A comparative study of conditional volatility patterns across developed, emerging and frontier markets using GARCH models

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Aalborg University Business School
Education: MSc in finance
Author: Pernille Petersen Bredall (20183015)
Supervisor: Cesario Mateus

Abstract

The globalization and liberalization of the stock market have eliminated cross border trading barriers around the world and have given investors the possibility to diversify their portfolios international. The integration of international stock markets has increased the importance of understanding how volatility differ across different markets to make informed trading decisions. Literature has established certain characteristics about volatility patterns in terms of persistence, asymmetry and mean reversion, but due to international trading there is a need for a comparable study across different markets, which this study aims to fill. The main objective of this study is to measure and examine how volatility patterns in terms of persistence, asymmetry and mean reversion differs across developed, emerging and frontier markets and which market has the fastest speed of mean reversion. For this purpose, we have selected market indices for developed, emerging and frontier markets for the period of 31-12-2012 to 31-12-2022 to examine historical trends and patterns of volatility in a comparative framework using GARCH and TGARCH models and to define if there are predictable patterns, and the half-life volatility model developed from GARCH to measure the speed of mean reversion. The study found evidence of volatility persistence, asymmetry and mean reversion in all three markets which suggests that there exist predictable patterns through financial assets volatility. The study also found that developed markets had the highest overall volatility and strongest volatility asymmetry in the selected period while emerging markets were more persistent. Frontier markets had the fastest speed of mean reversion, while emerging market had the slowest speed of mean reversion.

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1.0 Introduction

Volatility is a statistical measurement used to describe fluctuations or variations in the prices of financial assets. It quantifies the dispersion of the financial asset returns around its mean and measures how much the price changes over time. Volatility is often used to assess the risk associated with holding that asset. Stock return volatility has been extensively researched in the field of finance for many years, as it has significant implication for investment decisions and overall market stability (Adrian & Rosenberg, 2008). The volatility is not directly observable, so its measurement will always be an estimate and numerous of studies have attempted to comprehend it and developed estimators to enhance our ability to measure volatility in financial markets. Better measurements allow us to gain a deeper understanding of its dynamics and, in turn, predict future volatility (Patton & Sheppard, 2015).

In general, the literature describes certain characteristics about the patterns of volatility in terms of persistence, asymmetry and mean reversion. Persistence refers to the tendency of volatility to persist over time, which means that high volatility in financial markets is likely to be followed by high volatility in the future and low volatility vice versa (Adrian & Rosenberg, 2008; Tan & Khan, 2010; Tripathy, 2022; Ding & Granger, 1996; Su & Wang, 2020). Asymmetry refers to the asymmetric response to positive and negative shocks, which means that bad news has a larger impact on volatility than good news. Negative jumps lead to significantly higher future volatility, a phenomenon also known as the leverage effect. This empirical phenomenon states that conditional volatility is negatively related to returns (Adrian & Rosenberg, 2008; Tan & Khan, 2010; Campbell & Hentschel, 1992; Su & Wang, 2020). Mean reversion refers to the tendency of the volatility to return to its long-term mean value over time, which means that shocks to the market have a temporary effect, and it is possible to forecast future movements in stock prices based on historical data behaviour (Bali & Demirtas, 2008; Goudarzi, 2013). The last statement contradicts fundamental financial theory regarding the efficient market hypothesis developed by Eugene Fama, which state, that the market is efficient and current prices are incorporating all available information, and mispricing do not occur in predictable patterns (Fama, 1960). If markets are containing predictable patterns, it might give the investors the opportunity to gain abnormal returns by the understanding of how markets are reacting.

The general described characteristics of the volatility patterns occurring differently across different markets. The same shock in the market does not necessarily have the same influence on different markets since other factors plays a role in the reaction to identical shocks, such as the level of liquidity, the level of financial market development, and technology (Eizaguirre et al., 2004; Gniadkowska-Szymańska, 2017). The different characteristics in different markets are the reason why financial theory segments and draws a line between developed, emerging and frontier markets. Developed markets are often characterized as more liquid and efficient, while emerging markets are often characterized as more volatile (Goetzmann & Jorion, 1999). Frontier markets are described as less developed than the two other markets and is characterized by the presence of higher risk and volatility. At the same time, frontier markets are more exposed to exchange risk than the two other markets (Sukumaran, Gupta & Jithendranathan, 2015).

The global trading liberalization and the globalization of stock markets have eliminated trading barriers between countries and integrated stock markets around the world and have opened the possibilities for traders to diversify their portfolios for reducing risk (Kundu & Sarkar, 2016; Naghavi & Lau, 2016). Modern portfolio theory suggests that diversification benefits lead to higher expected returns especially among markets with lower correlation (Markowitz, 1952).

Even though different markets have country specific factors that make stock markets different from each other, globalization has integrated equity markets, and developed equity markets are sufficiently integrated which has led to high correlations between countries in developed markets (Sukumaran, Gupta & Jithendranathan, 2015). This has caused, that during the past few decades, global equities and specially emerging market equities have experienced an exponentially growth in terms of trading activity, market capitalization and number of listed companies (Mohtadi & Agarwal, 2001). This is because of investors seek to other markets to get benefits of diversifications from markets with lower correlation. The experience of exponentially growth in emerging markets have caused those emerging markets has been more integrated in recent years and the correlation has increased between emerging market itself and with developed markets as well. Frontier markets are smaller in market capitalization and less accessible than emerging markets, and they also have low annual turnover in comparison and the recent access has attracted more investors to frontier markets to seek for diversification benefits as well (Sukumaran, Gupta & Jithendranathan, 2015).

Volatile markets are a common phenomenon in today's global economy. Various factors can have a significantly impact on the stability and performance of different markets. Despite extensive research in market volatility, there is still gaps and challenges that needs to be addressed. Existing literature has predominantly focused on specific markets and only few studies have compared two different groups of markets, and there is need for a comprehensive and comparative analysis of market volatility across different groups markets. Understanding the nature and drivers of the volatility in different levels of developed markets is crucial for investors and financial analysts to make informed decisions. The ability to international investments has increased the importance of understanding volatility patterns across different markets since existing literature are generalizing on developed markets (Muguto & Muzindutsi, 2022). Furthermore, with changing market conditions, the determinants and dynamics may have evolved, and a fresh examination of this phenomenon needs in the current context. Since international equity markets are more integrated, and the economies have gone through some volatile periods in terms of recent times Covid-19, war and inflation. Therefore, international equity markets might be more efficient why it is also important to look at the mean reversion process to see if there still are scope for forecasting by predictable volatility patterns.

Based on this, the main objective of this thesis is to measure, examine and compare volatility patterns in terms of persistence, asymmetry and mean reversion across developed, emerging and frontier markets and measure the speed of mean reversion in different markets. This study will examine the historical trends and patterns of market volatility in a comparative framework to see if there is predictable patterns or just random walk movements. Findings of this research will contribute to existing literature on market volatility and provide valuable insight for investors in managing and mitigating its effects and for risk management. This leads to the following research question:

How do volatility patterns in terms of persistence, asymmetry, and mean reversion differ across developed, emerging, and frontier markets? Are there predictable volatility patterns, and which markets exhibit the fastest mean reversion?

To answer the research question, GARCH models will be developed to identify volatility patterns by historical data from developed, emerging and frontier markets. The speed of mean reversion in different markets will be measured by the half-life volatility method.

The rest of this thesis is organized as follows. In next section, we will review the relevant literature on market volatility, identify the research gap and outline the theoretical framework that will guide the analysis. Subsequently, we will go through and present the methodology for the research, the data sources, analytical techniques, and empirical models employed in this study. The following sections will present the results and findings of the empirical analysis, followed by a discussion. Finally, we will conclude with a summary of the main findings.

2.0 Literature review

Numerous of studies have addressed volatility for several decades and put forth various explanations to elucidate different features of volatility. A common statistical measurement of volatility is the standard deviation of stock markets daily returns. It is a simple measure of volatility that summarizes the probability of extreme values, and if the standard deviation is large, the probability of large positive or negative returns is also large and vice versa. This measurement suggests a constant variance. Since the variance of stock returns are time varying, further methods to modelling volatility is evolved, such as time series model as ARCH that suggests a serial uncorrelated process with a non-constant conditional variance and a constant unconditional variance. A further development of the ARCH model is the GARCH model which was developed later on and imply both ARCH and GARCH effects by Bollerslev (1986). These models suggest a negative relationship between returns and volatility (Daly, 2008; Engle, 1982).

2.1 Volatility Persistence

Various explanations have been used to describe the three volatility patterns, persistence, asymmetry, and mean reversion. Regarding persistence, there is an existence of dependency between distant observation which means that there is tendency of memory in the volatility, indicating the influence of shocks in the stock market persists over a period (Tripathy, 2022). Cheung and Lai (1995) found evidence of long memory in stock returns for 18 countries and several studies found similar evidence of long memory in other countries as well (Crato, 1994). Cavalcante and Assaf (2004) find evidence of long memory in both return and conditional volatility measured in the Brazilian stock market. The persistence is mainly attributed to be driven by returns, and large returns, positive and negative is highly depended on the forecasting volatility and is much more impacted by negative returns that increase volatility persistence (Wang & Yang, 2018).

Likewise, Patton and Sheppard (2015) find that realized negative volatility is of much greater importance for future volatility than positive volatility, and especially large jumps in returns have a strong influence, where large negative jumps lead to significantly higher future volatility. Other studies attribute the volatility to be mainly driven by the constant arrival of new information, which causes volatility persistence (Beg & Anwar, 2012). Another explanation is the overall business cycle, where research has shown that volatility in stock markets can be influenced by the alternating forces of bull and bear markets, and volatility especially persists in bearish market conditions. Here, the effect of news is highlighted as investors listen to news, especially in bearish times, which affects investors' beliefs, and the asymmetric volatility persistence is caused by market forces (Yaya & Gil-Alana, 2014; Jones et al., 2004).

2.2 Volatility Asymmetry

These studies all found evidence of persistence in volatility, but they also found that the persistence was asymmetric, that investors react more to either negative returns or negative news than to positive movements which leads to higher volatility when negative price movements occur and the relationship between volatility and returns are negative. This exhibits a phenomenon of asymmetry in volatility, and one of the first explanations to this phenomenon was the leverage effect. Black (1976) and Christie (1982) both explored the relationship between leverage and stock price volatility, and observed, that stock prices tend to be more sensitive to negative shocks when a company's leverage was higher, and higher leverage lead to higher volatility.

On the other hand, a later study provided by Bekaert & Wu (2000) rejected the pure leverage model of Christie (1982), they found evidence for asymmetric volatility, but it was supported by volatility feedback. They argue that the volatility asymmetry is affected by time varying market conditions rather than leverage. Hasanhodic & Lo (2011) argue for a behavioural interpretation of the leverage effect. In their study they isolate the effect of leverage and volatility on stock returns and still find asymmetric volatility, which means that the leverage effect is not solely driven by the level of leverage as previously suggested have implied. They also reject the explanation of time-varying expected return but suggest that human behaviour is shaped by recent experiences with current information, which can have an impact on our future behaviour. Another explanation is that volatility asymmetry causes asymmetric attention. Stocks that receive higher attention from the number of analysts giving forecasts show larger volatility asymmetry. Additionally, stocks with low institutional ownership, a larger proportion of retail investors, and stocks with more idiosyncratic volatility show a larger asymmetry since retail investors are more prone to paying attention (Dzieliński et al., 2018).

2.3 Volatility Mean Reversion

Regarding mean reversion, numerous of studies have found evidence for mean reversion through different empirical results, including Bali & Demirtas (2008), Randolph (1991). Where earlier time series models assumed that the variance was constant, Engle (1982) developed the ARCH model, which showed that the conditional variance changed over time as a function of past errors, leaving the unconditional variance as a constant. The negative relationship between volatility and returns makes it intuitive that volatility cannot go up to infinitive because then the stock prices would reach zero. When volatility is mean reverting, it means that shocks have a temporary effect, so they return to their trend path over time. If the volatility were not mean reverting, it would imply that shocks would have a permanent effect and keep increasing each time a shock occurs over time (Goudarzi, 2013; Hillebrand, 2003).

There are several explanations to the phenomenon of mean reversion as well. De Bondt & Thaler (1985) explains mean reversion as the overreaction hypothesis, which is explained as a psychological behaviour of individuals that tends to overweight recent data in their predictions, and then they tend to overreact to unexpected and dramatic news events. The results of their studies show that losers during the past three to five years tend to outperform prior period winners over the following three to five years. Another explanation is related to risk aversion, investors' risk aversion increases during crises, and investors are affected by fear, leading them to sell off their risky assets and move their capital into other assets (Guiso & Zingales, 2018). In this regard, when volatility is high and risk averse investors move their capital, liquidity decreases, which extends the period of high volatility (Kondor & Vayanos, 2019). Liquidity is important for price level correction, and after the extended period when bad news has passed, investors seek back to the market to earn profit from the lower price, which corrects the price level and leads to mean reversion in volatility.

2.4 Volatility in different markets

The level of liquidity and development varies among developed, emerging and frontier markets, and the degree of access to these markets also varies. Less developed markets have limited access to available information, which affects volatility differently across markets. As a country's level of development increases, the volatility of its macroeconomic shocks decreases (Eizaguirre et al., 2004; Gniadkowska-Szymańska, 2017; Koren & Tenreyro, 2007). Empirical examinations of volatility patterns have been conducted in different markets and market conditions. The most common method used to examine these patterns in volatility is the use of different types of GARCH models to capture different patterns. In the following, we will elaborate some of the literatures previously empirical findings in terms of volatility patterns in different markets.

Wang & Yang (2018) find evidence of persistence and asymmetry in conditional volatility in the period of 2000-2014 on the S&P 100 index and S&P 500 ETF. They find that the volatility is mainly driven by returns, but the asymmetric volatility persistence has a much higher impact and explanatory power than the return on the future volatility. They also find that the volatility persistence varies with the state of the market. In a later study, they find evidence of volatility persistence and asymmetry in the Chinese stock market in the period of 2007-2016 and highlight the impact of changing market states on volatility persistence (Su & Wang, 2020). De Santis (1997) finds evidence of predictability and persistence in emerging markets as well, and the level of volatility is higher in emerging markets than in developed at both conditional and unconditional levels. The study also suggest that volatility decreases with liberalization. Engle & Patton (2007) find evidence for persistence and asymmetry in conditional volatility in the US market over the period of 1988-2000, testing the volatility patterns on the Dow Jones index. Likewise, evidence for persistence and asymmetry was found in the S&P 500 and Nasdaq 100 in the period 1990-2007 (Kumar & Dhankar, 2010).

