Are equity returns still predictable? The case of industry portfolios and ARIMA models

With the application of the Box-Jenkins methodology

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0.1 Abstract

The efficient market hypothesis is a fundamental concept in financial economics that suggests asset prices fully reflect all available information. As a result, investors cannot consistently achieve higher returns than the market average by using historic and publicly available information. However, there is ongoing debate about the degree to which markets are efficient and whether it is possible to predict asset returns. This thesis contributes to the debate by investigating the weak-form of efficiency and the relevance of ARIMA models for predicting log returns of self-constructed industry portfolios in Sweden. The results indicate a rejection of the weak-form of efficiency for industry portfolios in Sweden. This finding suggests that past prices and returns is not fully reflected in today's prices, and that investors may be able to achieve excess returns by using historic information in active investment strategies. However, the economic-value add of ARIMA models for predicting log returns was found to be inconsistent across industries. This implies that the predictability of returns may vary depending on the specific industry or sector being analysed.

These results have significant implications for researchers and professionals interested in the predictability of industry returns and market efficiency. By shedding light on the limitations of market efficiency and the potential for return predictability in certain industries, this thesis contributes to the ongoing debate about the efficiency of financial markets and the potential for investors to achieve superior returns through active investment strategies.

Key words: *ARIMA, Autoregressive Integrated Moving average, market-efficiency, weak-form, predictability, Augmented Dickey fuller test, stationarity, Random walk, Ljung-Box test, autocorrelation.*

Table of contents

1. Introduction

Asset returns has undergone exhaustive research in recent years for whom the greatest interest comes from academics and professionals in the financial industry. The time period applied for this study is from in Jan. 2010 to Dec. 2022 for log returns of Swedish industry portfolios. To the best of my knowledge this period hasn't undergone similar research.

The great interest for predictability is due to the consequences of being able to continually make economic value-added forecast for investment purposes. The question if asset returns are predictable has developed into two main theories. The theory for asset returns to be unpredictable is that they follow a random walk, meaning that the behaviour of the returns is completely random. This is due to the market being efficient and the impossibility of earning an excess return relative to the market return by using historic- or public information for investment strategies (Fama, 1995). Originally Eugene Fama (1970) introduced his theory of market efficiency stating three levels hereof, for which a common tested form of market efficiency is the weak-form, stating all prior historic information can't be used for earning excess returns[1].

The common applied methods in research for testing the validity of the weak-form is the Augmented-Dickey Fuller- and Ljung-Box test (Shaker, 2013; Afeef, Ihsan & Zada, 2018; Worthington & Higgs, 2003). The null hypothesises for the tests is respectively, that the time series of returns follows a random walk and if rejected, then there is significant evidence for the time series is stationary and thereby mean-reverting. Whereas if rejecting the null hypothesis for the Ljung-Box, then significant evidence for autocorrelation is provided and returns thereby are dependent on each other. To the best of my knowledge the most recent study for the Swedish market was conducted by Shaker (2013) who found significant evidence for rejecting the null hypothesis for both the Augmented Dickey Fuller- and Ljung-Box test for the main index OMXS30. Thereby rejecting the validity of the weak-form of efficient market in Sweden, which gives an incentive to further study the predictability in the Swedish market.

[1] A detailed description of the Efficient Market Hypothesis is provided in section 2.1.1 The Efficient Market Hypothesis.

1.1 Swedish Industry Portfolios

The Swedish stock market is the largest in Scandinavia in terms of the number of listed companies, with 1068 investable equities for the period of investigation (see Chapter 4, section 4.1 of the present thesis). This provides a rich source of data for statistical analysis and the possibility of studying multiple industry portfolios. For this study, a total of five self-constructed industry portfolios consisting of 66 companies in total is used. The data collected contains both monthly and weekly observations from Jan. 2010 to Dec. 2022 to capture any potential patterns in returns. The use of monthly and weekly data has been applied in previous studies, which have similar methodology as to this study (Chowdhury, 1999; Frennberg & Hansson, 1993; Kim & Shamsuddin, 2008; Har, Sundaram & Ong, 2008).

