Pressure Investigation of Undisturbed Wave Field

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WAVESTAR

INORE

Introduction

Wave energy is generated through a wave-structure-interaction where the energy in the wave motion is transformed into electric energy through various devices. During this process the current, the generated dynamic pressure and the slamming forces from the waves strain the structure. In order to optimize the energy output and the structural design it is necessary to have a knowledge of the pressures and forces on the given structure, as was the case with the Wavestar point absorber that fractured in the float shell shortly after installed at the coast near Hanstholm in Denmark.

Figure 1: Wavestar at Hanstholm in Denmark.

Many available theories give a good description of the undisturbed wave field but in order to investigate the pressures in water disturbed by a given structure, a numerical approach is often needed in order to simulate the wave-structure- interference.

Methodology

The aim is to develop a design method to estimate the "real" wave forces from the knowledge of the undisturbed wave field at the desired location of the wave energy device. Therefore the many different theories on the undisturbed wave field are examined and their resulting total force on the structure is compared with numerical calculations of the disturbed wave field in order to find a design factor.

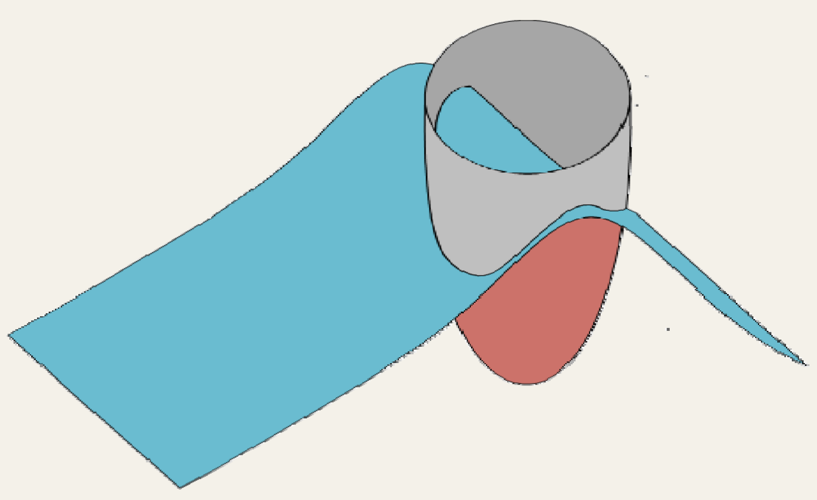

Figure 2: Concept of undisturbed wave field.

The investigation of the pressures in the undisturbed wave field is carried out using Stokes 1st order and 5th order theory together with Deans Stream function theory and used together with a discretized model as shown in Figure 2. The centroids of the discretized elements are used to find the position in the wave field and the pressure are here calculated from the given theories. The pressures on the discretized elements are integrated over the wetted surface of the structure in order to find the total forces and moments on the structure.

Definitions and Symbols

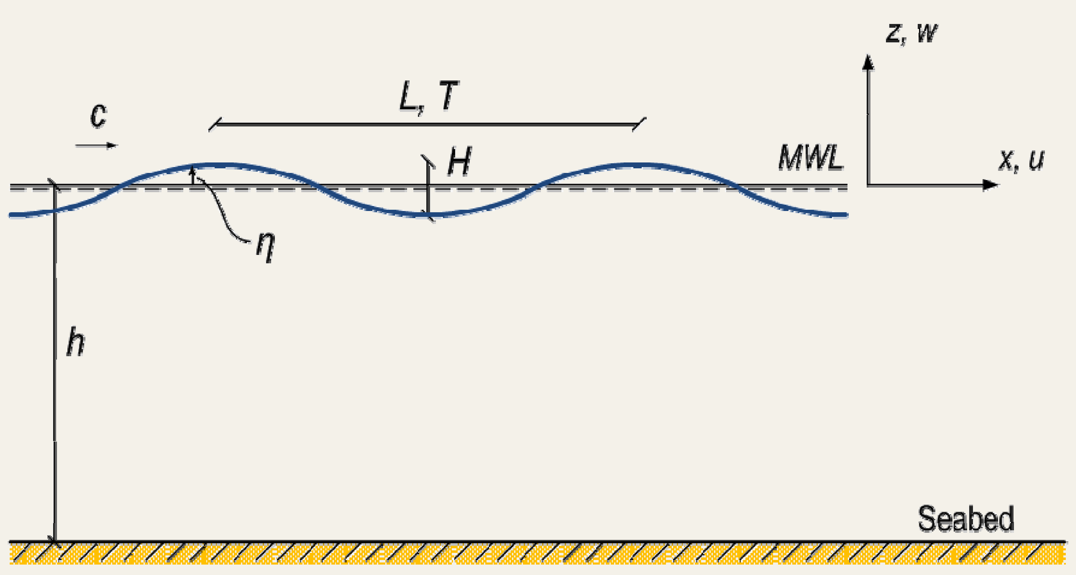

Figure 3: Definitions and symbols.