Dedi & Yavas (2016) found evidence of persistence and asymmetry in five markets in the period of 2011-2016 in Germany, UK, China, Russia and Turkey as a spill-over effect. They found an increasing correlation between equity markets, but the correlation is still separated based on the development level of the markets. Russia and Turkey were found to be more volatile than the other markets. In a later paper, they investigated the volatility before, during and after the financial crisis in 2007-2008 in Germany, France, Italy, UK and US. They found persistence in all the markets before the crisis, increased persistence during the crisis in all the markets, and a decline in persistence after the crisis in all the countries except the US (Dedi & Yavas, 2017). This suggests that persistence is related to the market state and the level of volatility, with higher persistence during a crisis.

Baur & Dimpfl (2018) argue that volatility asymmetry is only related to high volatility phenomena, as they only find evidence of asymmetry in high volatility regimes and not in low volatility regimes. Talpepp & Rieger (2010) conducted tests on different hypothesis proposed to explain the asymmetric volatility in 49 countries worldwide. They identified four factors that seem to increase volatility asymmetry:

1) GDP/capita as a proxy for economic development. This suggests that the asymmetric is not caused by a lack of market development, but rather it is related to the proportion of non-professional investors in the market, making it more likely in wealthier and developed markets.

2) Stock market participation as a percentage of private investors in the market, which shows a positive correlation with volatility asymmetry.

3) Analyst coverage, which is linked to asymmetric attention.

4) Short selling increases the volatility in bear markets. The evidence for the leverage effect was not as strong and the asymmetric is clearly related to private investors and behavioural sentiment according to this study.

Bekaert & Campbell (1997) investigate the volatility in 20 emerging markets over the period of 1976-1992 and find evidence for that the capital market liberalization have a decreasing effect on volatility and increases the correlation between local market returns and the world market. They also did an experiment with a poorly developed stock market in a relatively closed country, that experienced higher volatility. They also find evidence for persistence and asymmetry volatility in the majority of the markets. On the Indian stock market for the period of 2000-2007, Bose (2007) found evidence of persistence, asymmetry and mean reversion. The time it takes for reversion to the mean level is fast in this market. On the Russian stock market, Caporate et al. (2020) find evidence of mean reversion and persistence, but they find that the persistence tends to have a short effect of shocks. Yaya & Gil-Alana (2014) find evidence of persistency and asymmetry in the Nigerian stock market during bull and bear markets, which is influenced by investors beliefs based on both bad and good news they have listened. They also find that the Nigerian stock market are spending more time in bull market than in bear market phases and argue for that this is a general statement. The longer persistence of bull markets than bear markets is in line with other studies (Lunde & Timmermann, 2004).

2.5 Volatility overview

It is more common that previously studies have focused on individual stock markets and small individual groups of markets in terms of studies of volatility characteristics rather than different groups of markets. In particular, there has been significantly more focus on studies regarding volatility persistence and asymmetry. It is also common that studies mention the mean reversion process as the sum of alpha and beta, but they do not comment on it. Mallikarjuna & Rao (2019) is one of the few that group the study as developed, emerging and frontier markets in the period of 2000-2018. Here they find evidence of persistence in all the markets, and for asymmetry and leverage effect in developed and emerging markets, but they do not find evidence of asymmetry and leverage effect in frontier stock markets. They find that developed markets are more sensitive to information, showing asymmetric effects of volatility. Frontier markets, on the other hand, exhibit a symmetric effect, indicating that both bad and good news have an equal impact. Emerging markets are relatively more sensitive to information compared to frontier markets but less sensitive compared to developed markets. Another study that has compared the mean reversion in developed and emerging markets find evidence for mean reversion in both markets. The study also revealed higher volatility in emerging markets compared to developed markets. Additionally, the mean reverting process was found to be faster in emerging markets (Ahmed et al., 2018). Muguto and Muzindutsi (2022) conducted a study comparing volatility persistence, asymmetry, risk-return relationship, and mean reversion among BRICS and G7 markets in the period from 2003 to 2020. The study found that developed markets had higher volatility persistence compared to emerging markets. They also found evidence for mean reversion and that shocks decay faster in developed markets. Regarding asymmetry, they found evidence in some of the BRICS markets and all the G7 markets, where it was very strong in G7 markets.

In general, the literature is scant for frontier markets in terms of volatility patterns, and there is a need for evidence of how volatility patterns occur in countries in these markets as well. Emenike & Enock (2020) agree the lack of evidence from frontier markets and highlight the asymmetry. In their study, they analyse the leverage effect in frontier markets with the Ugandan Securities Exchange as a proxy for frontier markets in the period of 2011-2017. They find evidence of volatility shock persistence and a different reaction of good and bad news than earlier found in other markets, since they suggest that the market is more impacted by good news than bad news of the same magnitude. The study is only using a single country index as a proxy for frontier markets instead of using a group as a proxy. Similar, another study also find evidence of that positive return are associated with higher volatility than negative return in KSE-100 (Saleem, 2007).

According to all these studies provided above, it seems like there exist predictable patterns in the market which is contradicting to the efficient market hypothesis that states, that all available information is quickly and accurately reflected in asset prices. According to the efficient market hypothesis, all investors have access to the same information, and markets are highly competitive, so it is not possible to employ any investment strategy to get any abnormal return. The efficient market hypothesis has three different forms that describes the level of efficiency in the market, the weak, semi-strong and strong form. In the weak form, current prices reflect all past market level information in terms of price and volume, so it suggests that no investor can consistently earn abnormal profits by using only past market data. The semi-strong form consists that in addition to all historical information, prices also reflect all publicly available fundamental company and economic information. The strong form imply that the market is efficient if all information, including both public and private information, is reflected in the current market price. This mean that even insider information can be used to earn abnormal profits (Fama, 1970). The efficient market hypothesis is associated with the random walk hypothesis that states, if the price of a security contains all available information, the price change is independent and random variable, and the probability of a price movements is equal in each direction which means that the price is following a random walk (Fama, 1970). On the other hand, mean reversion theory consider there is inefficiency in the market in short term and the volatility has boundaries and predictable patterns.

Overall, the literature review conducted above indicates consensus among researchers that volatility patterns exist in various markets. However, there is ongoing debate surrounding the factors that contribute to these patterns and the implications they hold. While some studies have explored these issues extensively, the evidence is largely drawn from individual countries or stock markets, and there is a dearth of research on volatility patterns across developed, emerging and frontier markets. As a result, there is a need for more comprehensive and comparative studies to provide a deeper understanding of how volatility patterns occur in different groups of markets, a gap this thesis tries to fill.

3.0 Data and methodology

In this section, we describe the data and methodology used to analyse and answer the research question. The data is essential to provide empirical evidence and support the analysis, while the methodology is crucial to ensure accuracy and reliability of the results. The chosen data should be well justified and consistent with previous studies in the field. Additionally, we will provide a detailed overview of the statistical and econometric methods used to analyse the data and answer the research question.

3.1 Data collection

For the analysis, broad market indices are employed to measure and examine the volatility patterns in different markets. The different market categorizes are distinguished as the three categories: developed markets, emerging markets and frontier markets. To categorize the three markets, the MSCI World market classification is used to determine which countries belongs to which market category. The MSCI consider different factors to categorize the countries, such as GDP per capita income, economic development, size and liquidity and local government regulations (MSCI.com, 2022). Two countries are chosen from each continent in each category, where the biggest represented index for the given country is used for the data sample. The table below represents the countries and corresponding indices selected for developed markets, emerging markets and frontier markets.

Table 1: Market Classification

Developed markets			Emerging Markets			Frontier Markets			
Americas	**Europe & Middle East**	**Pacific**	**Americas**	**Europe & Middle East**	**Asia**	**Europe**	**Africa**	**Middle East**	**Asia**
Canada	United Kingdom	Australia	Chile	Hungary	China	Croatia	Morocco	Bahrain	Pakistan
S&P/TSX	FTSE 100	S&P/ASX 200	IPSA	BUX	SCI	CROBEX	MASI	Bahrain All Share index	KSE 100
United States	Germany	Japan	Brazil	Poland	Taiwan	Romania	Tunisia	Jordan	Vietnam
S&P 500	DAX	Nikkei 225	BOVESPA	WIG20	TWSE	BET	Tunindex	ASEX	VN-Index

The data sample provides the period from 31/12/2012 – 31/12/2022 and includes daily data. The period is particularly relevant as it includes significant events that have shaped the economic landscape of the past decade. Covid-19 pandemic, war in Ukraine, and high inflation are among the most notable events that have affected the global financial markets and caused significant volatility. By including this period in the study, the analysis captures the impact of these events on the global financial markets and how they have influenced the level of volatility. Furthermore, the period of the data ensures an up to date and relevant insights into the current state of the financial markets. Therefore, this period has been selected to provide a comprehensive understanding of the impact of recent events on the volatility on global financial markets. Since developed, emerging and frontier markets consists of data from countries worldwide, there is a difference in which days the respective markets are open. Due to varying number of opening days in different markets, the number of observations for each country is not consistent. In order to have an equal number of observations for each country, the data has been matched by dates. Some cells in the dataset represent closing days for the market, which are empty. To address this issue, the empty cells are filled by calculating the average price of the last five days before the specific closing days. This ensures that the dataset is more uniform, complete and comparable across different markets.

To examine the volatility of stock returns, the daily stock prices are computed as stock returns, which are calculated as the log return:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

Where r_t is the daily return, calculated as the percentage change in the price of the index's prices between period P_t and P_{t-1} expressed as the natural logarithm of the ratio of the final price to the initial price.

3.2 Diagnostics of time series

The first step in the analysis of the returns is to understand the returns characteristics by running diagnostics tests on the returns for each country, which is tests for normality, stationarity, autocorrelation and heteroscedasticity. Firstly, the data is organized as time series for each country. The countries are then categorized into three different market types, developed, emerging and frontier markets. All the tests and models used in the analysis are applied to the respective time series for each country and then compared and analysed as a group based on their market type.

Then normality test is used to test whenever the data of the time series are following a normal distribution. The normality test helps to assess whether the data is suitable for statistical analysis using methods that assume a normal distribution. The test statistic has the following equation:

$$JB = \frac{n}{6}(S^2 + 0{,}25(K - 3)^2) \tag{2}$$

Where n is the sample size, S is the skewness of the sample, and K is the kurtosis of the sample. If the p-value of the Jarque-bera test is less than the significant level at 0,05, then we reject the null hypothesis that the data follows a normal distribution. Conversely, if we do not reject the null hypothesis, then the data may be normal distributed (Tsay, 2010).

For the stationarity the ADF test (Augmented Dickey-Fuller Test) are used. The ADF test is a statistical test used in time series analysis to determine whether a time series is stationary or a random walk. The idea behind the test is to determine if the time series has a unit root which means that the time series are non-stationary and then follows a random walk, this means that the value at any given time is highly correlated with its value at a previous time. Under the null hypothesis, the time series follows a random walk and if we reject the null hypothesis, the time series are stationary (Tsay, 2010).

3.3 Autoregression Moving Average

If a time series is stationary, then we assume that the time series mean reverts since it follows a stationary pattern. In this case, the time series might have a fundamental value it reverts to and then it can be predicted. To forecast, we need a regression model where the variables are depending on its previous variables. This statement are we testing in the time series by the Ljung-Box test which are testing whether the past log returns help to explain future log returns, then the data has autocorrelation (Tsay, 2010). Under the null, the time series are white noise, which mean that the past value does not help to predict the future. If the null hypothesis is rejected, it means that the time series has autocorrelation. We also take a look at the plot of the ACF and PACF to define the orders of autocorrelation (Tsay, 2010). The autocorrelation indicates that there is a linear relationship between the variables in the past and in the future. We estimate a model with this predictive power:

$$r_t = \phi_0 + \phi_1 r_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2) \tag{3}$$

This model is an autoregressive process AR(1). The AR model relates the current value of a time series to its past values where the coefficient in the model determining the extent of this dependence (Tsay, 2010). The returns can also be indirectly dependent by their previous forecasting error, this is the moving average model:

$$r_t = c_0 + \varepsilon_t + \theta_1 \varepsilon_{t-1}, \quad \varepsilon_t \sim WN(0, \sigma^2) \tag{4}$$

The ARMA model combines autoregressive and moving average terms and we estimate how many autoregressive terms and how many moving average terms from the ACF and PACF where the ACF includes all in between observations of the correlation coefficients between times and the PACF excludes the effects of observations in between (Tsay, 2010). To do this, the auto ARIMA function in R is used for this purpose to get a mean model for the conditional mean for each of the time series. To define if the model captures the relation by diagnostics, we perform the Ljung-box test on the residuals. The error terms should behave as white noise, so if the LB test is rejected, then we should include more lags until the error terms behaves as white noise (Tsay, 2010).

3.4 Autoregressive Conditional Heteroscedasticity

When we have estimated the mean value of the time series, we can estimate the variance of the time series as the GARCH model. The GARCH model combines the ARCH effects and the GARCH effects where the ARCH effects are the shocks of volatility that tends to cluster, which means that periods with high volatility are followed by periods with low volatility. The assumption is that the variance of the returns is autocorrelated - higher than the returns itself, so it turns out that asset returns depend strongly on the past in a non-linear way, by the variance of the returns which is captured by the volatility clustering effects. If the returns were random, the variance should be the same regardless of the time, that's why we assume heteroscedasticity of the variance. The volatility clustering is the conditional variance and is represented by the ARCH effects (Tsay, 2010). The idea is that the volatility level today depends strongly on the level of volatility yesterday and we transform the sigma into a variable that depends on the time σ_t^2 and on previous values σ_{t-1}^2 so we get an autoregressive model for the variance:

$$\begin{gathered} y = E_{t-t}[y] + \varepsilon_t, \\ \varepsilon_t = \sigma_t v_t, \quad v_t \sim WN(0,1) \\ \sigma_t^2 = f(\sigma_{t-1}^2) \end{gathered} \tag{5}$$

Since the variance is unknown, we use the squared error terms and test for volatility clustering by the ARCH-LM test. The first step of this test is basically what we already have done - specified a mean equation and build an ARMA model to remove any linear dependence in the returns. The second step is to save the residuals from the ARMA model and square the residuals and propose an autoregressive model by the squared residuals.

