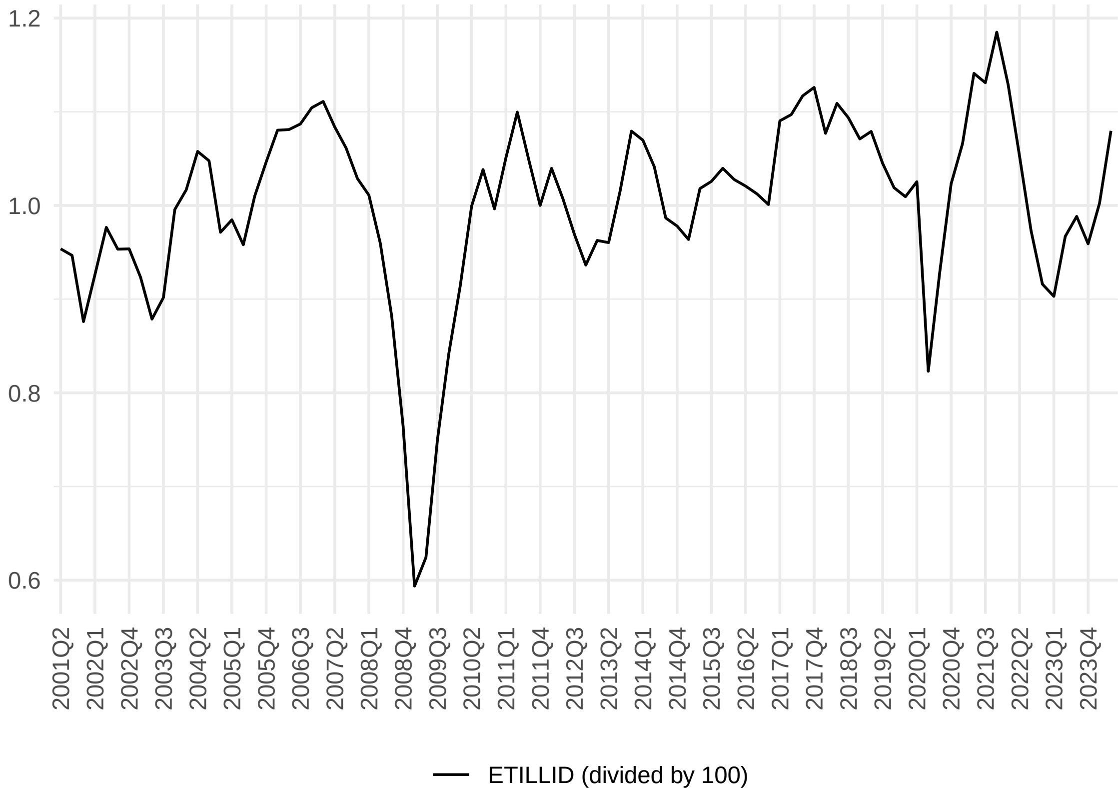


Figure 8: Deviations from 100 for the sentiment indicator.

The 'ETILLID'-series is to be used implicitly in the calculation of the desired level of investment in inventories, as exemplified by equation below at industry level. Implicitly, the desired level of investment in inventories for industries is obtained by isolation in the equation presented below:

$$y_P^i = y_e^i + i_{inv,d}^i.$$

If expected sales are higher than production, negative values will generate for the desired level of investment in inventories. Negative values occur when firms use their inventories (due to an economic boom) and therefore do not invest further. Positive values indicate that firms are investing more in inventories during bad times (due to an economic downturn), thereby preparing for better times.

To validate the method on current data, the deviation in 'ETILLID' has been applied to real sales at basic prices for the overall economy. As seen in figure 9, expected real sales are higher in periods when the economic confidence is positive, whereas the opposite is observed during downturns, as specifically seen in 2009Q1.

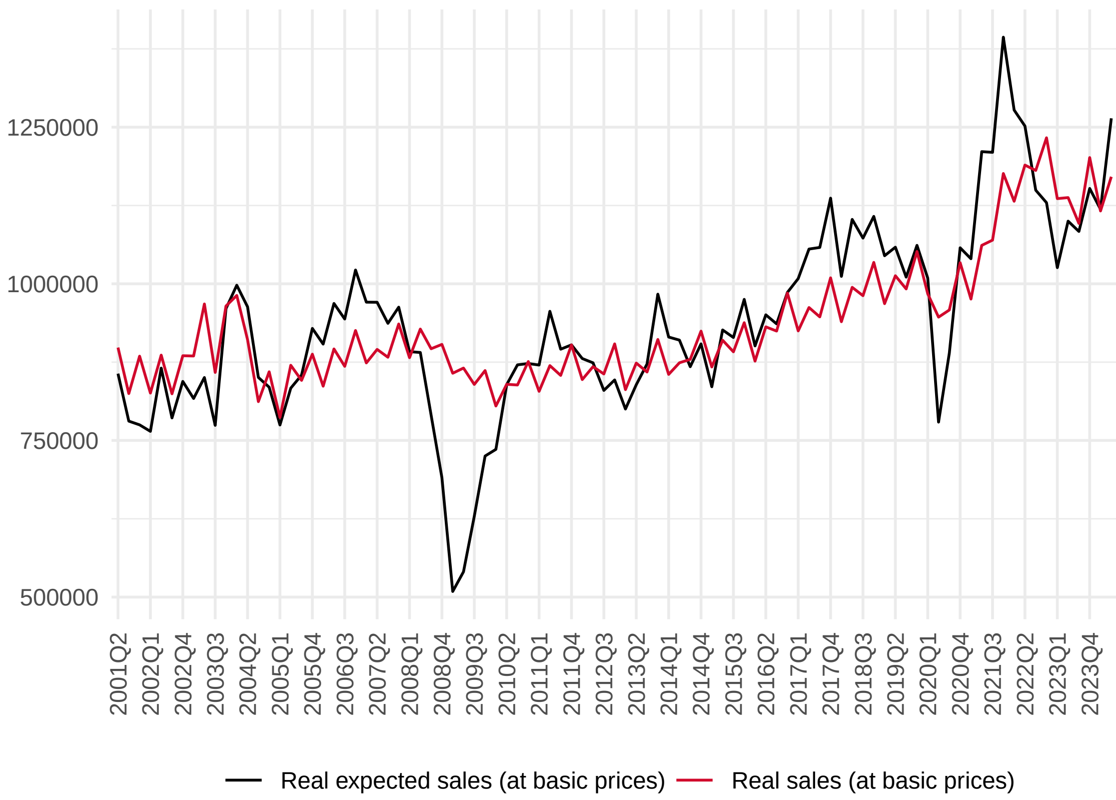


Figure 9: Actual real sales vs. expected real sales in millions.

6.1.7 Desired stock of inventories

In the system of equations describing the model's dynamics we envisage a process where firms define their production decision based on their expectations about future sales and a desired level of inventory accumulation. In order to incorporate the desired stock of real inventories, $inv_d^i = inv_{t-1}^i + i_{inv,d,t}^i$, certain conditions must be met.

The first step is to define the inventory levels for each industry at time zero, INV_0^i. This information is typically not reported in publicly available databases, such as DST, and therefore it is necessary to develop an alternative approach. In this case, the longest available time series, NAN1 (from 1966), on a yearly basis for changes in inventories has been chosen (DST, 2024b). Since the database generating the exogenous E-IO-SFC model starts in 2001Q1, it is chosen to accumulate the yearly series from 1966 to 2000[51] in order to obtain the best proxy for the initial inventory level[52]. This accumulated value for changes in real inventories, is distributed among the industries based on input-output shares as presented in section 5.3. The initial value for the level of real inventories, which is restricted to zero (meaning that the initial level can't be negative), in 2000Q4 is presented in table 19 as INV_0^i. To obtain INV_t^i, the following calculation is performed starting in nominal terms:

$$INV_t^i = INV_0^i + \sum_{t=0}^{t} \Delta INV_t^i,$$

$$inv_t^i = \frac{INV_t^i}{p_p^i}.$$

[51] This value is set to be the value corresponding to 2000Q4.

[52] Nominal data has been used to calculate these measures and thereafter divided by producer prices for each industry, cf. section 6.1.3.

	Energy	Transport	Agriculture	Manufacturing and construction	Other manufacturing and services
inv_0^i	0	0	15568.8	88236.7	6983.23

Table 19: Overview of the real stock of inventories for the industries.

The information presented in table 19 can be used to determine the desired level of real inventories for each industry presented in the start of this section.

6.1.8 Desired level of investment in inventories

In order to run the exogenous E-IO-SFC model, the β_{inv}^i-parameters in the equation presented below, must be obtained:

$$i_{inv,d}^i = \beta_{inv}^i \cdot (inv_d^i - inv^i).$$

By definition, these parameters should ideally be positive and lie between 0 and 1, or go higher than 1 if firms over-adjust. This is generally the case, but due to the way the system of equations is defined and the components of expected sales, cf. section 6.1.6 certain limitations arise. The parameters are obtained by isolating β_{inv}^i in the above equation. As an example the method is presented for the i-industry below[53]:

$$\beta_{inv}^i = \frac{i_{inv,d}^i}{(inv_d^i - inv^i)}.$$

In isolation, the β_{inv}^i-value would be negative in cases where expected sales exceed production, cf. section 6.1.6 or in situations where the stock of inventories is higher than the desired inventories. Moreover, the absolute values of the β-coefficients become high when the desired inventory level, inv_d^i approaches the actual inventory level inv^i, as this results in division by a very small number, which mathematically generates a high value for the β_{inv}^i-parameter. From an economic perspective, a negative β_{inv}^i-value does not make sense. This is happening because the current method used defining the IO structure is only consistent at the aggregated level, and thereby not being consistent at the industry level. Specifically the current IO coefficients are not consistent with the identity that says that sales have to equal to output, which is why adjustments is necessary. This is further commented in section 8. Values for the β_{inv}^i can be found from 2001Q2 to 2002Q1 below, and in this period the numbers are behaving as expected.

	β_{inv}^E	β_{inv}^T	β_{inv}^A	β_{inv}^{MC}	β_{inv}^{OMS}
2001Q2	0.3773772	1.8237852	0.0530802	0.09413872	0.8757564
2001Q3	0.4252423	2.0993211	0.0856113	0.14550388	1.0864406
2001Q4	0.7484958	4.8809216	0.1312236	0.21851386	2.4828166
2002Q1	0.3774995	2.9128081	0.0666512	0.12803207	1.4720977

Table 20: Overview of speed of adjustment for desired investment in inventories.

6.1.9 Reformulation of the tax equations

Originally, the tax equations regarding households and the rest of the world, equation 496 and 606 respectively, contained two unknown factors:

$$T^{H,RoW} = (WB^{H,RoW} + SB_r^{H,RoW} - SC_p^{H,RoW}) \cdot \tau_{H,RoW}^T + NI^{H,RoW} \cdot \tau_{H,RoW}^{NI}.$$

[53] $i_{inv,d}^i$ is obtained as a residual, by taking the difference between y_P^i and y_e^i.

Therefore, a reformulation was necessary to follow the same procedure as described in section 6.1.1. In an effort to approximate the ordinary income tax rate (τ_H^T) used in equation 496, a method based on data from PSKAT1 in DST and the aggregation of relevant tax components on an annual basis was developed. The aim was to derive a proxy for the average income tax rate by summing up the relevant components[54]. For households, the proxy was constructed using the following formula:

$$\tau_H^T = \frac{\text{Sum of the relevant PSKAT1 components}}{(WB - SC_p - SB_r)}.$$

With this definition, the tax on net interest income, τ_H^{NI}, remained the only unknown. However, when implicitly calculating this tax rate, the results were unrealistically high (e.g., to 355% in 2001). This could be due either to an underestimated proxy for the income tax or to a very low value of net interest income (NI). To improve this, the NI component was revised to include only receivable variables from D.41, D.44, and D.45, reflecting that tax should only be paid on what is received and not on what is paid. While this approach provided more realistic estimates, it still produced tax rates exceeding 100% in certain periods, which could again be due to low values of received interest. A plausible explanation is that D.41 does not include all actual interest payments received and paid by agents. Instead, it only accounts for the portion corresponding to the "reference interest rate." This follows the concept of Financial Intermediation Services Indireclty Measured (FISIM), where total interest payments are split into two parts: one recorded in D.41 and the other included in the gross value added of financial corporations (as production) and the consumption of financial services (by households, the government, and NFCs). Therefore, since D.41 includes only part of the actual interest payments, using received interest as the reference point for calculating the implicit tax rate on financial income may lead to an artificially low base for these interest payments (Danmarks Statistik, 2025).

To ensure consistent and realistic estimation of the tax burden over time, the equation was ultimately reformulated by assuming a single tax rate on total income, including received interest income (NI_r) for both households and the rest of the world:

$$T^{H,RoW} = (WB^{H,RoW} + SB_r^{H,RoW} - SC_p^{H,RoW} + NI_r^{H,RoW}) \cdot \tau_{H,RoW}^{NI},$$

which allows the procedure described in section 6.1.1 to be followed.

[54] The relevant components from PSKAT1 is: Ordinary income tax, lower limit (E.1.1.4), Additional income tax, intermediate limit (E.1.1.5), Additional income tax, upper limit (E.1.1.6), Equalization tax (E.1.1.7), Other central government tax (E.1.1.8), Church tax (E.1.2), County tax (E.1.3), Municipal tax (E.1.4), Healthcare contribution (E.1.10), Tax for limited taxation (E.1.11), Income tax for foreign scientists (E.1.13), and Labour market contributions (F).

6.2 Methodological considerations for the endogenous model

This subsection focuses on the endogenous model. For this part, data is constructed to support the modeling of behavioral relationships, with the data-generating process primarily centered on the variables included in the behavioral equations. Such variables could for instance be, cyclical deviations in unemployment, the targeted wage rate, and measures of productivity supported by relevant theoretical foundations. Additional variables used in the behavioral equations and present in the DST are discussed within the context of the specific model applications in which they are relevant in section 7.

6.2.1 Actual-, structural-, and the cyclical deviation in unemployment

The actual and structural unemployment rates are central to be able to construct the labor market structure in the E-IO-SFC model. Specifically, these relationships are formalized in equations 307 to 320 for the all sectors entering the model. For the purpose of reading flow the equations for production, employment, unemployment and is corresponding rates is presented below:

$$pr^i = \frac{y_P^i}{EMP^i},$$

$$EMP^i = \frac{y_P^i}{pr^i},$$

$$UE = LF - EMP^E - EMP^T - EMP^A - EMP^{MC} - EMP^{OMS} - EMP^{FC} - EMP^{CB},$$

$$UE_r = \frac{UE}{LF},$$

$$UE_s = UE_r \cdot \theta,$$

$$UR_rURs = UR_r - UR_s.$$

The above equations play a crucial role in modeling labor market behavior, as reflected in the behavioral equation for the general wage rate presented in estimated form in section 7.2.2. Time series for employment and unemployment, as described in section 5.4, have been constructed based on AKU tables provided by DST. These data series have been utilized for the creation of the actual and structural unemployment and furthermore the cyclical deviation in the unemployment rate. According to Gottfries (2013, p. 152), every individual at any given time can be categorized as either employed, unemployed, or outside the labor force. These stock variables are key determinants in characterizing the overall state of the labor market. The unemployment rate is typically measured as a percentage of the labor force, as illustrated above. This rate fluctuates with the business cycle and is presented as the black line in figure 10 below.

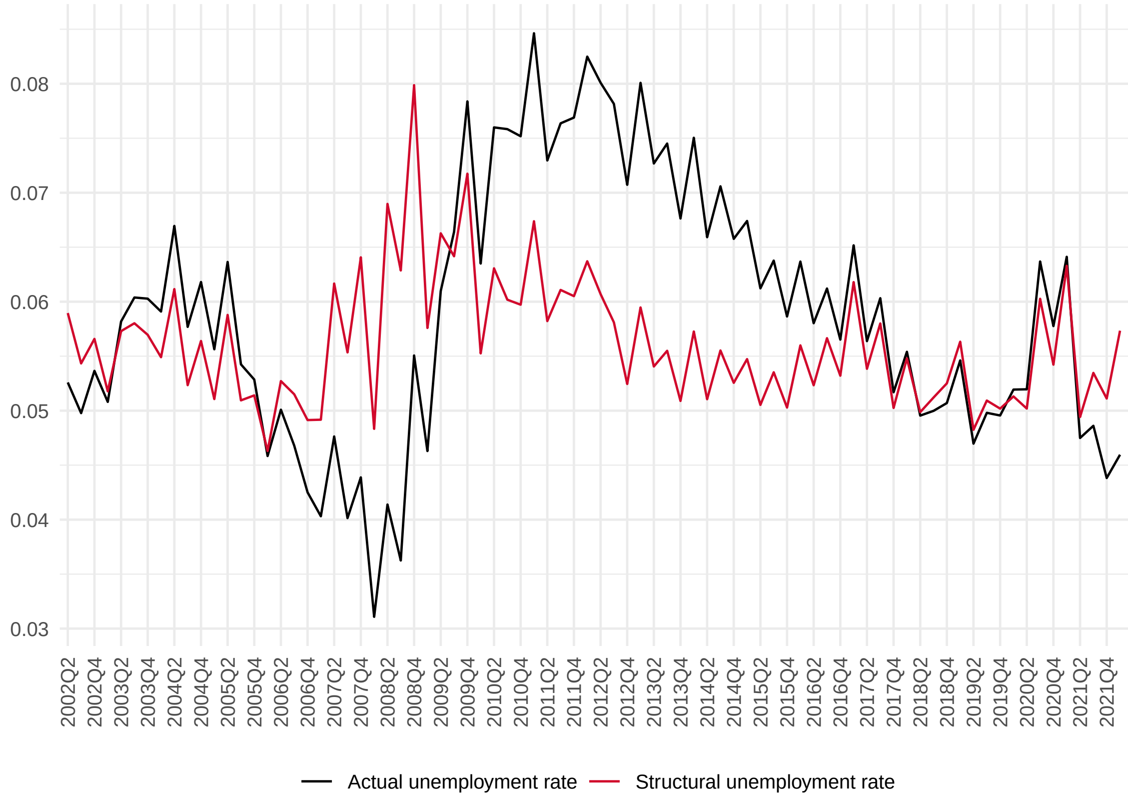


Figure 10: Actual and structural unemployment in percentage of the labor force.

In contrast, structural unemployment refers to an unemployment level consistent with a "normal" economic situation. That is, the level of unemployment compatible with stable price and wage developments. According to economic theory, this is also known as the "natural rate of unemployment", representing an equilibrium level. The structural unemployment rate is presented as the red line in figure 10. At this level, firms have no incentive to change relative wages, as unemployment is considered to be at its natural level (Gottfries, 2013, p. 165).

According to Bostrup (2020), structural unemployment cannot be directly observed and must instead be estimated. Consequently, there is some degree of uncertainty regarding its exact level. As shown in figure 11, structural unemployment has decreased by approximately 220.000 individuals from 1996 to 2020. Such reductions are often attributed to more stringent labor market regulation or less generous unemployment benefits [55].

[55] In the Danish context, such reforms could include, for instance, Helle Thorning's 2012 tax reform that included a reduction in the indexation of transfer incomes, including unemployment benefits, by 5 percent. This reduction was phased in over the period 2016–2023 (Bostrup, 2020).

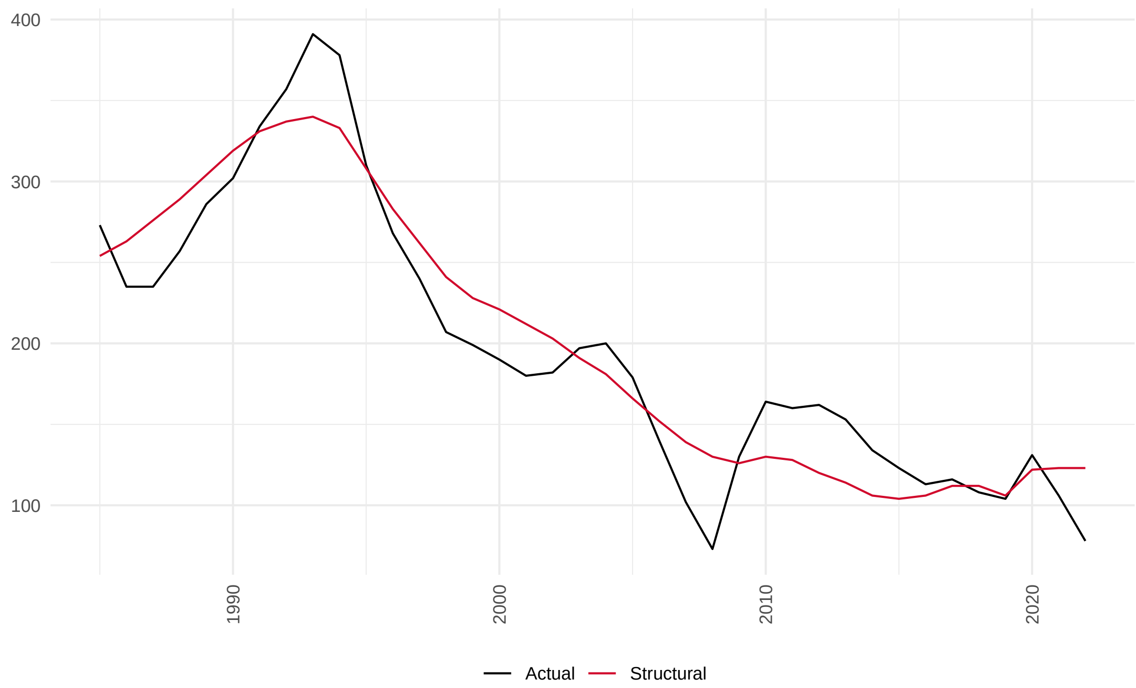


Figure 11: Data used for creating the structural unemployment rate.
Source: Calculations based on data from Bostrup (2020).

As shown in figure 11 above, Bostrup (2020) provides an estimate of the structural unemployment rate. This annual time series is used as input in the calculation of the structural unemployment rate in the E-IO-SFC model, cf. the UE_s-equation presented above. However, since E-IO-SFC is a quarterly model, the annual ultimo values are extracted and transformed into a quarterly time series through linear interpolation, cf. section 5.2.1. The purpose of using this data is solely to capture the structural component of unemployment. Therefore, the percentage deviation between actual and structural unemployment is calculated using the following formula:

$$\text{Pct. deviation} = \frac{\text{Structural Unemployment} - \text{Actual Unemployment}}{\text{Actual Unemployment}}$$

This percentage deviation is then multiplied by the observed unemployment rate (as shown in figure 10) to obtain a structural unemployment rate based on the labor market conditions from the AKU table. Since the original time series only extends to 2022, and the E-IO-SFC model requires data up to 2024Q4, the percentage deviation series is extended by eight quarters. This is done through extrapolation, with the applied method described in section 5.2.1. The resulting time series used as input in the E-IO-SFC model is illustrated in the previously shown figure 10. The structural unemployment will be exogenous in the model. Finally, the cyclical deviation serves as an explanatory variable in the behavioral equation for the wage rate in equation 216. This variable, defined as the difference between actual and structural unemployment, is plotted below. A positive value indicates that unemployment is above its structural level, typically reflecting an economic downturn, while a negative value suggests the opposite. The application of this time series in an econometric context is further discussed in section 7.2.2.

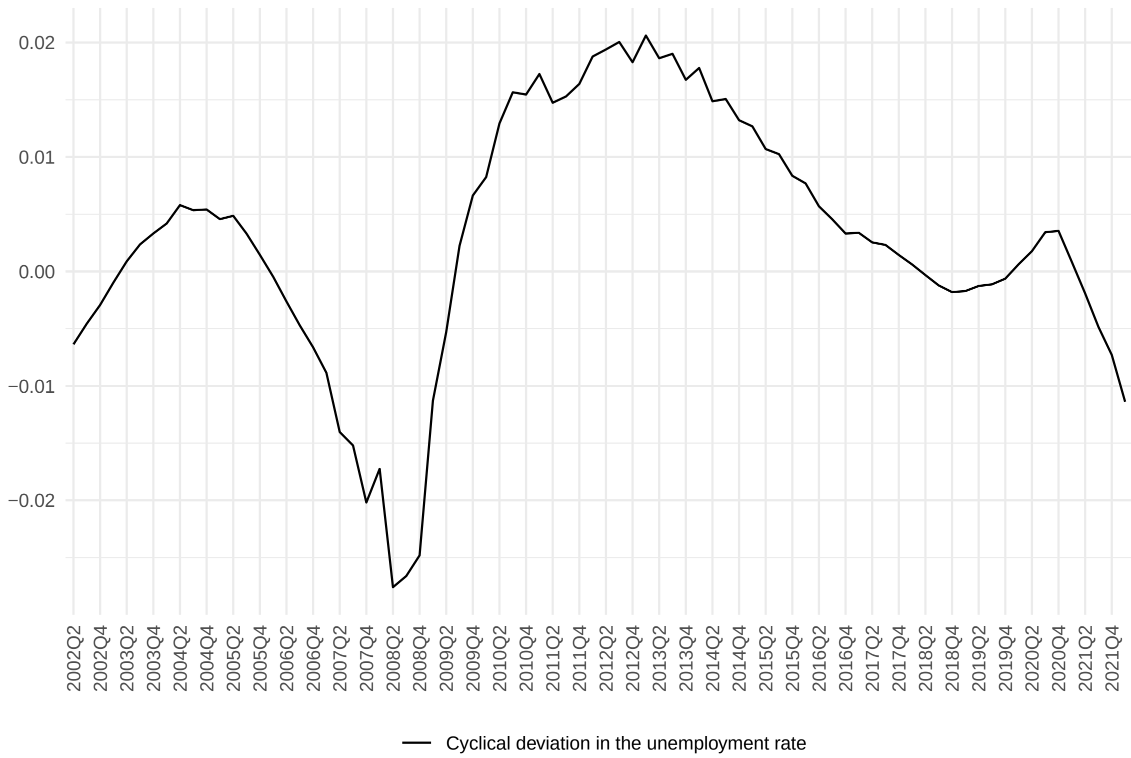


Figure 12: The cyclical deviation in unemployment.

6.2.2 Targeted wage rate, inflation and productivity

In relation to the behavioral equation for the wage rate presented in section 7.2.2 and also seen in equation 216 in the system of equations, it has been necessary to construct three different variables. One of these variables is the targeted wage rate that incorporates expectations in the wage setting process. According to Gottfries (2013, p. 239), the relationship between wage and price changes is best understood through the framework of the underlying Phillips curve, where inflation is defined as the difference between wage growth of the labor productivity:

$$\pi = \frac{\Delta W}{W} - \frac{\Delta E}{E}.$$

Since inflation is primarily driven by changes in wages, it is plausible to define the targeted wage as:

$$W_T = W_{t-1} \cdot (1 + \pi_{t-1})$$

This definition reflects a backward looking mechanism, where wage earners adjust nominal wages based on past inflation to maintain their real purchasing power. The targeted wage, W_T, can thus be interpreted as the wage level consistent with preserving the real wage, conditional on inflation being constant[56]. The time series for the actual and targeted wage rates is plotted below in figure 13. The red line, represents the targeted wage rate that would have been targeted by economic agents under the assumption that expectations are formed as defined above, solely based on observed inflation and previous wage levels. Deviations between the targeted and actual wage rates may be interpreted as a reflection of short, term wage rigidities, unexpected economic shocks, or

[56] Quarterly inflation is created as following which is used in the estimations for the wage rate:

$$\pi_t = \frac{P_{C,t} - P_{C,t-4}}{P_{C,t-4}}$$

.

other frictions in the wage, setting process. For instance, wage rigidities may stem from collective bargaining agreements, minimum wage laws, or long-term employment contracts that prevent immediate wage adjustments. Unexpected shocks could include sudden changes in productivity, financial crises, or global disruptions such as pandemics. Additionally, other frictions, such as delayed negotiations and information asymmetries further contribute to discrepancies between expected and realized wage developments.

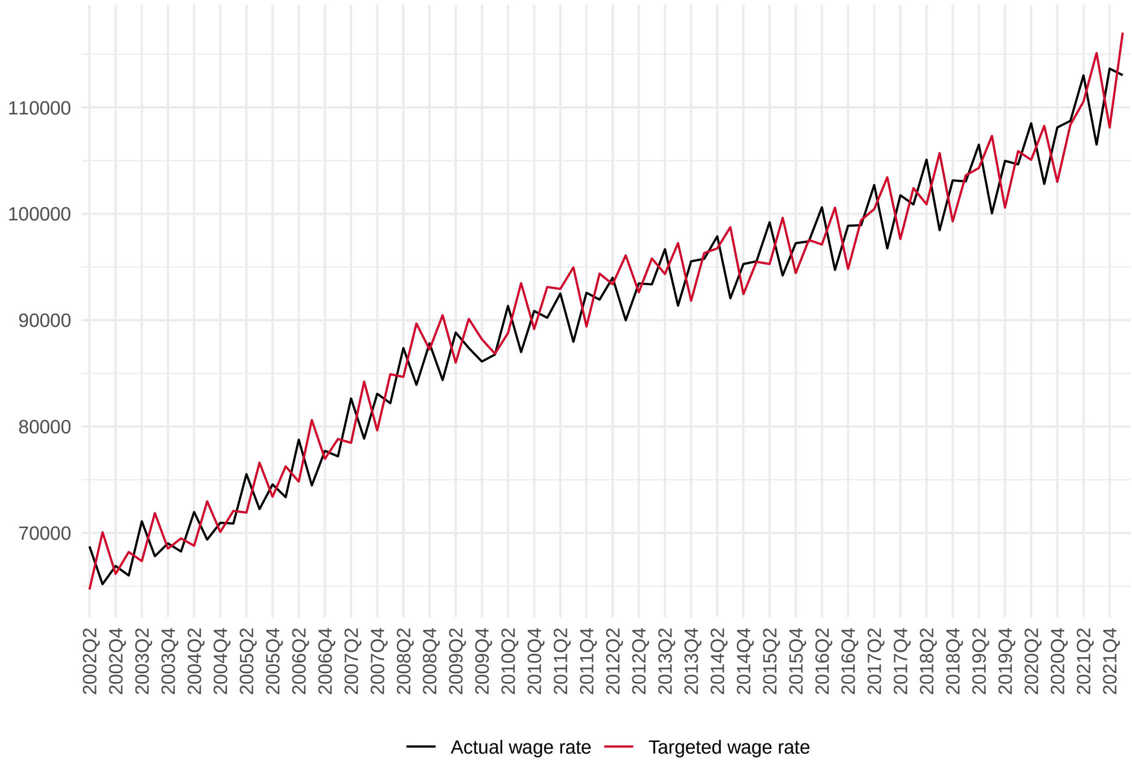


Figure 13: The actual and targeted wage rate in millions.

In relation to productivity, which is also included as one of the explanatory variables in the specification of the behavioral equation for the wage rate, the value added per employee for each industry is modeled as a positive function of market size, following the logic of the Smith-Kaldor-Verdoorn effect, in line with Canelli et al. (2021). The equation defining productivity at the industry level is shown in equations 307 to 311, and is generally defined as:

$$pr = \frac{y_p}{EMP},$$

and can be seen in figure 16 below. The productivity measure above describes a positive relationship between output growth and productivity growth, where the latter arises due to increasing returns to scale and learning-by-doing in production. Rising real production (y_p) with relatively smaller increases in employment (EMP) leads to higher labor productivity.

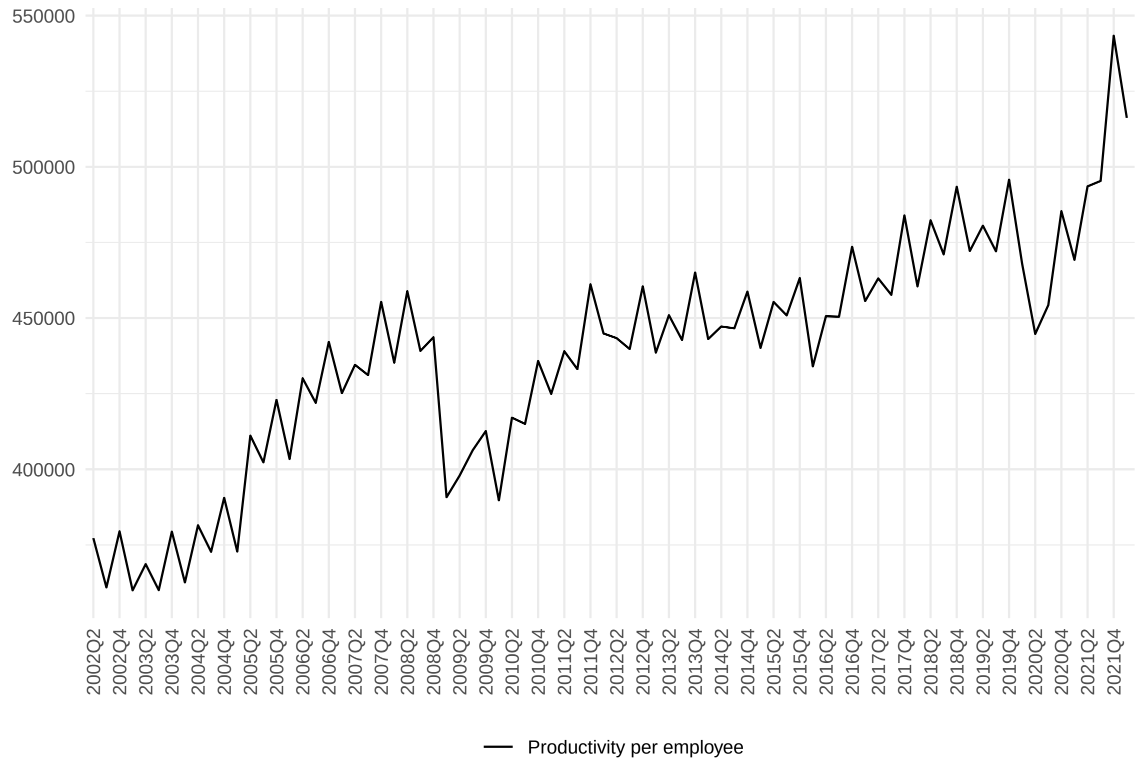


Figure 14: Aggregated labor productivity in DKK per employee.

6.2.3 Real exports

As outlined in section 2.2.1, the shift in the global agenda has raised new pressing questions such as tariffs and protectionism. It is therefore relevant to incorporate these aspects into a model framework that aims to assess industry-specific effects of policy measures or trade interventions. One way of doing so is by modeling real exports as a behavioral equation for a specific industry, allowing it to respond to changes in exogenous, as well as endogenous, factors. Additionally, efforts have been made to include foreign income. To operationalize this, we construct a proxy for income in the rest of the world (RoW), limited to Denmark's three main export destinations: the United States, Germany, and Sweden. According to DST (2025), these countries consistently rank as Denmark's three biggest export partners. For analytical consistency, these countries' GDP data must be harmonized in terms of price basis, currency, and scale to make them comparable. As shown in table 15, data from the Federal Reserve Bank of St. Louis (FRED) are used to retrieve nominal and real GDP series. U.S. GDP is available in billions of USD (Federal Reserve Bank of St. Louis, 2024c) and in constant 2017-prices (Federal Reserve Bank of St. Louis, 2024f); German GDP in millions of EUR (Federal Reserve Bank of St. Louis, 2024a), with real figures in 2010-prices (Federal Reserve Bank of St. Louis, 2024d); and Swedish GDP in millions of SEK (Federal Reserve Bank of St. Louis, 2024b), also in 2010-prices (Federal Reserve Bank of St. Louis, 2024e). This differs from the current database, where GDP is measured in DKK and 2020 prices. To make the series comparable, all GDP data are converted to DKK and expressed in 2020-prices. Using 2010 as an example base year, the procedure is as follows:

i) A deflator is calculated from the ratio of nominal to real GDP:

$$py_i^{2010} = \frac{GDP_i}{gdp_i}.$$

ii) The 2020-adjusted deflator is obtained by rescaling the series:

$$py_i^{2020} = \frac{py_i^{2010}}{py_{2020}^{2010}}.$$

iii) GDP in 2020-prices is calculated:

$$gdp_i^{2020} = \frac{GDP_i}{py_i^{2020}}.$$

Exchange rate data (DKK/USD, DKK/EUR, and DKK/SEK) are sourced from DST (2025a), available on a monthly basis from 2000M1 to 2024M12 and subsequently converted to quarterly frequency. Since the GDP figures from FRED are denominated in foreign currency, the exchange rates are inverted (e.g., $\frac{1}{DKK/EUR}$), to obtain domestic currency, before being applied[57]:

$$gdp^{GER} = \frac{902{,}704.11 \text{ EUR}}{0.134} = 6{,}743{,}741.30 \text{ DKK},$$

where $0,134 = \frac{1}{7{,}471}$ (i.e., the DKK/EUR rate for 2019Q4). To capture the relative importance of each country, export weights are calculated from UHM data (DST, 2025c), defined as the ratio between exports to a given country and the total exports to the three countries. UHM data is available monthly from 2010 to 2025 and subsequently converted to quarterly frequency. For earlier periods (2001Q1–2010Q4), the 2010Q1 shares are held constant. Figure 15 illustrates the evolution of export weights from 2010Q1 to 2024Q4.

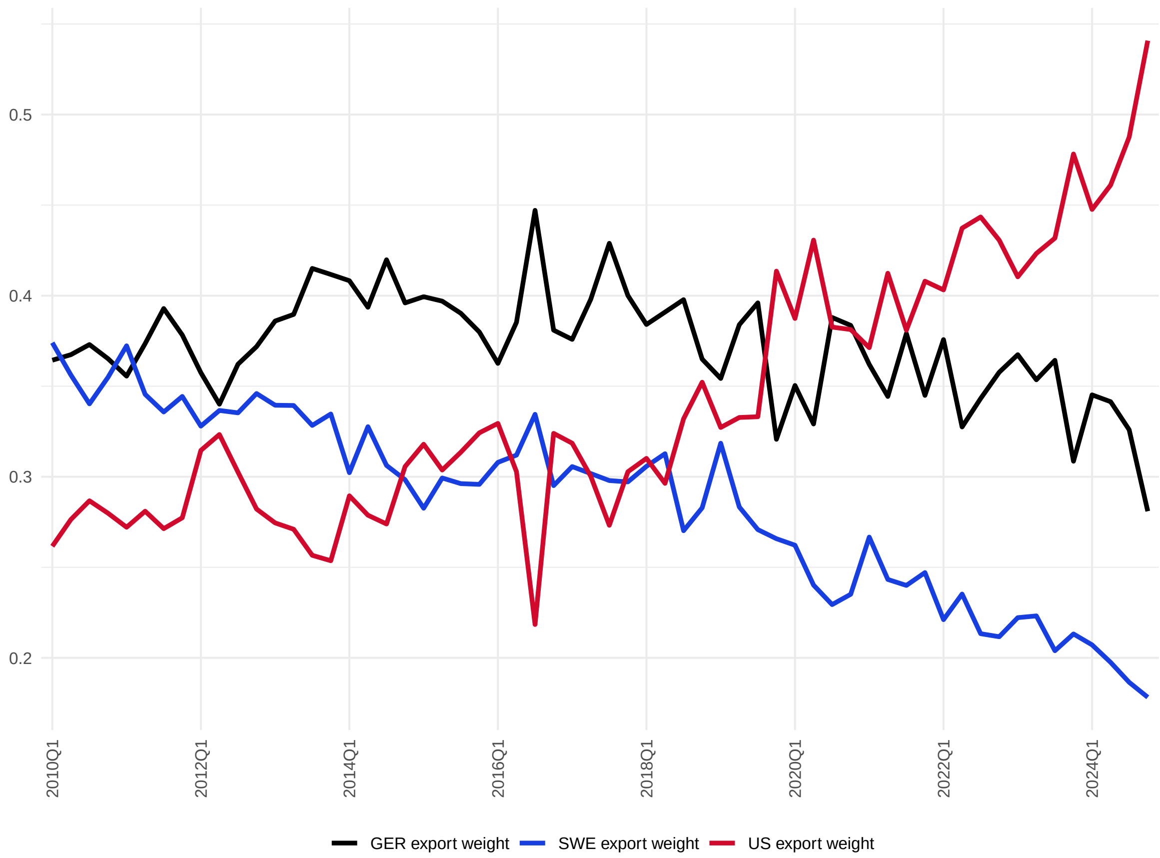


Figure 15: The evolution of export weights from 2010Q1 to 2024Q4.

[57]Ideally, purchasing power parity (PPP)-adjusted nominal exchange rates would be used to convert real GDP figures into a consistent domestic currency measure. This would provide a more accurate cross-country comparison of economic size. However, due to the lack of available quarterly PPP-adjusted series, standard nominal exchange rates are used. While this introduces some distortion, it is considered an acceptable approximation in the absence of better alternatives.

These weights are then applied to the real GDP series (converted to DKK and 2020-prices) and summed to construct the final proxy for RoW income:

$$\text{Proxy real GDP}^{\text{RoW}} = gdp^{GER} \cdot x^{GER}_{\text{weight}} + gdp^{SWE} \cdot x^{SWE}_{\text{weight}} + gdp^{USA} \cdot x^{USA}_{\text{weight}},$$

which serves as an exogenous determinant of real exports.

In order to include an exogenous factor related to tariffs, a τ-term is introduced in the equation governing the real exchange rate:

$$e = \frac{p_Y \cdot (1 + \tau^{MC}_{RoW})}{p_{IM}},$$

where p_{IM} denotes the price deflator for imports (i.e., the ratio of nominal to real imports) and py represents the GDP deflator, with both expressed in DKK[58].

The τ-parameter represents the United States AHS Weighted Average Tariff Rate, which is defined as the average of applied tariff rates weighted by the value of imported goods. The data are sourced from the World Integrated Trade Solution (WITS) and are available annually from 2000 to 2022. The annual series is converted into quarterly frequency by assuming that the tariff rate remains constant within each year, and the most recent annual value is held constant beyond 2022.

Finally, CPB Netherlands Bureau for Economic Policy Analysis (2025) is utilized to include the global trade indicator as an additional exogenous variable, as it may allow to capture the effects of some specific events which are more directly connected to export dynamics. This series is available monthly from 2000 to 2025 and subsequently converted to quarterly frequency.

The application of the behavioral equation for real exports in an industry-specific context, focusing on the manufacturing and construction industry, is further examined in section 7.2.2, in an econometric context.

6.2.4 The real interest rate

In relation to the behavioral equation defining real household consumption presented in section 7.2.2, and also shown in equation 471, it has been necessary to construct a variable that accounts for inflation dynamics in interest rate movements, thereby capturing the "real" interest effect on household consumption. According to Gottfries (2013) and standard economic theory, the real interest rate is defined as the difference between the nominal interest rate and the inflation rate, $r = i - \pi$. To obtain this definition, quarterly inflation has been used alongside the nominal interest rate, which is represented by the deposit rate from Danmarks Nationalbank. This rate can be downloaded from DST under the code DNRENTM. The resulting real interest rate used in equation 471 is illustrated below.

[58]Since both pim and p_Y are denominated in DKK, the nominal exchange rate (E) is omitted. If, however, import prices were expressed in foreign currency, the inclusion of E would be required. In such a case, the construction of a weighted exchange rate would be necessary to consolidate the three currencies (USD, EUR, and SEK) into a single foreign-currency proxy. This is particularly complex due to Denmark's fixed exchange rate regime with the euro, while the USD and SEK follow floating regimes. Thus, expressing all values in DKK simplifies the framework while assuming that currency dynamics are adequately captured through the conversion of individual GDP series.

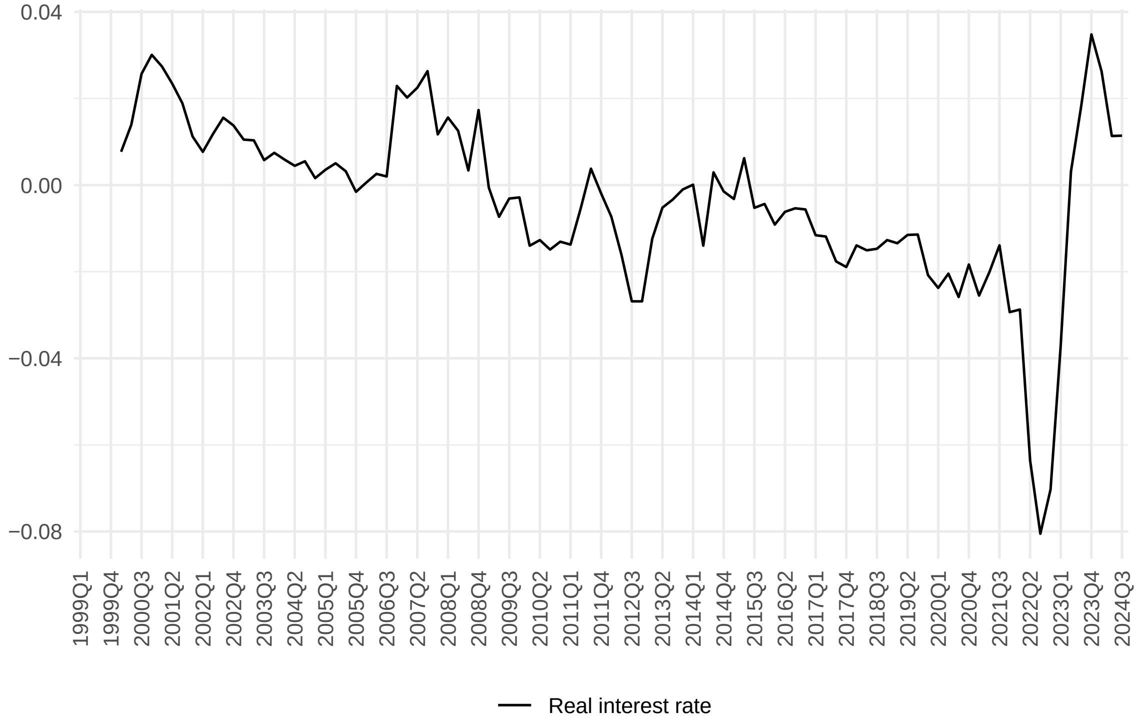


Figure 16: Real interest rate.

7 E-IO-SFC system of equations

The most salient feature of SFC models is their ability to capture complex inter-linkages among different sectors of the economy through detailed transaction and balance sheet interactions. The infographic in figure 17 illustrates the primary flows associated with various economic activities and financial transactions across sectors[59]. The arrows in the infographic indicate the direction of causality, pointing towards the receiving sector. For example, wages are received by households from NFC's, FC's, and the Central Bank.

The figure highlights the key income and expenditure components exchanged between sectors, including wages, consumption, investment, exports, imports, intermediate transactions, distribution of income (DI) and gross operating surplus (GOS), social contributions (SC), social benefits (SB), pension adjustments and taxes on income, production (NIT) and imports (NTI). Beyond real economic transactions, the figure also emphasizes the link between the real and financial sides of the economy by incorporating the allocation of financial wealth within the flow of funds, as well as net interest payments and expenditures associated with financial instruments.

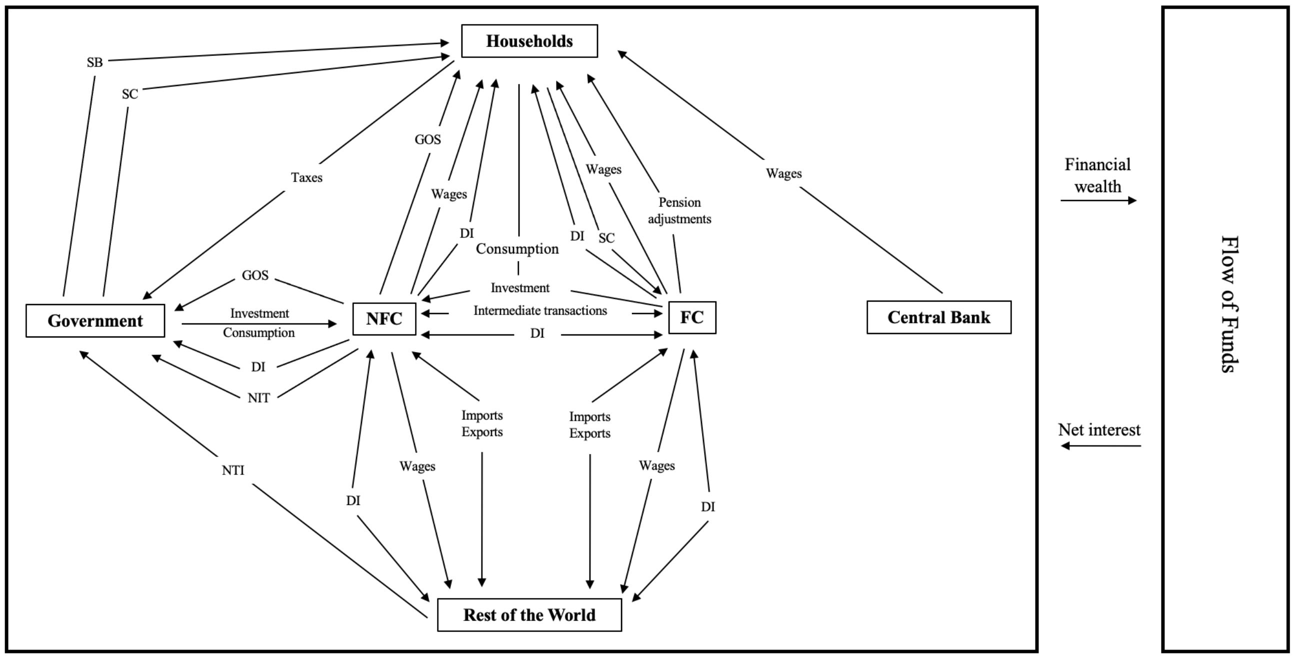


Figure 17: Overview of the E-IO-SFC model.

7.1 Key equations and the closure of the model

According to Valdecantos (2023b), the fourth step in developing an empirical SFC model is to construct a fully exogenous version of the model and verify that all accounting identities are satisfied. In this section, the model's most essential equations will be explained from the perspective of each sector. The complete system of equations which the E-IO-SFC model is build upon is presented in the appendix section B. In the framework of the E-IO-SFC model it is assumed that the NFC's and financial corporations are the sectors producing all goods and services, however NFCs are the only one producing capital goods. One of the main contributions of the model is the disaggregation of the NFC sector into five industries: energy, transport, agriculture, manufacturing and construction, and other manufacturing and services. The equations presented in this sector are therefore indexed by i, as they apply to all industries. While the model comprises both accounting identities and behavioral relationships, the behavioral components are presented in detail in section 7.2.2, section 7.2.3, and

[59]For simplicity, the disaggregated NFC sector is presented in an aggregated form.

appendix C.

7.1.1 Presentation of key equations

The structure of the E-IO-SFC relies to some extent on the theoretical building blocks of post-Keynesian theory: however, we also contribute by incorporating several novel features. We now proceed to present the key equations defining the whole system of equations in the E-IO-SFC model. The equations are organized by sector, and thereby reflecting the structure of the economy as presented in the figure 4 and 17, respectively. The overall aggregate demand for the economy in the E-IO-SFC model is given by the following equation:

$$AD = C^H + I_a^{NFC} + G + X - IM + \Delta INV,$$

where C^H denotes private consumption by the household sector, I_a^{NFC} represents the total accumulated investments by all sectors in the economy. G stands for total government consumption. X refers to total exports of goods and services, which are exclusively carried out by the producing sectors[60]. IM captures total imports, including intermediate goods, investment goods, and both private and public consumption imports. The final component of aggregate demand is changes in inventories. This element is included in the model because producers accumulate inventories in order to meet demand, as they are unable to instantly produce the exact amount of goods and services demanded. Inventory accumulation may also result from lower demand compared to producers' expectations. This also implies that demand does not always equal supply at every point in time, which is a key feature of SFC-models.

Non-financial corporations

As stated, NFCs produce goods and services, making it the primary source of income for this part of the economy. Nominal sales for each industry Y^i are defined as the household consumption C^i, government consumption G^i, exports X^i, intermediate sales IG_s^i, and current investments I_c^i:

$$Y^i = C^i + G^i + X^i + IG_s^i + I_c^i.$$

Since the production process takes time, firms must decide in each period how much to produce. It is assumed that economic agents' expectations about future real sales are a key determinant of current production decisions. This relationship is presented below:

$$\begin{aligned}\Delta\ln(y_{e,t}) = &-\beta_{0,y_e} + \beta_{1,y_e}\cdot\Delta\ln(y_{e,t-1}) + \beta_{2,y_e}\cdot\Delta\ln(CI_t) + \beta_{3,y_e}\cdot\Delta\ln\left(\frac{y_t}{k_t}\right)\\ &+ \beta_{4,y_e}\cdot\Delta\ln(yd_{t-2}^H) + \beta_{5,y_e}\cdot\Delta\ln(c_t) + \beta_{6,y_e}\cdot\Delta\ln(i_t)\\ &- \beta_{7,y_e}\cdot\ln(y_{e,t-1}) + \beta_{8,y_e}\cdot\ln(c_{t-1}) + \beta_{9,y_e}\cdot\ln(CI_{t-1}).\end{aligned}$$

This behavioral equation models the logarithmic differences of expected sales, thereby capturing the percentage change in expected sales over time. The variation is explained by changes in the capacity utilization rate, real disposable income of households, real consumption, a consumer confidence indicator in the long-run, and real investment. The change in the capacity utilization rate reflects how intensively firms are using their available production resources. All of these factors are expected to have a positive impact on real expected sales and, consequently, on firms' production decisions. The estimation follows the procedure outlined in section 7.2.2, which combines economic theory with empirical evidence.

After expectations about sales has been determined production for each industry can take place and is defined as the following relationship:

[60] All exports in the economy are undertaken by the producing sectors.

$$y_P^i = y_e^i + i_{inv,d}^i,$$

where each producing industry incorporates their expectations for real sales, capturing the demand to be met by the supply. Furthermore, the desired level of inventory accumulation appears as an additional component to align with the assumption that firms invest in inventories as a buffer stock to smooth production and accommodate unexpected fluctuations in demand or supply. Hence, while the E-IO-SFC model is heavily demand driven, we assume that the production decision is autonomous and mostly based on producers' expectations about the future.

The next behavioral relationship to be presented is firms' total real investment to capital ratio, which is a function of its own past values and a range of macroeconomic variables. The relationship is defined as follows:

$$\begin{aligned}\Delta \ln\left(\frac{i_t}{k_t}\right) &= \beta_{0,\frac{i}{k}} - \beta_{1,\frac{i}{k}} \cdot \Delta \ln\left(\frac{i_{t-1}}{k_{t-1}}\right) - \beta_{2,\frac{i}{k}} \cdot \Delta \ln\left(\frac{i_{t-2}}{k_{t-2}}\right) + \beta_{3,\frac{i}{k}} \cdot \Delta \ln\left(\frac{i_{t-4}}{k_{t-4}}\right) \\ &+ \beta_{4,\frac{i}{k}} \cdot \Delta \ln(g_t) + \beta_{5,\frac{i}{k}} \cdot \Delta \ln(g_{t-1}) - \beta_{6,\frac{i}{k}} \cdot \Delta \ln\left(\frac{x_t}{im_t}\right) + \beta_{7,\frac{i}{k}} \cdot \ln\left(\frac{y_t}{k_t}\right) \\ &- \beta_{8,\frac{i}{k}} \cdot \ln\left(\frac{i_{t-1}}{k_{t-1}}\right) - \beta_{9,\frac{i}{k}} \cdot \ln\left(g_{t-1}\right) + \beta_{10,\frac{i}{k}} \cdot \ln\left(\frac{y_{t-1}}{k_{t-1}}\right) - \beta_{11,\frac{i}{k}} \cdot \ln(i_{\mathrm{NFC}}).\end{aligned}$$

To capture the short-term dynamics and momentum in investment behavior the investment to capital ratio at one, two, and four periods is included as independent variables reflecting an auto-regressive process. Furthermore the logarithmic differences of government expenditure is included in both the short and the long run. According to Afonso and Aubyn (2009) government investments crowds-in private investments. We find this relationship in the short run, while in the long run it crowds-out private investments. Besides this, external conditions are captured through changes in trade openness, as approximated by the export-import ratio. This indicator has an unexpected relationship with investment decisions, which is further commented in appendix C.3 with an econometric viewpoint. The short-run and long-run effects of the capacity utilization capture supply-side conditions as firms are more likely to invest when they are operating near or above capacity. According to Dutt (2011), firms make investment decisions based on the capacity utilization rate, as it signals the buoyancy of aggregate demand[61] and shapes their expectations regarding future demand. Finally, the effects of the interest rate faced by firms, i_{NFC}, is incorporated and is affecting investments in the long-run. A higher interest rate increases the cost of financing, thereby reducing firms' demand for capital. In general, the behavioral equations for real investment incorporate both demand- and supply-side determinants, along with financial conditions, to explain fluctuations and trends in investment activity. The significance of the explanatory variables, as well as the underlying theoretical framework and diagnostic tests, are presented in detail in the appendix C.3. In the model the investment equation presented above is estimated at the aggregate level. To obtain the accumulation rate at the industry level we use shares derived from the IO tables.

The intermediate consumption from one industry to another highlights one of the E-IO-SFC models central contributions namely the input-output structure. The level of intermediate purchases from one industry to another is determined by the real production of the industry in question multiplied by a technical coefficient ($\alpha_p^{i,i}$), obtained by the input-output tables from DST, cf. section 5.3.1.

$$ig_p^{i,i} = y_P^i \cdot \alpha_p^{i,i}.$$

Overall, the input-output structure increases the model's ability to trace the distributional and dynamic effects

[61]The buoyancy of aggregate demand can be interpreted as an indicator of the economy's position in the business cycle, and thus serves as a relevant input for firms' investment decisions.

of shocks and policies across sectors, making it a more effective tool for scenario analysis and macro-financial policy evaluation, as already pointed out in section 2.2.

$$ig_s^i = ig_p^{E,i} + ig_p^{T,i} + ig_p^{A,i} + ig_p^{MC,i} + ig_p^{OMS,i} + ig_p^{FC,i}.$$

The above equation for each industry's intermediate sales are determined by the total intermediate purchases by all industries from that industry, as specified by the input-output structure. This ensures that the sum of intermediate inputs across all purchasing industries matches the total intermediate output supplied by each selling industry. As a result, the E-IO-SFC model preserves consistency between intermediate demand and supply at the sectoral level, which is crucial for maintaining stock-flow coherence.

To reflect the role of international trade within the E-IO-SFC model, the exports equation for the MC industry is highlighted. This behavioral equation captures the responsiveness of exports to both domestic and global economic conditions and is presented below:

$$\begin{aligned}\Delta \ln(x_t^{MC}) = \beta_{0,x} - \beta_{1,x} \cdot \Delta \ln(p_{X_t}^{MC}) + \beta_{2,x} \cdot \Delta \ln(p_{X_{t-1}}^{MC}) - \beta_{3,x} \cdot \Delta \ln(e_t) + \beta_{4,x} \cdot \Delta \ln(gdp_t^{RoW}) \\ - \beta_{5,x} \cdot \Delta \ln(gdp_{t-4}^{RoW}) + \beta_{6,x} \cdot \Delta \ln(WT_t^{indicator}) - \beta_{7,x} \cdot \ln(x_{t-1}^{MC}) + \beta_{8,x} \cdot \ln(gdp_{t-1}^{RoW}).\end{aligned}$$

This relationship captures changes in the export price deflator for the MC industry, both current and lagged, price competitiveness, and global demand conditions jointly shape export dynamics in the MC industry. By incorporating both short-run adjustments and long-run equilibria, the equation provides a comprehensive representation of trade behavior within an open economy like Denmark's. For a detailed explanation of the independent variables, their statistical significance, and theoretical underpinnings, the reader is referred to section 7.2.2.

In order to incorporate labor market dynamics into the E-IO-SFC model, the next equation presented is that of the wage rate which is functioning as a behavioral equation. It is modeled as a function of auto-regressive components, labor market indicators such as productivity and unemployment, as well as cyclical factors.

$$\begin{aligned}W = \beta_{0,W} + \beta_{1,W} \cdot \ln(W_{t-2}) - \beta_{2,W} \cdot \Delta \ln(W_{t-3}) + \beta_{3,W} \cdot \Delta \ln(PR_t) \\ - \beta_{4,W} \cdot \ln(W_{t-1}) - \beta_{5,W} \cdot \ln(UR_r UR_{s,t-1}) + \beta_{6,W} \cdot \ln(W_{T_{t-1}}).\end{aligned}$$

In this specification, W denotes the total wage rate at time t and serves as the dependent variable, as well as an auto-regressive component in the short run. This allows the E-IO-SFC model to reflect delayed responses to previous changes in the wage rate. The total wage rate is influenced by productivity, denoted by PR, which captures changes in labor productivity. This variable is included to account for how improvements or declines in productivity affect wage dynamics. Higher productivity increases the scope for wage growth without inducing inflationary pressure, while lower productivity tends to limit wage increases. Moreover, the wage rate is negatively affected by a cyclical component defined as the deviation of the actual unemployment rate from the structural rate, denoted by $UR_r UR_s$. This relationship implies that when unemployment exceeds its structural level, indicating slack in the labor market, wage growth tends to decelerate due to weaker labor demand. Finally, WT_{t-1} captures the long-run target wage rate. This represents the level of wages that labor unions or institutions aim to achieve, based on long-term contracts, expectations, or institutional norms. Including this variable enables the model to reflect gradual adjustments of actual wages toward a desired or negotiated benchmark over time. For a more detailed discussion of the included variables, their theoretical justification, and statistical significance, the reader is referred to appendix C.2.

Financial corporations

Given that financial corporations operate in a similar manner to NFC's within the structure of the model, the same equations are applicable. For a comprehensive and technical exposition of the underlying behavioral equations, the reader is referred to the appendix B where the financial corporations is described in detail.

Households

The first equation to be presented for households is the behavioral equation defining real consumption:

$$c^H = \beta_{0,c} + \beta_{1,c} \cdot \Delta \ln(c_{t-4}) + \beta_{2,c} \cdot \Delta \ln(yd_t^H) - \beta_{3,c} \cdot \Delta \ln(yd_{t-2}^H) + \beta_{4,c} \cdot \Delta \ln(fw_t^H) \\ + \beta_{5,c} \cdot \Delta r_t - \beta_{6,c} \cdot \ln(c_{t-1}) + \beta_{7,c} \cdot \ln(yd_{t-1}^H) + \beta_{8,c} \cdot \ln(fw_{t-1}^H).$$

Real consumption for households is modeled as a function of its own lag in the short run, real disposable income, household financial wealth fw^H, and the real interest rate. In addition to short-run fluctuations, consumption also adjusts toward a long-run equilibrium through the inclusion of an error correction mechanism. This implies that both short-term volatility and long-term trends influence current consumption behavior. Real disposable income has a positive short-run effect on household consumption, reflecting immediate income responses. Furthermore, since real disposable income also affects consumption decisions in the long run, this indicates a cointegrated relationship between real consumption and real disposable income. The relationship between household financial wealth and consumption is captured through fw^H, which positively influences consumption. This enables the E-IO-SFC model to account for the role of financial assets in shaping consumption behavior. Finally, households consumption is influenced by the real interest rate in the short run. According to Godley and Lavoie (2012, p. 114), higher interest rates can lead to increased disposable income, thereby stimulating consumption through a wealth effect. While this result may initially appear counterintuitive from a mainstream perspective, it reflects the idea that higher interest payments on government debt raise income for households holding government securities.

Overall, the specification combines income and wealth effects with interest rate sensitivity and consumption inertia to explain the dynamics of household consumption. All independent variables show a positive and economically meaningful impact on real consumption. For a full specification of the real consumption model, including theoretical background and econometric results, refer to appendix C.1.

Due to the model's assumption that only NFC's and FC's constitute the producing sectors of the economy, it is necessary to reallocate a share of the gross operating surplus (GOS) to households and the government, both of which, in reality, contribute to income generation through production[62].The equation defining this relationship is presented below:

$$GOS^{H,G} = GOS^E_{H,G} + GOS^T_{H,G} + GOS^A_{H,G} + GOS^{MC}_{H,G} + GOS^{OMS}_{H,G} + GOS^{FC}_{H,G}.$$

This adjustment allows the model to replicate empirically the net lending positions reported by DST across sectors, thereby preserving accounting consistency and ensuring a more accurate macroeconomic representation. For a more detailed explanation we refer to section 5.1.

Government

The most relevant equation to present for the government is the one determining its revenue. This equation highlights the various income components accruing to the government sector:

[62]The superscript denotes that a share of the producing sectors' GOS, is distributed to households and government.

$$R^G = T_s^G + GOS^G + SC_r^G + NI^G + DI_r^G + OC_r^G.$$

Government income consists of the sum of taxes paid by the remaining sectors. This includes both taxes on income and wealth as well as net indirect taxes, the latter being paid exclusively by the producing sectors. The next income component is the gross operating surplus, which is redistributed by the producing sectors for the same purpose as in the case of the household sector. Additionally, the government receives social contributions from households and the rest of the world (RoW), as well as other current transfers from all sectors of the economy. It also receives distributed income from the producing sectors, the RoW, and notably from the central bank, which redistributes part of its profit to the government. Finally, the government earns net interest income, reflecting the net returns on its portfolio of financial instruments.

The wide range of income sources ensures that the government is financially linked to all other sectors in the economy. This interconnection is a key feature of SFC models, where each sector's transactions must balance with the rest. It also implies that if one sector experiences a downturn, for example, households reducing consumption or firms lowering output, this will have a direct impact on government revenue. Thus, the model captures how macroeconomic fluctuations propagate through the system and affect public finances in a coherent and consistent manner.

Central Bank

The Central Bank plays a fundamental but relatively limited role in the Danish economy, as Denmark operates under a fixed exchange rate regime with a strong commitment to monetary stability, the central bank primarily conducts monetary policy to maintain the krone's peg[63] to the euro, rather than actively steering domestic demand or output through interest rate manipulation.

Furthermore, the Central Bank's direct interactions with the other sectors, such as households, firms, or the rest of the world, are relatively limited. Its main functions include issuing currency, managing foreign exchange reserves, and settling payments between banks, which are mostly institutional and do not directly affect sectoral income in the same way as, for example, government transfers or corporate profits do. For this reason, there are no equations for the central bank that require further explanation beyond what is already covered in the other sectors.

Rest of the World

The rest of the world balance highlights the interconnection between the domestic sectors and the rest of the world. Here, some components take on the opposite meaning compared to what was previously presented. For example, IM are now recorded as income, since imports are defined as domestic purchases from abroad. Conversely, X are treated as expenditures for the same reason. The remaining components, whether recorded as income or expenses in the balance, relate to Danish agents operating internationally.

Net interest, financial wealth and Flow of Funds

The equations governing net interest, financial wealth, and the flow of funds, all of which exhibit a common structural form across the different sectors and industries within the framework of the E-IO-SFC model is now presented:

[63] Denmark maintains a fixed exchange rate policy by pegging the krone to the euro within a narrow fluctuation band as part of the Exchange Rate Mechanism II (ERM II). This means the central bank intervenes in currency markets to keep the exchange rate stable.

$$NI^i = F.2_s^i \cdot \iota_{F.2}^i + F.3_s^i \cdot \iota_{F.3}^i + F.4_s^i \cdot \iota_{F.4}^i + F.5_s^i \cdot \iota_{F.5}^i + F.6_s^i \cdot \iota_{F.6}^i + Interest_{F.7}^i + Interest_{F.8}^i.$$

The equation for net interest ensures the crucial link between the real and financial sides of the E-IO-SFC model. By introducing this equation, we ensure that the allocation of real wealth across different financial instruments, such as loans, deposits, and bonds, has a direct impact on the distribution of income across industries. This mechanism captures how financial decisions, including portfolio choices and interest rate developments, feed back into the real economy by influencing production costs, profitability, and investment incentives in various sectors. In this way, the equation plays a central role in integrating financial dynamics into macroeconomic modeling, further explanation of the interest rates are to be found in section 6.1.5.

The final common equations to be presented from the system of equations are financial wealth and the flow of funds. By the inclusion of these equations we ensure the interconnection between sectoral and industry profits and the corresponding decisions regarding the allocation of financial instruments:

$$V^i = NL_t^i + V_{t-1}^i + Rev^i.$$

Financial wealth in each period is determined by the sector's and industries net lending positions, the stock of financial wealth from the previous period, and revaluations or other changes in the value of financial instruments. These revaluations may result from changes in exchange rates or asset prices, which in turn reflect broader macroeconomic conditions and market expectations. Consequently, the evolution of financial wealth not only captures past saving behavior but also incorporates the impact of financial market dynamics and policy shifts.

The level of financial wealth influences decision-making, as shown, for instance, in the equation for private consumption. When the financial result is positive (the sector is in surplus), the sector faces two main options: either to increase its holdings of financial assets or to reduce its outstanding liabilities. These choices reflect the dual nature of financial management, balancing asset accumulation with debt reduction, and are influenced by factors such as expected returns, interest rates, and risk preferences.

$$F.j_{s(t)}^i = V^i \cdot \beta_{F.j}^i.$$

The desired allocation of financial wealth to each financial instrument is governed by its β value, which reflects the target proportion of total financial wealth to be held in that specific instrument. These β values can be interpreted as portfolio preferences, potentially influenced by factors such as risk, return expectations, and institutional constraints. These portfolio choice elements are estimated behaviorally as shown in section 7.2.3.

The equations presented above, just a sample of the full model, illustrate how the proposed integrated approach captures the interaction of multiple transmission channels. Production is largely driven by (expected) demand, which is itself shaped by the decisions of various agents (households, firms, the rest of the world, etc.). These decisions propagate through the economy via the interconnections between industries and their relationships with sectors. For instance, different industries have varying labor requirements, which affect the unemployment rate and, in turn, the wage rate—one of the key components of disposable income and consumption. The resulting dynamics of economic activity give rise to surpluses and deficits, reflected in changes to agents' balance sheets. These balance sheets can also shift due to portfolio decisions—another form of agent behavior. Such financial changes feed back into the real economy, for example through disposable income in the consumption equation, thereby influencing demand and production once again.

7.1.2 Redundant equation - closure of the model

A fundamental characteristic of SFC models is the presence of a redundant equation, which emerges as a natural consequence of the model's accounting structure. SFC models are built on rigorous national accounting principles and rely on a complete articulation of the financial and real transactions between institutional sectors.

Because of this structure, one of the model's equations will inevitably be redundant. That is, it will not provide any additional information beyond what is already implied by the rest of the system. The redundancy is not due to poor model specification, but rather reflects the internal consistency of the framework. It mirrors Walras' Law in general equilibrium theory, which states that if all but one market in an economy are in equilibrium, the final market must also be in equilibrium, provided that agents' budget constraints are satisfied. Similarly, in an SFC context, if all but one of the accounting and behavioral equations are satisfied, the final one will be automatically satisfied by construction (Godley and Lavoie, 2012). In this model, this redundancy arises in the context of government securities ($F.3^G$), which are defined as the 'buffer variable', capturing the difference between the government's net lending position and its transactions in other financial instruments thereby capturing vertical consistency:

$$F.3^G = NL^G - F.2^G - F.4^G - F.5^G - F.6^G - F.7^G - F.8^G,$$

where NL^G is government net lending, and $F.2^G$ through $F.8^G$ represent transactions in other financial instruments. All transactions of the financial instruments are recorded as net assets, which explains the signs used in the equation. For instance, in a period where a given instrument shows a negative value for net assets, this would correspond to a positive contribution to the value of $F.3^G$. This equation ensures that the financial side of the government's budget is fully closed and internally consistent.

Now, when looking at the TFM there is also an identity representing the ex post balancing of the securities' market.

$$F.3^G = F.3^H + F.3^E + F.3^T + F.3^A + F.3^{MC} + F.3^{OMS} + F.3^{FC} + F.3^{CB} + F.3^{RoW}.$$

This equation states that the total quantity of government securities issued must be equal to the total transactions of securities (issued by the government as well as the other sectors) by the remaining sectors and industries. As this identity follows directly from the comprehensive double-entry accounting structure, it is logically redundant. Once all other equations in the model have been defined (including the elements on the right-hand side of the equation above) and accounting identities have been satisfied, this equation will hold automatically. Therefore, to avoid over-determination when solving the system, this redundant identity is removed from the model. Nevertheless, it plays an important role as a consistency check: if the model solution violates this equation, it signals a fundamental inconsistency in the accounting framework or numerical implementation.

Another salient feature of the SFC approach is the achievement of vertical and horizontal consistency[64], which is ensured by equations for the transactions of different financial instruments respect to the sector/industry. The equations that ensure vertical and horizontal consistency for the government is described by the two previous equations. Whereas the first presented equation secures the vertical consistency by determining securities, say buffer-variable, as the residual of net lending position and the remaining transactions of the financial instruments (this will by definition "close" the column for this instrument and satisfy the budget constraint for the government). The second equation secures the horizontal consistency by determining the supply of government securities (in flow terms) based on the transactions of the remaining sectors in the same instrument. The

[64] Recall the first two principles described in section 3.1

same method is applied to the remaining sectors and industries, each of which includes an equation with the same structure to ensure vertical consistency.

When it comes to horizontal consistency, however, we adopt a more pragmatic approach, as it is strictly necessary to ensure consistency for all of the instruments. In our closure, horizontal consistency is achieved by assuming a quantity adjustment in each market, whereby a specific sector is endogenously determined to supply or absorb exactly as much of a given financial instrument as is demanded by the other sectors. Thus, we do not impose formal constraints across all sectors, but instead rely on economic intuition regarding which sector is best suited to act as the residual in each market. This also means that not all sectors are treated symmetrically.

For example, horizontal consistency in the market for trade credits (F.8) is ensured through NFC's, reflecting their central role in granting and receiving short-term inter-company credit. For FC's, horizontal consistency is maintained for four financial instruments: deposits (F.2), loans (F.4), insurance technical reserves (F.6), and financial derivatives (F.7). This reflects economic intuition, as FC's are typically the primary agent managing these instruments.

Monetary gold (F.1) is treated separately, with horizontal consistency ensured by the CB, in line with its exclusive role in managing the country's official gold reserves as part of monetary policy and reserve management. Finally, the market for equities (F.5) is balanced through the RoW sector, reflecting the fact that a substantial portion of equity transactions take place across borders, especially relevant in open economies with integrated capital markets.

7.2 Behavioral equations

According to Valdecantos (2023b), the fifth step in developing an empirical SFC model is to calibrate and estimate behavioral equations entering the model. This section describes the econometric estimations of the behavioral equations entering the E-IO-SFC model and the rationale behind them. First, in section 7.2.1, a step-by-step approach to estimating the behavioral equations is presented, taking into account standard time series assumptions and ensuring that general econometric principles are followed. Second, overall we estimate 11 different behavioral equations for the income side of the economy of which four are presented in section 7.2.2. Subsequently, in section 7.2.3, the endogenous structure of the financial part of the model is outlined, along with the methodological choices and assumptions made during its construction. For the financial part one financial-beta, for the households, is estimated for illustrative and methodological purposes.

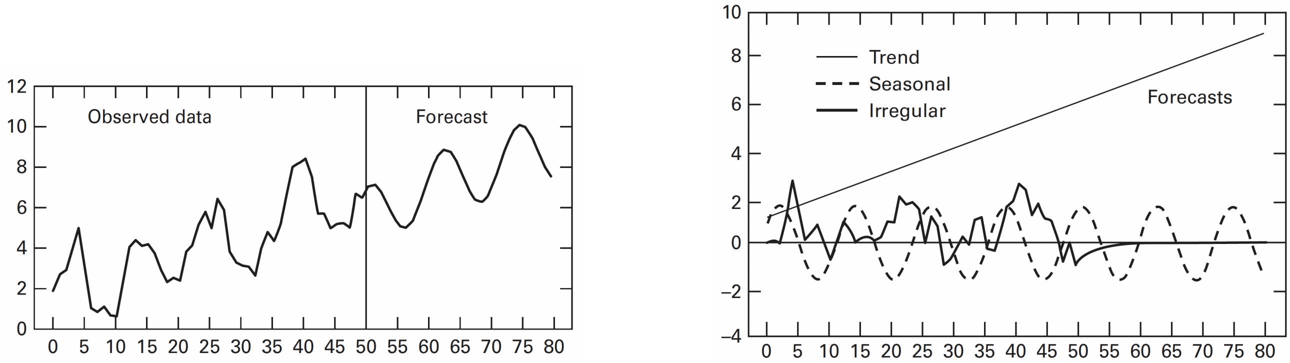
7.2.1 Econometric estimations at the general level

Following the construction and presentation of the database in section 5, and the core matrices that structure the model and ensure the link between stocks (the balance sheet) and flows (the income side) of the Danish economy in section 4, as well as the definition of key accounting identities in section 7.1, this section outlines the choices that shape the behavioral equations and structural specifications of the E-IO-SFC model. For the behavioral equations that include structural parameters, these are estimated using econometric techniques and dynamic regressions in order to identify the underlying empirical relationships in each behavioral equation. The following bullet points provide an overview of the estimation procedure, which is subsequently described in detail.

Step 1: Remove seasonal effects and unit root testing
Step 2: Log-linearize relevant relations and variables
Step 3: Test for cointegration
Step 4: Run the specific model
Step 5: Control for structural breaks and outliers
Step 6: Run diagnostic tests
Step 7: Evaluate static and dynamic forecasts.

Removal of seasonal effects and unit root testing

A time series can be decomposed into a trend, a seasonal, a cyclical, and an irregular component. The trend represents the long-term behavior of the series and the cyclical component represents the regular periodic movements. The primary task of the econometrician, according to Enders (2015, p. 2), is to estimate and forecast the irregular component of a time series. The three components of a time series are illustrated in figure 18, with the objective of this section being to isolate the irregular component, allowing us to safely proceed to step 2.



(a) A illustrative time-series. (b) A decomposed time-series.

Figure 18: Inspiration taken from Enders (2015, p. 2).

