Fostering Autonomy and Engagement
The Flipped Classroom in Danish Upper Secondary Mathematics

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Abstract

This master's thesis explores the flipped classroom model in a Danish upper secondary mathematics class, guided by Self-Determination Theory (SDT), Cognitive Load Theory (CLT), Bloom's Taxonomy, and Vygotsky's Zone of Proximal Development (ZPD). The study investigates the transition from traditional, teacher-centric teaching methods to a more dynamic, student-engaged approach. It examines the ability of flipped classroom to address misconceptions in mathematics education and its potential in steering students away from surface learning. Utilizing a mix of pre- and post-implementation data, including questionnaires and interviews, the research evaluates the impact on student engagement, comprehension, and preparedness for academic challenges. The findings point to the effectiveness of a blended approach that integrates traditional teaching with flipped classroom strategies, suggesting this as a viable path to optimize mathematical qualifications in line with the progressive goals of Danish upper secondary education.

Preface

This master's thesis, developed during the autumn semester of 2023, marks the culmination of my academic journey as a Master's student in Mathematics at the Department of Mathematical Sciences, along with a minor in English at the Faculty of Humanities, Aalborg University. The process of delving into this research and writing has been an enriching experience, intertwining my passion for mathematics with innovative educational methodologies.

I would like to express my sincere thanks to my supervisor, Hans Hüttel, whose guidance and support were indispensable throughout this journey. Their expertise profoundly influenced my research approach and the development of this thesis.

I also owe a special thank you to the teacher, Joana Angelica Rodzewicz Sohnesen, and the students from Class 1.a at Aalborghus Gymnasium. Their participation in this research was invaluable, and this thesis could not have been realized without their collaboration and commitment. Their openness and active involvement were crucial to the success of this project.

Additionally, supplementary data is provided in a master zip file labeled 'Appendices.' Within this file, there are individual zip files for each data type and for each lesson. These include raw data from surveys and interviews, and other relevant supporting materials. Each zip file is clearly labeled and organized to facilitate easy reference.

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Contents

1 Introduction

This introductory chapter lays the foundation for the research question of this master's thesis by introducing key concepts. I start with traditional teaching methods in mathematics education, known for their teacher-centric, lecture-based approach, and explore their impact in the context of upper secondary schools. Alongside, I introduce the term *signature pedagogies*, as defined by Lee S. Shulman, particularly in response to common misconceptions in mathematics education. Additionally, the *flipped classroom* method is presented as a potential innovative approach to teaching mathematics.

1.1 Traditional Teaching Method

In the realm of upper secondary mathematics education, traditional teaching methods have long been the norm. These methods are predominantly characterized by the teacher's central role as an instructor, where the focus is on delivering content to the students and emphasizing a non-interactive style, where the student primarily acts as a passive recipient of that knowledge or content (Anku, 1996; Ardeleanu, 2019; Brito et al., 2009; Smith and Wood, 2000). Typically, such methods utilize standard tools like blackboards or whiteboards, with the teacher leading the instruction and deciding on the teaching-learning methods to be employed. In a traditional mathematics classroom, the teaching is often detached from students' daily experiences, leading to a disconnection where students struggle to see the relevance of the material to their lives. Teachers, while expert in their subject matter, frequently encounter questions from students like "Why do we need to learn this?" which highlights a gap in engagement and real-world application (Ardeleanu, 2019).

The layout of a traditional classroom itself speaks volumes about the teaching approach. Students are usually seated in a way that everyone has a clear view of the teacher, who leads the lesson from the front of the class. The teacher introduces new concepts, relating them to previously learned material, often using a black or whiteboard for demonstrations. This approach includes a mix of lecturing and solving problems, with the teacher guiding the students through a set curriculum of mathematical concepts (Ardeleanu, 2019). While the teacher may seek to involve students by asking questions during problem-solving, the interaction is typically limited. Students work individually on exercises, either in their manuals or worksheets, and the teacher provides additional explanations as needed. The lesson often concludes with the teacher addressing more complex problems and reinforcing the new concepts, followed by assigning homework for further practice (Ardeleanu, 2019).

This model, despite its widespread use, has been critiqued for its lack of experiential learning and limited student engagement. Modern pedagogical strategies suggest a shift towards methods that allow students to discover knowledge and develop problem-solving skills through real-world applications (Abdulwahed et al., 2012). Yet, traditional methods like didactic exposition and exercises remain prevalent, forming the backbone of mathematics instruction in many upper secondary schools.

While the majority of the sources referenced in this section primarily focus on traditional teaching methods within the context of higher education, their core principles and characteristics are notably aligned with those observed in upper secondary school settings. This alignment allows for a meaningful application and analysis of these methods in the context of upper secondary mathematics education. It is important to note this contextual difference as it underscores the versatility and persistence of traditional teaching approaches across various educational levels.

The persistence of traditional teaching methods in upper secondary mathematics education, characterized by a teacher-centric, non-interactive approach, raises important questions about the effectiveness and relevance of these methods in today's educational landscape. This concern is echoed in the dynamic landscape of Danish upper secondary education, where the Ministry of Children and Education emphasizes the importance of diverse and adaptive teaching methods. As outlined in their educational guidelines, the goal is to cater to the varied starting points of students, focusing on their progression and using formative evaluations to guide teaching. This approach underscores the necessity of evolving teaching methods that are not only responsive to student needs but also aimed at preparing them for future academic challenges (of Children and Education, 2023).

Against this backdrop, this master's thesis situates itself within the context of examining traditional teaching methods and their alignment with the Ministry's vision of progressive and student-centered education. In particular, it questions how these traditional approaches – characterized by teacher-led instruction and a focus on rote learning – fit into the modern educational paradigm that seeks to foster greater student independence and readiness for further studies. Thus, moving beyond the traditional paradigm, the concept of *signature pedagogies* as introduced by Lee S. Shulman becomes pertinent. These pedagogies encompass the distinct teaching styles inherent to different professions, including mathematics. Understanding these pedagogies, particularly in the context of mathematics, opens the door to reimagining how mathematics can be taught more effectively, moving away from surface-level learning towards a more experiential and student-centered approach.

To explore alternatives, this master's thesis delves into the concept of *flipped classroom* methodology. This innovative approach, which reverses the traditional model of classroom instruction, offers a potentially transformative solution, as it, among others, aligns with the Ministry's call for varied teaching methods and progression towards greater student autonomy. The flipped classroom's emphasis on active, student-led learning and formative assessment resonates with the educational objectives outlined by the Danish Ministry of Children and Education.

In the following sections, I will examine the nuances of signature pedagogies in mathematics, the intricacies of the flipped classroom method and its applicability in mathematics education, and ultimately, frame the central research question that guides this thesis.

1.2 Signature pedagogies in Mathematics

Lee S. Shulman presented the term *Signature pedagogy* and defines it as "types of teaching that organize the fundamental ways in which practitioners are educated for their new professions" (Shulman, 2005). Every academic subject is submitted to so-called signature pedagogy, yet the teaching activities within every subject adhere to the signature pedagogy in different ways (Gnaur and Hüttel). Nevertheless, Shulman argues, that the signature pedagogies have three fundamental structures, namely surface structure, deep structure, and implicit structure Shulman (2005). These three structures are to be understood as follows:

- **Surface structure**: The surface structure is about the visible and practical aspects of teaching and learning (Shulman, 2005). It includes the actual actions and behaviors that happen in the classroom. These actions include teaching methods, demonstrations, asking and answering questions, interactions between the teacher and students, and even when the teacher decides to be more or less involved in the learning process. Thus, my interpretation of the surface structure, in the context of mathematics, would involve how the teacher explains mathematical concepts, how students practice solving mathematical problems, and how they interact during mathematical lessons.

- **Deep structure**: Any signature pedagogy also has a "deep structure", which is a "set of assumptions about how best to impart a certain body of knowledge and know-how" (Shulman, 2005). That is to say that these special ways of teaching is based on some fundamental beliefs about the best ways to teach that subject.

In the context of mathematics, I would interpret the "deep structure" as to mean that there are particular ways of teaching mathematics that are considered very effective. These teaching approaches are built on some core ideas about how to help students understand mathematical concepts and problem-solving skills deeply. These ideas guide how mathematics is taught in ways that are most beneficial for students. In other words, the "deep structure" in mathematics education would involve those fundamental principles or strategies that guide how mathematics is taught effectively, ensuring that students really understand the mathematics they are learning.

- **Implicit structure**: When Shulman argues that signature pedagogies have an "implicit structure" and a "moral dimension" that includes "beliefs about professional attitudes, values, and dispositions," Shulman (2005), he is referring to the hidden, unspoken aspects of teaching and learning. To put it in the light of mathematics, I would argue that this structure means that not only do we have effective ways of teaching mathematics, but there are also unwritten rules about how mathematics should be approached professionally. in this case, these unspoken rules involve beliefs about how mathematics educators and students should behave and what values are important.

To this day, there are common misconceptions of mathematics by students through different educational levels. Among the common misconceptions held by students is the belief that learning mathematics primarily involves memorization. This misconception is that students can effectively learn mathematics simply by memorizing formulas and procedures, without understanding the underlying concepts and principles (Gurung et al., 2009). That is to say, students solve mathematical problems by reproducing the steps and signs presented by the teacher without understanding what they actually mean (King, 1993)(Hüttel and Gnaur, 2016). This type of learning is also referred to what Marton and Säljö (1976) define as *surface learning* and which means that the learning process is in a form of memorization of what the teacher presents to the students (Hüttel and Gnaur, 2016). When mathematics instruction focuses primarily on memorizing formulas and procedures, students may come to believe that mathematics is all about memorization and that they do not need to understand the underlying concepts.

Moreover, relying solely on surface-level learning can result in students who can perform calculations but struggle to explain the meaning or concept behind those calculations. Hence, they may lack a deep understanding of why certain mathematical operations work. Nonetheless, students who have been taught to memorize specific procedures may find it challenging to apply their knowledge to new or unfamiliar problems. They may struggle when confronted with variations of problems that differ slightly from those they have memorized. This leads to another misconception, which is the students belief that mathematics is assumable done quickly from knowing the facts and the process (Gurung et al., 2009), hence the they would not struggle with solving a problem for long before giving up.

An additional issue which memorization-based learning might prevent, is the students' ability to transfer their mathematical knowledge to real-world situations or other disciplines. They may view mathematics as a set of isolated rules rather than a versatile tool for solving a wide range of problems. Talbert (2017) and Gurung et al. (2009) argue that the traditional mathematical pedagogy is what reinforces these misconceptions within the students. The traditional mathematical pedagogy, as described in the previous section, is a teacher-centered lecture; the teacher takes the role as the "sage on the stage" (King, 1993). In case of a mathematics lesson, the teacher is carefully writing on the chalkboard, lecturing on facts, theorems, and presents examples that, most often, are related to the practices/problems that are assigned as activities for the students. These activities are done individually or in groups if allowed, and if the students want help, they will either have to rely on their peers or interact with their teacher, which possibly is time delayed (Talbert, 2017). The students are thus passive listeners, rather than active ones (King, 1993).

With that said, it is stressed by authors that there is a need to alter the students' beliefs about mathematics, by developing new pedagogies and taking into account emerging signature pedagogies. Gurung et al. (2009) present some of the emerging signature pedagogies in mathematics that include,

- encouragement of dynamic and active learning that engages students in *doing* mathematics, and not just *hearing* about it,
- instructional and assessment techniques that address conceptual knowledge and promote perseverance in problem-solving situations,
- instructional technique that involve collaborative work, both inside and outside of the classroom (Gurung et al., 2009).

Structuring mathematics lessons based on these signature pedagogies, indicate that students should not be surface learners in order to become practitioners of mathematics. Thus, one might ask, what are the expectations of the students' learning, or rather, what are the qualifications that students should gain from learning mathematics in order to become practitioners of the academic subject? What does *doing mathematics* mean? Nevertheless, surface-level learners are negatively associated with both comprehension and students' self-determination of learning (Schommer et al., 1992). Hence, what possibilities are there to assert these signature pedagogies in mathematics as well as to avoid students to become surface learners, thus optimizing the so-called qualifications? To investigate that, this master's thesis explores the teaching method called *flipped classroom*, and is presented in the following section.

1.3 The Flipped Classroom Method and Mathematics

Throughout the years, there has been drawn a lot of attention to the teaching method *flipped classroom* or *flipped learning*. Despite the fact that it does not have a single definition, it generally involves changes to the ways of which classrooms are usually structured, comprising in-class and out-of-class activities (Talbert, 2017; Novak et al., 2017). More specifically, flipped classroom is characterized by setting up ways for exposing the students to new materials and concepts in advance of class instead of in-class, something that would otherwise be done by means of lectures. As a consequence of this "flip", more time is spared during class which can be devoted to activities (Talbert, 2017; LaFee, 2013; Nielsen, 2023; Fulton, 2014), namely engaging in deeper discussions and implementing the new materials that the students have been preparing prior to class. The traditional lectures are thus replaced by, for example, videos, games, simulations, websites etc. Hence, the aim of the flipped classroom approach is to exploit time in the classroom for interactive learning activities (Song et al., 2017), in order to engage the students in collaborative and problem-based learning, as it develops their higher order thinking skills (Prince, 2004).

With that said, the flipped classroom method refers to inverting the so-called traditional method of teaching, which is similar to what is described in the previous section; a teacher-centered lesson, where the students are, most of the time, passive listeners (King, 1993; Talbert, 2017; Gurung et al., 2009). Figure 1.3.1 explicitly shows the general principles of the flipped classroom method;

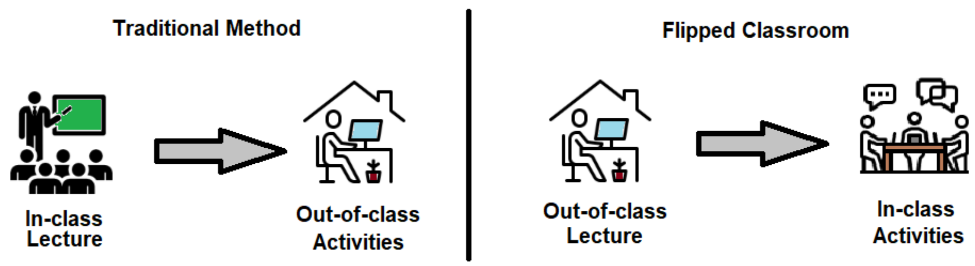

Figure 1.3.1: *The principles of flipped classroom*

As shown in Figur 1.3.1, students are exposed to new materials and concepts before class, usually at home, instead of in-class through a lecture as seen in the traditional model. Henceforth, the in-class session is devoted to active learning and to facilitate the interaction between students and teachers, instead of assigning the active learning as homework. Hence, this "flip" results in the paradigm shift where the teacher goes from being what King (1993) describes as a "sage on the stage" to a "guide on the side." This is the typical understanding of what the flipped classroom principles are, which in fact would seem as rather *simple* ideas.

However, Talbert (2017) stresses in his book that limiting the flipped classroom method to the concept of being a video-dependent method that should work as a substitution to the in-class lecture, and students working on what is supposed to be their homework during class time, is a misconception of the method. He argues that the main difference between the traditional method and flipped classroom is the *guidance* from the teacher (Talbert, 2017), which will be further explored in this master's thesis. That is to say, when it comes to implementing the flipped classroom method, it might not be as simple as it did sound. Nevertheless, the ways in which this method should be implemented also depend on which academic subject is being taught. The aim of this master's thesis is to focus on the implementation of the flipped classroom method in mathematics lessons in a Danish upper secondary class.

Hence, pondering the best ways to teach mathematics for students to become true practitioners, I consider the flipped classroom method. This method, which emphasizes the action of 'doing mathematics', may support and even transcend the emerging signature pedagogies. As this master's thesis seeks to understand and define what 'doing mathematics' truly entails, a critical examination of the flipped classroom's effectiveness becomes essential. This leads me to the central inquiry of this research, formulated to assess the practical implications of the flipped classroom within a Danish upper secondary mathematics class.

1.4 Research Question

To summarize, there are emerging pedagogies in mathematics due to misconceptions of the academic subject, indicating that students learn mathematics by merely being surface learners. These signature pedagogies include engaging the students into doing mathematics rather than hearing about it. They also include techniques that address conceptual knowledge, collaborative work, and promote perseverance in problem-solving. That is to say, the emerging signature pedagogies in mathematics seek to challenge these beliefs about mathematics and such that students do not fall into becoming surface learners. Nevertheless, these signature pedagogies demonstrate that the students are to acquire certain qualifications in order to become practitioners within mathematics. With the flipped classroom method as focus, the research question of this master's thesis is thus the following:

- What qualifications should students gain from mathematics lessons in a Danish upper secondary school and can the flipped classroom method be of help in optimizing these qualifications within a particular class, and if so, how?

To explore the qualifications students should gain in mathematics, this thesis not only considers the three emerging signature pedagogies in mathematics, as presented in Section 1.2, but also aligns with the educational standards and objectives set by the Danish Ministry of Children and Education. This holistic approach allows for a comprehensive understanding of 'what does it mean to do mathematics?', examining both pedagogical theories and national educational guidelines. Additionally, this master's thesis explores whether the flipped classroom method could be a possibility in preventing students from being surface learners in a particular class, hence optimizing these students' mathematical qualifications. In this case, a mathematics flipped classroom is constructed for a class of first years students in a Danish upper secondary school.

The overall objective of this master's thesis is to attempt to construct mathematics lessons for a first year upper secondary class using the flipped classroom method. The aim is to explore whether it could be a possibility, in this class, to avoid students being surface learners (if they are) and to (further) optimize their mathematical qualifications. To address this objective, it is imperative to first understand the methodological approach, followed by a review of existing literature on the subject, and finally delving into the foundational theories that underpin my study.

In the subsequent methodology chapter, the specific procedures, methods of data collection, and analytical strategies that will be employed in this master's thesis are detailed. After mapping out the methodological landscape, I will explore the existing literature in the field, gauging the current state of knowledge and identifying gaps. This leads to the theory chapter, where I immerse in the core theoretical framework of my study, culminating in the construction of hypotheses which will guide the empirical testing phase.

2 Methodology

The aim of this chapter is to present the methodology of this master's thesis, with Alan Bryman's book, "Social Research Methods", as basis. In his book, Bryman presents how to conduct a research, along with how to make preparations for the research and how authors should handle the collected data.

2.1 The Deductive and Inductive Approach

First and foremost, it is necessary to decide whether to carry out the research deductively, inductively, or perhaps a mixture of both. According to Bryman (2016), the deductive approach constructs hypotheses based on existing theories. The author will either accept or reject the hypotheses as a result of the carried out research. In other words, the deductive approach is theory driven, as the stated hypotheses or predictions are based on this theory. These statements are thus tested through empirical research. That is to say, the deductive approach involves moving from the general (the theory) to the specific (the hypotheses) and then to the empirical testing of those hypotheses.

The inductive approach, on the other hand, starts with specific observations or data, and not with a preconceived theory or hypotheses (Bryman, 2016). Instead, based on the observation achieved from the data analysis, theory is developed that can account for the observed phenomena. Thus, in this case, the theory is generated from the data rather than being pre-existing. Particularly, the inductive approach can lead to the refinement or modification of the theory as more data is collected and analyzed, as it is an iterative process where theory evolves with ongoing research.

In this master's thesis, I employ deductive research methods, initially focusing on formulating hypotheses based on existing theories and knowledge related to the flipped classroom method. These hypotheses, detailed in Chapter 4, are constructed to explore and interpret how students experience the flipped classroom approach. While the study begins with a deductive approach, it will also incorporate a qualitative perspective, as will be discussed in Section2.5. This structure allows for an initial theoretical framing, followed by an in-depth exploration of these theories in a specific educational context. The aim is not to definitively accept or reject these hypotheses but rather to investigate their plausibility through both theoretical and empirical lenses.

The research of this master's thesis is designed to shed light on the extent to which the hypotheses are supported by the data collected. Acknowledging the complexity and context-dependence of educational research, this master's thesis seeks to provide insights that contribute to the ongoing discourse on the effectiveness and applicability of the flipped classroom method. The findings are intended to enrich our understanding and inform future research and practice, blending deductive reasoning with qualitative analysis to offer a comprehensive view of the topic.

2.2 Literature review

Bryman (2016) introduces in his book, elements that are considered as essential when conducting a study. The first element is *literature review* in which he states that a thorough literature review is necessary for any research (Bryman, 2016). Through the literature review, the master's thesis brings into light the concepts within the field of research, as well as what other authors have applied as research methods within this field. Additionally, a literature review takes into account controversies and disagreements occurring between different researchers of the field (Bryman, 2016). However, Bryman (2016) emphasizes that it is rarely possible to establish every existing theory concerning the field of research, which is why the literature review needs to include sources that are considered as most crucial for the research, along with arguments why it is crucial. The literature review done in this master's thesis, is presented in Chapter 3.

2.3 Concepts and Theories

The importance of grounding our understanding in the field of flipped classroom research, as laid out in the literature review, naturally leads us into the exploration of specific concepts and theories that will frame this study. In this deductive study, the application of relevant theories is intrinsic to the research process. As outlined by Alan Bryman, concepts and theories are crucial in shaping and guiding research. Concepts serve as the building blocks of knowledge, representing categories, objects, phenomena, or principles that help in organizing and classifying information. This aids in comprehending complex subjects and facilitates deeper understanding Bryman (2016).

Theory, on the other hand, is a framework for understanding (Bryman, 2016). That is to say, theories are overarching frameworks or explanations that attempt to make sense of a set of related concepts. They provide a structured way of understanding and explaining a particular phenomenon or set of phenomena. Theories often involve relationships, causality, and principles that connect concepts (Bryman, 2016). Moreover, Theories generalize and synthesize knowledge about a specific area of study. They help researchers and scholars see patterns, regularities, and connections among various concepts. Nevertheless, theories often have predictive power. They allow researchers to make predictions or rather hypotheses about future events or outcomes based on the relationships and principles embedded within the theory (Bryman, 2016).

According to my interpretation, concepts can thus be thought of as the individual pieces of a puzzle, while theories are the assembled puzzle that shows how these pieces fit together to create a coherent picture. Moreover the development and refinement of theories often involve the exploration, clarification, and redefinition of underlying concepts. In this sense, theories can evolve as our understanding of concepts deepens.

In case of this master's thesis, the overall concept that is explored is the flipped classroom method. Thus, the aim is to present educational theoretical frameworks, as they offer a foundation for understanding how learning and teaching work. By presenting these theories, I establish a theoretical context for my study, helping to justify hypotheses; as mentioned earlier, these theories assist in formulating hypotheses about the expected outcomes of using the flipped classroom method in a mathematics class. Moreover, theories offer insights into effective teaching practices. By drawing on these theories, the design of the flipped classroom can be approached in a way that aligns with pedagogical principles. The presentation of what theories this master's thesis will consider, is presented in Chapter 4. After conducting the experiment, it is possible to analyze and interpret the data in light of the presented theories, by explaining how my findings align with or challenge the theoretical expectations.

2.4 Sampling Cases

The next research element is that of *sampling cases*, which is where the author chooses 'cases'(Bryman, 2016). According to Bryman (2016), in this phase of a research, authors choose what they consider as relevant data that are possible to include. Yet, it is highly impossible to include all types of relevant data, due to, among others, time constraint. It can also be due to the big amount of data sources, or what bryman referred to as cases. Hence, one needs to limit the amount of data by deselecting, or 'sampling', the cases (Bryman, 2016).

The primary case of this master's thesis is a first year upper secondary mathematics class. This data source, or case, was obtained through reaching out to an upper secondary school through email, where only one mathematics teacher, along with their students, whom was willing to cooperate and be a part of this research. However, since there was only one teacher that accepted my request, there was no need to sample the case.

2.5 Data Collection

Bryman (2016) introduces the next research element, namely the *data collection* phase. In this phase, the researchers need to choose how to collect the data. Bryman (2016) emphasizes that the level of structure within the data collection depends on the research, as a structured and a less structured data collection can both be of advantage or disadvantage. In case of a highly structured data collection, it can be in the form of structured interviews or questionnaire surveys (Bryman, 2016). Examples of less structured data collections can be, among others, observing the participants of the research or produce semi-structured interviews (Bryman, 2016).

However, the nature of the data collection phase largely depends on whether the study is *qualitative*, *quantitative*, or a combination of both. Bryman (2016) presents the contrasts between the two types of researches as well as their similarities. Among the contrasting features, is that a quantitative research is seen as being centered with numbers and measurements, while a qualitative research is using words in the presentation of analyses of society (Bryman, 2016). Moreover, Bryman highlights that a key distinction between quantitative and qualitative research lies in the focus on the researcher's perspective in the former and the participants' perspectives in the latter. In other words, in a quantitative research, the investigator is the one who takes the lead. That is to say, it is the investigators concerns that (s)he brings which drive the investigation (Bryman, 2016). In a qualitative research, it is the participants who drive the investigation, as it is their opinion about what they see as important and significant that provides the point of orientation (Bryman, 2016).

As mentioned earlier, the data collection's structure-level depended on what type of research that is conducted. According to Bryman (2016), a quantitative research is typically highly structured, in order for the investigator to be able to examine the exact concepts and issues that is of the study's focus. A qualitative research, on the other hand, is seen as invariably unstructured, due to the fact that it provides the possibility of emerging concepts out of data collection (Bryman, 2016). Nevertheless, a quantitative researcher often want their findings to be generalizable, whereas the qualitative researcher pursues an understanding of behaviour, values, and beliefs in terms of the context in which the research is conducted (Bryman, 2016).

Considering these contrasts, it is safe to assume that a qualitative research typically emphasizes an inductive inquiry, and a quantitative research a deductive inquiry. Yet, Bryman (2016) stresses that despite these contrasts, depicting the differences between quantitative and qualitative research, they should not be considered as absolute distinctions. For example, a qualitative research can also be employed to test theories. That is to say, it is possible to conduct a qualitative research and follow a deductive approach, however, the researcher has to keep in mind that (s)he has to remain open to emergent themes or patterns that may not fit within the initial framework, as a qualitative research is typically seen as unstructured (Bryman, 2016). Thus, in terms of the deductive approach, there are possibilities of modifying the hypotheses based on the qualitative data.

In context of this master's thesis, the data collection is through constructing open-ended questionnaires for students in the mathematics upper secondary class, as well as interviewing the participants of the research. These two methods will be further elaborated in Subsection 2.5.1 and 2.5.2, respectively. Moreover, throughout the implementation of the flipped classroom method, students will be provided with quizzes at the end of most lectures to evaluate their understanding. Thus, the primary data for this master's thesis is derived from the students' responses to questionnaires, quizzes, and interviews. This qualitative data set allows for an analysis of students' overall mathematical qualifications, primarily based on their self-reported experiences and perceptions. Details of this analysis method will be elaborated in Section 2.6. Nevertheless, such a data collection approach allows me also to take into account the students' perception of a mathematics flipped classroom, facilitating a comparative analysis of its effectiveness.

2.5.1 Open-ended Questionnaires

Questionnaires are a well known data collection method, which can be used in both qualitative and quantitative researches. In a qualitative research, questionnaires may be used to collect narrative or descriptive data from participants, and the responses are typically open-ended. This is based on Alan Bryman's arguments, which is that open-ended questionnaires allow respondents to answer in their own terms (Bryman, 2016). He also adds that open-ended questionnaires opens up for unusual responses to be derived, and they do not suggest certain kind of answers to respondents. Thus, with such a data collection, it is possible to evaluate students' knowledge and understanding of issues (Bryman, 2016). This use of questionnaires aligns with a qualitative approach because it aims to capture rich, contextual, and often non-numeric data (Bryman, 2016).

With the aim of answering the research question of this master's thesis, questionnaires with open-ended questions are designed, which encourage participants to provide detailed, narrative responses. These responses will allow me to capture rich qualitative data about their experiences with the flipped classroom method. That is to say, questionnaire items are crafted and which center on exploring participants' perceptions, experiences, and insights related to the flipped classroom method and ensure that the questions are open-ended and flexible to capture the nuances of their experiences. My objective with using open-ended questionnaires also includes to gather the participants' perspectives on the impact of the flipped classroom method on their mathematical qualifications, where I ask about changes in their learning, challenges they have encountered, and any benefits they have experienced. Hence, the design of the questionnaires include follow-up probes, such that I can dig deeper into the participants' responses, as these probes can encourage participants to elaborate on their answers and provide more context. Moreover, given that I am working within a specific classroom, my sample size may be relatively small, which aligns with the qualitative approach, as it often focuses on depth rather than breadth understanding.

However, despite the advantages of open-ended questionnaires presented above, it is important to also highlight disadvantages of this method of collecting data. Bryman (2016) stresses that main disadvantage of this method, is that they require great effort from respondents. In other words, many prospective respondents may be put off by the prospect of having to write extensively. This can lead to the problem of low response rates, as answering the open-ended questionnaires can be time-consuming and might deter some participants from completing the questionnaire (Bryman, 2016).

Nevertheless, as mentioned in Section 2.5, this master's thesis also collects data through interviews with the participants. Transitioning from open-ended questionnaires to interviews reduces the burden on participants, as they can express their thoughts verbally, which can be more convenient and less time-intensive. Additionally, using interviews as an adaptation to questionnaires allow me to explore the unexpected findings in a more personalized and in-depth manner. It adds a qualitative dimension to the research, which can provide valuable context and deeper insights into the experiences of the participants. Constructing such interviews, with focus in conducting a qualitative research, is presented in the following subsection.