This study opted to construct its own industry portfolios rather than relying on industry indexes, as this approach provides greater investability and allows for a more direct comparison of returns across industries. By using equal-weighting rather than value-weighted the reflection of the idiosyncratic risk of each industry is clearer (Kittsley, 2006). This could become useful for professional portfolio managers in enhancing their decision making for purposes such as tactical asset allocation. The five industries of interest are Biotechnology, Information Technology, Packaged Software, Real Estate Development, and Industrial Machinery[2]. These industries were selected based on their distinct characteristics in terms of idiosyncratic risk.

1.2 Predictability of industry portfolios

For predicting the returns of the industry portfolios, the Autoregressive Integrated Moving average (ARIMA) model is applied. It was presented by George Box and Gwilym Jenkins in 1970 and is a study of autocorrelation for a time series, where the dependent- and independent variable is a part of the same data generating process (Box, Jenkins, Reinsel & Jung., 2015). For the usage of ARIMA models the Box-Jenkins methodology was introduced, which is a 3-step framework for model identification, estimation, and diagnostics (Box et al., 2015)[3]. In time series analysis the application of ARIMA is commonly applied across industries and academia for forecasting. Within

[2] Further specification and description of the industry portfolios is provided in chapter 4. Data collection.

[3] The Box-Jenkins framework and ARIMA model is respectively specified in section 2.4 Prediction of asset returns and 3.6 Autoregressive Integrated Moving average model.

financial research it has been heavily applied for stock returns for the creation of investment strategies and enhancement of decision making. Examples of evidence for predictability using ARIMA models is found by Bakar, Rosbi and Uzaki (2018) for Malaysian oil and gas sector, and Rounaghi and Zadeh (2016) for stocks on the London Stock Exchange and in the SP500 Index.

1.3 Research questions

In this thesis, the objective is to investigate the weak-form of market efficiency and predictability of self-constructed Swedish industry portfolios using ARIMA models. The research questions are as follows:

1) *Are the weak-form of market efficiency valid for the log returns of specific industry portfolios in Sweden?*

2) *Are ARIMA models still relevant in predicting the log returns of self-constructed Swedish industry portfolios?*

To the best of my knowledge prior research about the validity of weak-form market efficiency and predictability using ARIMA is highly limited for industry portfolios, while similar studies for main indexes has undergone much research. Therefore, the results from this study provides new evidence for market efficiency and predictability for the given industries in Sweden. The weak-form of market efficiency was rejected for all industry portfolios by using the Augmented Dickey Fuller test, therefore significant evidence for rejecting the random walk model is provided, which is in support of previous studies conducted for the Swedish market. Regarding the results for the Ljung-Box test and predictability using ARIMA models, these aren’t homogenous across industries and observation frequency. This can be due to the level of homogeneity for companies in each industry vary and autocorrelation is less frequent on a weekly and monthly basis.

1.4 Thesis structure

To investigate the research questions and provide the reader with sufficient knowledge of the field the following structure of the thesis has been formed.

Chapter 2. Literature review introduces the debate of predictability of asset returns with regard to the random walk model and efficient market hypothesis, followed by an introduction to the Box-Jenkins framework for the application of ARIMA models.

Chapter 3. Methodology explains the concept of stationarity and non-stationarity, including the method for testing it, and the importance hereof for a time series when doing ARIMA modelling. Besides this the constituents of the ARIMA model are presented, and the goodness of fit measurements applied in this study.

Chapter 4. Data collection presents the method used for stock selection for the industry portfolios, and a general introduction to each industry. Besides this the method for calculation of portfolio returns is presented followed by a section of descriptive statistics.

Chapter 5. Analysis of results presents the findings obtained from the application of the Box-Jenkins framework in a chronological order of identification, estimation, and diagnostics, followed by an evaluation off the model’s forecasting performance. Towards the end of the chapter, a summary of the results is provided. This summary consolidates the key findings and highlights the main findings drawn from the analysis

Chapter 6. Reflection aims to provide suggestions for future studies and address the limitations of this study. It discusses the applicability of the results in future research, highlighting potential implementations to the methodology.

Chapter 7. Conclusion applies the results of this study to synthesize the findings and their implications, providing a conclusive response to the research question and highlighting the study's contributions.

2. Literature review

This chapter introduces the existing literature about the predictability of asset returns and how this relies to the efficient market hypothesis, while bringing an in-depth overview of the studies of market efficiency in Sweden. Hereafter the concept of mean reversion and relevant studies in financial academia for ARIMA models is introduced, as well as the methodology of constructing this econometric model using stock returns.

