Advanced Analysis of Steel Structures

Master Thesis

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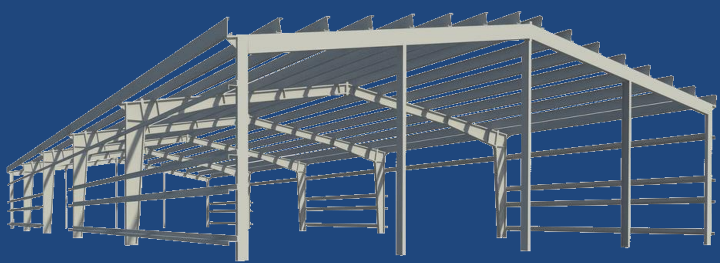
Appendix Report



Group B-204d | M.Sc.Structural and Civil Engineering | Aalborg University | 4th Semester | Spring 2013

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B Worked Example - Interaction Formulae

This chapter is an elaboration of the calculations found in Chapter 4 of the main report.

In European Standard [2005], the following equations must be satisfied satisfied when calculating by Interaction Formulae, see Equation (B.1) and (B.2).

$$\frac{N_{\text{Ed}}}{\frac{\chi_{\text{y}} f_{\text{y}} A_{\text{i}}}{\gamma_{\text{M1}}}} + k_{\text{yy}} \frac{M_{\text{y,Ed}} + \Delta M_{\text{y,Ed}}}{\chi_{\text{LT}} \frac{f_{\text{y}} W_{\text{i}}}{\gamma_{\text{M1}}}} + k_{\text{yz}} \frac{M_{\text{z,Ed}} + \Delta M_{\text{z,Ed}}}{\frac{f_{\text{y}} W_{\text{i}}}{\gamma_{\text{M1}}}} \leq 1 \tag{B.1}$$

$$\frac{N_{\text{Ed}}}{\frac{\chi_{\text{z}} f_{\text{y}} A_{\text{i}}}{\gamma_{\text{M1}}}} + k_{\text{zy}} \frac{M_{\text{y,Ed}} + \Delta M_{\text{y,Ed}}}{\chi_{\text{LT}} \frac{f_{\text{y}} W_{\text{i}}}{\gamma_{\text{M1}}}} + k_{\text{zz}} \frac{M_{\text{z,Ed}} + \Delta M_{\text{z,Ed}}}{\frac{f_{\text{y}} W_{\text{i}}}{\gamma_{\text{M1}}}} \leq 1 \tag{B.2}$$

N_{Ed} , $M_{\text{y,Ed}}$, $M_{\text{z,Ed}}$	Design values of the compression force and the maximum moments about the *y-y* and *z-z* axis along the member, respectively	[-]
$\Delta M_{\text{y,Ed}}$, $\Delta M_{\text{z,Ed}}$	Moments due to the shift of the centroidal axis for class 4 sections	[-]
χ_{y} , χ_{z}	Reduction factors due to flexural buckling	[-]
χ_{LT}	Reduction factor due to lateral torsional buckling	[-]
k_{yy} , k_{yz} , k_{zy} , k_{zz}	Interaction factors	[-]

The values shown in Table B.1 are given in European Standard [2005] for the different cross-section (C-S) classes.

Class	1	2	3	4
A_{i}	A	A	A	A_{eff}
W_{y}	$W_{\text{pl,y}}$	$W_{\text{pl,y}}$	$W_{\text{el,y}}$	$W_{\text{eff,y}}$
W_{z}	$W_{\text{pl,z}}$	$W_{\text{pl,z}}$	$W_{\text{el,z}}$	$W_{\text{eff,z}}$
$\Delta M_{\text{y,Ed}}$	0	0	0	$e_{\text{N,y}}\, N_{\text{Ed}}$
$\Delta M_{\text{z,Ed}}$	0	0	0	$e_{\text{N,z}}\, N_{\text{Ed}}$

Table B.1: Values for $N_{\text{Rk}} = f_{\text{y}}\, A_{\text{i}}$, $M_{\text{i,Rk}} = f_{\text{y}}\, W_{\text{i}}$ and $\Delta M_{\text{i,Ed}}$ [European Standard, 2005].

This appendix will give a full worked example of the calculation for a profile in regard of Interaction Formulae in the EC. In order to find the right profile for Interaction Formulae in EC, a MATLAB code has been developed. The code has input parameters as section and material properties and different assumptions such as effective buckling lengths. As output, the program states if the equations of Interaction Formulae are satisfied. If not, a new profile with different section properties must be entered. This makes the whole process an iterative process. This example is calculated for a HE320A profile where every calculation is based on European Standard [2005].

B.1 Section Properties

Some of the profile properties have been adjusted in order to fit with the results from Abaqus/CAE. In Abaqus/CAE, no radius for a rolled sections is taken into account, meaning that the area in Table

B.2 has been reduced. In addition, the moment of inertia is also calculated and reduced due to the removal of area of the rolled section. Whenever in this calculation a choice must be made between rolled or welded section, the welded section is always chosen in order to be able to compare the results in Abaqus/CAE.

HE320A			
Description	Symbol	Value	Unit
Height	h	310	[mm]
Width	b	300	[mm]
Thickness of web	d	9	[mm]
Thickness of flange	t	15.5	[mm]
Radius	r	27	[mm]
Cross-sectional area	A	$11.77 \cdot 10^3$	$\left[\mathrm{mm}^2\right]$
Second moment of area about y-y axis	I_y	$218.12 \cdot 10^6$	$\left[\mathrm{mm}^4\right]$
Elastic section modulus	$W_{\mathrm{el,y}}$	$1480 \cdot 10^3$	$\left[\mathrm{mm}^3\right]$
Plastic section modulus	W_{pl}	$1628 \cdot 10^3$	$\left[\mathrm{mm}^3\right]$
Second moment of area about z-z axis	I_z	$69.8 \cdot 10^6$	$\left[\mathrm{mm}^4\right]$
Torsional moment of inertia	I_v	$1080 \cdot 10^3$	$\left[\mathrm{mm}^4\right]$
Warping moment of inertia	I_w	$1510 \cdot 10^9$	$\left[\mathrm{mm}^6\right]$

Table B.2: Cross-sectional properties for HE320A [Mohr and et al., 2009].

B.2 Check of HE320A for Element 3

As HE320A is classified as cross-section (C-S) class 1, the following calculations will take this into account. In addition, the loads with snow as the dominant load can be seen for the reference frame in Figure B.1.

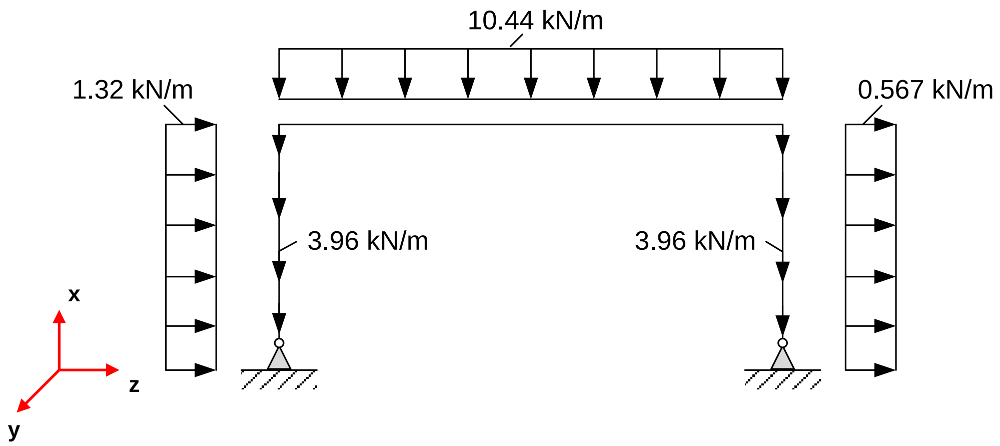


Figure B.1: Loads on the reference frame with snow as the dominant load.

From these loads, the moment diagram seen in Figure B.2 is calculated in the FEM software Autodesk Robot.

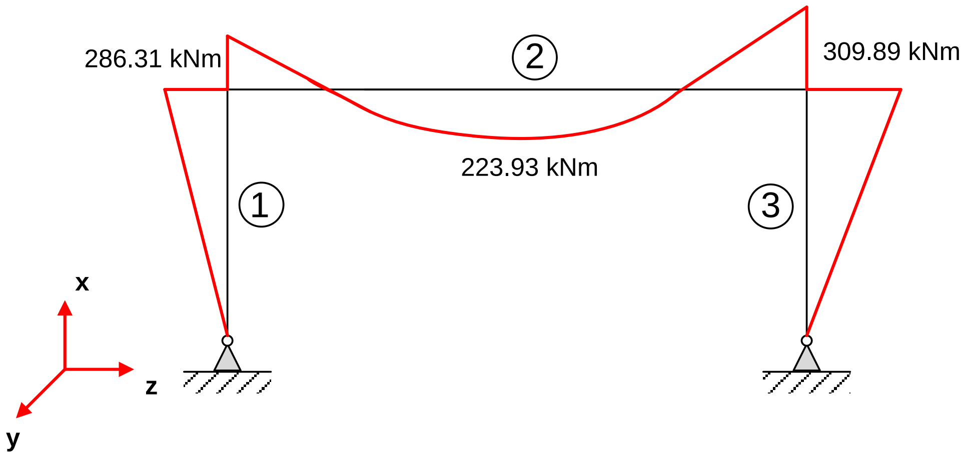


Figure B.2: Illustration of the moments on the frame.

The column checked is element 3 as this element will have the greatest moment and compression force out of the two columns.

B.2.1 Forces and Moments

The compression force, N_{Ed}, is determined by the applied loads on the roof of the frame and self-weight of the element. In addition,the wind load will induce a small compression force as well. The total reaction force in element 3 is found by Autodesk Robot. The total force is found to be $N_{\mathrm{Ed}} = 125.38$ kN.

The moment about the y-axis is also calculated by Autodesk Robot. Moments are occurring because of the loads on the roof and the loads from the sides due to wind actions. The moments are then added together, and the worst moment is calculated to be $M_{\mathrm{y,Ed}} = 309.89$ kNm.

Since no out-of-plane loads are applied, the moment about the z-axis is equal to zero ($M_{\mathrm{z,Ed}} = 0$ kNm). As there are no shift of the centroidal axis (not class 4 sections), there are no additional moments. Hence, $\Delta M_{\mathrm{y,Ed}} = \Delta M_{\mathrm{z,Ed}} = 0$.

B.2.2 Cross-Section Checks

Compression

The design resistance of the C-S for the uniform compression force, $N_{\mathrm{c,Rd}}$, should satisfy Eq. (B.3).

$$\frac{N_{\mathrm{Ed}}}{N_{\mathrm{c,Rd}}} \leq 1.0 \tag{B.3}$$

The design resistance of the C-S for uniform compression, $N_{\mathrm{c,Rd}}$, can be calculated as in Eq. (B.4).

$$\begin{aligned} N_{\mathrm{c,Rd}} &= \frac{A\, f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \\ &= \frac{11774\ \mathrm{mm}^2 \cdot 235\ \mathrm{N/mm}^2}{1.1} \\ &= 2515.35 \cdot 10^3\ \mathrm{N} \end{aligned} \tag{B.4}$$

The C-S check is shown in Eq. (B.5).

$$\frac{125.38\ \mathrm{kN}}{2515.35\ \mathrm{kN}} = 0.050 \leq 1.0 \tag{B.5}$$

Bending Moment

The design value of the bending moment, M_{Ed}, at each cross-section should satisfy Eq. (B.6).

$$\frac{M_{\text{Ed}}}{M_{\text{c,Rd}}} \leq 1.0 \tag{B.6}$$

The design resistance for bending, $M_{\text{c,Rd}}$, is calculated in Eq. (B.7).

$$\begin{aligned} M_{\text{c,Rd}} &= \frac{W_{\text{pl}}\, f_{\text{y}}}{\gamma_{\text{M0}}} \\ &= \frac{1628 \cdot 10^3 \text{ mm}^3 \cdot 235 \text{ N/mm}^2}{1.1} \\ &= 347.8 \cdot 10^6 \text{ Nmm} \end{aligned} \tag{B.7}$$

M_{Ed} is equal to $M_{\text{y,Ed}}$ = 309.89 kNm. This means that the C-S check will be as Eq. (B.8).

$$\frac{309.89 \text{ kNm}}{347.8 \text{ kNm}} = 0.89 \leq 1.0 \tag{B.8}$$

It can be seen that the element in regard of the bending moment has a high utilization ratio.