It is the objective of the estimated behavioral equations in this model to produce credible results. Therefore, it is important to ensure that the data used as input variables (both dependent and independent) meet the necessary standards. The first step in obtaining the irregular component of a time series is to "clean" the data of any seasonal effects. A typical time series experiences fluctuations in various metrics based on the time of a year or a given period. These fluctuations can sometimes be large, and at times so substantial that they obscure important characteristics of the time series. To reveal the "true" behavior of the series and to prevent the estimated structural parameters from being biased due to correlations arising from seasonal effects rather than the true data generating process, it is essential to remove seasonality. Seasonally adjusted time series allow for a clearer understanding of the underlying trends by removing the "noise" of seasonal fluctuations. For clarity, the seasonal component is reintroduced after estimation to ensure that the model's output reflects the observed seasonality in the data and that the values generated by the model are comparable to the officially reported series (this is particularly relevant for nominal variables, such as financial stocks).

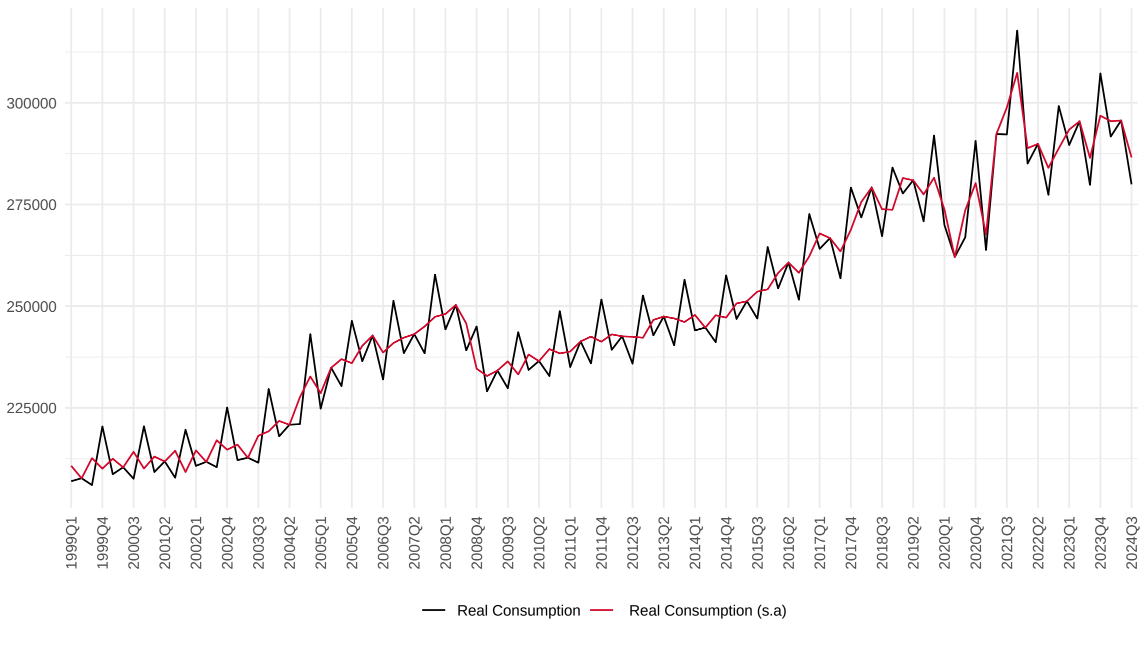


Figure 19: A time-series for real consumption in millions.

In figure 19, real consumption is shown as a seasonally adjusted time series[65], depicted in red. While the seasonal component has been removed, the series still exhibits an upward trend, indicating that it could follow a persistent,

[65]Seasonal adjustment is carried out using the `seas` function from the `seasonal` package in R.

non-stationary path. Just as seasonality was removed to avoid biased parameter estimates, it is also necessary to address the presence of a trend for the same reason and to prevent regression models from being spurious. To test for stationarity and identify the presence of a unit root, the Augmented Dickey-Fuller (ADF) test is applied, in line with the theoretical framework behind it, which is the final step in isolating the irregular component of real consumption[66].

According to Enders (2015, p. 207), there exist three versions of the ADF-test, each designed to account for autocorrelation and ensure the reliability of the resulting t-statistics. The first version tests for a pure random walk, the second includes an intercept (also referred to as a drift term), and the third incorporates both an intercept and a deterministic trend. Choosing the appropriate version of the test depends on the characteristics of the time series under examination. If the series visually appears to follow a random walk, the first version is appropriate. However, in the case of real consumption, the series clearly exhibits both an intercept and a trend. Therefore, the third version of the ADF-test is applied. For the given test, a τ test statistic is reported and compared to its corresponding critical value. If the test statistic is greater (i.e., less negative) than the critical value, reject the null hypothesis fails to reject the presence of a unit root, indicating that the time series follows an $I(1)$ process. In such cases, the variable is differenced to ensure that all variables used in the estimation are $I(0)$ processes, thereby avoiding the problem of spurious regression. Furthermore, the autocorrelation function (ACF) plot is examined to assess the degree of autocorrelation in the series. A high level of autocorrelation may indicate the presence of a unit root, suggesting that the series could be non-stationary. The ACF plot thereby serves as a complementary tool in the decision-making process regarding the presence of a unit root. Figure 20 presents the stationary irregular component of real consumption, obtained after removing both seasonal effects and deterministic trends.

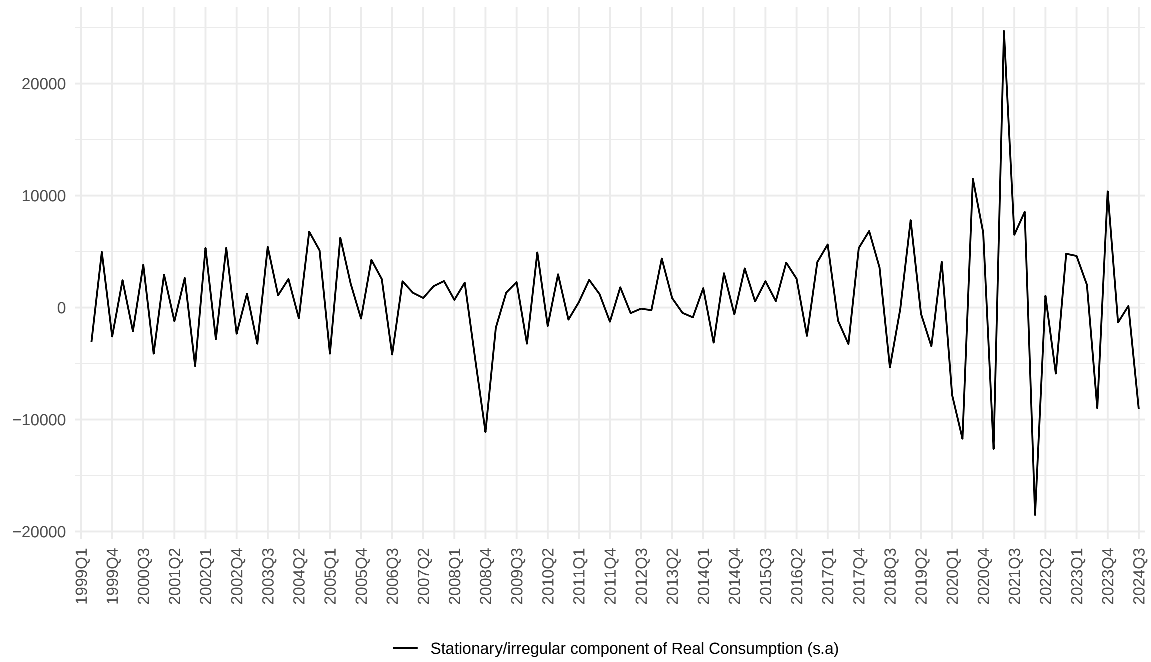


Figure 20: The final irregular time-series of real consumption.

[66]Even though the series exhibits an upward trend, it does not necessarily imply non-stationarity. If the trend is deterministic, that is, it can be captured by a linear time function, the underlying process may still be stationary after removing that trend. This is why the ADF test allows for the inclusion of a deterministic trend term when specifying the test regression. Failing to account for such a trend could otherwise lead to incorrect conclusions about the series' stationarity.

Log-linearize relevant relations and variables

After cleaning the input data to remove seasonal and trend components, the relevant relationships are selected and log-linearized. When it is theoretically intuitive to log-linearize a variable, for instance, real consumption, it is done to obtain proportional relationships between the dependent and independent variables. Log-linearization also helps stabilize the variance of the error term, which is desirable since economic data often exhibit heteroscedasticity. Therefore, a logarithmic transformation can help make the error terms more homoscedastic. Second, the logarithmic transformation can normalize skewed distributions. Many economic variables, such as consumption, tend to be positively skewed. Taking the logarithm of these variables typically produces more symmetric distributions that better approximate normality, which in turn enhances the validity of hypothesis testing and statistical inference. Overall, variables entering the different behavioral equations are log-linearized when it makes economic sense and when it is mathematically feasible[67] to do so (Benoit, 2011).

Test for cointegration

Cointegration refers to a situation in which a linear combination of two or more non-stationary variables, each integrated of the same order (commonly $I(1)$)[68], results in a stationary process. That is, while the individual series themselves may exhibit unit roots, the error term from their linear combination is integrated of a lower order, typically $I(0)$, indicating the presence of a long-run equilibrium relationship between the variables. This situation is depicted below in figure 21.

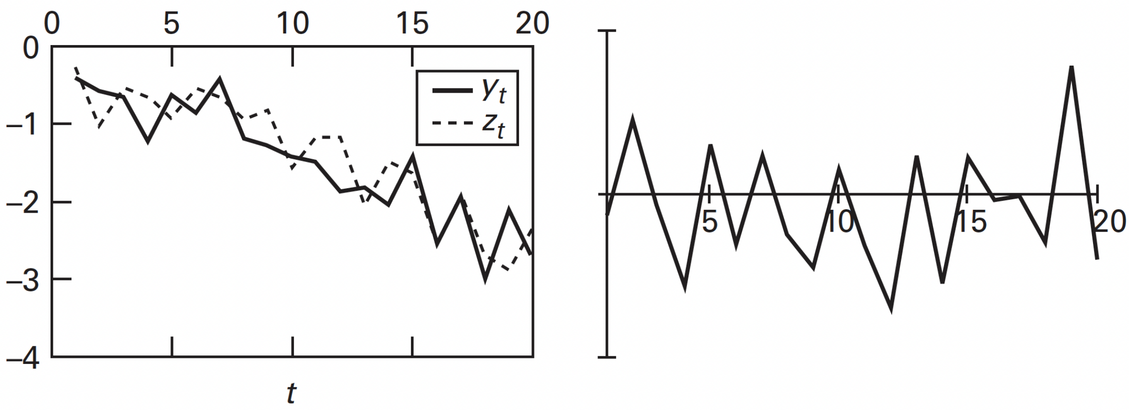


Figure 21: Illustrative cointegration example.

Source: Enders (2015, p. 348).

For illustrative purposes, consider two time series, y_t and z_t. Each series is non-stationary, as they share the same stochastic trend. Therefore, they are cointegrated, implying that a specific linear combination of them is stationary, as illustrated in the left panel of figure 21. Their long-run relationship can be expressed as follows:

$$y_t = \mu + \beta z_t + u_t,$$

where the equilibrium relationship is given by $\mu + \beta z_t$. In periods where y_t exceeds this equilibrium level, some form of error correction is expected to occur. Since deviations from a long-run equilibrium cannot persist indefinitely, an adjustment must take place such that the system reverts back to $y_t = \mu + \beta z_t$. Furthermore, as illustrated in the left panel, the error term from the linear combination is stationary, which implies that shocks to this component gradually fades out and the series reverts to its long-run path.

[67]In cases such as the one shown in figure 20, the logarithm is taken first, and then the differenced or irregular component is used in the model. This order is necessary because the logarithm cannot be applied to negative values.

[68]This can differ, and will be commented on later in this paragraph.

Now consider the following error correction models, which describe three possible adjustment mechanisms towards restoring the long-run equilibrium. The first model features adjustment through y_t, and is specified as follows:

$$\Delta y_t = \alpha_1(y_{t-1} - \mu - \beta z_{t-1}) + v_{1,t}, \quad \alpha_1 < 0$$

The second model reflects adjustment through z_t:

$$\Delta z_t = \alpha_2(y_{t-1} - \mu - \beta z_{t-1}) + v_{2,t}, \quad \alpha_2 > 0$$

Lastly, correction can also occur through both y_t and z_t. The speed-of-adjustment parameters, α_1 and α_2, determine how quickly the variables adjust to restore long-run equilibrium, and thus plays a crucial role in the system's dynamics. Specifically, the α_i parameters determine how quickly the variables revert to their long-run equilibrium.

If there is no cointegration between the variables (which can be tested, as will be elaborated upon later), then no long-run relationship exists to correct towards, implying that $\alpha_i = 0$. In this case, the error correction model (ECM) reduces to a short-run model in first differences (Turatti, 2024). This underscores the importance of identifying whether a stable long-run equilibrium relationship exists between non-stationary variables when estimating the model. Since the variables may be stationary, non-stationary, or a mix of both, it is essential to apply a testing framework that accommodates all possible integration properties. Correctly identifying the integration characteristics and the presence of cointegration is key for determining the appropriate model specification and for interpreting the estimated parameters meaningfully. Therefore, the next step is to present the test for cointegration: the Johansen test or the ARDL bounds test. The choice between them primarily depends on the integration order of the variables under consideration.

The ARDL bounds testing approach, developed by Pesaran et al. (2001), is a procedure used to examine long-run relationships between variables in levels. It is applicable regardless of whether the underlying regressors are purely $I(0)$, purely $I(1)$, or a mixture of both. In R, this test is implemented by estimating a level-based ARDL model[69], where the appropriate number of lags is selected to eliminate serial correlation in the residuals. Thereafter, an F-test is applied in a general ECM model, where lagged levels of the variables under consideration are included to test for cointegration. If the F-statistic is greater than the upper bound[70], the null hypothesis of no cointegration is rejected. If the F-statistic is less than the lower bound, the null hypothesis cannot be rejected. If the F-statistic falls between the bounds, the test is inconclusive.

If the ARDL-bounds test yields inconclusive results, we proceed by testing for cointegration using the Johansen method, which is more suitable when all variables are $I(1)$. According to Turatti (2024), Johansen's approach is based on the relationship between the rank of a matrix and its characteristic roots[71]. Once these two tests have been conducted, confidence in the assessment of cointegration can be established, allowing for the selection of the model that best fits the data.

Run the specific model

To determine the appropriate model for each variable, an Error Correction Model (ECM) is selected if cointegration is present among the variables. The primary objective is to capture long-term relationships, which are a

[69] In R, this is done using `auto.ardl()`, which selects the best model based on the provided input variables.

[70] According to Pesaran et al. (2001) - "Two sets of asymptotic critical values are provided for the two polar cases which assume that all the regressors are, on the one hand, purely $I(1)$ and, on the other, purely $I(0)$. Since these two sets of critical values provide critical value bounds for all classifications of the regressors into purely $I(1)$, purely $I(0)$ or mutually cointegrated, we propose a bounds testing procedure. If the computed Wald or F-statistic falls outside the critical value bounds, a conclusive inference can be drawn without needing to know the integration/cointegration status of the underlying regressors. However, if the Wald or F-statistic falls inside these bounds, inference is inconclusive and knowledge of the order of the integration of the underlying variables is required before conclusive inferences can be made."

[71] In R, the test is carried out using the `ca.jo()` function, which is applied with `type = "trace"` and can be used with either a trend or a constant.

defining feature of both cointegration and ECM frameworks. In the absence of cointegration, the focus shifts to modeling short-term dynamics using differenced variables.

Control for structural breaks and outliers

Additionally, structural breaks are tested for and, if present, incorporated into the model using dummy variables[72]. Structural breaks occur when a time series undergoes a sudden shift, often due to changes in the definition or measurement of a variable. These breaks can complicate interpretation, particularly in economic analyses where data consistency over time is essential. They may also lead to insignificant parameter estimates that would otherwise be significant, ultimately introducing bias in the regression model (Zeileis et al., 2003). It is therefore crucial to detect and account for such breaks in the analysis[73]. Furthermore, based on visual inspection, dummy variables are included to capture apparent outliers and structural breaks in the residuals, with the aim of achieving normally distributed residuals.

Run diagnostic tests

To obtain the best linear unbiased estimates, each model is tested against the standard Gauss–Markov assumptions in accordance to Wooldridge (2013) chapter 10.3. In particular, we test whether the error term ϵ_t is independent of the explanatory variables $\mathbf{X}$ and follows an i.i.d. process distributed as $\mathcal{N}(0, 1)$. To assess this, the residuals over time, their autocorrelation function (ACF), a histogram of the residuals, and a residuals-versus-fitted plot are examined. These graphical diagnostics help evaluate whether the residuals are approximately normally distributed and uncorrelated. In addition, two formal normality tests are conducted: the Shapiro–Wilk test, originally presented in Shapiro and Wilk (1965), and the Jarque–Bera test, originally introduced in Jarque and Bera (1980). Both tests have the null hypothesis that the residuals are normally distributed. If the reported p-value exceeds the 5% significance level, the null hypothesis cannot be rejected, suggesting that the residuals do not deviate significantly from normality.

Furthermore, serial correlation is tested for, which occurs when the errors conditional on $\mathbf{X}$ in two different periods are correlated. This can be expressed as $Corr(\epsilon_t, \epsilon_s \mid \mathbf{X}) = 0$ for all $t \neq s$. Serial correlation is detected through PACF and ACF plots, with first-order serial correlation tested using the Durbin-Watson test, originally presented in Durbin and Watson (1950) and higher-order serial correlation tested using the Breusch-Godfrey test, presented in Godfrey (1978). Both tests have the null hypothesis that there is no serial correlation. If the reported p-value exceeds the 5% significance level, the null hypothesis cannot be rejected, suggesting that the residuals are not serially correlated over time.

Heteroscedasticity, refers to the situation where the variance of the error term conditional on $\mathbf{X}$ is not constant over time. Heteroscedasticity is tested for by examining whether the variance of the error term, $Var(\epsilon_t \mid \mathbf{X})$, is constant across all t, i.e., $Var(\epsilon_t) = \sigma^2$ for $t = 1, \ldots, n$. This is assessed using residuals versus fitted values and the Breusch-Pagan test. The null hypothesis of the Breusch-Pagan test is that the error terms are homoscedastic. If the reported p-value exceeds the 5% significance level, the null hypothesis cannot be rejected, suggesting that the residuals are homoscedastic. Additionally, the presence of multicollinearity among the independent variables is examined to avoid spurious correlations. Furthermore, parameter stability is tested using the CUSUM test. If the CUSUM statistic remains within the critical bounds at the 5 percent significance level, the null hypothesis that all coefficients in the regression are stable cannot be rejected (Brown et al., 1975).

[72] While splitting the sample could be a possible solution, the dummy variable method is preferred, as the goal is to incorporate as many data points as possible in the estimations.

[73] This is done in R, using the `breakpoints()` function, which is to be found in the `strucchange` package. This specific breakpoint test is from Bai and Perron (1998).

Evaluate static and dynamic forecasts

In the final step before selecting the best model, we evaluate the static forecast by comparing the specific behavioral equations predicted values to the actual data at each time point, reporting the corresponding root mean square error (RMSE). Additionally, we assess the dynamic forecast, which examines how the model's predictions evolve over time relative to the observed values. RMSE is also reported for the dynamic forecast. In order to interpret the forecasts and evaluate the RMSE across different behavioral equations, we employ the normalized root mean square error (NRMSE).

7.2.2 Econometric estimations for the real variables

According to Valdecantos (2023b), the fifth step in constructing an empirical SFC model involves estimating and calibrating behavioral equations. The estimation procedure applied in this section follows the step-by-step approach outlined earlier in section 7.2.1. Each behavioral equation incorporated into the E-IO-SFC model is estimated based on a theoretical framework that is considered suitable for capturing the underlying economic mechanisms. The estimations presented in this section have been selected with two objectives in mind: to demonstrate the practical estimation procedure, and to present behavioral equations that represent a contribution to the existing literature in the SFC community. Specifically, this section highlights estimations for real expected sales, exports by the MC industry, a portfolio share (representing the financial account), and producer prices. The remaining estimated behavioral equations, along with their theoretical foundations, are provided in full in the appendix, cf. section C. The theoretical assumptions guiding the specification of each behavioral relationship are discussed in their respective subsections, beginning with real expected sales. After the presentation of each model, the estimation results are assessed in light of theoretical expectations. Finally, diagnostic tests, model fit statistics, and brief reflections on the model's behavior are discussed. Detailed diagnostics and plots for all input variables used in the estimation process are provided in the appendix as well.

Real expected sales

The behavioral equation defining real expected sales is modeled as a function of indicators that drive overall economic activity and a confidence indicator. Specifically, it depends on the capacity utilization rate, serving as a business cycle indicator, real consumption, and disposable income for households, real investments and a confidence indicator. Since real expected sales are defined as actual real sales multiplied by a sentiment indicator for the corporate sector in Denmark, cf. section 6.1.6, the focus has been on selecting variables that are widely recognized as business cycle drivers. While there is no single unified theory specifically explaining the formation of expected real sales, the modeling approach draws on economic reasoning about how firms form expectations based on observable macroeconomic conditions.

From a theoretical standpoint, private consumption is commonly regarded as the primary driver of business cycles, as it constitutes the largest component of aggregate demand, cf. Gottfries (2013, p. 86). Accordingly, real consumption is expected to be a key explanatory variable for expected sales. In line with the concept of backward-looking expectations, cf. Godley and Lavoie (2012, p. 81), it is further assumed that past realizations of disposable income influence current expectations of sales.

Moreover, reflecting standard business cycle dynamics, real investment and the capacity utilization rate are included as explanatory variables, both of which are expected to have a positive relationship with expected sales. The confidence indicator, which captures the overall consumer sentiment in the economy (as opposed to corporate sentiment 'ETILLID'), is also incorporated. This specific choice of confidence measure serves two purposes: first, it mitigates the risk of multicollinearity in the model, as the indicator is less likely to be directly correlated with firm-level variables; and second, it allows the model to account for sentiment-driven fluctuations in expected sales, thereby capturing potential demand-side shocks that are not explained by the fundamental macroeconomic variables alone. Together, these variables are chosen to reflect both theoretical foundations and practical considerations, and are all expected to exert a positive influence on real expected sales.

Since cointegration is identified according to the bounds and Johansen tests in section C.4 of the appendix, real expected sales are estimated as an ECM. The estimated behavioral equation below, with input data spanning from 2002Q1 to 2024Q4, illustrates the presented theoretically relationship within the context of the model.

$$\Delta \ln(y_{e,t}) = -2.24 + 0.21 \cdot \Delta \ln(y_{e,t-1}) + 0.005 \cdot \Delta \ln(CI_t) + 1.06 \cdot \Delta \ln(\frac{y_t}{k_t})$$
$$+ 0.18 \cdot \Delta \ln(yd^H_{t-2}) + 0.61 \cdot \Delta \ln(c_t) + 0.17 \cdot \Delta \ln(i_t)$$
$$- 0.31 \cdot \ln(\mathbf{y_{e,t-1}}) + 0.53 \cdot \ln(\mathbf{c_{t-1}}) + 0.001 \cdot \ln(\mathbf{CI_{t-1}})$$

The behavioral equation for real expected sales, as outlined above, is consistent with theoretical expectations in standard economic theory. Real expected sales depends positively in the short-run on capacity utilization rate, real consumption, disposable income, investments, the confidence indicator and it's own lags. The long-run consumption elasticity to real expected sales (1.69) suggests that household consumption is an important long-run driver of real expected sales. The long-run confidence elasticity to real expected sales (0.004) suggests that higher confidence is linked to higher expectations, but the magnitude is small. Furthermore, approximately 31% of any created disequilibrium generated by a shock is error corrected within one quarter.

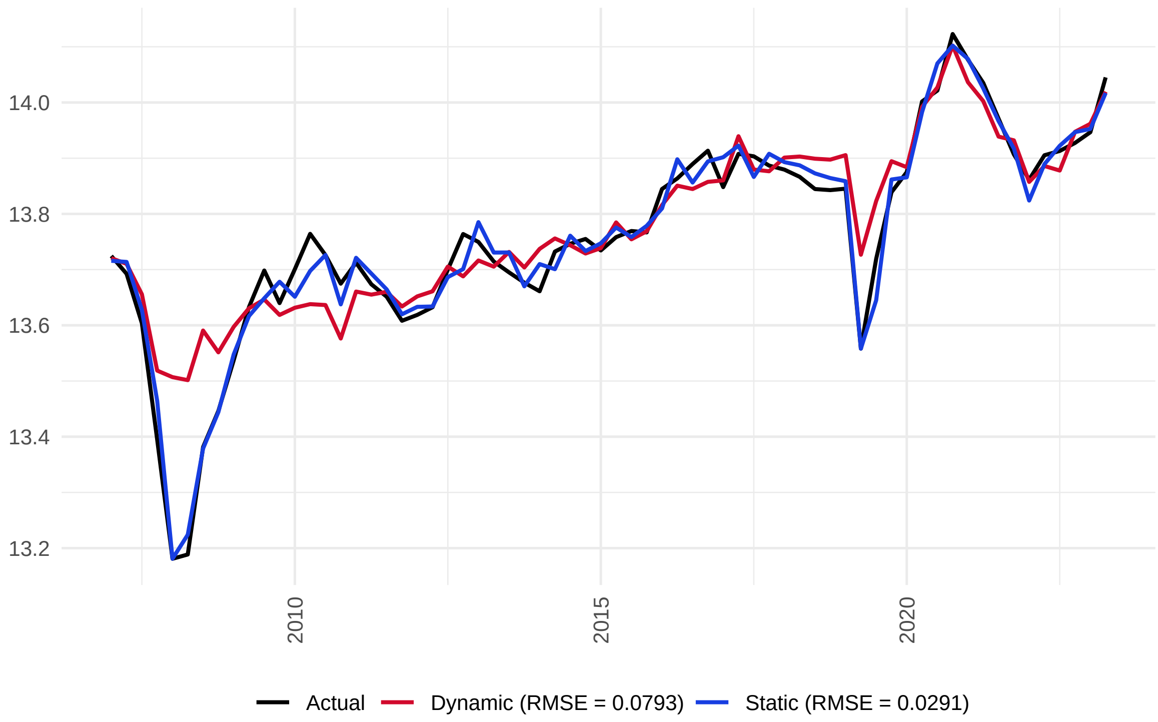


Figure 22: Overall fit of real expected sales in E-IO-SFC.

The behavioral equation for real expected sales passes all diagnostic tests, with detailed results presented in section C.4 of the appendix. The RMSE is 0.0793 for the dynamic forecast and 0.0291 for the static forecast. For interpretation purposes and to allow for comparability across models, the RMSE is transformed into a normalized root mean square error (NRMSE), by:

$$NRMSE = \frac{RMSE}{max(x) - min(x)},$$

where x is the dependent variable in the specific estimated behavioral equation. This implies that the static forecast has an average absolute error of approximately 3.0%, meaning that the model's predictions deviate, on average, by 3.0% from the actual observed values of real expected sales during the forecast period. The dynamic forecast shows a slightly higher average absolute error of approximately 8.4%, reflecting reduced accuracy due to the accumulation of forecast errors over time. The overall model fit is particularly strong in the post-crisis period after 2009–2010. In evaluating the reliability of an estimated model, especially for forecasting purposes, it is crucial to ensure a good fit toward the end of the sample period. Based on these criteria, the behavioral equation for real expected sales is assessed to perform well.

Real export as an industry-specific case

In light of the shift in the global trade agenda and the increasing relevance of protectionist policies such as tariffs, efforts have been made to model real exports for a specific industry. While the complete version of the model will entail specifying behavioral equations for all industries within the NFC sector, for illustrative purposes it is chosen to represent the exports of the MC industry. Moreover, this seems reasonable given that, according to figure 2b, the MC industry constitutes the largest portion of total exports[74]. Furthermore, incorporating an industry-specific behavioral export equation allows the E-IO-SFC model to exploit a key feature of the input-output structure, as further elaborated in section 7.3.

The behavioral equation for real exports by the MC industry is specified as a function of key determinants of external demand for a small open economy such as Denmark. These include the export price for MC, foreign income (proxied by a weighted real GDP measure based on Denmark's three largest export markets (USA, Germany, and Sweden))[75], the real exchange rate (inclusive of a tariff component), and a world trade indicator. From a theoretical perspective, two main factors determine external demand in a small open economy: i) the real exchange rate, and ii) foreign income. As discussed by Gottfries (2013, p. 330), an increase in foreign income leads to higher net exports due to international demand spill-overs, while an appreciation of the real exchange rate, making domestic goods more expensive relative to foreign goods, reduces export volumes.

For the specification of the model, although the ARDL bounds and Johansen cointegration tests reported in section C.6 do not indicate a long-run equilibrium relationship between the variables, the estimated long-run coefficients are statistically significant and contribute meaningfully to the model's fit. Therefore, despite the formal test outcomes, an ECM specification is retained. The estimated equation over the sample period 2002Q2–2024Q2 is as follows:

$$\begin{aligned}\Delta \ln(x_t^{MC}) = {} & 0.38 - 0.91 \cdot \Delta \ln(p_{X_t}^{MC}) + 0.49 \cdot \Delta \ln(p_{X_{t-1}}^{MC}) - 0.43 \cdot \Delta \ln(e_t) + 0.13 \cdot \Delta \ln(gdp_t^{RoW}) \\ & - 0.05 \cdot \Delta \ln(gdp_{t-4}^{RoW}) + 0.49 \cdot \Delta \ln(WT_t^{indicator}) - 0.11 \cdot \ln(\mathbf{x_{t-1}^{MC}}) + 0.05 \cdot \ln(\mathbf{gdp_{t-1}^{RoW}})\end{aligned}$$

The estimated relationship aligns with the presented economic theory. Real exports by the MC industry respond negatively to short-run changes in the export price deflator (P_X^{MC}) for MC and the real exchange rate. Conversely, foreign income and the world trade volume, captured by the World Trade Indicator, exert positive effects, which also aligns with standard economic theory. The inclusion of the World Trade Indicator was motivated by the model's inability to capture major exogenous shocks such as the COVID-19 pandemic and the energy crisis. A plausible explanation is that the input data may be significantly influenced by political interventions, such as emergency fiscal measures or extensive state aid implemented in Denmark during the COVID-19 pandemic (European Commission, 2025). These interventions may obscure the true dynamics of market adjustment, thereby limiting the model's ability to predict crises of this nature. Including this indicator improved model performance, as reflected in an increase in the adjusted R^2 from 0.57 to 0.61 and reductions were seen in both static and dynamic RMSE values[76]. The long-run foreign income elasticity to real export by MC (0.48) suggests that foreign income is an important long-run driver of MC export, indicating that real export adjusts moderately in response to foreign income increases. Furthermore, approximately 11% of any disequilibrium generated by a shock is error corrected within one quarter, indicating a relatively adequate speed of adjustment.

[74]To maintain consistency within the framework, real export at the aggregate level is still retained in the system of equations and subsequently distributed to industries based on their export shares.

[75]See section 6.2.3 for the construction of the foreign income proxy.

[76]Static RMSE declined to 4.4%, and dynamic RMSE to 10.04% with the inclusion of the trade indicator.

Figure 23 presents the overall fit of the model. The RMSE equals 0.0205 for the static forecast and 0.0481 for the dynamic forecast, indicating an adequate predictive performance of the model. The static forecast corresponds to an average absolute error of approximately 3.89%, meaning that the model's predictions deviate, on average, by 3.89% from the actual observed values of real exports by MC during the forecast period. In comparison, the dynamic forecast yields a slightly higher average absolute error of 9.13%. The static forecast exhibits good accuracy, whereas the dynamic forecast indicates that further refinements are needed, particularly with respect to capturing both short-run dynamics and longer-run adjustments. Overall, however, the model adequately reproduces the trend in real exports from the MC industry. Although noticeable deviations are present at the beginning of the sample, they are gradually corrected. Future work should extend the specification with additional explanatory variables to enhance the model's ability to track macroeconomic fluctuations in the export series.

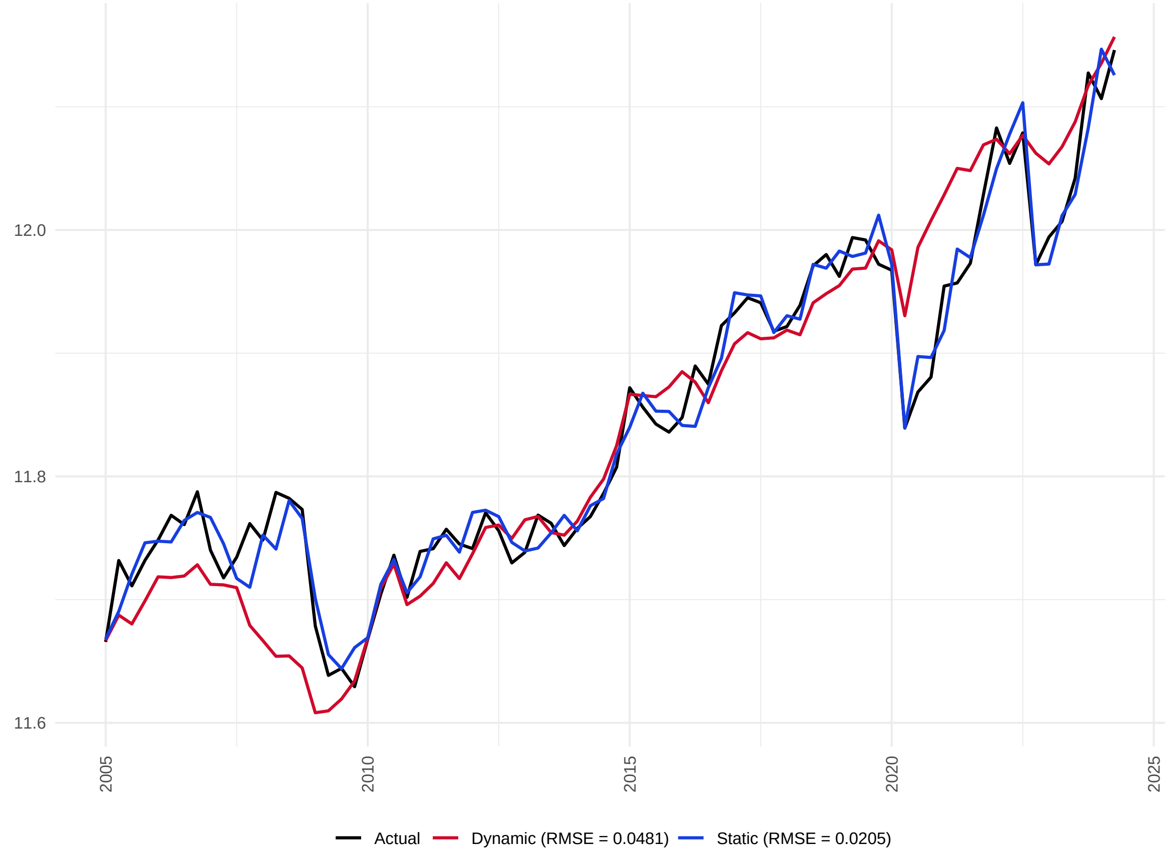


Figure 23: Overall fit of real export for MC in E-IO-SFC.

Producer price for the transport industry

To account for inter-industry dependencies which might be driven by producer prices, the E-IO-SFC model includes industry-level price behavioral relationships. These prices may have substantial spillover effects across sectors and the overall economy. Rising producer prices (PP) can increase production costs in the industry, thereby affecting competitiveness, profitability, and ultimately the financial position of the firms involved. Moreover, an increase in producer prices may ripple through the supply chain: if one industry experiences a rise in production costs, it may pass these costs on to its output prices, thereby affecting downstream industries that rely on intermediate inputs. This ripple effect may ultimately influence consumer prices and aggregate demand in other sectors or industries. The inclusion of producer prices in the E-IO-SFC model allows the identification of supply chain disruptions and spillovers between industries, as also demonstrated by Platitas and Ocampo

(2025) in the context of inflation dynamics. This section specifically examines the producer price dynamics of the transport industry, while results for the remaining industries can be found in section C.7 tosection C.11 of the appendix.

From a theoretical standpoint, as outlined in Gottfries (2013, pp. 241–243), prices may increase as a result of cost-push shocks, which raise production costs and thereby contribute to inflation. Consequently, unit costs are expected to exert a positive influence on the producer price in the transport industry. Furthermore, the producer price in the transport industry is assumed to be influenced by the producer prices in other industries within the E-IO-SFC model, reflecting inter-industry linkages. This assumption is also supported by the findings of Platitas and Ocampo (2025), who emphasize such spillover effects from an inflationary perspective.

Given the presence of cointegration identified via both the ARDL bounds and the Johansen tests (see section C.8), the producer price for the transport industry is estimated using an ECM. The estimated behavioral equation below, based on quarterly data from 2001Q1 to 2024Q2, illustrates the aforementioned mechanisms:

$$\begin{aligned}\Delta \ln(pp_t^T) = & -0.01 + 0.43 \cdot \Delta \ln(p_{IM_t}) + 0.12 \cdot \Delta \ln(p_{IM_{t-1}}) + 0.06 \cdot \Delta \ln(UC_{t-3}^T) + 0.29 \cdot \Delta \ln(p_{I_{t-1}}) \\ & + 2.09 \cdot \Delta \ln(pp_t^{OMS}) - 0.11 \cdot \ln(\mathbf{pp_{t-1}^T}) + 0.04 \cdot \ln(\mathbf{pp_{t-1}^E}) + 0.05 \cdot \ln(\mathbf{pp_{t-1}^A})\end{aligned}$$

The behavioral equation for the transport industry's producer price aligns well with theoretical expectations. In the short run, producer prices are positively influenced by the import deflator (p_{IM}), the industry's own unit labor costs (UC^T), the investment deflator (p_I), and the producer prices of the OMS industry. The significant short-run and long-run coefficients on OMS, energy, and agriculture underscore the inter-industry dependencies and transmission mechanisms operating through intermediate input linkages.

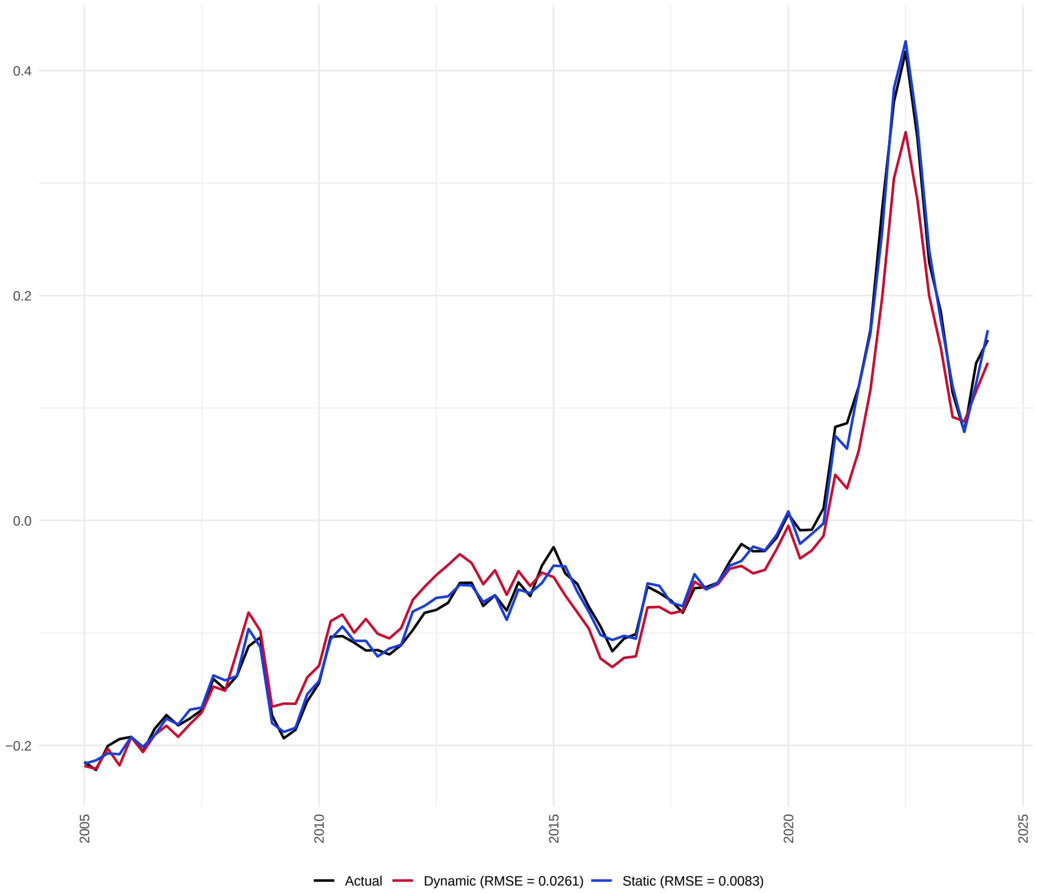


Figure 24: Overall fit of producer prices for the transport industry in E-IO-SFC.

The long-run elasticity of the transport industry's producer price with respect to energy industry prices is 0.36, suggesting that energy prices are an important long-run determinant. This implies that transport prices adjust moderately in response to persistent changes in energy costs. Similarly, the long-run elasticity with respect to agricultural producer prices is 0.44, indicating a comparable influence. Furthermore, the error correction term implies that approximately 11% of any disequilibrium caused by a shock is corrected each quarter, indicating a moderate speed of adjustment toward long-run equilibrium.

In figure 24 above, the overall fit of the model is presented. The RMSE is 0.0261 for the dynamic forecast and 0.0083 for the static forecast, indicating that the model's predictive performance is satisfactory. The static forecast corresponds to an average absolute error of approximately 1.17%, meaning that the model's predictions deviate, on average, by 1.17% from the actual observed values of the producer price, by the transport industry, during the forecast period. In comparison, the dynamic forecast yields a slightly higher average absolute error of 3.69%. Overall, the model fit is satisfactory, as it successfully reproduces the underlying trend over the estimation period. Additionally, it captures the short- and long-run fluctuations.

7.2.3 Econometric estimations for the financial variables

A key feature of the stock-flow consistent approach is the fulfillment of agents' budget constraints. Although agents' plans may not be entirely consistent *ex ante*, it must always be the case that budget constraints are satisfied *ex post*. As it happens in the real world, in SFC models this is represented through a "buffer variable", i.e., a financial asset/liability that adjusts for any discrepancies between agents' incomes (resources) and expenditures (uses of funds). For example, in the case of households, if the buffer consists of deposits, this implies that, in the event of a deficit in a given period, households will draw on their stock of deposits to cover this deficit. In order to ensure the fulfillment of each agents' budget constraint, the "buffer" variable is normally represented as a residual of all income and expenditure transactions, thereby serving as a link between the real and financial sides of the economy.

Once the "buffer" variable for each agent has been defined, the remaining financial instruments can be specified as a portfolio allocation process. This applies to both sectors and industries across the economy. Given that the primary objective of this paper is to lay the empirical foundations for constructing the E-IO-SFC model, here we focus on the estimation of a representative financial β, which serves as a demonstration of the methodology used to endogenize financial behavior in the model. The variable presented below, which we estimate econometrically, reflects the share of total financial wealth that the household sector allocates in the form of securities:

$$\beta^H_{F.3} = \frac{F.3^H_S}{FW^H},$$

where $\beta^H_{F.3}$ denotes the proportion of financial wealth (FW^H) held by households in securities ($F.3^H_S$). To effectively model the inter-dependencies between the financial and real sectors of the economy, it is necessary not only to endogenize the real side through behavioral equations, as outlined in section 7.2.2, but also to incorporate endogenous financial behavior within sectors and industries. This financial behavior is both influenced by and influences the real side of the economy, highlighting the inter-dependencies. For example, households consumption depends on disposable income, which in turn depends on interest earning on securities, which are a function of the past accumulated stocks of securities.

This section therefore follows the standard SFC models perspective to identify plausible relationships linking the real and financial sides of the economy. These relationships are intended to serve as inputs in the E-IO-SFC model (when running endogenously), with particular emphasis placed on the behavioral dynamics of the financial side of the economy. According to Zezza and Zezza (2020), one of the early proponents of the SFC framework was James Tobin, whose portfolio theory of investment was later adopted and extended by Wynne Godley, although within a post-Keynesian framework, and subsequently integrated into SFC modeling by Godley and Lavoie (2012). The existing proposals that model the Tobinesque portfolio equations proposed in Tobin (1969)[77] are theoretically outlined in model PC in Godley and Lavoie (2012). From an empirical standpoint Zezza and Zezza (2020) attempt to adapt the structure of the financial side of their model by keeping the main concepts embedded in Tobins' theory. The authors state themselves that it is difficult to estimate the appropriate relations, if they exist, between the relative rate of returns and the demand and supply for different assets and liabilities.

We try to fully adapt Tobin's theory in a practical manner, and show that the principles behind it also hold empirically. Following the portfolio allocation presented in model PC, $\beta^H_{F.3}$ is modeled as a function of disposable income in relation to household wealth, the rate of return on securities and deposits, and a consumer confidence indicator[78] reflecting risk tolerance in portfolio allocation. It is therefore expected that, if disposable income increases relative to financial wealth, households will allocate a larger share of their income to securities. Regarding the confidence indicator, a negative sign is expected. The rationale is that when households anticipate a positive economic outlook over the coming quarters, they may exhibit greater risk tolerance and therefore shift their portfolio allocation from safe assets such as securities to riskier assets like equities. For asset prices, we expect a Tobin-style substitution mechanism: an increase in the rate of return on securities induces households to hold more bonds, while an increase in the deposit rate makes deposits more attractive, leading to a reallocation away from securities and toward deposits. This reflects the $n-1$ relationship described in Tobin (1969), where increasing the allocation towards one asset necessarily implies reducing the allocation of at least one other asset in a constrained portfolio setting.

[77] Tobin's general equilibrium approach models economic agents as choosing a portfolio allocation among various assets (e.g., money, bonds, deposits and capital) based on relative returns and risk (Tobin, 1969).

[78] The confidence indicator is currently treated as exogenous. However, it is intended to be endogenize in future work, following a similar approach to that used for real expected sales, as presented in section C.4 in the appendix.

According to the Johansen cointegration test presented in section C.5 of the appendix, together with the regression output and diagnostic tests, evidence of cointegration is found. Therefore, the share of securities out of total financial wealth for households is modeled using an ECM model. The estimated relationship is presented below, with the sample spanning from 2002Q1 to 2024Q3:

$$\begin{aligned}\Delta \ln(\beta^H_{F.3,t}) &= -0.31 - 0.25 \cdot \Delta \ln(\beta^H_{F.3,t-2}) - 0.30 \cdot \Delta \ln(\beta^H_{F.3,t-3}) + 0.43 \cdot \Delta \ln(\beta^H_{F.3,t-4}) \\ &- 0.005 \cdot \Delta(CI_{t-3}) + 0.41 \cdot \Delta \ln(\frac{YD^H_t}{FW^H_t}) + 0.33 \cdot \Delta \ln(\frac{YD^H_{t-3}}{FW^H_{t-3}}) \\ &- 0.05 \cdot \ln(\boldsymbol{\beta^H_{F.3,t-1}}) + 20.65 \cdot (\boldsymbol{\iota^{F.3}_{t-1}}) - 9.82 \cdot (\boldsymbol{\iota^{F.2}_{t-1}})\end{aligned}$$

The presented theoretical relationships are supported empirically, and the signs of the explanatory variables behave as expected. The positive relationship between $\beta^H_{F.3}$ and $\frac{YD_H}{FW^H}$ suggests that, as households' income increases relative to their financial wealth, they tend to allocate a larger share of their portfolio to bonds, both contemporaneously and with a lag of three quarters. As expected, the confidence indicator exhibits a negative short-run effect. Using a linear downscaling method[79], the estimated long-run semi-elasticity between $\beta^H_{F.3}$ and $\iota^{F.3}$ is 3.8%, indicating a strong positive relationship. That is, a 1 percentage point increase in the rate of return on securities leads to a 3.8% increase in the share of financial wealth allocated to securities by households. The long-run semi-elasticity between $\beta^H_{F.3}$ and $\iota^{F.2}$ is estimated at -1.81, which is both expected and economically reasonable. Furthermore, approximately 5% of any disequilibrium caused by a shock is corrected within one quarter, indicating a relatively slow but stable adjustment process back to the long-run equilibrium. In figure 25 below, the overall fit of the model is presented.

The RMSE is 0.13 for the dynamic forecast and 0.036 for the static forecast, indicating that the model's predictive performance is sufficient. The static forecast corresponds to an average absolute error of approximately 1.2%[80], meaning that the model's predictions deviate, on average, by 1.2% from the actual observed values of the ratio $\beta^H_{F.3}$ during the forecast period. In comparison, the dynamic forecast yields a higher average absolute error of 4.5%. Overall, the model fit is adequate, as it successfully reproduces the underlying trend over the estimation period. However, the performance of the dynamic forecast suggests room for improvement, particularly in capturing short-run fluctuations. Future work with the fully endogenous E-IO-SFC model should aim to incorporate short-run explanatory variables to enhance the model's ability to track short-term dynamics in the time series.

[79] A 1 percentage point increase in $\iota^{F.3}$, e.g., from 0.4% to 1.4%, implies an effect of $20.65\% \cdot 0.01 = 0.2065\%$.

[80] Reported as the normalized RMSE.

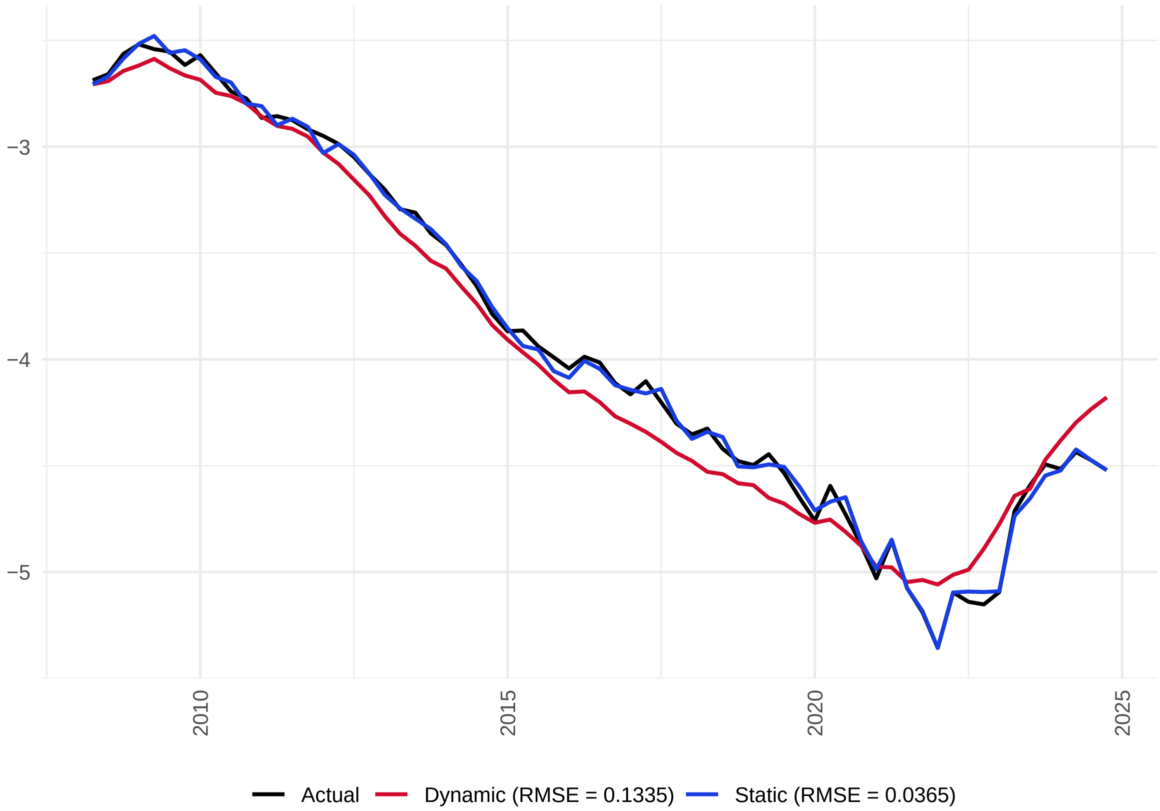


Figure 25: Overall fit of $\beta^{H}_{F.3}$ in E-IO-SFC.

7.3 Directed Acyclic Graph

Having introduced the core behavioral equations of the model, this subsection illustrates how a policy-relevant shock propagates through the full system of equations in this E-IO-SFC model. Specifically, we simulate a tariff shock imposed by the rest of the world on the MC industry, not as a comprehensive scenario analysis, but to demonstrate, clearly and intuitively, how such a disturbance spreads through the model's structural and financial linkages.

To trace the dynamic transmission of the shock, we present a Directed Acyclic Graph (DAG), which maps the causal pathways through the system of equations. The DAG highlights the model's internal consistency by showing how changes in one part of the economy affect others via production inter-dependencies, behavioral responses, and financial adjustments. In particular, it illustrates (i) the role of the input–output structure in transmitting real-side shocks across sectors, and (ii) the financial channels through which income and portfolio effects feed back into the broader economy.

By linking together the relevant equations and sectoral balances, this section illustrates explicit how the model responds dynamically to shocks. This lays the foundation for understanding both short-term instability and the longer-term stabilizing effects that emerge through trade, prices, and financial behavior. Figure 26 provides a visual overview of the propagation mechanism described in this section. Each arrow corresponds to a causal link derived from the model's equations or accounting identities.

A tariff shock on MC products, denoted as τ_{RoW}^{MC}, leads to a appreciation in the real exchange rate for MC goods through the following equation:

$$e^{MC} = \frac{p_Y \cdot (1 + \tau_{RoW}^{MC})}{p_{IM}}.$$

As shown in the estimated export equation in section 7.2.2, the real exchange rate enters with a negative coefficient, implying that a deterioration in competitiveness leads to a decline in exports. This is, in fact, what the tariffs currently debated in the US are meant to generate. As a consequence, the decline in exports leads to lower sales volumes for the MC industry. Real sales for each NFC industry, including MC, are defined as:

$$y^{MC} = c^{MC} + g^{MC} + x^{MC} + ig_s^{MC} + i_c^{MC}.$$

Lower exports thus reduce y^{MC}, leading to an accumulation of inventories as the gap between expected and realized sales widens. In the subsequent period, firms reduce their production due to excessive inventory accumulation. The desired level of real inventories is given by:

$$inv_d^{MC} = inv_{t-1}^{MC} + i_{inv,d,t}^{MC} \tag{1}$$

A decline in y^{MC} also implies a change in inventories. This dual adjustment, both from actual inventory accumulation and a downward revision in desired stock levels, contributes to a significant reduction in real production. Accordingly, real production in the MC industry is defined as:

$$y_P^{MC} = y_e^{MC} + i_{inv,d}^{MC}.$$

This mechanism illustrates how sales shocks not only affect current production decisions through unsold goods, but also influence forward-looking expectations about optimal inventory levels and expected sales.

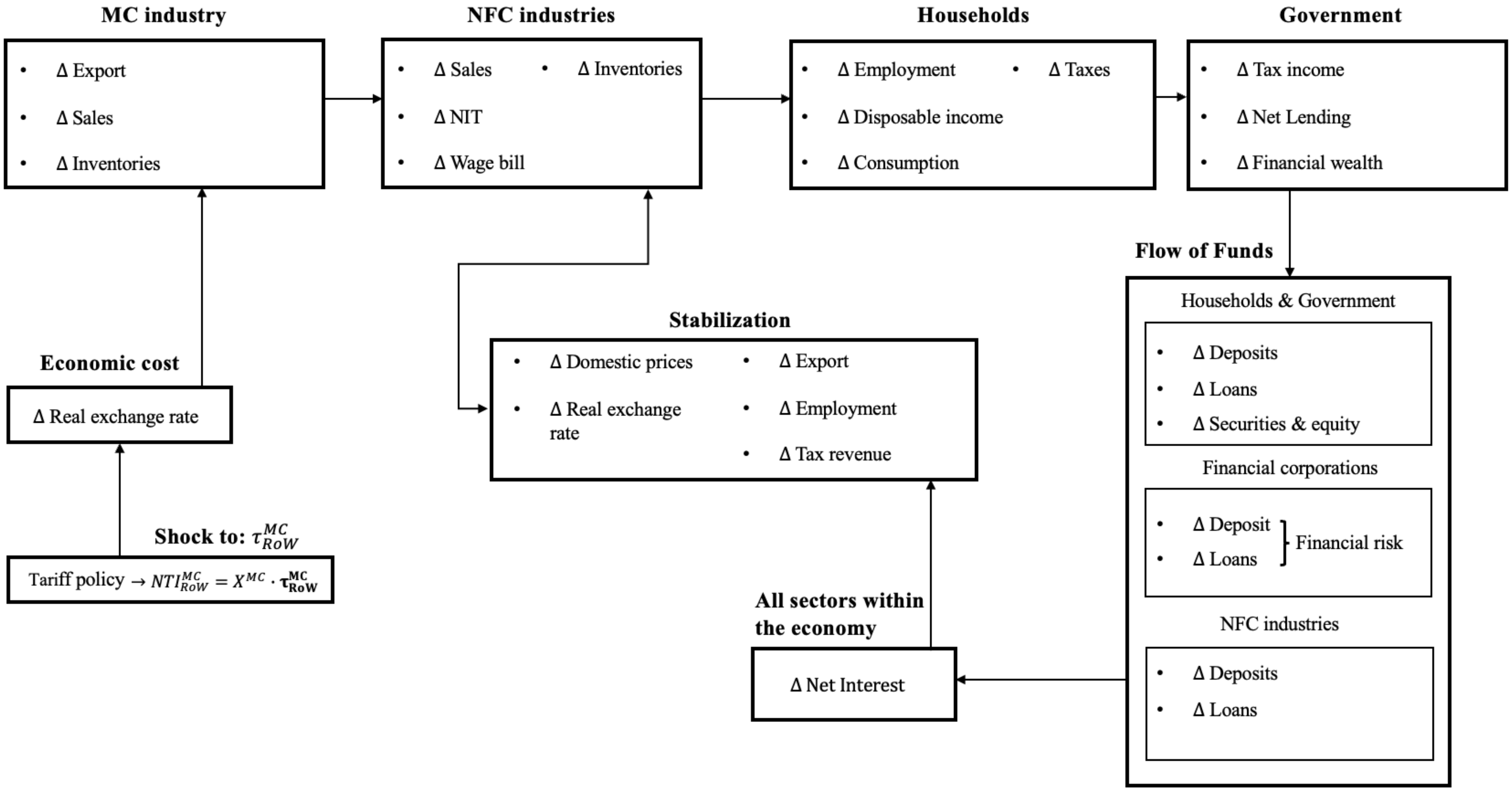


Figure 26: Directed Acyclic Graph (DAG).

Given the structure of the input–output relationship with in model, this decline in production leads to a fall in intermediate input purchases, not only within MC but also in other industries with strong inter-linkages, most notably OMS and MC itself.

$$ig_p^{MC,MC} = y_P^{MC} \cdot \alpha_p^{MC,MC},$$

$$ig_p^{MC,OMS} = y_P^{MC} \cdot \alpha_p^{MC,OMS}.$$

The reduction in production reduces employment, thereby lowering the wage bill for households. The following equation illustrates the mechanism for the MC industry, where the impact is expected to be highest, though the same formulation applies to other industries as well.

$$EMP^{MC} = \frac{y_P^{MC}}{pr^{MC}}.$$

As discussed in section C.2, wages are negatively affected by the cyclical component of unemployment. Consequently, rising unemployment puts downward pressure on the wage bill, amplifying the decline in household income.

$$YD^H = WB^H + GOS^H + DI_r^H + NI^H + SB^H + OC^H - SC_p^H - T^H.$$

Simultaneously, rising unemployment increases government expenditure on social benefits, as these transfers are assumed to respond positively to the level of unemployment. The combined effect is a reduction in household disposable income (YD^H), which further reduces consumption, particularly of goods produced by the most exposed industries.

$$c^{MC} = c^H \cdot \alpha_c^{MC}.$$

$$c^{OMS} = c^H \cdot \alpha_c^{OMS}.$$

This transmission is made possible by the input–output structure of the model, which links household demand to industry-specific sales through time-varying technical coefficients. As a result, a decline in YD^H feeds back into the production system with industry-specific intensity. As consumption drops, production contracts further, reinforcing the negative spiral.

As the wages fall, due to lower employment, then the prices will decrease (or increase in a slower pace). This will make the real exchange rate depreciate, making the domestic goods more competitive and allowing exports to recover. Whether they can return to their pre-tariff shock level depends on how strong these adjustment processes are (namely, how much downward flexible prices are, and how sensitive manufacturing exports are to the real exchange rate). These stabilizing forces operate with a lag, as prices, wages, and investment plans adjust only gradually to new conditions.

On the government side, higher social benefit payments (e.g. unemployment benefits) and lower tax revenues worsen the government sector's net lending position. To finance the resulting deficit, the government issues additional bonds, increasing the public debt and, consequently, future interest payments:

$$F.3^G = NL^G - F.2^G - F.4^G - F.5^G - F.6^G - F.7^G - F.8^G.$$

For the sectors or industries that hold these newly issued bonds, such as financial corporations or the rest of the world, this results in higher net interest income, thereby strengthening their financial position. For households, the effect is the opposite, because a decrease in YD^H, reduces their holdings of securities:

$$\Delta \ln(\beta^H_{F.3,t}) = -\beta_{0,\beta^H_{F.3}} - \beta_{1,\beta^H_{F.3}} \cdot \Delta \ln(\beta^H_{F.3,t-2}) - \beta_{2,\beta^H_{F.3}} \cdot \Delta \ln(\beta^H_{F.3,t-3}) + \beta_{3,\beta^H_{F.3}} \cdot \Delta \ln(\beta^H_{F.3,t-4})$$
$$- \beta_{4,\beta^H_{F.3}} \cdot \Delta(CI_{t-3}) + \beta_{5,\beta^H_{F.3}} \cdot \Delta \ln\left(\frac{YD^H_t}{FW^H_t}\right) + \beta_{6,\beta^H_{F.3}} \cdot \Delta \ln\left(\frac{YD^H_{t-3}}{FW^H_{t-3}}\right)$$
$$- \beta_{7,\beta^H_{F.3}} \cdot \ln(\beta^H_{F.3,t-1}) + \beta_{8,\beta^H_{F.3}} \cdot (\iota^{F.3}_{t-1}) - \beta_{9,\beta^H_{F.3}} \cdot (\iota^{F.2}_{t-1}).$$

For FC, the decline in YD^H may lead households to increase borrowing, resulting in higher loan issuance and associated interest income. At the same time, household deposits fall as savings are drawn down. The combined effect raises FC income through higher net interest. As shown in section 6.1.5, the spread between lending and deposit rates for households is relatively large. This implies that a shift from deposits to loans significantly increases FC income. Since FC's are taxed on profits, this creates a secondary fiscal feedback: higher tax revenue from FC's partially offsets the decline in tax payments from households.

For NFC's, falling sales reduce operating income, potentially pushing firms into a negative cash flow position. In such cases, firms may draw down accumulated financial assets or resort to borrowing in order to cover fixed costs and financial obligations. This increases their debt exposure and amplifies the financial vulnerability of the corporate industry. As debt rises, so do future interest payments, which may further strain firm-level balance sheets if revenues do not recover in time. Moreover, if declining revenues force some firms into bankruptcy, FC's may incur losses from defaults, reducing their profits. In severe cases, this could lead to broader financial instability, where the 2007–08 subprime crisis in the U.S. stands as a cautionary example.

Additionally, the tariff shock may also affect equity valuations, as observed in recent financial market reactions. This can be captured through revaluation variables such as $F.5$. A drop in $F.5$ lowers the financial wealth of all agents holding this asset, thereby reducing consumption and investment through negative wealth effects and potentially deepening the economic downturn.

Despite the initial contractionary effects of the tariff shock, the model exhibits stabilizing dynamics over time. As falling employment and demand put downward pressure on prices and wages, the real exchange rate gradually improves, potentially restoring international competitiveness. This boosts exports and revitalizes production. Simultaneously, financial feedback mechanisms, such as increased interest income for bond holders and partial fiscal offset from financial corporations, contribute to mitigating the overall impact. However, the eventual improvement in household finances is more likely to result from the recovery of employment and wages as the economy gradually rebounds. Together, these mechanisms contribute to closing the loop and potentially support a recovery path for the economy. However, if the negative transmission effects, including financial channels such as balance sheet deterioration, losses on financial assets, and firm bankruptcies, are sufficiently strong, the economy may remain stuck in a prolonged recession, with export recovery proving insufficient. Ultimately, the strength of this modeling framework lies in its ability to simulate and explore alternative macroeconomic trajectories that may unfold in response to an external shock.

8 Where to go from now?

Having established all the essential building blocks in the previous sections, we are now in a position to construct a quarterly E-SFC-IO model for Denmark. Up to this point, we have laid out the theoretical framework, compiled the necessary data, and outlined the methodological choices that underpin the model components. The natural next step is to integrate these components, beginning with the construction of the exogenous model, followed by the endogenous model that incorporates the behavioral equations presented.

This integration has already been initiated in Bimets[81]. However, during this process, several significant issues emerged, including problems related to the annual input-output series, DST (2025b): From the input-output (IO) tables, it is possible to compute annual technical coefficients, which reflect: (i) the inter-industry flows of intermediate consumption (e.g., how much the energy sector purchases from other industries), and (ii) the distribution of final demand across industries, as seen in section 5.3.3. These coefficients provide annual targets for the corresponding weights and are consistent with aggregate annual data.

However, applying these annual coefficients directly to quarterly data introduces inconsistencies at industrial level. This stems from the fact that annual technical coefficients, which capture intermediate and final transactions between industries, do not necessarily align with the intra-annual dynamics. For example, if the energy sector purchased 0.1 units from itself in 2019, this does not imply the same transaction occurred uniformly across each quarter of that year.

The current computation of the IO-coefficients

The IO table form DST (NAIO1F) provides information on intermediate domestic transactions. This data makes it possible to understand how resources are allocated and goods are exchanged among industries. After allocating all sectors into the selected industry classification, as described in section 5.3.2, the following selections are made in the NAIO1F table:

1. 'Domestic production' is chosen as the type of supply.
2. 'Total-(Supply)' is selected in 'Supplying industries'.
3. The industry in interest is selected in 'Use'.

Supplying industry	Purchases (E)
E	6.600
T	436
A	1.317
MC	6.593
OMS	7.589
FC	970

(a) Purchases by E from supplying i industry

Supplying industry	Purchases (MC)
E	6.734
T	12.679
A	44.758
MC	178.847
OMS	156.074
FC	6.631

(b) Purchases by MC from supplying i industry

Table 21: Intermediate purchases by E (a) and MC (b) from each industry in mio DKK (2019) (DST, 2025b).

To illustrate the process, table 21 presents an example for 2019. Table 21 (a) and (b) show the amount of intermediate purchases by the E and the MC from all other industries. This procedure is repeated for all other industries, including financial corporations.

[81] Bimets is a software framework developed using the R programming language. It is designed for time series analysis and econometric modeling, enabling users to create and manipulate time series, specify simultaneous equation models of any size using a high-level description language, and perform model estimation, structural stability analysis, deterministic and stochastic simulations, as well as forecasting (Luciani and Stok, 2024).

Furthermore the total output for the different industries, obtained from NKBP10, are presented in table 22.

Industry	**Total output**
E	55.483
T	432.065
A	86.158
MC	1.118.842
OMS	2.203.551
FC	194.777

Table 22: Overview of total output (2019) (DST, 2025b).

Due to discrepancies between the figures of intermediate consumption (P.2) in the NKBP10 dataset (DST, 2024e) and the NAIO1F dataset, the value $IG_p^{i,j}$ from NAIO1F cannot be used directly. Consequently, the NAIO1F dataset is used exclusively for calculating the technical coefficients of intermediate inputs. Based on the two preceding tables, we can compute the technical coefficients that reflect the input intensity from industry i required to produce one unit of output in industry j. It is important to note that $IG_p^{i,j}$ originates from table 21, while Y_P^i is shown in table 22.

$$\alpha^{i,j} = \frac{IG_p^{i,j}}{Y_P^i}$$

For instance, $alpha^{E,MC} = \frac{6.593}{55.483} = 0.137$, highlighted in table 23, shows the amount of units E are purchasing from MC to produce one unit.

Industry	E	T	A	MC	OMS	FC
E	0.119	0.001	0.015	0.006	0.005	0.001
T	0.008	0.090	0.003	0.011	0.0028	0.005
A	0.024	0.000	0.128	0.040	0.001	0.000
MC	0.119	0.029	0.189	0.160	0.043	0.020
OMS	**0.137**	0.075	0.110	0.139	0.197	0.116
FC	0.017	0.010	0.067	0.006	0.020	0.228

Table 23: Share of intermediate inputs sourced from each supplying industry in 2019

Operational issues in handling IO-shares and IO-technical coefficients

As mentioned in the beginning of this section, challenges have arisen regarding the use of input–output coefficients, leading to internal inconsistencies. These issues affect all final demand components contributing to sales and production figures at the industry level where IO-shares[82] and IO technical coefficients are applied. In particular, inconsistencies also appear in intermediate sales and production figures.

From a practical viewpoint the problem can for instance be illustrated by the mismatch between sales and production figures at the industry level. Despite the accounting identity stating that production should equal sales plus the change in inventories, this identity is not satisfied in the current setup at the industry level. For instance, in the MC industry, sales are consistently lower than production, while the opposite holds for the transport sector. This systematic divergence introduces an inconsistency in the production account at the industry level.

[82]By IO shares we mean the shares used to distribute the aggregate demand components.

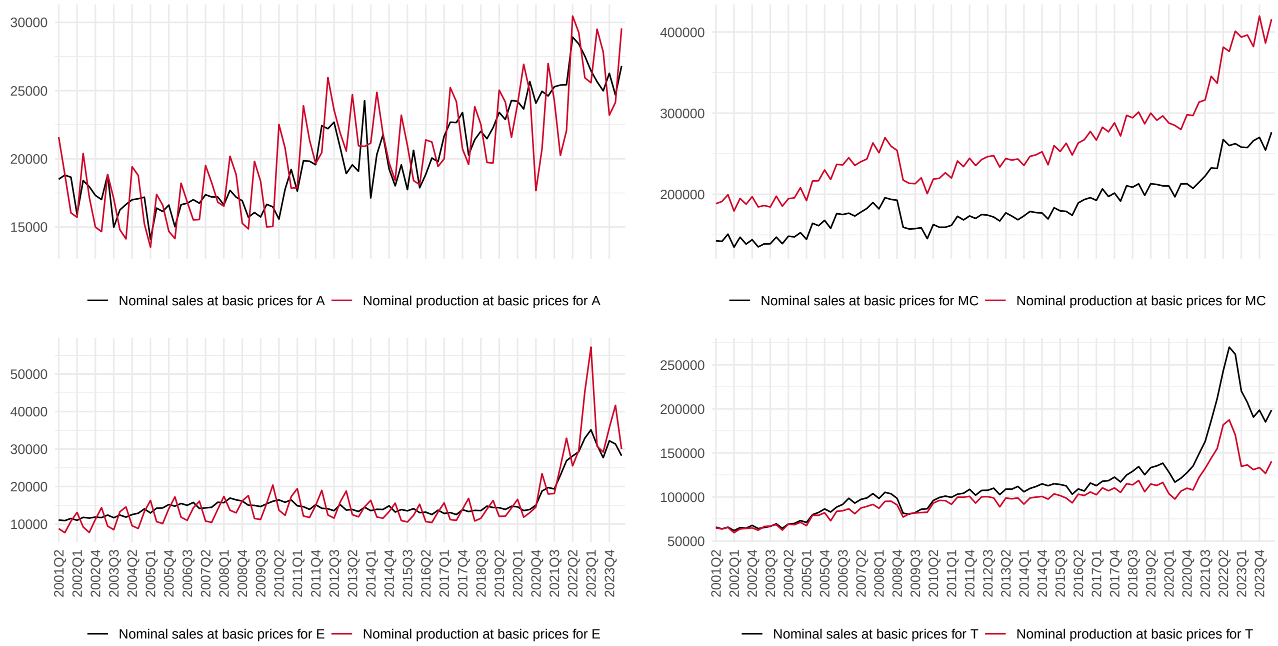


Figure 27: Nominal sales at basic prices vs. nominal production at industry level.

The above inconsistencies do not appear at the aggregated level because the IO-shares sum to one. Since the IO-coefficients and shares are calculated using annual data it is expected to see discrepancies at the quarterly industry level, as described above. The aggregated figure for nominal sales at basic prices and nominal production is presented below. Here it is seen that the aforementioned identity holds.

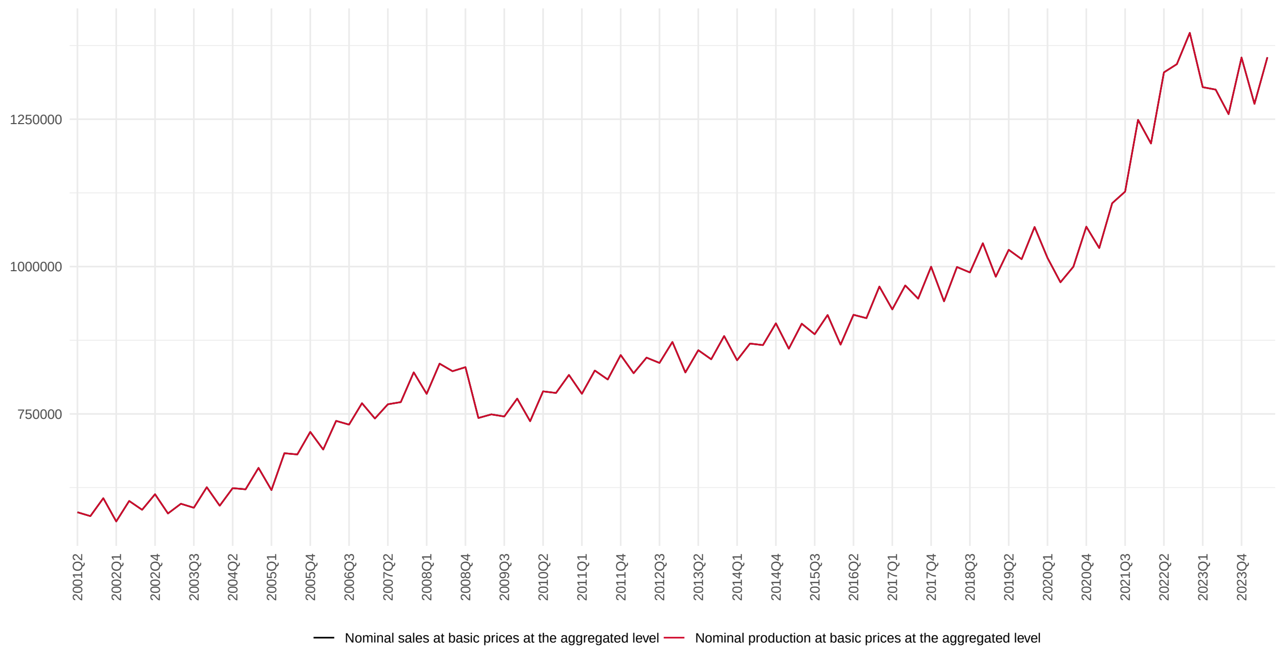


Figure 28: Nominal sales at basic prices vs. nominal production at the aggregated level.

To the best of our knowledge, this represents the first attempt, both within the SFC community and the broader CGE modeling literature, to integrate these specific data sources, particularly input–output data, into a quarterly framework. Consequently, the data inconsistencies discussed in this section only became apparent during the operationalization of the exogenous model. The question that follows is: where do we go from here?

Since IO coefficients are not available at quarterly frequency, it becomes necessary to develop a method that adjusts the annual coefficients to a quarterly scale, ensuring consistency with the final demand components and intermediate transactions at the industry level and alignment with aggregate figures. This method needs to incorporate some kind of quarterly fluctuations into the annual coefficients to obtain the intra-annual dynamics. A potential method has already been applied in section 6.1.4.1. Therefore, the next step is to develop a method for computing quarterly IO-shares and technical coefficients that ensures consistency with the identity stating that output equals sales (including inventories) at the industry level, as is the case at the aggregate level shown in figure 28.

9 Discussion

This discussion reflects on the initial process of developing the foundational framework for the E-IO-SFC model. The aim here is to discuss the various approaches and alternatives that could have been employed at the outset of the different building blocks presented by Valdecantos (2023b). The discussion can be divided into several parts, where the first part focuses on reflections regarding the process of developing the foundations using Denmark as a case study, exploring what could have been done differently, and highlighting lessons learned. These insights aim to provide guidance for new developers undertaking the construction of quarterly empirical SFC models that incorporate input-output structures. Next, the discussion will touch on reflections concerning the data work from DST and how this data integration is coupled with the structure of the E-IO-SFC model. The process of aligning and harmonizing DST data with the requirements of the model presents both challenges and opportunities. Insights from this experience may provide valuable guidance for others aiming to develop similar models. Lastly, the future work of the model will be discussed.

The foundational framework for the E-IO-SFC model is based on a set of empirically grounded behavioral equations. Its development was motivated by the recognition of structural limitations in contemporary macroeconomic models, particularly the treatment of the NFC sector as an aggregated entity. To address this, the E-IO-SFC model applied here disaggregates the NFC sector using input–output tables, thereby capturing interdependencies between industries. These inter-industry linkages are not only conceptually important, but are also supported by empirical evidence: econometric estimations of producer prices reveal that prices in one industry are influenced by price developments in others. For instance, the producer price for the transport industry is found to depend on price dynamics in the agriculture and energy industries. This empirical inter-dependence supports the argument made by Passarella (2023a) that the IO structure should be integrated into macroeconomic models not solely for theoretical coherence, but also due to its statistically verifiable relevance. Moreover, the identified economic problem revealed a persistent lack of integration between the real and financial spheres in existing macroeconomic models, along with a limited treatment of sectoral and industry-level financial balances. In current approaches, including our own specification, this integration is often confined to the consumption function, which depends on financial wealth and other variables. To address this shortcoming, a Tobinesque relationship has been incorporated into the household financial balances, specifically governing portfolio allocation. This formulation captures the interaction between real and financial variables, as households adjust their asset composition based on both disposable income (real fundamentals) and the intertemporal returns on interest-bearing assets (financial incentives). Together, these specifications constitute a significant step toward a more realistic macroeconomic framework, in which financial balances and their feedback effects on real activity are increasingly endogenized. Ongoing work aims to further strengthen this integration and refine the dynamic interplay between financial structures and the real economy.