2.5.2 Qualitative Interviews

According to Bryman (2016), there are two main types of interviews in qualitative research, namely the *unstructured interview* and the *semi-structured interview*. In fact, the term *qualitative interview* is used to capture these two types of interviews. Bryman (2016) argues, that what defines a qualitative interview is that the initial research ideas are more open-ended compared to a structured interview, which is designed to answer specific questions. Moreover, there is an emphasis on interviewees' own perspectives in a qualitative interview, as there is a greater interest in their point of view and gives insight into what they see as relevant and important (Bryman, 2016). Thus, in a qualitative interview, it is often encouraged to 'go off track'. That is to say, the interviewer can depart significantly from the interview guide; "They can ask new questions that follow up interviewees' replies and can vary the order and even the wording of questions" (Bryman, 2016)[s. 467].

The aim in this master's thesis is to construct qualitative interview guides that include open-ended questions specifically tailored to explore the themes or areas of interest that have emerged from the questionnaire data. The questionnaires include probes to encourage participants to elaborate on their responses. Moreover, the interviews are conducted through one-on-one interviews with the selected participants, as well as creating a comfortable and open atmosphere where they can freely discuss their experiences, challenges, and any benefits they have experienced with the flipped classroom. Thus, the objective is to use interviews to explore the unexpected themes or insights in greater detail and encourage participants to provide narratives, context, and real-life examples to support their responses.

2.6 Data Analysis

After collecting the data, it is then time to consider how to analyze them. Bryman (2016) stresses that the main objective of this phase is to consolidate the collected data such that it is manageable. However, it is crucial to do it carefully such that it does not lead to misinterpretation of the data or perhaps mistakenly remove some of the data that is significant for the research (Bryman, 2016).

In a qualitative research, the most frequent way in consolidating the collected data, is through *coding*; it serves as a shorthand device to label, separate, compile, and organize data (Bryman, 2016). The way in which this qualitative device is typically developed, which Bryman (2016) introduces, is elaborated in the following subsection. Nevertheless, another tool that is considered to serve as a complementary role in qualitative data analysis is also presented in Subsection 2.6.2, namely a *thick description*.

2.6.1 Coding

According to Bryman (2016), coding (also known as *indexing*) is the starting point in most forms of qualitative data analysis. He argues that among the considerations in developing codes are cited as follows:

- "Of what general category is this item of data an instance?
- What does this item of data represent?
- What is this item of data about?
- Of what topic is this item of data an instance?

- What question about a topic does this item of data suggest?
- What sort of answer to a question about a topic does this item of data imply?
- What is happening here?
- What are people doing?
- What do people say they are doing?
- What kind of event is going on?" (Bryman, 2016)[s. 581].

In this master's thesis, coding serves as a foundational step in qualitative data analysis. By systematically categorizing and labeling data — specifically from open-ended questionnaires — I can identify patterns and concepts within the data. This process of coding not only organizes the data into manageable units but also facilitates the recognition of both similarities and distinctions among the responses.

Transitioning from coding to the next stage of qualitative analysis, I approach the concept of *thick description*. As noted by Bryman (2016), qualitative findings often focus on the unique context and significance of a specific aspect of the social world being studied. This approach typically involves an in-depth examination of a small group, emphasizing depth of understanding over breadth. Following the systematic organization of data through coding, a *thick description* enables the researcher to provide rich, detailed accounts of certain nuances and complexities. In this master's thesis, thick description will be used to enrich the interpretation of the coded data, offering a comprehensive and contextual understanding of the students' experiences and perceptions in the flipped classroom environment

2.6.2 Thick Description

According to Bryman (2016), qualitative researchers are encouraged to construct a so-called *thick description*, as it is a method that allows the researcher to provide with rich accounts of the details of a culture (Bryman, 2016). Ponterotto (2015) discusses the concept of *thick description* in qualitative research as well, emphasizing its use beyond life-story analysis. He suggests how thick description can be applied in various disciplines and research approaches, such as sociology, psychology, and education, where in such cases there is relied on interviews (Ponterotto, 2015). In advocating for the method of thick description, Ponterotto (2015) argues that it involves providing detailed accounts of the research participants, encompassing both their demographic and psychological characteristics. This provides a vivid and detailed understanding of the sample without compromising anonymity (Ponterotto, 2015). Moreover, one should describe the research setting and procedures in detail, include information about the interview context, such as location, length, recording procedures, and reactions of both the interviewer and interviewee. This context enhances the reader's comprehension and ability to critique the study (Ponterotto, 2015).

In this master's thesis, the intertwining of coding and thick description is central to the qualitative data analysis. Initially, I will engage in coding to categorize and label the data, identifying themes and patterns within the interview transcripts and open-ended questionnaire responses. Following this, thick description will be employed to provide context and depth to these identified themes. Narrative excerpts from interviews will be incorporated to vividly illustrate and support the themes. Furthermore, this master's thesis will utilize the concept of triangulation, which Bryman (2016) defines as "[t]he use of more than one method or source of data in the study of a social phenomenon so that the findings may be cross-checked" [s. 697]. The combination of coding and thick description will not only structure and categorize the data but also bring it to life, allowing readers to connect deeply with the experiences of the participants.

3 Flipped Classrooms in Mathematics Education

Having established the research methods and approaches in the previous chapter, I now turn the attention to a broader exploration of the educational landscape. In this chapter, I delve into the diverse landscape of mathematics education, contrasting traditional teaching methods with the innovative flipped classroom approach. The objective is to unravel the complexities of these pedagogical strategies, examining their impact on student learning and engagement in mathematics.

This chapter begins by reassessing traditional teaching methods in mathematics, critically examining the structured, teacher-centric approaches that have long dominated the educational landscape. I explore how these methods, rooted in educational history, have influenced mathematics teaching and learning. The discussion then shifts to explore emerging signature pedagogies that emphasize dynamic and active learning, conceptual knowledge development, and collaborative work. This exploration seeks to understand how innovative teaching methodologies, like the flipped classroom method, can realign mathematics education with the needs of contemporary learners, fostering deeper understanding and practical skill development.

Next, I move to critically examine the flipped classroom method, drawing extensively from Robert Talbert's influential book "Flipped Learning." This text serves as a foundational reference, providing valuable insights into the restructuring of the mathematics learning process through active student participation and independent exploration. It plays a key role in distinguishing the flipped classroom approach from traditional educational models, particularly highlighting the pivotal shift towards teacher guidance inherent in the flipped classroom methodology.

Finally, the chapter addresses controversies and differing perspectives on the flipped classroom, focusing on the influence of student preparedness and individual learning preferences. By exploring existing studies on this teaching method, the discussion aims to offer a nuanced perspective on optimizing student qualifications and learning outcomes through the flipped classroom, especially within the context of Danish upper secondary education. This exploration aims to deepen our understanding of how the flipped classroom approach can effectively enhance mathematics education.

3.1 Reassessing Traditional Teaching Methods in Mathematics

The landscape of mathematics education has long been dominated by traditional teaching methods, a paradigm characterized by its structured, teacher-centric approach. These methods, deeply rooted in educational history, have shaped the way mathematics is taught and understood in various settings, including Danish upper secondary schools. As mentioned in the introductory chapter, traditional teaching in mathematics is marked by a distinct focus on the teacher as the primary source of knowledge and authority in the classroom. As highlighted by Abdulwahed et al. (2012), this method often employs standard tools such as blackboards or whiteboards, with the teacher leading the instruction from the front of the classroom. Students, arranged to have a clear view of the teacher, typically engage in a passive learning role, receiving information rather than actively participating in its construction. Thus, lectures and didactic instruction form the core of this approach, where the emphasis is placed on delivering content, often leading to a reliance on rote memorization and procedural understanding. In these settings, the teacher's explanations and demonstrations take precedence, with students primarily engaged in listening and note-taking. The interaction is generally one-way, with limited opportunities for students to contribute or engage in discussions (Abdulwahed et al., 2012; Ardeleanu, 2019).

The implications of such a teaching approach extend beyond the classroom layout. As Abdulwahed et al. (2012) suggest, this method can significantly impact students' engagement and understanding of mathematics. One of the critical limitations is the lack of real-world application and context in the learning process, which can lead to a disconnection between the subject matter and students' everyday experiences. While students may achieve a certain level of proficiency in mathematical procedures and calculations, they often lack a deeper understanding of the underlying principles and real-world applications of mathematics (Ardeleanu, 2019). This gap often results in misconceptions about the nature of mathematics, viewing it as a set of isolated procedures rather than a dynamic and applicable field of study. Moreover, a focus on memorization and procedural learning in traditional teaching methods can hinder the development of critical thinking and problem-solving skills. Students may become adept at following specific steps to solve problems but may struggle to understand or apply mathematical concepts in new or varied contexts. This approach can also stifle creativity and limit opportunities for deeper exploration of the subject (Abdulwahed et al., 2012). In fact, the traditional teaching methods are increasingly being critiqued in light of contemporary educational needs and pedagogical research. Experts argue, as Abdulwahed et al. (2012) elucidate, that these methods may no longer suffice in fostering a comprehensive and deep understanding of mathematics. The critique centers around the need for more interactive, student-centered approaches that actively involve learners in the construction of their knowledge and encourage exploration beyond the confines of predetermined algorithms and formulas (Abdulwahed et al., 2012).

In recognizing the limitations of traditional teaching methods in mathematics, there emerges a compelling argument for alternative pedagogical strategies. These strategies, which we will explore in the subsequent sections, are geared towards enhancing student engagement, deepening conceptual understanding, and aligning mathematics education with real-world applications. This shift in focus is crucial as we consider the evolving landscape of mathematics education and seek methods, such as the flipped classroom, that could potentially optimize student qualifications and learning outcomes in Danish upper secondary schools. Thus, recognizing these shortcomings leads us to the emerging pedagogical strategies that promise a more dynamic and participatory form of learning. This exploration is crucial, not only to address the gaps left by traditional methods but also to align with contemporary educational needs that prioritize student engagement and conceptual understanding.

3.2 Mathematical Engagement: Learning, Skills, and Practice

As mentioned in the introductory chapter, Section 1.2, the discussion on altering students' beliefs about mathematics through emerging signature pedagogies is crucial for effective learning. The key aspects of these pedagogies that were presented, as outlined in Gurung et al. (2009), focus on:

- **Encouragement of Dynamic and Active Learning**: This approach emphasizes the importance of students actively engaging in the process of doing mathematics. Active engagement contrasts with various forms of passive learning, where students might listen to lectures without direct involvement, merely observe, or mechanically follow procedures without deeper understanding. The active participation involves activities where students explore, experiment, and apply mathematical concepts in practical contexts, thus deepening their understanding and making the learning process more engaging and meaningful.

- **Instructional and Assessment Techniques for Conceptual Knowledge and Perseverance**: These techniques aim to develop a deep understanding of mathematical concepts, going beyond rote memorization. Students are encouraged to persevere through challenging problem-solving situations, fostering a growth mindset and resilience. The assessments in this approach are designed to evaluate conceptual understanding and the application of knowledge, rather than just testing memory.

- **Collaborative Work Inside and Outside the Classroom**: Collaborative learning plays a significant role in this pedagogical approach. It involves students working together on mathematical problems, sharing ideas, and learning from each other. This not only enhances understanding but also develops essential skills like communication, teamwork, and critical thinking.

Based on these signature pedagogies, the student expectations and qualifications can thus be argued to be as follows:

- **Expectations of Student Learning**: Students are expected to become active learners who engage deeply with mathematical concepts. They should be able to apply their knowledge to solve problems, think critically, and work collaboratively. The focus is on developing a holistic understanding of mathematics as a discipline, rather than just acquiring factual knowledge.

- **Qualifications for Practitioners**: The qualifications here refer to the skills and competencies that students gain through this approach. This includes a strong conceptual understanding of mathematics, problem-solving skills, the ability to collaborate effectively, and perseverance in challenging situations. These are essential qualifications for anyone wishing to become a practitioner in the field of mathematics.

Nevertheless, Gurung et al. (2009) emphasize the importance of understanding what 'doing mathematics' means. They argue that 'doing mathematics' involves actively engaging with mathematical concepts, exploring them through problem-solving, critical thinking, and practical application. It is about understanding the 'why' behind the 'how,' which leads to a deeper and more comprehensive understanding of the subject (Gurung et al., 2009).

However, in the context of this master's thesis, it is essential to acknowledge the specific scope and limitations of the study. The application of these emerging signature pedagogies is explored within a singular classroom setting, focusing exclusively on the subject of descriptive statistics. This particular concentration, detailed further in Chapter 5, provides a specific lens through which these pedagogical strategies are examined but may limit the generalizability of the findings across other mathematical domains. Furthermore, the proficiency level of the students involved in this study is at level B, as outlined in Chapter 5. This level of proficiency inherently shapes the depth and approach to the mathematical engagement in this context. The expectations and qualifications for learning and practice in mathematics, as previously discussed, might therefore need to be adapted to suit this specific proficiency level.

According to the Danish Ministry of Children and Education's guidelines for Mathematics Level B, the course aims to provide students with a comprehensive, application-oriented, and study-preparatory mathematical insight. According to this decree, the curriculum is designed to help students develop a solid foundation for active, constructive, and innovative participation in a democratic society. Specifically, students are expected to gain the competence to understand, formulate, and solve problems related to real-world phenomena, employing mathematical reasoning and logical thinking. This prepares them for discussions on mathematical applications and equips them with the necessary academic competencies for higher education (of Children and Education, 2017). Of particular relevance to this master's thesis is the qualification in descriptive statistics. This subject is central as it represents the area of focus for students during the implementation of the flipped classroom method. The decree emphasizes the importance of applying statistical and probability-theoretic models to analyze data from various fields, perform simulations, conduct hypothesis testing, and establish confidence intervals. Students are expected to develop the ability to question and interpret these models and formulate conclusions in clear language. The goal is to enable students to use mathematical tools for understanding and solving problems in descriptive statistics, thereby preparing them for practical applications in their academic and future professional life (of Children and Education, 2017). This focus aligns well with the flipped classroom approach, as it encourages students to engage with real-world data and apply mathematical concepts in meaningful ways.

Having explored the key aspects of dynamic and active learning, conceptual knowledge development, and collaborative work in the context of mathematical engagement, it becomes clear that innovative teaching methodologies play a crucial role in actualizing these pedagogical strategies. One such methodology that aligns with the principles of active learning and student engagement is the flipped classroom method. This approach, which inverts the traditional classroom model, complements the pedagogical goals discussed earlier by fostering an environment where students are prepared to actively participate and engage in mathematical problem-solving and conceptual exploration.

Thus, in the following section I delve into how this teaching approach reorganizes the conventional structure of mathematics education. I examine its principles, the potential misconceptions surrounding its implementation, and how it can be effectively utilized to enhance students' learning experiences in alignment with the emerging signature pedagogies. This exploration will provide a comprehensive understanding of how the flipped classroom method can serve as a practical application of the theoretical pedagogical concepts discussed and contribute to the development of students as active learners and practitioners in mathematics.

3.3 The Concept of the Flipped Classroom Method

As presented in the introductory chapter, the main principles of the flipped classroom method are inverting the traditional model, in the sense that the in-class lecture becomes the students' homework to prepare prior to class, in order to participate in the in-class learning activities where the students engage in active learning. The ways in which the teachers *can* implement the flipped classroom method, are through, among others, using certain types of technology; for example, using videos for students to watch as preparation for the in-class activities.

However, Talbert (2017) emphasizes that limiting the flipped classroom method to a method that is only about putting video lectures outside class an doing homework in-class is among the common misconceptions of the overall mentioned method. In fact, there is an assumption that flipped classroom *requires* recorded video lectures in order to construct a 'true' flipped learning environment (Talbert, 2017). However, Talbert (2017) argue that it is common among flipped courses to have videos that are available, as there also are real advantages of doing so, yet it is neither necessary nor is it always the case that videos need to be used. One of the original implementations of what defines flipped classroom is in fact *peer instruction* and not videos; what determines the flipped classroom is not the technology that is used, but the activities that take place out-of-class and in-class (Talbert, 2017).

Talbert (2017) discusses that it is not necessary for the teacher to provide the students with direct instructions to prepare them for the in-class activities. For example, there can be other activities than to just watch video lectures or doing guided readings. Talbert (2017) argues that in some cases students' preparation for in-class activities might be best accomplished by leaving direct instructions out of the picture. Instead, direct instructions can be preceded by guided "discovery learning" activities, as it has shown, in some cases, to improve student learning (Talbert, 2017). That is to say, instead of students' simply being assigned to watch, for example, a video lecture, the teacher can provide the students with structured/guided activities to engage in a so-called "discovery learning". Talbert (2017) has thus attempted to formulate a definition of flipped classroom, which is the following:

> *Flipped Learning is a pedagogical approach in which first contact with new concepts moves from the group learning space to the individual learning space in the form of structured activity, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter* (Talbert, 2017)[s. 20].

For the reader's awareness, this master's thesis uses the term *flipped classroom*, yet as mentioned in the introductory chapter, the term *flipped learning* is also used for the same concept by some authors, and this is the case with Robert Talbert in his book. In essence, Talbert (2017) is describing how flipped classroom reorganizes the traditional teaching and learning process. He emphasizes students' independent exploration of new concepts outside the classroom, allowing in-class time to be used for active, collaborative, and creative activities with the guidance of the educator. Thus, in my interpretation, the main difference between 'structured' and 'direct structured' lies in the level of guidance and facilitation provided by the educator. 'Structured activities' are organized and purposeful tasks for students, while 'direct structured activities' are those structured activities where the teacher plays an active role in guiding and supporting the students as they complete the tasks. Thus, the term 'direct' implies a more hands-on teaching approach in these activities. In this master's thesis, the aim is to follow Talbert's presented definition, hence emphasizing the structured method of guiding the students. In the next section, the master's thesis explores existing researches considering the flipped classroom method in teaching mathematics, taking into account their methods of conducting it.

3.4 Existing Research on the Flipped Classroom

The flipped classroom method has seen burgeoning interest in academia. As emphasized by Bryman (2016), a literature review serves not only to shed light on key concepts within the field of research but also to present the methods other researchers have employed. In this light, this section presents an analysis of pivotal studies on the flipped classroom method in mathematics, elucidating the concepts, methodologies, and any controversies within this domain. In addition, the selection of these studies is carefully justified, underscoring their significance and relevance to this master's thesis.

3.4.1 The Study by Novak et al. (2017)

Novak et al. (2017) have conducted a study, which is a case study of implementing the flipped classroom method in a mathematics lecture with 300 undergraduate students. The study aimed to reinvent the traditional lecture on matrix determinants by having students prepare for the class in advance, thereby freeing up in-class time for active and engaging learning activities, including discussions and debates. The study discusses students' perspectives on this flipped lecture approach and includes survey data and focus group themes. The survey results indicate that a significant portion of students found the flipped lecture more effective in developing their ability to master matrix determinants and more engaging than a standard lecture (Novak et al., 2017). However, the survey also showed that a considerable number of students spent minimal time preparing for the flipped lecture. In fact, the data from this research suggests that poorly prepared students perceived greater benefits from the flipped lecture compared to well-prepared students. Well-prepared students had mixed opinions, with about 50% finding the flipped lecture less effective (Novak et al., 2017). However, this study does not elaborate on the specific reasons behind this perception or provide data on in-class performance based on preparation levels.

3.4.2 The Study by Sun and Xie (2020)

The work of Sun and Xie (2020) stands out for its emphasis on student preparedness, a crucial aspect of the flipped classroom method. By adopting a longitudinal study design, they could track student performance across a semester, correlating it with pre-class preparation levels. Their findings underscore the importance of student readiness for optimal outcomes in flipped classrooms. In this study, they suggest that it is crucial to identify students who may be at risk of underperforming or struggling in the flipped classroom. By "early identification," they mean recognizing these students at the beginning of the course or as soon as possible. These students are at risk because they might not adequately prepare for the flipped classroom, and this can impact their performance. However, the study found that students who were well-prepared before class performed better in in-class activities and achieved higher grades in the course. This indicates the importance of pre-class preparation in a flipped classroom setting.

3.4.3 Addressing Controversies

Both studies are centered around the concept of the flipped classroom and they both investigate the effectiveness and outcomes of using the flipped classroom approach in mathematics education. Moreover, the studies explore the aspect of student engagement in a flipped classroom. The study, written by Novak et al. (2017), discuss student engagement in terms of participation in class debate and group discussions, while the second paper by Sun and Xie (2020) delve into the relationship between pre-class preparedness and student engagement. Both studies seek to determine the effectiveness of the flipped classroom model, where the Novak et al. (2017) assess the students' perspectives on the flipped lecture approach, while Sun and Xie (2020) examine the impact of pre-class preparedness on student performance in a flipped mathematics course. In addition to that, both Novak et al. (2017) and Sun and Xie (2020) consider student perceptions and opinions regarding the flipped classroom approach. Novak et al. (2017) explore how students responded to the flipped lecture model, while Sun and Xie (2020) investigate how students' attitudes and goals influence their performance in a flipped classroom.

However, an intriguing divergence between the two studies is the perception of less prepared students. While Novak et al. (2017) found that underprepared students perceived greater benefits from the flipped method, Sun and Xie (2020) highlighted the advantages of being well-prepared. This discrepancy points to a potential area of contention in the literature that may warrant further exploration. Yet, in essence, both studies are centered around the flipped classroom approach, which aligns with the focus of my study. They provide insights into the application of the flipped classroom method, student engagement, and outcomes. These studies investigate the effectiveness of the flipped classroom method in mathematics education, which directly relates to my research on optimizing mathematical qualifications in a specific mathematics classroom.

3.4.4 Justification for Selection

In justifying the choice of reviewing these studies, both of them while primarily focused on mathematics education in higher education settings, have been selected for their potential applicability and valuable insights into the context of upper secondary school mathematics education. Their findings, and emphasis on student engagement and perceptions, though originally situated in higher education, offer perspectives that can be adapted and contextualized for upper secondary settings in this master's thesis.

The study, conducted by Novak et al. (2017), delves into the experiences of students and educators in a higher education context who have engaged with the flipped classroom model. The findings underscore the impact of this pedagogical approach on student engagement, understanding, and performance. By exploring the effectiveness and challenges of flipped learning, the paper lays a foundation upon which I can build in my master's thesis. While this paper has focused on higher education, it has raised questions about the applicability of the flipped classroom method in the context of upper secondary mathematics education, which is a question I aim to address.

Sun and Xie (2020) have also contributed significantly to my understanding of the flipped classroom method in the context of mathematics education. This study, conducted in a tertiary setting, offers insights into the impact of student preparedness on the effectiveness of the flipped classroom. The study highlights that poorly prepared students may find flipped classrooms more effective for their learning preferences, thereby emphasizing the potential variability in students' responses to this teaching approach. This master's thesis aims to extend this understanding by examining how this variability plays out in an upper secondary mathematics classroom.

3.5 Evaluating the Flipped Classroom in Mathematics

This section of the chapter delves into a critical examination of the two key studies on the flipped classroom method, with an emphasis on how these insights can be utilized to optimize mathematics education. The goal here is to understand the flipped classroom's impact on student engagement, preparedness, and overall academic performance in mathematics, particularly in the context of Danish upper secondary education.

The study by Novak et al. (2017) reveals the potential of the flipped classroom to increase student engagement and understanding in mathematics. Their approach, which involves active learning strategies such as debates and group discussions, aligns well with the educational goal of deepening students' comprehension of mathematical concepts. Moreover, such an approach aligns with the principles of signature pedagogies, which advocate for an active learning process rather than passive absorption of information, as presented in Section 3.2.

The research by Sun and Xie (2020) emphasizes the significance of student preparedness in a flipped classroom. Their findings suggest that students who engage more thoroughly with pre-class activities tend to have better academic outcomes. This underscores the importance of pre-class engagement as a key factor in optimizing student performance in mathematics education. Nevertheless, The study's emphasis on pre-class preparedness aligns with the pedagogical focus on active student participation. In a flipped classroom setting, the responsibility for initial learning shifts to the student, which resonates with the idea of encouraging students to be self-motivated learners, a key aspect of the signature pedagogies.

Additionally, the two studies lead to a conclusion that underscores the importance of considering individual learning preferences and preparation levels in implementing flipped classroom. The study by Novak et al. (2017) highlights varied student responses to the flipped classroom method, particularly noting that students who were less prepared perceived greater benefits compared to well-prepared students. This variation in perception and benefit from the flipped classroom approach suggests that individual differences in students' preparedness levels and possibly their learning preferences play a significant role in how they experience and benefit from this pedagogical approach.

The study by Sun and Xie (2020) emphasizes the importance of pre-class preparedness and identifies different learning profiles based on students' engagement with pre-class activities. The finding that well-prepared students tend to perform better academically in a flipped classroom setting implies that individual differences in students' preparation levels significantly affect their learning outcomes. This further suggests the need to consider these differences when implementing flipped classroom methodologies. However, a notable consideration which need be made is the difference in the maturity levels and learning motivations of younger students within the case of an upper secondary class.

While each study offers unique perspectives, together they provide a compelling argument that the flipped classroom can enhance student qualifications in mathematics. By promoting a more interactive and engaging learning environment and emphasizing the importance of student readiness, the flipped classroom method appears to foster a deeper understanding and stronger performance in mathematics. Moreover, the mixed responses from students in the study by Novak et al. (2017) underscore the need to consider individual learning preferences and preparation levels when implementing flipped classrooms. This insight is crucial for adapting the flipped classroom method to upper secondary mathematics education, where students' backgrounds and preparation levels can vary widely.

As we transition to the next chapter, the focus will shift to the theoretical underpinnings of this master's thesis. The chapter involves a detailed exploration of relevant theories that inform the master's thesis' framework and the formulation of hypotheses. These hypotheses, derived from both the literature reviewed and the theoretical perspectives, will guide the subsequent empirical investigation into the efficacy of the flipped classroom method within a specific mathematics classroom setting.

4 Theorizing Flipped Mathematics

I this chapter, I delve into four key theoretical frameworks that underpin this master's thesis, namely *Self-Determination Theory*, *Cognitive Load Theory*, *Bloom's Taxonomy*, and the *Zone of Proximal Development*. These theories have been carefully chosen for their relevance to different aspects of the flipped classroom method in mathematics education. Together, they provide a holistic view, addressing student motivation, cognitive challenges, skill development, and the needed support for group learning. By integrating these theories, the goal is to create a clear and comprehensive understanding of how flipped classrooms can enhance students' mathematical qualifications. The insights from these theories will form the foundation for the hypotheses that guide the subsequent investigation into the effectiveness of the flipped classroom approach.

4.1 Self-Determination Theory

In this section, I explore the first theoretical framework, *Self-Determination Theory*, which will be abbreviated as SDT throughout the rest of this master's thesis. While SDT was originally developed by Deci and Ryan (1980), my interpretation and application of the theory in this context are significantly informed by insights and explanations from Robert Talbert's book "Flipped Learning." Therefore, the ensuing discussion primarily draws upon and focuses on Talbert's perspective, as detailed in (Talbert, 2017).

SDT is a theory of human motivation, which focuses on the type of motivation a person has. Since the flipped classroom method depends to a great deal on student motivation, specifically the motivation to complete preparatory activities in order to participate actively in-class and so forth, SDT has alot to say in terms of flipped classroom. Generally, SDT makes two different distinctions between motivation types, namely the distinction between *intrinsic* and *extrinsic* motivation and the distinction between *autonomous* and *controlled* motivation (Talbert, 2017).

Intrinsic motivation refers to the motivation that comes from within an individual. It is driven by personal satisfaction, interest, and a genuine enjoyment of an activity. That is to say, people who are intrinsically motivated engage in an activity because they find it inherently rewarding or fulfilling. It often leads to a higher level of engagement, creativity, and persistence, as the individual is motivated by their own internal desires rather than external factors. Extrinsic motivation, on the other hand, is driven by external factors such as rewards, punishments, or social pressure. People who are extrinsically motivated engage in an activity to achieve a specific outcome or to avoid negative consequences, rather than for the inherent enjoyment of the task. Thus, the main difference between intrinsic and extrinsic motivation is the source of motivation. Intrinsic motivation arises from internal factors like personal interest and satisfaction, while extrinsic motivation is based on external factors like rewards, punishments, or social expectations. The two types of motivation can coexist, and the degree to which each type is present can vary depending on the individual and the specific context (Talbert, 2017).

Autonomous motivation is a type of motivation characterized by a high degree of self-determination. It means that individuals engage in an activity because they genuinely want to do it, find personal value or satisfaction in it, and feel a sense of choice and control over their action. Moreover, autonomous motivation includes or consists of both intrinsic motivation and types of extrinsic motivation. That is to say, intrinsic motivation is a specific type of autonomous motivation, and that within autonomous motivation, there are specific types of extrinsic motivation that are also included. Nevertheless, Talbert (2017) emphasizes that the ultimate goal, within autonomous motivation, is for individuals to integrate the activity into their sense of self. This means that they not only value and understand the task but also see it as a part of who they are and something that aligns with their personal identity and values.

By contrast to autonomous motivation, controlled motivation is a type of motivation characterized by a lack of self-determination (Talbert, 2017). It means that individuals engage in an activity due to external factors, rather than their own intrinsic desires or interests. They feel a sense of external pressure or control. Within controlled motivation, there are different components. The first component Talbert (2017) mentions is *external regulation.* In this context, external regulation refers to motivation that is entirely driven by external contingencies, such as rewards or punishments. Individuals engage in an activity because they are rewarded for it or to avoid punishment. The second component is *introjected regulation.* This refers to a situation where the regulation of one's actions has been partially internalized, but it is still motivated by factors such as the desire for approval, avoidance of shame, contingent self-esteem (self-worth that depends on meeting certain conditions), and ego-involvements (self-identity concerns). Thus, the key characteristic of controlled motivation is that individuals experience a sense of pressure to think, feel, or behave in specific ways. They are not engaging in the activity because they genuinely want to or find it personally satisfying, but rather because they feel compelled to do so for external reasons or to meet certain internal standards (Talbert, 2017).