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Thoughts on Theory Application

The application of the various theories is valid for certain ocean conditions due to the assumptions related to the derivation of the theories. Figure 4 shows the ranges of the application for the different theories, described by the wave height, water depth, wave period and the gravitational acceleration.

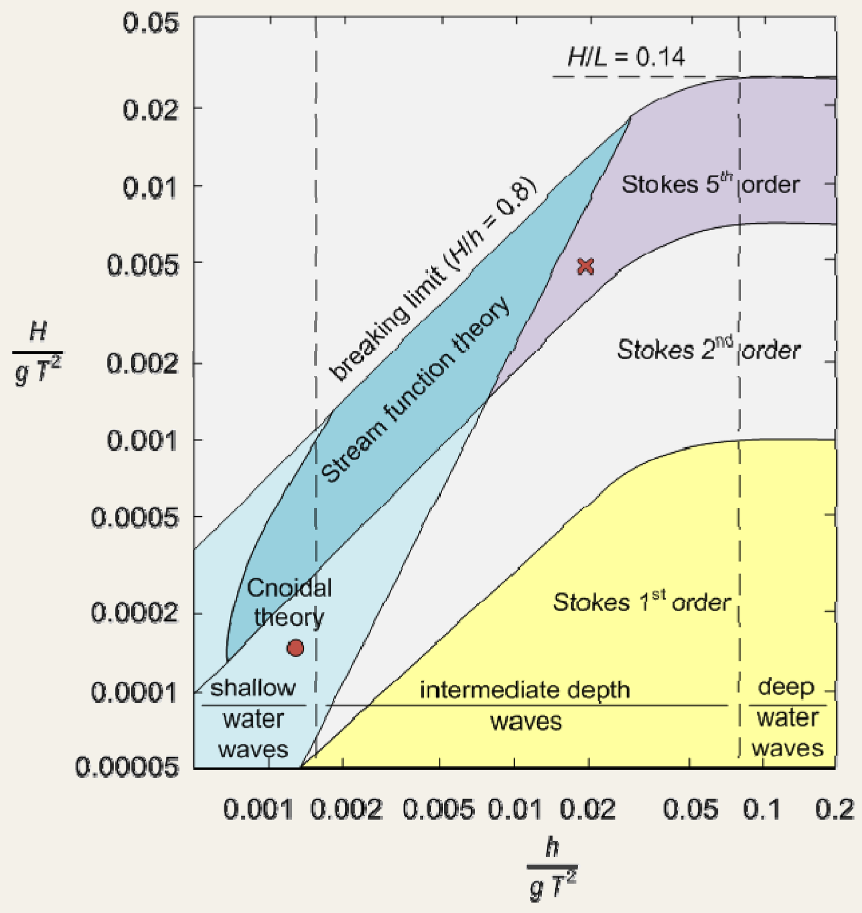

Figure 4: Overview of valid areas of application of the different theories. The red marks represents test cases.

All Stoke theories can be applied for deep water waves. The higher order theory the higher are the waves that can be described by the theory. When going from deep water to shallow water the physics fall apart and the surface elevation using any non-linear Stoke theory starts showing secondary crests in the wave trough. This "malfunction" in the theory is caused by the assumptions in the approximation of the surface boundary conditions when solving the partial differential equation for the undisturbed wave field.

A demonstration of the range in application of the various theories is made with two wave scenarios listed in Table 1 and shown in Figure 4. The illustrations in Figure 5 and 6 show how Stokes 5th order theory breaks down when outside the range of its application. Stokes 1st order theory does not break down as such but the description of the wave field is no longer in compliance with the real wave behavior.

Test case	Notation in Figure 4	H [m]	T [s]	h [m]
No. 1	✖	0.41	2.0	0.75
No. 2	●	0.01	2.5	0.08

Table 1: Wave specifics for test cases.