Reflections on the data-generating process and its application as input to the E-IO-SFC model highlight the essential need for consistent and coherent data sources. To ensure internal consistency across the model, the use of a single, comprehensive data provider, such as DST, is necessary. This consistency yields two key advantages: (i) the use of a fully consistent and comprehensive dataset, and (ii) direct alignment with the official Danish national accounts, ensuring full transparency and traceability, as DST is the sole data source used. Together, these two key advantages made the development of a quarterly E-IO-SFC framework for the Danish economy possible. During the construction of the dataset, several gaps were explored in the published DST tables, most notably the absence of consistent employment figures at the desired level of disaggregation. Crucially, it is essential to maintain the methodological integrity of each dataset to avoid combining observations based on different accounting conventions. In this project, such inconsistencies were resolved using back-dating techniques to extend series consistently within the DST framework. For variables not available at quarterly frequency, interpolation methods were applied. These challenges underscore the complexity of the data-generating process, even when relying on one of the most detailed and methodologically advanced national statistical agencies in the world. The fact that such issues arise even in the Danish context highlights the degree of thoroughness required to ensure data coherence and validity. This is particularly critical for SFC models, which by definition demand full stock-flow consistency and therefore impose strict requirements on the internal consistency, frequency alignment, and reliability of the underlying data. Beyond working with already established statistical tables, the construction of an SFC model also requires creative and practically grounded economic thinking to develop non-observable variables. When such variables are conceptually important but not directly measurable, the modeler must construct proxies or inferred series that allow these dimensions to be meaningfully integrated into the framework. This aspect is particularly relevant when aiming to endogenize behavioral mechanisms or sector-specific balances that are not explicitly reported in official statistics, but which are crucial for capturing realistic macroeconomic dynamics.

From a broader perspective, and taking the learning process into consideration, one key lesson has emerged that should be communicated for future research purposes. When embarking on the development of an empirical IO-SFC model, it is advisable, beyond adhering to the necessary building blocks identified by Valdecantos (2023b), to begin with a fully aggregated version of the model (without disaggregating the NFC sector) and ensure that this core structure is functioning correctly and remains internally consistent. It is therefore recommended to begin with an aggregated version of a given model and to progressively expand its complexity through a step-by-step approach, continuously simulating as new components are added. Failing to follow this sequentially development process can lead to a loss of oversight, where emerging problems become numerous and difficult to disentangle. This issue has been observed during the development of the foundational E-IO-SFC model, and thus deserves to be explicitly highlighted for future modelers.

These challenges have manifested most notably in the baseline simulation of the E-IO-SFC model, which was developed in its full form rather than through the recommended step-by-step procedure. When running the model, it became evident that the implementation of the input–output structure had introduced significant complications. This issue is further elaborated in section 8 and could likely have been avoided by adopting a more incremental and modular development strategy. In addition, valuation issues have emerged, especially in the context of aligning the system of equations with empirical data. While equations in a stock-flow consistent framework are typically formulated based on conceptual relationships between variables, empirical implementation requires that one carefully considers how these variables are actually reported in national accounts. Differences in valuation methods - for instance, between market prices and basic prices - can result in mismatches that undermine the internal coherence of the model. These issues have been particularly pronounced in components related to inventories and expected sales, where the valuation basis has, in some cases, led to unrealistic or inconsistent data series. This experience underscores the importance of resolving such valuation mismatches before formulating the model equations, ensuring that the model structure reflects the way in which the data are reported. Taking such steps early in the process may reduce the risk of misalignment and improve both model interpretability and empirical relevance.

In hindsight, it is clear that beginning from a simplified baseline and gradually introducing complexity only after establishing a consistent core framework is a more robust approach. This minimizes technical errors, ensures interpretability of results, and supports a transparent modeling process that remains grounded in macroeconomic consistency. Lastly, future work on the E-IO-SFC model will focus on resolving remaining inconsistencies and further endogenizing the system of equations.

This discussion has demonstrated the importance of maintaining consistency in data, gradually expanding model complexity, and integrating financial and real spheres more thoroughly than is typically done in conventional macroeconomic modeling. These reflections not only inform the methodological choices made throughout this project, but also offer broader guidance for future empirical IO-SFC modeling efforts. By documenting the lessons learned, from data construction to structural implementation, this discussion contributes to the emerging literature on E-IO-SFC models and paves the way for more robust, transparent, and policy-relevant macroeconomic tools.

10 Conclusion

This project has examined how to lay the foundations for the construction of a quarterly E-IO-SFC model, using Denmark as a case study. This is highly relevant both globally and nationally, given the identified gaps in the current macroeconomic modeling landscape. Our initial analysis shows that the current SFC modeling landscape lacks sufficient disaggregation of the NFC sector. In addition, it highlights the need for a more detailed representation of the financial balances of both sectors and industries. Based on this, we conclude that the incorporation of input-output structures is essential. With regard to financial balances, our analysis shows that accounting statistics are a useful tool for disaggregating financial data at the industry level. The result of this process is a quarterly, industry-level database comprising real and financial variables aligned with national accounting principles, providing a solid foundation for the development of SFC models in the Danish context.

Furthermore, it can be concluded that it is necessary to partially transform the reported data from DST, due to different time frequencies and assumptions embedded in the current model. Consequently, to construct a coherent quarterly database suited for the E-IO-SFC model, it is essential to apply a range of econometric techniques and economic assumptions. These are required to address data gaps, reconcile inconsistencies, and build a structure that is empirically robust.

Moreover, a deep and comprehensive understanding of the SNA proves crucial, not only for designing the database architecture but also for developing the system of equations in order to maintain internal consistency. These equations must accurately reflect the definitions and inter-dependencies of variables as specified in the SNA, to maintain coherence between accounting identities and behavioral relationships. This highlights the technical and conceptual demands associated with developing a fully exogenous model. In addition, it is essential to identify the key behavioral equations that underpin the model and to estimate them using appropriate econometric methods, grounded in sound economic reasoning. Ultimately, combining a consistent accounting framework with robust behavioral specifications is what allows for the construction of a well-functioning E-IO-SFC model.

From working with annual input-output tables, we conclude that data adjustments must be handled with great care to ensure alignment with the quarterly industry-level framework. This remains an area in need of further refinement. Drawing on the overall experience by laying the foundations, it is advisable to begin with an aggregated version of the model and expand it gradually through a structured, step-by-step approach.

In summary, the development of the E-IO-SFC model has revealed both conceptual advances and practical challenges in constructing a quarterly, empirically grounded, stock-flow consistent framework with an integrated input–output structure.

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Appendix

A Balancesheet and Transactions-Flow matrix with values

	Households	Energy	Transport	Agriculture	Manufacturing and Construction	Other manufacturing and services
Monetary gold and SDR (F.1)						
Currency and deposits (F.2)	1219784	36124	19627	4208	58507	219343
Securities (F.3)	54187	−1722	−935	−201	−2788	−10453
Loans (F.4)	−2377682	−226423	−205392	−155032	−327485	−1059797
Equity (F.5)	3036180	−214722	−146222	−148386	−579994	−2169444
Insurance tech. reserves (F.6)	3936370	111	422	101	1984	5697
Financial derivatives (F.7)	−1718	−374	−1420	−340	−6676	−19168
Trade credits (F.8)	18072	714	468	37	563	1223
Net Worth	5885193	−406292	−333452	−299612	−855888	−3032599

	Financial corporations	Government	Central Bank	Rest of the world	$\sum$
Monetary gold and SDR (F.1)			20645	991	21636
Currency and deposits (F.2)	−1708595	87916	69307	−6221	0
Securities (F.3)	525735	−740542	−28291	205011	1
Loans (F.4)	4659293	193684	−5821	−495344	1
Equity (F.5)	954386	549583	−67107	−1214275	0
Insurance tech. reserves (F.6)	−4002549	1418	0	56446	0
Financial derivatives (F.7)	90776	−1152	368	−60295	0
Trade credits (F.8)	8618	36681	−490	−65887	0
Net Worth	527664	127588	−32034	−1580565	2

Table 24: Balancesheet (BS). The BS is showing all the stocks for the financial instruments between the economic institutions in 2019Q4. The values are represented in millions DKK in current prices and non-seasonally adjusted.

B System of equations

Some of the common equations are repeated in the case of the different sectors and industries, these descriptions would only appear once in the equation section for Non-financial components. Note: The undefined components are exogenously given.

Non-financial corporations

Aggregate demand: Aggregate demand represents the total nominal demand for goods and services in an economy at a given time. It is composed of household consumption C^H, current investments by non-financial corporations I_a^{NFC}, government spending G, exports X, imports IM, and changes in inventories ΔINV. This formulation reflects the expenditure-based approach to measuring economic activity, where aggregate demand captures all final demand components, including external[83] and inventory adjustments.

$$AD = C^H + I_a^{NFC} + G + X - IM + \Delta INV \tag{2}$$

Nominal sales for each NFC sector: Represented by the sum of private consumption, government consumption, exports, intermediate sales, and current investments. All in nominal terms and for each industry.

$$Y^E = C^E + G^E + X^E + IG_s^E + I_c^E \tag{3}$$

$$Y^T = C^T + G^T + X^T + IG_s^T + I_c^T \tag{4}$$

$$Y^A = C^A + G^A + X^A + IG_s^A + I_c^A \tag{5}$$

$$Y^{MC} = C^{MC} + G^{MC} + X^{MC} + IG_s^{MC} + I_c^{MC} \tag{6}$$

$$Y^{OMS} = C^{OMS} + G^{OMS} + X^{OMS} + IG_s^{OMS} + I_c^{OMS} \tag{7}$$

Real sales for each NFC industry: Represented by the same variables as in the nominal sales equations, but in real terms.

$$y^E = c^E + g^E + x^E + ig_s^E + i_c^E \tag{8}$$

$$y^T = c^T + g^T + x^T + ig_s^T + i_c^T \tag{9}$$

$$y^A = c^A + g^A + x^A + ig_s^A + i_c^A \tag{10}$$

$$y^{MC} = c^{MC} + g^{MC} + x^{MC} + ig_s^{MC} + i_c^{MC} \tag{11}$$

$$y^{OMS} = c^{OMS} + g^{OMS} + x^{OMS} + ig_s^{OMS} + i_c^{OMS} \tag{12}$$

[83] refers to international trade

Total expected real sales: The description of how the total expected real sales is estimated, can be found in section 7.2.2

$$\begin{aligned}\Delta\ln(y_{e,t}) = \beta_{0,y_e} &+ \beta_{1,y_e}\cdot\Delta\ln(y_{e,t-1}) + \beta_{2,y_e}\cdot\Delta\ln(CI_t) + \beta_{3,y_e}\cdot\Delta\ln\left(\frac{y_t}{k_t}\right)\\ &+ \beta_{4,y_e}\cdot\Delta\ln(yd^H_{t-2}) + \beta_{5,y_e}\cdot\Delta\ln(c_t) + \beta_{6,y_e}\cdot\Delta\ln(i_t)\\ &+ \beta_{7,y_e}\cdot\ln\left(\mathbf{y_{e,t-1}}\right) + \beta_{8,y_e}\cdot\ln\left(\mathbf{c_{t-1}}\right) + \beta_{9,y_e}\cdot\ln\left(\mathbf{CI_{t-1}}\right)\end{aligned} \tag{13}$$

Expected real sales for each industry: The relevance of this formulation lies in the fact that industries cannot produce instantly. Instead, they must decide production levels based on their expected real sales. These expectations are captured by a behavioral equation estimated at the level of the overall economy, reflecting aggregate expected demand. Details on the computation of in-sample expected sales are provided in section 6.1.6. This estimated expected value is then distributed among the different selling industries using predetermined alpha shares(α_P^i), representing each industry's historical contribution to production.

$$y_e^E = y_e \cdot \alpha_P^E \tag{14}$$

$$y_e^T = y_e \cdot \alpha_P^T \tag{15}$$

$$y_e^A = y_e \cdot \alpha_P^A \tag{16}$$

$$y_e^{MC} = y_e \cdot \alpha_P^{MC} \tag{17}$$

$$y_e^{OMS} = y_e \cdot \alpha_P^{OMS} \tag{18}$$

Real production of NFC industries: This is given by the real expected sales and the desired investment in inventories.

$$y_P^E = y_e^E + i_{inv,d}^E \tag{19}$$

$$y_P^T = y_e^T + i_{inv,d}^T \tag{20}$$

$$y_P^A = y_e^A + i_{inv,d}^A \tag{21}$$

$$y_P^{MC} = y_e^{MC} + i_{inv,d}^{MC} \tag{22}$$

$$y_P^{OMS} = y_e^{OMS} + i_{inv,d}^{OMS} \tag{23}$$

Desired level of real inventories: Determined by a constant α-parameter of desired inventory to real sales ratio, which secures an expected positive correlation.

$$inv_d^E = inv_{t-1}^E + i_{inv,d,t}^E \tag{24}$$

$$inv_d^T = inv_{t-1}^T + i_{inv,d,t}^T \tag{25}$$

$$inv_d^A = inv_{t-1}^A + i_{inv,d,t}^A \tag{26}$$

$$inv_d^{MC} = inv_{t-1}^{MC} + i_{inv,d,t}^{MC} \tag{27}$$

$$inv_d^{OMS} = inv_{t-1}^{OMS} + i_{inv,d,t}^{OMS} \tag{28}$$

Desired investment in inventories: This is determined by the deviation between desired and actual stock of inventories, multiplied by a speed of adjustment coefficient, β_{inv}^i.

$$i_{inv,d}^E = \beta_{inv}^E \cdot (inv_d^E - inv^E) \tag{29}$$

$$i_{inv,d}^T = \beta_{inv}^T \cdot (inv_d^T - inv^T) \tag{30}$$

$$i_{inv,d}^A = \beta_{inv}^A \cdot (inv_d^A - inv^A) \tag{31}$$

$$i_{inv,d}^{MC} = \beta_{inv}^{MC} \cdot (inv_d^{MC} - inv^{MC}) \tag{32}$$

$$i_{inv,d}^{OMS} = \beta_{inv}^{OMS} \cdot (inv_d^{OMS} - inv^{OMS}) \tag{33}$$

Change of actual level of real inventories: Given by the gap between real production and real sales.

$$\Delta inv^E = y_P^E - y^E \tag{34}$$

$$\Delta inv^T = y_P^T - y^T \tag{35}$$

$$\Delta inv^A = y_P^A - y^A \tag{36}$$

$$\Delta inv^{MC} = y_P^{MC} - y^{MC} \tag{37}$$

$$\Delta inv^{OMS} = y_P^{OMS} - y^{OMS} \tag{38}$$

Change in nominal inventories: Given by the change in real inventories multiplied by the price on inventories for each industry. It is assumed, that the prices on inventories is the producer price for each industry.

$$\Delta INV^E = \Delta inv^E \cdot pp_E \tag{39}$$

$$\Delta INV^{T} = \Delta inv^{T} \cdot pp_{T} \tag{40}$$

$$\Delta INV^{A} = \Delta inv^{A} \cdot pp_{A} \tag{41}$$

$$\Delta INV^{MC} = \Delta inv^{MC} \cdot pp_{MC} \tag{42}$$

$$\Delta INV^{OMS} = \Delta inv^{OMS} \cdot pp_{OMS} \tag{43}$$

$$\Delta INV = \Delta INV^{E} + \Delta INV^{T} + \Delta INV^{A} + \Delta INV^{MC} + \Delta INV^{OMS} \tag{44}$$

Total real current investment to capital ratio: The description of how the total real current investment to capital ratio is estimated, can be found in section C.3

$$\begin{aligned}\Delta \ln(\frac{i_t}{k_t}) &= \beta_{0,\frac{i}{k}} + \beta_{1,\frac{i}{k}} \cdot \Delta \ln(\frac{i_{t-1}}{k_{t-1}}) + \beta_{2,\frac{i}{k}} \cdot \Delta \ln(\frac{i_{t-2}}{k_{t-2}}) \\ &+ \beta_{3,\frac{i}{k}} \cdot \Delta \ln(\frac{i_{t-4}}{k_{t-4}}) + \beta_{4,\frac{i}{k}} \cdot \Delta \ln(g_t) + \beta_{5,\frac{i}{k}} \cdot \Delta \ln(g_{t-1}) + \beta_{6,\frac{i}{k}} \cdot \Delta \ln(\frac{x_t}{im_t}) + \beta_{7,\frac{i}{k}} \cdot \ln(\frac{\mathbf{y_t}}{\mathbf{k_t}}) \\ &+ \beta_{8,\frac{i}{k}} \cdot \ln(\frac{\mathbf{i_{t-1}}}{\mathbf{k_{t-1}}}) + \beta_{9,\frac{i}{k}} \cdot \ln(\mathbf{g_{t-1}}) + \beta_{10,\frac{i}{k}} \cdot \ln(\frac{\mathbf{y_{t-1}}}{\mathbf{k_{t-1}}}) + \beta_{11,i} \cdot \mathbf{i_t^{NFC}}\end{aligned} \tag{45}$$

Real accumulated investment: The estimated figures of real accumulated investments is distributed based on the industry's respective α-shares calculated from the IO-table described as gross fixed capital formation, cf. section 6.1.1. This reflects the historical contribution of the different production industries to current investments.

$$i_a^E = i \cdot \alpha_{I_a}^E \tag{46}$$

$$i_a^T = i \cdot \alpha_{I_a}^T \tag{47}$$

$$i_a^A = i \cdot \alpha_{I_a}^A \tag{48}$$

$$i_a^{MC} = i \cdot \alpha_{I_a}^{MC} \tag{49}$$

$$i_a^{OMS} = i \cdot \alpha_{I_a}^{OMS} \tag{50}$$

Imported investments: $\alpha_{i_{IM}}^{i}$ represents the share of imported investments of the total investments which is, for simplicity, the same among the industries. The computation of the *α-shares* can be found, cf. section 6.1.1

$$i_{IM}^E = i^E \cdot \alpha_{I_{IM}}^E \tag{51}$$

$$i_{IM}^T = i^T \cdot \alpha_{I_{IM}}^T \tag{52}$$

$$i^{A}_{IM} = i^{A} \cdot \alpha^{A}_{I_{IM}} \tag{53}$$

$$i^{MC}_{IM} = i^{MC} \cdot \alpha^{MC}_{I_{IM}} \tag{54}$$

$$i^{OMS}_{IM} = i^{OMS} \cdot \alpha^{OMS}_{I_{IM}} \tag{55}$$

Nominal accumulated investments: The total level of real investments multiplied by the exogenous investment price.

$$I^{E} = i^{E}_{a} \cdot p^{E}_{a} \tag{56}$$

$$I^{T} = i^{T}_{a} \cdot p^{T}_{a} \tag{57}$$

$$I^{A} = i^{A}_{a} \cdot p^{A}_{a} \tag{58}$$

$$I^{MC} = i^{MC}_{a} \cdot p^{MC}_{a} \tag{59}$$

$$I^{OMS} = i^{OMS}_{a} \cdot p^{OMS}_{a} \tag{60}$$

The total accumulated investments:

$$I^{NFC}_{a} = I^{E} + I^{T} + I^{A} + I^{MC} + I^{OMS} + I^{FC} + I^{H} + I^{G} \tag{61}$$

Current nominal investments for every industry: Determined by a share of the total accumulated investments, to distribute the accumulated investments in the current account of the industries that carry out the investments.

$$I^{E}_{c} = I^{NFC}_{a} \cdot \alpha^{E}_{I} \tag{62}$$

$$I^{T}_{c} = I^{NFC}_{a} \cdot \alpha^{T}_{I} \tag{63}$$

$$I^{A}_{c} = I^{NFC}_{a} \cdot \alpha^{A}_{I} \tag{64}$$

$$I^{MC}_{c} = I^{NFC}_{a} \cdot \alpha^{MC}_{I} \tag{65}$$

$$I^{OMS}_{c} = I^{NFC}_{a} \cdot \alpha^{OMS}_{I} \tag{66}$$

Current real investments for every industry: Given by the nominal current investments divided by the price for each industry.

$$i_c^E = \frac{I_c^E}{p^E} \tag{67}$$

$$i_c^T = \frac{I_c^T}{p^T} \tag{68}$$

$$i_c^A = \frac{I_c^A}{p^A} \tag{69}$$

$$i_c^{MC} = \frac{I_c^{MC}}{p^{MC}} \tag{70}$$

$$i_c^{OMS} = \frac{I_c^{OMS}}{p^{OMS}} \tag{71}$$

Nominal foreign investments: Given by the real foreign investments multiplied by the price of imports.

$$I_{IM}^E = i_{IM}^E \cdot p_{IM} \tag{72}$$

$$I_{IM}^T = i_{IM}^T \cdot p_{IM} \tag{73}$$

$$I_{IM}^A = i_{IM}^A \cdot p_{IM} \tag{74}$$

$$I_{IM}^{MC} = i_{IM}^{MC} \cdot p_{IM} \tag{75}$$

$$I_{IM}^{OMS} = i_{IM}^{OMS} \cdot p_{IM} \tag{76}$$

Nominal stock of capital: The nominal capital stock in the current period is defined as the sum of the nominal capital stock from the previous period, plus current nominal investments, minus depreciations. This reflects the accumulation process of capital over time.

$$K^E = K_{t-1}^E + I^E - D^E \tag{77}$$

$$K^T = K_{t-1}^T + I^T - D^T \tag{78}$$

$$K^A = K_{t-1}^A + I^A - D^A \tag{79}$$

$$K^{MC} = K_{t-1}^{MC} + I^{MC} - D^{MC} \tag{80}$$

$$K^{OMS} = K_{t-1}^{OMS} + I^{OMS} - D^{OMS} \tag{81}$$

Real stock of capital: Given by the nominal stock of capital divided by the investment price.

$$k^E = \frac{K^E}{p_I} \tag{82}$$

$$k^T = \frac{K^T}{p_I} \tag{83}$$

$$k^A = \frac{K^A}{p_I} \tag{84}$$

$$k^{MC} = \frac{K^{MC}}{p_I} \tag{85}$$

$$k^{OMS} = \frac{K^{OMS}}{p_I} \tag{86}$$

Depreciations: δ^i represents the rate of depreciation of the capital stock in the previous period.

$$D^E = \delta^E \cdot (K^E_{t-1}) \tag{87}$$

$$D^T = \delta^T \cdot (K^T_{t-1}) \tag{88}$$

$$D^A = \delta^A \cdot (K^A_{t-1}) \tag{89}$$

$$D^{MC} = \delta^{MC} \cdot (K^{MC}_{t-1}) \tag{90}$$

$$D^{OMS} = \delta^{OMS} \cdot (K^{OMS}_{t-1}) \tag{91}$$

Intermediate real consumption transactions: The intermediate purchases from every industry, $ig_p^{p,s}$ where p denotes the purchasing industry and s denotes selling industry. The $\alpha_p^{i,i}$ value denotes a technical coefficient of intermediate goods purchased from every industry to production. Because of the value added through production, the α-coefficients will not sum to one industrywise.

$$ig_p^{E,E} = y_P^E \cdot \alpha_p^{E,E} \tag{92}$$

$$ig_p^{E,T} = y_P^E \cdot \alpha_p^{E,T} \tag{93}$$

$$ig_p^{E,A} = y_P^E \cdot \alpha_p^{E,A} \tag{94}$$

$$ig_p^{E,MC} = y_P^E \cdot \alpha_p^{E,MC} \tag{95}$$

$$ig_p^{E,OMS} = y_P^E \cdot \alpha_p^{E,OMS} \tag{96}$$

$$ig_p^{E,FC} = y_P^E \cdot \alpha_p^{E,FC} \tag{97}$$

$$ig_p^{T,E} = y_P^T \cdot \alpha_p^{T,E} \tag{98}$$

$$ig_p^{T,T} = y_P^T \cdot \alpha_p^{T,T} \tag{99}$$

$$ig_p^{T,A} = y_P^T \cdot \alpha_p^{T,A} \tag{100}$$

$$ig_p^{T,MC} = y_P^T \cdot \alpha_p^{T,MC} \tag{101}$$

$$ig_p^{T,OMS} = y_P^T \cdot \alpha_p^{T,OMS} \tag{102}$$

$$ig_p^{T,FC} = y_P^T \cdot \alpha_p^{T,FC} \tag{103}$$

$$ig_p^{A,E} = y_P^A \cdot \alpha_p^{A,E} \tag{104}$$

$$ig_p^{A,T} = y_P^A \cdot \alpha_p^{A,T} \tag{105}$$

$$ig_p^{A,A} = y_P^A \cdot \alpha_p^{A,A} \tag{106}$$

$$ig_p^{A,MC} = y_P^A \cdot \alpha_p^{A,MC} \tag{107}$$

$$ig_p^{A,OMS} = y_P^A \cdot \alpha_p^{A,OMS} \tag{108}$$

$$ig_p^{A,FC} = y_P^A \cdot \alpha_p^{A,FC} \tag{109}$$

$$ig_p^{MC,E} = y_P^{MC} \cdot \alpha_p^{MC,E} \tag{110}$$

$$ig_p^{MC,T} = y_P^{MC} \cdot \alpha_p^{MC,T} \tag{111}$$

$$ig_p^{MC,A} = y_P^{MC} \cdot \alpha_p^{MC,A} \tag{112}$$

$$ig_p^{MC,MC} = y_P^{MC} \cdot \alpha_p^{MC,MC} \tag{113}$$

$$ig_p^{MC,OMS} = y_P^{MC} \cdot \alpha_p^{MC,OMS} \tag{114}$$

$$ig_p^{MC,FC} = y_P^{MC} \cdot \alpha_p^{MC,FC} \tag{115}$$

$$ig_p^{OMS,E} = y_P^{OMS} \cdot \alpha_p^{OMS,E} \tag{116}$$

$$ig_p^{OMS,T} = y_P^{OMS} \cdot \alpha_p^{OMS,T} \tag{117}$$

$$ig_p^{OMS,A} = y_P^{OMS} \cdot \alpha_p^{OMS,A} \tag{118}$$

$$ig_p^{OMS,MC} = y_P^{OMS} \cdot \alpha_p^{OMS,MC} \tag{119}$$

$$ig_p^{OMS,OMS} = y_P^{OMS} \cdot \alpha_p^{OMS,OMS} \tag{120}$$

$$ig_p^{OMS,FC} = y_P^{OMS} \cdot \alpha_p^{OMS,FC} \tag{121}$$

Intermediate real sales transactions: The intermediate sales from every industry are determined by the total purchases from the industry on the right-hand side of the equation.

$$ig_s^{E} = ig_p^{E,E} + ig_p^{T,E} + ig_p^{A,E} + ig_p^{MC,E} + ig_p^{OMS,E} + ig_p^{FC,E} \tag{122}$$

$$ig_s^{T} = ig_p^{E,T} + ig_p^{T,T} + ig_p^{A,T} + ig_p^{MC,T} + ig_p^{OMS,T} + ig_p^{FC,T} \tag{123}$$

$$ig_s^{A} = ig_p^{E,A} + ig_p^{T,A} + ig_p^{A,A} + ig_p^{MC,A} + ig_p^{OMS,A} + ig_p^{FC,A} \tag{124}$$

$$ig_s^{MC} = ig_p^{E,MC} + ig_p^{T,MC} + ig_p^{A,MC} + ig_p^{MC,MC} + ig_p^{OMS,MC} + ig_p^{FC,MC} \tag{125}$$

$$ig_s^{OMS} = ig_p^{E,OMS} + ig_p^{T,OMS} + ig_p^{A,OMS} + ig_p^{MC,OMS} + ig_p^{OMS,OMS} + ig_p^{FC,OMS} \tag{126}$$

Real intermediate imports: The intermediate imports by every industry. The first bracket represents the total amount of domestic intermediate purchases, which is multiplied by the share of imported intermediate goods defined as $\alpha_{IM_p}^{p,s} = \frac{ig_{IM_p}^{p,s}}{ig_p^{p,s}}$.

$$ig_{IM_p}^{E,E} = (y_P^E \cdot \alpha_p^{E,E}) \cdot \alpha_{IM_p}^{E,E} \tag{127}$$

$$ig_{IM_p}^{E,T} = (y_P^E \cdot \alpha_p^{E,T}) \cdot \alpha_{IM_p}^{E,T} \tag{128}$$

$$ig_{IM_p}^{E,A} = (y_P^E \cdot \alpha_p^{E,A}) \cdot \alpha_{IM_p}^{E,A} \tag{129}$$

$$ig_{IM_p}^{E,MC} = (y_P^E \cdot \alpha_p^{E,MC}) \cdot \alpha_{IM_p}^{E,MC} \tag{130}$$

$$ig_{IM_p}^{E,OMS} = (y_P^E \cdot \alpha_p^{E,OMS}) \cdot \alpha_{IM_p}^{E,OMS} \tag{131}$$

$$ig_{IM_p}^{E,FC} = (y_P^E \cdot \alpha_p^{E,FC}) \cdot \alpha_{IM_p}^{E,FC} \tag{132}$$

$$ig_{IM_p}^{T,E} = (y_P^T \cdot \alpha_p^{T,E}) \cdot \alpha_{IM_p}^{T,E} \tag{133}$$

$$ig_{IM_p}^{T,T} = (y_P^T \cdot \alpha_p^{T,T}) \cdot \alpha_{IM_p}^{T,T} \tag{134}$$

$$ig_{IM_p}^{T,A} = (y_P^T \cdot \alpha_p^{T,A}) \cdot \alpha_{IM_p}^{T,A} \tag{135}$$

$$ig_{IM_p}^{T,MC} = (y_P^T \cdot \alpha_p^{T,MC}) \cdot \alpha_{IM_p}^{T,MC} \tag{136}$$

$$ig_{IM_p}^{T,OMS} = (y_P^T \cdot \alpha_p^{T,OMS}) \cdot \alpha_{IM_p}^{T,OMS} \tag{137}$$

$$ig_{IM_p}^{T,FC} = (y_P^T \cdot \alpha_p^{T,FC}) \cdot \alpha_{IM_p}^{T,FC} \tag{138}$$

$$ig_{IM_p}^{A,E} = (y_P^A \cdot \alpha_p^{A,E}) \cdot \alpha_{IM_p}^{A,E} \tag{139}$$

$$ig_{IM_p}^{A,T} = (y_P^A \cdot \alpha_p^{A,T}) \cdot \alpha_{IM_p}^{A,T} \tag{140}$$

$$ig_{IM_p}^{A,A} = (y_P^A \cdot \alpha_p^{A,A}) \cdot \alpha_{IM_p}^{A,A} \tag{141}$$

$$ig_{IM_p}^{A,MC} = (y_P^A \cdot \alpha_p^{A,MC}) \cdot \alpha_{IM_p}^{A,MC} \tag{142}$$

$$ig_{IM_p}^{A,OMS} = (y_P^A \cdot \alpha_p^{A,OMS}) \cdot \alpha_{IM_p}^{A,OMS} \tag{143}$$

$$ig_{IM_p}^{A,FC} = (y_P^A \cdot \alpha_p^{A,FC}) \cdot \alpha_{IM_p}^{A,FC} \tag{144}$$

$$ig_{IM_p}^{MC,E} = (y_P^{MC} \cdot \alpha_p^{MC,E}) \cdot \alpha_{IM_p}^{MC,E} \tag{145}$$

$$ig_{IM_p}^{MC,T} = (y_P^{MC} \cdot \alpha_p^{MC,T}) \cdot \alpha_{IM_p}^{MC,T} \tag{146}$$

$$ig_{IM_p}^{MC,A} = (y_P^{MC} \cdot \alpha_p^{MC,A}) \cdot \alpha_{IM_p}^{MC,A} \tag{147}$$

$$ig_{IM_p}^{MC,MC} = (y_P^{MC} \cdot \alpha_p^{MC,MC}) \cdot \alpha_{IM_p}^{MC,MC} \tag{148}$$

$$ig_{IM_p}^{MC,OMS} = (y_P^{MC} \cdot \alpha_p^{MC,OMS}) \cdot \alpha_{IM_p}^{MC,OMS} \tag{149}$$

$$ig_{IM_p}^{MC,FC} = (y_P^{MC} \cdot \alpha_p^{MC,FC}) \cdot \alpha_{IM_p}^{MC,FC} \tag{150}$$

$$ig_{IM_p}^{OMS,E} = (y_P^{OMS} \cdot \alpha_p^{OMS,E}) \cdot \alpha_{IM_p}^{OMS,E} \tag{151}$$

$$ig_{IM_p}^{OMS,T} = (y_P^{OMS} \cdot \alpha_p^{OMS,T}) \cdot \alpha_{IM_p}^{OMS,T} \tag{152}$$

$$ig_{IM_p}^{OMS,A} = (y_P^{OMS} \cdot \alpha_p^{OMS,A}) \cdot \alpha_{IM_p}^{OMS,A} \tag{153}$$

$$ig_{IM_p}^{OMS,MC} = (y_P^{OMS} \cdot \alpha_p^{OMS,MC}) \cdot \alpha_{IM_p}^{OMS,MC} \tag{154}$$

$$ig_{IM_p}^{OMS,OMS} = (y_P^{OMS} \cdot \alpha_p^{OMS,OMS}) \cdot \alpha_{IM_p}^{OMS,OMS} \tag{155}$$

$$ig_{IM_p}^{OMS,FC} = (y_P^{OMS} \cdot \alpha_p^{OMS,FC}) \cdot \alpha_{IM_p}^{OMS,FC} \tag{156}$$

Intermediate nominal consumption transactions: The nominal intermediate purchases from every industry multiplied by the producer price, respect to the industry.

$$IG_p^E = ig_p^{E,E} \cdot p_p^E + ig_p^{E,T} \cdot p_p^T + ig_p^{E,A} \cdot p_p^A + ig_p^{E,MC} \cdot p_p^{MC} + ig_p^{E,OMS} \cdot p_p^{OMS} + ig_p^{E,FC} \cdot p_p^{FC} \tag{157}$$

$$IG_p^T = ig_p^{T,E} \cdot pp^E + ig_p^{T,T} \cdot pp^T + ig_p^{T,A} \cdot pp^A + ig_p^{T,MC} \cdot pp^{MC} + ig_p^{T,OMS} \cdot pp^{OMS} + ig_p^{T,FC} \cdot pp^{FC} \tag{158}$$

$$IG_p^A = ig_p^{A,E} \cdot pp^E + ig_p^{A,T} \cdot pp^T + ig_p^{A,A} \cdot pp^A + ig_p^{A,MC} \cdot pp^{MC} + ig_p^{A,OMS} \cdot pp^{OMS} + ig_p^{A,FC} \cdot pp^{FC} \tag{159}$$

$$IG_p^{MC} = ig_p^{MC,E} \cdot pp^E + ig_p^{MC,T} \cdot pp^T + ig_p^{MC,A} \cdot pp^A + ig_p^{MC,MC} \cdot pp^{MC} + ig_p^{MC,OMS} \cdot pp^{OMS} + ig_p^{MC,FC} \cdot pp^{FC} \tag{160}$$

$$IG_p^{OMS} = ig_p^{OMS,E} \cdot pp^E + ig_p^{OMS,T} \cdot pp^T + ig_p^{OMS,A} \cdot pp^A + ig_p^{OMS,MC} \cdot pp^{MC} + ig_p^{OMS,OMS} \cdot pp^{OMS} + ig_p^{OMS,FC} \cdot pp^{FC} \tag{161}$$

Intermediate nominal sales transactions: The nominal intermediate sales from every industry are determined by the total sales multiplied by the producer price for the respective industry.

$$IG_s^E = ig_s^E \cdot pp^E \tag{162}$$

$$IG_s^T = ig_s^T \cdot pp^T \tag{163}$$

$$IG_s^A = ig_s^A \cdot pp^A \tag{164}$$

$$IG_s^{MC} = ig_s^{MC} \cdot pp^{MC} \tag{165}$$

$$IG_s^{OMS} = ig_s^{OMS} \cdot pp^{OMS} \tag{166}$$

Nominal intermediate imports: Given by the real imported intermediate goods from every industry multiplied by the import price.

$$IG_{IM}^{E} = (ig_{IMp}^{E,E} + ig_{IMp}^{E,T} + ig_{IMp}^{E,A} + ig_{IMp}^{E,MC} + ig_{IMp}^{E,OMS} + ig_{IMp}^{E,FC}) \cdot p_{IM} \tag{167}$$

$$IG_{IM}^{T} = (ig_{IMp}^{T,E} + ig_{IMp}^{T,T} + ig_{IMp}^{T,A} + ig_{IMp}^{T,MC} + ig_{IMp}^{T,OMS} + ig_{IMp}^{T,FC}) \cdot p_{IM} \tag{168}$$

$$IG_{IM}^{A} = (ig_{IMp}^{A,E} + ig_{IMp}^{A,T} + ig_{IMp}^{A,A} + ig_{IMp}^{A,MC} + ig_{IMp}^{A,OMS} + ig_{IMp}^{A,FC}) \cdot p_{IM} \tag{169}$$

$$IG_{IM}^{MC} = (ig_{IMp}^{MC,E} + ig_{IMp}^{MC,T} + ig_{IMp}^{MC,A} + ig_{IMp}^{MC,MC} + ig_{IMp}^{MC,OMS} + ig_{IMp}^{MC,FC}) \cdot p_{IM} \tag{170}$$

$$IG_{IM}^{OMS} = (ig_{IMp}^{OMS,E} + ig_{IMp}^{OMS,T} + ig_{IMp}^{OMS,A} + ig_{IMp}^{OMS,MC} + ig_{IMp}^{OMS,OMS} + ig_{IMp}^{OMS,FC}) \cdot p_{IM} \tag{171}$$

Private and public imports: The private and public imports are determined by an α-share, $\alpha_{C_{IM}}$, of the total private and public consumption.

$$C_{IM} = (C^H + G) \cdot \alpha_{C_{IM}} \tag{172}$$

$$C_{IM}^{E} = C_{IM} \cdot \alpha_{C_{IM}^{E}} \tag{173}$$

$$C_{IM}^{T} = C_{IM} \cdot \alpha_{C_{IM}^{T}} \tag{174}$$

$$C_{IM}^{A} = C_{IM} \cdot \alpha_{C_{IM}^{A}} \tag{175}$$

$$C_{IM}^{MC} = C_{IM} \cdot \alpha_{C_{IM}^{MC}} \tag{176}$$

$$C_{IM}^{OMS} = C_{IM} \cdot \alpha_{C_{IM}^{OMS}} \tag{177}$$

Nominal imports: The total nominal imports is given by the intermediate imported goods, foreign investments and the private and public imports.

$$IM = IG_{IM}^{E} + IG_{IM}^{T} + IG_{IM}^{A} + IG_{IM}^{MC} + IG_{IM}^{OMS} + IG_{IM}^{FC} + I_{IM}^{E} + I_{IM}^{T} + I_{IM}^{A} + I_{IM}^{MC} + I_{IM}^{OMS} + C_{IM} \tag{178}$$

$$IM^{E} = IG_{IM}^{E} + I_{IM}^{E} + C_{IM}^{E} \tag{179}$$

$$IM^{T} = IG_{IM}^{T} + I_{IM}^{T} + C_{IM}^{T} \tag{180}$$

$$IM^A = IG_{IM}^A + I_{IM}^A + C_{IM}^A \tag{181}$$

$$IM^{MC} = IG_{IM}^{MC} + I_{IM}^{MC} + C_{IM}^{MC} \tag{182}$$

$$IM^{OMS} = IG_{IM}^{OMS} + I_{IM}^{OMS} + C_{IM}^{OMS} \tag{183}$$

Real export: The description of how the real export equations are made, can be seen in section 7.2.2. In the future, the remaining export equations will be estimated.

$$\begin{aligned}\Delta \ln(x_t) = \beta_{0,x} + \beta_{1,x} \cdot \Delta \ln(x_{t-1}) + \beta_{2,x} \cdot \Delta \ln(\epsilon^{84}) + \beta_{3,x} \cdot \Delta \ln(\epsilon_{t-1}) + \beta_{4,x} \cdot \Delta \ln(\epsilon_{t-2}) + \\ \beta_{5,x} \cdot \Delta \ln(gdp^{RoW}) + \beta_{6,x} \cdot \Delta \ln(gdp_{t-4}^{RoW}) + \beta_{7,x} \cdot \ln\left(\mathbf{gdp_{t-1}^{RoW}}\right) + \beta_{8,x} \cdot \ln\left(\mathbf{x_{t-1}}\right)\end{aligned} \tag{184}$$

$$x^E = x \cdot \alpha_x^E \tag{185}$$

$$x^T = x \cdot \alpha_x^T \tag{186}$$

$$x^A = x \cdot \alpha_x^A \tag{187}$$

$$\begin{aligned}\Delta \ln(x^{MC}) = {} & \beta_{0,x^{MC}} + \beta_{1,x^{MC}} \cdot \Delta \ln(P_{X_t}^{MC}) + \beta_{2,x^{MC}} \cdot \Delta \ln(P_{X_{t-1}}^{MC}) + \beta_{3,x^{MC}} \cdot \Delta \ln(e) \\ & + \beta_{4,x^{MC}} \cdot \Delta \ln(gdp^{RoW}) + \beta_{5,x^{MC}} \cdot \Delta \ln(gdp_{t-4}^{RoW}) + \beta_{6,x^{MC}} \cdot \Delta \ln(WT) \\ & + \beta_{7,x^{MC}} \cdot \ln(\mathbf{x_{t-1}^{MC}}) + \beta_{8,x^{MC}} \cdot \ln(\mathbf{gdp_{t-1}^{RoW}})\end{aligned} \tag{188}$$

$$x^{OMS} = x \cdot \alpha_x^{OMS} \tag{189}$$

Nominal exports: The nominal export is obtained by multiplying the real exports by the consumer price for each industry.

$$X^E = x^E \cdot p^E \tag{190}$$

$$X^T = x^T \cdot p^T \tag{191}$$

$$X^A = x^A \cdot p^A \tag{192}$$

$$X^{MC} = x^{MC} \cdot p^{MC} \tag{193}$$

$$X^{OMS} = x^{OMS} \cdot p^{OMS} \tag{194}$$

[84] $\epsilon = \frac{p_y}{p_{im}}$

$$X = X^E + X^T + X^A + X^{MC} + X^{OMS} + X^{FC} \tag{195}$$

Gross value added: Determined by each industry's total sales minus intermediate goods purchased from other industries, plus changes in inventories, and minus imports.

$$GVA^E = Y^E - IG_p^E + \Delta INV^E - IM^E \tag{196}$$

$$GVA^T = Y^T - IG_p^T + \Delta INV^T - IM^T \tag{197}$$

$$GVA^A = Y^A - IG_p^A + \Delta INV^A - IM^A \tag{198}$$

$$GVA^{MC} = Y^{MC} - IG_p^{MC} + \Delta INV^{MC} - IM^{MC} \tag{199}$$

$$GVA^{OMS} = Y^{OMS} - IG_p^{OMS} + \Delta INV^{OMS} - IM^{OMS} \tag{200}$$

Net indirect taxes less subsidies on production: Determined by the total nominal sales multiplied by a coefficient which replicates a calculated tax.

$$NIT^E = Y^E \cdot \tau_{NIT}^E \tag{201}$$

$$NIT^T = Y^T \cdot \tau_{NIT}^T \tag{202}$$

$$NIT^A = Y^A \cdot \tau_{NIT}^A \tag{203}$$

$$NIT^{MC} = Y^{MC} \cdot \tau_{NIT}^{MC} \tag{204}$$

$$NIT^{OMS} = Y^{OMS} \cdot \tau_{NIT}^{OMS} \tag{205}$$

Nominal value added at factor cost: Gross value added subtracted by net indirect taxed

$$GVA_F^E = GVA^E - NIT^E \tag{206}$$

$$GVA_F^T = GVA^T - NIT^T \tag{207}$$

$$GVA_F^A = GVA^A - NIT^A \tag{208}$$

$$GVA_F^{MC} = GVA^{MC} - NIT^{MC} \tag{209}$$

$$GVA_F^{OMS} = GVA^{OMS} - NIT^{OMS} \tag{210}$$

Compensation of employees in each industry: The wage bill includes the level of employment multiplied by a estimated wage rate, which includes social contributions.

$$WB^E = EMP^E \cdot W^E \tag{211}$$

$$WB^T = EMP^T \cdot W^T \tag{212}$$

$$WB^A = EMP^A \cdot W^A \tag{213}$$

$$WB^{MC} = EMP^{MC} \cdot W^{MC} \tag{214}$$

$$WB^{OMS} = EMP^{OMS} \cdot W^{OMS} \tag{215}$$

Total wage rate: The description of total wage rate and how it is estimated, can be found in with the underlying parameters dynamics section C.2.

$$\begin{aligned} W = \beta_{1,W} \cdot \ln(W_{t-1}) + \beta_{2,W} \cdot \Delta \ln(W_{t-2}) + \beta_{3,W} \cdot \Delta \ln(W_{t-3}) \\ + \beta_{4,W} \cdot \Delta \ln(pr) + \beta_{5,W} \cdot UE_{r_{t-1}} + \beta_{6,W} \cdot \ln(W_{T_{t-1}}) \end{aligned} \tag{216}$$

$$W^E = \alpha_W^E \cdot W \tag{217}$$

$$W^T = \alpha_W^T \cdot W \tag{218}$$

$$W^A = \alpha_W^A \cdot W \tag{219}$$

$$W^{MC} = \alpha_W^{MC} \cdot W \tag{220}$$

$$W^{OMS} = \alpha_W^{OMS} \cdot W \tag{221}$$

Gross operating surplus and mixed income: Simply computed by the gross value added at factor cost subtracted by the wage bill in each industry.

$$GOS^E = GVA_F^E - WB^E \tag{222}$$

$$GOS^T = GVA_F^T - WB^T \tag{223}$$

$$GOS^A = GVA_F^A - WB^A \tag{224}$$

$$GOS^{MC} = GVA_F^{MC} - WB^{MC} \tag{225}$$

$$GOS^{OMS} = GVA_F^{OMS} - WB^{OMS} \tag{226}$$

Gross operating surplus and mixed income distributed: A share of GOS is distributed to households and government, respectively. The $\theta^i_{H,G}$ values are obtained from DST in the **attached document** titled "Distribution & GVA by industries and institutional sectors 2016".

$$GOS_H^E = \theta_H^E \cdot GOS^E \tag{227}$$

$$GOS_G^E = \theta_G^E \cdot GOS^E \tag{228}$$

$$GOS_H^T = \theta_H^T \cdot GOS^T \tag{229}$$

$$GOS_G^T = \theta_G^T \cdot GOS^T \tag{230}$$

$$GOS_H^A = \theta_H^A \cdot GOS^A \tag{231}$$

$$GOS_G^A = \theta_G^A \cdot GOS^A \tag{232}$$

$$GOS_H^{MC} = \theta_H^{MC} \cdot GOS^{MC} \tag{233}$$

$$GOS_G^{MC} = \theta_G^{MC} \cdot GOS^{MC} \tag{234}$$

$$GOS_H^{OMS} = \theta_H^{OMS} \cdot GOS^{OMS} \tag{235}$$

$$GOS_G^{OMS} = \theta_G^{OMS} \cdot GOS^{OMS} \tag{236}$$

Distributed income of corporation, payments: A share of every industry's profit is distributed among the other sectors in the economy. The α_p^i are computed by dividing the known values for DI_p^i and F^i, $\frac{DI_p}{F} = \alpha_p^i$.

$$DI_p^E = F^E \cdot \alpha_p^E \tag{237}$$

$$DI_p^T = F^T \cdot \alpha_p^T \tag{238}$$

$$DI_p^A = F^A \cdot \alpha_p^A \tag{239}$$

$$DI_p^{MC} = F^{MC} \cdot \alpha_p^{MC} \tag{240}$$

$$DI_p^{OMS} = F^{OMS} \cdot \alpha_p^{OMS} \tag{241}$$

Distributed income of corporation, receivable: The β_r^i values indicates the received share of the total distributed income paid and is defined as the sum of all payments divided by the amount recieved in industry i, $\frac{\sum DI_p}{DI_r^i} = \alpha_r^i$. This amount is shared between NFCs, FC, G, H and RoW.

$$DI_r^E = (DI_p^E + DI_p^T + DI_p^A + DI_p^{MC} + DI_p^{OMS} + DI_p^{FC} + DI_p^{RoW}) \cdot \alpha_r^E \tag{242}$$

$$DI_r^T = (DI_p^E + DI_p^T + DI_p^A + DI_p^{MC} + DI_p^{OMS} + DI_p^{FC} + DI_p^{RoW}) \cdot \alpha_r^T \tag{243}$$

$$DI_r^A = (DI_p^E + DI_p^T + DI_p^A + DI_p^{MC} + DI_p^{OMS} + DI_p^{FC} + DI_p^{RoW}) \cdot \alpha_r^A \tag{244}$$

$$DI_r^{MC} = (DI_p^E + DI_p^T + DI_p^A + DI_p^{MC} + DI_p^{OMS} + DI_p^{FC} + DI_p^{RoW}) \cdot \alpha_r^{MC} \tag{245}$$

$$DI_r^{OMS} = (DI_p^E + DI_p^T + DI_p^A + DI_p^{MC} + DI_p^{OMS} + DI_p^{FC} + DI_p^{RoW}) \cdot \alpha_r^{OMS} \tag{246}$$

Distributed income of corporation: The net value for DI is obtained by the residual of payments and receiving by every sector.

$$DI^E = DI_r^E - DI_p^E \tag{247}$$

$$DI^T = DI_r^T - DI_p^T \tag{248}$$

$$DI^A = DI_r^A - DI_p^A \tag{249}$$

$$DI^{MC} = DI_r^{MC} - DI_p^{MC} \tag{250}$$

$$DI^{OMS} = DI_r^{OMS} - DI_p^{OMS} \tag{251}$$

Net Interest + Other investments: Determined by the stock of funds multiplied by a unique interest, $\iota_{F.}^i$, on interest bearing assets plus the interest expenditures on loans. According to section 6.1.5, $F.7$ and $F.8$ are not associated with explicit interest rates and are therefore treated as fixed income values.

$$NI^E = F.2_s^E \cdot \iota_{F.2}^E + F.3_s^E \cdot \iota_{F.3}^E + F.4_s^E \cdot \iota_{F.4}^E + F.6_s^E \cdot \iota_{F.6}^E + interest_{F.7}^E + interest_{F.8}^E \tag{252}$$

$$NI^T = F.2_s^T \cdot \iota_{F.2}^T + F.3_s^T \cdot \iota_{F.3}^T + F.4_s^T \cdot \iota_{F.4}^T + F.6_s^T \cdot \iota_{F.6}^T + interest_{F.7}^T + interest_{F.8}^T \tag{253}$$

$$NI^A = F.2_s^A \cdot \iota_{F.2}^A + F.3_s^A \cdot \iota_{F.3}^A + F.4_s^A \cdot \iota_{F.4}^A + F.6_s^A \cdot \iota_{F.6}^A + interest_{F.7}^A + interest_{F.8}^A \tag{254}$$

$$NI^{MC} = F.2_s^{MC} \cdot \iota_{F.2}^{MC} + F.3_s^{MC} \cdot \iota_{F.3}^{MC} + F.4_s^{MC} \cdot \iota_{F.4}^{MC} + F.6_s^{MC} \cdot \iota_{F.6}^{MC} + interest_{F.7}^{MC} + interest_{F.8}^{MC} \tag{255}$$

$$NI^{OMS} = F.2_s^{OMS} \cdot \iota_{F.2}^{OMS} + F.3_s^{OMS} \cdot \iota_{F.3}^{OMS} + F.4_s^{OMS} \cdot \iota_{F.4}^{OMS} + F.6_s^{OMS} \cdot \iota_{F.6}^{OMS} + interest_{F.7}^{OMS} + interest_{F.8}^{OMS} \quad (256)$$

Reinvested earnings on FDI: RoW is reinvesting a share $\frac{F^i_{ReFI}}{DI^{RoW}} = \alpha^i_{ReFI}$ of the net distributed income. The reinvested amount is included in the respective industries retained earnings.

$$F^E_{ReFI} = \alpha^E_{ReFI} \cdot DI^{RoW} \quad (257)$$

$$F^T_{ReFI} = \alpha^T_{ReFI} \cdot DI^{RoW} \quad (258)$$

$$F^A_{ReFI} = \alpha^A_{ReFI} \cdot DI^{RoW} \quad (259)$$

$$F^{MC}_{ReFI} = \alpha^{MC}_{ReFI} \cdot DI^{RoW} \quad (260)$$

$$F^{OMS}_{ReFI} = \alpha^{OMS}_{ReFI} \cdot DI^{RoW} \quad (261)$$

Retained earnings: Determined by earnings, reinvested earnings and the net distributed income to every industry.

$$RE^E = F^E + F^E_{ReFI} + DI^E \quad (262)$$

$$RE^T = F^T + F^T_{ReFI} + DI^T \quad (263)$$

$$RE^A = F^A + F^A_{ReFI} + DI^A \quad (264)$$

$$RE^{MC} = F^{MC} + F^{MC}_{ReFI} + DI^{MC} \quad (265)$$

$$RE^{OMS} = F^{OMS} + F^{OMS}_{ReFI} + DI^{OMS} \quad (266)$$

Gross profits: Sum of gross operating surplus and net interest payments for each industry minus the amount of gross operation surplus distributed to households and government.

$$GF^E = GOS^E + NI^E - GOS^E_H - GOS^E_G \quad (267)$$

$$GF^T = GOS^T + NI^T - GOS^T_H - GOS^{OMS}_G \quad (268)$$

$$GF^A = GOS^A + NI^A - GOS^A_H - GOS^A_G \quad (269)$$

$$GF^{MC} = GOS^{MC} + NI^{MC} - GOS^{MC}_H - GOS^{MC}_G \quad (270)$$

$$GF^{OMS} = GOS^{OMS} + NI^{OMS} - GOS_H^{OMS} - GOS_G^{OMS} \quad (271)$$

Profits: Final profits are determined by income taxes subtracted from the gross profit.

$$F^E = GF^E - T^E \quad (272)$$

$$F^T = GF^T - T^T \quad (273)$$

$$F^A = GF^A - T^A \quad (274)$$

$$F^{MC} = GF^{MC} - T^{MC} \quad (275)$$

$$F^{OMS} = GF^{OMS} - T^{OMS} \quad (276)$$

Current taxes on income and wealth: The current taxes on income is paid by all industries to the government. The τ coefficients are different tax rates for the industries.

$$T^E = GF^E \cdot \tau^E \quad (277)$$

$$T^T = GF^T \cdot \tau^T \quad (278)$$

$$T^A = GF^A \cdot \tau^A \quad (279)$$

$$T^{MC} = GF^{MC} \cdot \tau^{MC} \quad (280)$$

$$T^{OMS} = GF^{OMS} \cdot \tau^{OMS} \quad (281)$$

Other current transfers paid: Reflects all the current transfers not included in the previous items. All the sectors and industries both receive and pays other transfers except for the CB, which only pays. The payments of other current transfers are determined by a α-share, $\frac{OC_p^i}{GOS^i}$ of GOS, which is used to distribute it for each industry.

$$OC_p^E = GOS^E \cdot \alpha_{OC_p}^E \quad (282)$$

$$OC_p^T = GOS^T \cdot \alpha_{OC_p}^T \quad (283)$$

$$OC_p^A = GOS^A \cdot \alpha_{OC_p}^A \quad (284)$$

$$OC_p^{MC} = GOS^{MC} \cdot \alpha_{OC_p}^{MC} \quad (285)$$

$$OC_p^{OMS} = GOS^{OMS} \cdot \alpha_{OC_p}^{OMS} \tag{286}$$

Other current transfers, received: A share, $\alpha_{OC_r}^i$, of the total other current transfers paid are distributed to every sector and industry. The α-share is obtained by the following calculation: $\frac{OC_r^i}{\sum OC_p^i}$.

$$OC_r^E = (OC_p^H + OC_p^E + OC_p^T + OC_p^A + OC_p^{MC} + OC_p^{OMS} + OC_p^{FC} + OC_p^G + OC_p^{CB} + OC_p^{RoW}) \cdot \alpha_{OC_r}^E \tag{287}$$

$$OC_r^T = (OC_p^H + OC_p^E + OC_p^T + OC_p^A + OC_p^{MC} + OC_p^{OMS} + OC_p^{FC} + OC_p^G + OC_p^{CB} + OC_p^{RoW}) \cdot \alpha_{OC_r}^T \tag{288}$$

$$OC_r^A = (OC_p^H + OC_p^E + OC_p^T + OC_p^A + OC_p^{MC} + OC_p^{OMS} + OC_p^{FC} + OC_p^G + OC_p^{CB} + OC_p^{RoW}) \cdot \alpha_{OC_r}^A \tag{289}$$

$$OC_r^{MC} = (OC_p^H + OC_p^E + OC_p^T + OC_p^A + OC_p^{MC} + OC_p^{OMS} + OC_p^{FC} + OC_p^G + OC_p^{CB} + OC_p^{RoW}) \cdot \alpha_{OC_r}^{MC} \tag{290}$$

$$OC_r^{OMS} = (OC_p^H + OC_p^E + OC_p^T + OC_p^A + OC_p^{MC} + OC_p^{OMS} + OC_p^{FC} + OC_p^G + OC_p^{CB} + OC_p^{RoW}) \cdot \alpha_{OC_r}^{OMS} \tag{291}$$

Other current transfers, net: Net other current transfers is made by subtracting payed from received other current transfers.

$$OC^E = OC_r^E - OC_p^E \tag{292}$$

$$OC^T = OC_r^T - OC_p^T \tag{293}$$

$$OC^A = OC_r^A - OC_p^A \tag{294}$$

$$OC^{MC} = OC_r^{MC} - OC_p^{MC} \tag{295}$$

$$OC^{OMS} = OC_r^{OMS} - OC_p^{OMS} \tag{296}$$

Savings: Determined by retained earnings plus the net value of other current transfers.

$$S^E = RE^E + OC^E \tag{297}$$

$$S^T = RE^T + OC^T \tag{298}$$

$$S^{A} = RE^{A} + OC^{A} \tag{299}$$

$$S^{MC} = RE^{MC} + OC^{MC} \tag{300}$$

$$S^{OMS} = RE^{OMS} + OC^{OMS} \tag{301}$$

Net lending: The net lending position for every industry depends on savings, investments, capital and non-financial and non-produced assets transfers, change in inventories plus the adjustment variable[85]. NPL are determined exogenously.

$$NL^{E} = S^{E} - I^{E} - NPL^{E} - \Delta INV^{E} + Adj^{E} \tag{302}$$

$$NL^{T} = S^{T} - I^{T} - NPL^{T} - \Delta INV^{T} + Adj^{T} \tag{303}$$

$$NL^{A} = S^{A} - I^{A} - NPL^{A} - \Delta INV^{A} + Adj^{A} \tag{304}$$

$$NL^{MC} = S^{MC} - I^{MC} - NPL^{MC} - \Delta INV^{MC} + Adj^{MC} \tag{305}$$

$$NL^{OMS} = S^{OMS} - I^{OMS} - NPL^{OMS} - \Delta INV^{OMS} + Adj^{OMS} \tag{306}$$

Labor productivity: Based on all production divided by the employment in each industry.

$$pr^{E} = \frac{y_P^{E}}{EMP^{E}} \tag{307}$$

$$pr^{T} = \frac{y_P^{T}}{EMP^{T}} \tag{308}$$

$$pr^{A} = \frac{y_P^{A}}{EMP^{A}} \tag{309}$$

$$pr^{MC} = \frac{y_P^{MC}}{EMP^{MC}} \tag{310}$$

$$pr^{OMS} = \frac{y_P^{OMS}}{EMP^{OMS}} \tag{311}$$

Employment by industry: Employment by industry, which, like in most Post-Keynesian models, are determined by two fundamental factors; the level of economic activity, y_P^n and productivity, pr^i, for the specific industry. Applying this approach, with an adjustment for sales to productivity, at an industrial level, employment level in each industry can be defined as a function of total production of industry i to the corresponding productivity ratio:

[85] a variable capturing the discrepancy between net lending and the sum of the transactions of financial instruments

$$EMP^E = \frac{y_P^E}{pr^E} \tag{312}$$

$$EMP^T = \frac{y_P^T}{pr^T} \tag{313}$$

$$EMP^A = \frac{y_P^A}{pr^A} \tag{314}$$

$$EMP^{MC} = \frac{y_P^{MC}}{pr^{MC}} \tag{315}$$

$$EMP^{OMS} = \frac{y_P^{OMS}}{pr^{OMS}} \tag{316}$$

Unemployment: Determined by the labor in the different sectors subtracted from the total labor force which is exogenously given.

$$UE = LF - EMP^E - EMP^T - EMP^A - EMP^{MC} - EMP^{OMS} - EMP^{FC} - EMP^{CB} \tag{317}$$

Unemployment rate: The unemployment rate is computed by the difference between the unemployment and the labor force.

$$UE_r = \frac{UE}{LF} \tag{318}$$

Structural unemployment rate: The structural unemployment rate is computed by multiplying the cyclical unemployment rate by a time-varying factor that captures structural deviations in unemployment, θ. For a detailed description, see section 6.2.

$$UE_s = UE_r \cdot \theta \tag{319}$$

The cyclical deviation in the unemployment rate: The cyclical unemployment rate is computed as the difference between the unemployment rate and the structural unemployment rate, see section 6.2.

$$UR_rURs = UR_r - UR_s \tag{320}$$

Financial wealth: Determined by the net lending position plus the financial wealth in the previous period and the revaluations of the financial instruments.

$$V^E = NL_t^E + V_{t-1}^E + Rev^E \tag{321}$$

$$V^T = NL_t^T + V_{t-1}^T + Rev^T \tag{322}$$

$$V^A = NL_t^A + V_{t-1}^A + Rev^A \tag{323}$$

$$V^{MC} = NL_t^{MC} + V_{t-1}^{MC} + Rev^{MC} \tag{324}$$

$$V^{OMS} = NL_t^{OMS} + V_{t-1}^{OMS} + Rev^{OMS} \tag{325}$$

Flow of Funds for non-financial corporations at industry level

The financial wealth of each industry is allocated to six different funds. Loans (F.4) function as a buffer to ensure the fulfillment of each industry's budget constraint. This implies that if an industry experiences a deficit in a given period, it will increase its borrowing from the financial sector, and vice versa. Revaluations and other changes used to compute transactions of funds are determined exogenously for each industry. The transactions for all financial instruments, except F.4, are given by the change in stocks minus revaluations and other changes. Transactions for F.4 are defined endogenously as net lending minus assets plus liabilities.

Currency and deposits (F.2) stock:

$$F.2^{E}_{s(t)} = V^{E} \cdot \beta^{E}_{F.2} \tag{326}$$

$$F.2^{T}_{s(t)} = V^{T} \cdot \beta^{T}_{F.2} \tag{327}$$

$$F.2^{A}_{s(t)} = V^{A} \cdot \beta^{A}_{F.2} \tag{328}$$

$$F.2^{MC}_{s(t)} = V^{MC} \cdot \beta^{MC}_{F.2} \tag{329}$$

$$F.2^{OMS}_{s(t)} = V^{OMS} \cdot \beta^{OMS}_{F.2} \tag{330}$$

Securities (F.3) stock:

$$F.3^{E}_{s(t)} = V^{E} \cdot \beta^{E}_{F.3} \tag{331}$$

$$F.3^{T}_{s(t)} = V^{T} \cdot \beta^{T}_{F.3} \tag{332}$$

$$F.3^{A}_{s(t)} = V^{A} \cdot \beta^{A}_{F.3} \tag{333}$$

$$F.3^{MC}_{s(t)} = V^{MC} \cdot \beta^{MC}_{F.3} \tag{334}$$

$$F.3^{OMS}_{s(t)} = V^{OMS} \cdot \beta^{OMS}_{F.3} \tag{335}$$

Loans (F.4) stock:

$$F.4^{E}_{s(t)} = F.4^{E}_{s(t-1)} + F.4^{E} + F.4^{E}_{roc(t)} \tag{336}$$

$$F.4^{T}_{s(t)} = F.4^{T}_{s(t-1)} + F.4^{T} + F.4^{T}_{roc(t)} \tag{337}$$

$$F.4^{A}_{s(t)} = F.4^{A}_{s(t-1)} + F.4^{A} + F.4^{A}_{roc(t)} \tag{338}$$

$$F.4^{MC}_{s(t)} = F.4^{MC}_{s(t-1)} + F.4^{MC} + F.4^{MC}_{roc(t)} \tag{339}$$

$$F.4^{OMS}_{s(t)} = F.4^{OMS}_{s(t-1)} + F.4^{OMS} + F.4^{OMS}_{roc(t)} \tag{340}$$

Equity (F.5) stock:

$$F.5^{E}_{s(t)} = V^{E} \cdot \beta^{E}_{F.5} \tag{341}$$

$$F.5^{T}_{s(t)} = V^{T} \cdot \beta^{T}_{F.5} \tag{342}$$

$$F.5^{A}_{s(t)} = V^{A} \cdot \beta^{A}_{F.5} \tag{343}$$

$$F.5^{MC}_{s(t)} = V^{MC} \cdot \beta^{MC}_{F.5} \tag{344}$$

$$F.5^{OMS}_{s(t)} = V^{OMS} \cdot \beta^{OMS}_{F.5} \tag{345}$$

Insurance tech. reserves (F.6) stock:

$$F.6^{E}_{s(t)} = V^{E} \cdot \beta^{E}_{F.6} \tag{346}$$

$$F.6^{T}_{s(t)} = V^{T} \cdot \beta^{T}_{F.6} \tag{347}$$

$$F.6^{A}_{s(t)} = V^{A} \cdot \beta^{A}_{F.6} \tag{348}$$

$$F.6^{MC}_{s(t)} = V^{MC} \cdot \beta^{MC}_{F.6} \tag{349}$$

$$F.6^{OMS}_{s(t)} = V^{OMS} \cdot \beta^{OMS}_{F.6} \tag{350}$$

Financial derivatives (F.7) stock:

$$F.7^{E}_{s(t)} = V^{E} \cdot \beta^{E}_{F.7} \tag{351}$$

$$F.7^{T}_{s(t)} = V^{T} \cdot \beta^{T}_{F.7} \tag{352}$$

$$F.7^{A}_{s(t)} = V^{A} \cdot \beta^{A}_{F.7} \tag{353}$$

$$F.7^{MC}_{s(t)} = V^{MC} \cdot \beta^{MC}_{F.7} \tag{354}$$

$$F.7^{OMS}_{s(t)} = V^{OMS} \cdot \beta^{OMS}_{F.7} \tag{355}$$

Trade credits (F.8) stock:

$$F.8^{E}_{s(t)} = V^{E} \cdot \beta^{E}_{F.8} \tag{356}$$

$$F.8^{T}_{s(t)} = V^{T} \cdot \beta^{T}_{F.8} \tag{357}$$

$$F.8^{A}_{s(t)} = V^{A} \cdot \beta^{A}_{F.8} \tag{358}$$

$$F.8^{MC}_{s(t)} = V^{MC} \cdot \beta^{MC}_{F.8} \tag{359}$$

$$F.8^{OMS}_{s(t)} = V^{OMS} \cdot \beta^{OMS}_{F.8} \tag{360}$$

Currency and deposits (F.2) transactions:

$$F.2^{E} = F.2^{E}_{s(t)} - F.2^{E}_{s(t-1)} - F.2^{E}_{roc(t)} \tag{361}$$

$$F.2^{T} = F.2^{T}_{s(t)} - F.2^{T}_{s(t-1)} - F.2^{T}_{roc(t)} \tag{362}$$

$$F.2^{A} = F.2^{A}_{s(t)} - F.2^{A}_{s(t-1)} - F.2^{A}_{roc(t)} \tag{363}$$

$$F.2^{MC} = F.2^{MC}_{s(t)} - F.2^{MC}_{s(t-1)} - F.2^{MC}_{roc_{(t)}} \tag{364}$$

$$F.2^{OMS} = F.2^{OMS}_{s_{(t)}} - F.2^{OMS}_{s_{(t-1)}} - F.2^{OMS}_{roc_{(t)}} \tag{365}$$

Securities (F.3) transactions:

$$F.3^{E} = F.3^{E}_{s_{(t)}} - F.3^{E}_{s_{(t-1)}} - F.3^{E}_{roc_{(t)}} \tag{366}$$

$$F.3^{T} = F.3^{T}_{s_{(t)}} - F.3^{T}_{s_{(t-1)}} - F.3^{T}_{roc_{(t)}} \tag{367}$$

$$F.3^{A} = F.3^{A}_{s_{(t)}} - F.3^{A}_{s_{(t-1)}} - F.3^{A}_{roc_{(t)}} \tag{368}$$

$$F.3^{MC} = F.3^{MC}_{s_{(t)}} - F.3^{MC}_{s_{(t-1)}} - F.3^{MC}_{roc_{(t)}} \tag{369}$$

$$F.3^{OMS} = F.3^{OMS}_{s_{(t)}} - F.3^{OMS}_{s_{(t-1)}} - F.3^{OMS}_{roc_{(t)}} \tag{370}$$

Loans (F.4) transactions:

$$F.4^{E} = NL^{E} - F.2^{E} - F.3^{E} - F.5^{E} - F.6^{E} - F.7^{E} - F.8^{E} \tag{371}$$

$$F.4^{T} = NL^{T} - F.2^{T} - F.3^{T} - F.5^{T} - F.6^{T} - F.7^{T} - F.8^{T} \tag{372}$$

$$F.4^{A} = NL^{A} - F.2^{A} - F.3^{A} - F.5^{A} - F.6^{A} - F.7^{A} - F.8^{A} \tag{373}$$

$$F.4^{MC} = NL^{MC} - F.2^{MC} - F.3^{MC} - F.5^{MC} - F.6^{MC} - F.7^{MC} - F.8^{MC} \tag{374}$$

$$F.4^{OMS} = NL^{OMS} - F.2^{OMS} - F.3^{OMS} - F.5^{OMS} - F.6^{OMS} - F.7^{OMS} - F.8^{OMS} \tag{375}$$

Equity (F.5) transactions:

$$F.5^{E} = F.5^{E}_{s_{(t)}} - F.5^{E}_{s_{(t-1)}} - F.5^{E}_{roc_{(t)}} \tag{376}$$

$$F.5^{T} = F.5^{T}_{s_{(t)}} - F.5^{T}_{s_{(t-1)}} - F.5^{T}_{roc_{(t)}} \tag{377}$$

$$F.5^{A} = F.5^{A}_{s_{(t)}} - F.5^{A}_{s_{(t-1)}} - F.5^{A}_{roc_{(t)}} \tag{378}$$

$$F.5^{MC} = F.5^{MC}_{s_{(t)}} - F.5^{MC}_{s_{(t-1)}} - F.5^{MC}_{roc_{(t)}} \tag{379}$$

$$F.5^{OMS} = F.5^{OMS}_{s_{(t)}} - F.5^{OMS}_{s_{(t-1)}} - F.5^{OMS}_{roc_{(t)}} \tag{380}$$

Insurance tech. reserves (F.6) transactions:

$$F.6^{E} = F.6^{E}_{s_{(t)}} - F.6^{E}_{s_{(t-1)}} - F.6^{E}_{roc_{(t)}} \tag{381}$$

$$F.6^{T} = F.6^{T}_{s_{(t)}} - F.6^{T}_{s_{(t-1)}} - F.6^{T}_{roc_{(t)}} \tag{382}$$

$$F.6^{A} = F.6^{A}_{s_{(t)}} - F.6^{A}_{s_{(t-1)}} - F.6^{A}_{roc_{(t)}} \tag{383}$$

$$F.6^{MC} = F.6^{MC}_{s_{(t)}} - F.6^{MC}_{s_{(t-1)}} - F.6^{MC}_{roc_{(t)}} \tag{384}$$

$$F.6^{OMS} = F.6^{OMS}_{s_{(t)}} - F.6^{OMS}_{s_{(t-1)}} - F.6^{OMS}_{roc_{(t)}} \tag{385}$$

Financial derivatives (F.7) transactions:

$$F.7^{E} = F.7^{E}_{s_{(t)}} - F.7^{E}_{s_{(t-1)}} - F.7^{E}_{roc_{(t)}} \tag{386}$$

$$F.7^{T} = F.7^{T}_{s_{(t)}} - F.7^{T}_{s_{(t-1)}} - F.7^{T}_{roc_{(t)}} \tag{387}$$

$$F.7^{A} = F.7^{A}_{s_{(t)}} - F.7^{A}_{s_{(t-1)}} - F.7^{A}_{roc_{(t)}} \tag{388}$$

$$F.7^{MC} = F.7^{MC}_{s_{(t)}} - F.7^{MC}_{s_{(t-1)}} - F.7^{MC}_{roc_{(t)}} \tag{389}$$

$$F.7^{OMS} = F.7^{OMS}_{s_{(t)}} - F.7^{OMS}_{s_{(t-1)}} - F.7^{OMS}_{roc_{(t)}} \tag{390}$$

Trade credits (F.8) transactions:

$$F.8^{E} = F.8^{E}_{s_{(t)}} - F.8^{E}_{s_{(t-1)}} - F.8^{E}_{roc_{(t)}} \tag{391}$$

$$F.8^{T} = F.8^{T}_{s_{(t)}} - F.8^{T}_{s_{(t-1)}} - F.8^{T}_{roc_{(t)}} \tag{392}$$

$$F.8^{A} = F.8^{A}_{s_{(t)}} - F.8^{A}_{s_{(t-1)}} - F.8^{A}_{roc_{(t)}} \tag{393}$$

$$F.8^{MC} = F.8^{MC}_{s_{(t)}} - F.8^{MC}_{s_{(t-1)}} - F.8^{MC}_{roc_{(t)}} \tag{394}$$

$$F.8^{OMS} = F.8^{OMS}_{s_{(t)}} - F.8^{OMS}_{s_{(t-1)}} - F.8^{OMS}_{roc_{(t)}} \tag{395}$$

To ensure horizontal consistency for transactions and to ensure that the demand for trade credit transactions equals the supply, the following equations must be specified. $F.8^{NFC}$ is the sum of $F.8$ for the industries:

$$F.8^{NFC} = F.8^{H} + F.8^{G} + F.8^{FC} + F.8^{CB} + F.8^{RoW} \tag{396}$$

Financial corporations

Nominal sales:

$$Y^{FC} = C^{FC} + G^{FC} + X^{FC} + IG_s^{FC} \tag{397}$$

Real sales:

$$y^{FC} = c^{FC} + g^{FC} + x^{FC} + ig_s^{FC} \tag{398}$$

Nominal investment:

$$I^{FC} = i^{FC} \cdot p_i^{FC} \tag{399}$$

Real investment:

$$i^{FC} = i \cdot \alpha_{I_a}^{FC} \tag{400}$$

Nominal stock of capital:

$$K^{FC} = K_{t-1}^{FC} + I^{FC} - D^{FC} \tag{401}$$

Real stock of capital:

$$k^{FC} = \frac{K^{FC}}{p_i} \tag{402}$$

Depreciations:

$$D^{FC} = \delta \cdot (K_{t-1}^{FC}) \tag{403}$$

Intermediate real consumption transactions:

$$ig_p^{FC,E} = y^{FC} \cdot \alpha_p^{FC,E} \tag{404}$$

$$ig_p^{FC,T} = y^{FC} \cdot \alpha_p^{FC,T} \tag{405}$$

$$ig_p^{FC,A} = y^{FC} \cdot \alpha_p^{FC,A} \tag{406}$$

$$ig_p^{FC,MC} = y^{FC} \cdot \alpha_p^{FC,MC} \tag{407}$$

$$ig_p^{FC,OMS} = y^{FC} \cdot \alpha_p^{FC,OMS} \tag{408}$$

$$ig_p^{FC,FC} = y^{FC} \cdot \alpha_p^{FC,FC} \tag{409}$$

Intermediate real sales transactions: The intermediate sales from FC. Determined by the sum of the other sectors purchases from FC.

$$ig_s^{FC} = ig_p^{E,FC} + ig_p^{T,FC} + ig_p^{A,FC} + ig_p^{MC,FC} + ig_p^{OMS,FC} + ig_p^{FC,FC} \tag{410}$$

Real intermediate imports: The intermediate imports by FCs, the $\alpha_{ig_p}^{FC,s}$ represents the share of imported intermediate goods.

$$ig_{p_{IM}}^{FC,E} = (y^{FC} \cdot \alpha_p^{FC,E}) \cdot \alpha_{ig_p}^{FC,E} \tag{411}$$

$$ig_{p_{IM}}^{FC,T} = (y^{FC} \cdot \alpha_p^{FC,T}) \cdot \alpha_{ig_p}^{FC,T} \tag{412}$$

$$ig_{p_{IM}}^{FC,A} = (y^{FC} \cdot \alpha_p^{FC,A}) \cdot \alpha_{ig_p}^{FC,A} \tag{413}$$

$$ig_{p_{IM}}^{FC,MC} = (y^{FC} \cdot \alpha_p^{FC,MC}) \cdot \alpha_{ig_p}^{FC,MC} \tag{414}$$

$$ig_{p_{IM}}^{FC,OMS} = (y^{FC} \cdot \alpha_p^{FC,OMS}) \cdot \alpha_{ig_p}^{FC,OMS} \tag{415}$$

$$ig_{p_{IM}}^{FC,FC} = (y^{FC} \cdot \alpha_p^{FC,FC}) \cdot \alpha_{ig_p}^{FC,FC} \tag{416}$$

Intermediate nominal consumption transactions: The nominal intermediate purchases from FC.

$$IG_p^{FC} = ig_p^{FC,E} \cdot pp^E + ig_p^{FC,T} \cdot pp^T + ig_p^{FC,A} \cdot pp^A + ig_p^{FC,MC} \cdot pp^{MC} + ig_p^{FC,OMS} \cdot pp^{OMS} + ig_p^{FC,FC} \cdot pp^{FC} \tag{417}$$

Intermediate nominal sales transactions: The real intermediate sales from FC multiplied by the producer price for the financial corporation sector.

$$IG_s^{FC} = ig_s^{FC} \cdot pp^{FC} \tag{418}$$

Nominal intermediate imports: Determined by the real intermediate imports times a given import price.

$$IG_{IM}^{FC} = ig_{IM}^{FC} \cdot p_{IM} \tag{419}$$

Net intermediate for FCs: Given by the difference between intermediate sales and purchases

$$IG_{net}^{FC} = IG_s^{FC} - IG_p^{FC} - IG_{IM}^{FC} \tag{420}$$

Nominal export:

$$X^{FC} = x^{FC} \cdot p^{FC} \tag{421}$$

Real export:

$$x^{FC} = x \cdot \alpha_x^{FC} \tag{422}$$

Net indirect taxes less subsidies on production: A given tax paid on the total sales.

$$NIT^{FC} = Y^{FC} \cdot \tau^{FC}_{NIT} \tag{423}$$

Private and public imports:

$$C^{FC}_{IM} = C_{IM} \cdot \alpha_{C^{FC}_{IM}} \tag{424}$$

Nominal imports:

$$IM^{FC} = IG^{FC}_{IM} + C^{FC}_{IM} \tag{425}$$

Nominal value added at factor cost: Total sales subtracted by intermediate purchases and net indirect taxes determines the nominal value added.

$$GVA^{FC}_{F} = Y^{FC} - IG^{FC}_{p} - NIT^{FC} - IM^{FC} \tag{426}$$

Compensation of employees:

$$WB^{FC} = EMP^{FC} \cdot W^{FC} \tag{427}$$

Wage rate:

$$W^{FC} = \alpha^{FC}_{W} \cdot W \tag{428}$$

Gross operating surplus and mixed income:

$$GOS^{FC} = GVA^{FC}_{F} - WB^{FC} \tag{429}$$

Gross operating surplus and mixed income distributed: A share of GOS is distributed to households and government.

$$GOS^{FC}_{H} = \theta^{FC}_{H} \cdot GOS^{FC} \tag{430}$$

$$GOS^{FC}_{G} = \theta^{FC}_{G} \cdot GOS^{FC} \tag{431}$$

Distributed income of corporation, payments (D.42): A share of gross operating surplus are paid as distributed income.

$$DI^{FC}_{p} = F^{FC} \cdot \alpha^{FC}_{p} \tag{432}$$

Distributed income of corporation, receivable (D.42): The α-coefficient is represents the share of total distributed income paid, received by FC.

$$DI^{FC}_{r} = (DI^{E}_{p} + DI^{T}_{p} + DI^{A}_{p} + DI^{MC}_{p} + DI^{OMS}_{p} + DI^{FC}_{p} + DI^{RoW}_{p}) \cdot \alpha^{FC}_{DI_r} \tag{433}$$

Distributed income of corporations (D.42): The net value is obtained by receivings minus payments.

$$DI^{FC} = DI_r^{FC} - DI_p^{FC} \tag{434}$$

Net interest: The same determination applies as for the NFCs

$$NI^{FC} = F.2_s^{FC} \cdot \iota_{F.2}^{FC} + F.3_s^{FC} \cdot \iota_{F.3}^{FC} + F.4_s^{FC} \cdot \iota_{F.4}^{FC} + F.6_s^{FC} \cdot \iota_{F.6}^{FC} + interest_{F.7}^{FC} + interest_{F.8}^{FC} \tag{435}$$

Reinvested earnings on FDI: A share of distributed income to RoW are reinvested in FCs

$$F_{ReFI}^{FC} = \alpha_{ReFI}^{FC} \cdot DI^{RoW} \tag{436}$$

Retained earnings: Consists by profits, reinvested earnings, net distributed income and the negative values of investments and NPL which are exogenously determined.

$$RE^{FC} = F^{FC} + F_{ReFI}^{FC} + DI^{FC} \tag{437}$$

Gross profits: Determined by gross operation surplus plus net interest minus the amount of gross operation surplus distributed to households and government, respectively.

$$GF^{FC} = GOS^{FC} + NI^{FC} - GOS_H^{FC} - GOS_G^{FC} \tag{438}$$

Profits: Gross profits subtracted by income taxes constitutes the final profits.

$$F^{FC} = GF^{FC} - T^{FC} \tag{439}$$

Taxes on income and wealth: Determined by a given tax rate multiplied by gross profits

$$T^{FC} = GF^{FC} \cdot \tau^{FC} \tag{440}$$

Social contributions received: A share of the social contributions paid by households and RoW are received by FCs. This share is given by, $\frac{SC_r^{FC}}{SC^H + SC^{RoW}}$.

$$SC_r^{FC} = (SC_p^H + SC_p^{RoW}) \cdot \alpha_{SC_r}^{FC} \tag{441}$$

Social benefits paid: Given by the total amount of social benefits received minus the social benefits paid by the government.

$$SB_p^{FC} = SB_r^H + SB_r^{RoW} - SB_p^G \tag{442}$$

Other current transfers paid:

$$OC_p^{FC} = GOS^{FC} \cdot \alpha_{OC_p}^{FC} \tag{443}$$

Other current transfers received:

$$OC_r^{FC} = (OC_p^E + OC_p^T + OC_p^A + OC_p^{MC} + OC_p^{OMS} + OC_p^{FC} + OC_p^H + OC_p^G + OC_p^{CB} + OC_p^{RoW}) \cdot \alpha_{OC_r}^{FC} \quad (444)$$

Other current transfers, net: Net other current transfers is made by subtracting payed from received other current transfers.

$$OC^{FC} = OC_r^{FC} - OC_p^{FC} \quad (445)$$

Adjust for change in the pension entitlements: The change in pension entitlements consists of a share, α_{SC}^{FC}, of social contributions received by financial corporations.

$$PE^{FC} = SC_r^{FC} \cdot \alpha_{SC}^{FC} \quad (446)$$

Savings:

$$S^{FC} = RE^{FC} + OC^{FC} \quad (447)$$

Net lending:

$$NL^{FC} = S^{FC} - I^{FC} - NPL^{FC} - PE^{FC} + Adj^{FC} \quad (448)$$

Labor productivity: The value added per employee in financial corporations is defined as a positive function of market size, following the Smith-Kaldor-Verdoorn effect and the approach commonly used at the industry level as referenced in Canelli et al. (2021). Since production is not explicitly modeled for financial corporations in this framework, we lack a direct measure of market size, often represented by real production. Therefore, a proxy must be established. This proxy is defined as the sum of the wage bill paid by financial corporations and the gross operating surplus (GOS) received, deflated by a price index: $y_P^{FC} = \frac{(WB^{FC} + GOS^{FC})}{p^{FC}}$, where y_P^{FC} is the value added of financial corporations. The value added per employee is then defined as pr^{FC}. This formulation includes an autonomous component, the productivity elasticity with respect to the real wage, and the elasticity with respect to the proxy for real output.

$$y_P^{FC} = \frac{(WB^{FC} + GOS^{FC})}{pp^{FC}} \quad (449)$$

$$pr^{FC} = \frac{y_P^{FC}}{EMP^{FC}} \quad (450)$$

Employment:

$$EMP^{FC} = \frac{y_P^{FC}}{pr^{FC}} \quad (451)$$

Financial wealth:

$$V^{FC} = NL^{FC} + V_{t-1}^{FC} + Rev^{FC} \quad (452)$$

Flow of Funds financial corporations

The financial wealth of the financial corporations is allocated to six different funds. Equities (F.5) function as a buffer to ensure the fulfillment of the financial corporation's budget constraint. This means that if an industry runs a deficit in a given period, the imbalance will be absorbed through adjustments in equities (F.5), which serve as a residual item to ensure the budget constraint is satisfied. Revaluations and other changes used to compute transactions of funds are determined exogenously for each industry. The transactions for all financial instruments, except F.5, are given by the change in stocks minus revaluations and other changes. Transactions for F.5 are defined endogenously as net lending minus assets plus liabilities.