Shear

The design value of the shear force, V_{Ed}, at each C-S should satisfy Eq. (B.9).

$$\frac{V_{\text{Ed}}}{V_{\text{c,Rd}}} \leq 1.0 \tag{B.9}$$

$V_{\text{c,Rd}}$ is the design shear resistance. As HE320A is classified as C-S class 1, the verification of the shear force should be plastic. In addition, it is so that as there is no torsion in the frame, hence the plastic shear resistance is given as seen in Eq. (B.10).

$$V_{\text{pl,Rd}} = \frac{A_{\text{v}}\, \left(f_{\text{y}}/\sqrt{3}\right)}{\gamma_{\text{M0}}} \tag{B.10}$$

A_{v} is the shear area and will be taken as the expression shown in Eq. (B.11) for sections with load parallel to web.

$$\begin{aligned} A_{\text{v}} &= A - 2b\, t_{\text{f}} + (t_{\text{w}} \cdot t_{\text{f}}) \\ &= (11.77 \cdot 10^3 \text{ mm}^2) - 2 \cdot 300 \text{ mm} \cdot 15.5 \text{ mm} + (9 \text{ mm} \cdot 15.5 \text{ mm}) \\ &= 2610 \text{ mm}^2 \end{aligned} \tag{B.11}$$

It should also be noticed that Eq. (B.11) is based on a profile without the welds of the section,cf. Section B.1. In addition, A_{v} should not be less than Eq. (B.12).

$$\begin{aligned} &\eta h_{\text{w}} t_{\text{w}} \\ &\eta h_{\text{w}} t_{\text{w}} = 1.0 \cdot 300 \text{ mm} \cdot 9 \text{ mm} = 2700 \text{ mm}^2 \end{aligned} \tag{B.12}$$

The factor for shear area, η, may conservatively be taken equal to 1.0 according to European Standard [2005].

Thereby, A_{v} is taken as 2700 mm^2. Now, $V_{\text{pl,Rd}}$ can be calculated as shown in Eq. (B.13).

$$\begin{aligned} V_{\text{pl,Rd}} &= \frac{2700 \text{ mm}^2 \left(235 \text{ N/mm}^2/\sqrt{3}\right)}{1.1} \\ &= 524.38 \cdot 10^3 \text{ N} \end{aligned} \tag{B.13}$$

The C-S check can then be made as the shear force is found to be 63.4 kN in Autodesk Robot in Eq. (B.14).

$$\frac{63.4\text{ kN}}{524.38\text{ kN}} = 0.12 \ \leq \ 1.0 \tag{B.14}$$

Linear summation of the utilization ratios:

According to Eurocode 3 [European Standard, 2005], the following is stated about general resistance of cross-sections. "As a conservative approximation for all cross-sectional classes a linear summation of the utilization ratios for each stress resultant may be used". For cross-section classes 1, 2 and 3 the following criteria must be verified, see Eq. (B.15).

$$\frac{N_{\text{Ed}}}{N_{\text{Rd}}} + \frac{M_{\text{y,Ed}}}{M_{\text{y,Rd}}} + \frac{M_{\text{z,Ed}}}{M_{\text{z,Rd}}} \leq 1.0 \tag{B.15}$$

By inserting the values previously calculated into Eq.(B.15), the verification can be done as shown in Eq. (B.16).

$$\begin{aligned} \frac{125.38\text{ kN}}{2515.35\text{ kN}} + \frac{309.89\text{ kNm}}{347.8\text{ kNm}} + 0 &\leq 1.0 \\ 0.94 \leq 1.0 &\Rightarrow \text{OK} \end{aligned} \tag{B.16}$$

B.2.3 Buckling Curves

The reduction factor for the relevant buckling curve, χ, is calculated in Eq. (B.17).

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \quad , \quad \chi \leq 1 \tag{B.17}$$

The value, Φ, to determine χ is calculated in Eq. (B.18).

$$\Phi = 0.5 \ \left(1 + \alpha \ \left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right) \tag{B.18}$$

α is an imperfection factor and is determined by a buckling curve that is defined by the profile used. For HE320A, the relationship between the hight and the width of the section is calculated in Eq. (B.19).

$$h/b = 310\text{ mm}/300\text{ mm} = 1.03 \tag{B.19}$$

Since the relationship is 1.03, the web thickness, t_{w}, is equal to 15.5 mm and the section is assumed to be a welded section, the buckling curves will be according to Table B.3.

Buckling about axis	Buckling curve for S235	Imperfection factor α
y-y	*b*	0.34
z-z	*c*	0.49

Table B.3: Imperfection factors based on buckling curves.

The non-dimensional slenderness, $\overline{\lambda}$ is calculated in Eq. (B.20).

$$\overline{\lambda} = \sqrt{\frac{A\ f_{\text{y}}}{N_{\text{cr}}}} \tag{B.20}$$

N_{cr} is the elastic critical force for the relevant buckling mode based on a the gross cross sectional properties. N_{cr} is calculated as seen in Eq. (B.21).

$$N_{cr} = \frac{\pi^2 E I}{l_s^2} \tag{B.21}$$

l_s is the effective length of element 3. The effective length will depend on the different support conditions of the element. An example of the effective length with different support conditions can be seen in Figure B.3. Here, it is shown which factor the original length of the element should be multiplied by in order to give the effective length, l_s.

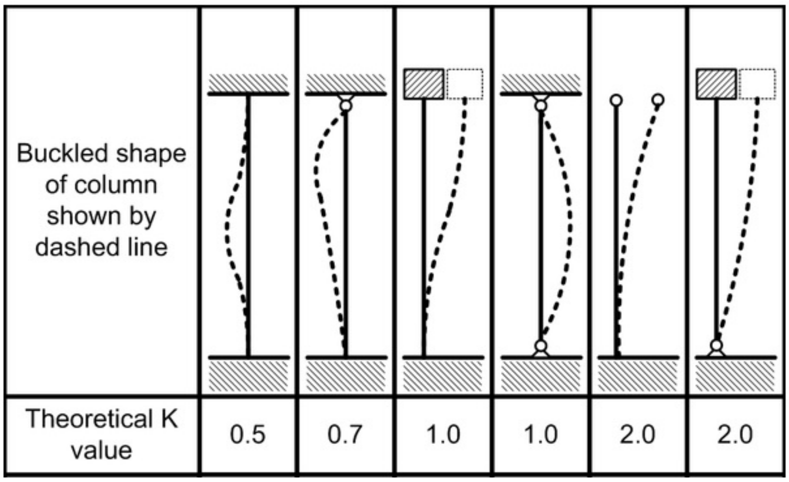


Figure B.3: Effective length factors, K [Wikipedia].

The elastic critical force, N_{cr}, is calculated for both $y-y$ axis and $z-z$ axis and their respective effective length. In the end, this will lead to a difference in the reduction factor, χ, in regard of the different axis. Therefore, a reduction factor, χ_y and χ_z, for the y and the z axis, respectively, is calculated separately.

Reduction factor, χ_y, for the y axis:

First N_{cr} is calculated and the effective length, l_s, should be found. From Force-Euro, an effective length equation is given for element 3 about the strong axis. The effective length, l_s, for element 3 is calculated in Eq. (B.22), and an illustration of the parameters can be seen in Figure B.4.

$$l_s = h \sqrt{4 + 3.2 \, \frac{s\, I}{h\, I_o}} \tag{B.22}$$

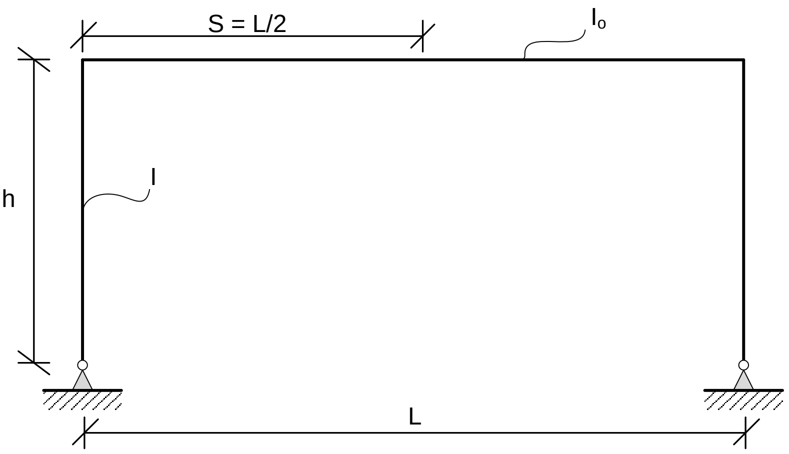


Figure B.4: Illustration of parameters for Eq. (B.22) [Force-Euro].

By inserting the same moment of inertia for both element 2 and 3, the height of the structure h = 5000 mm and half of the length of element 2, the effective length, l_s, is calculated to be l_s = 16120 mm. The elastic critical normal force, N_{cr}, can then be calculated as in Eq. (B.23).

$$N_{cr} = \frac{\pi^2 \, E \, I_y}{l_s^2} = \frac{\pi^2 \cdot 2.1 \cdot 10^{)} \text{ N/mm}^2 \cdot 218.12 \cdot 10^6 \text{ mm}^4}{16120 \text{ mm}^2} = 1744.1 \cdot 10^3 \text{N} \tag{B.23}$$

Then the reduction factor, χ_y, can be calculated by Eq. (B.24), (B.25) and (B.26).

$$\overline{\lambda} = \sqrt{\frac{11.77 \cdot 10^3 \text{ mm}^2 \ \cdot 235 \text{ N/mm}^2}{1744.1 \cdot 10^3 \text{ N}}} = 1.26 \tag{B.24}$$

$$\Phi = 0.5 \ \left(1 + 0.34 \ (1.26 - 0.2) + 1.26^2\right) = 1.47 \tag{B.25}$$

The reduction factor, χ_y, for buckling curve b in y-y axis will then be:

$$\chi_y = \frac{1}{1.47 + \sqrt{1.47^2 - 1.26^2}} = 0.447 \tag{B.26}$$

The calculated χ_y is within the limits given in Equation (B.17), so nothing else is done.

Reduction factor, χ_z, for the *z* axis:

The effective length, l_s, about the z-axis is based on a length between a pinned and fixed support in the top of element 3, which is were element 3 meets element 2. At the bottom of element 3, the support will still be pinned. To see the difference by using the two types of supports at the top, the calculation for both has been done and the results are shown Table B.4.

Type of support	Effective length, l_s [mm]	N_{cr} [N]	χ_z [-]
Fixed end	3500	$11810.0 \cdot 10^3$	0.85
Pinned end	5000	$5786.7 \cdot 10^3$	0.73

Table B.4: The results of different effective length about the z axis for element 3.

In the end, the support condition of element 2 is something in between fixed and pinned, but it is difficult to say exactly. Therefore, the worst case is chosen. Hence pinned end is chosen giving element (3) an effective length of l_s = 5000 mm. The results based on this effective length can be seen in Table B.5.

Type of support at the ends	Effective length, l_s [mm]	$\bar{\lambda}$ [-]	N_{cr} [N]	χ_z [-]
Pinned	5000	0.69	$5786.7 \cdot 10^3$	0.73

Table B.5: Results for element 3 about the z axis.

Checks of Buckling Effects

There are two different checks which can be performed to check if buckling effects may be ignored and that only cross-sectional checks will apply. The checks are:

1. $\bar{\lambda} \leq 0.2$
2. $\frac{N_{Ed}}{N_{cr}} \leq 0.04$

For the reduction factor, χ_y:

1. $\overline{\lambda} = 0.78 \leq 0.2 \Rightarrow$ NOT OK
2. $\frac{N_{Ed}}{N_{cr}} = \frac{125.38 \cdot 10^3 \text{ N}}{4520.8 \cdot 10^3 \text{ N}} = 0.028 \leq 0.04 \Rightarrow$ OK

For the reduction factor, χ_z:

1. $\overline{\lambda} = 0.69 \leq 0.2 \Rightarrow$ NOT OK
2. $\frac{N_{Ed}}{N_{cr}} = \frac{125.38 \cdot 10^3 \text{ N}}{5786.7 \cdot 10^3 \text{ N}} = 0.022 \leq 0.04 \Rightarrow$ OK

Based on these results, it can be seen that the buckling effects may not be ignored for this section due to the slenderness of the element.

In addition to the checks just made, the European Standard [2005] states another check needed for a compressed element. It states that the element should be verified against buckling as shown in Eq. (B.27).

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1.0 \tag{B.27}$$

The buckling resistance, $N_{b,Rd}$, for the compressed element is calculated in Eq. (B.28).

$$N_{b,Rd} = \frac{\chi\, A\, f_y}{\gamma_{M1}} = \frac{0.447 \cdot 11.77 \cdot 10^3 \text{ mm}^2 \cdot 235 \text{ N/mm}^2}{1.2} = 1030.32 \cdot 10^3 \text{ N} \tag{B.28}$$

The reduction factor, χ, is taken as the most unfavorable result of the two axis, hence χ_y is used.

The relationship between the compression force and buckling resistance will then be as shown in Eq. (B.29).

$$\frac{N_{\mathrm{Ed}}}{N_{\mathrm{b,Rd}}} = \frac{125.38 \cdot 10^3\ \mathrm{N}}{1030.32 \cdot 10^3\ \mathrm{N}} = 0.12 \leq 1.0 \Rightarrow OK \tag{B.29}$$

B.2.4 Lateral Torsional Buckling Curves for Rolled and Equivalent Sections

The EC gives a general case when it comes to lateral torsional buckling curves and an option for rolled or equivalent sections. Therefore, it is assumed that the element has rolled or equivalent sections, and the last option of the lateral torsional buckling is chosen.