Talbert (2017) provides with following example to explicitly understand the overall difference between autonomous motivation and controlled motivation:

> a person can feel extrinsic motivation that is autonomous, such as when we complete work on a research paper because we need to finish that paper in order to get a positive performance evaluation (the "extrinsic" part) and when we personally place a high value on professional advancement and doing well in our jobs (the "autonomous" part). If, however, we are completing that paper not because we value professional advancement but because we have been made to feel inferior to our colleagues if we don't publish, this is extrinsic motivation that is controlled in nature [s. 43].

However, in Robert Talbert's book, it is stated that the psychologists, Deci and Ryan, point out that although people are naturally strongly intrinsically motivated from birth, the intrinsic motivation can be affected by the external conditions and the satisfaction of certain needs (Talbert, 2017). In fact, in order to maintain intrinsic motivation, one needs to satisfy three cognitive needs, namely *competence*, *autonomy*, and *relatedness*. Competence refers to the need to believe that one has acquired the necessary knowledge, skills, and behaviors to be successful in a particular social context. In an educational setting, it means feeling capable and confident in one's ability to understand and excel in the subject matter. Autonomy relates to the need for a sense of control and independence over one's own knowledge and actions. It involves having the freedom to make choices, set goals, and take ownership of one's learning process. Relatedness pertains to the need to feel connected and a sense of belonging to a social group within a given context. In an educational context, it means feeling part of a community or group of learners. It emphasizes the importance of social interactions, supportive relationships, and a sense of being valued within the learning environment (Talbert, 2017).

Talbert (2017) argues that, ideally, the students are preferred to be intrinsically motivated by the subjects they are being taught in, or at least have them to experience autonomous motivation. In other words, teachers want students to be "engaged" in their work. Henceforth, the aim is to consider what kinds of course designs provide a learning environment that promote the satisfaction of the above mentioned needs for competence, autonomy, and relatedness (Talbert, 2017).

The flipped classroom method uniquely aligns with the principles of SDT by fostering an environment conducive to intrinsic motivation and autonomy. This method involves students exploring content independently before in-class sessions, providing a flexible learning framework. The choice and control granted to students inherently support autonomy, a key component of SDT. In a flipped classroom, students are free to make decisions about when, where, and how they interact with the learning materials. Moreover, the design of pre-class materials is paramount in supporting competence, another fundamental need outlined in SDT. Clear, well-structured, and goal-aligned materials help students feel competent, as they can better understand and grasp the subject matter. Incorporating assessments or quizzes related to these materials further reinforces this sense of competence. When students can assess their understanding and progress, it builds their confidence and motivates them to engage deeper with the content. In addition to autonomy and competence, the flipped classroom addresses the need for relatedness (Talbert, 2017). This instructional approach transforms the traditional classroom into a space for collaboration and interaction. By emphasizing group discussions, peer feedback, and teamwork during in-person sessions, students experience a sense of belonging and community. This feeling of being connected and valued within a learning environment is crucial for fostering positive educational experiences and is a core component of SDT.

In summary, SDT provides the theoretical foundation for understanding why students might be motivated to engage with pre-class materials and take control of their learning in a flipped classroom. It emphasizes the significance of intrinsic motivation, autonomy, competence, and relatedness, all of which are integral components of student engagement and success within this instructional approach.

4.2 Cognitive Load Theory

The second theoretical framework for this master's thesis is *Cognitive Load Theory*, which will be abbreviated CLT throughout the rest of this master's thesis. CLT is a concept often used in psychology and education to understand how the human mind manages and processes information. The core idea of cognitive load refers to the amount of mental effort or mental 'work' that an individual's working memory must undertake to process and make sense of information (Talbert, 2017). In this master's thesis, the insights and application of CLT, especially in the context of instructional design and the flipped classroom method, are informed substantially by the work of Talbert (2017). Talbert's interpretation provides a contemporary perspective that is particularly relevant to the flipped classroom approach being examined here.

Based on CLT, learning tasks can have three forms of cognitive load, namely *intrinsic load*, *extrinsic load*, and *germane load*. Intrinsic load is the cognitive load that is inherent to a task or the complexity of the information itself. Some tasks or content are naturally more complex and, therefore, have a higher intrinsic cognitive load. For example, solving a complex mathematical problem will inherently have a higher cognitive load compared to a simple arithmetic task. Extrinsic load, on the other hand, refers to the cognitive load imposed by the instructional design, materials, or methods rather than the content itself. For example, poorly designed instructional materials or overly complex presentation can add to the cognitive load and make learning more challenging. In effective teaching, efforts are made to minimize extrinsic load, allowing learners to focus on the content. Germane load is the mental effort required to understand and integrate new information into existing mental frameworks, helping individuals learn and build their knowledge (Talbert, 2017).

In terms of a flipped classroom, students engage with pre-class materials independently, such as watching video lectures or reading texts. This is where intrinsic cognitive load is significant because students are processing new information and trying to grasp the core concepts before the in-class activities. Moreover, the design of pre-class materials and activities is critical; if the materials are confusing, disorganized, or overly complex, students will experience a high extrinsic load. Therefore, the aim is to carefully design pre-class resources to minimize extrinsic load and make it easier for students to understand the content. Nevertheless, germane cognitive load can be seen as the mental effort students put into making connections between the pre-class materials and the in-class activities. When students actively engage with the pre-class content and then collaborate in the in-class activities to apply that knowledge, they are constructing a deeper understanding of the subject matter, which aligns with the germane load.

In summary, in a flipped classroom, intrinsic cognitive load is related to students' efforts to comprehend the new content before the in-class activities. Extrinsic load is linked to how well the pre-class materials are designed and whether they create unnecessary mental strain. Finally, germane load is associated with the mental effort put into connecting pre-class and in-class activities, leading to the formation of lasting mental schemas and deeper learning.

4.3 Bloom's Taxonomy

Bloom's Taxonomy, established by Benjamin S. Bloom and his colleagues in 1956, stands as a pivotal framework in education, offering a systematic categorization of educational goals and objectives. Since its introduction, it has been instrumental in enabling educators to create learning experiences tailored to various cognitive levels. This taxonomy transcends from being merely a tool for classifying educational aims, as it also serves as a comprehensive guide for designing learning activities that engage and challenge students at different stages of understanding and application. In a flipped classroom setting, this taxonomy becomes particularly vital, promoting active learning and deeper engagement both independently and within the classroom.

Originally, Bloom's Taxonomy concentrated on the 'Cognitive Domain,' organizing cognitive processes into six hierarchical levels, each signifying distinct intellectual capacities. In this master's thesis, the interpretation and application of Bloom's Taxonomy within the cognitive domain are significantly influenced by the work of Krathwohl (2002). Their contributions offer a modernized perspective and an expansion of Bloom's initial model, aligning it with contemporary educational practices and insights. The cognitive domain levels are as follows:

1. Knowledge: This level focused on the ability to recall or recognize facts and information.

2. Comprehension: This level involved understanding the meaning of information and being able to explain or interpret it.

3. Application: Students at this level could apply their knowledge and understanding to solve problems or carry out tasks.

4. Analysis: This level required breaking down information into its component parts and understanding the relationships between those parts.

5. Synthesis: At this level, students could combine elements to form a new whole and generate creative ideas.

6. Evaluation: This was the highest level, involving making judgments or assessments based on criteria and standards (Krathwohl, 2002).

This hierarchical framework is crucial in understanding how educational objectives evolve from simple to complex cognitive processes (Krathwohl, 2002). However, the Revised Taxonomy, elaborated by Krathwohl (2002), presents a two-dimensional framework comprising Knowledge and Cognitive Processes. Krathwohl (2002) presents that the Knowledge dimension retains four main categories, including a new category called Metacognitive Knowledge:

- *Factual Knowledge*: This category pertains to the basic elements that students need to know to grasp a specific subject or problem.

- *Conceptual Knowledge*: This category focuses on understanding the interrelationships between basic elements and their organization within a broader structure.

- *Procedural Knowledge*: This category addresses the practical knowledge of how to perform specific tasks, use skills, and apply methods.

- *Metacognitive Knowledge*: A significant addition to the Knowledge dimension and which deals with knowledge related to cognition itself, including awareness of one's own cognitive processes (Krathwohl, 2002).

The Cognitive Process dimension, as emphasized by Krathwohl (2002), features category renamings of the original taxonomy and the use of verb forms, as depicted in

Figure 4.3.1.

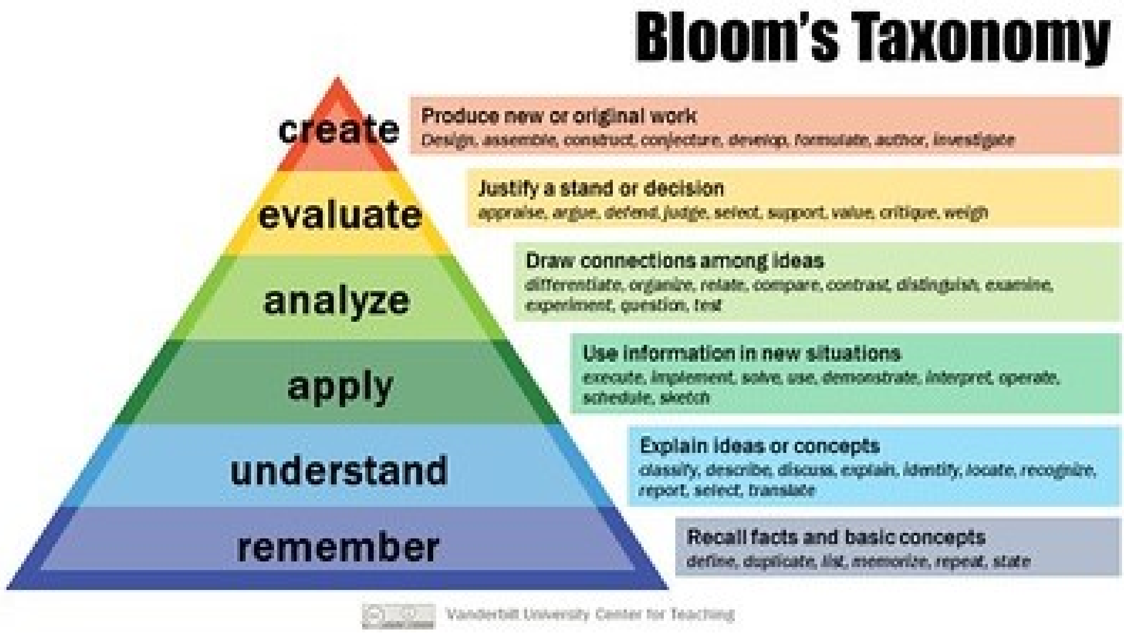

Figure 4.3.1: *Bloom's Taxonomy Pyramid (Armstrong, 2010)*

So, while the Cognitive Process dimension focuses on what students do with the knowledge, the Knowledge dimension deals with different types of knowledge that students can possess, including factual, conceptual, procedural, and metacognitive knowledge. The four categories within the Knowledge dimension provide a more comprehensive understanding of the knowledge aspects involved in the learning process (Krathwohl, 2002).

Applying Bloom's Taxonomy in the context of a flipped classroom can enhance the effectiveness of this teaching approach by promoting active and higher-order thinking among students. Here is how I can apply the theory:

1. Pre-Class Activities (Lower Levels of Bloom's Taxonomy):

    - **Remember:** Assign a short video or reading material about a new mathematical concept. Follow this with a simple quiz to help students recall key terms and basic facts.

    - **Understand:** Encourage students to summarize or outline the pre-class materials to ensure they comprehend the content. For example, after engaging with the pre-class materials, students can complete quizzes or answer reflective questions designed to assess their comprehension. These could be multiple-choice questions, short answer questions, or even open-ended questions that prompt students to explain concepts in their own words. The aim is to ensure that they have not just memorized the content but also understood it.

2. In-Class Activities (Higher Levels of Bloom's Taxonomy):

    - **Apply:** During in-class sessions, facilitate discussions, group activities, or problem-solving exercises that require students to apply the mathematical concepts they have learned.

    - **Analyze:** Encourage students to analyze real-world scenarios or case studies related to the topic. Ask them to break down complex issues into smaller components.

    - **Evaluate:** Have students critically assess information, arguments, or solutions. Engage them in debates to foster evaluation skills. For example, they could compare and contrast various approaches to solving a problem, justifying their preferences based on logical reasoning.

    - **Create:** In this phase, encourage students to create projects, presentations, or solutions that showcase their understanding of the topic. This could include designing experiments, developing solutions to problems, or creating multimedia presentations. For example, ask students to develop a short presentation or a poster where they explain a mathematical concept in a creative way, possibly relating it to real-life scenarios.

Furthermore, in the context of a flipped mathematics classroom, Bloom's Taxonomy not only shapes the structure of learning activities but also profoundly influences the design of assessments. These assessments move beyond mere recollection, urging students to engage with mathematical concepts at deeper cognitive levels. For example, quizzes and tests can encompass not just straightforward recall of pre-class material but also challenge students to elucidate mathematical concepts in more complex scenarios.

Analytical skills can be evaluated through case studies and discussions, perhaps based on real data. Students might be required to dissect this information and present their analysis, possibly as a written report or an oral presentation for their peers. This approach not only assesses their understanding but also enhances their ability to communicate complex ideas. Moreover, evaluative skills can be honed through debates where students critically assess a mathematical concept or methodology. These assessments demand logical reasoning and evidence-based arguments, fostering a deeper level of engagement and critical thinking. For the highest order of Bloom's Taxonomy, creation, students are encouraged to integrate multiple mathematical concepts into comprehensive projects. This could involve creating educational content like video tutorials, or undertaking small-scale research projects. Such tasks not only assess students' understanding but also their ability to synthesize and innovate.

In parallel to these cognitive assessments, the flipped classroom also places a significant emphasis on metacognitive knowledge. This involves encouraging students to reflect on their own learning processes, thereby enhancing their understanding and application of mathematical concepts. One effective method is reflective journaling, where students are encouraged to document not just what they learned, but also how they learned it. This reflection includes thoughts on pre-class materials, in-class activities, and the application of knowledge in different scenarios. Such practices help students gain insights into their learning preferences and areas that need improvement.

In summary, the flipped classroom, underpinned by Bloom's Taxonomy, transcends traditional educational paradigms. It fosters a learning environment where students are not mere passive recipients of knowledge but active participants in a journey that encompasses understanding, applying, analyzing, evaluating, creating, and reflecting upon mathematical concepts.

4.4 Zone of Proximal Development

The fourth and final theoretical framework of this master's thesis is the Zone of Proximal Development, abbreviated as ZPD. The ZPD is a concept that has garnered significant attention in the fields of education and cognitive development. In this section, I delve into the essence of ZPD, combining insights from Fani and Ghaemi (2011) to provide a comprehensive view of its definition, historical evolution, importance in education, and cognitive development.

For starters, Vygotsky's definition of ZPD is elucidated as:

> *the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers* (Vygotsky, 1978) [p. 89].

This definition, framed with students in mind, also acknowledges the role of "adult guidance" as a crucial element in the ZPD (Fani and Ghaemi, 2011). Furthermore, it recognizes the influence of more capable peers in providing the necessary assistance for a student's development. This assistance, commonly referred to as 'scaffolding,' is instrumental in helping learners bridge the gap between their current level of understanding and their potential developmental level (Fani and Ghaemi, 2011).

Vygotsky argued that it was insufficient to understand student's developmental level based solely on their independent performance (Vygotsky, 1978). Instead, he proposed that one should also consider the upper boundary of development, defined by the kind of tasks that students could accomplish with the assistance of someone more knowledgeable (Vygotsky, 1978). This upper boundary became what he termed the Zone of Proximal Development (ZPD). The ZPD reflects the difference between what a student can achieve alone and what they can attain with guidance and collaboration from a more capable individual. Importantly, Vygotsky's critical insight was that instruction and learning were not merely following development but rather paving the way for development to follow (Fani and Ghaemi, 2011). Fani and Ghaemi (2011) have thus highlighted the paramount importance of ZPD in education and cognitive development. As they discuss, ZPD serves as a scaffold for learners to progress and broaden their knowledge base, promoting a socio-cultural approach to learning.

Now that the essential concepts of ZPD have been explored, it is time to examine how this theory can be applied within the context of the flipped classroom method. My research focuses on whether the flipped classroom can optimize students' mathematical qualifications in a specific mathematics classroom, and here is how ZPD plays a pivotal role in this endeavor. Since students in a flipped classroom are introduced to new content independently before coming to class, the classroom becomes a dynamic space for engagement, interaction, and collaboration. This restructuring of learning activities presents a unique opportunity to integrate ZPD effectively.

Vygotsky's ZPD concept posits that learning and development are not restricted to individual capabilities but can be significantly enhanced through collaborative interactions with more knowledgeable others. In the context of a flipped classroom, these 'more knowledgeable others' can be both the instructor and peers. As students engage with pre-class materials and arrive with varying levels of understanding, ZPD comes into play. When students are working together on complex mathematical problems, they are not only consolidating their own understanding but also actively participating in the ZPD of their peers.

In the flipped classroom, the instructor embodies Vygotsky's concept of 'adult guidance' by adopting the role of a facilitator, acknowledging the diverse readiness levels of students and providing tailored support. This approach resonates with Alison King's idea of the instructor as a 'guide on the side,' where the teacher's role shifts from being the primary knowledge dispenser to a supportive guide (King, 1993). This facilitates a learning environment where students actively engage in constructing knowledge, which is in line with Vygotsky's emphasis on the ZPD and King's advocacy for interactive, student-centered learning. The flipped classroom thus becomes a practical application of these educational theories, emphasizing collaborative problem-solving and active engagement in learning.

4.5 Hypotheses

After conducting a review of relevant literature in Chapter 3 and constructing a robust theoretical framework, I have developed a profound understanding of the dynamics of flipped classrooms and their potential influence on students' mathematical qualifications. In this context, I have formulated a set of hypotheses that will guide this master's thesis in evaluating the effectiveness of the flipped classroom method. Each hypothesis is introduced, accompanied by a rationale and theoretical foundation, and is aligned with the research objectives. The expected outcomes and potential contributions of each hypothesis are also discussed.

- **Main Hypothesis**: The active participation of students in in-class activities in the flipped classroom significantly enhances their mathematical qualifications.

The main hypothesis suggests that implementing flipped classrooms can significantly enhance students' mathematical qualifications. It serves as the core research question, aiming to determine if adopting flipped classrooms has a noticeable impact on students' mathematical proficiency. This hypothesis is rooted in SDT, which emphasizes the importance of intrinsic motivation, autonomy, competence, and relatedness. In a flipped classroom, students' active participation during in-class activities aligns with SDT principles, potentially boosting their mathematical skills through increased intrinsic motivation, competence, and a sense of belonging within the learning community. If this hypothesis is supported within the context of the specific class studied, it may suggest that active learning strategies in flipped classrooms can be effective for enhancing mathematical qualifications in similar educational settings. While these findings may not be universally applicable, they could offer valuable insights for educators in similar upper secondary school environments or with similar student demographics.

The subsequent sub-hypotheses delve into specific facets of this central hypothesis, dissecting the nuanced elements of student motivation, cognitive load, cognitive processes, and the ZPD.

- **Sub-Hypothesis 1 (Motivation)**: Students exposed to flipped classrooms exhibit higher intrinsic motivation than those in traditional classrooms.
  - Rationale: This sub-hypothesis delves into the domain of student motivation. It aims to explore whether the flipped classroom model engenders a heightened level of intrinsic motivation among students. Enhanced motivation is considered a precursor to the overall improvement of mathematical qualifications, as it drives active participation and engagement. Positive findings in this specific class context could indicate the potential benefits of intrinsic motivation in similar educational settings. This could encourage educators in upper secondary schools to consider more student-driven approaches.

- **Sub-Hypothesis 2 (Cognitive Load)**: Flipped classrooms lead to a reduction in extrinsic cognitive load, allowing students to allocate cognitive resources more effectively to mathematical learning.
  - Rationale: Cognitive load theory provides the framework for this hypothesis, examining the effects of flipped classrooms on cognitive load. It focuses on the reduction of extrinsic cognitive load, a critical element that can facilitate improved mathematical qualifications. By optimizing cognitive resources, students may better comprehend and retain mathematical content. Confirmation of this hypothesis in the context of this master's thesis could highlight the role of instructional design in managing cognitive load for a similar student demographic. This might inspire other educators to explore and adapt their instructional materials accordingly.
- **Sub-Hypothesis 3 (Cognitive Processes)**: Flipped classrooms promote higher-order cognitive processes, leading to improved mathematical problem-solving abilities.
  - Rationale: This sub-hypothesis investigates the cognitive processes involved in learning and how flipped classrooms can enhance them. The development of higher-order thinking skills and advanced problem-solving abilities is expected to contribute to improved mathematical qualifications. If validated within the specific class, this hypothesis could suggest the effectiveness of flipped classrooms in promoting higher-order cognitive processes in a similar educational context.
- **Sub-Hypothesis 4 (ZPD)**: The flipped classroom model effectively utilizes the ZPD to provide targeted support and enhance students' mathematical learning.
  - Rationale: Drawing on Vygotsky's ZPD, this sub-hypothesis examines the instructional approach of flipped classrooms. It explores how scaffolding and collaborative learning within the flipped classroom context foster the development of mathematical qualifications by providing the necessary support. Postive findings for this hypothesis in the context of my master's thesis might reinforce the value of tailored instructional strategies using the ZPD concept in similar educational settings.

In summary, the main hypothesis and its corresponding sub-hypotheses present a comprehensive framework for the investigation. These hypotheses are designed to collectively address the central research question, forming a cohesive narrative that guides the study on the impact of flipped classrooms on students' mathematical qualifications.

5 From Theory to Practice

As we transition from the exploration of theoretical frameworks to their practical applications in this chapter, reflecting on the foundational theories discussed previously is crucial. The integration of SDT, CLT, Bloom's Taxonomy, and the ZPD has shed light on the multifaceted dynamics of flipped classrooms in mathematics education. These theories have provided valuable insights into student motivation, cognitive challenges, skill development, and the support needed for effective group learning. This chapter aims to bridge the gap between these theoretical underpinnings and their real-world application in classroom settings. It outlines the methodologies, strategies, and tools chosen for this exploration, demonstrating how these principles can be actualized to enhance students' mathematical qualifications. In alignment with the data collection methods outlined in Chapter 2, this chapter elaborates on the application of these methods for data gathering and analysis, thus linking theory with the practical aspects of my master's thesis.

5.1 Operational Definitions and Method of Inquiry

As presented in Chapter 2, in the realm of qualitative research, the understanding of constructs or phenomena is anchored in the lived experiences and perceptions of participants. As such, rather than relying on quantitative metrics or scales, I operationalize the hypotheses, presented in Section 4.5, through rich, descriptive definitions that capture the essence of what I am investigating. These operational definitions guide my inquiry, ensuring that the data collection tools are congruent with the facets of the hypotheses I aim to explore. To unpack the layers of each hypothesis, I have identified specific methods of inquiry. These methods — framed as open-ended questions or prompts — seek to solicit detailed and reflective responses from participants. By drawing out their experiences, feelings, and perceptions related to the flipped classroom and its potential impacts, I aim to construct a nuanced understanding of the core research areas.

Below, each hypothesis is paired with its operational definition and the corresponding method of inquiry:

- Main Hypothesis:
  - **Operational Definition**: Investigate students' perceptions of their own mathematical qualifications and how active participation in the flipped classroom contributes to these perceptions.
  - **Method of Inquiry**: Exploring students' experiences, feelings, and thoughts about the flipped classroom through open-ended questionnaires and follow-up interviews. This is done through, for example, asking questions related to how they feel the flipped classroom has impacted their mathematical abilities and confidence.
- Sub-Hypothesis 1 (Motivation):
  - **Operational Definition**: Delving into students' intrinsic motivation levels in relation to their learning experiences in the flipped classroom.
  - **Method of Inquiry**: Here I pose questions that help understand their motivation levels such as, "How do you feel about learning mathematics in a flipped classroom?" or "What drives you to engage in in-class activities?"
- Sub-Hypothesis 2 (Cognitive Load):
  - **Operational Definition**: Exploring students' feelings and experiences about cognitive load and how the flipped classroom model affects this load.
  - **Method of Inquiry**: In this case, questions might include, "How do you manage your study time in the flipped classroom?" or "Do you feel overwhelmed with the amount of content you need to self-study in the flipped classroom format?"
- Sub-Hypothesis 3 (Cognitive Processes):
  - **Operational Definition**: Understanding the depth and nature of cognitive processes students undergo while engaging in the flipped classroom.
  - **Method of Inquiry**: Here I ask questions related to problem-solving experiences, such as, "Describe a time when you felt challenged in class and how you approached the problem?" or "How does the flipped classroom approach help you think differently about mathematical problems?"
- Sub-Hypothesis 4 (ZPD):
  - **Operational Definition**: Investigating how the flipped classroom model provides support to students, potentially fitting within their ZPD.

  - **Method of Inquiry**: In this case, I ask questions like "How do in-class activities support your learning?" or "Describe a time when collaborative activities helped you understand a complex topic better."

5.2 From Pre-class Preparation to In-class Application

This section delves into the specific design choices made to implement the flipped classroom for this research. I explore the resources and activities assigned for pre-class preparations, the nature of in-class collaborative tasks, and the mechanisms in place for summative reflections, each tailored to optimize student engagement and learning outcomes.

5.2.1 Flipped Classroom in Class 1.a

This master's thesis delves into the application of the flipped classroom model in teaching descriptive statistics, focusing on Class 1.a from Aalborghus Gymnasium. As mentioned in Chapter 2, the selection of this class was a fortunate outcome, as it was the only one available following a search for willing participants. The students in this class have opted for a curriculum with a strong emphasis on social sciences and languages, while also studying mathematics at B-level. This dynamic poses interesting questions about how the flipped classroom approach can be effective in a setting where mathematics might not be the primary interest of the students.

Before implementing the flipped classroom model in Class 1.a, it was essential to establish a foundational understanding among the students about the purpose and nature of the study. To achieve this, I introduced myself to the class and presented the objectives and significance of my master's thesis, clarifying the role that the students would play in this master's thesis. This introductory session was critical to ensure that the students were aware of the master's thesis' context and their contribution to it. Accompanying this introduction were PowerPoint slides, which were used to effectively communicate the goals of the research and the methodology of the flipped classroom approach. These slides are an integral part of the master's thesis preparatory phase and are available for review in the supplementary materials. For those interested in exploring these presentation details, the PowerPoint slides, along with other educational resources, are compiled in the zip file, Appendices, as PDF that accompanies this master's thesis.

Following this initial introduction, this master's thesis delves into the application of the flipped classroom model in teaching descriptive statistics to Class 1.a. In their exploration of descriptive statistics, the students in Class 1.a will engage with a variety of key topics, vital for a foundational understanding of the subject. These include Middelværdi/middeltal/gennemsnit (Mean/Average), Kvartilsættet (Quartile Set) comprising the lower quartile, median, upper quartile, and the extended quartile set, Boksplot (Box Plot), Kvartilbredde og Variationsbredde (Quartile and Range Width), Outlier analysis, Venstreskæv og Højreskæv (Left-skewed and Right-skewed Distributions), Hyppighed, Frekvens, Kumuleret Frekvens (Frequency, Relative Frequency, Cumulative Frequency), Intervalhyppighed, Intervalfrekvens, Kumuleret Intervalfrekvens (Interval Frequency, Interval Relative Frequency, Cumulative Interval Frequency), and Sumkurve og Fraktiler (Cumulative Curve and Percentiles). This is the curriculum the students are required to complete, covering essential aspects of descriptive statistics that are integral to their overall academic development.

Nevertheless, an integral part of the students' curriculum in descriptive statistics involves learning to work with Maple, a software tool for mathematical computation. This includes, among others, teaching them Maple functions like data handling, creating box plots, and importing large datasets from Excel to Maple. These resources are designed to equip students with practical skills in using Maple for their statistical analysis, complementing their theoretical understanding of the subject.

5.2.2 Pre-class Preparation

This master's thesis centralizes on the flipped classroom model implemented in Class 1.a at Aalborghus Gymnasium. This approach, pivoting the initial learning phase outside the classroom, allows for more interactive and application-focused sessions during class time. It is noteworthy that the actual implementation, including the creation of learning materials, was undertaken by the class teacher, not by myself as the author. In this particular execution, the teacher of Class 1.a has played an instrumental role in creating initial learning materials, primarily through a series of short, custom-made videos available on YouTube. These videos, tailored to the students' operating systems (Mac or Windows), guide them through various Maple functions and form the crux of content delivery, supplementing traditional teaching methods. The creation of these short videos, each culminating at no more than 15 minutes, is rooted in CLT. This theory emphasizes the importance of managing cognitive load for effective learning. By compartmentalizing information into digestible segments, these videos aim to minimize extrinsic load, thereby enhancing students' capacity to process and understand new information.