Currency and deposits (F.2) stock:

$$F.2^{FC}_{s(t)} = V^{FC} \cdot \beta^{FC}_{F.2} \tag{453}$$

Securities (F.3) stock:

$$F.3^{FC}_{s(t)} = V^{FC} \cdot \beta^{FC}_{F.3} \tag{454}$$

Loans (F.4) stock:

$$F.4^{FC}_{s(t)} = F.4^{FC}_{s(t-1)} + F.4^{FC} + F.4^{FC}_{roc_{(t)}} \tag{455}$$

Equity (F.5) stock:

$$F.5^{FC}_{s(t)} = V^{FC} \cdot \beta^{FC}_{F.5} \tag{456}$$

Insurance tech. reserves (F.6) stock:

$$F.6^{FC}_{s(t)} = V^{FC} \cdot \beta^{FC}_{F.6} \tag{457}$$

Financial derivatives (F.7) stock:

$$F.7^{FC}_{s(t)} = V^{FC} \cdot \beta^{FC}_{F.7} \tag{458}$$

Trade credits (F.8) stock:

$$F.8^{FC}_{s(t)} = V^{FC} \cdot \beta^{FC}_{F.8} \tag{459}$$

Currency and deposits (F.2) transactions:

$$F.2^{FC} = F.2^{FC}_{s_{(t)}} - F.2^{FC}_{s_{(t-1)}} - F.2^{FC}_{roc_{(t)}} \tag{460}$$

Securities (F.3) transactions:

$$F.3^{FC} = F.3^{FC}_{s_{(t)}} - F.3^{FC}_{s_{(t-1)}} - F.3^{FC}_{roc_{(t)}} \tag{461}$$

Loans (F.4) transactions:

$$F.4^{FC} = F.4^{FC}_{s_{(t)}} - F.4^{FC}_{s_{(t-1)}} - F.4^{FC}_{roc_{(t)}} \tag{462}$$

Equity (F.5) transactions:

$$F.5^{FC} = NL^{FC} - F.2^{FC} - F.3^{FC} - F.4^{FC} - F.6^{FC} - F.7^{FC} - F.8^{FC} \tag{463}$$

Insurance tech. reserves (F.6) transactions:

$$F.6^{FC} = F.6^{FC}_{s_{(t)}} - F.6^{FC}_{s_{(t-1)}} - F.6^{FC}_{roc_{(t)}} \tag{464}$$

Financial derivatives (F.7) transactions:

$$F.7^{FC} = F.7^{FC}_{s_{(t)}} - F.7^{FC}_{s_{(t-1)}} - F.7^{FC}_{roc_{(t)}} \tag{465}$$

Trade credits (F.8) transactions:

$$F.8^{FC} = F.8^{FC}_{s_{(t)}} - F.8^{FC}_{s_{(t-1)}} - F.8^{FC}_{roc_{(t)}} \tag{466}$$

To ensure horizontal consistency for transactions and to ensure that the demand for currency and deposits, loans, insurance technical reserves and financial derivatives transactions equals the supply, the following equations must be specified:

$$F.2^{FC} = F.2^{H} + F.2^{E} + F.2^{T} + F.2^{A} + F.2^{MC} + F.2^{OMS} + F.2^{G} + F.2^{CB} + F.2^{RoW} \tag{467}$$

$$F.4^{FC} = F.4^{H} + F.4^{E} + F.4^{T} + F.4^{A} + F.4^{MC} + F.4^{OMS} + F.4^{G} + F.4^{CB} + F.4^{RoW} \tag{468}$$

$$F.6^{FC} = F.6^{H} + F.6^{E} + F.6^{T} + F.6^{A} + F.6^{MC} + F.6^{OMS} + F.6^{G} + F.6^{CB} + F.6^{RoW} \tag{469}$$

$$F.7^{FC} = F.7^{H} + F.7^{E} + F.7^{T} + F.7^{A} + F.7^{MC} + F.7^{OMS} + F.7^{G} + F.7^{CB} + F.7^{RoW} \tag{470}$$

Households

Real consumption: The explanation for the real consumption, can be seen in section C.1.

$$\begin{aligned} c^H = {} & \beta_{0,c^H} + \beta_{1,c^H} \cdot \Delta \ln(c_{t-4}) + \beta_{2,c^H} \cdot \Delta \ln(yd_t^H) + \beta_{3,c^H} \cdot \Delta \ln(yd_{t-2}^H) \\ & + \beta_{4,c^H} \cdot \Delta \ln(fw_t^H) + \beta_{5,c^H} \cdot \Delta r_t + \beta_{6,c^H} \cdot \ln(\mathbf{c_{t-1}}) \\ & + \beta_{7,c^H} \cdot \ln(\mathbf{yd^H_{t-1}}) + \beta_{8,c^H} \cdot \ln(\mathbf{fw^H_{t-1}}) \end{aligned} \tag{471}$$

$$c^E = c^H \cdot \alpha_c^E \tag{472}$$

$$c^T = c^H \cdot \alpha_c^T \tag{473}$$

$$c^A = c^H \cdot \alpha_c^A \tag{474}$$

$$c^{MC} = c^H \cdot \alpha_c^{MC} \tag{475}$$

$$c^{OMS} = c^H \cdot \alpha_c^{OMS} \tag{476}$$

$$c^{FC} = c^H \cdot \alpha_c^{FC} \tag{477}$$

Nominal consumption: Determined by the real aggregate consumption multiplied by a beta coefficient, which represents the share of household consumption allocated to every single industry times the price level of the respective industry

$$C^E = c^E \cdot p^E \tag{478}$$

$$C^T = c^T \cdot p^T \tag{479}$$

$$C^A = c^A \cdot p^A \tag{480}$$

$$C^{MC} = c^{MC} \cdot p^{MC} \tag{481}$$

$$C^{OMS} = c^{OMS} \cdot p^{OMS} \tag{482}$$

$$C^{FC} = c^{FC} \cdot p^{FC} \tag{483}$$

$$C^{H} = C^{E} + C^{T} + C^{A} + C^{MC} + C^{OMS} + C^{FC} \tag{484}$$

Disposable income: This equation consists of the households wage bill, gross operating surplus and distributed income to households, net interest, social benefits and the net value of other current transfers minus social contributions, taxes on income.

$$YD^{H} = WB^{H} + GOS^{H} + DI_{r}^{H} + NI^{H} + SB^{H} + OC^{H} - SC_{p}^{H} - T^{H} \tag{485}$$

Real disposable income: Represents the real disposable income by dividing the nominal disposable income by the overall price index for consumption

$$yd^{H} = \frac{YD^{H}}{p_{Y}} \tag{486}$$

Real investments:

$$i^{H} = i \cdot \alpha_{I_a}^{H} \tag{487}$$

Nominal investments:

$$I^{H} = i^{H} \cdot p_{a}^{H} \tag{488}$$

Nominal stock of capital:

$$K^{H} = K_{t-1}^{H} + I^{H} - D^{H} \tag{489}$$

Real stock of capital:

$$k^{H} = \frac{K^{H}}{p_{I}} \tag{490}$$

Depreciations:

$$D^{H} = \delta \cdot (K_{t-1}^{H}) \tag{491}$$

Total compensation for households: Determined by the aggregate compensation to households from the employing sectors minus the share of wages to RoW.

$$WB^{H} = WB^{E} + WB^{T} + WB^{A} + WB^{MC} + WB^{OMS} + WB^{FC} + WB^{CB} - WB^{RoW} \tag{492}$$

Gross operating surplus and mixed income: Distributed to households from the producing sectors.

$$GOS^H = GOS^E_H + GOS^T_H + GOS^A_H + GOS^{MC}_H + GOS^{OMS}_H + GOS^{FC}_H \tag{493}$$

Net Interest: Income through interest bearing assets plus interest expenditures on loans, which will appear with a negative value.

$$NI^H = F.2^H_s \cdot \iota^H_{F.2} + F.3^H_s \cdot \iota^H_{F.3} + F.4^H_s \cdot \iota^H_{F.4} + interest^H_{F.7} + interest^H_{F.8} \tag{494}$$

Distributed income of corporations, receivable: Households are receiving a share of the distributed income paid.

$$DI^H_r = (DI^E_p + DI^T_p + DI^A_p + DI^{MC}_p + DI^{OMS}_p + DI^{FC}_p + DI^{RoW}_p) \cdot \alpha^H_r \tag{495}$$

Current taxes on income: A common tax on wage, social benefits plus a tax on wealth and net interest. The social contributions are subtracted since these must be passed to the government and financial corporations.

$$T^H = (WB^H - SC^H_p + SB^H_r + NI^H) \cdot \tau^T_H \tag{496}$$

Social contributions paid: Social contributions are determined by the sum of all contributions paid through wages by households, which is represented by α^H_{SC}.

$$SC^H_p = WB^H \cdot \alpha^H_{SC} \tag{497}$$

Social benefits: The only receivers of social benefits are households and RoW.

$$SB^H_r = (SB^{FC}_p + SB^G_p) \cdot \alpha^H_{SB_r} \tag{498}$$

Other current transfers paid: A computed share $\beta^H_{OC_p}$ of the wage bill is paid as other current transfers .

$$OC^H_p = WB^H \cdot \alpha^H_{OC_p} \tag{499}$$

Other current transfers received:

$$OC^H_r = (OC^E_p + OC^T_p + OC^A_p + OC^{MC}_p + OC^{OMS}_p + OC^{FC}_p + OC^H_p + OC^G_p + OC^{CB}_p + OC^{RoW}_p) \cdot \alpha^H_{OC_r} \tag{500}$$

Other current transfers net:

$$OC^H = OC^H_r - OC^H_p \tag{501}$$

Adjustment for change in the pension entitlements:

$$PE^H = PE^{FC} \tag{502}$$

Savings: The periodically savings allocated to the transactions are defined as the disposable income minus consumption.

$$S^H = YD^H - C^H \tag{503}$$

Net lending:

$$NL^H = S^H - I^H - NPL^H + PE^H + Adj^H \tag{504}$$

Financial wealth: Nominal wealth is defined by savings and wealth in the previous period.

$$V^H = NL^H + V^H_{t-1} + Rev^H \tag{505}$$

Flow of Funds for households

The financial wealth of households is allocated between six different funds. Deposits (F.2) function as a buffer to ensure the fulfillment of households budget constraint. This implies that if households experiences a deficit in a given period, they will withdraw from its deposits in the financial sector, and vice versa. Revaluations and other changes used to compute transactions of funds are determined exogenously for each industry. The transactions for all financial instruments, except F.2, are given by the change in stocks minus revaluations and other changes. Transactions for F.2 are defined endogenously as net lending minus assets plus liabilities.

Currency and deposits (F.2) stock:

$$F.2^{H}_{s(t)} = V^{H} \cdot \beta^{H}_{F.2} \tag{506}$$

Securities (F.3) stock:

$$F.3^{H}_{s(t)} = V^{H} \cdot \beta^{H}_{F.3} \tag{507}$$

$$\begin{aligned}\Delta \ln(\beta^{H}_{F.3,t}) = {} & \beta_{0,\beta^{H}_{F.3}} + \beta_{1,\beta^{H}_{F.3}} \cdot \Delta \ln(\beta^{H}_{F.3,t-2}) + \beta_{2,\beta^{H}_{F.3}} \cdot \Delta \ln(\beta^{H}_{F.3,t-3}) + \beta_{3,\beta^{H}_{F.3}} \cdot \Delta \ln(\beta^{H}_{F.3,t-4}) \\ & + \beta_{4,\beta^{H}_{F.3}} \cdot \Delta(CI_{t-3}) + \beta_{5,\beta^{H}_{F.3}} \cdot \Delta \ln\left(\frac{YD^{H}_{t}}{FW^{H}_{t}}\right) + \beta_{6,\beta^{H}_{F.3}} \cdot \Delta \ln\left(\frac{YD^{H}_{t-3}}{FW^{H}_{t-3}}\right) \\ & + \beta_{7,\beta^{H}_{F.3}} \cdot \ln(\beta^{H}_{F.3,t-1}) + \beta_{8,\beta^{H}_{F.3}} \cdot (\iota^{\mathbf{F.3}}_{\mathbf{t-1}}) + \beta_{9,\beta^{H}_{F.3}} \cdot (\iota^{\mathbf{F.2}}_{\mathbf{t-1}})\end{aligned} \tag{508}$$

Loans (F.4) stock:

$$F.4^{H}_{s(t)} = F.4^{H}_{s(t-1)} + F.4^{FC} + F.4^{FC}_{roc_{(t)}} \tag{509}$$

Equity (F.5) stock:

$$F.5^{H}_{s(t)} = V^{H} \cdot \beta^{H}_{F.5} \tag{510}$$

Insurance tech. reserves (F.6) stock:

$$F.6^{H}_{s(t)} = V^{H} \cdot \beta^{H}_{F.6} \tag{511}$$

Financial derivatives (F.7) stock:

$$F.7^{H}_{s(t)} = V^{H} \cdot \beta^{H}_{F.7} \tag{512}$$

Trade credits (F.8) stock:

$$F.8^{H}_{s(t)} = V^{H} \cdot \beta^{H}_{F.8} \tag{513}$$

Currency and deposits (F.2) transactions:

$$F.2^H = NL^H - F.3^H - F.4^H - F.5^H - F.6^H - F.8^H - F.7^H \tag{514}$$

Securities (F.3) transactions:

$$F.3^H = F.3^H_{s_{(t)}} - F.3^H_{s_{(t-1)}} - F.3^H_{roc_{(t)}} \tag{515}$$

Loans (F.4) transactions:

$$F.4^H = F.4^H_{s_{(t)}} - F.4^H_{s_{(t-1)}} - F.4^H_{roc_{(t)}} \tag{516}$$

Equity (F.5) transactions:

$$F.5^H = F.5^H_{s_{(t)}} - F.5^H_{s_{(t-1)}} - F.5^H_{roc_{(t)}} \tag{517}$$

Insurance tech. reserves (F.6) transactions:

$$F.6^H = F.6^H_{s_{(t)}} - F.6^H_{s_{(t-1)}} - F.6^H_{roc_{(t)}} \tag{518}$$

Financial derivatives (F.7) transactions:

$$F.7^H = F.7^H_{s_{(t)}} - F.7^H_{s_{(t-1)}} - F.7^H_{roc_{(t)}} \tag{519}$$

Trade credits (F.8) transactions:

$$F.8^H = F.8^H_{s_{(t)}} - F.8^H_{s_{(t-1)}} - F.8^H_{roc_{(t)}} \tag{520}$$

Government

Government Revenue: Government revenue is determined by income taxes, gross operation surplus from different sectors, social contributions received, net interest, distributed income and other current transfers received.

$$R^G = T_s^G + GOS^G + SC_r^G + NI^G + DI_r^G + OC_r^G \tag{521}$$

Gross operating surplus and mixed income: Government is allocated a share of the total gross operation surplus.

$$GOS^G = GOS_E^G + GOS_T^G + GOS_A^G + GOS_{MC}^G + GOS_{OMS}^G + GOS_{FC}^G \tag{522}$$

Social contributions received:

$$SC_r^G = (SC_p^H + SC_p^{RoW}) \cdot \alpha_{SC_r}^G \tag{523}$$

Net interest: Given the way interest rates is defined, in section 6.1.5, both F.7 and F.8 is a exogenous.

$$NI^G = F.2_s^G \cdot \iota_{F.2}^G + F.3_s^G \cdot \iota_{F.3}^G + F.4_s^G \cdot \iota_{F.4}^G + F.6_s^G \cdot \iota_{F.6}^G + interest_{F.7}^G + interest_{F.8}^G \tag{524}$$

Distributed income of corporation, receivable (D.42):

$$DI_r^G = (DI_p^E + DI_p^T + DI_p^A + DI_p^{MC} + DI_p^{OMS} + DI_p^{FC} + DI_p^{RoW}) \cdot \alpha_{DI_r}^G + DI_p^{CB} \tag{525}$$

Distributed income of corporation (D.42):

$$DI^G = DI_r^G \tag{526}$$

Other current transfers paid:

$$OC_p^G = R^G \cdot \alpha_{OC_p}^G \tag{527}$$

Other current transfers received:

$$OC_r^G = (OC_p^E + OC_p^T + OC_p^A + OC_p^{MC} + OC_p^{OMS} + OC_p^H + OC_p^{FC} + OC_p^G + OC_p^{CB} + OC_p^{RoW}) \cdot \alpha_{OC_r}^G \tag{528}$$

Indirect taxes on production: All the indirect taxes paid by the producing sectors is received by the government

$$NIT^G = NIT^E + NIT^T + NIT^A + NIT^{MC} + NIT^{OMS} + NIT^{FC} \tag{529}$$

Net taxes on import. The industries pays a tax on all imported goods. This is a directly income to the government. In model, imports are payed to the RoW, and the tariff is already embedded in the value of imported goods. To maintain accounting consistency, it is assumed that RoW transfers the collected tariff revenue back to the domestic government. Even though this assumption does not reflect how trade and tariffs work in reality, it is used in the model to ensure that the government receives the correct tax revenue from imports.

$$NTI^G = IM \cdot \tau_{RoW} \tag{530}$$

Taxes from all sectors: All taxes on income and wealth are received by the government.

$$T^G = T^H + T^E + T^T + T^A + T^{MC} + T^{OMS} + T^{FC} + T^{RoW} \tag{531}$$

Sum of taxes to the government

$$T_s^G = T^G + NIT^G + NTI^G \tag{532}$$

Total government expenditures: Determined by government consumption, social benefits paid and other current transfers paid.

$$GE^G = G + SB_p^G + OC_p^G \tag{533}$$

Government consumption: Government consumption is determined by a fraction of aggregate demand. Total government consumption is distribute between the NFCs and FC. The alpha coefficient denotes the share of consumption respect to the sectors.

$$G^E = G \cdot \alpha_E^G \tag{534}$$

$$G^T = G \cdot \alpha_T^G \tag{535}$$

$$G^A = G \cdot \alpha_A^G \tag{536}$$

$$G^{MC} = G \cdot \alpha_{MC}^G \tag{537}$$

$$G^{OMS} = G \cdot \alpha_{OMS}^G \tag{538}$$

$$G^{FC} = G \cdot \alpha_{FC}^G \tag{539}$$

Real government consumption:

$$g^E = \frac{G^E}{p^E} \tag{540}$$

$$g^T = \frac{G^T}{p^T} \tag{541}$$

$$g^A = \frac{G^A}{p^A} \tag{542}$$

$$g^{MC} = \frac{G^{MC}}{p^{MC}} \tag{543}$$

$$g^{OMS} = \frac{G^{OMS}}{p^{OMS}} \tag{544}$$

$$g^{FC} = \frac{G^{FC}}{p^{FC}} \tag{545}$$

$$g = g^E + g^T + g^A + g^{MC} + g^{OMS} + g^{FC} \tag{546}$$

Real investments:

$$i^G = i \cdot \alpha^G_{i_a} \tag{547}$$

Nominal investments: Real government investments is given exogenous so it can be used as a policy variable.

$$I^G = i^G \cdot p^G_a \tag{548}$$

Nominal stock of capital:

$$K^G = K^G_{t-1} + I^G - D^G \tag{549}$$

Real stock of capital:

$$k^G = \frac{K^G}{p_I} \tag{550}$$

Depreciations:

$$D^G = \delta^G \cdot (K^G_{t-1}) \tag{551}$$

Social benefits paid: Determined by a share of the total recieved social benefits

$$SB^G_p = (SB^H_r + SB^{RoW}_r) \cdot \alpha^G_{SB_p} \tag{552}$$

Saving: Its assumed that a share of the profits from the CB (DI^{CB}_p) goes to the government.

$$S^G = R^G - GE^G \tag{553}$$

Net lending:

$$NL^G = S^G - I^G - NPL^G + Adj^G \tag{554}$$

Financial wealth:

$$V^G = NL^G + V^G_{t-1} + Rev^G \tag{555}$$

Flow of Funds for the government

The financial wealth for the government is allocated between six different funds. Loans (F.4) function as a buffer to ensure the fulfillment of governments budget constraint. This implies that if the government experiences a deficit in a given period, they will increase its borrowing from the financial sector, and vice versa. Revaluations and other changes used to compute transactions of funds are determined exogenously for each industry. The transactions for all financial instruments, except F.4, are given by the change in stocks minus revaluations and other changes. Transactions for F.4 are defined endogenously as net lending minus assets plus liabilities.

Currency and deposits (F.2) stock:

$$F.2^G_{s(t)} = V^G \cdot \beta^G_{F.2} \tag{556}$$

Securities (F.3) stock:

$$F.3^G_{s(t)} = V^G \cdot \beta^G_{F.3} \tag{557}$$

Loans (F.4) stock:

$$F.4^G_{s(t)} = F.4^G_{s(t-1)} + F.4^G + F.4^G_{roc_{(t)}} \tag{558}$$

Equity (F.5) stock:

$$F.5^G_{s(t)} = V^G \cdot \beta^G_{F.5} \tag{559}$$

Insurance tech. reserves (F.6) stock:

$$F.6^G_{s(t)} = V^G \cdot \beta^G_{F.6} \tag{560}$$

Financial derivatives (F.7) stock:

$$F.7^G_{s(t)} = V^G \cdot \beta^G_{F.7} \tag{561}$$

Trade credits (F.8) stock:

$$F.8^G_{s(t)} = V^G \cdot \beta^G_{F.8} \tag{562}$$

Currency and deposits (F.2) transactions:

$$F.2^G = F.2^G_{s_{(t)}} - F.2^G_{s_{(t-1)}} - F.2^G_{roc_{(t)}} \tag{563}$$

Securities (F.3) transactions:

$$F.3^G = NL^G - F.2^G - F.4^G - F.5^G - F.6^G - F.7^G - F.8^G \tag{564}$$

Loans (F.4) transactions:

$$F.4^G = F.4^G_{s_{(t)}} - F.4^G_{s_{(t-1)}} - F.4^G_{roc_{(t)}} \tag{565}$$

Equity (F.5) transactions:

$$F.5^G = F.5^G_{s_{(t)}} - F.5^G_{s_{(t-1)}} - F.5^G_{roc_{(t)}} \tag{566}$$

Insurance tech. reserves (F.6) transactions:

$$F.6^G = F.6^G_{s_{(t)}} - F.6^G_{s_{(t-1)}} - F.6^G_{roc_{(t)}} \tag{567}$$

Financial derivatives (F.7) transactions:

$$F.7^G = F.7^G_{s_{(t)}} - F.7^G_{s_{(t-1)}} - F.7^G_{roc_{(t)}} \tag{568}$$

Trade credits (F.8) transactions:

$$F.8^G = F.8^G_{s_{(t)}} - F.8^G_{s_{(t-1)}} - F.8^G_{roc_{(t)}} \tag{569}$$

Central bank

Revenue: Determined by net interest and distributed income to the Central Bank.

$$R^{CB} = NI^{CB} + DI_r^{CB} \tag{570}$$

Wage rate:

$$W^{CB} = \alpha_W^{CB} * W \tag{571}$$

Compensation of employees:

$$WB^{CB} = EMP^{CB} \cdot W^{CB} \tag{572}$$

Employment: Employment in CB is determined by an α_{EMP}^{CB} share, computed by the share of CB wage out of the total wage to CB and FC.

$$EMP^{CB} = \alpha_{EMP}^{CB} \cdot EMP^{FC} \tag{573}$$

Net interest:

$$NI^{CB} = F.2_s^{CB} \cdot \iota_{F.2}^{CB} + F.3_s^{CB} \cdot \iota_{F.3}^{CB} + F.4_s^{CB} \cdot \iota_{F.4}^{CB} + interest_{F.7}^{CB} + interest_{F.8}^{CB} \tag{574}$$

Distributed income of corporation, payments (D.42):

$$DI_p^{CB} = R^{CB} \cdot \alpha_p^{CB} \tag{575}$$

Distributed income of corporation, receivable (D.42):

$$DI_r^{CB} = (DI_p^E + DI_p^T + DI_p^A + DI_p^{MC} + DI_p^{OMS} + DI_p^{FC} + DI_p^{RoW}) \cdot \alpha_{DI_r}^{CB} \tag{576}$$

Distributed income of corporations:

$$DI^{CB} = DI_r^{CB} - DI_p^{CB} \tag{577}$$

Other current transfers paid:

$$OC_p^{CB} = R^{CB} \cdot \alpha_{GOS}^{CB} \tag{578}$$

Profits:

$$F^{CB} = R^{CB} - WB^{CB} - OC_p^{CB} - DI_p^{CB} \tag{579}$$

Savings:

$$S^{CB} = F^{CB} \tag{580}$$

Net lending:

$$NL^{CB} = S^{CB} + Adj^{CB} \tag{581}$$

Financial wealth:

$$V^{CB} = NL^{CB} + V_{t-1}^{CB} + Rev^{CB} \tag{582}$$

Flow of Funds for the Central Bank

The financial wealth for the Central Bank is allocated between six different funds. Loans (F.4) function as a buffer to ensure the fulfillment of Central Banks budget constraint. This implies that if the Central Bank experiences a deficit in a given period, they will increase its borrowing from the financial sector, and vice versa. Revaluations and other changes used to compute transactions of funds are determined exogenously for each industry. The transactions for all financial instruments, except F.4, are given by the change in stocks minus revaluations and other changes. Transactions for F.4 are defined endogenously as net lending minus assets plus liabilities.

Monetary gold and SDR (F.1) stock:

$$F.1^{CB}_{s(t)} = V^{CB} \cdot \beta^{CB}_{F.1} \tag{583}$$

Currency and deposits (F.2) stock:

$$F.2^{CB}_{s(t)} = V^{CB} \cdot \beta^{CB}_{F.2} \tag{584}$$

Securities (F.3) stock:

$$F.3^{CB}_{s(t)} = V^{CB} \cdot \beta^{CB}_{F.3} \tag{585}$$

Loans (F.4) stock:

$$F.4^{CB}_{s(t)} = F.4^{CB}_{s(t-1)} + F.4^{CB} + F.4^{CB}_{roc_{(t)}} \tag{586}$$

Equity (F.5) stock:

$$F.5^{CB}_{s(t)} = V^{CB} \cdot \beta^{CB}_{F.5} \tag{587}$$

Financial derivatives (F.7) stock:

$$F.7^{CB}_{s(t)} = V^{CB} \cdot \beta^{CB}_{F.7} \tag{588}$$

Trade credits (F.8) stock:

$$F.8^{CB}_{s(t)} = V^{CB} \cdot \beta^{CB}_{F.8} \tag{589}$$

Monetary gold and SDR (F.1) transactions:

$$F.1^{CB} = F.1^{CB}_{s_{(t)}} - F.1^{CB}_{s_{(t-1)}} - F.1^{CB}_{roc_{(t)}} \tag{590}$$

Currency and deposits (F.2) transactions:

$$F.2^{CB} = F.2^{CB}_{s_{(t)}} - F.2^{CB}_{s_{(t-1)}} - F.2^{CB}_{roc_{(t)}} \tag{591}$$

Securities (F.3) transactions:

$$F.3^{CB} = F.3^{CB}_{s_{(t)}} - F.3^{CB}_{s_{(t-1)}} - F.3^{CB}_{roc_{(t)}} \tag{592}$$

Loans (F.4) transactions:

$$F.4^{CB} = NL^{CB} - F.1^{CB} - F.2^{CB} - F.3^{CB} - F.5^{CB} - F.6^{CB} - F.7^{CB} - F.8^{CB} \tag{593}$$

Equity (F.5) transactions:

$$F.5^{CB} = F.5^{CB}_{s_{(t)}} - F.5^{CB}_{s_{(t-1)}} - F.5^{CB}_{roc_{(t)}} \tag{594}$$

Financial derivatives (F.7) transactions:

$$F.7^{CB} = F.7^{CB}_{s_{(t)}} - F.7^{CB}_{s_{(t-1)}} - F.7^{CB}_{roc_{(t)}} \tag{595}$$

Trade credits (F.8) transactions:

$$F.8^{CB} = F.8^{CB}_{s_{(t)}} - F.8^{CB}_{s_{(t-1)}} - F.8^{CB}_{roc_{(t)}} \tag{596}$$

To ensure horizontal consistency for transactions and to ensure that the demand for monetary gold and SDR transactions equals the supply, the following equation must be specified:

$$F.1^{CB} = F.1^{RoW} \tag{597}$$

Rest of the World

RoW balance: Determined by the income components, import, wage bill, other current transfers received, net interest and distributed income received of the RoW subtracted by the expenditures, net taxes on imports, income taxes, export, other current transfers paid, distributed income paid and reinvested earnings on FDI.

$$B^{RoW} = IM + WB^{RoW} + SB_r^{RoW} + OC_r^{RoW} + NI^{RoW} + DI_r^{RoW} - T^{RoW} - X - OC_p^{RoW} - DI_p^{RoW} - ReFI^{RoW} - NTI^G \tag{598}$$

Total compensation for RoW: Determined by the total wage bill multiplied with a coefficient that shows how large the share of wage bills are paid to the RoW.

$$WB^{RoW} = (WB^E + WB^T + WB^A + WB^{MC} + WB^{OMS} + WB^{FC} + WB^{CB}) \cdot \alpha_{WB}^{RoW} \tag{599}$$

Net taxes on imports: This accounts for the taxes RoW pays on their imported goods. This goes directly to the government. All the taxes from the RoW is directly distributed to the Government.

$$NTI^{RoW} = NTI^G \tag{600}$$

Net interest (D.41 + D.44 + D.45):

$$NI^{RoW} = F.2_s^{RoW} \cdot \iota_{F.2}^{RoW} + F.3_s^{RoW} \cdot \iota_{F.3}^{RoW} + F.4_s^{RoW} \cdot \iota_{F.4}^{RoW} + F.6_s^{RoW} \cdot \iota_{F.6}^{RoW} + interest_{F.7}^{RoW} + interest_{F.8}^{RoW} \tag{601}$$

Distributed income of corporation, payments (D.42p):

$$DI_p^{RoW} = (IM + WB^{RoW} + SB_r^{RoW} + OC_r^{RoW} + NI^{RoW}) \cdot \alpha_{DI_p}^{RoW} \tag{602}$$

Distributed income of corporation, receivable (D.42r):

$$DI_r^{RoW} = (DI_p^E + DI_p^T + DI_p^A + DI_p^{MC} + DI_p^{OMS} + DI_p^{FC} + DI_p^{RoW}) \cdot \alpha_{DI_r}^{RoW} \tag{603}$$

Distributed income of corporation (D.42):

$$DI^{RoW} = DI_r^{RoW} - DI_p^{RoW} \tag{604}$$

Reinvested earnings on FDI: The reinvested earnings on FDI from RoW are determined by the sum of reinvested earnings in the relevant sectors.

$$ReFI^{RoW} = F_{ReFI}^E + F_{ReFI}^T + F_{ReFI}^A + F_{ReFI}^{MC} + F_{ReFI}^{OMS} + F_{ReFI}^{FC} \tag{605}$$

Current taxes on income:

$$T^{RoW} = (WB^{RoW} + SB_r^{RoW} - SC_p^{RoW} + NI^{RoW}) \cdot \tau_{RoW}^{Total} \tag{606}$$

Social contributions received: Social contributions to the rest of the world, is made by the residual between the social contribution from the households, government and financial corporations.

$$SC_r^{RoW} = (SC_p^H + SC_p^{RoW}) \cdot \alpha_{SC_r}^{RoW} \tag{607}$$

Social contributions paid:

$$SC_p^{RoW} = WB^{RoW} \cdot \alpha_{SC}^{RoW} \tag{608}$$

Social benefits recieved:

$$SB_r^{RoW} = SB_p^G + SB_p^{FC} - SB_r^H \tag{609}$$

Other current transfers paid:

$$OC_p^{RoW} = (IM + WB^{RoW} + SB_r^{RoW} + OC_r^{RoW} + NI^{RoW}) \cdot \alpha_{OC_p}^{RoW} \tag{610}$$

Other current transfers received: Determined by a share of the sum of other current transfers paid by all sectors in the economy.

$$OC_r^{RoW} = (OC_p^E + OC_p^T + OC_p^A + OC_p^{MC} + OC_p^{OMS} + OC_p^H + OC_p^{FC} + OC_p^G + OC_p^{CB} + OC_p^{RoW}) \cdot \alpha_{OC_r}^{RoW} \tag{611}$$

Savings:

$$S^{RoW} = B^{RoW} \tag{612}$$

Net lending:

$$NL^{RoW} = S^{RoW} - NPL^{RoW} + Adj^{RoW} \tag{613}$$

Financial Wealth:

$$V^{RoW} = NL^{RoW} + V_{t-1}^{RoW} + Rev^{RoW} \tag{614}$$

Flow of Funds for the Rest of the World

The financial wealth for the RoW is allocated between seven different funds. Loans (F.4) function as a buffer to ensure the fulfillment of RoW's budget constraint. This implies that if the Central Bank experiences a deficit in a given period, they will increase its borrowing from the financial sector, and vice versa. Revaluations and other changes used to compute transactions of funds are determined exogenously for each industry. The transactions for all financial instruments, except F.4, are given by the change in stocks minus revaluations and other changes. Transactions for F.4 are defined endogenously as net lending minus assets plus liabilities. Furthermore, one of the parameters $\beta^{i}_{F.i}$ for one financial instrument will be estimated for RoW.

Monetary gold and SDR (F.1) stock:

$$F.1^{RoW}_{s(t)} = V^{RoW} \cdot \beta^{RoW}_{F.1} \tag{615}$$

Currency and deposits (F.2) stock:

$$F.2^{RoW}_{s(t)} = V^{RoW} \cdot \beta^{RoW}_{F.2} \tag{616}$$

Securities (F.3) stock:

$$F.3^{RoW}_{s(t)} = V^{RoW} \cdot \beta^{RoW}_{F.3} \tag{617}$$

Loans (F.4) stock:

$$F.4^{RoW}_{s(t)} = F.4^{RoW}_{s(t-1)} + F.4^{RoW} + F.4^{RoW}_{roc_{(t)}} \tag{618}$$

Equity (F.5) stock:

$$F.5^{RoW}_{s(t)} = V^{RoW} \cdot \beta^{RoW}_{F.5} \tag{619}$$

Insurance tech. reserves (F.6) stock:

$$F.6^{RoW}_{s(t)} = V^{RoW} \cdot \beta^{RoW}_{F.6} \tag{620}$$

Financial derivatives (F.7) stock:

$$F.7^{RoW}_{s(t)} = V^{RoW} \cdot \beta^{RoW}_{F.7} \tag{621}$$

Trade credits (F.8) stock:

$$F.8^{RoW}_{s(t)} = V^{RoW} \cdot \beta^{RoW}_{F.8} \tag{622}$$

Monetary gold and SDR (F.1) transactions:

$$F.1^{RoW} = F.1^{RoW}_{s_{(t)}} - F.1^{RoW}_{s_{(t-1)}} - F.1^{RoW}_{roc_{(t)}} \tag{623}$$

Currency and deposits (F.2) transactions:

$$F.2^{RoW} = F.2^{RoW}_{s_{(t)}} - F.2^{RoW}_{s_{(t-1)}} - F.2^{RoW}_{roc_{(t)}} \tag{624}$$

Securities (F.3) transactions:

$$F.3^{RoW} = F.3^{RoW}_{s_{(t)}} - F.3^{RoW}_{s_{(t-1)}} - F.3^{RoW}_{roc_{(t)}} \tag{625}$$

Loans (F.4) transactions:

$$F.4^{RoW} = NL^{RoW} - F.1^{RoW} - F.2^{RoW} - F.3^{RoW} - F.5^{RoW} - F.6^{RoW} - F.7^{RoW} - F.8^{RoW} \tag{626}$$

Equity (F.5) transactions:

$$F.5^{RoW} = F.5^{RoW}_{s_{(t)}} - F.5^{RoW}_{s_{(t-1)}} - F.5^{RoW}_{roc_{(t)}} \tag{627}$$

Insurance tech. reserves (F.6) transactions:

$$F.6^{RoW} = F.6^{RoW}_{s_{(t)}} - F.6^{RoW}_{s_{(t-1)}} - F.6^{RoW}_{3} 8oc_{(t)} \tag{628}$$

Financial derivatives (F.7) transactions:

$$F.7^{RoW} = F.7^{RoW}_{s_{(t)}} - F.7^{RoW}_{s_{(t-1)}} - F.7^{RoW}_{roc_{(t)}} \tag{629}$$

Trade credits (F.8) transactions:

$$F.8^{RoW} = F.8^{RoW}_{s_{(t)}} - F.8^{RoW}_{s_{(t-1)}} - F.8^{RoW}_{roc_{(t)}} \tag{630}$$

To ensure horizontal consistency for transactions and to ensure that the demand for equities transactions equals the supply, the following equation must be specified:

$$F.5^{RoW} = F.5^{H} + F.5^{E} + F.5^{T} + F.5^{A} + F.5^{MC} + F.5^{OMS} + F.5^{FC} + F.5^{G} + F.5^{CB} \tag{631}$$

Prices

Producer prices:

$$\begin{aligned}\Delta\ln(pp^E_t) = \beta_{0,pp^E} + \beta_{1,pp^E}\cdot\Delta\ln(pp^E_{t-1}) + \beta_{2,pp^E}\cdot\Delta\ln(pp^E_{t-4}) + \beta_{3,pp^E}\cdot\Delta\ln(WB^E_t) + \beta_{4,pp^E}\cdot\Delta\ln(pp^{MC}_t)\\ + \beta_{5,pp^E}\cdot\Delta\ln(pp^{OMS}_t) + \beta_{6,pp^E}\cdot\ln(\mathbf{pp^E_{t-1}}) + \beta_{7,pp^E}\cdot\ln(\mathbf{pp^{MC}_{t-1}})\end{aligned} \tag{632}$$

$$\begin{aligned}\Delta\ln(pp^T_t) = \beta_{0,pp^T} + \beta_{1,pp^T}\cdot\Delta\ln(p_{IM_t}) + \beta_{2,pp^T}\cdot\Delta\ln(p_{IM_{t-1}}) + \beta_{3,pp^T}\cdot\Delta\ln(UC^T_{t-3}) + \beta_{4,pp^T}\cdot\Delta\ln(p_{I_{t-1}})\\ + \beta_{5,pp^T}\cdot\Delta\ln(pp^{OMS}_t) + \beta_{6,pp^T}\cdot\ln(\mathbf{pp^T_{t-1}}) + \beta_{7,pp^T}\cdot\ln(\mathbf{pp^E_{t-1}}) + \beta_{8,pp^T}\cdot\ln(\mathbf{pp^A_{t-1}})\end{aligned} \tag{633}$$

$$\Delta\ln(pp^A_t) = \beta_{0,pp^A} + \beta_{1,pp^A}\cdot\Delta\ln(P_{rape_t}) + \beta_{2,pp^A}\cdot\ln(\mathbf{pp^A_{t-1}}) + \beta_{3,pp^A}\cdot\ln(\mathbf{pp^{MC}_{t-1}}) + \beta_{4,pp^A}\cdot\ln(\mathbf{P_{rape_{t-1}}}) \tag{634}$$

$$\begin{aligned}\Delta\ln(pp^{MC}_t) = \beta_{0,pp^{MC}} + \beta_{1,pp^{MC}}\cdot\Delta\ln(p_{IM_t}) + \beta_{2,pp^{MC}}\cdot\Delta\ln(p_{I_{t-4}}) + \beta_{3,pp^{MC}}\cdot\Delta\ln(UC^{MC}_{t-2})\\ + \beta_{4,pp^{MC}}\cdot\ln(\mathbf{pp^{MC}_{t-1}}) + \beta_{5,pp^{MC}}\cdot\ln(\mathbf{p_{IM_{t-1}}})\end{aligned} \tag{635}$$

$$\begin{aligned}\Delta\ln(pp^{OMS}_t) = \beta_{0,pp^{OMS}} + \beta_{1,pp^{OMS}}\cdot\Delta\ln(WB^{OMS}_t) + \beta_{2,pp^{OMS}}\cdot\Delta\ln(pp^E_t) + \beta_{3,pp^{OMS}}\cdot\Delta\ln(pp^T_t)\\ + \beta_{4,pp^{OMS}}\cdot\Delta\ln(pp^{MC}_t) + \beta_{5,pp^{OMS}}\cdot\Delta\ln(P_{I_{t-1}}) + \beta_{6,pp^{OMS}}\cdot\Delta\ln(P_{G_t})\\ + \beta_{7,pp^{OMS}}\cdot\Delta\ln(P_{G_{t-2}}) + \beta_{8,pp^{OMS}}\cdot\ln(\mathbf{pp^{OMS}_{t-1}}) + \beta_{9,pp^{OMS}}\cdot\ln(\mathbf{WB^{OMS}_{t-1}})\\ + \beta_{10,pp^{OMS}}\cdot\ln(\mathbf{pp^T_{t-1}}) + \beta_{11,pp^{OMS}}\cdot\ln(\mathbf{pp^{MC}_{t-1}})\end{aligned} \tag{636}$$

$$\begin{aligned}\Delta\ln(pp^{FC}_t) = \beta_{0,pp^{FC}} + \beta_{1,pp^{FC}}\cdot\Delta\ln(WB^{FC}_{t-4}) + \beta_{2,pp^{FC}}\cdot\Delta\ln(p_{G_{t-4}}) + \beta_{3,pp^{FC}}\cdot\Delta\ln(p_{C_t})\\ + \beta_{4,pp^{FC}}\cdot\ln(\mathbf{pp^{FC}_{t-1}}) + \beta_{5,pp^{FC}}\cdot\ln(\mathbf{pp^E_{t-1}})\end{aligned} \tag{637}$$

Consumer prices: Consumer prices at industry level: Consumer prices are obtained by adding one plus the percentage of net indirect taxes from consumer prices.

$$p^E = pp^E\cdot(1+\tau^E) \tag{638}$$

$$p^T = pp^T\cdot(1+\tau^T) \tag{639}$$

$$p^{A} = pp^{A} \cdot (1 + \tau^{A}) \tag{640}$$

$$p^{MC} = pp^{MC} \cdot (1 + \tau^{MC}) \tag{641}$$

$$p^{OMS} = pp^{OMS} \cdot (1 + \tau^{OMS}) \tag{642}$$

$$p^{FC} = pp^{FC} \cdot (1 + \tau^{FC}) \tag{643}$$

Real exchange rate:

$$e = \frac{p_{Y} \cdot (1 + \tau^{RoW})}{p^{\mathrm{RoW}}} \tag{644}$$

C Econometric estimations

In this section, the data used in the regressions will be presented in detail to ensure transparency regarding the econometric estimations for the reader. The tests and their corresponding plots will be presented in either tables or figures.

C.1 Real consumption

Real Consumption

The real consumption function is assumed to be a function of real disposable income for households, its own lags, the real interest rate denoted as r[86], and the financial wealth of the households denoted as, fw. Real disposable income is constructed as in equation 485, incorporating wages, gross operating surplus, distributed income, net interest, social benefits, other current transfers, social contributions and taxes. This construction ensures that disposable income is theoretically consistent with the definition provided by Godley and Lavoie (2012), since it reflects household income after tax payments. Financial wealth is defined as the difference between financial assets and liabilities of households. Disposable income is expected to exert a positive effect on real consumption and so is financial wealth and the real interest rate.

According to Godley and Lavoie (2012, p. 113), in their portfolio choice model (model PC), where no saving occurs in the long run, consumption in the steady-state is equal to disposable income. This suggests that an increase in disposable income leads to a corresponding increase in consumption. However, causality might also work in another direction. The consumption function, presented below, also incorporates the real interest rate and financial assets as explanatory variables. According to Godley and Lavoie (2012, p. 114), higher interest rates can lead to an increase in disposable income and thereby consumption presenting some kind of a wealth effect. This result may initially seem counterintuitive from a mainstream perspective. But, higher interest payments on government debt may increase disposable income for households holding government securities, thereby stimulating consumption. However, this effect may be offset if a significant share of households carry liabilities with flexible interest rates, as their debt servicing costs would also rise and lower disposable income. This asymmetry is not captured in the PC model from Godley and Lavoie (2012), which does not include household liabilities. As disposable income grows, households are induced to accumulate greater financial wealth, particularly in the form of government bills or other financial assets. Furthermore, higher interest rates can incentivize households to allocate a larger share of their wealth to interest-bearing assets. Contrary to standard mainstream economics, this may not always stimulate consumption. As households divert a greater portion of their income toward saving motivated by higher returns on financial assets consumption and overall economic activity may decline.

Since cointegration is identified according to the bounds and Johansen test in section C.1 of the appendix, the real consumption equation is estimated as an ECM model with both short-run and long-run adjustments. The estimated relationship is presented below, with the sample spanning from 1999Q3 to 2024Q3. The long-run variables are presented in bold.

$$
\begin{aligned}
c^H &= 1.60 + 0.40 \cdot \Delta \ln(c_{t-4}) + 0.08 \cdot \Delta \ln(yd_t^H) - 0.068 \cdot \Delta \ln(yd_{t-2}^H) \\
&\quad + 0.13 \cdot \Delta \ln(fw_t^H) + 0.34 \cdot \Delta r_t \\
&\quad - 0.31 \cdot \ln(\mathbf{c_{t-1}}) + 0.13 \cdot \ln(\mathbf{yd_{t-1}^H}) + 0.05 \cdot \ln(\mathbf{fw_{t-1}^H}).
\end{aligned}
$$

The estimated behavioral equation for real consumption aligns with economic theory presented above, indicating that consumption responds positively to changes in both real disposable income and financial wealth, in the

[86]The computation of the real interest rate can be seen in section 6.2.

short run as well as in the long run. The long-run income elasticity (0.41)[87], suggesting that consumption adjusts moderately in response to income increases. In other words, a 1% increase in disposable income leads to a 0.41% increase in real consumption in the long run. The elasticity to consume out of financial wealth is lower but positive. Specifically, a 1% increase in financial wealth leads to a 0.15% increase in real consumption. Furthermore, approximately 31% of any disequilibrium generated by a shock is error corrected within one quarter in this model, indicating a relatively moderate speed of adjustment.

The short-run effects in the estimated consumption equation are captured by the coefficients on the differenced variables, that is, the terms involving a Δ. These coefficients measure the immediate impact of changes in the explanatory variables on changes in real consumption within the same quarter, and do not entail a long-lasting effect. For example past changes in consumption ($\Delta \ln(c_{t-4})$), specifically four quarters in the past, have a positive and significant effect, suggesting lagged effects in consumption behavior over time. An immediate and lagged effect is observed for disposable income, but the overall effect on real consumption in period, t is positive and is in line with the expectations. As pointed out by Godley and Lavoie (2012) in the short run it is empirically justified by this model, that higher interest rates lead to increases in consumption. Using a linear downscaling method, a one percentage point increase in the real interest rate increases consumption by 0.0034%.The similar effect is observed in the case of financial wealth. An increase in financial wealth of 1% increases consumption by 0.13% in the short run. In general, the short-run dynamics indicate that real consumption is moderately responsive to contemporaneous changes in real interest rates and wealth.

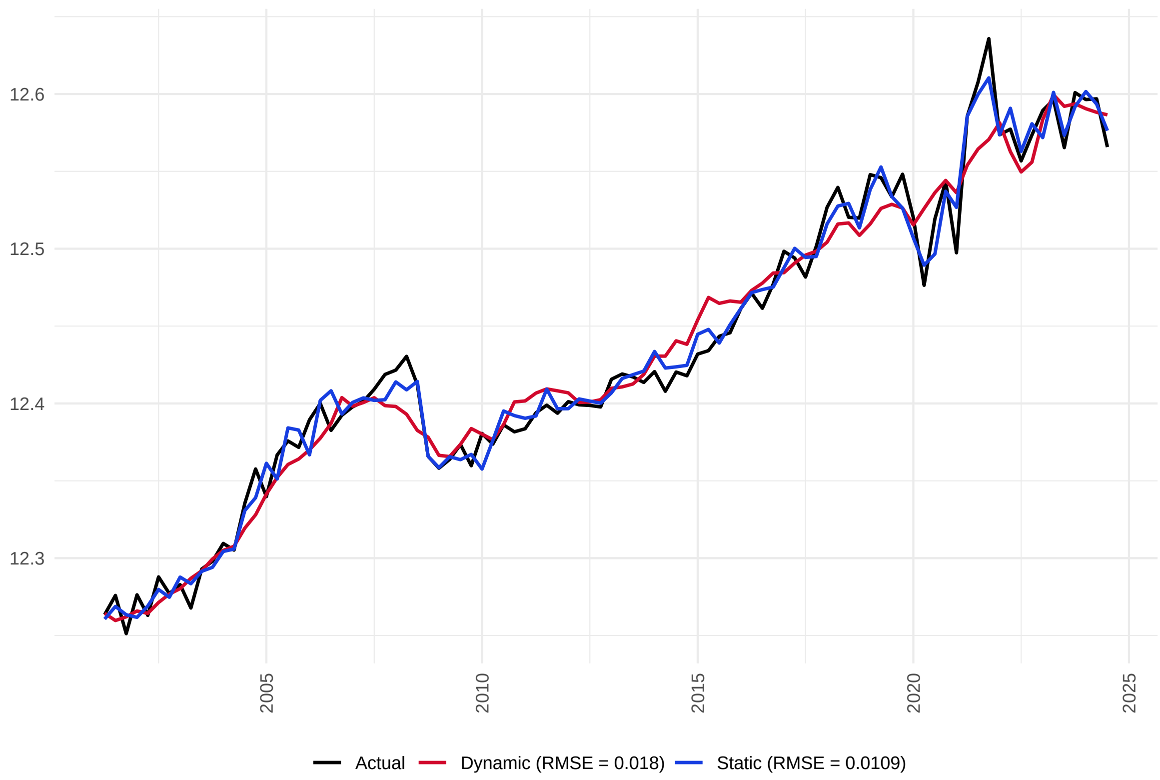


Figure 29: Overall fit of real consumption in E-IO-SFC.

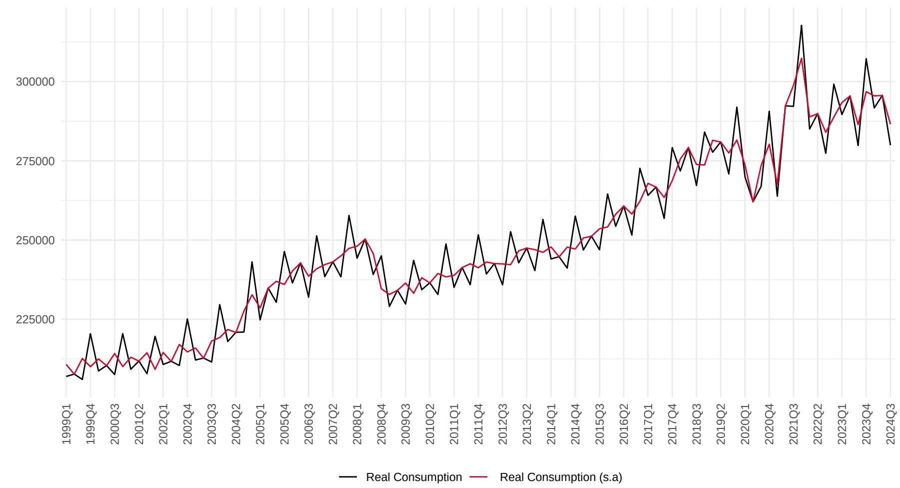
The estimated behavioral equation satisfies all the diagnostic tests presented in section 7.2.1, and the results of these tests are reported in section C.1. The overall fit of the ECM model for real consumption is presented in figure 29. The model yields an RMSE of 0.0109 for the static forecast and 0.018 for the dynamic forecast. Based on the transformed RMSE value, the static forecast has an average absolute error of approximately 2.9%,

[87] $= \frac{0.13}{0.31}$.

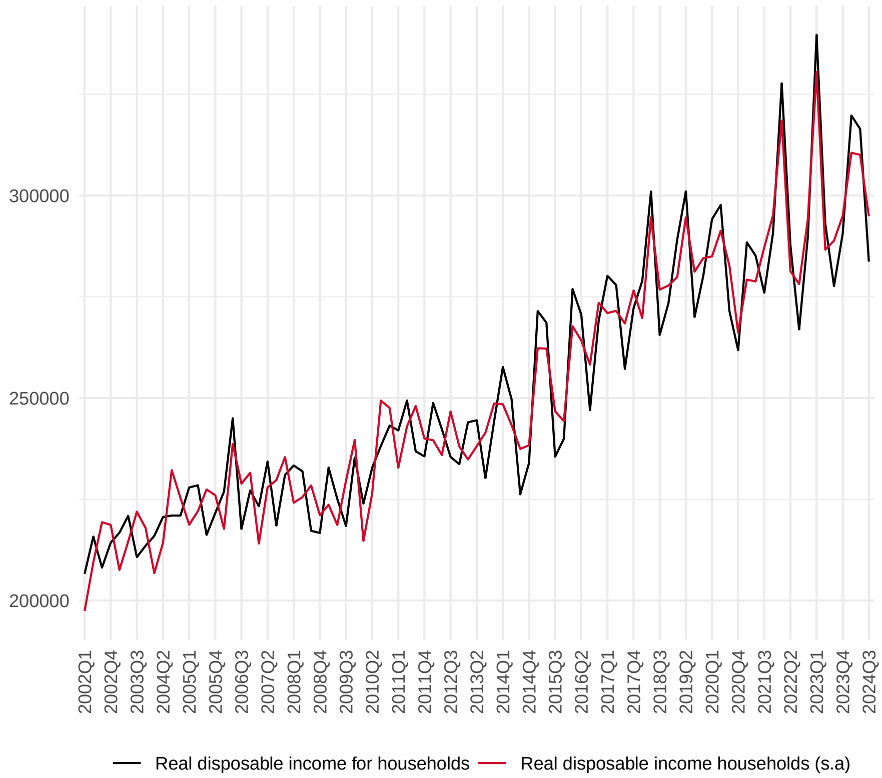
meaning that the model's predictions deviate, on average, by 2.9% from the actual observed values of real consumption during the forecast period. The dynamic forecast exhibits a slightly higher average absolute error of approximately 4.6%, reflecting reduced accuracy due to the accumulation of forecast errors over time. This model is therefore used as the behavioral equation in the E-IO-SFC model and illustrates how households make their consumption decisions within the model[88].

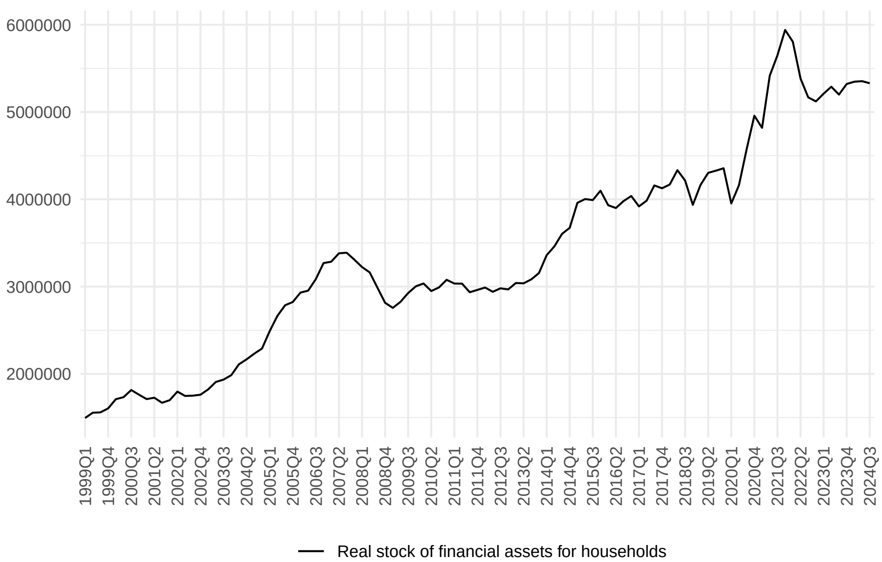
The model exhibits a satisfactory overall fit to the data, as it successfully captures the general business cycle fluctuations in real consumption. Both the static and dynamic forecasts broadly follow the observed time series, indicating that the model has captured the underlying economic mechanisms during stable or "normal" periods. Noticeable deviations occur in the dynamic forecasts during turbulent periods, such as the financial crisis in 2008 and the energy crisis in 2021. These episodes are marked by strong exogenous shocks, which the model is not designed to capture. Rather than forecasting rare or extreme events, the model is constructed to reflect the expected trajectory of real consumption based on standard macroeconomic behavior and historically observed patterns. Consequently, the model is considered to provide a reliable representation of real consumption dynamics under typical economic conditions and can therefore be used as a behavioral equation for real consumption in the E-IO-SFC model.

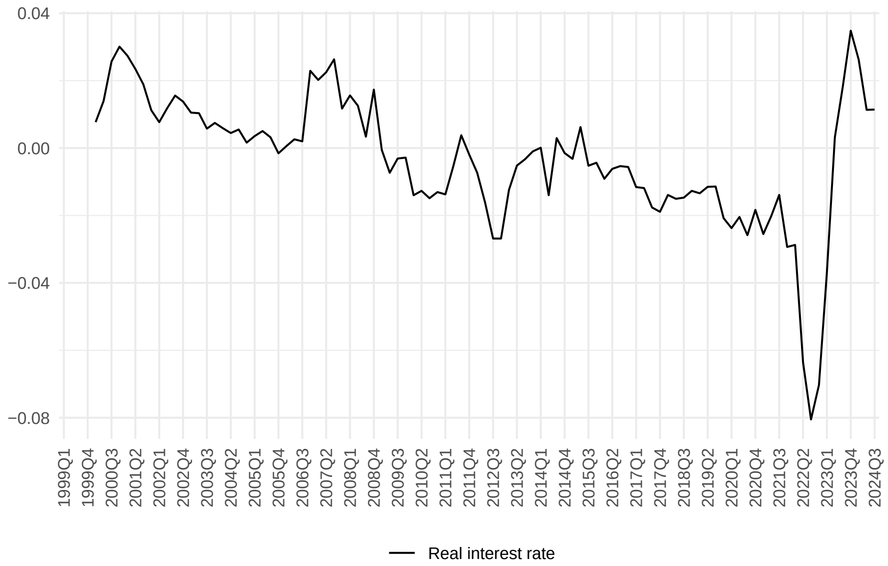
Step 1: Data and unit root testing



[88]In section C.1 of the appendix, the detailed regression output can be found under step 4.


300000
250000
200000
2002Q1
2002Q4
2003Q3
2004Q2
2005Q1
2005Q4
2006Q3
2007Q2
2008Q1
2008Q4
2009Q3
2010Q2
2011Q1
2011Q4
2012Q3
2013Q2
2014Q1
2014Q4
2015Q3
2016Q2
2017Q1
2017Q4
2018Q3
2019Q2
2020Q1
2020Q4
2021Q3
2022Q2
2023Q1
2023Q4
2024Q3
Real disposable income for households
Real disposable income households (s.a)


6000000
5000000
4000000
3000000
2000000
1999Q1
1999Q4
2000Q3
2001Q2
2002Q1
2002Q4
2003Q3
2004Q2
2005Q1
2005Q4
2006Q3
2007Q2
2008Q1
2008Q4
2009Q3
2010Q2
2011Q1
2011Q4
2012Q3
2013Q2
2014Q1
2014Q4
2015Q3
2016Q2
2017Q1
2017Q4
2018Q3
2019Q2
2020Q1
2020Q4
2021Q3
2022Q2
2023Q1
2023Q4
2024Q3
Real stock of financial assets for households



Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
$assets_{sa}^{H}$	-2.7681	-3.43	trend	I(1)
c_{sa}	-0.8461	-2.88	drift	I(1)
yd_{sa}^{H}	-1.6048	-2.88	drift	I(1)
r	-3.3927	-1.95	none	I(0)

Table 25: Results from ADF Tests for variables entering Real Consumption estimation.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
3.4746	4	Case II (Restricted intercept and no trend)	[2.56 ; 3.49]	Inconclusive
4.0645	4	Case III (Unrestricted intercept and no trend)	[2.86 ; 4.01]	Cointegration

Table 26: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 3$	5.80	12.25
$r \leq 2$	20.08	25.32
$r \leq 1$	34.97	42.44
$r = 0$	69.07	62.99

Table 27: Johansen Trace Test Results with trend.

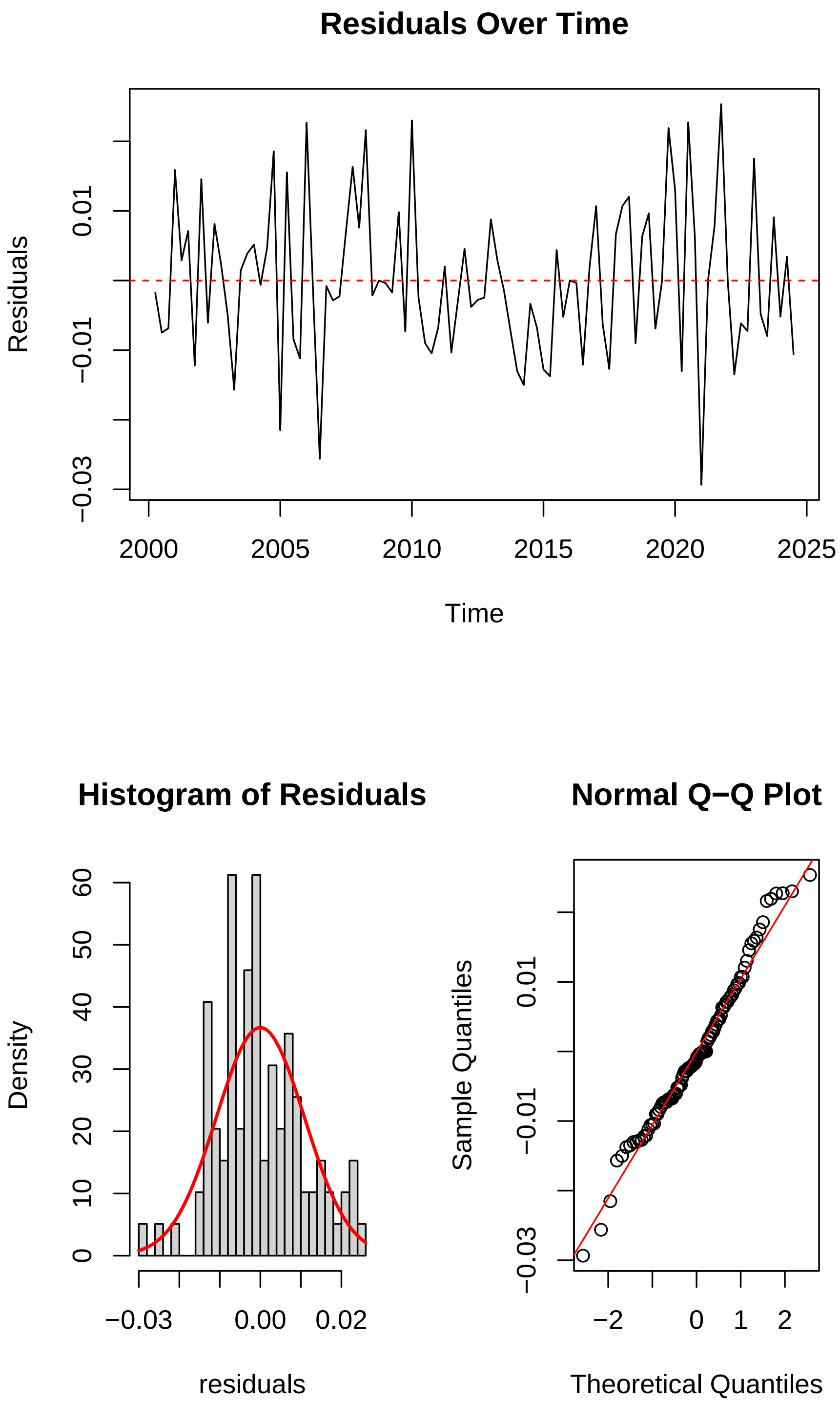
Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 3$	3.00	9.24
$r \leq 2$	11.85	19.96
$r \leq 1$	28.65	34.91
$r = 0$	58.31	53.12

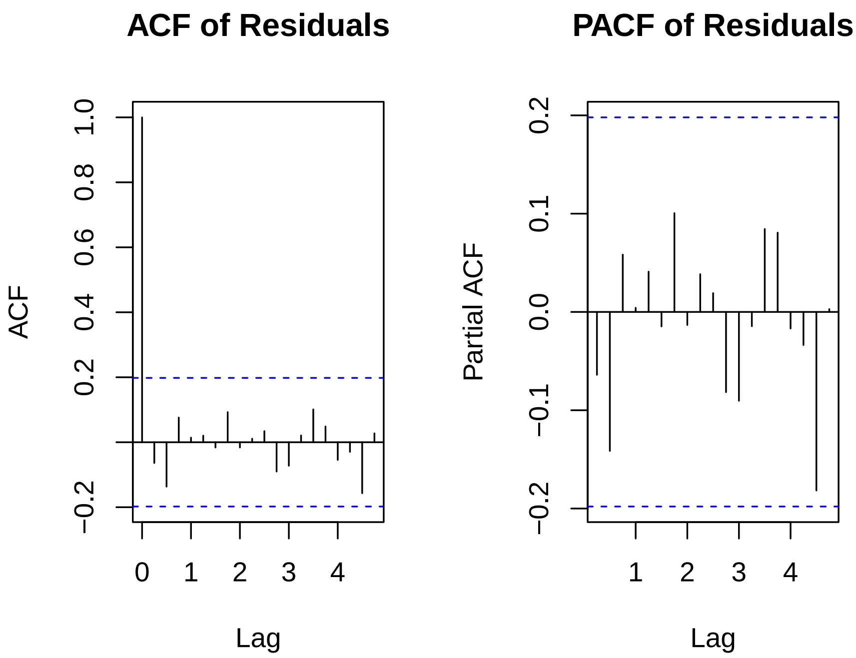
Table 28: Johansen Trace Test Results without trend and constant.

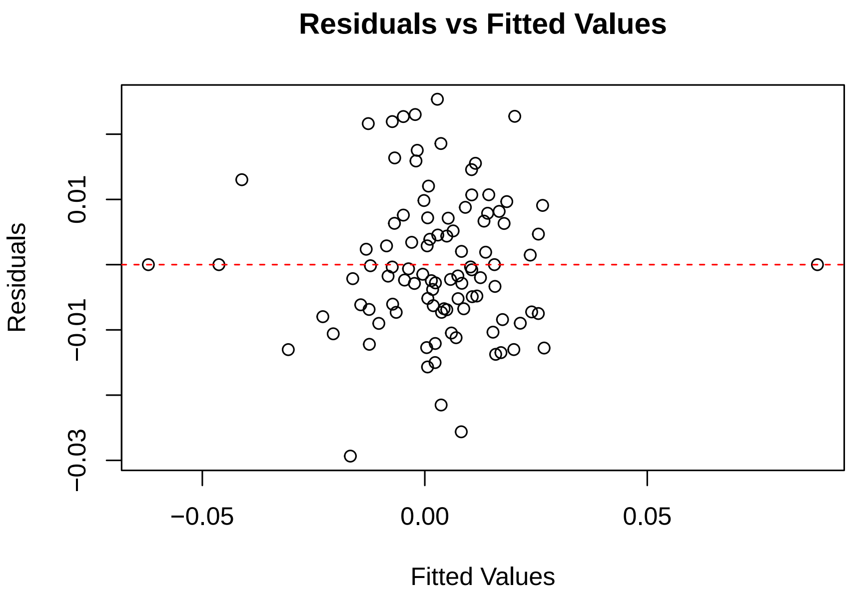
Step 4: Run the specific model

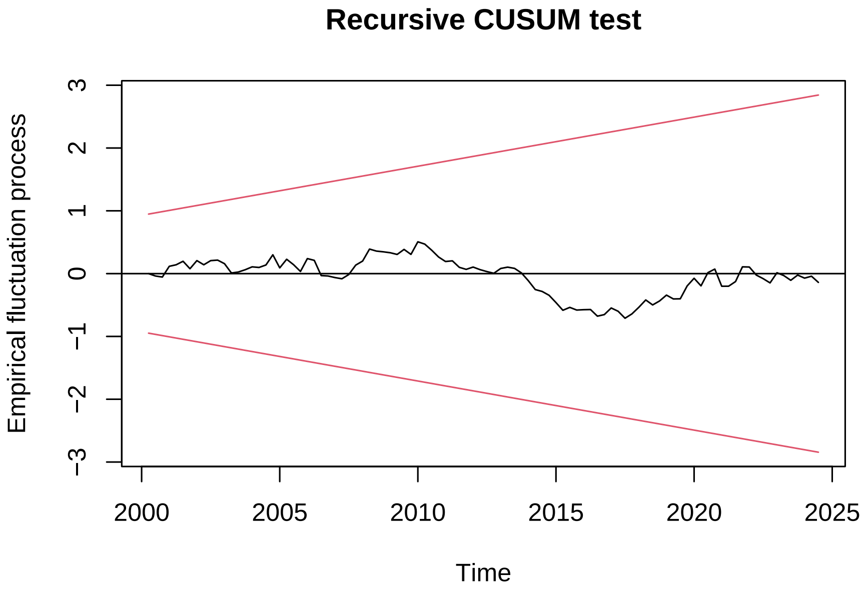
```
Call:
dynlm(formula = d(lc_sa) ~ L(d(lc_sa), 4) + d(lyd_H_sa) + L(d(lyd_H_sa), 2) +
d(lassets_H) + d(r) + L(lc_sa, 1) + L(lyd_H_sa, 1) + L(lassets_H, 1) +
dummy08 + dummy20 + dummy21Q2 + dummy22, data = data)

Residuals:
    Min       1Q   Median       3Q      Max
-0.02935 -0.00716 -0.00113 0.00701 0.02536

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      1.59880    0.44445   3.597 0.000539 ***
L(d(lc_sa), 4)   0.40038    0.06953   5.758 0.000000132 ***
d(lyd_H_sa)      0.07939    0.03495   2.272 0.025633 *
L(d(lyd_H_sa),2) -0.06763    0.02821  -2.397 0.018705 *
d(lassets_H)     0.12559    0.03697   3.397 0.001039 **
d(r)             0.34440    0.13590   2.533 0.013156 *
L(lc_sa,1)      -0.31094    0.06939  -4.481 0.000023054 ***
L(lyd_H_sa,1)    0.12638    0.03615   3.496 0.000753 ***
L(lassets_H,1)   0.04650    0.01487   3.127 0.002419 **
dummy08         -0.03653    0.01257  -2.907 0.004655 **
dummy20         -0.03810    0.00880  -4.328 0.000004082 ***
dummy21Q2        0.07191    0.01316   5.460 0.000000468 ***
dummy22         -0.03247    0.01280  -2.536 0.013033 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.01162 on 85 degrees of freedom
Multiple R-squared: 0.7156,   Adjusted R-squared: 0.6755
F-statistic: 17.82 on 12 and 85 DF,  p-value: < 0.0000000000000000022
```

Step 6: Run diagnostic tests




ACF of Residuals
PACF of Residuals
ACF
Partial ACF
Lag
Lag
1.0
0.8
0.6
0.4
0.2
−0.2
0.2
0.1
0.0
−0.1
−0.2
0
1
2
3
4


Residuals vs Fitted Values
Residuals
Fitted Values
0.01
−0.01
−0.03
−0.05
0.00
0.05



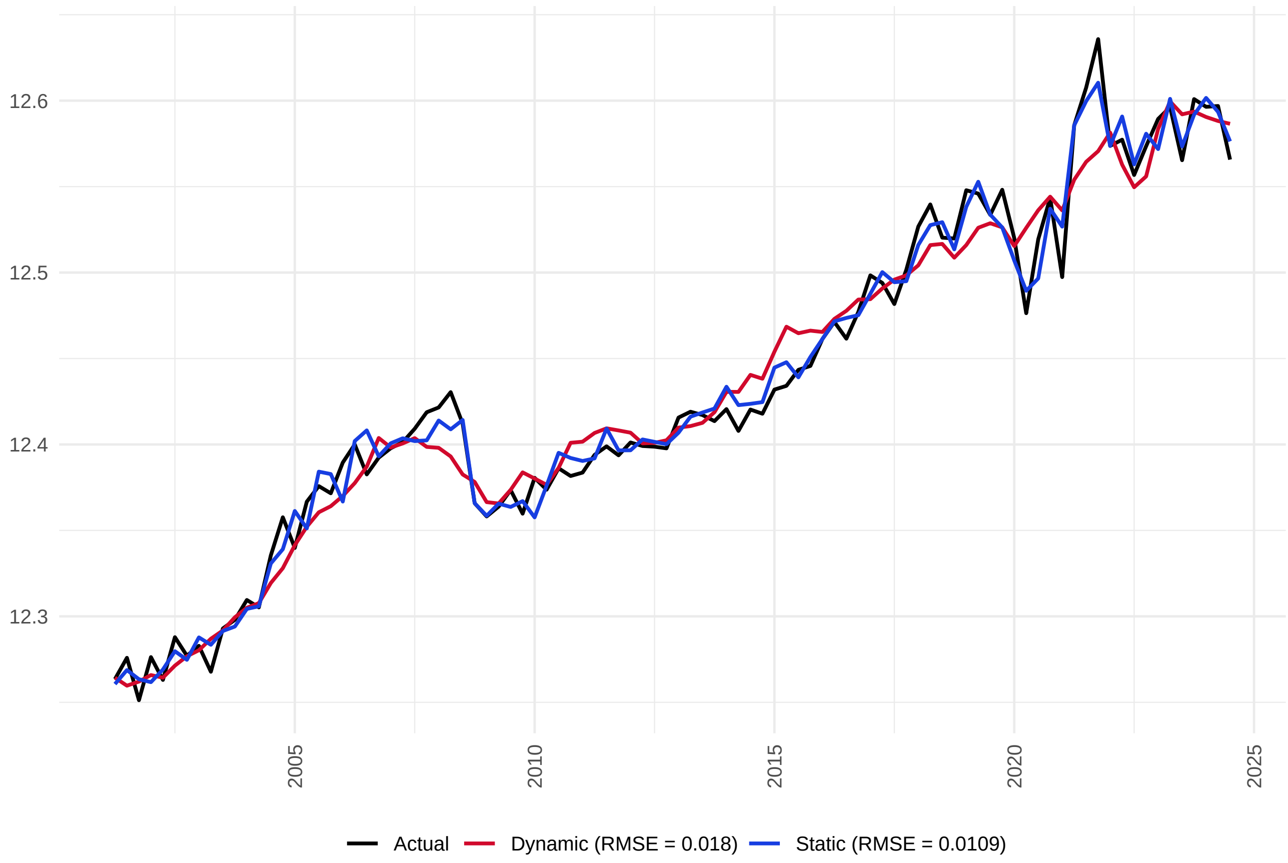
Test	**p-value**
Shapiro-Wilk	0.2139
Jarque Bera	0.72
Durbin-Watson	0.6246
Breusch-Godfrey, order 4	0.4302

Table 29: Diagnostics statistics.