2.1 The predictability of asset returns

Market participants have endeavoured to predict the fluctuations of stock prices through a variety of methods. The methods have ranged from analysing financial reports and estimating intrinsic values of stocks to applying technical analysis to identify price patterns (Fama, 1995). In academia, the predictability of asset returns has led to the development of several quantitative models, where this study applies the ARIMA model. To understand the reason for why stock prices can be predicted the efficient markets hypothesis originated, stating predictability depends upon the market efficiency.

2.1.1 The Efficient Market Hypothesis

In 1970 Eugene Fama published the paper *Efficient capital markets: A review of theory and empirical work,* where he presents his theory of the efficient market hypothesis (EMH), which tries to explain different levels of efficiency dependent upon the information priced into the market, and how come prices can deviate from an equilibrium. This resulted in the introduction of three forms of market efficiency namely.

- *The weak-form* of market efficiency states that all historic price data is reflected in current prices, therefore investment strategies incorporating this wouldn't be beneficent.
- *The semi-strong* form states that all public information available is reflected in the market prices and assumes that all new information released to the public quickly will be priced correctly. Thereby any investment strategy using fundamental analysis wouldn't be beneficiary.

- *The strong-form* states that all insider information is efficiently priced into the market, and thereby insiders having monopoly on their information would not be able to generate excess returns on behalf of this (Fama, 1970).

The origin of the EMH is due to the random walk model. According to Fama the random walk model was part of the idea of financial markets being a “fair game”, and equivalent to a coin-flip contest, where the outcome is random. Though non sought to find the rationale of the randomness, which his EMH was set to explain (Fama, 1970;1995).

Prior research has found evidence for multiple anomalies to the efficient market hypothesis, which contradicts that markets are efficient. For example, Wu and Mazouz (2016) who found significant evidence for industry reversals for the UK markets using monthly and daily data from Jan. 1970 to Dec. 2011. The results indicate losing industries outperforms winning industries in the following five years, and that industry reversals are robust through good- and bad periods of the economy. These results were based upon the methodology introduced by Fama and French (1993) with the applications of factors using multiple linear regression. In the same article Fama and French found evidence for size- and value effects could explain stock returns.

For evaluating market efficiency, time series analysis is a commonly applied method for testing the weak- and semi-strong form. The weak-form is tested for significant evidence against the random walk model using the Augmented Dickey-Fuller test, and the Ljung-Box test for autocorrelation in the time series of returns[4]. The semi-strong form is tested using event studies of news releases, while the *strong* form of efficiency shall be viewed as a theoretical benchmark of market efficiency (Fama, 1995). In this study, the is focus on testing the weak-form of market efficiency and therefore the application of ARIMA models for prediction is appropriate.

2.1.2 Prior research about market efficiency in Sweden

To the best of my knowledge, the most comprehensive studies for testing the weak-form of efficiency in the Swedish market are shown in table 1. A study by Jennergren and Korsvold (1974)

[4] The tests are further explained in section 2.2.1 Testing for random walk and 3.8 Ljung-Box test respectively.

tested 30 stocks on the Swedish exchange from 1967-1971 using daily closing prices and their log returns. Their study found evidence for rejecting the weak-form of market efficiency due to autocorrelation in the returns, but proposed others to test if the significance of continuous abnormal returns could be obtained, in order to verify the rejection of the weak-form of efficiency. The study by Frennberg and Hansson (1993) applied the Variance Ratio test and Autoregressive model test on their data consisting of monthly real- and excess return data from 1919-1990 for a Value-weighted Swedish stock index. Their findings rejected the weak-form of efficiency in Sweden and found short term autocorrelation with a maximum of twelve months, and negative auto correlation on longer horizons, but insignificant within the 5% level for the latter. A more recent study was conducted by Shaker (2013), where he used the Ljung-Box-, Augmented Dickey-Fuller-, and Variance Ratio test for the log returns of daily price data for the main index OMXS30. In the article significant evidence is found for autocorrelation at the 1% significance level for the first 10 lags (Shaker, 2013). Besides this evidence is found for rejecting the weak-form of market efficiency according to the Augmented Dickey-Fuller test, which is supported by the results from the Variance Ratio test.