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_{t-1}^2 + \cdots + \alpha_p \hat{\varepsilon}_{t-p}^2 + e_t \tag{6}$$

Under the null hypothesis, we have no ARCH effects. We apply the Ljung-Box test on the squared residuals to test for autocorrelation. In case we reject, we have volatility clustering captured by ARCH effects, which is also represented as the shocks in volatility often provided by bad news in the market (Tsay, 2010). We can then estimate a volatility model, which are composed of two interdependent equations, which are the mean equation that related the observations to some conditional mean and error terms:

$$r_t = E_{t-1}[r_t] + \varepsilon_t, \qquad \varepsilon_t = \sigma_t v_t \quad v_t \sim WN(0,1) \tag{7}$$

Where $E_{t-1}[r_t]$ can be some ARMA model or the mean equation can just be white noise. The volatility model also implies the variance equation, which can be written as:

$$\sigma_t^2 = f(\sigma_{t-1}^2) \tag{8}$$

Where the variance relates to previous values as a function (Tsay, 2010).
The ARCH model is developed from the ARCH-LM test and the foundation is that the conditional expectations of the squared error terms must be equal to the conditional variance.

$$E_{t-1}[\varepsilon_t^2] = \sigma_t^2 \tag{9}$$

We apply the conditional expectations to the ARCH-LM test regression and then we get the ARCH(p) model where p is the number of past error terms. An ARCH (1) can then be written as:

$$r_t = E_{t-1}[r_t] + \varepsilon_t, \quad \varepsilon_t = v_t\sqrt{\sigma_t^2} \quad v_t \sim WN(0,1) \tag{10}$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2$$

The variance at time t is the conditional variance.

3.5 General Autoregressive Conditionally Heteroscedasticity

Some weaknesses of the ARCH model are that the model assumes that positive and negative shocks have the same effects on volatility, and it does not account for persistence. The lack of the ability to capture persistence in the ARCH model leads us to the estimation of a GARCH model to capture the persistence, and the GARCH model at the same time also implies the ARCH effects (Tsay, 2010).

The GARCH (p,q) model is represented as follows:

$$r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t = v_t\sqrt{\sigma_t^2} \quad v_t \sim N(0,1) \tag{11}$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-q}^2$$

Where P is the number of ARCH terms ε_{t-i}^2, and Q is the number of GARCH terms σ_{t-i}^2.

$\alpha_p \varepsilon_{t-p}^2$ represents the number of shocks in the volatility.
$\beta_q \sigma_{t-q}^2$ represents persistence in volatility.

GARCH also define a long-term variance that for instance for a GARCH (1,1) can be written as:

$$\bar{\sigma}^2 = \frac{\alpha_0}{1 - \alpha_1 - \beta_1} \tag{12}$$

The GARCH models are estimated by maximum likelihood. By estimating coefficients for alpha and beta for each country in the three markets respectively, we are able to measure and examine which country are more exposed to large shocks in volatility by the alpha coefficients, and which country are more persistent after volatility shocks by the beta coefficients. From the sum of ARCH and GARCH effects (Alpha + beta) we are able to define which country has the highest overall volatility in general and we can also look at the overall average volatility level in the three markets to define which market are more exposed to risk by its volatility. Significant coefficients indicate the presence of volatility effects and higher values of these coefficients generally indicate higher volatility (Tsay, 2010).

To validate the presence of volatility effects, we look at the t-value and p-value for each of the estimated alpha and beta coefficients in each GARCH model which must be significant for the validation of the explanatory power. We also apply a test to validate for whether the volatility model has adequately captured the persistence in the variance of the returns for the different markets by looking at the standardized squared residuals that should be serial uncorrelated (Engle & Patton, 2001). We apply the Ljung-box test on the standardized squared residuals which imply a null hypothesis of no autocorrelation and if we reject the null hypothesis, then there is still serial autocorrelation in the standardized squared residuals. In this case, the estimated coefficient is not efficient enough to capture all the volatility and then we need either another GARCH model or more orders of ARCH and GARCH effects.

3.6 Mean-reversion

The mean reversion process is given as $\alpha_1 + \beta_1 < 1$ then the volatility reverts (Tsay, 2010). So, the estimated coefficients in the GARCH models must be less than one to follow a stationary pattern and then have a mean reversion process, and if the sum of alpha and beta is one or above one, the time series are non-stationary and follows a random walk which means in this case, it does not have a mean reverting process (Ahmed et al., 2018). The mean reversion process is also implied in the volatility clustering effect mentioned before, where volatility comes and goes, and in the long run the volatility will revert to a long-term value of volatility and current information has no effect on the unconditional volatility (Engle & Patton, 2001). To find if there is evidence of a mean reversion process for the time series of indices in each country, we are looking at the sum of estimated alpha and beta coefficients for all the GARCH models.

3.7 Half-life of volatility

Half-life of volatility was defined by Engle & Patton (2001) as the required time it takes for volatility to move half-way back towards its unconditional mean and the mean reverting form of the GARCH (1,1) model is given as:

$$(\varepsilon_t^2 - \bar{\sigma}^2) = (\alpha + \beta)(\varepsilon_{t-1}^2 - \bar{\sigma}^2) + \mu - \beta\mu_{t-1} \tag{13}$$

Where $\bar{\sigma}^2 = \frac{\alpha_0}{1-\alpha_1-\beta_1}$ is the unconditional variance or persistence of volatility and $\mu_t = (\varepsilon_t^2 - \bar{\sigma}^2)$. The magnitude of the mean reverting rate defined by the sum of alpha and beta, which is used to be close to 1 in a good, fitted model, and is controlling the speed of mean reversion in days in the model. If volatility shocks spikes during crisis, the number of days it takes until it has reverted from the first forecast and the half-way towards the unconditional variance is $(\alpha + \beta)^k = 0{,}5$ where k is expressed as days, thus we can express the half-life of volatility models as:

$$L_{half} = \frac{\ln\left(\frac{1}{2}\right)}{\ln(\alpha_1 + \beta_1)} \tag{14}$$

The model estimates the average mean time it takes for the volatility to revert the halfway back toward its unconditional mean. The higher the sum of alpha and beta is, the longer will the half-life of volatility be.

3.8 Threshold General Autoregressive Conditionally Heteroscedasticity

To capture the effect of asymmetry, we estimate a TGARCH model. There is a number of ways of parametrizing this idea, one of them is the Threshold GARCH model, which was proposed by Zakoian (1994) and was motivated and evolved upon the EGARCH model of Nelson (1991). The TGARCH provide and effective tool to estimate and reflect the asymmetric relation between volatility and past returns (Zakoian, 1994). A TGARCH (m,s) assumes the form:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{s} (\alpha_i + \gamma_i N_{t-1}) a_{t-1}^2 + \sum_{j=1}^{m} \beta_j \sigma_{t-j}^2 \tag{15}$$

Where $N_{t-1} = \begin{cases} 1 \; if \; a_{t-i} < 0 \\ 0 \; if \; a_{t-i} \geq 0 \end{cases}$

From the model, a positive a_{t-i} contributes $\alpha_i a_{t-1}^2$ to σ_t^2 whereas a negative a_{t-i} has a larger impact $(\alpha_i + \gamma_i) a_{t-1}^2$ with $\gamma_i > 0$. The model uses zero as a threshold to separate the impact of past shocks (Tsay, 2010). The estimated gamma coefficients for each country are then a parameter to capture asymmetry in their volatility. It measures the impact of positive and negative shocks on volatility differently, reflecting the idea that people often react more strongly to negative news than positive news. A positive gamma coefficient indicates that negative shocks have a greater impact on volatility than positive shocks, whereas if it is negative, it suggests the opposite. The magnitude of the coefficient indicates the strength of the asymmetry effect. To validate the evidence of asymmetry effects in countries volatility, we are looking at the t-values and p-values for each country to measure if the estimated gamma coefficients are significant, if they are not significant, we will not apply the coefficient in the model since it does not have an explanatory power.

To summarize, in this study we utilized broad market indices to examine the volatility patterns in developed, emerging and frontier markets categorized by the MSCI World market classifications in the period from 31/12/2012 to 31/12/2022. To capture the shocks and persistence of overall volatility and the effect of mean reversion, we utilized the GARCH model. The estimated coefficients from the GARCH model enabled us to measure the time in days it takes for volatility to revert halfway back to its unconditional mean using the half-life volatility model. Additionally, we examined whether shocks in volatility had an asymmetric reaction to positive and negative news by estimating the TGARCH model to capture asymmetry in volatility. In the following section we will present the results and findings from the empirically study.

4.0 Results and findings

The following three tables display the descriptive statistics and test results from the ADF and Jarque-Bera tests for the returns of developed markets, emerging markets and frontier markets indices respectively. The descriptive statistics exhibits how the returns are distributed in the different countries.

4.1 Descriptive statistics

From table 2 we can see that developed markets all have positive mean returns, with the highest mean return in USA and the lowest in UK over the selected time period. Many factors might have had an impact of the returns in different countries, and some of the reasons for that USA have the highest mean return, and UK the lowest, might be that USA have had a relatively stable economic growth in the selected period.

Table 2: Descriptive statistic and test results for Developed markets.

	Developed Markets					
Country	**Mean**	**SD**	**Skewness**	**Kurtosis**	**ADF P-value**	**JB P-value**
Canada	1,75E-04	0,0096507	-1,60526	41,83022	0,01	0
USA	3,90E-04	0,0111937	-0,86410	19,08553	0,01	0
UK	9,22E-05	0,0100778	-0,84706	15,35152	0,01	0
Germany	2,38E-04	0,0124876	-0,53504	12,41564	0,01	0
Australia	1,63E-04	0,0099081	-1,00149	14,95232	0,01	0
Japan	3,52E-04	0,0138474	-0,30892	7,85311	0,01	0

According to OECD, the US have had a GDP annual growth on 2,2%, 2,9% and 2,3% in 2017-2019 respectively. On the other hand, UK have had a more uncertainty economic growth, partly due to the uncertainty surroundings of Brexit, and they have had a GDP growth in the same period of 2,4%, 1,7% and 1,6%. In 2020 where Covid-19 happened, the GDP of US felt by -2,8% while the GDP of UK felt by -11% which is also accelerated by the Brexit in the same year (OECD, 2023).

The economy in UK have been exposed to uncertainty caused by political factors during Brexit while the USA have had a more stable political period in comparison. The Brexit have had an impact of the UK trade, investment level and labour market, which have caused lower economic growth and might have led to lower returns in UK (Dharshini, 2023). Another reason for a higher mean returns in US might be that during the pandemic, the tech sector experienced an exponential market growth, in this sector, the major companies are American and are represented with high weight of tech companies in the S&P 500 index (Poletti & Owens, 2022).

All the countries in developed markets are negative skewed indicating the average value are less than the median value. Especially we can see Canada are more negative skewed than the other countries, and the kurtosis are higher for Canada as well. The higher kurtosis indicates that the return distribution has a more concentrated, peaked top and thicker tails than a normal distribution, indicating that Canada has heavier tails and experienced more returns far from the mean relatively to the other countries. When we are looking at the standard deviation, Japan has the highest standard deviation and the lowest kurtosis, indicating that there are a few extreme outliers, but mainly the returns are close to the mean. Canada has the lowest standard deviation, indicating that the returns are closer to the mean. Interestingly, Canada has the highest kurtosis and are more negative skewed, while it has the lowest standard deviation, indicating that the observations are closer to the mean than the other countries observations, but the tails are heavier, mainly the left tail and there is a large possibility for returns away from the mean. It is observed that all the countries have a high kurtosis value relatively to a normal distribution that has a kurtosis value of 3. Canada has a kurtosis value of 41,83 this is mainly due to large negative returns in the days of covid where the stock market felt with large shocks.

Table 3: Descriptive statistic and test results for Emerging markets.

	Emerging Markets					
Country	**Mean**	**SD**	**Skewness**	**Kurtosis**	**ADF P-value**	**JB P-value**
Chile	7,51E-05	0,0118059	-1,23155	26,36490	0,01	0
Brazil	2,21E-04	0,0161959	-0,80567	14,96236	0,01	0
Hungary	3,40E-04	0,0126291	-1,08697	13,67520	0,01	0
Poland	-1,51E-04	0,0134608	-0,70299	12,29823	0,01	0
China	1,22E-04	0,0134766	-0,88520	10,62913	0,01	0
Taiwan	2,74E-04	0,0103112	-0,18452	11,61925	0,01	0

Table 3 depicts the descriptive statistics and test results for emerging markets, where the highest mean return was in Hungary and the lowest in Poland. Poland had a negative mean return in the period. A political factor that could have had a negative influence of the returns in Poland could be the polish parliamentary election in 2015 where PiS won the majority of the votes which led to tensions between Poland and EU and might have affected the investors (Skaaning, 2021). Chile is more negative skewed and has higher kurtosis than the other countries in emerging market, indicating more extreme values in Chile, they also have the second lowest mean return in the period. Brazil has the highest standard deviation, indicating that the observations is far from the mean relatively to the other countries.

Table 4: Descriptive statistic and test results for Frontier markets.

	Frontier Markets					
Country	**Mean**	**SD**	**Skewness**	**Kurtosis**	**ADF P-value**	**JB P-value**
Croatia	4,92E-05	0,0072046	-2,99731	47,04323	0,01	0
Romania	2,91E-04	0,0094674	-1,60897	22,58856	0,01	0
Morocco	5,35E-05	0,0069188	-1,31775	27,58247	0,01	0
Tunisia	-8,69E-05	0,0084720	-2,86016	54,59166	0,01	0
Bahrain	2,76E-04	0,0073498	-2,33361	47,00109	0,01	0
Jordan	9,53E-05	0,0057794	0,44244	15,39048	0,01	0
Pakistan	3,44E-04	0,0108270	-0,68090	8,87479	0,01	0
Vietnam	3,46E-04	0,0117055	-0,85523	7,32813	0,01	0

In frontier markets, Vietnam shows the highest mean return during the period. A major reason for this might be the stock market liberalization in Vietnam. Vietnam have had high economic growth rates in recent years which has attracted more investments to the country, they have gone through a lot of reforms to liberalize the economy in 2013 which has attracted more foreign investors, and Vietnam have had a stable political and economic situation in recent years which may have increased investor confidence and contributed to a higher mean return (Mateus & Hoang, 2021). Vietnam has a very low kurtosis value due to the relatively more stable economy and political factors during recent years, in comparison with conventional frontier markets. Croatia is more negative skewed than the other countries and have a relatively high kurtosis indicating that Croatia has experienced more returns far from the mean in the selected period. Vietnam at the same time has a higher standard deviation than the other countries, indicating that there might be a few outliers in the observations that is far from the mean, but due to the low kurtosis more observations are close to the mean.