Variable	**VIF Value**
L(d(lc_sa), 4)	1.400377
d(lyd_H_sa)	1.947284
L(d(lyd_H_sa), 2)	1.295293
d(lassets_H)	1.320169
d(r)	1.255488
L(lc_sa, 1)	35.399610
L(lyd_H_sa, 1)	14.848553
L(lassets_H, 1)	19.397240
dummy08	1.158212
dummy20	1.125212
dummy21Q2	1.272097
dummy22	1.202471

Table 30: Variance Inflation Factor (VIF) Results.

Step 7: Evaluate static and dynamic forecasts



C.2 Wage rate

Wage rate

The aggregate wage rate is modeled as a function of labor productivity, pr, its own lags, the targeted wage rate, WT, and the cyclical deviation in the unemployment rate, UR_rUR_s. Labor productivity is defined as described in section 6.2.2, in accordance with the theoretical framework outlined by Canelli et al. (2021) and Godley and Lavoie (2012, p. 287). The targeted wage rate is constructed following the methodology in section 6.2.2, drawing on the approach of Thomsen et al. (2024) and the theoretical background presented in Gottfries (2013). The cyclical component of unemployment, defined as the gap between the actual unemployment rate and the structural unemployment rate, is expected to exhibit a negative relationship with wage growth. The construction of the cyclical unemployment is further detailed in section 6.2.1.

Godley and Lavoie (2012, p. 302) present a simplified theory of wage inflation within their model, which incorporates private bank money, inventories, and inflation. Also known as model DIS. This theory is used in this paper to theoretically determine how wages are determined in the E-IO-SFC model. Godley and Lavoie argue that workers target a wage rate that depends on productivity and effective demand, the latter being approximated in this model by the cyclical deviation of unemployment. In model DIS, Godley and Lavoie (2012) use the ratio $\frac{N}{N_{fe}}$ as a proxy of effective demand, where N represents employment and N_{fe} denotes the full-employment level of employment. Although $\frac{N}{N_{fe}}$ and UR_rUR_s are defined differently, both reflect the cyclical position of the labor market. When employment is below its full-employment level ($N < N_{fe}$), the actual unemployment rate exceeds the structural rate, resulting in a positive $UR_r - UR_s$, knowing that structural unemployment does not imply full employment. This indicates that the supply of labor exceeds demand, meaning there are more people willing to work than there are available jobs. In such a situation, workers have less bargaining power, which puts downward pressure on wage growth. Conversely, when employment is close to or above the full-employment level, UR_rUR_s becomes negative, signalling stronger labor demand relative to supply, which tends to increase wage growth. Therefore, UR_rUR_s can serve as a proxy for effective demand, and its expected sign in the behavioral equation for the wage rate is negative. Moreover, it is stated that the wage rate is influenced by the a targeted wage rate which incorporates inflation expectations and productivity. This statement can be reflected as a simplified version of the endogenous wage-price spiral, where the wage rate is connected to inflation expectations and productivity. This dynamic is also known as the Philips-curve related to unemployment and wage inflation. The sign is expected to be positive. The underlying theory, which posits that workers seek fair pay, is based on the work of (Wood, 1978), to which Godley and Lavoie (2012) refer. According to this theory, as workers become more effective and productivity increases, they are likely to target a higher wage rate, as they may perceive these productivity gains as a result of their increased effort. Therefore, productivity is expected to have a positive impact on the wage rate. The estimated behavioral equation below illustrates the presented theoretical relationship within the context of the model.

Since cointegration is identified according to the bounds and Johansen test in section C.2 of the appendix, the wage rate equation is estimated as an ECM model with both short-run and long-run adjustments. The input data used is spanning from 2002Q1 to 2022Q1.

$$\Delta \ln(W_t) = 0.22 - 0.53 \cdot \Delta \ln(W_{t-2}) - 0.74 \cdot \Delta \ln(W_{t-3}) + 0.35 \cdot \Delta \ln(PR_t)$$
$$- 0.46 \cdot \ln(\mathbf{W}_{t-1}) - 0.58 \cdot \ln(\mathbf{UR_r UR_s}_{t-1}) + 0.44 \cdot \ln(\mathbf{WT}_{t-1})$$

The estimated behavioral equation for the overall wage rate aligns with economic theory presented above, indicating that the wage rate responds positively to changes in production in the short run and its targeted level in the long run. Furthermore, the wage rate exhibits a negative long-run relationship with the cyclical component of unemployment, reflecting the downward pressure on wages when unemployment exceeds its structural level.

Additionally, short-run dynamics are captured through negative coefficients on lagged wage growth, indicating inertia in wage adjustments, reflecting that past wage changes dampen current wage adjustments. The long-run unemployment elasticity (-1.26), suggests that the wage rate is highly sensitive to changes in the cyclical component of unemployment and reacts more than proportionally. In other words a 1% increase in cyclical unemployment (relative to structural) leads to a 1.26% decrease in the wage level in the long run, holding other factors constant. The long-run targeted wage elasticity (0.96) indicates that a 1% increase in the targeted wage rate is associated with a 0.96% increase in the actual wage rate in the long run, suggesting a strong alignment between actual wages and their targeted level over time. Furthermore, approximately 46% of any disequilibrium generated by a shock is error corrected within one quarter in this model, indicating a relatively moderate speed of adjustment.

The estimated behavioral equation satisfies all the diagnostic tests presented in section 7.2.1, and the results of these tests are reported in section C.2. Below, the overall fit of the ECM model for the wage rate is presented in figure 30. The model yields a RMSE of 0.007 for the static forecast and 0.022 for the dynamic forecast. This implies that the static forecast has an average absolute error of approximately 1.3%, meaning that the model's predictions deviate, on average, by 1.3% from the actual observed values of the aggregated wage rate during the forecast period. The dynamic forecast shows a slightly higher average absolute error of approximately 4.1%, reflecting reduced accuracy due to the accumulation of forecast errors over time. This model is therefore used as the behavioral equation in the E-IO-SFC model and illustrates how firms set their wages within the model[89].

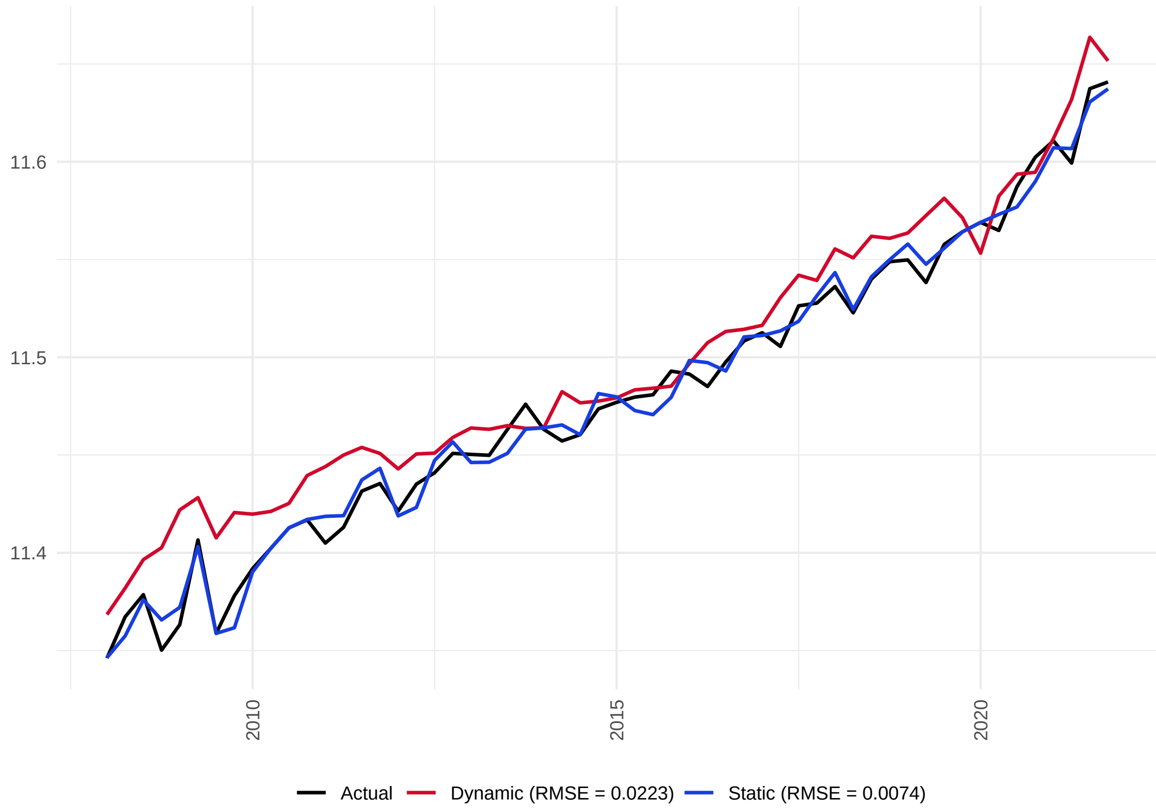


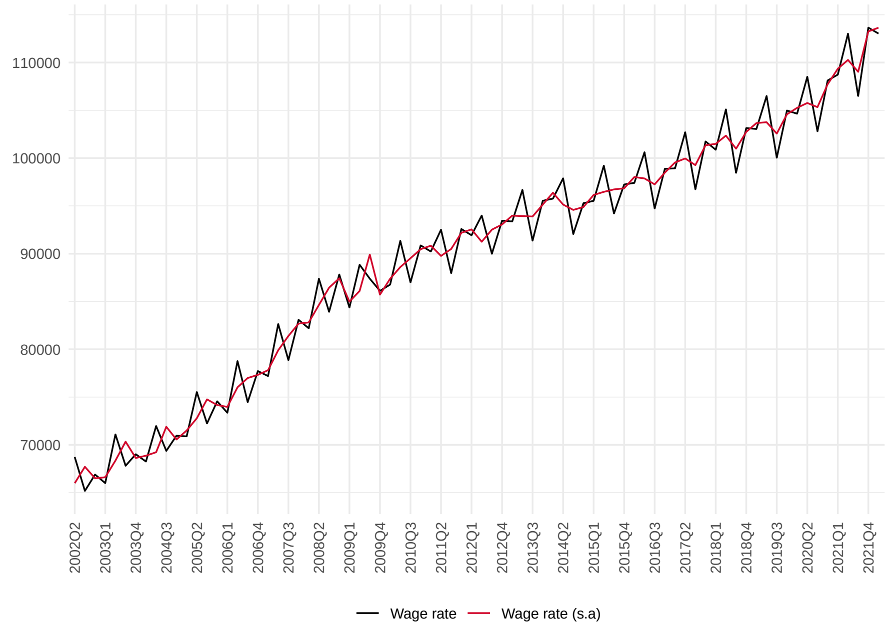
Figure 30: Overall fit of the wage rate in E-IO-SFC.

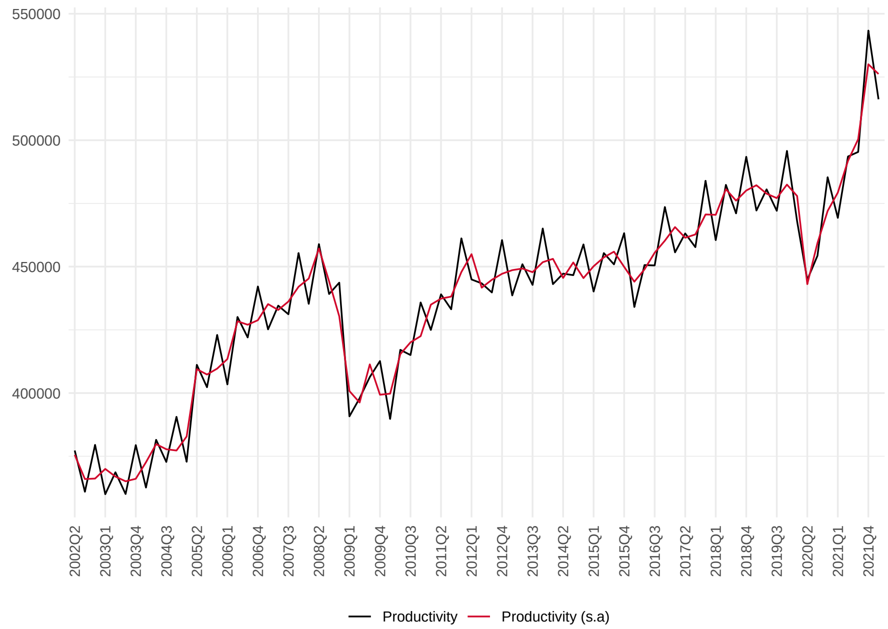
The model demonstrates a satisfactory fit to the data, effectively capturing the general business cycle fluctuations in the wage rate. Both the static and dynamic forecasts closely align with the observed time series, suggesting that the model accurately represents the underlying economic mechanisms during stable or "normal" periods. While

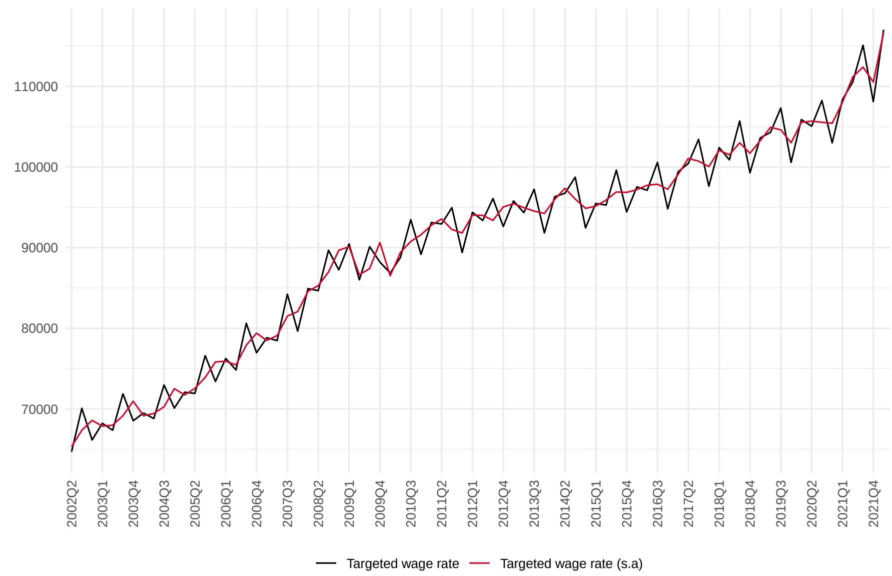
[89]In section C.2 of the appendix, the detailed regression output can be found under step 4.

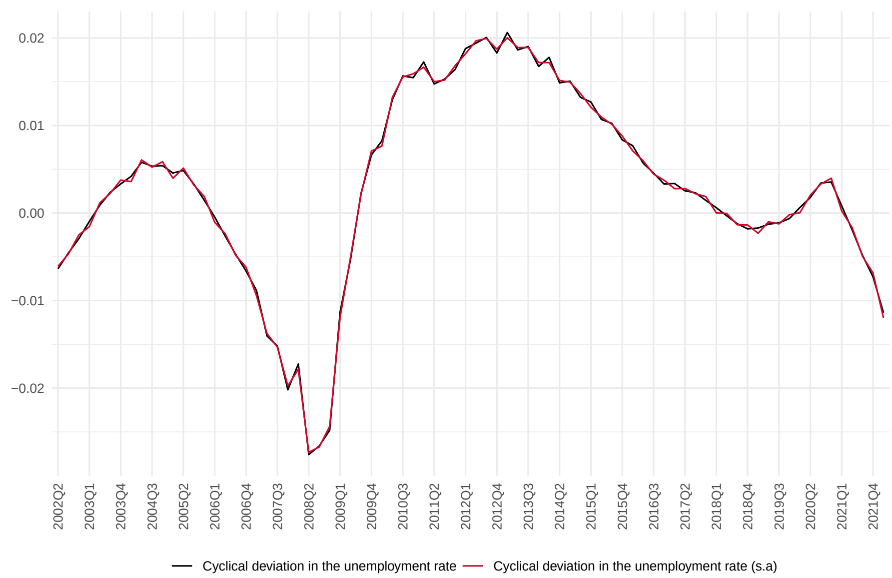
noticeable deviations appear at the beginning of the sample, particularly a level effect, the model subsequently corrects itself. However, the dynamic forecast slightly remains above the observed values. On a positive note, the forecasts do not diverge significantly toward the end of the sample. Overall, the model provides a reliable representation of wage dynamics under typical economic conditions and can thus be used as the behavioral equation for the wage rate within the E-IO-SFC model.

Step 1: Data and unit root testing









Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
W_{sa}	-1.9429	-3.45	trend	I(1)
PR_{sa}	-2.7141	-3.45	trend	I(1)
WT_{sa}	1.8943	-3.45	trend	I(I)
$UR_rUR_s^{sa}$	-1.5641	-2.89	drift	I(1)

Table 31: Results from ADF Tests for the variables entering Wage Rate estimation.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
24.627	3	Case II (Restricted intercept and no trend)	[2.79 ; 3.67]	Cointegration
13.16	3	Case III (Unrestricted intercept and no trend)	[3.23 ; 4.35]	Cointegration

Table 32: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 3$	4.41	12.25
$r \leq 2$	17.31	25.32
$r \leq 1$	41.80	42.44
$r = 0$	94.07	62.99

Table 33: Johansen Trace Test Results with trend.

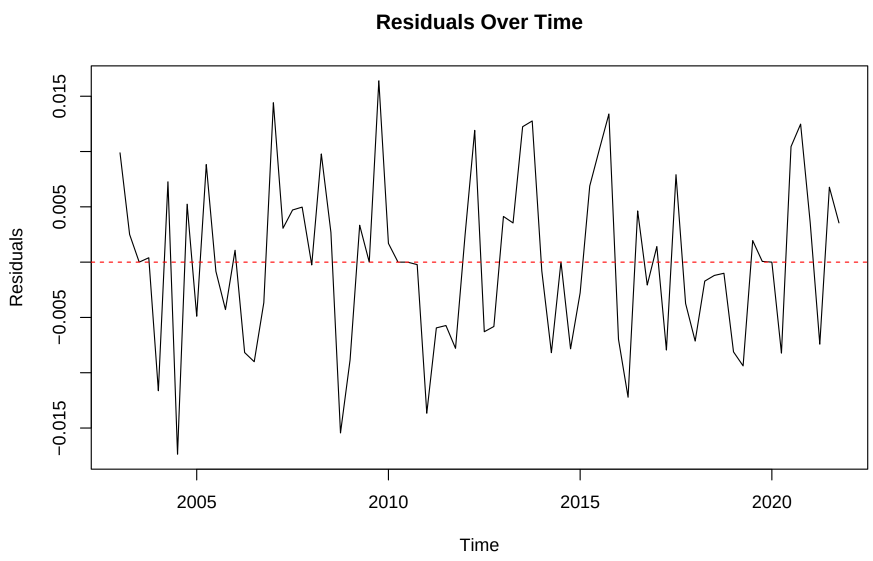
Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 3$	6.59	9.24
$r \leq 2$	18.77	19.96
$r \leq 1$	42.41	34.91
$r = 0$	115.96	53.12

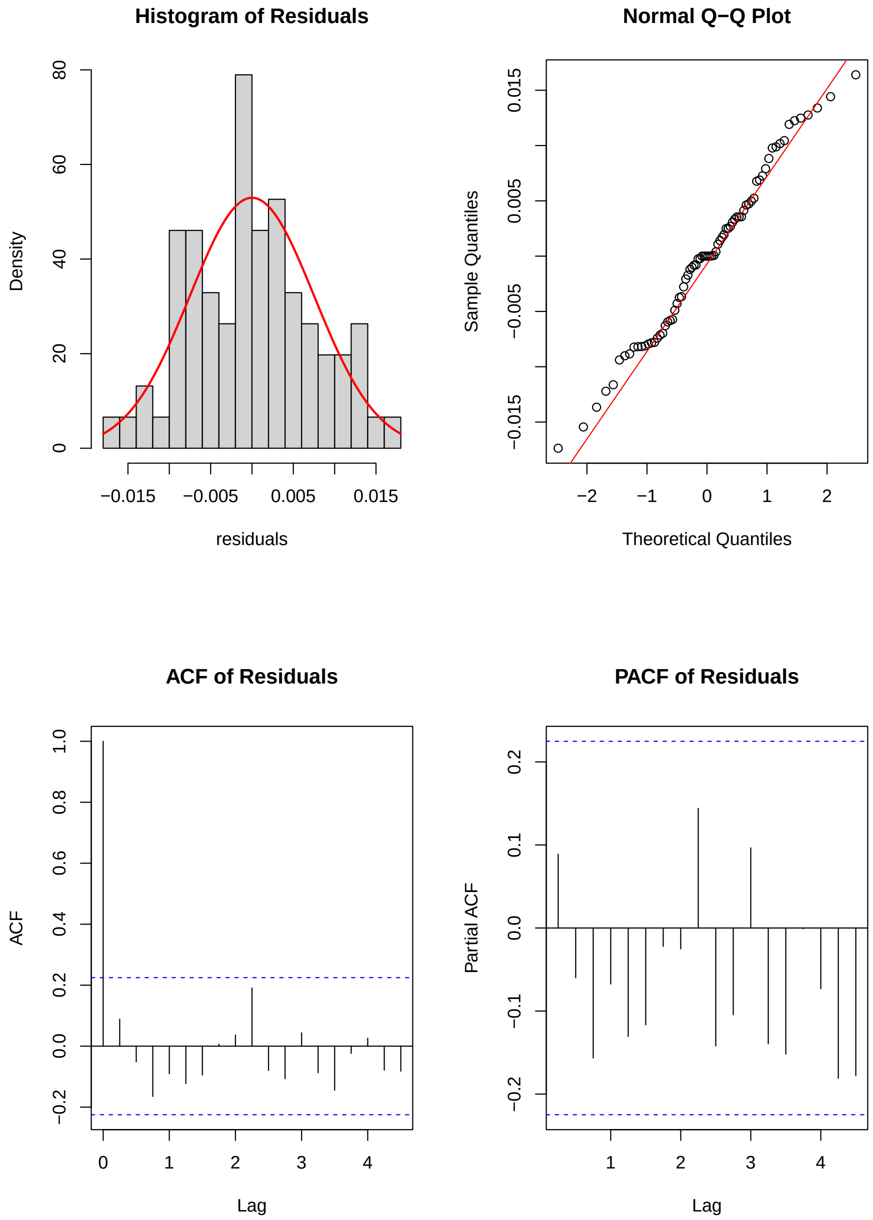
Table 34: Johansen Trace Test Results without trend and constant.

Step 4: Run the specific model

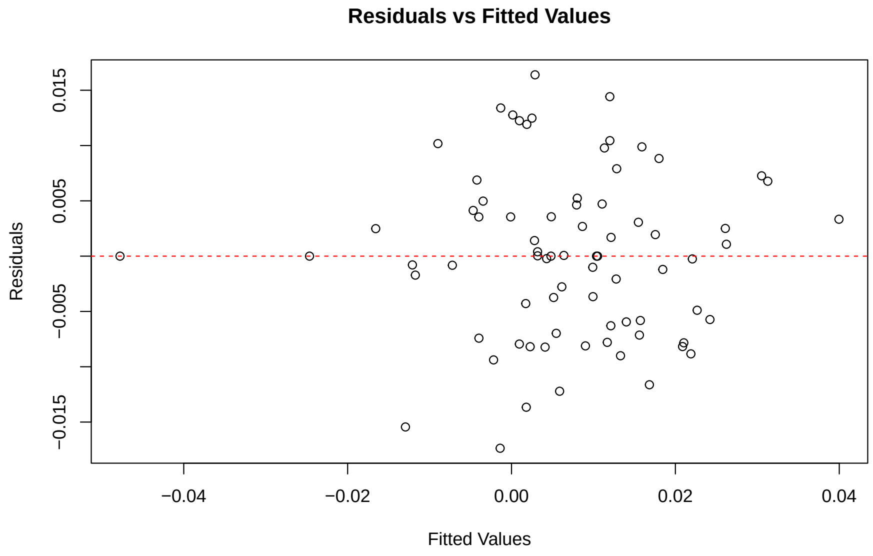
```
Call:
dynlm(formula = d(lW_sa) ~ L(d(lW_sa), 2) + L(d(lW_sa), 3) +
    d(lPR_sa) + L(lW_sa, 1) + L(UR_URs_sa, 1) + L(lW_T_sa, 1) +
    dummy03 + dummy09 + dummy10Q2 + dummy10Q3 + dummy20, data = data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.01737 -0.00603  0.00000  0.00465  0.01639

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)        0.22134    0.08310   2.664  0.00977 **
L(d(lW_sa), 2)    -0.53011    0.07865  -6.740 5.32e-09 ***
L(d(lW_sa), 3)    -0.73793    0.09360  -7.884 5.17e-11 ***
d(lPR_sa)          0.35312    0.05279   6.689 6.51e-09 ***
L(lW_sa, 1)       -0.46068    0.07117  -6.473 1.55e-08 ***
L(UR_URs_sa, 1)   -0.58013    0.09645  -6.015 9.55e-08 ***
L(lW_T_sa, 1)      0.44247    0.07222   6.127 6.15e-08 ***
dummy03           -0.01994    0.00887  -2.249  0.02799 *
dummy09           -0.05122    0.00884  -5.793 2.28e-07 ***
dummy10Q2         -0.02889    0.00952  -3.033  0.00349 **
dummy10Q3          0.01923    0.00845   2.276  0.02623 *
dummy20            0.02143    0.00922   2.325  0.02328 *
---

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.008154 on 64 degrees of freedom
Multiple R-squared:  0.7509,  Adjusted R-squared:  0.7081
F-statistic: 17.54 on 11 and 64 DF,  p-value: 2.29e-15
```

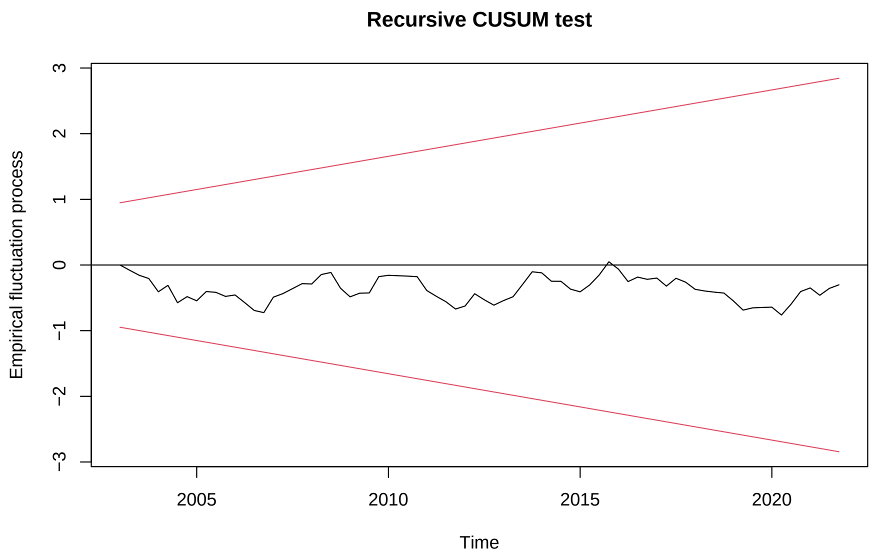
Step 6: Run diagnostic tests





Histogram of Residuals
Density
residuals
0
20
40
60
80
−0.015
−0.005
0.005
0.015
Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
−0.015
−0.005
0.005
0.015
−2
−1
0
1
2
ACF of Residuals
ACF
Lag
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
PACF of Residuals
Partial ACF
Lag
−0.2
−0.1
0.0
0.1
0.2
1
2
3
4





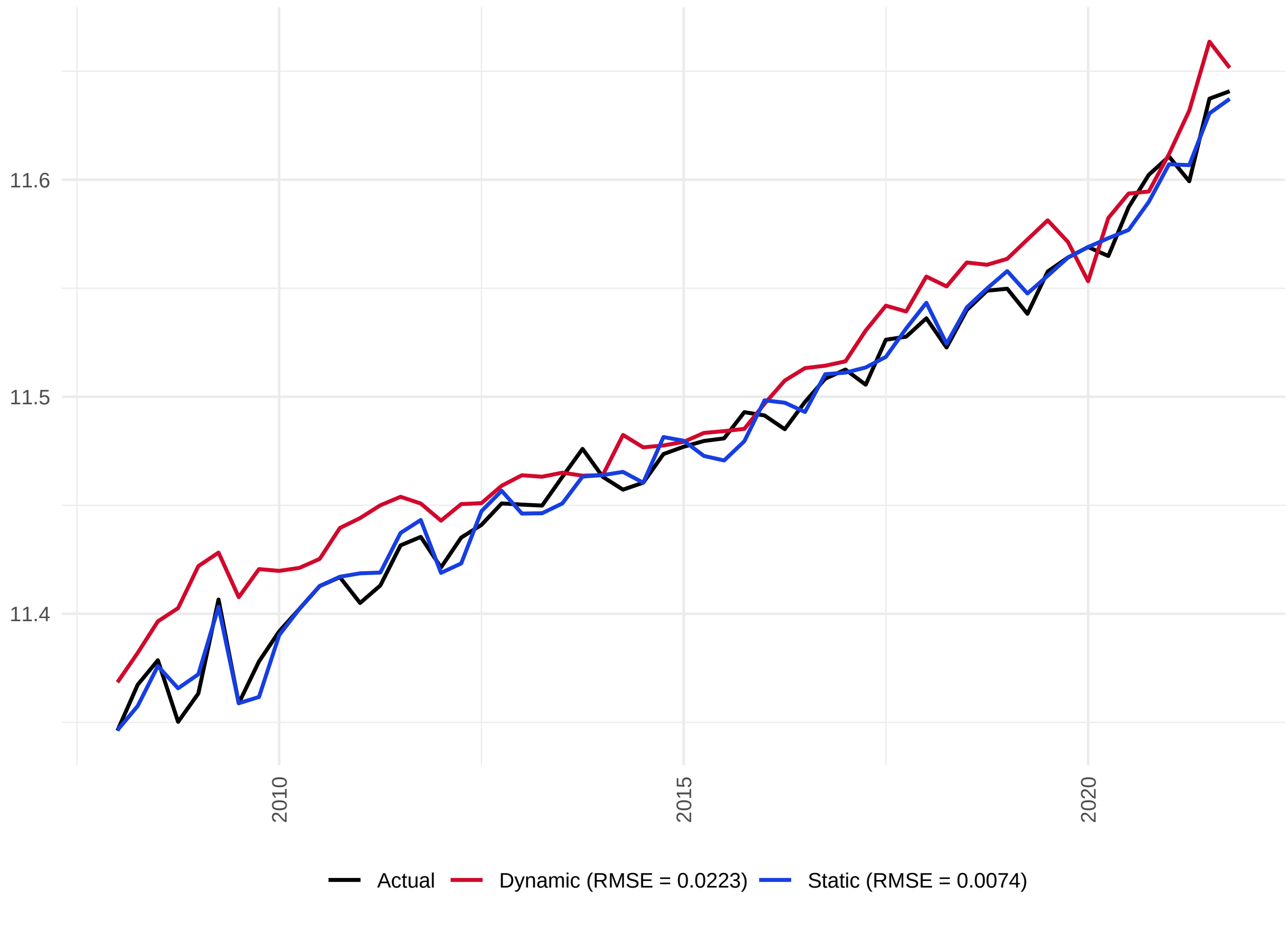
Test	p-value
Shapiro-Wilk	0.5498
Jarque Bera	0.6409
Durbin-Watson	0.1304
Breusch-Godfrey, order 4	0.3537

Table 35: Diagnostics statistics.

Variable	VIF Value
L(d(lW_sa), 2)	1.555521
L(d(lW_sa), 3)	2.209065
d(lPR_sa)	1.491044
L(lW_sa, 1)	110.023993
L(UR_URs_sa, 1)	1.282648
L(lW_T_sa, 1)	110.278678
dummy03	1.166649
dummy09	1.160063
dummy10Q2	1.346060
dummy10Q3	1.059519
dummy20	1.261692

Table 36: Variance Inflation Factor (VIF) Results.

Step 7: Evaluate static and dynamic forecasts



C.3 Investment to capital ratio

Real investment to capital ratio

The real investment-to-capital ratio is modeled as a function of government expenditures, g, its own lags, the capacity utilization rate, $\frac{y}{k}$, the lending rate faced by the NFC-sector,i_{NFC}, and the export-to-import ratio, $\frac{x}{im}$. Real investment, which constitutes the numerator in the calculation of the investment-to-capital ratio, consists of gross fixed capital formation[90] and net acquisitions of valuables[91]. The denominator, real capital stock, is defined as described in section 6.1.4.1. This ratio serves as an indicator of the investment rate or the investment intensity within the economy and is defined as in Dutt (2011), which is furthermore the reason why it is chosen to model investment as a ratio of the real stock of capital. The interest rate, i_{NFC}, which enters this behavioral equation, is defined as the interest rate faced by firms-those that undertake investment in the E-IO-SFC model and thus drive capital accumulation over time.

According to Dutt (2011), firms make investment decisions based on the capacity utilization rate, as it signals the buoyancy of aggregate demand[92] and shapes their expectations regarding future demand. The capacity utilization rate is conceptually similar to the output gap, a more standard measure in mainstream economics. However, a key advantage of using the capacity utilization rate is that it does not require assuming a specific production function to estimate the trend output. A high ($\frac{y}{k}$) reflects strong capacity utilization, which positively influences firms' investment decisions by signaling robust demand and improving expectations about future sales. Therefore, the expected sign for the capacity utilization rate is positive. While the Post-Keynesian perspective emphasizes the role of capacity utilization, the mainstream economic perspective, as outlined by Gottfries (2013, p. 74), focuses on the interplay between the interest rate and capital accumulation. Capital is a key factor of production, and changes in the capital stock are directly influenced by investment, as investment adds new buildings and machinery to the capital base. To finance such investments, firms typically need to borrow funds, making the real interest rate the intertemporal price of investment. Consequently, in the long run, capital accumulation is negatively affected by the real interest rate. A higher interest rate increases the cost of financing, thereby reducing firms' demand for capital. The investment function presented in Gottfries (2013, p. 79) explicitly incorporates this mechanism, stating that a lower real interest rate will stimulate investment. Hence, the interest rate faced by firms,$iNFC$, is expected to have a negative effect on investment in the long run.

Beyond firm-level determinants, public investment can be an important macroeconomic driver of the investment-to-capital ratio. From a theoretical perspective, and as outlined by Afonso and Aubyn (2009), an increase in public investment may result in two opposing effects on private investment. On the one hand, higher public investment can lead to increased government borrowing or taxation, potentially raising interest rates and reducing the availability of funds for private investors, thus crowding out private investment. In contrast, public investment can improve overall economic conditions, improve infrastructure, and increase private capital productivity, thereby fostering private investment through higher profitability and greater investment incentives, and thus a crowd-in effect. In the context of the E-IO-SFC model, public investment is therefore included as an explanatory variable in the investment-to-capital ratio equation to capture its potential role in influencing private investment behavior. Given the findings of Afonso and Aubyn (2009), which indicate a crowding-in effect of public investment in Denmark, the expected sign of the public investment variable is positive, reflecting the notion that higher government investment tends to stimulate, rather than displace, investments. The last explanatory variable in the investment to capital equation is the export to import ratio, $\frac{x}{im}$, which serves as a proxy for foreign demand conditions. While foreign trade components are already embedded in output and thereby indirectly in the capacity utilization rate, this variable is included to capture additional information related to international trade

[90]Which is investment in machinery and equipment as well as in buildings and dwellings.

[91]Valuables are non-financial assets that are not consumed or used in production, do not physically deteriorate under normal conditions, and are primarily acquired and held as stores of value.

[92]The buoyancy of aggregate demand can be interpreted as an indicator of the economy's position in the business cycle, and thus serves as a relevant input for firms' investment decisions.

dynamics. Specifically, the ratio may reflect relative competitiveness and the strength of foreign demand in a way not fully encapsulated by aggregate output[93]. From a theoretical perspective, a higher $\frac{x}{im}$ reflects stronger net exports, indicating robust foreign demand and competitiveness in international markets. This can foster investment activity through increased revenues and improved business expectations, especially in a small open economy such as the Danish economy. Given that Denmark consistently maintains a positive trade balance, the expected sign of $\frac{x}{im}$ is positive, as net exports are likely to stimulate private investment and thereby contribute to capital accumulation.

The estimated behavioral equation below illustrates the presented theoretical relationship within the context of the model[94]. Since cointegration is identified according to the bounds and Johansen tests in section C.3 of the appendix, the investment-to-capital ratio equation is estimated as an ECM model with both short-run and long-run adjustments. The input data used is spanning from 2002Q1 to 2024Q4.

$$\begin{aligned}\Delta \ln\left(\frac{i_t}{k_t}\right) &= 2.06 - 0.46 \cdot \Delta \ln\left(\frac{i_{t-1}}{k_{t-1}}\right) - 0.17 \cdot \Delta \ln\left(\frac{i_{t-2}}{k_{t-2}}\right) + 0.13 \cdot \Delta \ln\left(\frac{i_{t-4}}{k_{t-4}}\right) \\ &+ 0.41 \cdot \Delta \ln(g_t) + 0.49 \cdot \Delta \ln(g_{t-1}) - 0.65 \cdot \Delta \ln\left(\frac{x_t}{im_t}\right) + 0.65 \cdot \ln\left(\frac{\mathbf{y_t}}{\mathbf{k_t}}\right) \\ &- 0.14 \cdot \ln\left(\frac{\mathbf{i_{t-1}}}{\mathbf{k_{t-1}}}\right) - 0.19 \cdot \ln\left(\mathbf{g_{t-1}}\right) + 0.17 \cdot \ln\left(\frac{\mathbf{y_{t-1}}}{\mathbf{k_{t-1}}}\right) - 1.58 \cdot \ln(\mathbf{i}_{\text{NFC}})\end{aligned}$$

The estimated behavioral equation for the investment to capital ratio aligns mostly with economic theory and expectations presented above, indication that the investment positively responds to changes in public investments reflecting a crowding-in effect in period, t and $t-1$ in the short run. These effects suggest that public investment stimulates private investment both immediately and with a lag. From an autoregressive perspective, investment is also influenced by its own past values. This dynamic behavior reflects the fact that investment decisions are typically subject to adjustment costs, planning horizons, and expectations based on past activity. The estimated sign for the trade balance proxy contradicts with economic theory, but is included in the equation because it significantly improves the dynamic forecast and the overall fit. The decision is somewhat split: Should one exclude a variable because of non-theoretical relevance or include it because it improves to overall dynamics in the model? To this stage of the process it is chosen to keep the variable in the investment equation, but in the future when validating the entire model, it will be tested whether it brings inappropriate dynamics in the model. This reflects a common trade-off in empirical modeling between theoretical consistency and empirical performance. At this stage, model selection favors empirical adequacy, recognizing that theoretical alignment remains important. Therefore, the variable is included, with the understanding that its role will be reevaluated during model validation, particularly regarding endogenous dynamics and out-of-sample forecasting performance.

The long-run elasticity of public investment, which is approximately (-1.42), suggests the presence of a crowding-out effect in the long run. This implies that a one percent increase in public investment is associated with a 1.42 percent decrease in the private investment-to-capital ratio in the long run. In other words, although public investment may stimulate private investment in the short run, it appears to displace it in total. This could be due to diminishing marginal returns to capital in the private sector. However, the impact of interest rates has already been accounted for in the model, so it cannot be the sole reason for the crowding-out effect, even though theory suggests it. The long-run elasticity of the capacity utilization rate (1.31) aligns with theory and states that an increase in the utilization rate affects private investments-to-capital ratio by 1.31 percent. As stated by Gottfries (2013), the interest rate negatively affects investment. The long-run interest elasticity of the interest rate, indicate that investments are sensitive to changes in the interest rate. A one percentage point increase

[93]However, it is still necessary to be aware of multicollinearity problems, as the inclusion of multiple correlated explanatory variables can lead to unreliable coefficient estimates and inflated standard errors. But this will be tested for.

[94]The behavior of $\frac{x}{im}$ will be commented on later in this section, as it is at odds with the economic rationale.

in the lending rate increases real investment to capital ratio by 0.0113[95] percent. However, this result should be interpreted with caution, as the model has been estimated during a period of historically low interest rates in the Danish economy. When interest rates have been low for an extended period, the economy may become particularly sensitive to interest rate changes, as firms and investors are accustomed to cheap financing. This context may affect the observed effect of interest rates on investment. Therefore, while the estimated elasticity indicates a strong relationship between interest rates and investment, it could be more pronounced than what would be observed in a higher interest rate environment. This is an important consideration for future analysis with the E-IO-SFC model. Lastly, approximately 13% of any disequilibrium generated by a shock is corrected within one quarter in this model, indicating a relatively moderate speed of adjustment.

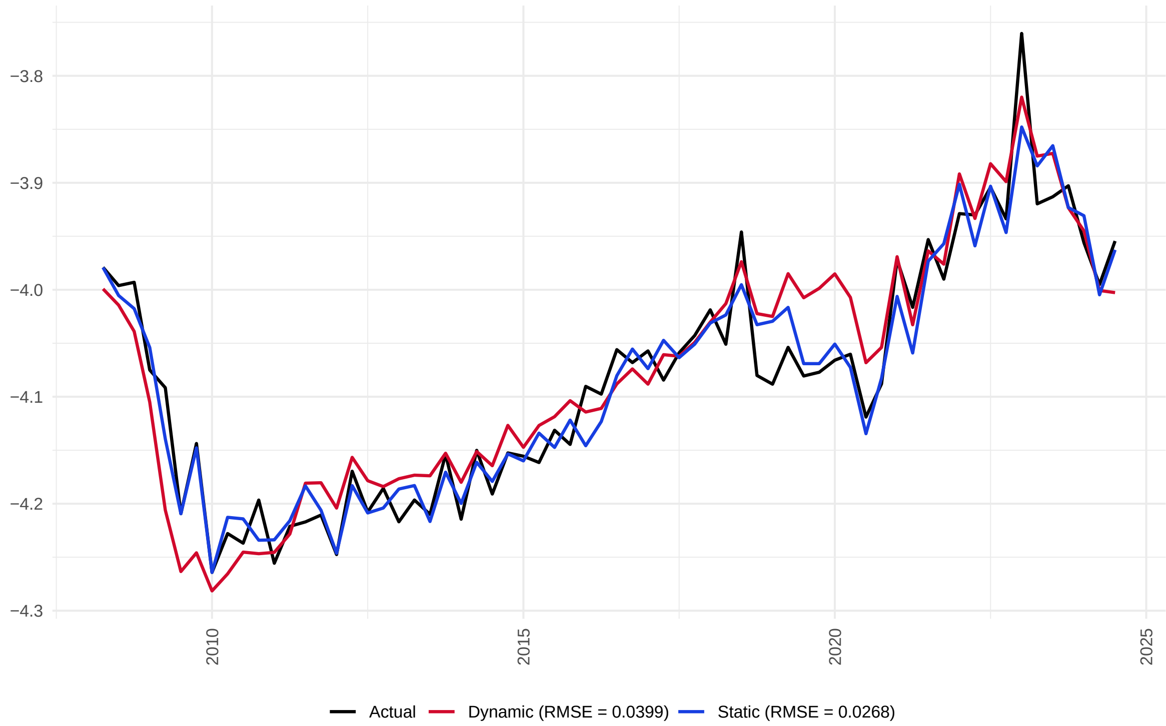


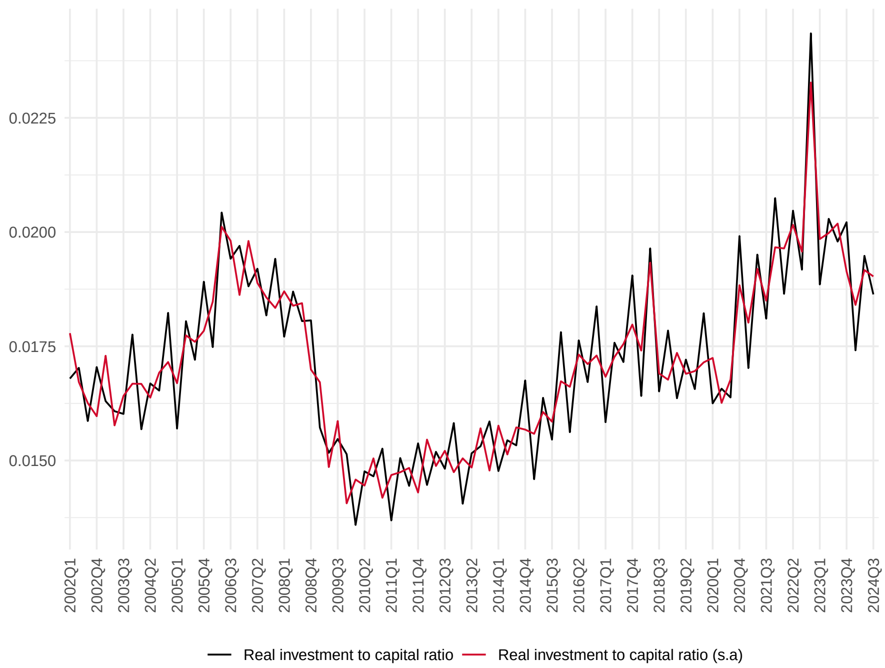
Figure 31: Overall fit of investment to capital ratio in E-IO-SFC.

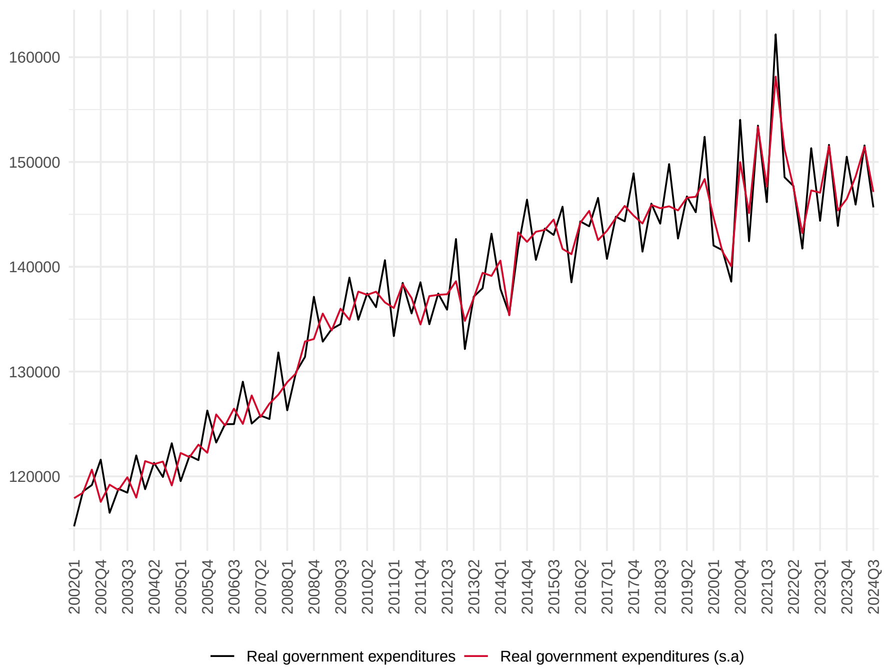
The estimated behavioral equation satisfies all the diagnostic tests presented in section 7.2.1, and the results of these tests are reported in section C.3. Below, the overall fit of the ECM model for the investment to capital ratio is presented in figure 31. The model yields a RMSE of 0,026 for the static forecast and 0,039 for the dynamic forecast. This implies that the static forecast has an average absolute error of approximately 5,3%, meaning that the model's predictions deviate, on average, by 5,3% from the actual observed values for the investment to capital ratio. The dynamic forecast shows a slightly higher average absolute error of approximately 7.9%, reflecting reduced accuracy due to the accumulation of forecast errors over time. Based on these diagnostics this model for the investment to capital ratio is used as the behavioral equation in the E-IO-SFC model[96].

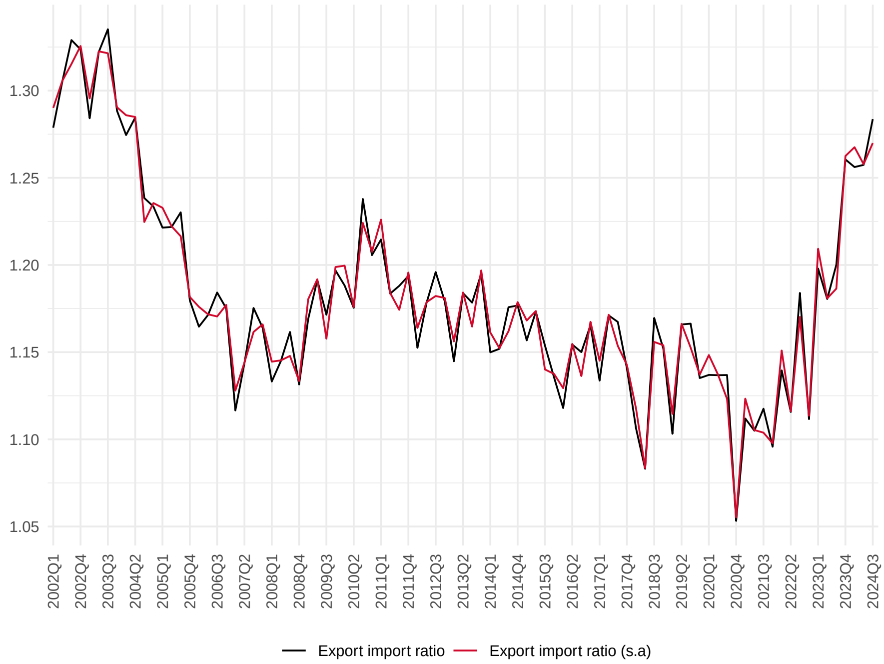
[95] Using a linear-dowscaling method a 1 percentage point increase in $iNFC$, e.g., from 0.4% to 1.4%, implies an effect of $-11.3\% \cdot 0.01 = 0.0113\%$.

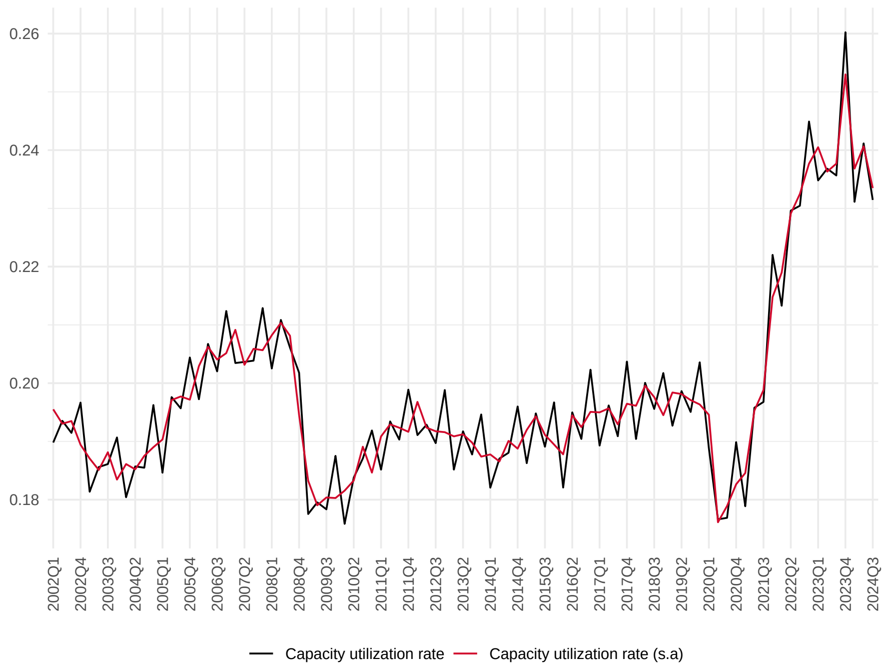
[96] In section C.3 of the appendix, the detailed regression output can be found under step 4.

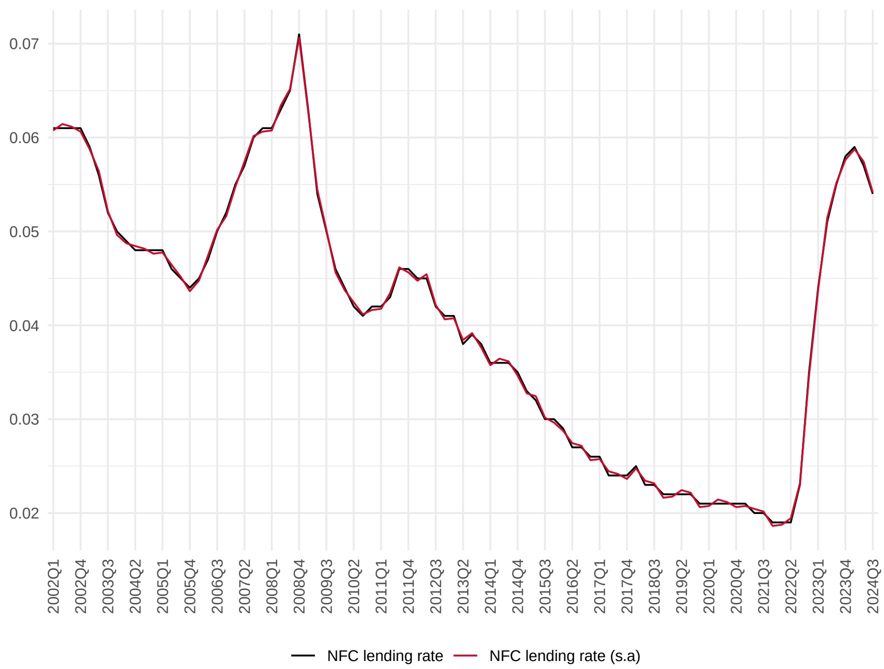
Step 1: Data and unit root testing






1.30
1.25
1.20
1.15
1.10
1.05
2002Q1
2002Q4
2003Q3
2004Q2
2005Q1
2005Q4
2006Q3
2007Q2
2008Q1
2008Q4
2009Q3
2010Q2
2011Q1
2011Q4
2012Q3
2013Q2
2014Q1
2014Q4
2015Q3
2016Q2
2017Q1
2017Q4
2018Q3
2019Q2
2020Q1
2020Q4
2021Q3
2022Q2
2023Q1
2023Q4
2024Q3
Export import ratio
Export import ratio (s.a)


0.26
0.24
0.22
0.20
0.18
2002Q1
2002Q4
2003Q3
2004Q2
2005Q1
2005Q4
2006Q3
2007Q2
2008Q1
2008Q4
2009Q3
2010Q2
2011Q1
2011Q4
2012Q3
2013Q2
2014Q1
2014Q4
2015Q3
2016Q2
2017Q1
2017Q4
2018Q3
2019Q2
2020Q1
2020Q4
2021Q3
2022Q2
2023Q1
2023Q4
2024Q3
Capacity utilization rate
Capacity utilization rate (s.a)



Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
g_{sa}	-2.6038	-3.45	trend	I(1)
$\frac{i}{k}_{sa}$	-1.3237	-2.89	drift	I(1)
$\frac{x}{im}_{sa}$	-1.9904	-2.89	drift	I(1)
$\frac{y}{k}_{sa}$	-0.7842	-2.89	drift	I(I)
$iNFC_{sa}$	-2.2662	-2.89	drift	I(I)

Table 37: Results from ADF Tests for variables entering investment to capital estimation.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
8.8864	4	Case II (Restricted intercept and no trend)	[2.56 ; 3.49]	Cointegration
10.663	4	Case III (Unrestricted intercept and no trend)	[2.86 ; 4.01]	Cointegration

Table 38: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 4$	7.92	12.25
$r \leq 3$	21.98	25.32
$r \leq 2$	37.87	42.44
$r \leq 1$	60.73	62.99
$r = 0$	107.26	87.31

Table 39: Johansen Trace Test Results with trend.

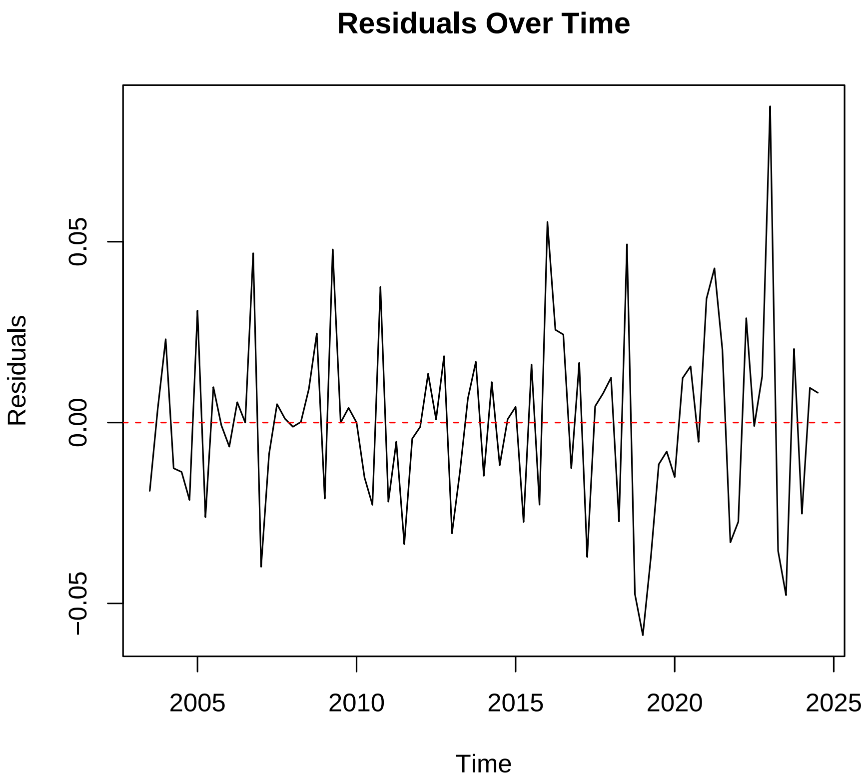
Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 4$	2.89	9.24
$r \leq 3$	8.30	19.96
$r \leq 2$	20.13	34.91
$r \leq 1$	45.04	53.12
$r = 0$	77.01	76.07

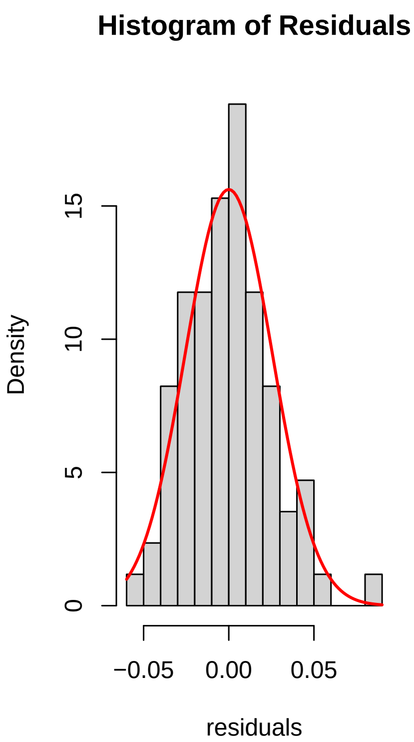
Table 40: Johansen Trace Test Results without trend and constant.

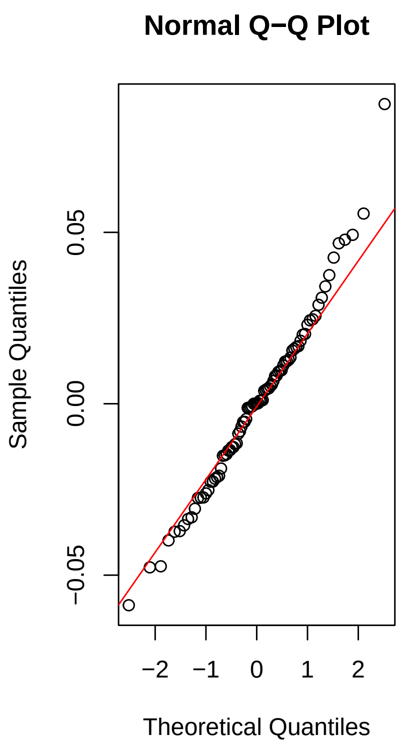
Step 4: Run the specific model

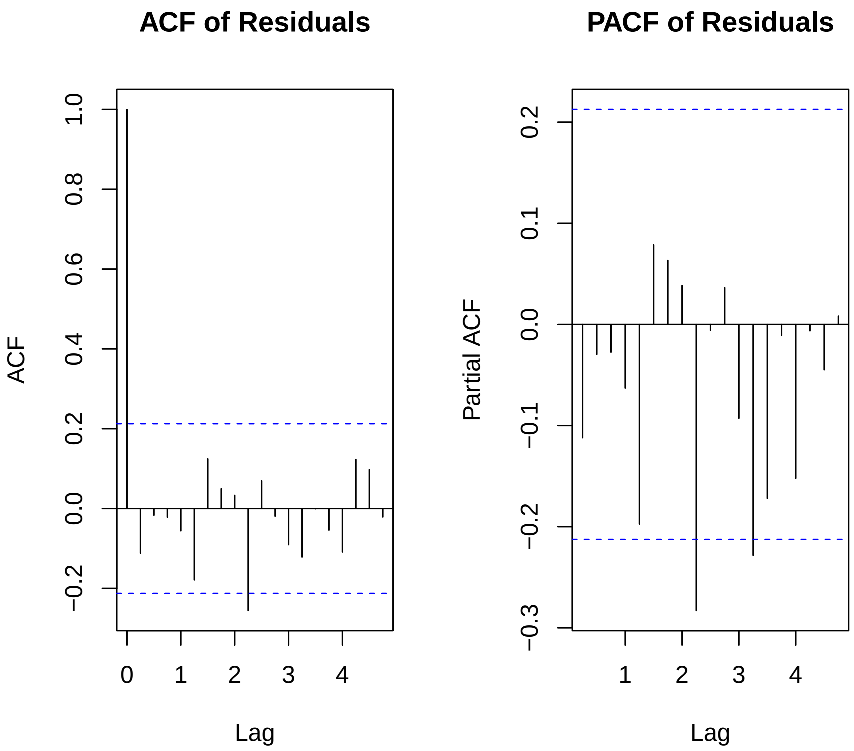
```
Call:
dynlm(formula = d(l_i_k_sa) ~ L(d(l_i_k_sa), 1) + L(d(l_i_k_sa), 2) +
  L(d(l_i_k_sa), 4) + d(l_g_sa) + L(d(l_g_sa), 1) +
  d(lx_im_ratio_sa) + d(l_cap_ul_sa) + L(l_i_k_sa, 1) + L(l_g_sa, 1) +
  L(l_cap_ul_sa, 1) + L(ci_S11_sa, 1) +
  dummy06Q3 + dummy09Q3 + dummy10Q1,
  data = data)

Residuals:
     Min        1Q    Median       3Q      Max
-0.05878 -0.01518  0.00000  0.01350  0.08743

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)          2.06316    1.13586   1.816  0.07359 .
L(d(l_i_k_sa), 1)   -0.45863    0.09156  -5.009 3.94e-06 ***
L(d(l_i_k_sa), 2)   -0.17246    0.08318  -2.073  0.04182 *
L(d(l_i_k_sa), 4)    0.13089    0.06544   2.000  0.04936 *
d(l_g_sa)            0.40487    0.18202   2.224  0.02935 *
L(d(l_g_sa), 1)      0.49523    0.19711   2.512  0.01430 *
d(lx_im_ratio_sa)   -0.64603    0.15064  -4.289 5.65e-05 ***
d(l_cap_ul_sa)       0.67477    0.13936   4.641 1.57e-05 ***
L(l_i_k_sa, 1)      -0.13444    0.05530  -2.431  0.01761 *
L(l_g_sa, 1)        -0.19139    0.09414  -2.033  0.04583 *
L(l_cap_ul_sa, 1)    0.17632    0.09123   1.933  0.05733 .
L(ci_S11_sa, 1)     -1.57819    0.50623  -3.118  0.00265 **
dummy06Q3            0.07128    0.02975   2.396  0.01924 *
dummy09Q3           -0.07340    0.03174  -2.313  0.02369 *
dummy10Q1           -0.05828    0.03072  -1.897  0.06195 .

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.02798 on 70 degrees of freedom
Multiple R-squared: 0.773,  Adjusted R-squared: 0.7276
F-statistic: 17.03 on 14 and 70 DF, p-value: < 2.2e-16
```

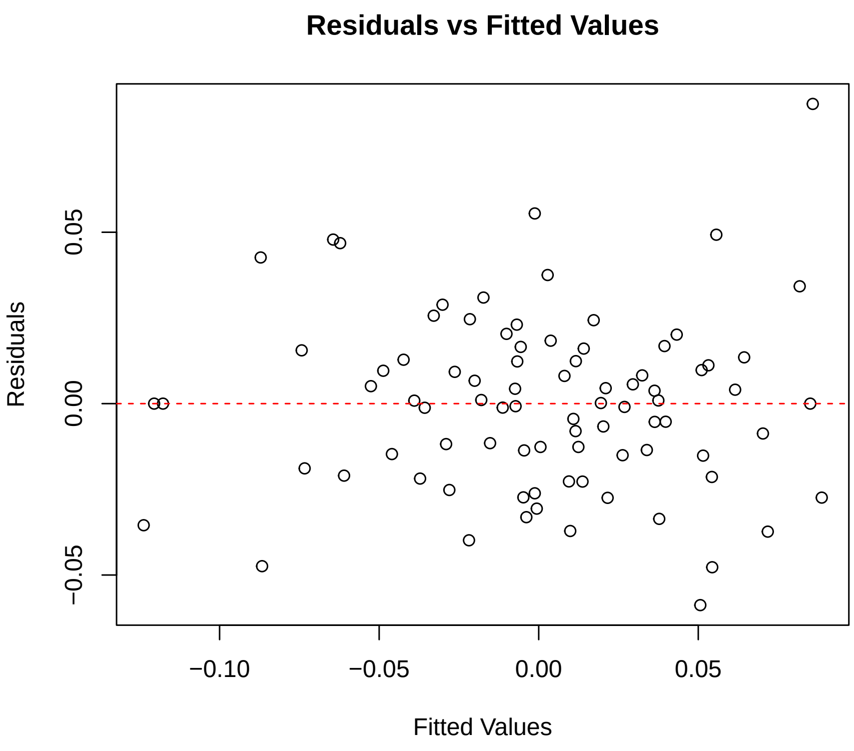
Step 6: Run diagnostic tests

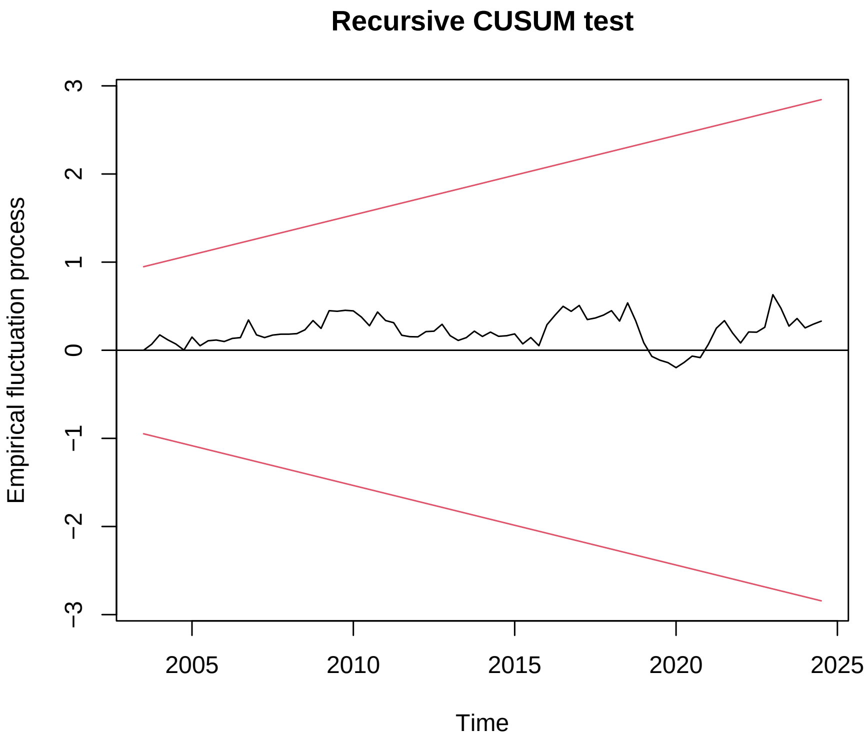







ACF of Residuals
ACF
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
1
2
3
4
Lag
PACF of Residuals
Partial ACF
0.2
0.1
0.0
−0.1
−0.2
−0.3
1
2
3
4
Lag


Residuals vs Fitted Values
Residuals
0.05
0.00
−0.05
−0.10
−0.05
0.00
0.05
Fitted Values



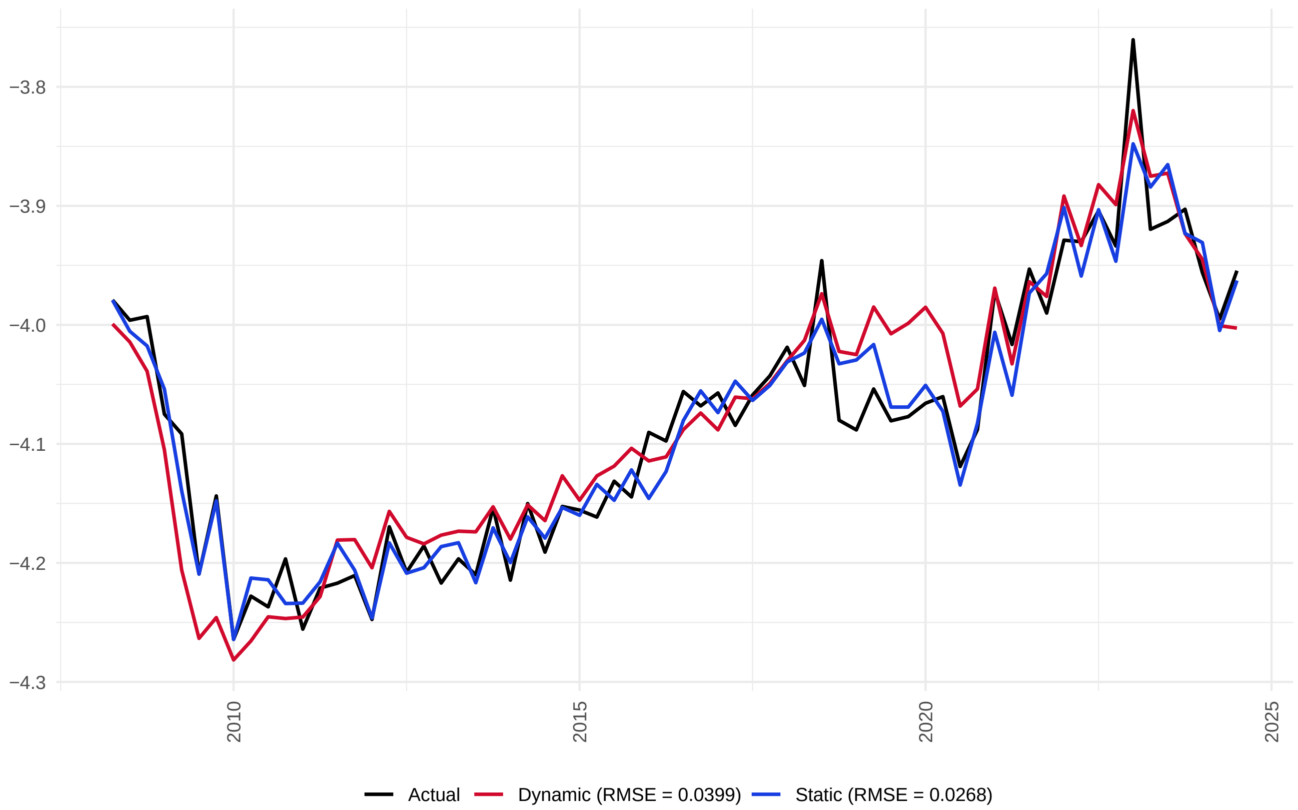
Test	**p-value**
Shapiro-Wilk	0.3997
Jarque Bera	0.1033
Durbin-Watson	0.7816
Breusch-Godfrey, order 4	0.4854

Table 41: Diagnostics statistics.

Variable	VIF Value
L(d(l_i_k_sa), 1)	2.634039
L(d(l_i_k_sa), 2)	2.162369
L(d(l_i_k_sa), 4)	1.348480
d(l_g_sa)	1.642618
L(d(l_g_sa), 1)	1.919433
d(lx_im_ratio_sa)	1.733000
d(lcap_ul_sa)	1.364025
L(l_i_k_sa, 1)	3.579507
L(l_g_sa, 1)	4.899414
L(lcap_ul_sa, 1)	4.955867
L(ci_S11_sa, 1)	5.319247
dummy06Q3	1.117165
dummy09Q3	1.271809
dummy10Q1	1.191679

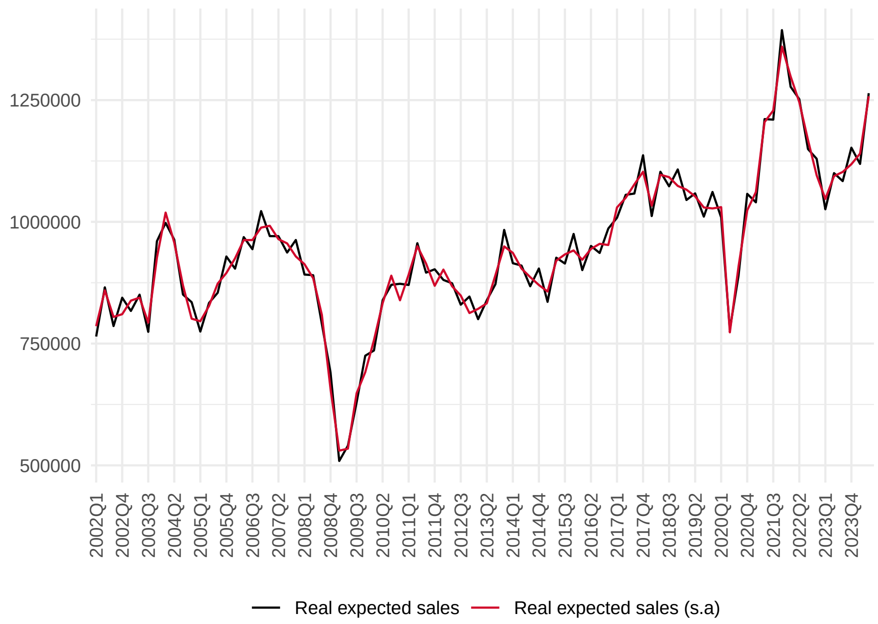
Table 42: Variance Inflation Factor (VIF) Results for investment equation.