In complement to video viewing, students are tasked with answering a limited set of questions – a maximum of three. These queries, formulated by the teacher, are designed to foster active engagement with the material. This strategy is pivotal for deeper processing of the video content, and it sets the stage for more concentrated and fruitful discussions during in-class sessions. Encouraging critical thinking and reflection, these tasks align with managing intrinsic load and ensure that students are not just passive receivers but active participants in the learning process. These activities predominantly cater to the **remembering** and **understanding** levels of Bloom's Taxonomy, forming a foundation for recalling and comprehending content. This groundwork is essential for facilitating more advanced, interactive learning activities in the classroom, thus enabling progression towards higher-order cognitive skills.

As discussed in Section 4.1, the maintenance of students' intrinsic motivation is contingent upon satisfying three cognitive needs:

1. **Autonomy**: Students exercise control over their learning by engaging with educational content, like videos, at their own pace and place, which can be especially motivating and aligns with the autonomy component of SDT.

2. **Competence**: The pre-class activities, including watching short videos and answering questions, prepare students for more complex in-class discussions and activities, enhancing their sense of competence.

3. **Relatedness**: The collaborative and interactive elements of the in-class sessions, such as group discussions and problem-solving, cater to this need and will be further elaborated in the subsequent subsection.

The decision to create unique videos stemmed from a desire to provide material that closely aligns with the specific learning objectives of Class 1.a and addresses the unique aspects of their curriculum. For those interested in a deeper exploration of the educational resources used in this study, the zip file, Appendices, accompanying the master's thesis includes links to the teacher-created videos and the designed student questions.

5.2.3 In-class Activities

The in-class sessions for Class 1.a commence with a dedicated phase for discussion and reflection, building upon the knowledge gained from pre-class preparations. Each lesson in the flipped classroom setting starts with a variety of interactive exercises. Sometimes, students engage in 'walk-and-talk' activities, where they discuss the pre-class content while moving around, which aids in energizing the learning environment. At other times, students work in small groups on 'exercises of the day,' with each group utilizing a whiteboard to facilitate collaboration and discussion. This approach encourages active participation and allows for a visual representation of their thought processes. Additionally, there are occasions where they engage in focused discussions in pairs, allowing for in-depth exploration of the topics. This structured discussion time enables students to share their understanding, clarify doubts, and engage in meaningful dialogues about new concepts. This method not only reinforces comprehension but also cultivates a collaborative learning environment where ideas and interpretations are freely exchanged.

Following these initial discussions, the class transitions to more structured activities that aim to deepen understanding and apply the concepts learned. Exercises and problems, closely aligned with the video content, are introduced to solidify understanding, enhance critical thinking, and encourage the practical application of statistical concepts. The emphasis on collaborative learning encourages students to work together, integrating insights from their discussions into these group activities. These structured activities correlate with the **applying** level of Bloom's Taxonomy, where students use information in novel ways to solve problems (see Figure 4.3.1). Additionally, during discussions and collaborative exercises, students engage in the **analyzing** process, dissecting information to explore relationships and concepts. When students assess each other's ideas, offer feedback, or make judgments based on criteria during discussions, they reach the **evaluating** level of Bloom's Taxonomy. Furthermore, the use of pair discussions and collaborative activities aligns with Vygotsky's concept of the ZPD. In this zone, learners are able to perform tasks with assistance that they could not accomplish independently. Peer discussions and guided activities provide essential support, helping students reach levels of understanding they might not achieve alone. These activities also enhance students' intrinsic motivation by fostering a sense of **relatedness** among them and with the teacher, which is a key component of motivation according to SDT.

To offer a comprehensive view of the teaching approach, all exercises, discussion questions, and materials used in each lecture are compiled in the zip file, Appendices, accompanying this master's thesis. This compilation provides an in-depth insight into the educational resources and strategies employed in the flipped classroom setting, demonstrating the strategic use of classroom time for discussion, collaboration, and application of knowledge.

5.2.4 Summative Reflections

Each session in Class 1.a typically concludes with a structured activity designed to reinforce and assess the day's learning. In the final 10 to 15 minutes of most sessions, students are presented with a quiz that I, as the author of this master's thesis, developed using Google Forms. These quizzes serve as a tool for summarizing and consolidating their understanding of the day's topics. It acts not only as a means to gauge the students' grasp of key concepts and terms but also as a reflective exercise, allowing them to review and internalize what they have learned.

However, it is important to note that in sessions where the self-study component at home was focused on using the Maple software, quizzes were not administered. This decision was made to align with the specific learning objectives of these sessions, which were more practical and application-oriented in nature. By omitting quizzes in these instances, the approach aimed to avoid overwhelming students with assessments and instead allowed them to concentrate on mastering the practical aspects of the software.

After each quiz, the teacher provides information about the next class, including details on the length and content of the upcoming videos. This preparation ensures that students are well-equipped for their pre-class activities. The structure of concluding sessions with a quiz and a preview of the next class helps maintain a consistent and focused learning trajectory.

The use of quizzes aligns with CLT, as they offer a structured method for students to summarize and reinforce the day's learning. This strategy effectively manages intrinsic load by helping students integrate new information. It also allows for paced review of key concepts, aiding in reducing cognitive overload. The selective use of quizzes, particularly in avoiding them during Maple-focused sessions, demonstrates an adaptive teaching approach, catering to the varying cognitive demands of different learning activities. As a final note, I would like to mention that comprehensive access to links for both the quizzes I developed and the students' responses, is provided. These materials are compiled into the zip file, Appendices, which will be submitted alongside this master's thesis.

5.3 Data Collection: Timing and Tools

For the purpose of understanding the depth and breadth of students' experiences and perceptions related to the flipped classroom method, a qualitative approach encompassing open-ended questionnaires and interviews was employed. Here is an overview of when and how data was collected:

- **Open-ended Questionnaires**
  - *Before Implementation*: Prior to implementing the flipped classroom method, students were given open-ended questionnaires. These were designed to capture their initial perceptions, expectations, and baseline experiences concerning their mathematical learning process. For readers interested in the specifics of these pre-implementation questionnaires, they are included in the supplementary materials. These questionnaires, along with the students' answers, can be found in the zip file, Appendices, that accompanies this master's thesis.
  - *After Implementation*: Upon concluding the flipped classroom phase, the post-questionnaires were carefully constructed with a specific focus on the hypotheses established in this study. These questionnaires, while similar in

structure to the pre-intervention ones, were modified to enable a targeted assessment of the reflections and outcomes pertinent to my hypotheses from Section 4.5. This facilitated a comparison of students' perceptions pre- and post-intervention, aiding in assessing the impact of the flipped classroom approach. For a detailed review of this post-intervention questionnaire, it is included in the supplementary materials. Readers can find this questionnaire, along with the students' answers, in the zip file, Appendices, accompanying this master's thesis.

- **Qualitative Interviews**
  - *Pre-Implementation Interviews*: After the initial questionnaires were completed, selected students were interviewed to delve deeper into their baseline perspectives. These conversations offered richer context, fleshing out the written responses and capturing the nuances of their expectations and apprehensions. It is important to note that participation in these interviews was voluntary, and some students opted out, respecting their preference for not engaging in this aspect of the study. The transcriptions of these interviews, offering an in-depth look into the students' initial perspectives, are included in the supplementary materials. Readers can access these transcriptions in the zip file, Appendices, that accompanies this master's thesis.
  - *Post-Implementation Interviews*: Similarly, after the post-implementation questionnaires, a round of interviews was conducted. This provided a platform for students to elaborate on their experiences, share anecdotes, and discuss the challenges and triumphs they encountered during the flipped classroom phase. Once again, for those interested in these detailed accounts, the transcriptions can be found in the supplementary materials. They are included in the zip file, Appendices, that accompanies this master's thesis, providing a holistic view of the students' experiences and feedback after the intervention.

By triangulating data from these sources, a thorough understanding of the students' journey —from initial expectations to final reflections — was obtained, highlighting the transformative potential and challenges of the flipped classroom method.

6 Unveiling Student Narratives

In this chapter, I dissect the implementation and impact of the flipped classroom model in a Danish upper secondary mathematics class, following a structured sequence of analysis. Initially, I start by coding the pre-implementation questionnaire data to gauge students' initial attitudes towards traditional teaching methods and their receptiveness to the flipped classroom approach. This is followed by a thick description of the pre-interviews, providing a detailed exploration of students' perspectives before the implementation. I then progress to coding the post-implementation questionnaire responses, analyzing changes and continuities in student perceptions. This analysis is complemented by a thick description of the post-interviews, delving deeper into students' experiences and reflections after experiencing the flipped classroom model.

The chapter further extends to align the findings from the post-data analysis with existing studies on the flipped classroom model, offering a comparative and contextual understanding of the results. Finally, I compare the insights from the pre- and post-data analyses, drawing conclusions on the flipped classroom model's effectiveness in enhancing mathematical qualifications, student engagement, and overall educational experience in the context of Danish upper secondary education. Note: Throughout the analysis chapter, 'S1', 'S2', etc., refer to 'Student 1', 'Student 2', and so on, respectively.

6.1 Coding of Pre-Questionnaire Data

This section details the process undertaken for coding the pre-implementation questionnaire data, gathered from students before the application of the flipped classroom method in 1.a. In this analysis, I delve into the students' perspectives on their current experiences in mathematics classes. The focus is to gauge the level of satisfaction among students with the existing teaching methods. To do this, I employ a thematic coding approach, as presented in Chapter 2. Initially, I read through all responses to familiarize myself with the students' perspectives. Subsequently, a set of codes was developed to categorize these perspectives into distinct themes. These codes were meticulously applied to each response, enabling a systematic analysis. Moreover, the coding was performed manually, ensuring a nuanced understanding of the data.

The codes identified from the students' responses are presented one by one, elaborating on each one of them before moving to the next code.

- **Current Learning Experience**: This code encompasses students' overall sentiments towards their mathematics classes. For instance, one student stated, "I think the mathematics classes are generally fine [...]" (S4), reflecting a general satisfaction with the current setup.

Several students express overall satisfaction with the current teaching methods, mentioning aspects like the effectiveness of the lessons, enjoying certain teaching aspects, for example solving problems in groups or the structured approach, and feeling that they are learning and benefiting from the classes. For example, S2 wrote "I think the mathematics classes work well and I get something out of the classes every time." This expresses satisfaction with the mathematics classes, feeling that they are productive and beneficial. S4 expressed "I think the mathematics classes are generally fine. I like that we get explained how to solve the assignments before we have to do them," appreciating the structured approach where concepts are explained before solving problems. In addition to that, S6 wrote "I think mathematics is okay, it is great that there is a structure laid out on the board for the block." In other words, S6 likes the structured format of the lessons with a clear outline on the board.

However, Some students have mixed feelings about the current setup. They appreciate certain aspects of the teaching but also point out areas that could be improved, such as reducing long lecture periods, incorporating more engaging methods like group work, or addressing the repetitiveness of lessons. For example, S5 expresses "Sometimes I think it becomes very drawn-out when the teacher just talks and we have to sit still and just listen. So if there was more group work and less board teaching, it would be better." This indicates that S5 appreciates aspects of teaching but suggests reducing long lectures and incorporating more group work. Another student, namely S13, expresses that "the classes become very similar, as it is just theory, tasks for the theory, new theory, and then tasks for the theory again, so much the same." S13 acknowledges a repetitive pattern in the lessons, suggesting a desire for more variety in teaching methods.

In terms of SDT, the students' satisfaction with structured approaches aligns with competence, as students feel capable and effective in their learning. Moreover, the desire for more group work (S5) and variety (S13) could be seen as a need for autonomy, where students seek more control and choice in their learning. However, students' familiarity with traditional teaching methods might influence their perception of autonomy. While some students express a desire for more group work and varied teaching methods (indicating a need for autonomy and control in their learning process), their familiarity with traditional methods might limit their ability to envision or feel comfortable with alternative approaches. This can lead to a preference for the known structure and predictability of traditional methods, which they may associate with competence and effectiveness. Students may feel more competent with traditional methods simply because they are accustomed to them. This familiarity can create a sense of security and competence, as they know what to expect and how to navigate the learning environment. Introducing new methods could initially challenge this sense of competence until they adapt and develop proficiency in these new methods.

Nevertheless, in terms of CLT, it might seem that the students' satisfaction with structured and clear teaching approaches also suggests that these methods help manage cognitive load effectively, making it easier for students to process and understand complex information. Yet again, traditional teaching methods, being familiar, may impose less cognitive load on students as they are well-versed in how to engage with these methods. This familiarity can make it easier for them to process and understand information, as they are not simultaneously grappling with unfamiliar teaching styles.

- **Learning Challenges**: This code is applied to responses indicating difficulties faced in learning. A student mentioned, "it is mostly when I have to do assignments alone that I do not really get started," (S1) highlighting challenges in self-paced tasks.

Several students mention specific challenges they face with the current teaching method. These include difficulties in understanding material presented on the board, becoming disengaged or bored during long lectures, and feeling overwhelmed by the pace or volume of information. S7 responded "that I do not dare to ask questions when the theory is being explained on the board in front of the class." In this case, S7 faces the challenge of feeling hesitant to ask questions in a class setting, thus indicating a barrier in understanding the material. Another example is S12 who notes that "I can not concentrate because I cannot sit and listen to a teacher for such a long time without falling asleep." Based on this, S12 struggles with maintaining concentration during long teaching sessions, which affects their learning. In addition to that, S15 wrote "I quickly lose concentration in class or that many of us are nervous about explaining if we do not understand what the teacher is explaining on the board, in class we tend to not raise our hand even though we might not understand it." Similar to S7 and S12, S15 describes difficulties with maintaining concentration and a reluctance to ask for clarification, which hampers their learning.

In essence, these challenges underscore the need for more interactive, engaging, and student-centered teaching approaches that encourage active participation, question-asking, and cater to different learning preferences and attention spans. The feedback suggests a potential reevaluation of the teaching methods to make them more inclusive and responsive to the diverse needs of students. Thus, in terms of SDT, students who struggle to understand material or stay engaged (like S7, who hesitates to ask questions, or S12, who has difficulty concentrating) may feel less competent in their learning environment. This lack of understanding can lead to feelings of inadequacy or frustration, which in turn, affects their motivation and engagement. Moreover, when students are presented with too much information at once or teaching methods are too demanding (like long lectures without breaks), it can lead to increasing the extrinsic load, hence it makes it difficult for students to process and retain information effectively.

- **Learning Preferences and Strategies**: This code captures students' expressed preferences or strategies that they find beneficial in the learning process. For example, a student's preference for group work could indicate that they find collaborative learning environments more engaging or effective. It is about identifying the methods, environments, or approaches that students feel support their learning process. For example, a student's response, "I prefer it when we have to solve tasks in groups," indicates a preference for group work, suggesting that collaborative learning is more effective for some students.

In terms of this code, several students express a clear preference for group work, highlighting its effectiveness in their learning process. For instance, S1, S4, S5, and S13 prefer solving tasks in groups, indicating a leaning towards collaborative learning environments. This preference suggests that these students find engagement and perhaps better understanding through interaction and discussion with peers. For example, S1 wrote "I learn best by sitting in a group so that some of the other group members can elaborate more on what the different things mean for me, or they can explain the task to me so I understand it better." Moreover, S4 wrote "I think I learn best with a mix of everything. However, there should not be too much board teaching because I really feel most confident that I learn the mathematical theories when I do them practically." This indicates that S4 prefers a diverse learning approach, combining theory with practical application.

Nevertheless, S7's response highlights a preference for learning at their own pace, possibly in a less pressured environment than traditional classroom settings. This preference for a self-paced approach points towards a need for flexibility in the learning process, allowing students to digest and understand material in a manner that suits their individual pace: "I learn mathematical theories best with a bit of lecture and then working on my own. There I can take it at my own pace and thereby ask the teacher for help when I need it" (S7).

Preferences for group work (S1, S4, S5, S13) and self-paced learning (S7) align with relatedness and autonomy from SDT, as students seek collaborative environments and control over their learning pace. Moreover, their preference for group work and practical application (S4) suggests these methods help students learn within their ZPD, where they can tackle more challenging tasks with peer support.

- **Perceptions of Qualifications**: This code relates to students' views on what essential skills or knowledge they should acquire from their mathematics lessons. For example, S2 expressed, "I believe that one should learn new things often," illustrating the expectation for frequent learning of new concepts.

In case of this code, many students emphasize the importance of learning new things often in mathematics classes. This is reflected in statements like those from S2 who says, "I believe that one should learn new things often," and S16 who mentions understanding the method being taught for solving problems, indicating an emphasis on acquiring new skills and knowledge. Students like S7 and S11 express a desire for mathematics lessons to include skills that are applicable in real life. S7 wrote about wanting to learn basic mathematics for everyday life, while S11 noted the importance of understanding methods and formulas, suggesting a need for practical skills that can be applied beyond the classroom: "I would say that I just need to be able to do the basic mathematics to get through adult life. I would like to be able to calculate" (S7), "understand methods or formulas for the things we do"(S11).

Responses like those from S10 and S18 focus on solving problems and understanding problem-solving as key skills to be gained from mathematics lessons: "How to work with the tasks/solve the tasks, and being able to somehow remember the tasks for another time"(S10), "understanding problem-solving" (S18). These responses suggest a perception that mathematics education should foster critical thinking and problem-solving abilities. However, a common theme among the students is the importance of acquiring skills necessary to pass exams and meet educational standards. For example, S1 talks about learning most of the material to get through the final exam, while S6 emphasizes learning what is needed to pass: "I feel I need to learn most of it to get through the final exam in 2nd year, it is important to get everything from the lessons"(S1), "that I learn what I need to in order to pass"(S6). This reflects a practical perspective on the qualifications necessary from mathematics classes.

In summary, the students' perceptions of qualifications in mathematics education revolve around acquiring new concepts, practical application, a diverse set of mathematical skills, problem-solving abilities, and meeting educational standards through the curriculum.

Relating this to Bloom's Taxonomy, the students emphasis on problem-solving and applying methods aligns with higher-order thinking skills in Bloom's Taxonomy, suggesting that students value these advanced cognitive processes in their learning.

- **Views on Flipped Classroom**: This code identifies students' opinions or expectations about the flipped classroom method, including any perceived benefits or concerns. When a student says, "I think it is great to get the opportunity to try a different method than the one we are used to" (S4), it highlights curiosity and openness towards experimenting with the flipped classroom approach.

A number of students, like, S1, S6, S9, S10, S15, and S17, have shown interest in the flipped classroom model, finding it more engaging and conducive to independent study. However, this preference is not universal, and some students prefer traditional methods or a blend of both. S significant concern among the students was their ability to understand and retain the material prepared at home. S4 articulated apprehension about comprehending preparatory material: "A bit unsure if it will be a too big challenge if I do not understand what we have to prepare at home." Similarly, S7 was worried about remembering the theory learned independently: "It will clearly be a challenge to remember the theory [...] something one has to get used to." The transition to a flipped classroom model could pose adaptation challenges, particularly for students more accustomed to traditional teaching methods. S12's response, "I am not so interested in mathematics, so I do not know how much it will help," suggests a potential struggle in engaging with the new format. Meanwhile, S17's comment, "It might be exciting to try, but I am not convinced it is better," reflects uncertainty about the effectiveness of this new approach.

In addition to that, time management emerged as a crucial concern. S6 expressed fear about keeping up without sufficient time for homework: "If I do not have time or the opportunity to do homework, I am afraid I would not be able to keep up." This sentiment was echoed by S11, who worried about balancing multiple tasks: "It might become difficult in the long run, if I have a lot of work, homework, and submissions." S4 also showed a concern about understanding the material: "I am somewhat unsure whether it will become too big of a challenge if we do not understand what we are supposed to prepare at home (like the videos we have to watch), because then it might become too difficult when we show up and have to do tasks about it." Moreover, some students showed concerns about managing time effectively for home preparations, here among them S8 who wrote "I am a bit afraid if there are many videos to watch at home while also having many assignments in other subjects."

There are also responses that indicate a need for a balanced approach in implementing the flipped classroom model. S5 suggested alternating between traditional and flipped classroom methods: "I do not think only the flipped classroom should be used, but maybe vary between flipped classroom and normal teaching." Additionally, S10 states "If the teacher is prepared for the lessons, makes the lessons fun/cozy - but not too much. Then I am ready to learn [...] I would like to try it, as then we can do more tasks in class, and then ask the teacher about what we understand. In that way, one also gets a 'hang' of what one is doing." These responses show their inclination towards a blended learning approach. They appreciate aspects of the traditional teaching methods but also recognize the potential benefits of incorporating elements of the flipped classroom, indicating a preference for a balanced educational experience. Furthermore, S14's willingness to try the new method, "Willing to try it and excited to see the results of it," underscores the potential benefits of experimenting with different teaching styles.

The students curiosity and openness to trying the flipped classroom, here among them S15 and S17, reflect a desire for autonomy in learning. However, their concerns about understanding the material and managing time effectively highlight a need for teaching methods that support their sense of competence, another key component of SDT. These concerns suggest that while students are interested in more autonomous learning methods, they also want to feel capable and effective in their learning process. They might be apprehensive about whether they can successfully adapt to and manage the demands of a flipped classroom without adequate support. The point about students being primarily familiar with traditional teaching methods is crucial here. Their exposure to and experience with traditional classroom settings might shape their expectations and perceptions of what effective learning looks like. When introduced to a new method like the flipped classroom, they may feel uncertain or anxious about their ability to adapt, which can impact their sense of competence.

Moreover, their limited exposure to alternative teaching methods could also mean that they might not fully appreciate the potential benefits of these methods or know how to navigate them effectively. This lack of familiarity might contribute to their concerns about understanding and managing time, as they are unsure of what to expect and how to prepare. In addition to that, in terms of CLT, concerns about retaining material prepared at home, such as S4 and S7, aligns with the fact that when new teaching methods are introduced (like the flipped classroom), students might experience an increase in cognitive load initially. This is because they are not only trying to learn the content but also adjusting to a new way of learning. Over time, as they become more familiar with these new methods, the cognitive load may decrease, and these methods might become as effective, if not more so, than traditional methods.

- **Engagement and Participation**:This code reflects how students feel about their level of engagement and participation in the current class format. A response like, "I like it when we delve deep into the subject [...]"(S2), suggests that in-depth exploration of topics can foster greater student engagement.

In terms of this code, students such as S2 expressed "I like it when we delve deep into the subject on the board so I can take notes on it and remember it better." This response suggests a preference for in-depth exploration of topics, indicating that such a detailed approach engages S2 more in the learning process. Moreover, the statement by S4 "I like it best when we have to solve tasks in groups" further indicates that collaborative learning environments can enhance their engagement and participation in the class.

Nevertheless, S10 wrote "I like it best when the teacher shows us how to do the tasks and explains the subject, and then lets us work on it in class for more than 30 minutes." This response shows that S10 appreciates the combination of teacher-led explanation followed by ample time to work on tasks, suggesting that this blend of teaching and active participation increases their engagement. In addition to that, S13 wrote "I like it when all the theory is written up either on the board, or we get the notes for it, so you do not have to listen to what the teacher is saying yourself." S13's response indicates a preference for visual learning aids, such as written notes or board work, which according to them, helps them to engage more effectively with the material.

However, while these preferences are indicative of what engages these students currently, it is important to consider the role of familiarity and exposure in shaping these opinions. Students often gravitate towards teaching methods they have experienced the most. This familiarity can sometimes overshadow other effective teaching styles that they have not yet encountered. Therefore, introducing students to a broader spectrum of teaching approaches might not only expand their understanding of different learning methods but also potentially reveal new preferences. This expanded exposure could lead to a more adaptable and versatile approach to learning, accommodating diverse learning needs and preferences.

- **Suggestions for Improvement**: This code is for responses where students propose ideas or changes to improve their learning experience. For example, a student's suggestion, "One could employ a more visual method" (S18) points to a desire for more visual learning methods in teaching, indicating a way to enhance their understanding.

In terms of this code, students have suggested improvements such as shorter, more interactive segments of lectures, more frequent breaks, and incorporating various teaching methods to cater to different preferences. For instance, student S14 suggested, "[...] Moreover, the classes could be more visual," highlighting a preference for increased visual components in teaching. This inclination towards visual aids, such as diagrams or illustrative examples, aligns with strategies aimed at reducing extrinsic cognitive load. By simplifying and clarifying instructional design, these visual elements can facilitate more efficient information processing. The recommendations from students, including S14 and S12, for more visually-driven and interactive teaching methods, underscore the importance of optimizing extrinsic load to improve learning outcomes. Nevertheless, several students, like S5 and S12, suggested modifications to the teaching style. S5 stated, "Sometimes I think it gets very drawn-out when the teacher just talks and we have to sit still and just listen. So if there was more group work and less chalkboard teaching, it would be better." This implies a desire for a teaching style that is more engaging, where students are actively participating and collaborating with their peers, rather than passively listening to lengthy lectures. Similarly, S12 noted, "I think it is boring, but mostly because I have already learned the things we are currently learning. We have to listen a lot and do not get much time for tasks, so it quickly becomes boring." Here, it could indicate that S12 is essentially asking for a more tailored approach that considers the students' familiarity with the material, avoids repetition, and allocates more time for hands-on tasks and problem-solving activities.

In addition, S3 expressed a preference for the current teaching method, stating, '"Probably when we are allowed to work independently." This response suggests a fondness for a teaching approach that emphasizes independence, potentially indicating an inclination towards self-guided learning. S4, on the other hand, wrote "I like it best when we solve tasks in groups." This highlights a preference for collaborative learning environments, suggesting that group activities could make the classes more engaging and interactive.

To summarize, in analyzing students' responses from the pre-implementation questionnaires on their current experiences in mathematics classes, several themes emerged, which can be interpreted through the lenses of SDT, CLT, Bloom's Taxonomy, and ZPD. Students generally expressed satisfaction with structured teaching approaches, aligning with the competence aspect of SDT. This suggests they feel capable and effective in understanding the material. However, their familiarity with traditional methods could be limiting their perception of autonomy, as they are accustomed to a certain style of teaching. This familiarity might also mean that the cognitive load is effectively managed, as they are well-versed in engaging with these methods. However, challenges in understanding material, staying engaged during long lectures, and feeling overwhelmed by the pace or volume of information were also highlighted. These challenges underscore a need for more engaging and varied teaching approaches. According to SDT, these difficulties might be affecting students' sense of competence, impacting their motivation and engagement. Additionally, these challenges suggest that cognitive load is not optimally managed, leading to an extrinsic load that hinders effective learning.

Many students expressed a preference for collaborative learning environments and group work, aligning with the relatedness component of SDT and indicating effective learning within their ZPD. This suggests that they find engagement and perhaps better understanding through interaction and discussion with peers. Nevertheless, the preference for self-paced learning reflects a desire for autonomy in learning processes. Students also emphasized the importance of learning new concepts regularly and acquiring practical skills, aligning with higher-order cognitive processes in Bloom's Taxonomy. This indicates that students value the application of knowledge and problem-solving skills.

Regarding views on the flipped classroom, curiosity and openness to trying new methods reflect a desire for autonomy in learning. However, concerns about understanding and time management highlight a need for teaching methods that support their sense of competence. The initial increase in cognitive load when adapting to new methods like the flipped classroom might pose challenges, but over time, as students become more familiar, these methods might become more effective.

Finally, suggestions for improvements, such as incorporating more visual elements and interactive methods, indicate a need for strategies that reduce unnecessary cognitive load and enhance information processing. The next section will analyze the pre-interviews conducted with the students, elaborating on their responses from the pre-questionnaire. This analysis will present a thick description of the collective insights from all interviews, connecting them to the questionnaire themes. It aims to provide a comprehensive understanding of student perspectives on current mathematics lessons and their attitudes towards the flipped classroom method.

6.2 In-depth Analysis of Student Pre-Interviews

In this part of the master's thesis, I explore the pre-interviews conducted with students from the Danish upper secondary class, 1.a. These interviews were intended to enhance and build upon the insights gained from the previously conducted questionnaire. The questionnaire initially shed light on the students' perceptions and attitudes towards mathematics lessons and the prospective adoption of the flipped classroom approach. However, the interviews delve deeper, providing a platform for students to further clarify and expand upon their responses to the questionnaire, thereby offering a more nuanced and detailed understanding of their perspectives.

The methodology for analyzing these interviews, as mentioned in Chapter 2, centers on the concept of 'thick description.' This approach involves a detailed and interpretative presentation of the pre-interview data, emphasizing the depth and the context of each student's experiences and viewpoints. Through this nuanced analysis, I aim to capture the complexities and subtleties of the students' attitudes towards their current learning experiences, their perceptions of the qualifications they should gain from mathematics lessons, and their views on the flipped classroom method. This analysis is vital for addressing the central research question: evaluating the potential effectiveness of the flipped classroom in enhancing student qualifications in mathematics within this specific educational context.

To start, I conducted individual interviews with students to gain deeper insights into their perspectives on mathematics education. The interviews were arranged in a private setting, in either an empty classroom or a meeting room. This was made possible through the assistance of their teacher, who kindly reserved these spaces for us. Her support was instrumental in creating an environment conducive to open and honest communication, allowing the students to feel comfortable and undisturbed during our discussions. For each interview, it was just the student and me, fostering a personal and direct dialogue. The one-on-one setting allowed each student to express their thoughts and opinions freely without the influence or presence of their peers. Moreover, I used my phone to record our conversations, ensuring an accurate capture of the nuances of these discussions. This method allowed me to focus entirely on the conversation at hand, rather than being distracted by note-taking. Nevertheless, throughout these interviews, ethical research practices were maintained. Each participant was aware of the purpose of the study and the use of the recordings. Confidentiality and anonymity were upheld, respecting the privacy and comfort of each student.