The reduction factor, χ_{LT}, for lateral-torsional buckling is determined in Eq. (B.30).

$$\chi_{\mathrm{LT}} = \frac{1}{\Phi_{\mathrm{LT}} + \sqrt{\Phi_{\mathrm{LT}}^2 - \beta\, \overline{\lambda}_{\mathrm{LT}}^2}} \quad \text{but} \quad \begin{cases} \chi_{\mathrm{LT}} \leq 1.0 \\ \chi_{\mathrm{LT}} \leq \frac{1}{\overline{\lambda}_{\mathrm{LT}}^2} \end{cases} \tag{B.30}$$

According to European Standard [2005], the following values are recommended for rolled sections:

$$\begin{aligned} \overline{\lambda}_{\mathrm{LT,0}} &= 0.4 \quad && \text{(Maximum value)} \\ \beta &= 0.75 \quad && \text{(Minimum value)} \end{aligned}$$

The value, Φ_{LT}, to determine the reduction factor, χ_{LT}, for lateral torsional buckling is calculated in Eq. (B.31).

$$\Phi_{\mathrm{LT}} = 0.5 \left(1 + \alpha_{\mathrm{LT}} \left(\overline{\lambda}_{\mathrm{LT}} - \overline{\lambda}_{\mathrm{LT,0}}\right) + \beta\, \overline{\lambda}_{\mathrm{LT}}^2\right) \tag{B.31}$$

The imperfection factor, α_{LT}, for lateral torsional buckling is found from the buckling curve. For HE320A the h/b relationship is as seen in Eq. (B.32).

$$h/b = 310\ \mathrm{mm}/300\ \mathrm{mm} = 1.03 \leq 2 \Rightarrow \text{Buckling curve } c \tag{B.32}$$

Buckling curve c gives α_{LT} = 0.49 as the profile is assumed to be welded.

Then the non-dimensional slenderness, $\overline{\lambda}_{\mathrm{LT}}$, can be calculated as in Eq. (B.33).

$$\overline{\lambda}_{\mathrm{LT}} = \sqrt{\frac{W_{\mathrm{pl,y}}\, f_{\mathrm{y}}}{M_{\mathrm{cr}}}} \tag{B.33}$$

The critical moment, M_{cr}, is calculated by the following general expression, see Eq. (B.34).

$$M_{\mathrm{cr}} = m_{\mathrm{n}}\, \frac{E\, I_{\mathrm{z}}}{l^2}\, h_{\mathrm{t}} \tag{B.34}$$

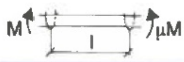
m_{n}	A value given by a table for the case investigated	[-]
h_{t}	Height of C-S from the middle of top flange to middle of bottom flange	[mm]

In order to determine a value for m_{n}, the relationship shown in Equation (B.35) must be calculated.

$$kl = \sqrt{\frac{G\, I_{\mathrm{v}}\, l^2}{E\, I_{\mathrm{w}}}} \tag{B.35}$$

kl is an input parameter that is used in different tables. The different tables reflect different loads and boundary conditions for the element analyzed. The table concerning element 3 is m_1 and the equation used to calculate m_1 can be seen in Figure B.5. From Figure B.5, it can be seen that μ needs to be determined. μ is in this case equal to zero as the moment of the bottom support is equal to zero.

m_1



$$m_1 = (9{,}22 - 4{,}29\,\mu)\sqrt{1 + \left(\frac{kl}{\pi}\right)^2}$$

$-1 \leq \mu \leq 1$

Figure B.5: Table 1 for an Eulerload [Mohr and et al., 2009].

In order to calculate m_1, kl must be calculated. It is calculated as seen in (B.36).

$$kl = \sqrt{\frac{8.1 \cdot 10^4\ \mathrm{N/mm^2} \cdot 1080 \cdot 10^3\ \mathrm{mm^4} 5000\ \mathrm{mm^2}}{2.1 \cdot 10^5\ \mathrm{N/mm^2} \cdot 1510 \cdot 10^9\ \mathrm{mm^6}}} = 2.63 \tag{B.36}$$

By inserting kl into Figure B.5, m_1 is determined to be 16.94. Next, M_{cr} can be calculated as shown in Eq. (B.37).

$$M_{\mathrm{cr}} = 19.94 \frac{2.1 \cdot 10^5\ \mathrm{N/mm^2} \cdot (69.8 \cdot 10^6\ \mathrm{mm^4})}{(5000\ \mathrm{mm})^2} \cdot 294.5\ \mathrm{mm} = 2925.1 \cdot 10^6\ \mathrm{Nmm} \tag{B.37}$$

The reduction factor, χ_{LT}, for lateral-torsional buckling can be calculated by using Eq. (B.38), (B.39) and (B.40).

$$\overline{\lambda}_{\mathrm{LT}} = \sqrt{\frac{W_{\mathrm{pl,y}}\, f_{\mathrm{y}}}{M_{\mathrm{cr}}}} = \sqrt{\frac{1628 \cdot 10^3\ \mathrm{mm^3} \cdot 235\ \mathrm{N/mm^2}}{2925.1 \cdot 10^6\ \mathrm{Nmm}}} = 0.36 \tag{B.38}$$

$$\Phi_{\mathrm{LT}} = 0.5\ \left(1 + 0.49\ (0.36 - 0.4) + 0.75 \cdot 0.36^2\right) = 0.527 \tag{B.39}$$

$$\chi_{\mathrm{LT}} = \frac{1}{0.527 + \sqrt{0.527^2 - (0.75 \cdot 0.36^2)}} = 1.05 \tag{B.40}$$

$$\begin{gathered} \chi_{\mathrm{LT}} \leq 1.0 \Rightarrow NOT OK \\ \chi_{\mathrm{LT}} \leq \frac{1}{\overline{\lambda}_{\mathrm{LT}}^2} = \frac{1}{0.36^2} = 7.72 \Rightarrow OK \end{gathered} \tag{B.41}$$

Since $\chi_{\mathrm{LT}} \geq 1.0$, χ_{LT} is set to 1.0 for the further calculations.

Check of Lateral Torsional Buckling

In some cases, lateral torsional buckling effects may be ignored and only cross-sectional checks will therefore apply. These cases are when:

$$\overline{\lambda}_{\mathrm{LT}} \leq \overline{\lambda}_{\mathrm{LT,0}}$$

$$\frac{M_{\mathrm{Ed}}}{M_{\mathrm{cr}}} \leq \overline{\lambda}_{\mathrm{LT,0}}^2$$

For HE320A with its loads applied, the checks can be calculated.

$$0.36 \leq 0.4 \Rightarrow OK$$

$$\frac{309.89 \cdot 10^6 \text{ Nmm}}{2925.1 \cdot 10^6 \text{ Nmm}} = 0.106 \leq 0.4^2 = 0.16 \Rightarrow OK$$

Based on these results, it can be seen that lateral torsional buckling effects may be ignored and only cross-sectional checks will apply.

In addition to the previous checks, another check of the buckling resistance must be done. It is stated in European Standard [2005] that "A laterally unrestrained member subjected to major axis bending should be verified against lateral torsional buckling". This is done in Eq. (B.42).

$$\frac{M_{\text{Ed}}}{M_{\text{b,Rd}}} \leq 1.0 \tag{B.42}$$

The design resistance for bending, $M_{\text{b,Rd}}$, is calculated in Eq. (B.43).

$$M_{\text{b,Rd}} = \chi_{\text{LT}}\, W_{\text{pl}}\, \frac{f_y}{\gamma_{\text{M1}}} = 1.0 \cdot 1628 \cdot 10^3 \text{ mm}^3 \frac{235 \text{ N/mm}^2}{1.2} = 318.82 \cdot 10^6 \text{ Nmm} \tag{B.43}$$

$$\frac{M_{\text{Ed}}}{M_{\text{b,Rd}}} = \frac{309.89 \cdot 10^6 \text{ Nmm}}{318.82 \cdot 10^6 \text{ Nmm}} = 0.97 \leq 1.0 \Rightarrow OK$$

B.2.5 Interaction Factors

There are two different methods in order to calculate the interaction factors, k_{ij}. As stated in Chapter 4 of the main report, the calculations will only be performed by method 2 for I-sections in C-S class 1 and 2. The interaction factors for I-sections in C-S class 1 and 2 are stated in Annex B in European Standard [2005] and shown in Eq. (B.44), (B.45), (B.46) and (B.47).

$$k_{\text{yy}} = C_{\text{my}} \left(1 + \left(\overline{\lambda}_{\text{y}} - 0.2\right) \frac{N_{\text{Ed}}}{\chi_{\text{y}}\, f_{\text{y}}\, A/\gamma_{\text{M1}}}\right) \leq C_{\text{my}} \left(1 + 0.8 \frac{N_{\text{Ed}}}{\chi_{\text{y}}\, f_{\text{y}}\, A/\gamma_{\text{M1}}}\right) \tag{B.44}$$

$$k_{\text{zy}} = 0.6 \cdot k_{\text{yy}} \tag{B.45}$$

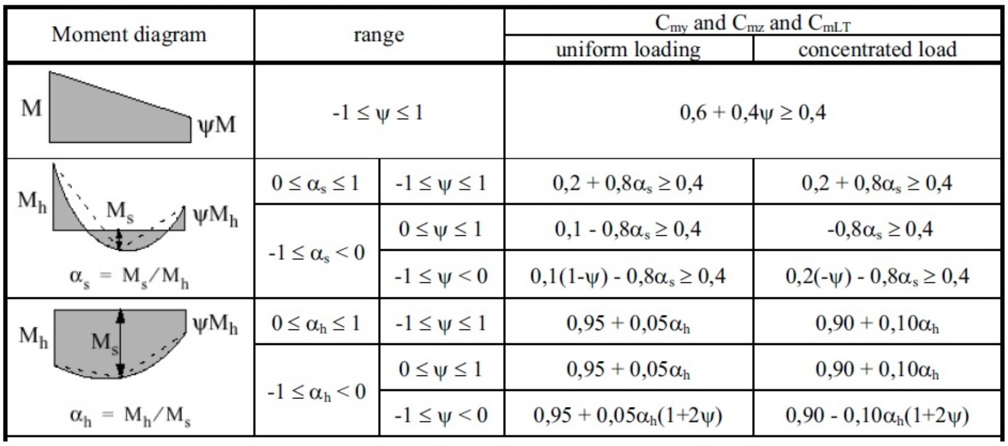
$$k_{\text{zz}} = C_{\text{mz}} \left(1 + 2\left(\overline{\lambda}_{\text{z}} - 0.6\right) \frac{N_{\text{Ed}}}{\chi_{\text{z}}\, f_{\text{y}}\, A/\gamma_{\text{M1}}}\right) \leq C_{\text{mz}} \left(1 + 1.4 \frac{N_{\text{Ed}}}{\chi_{\text{z}}\, f_{\text{y}}\, A/\gamma_{\text{M1}}}\right) \tag{B.46}$$

$$k_{\text{yz}} = 0.6 \cdot k_{\text{zz}} \tag{B.47}$$

Moment factor	Bending axis	Points braced in direction
C_{my}	*y-y*	*z-z*
C_{mz}	*z-z*	*y-y*

Table B.6: Moment factors.

The moment factors are depending on the shape of the bending moment about the bending axis. These can be seen in Figure B.6. It should also be stated that as there are no moments about the z axis, then $C_{\text{mz}} = 0$.



Moment diagram	range		C_{my} and C_{mz} and C_{mLT} uniform loading	concentrated load
M … ψM	$-1 \le \psi \le 1$		$0,6 + 0,4\psi \ge 0,4$	
M_h, M_s, ψM_h	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0,2 + 0,8\alpha_s \ge 0,4$	$0,2 + 0,8\alpha_s \ge 0,4$
	$-1 \le \alpha_s < 0$	$0 \le \psi \le 1$	$0,1 - 0,8\alpha_s \ge 0,4$	$-0,8\alpha_s \ge 0,4$
$\alpha_s = M_s/M_h$		$-1 \le \psi < 0$	$0,1(1-\psi) - 0,8\alpha_s \ge 0,4$	$0,2(-\psi) - 0,8\alpha_s \ge 0,4$
M_h, M_s, ψM_h	$0 \le \alpha_h \le 1$	$-1 \le \psi \le 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
	$-1 \le \alpha_h < 0$	$0 \le \psi \le 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
$\alpha_h = M_h/M_s$		$-1 \le \psi < 0$	$0,95 + 0,05\alpha_h(1+2\psi)$	$0,90 - 0,10\alpha_h(1+2\psi)$

Figure B.6: Equivalent uniform moment factors, C_m [European Standard, 2005].

As for the moment of element 3, it will have more or less the shape of a triangle, as the wind is small and will not give a large contribution to the moment distribution. The triangular moment diagram will have the greatest moment on top, ($M_s = 309.89$ kNm), and a moment of zero at the supports. This means that the moment diagram shown at the top of Figure B.6 will give the approximate moment distribution as seen in element 3. However, for the moment about the y axis, the effective length, l_s, is twice the length of the original length of element 3. The question then becomes if the moment distribution should be according to the effective length of the element, see Figure B.7.