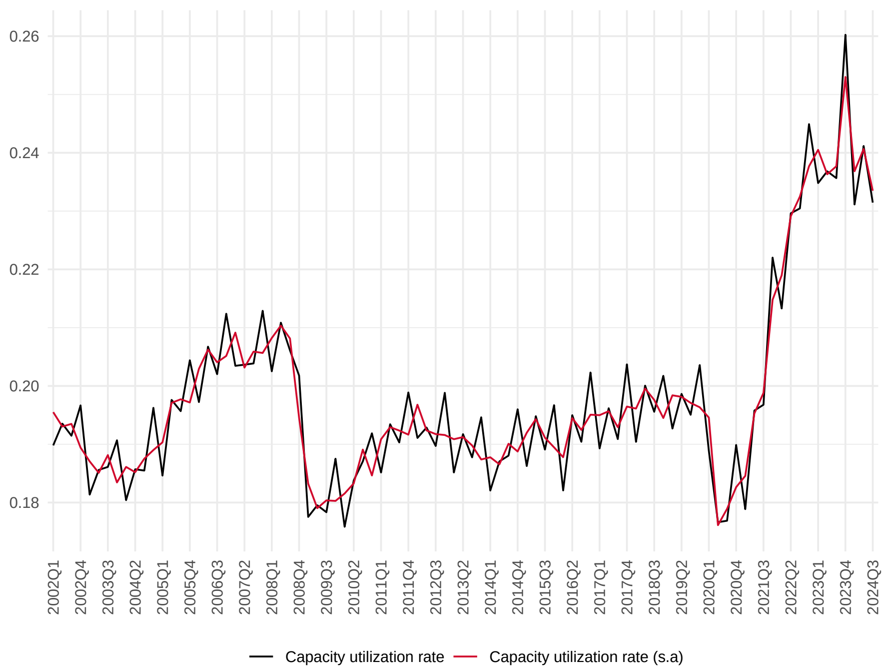
Step 7: Evaluate static and dynamic forecasts

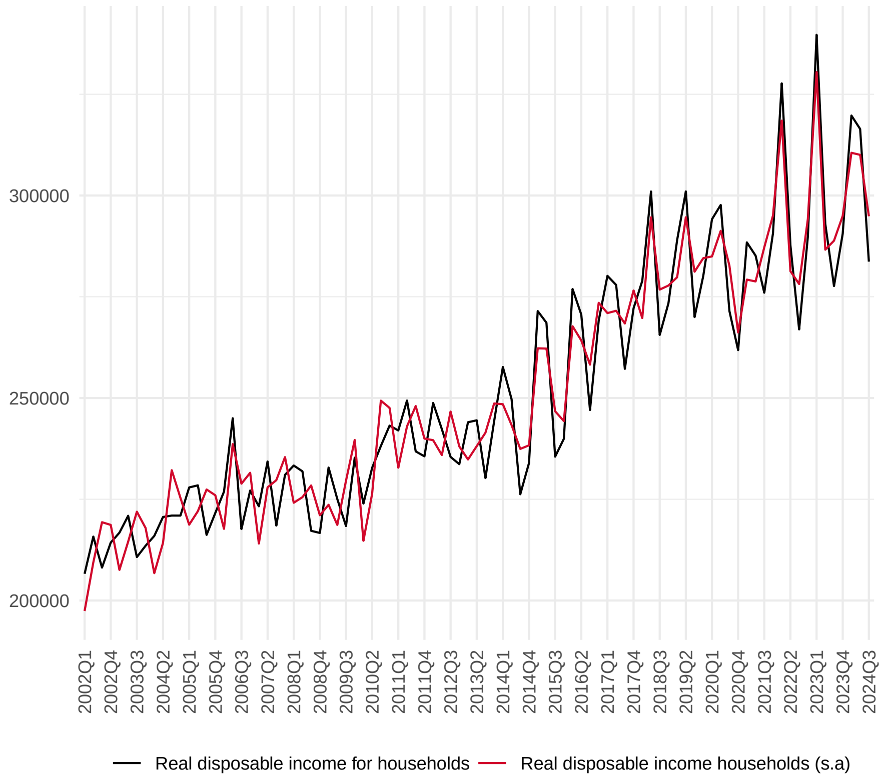


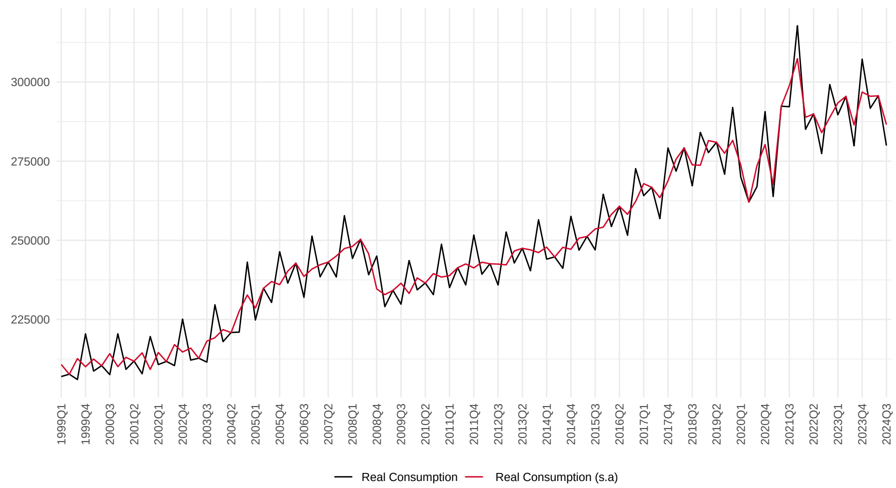
C.4 Real expected sales

Step 1: Data and unit root testing









Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
yd_H^{sa}	-1.262	-2.89	drift	I(1)
c_{sa}	-2.5656	-3.45	trend	I(I)
i_{sa}	-1.7808	-3.45	trend	I(I)
$\frac{y}{k}_{sa}$	-0.6628	-2.89	drift	I(1)
y_e^{sa}	-1.972	-2.89	drfit	I(1)
CI_{sa}	-3.0649	-2.89	drift	I(0)

Table 43: Results from ADF Tests for variables entering real expected sales estimation.

Step 3: Conintegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
5.406	5	Case II (Restricted intercept and no trend)	[2.39 ; 3.38]	Cointegration
6.3	5	Case III (Unrestricted intercept and no trend)	[2.62 ; 3.79]	Cointegration

Table 44: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 5$	4.26	12.25
$r \leq 4$	14.05	25.32
$r \leq 3$	28.52	42.44
$r \leq 2$	43.69	62.99
$r \leq 1$	76.66	87.31
$r = 0$	127.09	114.90

Table 45: Johansen Trace Test Results with trend.

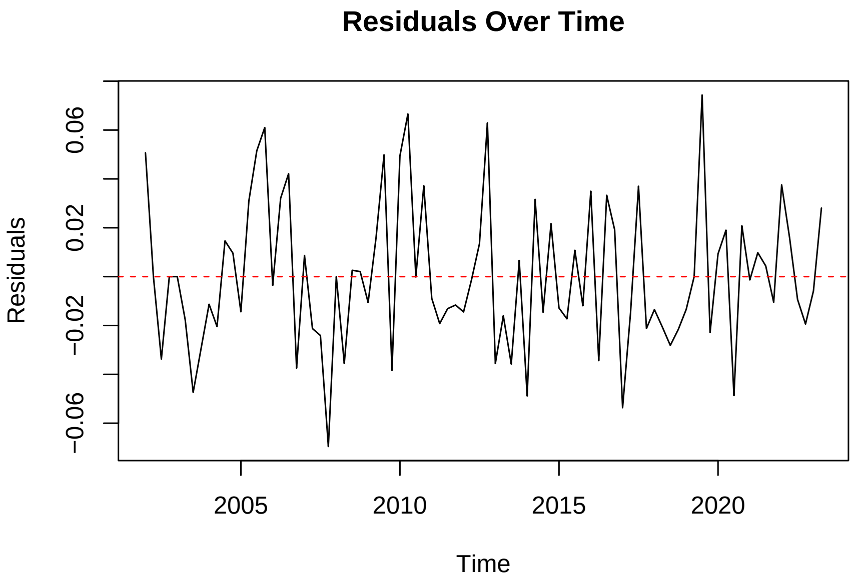
Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 5$	4.04	9.24
$r \leq 4$	8.81	19.96
$r \leq 3$	19.14	34.91
$r \leq 2$	38.80	53.12
$r \leq 1$	67.49	76.07
$r = 0$	121.28	102.14

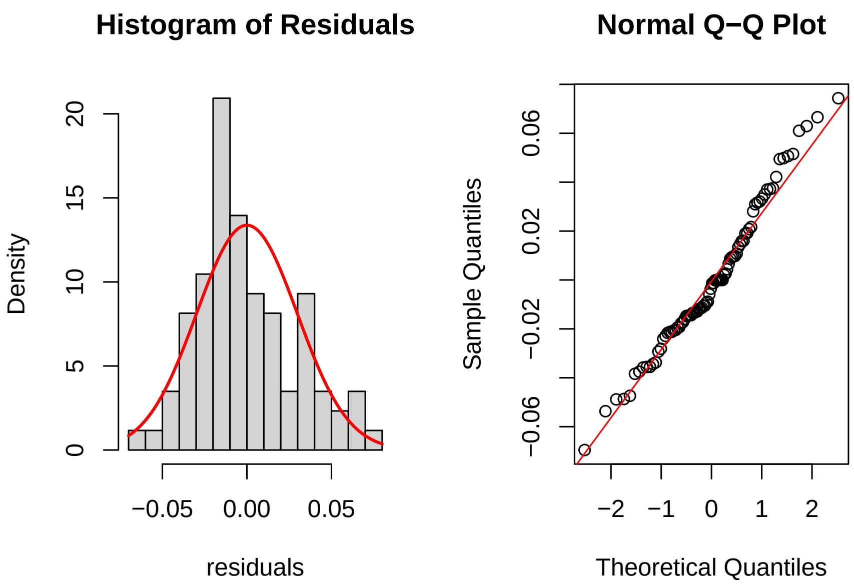
Table 46: Johansen Trace Test Results without trend and constant.

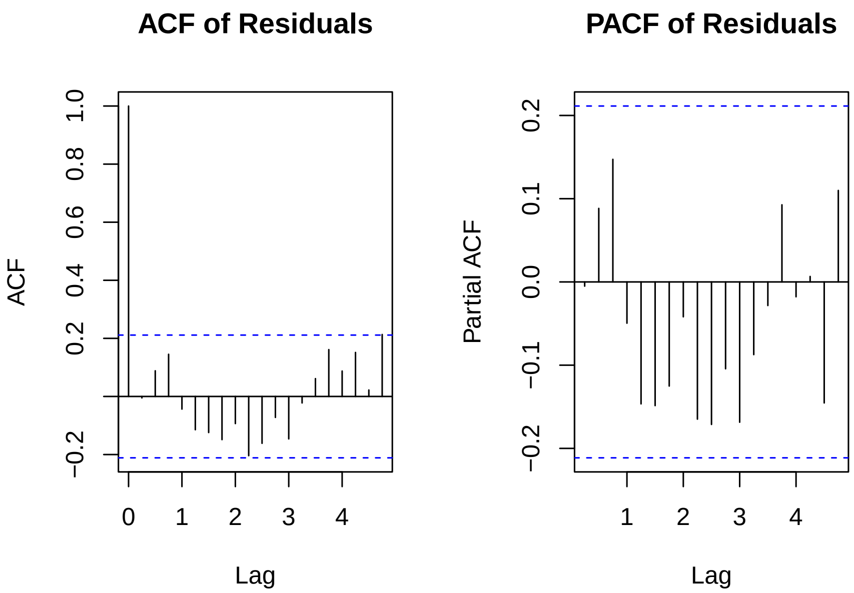
Step 4: Run the specific model

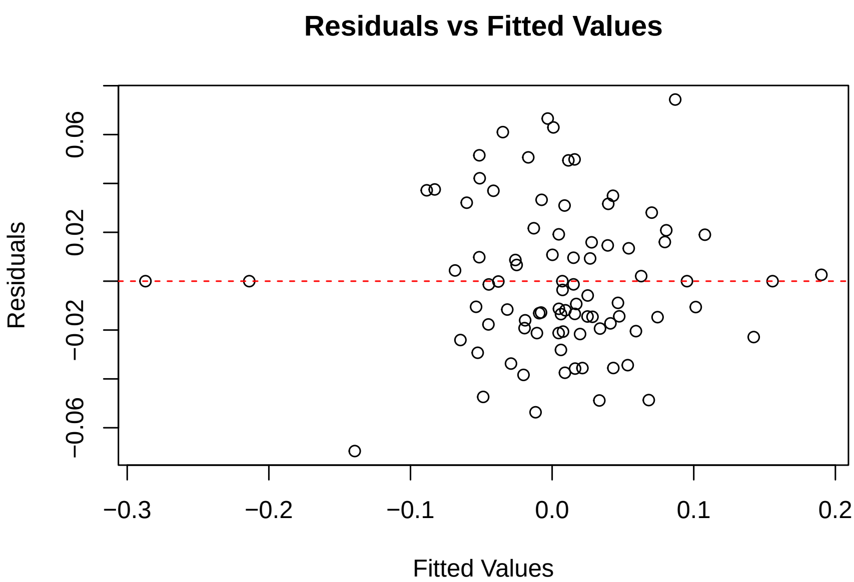
```
Call:
dynlm(formula = d(ly_e_sa) ~ L(d(ly_e_sa), 1) + d(C_TILLID_t) +
  d(lcap_ul_sa) + L(d(lyd_h_sa), 2) + d(lc_sa) + d(li_sa) +
  L(ly_e_sa, 1) + L(lc_sa, 1) + L(C_TILLID_t, 1) + dummy08 +
  dummy19 + dummy03 + dummy02Q4 + dummy02Q1, data = data)

Residuals:
     Min        1Q    Median        3Q       Max
-0.06954 -0.01937 -0.00247  0.01827  0.07434

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)
(Intercept)           -2.241010   0.610248   -3.67  0.00046 ***
L(d(ly_e_sa), 1)       0.216493   0.058796    3.68  0.00045 ***
d(C_TILLID_t)          0.004854   0.001184    4.10  0.00011 ***
d(lcap_ul_sa)          1.066221   0.220667    4.83 0.0000076 ***
L(d(lyd_h_sa), 2)      0.184608   0.078591    2.35  0.02161 *
d(lc_sa)               0.609943   0.226823    2.70  0.00877 **
d(li_sa)               0.173665   0.071500    2.43  0.01768 *
L(ly_e_sa, 1)         -0.309689   0.035735   -8.67 <0.000000001 ***
L(lc_sa, 1)            0.525986   0.070190    7.49 <0.000000001 ***
L(C_TILLID_t, 1)       0.001353   0.000502    2.69  0.00879 **
dummy08               -0.197679   0.036026   -5.49 <0.0000010 ***
dummy19               -0.129737   0.040950   -3.17  0.00262 **
dummy03                0.123515   0.036079    3.42  0.00103 **
dummy02Q4              0.204773   0.034104    6.00 <0.0000008 ***
dummy02Q1              0.131060   0.035533    3.69  0.00044 ***

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.0326 on 71 degrees of freedom
Multiple R-squared: 0.836,  Adjusted R-squared: 0.803
F-statistic: 25.8 on 14 and 71 DF,  p-value: <0.00000000000000000002
```

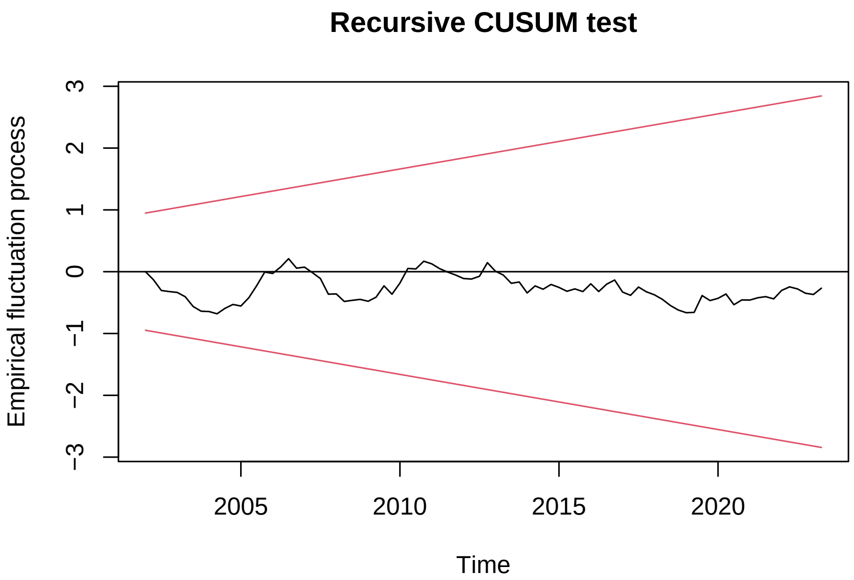
Step 6: Run diagnostic tests






ACF of Residuals
ACF
1.0
0.8
0.6
0.4
0.2
−0.2
0
1
2
3
4
Lag
PACF of Residuals
Partial ACF
0.2
0.1
0.0
−0.1
−0.2
1
2
3
4
Lag


Residuals vs Fitted Values
Residuals
0.06
0.02
−0.02
−0.06
−0.3
−0.2
−0.1
0.0
0.1
0.2
Fitted Values



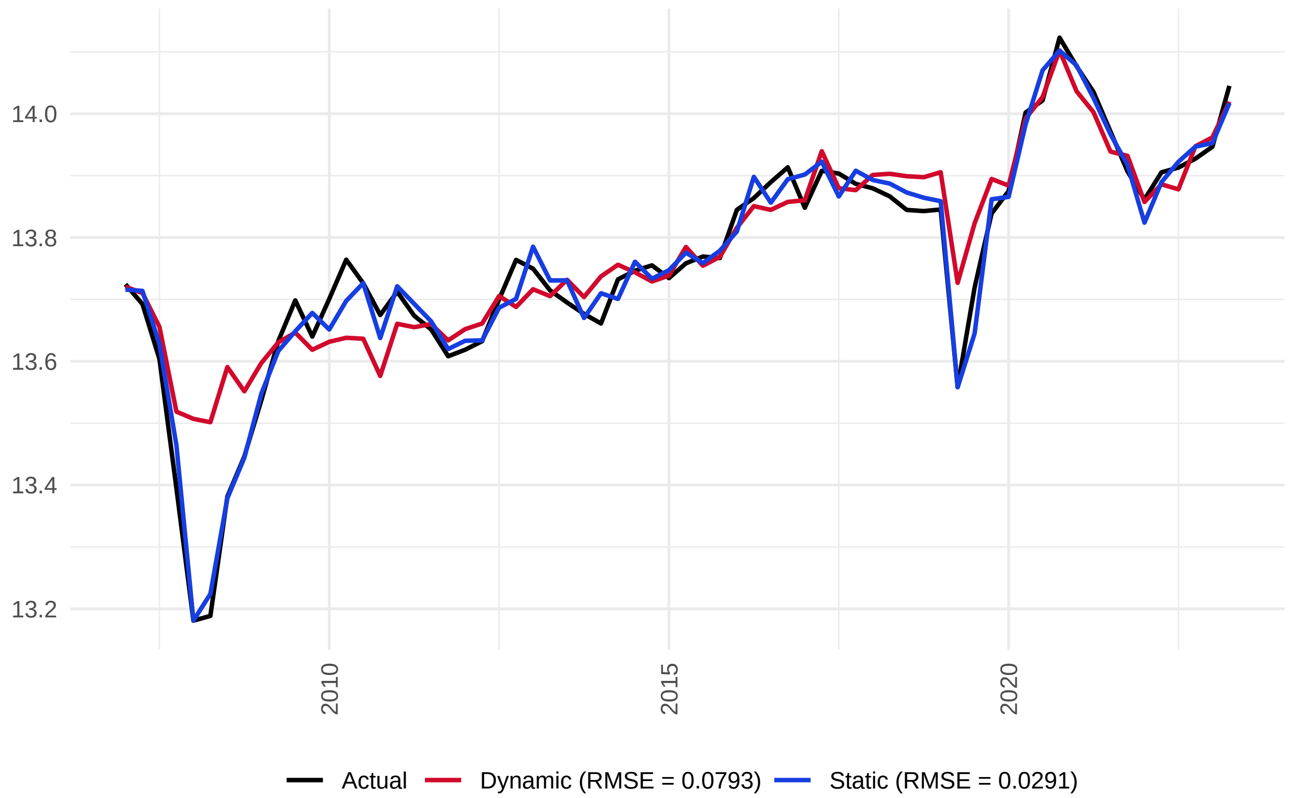
Test	p-value
Shapiro-Wilk	0.183
Jarque Bera	0.338
Durbin-Watson	0.344
Breusch-Godfrey, order 4	0.492

Table 47: Diagnostics statistics.

Variable	VIF Value
L(d(ly_e_sa), 1)	1.46439
d(C_TILLID_t)	1.62400
d(lcap_ul_sa)	2.18765
L(d(lyd_h_sa), 2)	1.14701
d(lc_sa)	1.76835
d(li_sa)	1.42717
L(ly_e_sa, 1)	2.74593
L(lc_sa, 1)	3.33974
L(C_TILLID_t, 1)	1.56716
dummy08	1.20462
dummy19	1.55647
dummy03	1.20818
dummy02Q4	1.07955
dummy02Q1	1.17187

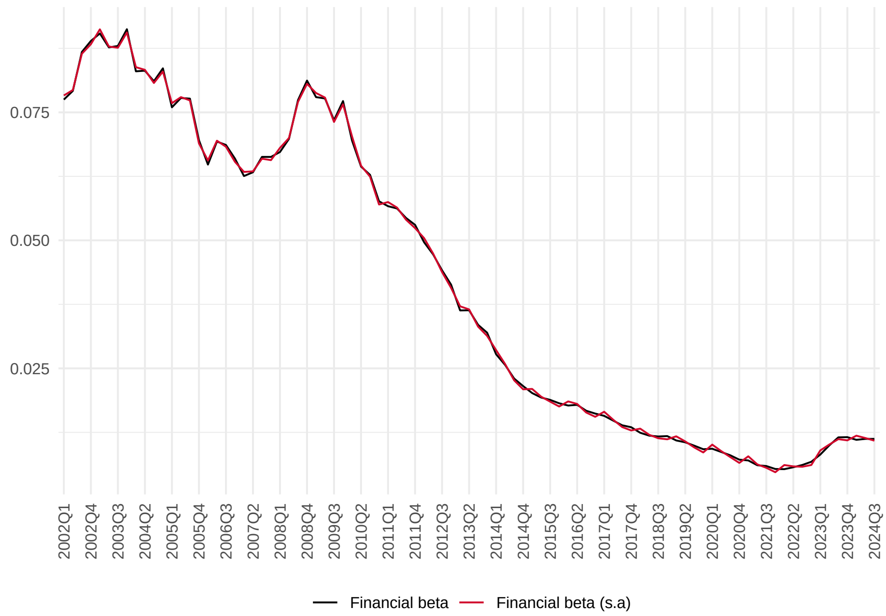
Table 48: Variance Inflation Factor (VIF) Results.

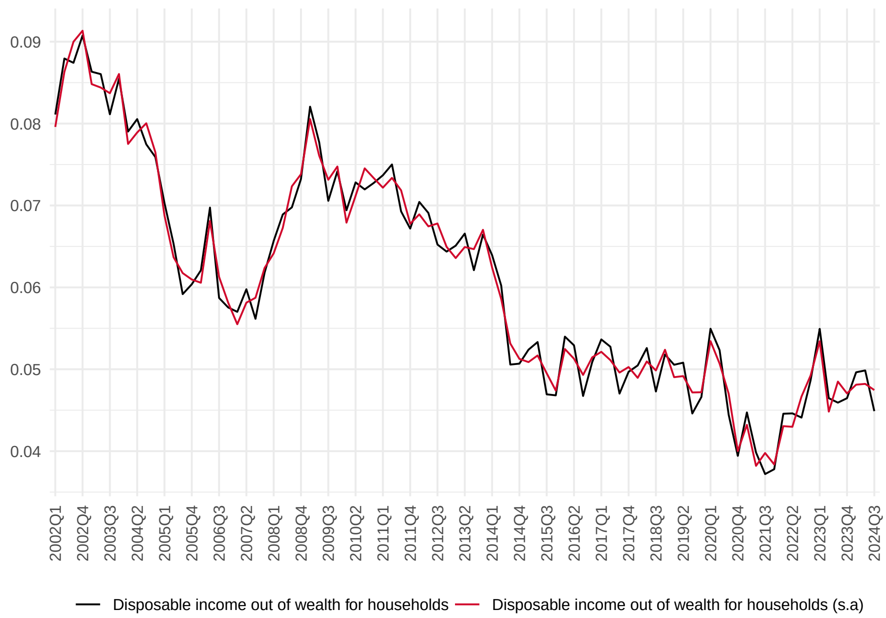
Step 7: Evaluate static and dynamic forecasts

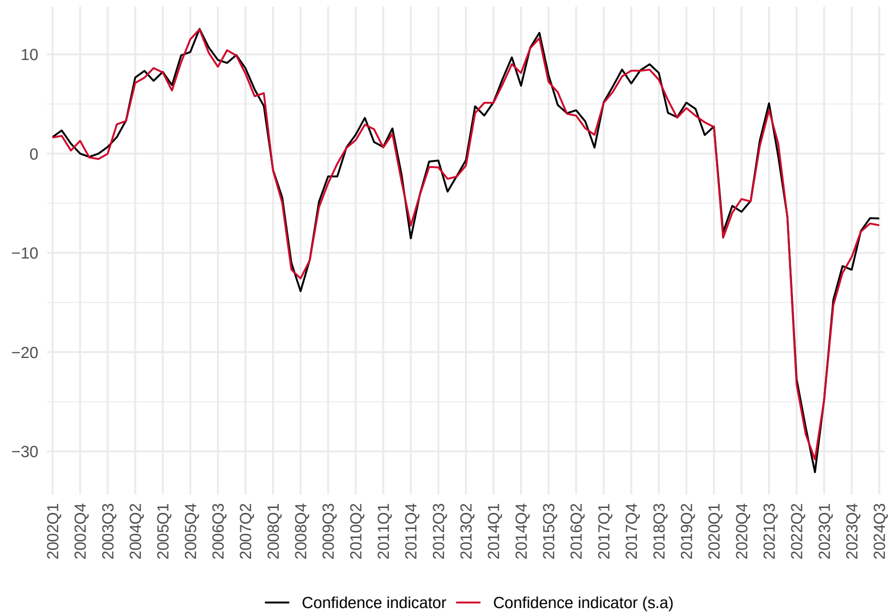


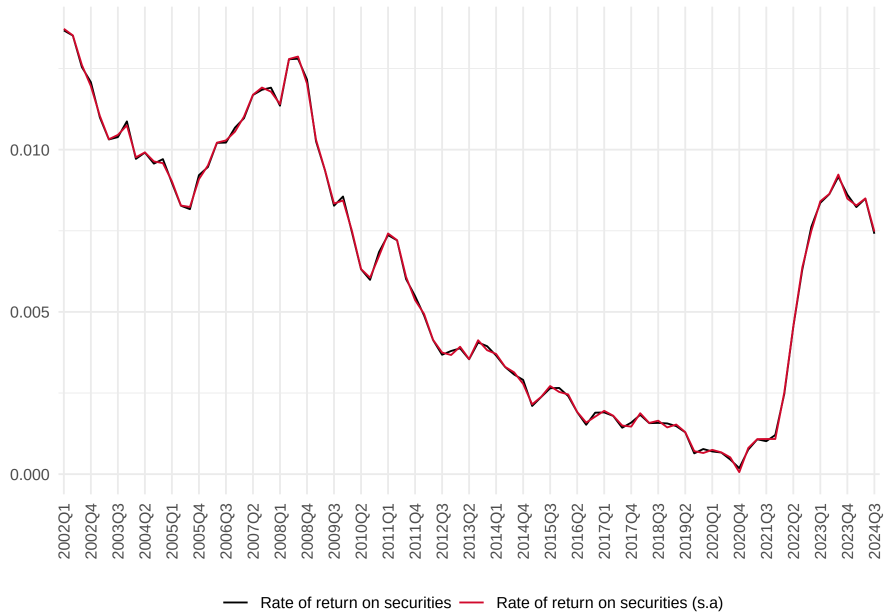
C.5 Financial beta (F.3) estimation

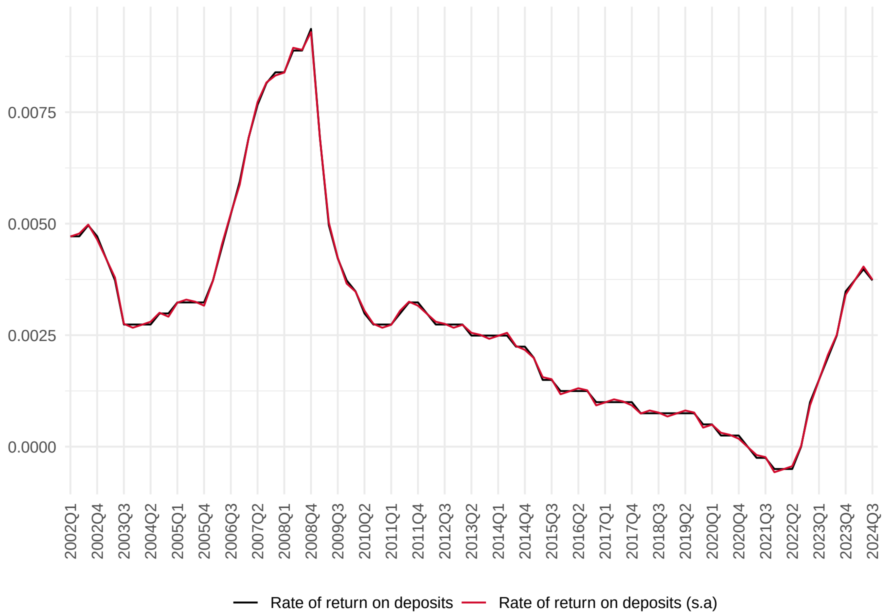
Step 1: Data and unit root testing






10
0
−10
−20
−30
2002Q1
2002Q4
2003Q3
2004Q2
2005Q1
2005Q4
2006Q3
2007Q2
2008Q1
2008Q4
2009Q3
2010Q2
2011Q1
2011Q4
2012Q3
2013Q2
2014Q1
2014Q4
2015Q3
2016Q2
2017Q1
2017Q4
2018Q3
2019Q2
2020Q1
2020Q4
2021Q3
2022Q2
2023Q1
2023Q4
2024Q3
Confidence indicator
Confidence indicator (s.a)


0.010
0.005
0.000
2002Q1
2002Q4
2003Q3
2004Q2
2005Q1
2005Q4
2006Q3
2007Q2
2008Q1
2008Q4
2009Q3
2010Q2
2011Q1
2011Q4
2012Q3
2013Q2
2014Q1
2014Q4
2015Q3
2016Q2
2017Q1
2017Q4
2018Q3
2019Q2
2020Q1
2020Q4
2021Q3
2022Q2
2023Q1
2023Q4
2024Q3
Rate of return on securities
Rate of return on securities (s.a)



Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
$\beta^{H}_{F.3_{sa}}$	-1.1704	-3.45	trend	I(1)
$\frac{YD_H}{FW_H}{}_{sa}$	-1.7621	-2.89	drift	I(1)
$\iota_{F.2_{sa}}$	-2.1057	-2.89	drift	I(I)
$\iota_{F.3_i sa}$	-2.0074	-2.89	drift	I(I)
CI_{sa}	-3.049	-2.89	drift	I(0)

Table 49: Results from ADF Tests for variables entering financial beta estimation.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
2.5179	4	Case II (Restricted intercept and no trend)	[2.56 ; 3.49]	No cointegration
2.3396	4	Case III (Unrestricted intercept and no trend)	[2.86 ; 4.01]	No cointegration

Table 50: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 4$	12.89	12.25
$r \leq 3$	26.41	25.32
$r \leq 2$	45.25	42.44
$r \leq 1$	72.59	62.99
$r = 0$	129.15	87.31

Table 51: Johansen Trace Test Results with trend.

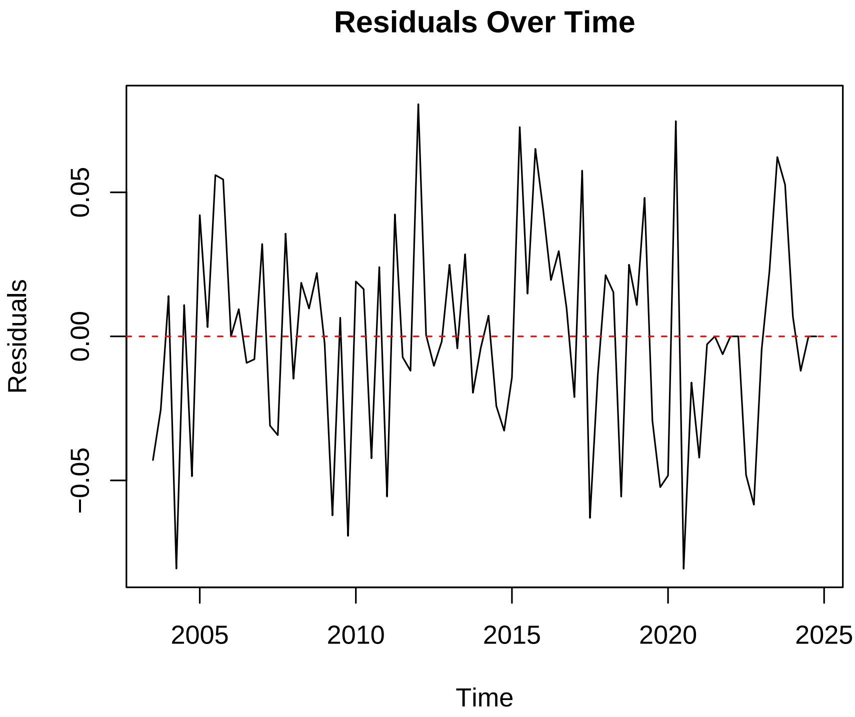
Null Hypothesis	**Test Statistic**	**5% Critical Value**
$r \leq 4$	5.86	9.24
$r \leq 3$	19.41	19.96
$r \leq 2$	35.67	34.91
$r \leq 1$	60.22	53.12
$r = 0$	118.46	76.07

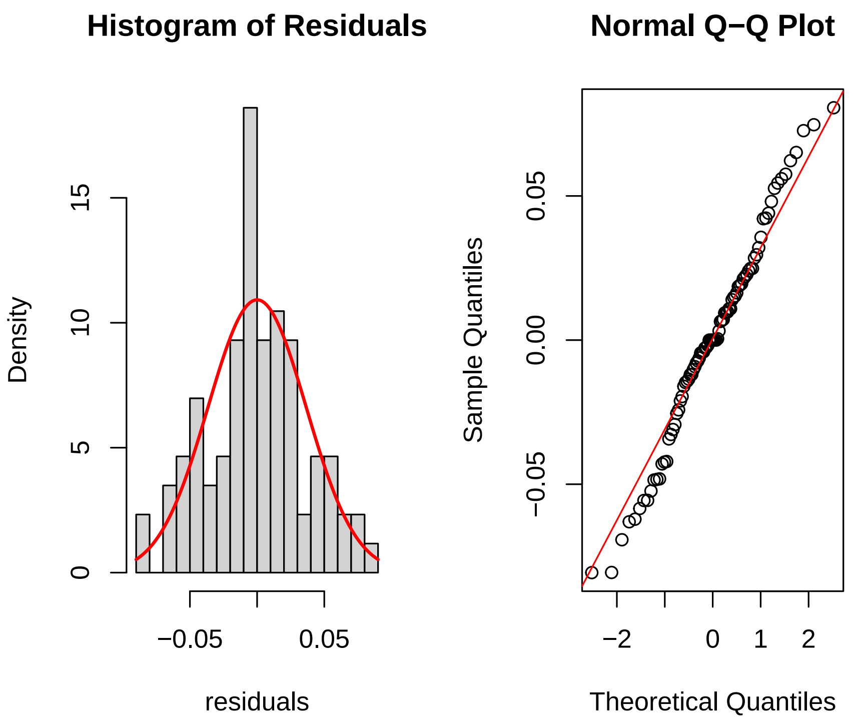
Table 52: Johansen Trace Test Results without trend and constant.

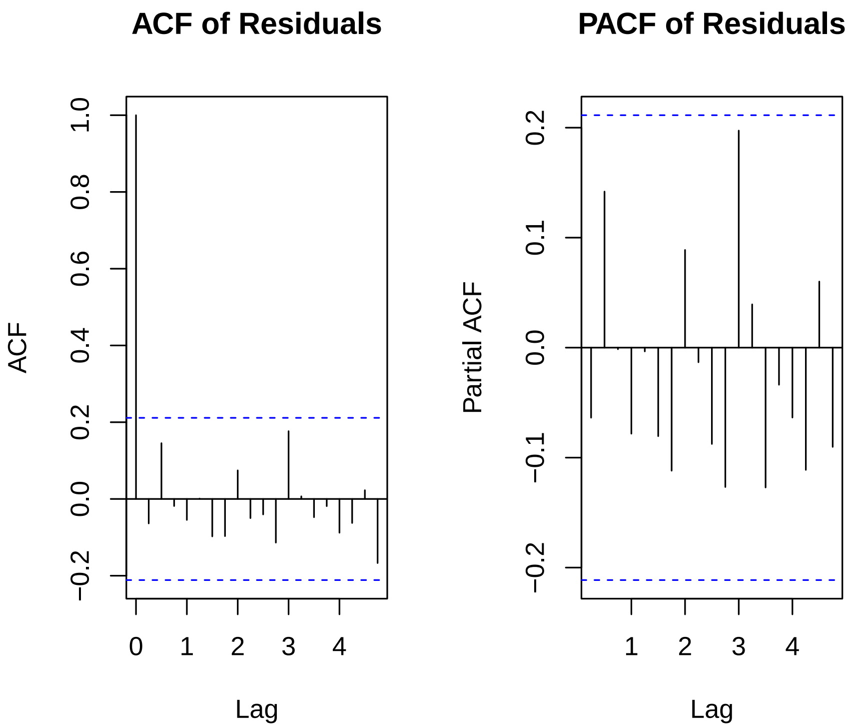
Step 4: Run the specific model

```
Call:
dynlm(formula = d(lbeta_F3_H_sa) ~ L(d(lbeta_F3_H_sa), 2) +
  L(d(lbeta_F3_H_sa), 3) + L(d(lbeta_F3_H_sa), 4) +
  L(d(C_TILLID_t_sa), 3) + d(lYD_H_W_sa) +
  L(d(lYD_H_W_sa), 3) + L(lbeta_F3_H_sa, 1) +
  L(Iota_F3_i_sa, 1) + L(Iota_F2_H_sa, 1) +
  dummy06Q1 + dummy21Q3 + dummy22Q1 +
  dummy22Q2 + dummy24Q3 + dummy24Q4,
  data = data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.08066 -0.02071  0.00000  0.02183  0.08060

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)            -0.304739   0.050190  -6.072 5.85e-08 ***
L(d(lbeta_F3_H_sa), 2) -0.253313   0.061489  -4.120 1.03e-04 ***
L(d(lbeta_F3_H_sa), 3) -0.301420   0.068778  -4.383 4.04e-05 ***
L(d(lbeta_F3_H_sa), 4)  0.433896   0.061863   7.014 1.18e-09 ***
L(d(C_TILLID_t_sa), 3) -0.004671   0.001378  -3.389 1.15e-03 **
d(lYD_H_W_sa)           0.417635   0.083924   4.976 4.46e-06 ***
L(d(lYD_H_W_sa), 3)     0.337753   0.096752   3.491 8.38e-04 ***
L(lbeta_F3_H_sa, 1)    -0.054279   0.010018  -5.418 8.05e-07 ***
L(Iota_F3_i_sa, 1)     20.650061   2.783258   7.419 2.14e-09 ***
L(Iota_F2_H_sa, 1)     -9.825514   3.664808  -2.681 9.15e-03 **
dummy06Q1              -0.079177   0.041959  -1.887 6.33e-02 .
dummy21Q3              -0.135598   0.044465  -3.050 3.24e-03 **
dummy22Q1              -0.115488   0.045712  -2.526 1.38e-02 *
dummy22Q2               0.092422   0.047913   1.929 5.78e-02 .
dummy24Q3              -0.150202   0.044135  -3.403 1.10e-03 **
dummy24Q4              -0.124642   0.043358  -2.875 5.35e-03 **

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.04026 on 70 degrees of freedom
Multiple R-squared:  0.8344,  Adjusted R-squared:  0.7989
F-statistic: 23.51 on 15 and 70 DF,  p-value: < 2.2e-16
```

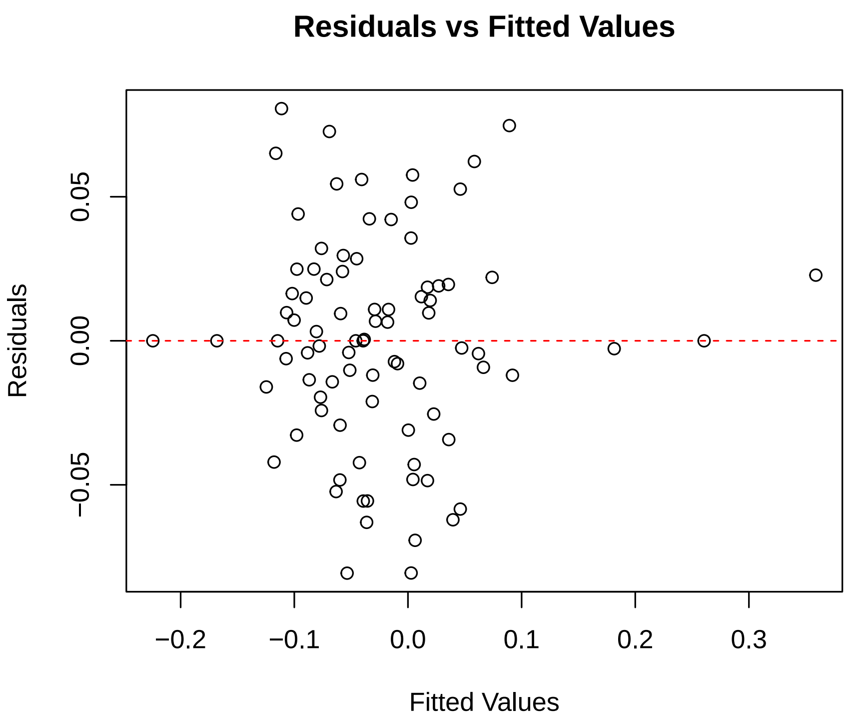
Step 6: Run diagnostic tests



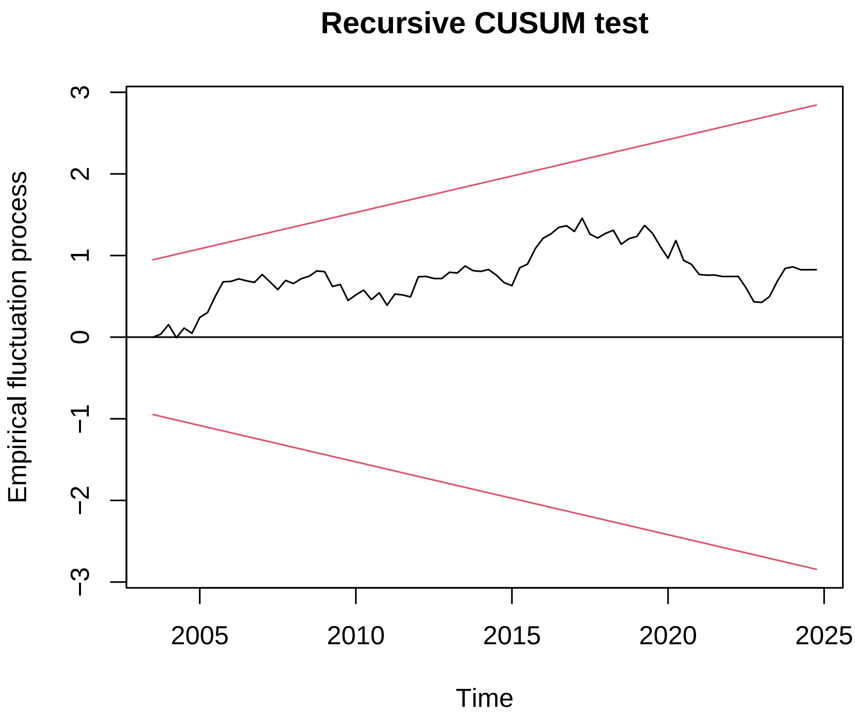




ACF of Residuals
PACF of Residuals
ACF
Partial ACF
Lag
Lag



Residuals vs Fitted Values
Residuals
Fitted Values



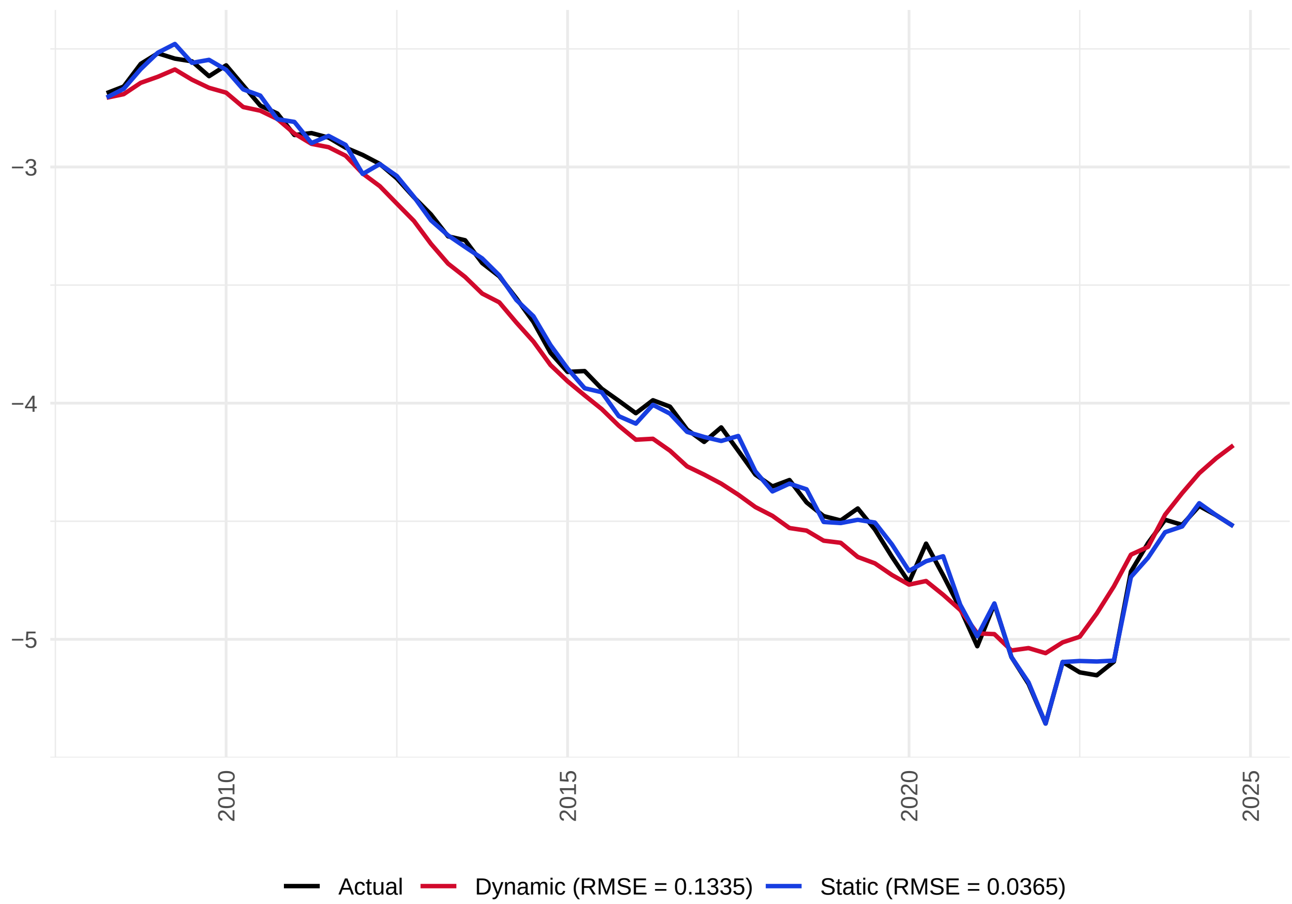
Test	p-value
Shapiro-Wilk	0.4965
Jarque Bera	0.8186
Durbin-Watson	0.5391
Breusch-Godfrey, order 4	0.4381

Table 53: Diagnostics statistics.

Variable	VIF Value
L(d(lbeta_F3_H_sa), 2)	1.608255
L(d(lbeta_F3_H_sa), 3)	2.015541
L(d(lbeta_F3_H_sa), 4)	1.633767
L(d(C_TILLID_t_sa), 3)	1.283498
d(lYD_H_W_sa)	1.314631
L(d(lYD_H_W_sa), 3)	1.783823
L(lbeta_F3_H_sa, 1)	4.643468
L(Iota_F3_i_sa, 1)	6.117724
L(Iota_F2_H_sa, 1)	3.838676
dummy06Q1	1.073409
dummy21Q3	1.205452
dummy22Q1	1.273998
dummy22Q2	1.399646
dummy24Q3	1.187599
dummy24Q4	1.146156

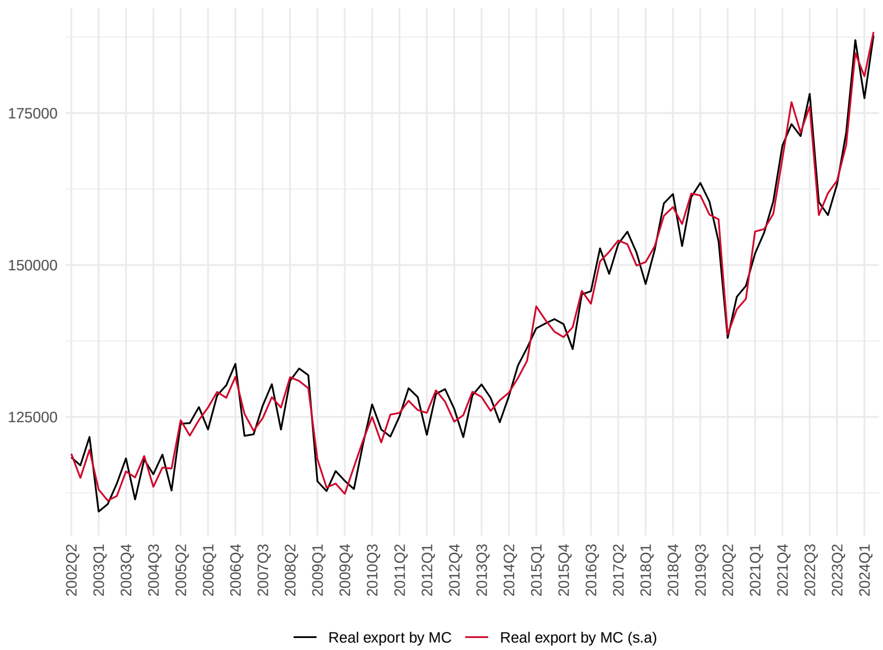
Table 54: Variance Inflation Factor (VIF) results for the financial β equation.

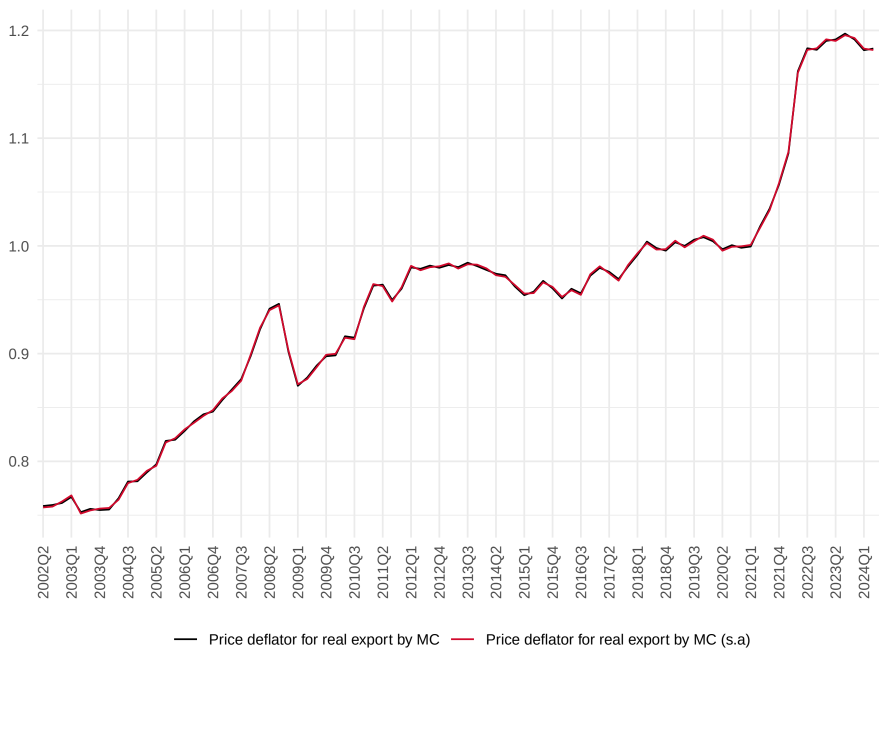
Step 7: Evaluate static and dynamic forecasts

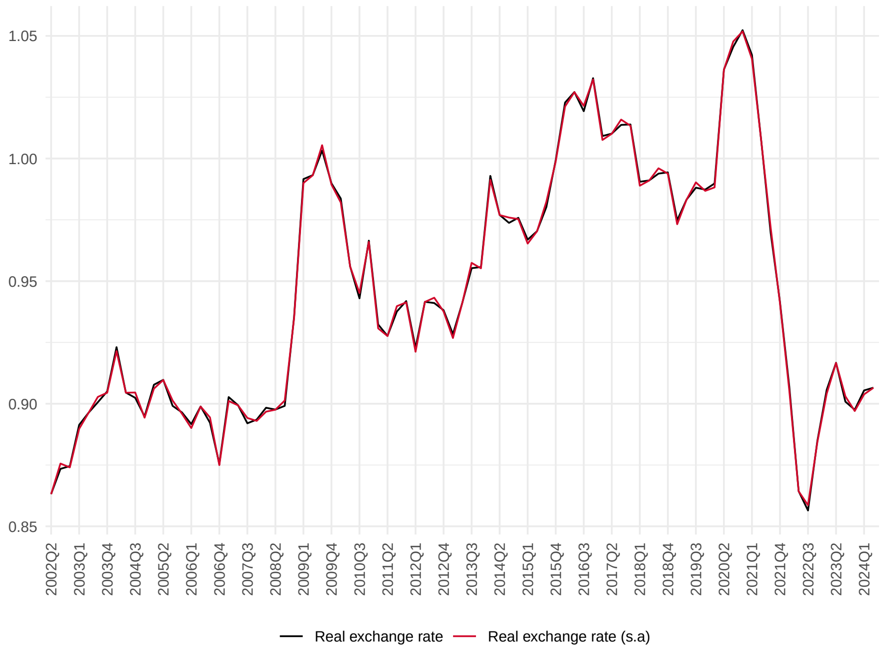


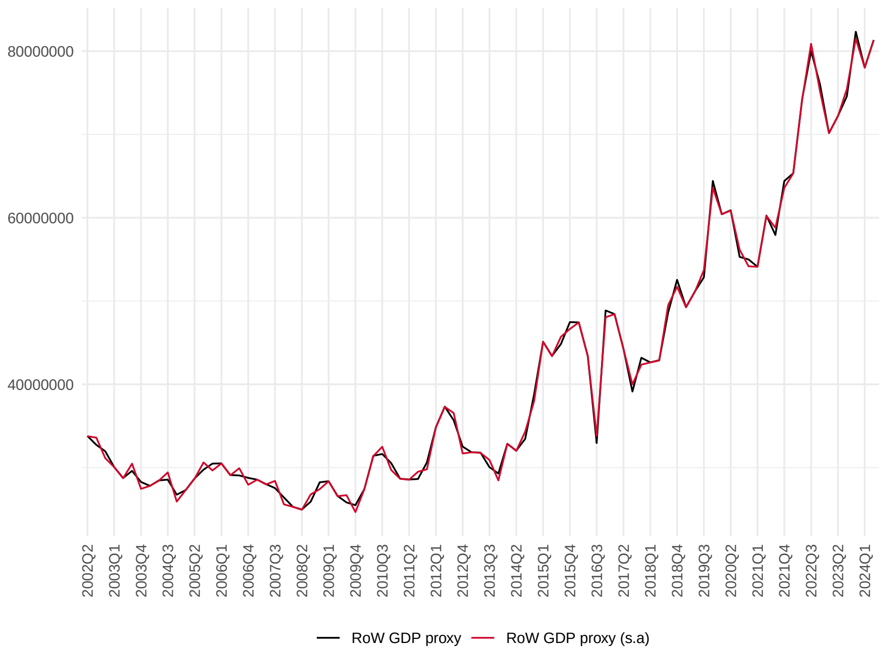
C.6 Real export by the MC industry

Step 1: Data and unit root testing






1.05
1.00
0.95
0.90
0.85
2002Q2
2003Q1
2003Q4
2004Q3
2005Q2
2006Q1
2006Q4
2007Q3
2008Q2
2009Q1
2009Q4
2010Q3
2011Q2
2012Q1
2012Q4
2013Q3
2014Q2
2015Q1
2015Q4
2016Q3
2017Q2
2018Q1
2018Q4
2019Q3
2020Q2
2021Q1
2021Q4
2022Q3
2023Q2
2024Q1
Real exchange rate
Real exchange rate (s.a)


80000000
60000000
40000000
2002Q2
2003Q1
2003Q4
2004Q3
2005Q2
2006Q1
2006Q4
2007Q3
2008Q2
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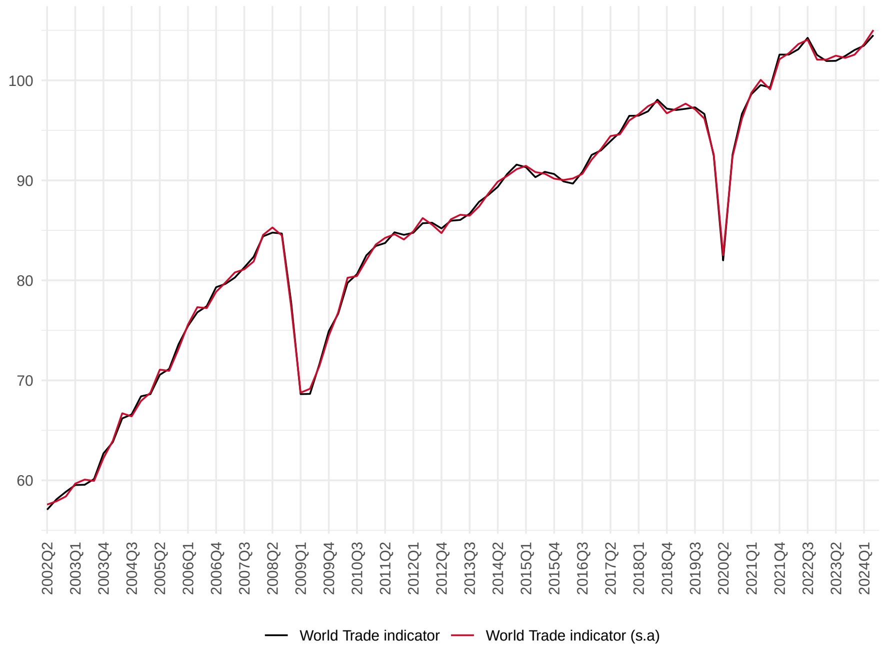
2010Q3
2011Q2
2012Q1
2012Q4
2013Q3
2014Q2
2015Q1
2015Q4
2016Q3
2017Q2
2018Q1
2018Q4
2019Q3
2020Q2
2021Q1
2021Q4
2022Q3
2023Q2
2024Q1
RoW GDP proxy
RoW GDP proxy (s.a)



Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
x_{sa}^{MC}	-2.2314	-3.45	trend	I(1)
$p_{X\,sa}^{MC}$	-2.3202	-2.89	trend	I(1)
GDP_{sa}^{RoW}	-2.1234	-2.89	trend	I(I)
e_{sa}^{MC}	-2.3102	-2.89	drift	I(1)
$WT_{sa}^{indicator}$	-1.6621	-2.89	drift	I(I)

Table 55: Results from ADF Tests for variables entering real export by MC estimation.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
3.5263	4	Case II (Restricted intercept and no trend)	[2.56 ; 3.49]	Cointegration
3.5429	4	Case III (Unrestricted intercept and no trend)	[2.86 ; 4.01]	Inconclusive

Table 56: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 4$	5.89	12.25
$r \leq 3$	14.17	25.32
$r \leq 2$	27.77	42.44
$r \leq 1$	51.62	62.99
$r = 0$	87.58	87.31

Table 57: Johansen Trace Test Results with trend.

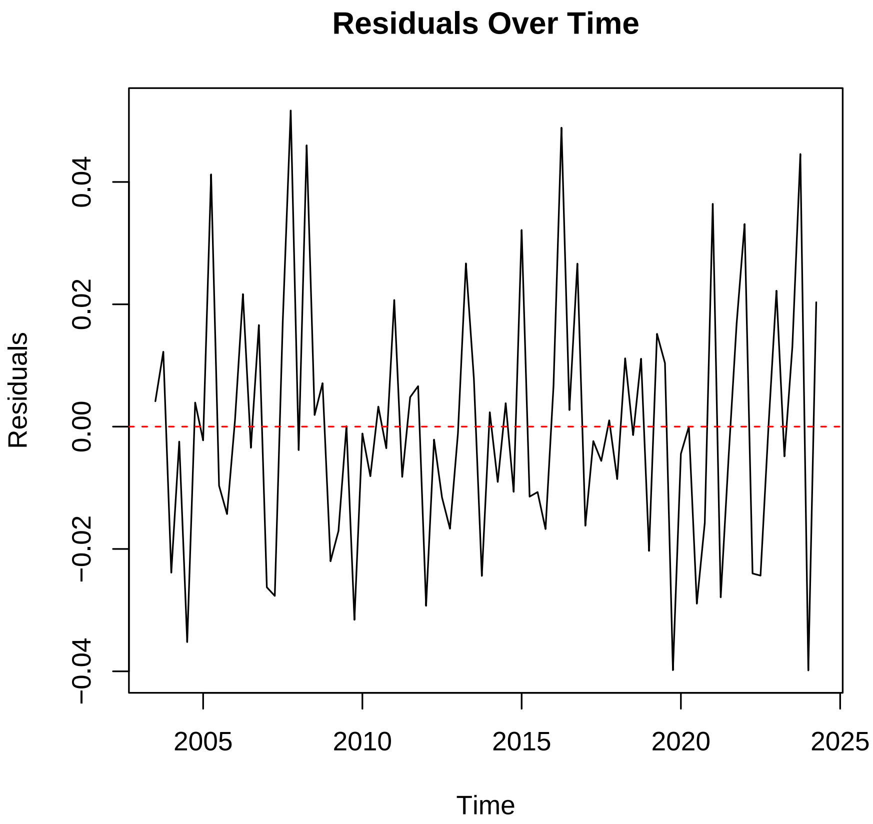
Null Hypothesis	**Test Statistic**	**5% Critical Value**
$r \leq 4$	5.62	9.24
$r \leq 3$	12.96	19.96
$r \leq 2$	23.88	34.91
$r \leq 1$	45.20	53.12
$r = 0$	77.90	76.07

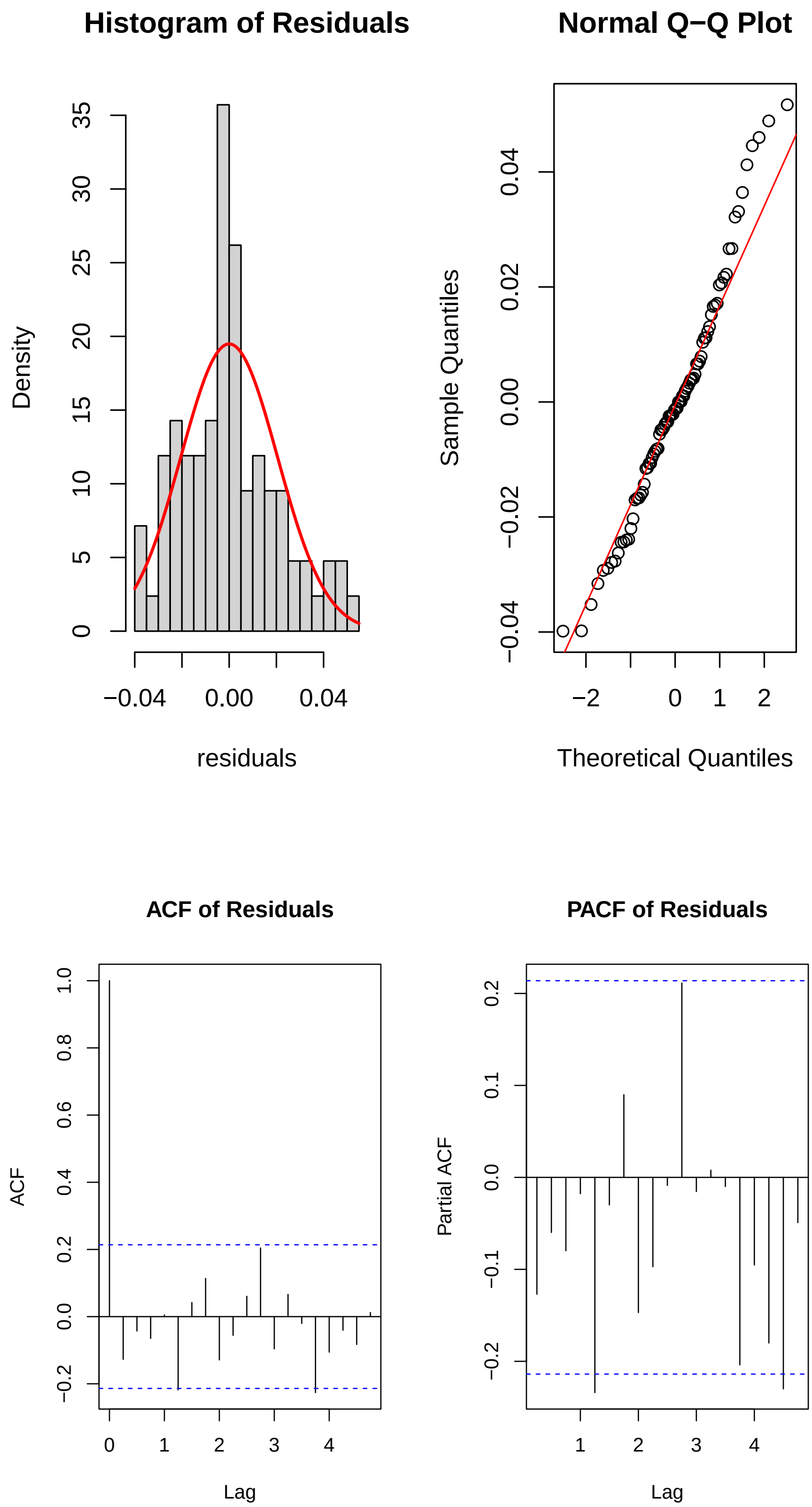
Table 58: Johansen Trace Test Results without trend and constant.

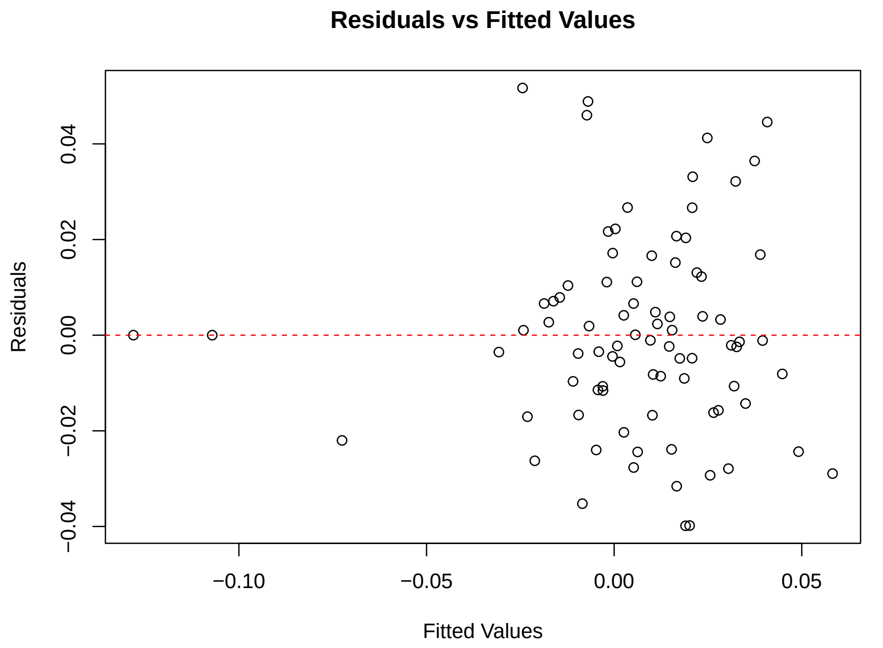
Step 4: Run the specific model

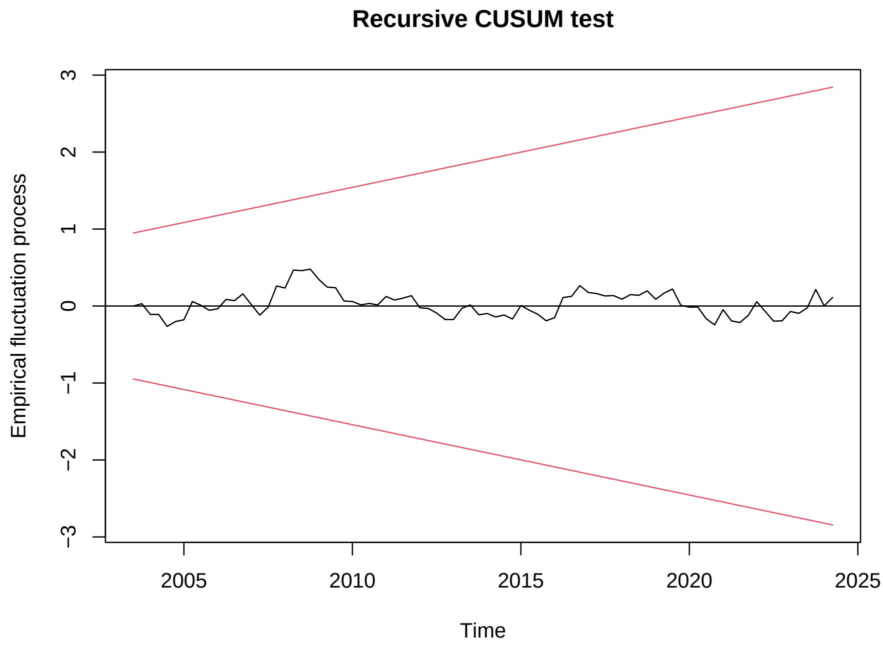
```
Call:
dynlm(formula = d(lx_MC_sa) ~ d(lpx_MC_sa) + L(d(lpx_MC_sa), 1) +
  d(le_sa) + d(lRoW_GDP_sa) + L(d(lRoW_GDP_sa), 4) + d(lWT_indicator_sa) +
  L(lx_MC_sa, 1) + L(lRoW_GDP_sa, 1) + dummy20 + dummy22, data = data_x)

Residuals:
      Min        1Q    Median        3Q       Max
-0.039855 -0.012259 -0.001257  0.011119  0.051682

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)               0.38233    0.34282   1.115 0.268394
d(lpx_MC_sa)             -0.91268    0.23405  -3.899 0.000212 ***
L(d(lpx_MC_sa), 1)        0.49307    0.19526   2.525 0.013737 *
d(le_sa)                 -0.43273    0.17740  -2.439 0.017147 *
d(lRoW_GDP_sa)            0.12986    0.03108   4.178 0.0000805 ***
L(d(lRoW_GDP_sa), 4)     -0.05118    0.02977  -1.719 0.089856 .
d(lWT_indicator_sa)       0.48571    0.12104   4.013 0.000144 ***
L(lx_MC_sa, 1)           -0.11205    0.05451  -2.056 0.043396 *
L(lRoW_GDP_sa, 1)         0.05433    0.02009   2.704 0.008512 **
dummy20                  -0.07297    0.02682  -2.721 0.008134 **
dummy22                  -0.09738    0.02407  -4.045 0.000129 ***

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.02183 on 73 degrees of freedom
Multiple R-squared: 0.6587,    Adjusted R-squared: 0.6119
F-statistic: 14.09 on 10 and 73 DF,  p-value: 0.00000000000000001748
```

Step 6: Run diagnostic tests




Histogram of Residuals
Density
residuals
Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
ACF of Residuals
ACF
Lag
PACF of Residuals
Partial ACF
Lag





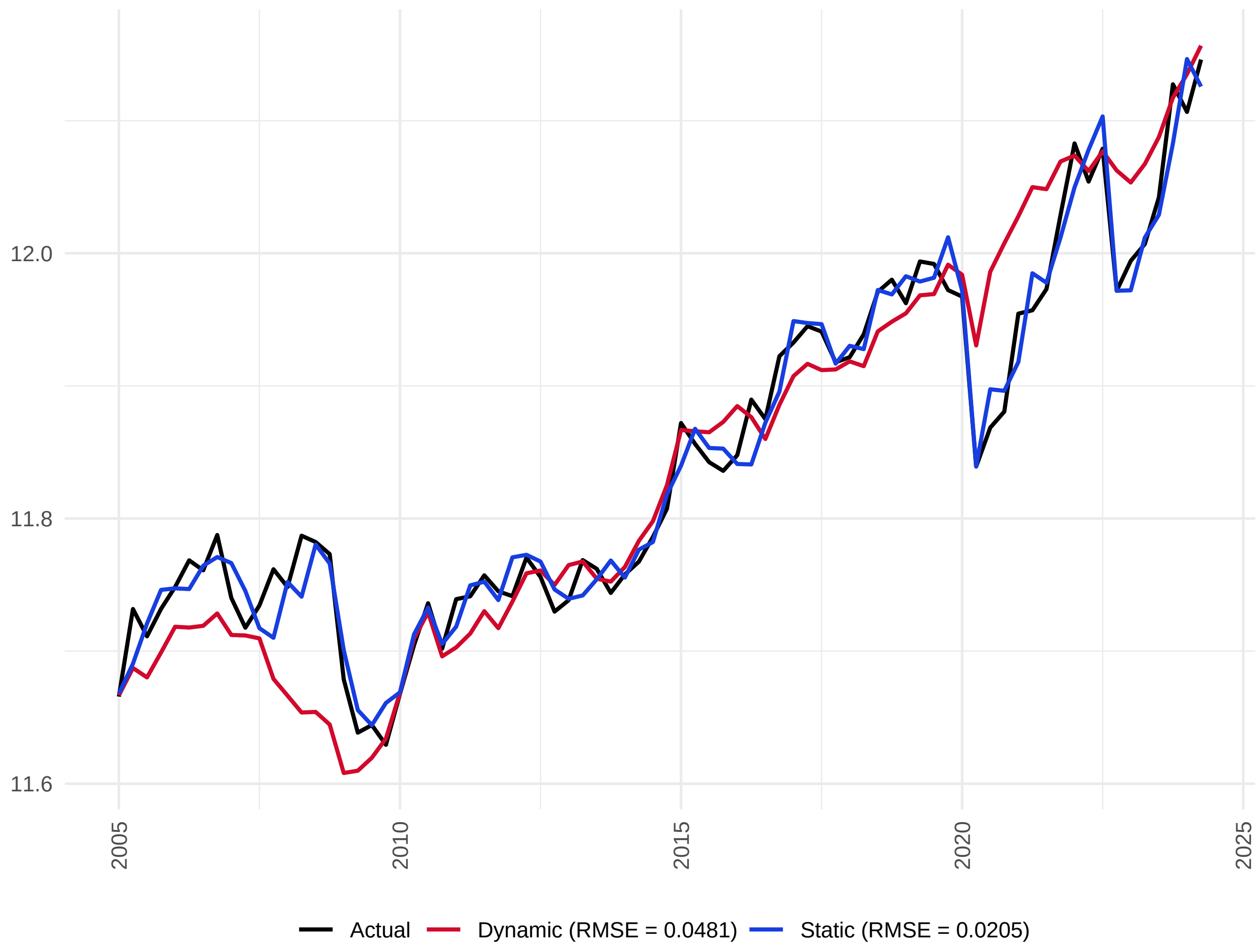
Test	p-value
Shapiro-Wilk	0.1023
Jarque Bera	0.2602
Durbin-Watson	0.7931
Breusch-Godfrey, order 4	0.4342

Table 59: Diagnostics statistics.