The skewnesses are negative for all countries in all the markets, except for Jordan. The negative skewness indicates that the returns are more left to the median returns, so Jordan has more returns higher than the median returns. In Jordan, the banking sector is the primary sector within the financial system, which plays a pivotal role in the country's economy. The banking industry development has affected the economic growth in the country positive, and this has led to more stability in the country's financial markets, which might have led to less volatility and a positive mean return (Almahadin et al., 2021). Overall, the frontier markets have the highest negative skewness of the three markets which indicates that the returns are lower than the median return. The emerging markets have less skewed value of the three markets.

All markets have a relatively high kurtosis value which indicates that a return distribution has a more concentrated, peaked top and thicker tails than a normal distribution. The frontier markets have on average the highest kurtosis values which indicates that frontier markets have heavier tails due to more returns in positive and negative direction and there is a greater risk of experiencing large losses, and on the other hand, it can also indicate a higher opportunity to achieve higher returns. Emerging markets have the lowest kurtosis value, so emerging markets have less data points in the tails and the returns are less exposed to extreme values than the other market groups. Emerging markets have the highest standard deviation and together with lowest kurtosis, this is indicating that emerging markets have observations closest to the mean with few extreme outliers. Frontier markets have the lowest standard deviation and together with high kurtosis, indicating that the returns are closer to the mean relatively to the other markets, but it has heavier tails, so there is more possibility for returns far from the mean.

If we look at all the descriptive statistics together, we can interpret from this that developed markets have in general on average the highest mean return compared to the other markets and a medium variance of the returns with less possibility to returns far from the mean than frontier markets, but more possibility than for emerging markets. The emerging markets have on average the lowest return and the highest standard deviation but at the same time lower kurtosis, indicating that returns are closer to the mean and there are few extreme returns in these markets and the returns are less skewed than the other markets. Frontier markets have the lowest standard deviation among the three markets, but are more skewed and have higher kurtosis, indicating that there are more returns lower than the median return and even though the returns are closer to the mean, there is heavier tails, so there is more possibility for returns far from the mean. This gives us an indication of that frontier markets are the riskiest market, followed by developed and then emerging markets looks like to be less risky, but they have possibility for few extreme outliers.

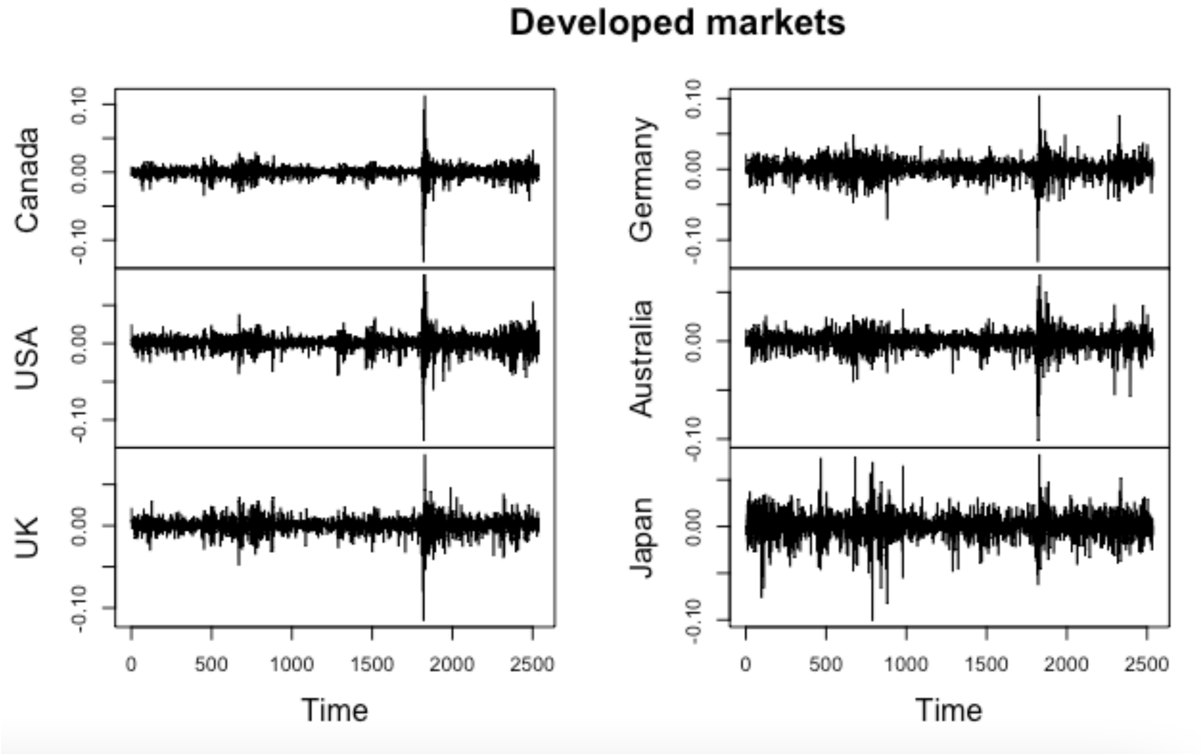
4.2 Diagnostic tests

The test results of the ADF and Jarque-Bera shows that all the countries return are stationary and has no unit root and they are not normal distributed which also was indicated by the values of the skewness and kurtosis. The null hypothesis of a normal distribution was rejected for all countries based on the Jarque-Bera tests. This suggests that the time series were leptokurtic, and investors were more exposed to extremely low or high returns. The rejection of the null hypothesis of the ADF shows that all returns are stationary. When the returns follow a stationary pattern, then the returns are mean reverting and might have a fundamental value it reverts to.

This suggests that the data has autocorrelation, which we test with Ljung-box, and for all countries in each market, the null hypothesis are rejected which means, that there is a significant autocorrelation, so past log returns help to explain future log returns. The evidence of autocorrelation at the returns are in contrasts to the efficient market hypothesis that states that financial markets are efficient and there should be no predictable patterns or trends in asset returns (Fama, 1970). So, the evidence of autocorrelation suggests that there are predictable patterns in the data, which indicates that there exists inefficiency in the market that can be predicted.

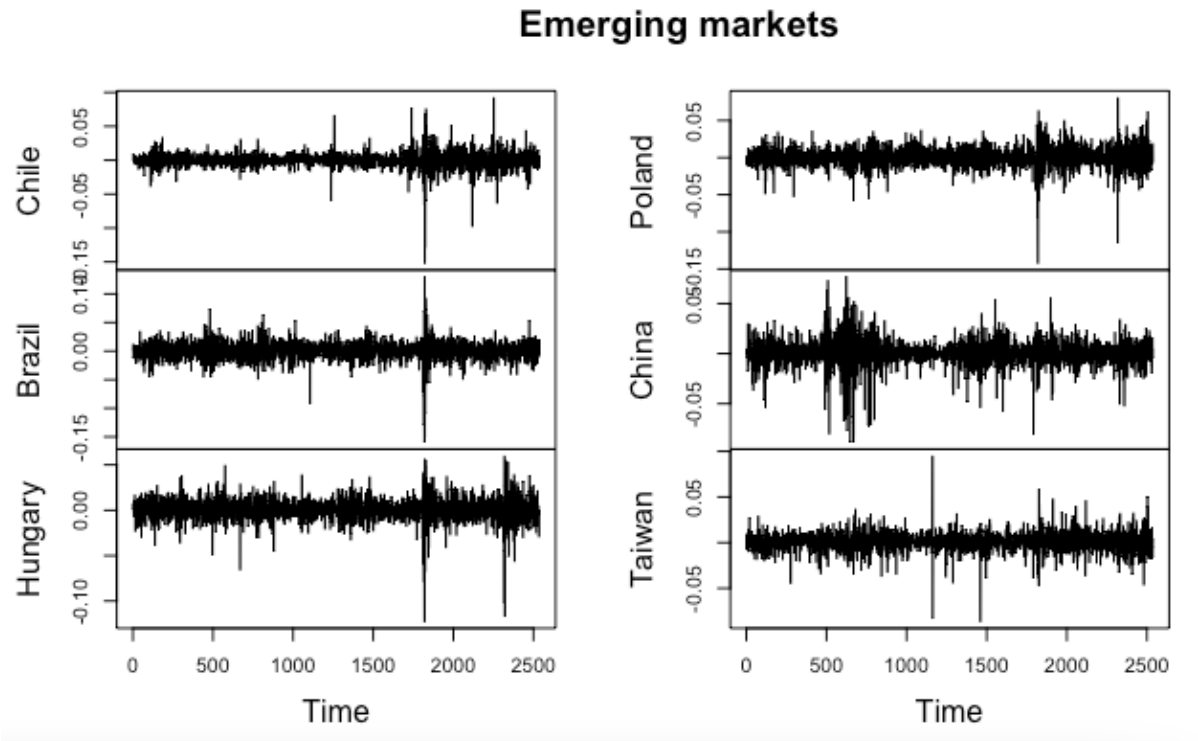
Figure 1, 2 and 3 shows the daily return plots for developed markets, emerging markets and frontier markets respectively. The plots shows that some periods are riskier than other, depicted by higher volatility of returns in those periods. It can be noticed that there is coincide between events that happens in all the markets at the same time, the effect of Covid-19 pandemic is clearer in developed markets compared to the two other markets. However, all the time series exhibit a constant mean over the sample period and some of the indices exhibits more risk than other.

Figure 1: Time series plot of developed markets.



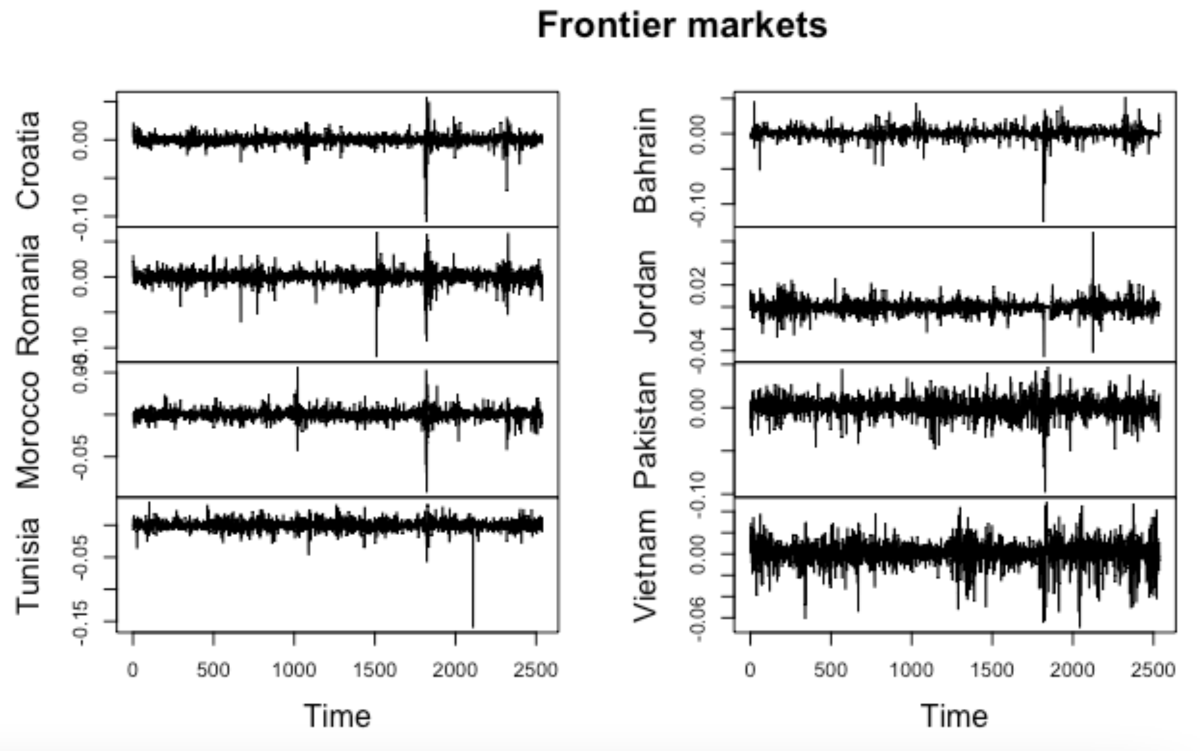
For the countries in developed markets, the time series plot show, that Covid-19 forced a huge spike for all the markets. For all the markets with exception of Japan, the time before the Covid-19 pandemic is relatively more stable, especially in Canada and USA, where the period after the crisis seems more turbulent. For Japan, the returns are more volatile in a period before Covid-19 crisis, and they are less exposed to this event. Some events that have happened in the period before the pandemic, which might have had an influence of the volatility in Japan, is the fact that Japan has been characterized by high political uncertainty, herby in the economic politic, structural reforms, and trade politics. In January-February 2016, the BOJ announced negative interest rates which lowered the yield curve and led to yen depreciation. In 2017 there was uncertainty about consumption tax high. This uncertainty events might have affected the stock market volatility in the period (Saxegaard et al., 2022).

Figure 2: Time series plot of emerging markets.



In China, we can see a lot of volatility in the years of 2015-2016, it causes that the Chinese stock market was overheated because before the turbulence began, the stock market had had a huge increase and many investors had invested by borrowed money which led to even more volatility. When the bubble burst, the Chinese stock market had lost 30% of its value over a period of three weeks (Encyclopedia, 2022). In Chile, we can see from the plot that the stock market was more stable and less volatile and small returns in general before Covid-19 happened, and after the event, the market exhibits a huge more volatile pattern. For frontier markets, it is noticeable that the stock market in Pakistan and Vietnam exhibits more returns in both direction in comparison with the other frontier markets. It also seems like frontier markets are less exposed to extreme returns during the pandemic as we could see in emerging and especially developed markets.

Figure 3: Time series plot of frontier markets.