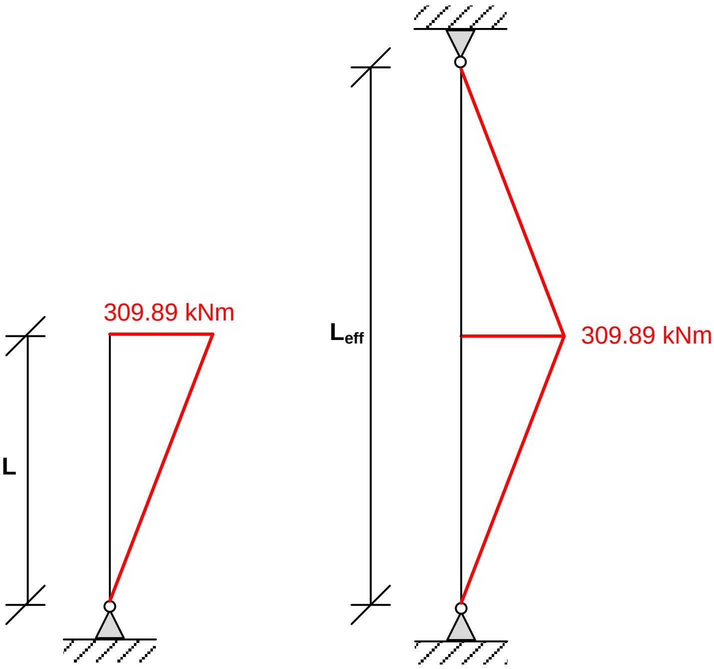


Figure B.7: Different moment distribution according to original length and effective length of element 3.

It is in this case considered that the moment will be distributed for the effective length about the y axis. This means that the moment used to calculate C_{my} is shown at the bottom of Figure B.6. In this moment diagram, $M_h = 0$ and $M_s = 309.89$ kNm. The calculation of α_h will be zero. Then

C_{my} can be calculated as shown in Eq. (B.48).

$$0.95 + 0.05\,\alpha_h \tag{B.48}$$

The moment multiplier, ψ, is equal to zero in this case, which means that C_{my} = 0.95. The interaction factors can then be calculated by using Eq. (B.44), (B.45), (B.46) and (B.47).

$$\begin{aligned}
k_{yy} &= 0.984 \leq 0.997 \Rightarrow OK \\
k_{zy} &= 0.6 \cdot 0.984 \\
k_{zz} &= 0 \leq 0 \Rightarrow OK \\
k_{yz} &= 0.6 \cdot 0 = 0
\end{aligned} \tag{B.49}$$

B.2.6 Interaction Formulae EC-Verification of Element 3

As stated in the beginning of this appendix, the following equation must be verified from the EC, see Eq. (B.50) and (B.51).

$$\frac{N_{Ed}}{\frac{\chi_y f_y A_i}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{f_y W_i}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{f_y W_i}{\gamma_{M1}}} \leq 1 \tag{B.50}$$

$$\frac{N_{Ed}}{\frac{\chi_z f_y A_i}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{f_y W_i}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{f_y W_i}{\gamma_{M1}}} \leq 1 \tag{B.51}$$

In addition to the properties of the C-S, see Table B.2, the previous calculations have the following results that is used in EC Interaction Formulae, see Table B.7.

Factor	Value	Unit
N_{Ed}	$125.38 \cdot 10^3$	[N]
$M_{y,Ed}$	$309.89 \cdot 10^6$	[Nmm]
$M_{z,Ed}$	0	[Nmm]
$\Delta M_{y,Ed}$	0	[Nmm]
χ_y	0.447	[-]
χ_z	0.73	[-]
χ_{LT}	1.0	[-]
k_{yy}	0.984	[-]
k_{yz}	0	[-]
k_{zz}	0	[-]
k_{zy}	0.59	[-]

Table B.7: Summarize of the results for values in EC Interaction Formulae.

With every value inserted into Eq. (B.50) and (B.51), each part of the equation is calculated. The results of the different parts are as follows:

$$0.1217 + 0.972 + 0 = 1.09 \leq 1 \Rightarrow NOT\,OK$$
$$0.0745 + 0.5832 + 0 = 0.66 \leq 1 \Rightarrow OK$$

This means that the utilization ratio about the y axis of element 3 is above 100%. In reality, a new profile would be chosen, and the whole procedure would start all over again to gain a utilization ratio less than 100%. However, for this project, the utilization ratio will be compared with the utilization ratio given for the General Method, see Chapter 6 .

B.3 Check of HE320A for element 2

The analysed element of the frame is element 2, which can be seen in Chapter 2.

B.3.1 Forces and Moments

The compression force, N_{Ed}, of element 2 is taken from Autodesk Robot, where it can be seen that $N_{\mathrm{Ed}} = 60.56$ kN. The moments acting on this element will be the same as element 3 $\left(M_{\mathrm{y,Ed}} = 309.89 \text{ kNm}, M_{\mathrm{z,Ed}} = \Delta M_{\mathrm{y,Ed}} = \Delta M_{\mathrm{z,Ed}} = 0 \text{ kNm}\right)$.

B.3.2 Cross-Section Checks

Compression

The design resistance of the C-S for the uniform compression force,$N_{\mathrm{c,Rd}}$, should satisfy Eq. (B.52).

$$\frac{N_{\mathrm{Ed}}}{N_{\mathrm{c,Rd}}} \leq 1.0 \tag{B.52}$$

$$\begin{aligned} N_{\mathrm{c,Rd}} &= \frac{A\, f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \\ &= \frac{11774 \text{ mm}^2 \cdot 235 \text{ N/mm}^2}{1.1} \\ &= 2515.35 \cdot 10^3 \text{ N} \end{aligned} \tag{B.53}$$

The C-S check will then be shown in Eq. (B.54).

$$\frac{60.56 \text{ kN}}{2515.35 \text{ kN}} = 0.024 \leq 1.0 \tag{B.54}$$

Bending Moment

The design value of the bending moment,M_{Ed}, at each C-S should satisfy Eq. (B.55).

$$\frac{M_{\mathrm{Ed}}}{M_{\mathrm{c,Rd}}} \leq 1.0 \tag{B.55}$$

The design resistance from bending, $M_{\mathrm{c,Rd}}$, is calculated in Eq. (B.56).

$$M_{\mathrm{c,Rd}} = \frac{W_{\mathrm{pl}}\, f_{\mathrm{y}}}{\gamma_{\mathrm{M0}}} \tag{B.56}$$

$$\begin{aligned} &= \frac{1628 \cdot 10^3 \text{ mm}^3 \cdot 235 \text{ N/mm}^2}{1.1} \\ &= 347.8 \text{ kNm} \end{aligned} \tag{B.57}$$

M_{Ed} is equal to $M_{\mathrm{y,Ed}}$ = 309.89 kNm. This means that the C-S check will be as shown in Eq. (B.58).

$$\frac{309.89 \text{ kNm}}{347.8 \text{ kNm}} = 0.89 \leq 1.0 \tag{B.58}$$

Shear

The design value of the shear force,V_{Ed}, at each C-S should satisfy Eq. (B.59).

$$\frac{V_{\mathrm{Ed}}}{V_{\mathrm{c,Rd}}} \leq 1.0 \tag{B.59}$$

where $V_{\mathrm{c,Rd}}$ is the design shear resistance. As HE320A is classified as C-S class 1, the verification of the shear force should be plastic. In addition, it is so that as there is no torsion in the frame, hence the plastic shear resistance is given as shown in Eq. (B.60).

$$V_{\mathrm{pl,Rd}} = \frac{A_{\mathrm{v}}\left(f_{\mathrm{y}}/\sqrt{3}\right)}{\gamma_{\mathrm{M0}}} \tag{B.60}$$

A_{v} is the shear area and will be taken as the following for sections with load parallel to web as seen in Eq. (B.61).

$$\begin{aligned} A_{\mathrm{v}} &= A - 2\,b\,t_{\mathrm{f}} + (t_{\mathrm{w}} \cdot t_{\mathrm{f}}) \\ &= (11.77 \cdot 10^3 \text{ mm}) - 2 \cdot 300 \text{ mm} \cdot 15.5 \text{ mm} + (9 \text{ mm} \cdot 15.5 \text{ mm}) \\ &= 2610 \text{ mm}^2 \end{aligned} \tag{B.61}$$

It should also be noticed that this equation is based on a profile without any welds of the section, see Section B.1. In addition, A_{v} should not be less than what is shown in Eq. (B.62).

$$\begin{aligned} &\eta\ h_{\mathrm{w}}\ t_{\mathrm{w}} \\ &\eta h_{\mathrm{w}} t_{\mathrm{w}} = 1.0 \cdot 300 \text{ mm} \cdot 9 \text{ mm} = 2700 \text{ mm}^2 \end{aligned} \tag{B.62}$$

The factor for shear area, η, may conservatively be taken equal to 1.0 as stated in European Standard [2005]. Thereby, A_{v} is taken as 2700 mm^2. Now, $V_{\mathrm{pl,Rd}}$ can be calculated as shown in Eq. (B.63).

$$\begin{aligned} V_{\mathrm{pl,Rd}} &= \frac{2700 \text{ mm}^2 \left(235 \text{ N/mm}^2/\sqrt{3}\right)}{1.1} \\ &= 524.38 \cdot 10^3 \text{ N} \end{aligned} \tag{B.63}$$

The C-S check can then be made, and the shear force is found from Autodesk Robot to be 105.58 kN.

$$\frac{105.58 \text{ kN}}{524.38 \text{ kN}} = 0.20 \leq 1.0$$

Linear summation of the utilization ratios:

According to Eurocode 3 [European Standard, 2005], the following is stated about general resistance of cross-sections. "As a conservative approximation for all cross-sectional classes a linear summation of the utilization ratios for each stress resultant may be used". For cross-section classes 1, 2 and 3 the following criteria must be verified, see Eq. (B.64).

$$\frac{N_{\mathrm{Ed}}}{N_{\mathrm{Rd}}} + \frac{M_{\mathrm{y,Ed}}}{M_{\mathrm{y,Rd}}} + \frac{M_{\mathrm{z,Ed}}}{M_{\mathrm{z,Rd}}} \leq 1.0 \tag{B.64}$$

By inserting the values previously calculated into Eq.(B.64), the verification can be done as shown in Eq. (B.65).

$$\begin{aligned} \frac{60.56 \text{ kN}}{2515.35 \text{ kN}} + \frac{309.89 \text{ kNm}}{347.8 \text{ kNm}} + 0 \leq 1.0 \\ 0.914 \leq 1.0 \Rightarrow \text{OK} \end{aligned} \tag{B.65}$$

B.3.3 Buckling Curves

The reduction factor for the relevant buckling curve χ is calculated in Eq. (B.66).

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \overline{\lambda}^2}} \quad , \quad \chi \leq 1 \tag{B.66}$$

The value, Φ, to determine χ is calculated in Eq. (B.67).

$$\Phi = 0.5 \, \left(1 + \alpha \, \left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right) \tag{B.67}$$

α is an imperfection factor and is determined by a buckling curve that is defined by the profile used. For HE320A the relationship between the height and the width of the section is calculated as shown in Eq. (B.68).

$$\frac{h}{b} = \frac{310 \text{ mm}}{300 \text{ mm}} = 1.03 \tag{B.68}$$

As the relationship is 1.03, the web thickness is equal to 15.5 mm and the section is assumed to be a welded section, the buckling curves will be according to Table B.8.

Buckling about axis	Buckling curve for S235	Imperfection factor α
y-y	*b*	0.34
z-z	*c*	0.49

Table B.8: Imperfection factors based on buckling curves.

The non-dimensional slenderness, $\overline{\lambda}$ is calculated as shown in Eq. (B.69).

$$\overline{\lambda} = \sqrt{\frac{A \, f_y}{N_{cr}}} \tag{B.69}$$

N_{cr} is the elastic critical force for the relevant buckling mode based on a the gross cross-sectional properties. N_{cr} is calculated as shown in Eq. (B.70).

$$N_{cr} = \frac{\pi^2 \, E \, I}{l_s^2} \tag{B.70}$$

Reduction factor, χ_y, for the *y* axis:

l_s is the effective length of element 2. From Force-Euro, the following equation is used to calculate the effective length of element 2, see Eq. (B.71). This equation only applies for the effective length about the strong axis.

$$l_s = l_{s,elem.3} \sqrt{\frac{N_{Ed,\ elem.3}}{N_{Ed,\ elem.2}}} \tag{B.71}$$

$l_{s,elem.3}$	Effective length of element 3	[mm]
$N_{Ed,\ elem.3}$	Internal normal force of element 3	[mm]
$N_{Ed,\ elem.2}$	Internal normal force of element 2	[mm]

The effective length, l_s is then calculated as shown in Eq. (B.72).

$$l_s = 16120 \text{ mm} \sqrt{\frac{125.38 \text{ kN}}{60.56 \text{ kN}}} = 23194.57 \text{ mm} \tag{B.72}$$

The effective length has been inserted into the MATLAB-file and the results can be seen in Table B.9.

Effective length, l_s [mm]	$\overline{\lambda}$	N_{cr} [N]	χ_y [-]
23194.57	1.81	$840.31 \cdot 10^3$	0.249

Table B.9: Results for element 2 about the *y* axis.