Table 1: Summary of EMH literature for Sweden

	Jennergren and Korsvold (1974)	**Frennberg and Hansson (1993)**	**Shaker (2013)**
Sample	30 single stocks	Main index	Main index
Time period	1967-1971	1919-1990	2003-2013
Frequency	Daily	Monthly	Daily

From previous studies in Sweden there is significant evidence for autocorrelation and stationarity in asset returns using historic price data. Therefore, all studies in Table 1 provides evidence for rejection of the random walk model and weak-form of efficiency.

2.2 Random walk model & White Noise

The Random walk model assumes stock prices are in equilibrium implying historic- and public information is priced into the market efficiently. This means the current market price is the best estimate of a stocks fair value, and deviations is due to market participants with different strategies will move the prices randomly (Fama, 1995). Different random walk models have evolved through time.

One such model is the *The Martin Gale model* Originated in the 15th century from Girolamo Gardano an Italian mathematician, who inspired by the theory of gambling proposed the principle of equal conditions for all participants in order for a game to be fair (Campell, Lo & MacKinlay, 1997). Therefore, the process of the game inherently must be a stochastic process defined in equation 1.

$$E[P_{t+1} \mid P_t , P_{t-1} , \dots , P_{t-k}] = P_t \tag{1}$$

In equation 1, *P* represents the price of an asset and *t* denotes the time of today whereas P_{t-1} is price at the time of the previous observation i.e., last month for a monthly series (Campell et al., 1997). Equation 1 shows that the best prediction for the price at time $t+1$ is the price at time t, therefore its current price. This is commonly referred to as the Martingale property. This implies market participants applying linear econometric models doesn't a gain sustainable advantage (Campell et al., 1997).

Another model is the *Random Walk with a drift*, which is similar to the Martingale model, though with the application of a drift μ and underlying assumptions of the error terms ε_j having a zero mean and nonzero variance. The model is defined in equation 2:

$$P_t = \mu + \ P_{t-1} \ + \varepsilon_t \ , \quad \varepsilon_j \sim WN(0, \sigma^2) \tag{2}$$

The random walk model is a stochastic process meaning the series is random, since the error term is assumed to be white noise (Tsay, 2005). In practice the randomness of asset returns is depended upon the efficiency of the market according to the EMH. For determining if the log returns follow a random walk the Augmented Dickey-Fuller (ADF) test is applied. Shaker (2013) applied the ADF-test to a series of the OMXS30 and found significant evidence for rejecting the null hypothesis. A rejection of the null hypothesis indicates stationarity and thereby the series to be mean reverting.

A White Noise process is a time series process where the returns are identically distributed and independent meaning the variance and mean are constant and the series are uncorrelated, therefore linear models can't help in predicting future returns. The returns are thereby described as $r_t = \varepsilon_t, \quad \varepsilon_t \sim WN(0, \sigma^2)$, where ε_t has a zero-mean, constant variance and the series is uncorrelated (Tsay, 2005). The series is stationary, but unpredictable using a linear model like ARIMA. The Ljung-Box test described in section 3.8 is commonly applied for testing a series for white noise behaviour. A White Noise series though can have non-linear dependence between observations.

2.2.1 Testing for random walk

For testing if asset returns follow a random walk the ADF test is applied. The regression formula in the ADF test is defined in equation 3. Where ΔP_t presents the change at time t, which for this study is the percentage change of the log price. Whereas gamma γ presents the presence of a unit-root in the time series (Tsay, 2005).

$$\Delta P_t = \phi_0 + \gamma P_{t-1} + \sum_{i=1}^{p} \phi_i \Delta P_{t-i} + \epsilon_t, \quad \epsilon_t \sim WN(0, \sigma^2) \tag{3}$$

The hypothesis of the test is:

$$H_0 : \gamma = 0$$
$$H_a : \gamma < 0$$

Under the null hypothesis the time series follows a random walk. The test performs a t-statistic for $\gamma = 0$ which is evaluated on the Dickey-Fuller distribution. One should reject the null, when the t-statistics is below the critical value of the Dickey-Fuller distribution on the chosen significance level (Tsay, 2005). If the null is rejected it's due to statistical evidence for the time series to be stationary, meaning the properties of the series is constant through time and mean reverting (Tsay, 2005).