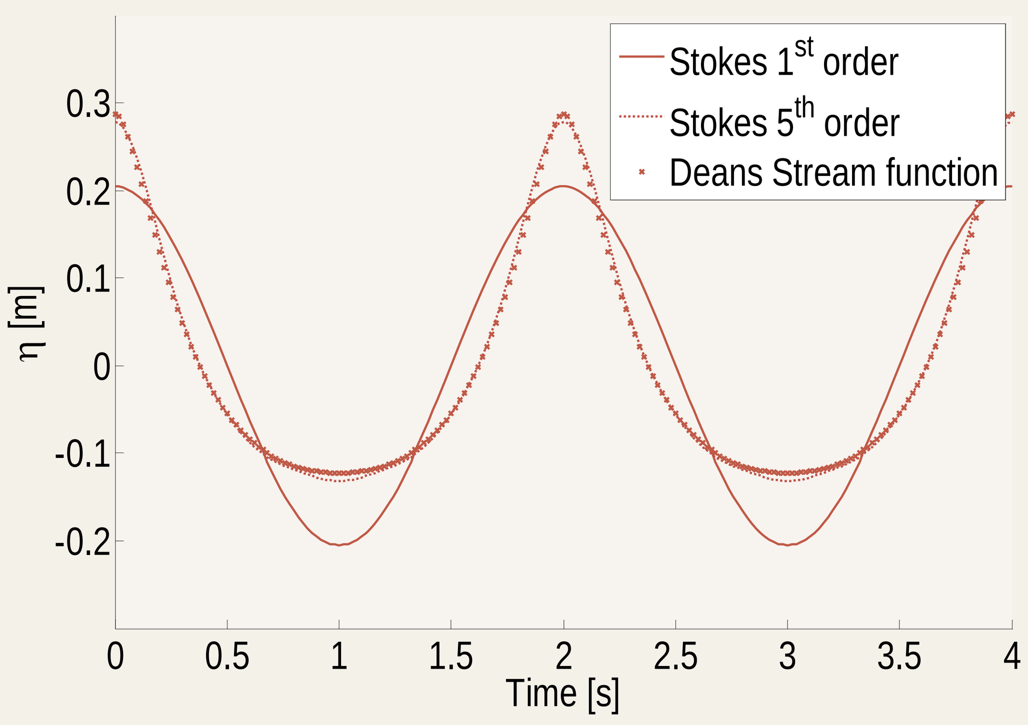

Figure 5: Surface elevation for test case no. 1.

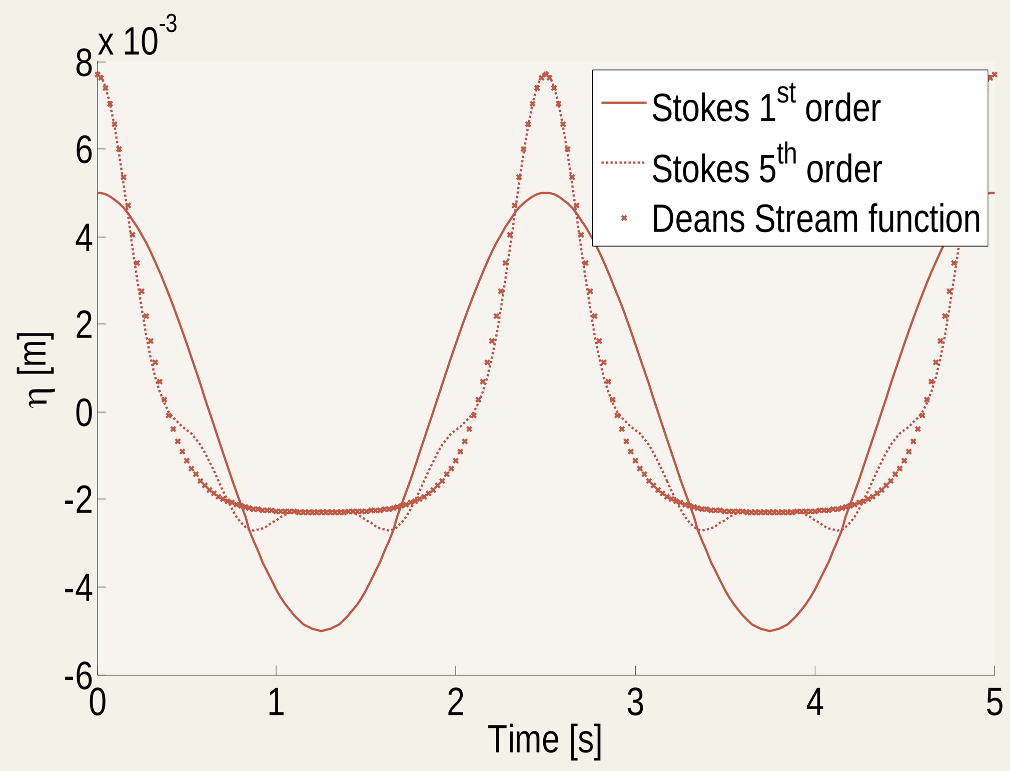

Figure 6: Surface elevation for test case no. 2.

Pressure Distribution

The total pressure in the water is composed of the hydrostatic pressure generated from the position relative to the mean water level and the dynamic pressure generated from the wave motion. The combination is written in the generalized Bernoulli Equation.

$$p = \underbrace{-\rho g z}_{\text{Hydrostatic pressure}} \underbrace{-\rho\left[\frac{1}{2}\left(u^2+w^2\right)+\frac{\partial \phi}{\partial t}\right]}_{\text{Dynamic pressure}}$$

The expression for the hydrostatic pressure is relatively simple and can be found from the position of the surface elevation. The dynamic part of the total pressure is depending on the velocity potential $\phi(z,x,t)$ and the velocities of the water particles.