$\underline{\chi_z}$

The support conditions would in reality most likely be between fixed and pinned supports. It is however, difficult to determine exactly what the value of effective length should be. Therefore, the worst case is chosen. Hence, the support conditions are assumed to be pinned in both ends giving element ∘ an effective length of l_s = 20000mm. The effective length, l_s has been inserted into the MATLAB-file and the results can be seen in Table B.10.

Type of support	Effective length, l_s [mm]	$\overline{\lambda}$	N_{cr} [kN]	χ_z [-]
Between fixed and pinned	20000	2.77	$361.67 \cdot 10^3$	0.11

Table B.10: Results for element 2 about the *z* axis.

Checks of Buckling Effects

There are two different checks which can be performed to check if buckling effects may be ignored and that only cross-sectional checks will apply. The checks are:

1. $\overline{\lambda} \leq 0.2$
2. $\frac{N_{Ed}}{N_{cr}} \leq 0.04$

For the reduction factor, χ_y:

1. $\overline{\lambda} = 1.81 \leq 0.2 \Rightarrow$ NOT OK
2. $\frac{N_{Ed}}{N_{cr}} = \frac{60.56 \cdot 10^3 \text{ N}}{840.31 \cdot 10^3 \text{ N}} = 0.0072 \leq 0.04 \Rightarrow$ OK

For the reduction factor, χ_z:

1. $\overline{\lambda} = 2.77 \leq 0.2 \Rightarrow$ NOT OK
2. $\frac{N_{Ed}}{N_{cr}} = \frac{60.56 \cdot 10^3 \text{ N}}{361.67 \cdot 10^3 \text{ N}} = 0.17 \leq 0.04 \Rightarrow$ NOT OK

Based on these results it can be seen that the buckling effects may not be ignored for this section due to the slenderness and the compression force of the element.

In addition to the checks just made, the European Standard [2005] states another check needed for a compressed element. It states that the element should be verified against buckling shown in Eq. (B.73).

$$\frac{N_{\mathrm{Ed}}}{N_{\mathrm{b,Rd}}} \leq 1.0 \tag{B.73}$$

The buckling resistance, $N_{\mathrm{b,Rd}}$, for the compressed element is calculated in Eq. (B.74).

$$N_{\mathrm{b,Rd}} = \frac{\chi\, A\, f_{\mathrm{y}}}{\gamma_{\mathrm{M1}}} = \frac{0.11 \cdot 11.77 \cdot 10^3\ \mathrm{mm}^2 \cdot 235\ \mathrm{N/mm}^2}{1.2} = 253.5 \cdot 10^3\ \mathrm{N} \tag{B.74}$$

The reduction factor, χ, is taken from the most unfavorable axis, hence χ_{z} is used.

The relationship between the compression force, N_{Ed} and buckling resistance, $N_{\mathrm{b,Rd}}$, will then be as shown in Eq. (B.75).

$$\frac{N_{\mathrm{Ed}}}{N_{\mathrm{b,Rd}}} = \frac{60.56 \cdot 10^3\ \mathrm{N}}{253.5 \cdot 10^3\ \mathrm{N}} = 0.24 \leq 1.0 \Rightarrow OK \tag{B.75}$$

B.3.4 Lateral Torsional Buckling Curves for Rolled and Equivalent Sections

The EC gives a general case when it comes to lateral torsional buckling curves and an option for rolled or equivalent sections. It is assumed that the element has rolled or equivalent sections, and the last option of the lateral torsional buckling is chosen.

The reduction factor, χ_{LT}, for lateral-torsional buckling is determined in Eq. (B.76).

$$\chi_{\mathrm{LT}} = \frac{1}{\Phi_{\mathrm{LT}} + \sqrt{\Phi_{\mathrm{LT}}^2 - \beta\, \overline{\lambda}_{\mathrm{LT}}^2}} \quad \text{but} \quad \begin{cases} \chi_{\mathrm{LT}} \leq 1.0 \\ \chi_{\mathrm{LT}} \leq \frac{1}{\overline{\lambda}_{\mathrm{LT}}^2} \end{cases} \tag{B.76}$$

According to European Standard [2005], the following values are recommended for rolled sections:

$$\begin{aligned} \overline{\lambda}_{\mathrm{LT,0}} &= 0.4 \quad \text{(Maximum value)} \\ \beta &= 0.75 \quad \text{(Minimum value)} \end{aligned}$$

The value, Φ_{LT}, to determine the reduction factor, χ_{LT}, for lateral torsional buckling is calculated in Eq. (B.77).

$$\Phi_{\mathrm{LT}} = 0.5\left(1 + \alpha_{\mathrm{LT}}\left(\overline{\lambda}_{\mathrm{LT}} - \overline{\lambda}_{\mathrm{LT,0}}\right) + \beta\, \overline{\lambda}_{\mathrm{LT}}^2\right) \tag{B.77}$$

The imperfection factor, α_{LT}, for lateral torsional buckling is found from the buckling curve. For HE320A the h/b relationship is as follows, see Eq. (B.78).

$$h/b = 310\ \mathrm{mm}/300\ \mathrm{mm} = 1.03 \leq 2 \Rightarrow \text{Buckling curve } b \tag{B.78}$$

Buckling curve b gives $\alpha_{\mathrm{LT}} = 0.34$.

Then the non-dimensional slenderness, $\overline{\lambda}_{\mathrm{LT}}$, can be calculated as shown in Eq. (B.79).

$$\overline{\lambda}_{\mathrm{LT}} = \sqrt{\frac{W_{\mathrm{pl,y}}\, f_{\mathrm{y}}}{M_{\mathrm{cr}}}} \tag{B.79}$$

The critical moment, M_{cr}, is calculated by the following general expression, see Eq.(B.80).

$$M_{\text{cr}} = m_{\text{n}} \frac{EI_{\text{z}}}{l^2} h_{\text{t}} \tag{B.80}$$

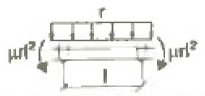
m_{n}	A value given by a table for the case investigated	[-]
h_{t}	Height of C-S from the middle of top flange to middle of bottom flange	[mm]

In order to determine a value for m_{n}, the relationship shown in Eq. (B.81) must be calculated.

$$kl = \sqrt{\frac{G\, I_{\text{v}}\, l^2}{E\, I_{\text{w}}}} \tag{B.81}$$

$$kl = \sqrt{\frac{8.1 \cdot 10^4\ \text{N/mm}^2 \cdot 1080 \cdot 10^3\ \text{mm}^4 20000\ \text{mm}^2}{2.1 \cdot 10^5\ \text{N/mm}^2 \cdot 1510 \cdot 10^9\ \text{mm}^6}} = 10.50$$

kl is an input parameter that is used in different tables. The different tables reflect different loads and boundary condition of the member analyzed. The table used for element 2 is m_5, and the table can be seen in Figure B.8. For element 2 there is a different moment acting on each end of the element. In order to make the calculation, the moments have to be symmetric. Therefore, m_5 will be calculated with both the highest and lowest end moment calculated for element 2, respectively'.



m_5

μ \ kl	0	1	2	3	4	6	8	10	15	20
0	28,7	30,7	36,3	44,6	54,7	78,0	103	130	198	267
1/24	36,1	38,9	47,0	59,1	74,1	110	149	190	296	405
1/16	40,5	44,0	53,9	69,1	88,4	135	187	243	389	539
1/12	44,7	48,8	60,9	80,1	105	169	244	326	546	777
1/8	46,0	50,4	63,3	84,0	112	182	268	361	610	865

Figure B.8: Table 5 for an Eulerload [Mohr and et al., 2009].

Before calculating m_5 for the moments, μ is determined as a factor of the relationship between the moment and the line load, r, over the length of the element. The relationship is expressed in Eq. (B.82).

$$M = \mu\, r\, l^2 \tag{B.82}$$

By inserting the highest end moment of M_{Ed} = 309.89 kNm, r = 10.44 kN/m and l = 20 m, μ is calculated to be $0.074 \approx \frac{1}{13.48}$. For the other end moment of M_{Ed} = 286.31 kNm, inserting M_{Ed} = 286.31 kNm, r = 10.44 kN/m and l = 20 m, μ is calculated to be $0.0686 \approx \frac{1}{14.59}$. A linear interpolation is then made between the values found in Figure B.8, namely the *kl* between the two μ values, $\mu = \frac{1}{12}$ and $\mu = \frac{1}{16}$ for the first case in Figure B.8. The results of m_5 for the different moments can be seen in Table B.11.

End moments	m_5
$M_{Ed} = 309.89$ kNm	307.07
$M_{Ed} = 286.31$ kNm	284.67

Table B.11: The results for m_5 of different end moments found by linear interpolation.

The worst case will be for $m_5 = 284.67$ as this will give a smaller critical moment, M_{cr}. This means that the following calculations will only take $m_5 = 284.67$ into account.

$$M_{cr} = 284.67 \, \frac{2.1 \cdot 10^5 \text{ N/mm}^2 \cdot 69.8 \cdot 10^6 \text{ mm}^4}{(20000 \text{ mm})^2} \cdot 294.5 \text{ mm} = 3072.1 \cdot 10^6 \text{ Nmm}$$

The reduction factor, χ_{LT}, for lateral torsional buckling can then be calculated by the following equations, see Eq. (B.83), (B.84) and (B.85).

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_{pl,y}\, f_y}{M_{cr}}} = \sqrt{\frac{(1628 \cdot 10^3) \text{ mm}^3 \cdot 235 \text{ N/mm}^2}{3072.1 \cdot 10^6 \text{ Nmm}}} = 0.35 \tag{B.83}$$

$$\Phi_{LT} = 0.5 \, \left(1 + 0.34 \, (0.35 - 0.4) + 0.75 \cdot 0.35^2\right) = 0.52 \tag{B.84}$$

$$\chi_{LT} = \frac{1}{0.52 + \sqrt{0.52^2 - (0.75 \cdot 0.35^2)}} = 1.05 \tag{B.85}$$

The checks can then be done shown in Eq. (B.86) and (B.87).

$$\chi_{LT} \leq 1.0 \Rightarrow \text{ NOT OK} \tag{B.86}$$

$$\chi_{LT} \leq \frac{1}{\overline{\lambda}_{LT}^2} = \frac{1}{0.35^2} = 8.16 \Rightarrow \text{OK} \tag{B.87}$$

Since $\chi_{LT} \geq 1.0$, χ_{LT} is set equal to 1.

B.3.4.1 Check of Lateral Torsional Buckling

In some cases, lateral torsional buckling effects may be ignored and only cross-sectional checks will therefore apply. These cases are when:

$$\overline{\lambda}_{LT} \leq \overline{\lambda}_{LT,0}$$
$$\frac{M_{Ed}}{M_{cr}} \leq \overline{\lambda}_{LT,0}^2$$

For HE320A with its loads applied, the checks can be calculated.

$$0.35 \leq 0.4 \Rightarrow \text{ OK}$$
$$\frac{309.89 \cdot 10^6 \text{ Nmm}}{3072.1 \cdot 10^6 \text{ Nmm}} = 0.10 \leq 0.4^2 = 0.16 \Rightarrow \text{ OK}$$

Based on these results, it can be seen that lateral torsional buckling effects may be ignored and only C-S checks will apply.

In addition to the previous checks, another check of the buckling resistance must be done. It is stated in European Standard [2005] that "A laterally unrestrained member subjected to major axis bending should be verified against lateral torsional buckling". This is done in Eq. (B.88).

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1.0 \tag{B.88}$$

The design resistance for bending, $M_{b,Rd}$, may be calculated as seen in Eq. (B.89).

$$M_{b,Rd} = \chi_{LT}\, W_{pl}\, \frac{f_y}{\gamma_{M1}} = 1.0 \cdot 1628 \cdot 10^3\ \text{mm}^3 \frac{235\ \text{N/mm}^2}{1.2} = 318.82 \cdot 10^6\ \text{Nmm} \tag{B.89}$$

$$\frac{M_{Ed}}{M_{b,Rd}} = \frac{309.89 \cdot 10^6\ \text{Nmm}}{318.82 \cdot 10^6\ \text{Nmm}} = 0.97 \leq 1.0 \Rightarrow\ \text{OK}$$

B.3.5 Interaction Factors

To calculate the interaction factors for element (2), the moment diagram in the middle of Figure B.6 is used. The α_s factor can be calculated based on the moments illustrated in Figure B.9.

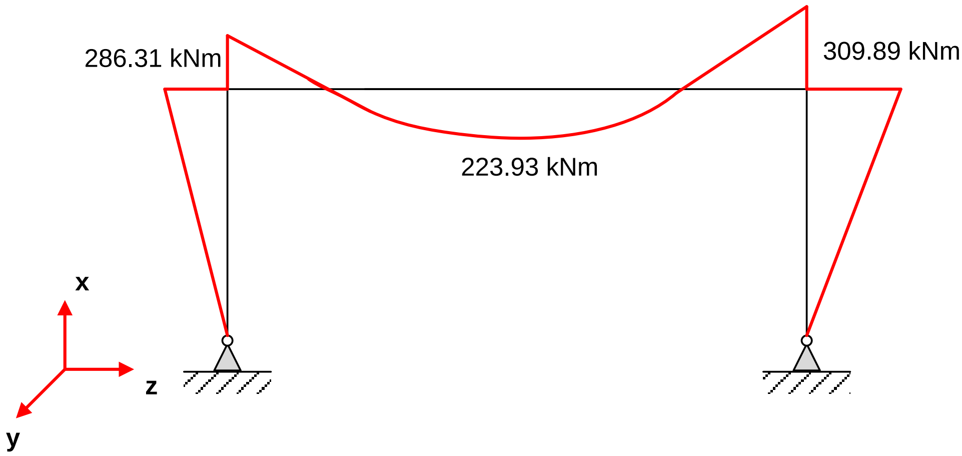


Figure B.9: Illustration of the moments of the reference frame.