2.3 Econometrics and mean reversion

The predictability of asset returns has been subject to extensive research, including the analysis and forecasting of time series of asset returns. In this thesis the linear regression model Auto Regressive Integrated Moving Average (ARIMA) is applied. The constituent of the model is the Autoregressive (AR) parameter with lag *p,* Integrated is the number of times the series has been differenced *d* to obtain stationarity, and the Moving Average with lag *q* is the number of lags for previous forecasting errors. The notation of ARIMA models is that an ARIMA (1,0,0) is equal to an ARMA(1,0) and AR(1) model. Going forward the terms ARIMA, ARMA, AR, and MA is going to be used interchangeably[5].

Hybrid models that incorporate both linear and nonlinear processes in a time series for returns have been developed, such as Hidden Markov Models, Artificial Neural Networks, and Genetic Algorithms, limitations of these models exist due to the lack of consistent methods for identifying changes in the time series between linearity and nonlinearity (Dong, Guo & Hu, 2020; Hassan, Nath & Kirley, 2007). The ARIMA model, remains a highly regarded model and is still widely applied in academic research about predictability of stock returns. For instance, the previous named studies of Bakar et al. (2018) and Rounaghi et al. (2016), but also the studies of Ariyo, Adewumi and Ayo (2014) and Affef, Ihsan and Zada (2018) for whom both studies found ARMIA models to have predictive power for stock returns.

2.3.1 The concept of mean reversion

The concept of mean reversion has a long history and has been applied in various contexts beyond financial academia. To the best of my knowledge Sir Francis Galton was the who first introduced the idea in 1886 when he observed that the height of children tended to revert towards an average, even when their parents were abnormally tall or short (Galton, 1886). In the world of finance, studies have investigated the effect of mean reversion in different situations and asset classes, with varying methods and objectives.

[5] Further specification for constituents and complete model is provided in section 3.6 Autoregressive Integrated Moving average model

One area of interest is asset allocation for investors. If asset prices followed a random walk model, there would be no incentive for investors to actively change their allocation. However, if asset returns reverted towards a fundamental value equivalent to a monthly or yearly expected return, investors would have incentives to adjust their portfolios.

Jaggia and Thosar (2005) studied the effect of mean reversion in a multiple asset portfolio consisting of large cap US stocks and T-bills. They found that, for an investment horizon of six years, it was beneficial to overweight equities for four years followed by a decrease in equity exposure for the remaining two years. This suggestion was made relative to a buy-and-hold strategy and tested on an autocorrelated sample and one that followed a random walk model (Jaggia & Thosar, 2005). It should be noted that the same suggestion clearly would not be advantageous if the time series followed a random walk model. Though the study found the strength of the reversion as weak, which according to the article should be expected, since otherwise market participants would quickly exploit the strong predictability of the asset returns (Jaggia & Thosar, 2005).

Another usage of mean reversion for investment strategies applies different market ratios for deciding whether a stock could be regarded as cheap or expensive. A method of the famous investor Benjamin Graham had a mean reverting characteristic, because of his focus towards stocks with low P/E ratios. This was due to his conviction that stocks with low multiples should revert towards a normalized level (Bondt and Thaler, 1989). A study of the P/E ratio for the SP500 index was made by Becker, Lee and Gup (2012), finding evidence for the P/E ratio to be a stationary process when allowing for structural breaks. Their study consisted of 100 years of monthly P/E observations for the SP500, where they for the whole period found evidence for a non-stationary process, but by using the unit-root test with Fourier function originally suggested by Enders and Lee (2004), which allows for structural breaks in the time series. It was shown that structural changes had provoked non-stationarity. After allowing for structural changes the P/E was a stationary process under 3.7 cycles averaging at 33 years (Becker et al. 2012). A similar study by Sauer and Chen (1996) have found mean reversion of stock prices to be a phenomenon of the pre-war era for UK-listed companies.

Another study examining mean reversion was conducted by Akarim and Sevim (2013), who aimed to test the market efficiency of 18 emerging markets between 1995 and 2010 using a mean reversion model for monthly data. The study found evidence for mean reversion in all countries. (Akarim & Sevim, 2013).