This observations of periods with higher and lower volatility relates to the phenomenon of volatility clustering, which is tested by the ARCH-LM test. Before testing for ARCH effects, we are testing for autocorrelation on the squared residuals on the ARMA models estimated for each country by the auto ARIMA function. The squared residuals perform as the variance of our mean model, so we test by the Ljung-box test if there is autocorrelation in the variances. We found evidence for autocorrelation on the squared residuals for all countries except for Tunisia that does not have a significant p-value for the Ljung-Box test which means that we do not have evidence for autocorrelation of the squared residuals in this time series of Tunisia, indicating that the variances of the returns in Tunisia might not have had sufficient variety in the selected period. We also find evidence for ARCH effects by the ARCH-LM test with a significant p-value in all countries except for Tunisia, indicating that the volatility does not tend to cluster in this country, but even though, we still estimate a GARCH model for this country to examine the patterns of volatility, but we have to analyse the results for Tunisia with that note in mind and read the results of Tunisia with care and not put too much weight on the results.

4.3 GARCH estimation

Table 5, 6 and 7 lists the estimated coefficients from the GARCH models for each country in each market respectively. The estimated coefficients are the omega which is the constant, the alpha1 represented as the shocks of volatility and the beta1 which is the persistence of volatility. The result in the tables shows that all the coefficients for alpha and beta are significant, demonstrating previous days affecting the current volatility of the three markets stock indices return series. The table also provides evidence for mean reversion process since the sum of alpha and beta coefficients are less than one ($\alpha + \beta < 1$) which means that the stock indices prices are mean reverting to their unconditional mean value after a certain period.

Table 5: GARCH coefficients of Developed markets

	Developed Markets					
	GARCH (1,1)	Omega	Alpha1	Beta1	Alpha+Beta	Ljung-Box Residuals
Canada	Coefficient	0,000002	0,201823	0,772864	0,974686	
	t-value		8,6741	30,3030		
	p-value		0	0		0,95220
USA	Coefficient	0,000004	0,192362	0,774024	0,966386	
	t-value		11,666	32,774		
	p-value		0	0		0,70550
UK	Coefficient	0,000005	0,143771	0,802779	0,946550	
	t-value		12,174	68,063		
	p-value		0	0		0,22260
Germany	Coefficient	0,000006	0,114792	0,848266	0,963058	
	t-value		11,136	79,086		
	p-value		0	0		0,11760
Australia	Coefficient	0,000003	0,126162	0,838100	0,964262	
	t-value		9,179	21,540		
	p-value		0	0		0,90850
Japan	Coefficient	0,000008	0,133165	0,828446	0,961611	
	t-value		10,350	80,379		
	p-value		0	0		0,05844

The sum of alpha and beta coefficients provides that for developed markets, Canada has the highest volatility in the selected period while UK has the lowest volatility. By the beta coefficients we can see that Germany is relatively more persistent while Canada is less persistent. Canada also has the highest alpha value which indicates that they have more sudden fluctuations in the volatility and Germany has less sudden fluctuations in the volatility. So, Canada has the highest overall volatility and abrupt swings in the volatility and is less persistence. In other words, the country is prone to experiencing short-term and sharp fluctuations in volatility, but these fluctuations are expected to subside relatively quickly. On the other hand, a country as Germany tends to have smaller shocks, but they persist for longer time. The sum of alpha and beta coefficients is less than one for all countries which constitutes evidence for the mean reversion process and the magnitude of the sum exhibits which country has the highest overall volatility. We are not rejecting the null hypothesis for the Ljung-Box test on the residuals, so the residuals behave as white noise and the GARCH (1,1) fully captures the volatility.

Table 6: GARCH coefficients of Emerging markets.

	Emerging Markets					
	GARCH (1,1)	Omega	Alpha1	Beta1	Alpha+Beta	Ljung-Box Residuals
	Coefficient	**0,000002**	**0,142791**	**0,855839**	**0,998630**	
Chile	t-value		5,555	36,304		
	p-value		0	0		0,85190
	Coefficient	**0,000013**	**0,086040**	**0,857560**	**0,943600**	
Brazil	t-value		17,578	102,237		
	p-value		0	0		0,65640
	Coefficient	**0,000006**	**0,102928**	**0,856705**	**0,959633**	
Hungary	t-value		12,088	91,280		
	p-value		0	0		0,12308
	Coefficient	**0,000005**	**0,083209**	**0,888212**	**0,971421**	
Poland	t-value		10,455	118,475		
	p-value		0	0		0,66700
	Coefficient	**0,000001**	**0,070035**	**0,924690**	**0,994724**	
China	t-value		5,616	77,819		
	p-value		0	0		0,63190
	Coefficient	**0,000016**	**0,099748**	**0,746542**	**0,846290**	
Taiwan	t-value		12,789	153,426		
	p-value		0	0		0,99020

Table 6 depicts the results for emerging markets, which exhibits that all the countries in emerging markets have evidence for mean reversion process by the sum of alpha and beta coefficients less than one, where Chile has the highest volatility and Taiwan has the lowest volatility. China is most persistence while Taiwan is less persistence. It indicates that Taiwan has short lives fluctuations in their volatility and the market returns tends to revert to their long-term average quickly. It means that Taiwan has relatively stable and predictable market conditions among the other countries. From the alpha coefficients we can see that Chile has more abrupt shocks in their volatility and China has less shocks in the volatility. The test results for the Ljung-Box test on the residuals are not rejecting the null hypothesis and there is no correlation between the coefficients and the residuals indicating that the residuals behave as white noise and the estimated coefficients in the model are fully capturing the volatility.

Table 7: GARCH coefficents of Frontier markets.

Frontier Markets						
	GARCH (1,1)	Omega	Alpha1	Beta1	Alpha+Beta	Ljung-Box Residuals
Croatia	Coefficient	0,000003	0,119394	0,802742	0,922136	
	t-value		14,236	81,680		
	p-value		0	0		0,99940
Romania	Coefficient	0,000008	0,249650	0,673781	0,923431	
	t-value		11,707	37,750		
	p-value		0	0		0,99890
Morocco	Coefficient	0,000005	0,221790	0,647210	0,869000	
	t-value		11,652	35,637		
	p-value		0	0		0,83950
Tunisia	Coefficient	0,000009	0,265172	0,658274	0,923446	
	t-value		12,225	36,349		
	p-value		0	0		0,99990
Bahrain	Coefficient	0,000005	0,128073	0,798660	0,926733	
	t-value		11,917	76,042		
	p-value		0	0		0,99070
Jordan	Coefficient	0,000005	0,271741	0,571775	0,843516	
	t-value		11,534	32,588		
	p-value		0	0		0,75330
Pakistan	Coefficient	0,000009	0,133114	0,789569	0,922683	
	t-value		12,480	66,729		
	p-value		0	0		0,30990
Vietnam	Coefficient	0,000003	0,096494	0,885275	0,981770	
	t-value		9,249	68,040		
	p-value		0	0		0,89840

For frontier market we can also see evidence for mean reversion process by the sum of alpha and beta coefficients. The sum of alpha and beta coefficients show that Vietnam has the highest volatility and Jordan has the lowest volatility. Vietnam is the most persistent country among the other countries in frontier markets while Jordan is less persistent. It indicates that Vietnam has relatively more uncertainty market conditions since it tends to have high fluctuations in their volatility which persist for longer time while Jordan has relatively more certain and predictable market conditions. But while Vietnam has more overall volatility, the country has the lowest sudden fluctuations, and when they occur, they persist for longer time and Jordan has more sudden fluctuations shocks in the volatility that tends to be short lived.

Overall, if we look at the estimated values for each market, frontier markets tend to have the highest alpha values on average. This suggests that these markets are more prone to sudden and large fluctuations in volatility. At the same time, frontier markets have the lowest beta value on average, indicating that higher volatility shocks tend not to persist for long periods of time in these markets. On the other hand, emerging markets tend to have the lowest alpha values on average, which means that they do not react as strongly to market fluctuations as the other markets. However, according to the beta coefficient, these markets tend to exhibit greater persistence in volatility relative to the other markets. Developed market, on average, have the highest sum of alpha and beta coefficients. This indicates that they have the highest overall volatility levels. This can be attributed to the fact that these markets tend to have a medium reaction to shocks in volatility and a medium level of persistence, compared to the other markets.

For all the markets we have tested the standardized squared residuals for serial autocorrelation by the Ljung-Box test to see whether the volatility model has adequately captured the persistence in the variance of the returns. On the tables above we can see the results that shows that all the countries have independent and random standardized squared residuals since we do not reject the null hypothesis in all cases. Then the GARCH model are adequately capturing the volatility and there is no serial autocorrelation left in the standardized squared residuals.

4.4 Speed of mean reversion

The results from the calculations of the speed of mean reversion process by the half-life method is shown in table 8. It exhibits how many days it takes for each country after a volatility shock has happened to revert the halfway back to its mean. We can see from the table that Chile has the slowest mean reversion process, thus it takes 505 days to revert back to half of its mean, which is the longest period in comparison to the other developed, emerging and frontier markets. In contrast, Jordan has the fastest mean reversion process, thus it takes only 4 days to revert back to half of its mean, which is the fastest mean reverting process in comparison to all the other countries. The length in days of the mean reverting process can give an indication of how long it takes if deviations from the unconditional mean occur to revert based on historical data. The shorter half-life, the faster will the variable return back to its long term mean value, the more effective is the given market to correct deviations. By comparing the half-life of volatility across all the markets, then we get an insight to how effective the financial markets of different countries are at absorbing and responding to shocks and volatility.

Table 8: Speed of mean reversion for all the markets.

Markets	Countries	Alpha	Beta	Alpha+Beta	Mean reversion in days	Mean reversion Rank	Volatility Ranking
Developed Markets	Canada	0,2018	0,7729	0,9747	27,0344	17	4
	USA	0,1924	0,7740	0,9664	20,2721	15	6
	UK	0,1438	0,8028	0,9465	12,6184	10	11
	Germany	0,1148	0,8483	0,9631	18,4144	13	8
	Australia	0,1262	0,8381	0,9643	19,0467	14	7
	Japan	0,1332	0,8284	0,9616	17,7069	12	9
Emerging Markets	Chile	0,1428	0,8558	0,9986	505,5730	20	1
	Brazil	0,0860	0,8576	0,9436	11,9398	9	12
	Hungary	0,1029	0,8567	0,9596	16,8221	11	10
	Poland	0,0832	0,8882	0,9714	23,9054	16	5
	China	0,0700	0,9247	0,9947	131,0315	19	2
	Taiwan	0,0997	0,7465	0,8463	4,1532	2	19
Frontier Markets	Croatia	0,1194	0,8027	0,9221	8,5507	4	17
	Romania	0,2497	0,6738	0,9234	8,7015	6	15
	Morocco	0,2218	0,6472	0,8690	4,9365	3	18
	Tunisia	0,2652	0,6583	0,9234	8,7032	7	14
	Bahrain	0,1281	0,7987	0,9267	9,1096	8	13
	Jordan	0,2717	0,5718	0,8435	4,0731	1	20
	Pakistan	0,1331	0,7896	0,9227	8,6138	5	16
	Vietnam	0,0965	0,8853	0,9818	37,6737	18	3

Within developed markets, we can see differences in the countries speed of mean reversion. UK has the fastest speed of mean reversion where it takes about 12 days to revert back to half of its mean, while it takes 27 days for Canadas volatility to revert back to half of its mean. Among countries within the group of emerging markets, we have two outliers which is Chile and China, here it takes respectively around 505 days and 131 days for the two countries to revert the half-way back to their mean, while it only takes around 4 days for Taiwan, which has the fastest mean reverting process among emerging markets. For Frontier markets, the fastest mean reversion process is in Jordan and the slowest is in Vietnam, which takes around 37 days to revert back to half of its mean value of volatility. Within emerging markets there is a huge difference between the number of days it takes, where it is more consistent in developed and frontier markets, and specially for frontier markets we can see that for all countries except Vietnam, the number of days is relatively close and consistent. If we look at the overall average of days it takes for the three markets respectively, frontier markets have the fastest speed of mean reversion of volatility and emerging markets have the slowest speed of mean reversion process of volatility on average. It is noticeable due to the ranking of mean reversion and volatility from the tables right side, that the higher the sum of ARCH and GARCH coefficients leads to higher volatility and thus slow the speed of mean reversion process and vice versa. It means that the speed of mean reversion and the level of volatility has the inverse relationship. Jordan has the fastest speed of mean reversion, and the lowest volatility and Chile has the highest volatility and the slowest mean reverting process.

4.5 TGARCH for asymmetry

To examine whether there is evidence for asymmetry in volatility for the three markets, we have estimated TGARCH models for all the countries. The results for the estimated coefficients are depicted in the following three tables respectively. In comparison to the conventional GARCH model, we have added two more coefficients in the TGARCH model, which is gamma and delta coefficients. The gamma coefficient is a parameter used to capture asymmetry in volatility. It measures the impact of positive and negative shocks on volatility differently, thus a positive gamma coefficient indicates that negative shocks have a greater impact on volatility than positive shocks, whereas if it is negative, it suggests the opposite. The magnitude of the coefficient indicates the strength of the asymmetry effect. The delta coefficient measures the persistence of the asymmetry provided by the gamma.

Table 9 presents the estimated TGARCH coefficients for developed markets, revealing the presence of significant gamma coefficients in all countries except for the UK. The non-significant p-value for UK coefficient suggests that we do not have sufficient evidence in our selected data to support the presence of an asymmetric effect on volatility in this country. Across all markets in the group of developed markets, the estimated gamma coefficient is positive, indicating that negative shocks have a greater impact on conditional volatility than positive shocks. Notably, Germany has the highest gamma value among the developed markets, suggesting that negative shocks have a stronger impact on volatility in this country than in others. Conversely, Canada has the lowest gamma value, indicating that the asymmetric effect on volatility is weakest in Canada relatively to the other developed markets. These findings provide insight into potential differences in volatility asymmetry within developed markets. The delta estimated coefficients indicates that when shocks in volatility are more affected by bad news, USA experiencing more persistency in the asymmetric effect on volatility among the countries in developed markets measured by the magnitude of the delta coefficient. Conversely, Canada is less persistent to bad news among developed markets.

Table 9: TGARCH coefficients for developed markets.