The α_s value is calculated in Eq. (B.90).

$$\alpha_s = \frac{M_s}{M_h} = \frac{223.93\ \text{kNm}}{-309.89\ \text{kNm}} = -0.72 \tag{B.90}$$

M_s is the moment in the middle of element (2), and M_h is the highest moment occurring on element (2). The result of α_s means that $-1 \leq \alpha_s \leq 0$. The reduction factor of the moment diagram, ψ, can then be calculated as shown in Eq. (B.91).

$$\psi = \frac{-286.31\ \text{kNm}}{-309.89\ \text{kNm}} = 0.92 \tag{B.91}$$

As ψ has the value of 0.92, the following equation, Eq. (B.92), should be used to calculate C_{my}, as C_{mz} will be zero due to no moment about the z-axis.

$$0.1 - 0.8 \cdot \alpha_s \geq 0.4 \tag{B.92}$$
$$0.1 - 0.8 \cdot (-0.72) = 0.676 \geq 0.4 \Rightarrow OK$$

This means that 0.676 is the value replacing C_{my} in the equations of the interaction factors. This gives the following interaction factors:

$$k_{yy} = 0.0.791$$
$$k_{yz} = 0.0$$
$$k_{zy} = 0.475$$
$$k_{zz} = 0.0$$

B.3.6 Interaction Formulae EC-Verification of Element 2

Element 2 must be verified by the equations shown in Eq. (B.93) and (B.94).

$$\frac{N_{\mathrm{Ed}}}{\frac{\chi_{\mathrm{y}}\, f_{\mathrm{y}} A_{\mathrm{i}}}{\gamma_{\mathrm{M1}}}} + k_{\mathrm{yy}} \frac{M_{\mathrm{y,Ed}} + \Delta M_{\mathrm{y,Ed}}}{\chi_{\mathrm{LT}} \frac{f_{\mathrm{y}} W_{\mathrm{i}}}{\gamma_{\mathrm{M1}}}} + k_{\mathrm{yz}} \frac{M_{\mathrm{z,Ed}} + \Delta M_{\mathrm{z,Ed}}}{\frac{f_{\mathrm{y}} W_{\mathrm{i}}}{\gamma_{\mathrm{M1}}}} \leq 1 \tag{B.93}$$

$$\frac{N_{\mathrm{Ed}}}{\frac{\chi_{\mathrm{z}}\, f_{\mathrm{y}} A_{\mathrm{i}}}{\gamma_{\mathrm{M1}}}} + k_{\mathrm{zy}} \frac{M_{\mathrm{y,Ed}} + \Delta M_{\mathrm{y,Ed}}}{\chi_{\mathrm{LT}} \frac{f_{\mathrm{y}} W_{\mathrm{i}}}{\gamma_{\mathrm{M1}}}} + k_{\mathrm{zz}} \frac{M_{\mathrm{z,Ed}} + \Delta M_{\mathrm{z,Ed}}}{\frac{f_{\mathrm{y}} W_{\mathrm{i}}}{\gamma_{\mathrm{M1}}}} \leq 1 \tag{B.94}$$

Table B.12 gives a summary of the values calculated for the beam by Interaction Formulae.

Factor	Value	Unit
N_{Ed}	60.56	[kN]
M_{Ed}	309.89	[kNm]
$N_{\mathrm{cr,y}}$	840.31	[kN]
$N_{\mathrm{cr,z}}$	361.67	[kN]
M_{cr}	104.22	[kNm]
χ_{y}	0.249	[-]
χ_{z}	0.11	[-]
χ_{LT}	1.0	[-]
k_{yy}	0.791	[-]
k_{yz}	0.0	[-]
k_{zy}	0.475	[-]
k_{zz}	0.0	[-]

Table B.12: The results for element 2 with HE320A for Interaction Formulae.

With every value inserted into the equation, the utilization ratio for each part of the expressions for Interaction Formulae is calculated, see Eq. (B.95) and (B.96).

$$0.1057 + 0.7692 + 0 = 0.87 \leq 1 \Rightarrow \mathrm{OK} \tag{B.95}$$

$$0.2381 + 0.4615 + 0 = 0.70 \leq 1 \Rightarrow \mathrm{OK} \tag{B.96}$$

As seen from the results of Interaction Formulae for element 2, the profile has a utilization ratio less than 100%, which means that the profile HE320A may be used for this element.

C Finite Element Method (FEM)

The following appendix is based on Cook and et al. [2002] and describes the *Finite Element Method* (FEM) which is a commonly used numerical method for analysing complicated engineering problems. The method is used to solve partial differential equations approximately. The structure, which is to be analysed, is divided into smaller parts, so-called *finite elements*. Approximate solutions to the differential equations describing each of the finite elements are determined and combined in relation to each other. The behaviour of each finite element is described by simple polynomial terms, while the actual behaviour of the entire structure is more complicated. The error made due to the approximation can be reduced by increasing the number of elements and thereby making a finer mesh, which is a particular arrangement of the elements.

The model on which the *Finite Element Method* (FEM) is applied to is an approximation too since it is not the actual physical model that is analysed. Assumptions concerning the geometry, material properties, loads and boundary conditions are made, and these are based on the features which are important and less important in obtaining the desired outcomes.

The advantages of using the *Finite Element Method* (FEM) compared to most other numerical analysis methods concerning versatility and physical appeal, are that there is no geometrical restriction which means that the shape of the structure analysed can be arbitrary. Nor the boundary conditions, the loading or the material properties are restricted, which makes it possible to support any part of the structure while applying distributed or concentrated forces to any other part and to change the material properties of each element and even within each element.

The analysis performed by the *Finite Element Method* (FEM) contains the following three steps:

- *Preprocessing*
- *Simulation*
- *Postprocessing*

The *Preprocessing* step deals with the input of the data for model. The input data describes the geometry, material properties, loads and boundary conditions of the structure. The software used for the *Finite Element Method* (FEM) generates the mesh of the model automatically, but during the *Preprocessing* step, the type and density of the finite elements must be chosen to fit the model best in all regions of it.

The actual analysis of the model is conducted at the step named *Simulation*. During this step, matrices describing the behaviour of each element are automatically generated by the *Finite Element* (FE) software. These matrices are combined to one matrix equation which describes the entire model. The time consumption for a computer performing the calculations during the *Simulation* step can vary from few seconds to weeks depending on how detailed the model is done.

In the *Postprocessing* step, the results of the analysis done by the *Finite Element Method* (FEM) are listed or shown graphically. It must be chosen in the software what results to list or display. Normally, the deformed shape of the model with exaggerated deformations is displayed and sometimes, the development of the deformations is animated. In addition, the stresses in various planes can be shown.

It is individually from one *finite element* (FE) software to another how these three steps are performed.

D Abaqus

Abaqus is a software package consisting of powerful engineering simulation programs developed by *SIMULIA*. These programs are based on the *Finite Element Method* (FEM), and they are able to do relavtively simple linear analyses to more challenging non-linear simulations. *Abaqus* has a comprehensive database of elements and material models which can model most geometries and simulate the behaviour of the most typical types of material used in engineering. The package of programs is designed as a tool to analyse not only structural problems but also engineering problems related to fluid dynamics, soil mechanics, thermal management, etc..

Generally, *Abaqus* is composed of three different products to do an analysis - a *Standard*, an *Explicit* and a *CFD* analysis. It is possible to extend the *Standard* and the *Explicit* analyses by different add-on analyses. The *Standard* analysis is applied in this project report, and the *Abaqus/CAE* (Complete Abaqus Environment) is used to setup and mesh a model. Using the *Abaqus/CAE*, the structure can be assigned physical and material properties together with loads and boundary conditions. It is possible to do the meshing of the model and a verification of the results in *Abaqus/CAE* by powerful options which are embedded in the software. When the analysis in *Abaqus/CAE* is accomplished, the analysed jobs can be submitted, monitored and controlled by the software, and an interpretation of the results can afterwards be shown in the associated *Visualization module* [SIMULIA, 2012].

E Imperfections of a Frame

Imperfections of a frame are described in Eurocode 3 [European Standard, 2005], and where relevant, these must be taken into account in the modelling of a frame. It can be done by either initially modelling the frame out of plumb or simpler, by using a system of equivalent horizontal forces (EHF).

As stated in Eurocode 3 [European Standard, 2005], "frames sensitive to buckling in a sway mode the effect of imperfections should be allowed for in frame analysis by means of an equivalent imperfection in the form of an initial sway imperfection and individual bow imperfection of members". The global initial sway imperfections are calculated according to Eq. (E.1) where the the initial imperfection, ϕ, is calculated.

$$\phi = \phi_0 \; \alpha_{\mathrm{h}} \; \alpha_{\mathrm{m}} \tag{E.1}$$

ϕ_0	Basic value: $\phi_0 = \frac{1}{200}$	[-]
α_{h}	Reduction factor for height h applicable to columns: $\alpha_{\mathrm{h}} = \frac{2}{\sqrt{h}}$ but $\frac{2}{3} \leq \alpha_{\mathrm{h}} \leq 1.0$	[-]
h	Height of the structure	[m]
α_{m}	Reduction factor for the number of columns in a row: $\alpha_{\mathrm{m}} = \sqrt{0.5 \; \left(1 + \frac{1}{m}\right)}$	[-]
m	Number of columns in a row - for a frame the number of columns in a single frame	[-]

The physical interpretation of the initial imperfection, ϕ, can be shown as an inclination from vertical in Figure E.1.

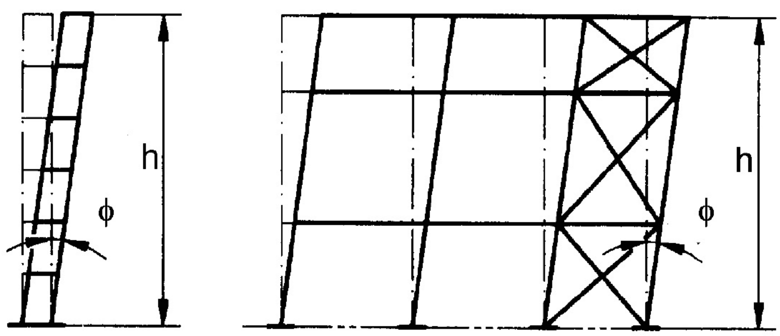


Figure E.1: Equivalent sway imperfections [European Standard, 2006].

The height of the structure, h, and the number of columns in a row, m, are for a single span frame equal to the height of the column and two columns, respectively.

The relative initial local bow imperfections is determined by Eq. (E.2).

$$\frac{e_0}{L} \tag{E.2}$$

e_0	Maximum amplitude of a member imperfection	[mm]
L	Length of the member	[mm]

According to Eurocode 3 [European Standard, 2005], recommended values for Eq. (E.2) is given in Table E.1 where the values recommended are based on buckling curves and wither the analysis is elastic or plastic.

Buckling curve acc. to Table 6.1	Elastic analysis e_0 / L	Plastic analysis e_0 / L
a_0	1/350	1/300
a	1/300	1/250
b	1/250	1/200
c	1/200	1/150
d	1/150	1/100

Table E.1: Design values of initial local bow imperfection e_0 / L

When implementing imperfections into the calculation, Eurocode 3 [European Standard, 2005] gives an option that is stated as the following. “The effects of initial sway imperfection and local bow imperfections may be replaced by systems of EHF, introduced for each column”. The EHF for both initial sway and local bow imperfections are illustrated in Figure E.2.

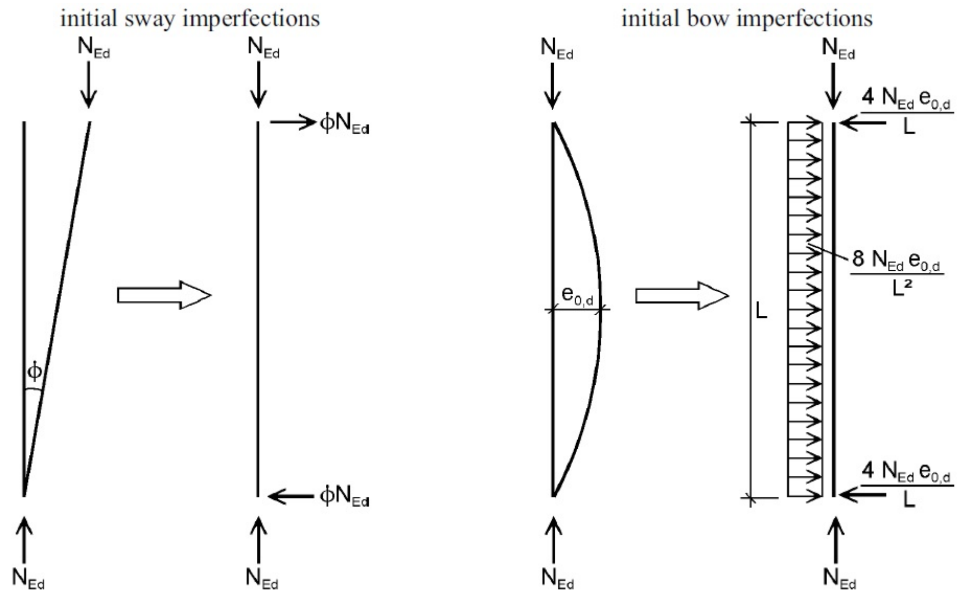


Figure E.2: Replacement of initial imperfections by equivalent horizontal forces [European Standard, 2006].