The concept of mean reversion is relevant for this thesis, since stationarity is required for making ARIMA models. Therefore, the null hypothesis of the ADF test must be rejected prior to model identification, and differencing can be applied to enhance stationarity. Differencing is done by subtracting the series such as $\Delta r_t = r_t - r_{t-1}$ for a series of returns. In addition to using ARIMA models, the variance ratio test has also been commonly applied to test for market efficiency and whether a time series of returns has a mean-reverting behaviour. The test was originally introduced by Lo and MacKinlay (1988) and has been widely used in academic research by Poterba and Summers (1988), Shaker (2013), Frennberg and Hansson (1993) etc. In simple terms, the variance ratio test has a null hypothesis stating that the autocorrelation of the time series is equal to zero and therefore uncorrelated (Charles & Darné, 2009).

2.4 Prediction of asset returns

In academia, multiple methods have evolved for forecasting returns, such as factor investing introduced by Fama and French (1993) have shown evidence for explanatory power for factors being proxies for value- and small cap companies, while in time series analysis ARIMA is highly regarded and commonly applied for forecasting. For this study ARIMA is the appropriate model for forecasting returns since prior research have found evidence for rejecting the weak-form of efficiency in Sweden due to stationarity and autocorrelation. The steps for fitting an ARIMA model to a time series were put into a framework by Box et al. (2015), where the iterative steps of the model fit can be defined as follows.

1) *Identification:* The objective is to identify appropriate ARIMA models for a time series. Though first the series is tested for stationarity using the ADF test. If evidence is against rejecting the null hypothesis for non-stationarity, then it's recommended to difference the series. But over-differencing can cause false autocorrelation. For determining a qualified order for the number of lags for AR(*p)* and *MA(q),* the autocorrelation and partial

autocorrelation function is used, since plotting these as correlograms can indicate the number of significant lags (Box et al., 2015). Besides applying correlograms the Akaike Information Criterion (AIC) presented by Akaike (1974a) shall be used. According to Box et al. (2015) and Tsay (2005) AIC is a supplementary tool in the identification phase, while for Mondal et al. (2014) a primary tool for model identification. The usage of AIC for model selection is also recommended by Dong, Guo and Reichgelt (2020). Using an information criterion for model identification and selection yields the possibility of doing parameter estimation of numerous models and further specify those with the greatest fit.

2) *Estimation:* Involves the usage of Maximum Likelihood to estimate the parameter values of the selected models. This means fitting the identified models with their given number of lags to the time series. As stated by Box et al. (2015) the identification and estimation stage are highly similar. In the context of this thesis the ARIMA models will be fitted and selected according to the identified models by AIC and correlograms.
3) *Diagnostics:* Concerns the evaluation of the fitted models to the time series and ultimately decide its adequacy, as Box et. al describe "*All models are approximations and no model form can ever represent the truth absolutely*" (Box et al., 2015, p. 285). The overall objective is to find models, which have evidence for being the best representation of the given time series. By estimating the parameters using Maximum Likelihood the residuals become the object of investigation in the diagnostics phase. The residuals should first be investigated for any patterns using ACF and PACF correlograms. Besides this the residuals are investigated for autocorrelation using the Ljung-Box test, which Box et al. (2015) recommends. The null hypothesis states there is no significant evidence for joint autocorrelation in the sample, meaning if the null fails to be rejected the model captures all the autocorrelation in the time series. Though if one rejects the null additional the model needs additional lags.

After estimation the models is diagnosed and redefined if necessary. Hereafter a forecast through the period Jan. 2020 to Dec. 2022 is made, where goodness of fit measurements is applied for model evaluation.

3. Methodology

The methodology applied in this thesis follows the approach of the Box-Jenkins framework, which was described in the literature review section in 2.4. This chapter aims to introduce the theoretical methods and concepts applied in this study with regard to calculation of returns, identification-, estimation- and diagnostics of ARIMA models, and out-of-sample 1-step ahead forecast.

3.1 Simple returns

Financial assets such as stocks and bonds are quoted in markets by their price, but when doing statistical analysis, the usage of prices can lead to spurious models. The simple return is the percentage return commonly shown by brokers and defined in equation 4.

$$R_t = \frac{P_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{4}$$

For a period consisting of multiple observations the simple return equals:

$$R_t = \frac{P_t}{P_{t-k}} - 1 = \frac{P_t - P_{t-k}}{P_{t-k}} \tag{5}$$

In the above formulas R_t is the simple return for a period. P_t is the price at time t and P_{t-1} and P_{t-k} is respectively the price at time *t-1* or *t-k* (Tsay, 2005). Though for statistical analysis log returns has attractive properties, and multiple of the referenced papers from the literature review applies this.