Variable	VIF Value
d(lpx_MC_sa)	1.852658
L(d(lpx_MC_sa), 1)	1.346782
d(le_sa)	1.886695
d(lRoW_GDP_sa)	1.187205
L(d(lRoW_GDP_sa), 4)	1.095833
d(lWT_indicator_sa)	2.128232
L(lx_MC_sa, 1)	8.982946
L(lRoW_GDP_sa, 1)	8.652930
dummy20	1.491511
dummy22	1.201965

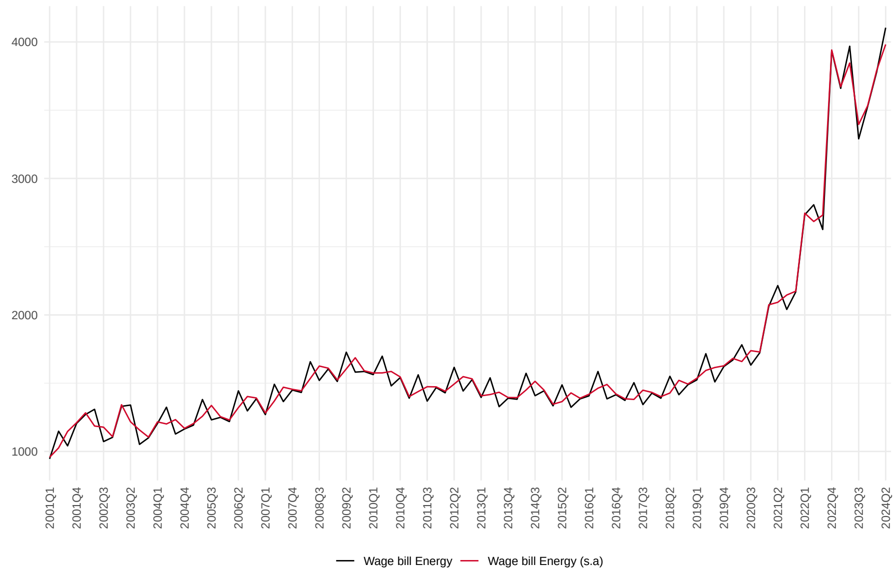
Table 60: Variance Inflation Factor (VIF) results for the MC export equation.

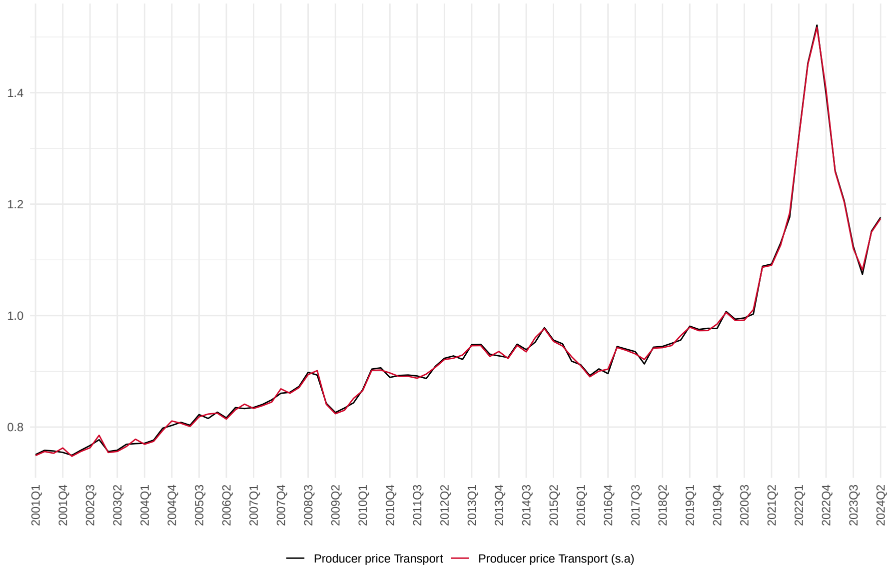
Step 7: Evaluate static and dynamic forecasts

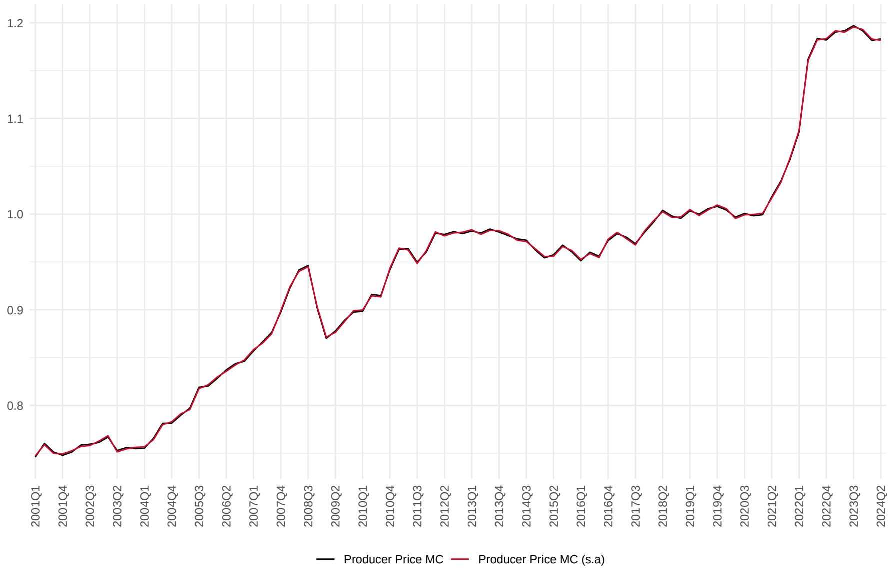


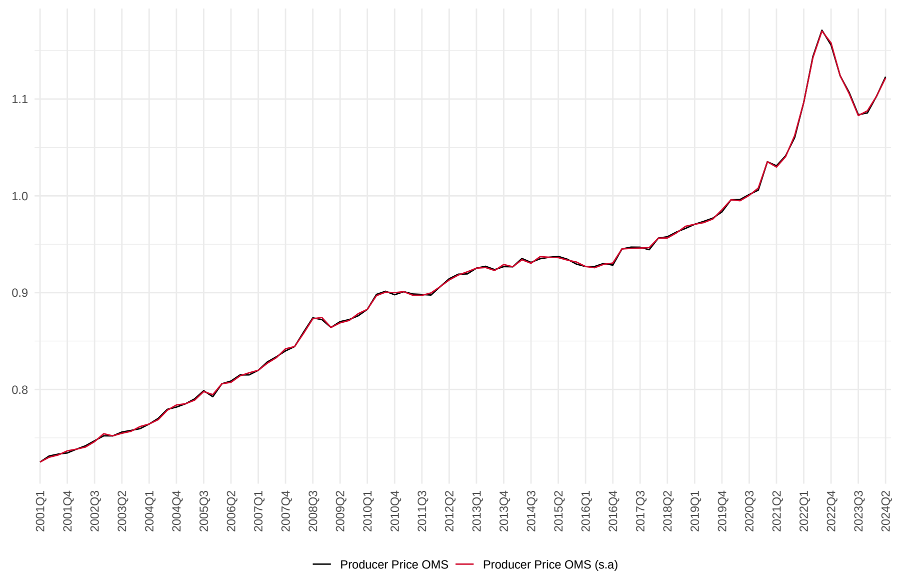
C.7 Producer prices - Energy

Step 1: Data and unit root testing




1.4
1.2
1.0
0.8
2001Q1
2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
2006Q2
2007Q1
2007Q4
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Producer price Transport
Producer price Transport (s.a)


1.2
1.1
1.0
0.9
0.8
2001Q1
2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
2006Q2
2007Q1
2007Q4
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Producer Price MC
Producer Price MC (s.a)



Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
pp^{E}_{sa}	-0.4511	-3.45	trend	I(1)
pp^{MC}_{sa}	-2.3694	-3.45	trend	I(1)
pp^{OMS}_{sa}	-3.0282	-3.45	trend	I(1)
pp^{T}_{sa}	-1.855	-2.89	drift	I(1)
$D1^{E}_{sa}$	2.7502	-1.95	none	I(1)

Table 61: Results from ADF Tests for variables entering estimation for producer prices for the energy industry.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
2.5049	4	Case II (Restricted intercept and no trend)	[2.56 ; 3.49]	No cointegration
1.6781	4	Case III (Unrestricted intercept and no trend)	[2.86 ; 4.01]	No cointegration

Table 62: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 4$	7.46	12.25
$r \leq 3$	21.58	25.32
$r \leq 2$	42.18	42.44
$r \leq 1$	84.99	62.99
$r = 0$	182.12	87.31

Table 63: Johansen Trace Test Results with trend.

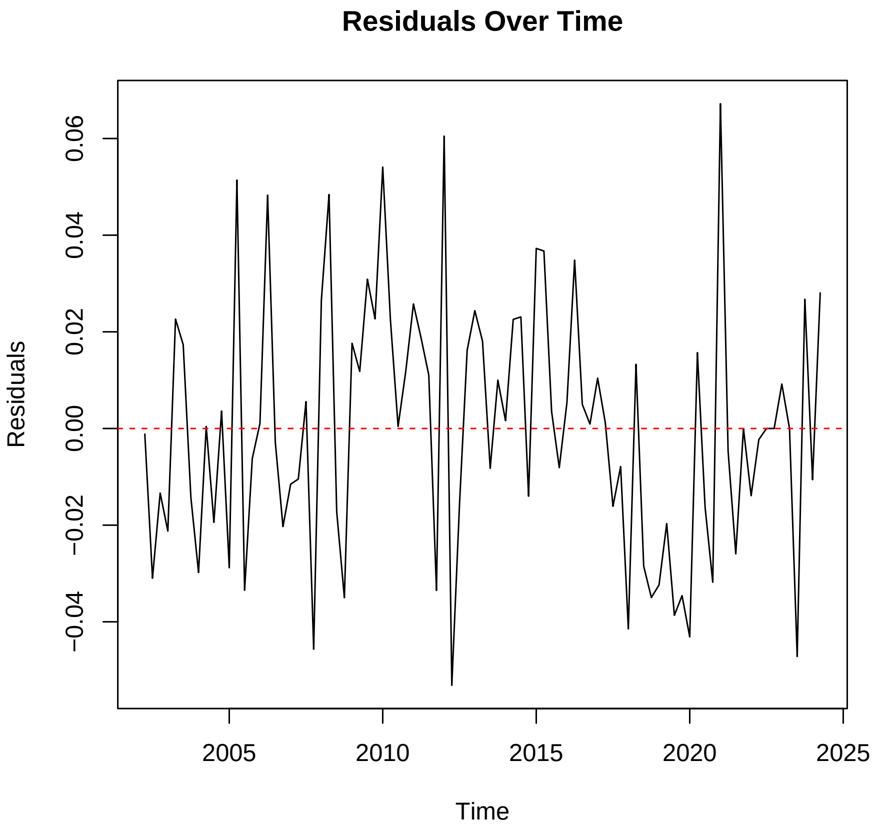
Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 4$	4.42	9.24
$r \leq 3$	20.12	19.96
$r \leq 2$	52.58	34.91
$r \leq 1$	106.35	53.12
$r = 0$	166.60	76.07

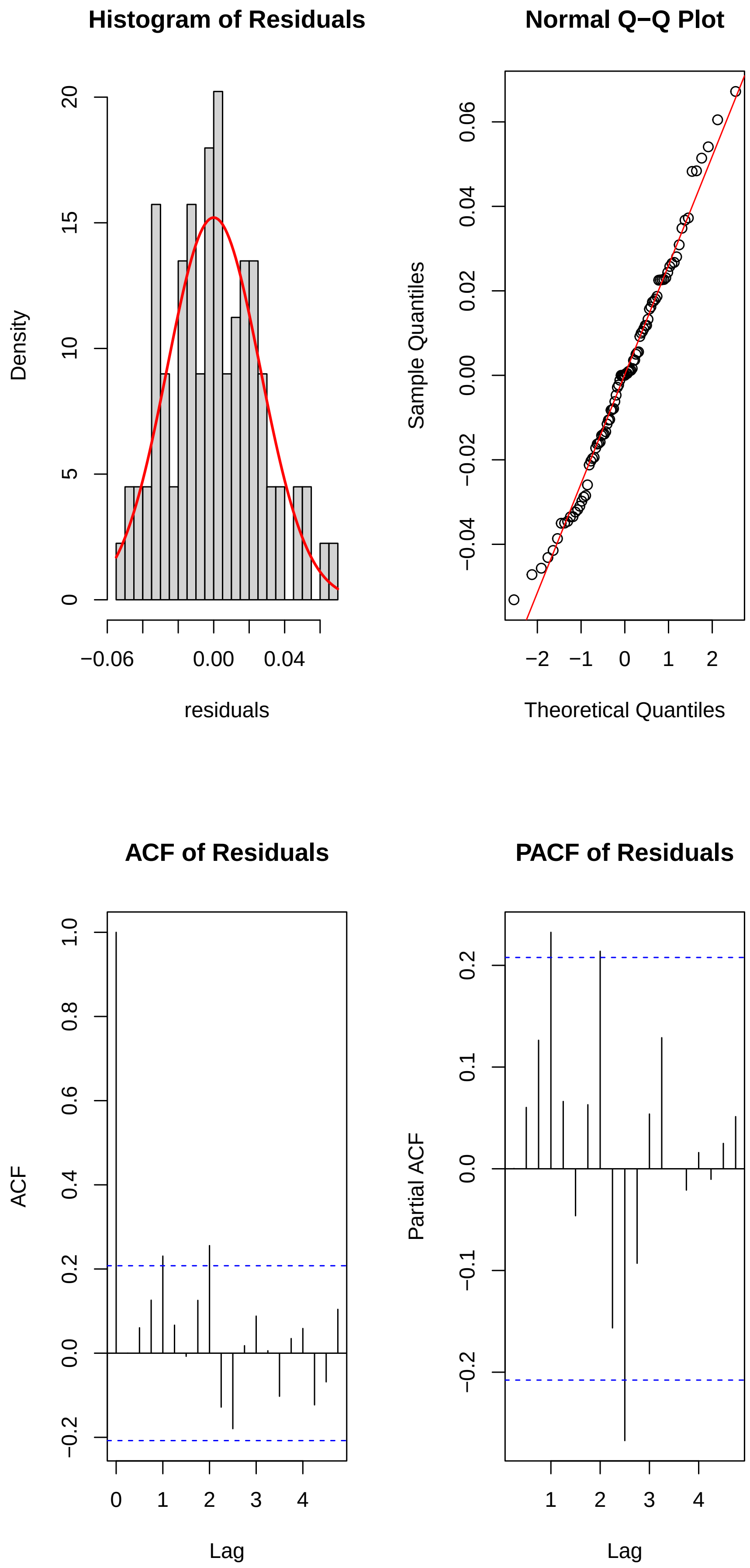
Table 64: Johansen Trace Test Results without trend and constant.

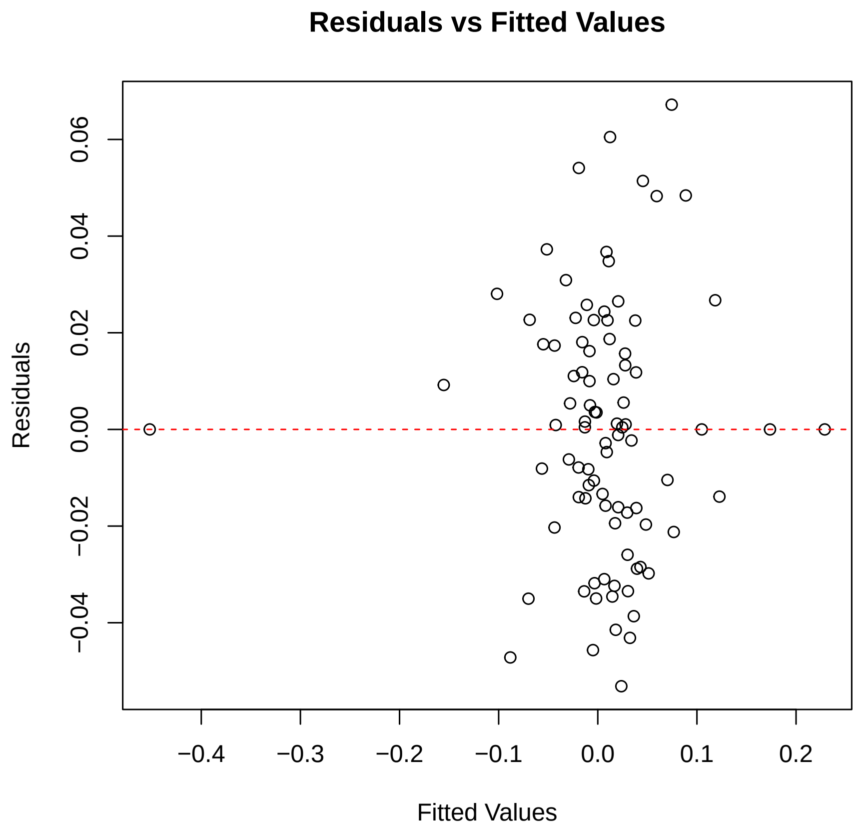
Step 4: Run the specific model

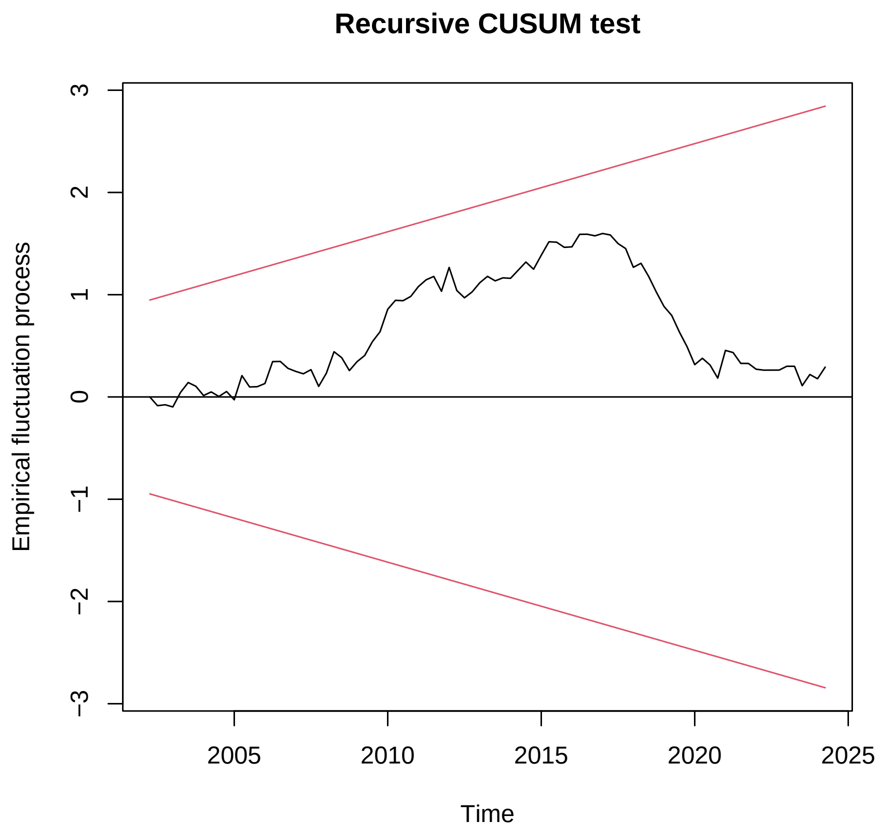
```
Call:
dynlm(formula = d(lpp_E_sa) ~ L(d(lpp_E_sa), 1) + L(d(lpp_E_sa), 4) +
  d(LD1_E_sa) + L(d(lpp_T_sa), 2) + d(lpp_MC_sa) + d(lpp_OMS_sa) +
  L(lpp_E_sa, 1) + L(lpp_MC_sa, 1) + dummy21 + dummy22 + dummy22.4 + dummy23,
  data = data_PP_E)

Residuals:
     Min       1Q   Median       3Q      Max
-0.05313 -0.01721  0.00000  0.01762  0.06720

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)            0.01029    0.00538   1.912  0.059648 .
L(d(lpp_E_sa), 1)      0.15617    0.06160   2.535  0.013290 *
L(d(lpp_E_sa), 4)      0.30141    0.04598   6.555 5.98e-09 ***
d(LD1_E_sa)            0.23792    0.06122   3.887 0.000216 ***
L(d(lpp_T_sa), 2)      0.41749    0.13669   3.054  0.003110 **
d(lpp_MC_sa)           0.59697    0.26348   2.266  0.026314 *
d(lpp_OMS_sa)          1.02584    0.46099   2.225  0.029027 *
L(lpp_E_sa, 1)        -0.36901    0.05611  -6.577 5.45e-09 ***
L(lpp_MC_sa, 1)        0.49418    0.08681   5.692 2.24e-07 ***
dummy21                0.10218    0.02949   3.466 0.000873 ***
dummy22                0.14350    0.03322   4.319 4.68e-05 ***
dummy22.4              0.16891    0.03974   4.251 5.99e-05 ***
dummy23               -0.25757    0.03939  -6.538 6.42e-09 ***

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.02823 on 76 degrees of freedom
Multiple R-squared:  0.8824,  Adjusted R-squared:  0.8638
F-statistic: 47.51 on 12 and 76 DF,  p-value: < 2.2e-16
```

Step 6: Run diagnostic tests




Histogram of Residuals
Density
residuals
0
5
10
15
20
−0.06
0.00
0.04
Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
−0.04
−0.02
0.00
0.02
0.04
0.06
−2
−1
0
1
2
ACF of Residuals
ACF
Lag
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
PACF of Residuals
Partial ACF
Lag
−0.2
−0.1
0.0
0.1
0.2
1
2
3
4


Residuals vs Fitted Values
0.06
0.04
0.02
0.00
−0.02
−0.04
Residuals
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
Fitted Values


Recursive CUSUM test
3
2
1
0
−1
−2
−3
Empirical fluctuation process
2005
2010
2015
2020
2025
Time

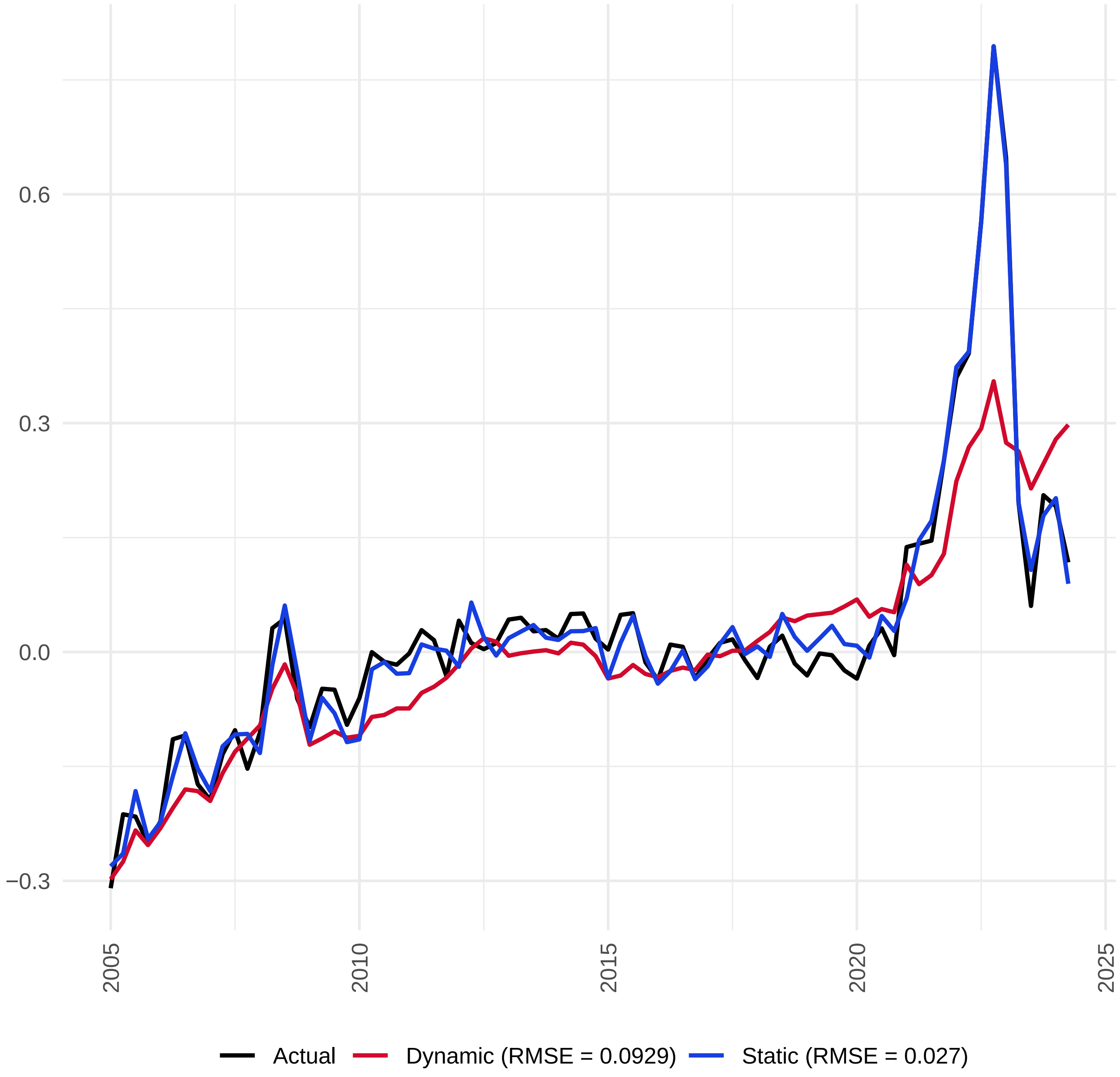
Test	**p-value**
Shapiro-Wilk	0.98653
Jarque Bera	0.5215
Durbin-Watson	0.3887
Breusch-Godfrey, order 4	0.09259

Table 65: Diagnostics statistics.

Variable	**VIF Value**
L(d(lpp_E_sa), 1)	2.423697
L(d(lpp_E_sa), 4)	1.254063
d(LD1_E_sa)	1.994610
L(d(lpp_T_sa), 2)	1.926409
d(lpp_MC_sa)	1.472704
d(lpp_OMS_sa)	2.205576
L(lpp_E_sa, 1)	14.974625
L(lpp_MC_sa, 1)	12.130784
dummy21	1.078968
dummy22	1.370004
dummy22.4	1.959806
dummy23	1.926058

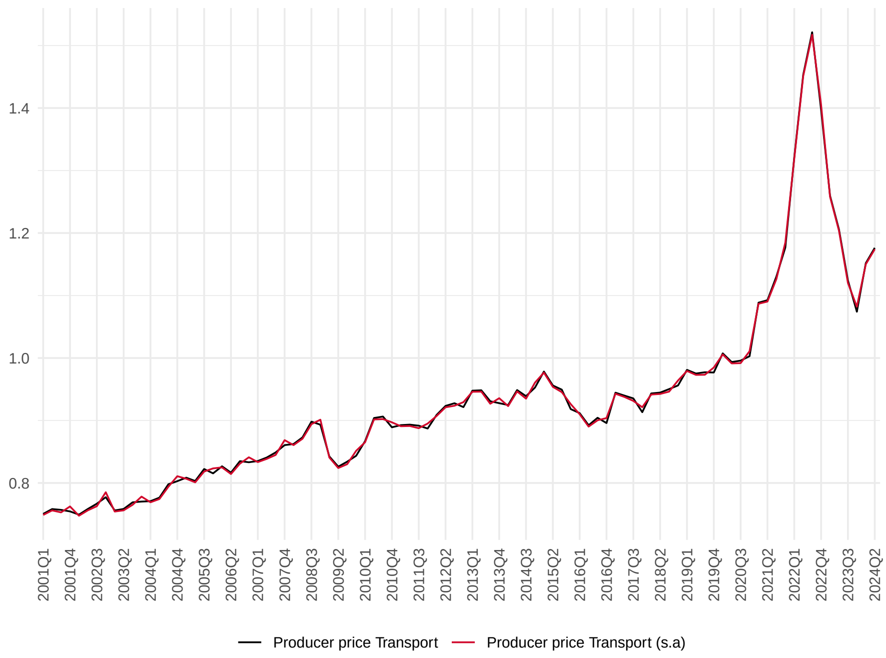
Table 66: Variance Inflation Factor (VIF) results for the pp^E regression.

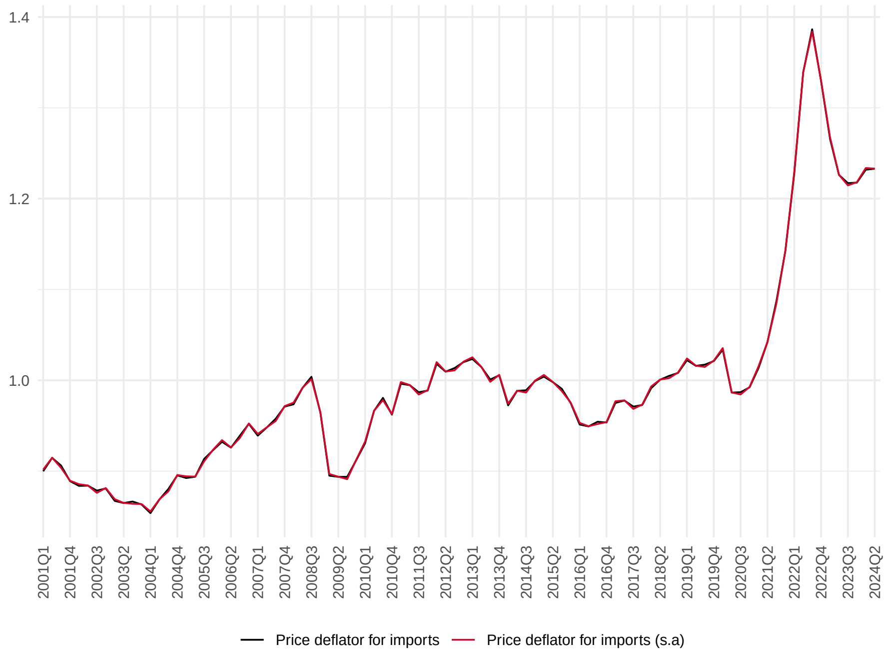
Step 7: Evaluate static and dynamic forecasts

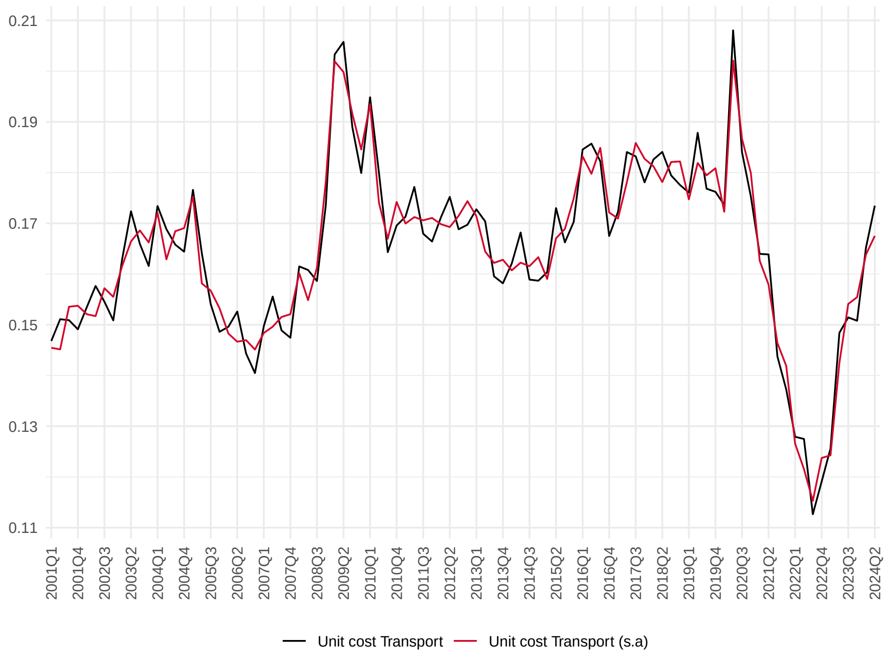


C.8 Producer prices - Transport

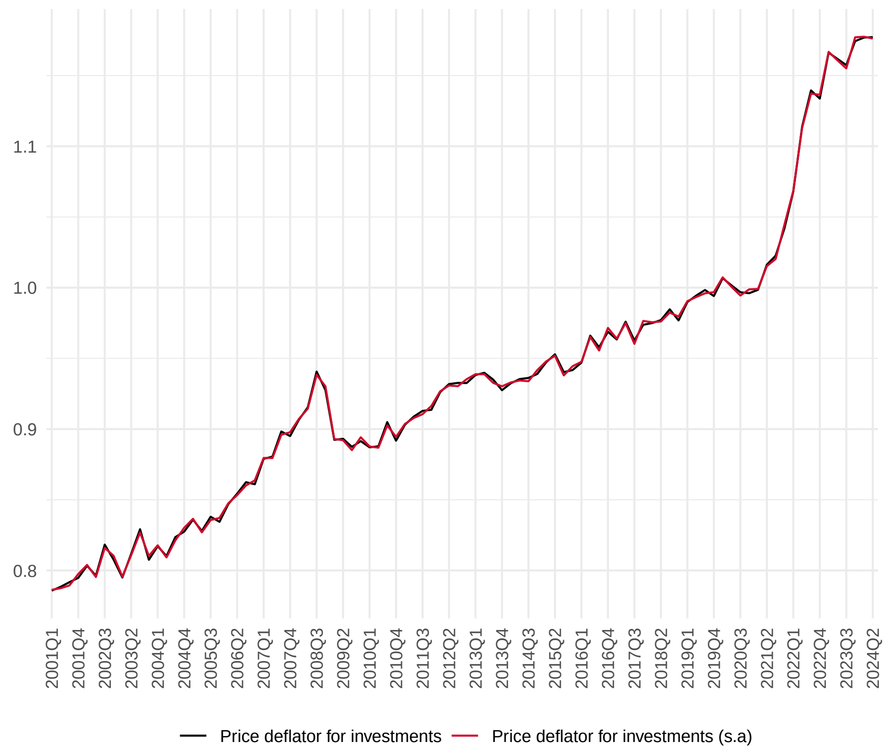
Step 1: Data and unit root testing



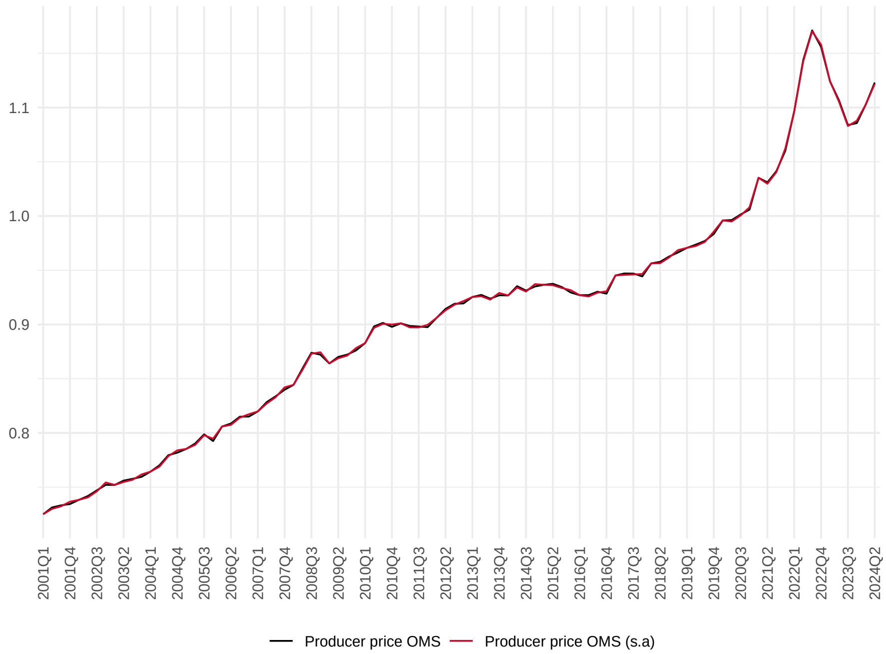


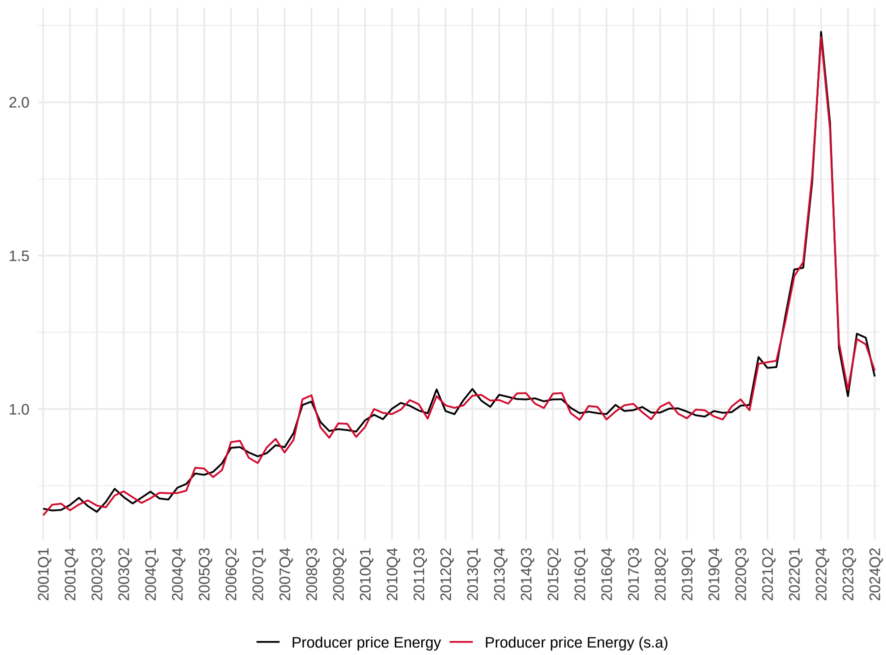


0.21
0.19
0.17
0.15
0.13
0.11
2001Q1
2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
2006Q2
2007Q1
2007Q4
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Unit cost Transport
Unit cost Transport (s.a)



1.1
1.0
0.9
0.8
2001Q1
2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
2006Q2
2007Q1
2007Q4
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Price deflator for investments
Price deflator for investments (s.a)


1.1
1.0
0.9
0.8
2001Q1
2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
2006Q2
2007Q1
2007Q4
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Producer price OMS
Producer price OMS (s.a)


2.0
1.5
1.0
2001Q1
2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
2006Q2
2007Q1
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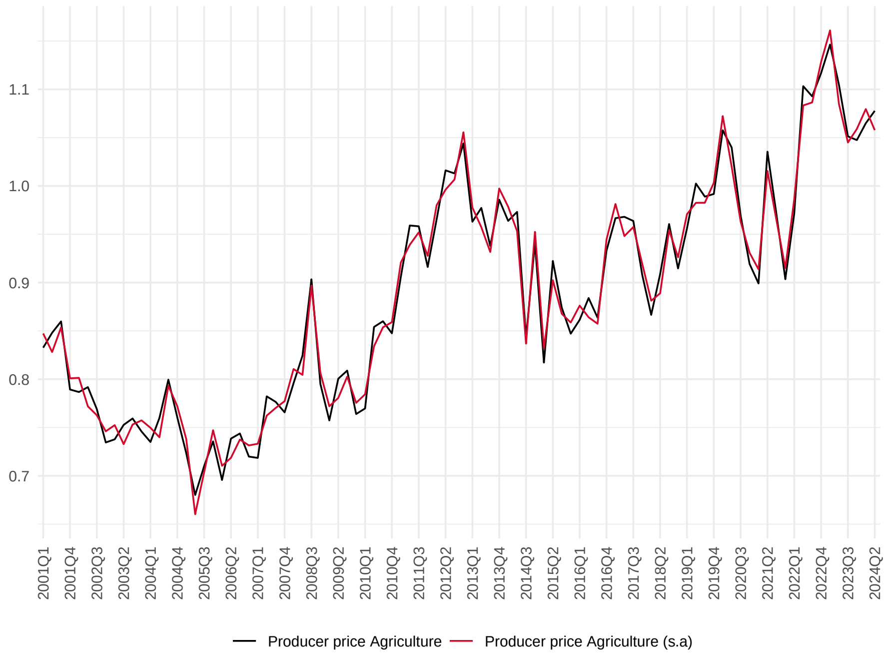
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Producer price Energy
Producer price Energy (s.a)



Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
pp_{sa}^{T}	-2.548	-3.45	trend	I(1)
$p_{IM_{sa}}$	-3.3032	-3.45	trend	I(1)
$p_{I_{sa}}$	-0.4928	-3.45	trend	I(1)
pp_{sa}^{A}	-3.3388	-3.45	trend	I(I)
UC_{sa}^{T}	-2.5651	-2.89	drift	I(I)
pp_{sa}^{E}	-0.4511	-1.95	none	I(1)

Table 67: Results from ADF Tests for variables entering transports producer prices estimation.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
13.7	6	Case II (Restricted intercept and no trend)	[2.27 ; 3.28]	Cointegration
11.723	6	Case III (Unrestricted intercept and no trend)	[2.45 ; 3.61]	Cointegration

Table 68: Results from ARDL Bounds Test.

Null Hypothesis	**Test Statistic**	**5% Critical Value**
$r \leq 6$	4.95	12.25
$r \leq 5$	21.75	25.32
$r \leq 4$	43.27	42.44
$r \leq 3$	69.65	62.99
$r \leq 2$	107.61	87.31
$r \leq 1$	160.02	114.90
$r = 0$	260.65	146.76

Table 69: Johansen Trace Test Results with trend.

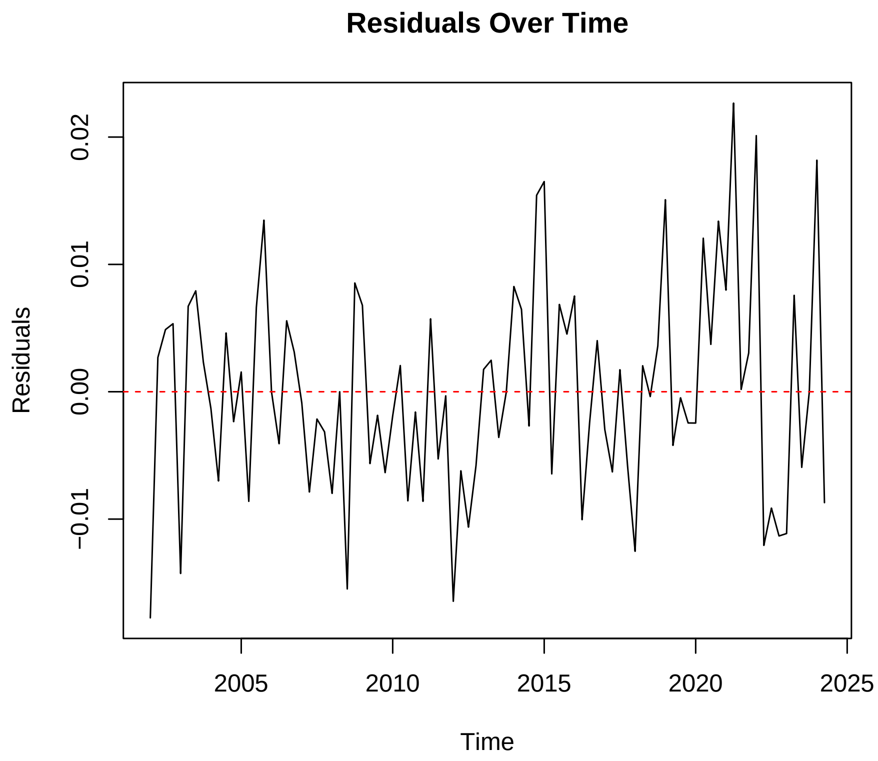
Null Hypothesis	**Test Statistic**	**5% Critical Value**
$r \leq 6$	5.94	9.24
$r \leq 5$	20.73	19.96
$r \leq 4$	39.94	34.91
$r \leq 3$	67.31	53.12
$r \leq 2$	104.27	76.07
$r \leq 1$	154.49	102.14
$r = 0$	265.44	131.70

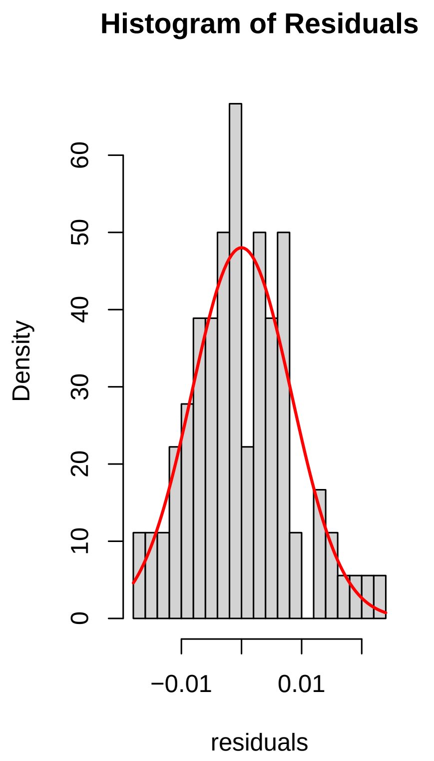
Table 70: Johansen Trace Test Results without trend and constant.

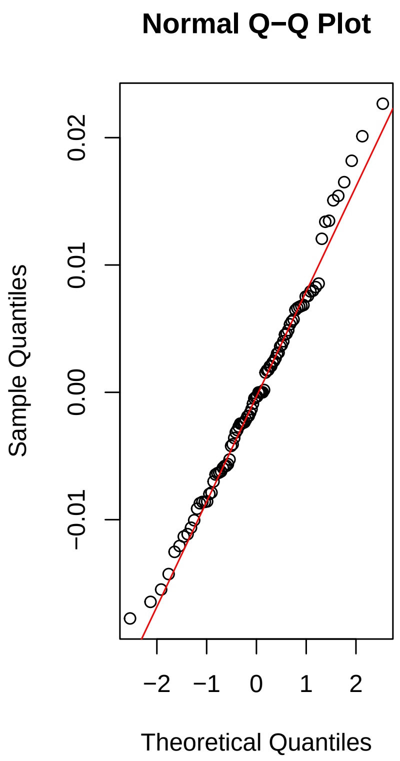
Step 4: Run the specific model

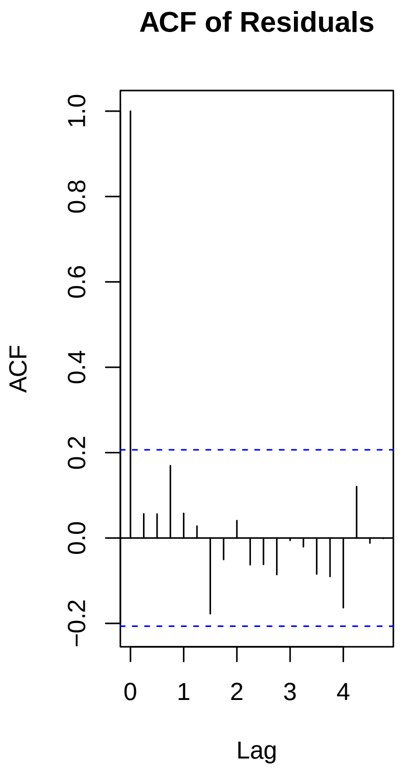
```
Call:
dynlm(formula = d(lpp_T_sa) ~ d(lpim_sa) + L(d(lpim_sa), 1) +
  L(d(lUC_T_sa), 3) + L(d(lP_I_sa), 1) + d(lpp_OMS_sa) +
  L(lpp_T_sa, 1) + L(lpp_E_sa, 1) + L(lpp_A_sa, 1) +
  dummy06 + dummy08 + dummy23, data = data_PP_T)

Residuals:
     Min        1Q    Median        3Q       Max
-0.0177562 -0.0059012 -0.0003502  0.0052279  0.0226657

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)             -0.008822   0.001989  -4.436 0.0000296589 ***
d(lpim_sa)               0.433003   0.067813   6.385 0.0000000113 ***
L(d(lpim_sa), 1)         0.123907   0.058450   2.120 0.037196 *
L(d(lUC_T_sa), 3)        0.066447   0.022062   3.012 0.003499 **
L(d(lP_I_sa), 1)         0.291727   0.090938   3.208 0.001940 **
d(lpp_OMS_sa)            2.095427   0.169845  12.337 < 0.0000000000000000002 ***
L(lpp_T_sa, 1)          -0.114448   0.020899  -5.476 0.0000005122 ***
L(lpp_E_sa, 1)           0.041461   0.013059   3.175 0.002147 **
L(lpp_A_sa, 1)           0.051211   0.012836   3.990 0.000148 ***
dummy06                 -0.015818   0.009433  -1.677 0.097573 .
dummy08                 -0.027139   0.009083  -2.988 0.003752 **
dummy23                 -0.025980   0.009444  -2.751 0.007387 **

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.008875 on 78 degrees of freedom
Multiple R-squared: 0.9282,    Adjusted R-squared: 0.918
F-statistic: 91.64 on 11 and 78 DF,  p-value: < 0.00000000000000000022
```

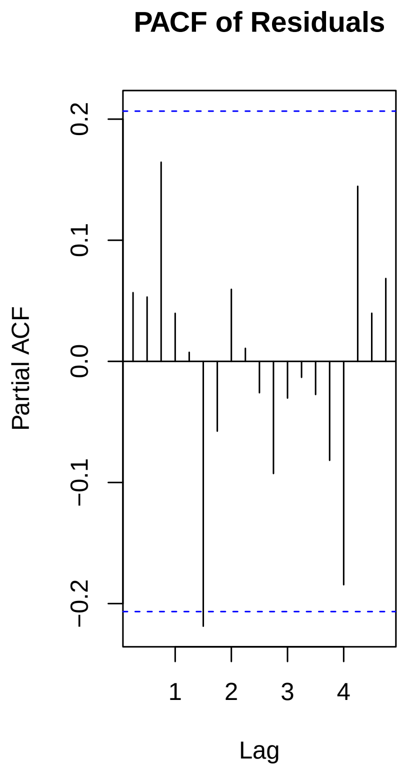
Step 6: Run diagnostic tests

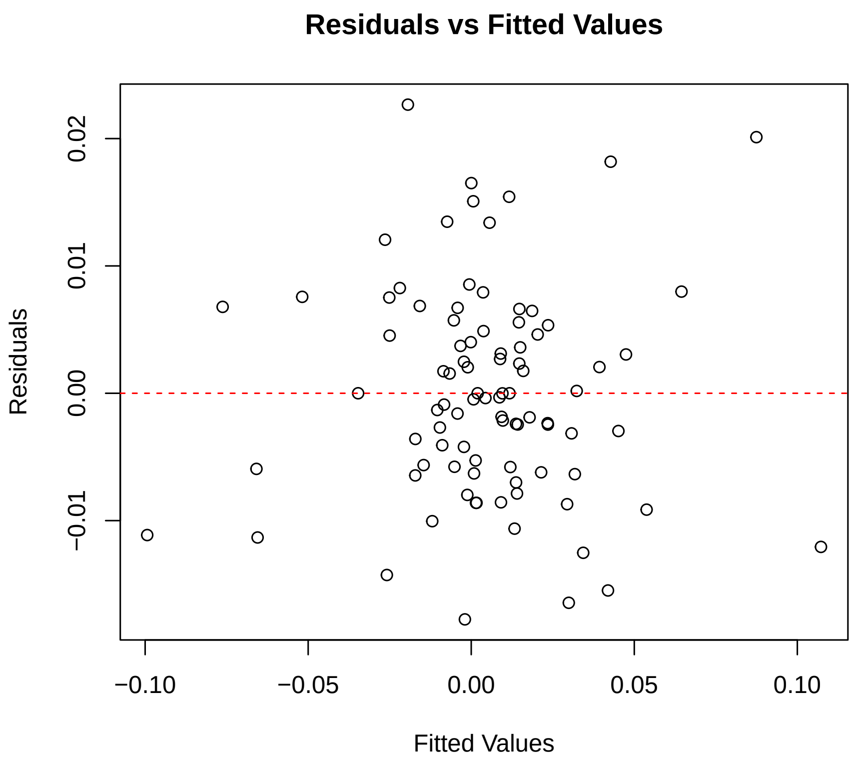


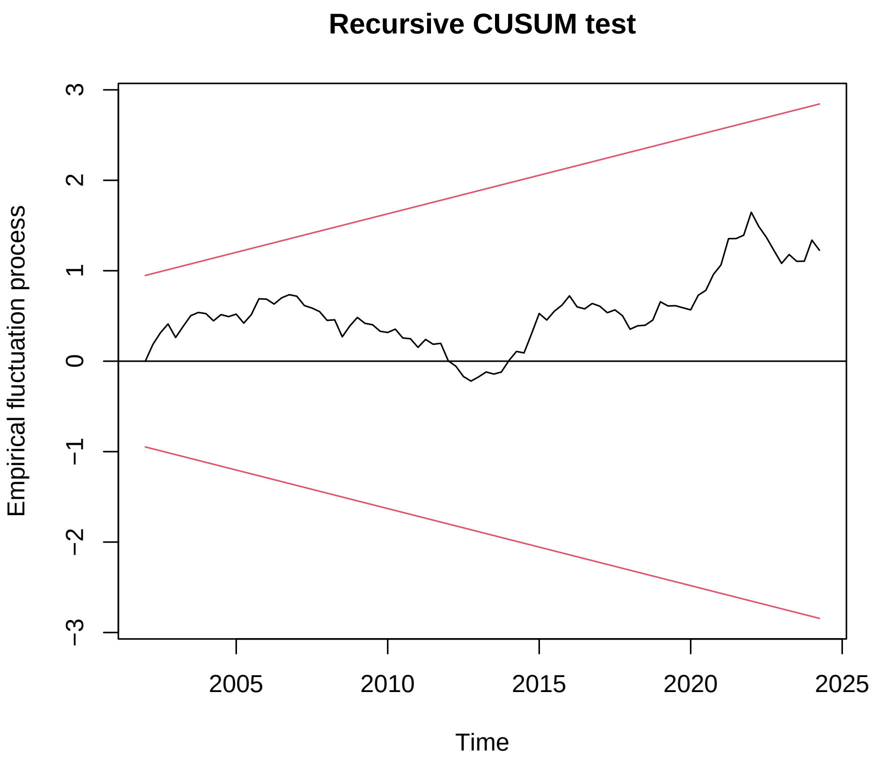





ACF of Residuals
ACF
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
1
2
3
4
Lag


PACF of Residuals
Partial ACF
0.2
0.1
0.0
−0.1
−0.2
1
2
3
4
Lag


Residuals vs Fitted Values
Residuals
0.02
0.01
0.00
−0.01
−0.10
−0.05
0.00
0.05
0.10
Fitted Values



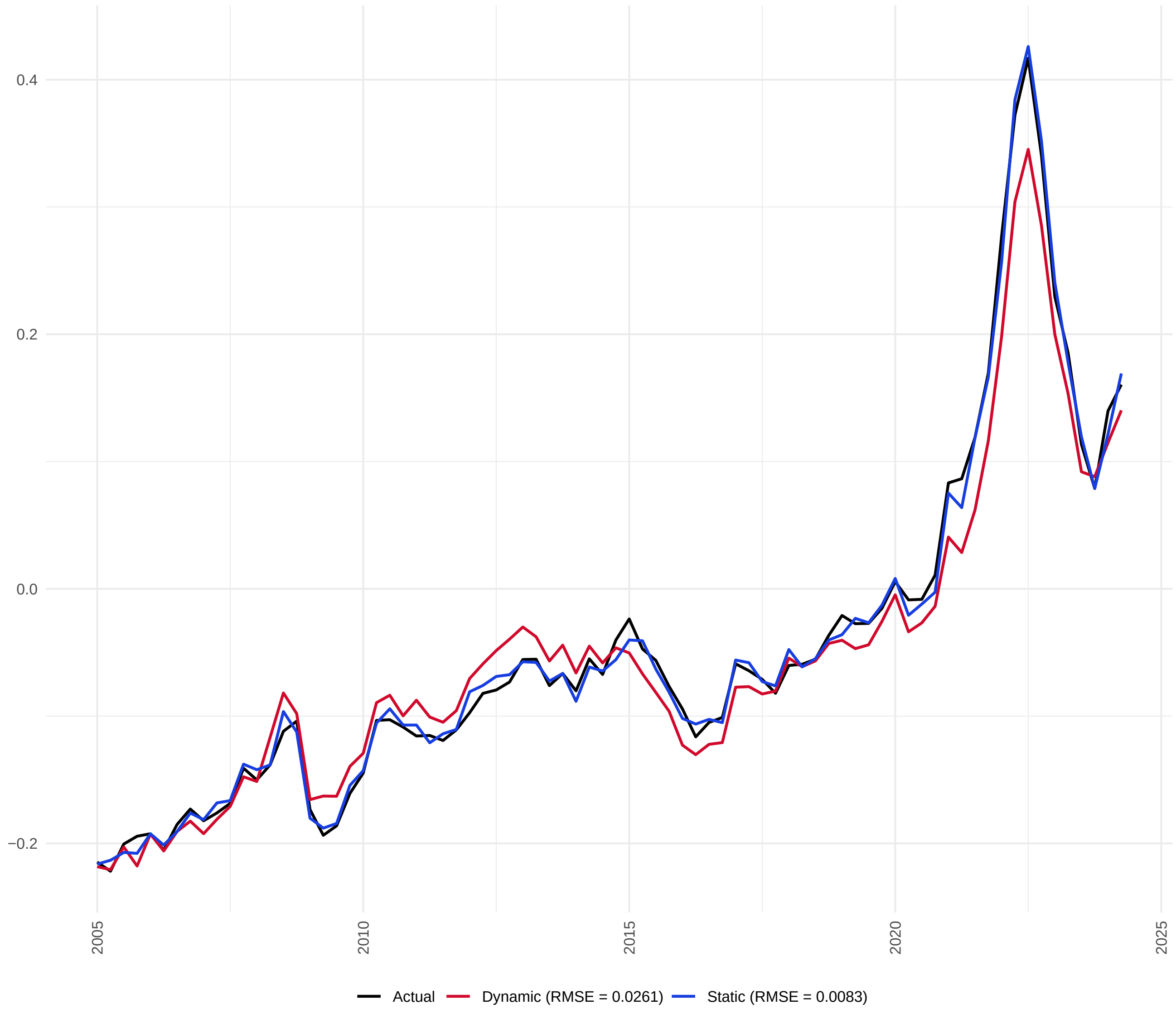
Test	**p-value**
Shapiro-Wilk	0.5089
Jarque Bera	0.4193
Durbin-Watson	0.1271
Breusch-Godfrey, order 4	0.3128

Table 71: Diagnostics statistics.

Variable	**VIF Value**
d(lpim_sa)	2.662464
L(d(lpim_sa), 1)	1.993466
L(d(lUC_T_sa), 3)	1.261681
L(d(lP_I_sa), 1)	1.356055
d(lpp_OMS_sa)	3.030579
L(lpp_T_sa, 1)	10.430771
L(lpp_E_sa, 1)	8.504214
L(lpp_A_sa, 1)	3.229775
dummy06	1.117221
dummy08	1.035733
dummy23	1.119896

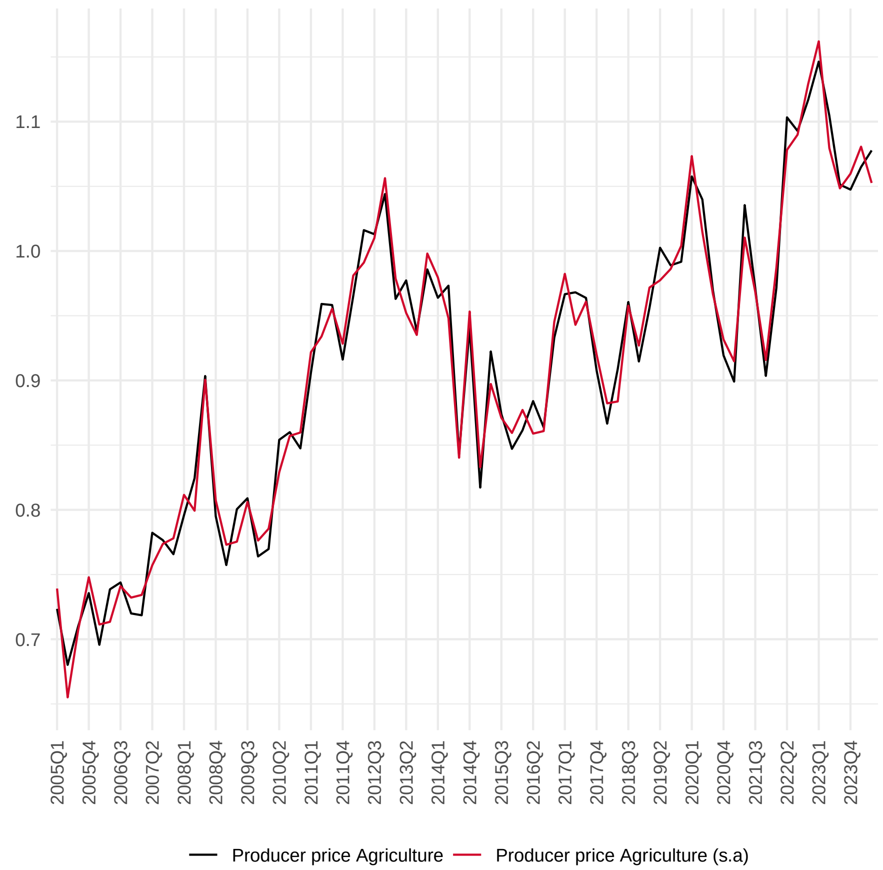
Table 72: Variance Inflation Factor (VIF) results for the pp^T regression.

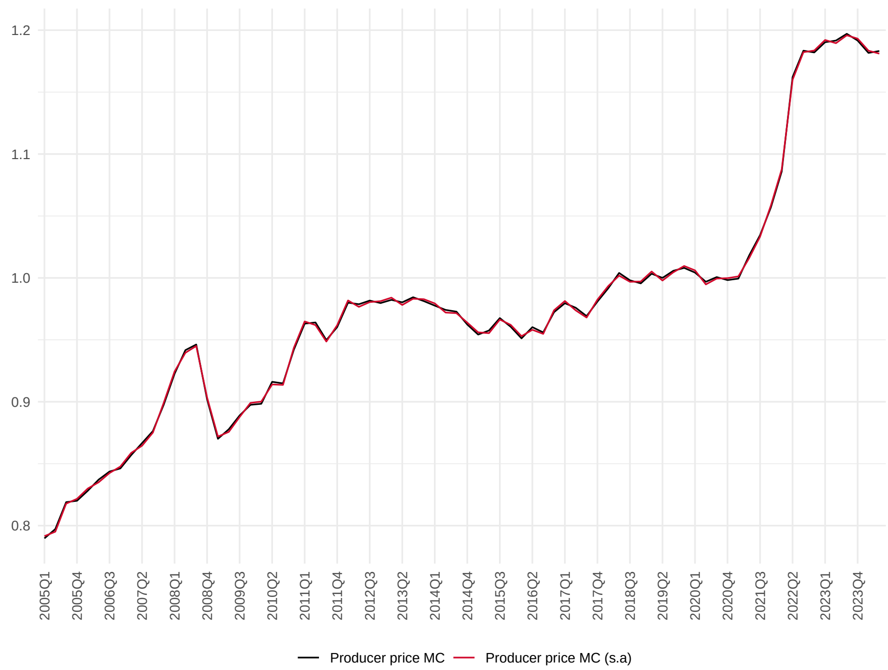
Step 7: Evaluate static and dynamic forecasts

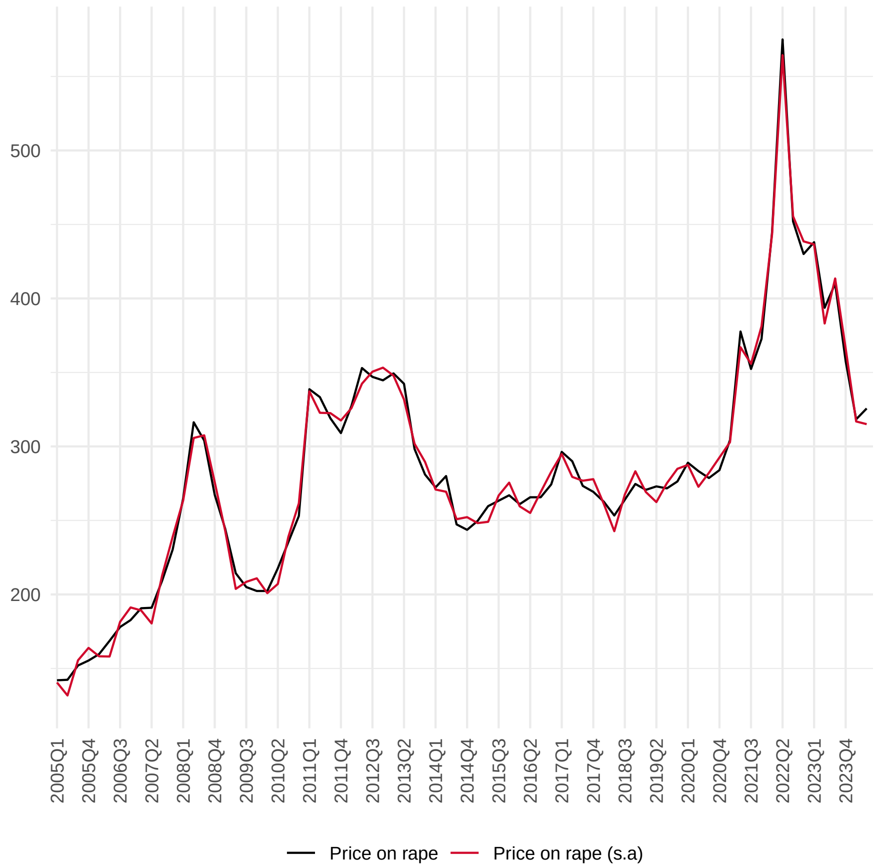


C.9 Producer prices - Agriculture

Step 1: Data and unit root testing







Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
pp_{sa}^{A}	-3.0074	-3.45	trend	I(1)
pp_{sa}^{MC}	-2.3301	-3.45	trend	I(1)
$P_{Rape_{sa}}$	-2.6582	-3.45	trend	I(1)

Table 73: Results from ADF Tests for variables entering estimation for producer prices for the agriculture industry.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
6.9813	2	Case II (Restricted intercept and no trend)	[3.10 ; 3.87]	Cointegration
9.2566	2	Case III (Unrestricted intercept and no trend)	[3.79 ; 4.85]	Cointegration

Table 74: Results from ARDL Bounds Test.

Null Hypothesis	**Test Statistic**	**5% Critical Value**
$r \leq 2$	3.36	12.25
$r \leq 1$	15.82	25.32
$r = 0$	30.76	42.44

Table 75: Johansen Trace Test Results with trend.

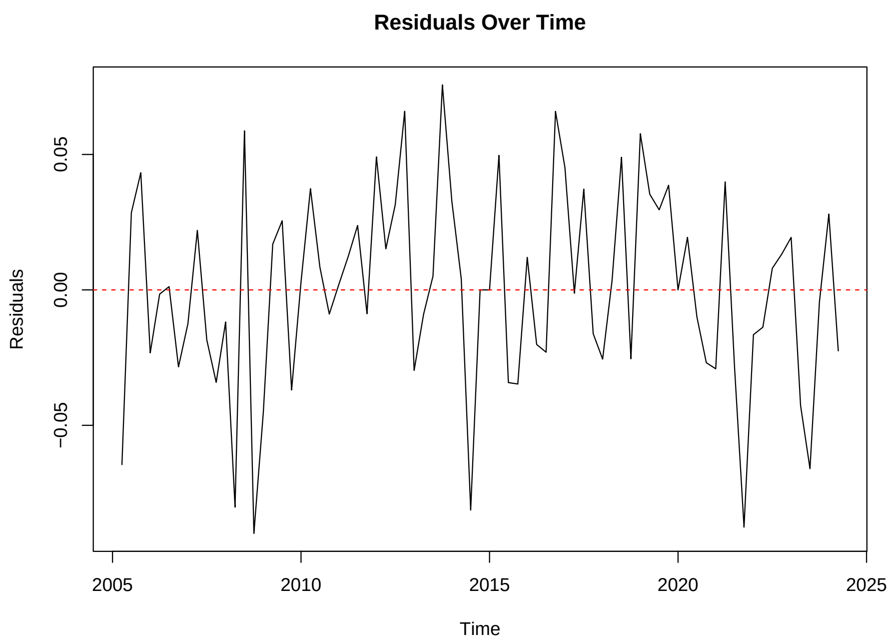
Null Hypothesis	**Test Statistic**	**5% Critical Value**
$r \leq 2$	3.97	9.24
$r \leq 1$	13.44	19.96
$r = 0$	28.40	34.91

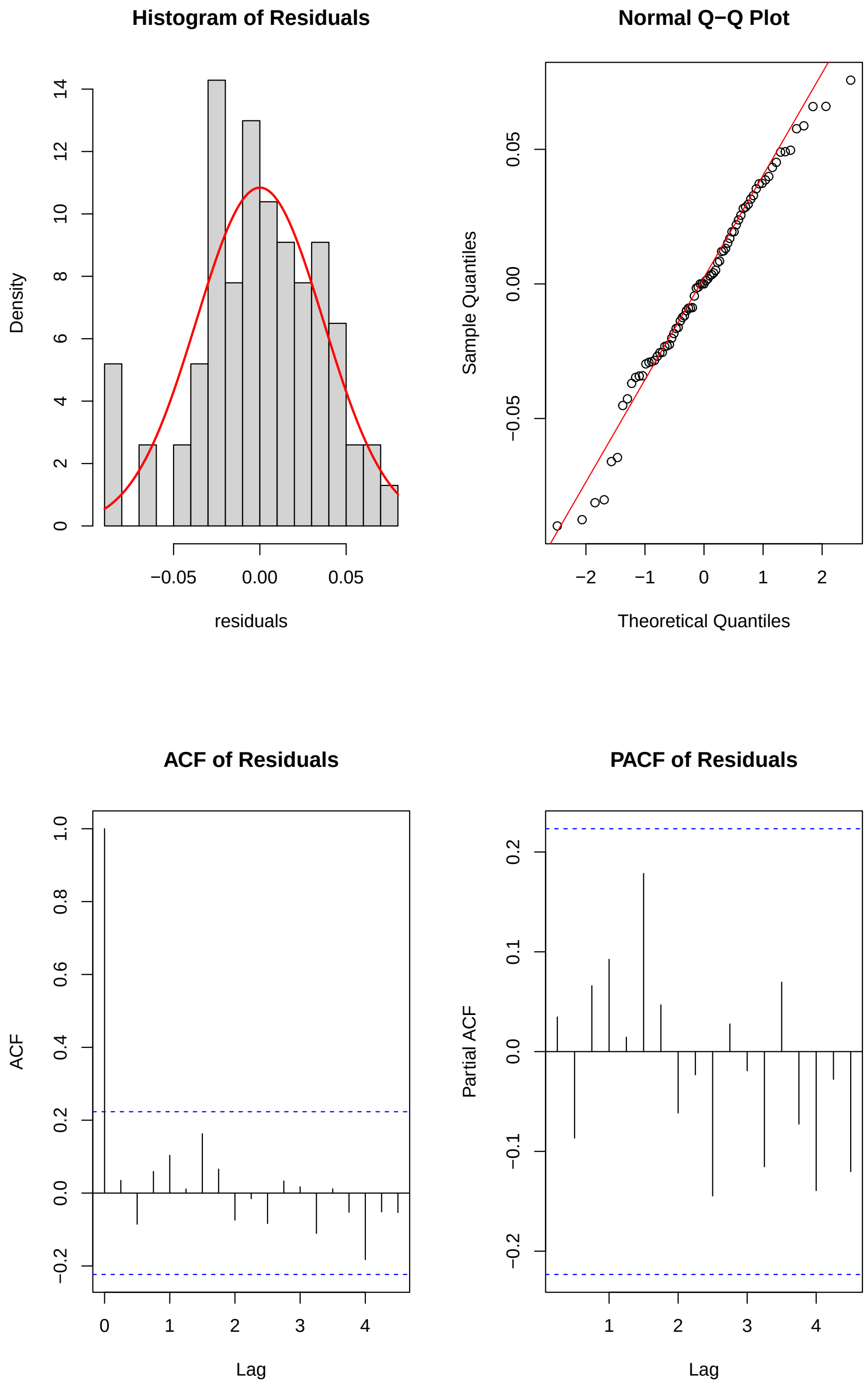
Table 76: Johansen Trace Test Results without trend and constant.

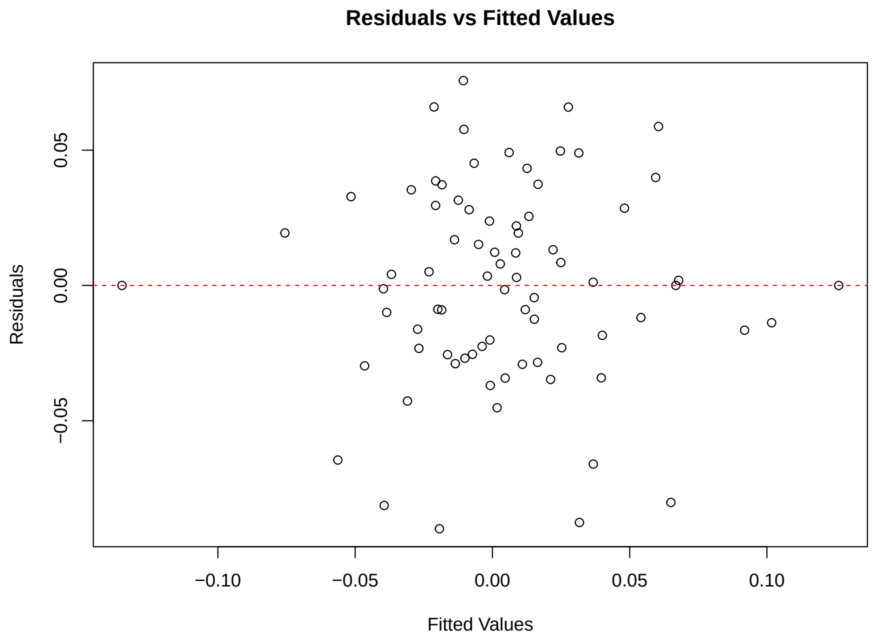
Step 4: Run the specific model

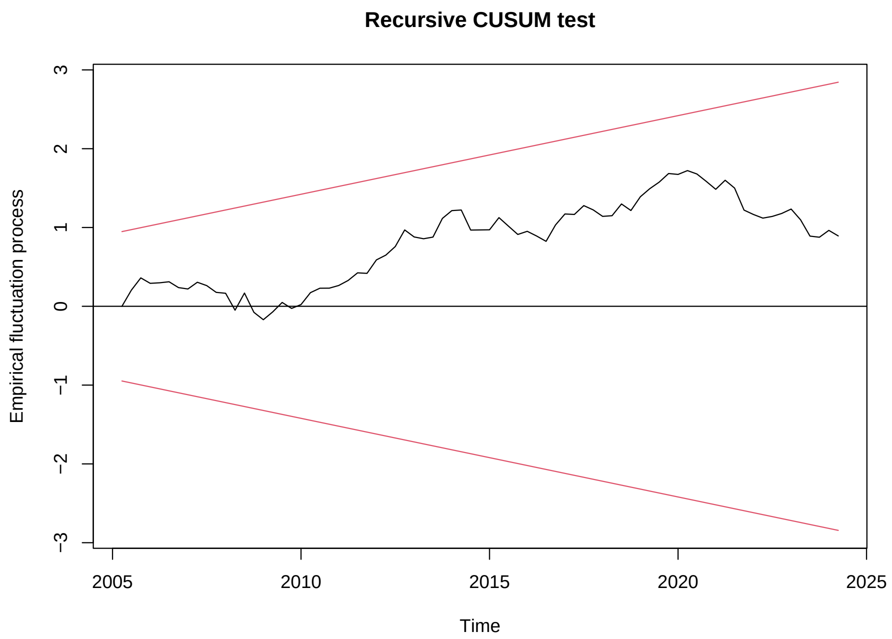
```
Call:
dynlm(formula = d(lpp_A_sa) ~ d(lprape_sa) + L(lpp_A_sa, 1) +
  L(lprape_sa, 1) + L(lpp_MC_sa, 1) + dummy14 + dummy15 + dummy20,
  data = data_PP_A)

Residuals:
     Min       1Q   Median       3Q      Max
-0.08991 -0.02325  0.00000  0.02800  0.07568

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)         -0.61775    0.21729  -2.843 0.00587 **
d(lprape_sa)         0.22765    0.05264   4.325 5.03e-05 ***
L(lpp_A_sa, 1)      -0.47245    0.08987  -5.257 1.55e-06 ***
L(lprape_sa, 1)      0.10344    0.03772   2.742 0.00777 **
L(lpp_MC_sa, 1)      0.33428    0.12211   2.737 0.00787 **
dummy14              0.09860    0.03936   2.505 0.01461 *
dummy15             -0.09589    0.03951  -2.427 0.01785 *
dummy20              0.09650    0.03950   2.443 0.01713 *
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.03861 on 69 degrees of freedom
Multiple R-squared:  0.5344,  Adjusted R-squared:  0.4872
F-statistic: 11.31 on 7 and 69 DF,  p-value: 1.859e-09
```

Step 6: Run diagnostic tests




Histogram of Residuals
Density
residuals
−0.05
0.00
0.05
0
2
4
6
8
10
12
14
Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
−0.05
0.00
0.05
−2
−1
0
1
2
ACF of Residuals
ACF
Lag
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
PACF of Residuals
Partial ACF
Lag
−0.2
−0.1
0.0
0.1
0.2
1
2
3
4





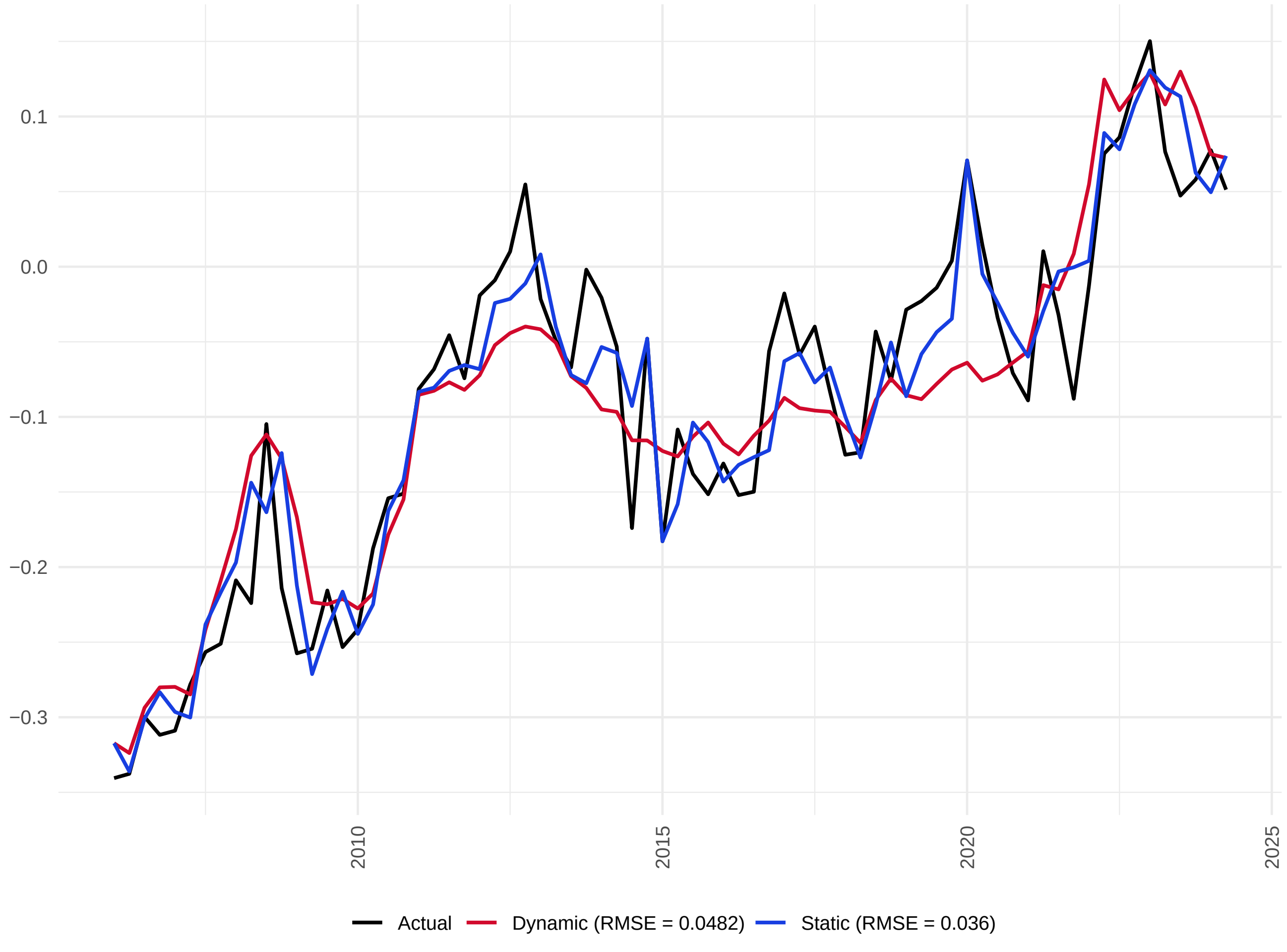
Test	**p-value**
Shapiro-Wilk	0.3206
Jarque Bera	0.5355
Durbin-Watson	0.2026
Breusch-Godfrey, order 4	0.6835

Table 77: Diagnostics statistics.