In this project, the EHF is applied to the reference frame where the following values of the global initial sway imperfections and initial local bow imperfections is displayed in Table E.2.

ϕ	Buckling curve	e_0 / L	L	e_0
0.0038537	b	1 / 200	5000 mm	25 mm

Table E.2: Values of imperfections applied to the reference frame.

F Bifurcation Buckling

The following appendix is based on Cook and et al. [2002]. Buckling is defined as the condition where the loads are sufficiently large to cause a loss of the stability of an equilibrium configuration without fracture or separation of the material. The buckling type called bifurcation buckling originates from the fundamental column theory where an axial compressive load with the size of the critical load, P_{cr}, results in that the straight pre-buckling configuration stops to be a stable state of equilibrium. Buckling without bifurcation can also occur which happens at a limit point where no alternative and infinitesimally close state of equilibrium is available. Buckling expressed by the limit point is non-linear whereas buckling with bifurcation is linear.

F.1 Linear Bifurcation Buckling

The following section contains a description of the analysis commonly performed on straight columns. Firstly, an arbitrary reference level of external load, $\{\mathbf{R}\}_{\text{ref}}$, is applied to the structure, and a standard linear analysis determining the element stresses is performed. The stiffness matrix for stresses related to the load, $\{\mathbf{R}\}_{\text{ref}}$, is consequently called $\{\mathbf{K}_\sigma\}_{\text{ref}}$. For some other load level, a scalar multiplier, λ, is applied as shown in Eq. (F.1).

$$[\mathbf{K}_\sigma] = \lambda\ [\mathbf{K}_\sigma]_{\text{ref}} \quad \text{when} \quad \{\mathbf{R}\} = \lambda\ \{\mathbf{R}\}_{\text{ref}} \tag{F.1}$$

$\{\mathbf{R}\}_{\text{ref}}$	Arbitrary reference level of external load	[-]
$\{\mathbf{R}\}$	Multiplication of all loads R_{i}	[-]
$[\mathbf{K}_\sigma]_{\text{ref}}$	Stress stiffness matrix	[-]
$[\mathbf{K}_\sigma]$	Multiplication of all stress stiffnesses $K_{\sigma,\text{i}}$	[-]
λ	Scalar multiplier	[-]

The two expressions in Eq. (F.1) show that a multiplication between a scalar multiplier, λ, and all loads, R_{i}, contained in $\{\mathbf{R}\}$ also applies for a multiplication between the same scalar multiplier, λ, and the intensity of the stress field, and this does not change the stress distribution.

The stiffness matrix, $[\mathbf{K}]$, is not changing by loading since the problem is assumed to be linear. Buckling displacements, $\{\delta\ \mathbf{D}\}$, relative to displacements of the reference configuration, $\{\mathbf{D}\}_{\text{ref}}$, are applied to the structure. The equation given in Eq. (F.2) are valid since external loads are not changing at a point of bifurcation.

$$\begin{aligned} ([\mathbf{K}] + \lambda_{\text{cr}}\ [\mathbf{K}_\sigma]_{\text{ref}})\ \{\mathbf{D}\}_{\text{ref}} &= \lambda_{\text{cr}}\ \{\mathbf{R}\}_{\text{ref}} \\ ([\mathbf{K}] + \lambda_{\text{cr}}\ [\mathbf{K}_\sigma]_{\text{ref}})\ \{\mathbf{D}_{\text{ref}} + \delta\ \mathbf{D}\} &= \lambda_{\text{cr}}\ \{\mathbf{R}\}_{\text{ref}} \end{aligned} \tag{F.2}$$

The two equations in Eq. (F.2) are subtracted from each other which results in the eigenvalue problem shown in Eq. (F.3).

$$([\mathbf{K}] + \lambda_{\text{cr}}\ [\mathbf{K}_\sigma]_{\text{ref}})\ \{\delta\ \mathbf{D}\} = \{\mathbf{0}\} \tag{F.3}$$

The lowest eigenvalue, λ_{cr}, is the smallest level of external load that results in bifurcation, and therefore, the expression in Eq. (F.4) is valid.

$$\{\mathbf{R}\}_{\text{cr}} = \lambda_{\text{cr}}\ \{\mathbf{R}\}_{\text{ref}} \tag{F.4}$$

The buckling mode is identified by the eigenvector, $\{\delta\ \mathbf{D}\}$, which is related to the lowest eigenvalue, λ_{cr}. The eigenvector, $\{\delta\ \mathbf{D}\}$, and thus also the buckling mode can be identified in *Abaqus/CAE*.

F.2 Non-Linear Buckling

Non-linear buckling is present when the prebuckling rotations are significant. This buckling problem can be solved by e.g. the Newton-Raphson method. A formation of a tangent-stiffness matrix, $[\mathbf{K}_\mathrm{t}]$, is thus possible, and this takes into account the effect of changing geometry and of stress stiffening. The calculation procedure is to solve Eq. (F.5) by using load increments, $\{\Delta\mathbf{R}\}$, where correction terms for the load and updates of the tangent-stiffness matrix, $[\mathbf{K}_\mathrm{t}]$, for each incremental step are incorporated.

$$[\mathbf{K}_\mathrm{t}]\ \{\Delta\mathbf{D}\} = \{\Delta\mathbf{R}\} \tag{F.5}$$

The displacement increments, $\{\Delta\mathbf{D}\}$, become very large when approaching a limit point. The tangent-stiffness matrix, $[\mathbf{K}_\mathrm{t}]$, will be singular at either a bifurcation or a limit point.

In general, non-linear calculations are composed of many small load steps, and the non-linear load-displacement curve is approximated by use of the *Element Method* as shown in Figure F.1. In each increment, a linear solution is established, and all these linear solutions give in total the approximated load-displacement curve. Therefore, the smaller increments, the closer to the real load-displacement curve.

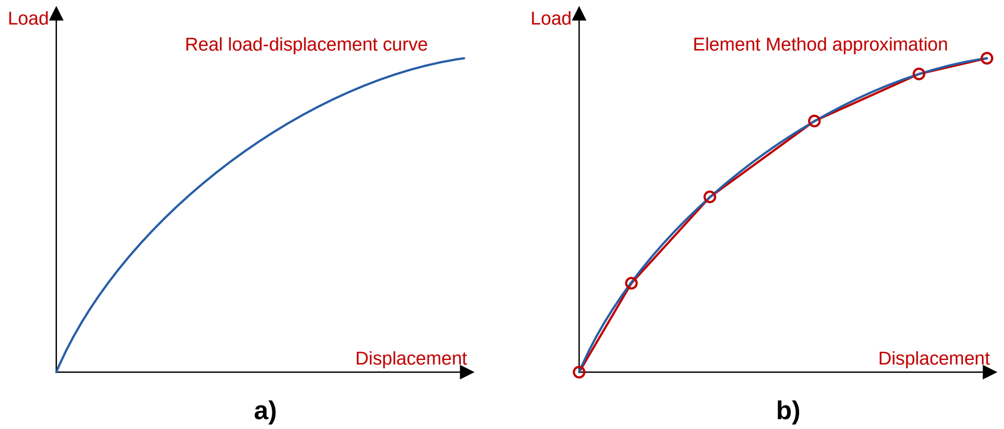


Figure F.1: Load-displacement curve - a) Real load-displacement curve; b) Element Method approximation

When bifurcation buckling is investigated, the calculations can be done in one single step because the load-displacement curve is linear. This step is the solution of the eigenvalue problem shown in Eq. (F.3).

The result is the instability load in a "perfect" world, cf. the Euler column, i.e. it is an upper bound solution for the bearing capacity. This value is included in several expressions in Eurocode, e.g. N_cr in Section 6.3.1 and M_cr in Section 6.3.2 in Eurocode 3 [European Standard, 2006]. It gives realistic results if displacements are small and linear up to the instability/buckling load.

If an Euler column is investigated for instability and the geometry and loads are symmetrical, no instability will occur in the calculations. Either the geometry or the loads have to be asymmetrical before calculations of instability can be performed. If the situation is that both the geometry and the loads are symmetrical, an asymmetrical contribution has to introduced and thereby, a forced instability. This can be done by introducing either a small asymmetrical load, a small eccentricity on the load or a small geometrical imperfection as shown in Figure F.2.

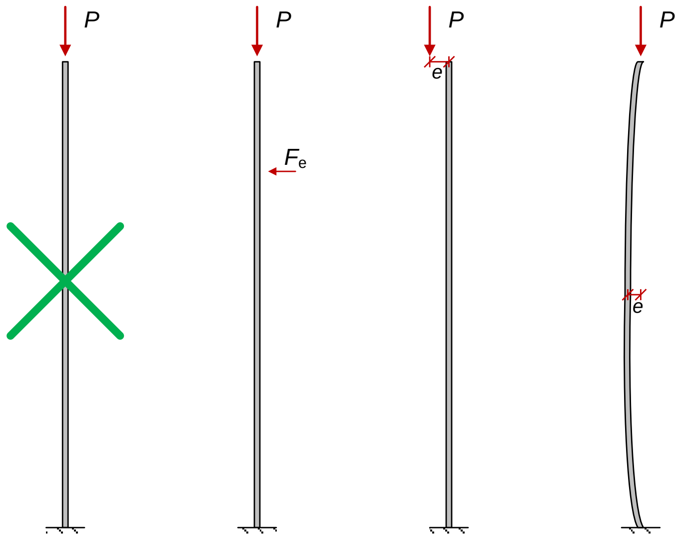


Figure F.2: Forced instability of Euler column.

It is of great importance to apply the smallest possible asymmetrical contribution to give the most realistic results. This is shown in Figure F.3. Here, the smallest eccentricity gives the most realistic results.

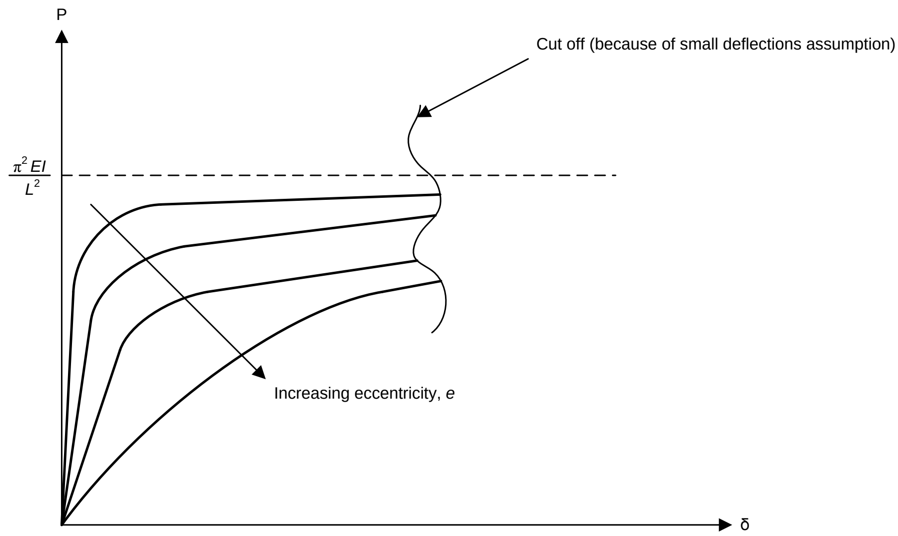


Figure F.3: P-δ diagram with eccentricity effect.

G Worked Example - General Method

This appendix contains a worked example by the General Method [European Standard, 2005] for the reference frame presented in Chapter 2. The minimum load amplifier, $\alpha_{\text{ult,k}}$, related to the in-plane behaviour of the frame is determined by the beam element model done in *Abaqus/CAE*. The result of this is further used in a combination with the other minimum load amplifier, $\alpha_{\text{cr,op}}$, related to the out-of-plane behaviour of the frame determined by an eigenvalue problem by the shell element model in *Abaqus/CAE*. Finally, the utilization ratio, UR, is determined by the inverse of the General Method.

G.1 Determination of the minimum load amplifier, $\alpha_{\text{ult,k}}$

The beam element model is used to determine the minimum load amplifier, $\alpha_{\text{ult,k}}$, for the in-plane behaviour of the reference frame. This is done by determining the relationship between the failure load, q_{max}, and the load actually applied, q_{actual}, to the reference frame as shown in Eq. (G.1).

$$\alpha_{\text{ult,k}} = \frac{q_{\text{max}}}{q_{\text{actual}}} \tag{G.1}$$

The load-displacement curve is then plotted as seen in Figure G.1 to determine the failure load, q_{max}, and the load actually applied, q_{actual}.