3.2 Log returns

In timeseries literature, log returns are preferred over simple returns when working with time series models. The equation for log returns is defined as:

$$\ln(r_t) = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(P_t) - \ln\,(P_{t-1}) \tag{6}$$

In equation 6 P_t is the price at time *t,* the denominator presents the price at the previous observation, and $\ln(r_t)$ is the log return at time t. Additionally, log returns tend to have a symmetric distribution around zero, making them more suitable for statistical analysis relative to simple returns (Tsay, 2005). The relationship between log- and simple returns can be summarized as:

$$\ln(r_t) = \ln(1 + R_t), \qquad R_t = e^{r_t} - 1 \tag{7 \& 8}$$

The property of log returns ultimately is of greatest use when working with time series models, and therefore it's commonly used in academia for time series analysis (Tsay, 2005). Log returns can be regarded as the first difference of a log price series as shown in equation 6, and often makes the time series stationary (Tsay, 2005).

3.3 Stationarity

Time series processes can be defined as strictly-, weakly- or nonstationary. A strictly stationary time series is defined by the joint distribution the time series is constant through time, whereas weakly stationarity is defined by:

1)The mean of the process is time invariant such that $E(r_T) = \mu$
2)The variance of the process is time invariant and non-zero: Var$(r_t) = \sigma^2$,
3)The covariance is a function of the lagged distance between the return of r_t and ,*k* and thereby not dependent on time *t* such that $Cov(r_t, r_{t-k}) = \gamma_k$ (Tsay, 2005).

These three properties of a timeseries must be valid before identifying lags to the ARIMA model according to the Box-Jenkins methodology. Generally, time series of raw macroeconomic data such as for the unemployment rate, consumer price index, GDP growth or stock prices doesn't have the properties of strict- or weak stationarity therefore data is converted to log returns or differencing of the time series is applied to stabilize the series for trends and seasonality (Tsay, 2005). A stationary time series has mean reverting behaviour towards the unconditional mean of the process. The test for stationarity is the described ADF test from section 2.2.1 in the literature review.

3.4 Unit-root nonstationary

A unit-root nonstationary process is a stochastic time series characterized by having means and variance that are time dependent, and the future value of the process is unpredictable (Tsay, 2005). The common example for a unit-root nonstationary process is the Random Walk model without a drift defined in equation 9.

$$P_t = P_0 + \sum_{i=1}^{t} \varepsilon_t, \qquad \varepsilon_t \sim WN(0, \sigma^2) \tag{9}$$

Where the price P_t is the price a time *t* and defined as the sum of $P_0 + \sum_{i=1}^{t} \varepsilon_t$, where ε_t is a white noise process (Box et al., 2015). Hereby the series doesn't have a long-term fixed value. Due to the properties of a unit-root nonstationary time series, which is common for price data it's not appropriate for ARIMA modelling and forecasting.

According to the Box-Jenkins methodology a time series must be differenced until stationarity. Generally, the methodology states the amount of differencing *d* should not exceed more than two, since over differencing would provoke additional autocorrelation and the number of Autoregressive- and Moving average lags would increase making the model unnecessary complex (Box et al., 2015). Though the usage of log returns generally makes financial data stationary (Tsay, 2005).

3.5 Autocorrelation Function (ACF) & Partial Autocorrelation Function (PACF)

As mentioned in the literature review the method of identifying ARIMA models implies the usage of the ACF and PACF correlograms. The general formula for estimating the correlation coefficient at lag-l is shown in equation 10 for a weakly stationary process.

$$\rho_l = \frac{Cov(r_t, r_{t-l})}{\sqrt{Var(r_t)Var(r_{t-l})}} = \frac{Cov(r_t, r_{t-l})}{\sqrt{Var(r_t)}} \tag{10}$$

The correlation relationship between variables can range between -1$\leq$ to ≤ 1 and captures the linear dependence (Tsay, 2005). If the estimated value is equal to -1 the relationship is said to be perfectly uncorrelated, if the correlation coefficient is equal to zero a correlation effect is non-existing, while if equal to 1 the variables are perfectly correlated (Tsay, 2005). For a data sample the estimated autocorrelation for lag-l becomes:

$$\hat{\rho}_l = \frac{\sum_{t=l+1}^{T}(r_t - \bar{r})(r_{t-l} - \bar{r})}{\sqrt{\sum_{t=1}^{T}(r_{t-l} - \bar{r})^2}} \tag{11}$$