Variable	VIF Value
d(lprape_sa)	1.103238
L(lpp_A_sa, 1)	6.702151
L(lprape_sa, 1)	5.678440
L(lpp_MC_sa, 1)	6.887952
dummy14	1.025765
dummy15	1.033775
dummy20	1.033225

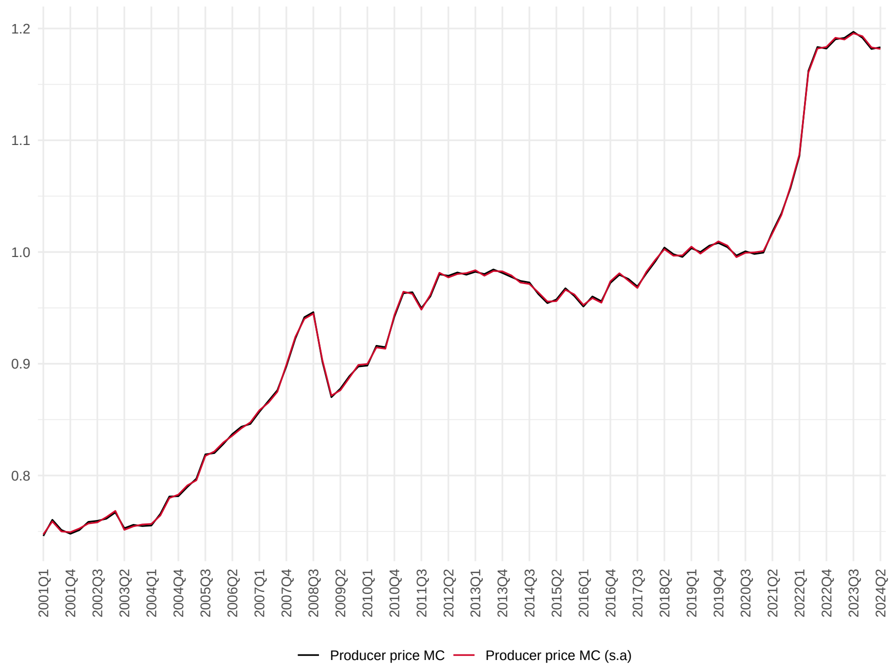
Table 78: Variance Inflation Factor (VIF) results for the pp^A regression.

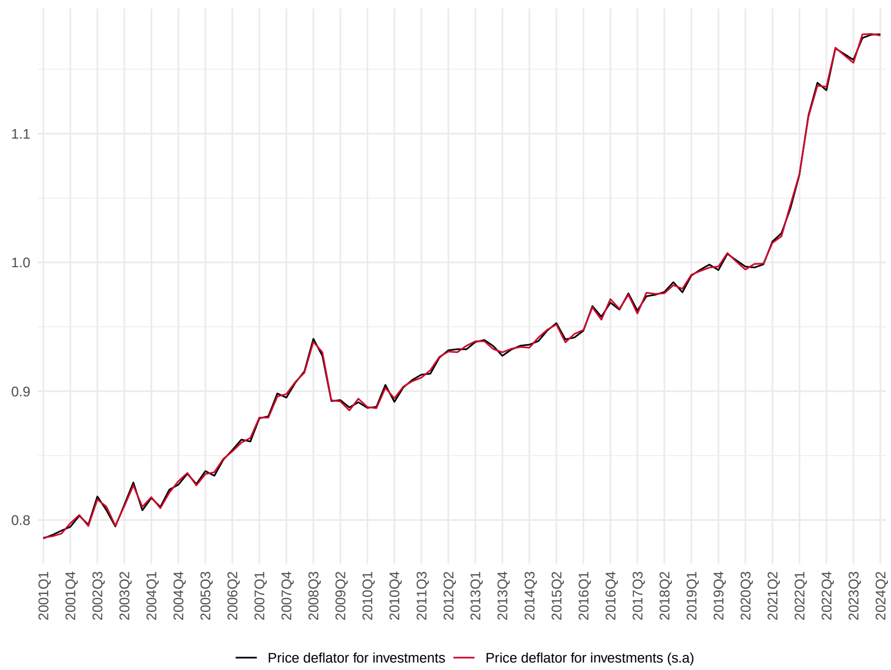
Step 7: Evaluate static and dynamic forecasts

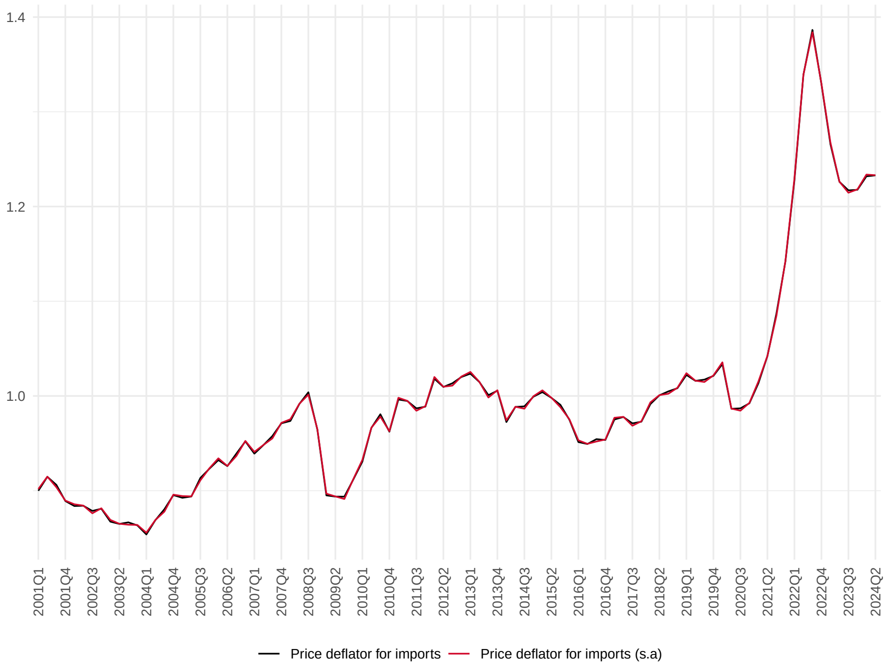


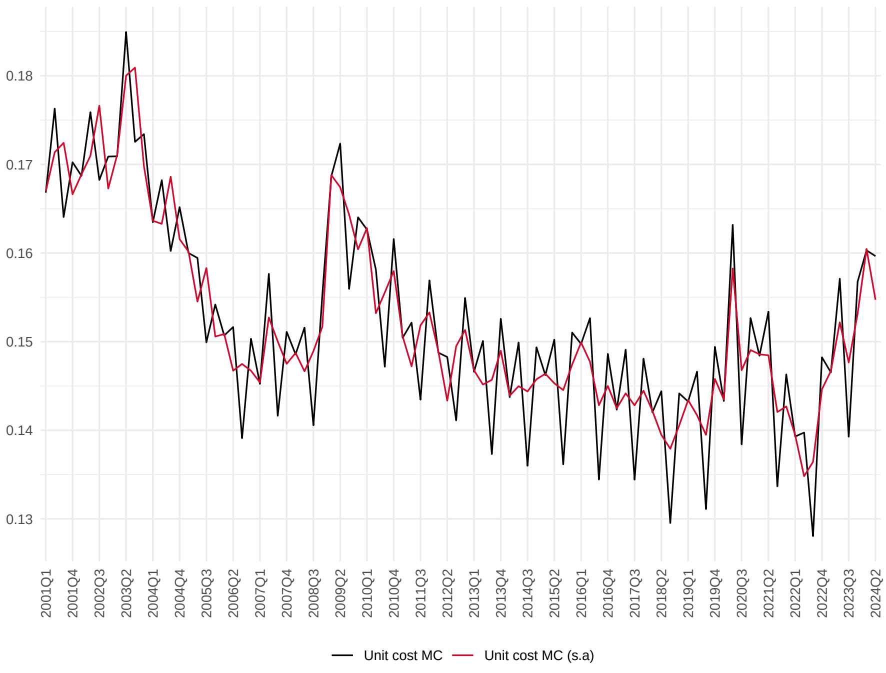
C.10 Producer prices - Manufacturing & Construction

Step 1: Data and unit root testing









Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
pp_{sa}^{MC}	-2.3694	-3.45	trend	I(1)
UC_{sa}^{MC}	-2.4653	-3.45	trend	I(1)
$p_{IM_{sa}}$	-3.3032	-3.45	trend	I(1)
$p_{I_{sa}}$	-0.4928	-3.45	trend	I(1)

Table 79: Results from ADF Tests for variables entering estimation for producer prices for the MC industry.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
5.57	3	Case II (Restricted intercept and no trend)	[2.79 ; 3.67]	Cointegration
3.9413	3	Case III (Unrestricted intercept and no trend)	[3.23 ; 4.35]	Inconclusive

Table 80: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 3$	4.15	12.25
$r \leq 2$	11.57	25.32
$r \leq 1$	22.66	42.44
$r = 0$	75.65	62.99

Table 81: Johansen Trace Test Results with trend.

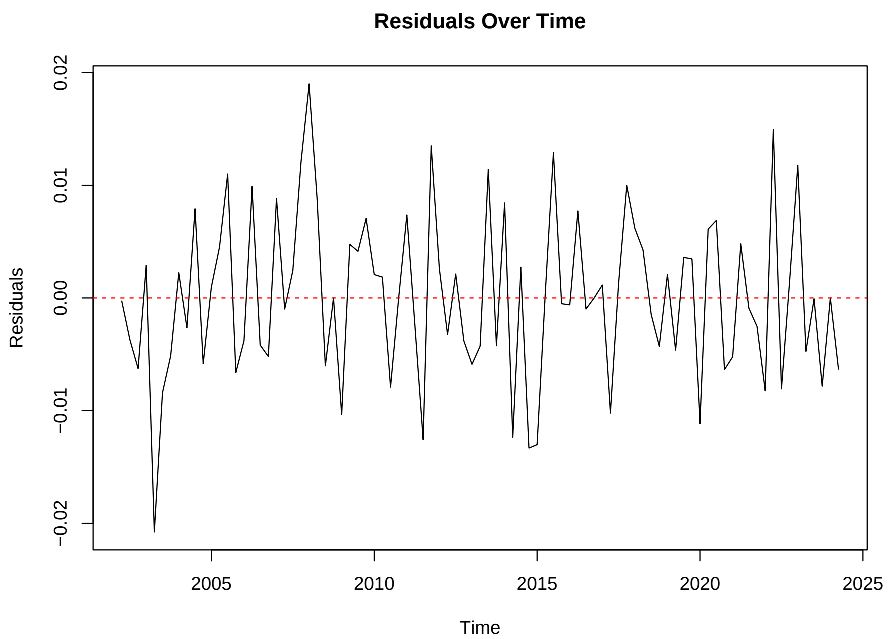
Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 3$	2.99	9.24
$r \leq 2$	10.64	19.96
$r \leq 1$	19.64	34.91
$r = 0$	81.32	53.12

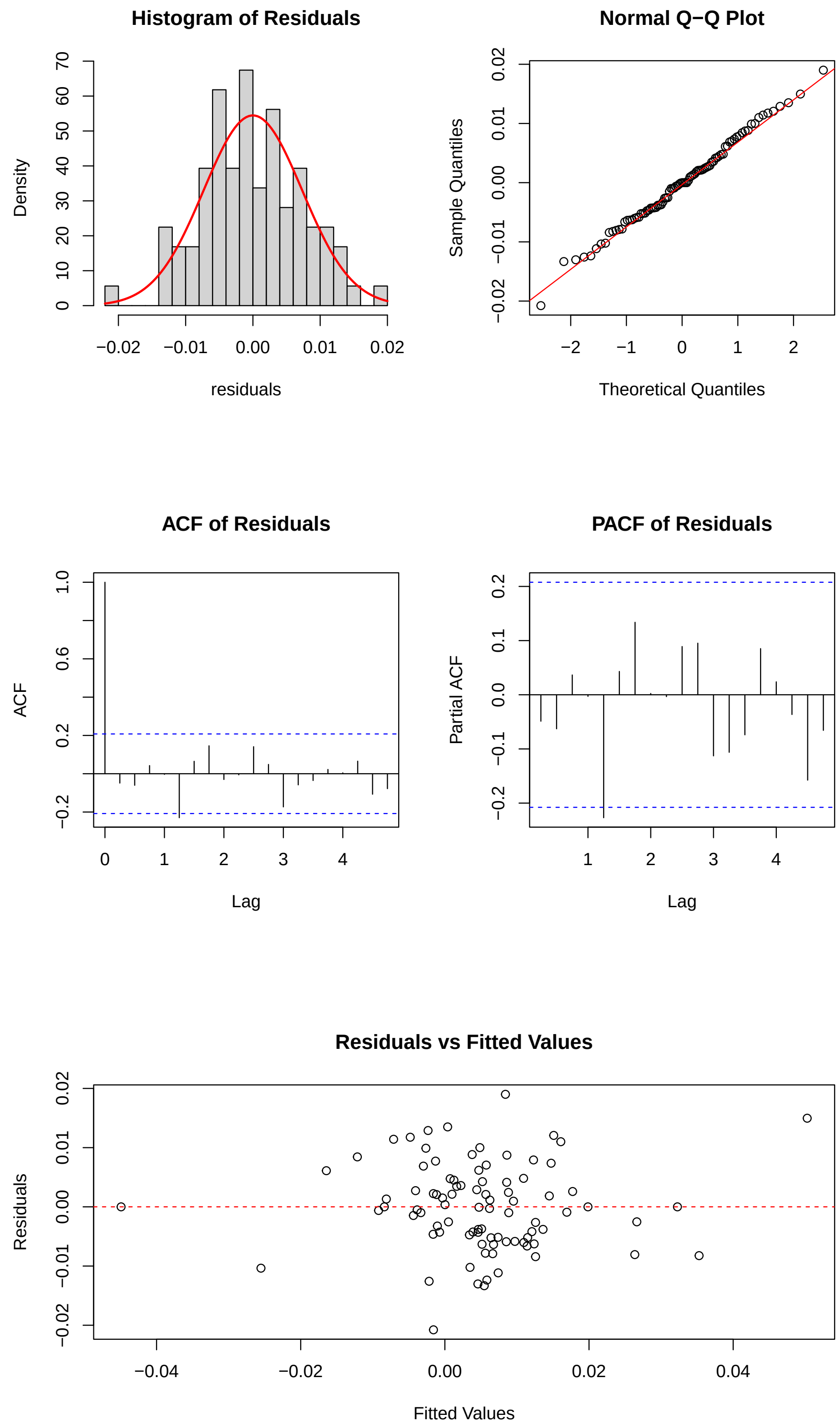
Table 82: Johansen Trace Test Results without trend and constant.

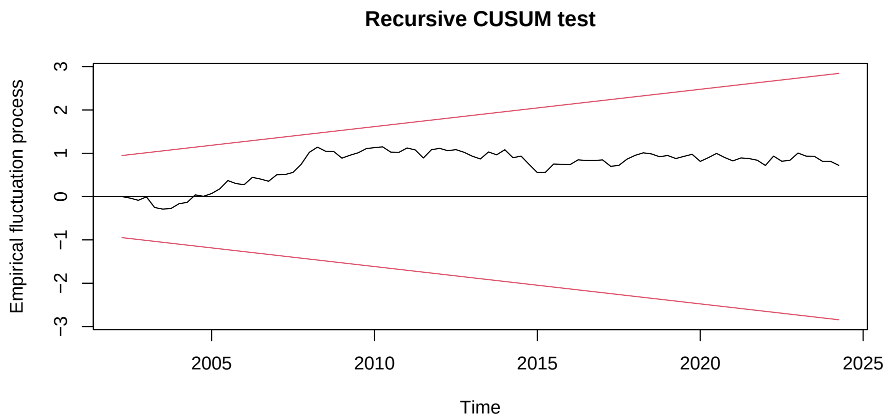
Step 4: Run the specific model

```
Call:
dynlm(formula = d(lpp_MC_sa) ~ d(lpim_sa) + L(d(lP_I_sa), 4) +
  L(d(lUC_MC_sa), 2) + L(lpp_MC_sa, 1) + L(lpim_sa, 1) +
  dummy08 + dummy10 + dummy16 + dummy24,
  data = data_PP_MC)

Residuals:
     Min        1Q    Median        3Q       Max
-0.020772 -0.005146  0.000000  0.004510  0.019007

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)            -0.0009425  0.0012672  -0.744 0.459207
d(lpim_sa)              0.4306772  0.0384502  11.201  < 2e-16 ***
L(d(lP_I_sa), 4)        0.2070363  0.0774937   2.672 0.009163 **
L(d(lUC_MC_sa), 2)      0.0521499  0.0280572   1.859 0.066795 .
L(lpp_MC_sa, 1)        -0.0668531  0.0170205  -3.928 0.000182 ***
L(lpim_sa, 1)           0.0796600  0.0206640   3.855 0.000234 ***
dummy08                -0.0315684  0.0079685  -3.962 0.000162 ***
dummy10                 0.0368419  0.0080436   4.580 1.71e-05 ***
dummy16                 0.0199915  0.0078901   2.534 0.013263 *
dummy24                -0.0207337  0.0081773  -2.536 0.013201 *

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.007732 on 79 degrees of freedom
Multiple R-squared:  0.7206,  Adjusted R-squared:  0.6888
F-statistic: 22.64 on 9 and 79 DF,  p-value: < 2.2e-16
```

Step 6: Run diagnostic tests




Histogram of Residuals
Density
residuals
Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
ACF of Residuals
ACF
Lag
PACF of Residuals
Partial ACF
Lag
Residuals vs Fitted Values
Residuals
Fitted Values



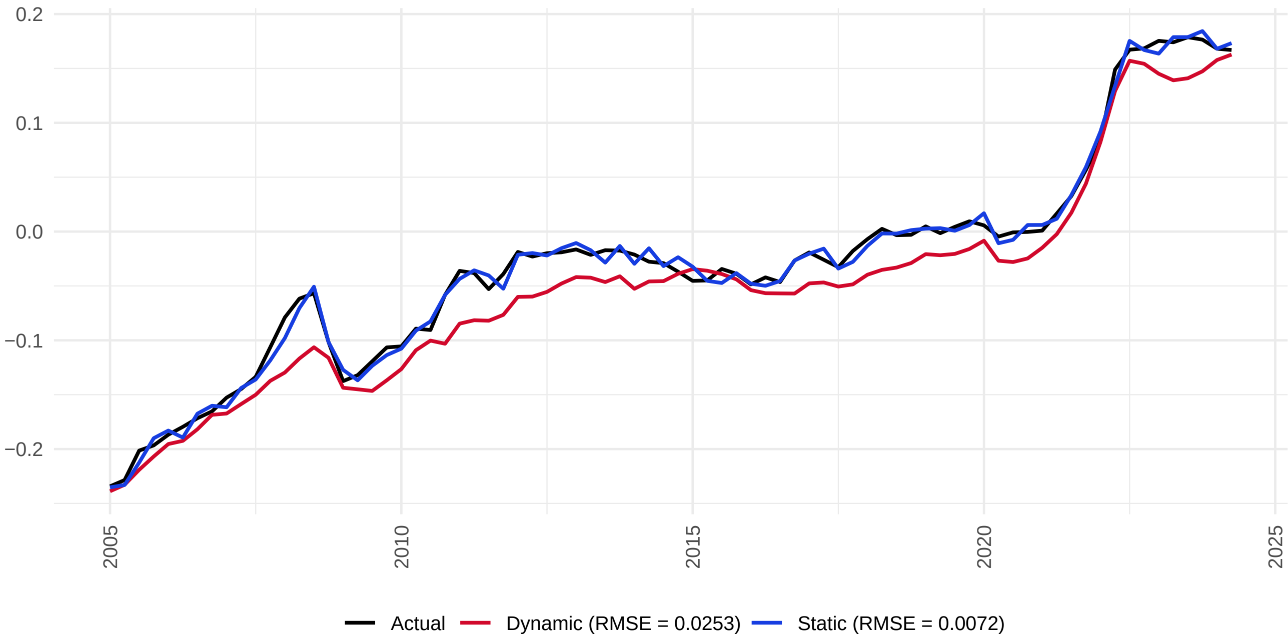
Test	**p-value**
Shapiro-Wilk	0.9711
Jarque Bera	0.9798
Durbin-Watson	0.5795
Breusch-Godfrey, order 4	0.9458

Table 83: Diagnostics statistics.

Variable	**VIF Value**
d(lpim_sa)	1.126198
L(d(lp_I_sa), 4)	1.267220
L(d(lUC_MC_sa), 2)	1.151309
L(lpim_sa, 1)	6.518508
L(lpp_MC_sa, 1)	6.214696
dummy08	1.050299
dummy10	1.070197
dummy16	1.029729
dummy24	1.106059

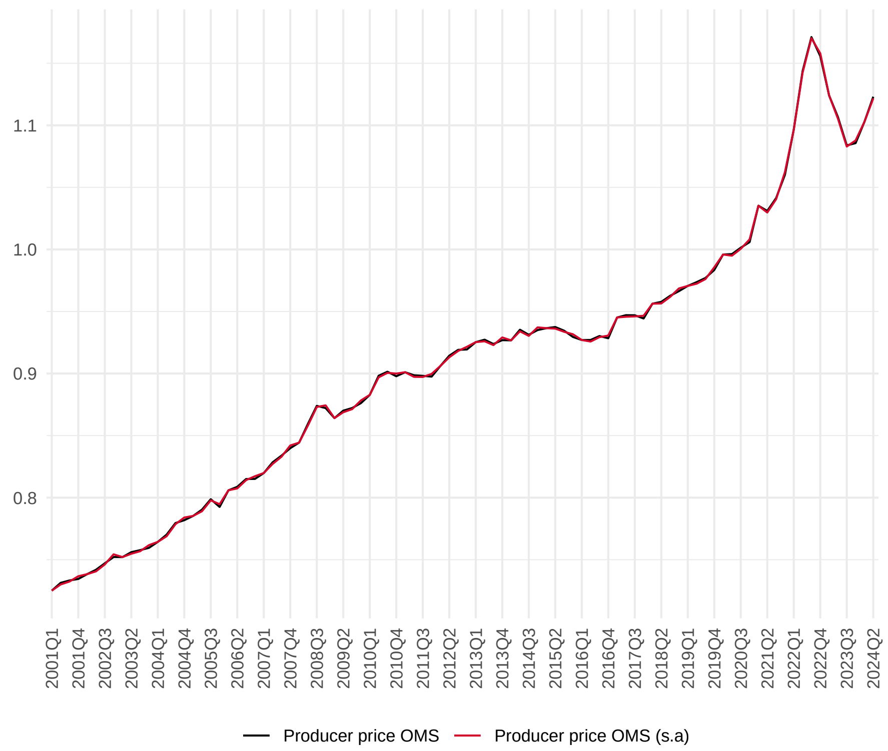
Table 84: Variance Inflation Factor (VIF) results for the pp^{MC} regression.

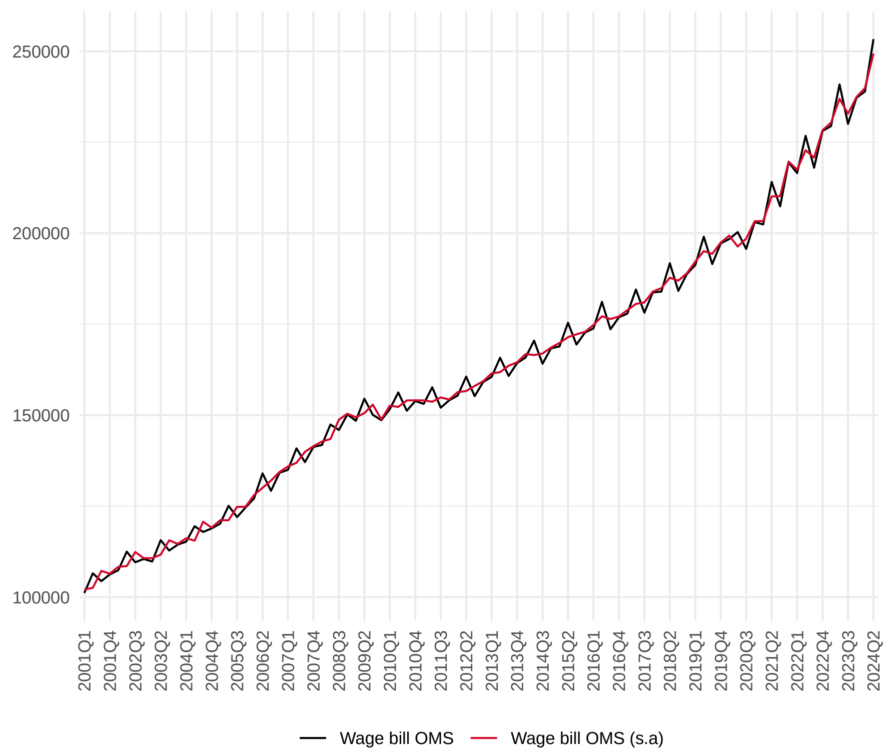
Step 7: Evaluate static and dynamic forecasts

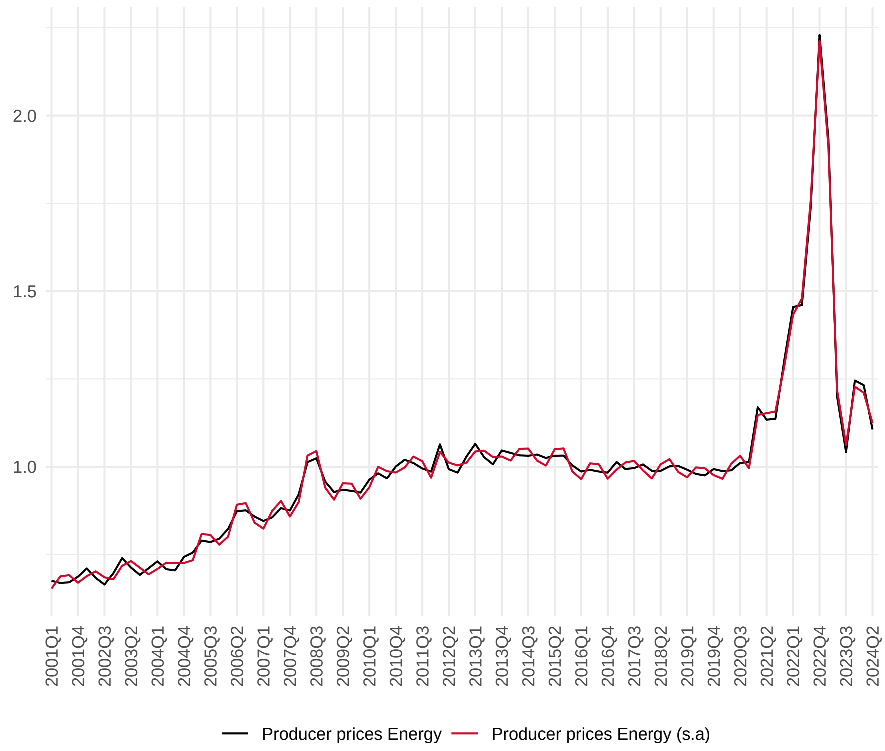


C.11 Producer prices - Other Manufacturing & Services

Step 1: Data and unit root testing



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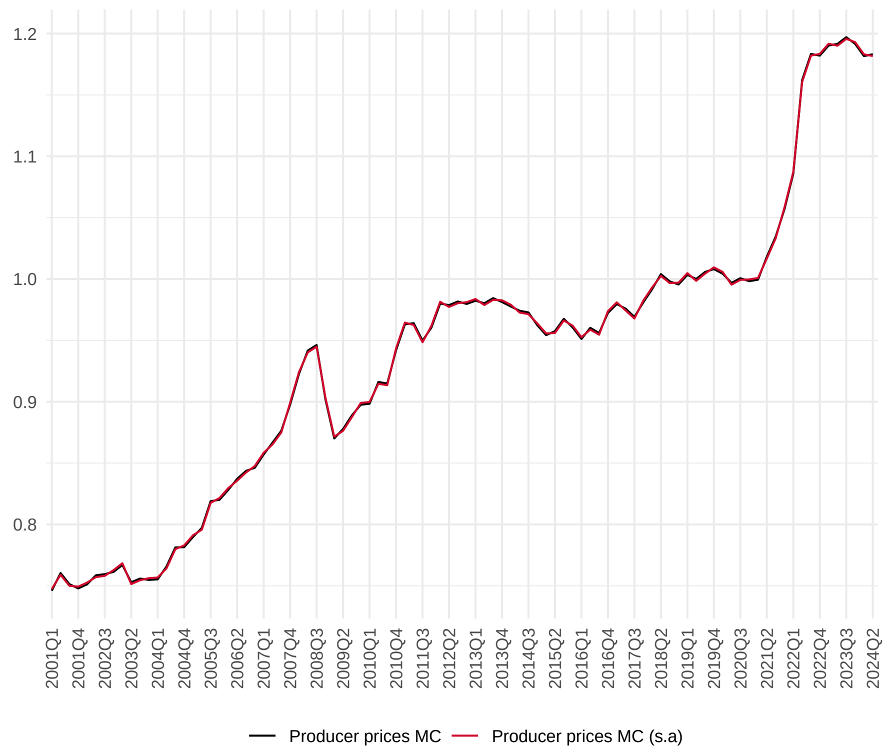
1.5
1.0
2001Q1
2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
2006Q2
2007Q1
2007Q4
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Producer prices Energy
Producer prices Energy (s.a)


1.2
1.1
1.0
0.9
0.8
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2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
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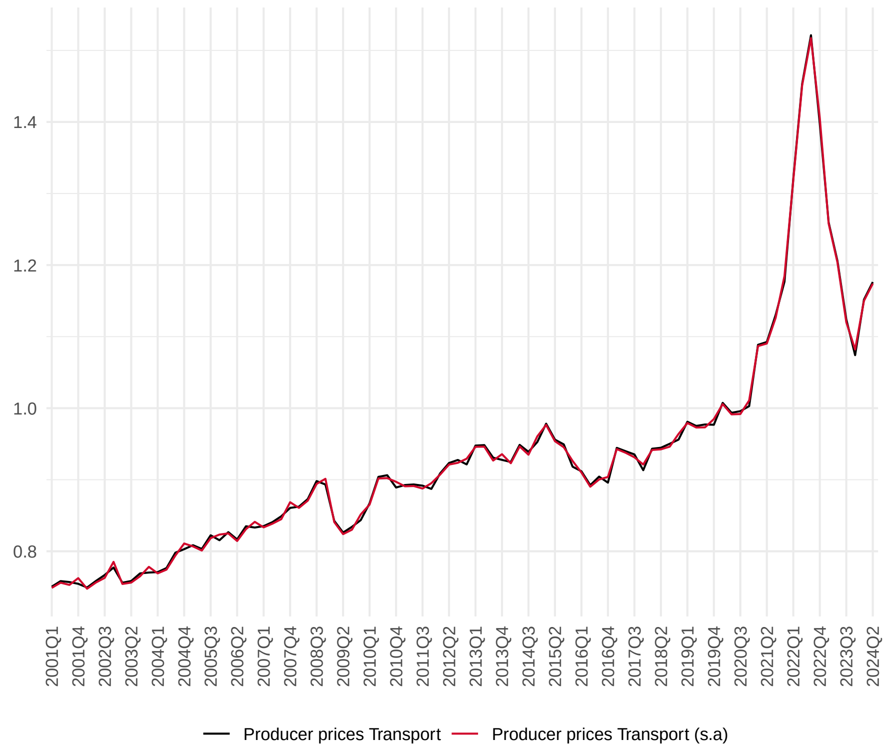
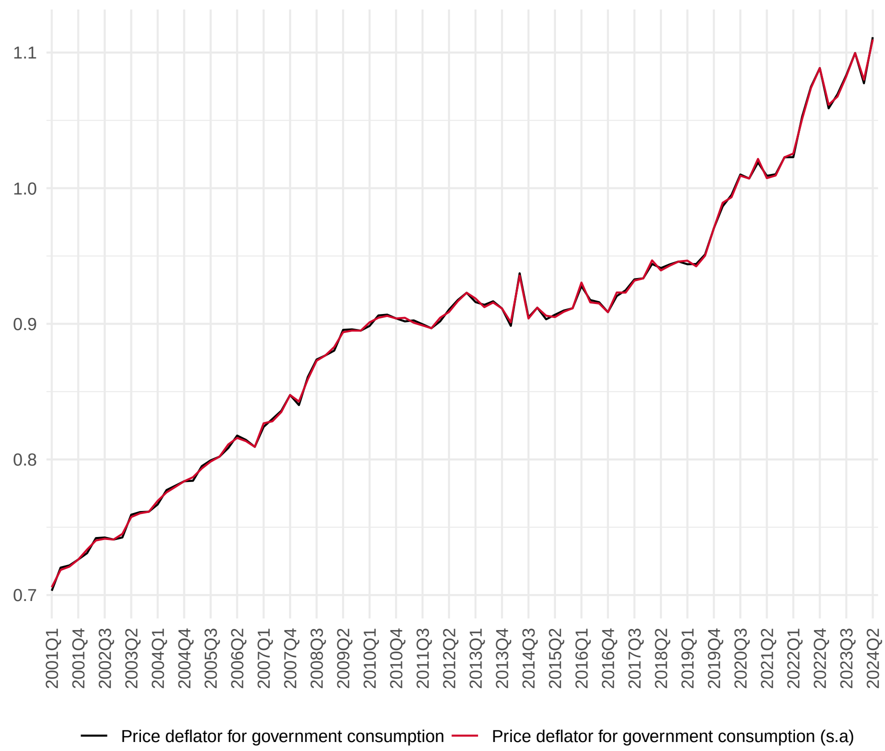
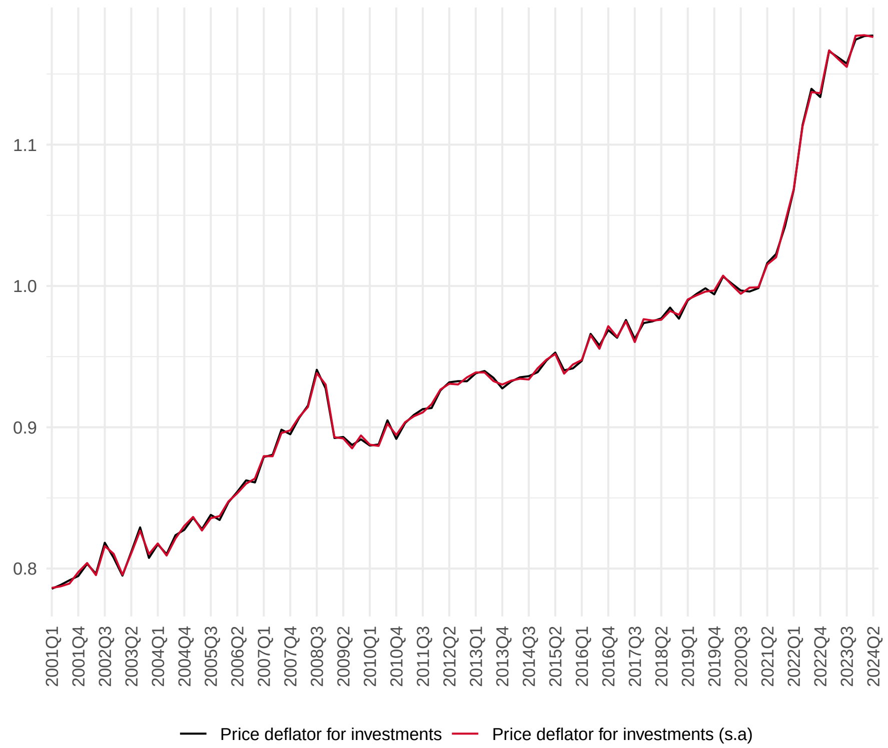
2007Q1
2007Q4
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Producer prices MC
Producer prices MC (s.a)


1.4
1.2
1.0
0.8
2001Q1
2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
2006Q2
2007Q1
2007Q4
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Producer prices Transport
Producer prices Transport (s.a)


1.1
1.0
0.9
0.8
0.7
2001Q1
2001Q4
2002Q3
2003Q2
2004Q1
2004Q4
2005Q3
2006Q2
2007Q1
2007Q4
2008Q3
2009Q2
2010Q1
2010Q4
2011Q3
2012Q2
2013Q1
2013Q4
2014Q3
2015Q2
2016Q1
2016Q4
2017Q3
2018Q2
2019Q1
2019Q4
2020Q3
2021Q2
2022Q1
2022Q4
2023Q3
2024Q2
Price deflator for government consumption
Price deflator for government consumption (s.a)



Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
pp_{sa}^{OMS}	-3.0282	-3.45	trend	I(1)
pp_{sa}^{MC}	-2.3694	-3.45	trend	I(1)
$P_{G_{sa}}$	-1.0705	-3.45	trend	I(1)
$P_{I_{sa}}$	-0.4928	-3.45	trend	I(1)
D1_{sa}^{OMS}	2.0535	-3.45	trend	I(1)
pp_{sa}^{T}	-1.855	-2.89	drift	I(1)
pp_{sa}^{E}	-0.4511	-1.95	none	I(1)

Table 85: Results from ADF Tests for variables entering estimation for producer prices for the OMS industry.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
9.359	6	Case II (Restricted intercept and no trend)	[2.27 ; 3.28]	Cointegration
7.1556	6	Case III (Unrestricted intercept and no trend)	[2.45 ; 3.61]	Cointegration

Table 86: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 6$	6.41	12.25
$r \leq 5$	16.21	25.32
$r \leq 4$	36.00	42.44
$r \leq 3$	59.74	62.99
$r \leq 2$	99.84	87.31
$r \leq 1$	178.16	114.90
$r = 0$	282.40	146.76

Table 87: Johansen Trace Test Results with trend.

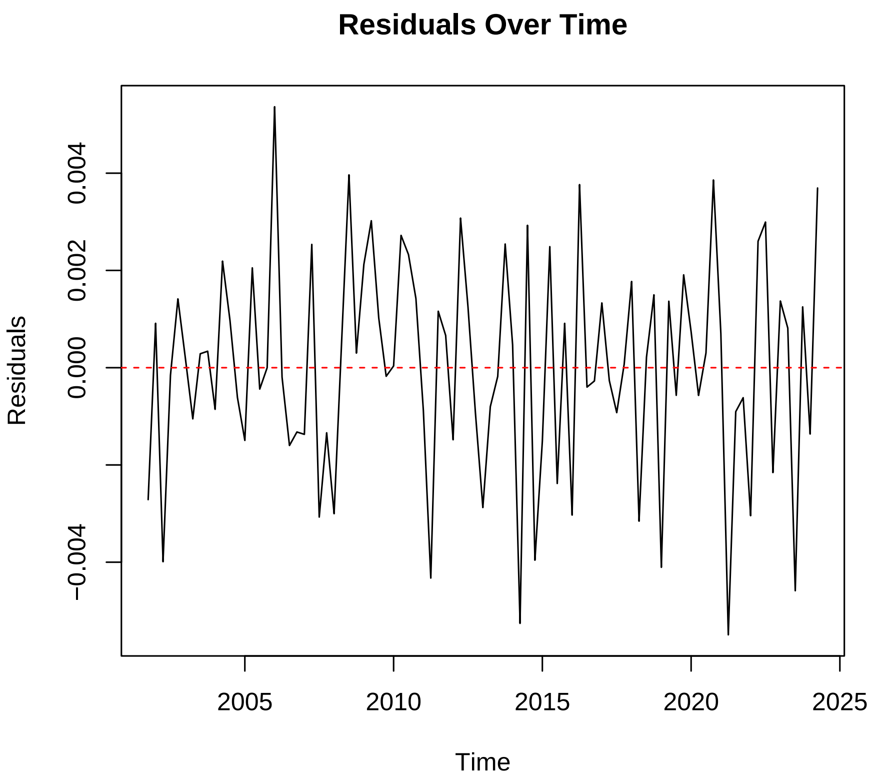
Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 6$	4.28	9.24
$r \leq 5$	10.88	19.96
$r \leq 4$	25.55	34.91
$r \leq 3$	48.09	53.12
$r \leq 2$	94.08	76.07
$r \leq 1$	172.93	102.14
$r = 0$	307.25	131.70

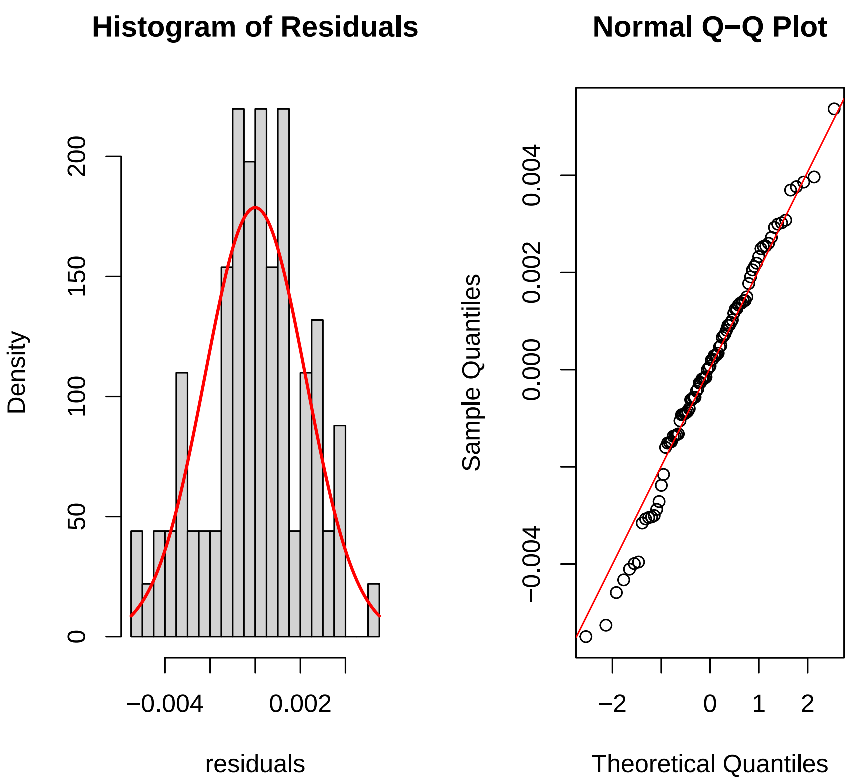
Table 88: Johansen Trace Test Results without trend and constant.

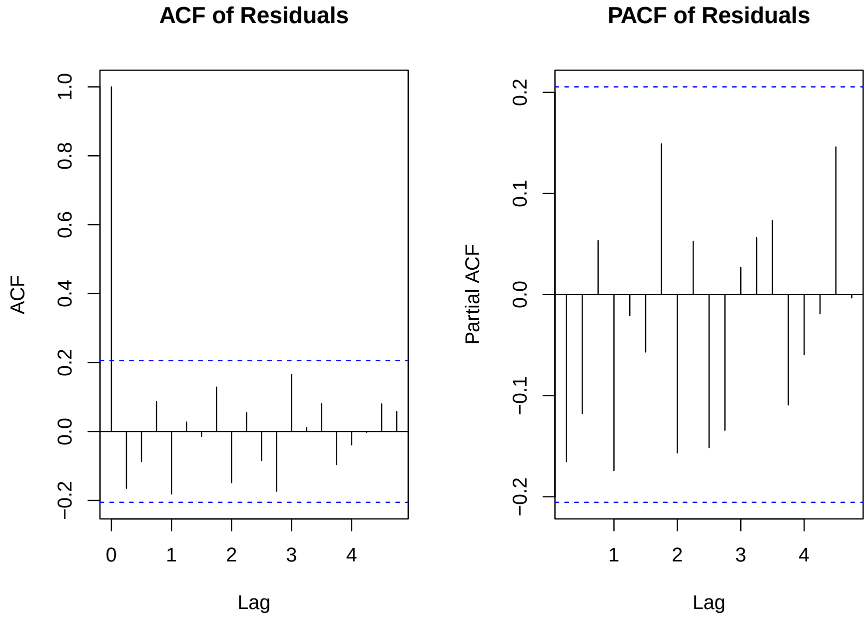
Step 4: Run the specific model

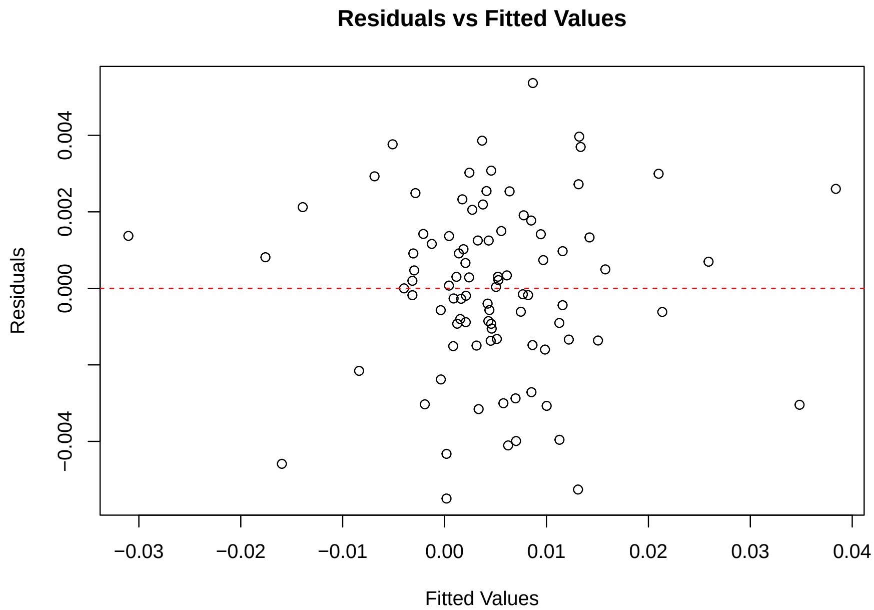
```
Call:
dynlm(formula = d(lpp_OMS_sa) ~ L(d(LD1_OMS_sa), 2) + d(lpp_E_sa) +
  d(lpp_T_sa) + d(lpp_MC_sa) + L(d(lP_I_sa), 1) + d(lP_G_sa) +
  L(d(lP_G_sa), 2) + L(lpp_OMS_sa, 1) + L(LD1_OMS_sa, 1) +
  L(lpp_T_sa, 1) + L(lpp_MC_sa, 1) + dummy05, data = data_PP_OMS)

Residuals:
     Min        1Q    Median        3Q       Max
-0.0054935 -0.0013310  0.0000715  0.0013928  0.0053658

Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
(Intercept)               -0.397099   0.132369  -3.000  0.003624 **
L(d(LD1_OMS_sa), 2)        0.050865   0.020409   2.492  0.014813 *
d(lpp_E_sa)                0.022988   0.003591   6.402  0.0000001016 ***
d(lpp_T_sa)                0.269407   0.011175  24.108  < 0.0000000000000002 ***
d(lpp_MC_sa)              0.055473   0.022509   2.465  0.015919 *
L(d(lP_I_sa), 1)          -0.097871   0.026697  -3.666  0.000448 ***
d(lP_G_sa)                 0.126440   0.025803   4.900  0.0000050921 ***
L(d(lP_G_sa), 2)           0.070663   0.025661   2.963  0.004039 **
L(lpp_OMS_sa, 1)         -0.153372   0.034837  -4.403  0.0000336037 ***
L(LD1_OMS_sa, 1)          0.032547   0.010853   2.999  0.003634 **
L(lpp_T_sa, 1)            0.043821   0.010219   4.288  0.0000511119 ***
L(lpp_MC_sa, 1)           0.040717   0.010693   3.757  0.000329 ***
dummy05                  -0.007820   0.002453  -3.187  0.002066 **

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.002398 on 78 degrees of freedom
Multiple R-squared: 0.9458,    Adjusted R-squared: 0.9375
F-statistic: 113.4 on 12 and 78 DF,  p-value: < 0.00000000000000022
```

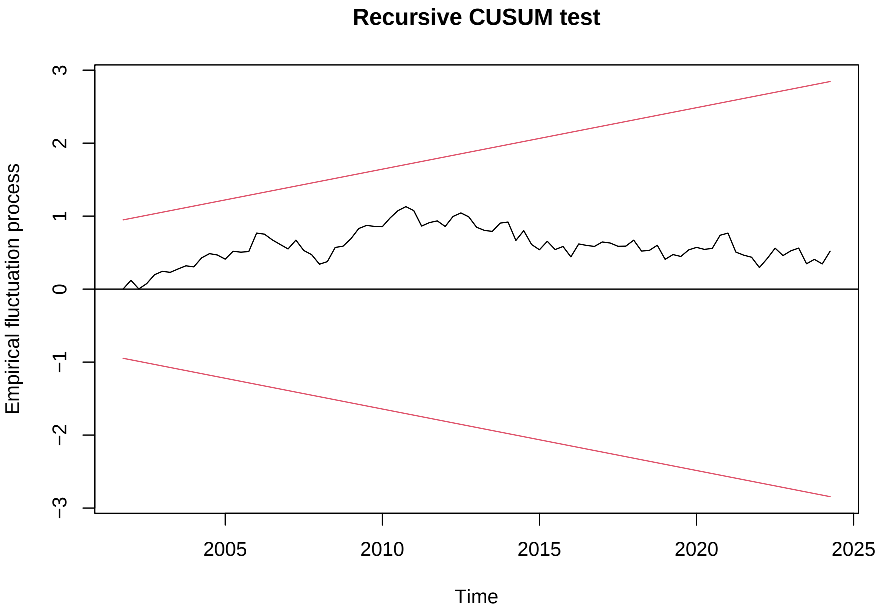
Step 6: Run diagnostic tests






ACF of Residuals
ACF
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
1
2
3
4
Lag
PACF of Residuals
Partial ACF
0.2
0.1
0.0
−0.1
−0.2
1
2
3
4
Lag


Residuals vs Fitted Values
Residuals
0.004
0.002
0.000
−0.004
−0.03
−0.02
−0.01
0.00
0.01
0.02
0.03
0.04
Fitted Values



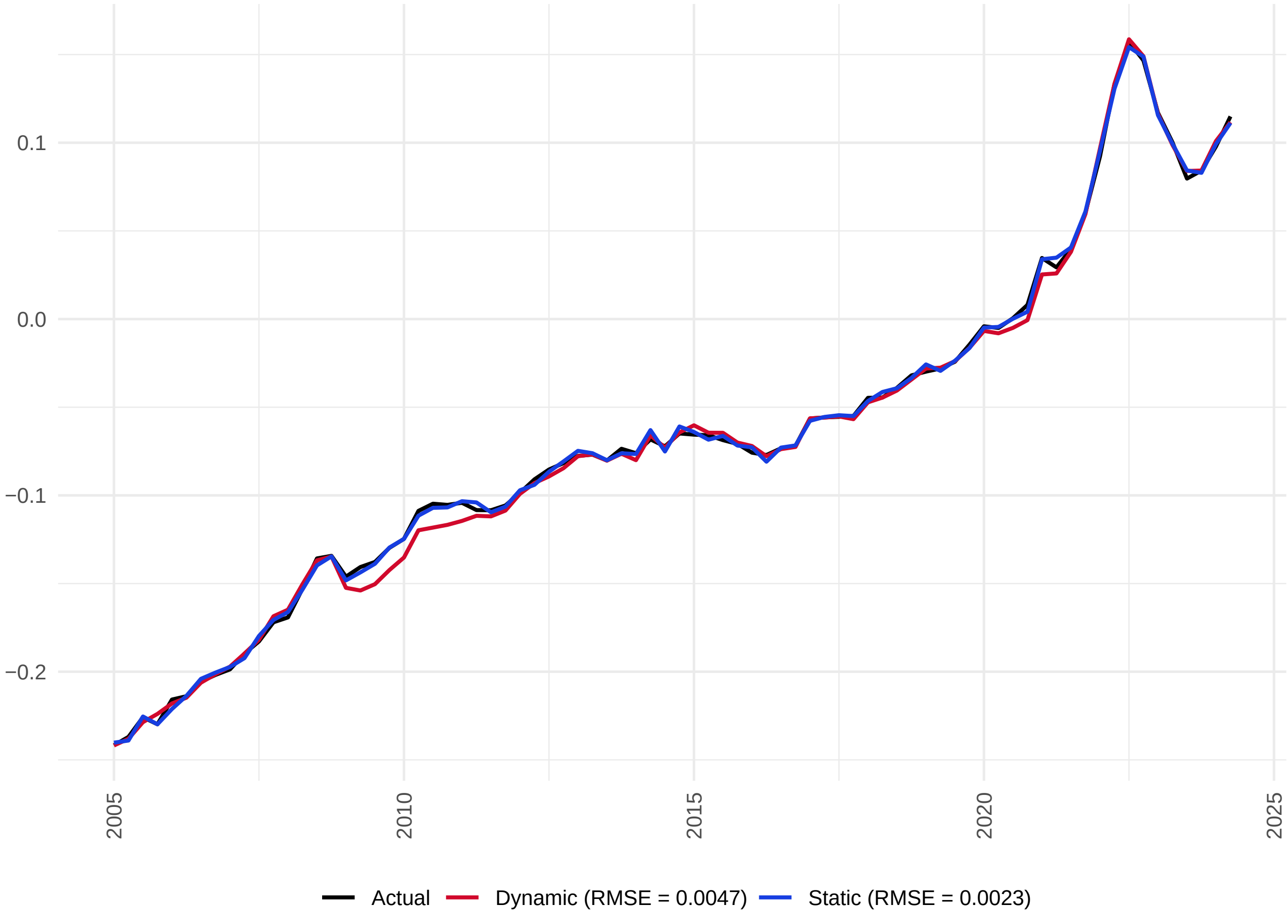
Test	p-value
Shapiro-Wilk	0.5044
Jarque Bera	0.5457
Durbin-Watson	0.839
Breusch-Godfrey, order 4	0.0692

Table 89: Diagnostics statistics.

Variable	VIF Value
L(d(LD1_OMS_sa), 2)	1.234074
d(lpp_E_sa)	1.158357
d(lpp_T_sa)	1.858746
d(lpp_MC_sa)	1.492247
L(d(lP_I_sa), 1)	1.601187
d(lP_G_sa)	1.217675
L(d(lP_G_sa), 2)	1.107354
L(lpp_OMS_sa, 1)	257.715813
L(LD1_OMS_sa, 1)	90.032042
L(lpp_T_sa, 1)	34.905300
L(lpp_MC_sa, 1)	27.429640
dummy05	1.035214

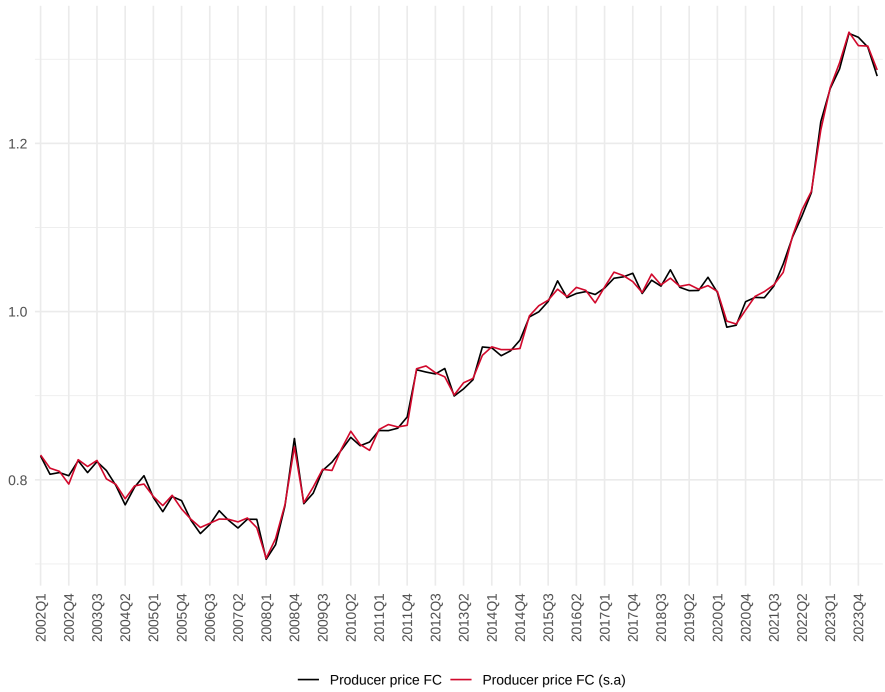
Table 90: Variance Inflation Factor (VIF) results for the pp^{OMS} regression.

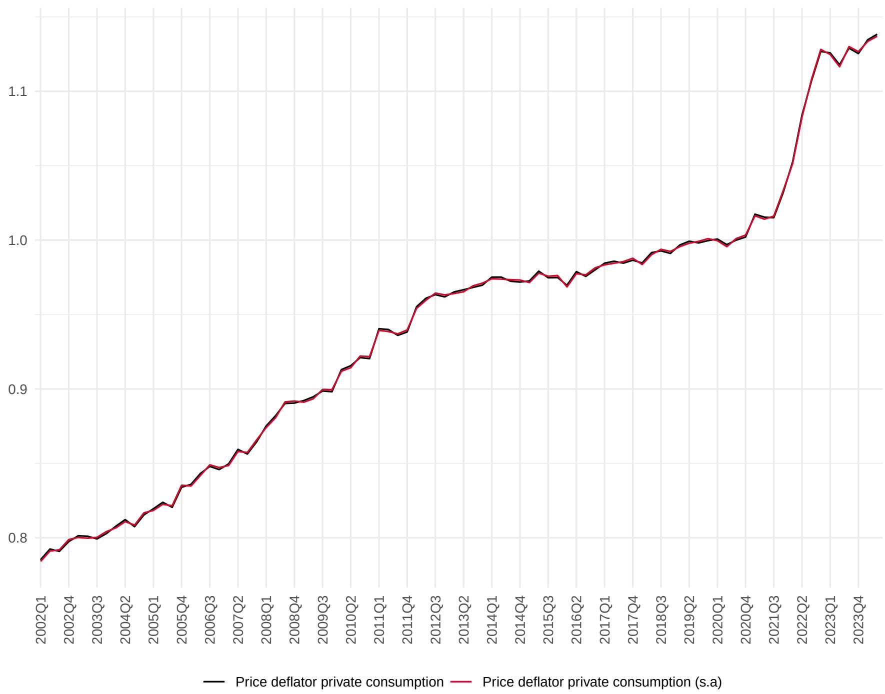
Step 7: Evaluate static and dynamic forecasts

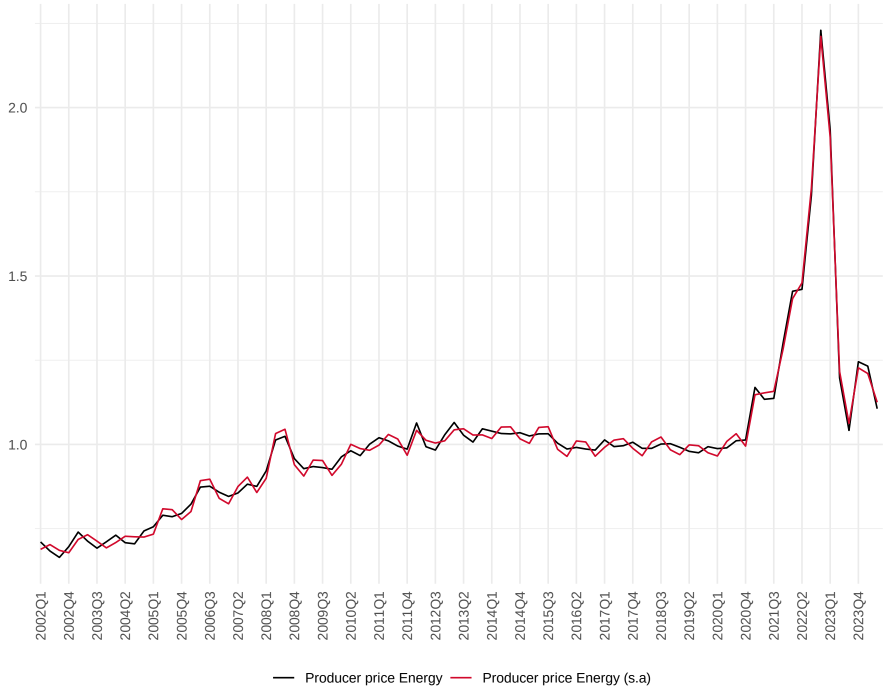


C.12 Producer prices - Financial corporations

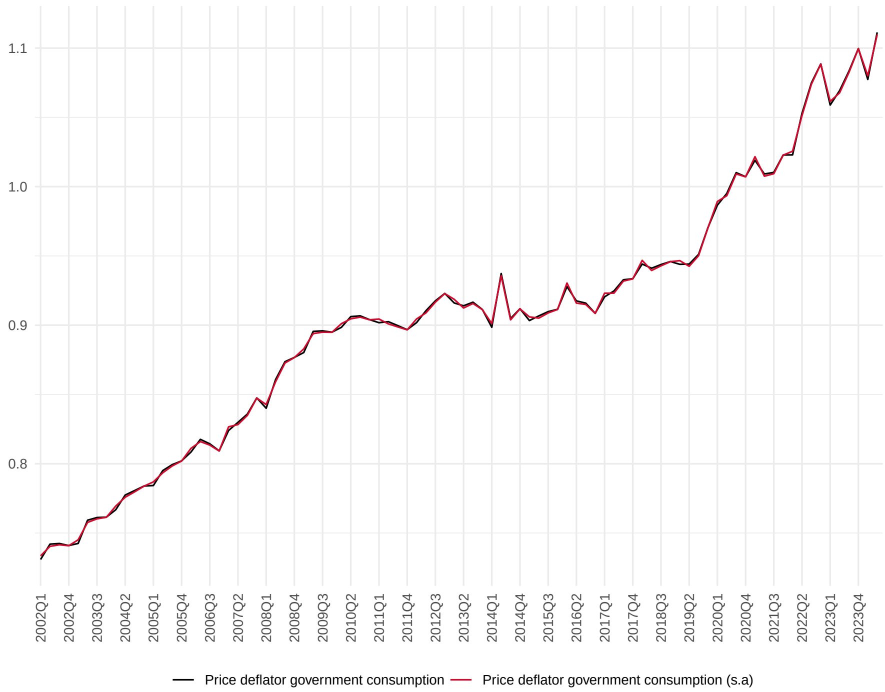
Step 1: Data and unit root testing



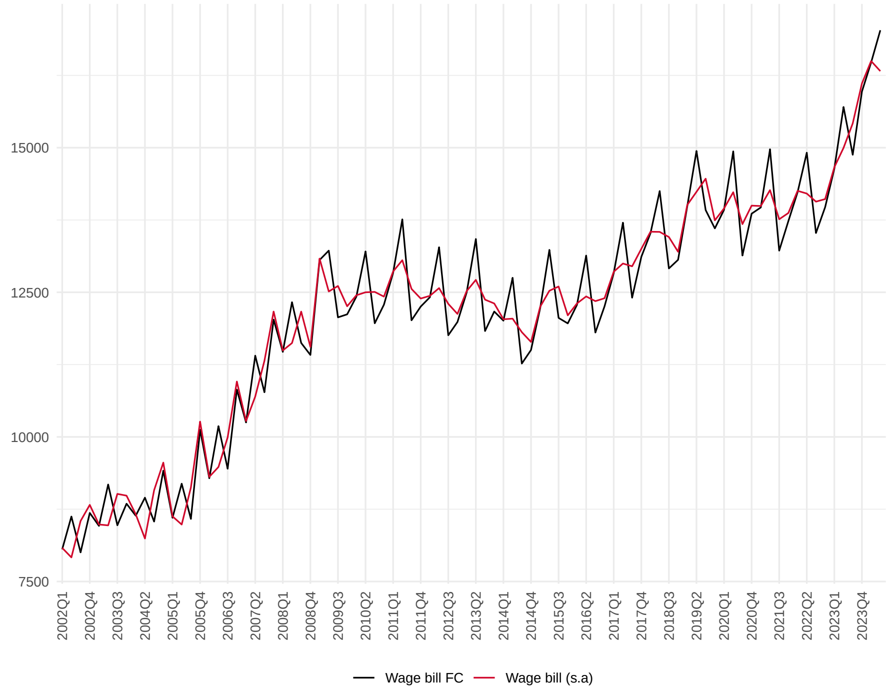




2.0
1.5
1.0
2002Q1
2002Q4
2003Q3
2004Q2
2005Q1
2005Q4
2006Q3
2007Q2
2008Q1
2008Q4
2009Q3
2010Q2
2011Q1
2011Q4
2012Q3
2013Q2
2014Q1
2014Q4
2015Q3
2016Q2
2017Q1
2017Q4
2018Q3
2019Q2
2020Q1
2020Q4
2021Q3
2022Q2
2023Q1
2023Q4
Producer price Energy
Producer price Energy (s.a)



1.1
1.0
0.9
0.8
2002Q1
2002Q4
2003Q3
2004Q2
2005Q1
2005Q4
2006Q3
2007Q2
2008Q1
2008Q4
2009Q3
2010Q2
2011Q1
2011Q4
2012Q3
2013Q2
2014Q1
2014Q4
2015Q3
2016Q2
2017Q1
2017Q4
2018Q3
2019Q2
2020Q1
2020Q4
2021Q3
2022Q2
2023Q1
2023Q4
Price deflator government consumption
Price deflator government consumption (s.a)



Variable	Test Statistic (t-value)	Critical Value (5%)	Specification	Conclusion
pp_{sa}^{FC}	-1.8617	-3.45	trend	I(1)
$D1_{sa}^{FC}$	-2.3487	-3.45	trend	I(1)
$p_{G_{sa}}$	-0.9432	-3.45	trend	I(1)
$p_{C_{sa}}$	-1.2949	-3.45	trend	I(1)
pp_{sa}^{E}	-0.4481	-1.95	none	I(1)

Table 91: Results from ADF Tests for variables entering estimation for producer prices for the FC sector.

Step 3: Cointegration test

F-statistic	(k)	Case	Bounds Interval (5%)	Conclusion
6.31	4	Case II (Restricted intercept and no trend)	[2.56 ; 3.49]	Cointegration
6.61	3	Case III (Unrestricted intercept and no trend)	[2.86 ; 4.01]	Cointegration

Table 92: Results from ARDL Bounds Test.

Null Hypothesis	Test Statistic	5% Critical Value
$r \leq 4$	5.23	12.25
$r \leq 3$	14.72	25.32
$r \leq 2$	30.65	42.44
$r \leq 1$	56.75	62.99
$r = 0$	117.54	87.31

Table 93: Johansen Trace Test Results with trend.

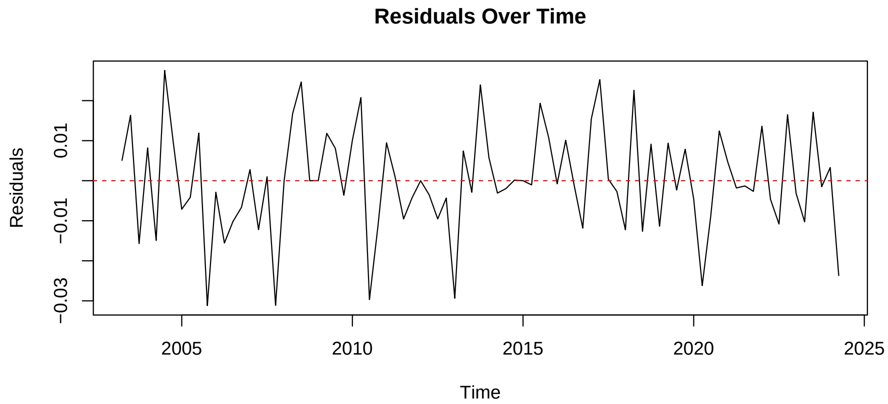
Null Hypothesis	**Test Statistic**	**5% Critical Value**
$r \leq 4$	4.66	9.24
$r \leq 3$	13.47	19.96
$r \leq 2$	39.40	34.91
$r \leq 1$	65.65	53.12
$r = 0$	140.47	76.07

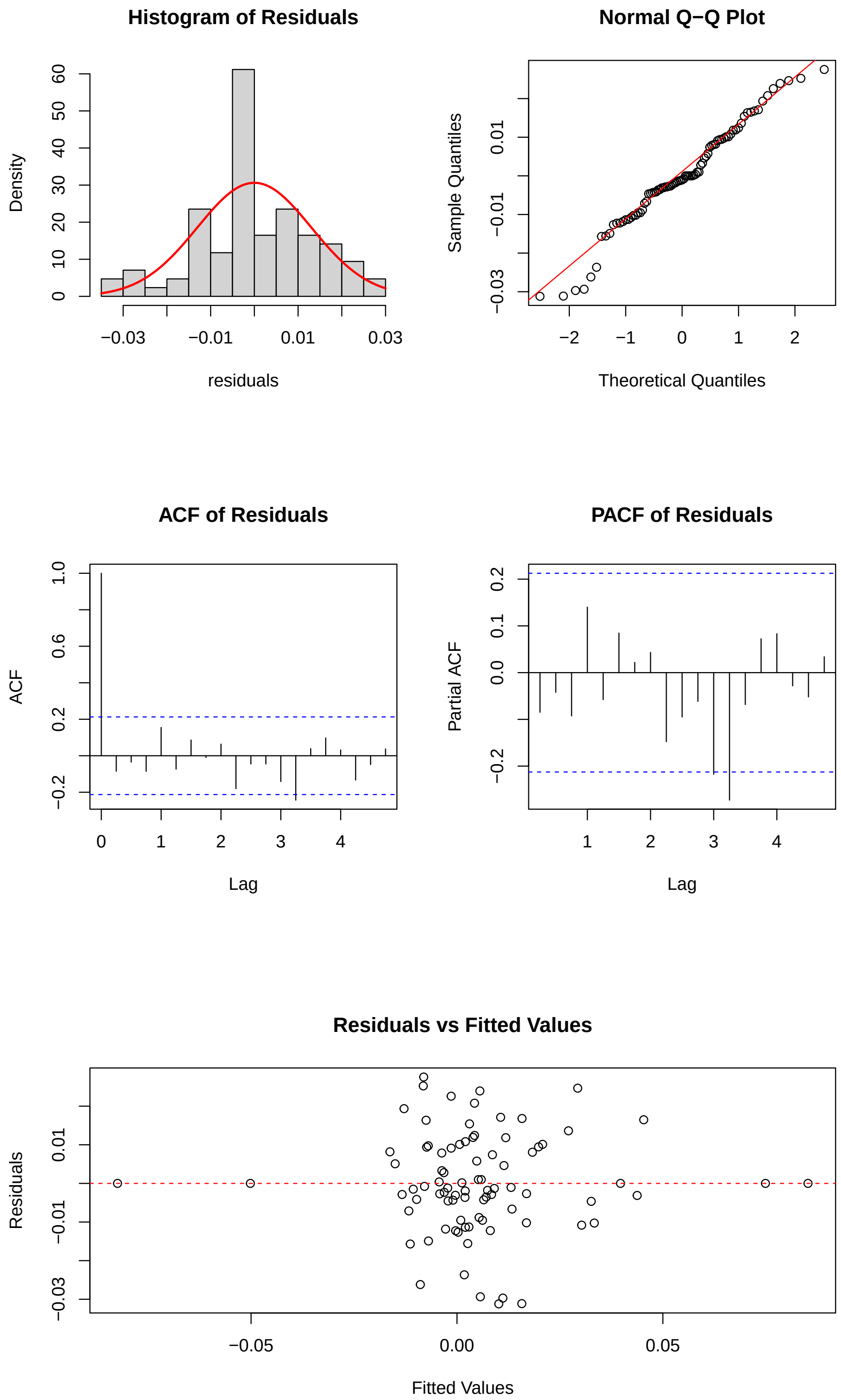
Table 94: Johansen Trace Test Results without trend and constant.

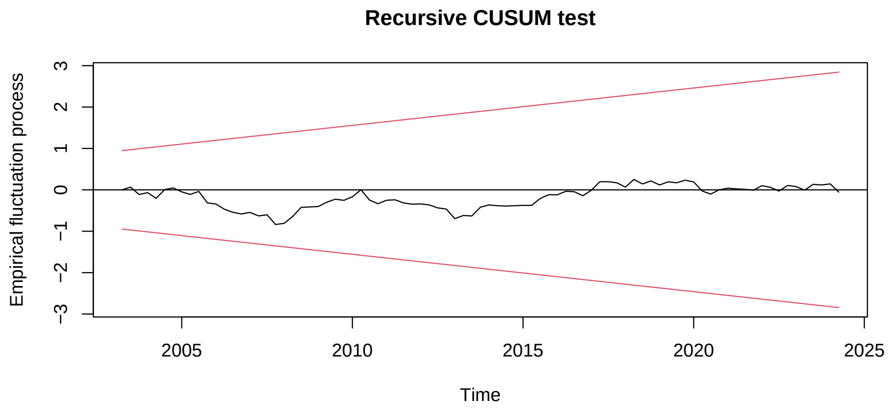
Step 4: Run the specific model

```
Call:
dynlm(formula = d(lpp_FC_sa) ~ L(d(LD1_FC_sa), 4) + L(d(lP_G_sa), 4) +
  d(lP_C_sa) + L(lpp_FC_sa, 1) + L(lpp_E_sa, 1) + dummy08 +
  dummy08.4 + dummy09 + dummy12 + dummy15, data = data_PP_FC)

Residuals:
     Min        1Q    Median        3Q       Max
-0.03120 -0.00712 -0.00105  0.00938  0.02754

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)
(Intercept)           -0.00271    0.00209  -1.297  0.19849
L(d(LD1_FC_sa), 4)     0.07408    0.03948   1.877  0.06453 .
L(d(lP_G_sa), 4)       0.30405    0.15246   1.994  0.04980 *
d(lP_C_sa)             0.59346    0.23962   2.477  0.01555 *
L(lpp_FC_sa, 1)       -0.05133    0.01496  -3.433  0.00098 ***
L(lpp_E_sa, 1)         0.06985    0.01155   6.050 5.47e-08 ***
dummy08               -0.05924    0.01469  -4.032  0.00013 ***
dummy08.4              0.06121    0.01468   4.171 8.17e-05 ***
dummy09               -0.07798    0.01445  -5.395 7.89e-07 ***
dummy12                0.06060    0.01426   4.251 6.13e-05 ***
dummy15                0.04504    0.01430   3.150  0.00236 **

---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.'

Residual standard error: 0.01388 on 74 degrees of freedom
Multiple R-squared: 0.7158,    Adjusted R-squared: 0.6774
F-statistic: 18.64 on 10 and 74 DF,  p-value: < 2.2e-16
```

Step 6: Run diagnostic tests




Histogram of Residuals
Density
residuals
Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
ACF of Residuals
ACF
Lag
PACF of Residuals
Partial ACF
Lag
Residuals vs Fitted Values
Residuals
Fitted Values



Test	p-value
Shapiro-Wilk	0.09541
Jarque Bera	0.7211
Durbin-Watson	0.6587
Breusch-Godfrey, order 4	0.3887

Table 95: Diagnostics statistics.

Variable	VIF Value
L(d(LD1_FC_sa), 4)	1.171150
L(d(lP_G_sa), 4)	1.117818
d(lP_C_sa)	1.140431
L(lpp_FC_sa, 1)	2.362142
L(lpp_E_sa, 1)	2.275007
dummy08	1.106745
dummy08.4	1.104770
dummy09	1.071340
dummy12	1.042034
dummy15	1.048361

Table 96: Variance Inflation Factor (VIF) results for the pp^{FC} regression.

Step 7: Evaluate static and dynamic forecasts