6.2.1 Views on Traditional Teaching

In the beginning, an examination of students' perspectives on traditional teaching methods in mathematics revealed both appreciation and suggestions for improvement. For example, S2 seems content with the traditional method, appreciates depth and note-taking in lessons, but mentions it can become tedious if the teacher talks for too long: "Well, I am quite happy with the traditional way, so I think that them standing there and talking for a long time could be the only issue [...]." In addition, S6 appreciates the structure of traditional methods, yet indicates a need for more real-world application and a desire for shorter, more focused teacher-led sessions, as they agreed that when doing that they will find the mathematics class to be more engaging: "Yes, I actually think so. It would at least make it more exciting [...]." Another example is S10 who enjoys the clarity and direct help available in traditional teaching but suggests it could be made more engaging. S20 is also generally comfortable with traditional teaching, emphasizing the importance of initial teacher-led instruction before moving to assignments. S20 prefers this method for its clarity and immediate access to assistance: "Yes, I think that works very well, the part where you just get told what the exercises you are about to do are about [...]."

However, some students like S14 expresses a need for more support from teachers, facing challenges when assistance is not readily available. S14 feels that traditional teaching often lacks sufficient examples and individual support. They prefer a blend of teacher-led examples and independent work, suggesting more interactive and practical approaches would enhance their learning experience: "I sometimes think the amount of examples, I often feel like we get it described on the board, and then we are kind of left to ourselves[...]." This indicates that S14 struggles with independently solving problems and navigating periods of waiting for help. In addition to that, S18 prefers a more visual and interactive approach in traditional teaching. S18 appreciates examples and guided problem-solving with the teacher, finding this method more beneficial for understanding and remembering concepts: "Personally, I think it has worked very well when it was like that, we solved several exercises on the board, where we could participate together with the teacher [...]."

Nevertheless, there are also students who express a desire for variation within this format. The notion of long lectures being monotonous surfaces frequently, pointing to a need for more engaging and dynamic classroom interactions. What the students defined as a long lecture was if the teacher spoke for more than 30 minutes. For instance, S15 finds traditional lectures can be lengthy and unengaging, leading to a loss of concentration and difficulty in retaining information for later application in assignments: "Well... I think it works much better because I often feel that they can stand and talk for an hour and a half at the board, and then there is just a quarter of an hour where you are allowed to do something yourself [...]."

Overall, while the traditional teaching methodology in mathematics – characterized by lectures followed by exercises – is generally well-received. However, students express a desire for more variety and engagement in the classroom. Moreover, a common theme is the need for more engaging and dynamic classroom interactions, with several students indicating that lectures exceeding 30 minutes tend to lose their attention.

6.2.2 Adaptation to Learning Preferences

In exploring the varied learning preferences of students in mathematics education, a common thread emerges: the appreciation for both collaborative and individual learning approaches. This diversity aligns well with the ZPD theory and the concept of relatedness from SDT. The ZPD suggests that learners can achieve more with guidance and collaboration than independently, fostering a sense of relatedness crucial for motivation and engagement. In addition to the ZPD and SDT's concept of relatedness, CLT and Bloom's Taxonomy offer further insight into the diverse learning preferences in class 1.a. CLT helps in understanding the efficacy of different teaching methods in managing cognitive load, thereby enhancing learning efficiency. For example, practical applications reduce extrinsic load, allowing students to focus more on the intrinsic aspects of the subject matter. Furthermore, Bloom's Taxonomy provides a framework for understanding how these teaching methods can help students achieve higher-order thinking skills (see Figure 4.3.1). Group discussions and individual studies, for example, encourage analysis, evaluation, and creation, which are critical for deep learning and understanding in mathematics.

For example, S2 prefers a blend of individual work and small group discussions, aligning with the ZPD concept where learning is optimized through social interaction and collaboration: "Well, I like that I sit together with someone, so I can ask if there is something" (S2). This setting allows students to work within their ZPD, extending their capabilities beyond what they can achieve alone, as well as extending their capabilities and fostering a sense of relatedness, crucial for motivation and engagement. Both S5 and S6 also indicate a preference for a mix of chalkboard instruction and group work, leaning towards practical, real-world applications to make learning more relatable.

S3 enjoys working in small groups to avoid boredom and enhance understanding through discussion, while S4 believes in the power of group discussions for deeper understanding and individual attention from the teacher: "there you just get the opportunity to talk together and kind of discuss what you, what kind of answers you have come up with and... Yes, how you should solve it and stuff like that, so there I feel that you learn more because you just get to discuss it with people" (S4). These collaborative environments not only exemplify the ZPD in action but also align with Bloom's Taxonomy, where higher-order thinking skills such as analyzing, evaluating, and creating are fostered. As students engage in discussions, they are encouraged to critically analyze problems, evaluate different solutions, and collaboratively create new approaches to solve these problems.

While some students find groups more effective for deeper discussions and understanding, others, such as S10, express a preference for individual work: "I like to do it myself, so I can have a better overview of it. And then if there is something I have difficulty with, I ask the teacher." This preference stems from challenges like varying levels of understanding within the group or differences in group dynamics.

The general consensus leans towards the value of collaborative learning, with many students recognizing that discussing and working through problems with peers can significantly clarify doubts and solidify their grasp of mathematical concepts, effectively utilizing their ZPD. S3's preference for working in small groups over large ones to prevent boredom and enhance understanding ("I like working in small groups. Otherwise, I get bored.") further emphasizes the role of social interaction in learning within the ZPD framework.

Practical application of knowledge is another aspect that resonates with many students. Students like S5 and S8 express a preference for active learning methods, such as applying concepts shortly after a theory session. For example, S5 responded with "it allows me to get started right away, so I can use the knowledge I just got." Similarly, S8 expressed "it is so I can get started right away, so I can use the knowledge I just got." Similar to S5, S8 emphasizes the importance of applying knowledge soon after it is acquired. This practice of immediate application reinforces learning and prevents forgetting. S8's comments suggest a balanced approach to theory and practice is beneficial. They appreciate being able to ask questions during theory sessions and value the opportunity to immediately apply what they have learned.

Nevertheless, the balance between theoretical instruction and practical exercises is crucial. Students appreciate being able to apply what they have learned in real-world scenarios, making the subject matter more relatable and understandable. For example, S5 agreed that providing them with real-life examples could help them with understanding the concepts better: "Yes, I think so, it would make it much easier to understand too." When students, like S5, engage with real-world examples, it helps in reducing extrinsic cognitive load, which is often associated with abstract concepts. This approach makes the subject matter more relatable and understandable, as it simplifies complex theoretical content by anchoring it in familiar contexts. This strategy effectively bridges the gap between theoretical knowledge and practical application, allowing students to focus on the intrinsic cognitive load – the essential aspect of learning the material.

6.2.3 Flipped Classroom Approach

As the students navigate the initial phase of the flipped classroom model, their reactions reveal a spectrum of experiences and apprehensions, influenced in part by their familiarity with traditional teaching methods reflect their levels of autonomy, a key concept in SDT. This innovative educational approach, experienced only once or twice in a series of five sessions, represents a significant shift from the conventional classroom setting to which many students are accustomed. For example, S4's positive view of the flipped classroom, appreciating the active engagement with tasks, illustrates not just an affinity for autonomy but also a sense of competence. S4's remark, "the fact that you get to work as much as you do with the tasks instead of just having chalkboard teaching, I think is really cool," suggests a preference for a learning environment where they can actively apply and test their understanding. This inclination indicates not only a desire for self-directed learning (autonomy) but also an environment where S4 feels capable and effective in handling the learning tasks (competence). Their enthusiasm for more engaging, hands-on experiences in the flipped classroom reveals a readiness to embrace both autonomy in directing their learning and a sense of competence in mastering the material

Similarly, students like S8, who appreciating the independence and self-directed learning opportunities offered by the flipped classroom, are exercising their autonomy in a learning environment that contrasts with the more structured traditional method. S8 appreciates the opportunity for self-paced learning, a stark contrast to the traditional classroom pace: "because it is not necessarily that you just sit and go through it with a teacher along with the rest of the class. But it is that you sit with it at home yourself, so you can, for example, the videos we have received, from Joana [the teacher], we can just watch as many times as we want to understand it... in that way, it could work much better." S8's appreciation for self-paced learning, signify their desire for more control and agency in their educational experiences.

Moreover, S7's perspective on the flipped classroom highlights its impact on the learning environment, particularly in terms of relatedness, one of the core components of SDT. Their comment sheds light on how the flipped classroom provides a more private and less intimidating atmosphere, contrasting with their past experiences in traditional classroom settings. S7 shares, "it is much better because, with the traditional method... I had a teacher where it was like, you get somewhat called out in front of the class if you answer wrong or something like you really do not want to answer there, but with this method, you can just get the help to yourself." This observation underscores the importance of a supportive and non-threatening educational environment. The flipped classroom model, as experienced by S7, seems to foster a sense of safety and belonging, where students can seek help and learn without the fear of public embarrassment or judgment. This nurturing atmosphere is conducive to building relatedness, a crucial element in SDT, where students feel connected and valued within the learning community, thus enhancing their overall engagement and motivation to learn.

However, students' prior experiences with traditional methods shape their concerns in the flipped classroom, particularly in terms of understanding and managing time. S2, for example, feels pressured mainly due to a heavy workload, a challenge that stems from managing responsibilities both in and outside the traditional classroom context: "I feel it has been a bit pressured lately. Now there are so many homework assignments and submissions, so it really has been 'last minute' that I just saw them and did not quite get to delve into it deeply, because I also had other things to take care of." This reflects the challenge of balancing the requirements of the flipped classroom model with the expectations set by traditional teaching methods. Additionally, S5's concerns about retaining information and S6's struggle with self-motivation further echo the challenges faced in adapting from a structured, teacher-led environment to one requiring greater personal initiative. S6's comment, "I think I find it a bit hard to concentrate when I might rather be doing something else than watching a mathematics video," underscores the adjustment required when moving away from the familiar territory of traditional teaching. In addition to that, it underscores the challenges of adjusting to a more autonomous learning environment. S6's comment about finding it hard to concentrate without the structure of a traditional classroom reflects a learning curve in adapting to the autonomy demanded by the flipped classroom.

As students are still in the early stages of experiencing the flipped classroom, their responses are likely influenced by their pre-existing comfort with traditional methods. The initial adaptation phase is critical, as it determines how well students can transition to a learning environment that requires a higher degree of autonomy. However, initial concerns might evolve as they become more accustomed to the new format, suggesting the need for educators to provide supportive strategies that bridge the gap between these distinct teaching approaches. The feedback underscores the importance of understanding students' educational backgrounds and designing flipped classroom content that is not only engaging but also aligns with their learning experiences and expectations.

6.2.4 Learning Objective and Motivation

In terms of learning objectives, students exhibit a marked preference for understanding concepts over mere memorization. This inclination resonates with Bloom's Taxonomy, which advocates progressing beyond basic recall of facts (lower-order thinking) to deeper cognitive levels such as understanding, application, analysis, evaluation, and creation (see Figure 4.3.1). Moreover, the students' preference for understanding concepts over mere memorization also aligns with the concept of competence in SDT. SDT posits that individuals are motivated by a need to feel competent and capable in their endeavors. For instance, S2 and S18's emphasis on grasping the reasoning behind mathematical methods and the importance of background knowledge align with the higher levels of Bloom's Taxonomy. They express that mere instruction on what to do without understanding the underlying reasons can be confusing and easily forgotten: "Yes, well I also think that background knowledge is important, because that is exactly how mathematics makes sense to me at least, I think. Because if I am just told, for example, to use this for that, then I do not know what I am going to use it for. It is a bit confusing, and then I can... I can easily forget it" (S18). This perspective underlines the move from mere knowledge (recalling facts) to understanding and applying concepts in different contexts. Moreover, their emphasizing of the importance of understanding the underlying reasons in mathematics rather than just being instructed on what to do, they are expressing a desire for competence - to truly grasp and apply mathematical concepts effectively.

Conversely, a few students like S5 view mathematics as primarily about remembering steps and formulas, "I think it is more like remembering it," aligning more with the lower levels of Bloom's Taxonomy focused on recall and understanding. This approach may limit the depth of learning and understanding, as it does not fully engage the higher cognitive processes encouraged in Bloom's Taxonomy. While it may lead to success in certain tasks, it does not necessarily engender a deep understanding or a sense of mastery, which are crucial for feeling competent.

Nevertheless, the variation in students' perspectives on the utility of mathematics education offers interesting insights. For instance, S2's interest in applying mathematical knowledge in higher education aligns with the higher levels of Bloom's Taxonomy, where the focus is on application and synthesis of knowledge in new contexts. This indicates a desire for competence and mastery which is a key aspect of SDT. In contrast, S3's focus on merely passing exams reflects a more utilitarian view, aligning with the lower levels of Bloom's Taxonomy that emphasize recall and basic understanding. This perspective might be rooted in extrinsic motivation, another aspect considered in SDT, where the goal is to achieve a specific outcome (in this case, passing exams) rather than mastering the subject for personal satisfaction or future application. These diverse viewpoints highlight the complexity of student motivations and learning objectives in the context of mathematics education, underscoring the need for a flipped classroom approach that can cater to these varied learning needs and aspirations.

The motivations driving students in mathematics are multifaceted, reflecting a mix of intrinsic and extrinsic factors. While some students exhibit a deep, personal interest in mathematics, seeking to understand and master the subject, others are motivated by external pressures such as the fear of falling behind, the need to achieve good grades, or expectations from teachers and parents. For many students, the primary impetus behind their engagement with mathematics stems from a fear of falling behind their peers. This sentiment is shared by students like S2, "I think it is mostly about being afraid of falling behind. Also because you feel very pressured to do homework, because then you feel like you are falling behind, you are afraid of being scolded, or that you do not understand it." This student indicates that a significant factor in their engagement with mathematics is the fear of falling behind others. The pressure of keeping up with homework and the class is a driving force behind their motivation. These factors relate to the SDT concept of controlled motivation, where actions are influenced by external pressures or rewards. For many students, the primary impetus behind their engagement with mathematics stems from a fear of falling behind their peers, indicative of extrinsic motivation driven by external factors, and which was the case of S2.

However, there were also students who were motivated due to self-interest in participation. For example, S4 says "it is probably because I want to learn it, but also to be sure about it... when you do the homework, then I also feel more confident that I have been able to understand it when I come to class... it is about making sure that you are prepared and have tried your best to understand it." S4's motivation appears to be driven by a desire for mastery rather than external factors like grades or approval. This intrinsic motivation is crucial for deep learning, as it encourages students to engage with material on a level that goes beyond surface understanding. Moreover, S4's comment about feeling more confident in class after completing homework illustrates the relationship between preparation and self-assurance in a learning environment. This confidence likely stems from a deeper understanding of the material, allowing S4 to engage more meaningfully in class discussions and activities.

In summary, this exploration of students' perspectives on mathematics learning reveals a spectrum of motivations and learning objectives. The majority exhibit a preference for deep understanding over simple memorization, aligning with higher levels of Bloom's Taxonomy and reflecting a desire for competence in SDT. This inclination towards understanding and applying mathematical concepts is crucial for the flipped classroom approach, which seeks to foster higher-order thinking and active learning. Conversely, some students, like S5, prioritize memorization that may limit deeper cognitive engagement. Meanwhile, students like S2 and S3 present varied motivations, from applying mathematical knowledge in higher education to focusing on passing exams. These insights underscore the diverse needs and aspirations of students in a mathematics classroom, reinforcing the importance of a flipped classroom methodology that can adapt to different learning styles and objectives. Moreover, the interplay of intrinsic and extrinsic motivations among students, such as S4's self-driven learning and S2's fear of falling behind, highlights the complex motivational landscape in mathematics education. This complexity supports the need for flexible and responsive teaching approaches like the flipped classroom, which can cater to a range of student motivations and learning preferences, ensuring a more inclusive and effective educational experience.

6.2.5 Summary

An overall summary of the thick description conducted in the above subsections is as follows:

1. **Traditional Teaching Views**: Students generally appreciate traditional teaching methods for their structure and clarity. However, they also express a desire for more interactive, engaging, and dynamic classroom experiences. Concerns about prolonged lectures and a lack of practical applications and frequent, suggesting a need for more variety and real-world relevance in teaching approaches.

2. **Adaptation to Learning Styles**: Students show a diverse range of learning preferences, favoring both collaborative and individual learning approaches. This diversity aligns with the ZPD and SDT's concept of relatedness. The integration of CLT and Bloom's Taxonomy provides further insights, emphasizing efficient cognitive load management and the development of higher-order thinking skills. Practical applications and real-world examples are favored for making learning more relatable and reducing extrinsic load.

3. **Flipped Classroom Approach**: In the early stages of experiencing the flipped classroom model, students' reactions are mixed and significantly influenced by their familiarity with traditional teaching methods. While some appreciate the autonomy and self-directed learning opportunities, others face challenges in time management and adapting to the new format.

4. **Learning Objectives and Motivation**: Students prefer understanding concepts over rote memorization, resonating with Bloom's Taxonomy and the SDT concept of competence. While some students are motivated by a desire to apply mathe-

matics in real-world scenarios and higher education, others are driven by extrinsic factors like the fear of falling behind or the pursuit of good grades.

In essence, these insights suggest the necessity for a mathematics curriculum that is adaptable and responsive to diverse student needs and preferences. Such a curriculum should blend theoretical instruction with practical application, foster independent and collaborative learning opportunities, and address the various motivational factors influencing students' engagement with mathematics.

When juxtaposing the findings from the pre-questionnaire and pre-interviews, it becomes evident that students are advocating for a transformation in their mathematics education experience. The diversity in their responses and the mixed reactions observed both in the questionnaires and interviews underscore the necessity of a more adaptive and inclusive teaching approach. This approach should thoughtfully combine elements of both traditional and flipped classroom methods to cater to various learning needs and preferences effectively. By doing so, it could enhance student engagement, motivation, and the overall quality of learning, thereby optimizing the qualifications students gain from their mathematics lessons in the Danish upper secondary setting. This blended approach, rooted in the insights provided by educational theories such as SDT, CLT, Bloom's Taxonomy, and ZPD, and informed by students' actual experiences and preferences, holds the potential to create a more dynamic, interactive, and effective learning environment.

6.3 Coding of Post-Questionnaire Data

This section presents an analysis of the responses collected from students after the implementation of the flipped classroom model, specifically in learning about descriptive statistics. The objective is to interpret these responses in the context of predefined hypotheses, which were formulated to guide this master's thesis, presented in Section 4.5. This approach ensures a focused exploration of key areas of interest: students' perceptions of their mathematical qualifications, motivation levels, cognitive load, cognitive processes, and their experiences within the ZPD in a flipped classroom setting. Thus, the goal is to systematically code these responses to align with the formulated hypotheses, providing a structured insight into the students' experiences and perceptions. However, it is worth noting that some questionnaires are missing due to student absence or non-completion.

For each hypothesis, codes are identified throughout the students overall answers to the questionnaire:

- **Main Hypothesis: Students' Perceptions of Mathematical Qualifications**:

1. *Complete Understanding*: Responses that show a strong grasp of the subject matter or indicate that the student feels they have learned almost everything. For example, S1 responded with "I have learned almost everything," suggesting a high level of comprehension and mastery of the material taught through the flipped classroom method.

2. *Partial Understanding*: Responses indicating that the student has learned key aspects of the subject but may not have a complete or thorough understanding. An example is the response by S2, "I learned what I needed to," which could indicate that the student learned the necessary material but does not necessarily convey a deep or comprehensive understanding of the entire subject.

3. *Struggle with Concepts*: Responses that reflect difficulties in understanding or engaging with the material taught in the flipped classroom. An example is S7's response, "I found it a bit harder to learn because there were many commands, etc., for Maple that you had to try at home, and when it did not work, you had to wait until the class." This response indicates a struggle with some aspects of the material, particularly the practical application of tools like Maple.

The code for complete understanding supports this hypothesis, indicating that students who have fully grasped the mathematical concepts likely benefited from the engaging and interactive nature of the flipped classroom. This reflects the principles of SDT, where high levels of intrinsic motivation and a sense of competence are evident. For example, S3 stated "It was great when I got to do more on my own," which demonstrates high intrinsic motivation and a sense of competence, both key elements of SDT that seem to have been fostered by the engaging and interactive qualities of the flipped classroom approach.

However, the 'Partial Understanding' code offers a more nuanced perspective. It indicates that while these students were actively participating, the flipped classroom model might not have fully met their specific learning needs. This is partly demonstrated by S11's comment, "It was fine, but probably because I already knew many of the concepts," which implies that prior knowledge played a significant role in their learning experience, rather than the method itself. Additionally, S7's feedback, "there were many commands etc. for Maple to try at home," highlights the challenges in managing cognitive load (CLT) and underscores the need for more tailored support (ZPD). These comments suggest a partial alignment with the hypothesis, indicating that while these students engaged with the material, the model may not have completely catered to their varied learning needs. This could be attributed to both the challenges in managing cognitive load and their familiarity with traditional teaching methods.

Students accustomed to more conventional classroom settings might find the active, self-directed nature of a flipped classroom a significant shift, impacting their ability to adapt and fully engage with this learning model.

For students struggling with concepts, this outcome presents a notable challenge to the main hypothesis. Feedback like "I do not like the flipped classroom" (S19) and "I had a bit of a hard time keeping up" (S18) highlights the struggles faced by some students. Additionally, S commented "I found it hard to maintain concentration when I had to watch the videos at home," highlighting difficulties in engaging with the flipped classroom model. These responses could indicate areas where the flipped classroom model fell short, possibly due to overwhelming cognitive load (CLT), lack of sufficient scaffolding (ZPD), or misalignment with motivational needs (SDT). The struggles of these students might also be amplified by their previous familiarity with traditional teaching methods, indicating a potential impediment in adapting to the autonomous and self-directed environment of a flipped classroom. For example, the challenges noted in students' responses, such as concentration difficulties (S6), might stem from their familiarity with traditional teaching methods. Accustomed to more passive learning, these students could find the active, self-directed nature of a flipped classroom a significant shift, impacting their ability to manage cognitive load and their motivation to engage with the material. This suggests that while the flipped classroom offers interactive and engaging learning, its effectiveness can be influenced by students' prior educational experiences.

In analyzing Sub-Hypothesis 1: Motivation in relation to the flipped classroom method, it is essential to consider the principles of SDT, which emphasizes three key needs: competence, autonomy, and relatedness, as these needs are crucial for fostering intrinsic motivation.

- **Sub-Hypothesis 1: Motivation Coding Approach**:

  1. *Increased Motivation*: Responses indicating a heightened interest or enthusiasm for learning due to the flipped classroom method. An example where this is evident, is S3's response, "It was great when I got to do more on my own," which suggests that the student found the self-directed aspect of the flipped classroom to be motivating.

  2. *Neutral Motivation*: Responses that suggest the student's level of motivation hasn't significantly changed or where the student expresses an indifferent attitude toward learning in the flipped classroom. For example, S5 responded with "I learned it fine and learned what I wanted to learn with it." This response indicates a level of satisfaction with the learning but does not explicitly express increased enthusiasm or interest.

3. *Decreased Motivation*: Responses indicating a reduction in interest or enthusiasm for learning due to the flipped classroom method. An example is where S19 responded "I do not think it is very good. I do not feel that it is a good way to learn." This response explicitly suggests a decrease in motivation or a lack of enthusiasm for the flipped classroom approach.

In analyzing Sub-Hypothesis 1: Motivation based on the pos-questionnaire, responses indicated a generally increased motivation among several students, appreciating the hands-on aspect of the flipped classroom, here among them S9, S10, S13, and S14. S9 commented "It was great, and I got a lot out of it," which suggests that the student found the flipped classroom method to be highly beneficial and motivating, indicating increased engagement and perceived value from this learning approach. Similarly S14 wrote "I thought it was really good and got a lot out of it." This student's response implies a positive and motivating experience with the flipped classroom, suggesting enhanced engagement and learning. Thus, S9 and S14 who found the method highly beneficial, likely experienced an increased sense of competence. Their responses suggest that the flipped classroom helped them feel more capable and confident in understanding the material, a crucial aspect of intrinsic motivation according to SDT.

Nevertheless, there were instances where students expressed neutral motivation, pointing to the method's varying impact on different learners. For example, S5's response: "I learned it fine and learned what I wanted to learn with it". Here, the student expresses satisfaction with their learning outcome but does not explicitly show increased excitement or interest, suggesting a neutral response to the method. Thus S5's comment may indicate a basic level of competence but does not necessarily reflect a strong sense of autonomy or relatedness. This neutrality could be due to the method meeting their academic needs without necessarily engaging them on a deeper, more intrinsic level. Additionally, S16 wrote "It was fine enough but it is not for me." This suggests that while the student found the method acceptable, it did not particularly resonate with them or significantly impact their motivation. That is to say, S16's comment suggests a limited connection with the method in terms of autonomy. While they might have been competent in the tasks, the lack of personal resonance with the method indicates a possible unfulfilled need for autonomy or relatedness. This neutral response could also be influenced by S16's familiarity with traditional teaching methods. Students accustomed to more structured, teacher-led classrooms might find the self-directed and independent nature of the flipped classroom challenging, affecting their sense of relatedness and undermining their motivation. Thus, While some students find it highly motivating, others may experience a more neutral response, not necessarily feeling a significant shift in their motivation or engagement levels.

However, S19's explicit dissatisfaction, 'I do not think it is very good. I do not feel that it is a good way to learn,' suggests a lack of fulfillment in one or more of the SDT needs, possibly indicating a mismatch between their expectations based on traditional teaching methods and the self-directed nature of the flipped classroom. This response highlights the need to consider individual learning backgrounds when implementing new teaching methods.

Overall, the analysis suggests that the impact of the flipped classroom method on student motivation varies. While it can significantly enhance motivation for some students by catering to their needs for autonomy and competence, as seen in responses like those from S3, S9, S10, S13, and S14, it may not universally appeal to all. Responses like S5's and S16's reflect a neutral stance, possibly due to a partial fulfillment of SDT needs or a preference for traditional teaching methods. S19's dissatisfaction further highlights this variability, suggesting a mismatch between the flipped classroom approach and their familiar learning environment. These findings underscore the importance of recognizing and addressing the diverse motivational needs of students in educational settings. The effectiveness of innovative teaching methods like the flipped classroom hinges not only on their alignment with motivational theories like SDT but also on their adaptability to individual student backgrounds and preferences.

In evaluating Sub-Hypothesis 2: Cognitive Load in the context of the flipped classroom, it is crucial to consider CLT, which focuses on how information processing demands can affect a learner's ability to effectively engage with educational material.

- **Sub-Hypothesis 2: Cognitive Load Coding Approach**:

  1. *Manageable Load*: Responses that indicate the student felt capable of handling the workload and the demands of the flipped classroom method without excessive stress. For example, S8 responded "It worked well, and it was reasonably easy to understand the videos," indicating that the student found the workload and the self-study component manageable and not overly challenging.

  2. *Overwhelmed by Load*: Responses indicating that the student felt stressed, burdened, or unable to cope with the amount of work required in the flipped classroom method. An example of such a case is S6' response, "I found it difficult to maintain concentration when I had to watch the videos at home." This could indicate a struggle with the self-study aspect, implying that the student might have felt overwhelmed by the workload.

Several students, such as S8 and S13, indicated that they found the cognitive load to be manageable within the flipped classroom setup. Their responses, like "It worked well, and it was reasonably easy to understand the videos" (S8) and "I was more engaged in learning descriptive statistics with this method" (S13), may reflect an optimal balance of intrinsic and extrinsic cognitive load. Other students commented that they did not experience any form of overload, namely S1, S3, S4 S9, S14, indicating that the workload was manageable and easy.

Conversely, responses from students like S2, S5, and S6 suggest an imbalance in this cognitive load, leading to feelings of being overwhelmed. For instance, S6's comment, "I found it difficult to maintain concentration when I had to watch the videos at home," implies a high extrinsic load where the format or amount of the self-study materials hindered effective learning. Similarly, S2's expression of feeling overwhelmed and unable to immerse themselves, "Yes, I often felt overwhelmed and therefore did not have the opportunity to immerse myself," could be indicative of an excessive overall cognitive load, surpassing their working memory's capacity to process the information effectively. Similar indications were provided by S6, S7, S10, S16, and S19. This suggests the need for a more tailored approach to accommodate different learning paces, meaning that a one-size-fits-all method in education, such as a standard flipped classroom approach, may not be effective for all students due to the diversity in how quickly or slowly they learn.

By analyzing these student responses through the lens of CLT, it becomes evident that the effectiveness of the flipped classroom method hinges on carefully managing the cognitive load. This requires educators to consider the intrinsic difficulty of the content and how the extrinsic load (such as the format and volume of pre-class materials) can be optimized to facilitate better comprehension and engagement. Furthermore, an important consideration in managing cognitive load in a flipped classroom is the familiarity of students with traditional teaching methods. Students who are more accustomed to conventional, teacher-led classrooms might experience an increased extrinsic cognitive load when adapting to the self-directed nature of the flipped classroom. This shift could potentially overwhelm their cognitive processing capabilities, affecting their learning efficiency. Therefore, understanding students' backgrounds and prior experiences with teaching methods is crucial in designing flipped classroom approaches that effectively manage cognitive load for all learners.

In analyzing Sub-Hypothesis 3: Cognitive Processes within the context of the flipped classroom, this section delves into how the method influences students' cognitive development in terms of Bloom's Taxonomy.