	Developed Markets					
	TGARCH (1,1)	omega	alpha1	beta1	gamma1	delta
Canada	Coefficient	2,76E-07	0,004418	0,069265	0,027247	0,010872
	p-value		0	0	0	0
USA	Coefficient	8,69E-08	0,139606	0,752404	0,261838	2,795962
	p-value		0	0	0	0
UK	Coefficient	8,31E-08	0,053577	0,831251	0,554568	2,783644
	p-value		0,25412	0	0,19761	0
Germany	Coefficient	4,07E-04	0,086778	0,890642	0,973073	1,045448
	p-value		0	0	0	0
Australia	Coefficient	1,01E-07	0,064799	0,861479	0,316485	2,674473
	p-value		0	0	0	0
Japan	Coefficient	3,18E-04	0,122114	0,860043	0,521960	1,150387
	p-value		0	0	0	0

Table 10: TGARCH coefficients for emerging markets.

	Emerging Markets					
	TGARCH (1,1)	omega	alpha1	beta1	gamma1	delta
Chile	Coefficient	1,73E-04	0,114318	0,902594	0,326208	0,949933
	p-value		0	0	0	0
Brazil	Coefficient	1,99E-04	0,081321	0,879373	0,478963	1,323945
	p-value		0	0	0	0
Hungary	Coefficient	1,74E-07	0,067933	0,842765	0,288714	2,804929
	p-value		0	0	0	0
Poland	Coefficient	2,85E-04	0,073858	0,915296	0,659294	1,040660
	p-value		0	0	0	0
China	Coefficient	1,32E-07	0,055603	0,920111	0,002467	2,537832
	p-value		0	0	0,94305	0
Taiwan	Coefficient	9,84E-08	0,036614	0,867472	0,139544	2,922044
	p-value		0	0	0,01112	0

Table 10 present the estimated TGARCH coefficients for countries in emerging markets. In terms of estimated gamma coefficients, all countries except China, have a significant p-value, suggesting evidence for asymmetric volatility in these markets. Specially, the positive gamma coefficients indicate that negative shocks have a stronger impact on volatility than positive shocks, which is consistent with an asymmetric effect. Poland has the highest gamma value among the emerging markets, indicating the strongest asymmetric effect on volatility compared to other countries in emerging markets. On the other hand, China has a very low estimated gamma value, suggesting that asymmetric effect is weak or almost absent in this market. In Taiwan, the persistency of asymmetric effects on volatility is higher among emerging markets, while Chile experiencing the lowest persistency of asymmetric effects among emerging markets.

Table 11: TGARCH coefficients for frontier markets.

	Frontier Markets					
	TGARCH (1,1)	omega	alpha1	beta1	gamma1	delta
Croatia	**Coefficient**	**1,55E-08**	**0,052843**	**0,838307**	**0,163994**	**2,956492**
	p-value		0	0	0,00044	0
Romania	**Coefficient**	**7,54E-08**	**0,126277**	**0,711819**	**0,388219**	**2,912567**
	p-value		0	0	0	0
Morocco	**Coefficient**	**1,61E-08**	**0,089631**	**0,818044**	**0,124380**	**2,916972**
	p-value		0,00033	0	0,00037	0
Tunisia	**Coefficient**	**1,40E-07**	**0,224492**	**0,556188**	**-0,266230**	**2,904482**
	p-value		0	0	0	0
Bahrain	**Coefficient**	**3,31E-08**	**0,056679**	**0,869947**	**0,186018**	**2,857036**
	p-value		0	0	0	0
Jordan	**Coefficient**	**6,60E-09**	**0,112315**	**0,799618**	**-0,030254**	**2,978364**
	p-value		0	0	0,41370	0
Pakistan	**Coefficient**	**1,12E-07**	**0,048731**	**0,833388**	**0,522202**	**2,850592**
	p-value		0	0	0	0
Vietnam	**Coefficient**	**1,38E-07**	**0,065445**	**0,873063**	**0,168632**	**2,707474**
	p-value		0	0	0,00018	0

Table 11 present the estimated TGARCH coefficients for frontier markets. The gamma coefficients are significant for all countries in this group except Jordan, indicating evidence of asymmetry in these countries. However, there is not enough evidence in the selected data to suggest asymmetry in Jordan. All the gamma coefficients are positive except in Tunisia[1] and Jordan, where the coefficients are negative. The negative gamma coefficients indicate that

[1] Tunisia did not exhibit significant ARCH effects and autocorrelation in the pre-estimation tests, so any finding that deviates relatively to the other countries should be interpreted with caution.

positive shocks have a larger impact on volatility in Tunisia than negative shocks, which contradicts the patterns observed in all other countries. Since the coefficient is not significant for Jordan, we cannot say with certainty whether positive shocks have a greater impact on volatility than negative shocks of the same magnitude in that country. In terms of persistency caused by the asymmetric effect, Jordan has more persistent volatility to asymmetric volatility among frontier markets, which means that if bad news appears, then Jordan is more persistent to this relatively to the other countries in frontier markets. On the other hand, Vietnam is less persistent to asymmetric volatility among frontier markets.

Based on our findings, we can deduce that the strength of the asymmetric effect on volatility varies across different markets. Developed markets have a stronger asymmetric effect on volatility compared to emerging and frontier markets, as indicated by the higher magnitude of their gamma coefficients. This suggests that the response of volatility to negative shocks is more pronounced in developed markets, which implies that negative shocks have a greater impact on volatility in developed markets than positive news of the same magnitude. Conversely, frontier markets have a weaker asymmetric effect on volatility than the other markets, with lower magnitude of their gamma coefficients. We find that one country in each market have a high p-value, indicating that there is no significant asymmetry in conditional volatility in those countries.

This suggest that historical volatility in those countries does not exhibit a significant asymmetric pattern in response to past deviations from expected volatility. In terms of the persistency of the asymmetric volatility, we find that overall, frontier markets are more persistent to asymmetric volatility among the three markets, followed by emerging markets and then developed markets, which is less persistent to asymmetric volatility. This may indicate that frontier markets are less sensitive to negative shocks, but when they occur, it takes longer to recover for them. Then developed markets seems to overreach more to negative news than the other markets, but it does not persist for longer time than in less developed markets.

Overall, these results from the TGARCH model suggests, that the volatility dynamics across the three markets are different. Developed markets exhibiting the strongest negative asymmetry effect, but the quickest decay after this effect, while frontier markets exhibit the weakest asymmetric effect, but the longest persistence of this effect. We have examined the persistence of volatility shocks, the presence of asymmetry in volatility, the existence of mean reversion in volatility, and the speed of mean reversion. Our results indicated that there are significant differences across markets in terms of these volatility patterns, but also within the three market groups.

We find that for the selected period, developed markets had the highest overall volatility measured by the magnitude of the sum of alpha and beta coefficients, emerging markets had the second highest overall volatility for the period and frontier markets had the lowest overall volatility. In terms of persistence, we find that all markets exhibit persistence in volatility, but the strength of persistence varies considerably across market groups, but also across countries within market groups. Emerging markets, in particular, exhibits the strongest persistence, drawn by two outlier countries, followed by developed markets and then frontier markets are less persistent. Frontier markets are corresponding more abruptly to shocks in volatility, followed by developed markets and emerging markets corresponding less to shocks is volatility.

Regarding asymmetry, our findings suggest that developed markets exhibits a stronger asymmetric effect on volatility compared to emerging and frontier markets, where frontier markets exhibit a weaker asymmetric effect on volatility in comparison. Negative shocks have a greater impact on volatility than positive shocks of the same magnitude in all countries except for Jordan. The impact of the asymmetric effect persists longer in frontier markets, and it decays quicker in developed markets. Finally, we find evidence for mean reversion process in all markets, although the speed of mean reversion varies across countries. Frontier markets exhibits the fastest speed of mean reversion, followed by developed markets and then emerging markets has the slowest speed of mean reversion process.

5.0 Discussion

The previous section has provided insight into volatility patterns in developed, emerging and frontier markets. The findings suggests that characteristics of volatility differ across these markets but also within the markets. In this section we will discuss the results of our findings and the implications and limitations for these findings.

5.1 Shocks in volatility

The ARCH effects measured by the estimated alpha coefficient in the GARCH (1,1) model is significant for all the markets and was also significant in the ARCH-LM test. This exhibits a tendency of volatility clustering which is memory patterns in volatility, where periods with high volatility often are followed by periods with low volatility, and the volatility periods tend to cluster. This suggests that past innovations explain current volatility and indicating the existence of a dependency between distant observations and memory patterns in volatility (Tripathy, 2022). The alpha coefficients exhibit the exposure to shocks in volatility, and regarding our results, frontier markets are more exposed to shocks in volatility than the two other markets. Frontier markets are followed by developed and then emerging markets that are less exposed to shocks in volatility.

This means that frontier markets are more sensitive to past shocks in volatility and past shocks have a stronger impact on the current volatility. It captures the short-term volatility dynamics and indicates how quickly the volatility respond to past shocks. The higher alpha coefficient of frontier markets may indicate that the market is more susceptible to volatility shocks and may be more inefficient in the process when news occurs and when they are being incorporated in the prices. That emerging markets are less sensitive to past shocks in volatility and have a weaker impact on the current volatility in comparison to the two other markets, may suggests that these markets are more efficient in the process of the reaction to new information into the stock market price and the market adjust to new information faster than the other markets with higher alpha coefficients.

In relation to previous theory, it is expected that frontier markets are more inefficient in the short-term dynamics of current volatility respond to past shocks than developed markets, but it is surprising that emerging markets should be more efficient when we are looking isolated to the alpha coefficient and the response to past shocks. The level of liquidity significantly affects both the stock prices and the rate of return and developed markets are more liquid than frontier and emerging markets. This means that it should be more reasonable when new information arrivals to the market and investors either buy or sell a large number of shares, that a less liquid market should be more affected in the price movements, since they cannot in terms of the liquidity buy or sell shares on the market at any given price, which might affect the stock price and rate of return (Gniadkowska-Szymańska, 2017).

The frontier market is characterized as less liquid than emerging markets, and they have less activity of investors in these markets and have a lack of easy access to information and market data in comparison to developed markets. This is why we would have expected that developed markets would have been measured with the lowest alpha coefficient, since they, given their easy access to information and market data and higher activity of investors and higher liquidity, should have been more efficient in the short-terms shock of volatility to quickly incorporate the new information to the stock market price and lack of liquidity in specially frontier market should cause more volatility when new information arrival in the short-term. Also, according to Koren & Tenreyro (2007), frontier markets should experience more abrupt fluctuations and shocks in volatility. On the other hand, Eizaguirre et al. (2004) suggests that growth in trading volume have a significant impact on stock market volatility, and the level of development increases the trading volume, so according to these findings, developed markets should have a higher volatility.

5.2 Volatility persistence

Regarding volatility persistency, we find evidence for volatility persistency in all the markets, which means that there are long memory patterns in volatility that affecting current and future volatility, indicating the influence of shocks in the stock market persist in a period, which is consistent with previously findings (Tripathy, 2022; Crato, 1994; Cheung & Lai, 1995). We find that emerging markets are more persistent relatively to developed and frontier markets. We also find that frontier markets are less persistent in comparison. It is surprising findings that developed markets should be the more persistent than frontier markets, but it is expected that emerging markets should be more persistent than developed markets.

According to the efficient market hypothesis, more efficient markets should be less persistent since they are adjusting quickly to new information. Concerning that developed markets should be less persistent among the three markets, since they have easy access to information and market data, and they have more activity of investors and are seen as more liquid relatively (Gniadkowska-Szymańska, 2017). Also, Eizaguirre et al. (2004) suggests that the level of development will decrease the persistence in volatility. Since frontier markets are less developed and have less easy access to new information and their liquidity is lower compared to the two other markets, then it is surprising that the effect of past shocks in volatility should be less persistent in frontier markets and according to this they should implement adjustments in stock prices quicker that the other markets.

On the other hand, Beg & Anwar (2012) suggests that the constant arrival of new information causes persistence, so, according to this statement, more availability to information and arrival of news should led to higher persistency and this can explain why developed markets are more persistent than frontier markets, but it cannot explain why emerging markets are more persistent than developed markets then.

An argument that could explain that emerging markets are more persistent than the other markets, is that there is more access to emerging markets than frontier markets, and investors have moved some capital to emerging markets for diversifying their portfolio and for higher risk adjusted return. Since our data sample period go through some crisis, investors get more risk averse during crisis, so they sell off their risky assets and moves their capital into more secure and well-known assets. When risk averse investors move their capital, the liquidity turns low, and emerging markets are already less liquid than developed market, which continue to influence volatility and extend the period with high volatility in emerging markets (Guiso & Zingales, 2018; Kondor & Vayanos, 2019). Also, the fact that our data sample containing more periods of crisis, herby the Covid-19 pandemic, the exceptional high inflation and the war in Ukraine, the uncertainty in these crises might have affected the volatility persistency as well, and the volatility persistence might increase during crisis (Dedi & Yavas, 2017).

5.3 Volatility asymmetry

Regarding asymmetry, we find evidence of volatility asymmetry in all markets except for UK in developed markets, China in emerging markets and Jordan in frontier markets. The evidence of asymmetry in volatility is consistent with previous findings (Bekaert & Wu, 2000; Hasanhodic & Lo, 2011; Su & Wang, 2020; Adrian & Rosenberg, 2008). We also find that developed markets have highest asymmetry among the three markets measured by the magnitude of the gamma coefficient in the TGARCH model. This measurement of the coefficient is positive for developed markets, suggesting that developed markets have an asymmetric reaction towards negative shocks in volatility, with a stronger reaction to negative news than frontier and emerging markets that mainly also have a positive gamma coefficient.

That our study suggests that negative news impacts stock market return volatility more than positive news is contradicting with the few previous studies that have analyzed the impact. Emenike & Enock (2020) suggests that positive news have a greater impact on stock markets return volatility than negative news, and Mallikarjuna & Rao (2019) does not find evidence of asymmetry, since they find that frontier markets have a more symmetric effect on stock market volatility. Our results also show that developed markets have a quicker decay of the asymmetric effect on volatility than emerging and frontier markets, where frontier markets are more persistent to asymmetric volatility.