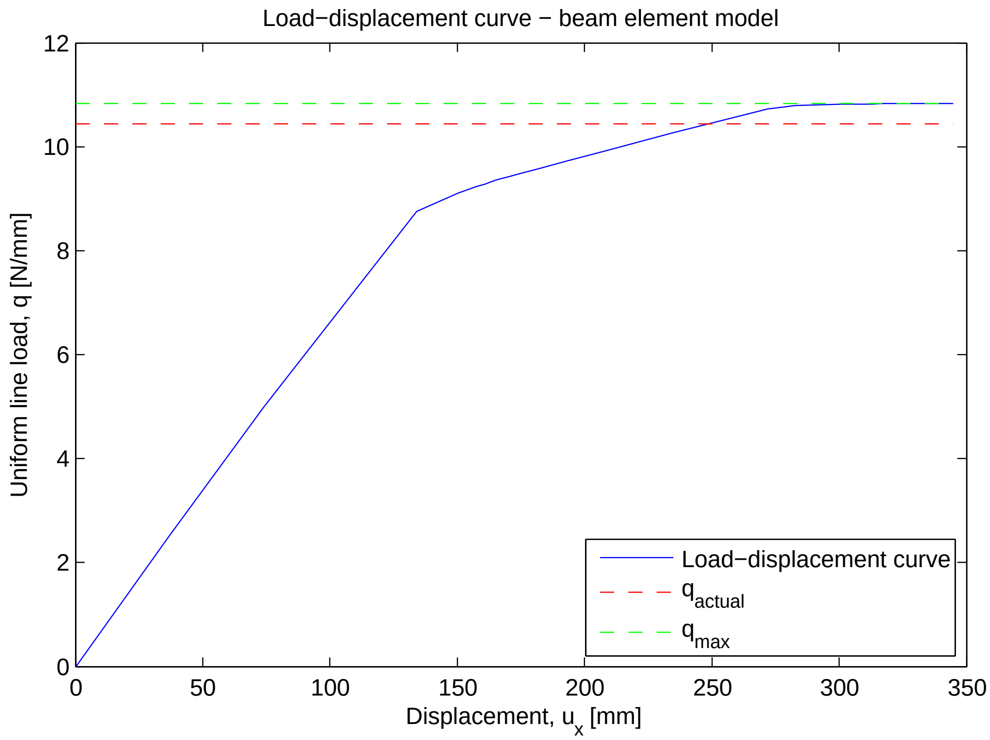


Figure G.1: Load-displacement curve for the beam element model.

The minimum load amplifier, $\alpha_{\text{ult,k}}$, is then determined in Eq. (G.2).

$$\alpha_{\text{ult,k,beam}} = \frac{10.84\ \text{N/mm}}{10.44\ \text{N/mm}} \tag{G.2}$$

$$\Updownarrow$$

$$\alpha_{\text{ult,k,beam}} = 1.0378 \tag{G.3}$$

The result of the determination of the minimum load amplifier, $\alpha_{\text{ult,k}}$, for the in-plane behaviour of the reference frame is shown in Table G.1.

Model used to determine $\alpha_{\text{ult,k}}$	$\alpha_{\text{ult,k}}$ [-]
Beam Element Model	1.0378

Table G.1: Minimum load amplifier, $\alpha_{\text{ult,k}}$, for in-plane behaviour determined by the beam element model.

G.2 Determination of the minimum load amplifier, $\alpha_{\text{cr,op}}$

This section describes how to determine the minimum load amplifier, $\alpha_{\text{cr,op}}$, related to the out-of-plane behaviour of the reference frame. This is done by solving the eigenvalue problem described in Appendix F and shown in Eq. (G.4) for the lowest eigenvalue, λ_{cr}, giving an out-of-plane buckling mode in the three-dimensional shell element model in *Abaqus/CAE*. The eigenvalue represents the minimum load amplifier, $\alpha_{\text{cr,op}}$, since the expression in Eq. (G.5) is valid and thereby, $\lambda_{\text{cr}} = \alpha_{\text{cr,op}}$.

$$([\mathbf{K}] + \lambda_{\text{cr}}\ [\mathbf{K}_\sigma]_{\text{ref}})\ \{\delta\ \mathbf{D}\} = \{\mathbf{0}\} \tag{G.4}$$

$$\alpha_{\text{cr,op}} = \frac{q_{\text{max}}}{q_{\text{actual}}} = \frac{\lambda_{\text{cr}}\ q_{\text{actual}}}{q_{\text{actual}}} \tag{G.5}$$

$[\mathbf{K}]$	Stiffness matrix	[-]
λ_{cr}	Eigenvalue - smallest level of external load for which there is bifurcation	[-]
$[\mathbf{K}_\sigma]_{\text{ref}}$	Stiffness matrix for stresses associated with load $\{\mathbf{R}\}_{\text{ref}}$	[-]
$\{\delta\ \mathbf{D}\}$	Eigenvector associated with λ_{cr} is the buckling mode (shown in Abaqus)	[-]

Abaqus/CAE is able to solve the eigenvalue problem in Eq. (G.4) and thereby show the lowest eigenvalue, λ_{cr}, giving an out-of-plane buckling mode. The lowest eigenvalue, λ_{cr}, is 2.8183 and thus is the minimum load amplifier, $\alpha_{\text{cr,op}} = 2.8183$ as well. It is the first buckling mode which gives this eigenvalue, and this buckling mode is shown in Figure G.2.

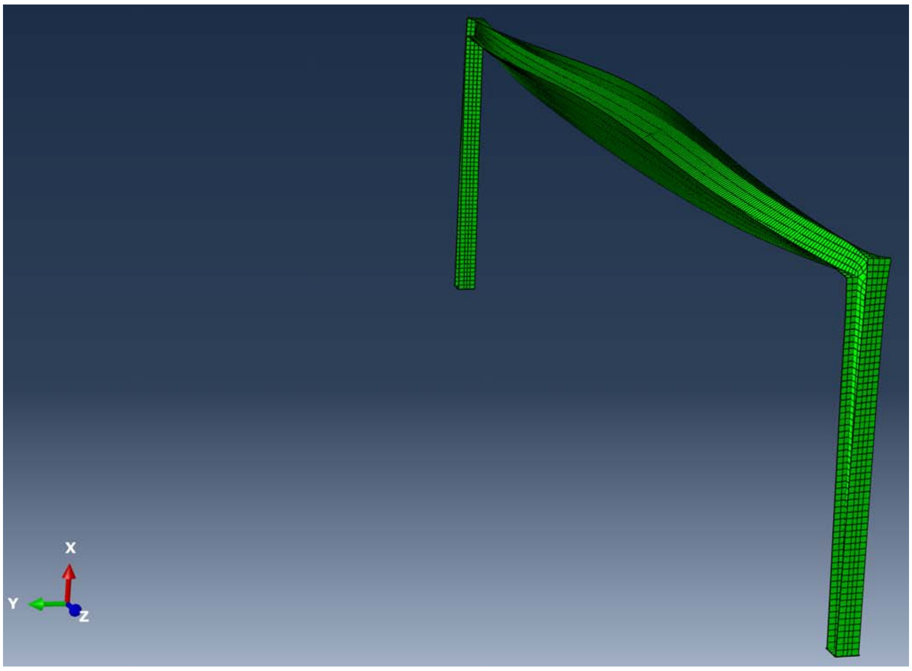


Figure G.2: First buckling mode shown in 3D.

G.3 Determination of the Utilization Ratio, UR

The utilization ratio, UR, is determined by the inverse of the result of the General Method as shown in Eq. (G.6).

$$UR = \frac{1}{\frac{\chi_{op}\,\alpha_{ult,k}}{\gamma_{M1}}} \tag{G.6}$$

Firstly, the global non-dimensional slenderness, $\overline{\lambda}_{op}$, is determined by Eq. (G.7) where the two minimum load amplifiers, $\alpha_{ult,k}$ and $\alpha_{cr,op}$, are used as input parameters.

$$\overline{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \tag{G.7}$$

$$\Downarrow$$

$$\overline{\lambda}_{op} = \sqrt{\frac{1.0378}{2.8183}} \tag{G.8}$$

$$\Updownarrow$$

$$\overline{\lambda}_{op} = 0.6068 \tag{G.9}$$

$\alpha_{ult,k}$	Minimum load amplifier for in-plane behaviour, $\alpha_{ult,k} = 1.0378$	[-]
$\alpha_{cr,op}$	Minimum load amplifier for out-of-plane behaviour, $\alpha_{cr,op} = 2.8183$	[-]

Next, the value, Φ_{LT}, to determine the reduction factor, χ_{LT}, for lateral torsional buckling is determined by Eq. (G.10).

$$\Phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\overline{\lambda}_{op} - \overline{\lambda}_{LT,0}\right) + \beta\,\overline{\lambda}_{op}^{2}\right] \tag{G.10}$$

$$\Downarrow$$

$$\Phi_{LT} = 0.5\left[1 + 0.49\cdot(0.6068 - 0.4) + 0.75\cdot 0.6068^2\right] \tag{G.11}$$

$$\Updownarrow$$

$$\Phi_{LT} = 0.6888 \tag{G.12}$$

α_{LT}	Imperfection factor for lateral torsional buckling, $\alpha_{LT} = 0.49$ (according to buckling curve *c*)	[-]
$\overline{\lambda}_{LT,0}$	Plateau length of the lateral torsional buckling curves for rolled sections, $\overline{\lambda}_{LT,0} = 0.4$ (Maximum value)	[-]
β	Correction factor for the lateral torsional buckling curves for rolled sections, $\beta = 0.75$ (Minimum value)	[-]

The value of Φ_{LT} is used to determine the reduction factor, χ_{LT}, for lateral torsional buckling in Eq. (G.13).

$$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\,\overline{\lambda}_{op}^2}} \tag{G.13}$$

$$\Downarrow$$

$$\chi_{LT} = \frac{1}{0.6888 + \sqrt{0.6888^2 - 0.75 \cdot 0.6068^2}} \tag{G.14}$$

$$\Updownarrow$$

$$\chi_{LT} = 0.8818 \tag{G.15}$$

Hereafter, a value, Φ, to determine the reduction factor, χ, for lateral buckling is calculated by Eq. (G.16).

$$\Phi = 0.5\,\left[1 + \alpha\,\left(\overline{\lambda}_{op} - 0.2\right) + \overline{\lambda}_{op}^2\right] \tag{G.16}$$

$$\Downarrow$$

$$\Phi = 0.5\,\left[1 + 0.34 \cdot (0.6068 - 0.2) + 0.6068^2\right] \tag{G.17}$$

$$\Updownarrow$$

$$\Phi = 0.7533 \tag{G.18}$$

α	Imperfection factor, $\alpha = 0.34$ (according to buckling curve *b*)	[-]

The result of Φ is used to determine the reduction factor, χ, for the relevant buckling mode in Eq. (G.19).

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \overline{\lambda}_{op}^2}} \tag{G.19}$$

$$\Downarrow$$

$$\chi = \frac{1}{0.7533 + \sqrt{0.7533^2 - 0.6068^2}} \tag{G.20}$$

$$\Updownarrow$$

$$\chi = 0.8336 \tag{G.21}$$

The reduction factor, χ_{op}, for the non-dimensional slenderness, $\overline{\lambda}_{op}$, can be determined by the minimum value of the reduction factor, χ_{LT}, for lateral torsional buckling and the reduction factor,

χ, for lateral buckling as shown in Eq. (G.22).

$$\chi_{\text{op}} = \min \begin{cases} \chi_{\text{LT}} & \text{for lateral torsional buckling} \\ \chi & \text{for lateral buckling} \end{cases} \tag{G.22}$$

$$\Downarrow$$

$$\chi_{\text{op}} = \min \begin{cases} 0.8818 & \text{for lateral torsional buckling} \\ 0.8336 & \text{for lateral buckling} \end{cases} \tag{G.23}$$

$$\Updownarrow$$

$$\chi_{\text{op}} = 0.8336 \tag{G.24}$$

The General Method is used to determine a value which has to be larger than one as seen in Eq. (G.25).

$$\frac{\chi_{\text{op}}\ \alpha_{\text{ult,k}}}{\gamma_{\text{M1}}} \geq 1.0 \tag{G.25}$$

$$\Downarrow$$

$$\frac{0.8336 \cdot 1.0378}{1.20} \geq 1.0 \tag{G.26}$$

$$\Updownarrow$$

$$0.7210 \geq 1.0 \tag{G.27}$$

The utilization ratio, UR, is determined by Eq. (G.28).

$$UR = \frac{1}{\frac{\chi_{\text{op}}\ \alpha_{\text{ult,k}}}{\gamma_{\text{M1}}}} \tag{G.28}$$

$$\Downarrow$$

$$UR = \frac{1}{\frac{0.8336\ 1.0378}{1.20}} \tag{G.29}$$

$$\Updownarrow$$

$$UR = 1.3870 \tag{G.30}$$

The utilization ratio, UR, is above 100% which means that the reference frame will be failing when investigating the frame with the General Method.

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