- **Sub-Hypothesis 3: Cognitive Processes Coding Approach**:

  1. *Enhanced Understanding*: Responses that show a clear improvement or deepening in understanding of the subject matter due to the flipped classroom method. S4 responded "I think it has provided a good understanding because you get to do many more exercises in class with the subject," which suggests that the student feels their understanding has been enhanced, particularly through the opportunity to work on more exercises in class.

2. *Challenge in Understanding*: Responses indicating difficulties in grasping or fully comprehending the subject matter as taught in the flipped classroom. In example of such a case is S6'2 response, "I found it difficult to maintain concentration when I had to watch the videos at home," which indicates a challenge in understanding due to difficulties in concentrating during the self-study phase, which is a crucial component of the flipped classroom model.

Students like S13 commented "I was more engaged in learning descriptive statistics with this method." This response suggests engagement with higher cognitive processes, as the student indicates a deeper involvement and engagement with the material. This enhanced engagement typically correlates with higher levels of cognitive processing like critical thinking and conceptual understanding. Moreover, S9 wrote "It was great, and I got a lot out of it," which indicates a positive interaction with the material at a level that goes beyond basic comprehension, suggesting engagement with more complex cognitive tasks and a deeper understanding of the subject matter.

However, other responses such as "I found it difficult to maintain concentration when I had to watch the videos at home" (S6), indicates challenges at the foundational cognitive processes. Difficulty in maintaining concentration can impede the ability to grasp basic concepts, which are essential for progressing to more complex cognitive tasks. In addition, S18's response "It was okay, but sometimes I was a bit pressed for time and found it hard to keep up," points to the challenges in keeping up with the foundational aspects of learning. Time pressure and difficulty in keeping pace can hinder the basic understanding and application of the material, which are lower-level cognitive processes in Bloom's Taxonomy. When students mention difficulties like maintaining concentration or managing time, these challenges can impede their ability to engage with deeper analysis or critical thinking. For instance, a student struggling to concentrate on video lectures might have difficulty synthesizing information, evaluating concepts, or applying knowledge in new contexts. This is due to the fact that these activities require focused attention and cognitive effort, which are compromised when a student is unable to concentrate effectively. Similarly, time management issues can lead to rushed learning, which often results in superficial understanding rather than in-depth exploration of topics. These individual experiences, ranging from enhanced engagement to challenges in concentration, not only reflect the diverse impact of the flipped classroom but also underscore the varying levels of cognitive engagement among students.

This variation in student experiences aligns with Bloom's Taxonomy, where the depth of understanding and cognitive engagement ranges from basic recall and comprehension to higher-order skills such as analysis and application. Students who exhibit greater engagement, like S13 and S9, seem to operate at these higher cognitive levels, while those facing concentration challenges, like S6 and S18, may struggle with even the foundational cognitive tasks

Nevertheless, students accustomed to traditional lecture-based learning might find the primary focus of the flipped classroom approach on self-directed study challenging. This shift from a more structured, teacher-led environment to one where students are responsible for their initial learning can significantly impact their adaptability and engagement with the material at various cognitive levels. For students who are familiar with and comfortable in traditional classrooms, the emphasis on active, self-directed learning represents a significant change. In traditional settings, students often rely on immediate feedback and direct instruction from teachers, and the structured nature of the learning environment provides a sense of predictability and security. The transition to a flipped classroom, which requires students to take a more proactive role in their learning, particularly during the self-study phase, can be daunting for those who are used to more passive learning approaches. That is to say, this adjustment to self-directed learning may affect students' ability to engage effectively with the material, especially when it comes to understanding and applying complex concepts. Without the real-time guidance and clarification that they are accustomed to, some students might struggle with the intrinsic difficulty of the content. Furthermore, preconceived notions of what constitutes 'effective learning' might lead them to view the self-study and interactive aspects of the flipped classroom as less legitimate or effective, thus influencing their motivation and confidence in this new learning format.

Additionally, the fact that the structure of a flipped classroom often varies, which includes diverse types of self-study materials and in-class activities, might seem less predictable to students who are comfortable with the structured and predictable nature of traditional teaching methods. This lack of familiarity can result in discomfort or resistance, impacting their overall learning experience. In other words, the lack of immediate feedback and guidance in a self-study environment, coupled with the challenges of managing self-directed learning, can hinder some students' ability to engage with foundational concepts, which are essential for progressing to more complex cognitive tasks within Bloom's Taxonomy.

Overall, students' responses have shown that the flipped classroom method enhanced the understanding of descriptive statistics for many students. However, challenges in understanding were noted, particularly where the method did not align with individual learning preferences. To put it differently, this teaching approach may not have been effective or preferred by some students due to their unique ways of learning. For example, some students might prefer traditional lecture-based learning where they listen to the instructor explain concepts in class rather than learning through videos or self-study materials at home. For instance, a student might find it easier to understand complex topics when they are explained in person, where they can immediately ask questions. However, in terms of the reliability of self-reported understanding, it is important to acknowledge the inherent limitations of this approach. While self-reports can provide valuable insights into students' perceptions of their learning experiences, they may not always accurately reflect true mastery of the subject matter. This discrepancy is due to a variety of factors, including cognitive biases and the individual's ability to assess their own understanding. Self-perception of understanding can sometimes be misleading, leading to overestimation or underestimation of actual competencies. Therefore, relying solely on self-reported data might not provide a complete or entirely accurate picture of students' true grasp of descriptive statistics within the flipped classroom context.

In addition to that, this discussion aligns with the concept of the *Pygmalion effect*, a psychological phenomenon where an individual's performance or perception of their own abilities is influenced by the expectations set by others, particularly figures of authority such as teachers (Rosenthal and Jacobson, 1968). In the context of this master's thesis, the Pygmalion effect might manifest in how teachers' expectations influence not only students' actual performance but also their self-assessment of understanding. For instance, if a teacher holds and communicates high expectations for students' understanding and performance in a flipped classroom setting, this could lead students to perceive themselves as more competent and well-prepared. While this belief can serve as a positive motivator and encourage a more engaged learning approach, it also introduces a potential bias in self-evaluation. Students may report a higher understanding based on their perception of meeting the teacher's expectations, rather than an objective measure of their learning. Moreover, if students believe that their teachers expect them to understand the material, they might be hesitant to admit when they do not. This could be due to fear of disappointing the teacher, as they either could have a desire to be seen in a positive light or a concern about the potential consequences of reporting a lack of understanding.

In exploring Sub-Hypothesis 4: Zone of Proximal Development (ZPD) within the context of the flipped classroom, this section examines how peer interactions and the need for support influence students' learning processes and their ability to reach potential development levels.

- **Sub-Hypothesis 4: Zone of Proximal Development (ZPD) Coding Approach**:

    1. *Positive Peer Interaction*: Responses indicating beneficial interactions with classmates that aided learning and understanding. One of the students, namely S7, responded, "It has been really good that we have been able to do exercises on small boards and in small groups." This response shows a positive experience with peer interaction in small group settings, indicating that such interactions were helpful in the learning process.

2. *Need for More Support*:Responses expressing a desire for additional assistance, guidance, or resources to better understand the material. An example of such a case is S18's respond, "It is also a bit difficult to get answers right away when you become unsure about something; I often have to wait until I come to school." This response indicates a need for more immediate support or clarification, especially when students encounter uncertainties during their independent study time at home.

A key element of the flipped classroom model is the opportunity for students to engage in peer interactions that significantly contribute to their learning within the Zone of Proximal Development (ZPD). This is evident in the responses of several students. For example, S10 wrote "Being able to work together on difficult subjects while also having the opportunity for self-study and self-learning." This response highlights the dual benefit of collaborative learning and independent work, where working with peers on challenging topics allows students to discuss, problem-solve, and learn from each other, effectively expanding their ZPD. In addition to that, S13 responded "It has helped my learning to stand at small boards in groups and help each other with tasks." This comment underlines the advantage of small group activities in a flipped classroom, as it promotes active engagement, where students can clarify concepts, share different perspectives, and support each other's understanding, fostering a deeper comprehension that might be challenging to achieve independently. Moreover, S7 commented "I have learned more through interactions." S7's simple yet profound statement underscores the importance of social interactions in the learning process. Engaging with peers in the flipped classroom creates a dynamic learning environment where students can enhance their understanding through shared insights and collaborative effort.

While the flipped classroom fosters beneficial peer interactions, it also reveals areas where students need additional support, particularly in self-directed learning aspects. This is evident in several student responses. For example, S7 expressed difficulties with independent learning, particularly when using complex tools like Maple at home. They noted, "It was a bit harder to learn because there were many commands etc. for Maple to try at home, and when it did not work, you had to wait until the class." This response underscores the challenges faced when students try to navigate new software or complex material without immediate guidance. Similarly, S18's feedback points to the need for timely support, "It is also a bit difficult to get answers right away when you become unsure about something; I often have to wait until I come to school." This illustrates the struggle with the delay in receiving feedback during self-study periods, a critical component of the flipped classroom. Such delays can hinder a student's progress within their ZPD, as they may need immediate clarification to move forward effectively with their learning. Furthermore, S19's response, "I do not think it is very good. I do not feel that it is a good way to learn," reflects a broader discomfort with the flipped classroom methodology. This sentiment may stem from a combination of factors including unfamiliarity with self-directed learning approaches and a preference for more structured, traditional teaching methods. It suggests that for some students, the flipped classroom model may stretch beyond their current ZPD, requiring more scaffolding and support to make the learning experience effective.

These responses collectively highlight the importance of considering individual learning needs and preferences in the flipped classroom. They underscore the necessity of providing appropriate support and guidance to ensure that all students can effectively navigate within their ZPD, thereby maximizing their learning potential in this innovative educational model. Thus, while the flipped classroom facilitated valuable peer interactions and teacher support for many, it is important to note that each student's ZPD is unique. This means that what is within one student's ZPD might be outside another's. Some students may require more scaffolding or different types of support to reach their potential in learning new concepts, which the standard flipped classroom model might not adequately provide.

Moreover, Many students are accustomed to traditional classroom settings where the teacher is the primary source of information, and learning is more passive. This familiarity can make transitioning to a flipped classroom, which requires more active and self-directed learning, challenging. Students may feel comfortable with the structure and predictability of traditional teaching methods. Changing to a flipped classroom approach requires them to step out of their comfort zone, which can be unsettling. They may resist or feel apprehensive about this change due to uncertainty or fear of not being able to adapt successfully.

In other words, students' preference for traditional teaching methods and their discomfort with the flipped classroom can stem from what they are used to and their unfamiliarity with self-directed learning. Recognizing and addressing these challenges is key to successfully implementing a flipped classroom approach. Providing clear instructions, additional support, and gradually introducing elements of self-directed learning can help ease this transition.

The following is a summary of key observations:

1. **Varied Levels of Understanding**:The responses indicate a spectrum of understanding among the students. While some have shown complete or enhanced understanding, attributing their success to the primary focus of the method, namely the self-study and in-class activities, others have struggled with concepts or found it challenging to adapt to this self-directed method.

2. **Diverse Motivational Responses**:The flipped classroom has had a varying impact on student motivation. Some students have experienced increased motivation,

finding the approach engaging and effective. However, there were also instances of neutral or decreased motivation, suggesting that the method does not uniformly influence all students positively.

3. **Workload Management**: Students' ability to manage the workload presented by the flipped classroom model also varied. While some found the workload manageable and conducive to learning, others felt overwhelmed, indicating the need for more tailored support.

4. **Peer Interaction and Teacher Support**:Positive peer interactions and teacher support were significant factors in the success of the flipped classroom. These elements were crucial in facilitating understanding and fitting well within the students' ZPD. However, the need for additional support in some cases pointed to the necessity of adaptability in teaching methods.

Building on the insights gained from the questionnaire analysis, the next phase of my exploration delves deeper into the students' experiences and perceptions through post-interviews. The aim of these interviews is to provide a platform for students to elaborate on their questionnaire responses, offering a richer, more nuanced understanding of their experiences with the flipped classroom method.

6.4 In-depth Analysis of Student Post-Interviews

This section is dedicated to analyzing the post-interviews, specifically examining their relevance to the main hypothesis and the four sub-hypotheses of the study. Through thick description, these interviews aim to enrich our comprehension of how students' experiences with the flipped classroom relate to their mathematical qualifications, motivation levels, cognitive load, cognitive processes, and their experiences within the ZPD. It is important to acknowledge that the group of students who participated in these post-interviews differed from those involved in the pre-interviews and post-questionnaire. This variance presents some constraints in terms of continuity and representational breadth, as not all perspectives captured in earlier phases could be further explored in these interviews.

6.4.1 Enhancing Mathematical Qualifications

Several students like S8, S9, S14, and S20 and others, appreciated the flipped classroom for promoting active participation, which aligns with the notion of enhancing mathematical qualifications, noting that they personally believe that they have achieved either a better understanding of the subject or just as much as if it was through traditional methods. In the context of the main hypothesis – that active participation in flipped classroom significantly enhances mathematical qualifications – the appreciation of active participation by several students can be interpreted as supporting this hypothesis.

As introduced in the introductory chapter, flipped classroom turns traditional teaching methods on their head. Instead of passive listening during class time, students engage in active learning through problem-solving, discussions, and hands-on activities. This shift from passive to active learning is crucial for deeper understanding and retention of mathematical concepts. Thus, when students are actively involved in their learning process, they tend to engage more deeply with the material. This is often because they have already familiarized themselves with the content through pre-class videos or readings. This preparatory work allows them to use class time more effectively for applying and extending their knowledge, which is critical in subjects like mathematics.

As demonstrated in the previous subsection, which analyzed responses from the post-questionnaire, several students appreciated the flipped classroom model for the autonomy it provided in managing their learning pace. This sentiment was particularly evident in the views of students like S8 and S9. Highlighting the aspect of autonomy in learning, S9 expressed satisfaction with the flipped classroom model "Yes, I just thought it was pretty cool that I could control the time myself [...]". This indicates that S9 appreciates the control over their learning pace, a key element of autonomy in SDT. The ability to revisit and understand content at their own pace indicates effective management of cognitive load, making complex mathematical concepts more accessible. This self-directed learning aspect also reflects the higher cognitive processes in Bloom's Taxonomy, as the student actively engages with the material, enhancing understanding and retention.

In addition, S8 responded "Yes, I think it has been fine because then you really get to try it, and the tasks get more difficult, so in the end, there is something you need a little help with, but that is also good." S8's response highlights the importance of active engagement and gradual progression in learning. By working on tasks that become increasingly challenging, the student experiences a sense of competence, which is a core component of SDT. This approach supports deep learning, aligning with higher-order thinking skills in Bloom's Taxonomy. Furthermore, the mention of needing help with complex tasks illustrates the ZPD concept, where student reaches a point where guided assistance becomes necessary for further learning.

Moreover, S14 commented "I was very worried about having to learn it beforehand [...] but I actually think the videos she posted were good... they were short, very precise, and I thought they were good." S14 appreciated the short and precise nature of the instructional videos, which effectively aligns with the concept of reducing extrinsic cognitive load. By presenting information in a concise and focused manner, these videos minimize unnecessary cognitive demands, allowing students to concentrate on the essential aspects of the mathematical concepts. This approach ensures that the intrinsic load, or the inherent difficulty of the material, remains the primary focus of the student's cognitive efforts. Furthermore, S14's initial apprehension about self-directed learning before engaging with the flipped classroom model offers insight into the adaptability of students to new educational paradigms. This transition from apprehension to appreciation suggests an evolution in S14's learning approach, possibly indicating a shift in their motivational factors as outlined in SDT. Initially, the sense of autonomy and responsibility in learning might have appeared daunting, but the effective design of the instructional materials (aligned with CLT) facilitated a smoother transition, thereby enhancing S14's sense of competence and autonomy. Certainly! To critically analyze student responses in light of educational theories, you'll want to delve into how each response not only supports but also illustrates these theories in practice. Here's an example of how you might approach this with S14's response:

"In analyzing S14's response, a critical lens reveals a deeper alignment with the principles of Cognitive Load Theory (CLT). S14 appreciated the short and precise nature of the instructional videos, which effectively aligns with the concept of reducing extrinsic cognitive load. By presenting information in a concise and focused manner, these videos minimize unnecessary cognitive demands, allowing students to concentrate on the essential aspects of the mathematical concepts. This approach ensures that the intrinsic load, or the inherent difficulty of the material, remains the primary focus of the student's cognitive efforts.

Moreover, S14's experience reflects the application of Bloom's Taxonomy in a flipped classroom. The preparatory work with videos enables students to cover lower-order cognitive tasks, such as understanding and remembering, outside the classroom. This preparation sets the stage for engaging in higher-order cognitive processes during class, like applying, analyzing, and creating, as students can utilize class time to delve into more complex problems with a better foundational understanding. Lastly, the ZPD is also evident in S14's experience. The initial self-study, supported by precise and concise videos, allows students to explore and understand concepts within their ZPD independently. The subsequent in-class activities then provide the necessary scaffolding, as students can seek clarification and assistance from teachers and peers on more challenging aspects of the material, pushing the boundaries of their ZPD.

Furthermore, the insights from students like S8, S9, and S14 provide an illustrative example of how various educational theories interconnect within the flipped classroom context, collectively contributing to enhanced mathematical qualifications. The autonomy and competence that students experience, central to SDT, are not standalone benefits. They are intricately linked to, and indeed facilitated by, the efficient management of CLT. For instance, the ability to control the learning pace and revisit complex concepts, as expressed by S9, demonstrates how reducing extrinsic cognitive load enhances understanding and fosters autonomy. Additionally, the engagement with higher-order cognitive processes, a cornerstone of Bloom's Taxonomy, is evident in the students' active participation and problem-solving during in-class activities. This advanced engagement, prompted by the preparatory work done at home, aligns with the deeper cognitive processes such as analysis and application. Moreover, the ZPD is also a significant factor, as highlighted by the students' appreciation for the balance of independent work and the need for occasional teacher support. This blend of autonomy and guided learning exemplifies the ZPD in action, where students navigate the space between what they can do independently and what they can achieve with assistance. Thus, the flipped classroom model, through its design, inherently encapsulates these theoretical elements, creating a rich environment for enhancing mathematical qualifications.

However, when exploring the flipped classroom model in relation to enhancing students' mathematical qualifications through the experiences of S2 and S5, it offers a nuanced understanding when analyzed against the backdrop of SDT, CLT, Bloom's Taxonomy, and ZPD. These perspectives, alongside the students' familiarity with traditional classroom methods, provide a comprehensive lens to evaluate the main hypothesis.

Student 2's experience in the flipped classroom model was marked by feelings of stress and being overwhelmed, as indicated by their statement, "...so it was really pressured at home, I had no time for anything else. It was only homework and assignments I did [...] it also became quite stressful. Like, oh shit now I have to remember this." This response highlights a critical aspect of CLT, which posits that excessive cognitive load can impede effective learning. In S2's case, the demanding structure of the flipped classroom, characterized by a significant amount of homework and assignments, resulted in cognitive overload. Such overload can be counterproductive, as it not only hampers the absorption and retention of mathematical concepts but also leads to negative emotions that can further detract from the learning experience. This situation also intersects with the principles of SDT, particularly the components of autonomy and competence. SDT emphasizes that for intrinsic motivation to be fostered in a learning environment, students must feel a sense of control over their learning process (autonomy) and believe in their ability to succeed in the tasks at hand (competence). In S2's scenario, the overwhelming workload may have diminished their sense of autonomy, making them feel more controlled by the demands of the course rather than being an active participant in their own learning. Additionally, the stress related to managing this workload could undermine their sense of competence, as they struggle to cope with the demands, thereby negatively affecting their intrinsic motivation to learn.

Similarly, Student 5's feedback about the workload and the length of preparatory videos, as illustrated by their comment, "...if you get too much, then already there - Yes, then I get a bit lost in it," also aligns with the insights of both SDT and CLT. S5's reaction suggests that the balance between workload and cognitive capacity is crucial. An excessive amount of preparatory material, especially if perceived as lengthy or complex, can lead to cognitive overload, aligning with CLT's assertion that learning is most effective when cognitive demands are kept within manageable limits. From an SDT perspective, this balance is also critical for maintaining student motivation. If students feel overwhelmed by the preparatory tasks, their sense of autonomy and competence could be compromised, leading to reduced motivation and engagement.

However, these reflections are further contextualized by the students' prior exposure to traditional teaching methods. This familiarity sets a benchmark against which the flipped classroom model is evaluated. Students' comfort with and expectations from conventional classroom settings play a significant role in how they adapt to and perceive the flipped classroom approach. The transition from a familiar educational environment to a novel one requires not only an adjustment in learning strategies but also a shift in mindset.

In summary, the responses from S8, S9, S14, and S20 support the main hypothesis that active participation in a flipped classroom environment enhances mathematical qualifications. The students' experiences reflect key educational theories like SDT, CLT, Bloom's Taxonomy, and ZPD. They demonstrate the effectiveness of the flipped classroom model in fostering autonomy, competence, reduced cognitive load, and higher-order cognitive processes, all of which contribute to improved mathematical understanding and skills. However, its success also depends on a balanced approach that considers cognitive load, nurtures autonomy, adapts to individual readiness levels, and smoothly transitions students from traditional to active learning methodologies. Integrating this understanding with educational theories and considering students' backgrounds is key to effectively leveraging the flipped classroom model to achieve its educational objectives.

6.4.2 Motivation

In relation to Sub-Hypothesis 1, which assesses motivation, S2 articulated challenges in managing time for pre-class video sessions within the flipped classroom model. S2's experiences highlight the significance of autonomy and competence, which are two crucial elements in SDT. S2 expressed feeling overwhelmed by the high workload and tight schedule due to other school subjects, training, and work, leading to stress and reduced autonomy in managing their learning. S2 stated, "It was really pressured at home, I had no time for anything else. It was only homework and assignments I did [...] it also became quite stressful." This scenario underscores the need for a balanced flipped classroom approach that not only fosters autonomy but also ensures that students do not become overwhelmed, thereby maintaining their sense of competence and intrinsic motivation. Moreover, S6's statement, "I find it easier to concentrate when I am at school... at home, there are so many things, I just get distracted," illustrates the challenges in maintaining autonomous engagement. It illustrates the importance of the learning environment in supporting or hindering students' autonomous engagement and perceived competence, essential for sustaining motivation. Therefore, the flipped model may not be universally motivating, particularly in less-than-ideal home learning environments.

Nevetheless, some students showed mixed feelings, here among them S18. They expressed "Yes, I have been very time-pressed, and there are also times when you forget to watch them. And then when I show up for class, I have no idea what I am supposed to do. That was the only problem. But otherwise, I think it was fine enough because we try to understand it ourselves." This shows the mixed feelings, noting that while the flipped classroom had certain motivational benefits, it was not universally motivating.

However, students like S9, S10, S11, and S14 enjoyed the fact that they had control over their learning space, the self-paced nature, video rewatching, and gaining a head start on topics, along with the effectiveness for comprehension of this method. This reflects increased motivation through autonomy, a key component of SDT. The ability to control their learning process and deeply understand concepts at their own pace is a clear manifestation of autonomy in learning. Reflecting on the autonomy aspect of the flipped classroom, S9 appreciated the control over their learning pace. They stated: "Yes, I just thought it was really cool that I could control the time myself, and I could rewind the videos a bit if I did not quite understand something. And then I could also repeat the video if I did not understand it." S9 highlights the their ability to control their learning pace and revisit concepts as needed, which is a key aspect of autonomy. Being able to learn at their own pace and review material multiple times supports their intrinsic motivation to learn and understand. Echoing the sentiment of autonomy and competence, S10 found value in the ability of flipped classroom to provide a head start on topics. They remarked: "It is about getting a little head start on what you are going to work with, knowing what you are working with. Yes, when she shows it in videos, Joana the teacher, it is great that you can write along over Maple and it goes at your own pace." Here, the student appreciates having a preliminary understanding of the topic before class, thanks to pre-class videos. This ability to prepare at their own pace and engage interactively with the material (like using Maple) also demonstrates autonomy, thus enhancing their motivation and engagement in the learning process.

Similarly, S11 also highlights the positive insight of being prepared and not coming to class 'empty-handed,' reflecting the importance of autonomy in learning: "It is about having an understanding of it yourself... like when you already know something about a topic or are familiar with it beforehand, it can be more fun and engaging to participate and be involved." The student's ability to engage with the material on a personal level, especially when they have prior knowledge, enhances their interest and engagement in the subject. This increased engagement is a direct result of feeling autonomous in their learning journey.

S10, S11, and S14 appreciated the ability to prepare in advance, stating that having prior knowledge made the in-class experience more engaging and effective. This aligns with the SDT component of competence, as the ability to feel prepared and capable enhances a student's sense of competence and intrinsic motivation. S14, discussing the benefits of self-preparation and its impact on classroom engagement, noted:

> I was very worried about having to learn it beforehand. Like having to take responsibility for learning it ourselves, but I actually found the videos she posted to be really good... they were short, very precise, and I think they were helpful, so you had a starting point when you started the exercises instead of just sitting in class maybe not following along and then just sitting with the assignments (S14).

That is to say the flipped classroom allowed them to feel prepared and capable, thereby enhancing their sense of competence and intrinsic motivation.

Highlighting the aspect of relatedness in the flipped classroom approach, a key component of SDT, S11 spoke to the benefits of collaborative activities following self-study. Their appreciation of collaborative in-class activities after self-study, and S8's emphasis on completing preparatory material for participation, show how the flipped classroom fosters a sense of community and collaboration, addressing the need for relatedness. For example, S11 expressed "It would have been easier if I had already done the homework before coming to school. Then I would always have a starting point and some knowledge about what I roughly should start with." In addition, S8 noted: "Yes, I think it would be a bit difficult because you do not get that information." These perspectives illustrate how collaboration, underpinned by prior individual preparation, can serve as a significant motivator. By engaging with the content beforehand, students are better equipped to interact meaningfully with peers and teachers, thereby enhancing their overall learning experience.

After analyzing the perspectives of students like S9, S10, S11, and S14, it becomes evident that the flipped classroom model can significantly foster autonomy and competence, two pivotal elements of Self-Determination Theory. Students who enjoyed the self-paced nature and the ability to control their learning process exhibit increased intrinsic motivation, an essential outcome predicted by SDT. However, the experiences of S2 and S6, who struggled with time management and distractions, highlight that the effectiveness of the flipped classroom in promoting SDT components is not universal. This mixed response among students supports Sub-Hypothesis 1 to a certain extent.

It suggests that while the flipped classroom can enhance intrinsic motivation through autonomy, competence, and relatedness, its efficacy is varied and can be influenced by individual circumstances and learning environments.

However, while the flipped classroom model effectively addresses key components of SDT, such as autonomy, competence, and relatedness, its intersection with traditional teaching methods brings forth unique challenges. These challenges are particularly evident in the transition phase where students, accustomed to traditional pedagogical approaches, find themselves adapting to a more self-directed learning. This transition can be daunting for some, initially hindering their motivation. Such a shift requires not only an adaptation to a new way of learning, which includes more self-directed study and pre-class preparation, but also a mental shift in how students perceive their role in the learning process. For students more comfortable with traditional methods, the change to a flipped classroom model may not immediately influence their motivation positively. However, over time, as they become more familiar and comfortable with this new approach, they may start to appreciate its benefits. This highlights the importance of considering students' prior educational experiences and the need for supportive measures during this transitional period.

6.4.3 Cognitive Load

In examining the impact of the flipped classroom model on cognitive load, Sub-Hypothesis 2 proposes that such an approach leads to a reduction in extrinsic cognitive load. This reduction is theorized to allow students to more effectively use their cognitive resources for understanding and mastering mathematical concepts. Central to this hypothesis is the premise that by alleviating the burden of extrinsic load - the load imposed by the manner of instruction rather than the content itself - students can engage more deeply and meaningfully with the subject matter. However, for some students, like S2, the requirement to engage in self-study through pre-class video sessions increased their extrinsic cognitive load. This was due to the addition of these sessions to their already busy schedules, which included other school subjects and personal responsibilities. The nature of self-study, requiring students to independently engage with and comprehend new content outside the traditional classroom setting, contributed to S2 feeling overwhelmed. This indicates that while the flipped classroom aims to enhance learning by providing materials beforehand, it can inadvertently add to the cognitive load for some students, particularly those struggling with time management or external distractions. However, despite S2's challenges with the self-study component, they recognized the value in the interactive learning opportunities afforded during class time. They appreciated the chance to discuss and consolidate their learning with peers and the instructor, stating, "It works very well...I think it has been good being able to talk to people about it." This indicates that the in-class portion of the flipped classroom, which often involves more active and collaborative learning, can mitigate the cognitive load by allowing students to process and apply the information they have studied independently.

Similarly, S6's experience highlights how external factors, like distractions at home, can exacerbate the cognitive load associated with pre-class video sessions: "[...] you just get distracted, then there is an assignment, or I have to cook dinner, or... there are so many things [...]." These distractions split their cognitive resources, making it challenging to focus on academic tasks. However, like S2, S6 might benefit from the interactive and hands-on nature of in-class activities, where the focus shifts to applying and discussing the learned material in a more structured and collaborative environment. This shift can help redistribute cognitive load more effectively, allowing students to engage more deeply with the material.

While pre-class self-study sessions are fundamental to this instructional strategy, they need to be carefully designed to avoid overwhelming students, specifically in terms of students who are not familiar in working in such a setting. Similarly, in-class activities should be structured to maximize student engagement and interaction, providing opportunities to alleviate cognitive load through collaborative and hands-on learning experiences.