It is surprising that developed markets are more asymmetric compared to the other markets, when we look at the efficient market hypothesis perspective, because developed markets should be less persistent and not have large fluctuations relatively to less efficient markets. But according to this, our findings of a quicker decay to asymmetric volatility in developed markets are consistent with the efficient market hypothesis. It is expected that emerging markets have a medium asymmetric volatility and persistency according to the order of development among the three markets. Another study that compares the asymmetric effect among developed, emerging and frontier markets during the period of 2000 to 2018 also find that developed markets corresponding to asymmetric volatility relatively more than emerging and frontier markets and they find no evidence at all in frontier markets which exhibits a symmetric effect on volatility (Mallikarjuna & Rao, 2019). We find the opposite to frontier markets, since they have an asymmetric effect on volatility in our research period.

There is a lot of explanations for asymmetric volatility in the literature, and a lot of them are consistent with our findings that is conversely to the efficient market hypothesis according to the gamma coefficients. Dzielinski et al. (2018) suggests that asymmetry is caused by asymmetric attention by the number of analysts, which can be an explanation since developed markets tends to have more attention by analysts which according to this can led to larger asymmetry in developed markets relatively. They also suggest that larger asymmetry is caused by the large percentage of retail investors, which also can be explained by that developed markets have experienced a large approach of individual private investors in recent years, and especially after the Covid-19 pandemic, which might have caused more irrational investors in developed markets.

Hasanhodic & Lo (2011) explains the volatility asymmetry with human behavioural, that is shaped by recent experiences with current information, which can have an impact on future behaviour and volatility. Which led to, that specially in developed markets, the access to online platforms through smartphones for individual investments has increased the trading volume, which has helped individual investors to enter the stock market as direct participants, this might have had an impact on the asymmetric volatility in developed markets (Nasdaq.com, 2022). It assumes that all else being equal, that non-professional, individual investors, might be more irrational in their decisions when they are trading on the stock market.

Another argument that supports the highest asymmetric volatility in developed markets is the overall business cycle, since the selected period for our data contains crisis, then investors are listening more to bad news in bearish times, which affects investors beliefs and the asymmetric volatility (Yaya & Gil-Alana, 2014; Jones et al., 2004). Then since developed markets have easier access to information, then bad news might affect them more than less developed markets. They also argue that the bad news in bearish times affecting the asymmetric volatility persistence, so when we find that frontier markets are more persistent to small asymmetric effects, it cannot be explained by the accessibility to news, but rather by the efficiency that cause developed markets to adjust quicker to asymmetric volatility relatively to emerging and frontier markets.

5.4 Overall volatility and mean reversion

Our results show that for the selected period, all markets are mean reverting. Reminding the mean reversion process can be measured by the sum of alpha and beta coefficients, that all should be less than one to be mean reverting. That we find evidence for mean reversion among all the markets, exhibit that there are predictable patterns in the volatility, and they are not just moving randomly. The magnitude of the sum of alpha and beta coefficients measures the overall level of volatility among the three markets. In our data sample, we find that developed markets have the highest overall volatility measured by the sum of alpha and beta coefficients.

That developed markets have the highest overall volatility among the three groups of markets are surprising since developed markets are always characterized as a more efficient market relatively to emerging and frontier markets, and according to the efficient market hypothesis, efficient markets are fully reflecting all available information and would then be expected to be less volatile. Thus, in efficient markets new information should be quickly and fully reflected in prices since market participants in developed markets have more access to information (Goetzmann & Jorion, 1999; Fama, 1960).

Frontier markets are measured to have the lowest overall volatility which is also contradicting to the efficient market hypothesis perspective, since frontier markets are characterized as the less developed market among the three markets, and they have less access to available information, and therefore, they should be more volatile. Also, in terms of liquidity, developed markets have more liquidity and which should cause that assets always can be sold on the market at any given price, where market with less liquidity tends to cause more price movements, and especially during crisis where investors risk aversion increases, and they move their capital to less risky markets. And when they move their capital, the liquidity in the less developed markets should according to this decrease and extend the persistency (Guiso & Zingales, 2018; Kondor & Vayanos, 2019). The liquidity of different developed markets should explain the mean reversion process as when bad news has passed, investors go back to a market which have decreased in value to gain abnormal profit at the lower price, which correct the price level and causes mean reversion. This statement can explain why we find that emerging markets are more persistent, since a lot of investors has moved capital to emerging markets for diversifying their portfolio, and during crisis investors are getting more risk averse and move their capital, which is decreasing the liquidity and extending the persistency as previous studies has suggested.

Developed markets are also characterized as more liberalized among the three markets, and according to Bekaert & Campbell (1997), capital market liberalization has a decreasing effect on volatility. Also, De Santis (1997) suggests that volatility decreases with liberalization, and they found that volatility is higher in emerging markets than in developed markets. This contrasts with our findings that suggests contradicting results if we look at the overall volatility. Koren & Tenreyro (2007) suggests that less developed countries experience more frequent and more severe aggregate shocks poor countries are working in less diversified sectors, where financial diversification is the mechanism that cause that the volatility decreases with the level of development, since they often work in more diversified sectors when they are more developed. This also contradicts to our findings.

On the other hand, our data is containing major recent crisis, and the volatility tends to increase and be more persistent during crisis that have affected more developed markets and since the persistency and asymmetric effect of volatility can be explained by spillover effects. (Dedi & Yavas, 2016; Dedi & Yavas, 2017). Since there is higher correlation among developed markets and they experience a higher spillover effect, then it can explain why recent crisis that has taken place in developed markets, also have had a larger impact on the overall volatility in developed markets (Dedi & Yavas, 2016; Sukumaran, Gupta & Jithendranathan, 2015). Also, the growth in trading volume especially in developed markets, but also in emerging markets, have a significant impact on stock market volatility and the level of development increases the trading volume, which led to higher volatility. This can explain the order of the overall volatility that is highest in developed markets, followed by emerging markets and then frontier markets (Eizaguirre et al., 2004).

As mentioned before, especially developed markets have experienced an increase in terms of private individual investors due to the easy access to trading platforms through investors smart phones. And many studies explain the asymmetric volatility and the mean reversion as human behavioural that all else being equal will increase if more individual and irrational investors enter the trading stock market. De Bondt & Thaler (1985) explains the mean reversion as an overreaction explained by psychological behaviour of individuals that tend to overweight recent data in their predictions and after a while, the shock has been observed in the data, and it reverts back towards its mean. In terms of mean reversion of volatility, the overreaction will then be expressed by patterns in volatility, large shocks, long persistency and asymmetric reaction to negative news, where they tend to overreach to unexpected and dramatic news. So, when we look at the results for the TGARCH of developed markets, then it seems like market participants overreach to bad news since they have the overall highest gamma coefficient, but the persistency in bad news in developed markets is fastest among the three markets to mean revert.

Also, Talpepp & Rieger (2010) find some factors that can explain and increase asymmetric volatility. They find that asymmetric volatility is more likely in wealthier and developed markets, not because of lack of development in less developed markets, but they find it is rather related to the proportion of non-professional investors on the market. They also find that stock market participants as a percentage of private investors in the market have a positive correlation to volatility asymmetry. The next factor they find is that if there is more coverage by analysts, it tends to have more asymmetric volatility, which is linked to the statement mentioned earlier about asymmetric attention cause increasing asymmetric volatility. The last factor they found as an explanation that can increase asymmetric volatility was short selling, and it especially increase volatility in bear markets. So, based on that, they reject the leverage effect as the pure explanation since they did not find strong enough evidence, so they conclude that the asymmetric volatility is clearly related to private investors and behavioural sentiments. These four factors that they find as an explanation that can increase asymmetric volatility, can help to explain why we find highest asymmetric volatility for our data in the selected period, and if we look at the kind of period we are analyzing and which events have taken place in the period, then it seems reasonable.

The patterns of volatility of developed, emerging and frontier markets are defined as the overall average for each market to make the comparison. It is noticeable that there is also different measurements in terms of the magnitude of the estimated coefficients within the respective groups of markets which can give an indication of that volatility patterns might be more country specific rather than group specific.

5.5 Speed of mean reversion

When measuring the speed of mean reversion in our data, we find that overall, frontier markets mean reverts fastest, followed by developed markets and then emerging markets. Emerging markets have two countries with a very slow mean reverting process which is Chile and China that draw the average of the mean reverting process in emerging markets in a slower direction. To measure the speed of mean reversion, we are using the estimated coefficients from the conventional GARCH model, which does not take into consideration whether the shocks in volatility are coming from either bad or good news but have a more symmetric influence. The speed of mean reverting is quite linked to the persistency and overall volatility that we estimated earlier, since we are using the same coefficients and then convert them into days as a better understandable measurement for investment decisions.

If developed markets should be more efficient relatively to emerging and frontier markets according to the efficient market hypothesis, then it is surprising that frontier markets have the fastest speed of mean reversion process since it indicates that frontier markets are quicker to absorb an implement movements and new information which is used to be related to efficient markets. Then it means, that based on our selected period for our data sample, frontier markets are more efficient among the three markets if we look at the speed of mean reversion. We also see that there is a huge difference in the persistence and thereby the speed of mean reversion among countries within a market type. Especially within emerging market we can see two outliers which is Chile and China with 505 and 131 days and in the same market we have Taiwan with only 4 days. This indicates that market types do not show the same patterns, but also differences can happen within the same group.

We find by ranking, that the higher overall volatility, the slower a mean reverting process and vice versa the lower volatility, the faster mean reversion which is consistent with Ahmed et al. (2018) findings. Emerging markets are containing some outliers that have a very slow mean reverting process and with exclusion, developed markets would then have the overall slowest mean reverting process. That higher volatility should lead to slower mean reverting process is consistent with the efficient market hypothesis, but it is just contradicting that we find the highest volatility and slowest mean reversion process in developed markets that we would have assumed to be more efficient. Converting the mean reversion into days can provide us with a more tangible measurement method for making investment decisions and a faster speed of mean reversion in frontier markets indicates that you should invest in frontier markets for a shorter investment horizon.

It is noticeable that developed markets have highest persistence if we look at the conventional GARCH model, while frontier markets are less persistent. When we are looking at the results for the TGARCH model capturing the effects of the asymmetry, it exhibits that developed markets tend to be affected more by negative news, but it does not tend to have a long-term persistency, and for frontier markets the reverse applies when frontier markets are less asymmetric and when bad news occur, it persists for longer time. It may indicate that developed markets are more persistent when good news happens and less persistent when bad news occurs while frontier markets is more persistent when bad news occur and less persistent when good news occur. Emerging markets have highest persistency when we are not taken asymmetry from bad news into account and a medium persistency when bad news appear. That developed markets should be more persistent to good news than bad news can be explained by the selected period, where we in developed markets have seen a tech-bubble and a huge increase in the stock market value of developed markets after private investors participating after the drop in the covid-crisis that has tend to increase the effect of an upward going market (Nobanee & Ellili, 2023).

5.6 Implications and limitations

Since our data sample is containing major recent crisis, our results might be affected by the recent events that have happened in the economy and the recent implications of the overall global stock market. Our results might have explained what happened to the volatility patterns during the selected period and not certainly an explanation and examination of the overall volatility patterns among different markets related to other periods. Major things have happened in the selected period, and it also contains implications from the increase of non-professional investors who has entered the market during the period. We have seen a large implication of social media to the movements of the stock market in mainly developed markets such as recent events as the tech bubble and the GameStop short squeeze, reddit comments and other meme stocks which is a newer phenomenon that have impacted the stock market recently (Nobanee & Ellili, 2023). The war in Ukraine and the sanctions followed herby from the western countries, might have had an impact on the volatility mainly in developed markets, that also have a higher correlation and spillover effect.

Conversely, the selected period was chosen to give an insight of the volatility patterns of recent data and how the market reach through different types of markets regarding current and recent volatility patterns to give a current insight. Financial markets are dynamic and recent implications such as social media and non-professional investors who has entered the market are implications which we should deal with. But the question is whether the recent period of volatility patterns can be used to predict future volatility patterns and movements, but there would also be implications by using a period excluding the recent events.

Another thing that is important to highlight, is the strengths and weaknesses by the applied methodology. Some of the potential strengths of the thesis and its methodology are the use of GARCH and TGARCH models which are well known and well tested methods for analyzing volatility patterns in financial markets. Also, the inclusion of group-based data from developed, emerging and frontier markets can give a more nuanced picture of how volatility exhibits in different groups of markets instead of the previous studies that mainly are driven by individual markets, and it can then contribute to the existing knowledge about volatility patterns. Some of the weaknesses are that there may be limitations in the data quality that can affect the validity of the results. The usage of GARCH and TGARCH models are assuming a normal distribution in the return, which we in the beginning of the basic statistical of the data rejected, since the return were more leptokusic, it is a limitation of the models since it is common that financial data are not normal distributed at all. Also, the fact that there is a lot of factors that are contributing to the volatility since volatility is a very complex phenomenon, so there might be factors affecting the volatility which have not been considered in this analysis.

5.7 Further research

Our study provides insight into the volatility patterns of the selected period, there are still gaps that needs to be investigated. Since the selected period containing the recent period with high inflation, further research could include the use of GARCH-X model for an exogenous variable such as macroeconomic factors as inflation or interest rates which may affect different types and levels of markets differently.

Another further research could be to test the ability of GARCH models to forecast volatility by applying the models to a test period and assessing the robustness of the estimated model. Additionally, since the liberalization has included international markets, further research could explore the conditional correlation between time series by using the DCC-GARCH model to investigate whether correlation vary through time. Mean reversion has been observed in various markets, but the literature is limited for explanation behind why mean reversion differ across different types of markets, therefor further research could examine why different markets exhibit mean reversion differently.

The pandemic has had a significant impact on the economy and financial markets recent years. Since we are including this period in our study, it could be interesting for further research to examine the implication of the pandemic as a structural break point, to investigate how volatility patterns differ across different types of markets through this crisis by the use of a BGARCH model. Finally, given the recent rise in non-professional investors and the growing impact of social media on financial markets, further studies could examine the effect of these factors on volatility.

6.0 Conclusions

This study has measured, examined and compared the volatility patterns in terms of persistence, asymmetry and mean reversion across developed, emerging and frontier markets using GARCH and TGARCH models. The study also compared the speed of mean reversion using the half-life volatility model. The historical period from 31/12/2012 to 31/12/2022 was used to analyse and examine the recent decades historical trends and patterns of market volatility in a comparative framework.