For Stoke waves the velocity potential is an analytical solution to the Laplace Equation, with approximated boundary conditions at the surface, Brorsen (2007) . For Stokes 1st order theory the approximation of the boundary condition assumes that the wave height is negligible compared to the wave length and the perturbation method is used in order to derive the theory behind the non-linear Stoke waves.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

For Deans Stream function theory the method is to find an approximate numerical solution to the partial differential equation with exact boundary conditions fulfilled at the surface. Since no terms are discarded in the boundary conditions there is no demands to H/L or h/L which makes the theory free of the limitations of the Stoke waves though the theory still breaks down at very low water depths and when close to the breaking limit, which Figure 4 also illustrates, Brorsen (2007) .

For Deans Stream function the velocity potential is zero due to the assumption of steady flow and instead the velocities in the expression for the total pressure are found from the stream function $\psi(z,x,t)$, Brorsen (2007) .

$$\frac{\partial^2 \psi}{\partial x_r^2} + \frac{\partial^2 \psi}{\partial z^2} = 0$$

Figure 7 shows the dynamic pressure given by Stokes 1st order theory and Deans Stream function theory for the same wave.

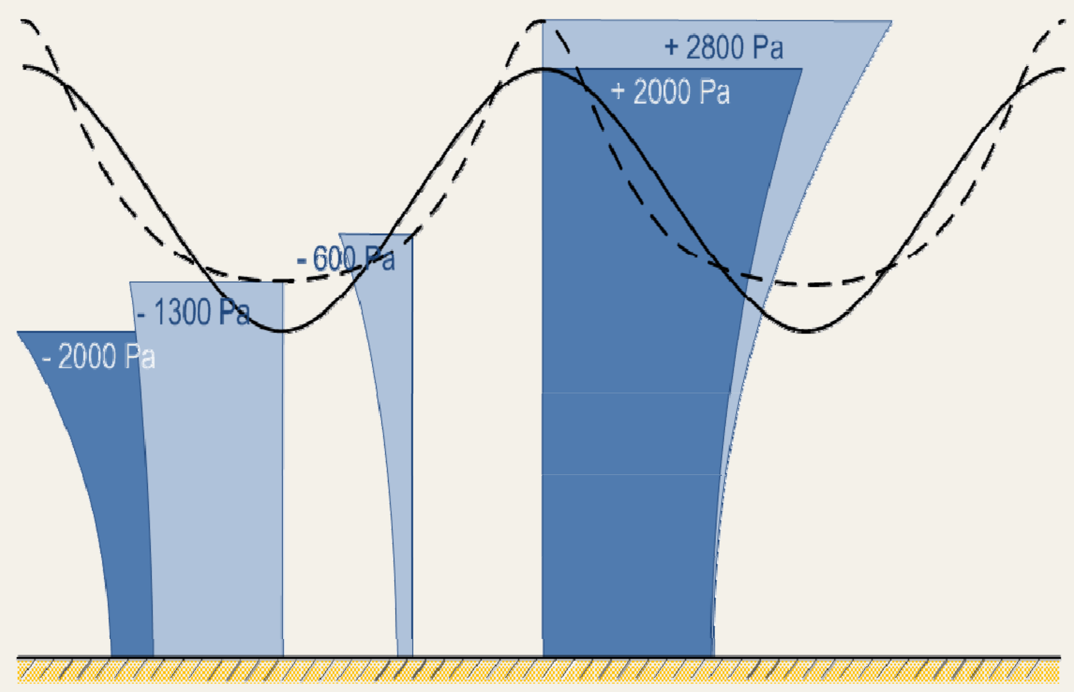

Figure 7: Relative distribution of dynamic pressure in water for test case no. 1. Light blue color represent results from Deans Stream function theory and dark blue color represent Stokes 1st order theory.

Conclusions and Further Plans

The calculations carried out in Matlab have shown good results Stokes 1st order and 5th order theory and Deans Stream function theory and a validation have been made against the computer software Wavelab at Aalborg University.

The pressure programs in Matlab have been employed together with the geometry shown in Figure 2 and they are being integrated in order to find the total forces on the point absorber.

Future plans include:

- Comparing with experimental test data from the wave basin at Aalborg University.
- Calculating the disturbed pressures, forces and moments on the point absorber in order to find the relationship between the forces from the two wave fields.

References

Brorsen, M. (2007). Non-linear Waves. DCE Lecture Notes No. 9, Aalborg University, Department of Civil Engineering.