Conversely, S3 and S4 showed positive experience with the flipped classroom model, offering insights into strategies that can effectively manage cognitive load. S3's feedback highlights the benefits of independent and active learning in reducing cognitive load. They noted, "We got to work independently, and we were not sitting still as much," emphasizing the advantage of engaging actively with the material rather than passively receiving information. Nevertheless, S3's perspective on the video format offers a nuanced approach to managing cognitive load. They pointed out that while a single long video of 15 minutes can be overwhelming, breaking it down into shorter segments that collectively make up the same duration is more manageable. This approach of segmenting longer content into shorter, more focused videos can help prevent students from feeling overwhelmed with a large amount of information at once. It allows for natural breaks where students can pause, reflect, and absorb the content before moving on to the next segment.

Furthermore, S9's positive response to the flipped classroom approach offers additional insight. They expressed, "It is really cool that I could control the timing myself." This ability to control the learning pace is a significant example of reduced extrinsic cognitive load in action. When students are given the autonomy to dictate the speed of their learning, they are likely to feel less burdened by the influx of new content. This self-paced approach empowers students to manage their cognitive resources more efficiently, leading to a more effective and less overwhelming learning experience. By adjusting the pace to suit their individual needs, students can engage with the material in a way that maximizes understanding and retention, while minimizing the risk of cognitive overload.

Thus, in analyzing student experiences within the flipped classroom model through the lens of CLT, we can gain valuable insights into their cognitive processing and learning effectiveness:

- **Intrinsic Load:** Students like S3 and S9, who positively engaged with the flipped classroom, demonstrated effective management of intrinsic load. Their ability to independently explore content and actively participate in class activities indicates that the complexity of the tasks was at an optimal level, facilitating effective information processing. This suggests that the flipped classroom model can adequately align with the inherent difficulty of the material, supporting effective learning.

- **Extrinsic Load:** Conversely, students such as S2 and S6 experienced an increase in extrinsic load, which CLT attributes to factors related to instructional design or external environmental factors, rather than the content complexity itself. S2's feeling of being overwhelmed by the volume of pre-class videos and S6's struggle with home distractions exemplify a high extrinsic load, pointing to the need for instructional adjustments.

- **Germane Load:** The positive experiences of students like S3 also indicate effective management of germane load. These students were able to actively construct knowledge by integrating pre-class materials with in-class activities, supporting deep learning and schema formation. This aligns with the CLT principle that emphasizes the importance of germane load for meaningful learning.

Therefore, the application of CLT in the context of the flipped classroom model highlights the critical need for careful instructional design. Optimizing pre-class materials and the learning environment is essential to balance intrinsic, extrinsic, and germane cognitive loads, thereby enhancing the overall effectiveness of the learning experience.

In summary, the analysis of Sub-Hypothesis 2 reveals mixed outcomes regarding cognitive load in the flipped classroom setting, highlighting the complexity of educational experiences. While some students, like S3 and S9, thrived in this setting, others, such as S2 and S6, faced challenges. These mixed outcomes underscore the importance of adaptable and student-centered instructional design in flipped classrooms. While the experiences of students like S3 suggest effective strategies for managing cognitive load, further exploration into optimizing the length and complexity of pre-class materials could offer solutions to the challenges faced by students like S2. Thus, future research could delve deeper into how specific elements of the flipped classroom, like the structuring of pre-class materials or the nature of in-class activities, can be fine-tuned to cater to diverse student needs. This could involve exploring methods to tailor content delivery to minimize extrinsic load, while simultaneously enriching intrinsic and germane load for a more effective learning experience.

Moreover, the role of individual differences, such as students' prior knowledge and personal circumstances, in shaping their cognitive load experiences within flipped classrooms deserves closer examination. Understanding these nuances can aid educators in creating more inclusive and effective learning environments.

Thus, an important consideration is the influence of familiarity with traditional teaching methods on students' perceptions of cognitive load. Students primarily accustomed to traditional instruction might initially perceive the flipped classroom as overwhelming, contributing to an increased cognitive load. Nevertheless, this perception may evolve over time as students become more familiar and adaptable to the flipped classroom approach. This highlights a potential initial adjustment period rather than an inherent limitation of the flipped classroom model, suggesting that with time and proper support, students can adapt to and potentially benefit from this innovative approach to learning. Overall, this analysis provides a foundational understanding of the cognitive load in flipped classrooms. However, it also opens the door to a deeper exploration of how instructional design can be fine-tuned to accommodate diverse learning needs, a topic ripe for further discussion and examination.

6.4.4 Cognitive Process

Sub-Hypothesis 3 of this master's thesis postulates that flipped classrooms promote higher-order cognitive processes, leading to improved mathematical problem-solving abilities. This hypothesis is anchored in the belief that such instructional models can enhance cognitive skills beyond basic understanding, fostering advanced thinking skills crucial for mathematical proficiency. Bloom's Taxonomy, as presented in Chapter 4, provides a structured framework to categorize these cognitive processes, offering insights into how educational objectives evolve from basic to complex cognitive skills, incorporating both the Knowledge and Cognitive Process dimensions.

In flipped classroom, the stages of **remembering** and **understanding** are impacted by the emphasis of the method on self-directed learning, which contrasts with the more structured, teacher-led approach of traditional settings. This is evident in several student responses, particularly when they discuss their experiences with the flipped classroom method. For example, S2 speaks about the pressure of having to handle a significant amount of homework and assignments, indicating a heavy reliance on remembering vast amounts of information: "It is a bit tough with all the homework, because you have so many subjects, and you have to remember to do all of it, and it is a bit stressful, especially when you have so many other things to think about too." The stress associated with remembering what needs to be done, especially with multiple subjects, suggests that the flipped classroom method is challenging their capacity to recall and manage information efficiently. This scenario exemplifies the foundational cognitive task of memorizing content, a crucial first step in the learning process.

Additionally, S7 discusses the difficulty in remembering mathematical concepts. Their statement, "It can be a bit like 'ah damn it', then there is a lot to remember," reveals the struggle with memorization, particularly in a subject they find challenging. This aligns with the remembering stage, where the primary focus is on recalling or recognizing facts and basic concepts. Students like S2 and S7, who exhibit challenges with managing and remembering a significant amount of information, may be facing these difficulties due to their familiarity with traditional teaching methods. In a traditional setting, the learning process is typically more structured and guided by the teacher, which differs significantly from the flipped classroom's approach of encouraging self-directed learning. This shift can impact foundational cognitive processes such as remembering and understanding, making it challenging for some students to adapt to the new learning environment, yet also encourages development in foundational cognitive skills, as the self-directed learning approach in flipped classrooms necessitates more active engagement in recalling and managing information.

Conversely, S5 states that "[...] It makes it easier to remember things because I am working with it at my own pace." This underscores how the self-paced nature of video learning aids in memory retention. The ability to control the pace of viewing allows S5 to engage more thoroughly with the material, leading to a better recollection of facts and concepts, which is foundational to the remember stage. Similarly, S9 and S10 both express positive views of the flipped classroom, indicating that this method aids in their remembering process, possibly due to the flexibility and control over their learning pace: "It is much better when you can control it yourself. Like, when I watch the videos, I can pause them and think about it, or watch them again if I need to. It makes it easier to remember things because I am working with it at my own pace" (S9), "I like that I can watch the videos at home and then work on it in class. It helps me to remember because I get a chance to go over it more than once" (S10). Hence, in the flipped classroom, the remember stage involves students independently recalling information from pre-class videos, which might foster better retention due to active engagement. In contrast, traditional settings often rely on teacher-led instruction for recall, which might not engage students as deeply.

Building upon the ability to recall information, the flipped classroom also facilitates deeper comprehension, as evidenced in the **understand** stage of Bloom's Taxonomy. When it comes to the understand level which refers to making sense of the material, interpreting it, and comprehending what is being communicated, S5 comments on the helpfulness of videos in understanding concepts. They imply that the format of the videos aids in making the recall of information more manageable along with the fact that the teacher explained the concepts really well: "I think the fact that it was a video that you had to sit and watch... She also explains it in such a good way, so it is easier to understand." S5 also noted that "I think the preparation with videos makes a big difference. I get a clearer understanding of the topic before we even start in class, and it helps me keep up better." As previously mentioned, S9 and S10 had positive insights of the flipped classroom method. They felt that the method helped them understanding the subject better, indicating that the format of the flipped classroom, with videos and self-paced study, contributes positively to their understanding. Additionally, their ability to control the pace of learning and revisit content suggests that this method enhances their comprehension and interpretation of the material. Another example, is S13 who also noted:

> I think I understood the topic we had a bit more. Where we prepared at home and watched those videos we were assigned, and then you could come up to school and talk with your seatmate about what you saw at home - and then work on tasks related to it. So, the thing about being prepared for what we were going to learn about and then doing tasks, and also it was not the same - so you could prioritize your time a bit because you do it at home, so you do not have to be 100% focused on understanding everything in school because you have already seen the videos at home.

This statement from Student 13 highlights how pre-class preparation through video, enabled them to engage more effectively in class discussions and group activities, enhancing their understanding of the subject matter. This demonstrates that the flipped classroom method effectively engages students at the foundational cognitive levels of remember and understand. Thus overall, students in a flipped classroom often report a deeper understanding of concepts, as they can learn at their own pace with videos and revisit confusing parts, unlike the fixed pace of traditional classrooms. However, this autonomy can also be overwhelming for some, potentially hindering their understanding if they lack self-discipline or motivation.

Having established a foundation in understanding, we observe how students then apply these concepts in practical settings, stepping into the **apply** stage. This is evident in the students' ability to use statistical concepts in practical contexts, as shown in their responses to quiz questions. For instance, when asked to provide real-world examples of how relative frequency can be used to interpret data, the students' responses varied from basic application to more nuanced interpretations. This variety in responses showcases their ability to apply abstract statistical concepts to everyday scenarios, an essential skill at the application level of cognitive processing. Additionally, in a question that required them to calculate and interpret relative frequency, cumulative frequency, and frequency for a given data set, the range of answers further illustrates their engagement with the application stage. Some students could accurately perform and explain these calculations, demonstrating a clear understanding and ability to apply these concepts. Others, however, possibly struggled with certain aspects, indicating areas where further instruction and practice might be beneficial.

In addition to S3's quote, about taking time at home to understand the content, implies that this pre-class preparation enables them to actively participate in class, which aligns with the application stage; students are expected to use the information they have understood in new and practical contexts. This active participation likely involves applying the concepts learned from the videos to class activities, discussions, or exercises, which is a direct application of their newly acquired knowledge. Similarly, S5's statement about getting a clearer understanding of the topic before class indicates that they are prepared to engage with the material in a more practical, applied manner during the class. This might include working on exercises that apply the concepts learned from the videos or collaborating with peers on solving various types of exercises or problems that require the application of these concepts.

Another example is S20, who also reflected on applying learned concepts in class. They mentioned, "It was again about these videos, because then you sort of already knew something about it... And then when you came to school, you got a bit more, so it was kind of summed up, sort of." This shows how the student applied the pre-class learning to engage more effectively in class discussions and activities. These examples illustrate how the flipped classroom method fosters the application of learned concepts in practical contexts and reflect the instructional goals aimed at enhancing problem-solving abilities and critical thinking in mathematics. Hence in an overall sense, the flipped classroom allows students to apply concepts in a practical context during class, providing immediate support from peers and teachers. However, this requires students to come prepared, and those who do not engage with pre-class materials might struggle more than in a traditional setting. For example, the few students like S2 and S7, struggled with managing and remembering a significant amount of homework across multiple subjects, hence it could lead to challenges of applying learned knowledge. That is to say, the transitioning to a flipped classroom can be challenging for students accustomed to traditional methods. The initial struggle with self-directed learning, especially in the understand and apply stages, highlights the need for gradual introduction and ongoing support.

Beyond application, the flipped classroom environment encourages students to engage in more complex cognitive tasks. This is particularly evident in the **analyze** stage, where students begin to dissect and critically engage with mathematical problems. Students like S13 reflected an engagement with the analyze stage of Bloom's Taxonomy;

they appreciated working in small groups where they could discuss and debate the problems: "I liked it because you had the problem already yourself and then you could talk to someone about it and discuss: How do you do it? What have we learned?" (S13). This process of discussing and dissecting problems with peers, rather than passively observing a single shared solution, highlights their analytical approach to understanding and solving mathematical problems. S4's comments also align with the analyze stage of Bloom's Taxonomy, as they mention, "Yes, exactly, because... I feel I was very well-prepared when we started working on tasks, and because I was, I just find it easy to solve the tasks since I know so much in advance. And working with others also makes it faster to get to the result, because you think about it together and discuss. And when I do it more independently, I really feel that I have understood it well." This reflects their ability to dissect and understand problems through collaborative discussion and independent thought, key aspects of analytical thinking.

However, it is important to note that these findings primarily rely on students' self-reports of their cognitive processes. While these insights are valuable, they are subjective and do not conclusively prove an actual improvement in cognitive skills. The potential unreliability of self-reported data and influences like the Pygmalion effect highlight areas for future research. Further studies could focus on more objective assessments of cognitive processing in flipped classroom settings to corroborate and expand upon these initial findings.

While acknowledging the subjective nature of students' self-reports and the need for further objective research, it is also important to explore how students critically assess and creatively contribute to the flipped classroom methodology. This exploration leads us into the higher-order cognitive stages of Bloom's Taxonomy, specifically the **evaluation** and **create** stages, where students' reflections move beyond mere understanding and application to critical analysis and innovative thinking.

In the **evaluation** phase of Bloom's Taxonomy, S4, S13, and S14 provide insightful perspectives on the flipped classroom method, as their responses illustrate a deep engagement with the learning process. For example, student S4 highlights how advanced preparation and collaborative learning in the flipped classroom enhance problem-solving skills. Their appreciation of peer discussions and self-study underscores an ability to evaluate the impact of the method on learning efficiency; "I felt very well-prepared for working on tasks [...] understanding it well due to more independent work." Additionally, student S13 evaluates the practicality of the method, specifically noting the benefits of group activities and interactive problem-solving at the blackboard; "I liked working in small groups at the blackboard, discussing and figuring out how to do it." This student's insights reflect a thoughtful assessment of how the method fosters understanding through hands-on experience and peer collaboration. Moreover, Student S14, initially skeptical, acknowledges the advantages of the flipped classroom over traditional methods. They provide a comparative evaluation, noting the concise and effective nature of video-based learning, demonstrating their ability to critique and judge different teaching strategies; "Initially worried about learning on my own, I found the videos concise and effective, better than just listening in class."

Finally, including the suggestions from students about modifying the flipped classroom method aligns with the **create** level of Bloom's Taxonomy. These suggestions demonstrate their ability to synthesize their experiences and propose new ideas. For instance:

1. **Reading Texts Instead of Videos**: Some students proposed replacing or supplementing videos with reading materials, indicating a preference for varied learning materials. For example, S2 suggested that reading texts instead of watching videos could be a better approach for some students, especially those who get easily distracted during videos. Nevertheless, while videos can provide visual explanations and dynamic illustrations, especially useful in conceptualizing abstract mathematical concepts, texts offer a more detailed and nuanced exploration of mathematical theories and problems. The ability to read and understand mathematical texts is a competency in itself, one that is essential for academic and professional success in the field of mathematics.

2. **Timing of Implementation**: A suggestion to introduce flipped classroom methods later in the educational journey, allowing students to initially adapt to a less independent method. Moreover, this suggestion was also to better prepare the students by letting them know beforehand in good time, that they are going to be more independent in their learning, as they move forward in their educational journey. This was suggested by S18, who advocates for introducing the flipped classroom method in a gradual manner, starting gently in the second year and intensifying in the final year of the upper secondary school, emphasizing the importance of preparing students in advance for the shift towards greater self-reliance in their learning journey.

3. **Mandatory Implementation from the Start**: This idea reflects a desire for consistency in teaching methods throughout the course. S6 discussed difficulties in maintaining concentration while studying at home due to distractions. They preferred traditional classroom teaching and expressed a preference for taking notes during lectures. However, S6 suggested that starting with flipped classroom from the beginning of their education might have been more effective.

4. **Mix of Traditional and Flipped Methods**: Students recommended a blended approach, combining the strengths of both traditional and flipped classroom methodologies. For example, S18 finds the flipped classroom effective with sufficient preparation time but challenging when time-constrained. Thus, they suggest

that a blend of methods could mitigate these challenges, providing flexibility and support when self-study is not feasible.

Nevertheless, it is crucial to contrast this approach with traditional learning methods in examining the cognitive processes fostered by flipped classrooms. In traditional settings, the learning process is often structured with a strong teacher-led component, which can support students in remembering and understanding mathematical concepts. However, this approach might not always encourage active engagement or deep comprehension. In contrast, flipped classrooms, with their self-paced video learning, empower students like S5 and S10 to engage more deeply with the material, enhancing both memory and understanding. While this autonomy in learning can foster better retention and comprehension, it also poses challenges. Students accustomed to structured learning, like S2 and S7, may find it overwhelming to manage and internalize vast amounts of information independently. Moreover, the flipped classroom model shines in its application and analysis stages. The model enables students to apply theoretical knowledge practically during class, providing immediate support from peers and teachers. This contrasts with traditional methods, where application might be limited to structured exercises with less real-time peer interaction. The group discussions and collaborative problem-solving highlighted by S13 and S4 illustrate how flipped classrooms encourage students to critically engage with mathematical problems. Thus, this active participation fosters a deeper understanding and practical application of concepts.

However, perhaps one of the most striking contrasts arises in the evaluation and creation stages of Bloom's Taxonomy. Traditional learning environments may not always provide ample opportunities for students to critically evaluate and creatively contribute to the learning methodology itself. In the flipped classroom context, students' engagement in metacognitive activities is an integral aspect, as evidenced by Bloom's Revised Taxonomy which includes Metacognitive Knowledge. This dimension deals with the awareness and understanding of one's own cognitive processes. In a flipped classroom, students like S4, S13, and S14 not only engage in foundational cognitive tasks but also demonstrate a heightened metacognitive awareness. They evaluate their learning strategies, reflecting on the efficacy of the flipped classroom method. This metacognitive engagement is a significant, yet indirect benefit of the flipped classroom model. It fosters a deeper sense of ownership and involvement in the learning process and encourages students to critically assess and suggest improvements to their learning journey. Thus, the flipped classroom model extends beyond enhancing understanding and application of concepts to cultivating advanced cognitive skills, including metacognition, evaluation, and creation, which could be in contrast to the traditional learning environmnets. Thus, it demonstrates how the flipped classroom can go beyond its core aims of enhancing understanding and application of concepts to also cultivate higher-order thinking skills like evaluation and creation. This level of engagement is indicative of higher-order cognitive skills, which are essential for advanced learning and problem-solving. Yet, it is important to note that while flipped classrooms can facilitate deeper analytical and evaluative thinking, these benefits are not automatically realized. The success of this model in fostering higher-order cognitive skills depends largely on the design of the classroom activities and the students' engagement with the preparatory material. If the in-class activities are not sufficiently challenging or if students are unprepared, the potential for developing advanced cognitive skills might not be fully tapped. That is to say, the flipped classroom model, through its emphasis on active learning and student autonomy, has the potential to enhance cognitive processes in learning mathematics. This model contrasts with traditional methods, offering a more engaging and participatory approach to education. However, the efficacy of this model in fostering higher-order cognitive processes also hinges on careful instructional design and the students' active participation in the learning process.

In summary, I have examined flipped classroom in mathematics using Bloom's Taxonomy, a framework categorizing cognitive processes from basic (remember, understand) to complex (apply, analyze, evaluate, create). Students' experiences align with lower and middle levels, reflecting enhanced comprehension, engagement, and problem-solving. They suggest modifications such as varied learning materials, timing implementation for adaptability, and blending traditional and flipped methods for flexibility. These insights demonstrate their cognitive engagement and creative thinking in improving educational approaches.

Thus, the analysis supports Sub-Hypothesis 3 to an extent, suggesting that the flipped classroom method fosters cognitive processes conducive to mathematical problem-solving and critical thinking. However, it is noteworthy that students' engagement in the higher-order cognitive processes of evaluation and create was more evident in their reflections on the teaching method itself rather than in their learning of mathematics. This could indicate that while the flipped classroom excels in engaging students in application and analysis, there might be limitations in nurturing synthesis and evaluation skills, especially if in-class activities are not sufficiently challenging or if students lack prior preparation. This observation indicates that while the flipped classroom can encourage critical engagement with instructional methods, its impact on fostering these advanced cognitive skills within the specific context of learning mathematics remains less explicit.

6.4.5 ZPD

Building upon the theoretical foundation laid in Chapter 4, this analysis delves into the practical application of Vygotsky's ZPD within the flipped classroom context. The focus is on how the flipped classroom model fits within the students' ZPD, providing the necessary support for enhancing mathematical learning. Through the lens of Sub-Hypothesis 4, this section critically examines student experiences and responses to identify how well the flipped classroom aligns with the principles of ZPD. This approach will help us understand if and how this educational model facilitates learning by appropriately challenging and supporting students.

The majority of students showed positivity towards the flipped classroom method. For example, S5 and S9 exhibited a clear preference for the flipped classroom approach, finding it instrumental in grasping more complex mathematical concepts. This preference suggests that video learning acts as an effective scaffold within their ZPD, providing a necessary support mechanism that enhances understanding before classroom engagement. Such pre-class preparation not only aligns with Vygotsky's theory but also underscores the potential flipped classroom to cater to individual learning needs within their developmental zone. For example, S5 appreciated the video component for making complex topics easier to understand, highlighting the effectiveness of visual and auditory learning aids in the ZPD: "I think it was good that it was a video we had to watch. She explains it in such a good way that it is easier to understand." S9 also found value in the self-paced nature of video learning, allowing for repeated viewing and better comprehension, a key aspect of scaffolding within the ZPD: "I just thought it was really cool that I could control the time myself, and I could rewind the videos a bit if I did not understand something." These insights suggest that the flipped classroom model, with its emphasis on pre-class video learning and allowing students control over their learning pace, effectively fits within students' ZPDs, enhancing their mathematical understanding.

In addition, students like S13 and S20 exemplify the positive impact of collaborative learning environments. As previously mentioned, S13 particularly appreciated group activities and interactive problem-solving at the blackboard, finding it easier to discuss, understand, and solve problems with peers. Similarly, S20 expressed enjoyment in group activities, particularly enjoying tasks where they could discuss and work out solutions together. This peer collaboration, supported by teacher guidance, effectively bridged gaps in understanding, aligning with the principles of ZPD by providing the necessary scaffolding and support within students' learning journeys.

The role of scaffolding is evident in pre-class materials like videos, bridging gaps between current abilities and potential learning. This scaffolding aligns perfectly with Vygotssky's ZPD concept, as it provides the necessary support to enhance understanding before students engage in classroom activities. As seen with students like S5 and S9, these materials allow for a self-paced approach to learning, enabling students to revisit and reinforce complex concepts, thereby fostering a deeper and more personal understanding. Additionally, the flipped classroom supports metacognitive development, as students actively engage in managing their learning journey. Metacognition refers to the awareness and understanding of one's own thought processes. In the context of a flipped classroom, students are not just passive recipients of information; instead, they are actively engaged in managing their learning journey. Through activities like watching videos, taking notes, and reflecting on their understanding, students like S13 and S20 are constantly evaluating and regulating their learning process. This self-reflection is a critical aspect of metacognition, aiding their development within their ZPD. For example, students' ability to control their learning pace with video lectures and the opportunity to engage in meaningful discussions and problem-solving activities during class allow them to reflect on their understanding and strategies. This process not only reinforces their comprehension of mathematical concepts but also encourages them to be more self-aware and reflective learners. By actively engaging with the learning material both before and during class, students are consistently assessing their understanding and identifying areas that require further exploration or clarification. This self-reflection is critical in aiding their development within their ZPD.

However, individual variations in the ZPD among students are evident. While some students found this method to be highly beneficial, aligning with their ZPD, others experienced challenges or found it less effective. This reflects the unique nature of each student's ZPD, where the same teaching method can be more conducive to learning for some students than for others. It underscores the importance of recognizing and addressing these individual differences in educational settings to optimize learning outcomes. Here among them were students like S2 and S11. For example, S2 found the flipped classroom method stressful due to the amount of homework and other commitments. They preferred traditional classroom settings where they could take notes during lectures. S2 felt that watching videos at home was challenging due to distractions and suggested that reading texts might be more effective for some students: "I found it okay not to sit and listen to the teacher for an entire hour, but it became really stressful at home because I did not have time for anything else, just homework and assignments." Another example, is S6, who found it challenging to maintain concentration while studying at home due to various distractions, like household chores and other assignments. They expressed a preference for classroom learning, where they could take notes during lectures, finding this method more conducive to concentration. S6 also mentioned that while videos are a comfortable format compared to text, they did not find them particularly engaging. They believed that being introduced to flipped classroom methods from the start of their education might have been more effective in adapting to this style of learning.

S11, on the other hand, had a better experience and suggests that the flipped classroom model can be particularly effective for students who have a foundational understanding of the subject matter. It allows these students to deepen their knowledge and develop additional skills that are pertinent to their field of study, as they appreciated being more independent and felt that the method helped them engage more with the content: "I liked the method; it was fine, especially because I already knew many of the things. It would have been different with a harder topic. The method helped me get better with software like Maple, which we used more than before. I felt more independent." However, S11's caveat about the potential challenges with more complex topics indicates that the effectiveness of this method may vary depending on the difficulty level of the content. This observation highlights the importance of considering the complexity of subject matter and the students' prior knowledge when implementing flipped classroom strategies, ensuring that the approach remains within the students' ZPD and effectively supports their learning journey.

These excerpts showcase the individual differences in how students perceived the flipped classroom model, reflecting their unique ZPD. S6 and S2's cases underscore the importance of recognizing and accommodating diverse learning needs and preferences in educational approaches, emphasizing that ZPD is not one-size-fits-all. The diverse responses of students to the flipped classroom model highlight the need for adaptable instructional strategies that cater to individual ZPDs. To address this, educators could consider varying the content and format of pre-class materials, perhaps incorporating a mix of videos, readings, and interactive modules. Offering additional support during in-class activities, like differentiated tasks or one-on-one guidance, can also help address diverse learning needs. Such flexibility ensures that each student receives the appropriate level of challenge and support, aligning with their unique developmental zone.

Nevertheless, students' familiarity and comfort with traditional methods of learning significantly influence their adaptation to flipped classrooms. Habitual learning patterns and established comfort zones play a significant role in how new educational approaches are perceived and assimilated by students. This aspect is particularly relevant when considering ZPD, as it not only involves the cognitive ability to grasp new concepts but also the adaptability to new methods of learning. Therefore, students' initial resistance or struggle with the flipped classroom model could partly stem from the discomfort of stepping out of their familiar learning habits, highlighting the need for a gradual and well-supported transition to new educational methodologies. To facilitate a smoother transition to the flipped classroom model, particularly for students accustomed to traditional teaching methods, educators could implement a phased approach. Gradually introducing elements of the flipped classroom, such as starting with shorter video segments or combining traditional and flipped methods, can ease students into this new learning environment. Providing structured guidance in the initial stages, such as clear instructions on how to engage with pre-class materials or offering support in developing self-study skills, can also be beneficial.

Considering the varied effectiveness of the flipped classroom in addressing different

ZPDs, future research could focus on differentiating instruction within this method. Additionally, research on methods to ease the transition for students more accustomed to traditional learning environments could provide valuable insights into enhancing the effectiveness of the flipped classroom method across diverse student populations.

Finally, integrating student feedback is essential for continuously refining the flipped classroom model, such as S5's and S9's positive experiences with video learning, can guide educators in selecting or designing content that is most effective for their student cohort. Similarly, feedback on in-class activities, as highlighted by S13 and S20's enjoyment of collaborative problem-solving, can inform the development of more engaging and interactive classroom sessions. Moreover, the feedback from students like S2, who found the flipped classroom method stressful due to the volume of homework, and S11, who appreciated the method for enhancing their independence and engagement, also provides invaluable insights into how the flipped classroom can be optimized. Thus, the process of integrating student feedback should be ongoing and dynamic, where educators can regularly solicit feedback through surveys, open discussions, or reflective assignments. This not only provides a continuous stream of insights into student experiences but also fosters a learning environment where students feel their opinions are valued and contribute to their educational journey.

In summary, the exploration of the flipped classroom model in the context of Vygotsky's ZPD has yielded insightful findings that align with Sub-Hypothesis 4. This hypothesis posits that the flipped classroom effectively utilizes the ZPD to provide targeted support and enhance students' mathematical learning. My analysis has shown that the flipped classroom model, through its integration of pre-class video learning and in-class collaborative activities, aligns well with the principles of ZPD. The model effectively acts as a scaffold, aiding students like S5 and S9 in understanding complex mathematical concepts. Furthermore, the emphasis of the model on metacognition encourages students to reflect on and regulate their learning processes, an essential aspect of development within the ZPD. This reflective practice was evident in students' ability to control their learning pace and engage in meaningful classroom discussions, thus enhancing their mathematical understanding.

However, the analysis also acknowledges the diverse experiences of students within the flipped classroom. While some, like S11, found the model highly effective, others, such as S2 and S6, faced challenges, highlighting the importance of adapting instructional strategies to cater to individual ZPDs. This diversity underscores the need for a flexible approach in the flipped classroom model to effectively meet the varied learning needs and preferences of students.

In light of these findings, Sub-Hypothesis 4 is supported to an extent. The flipped classroom model does utilize the ZPD concept effectively, providing necessary support and fostering mathematical learning. However, this effectiveness is contingent upon the adaptability of instructional strategies to individual student needs and the effective integration of scaffolding and metacognitive practices.