The findings conclude that developed markets exhibits the highest overall volatility, followed by emerging and then frontier markets. This implies that investors in developed markets may face more risk and uncertainty, but also potentially higher returns. In terms of persistency, emerging markets are more persistent among the three markets, followed by developed markets and then frontier markets are less persistent when asymmetric reaction is not considered. It was also observed that frontier markets have the highest speed of mean reversion followed by developed, and then emerging markets that have the slowest speed of mean reversion.

In terms of asymmetric volatility, all markets react more to bad news than to good news. Developed markets have the strongest asymmetric reaction to bad news among the three markets, followed by emerging and then frontier markets that are less asymmetric towards bad news. When bad news occurs, developed markets tends to have a short-term persistency to bad news, while developed markets are more persistent in terms of good news. Conversely, frontier markets have the weakest asymmetric reaction to bad news, and when they occur, they persist for a longer time, while the effect of good news in frontier markets have a short-term persistency in comparison. Emerging markets have a medium magnitude of asymmetry when bad news occurs, and also for the persistency towards bad news, while emerging markets have the highest persistency when asymmetry has not been considered.

That developed markets exhibits the highest overall volatility level among the three markets are inconsistent to the efficient market hypothesis and regarding the effect of asymmetric volatility, we reject the efficient market hypothesis. Our study suggests that developed markets are more inefficient due to the increase in trading volume and non-professional investors participating the stock market. The effect of recent crisis is better measured in developed market due to higher spillover effect in developed markets and developed markets containing more attention causing asymmetric volatility. Retail investors are reacting more to bad news, but the effect of bad news is not as persistent as good news in developed markets, where we have seen bubbles in the stock markets which has increased the volatility and persistency from good news in the stock markets of developed markets.

By the evidence of persistence, asymmetry and mean reversion, suggests that there are predictable volatility patterns in developed, emerging and frontier markets. There is evidence for significant different volatility patterns among developed, emerging and frontier markets by its magnitude, but volatility patterns also differ within the respective markets which indicates that volatility patterns are more country specific rather than group specific. The study suggesting that all the three markets are inefficient, and developed markets not necessarily are more efficient than less developed markets. High volatility asymmetry indicated a prevalence of irrational non-professional investors in developed markets that may influence changing characteristics of volatility. The fastest speed of mean reversion in frontier markets indicates that short term investments should be placed in frontier markets. The implications of these findings are important for investors, as they provide insight into the behaviour of volatility in different markets. By understanding these volatility patterns, investors are able to make more informed investment decisions and adjust their investment strategy for risk management.

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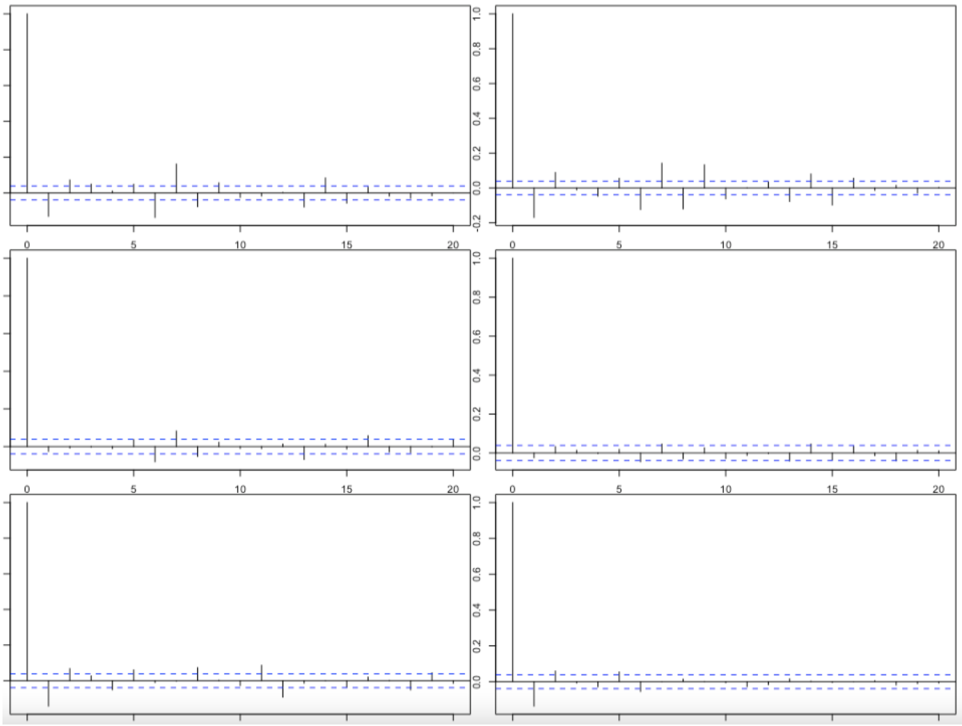
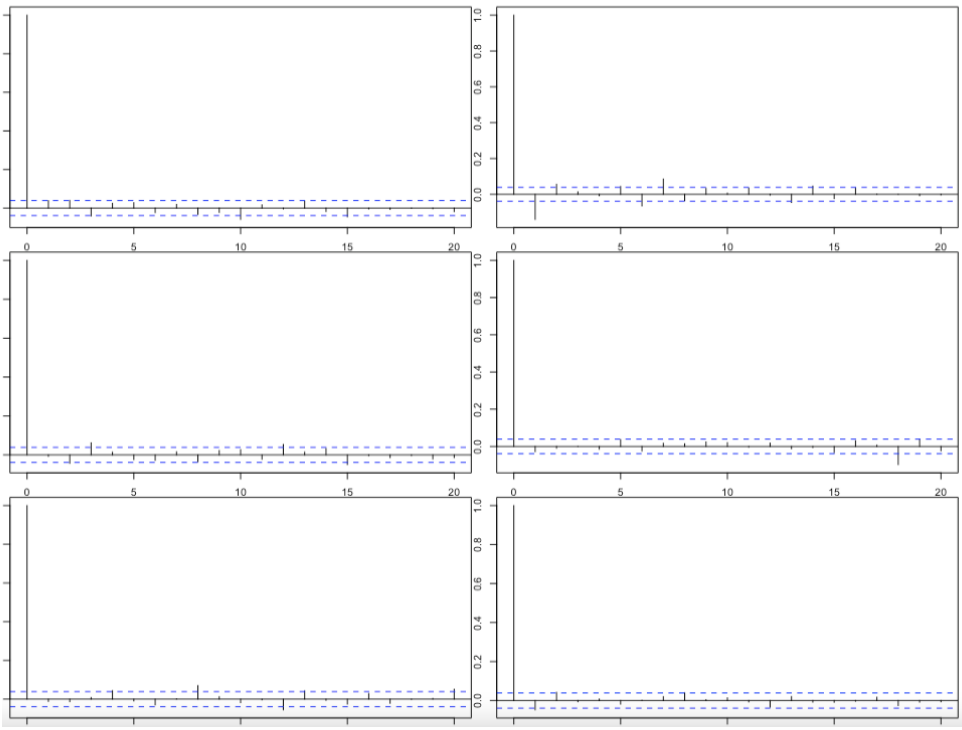
Appendix

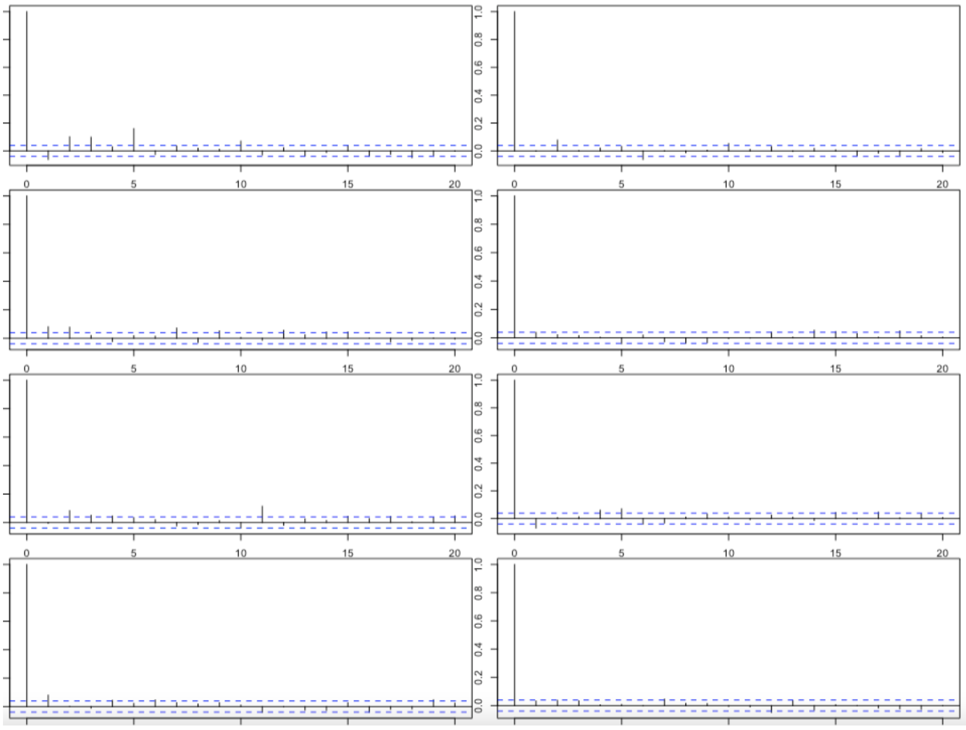
Appendix A: ACF plot developed markets



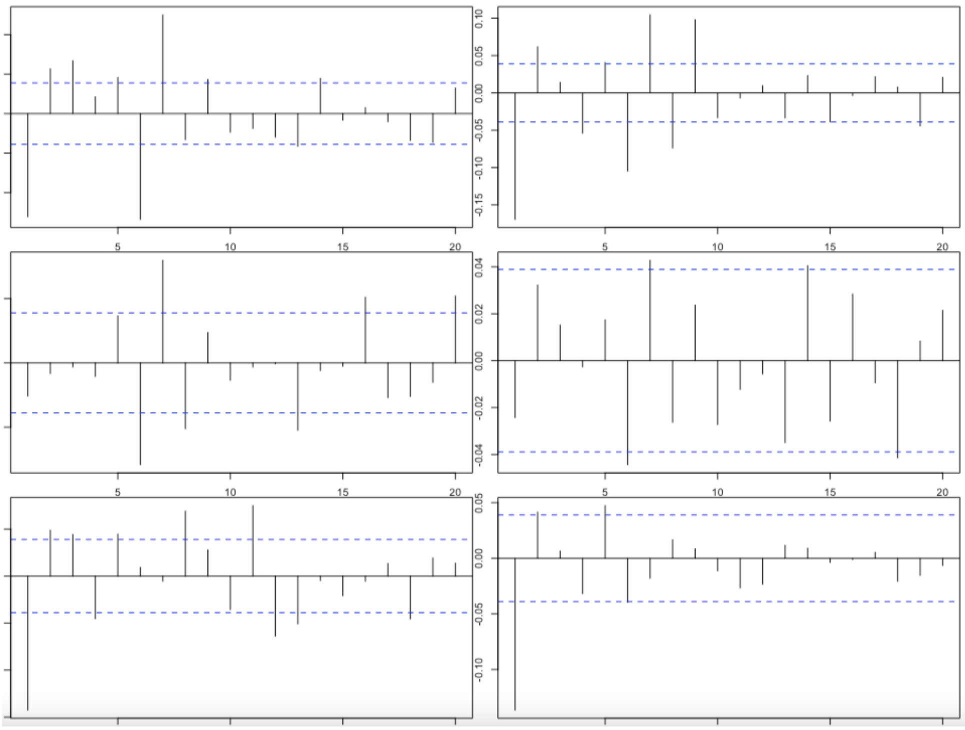
Appendix B: ACF plot emerging markets



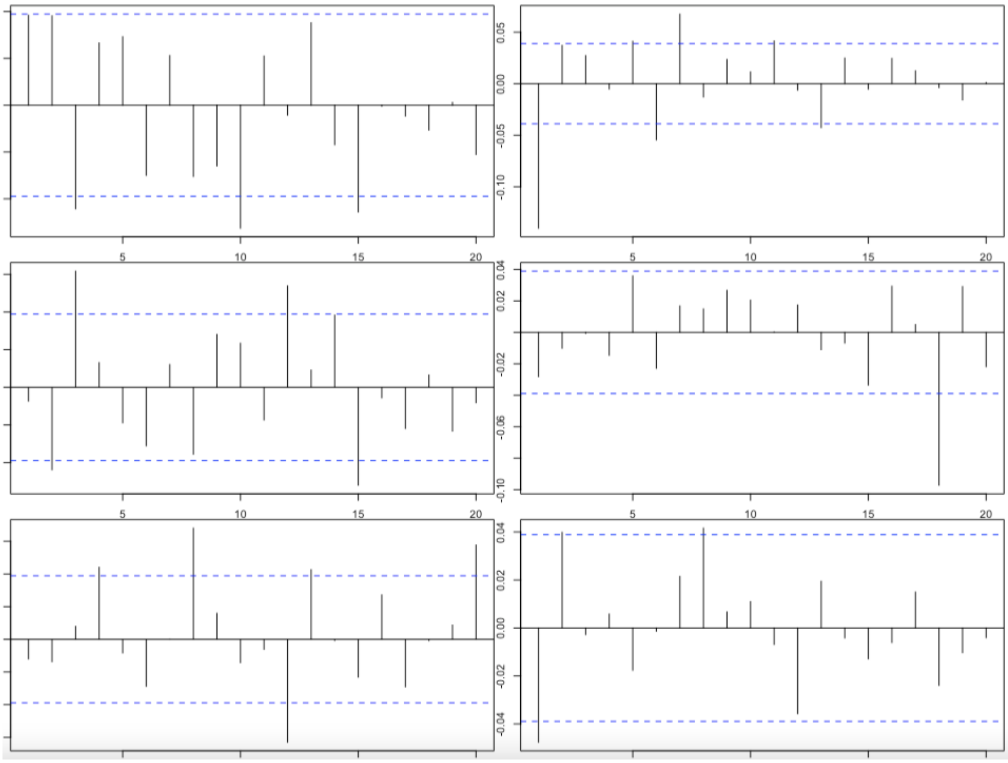
Appendix C: ACF plot frontier markets



Appendix D: PACF plot developed markets



Appendix E: PACF plot emerging markets



Appendix F: PACF plot frontier markets