6.4.6 Overall Summary

To encapsulate the outcomes of the post-implementation data analysis, the effects of the flipped classroom method on mathematical qualifications, motivation, cognitive load, cognitive processes, and the ZPD in class 1.a are multifaceted and diverse. The analysis reveals a spectrum of motivational responses among students. While the self-paced nature of the flipped classroom enhanced motivation for some, it posed challenges in time management and adaptation to autonomy for others, particularly those accustomed to traditional, teacher-led methods. This contrast necessitates the development of gradual transition strategies to nurture autonomy, competence, and relatedness, essential components of intrinsic motivation as outlined in SDT. Moreover, student responses indicate a wide range of experiences in managing cognitive load under the flipped classroom method. Those with a background in traditional educational settings showed varied capacities in handling intrinsic, extrinsic, and germane loads, pointing to the need for instructional designs that accommodate different cognitive abilities and ease the transition from traditional to flipped classroom.

In addition to that, the analysis, framed by Bloom's Taxonomy, suggests alignment predominantly at lower to middle cognitive levels, with limited evidence of higher-order cognitive skills, based on the students' self-report of their cognitive process. This observation is particularly pronounced in students more comfortable with traditional classrooms. Strategies to introduce and support advanced cognitive tasks gradually are thus essential to fully leverage the potential of the flipped classroom method. Moreover, it is shown that the flipped classroom method generally aligns well with students' ZPDs, yet this alignment varies based on individual adaptation to and familiarity with traditional teaching methods. Customized support and scaffolding are crucial for students transitioning from traditional methods of teaching, highlighting the importance of adaptive teaching approaches within the flipped mmodel.

In summary, while the transition from traditional to flipped classroom models in class 1.a shows promise in enhancing mathematical qualifications, it also brings forth challenges related to students' pre-existing comfort with traditional, teacher-centric educational approaches. These challenges influence students' cognitive, motivational, and developmental experiences within the new learning paradigm. The analysis underscores the importance of implementing thoughtful transition strategies by educators, taking into account students' prior learning experiences and providing necessary support to effectively adapt to this innovative teaching methodology.

In synthesizing the findings from the post-implementation analysis with relevant academic literature, it becomes evident that the relationship between student preparedness and the efficacy of the flipped classroom model is complex and multifaceted. My analysis indicates that in the context of class 1.a, students generally acknowledged the importance of pre-class preparation for effective participation and learning in the flipped classroom setting. This perspective aligns with the emphasis on student preparedness in flipped classrooms, as highlighted by Sun and Xie (2020). They observed that well-prepared students tend to perform better in in-class activities and have higher academic achievements, underscoring the significance of readiness in the flipped learning environment.

However, this emphasis on preparation contrasts with the observations made by Novak et al. (2017). In their study, underprepared students perceived greater benefits from the flipped classroom approach. This discrepancy might reflect a difference in student awareness or the educational contexts of the two studies. While the findings of Novak et al. (2017) suggest a possible lack of clarity among students about the role of preparation in flipped classrooms, my analysis indicates a greater recognition among class 1.a students of its importance for active engagement and learning success. This contrast raises important considerations for educators implementing the flipped classroom method. It suggests the need for clear communication about the expectations and benefits of pre-class preparation. Additionally, it highlights the importance of providing resources and support to help students develop effective preparatory habits, especially for those transitioning from more traditional, teacher-led instructional models.

The divergence in findings between my analysis and that of Novak et al. (2017) also points to the necessity of contextualized approaches in the application of flipped classroom methodologies. Factors such as student demographics, prior exposure to active learning methods, and the specific subject matter being taught can all influence how students perceive and benefit from flipped learning. Therefore, while the flipped classroom model holds promise for enhancing student engagement and learning outcomes, its implementation must be thoughtfully tailored to the unique needs and backgrounds of the student population.

Nevertheless, in examining the handling of cognitive load within the flipped classroom context, parallels can be drawn between the findings of the post-implementation analysis and the observations made by Novak et al. (2017). The study by Novak et al. (2017) highlighted a spectrum of student perceptions in regards to the effectiveness of the flipped classroom method. This spectrum suggests that students not only varied in their perceived benefits of the flipped classroom method but also experienced differing levels of cognitive load. In my analysis, it was noted that students more accustomed to traditional educational methods faced particular challenges in managing the cognitive load inherent in the flipped classroom method. This observation aligns with the findings of Novak et al. (2017), suggesting that cognitive load is a variable experience influenced by individual student backgrounds and learning habits. Novak et al. (2017) implied that to effectively cater to a diverse student population, differentiated instructional designs are necessary. This implication resonates strongly with the findings of this master's thesis; the variability in cognitive load management among students in class 1.a underscores the need for adaptive teaching strategies that can accommodate different levels of preparedness and familiarity with self-directed learning approaches.

In essence, the insights from Novak et al. (2017) complement our findings by reinforcing the notion that a one-size-fits-all approach is insufficient in the diverse landscape of student learning experiences. Instead, a nuanced understanding of cognitive load and its management is crucial for a successful implementation of flipped classroom, particularly in settings where students are transitioning from traditional to more active learning models.

Nevertheless, the focus was also placed on the impact of the flipped classroom on cognitive processes. This analysis predominantly revealed an alignment with lower to middle levels of Bloom's Taxonomy, a trend more noticeable among students accustomed to traditional classrooms. This finding primarily stems from subjective assessments and observations. In stark contrast, the study conducted by Sun and Xie (2020) employed a more robust methodology, utilizing a combination of trace data and self-report data. This approach provided a more objective analysis of pre-class preparedness and its subsequent effect on academic outcomes. While Sun and Xie (2020) underscore the tangible benefits of student readiness, this thesis suggests that mere preparedness might not be adequate to engage students in higher-level cognitive tasks, such as analysis and evaluation. This disparity underscores the necessity for more nuanced strategies within the flipped classroom model to foster these higher-order thinking skills. Potential strategies could include the integration of complex problem-solving tasks and the promotion of critical discussions, with a specific focus on aiding students in the transition from traditional to flipped classroom environments. Such an approach would not only address the gap in achieving higher cognitive levels but also enhance the overall effectiveness of the flipped classroom model.

Overall, the flipped classroom showed nuanced dynamics in class 1.a, revealing the intricate interplay of student motivation, cognitive load, cognitive processes, and their ZPD. It underscores the critical role of tailored teaching strategies in navigating these dynamics, particularly for students accustomed to traditional learning environments. As we transition to the next section, I delve deeper into the comparative analysis of pre- and post-implementation data, further exploring how these insights contribute to a more holistic understanding of the flipped classroom's impact on educational outcomes.

6.5 Comparing the Pre- and Post-Data

This section assesses the impact of the flipped classroom method on class 1.a's mathematical qualifications by comparing pre- and post-implementation data. Initially, students showed a preference for structured learning, a reflection of traditional teaching methods that align with the SDT aspects of competence and relatedness. Despite this, a notable number of students expressed a desire for more interactive and engaging learning experiences. They pointed out the limitations of long lectures and passive learning, suggesting a need for more practical applications and dynamic classroom interactions. This initial feedback set the stage for exploring alternative teaching methods that could address these expressed needs, suggesting a latent need for greater autonomy in learning, an essential component of SDT.

After implementing the flipped classroom, many students showed enhanced understanding and engagement based on their self-report, aligning with the autonomy and competence aspects of SDT. Students like S5 and S9, embraced the flipped classroom for its autonomy and active participation, demonstrating an alignment with intrinsic motivation and competence which a crucial aspects of SDT. However, challenges emerged, particularly in managing cognitive load and adapting to self-directed learning were notable, as observed with students like S2 and S6. The increase in extrinsic load, due to the demands of self-study and the management of multiple assignments, highlights the CLT's relevance in this context. This indicates the necessity of instructional design that carefully considers cognitive load management to facilitate effective learning.

Additionally, the analysis of the flipped classroom method in class 1.a, through the lens of Bloom's Taxonomy, suggests that while there was an enhancement in understanding and engagement in mathematics, the progression to higher-order cognitive processes was varied among students. While the method provided opportunities for applying and analyzing concepts, particularly through active participation and self-paced learning, the evidence of students consistently reaching the stages of evaluation and creation was less pronounced. This indicates that while the flipped classroom facilitated a move beyond the foundational stages of remembering and understanding for some students, the consistent engagement in higher-order thinking skills varied, reflecting the diverse responses and adaptability of students to this teaching method.

Nevertheless, in the post-implementation, the flipped classroom approach showed positive outcomes in enhancing students' understanding and engagement in mathematics, reflecting differences in students' ZPDs. While some students, such as S9, found the method to enhance their understanding and engagement, others like S2 struggled with the self-directed aspect. This variability underscores the need for tailored approaches that consider the unique ZPD of each student, offering appropriate scaffolding and support to optimize learning. However, as discovered in the thick description of the post-interviews, one of the students admitted that they initially were concerned about the self-study part, worried that they would struggle in understanding the concepts independently. It turned out that the student was able to understand the concepts within descriptive statistics better, as the student was able to focus on the videos in their own pace and thereafter working together with their peers in solving problems using the concepts they have prepared at home, letting them to fully grasp the concept.

Overall, the transition to the flipped classroom method revealed varied student responses, underlining the importance of individual learning preferences. While some thrived in this new environment, others, accustomed to traditional methods, faced disorientation. In analyzing the individual responses to the flipped classroom model, it is important to consider the influence of students' prior experiences with traditional teaching methods, as this background plays a significant role in shaping their adaptability to and perceptions of the flipped classroom. Thus, for students deeply accustomed to traditional teaching, a sudden shift to the flipped classroom method can be disorienting. It suggests the need for a gradual introduction of flipped methods, perhaps starting with blended methods that combine traditional and flipped elements. This approach can help ease the transition, allowing students to gradually develop the skills and confidence needed for more self-directed learning. Concerning cognitive load, the challenge for educators is to balance the enhancement of learning with manageable cognitive demands. This is particularly crucial for students new to self-directed learning.

In essence, these diverse experiences and the evolving nature of students' engagement with the flipped classroom, the integration of regular student feedback becomes paramount. This feedback, essential for dynamic teaching methods, offers real-time insights into students' experiences and challenges. Actively soliciting and responding to this feedback allows educators to adapt the instructional design to better align with students' cognitive load, motivation levels, and their zones of proximal development. Such a responsive approach not only enhances the effectiveness of the flipped classroom but also fosters a learning environment where students feel their voices are heard and valued. Regular feedback loops can lead to iterative improvements in the method, ensuring that it evolves to meet the diverse and changing needs of students in class 1.a. Integrating these insights with the principles of SDT, CLT, Bloom's Taxonomy, and ZPD, can further refine this approach, creating a more inclusive and effective learning environment. Therefore, harnessing student feedback will be crucial in continually optimizing the flipped classroom model to better serve the educational objectives and cater to the unique learning trajectories of each student.

7 Synthesis and Conclusion

In this concluding chapter, I critically examine the flipped classroom model in a Danish upper secondary mathematics class, focusing on exploring the validity and implications of the formulated hypotheses, focusing on the effectiveness of the method in enhancing mathematical qualifications and addressing diverse student needs. Finally, This chapter concludes with a summary of key findings and reflections on the broader implications of the study, thus wrapping up the master's thesis with comprehensive insights into the transformative potential of the flipped classroom method in mathematics education.

7.1 Discussion

This discussion critically examines the impact of the flipped classroom method in a specific Danish upper secondary mathematics class. It delves into the complexities of implementing this innovative teaching approach, while specifically assessing the validity and implications of the formulated hypotheses. Moreover, this discussion assesses how this innovative teaching approach aligns with established pedagogical strategies and curriculum guidelines, focusing on its effectiveness in enhancing mathematical qualifications and addressing student needs.

In terms of the key findings in the pre-analysis, the initial responses to traditional teaching methods showed a mix of satisfaction and desire for more interactive learning. Students' initial apprehension towards the flipped classroom model revealed a spectrum of learning preferences, influenced by their experiences with traditional teaching methods. This variation in readiness for new learning methods like the flipped classroom suggests a deeper challenge. As indicated by Hodge (2006), there are inherent difficulties in transitioning from traditional mathematics instruction to more student-centered approaches. Such a shift, being a departure from the norm, often encounters resistance from students and possibly even teachers (Hodge, 2006). This is primarily because student-centered methods, such as discovery-based learning, can initially feel uncomfortable and demand a significant amount of work and adaptation from both students and teachers (Hodge, 2006).

Furthermore, the mixed reactions to the flipped classroom suggest that while some students are open and adaptable to new learning methods, others show resistance or apprehension. This underscores the importance of recognizing and accommodating diverse learning needs and preferences. Moreover, the students' familiarity with traditional teaching methods can significantly influence their readiness for and response to innovative teaching methods like the flipped classroom. Adapting to a different approach, especially in a discipline like mathematics, which often has a strong tradition of conventional teaching methods, can be particularly challenging. Therefore, it is crucial to approach such transitions with an understanding of these inherent challenges and with strategies in place to support students and teachers through this change. This familiarity can create a comfort zone with traditional methods, making adaptation to innovative approaches more difficult. Recognizing this factor is crucial when implementing new teaching strategies, as it highlights the need for gradual transition, proper orientation, and additional support to help students acclimate to different learning environments.

Some of the difficulties which the students highlighted, and which also could be the result of their familiarity with the traditional methods, include their concerns about cognitive load, time management, and adapting to self-directed learning. This emphasize the need for supportive measures when implementing new teaching strategies. Thus, open communication and regular feedback mechanisms can help in understanding and addressing students' concerns. This can lead to continuous improvement of the teaching methods, making them more effective and acceptable to a broader range of students.

Based on the post-data analysis, the flipped classroom method demonstrated positive outcomes in student engagement and understanding. Yet, challenges in managing cognitive load and adapting to self-directed learning were notable and which suggests the need for a more nuanced approach that accommodates individual learning preferences. Then again, these challenges could be due to the students' constant familiarity with the traditional methods only, making it harder for them to adopt this new way of learning. However, as the flipped classroom model was introduced, a shift was witnessed in terms of perceptions; the post-data analysis illuminated how the model fostered enhanced understanding and engagement in mathematics. Students began to value the autonomy and active involvement in their learning process, a significant departure from their initial apprehensions. This shift is exemplified by students who overcame their initial worries about self-study and discovered a newfound ability to grasp complex concepts at their own pace, ultimately fostering deeper comprehension.

However, this evolution was not uniform across the cohort. The flipped classroom presented distinct challenges for some students, particularly in managing cognitive load and adapting to a more self-directed learning environment. These challenges echo their initial concerns but also point to the diverse learning needs within the class. While some students thrived, finding the method more motivating and engaging, others grappled with time management and external distractions. This disparity in experiences underlines the need for a tailored approach to flipped classroom implementation.

The evolution from pre- to post-implementation data revealed a significant shift in student perceptions. While initial apprehensions were evident, many students adapted to the flipped classroom, appreciating its benefits for active learning and autonomy. However, the effectiveness varied, emphasizing the need for tailored approaches. The findings align with SDT, showing that the flipped classroom can enhance intrinsic motivation through increased autonomy and engagement. However, not all students responded equally, indicating the varying nature of intrinsic and extrinsic motivational factors in such an educational setup. Moreover, the diverse experiences with cognitive load management echo the principles of CLT. While some students effectively managed intrinsic and extrinsic loads, others felt overwhelmed, suggesting the need for more tailored instructional designs to balance cognitive demands.

Nevertheless, in the context of class 1.a, the analysis through the lens of Bloom's Taxonomy indicated that the flipped classroom effectively supported lower to middle-level cognitive processes. However, the effectiveness of the method in fostering higher-order cognitive skills such as evaluation and creation was less evident. This lack of clarity in impact can be attributed partly to the nature of the data collected. The primary reliance on self-reported data from students introduces inherent limitations, as these responses may not fully capture the depth of engagement with these advanced cognitive processes. Self-assessment, while valuable, can be influenced by factors such as personal biases, interpretation of instructional experiences, and varying levels of self-awareness. Consequently, the students' perceptions of their engagement in evaluation and creation activities might not accurately represent their actual proficiency in these areas. This suggests a need for more objective measures or diverse data collection methods to gain a clearer understanding of the impact of the flipped classroom on higher-order thinking skills

Additionally, the Pygmalion effect is a vital consideration in this context. Teacher expectations and attitudes towards the flipped classroom model could significantly influence how students perceive and report their cognitive processes. If teachers convey high expectations and a positive attitude towards the development of higher-order thinking skills within the flipped classroom, students might self-report enhanced abilities in these areas, even if their actual proficiency is not commensurate.

Thus, future research should incorporate more objective measures to validate these self-reported findings. Performance assessments that specifically target higher-order cognitive skills, detailed classroom observations focusing on student engagement in complex problem-solving and critical thinking tasks, and thorough analyses of student work can provide more concrete evidence of the extent to which the flipped classroom model fosters advanced cognitive development. Hence, these objective measures might include:

- **Performance Assessments:** Analyzing students' academic performance through tests and assignments can offer objective data on their learning outcomes. This would help in assessing the efficacy of the flipped classroom method in enhancing students' understanding and mastery of mathematical concepts.
- **Classroom Observations:** Observing classroom dynamics, student-teacher interactions, and student engagement during both pre-class and in-class activities can provide insights into how effectively the flipped classroom is being implemented and experienced.
- **Analysis of Student Work:** Examining the quality and depth of student work, such as problem-solving approaches and the complexity of their answers, can offer objective evidence of cognitive processes and higher-order thinking skills development.

In fact, the quizzes administered throughout the mathematics lesson where the flipped classroom method was implemented, provide such an objective measure, offering insights into students' abilities to apply and analyze mathematical concepts. For example, their responses to questions requiring the calculation and interpretation of statistical measures like frequency, relative frequency, and cumulative frequency, and their application to real-world contexts, reveal varying levels of understanding and application skills. Some responses indicate a solid grasp of these concepts, while others point to areas needing further instruction. This discrepancy between self-reported understanding and actual performance on quizzes highlights the complexity of accurately assessing cognitive processes through self-report measures alone. Therefore, using quizzes adds an additional layer of data, complementing self-reported insights from students, as these quizzes provide a snapshot of students' abilities to apply and analyze mathematical concepts. However, they are not comprehensive enough to fully capture the depth and breadth of cognitive processes involved, which is why the above suggestion of further research is highlighted.

Moreover, future research should explore how teacher beliefs and behaviors, within the flipped classroom context, affect student engagement in and self-perceptions of higher-order cognitive tasks. Such studies shed light on the extent to which the Pygmalion effect shapes student responses in self-reported data, offering a more nuanced understanding of the impact of flipped classroom on cognitive skill development. In such cases, a longitudinal study is essential to understand the full impact of the flipped classroom model, particularly for students accustomed to traditional teaching methods.

While the impact of the flipped classroom model on developing higher-order cognitive skills for mathematical problem-solving warrants further exploration, it is notable that students did engage in such advanced cognitive processes when evaluating the flipped classroom method and suggesting improvements. This engagement demonstrates their ability to analyze the teaching method critically, evaluate its effectiveness, and creatively propose modifications – skills that align with the higher levels of Bloom's Taxonomy. For many students accustomed primarily to traditional teaching methods, the flipped classroom model can be their first significant exposure to an alternative educational approach. This new experience challenges them to actively evaluate and reflect on their own learning preferences and processes, potentially for the first time. Traditional methods often do not emphasize self-reflection or the critical evaluation of teaching methodologies to the same extent. Therefore, the process of assessing the flipped classroom and suggesting improvements could represent a significant step in developing their cognitive skills and awareness of their learning preferences. This shift from a passive to an active role in their education journey can be both challenging and enlightening, offering valuable insights into their personal learning needs and preferences.

Thus, based on some of the students' suggestions of improvement lead to the discussion of the effectiveness of the flipped classroom model being significantly influenced by the nature of its preparatory materials. As students have suggested the inclusion of reading texts alongside or in place of videos, future research should explore how different types of preparatory materials can cater to individual learning preferences and needs. This can be done through, for example, conducting studies that compare the impact of different types of materials like videos, reading texts, and interactive modules on student engagement and understanding in the flipped classroom. In addition to that, investigating how various materials support different aspects of learning, such as comprehension, application, and critical thinking, to tailor the flipped classroom experience effectively. As previously mentioned, implementing longitudinal studies can again be crucial in order to assess the long-term effects of diverse preparatory materials on student learning outcomes and cognitive development, as these studies would provide deeper insights into how students respond over time to different instructional approaches within the flipped classroom model.

Nevertheless, recognizing that the ability to read and understand mathematical texts is an essential skill, which future research should delve into how incorporating reading materials in the flipped classroom impacts student learning. This could involve, exploring how the integration of reading texts in flipped classroom preparation aids in developing students' comprehension skills, particularly for complex mathematical concepts.

Another suggestion proposed by the students was for a gradual introduction of the flipped classroom model throughout students' educational journey, which can highlight another area ripe for investigation. The timing of implementing the flipped classroom method is a critical factor, especially for students primarily exposed to traditional, lecture-based teaching methods. Thus, it can be argued that gradual and phased implementation strategies are crucial for these students, who may require more time and support to adapt to the more self-directed learning approach inherent in flipped classrooms. That is to say, future research should focus on the efficacy of gradually introducing the flipped classroom model, particularly for students accustomed to traditional teaching environments. Such an approach could involve starting with a combination of traditional methods and flipped classroom, where the conventional lecture-based approach is gradually supplemented with flipped classroom elements. This blended approach can help ease the transition for students, providing a familiar structure while introducing new learning paradigms. Thereafter, systematically increasing the proportion of flipped classroom elements over time, allowing students in a gradual shift to become increasingly comfortable with and adept at self-directed learning, minimizing the shock of a sudden transition from traditional methods, which in fact was the case of some students in this master's thesis.

Then again, this leads to the desire of conducting a longitudinal tracking, in order to conduct long-term studies to monitor how students with a traditional education background adjust to and engage with the flipped classroom model. This could help identify the stages in their educational journey where they are most receptive to this new method.

In terms of the students' ZPD, it was evident that there was a varied effectiveness of the flipped classroom in catering to individual ZPD. While some students thrived within this new learning environment, others required additional support, underscoring the importance of scaffolding and adaptable teaching methods.

Additionally, the findings align with signature pedagogies and the Danish Ministry of Children and Education's guidelines, advocating for dynamic learning, conceptual understanding, and collaborative practice. The flipped classroom model fostered these aspects, albeit with challenges in meeting diverse student preferences. Reflecting on the impact of the flipped classroom method, a notable shift was observed in students' engagement with mathematics. The approach aligns with expected outcomes of signature pedagogies, where students displayed enhanced problem-solving skills and a deeper conceptual understanding based on their self-report. Yet, challenges in adapting to self-directed learning underscore the need for more supportive measures to fully realize these pedagogical goals.

Lastly, comparing the existing studies on the flipped classroom method, namely

Novak et al. (2017) and Sun and Xie (2020), with the findings of the analysis in this master's thesis, reveals similarities in the impact in student preparedness on learning outcomes. My analysis extends these insights to a younger demographic, contributing to understanding the efficacy of the flipped classroom method at different educational stages. The analysis of the post-data echoes the emphasis, by Sun and Xie (2020), on student readiness, highlighting how varied levels of pre-class preparation among students influenced their overall engagement and learning outcomes. Just like their findings, I observed that well-prepared students reaped greater benefits from the flipped classroom, aligning with the theory that deeper engagement in pre-class activities correlates with enhanced academic performance. Furthermore, the diverse student responses captured in the analysis parallel those reported by Novak et al. (2017); individual learning preferences and prior exposure to traditional teaching methods significantly shaped students' experiences and perceptions of the flipped classroom, thereby offering insights into its nuanced impact at the upper secondary education level.

In essence, a flexible and phased implementation of the flipped classroom is recommended. This approach should blend traditional and innovative methods such as the flipped classroom, catering to diverse learning preferences and ensuring comprehensive development of mathematical competencies in line with Danish upper secondary education objectives. This is also to better comprehend students who are only familiar with the traditional methods, as it can help them to better move beyond their comfort zone.

7.2 Conclusion

This master's thesis explores the transformative potential of the flipped classroom method in Danish upper secondary mathematics education, guided by the ethos that "When the educational environment provides optimal challenges, rich sources of stimulation, and a context of autonomy, this motivation wellspring of learning is likely to flourish" (Deci, 1985, s. 245). This master's thesis addresses the pivotal question: What qualifications should students gain from mathematics lessons in a Danish upper secondary school, and can the flipped classroom method optimize these qualifications within a particular class, and if so, how?

Undertaking this research has been a journey of discovery and learning, not just in understanding the dynamics of the flipped classroom model but also in appreciating the complexities of educational change. This process has been illuminating, challenging preconceived notions, and deepening my understanding of what constitutes effective teaching and learning in contemporary education. It has reinforced my belief in the necessity for educational systems to evolve and adapt, ensuring they meet the needs of a diverse and changing student population.

The findings of this master's thesis clarified the significant impact of the flipped classroom model on student engagement, understanding, and overall mathematical qualifications. This pedagogical shift, from traditional teacher-led instruction to a more student-centered paradigm, resonates with global educational aspirations for fostering greater student autonomy and active involvement in learning. The study confirms the effectiveness of the flipped classroom in creating interactive learning environments but also highlights the diversity in student adaptability and the challenges posed by self-directed learning. Incorporating the principles of SDT and CLT, the thesis underscores the crucial role of intrinsic motivation and balanced instructional design in the learning process. While the flipped classroom adeptly supports lower to middle-level cognitive tasks, its efficacy in nurturing higher-order thinking skills is a subject for further exploration.

The research conducted in this master's thesis has unearthed key findings about the impact of the flipped classroom model in a Danish upper secondary mathematics class. Among the key findings are the following:

- A primary discovery is the students' self-report in terms of significant elevation in student engagement and understanding of mathematical concepts. According to most of them, the flipped classroom model, shifting from traditional teacher-led instruction to student-centered learning, has proven effective in fostering a more interactive and autonomous learning environment.
- In terms of their adaptability to the self-directed-learning approach, the research revealed varied student responses to this new learning paradigm. Some students thrived in the self-directed environment while others faced challenges, underscoring the need for adaptive teaching strategies that cater to diverse learning needs.
- In the light of the students' cognitive process, the study found that the flipped classroom model effectively supports lower to middle-level cognitive tasks. However, its role in facilitating higher-order thinking skills like evaluation and creation remains an area ripe for further investigation.

These insights not only confirm the effectiveness of the flipped classroom in certain domains but also highlight the complexities and nuances inherent in its implementation. The findings reflect a multifaceted impact of the flipped classroom method, providing valuable insights for educators in mathematics education.

Looking forward, several key areas for future research emerge as critical to deepening our understanding of the flipped classroom impact on mathematics education:

- **Longitudinal Studies:** Conducting longitudinal research is essential to assess the long-term effects of the flipped classroom on students' cognitive development, particularly higher-order thinking skills. Such studies will reveal how sustained exposure to this teaching method influences students' learning trajectories and adaptability, especially for those initially accustomed to traditional classroom settings.

- **The Effectiveness of Various Preparatory Materials:** Investigating the effectiveness of various preparatory materials in the flipped classroom will shed light on catering to diverse learning needs. This investigation could assess how these materials cater to diverse learning needs and preferences and their impact on enhancing student engagement and understanding. Such studies contributes with understanding the differential impacts of these materials, which can guide educators in designing more effective and inclusive flipped classroom experiences.

- **Phased Implementation Strategies:** Research on phased implementation strategies, especially for students transitioning from traditional teaching methods, will help identify effective ways to support students in adapting to self-directed learning, minimizing resistance, and maximizing engagement. This approach would help in understanding the most effective ways to support students in adapting to self-directed learning within the flipped classroom framework as well as facilitating a smooth transition to new learning paradigms and ensuring their acceptance and success.

- **Influence of Teacher Expectations and Attitudes:** Examining the role of teacher expectations and attitudes, including the Pygmalion effect, is important in both subjective and objective research contexts. This line of inquiry will reveal how teacher perceptions and behaviors influence student engagement and cognitive task performance in flipped classrooms. Understanding this dynamic can help in devising strategies to optimize teacher-student interactions for improved learning outcomes.

- **Incorporation of Objective Data:** Future studies should strive to include more objective measures, such as performance assessments, classroom observations, and analysis of student work. Such an approach will complement subjective self-reports and provide a more balanced and comprehensive understanding of the flipped classroom effectiveness. Particularly, it is important in validating the impact of the flipped classroom model on various aspects of student learning, reducing the reliance on potentially biased self-reports.

In essence, these future research avenues are vital for a nuanced understanding of the flipped classroom potential in redefining mathematics education. They will offer valuable insights for optimizing this teaching model across diverse student populations and educational settings.

In conclusion, this master's thesis contributes to the evolving landscape of pedagogical strategies in mathematics education, aligning closely with the Danish Ministry of Children and Education's focus on fostering diverse and adaptive teaching methods. This alignment underlines the importance of developing strategies that are not just tailored to varying student needs but are also designed to effectively prepare students for the increasingly complex academic and real-world challenges they will face. Although this master's thesis focused on a specific Danish upper secondary mathematics class, it presents insightful findings on the application and implications of the flipped classroom model. While the research was conducted within the confines of one class, it offers a glimpse into the transformative potential of this teaching approach. The study highlights how the flipped classroom can foster enhanced student engagement, deeper understanding of mathematical concepts, and adaptability to self-directed learning. Thus, it can be argued that this master's thesis holds a promise for enhancing students' mathematical qualifications in a manner that nurtures their intrinsic motivation for learning and equips them for the educational landscape of the 21st century.

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