In equation 11 $\bar{r}$ represents the sample mean for the given time series, while r_{t-l} and r_t is the log return at time t and $t - l$, and $\hat{\rho}_l$ is the estimated correlation coefficient (Tsay, 2005). The correlation between r_t and r_{t-l} is of great interest in time series analysis, since if estimated positive a positive return yesterday will on average be followed by a positive return today (Tsay, 2005). The question of interest hereafter becomes, how does one identify a statistically significant autocorrelation in a time series, and how can one model it?

For model identification the answer is correlograms, where the ACF shows the correlation coefficient between r_t and $r_{t-\ell}$, whereas the PACF shows the correlation coefficient between r_t and $r_{t-\ell}$, while removing the effect of intermediate lags (Box et al., 2015). The correlograms is appropriate for identification of trends, seasonality, and the number of potential lags in the model. The significance lines indicates that correlation coefficients falling outside the lines are significant. A significance level at the 95% level is presented in equation 12.

$$\left[\frac{-1,96}{\sqrt{n}}, \frac{1,96}{\sqrt{n}}\right] \tag{12}$$

In the above formula n represents the number of observations in the data sample. For a series to be white noise, then at all lags the correlation coefficients should be close to zero and therefore within the lines. Following parameter estimation, a correlogram of the residuals should be plotted, and analysed to identify misspecifications. The usage of correlograms ultimately is a critical way when inspecting time series and evaluating model fits, which according to Box et al. (2015) can't be substituted by statistical test.

3.6 Autoregressive Integrated Moving average model

As presented in the introduction the second research question regards the predictability of industry portfolios using ARIMA. The aim is to find an ARIMA model describing the time series most accurately.

3.6.1 Autoregressive model

The Autoregressive model denoted as AR(*p*), whereas *p* is the number of lags. This model assumes linearity between the return of today r_t the dependent variable, and previous returns r_{t-1} and r_{t-k} the independent variables. The parameter coefficient is named phi ϕ_k, and when the total absolute values of the parameters are <1 the process is mean reverting (Tsay, 2005). A generalized AR(*p)* model is presented in equation 13.

$$r_t = \phi_0 + \phi_1 r_{t-1} + \cdots + \phi_k r_{t-k} + \varepsilon_t, \qquad \varepsilon_t \sim WN(0, \sigma^2) \tag{13}$$

As the above formula describes r_t is a function of previous log returns (Tsay, 2005). In the model it's assumed ε_t is white noise process with a zero mean and a variance. For an AR(*p*) model it's assumed the log return of the time series has an expected value in the long run, which is defined as:

$$E(r_t) = \frac{\phi_0}{1 - \phi_1 - \cdots - \phi_P} \tag{14}$$

In the above equation ϕ_0 is the constant of the process and $\phi_1 - \cdots - \phi_P$ is the estimated Autoregressive lags of the model (Tsay, 2005).

3.6.2 Moving average model

The Moving Average model denoted MA(*q*) where *q* is the number of lags, assume a linear relationship between r_t the past error terms of the series. This indicates r_t being predictable by the historic forecasting errors, and the parameter θ_k can be interpreted as an error correction to the previous errors of (Tsay, 2005). The MA(*q*) model is defined in equation 15.

$$r_t = c_0 + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_k \varepsilon_{t-k}, \quad \varepsilon_t \sim WN(0, \sigma^2) \tag{15}$$

The unconditional mean of the MA(q) models is $E(r_t) = c_0$ (Tsay, 2005). Though some time series processes are complex and thereby require a combination of the AR and MA models.

3.6.3 The combined ARIMA model

The combined model of AR(p) and MA(q) is denoted as ARIMA(p,d,q), where I stand for *Integrated,* and the number of differencing's used to achieve stationarity. An ARIMA (p,d,q) is defined as:

$$r_t = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + \varepsilon_t - \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}, \quad \varepsilon_t \sim WN(0, \sigma^2). \tag{16}$$

The above definition of the general ARIMA(p,d,q) conditions that the parameters of the model is <0. Besides this the property of the unconditional mean is similar to the one of AR models in equation 14 (Tsay, 2005). When performing identification of AR, MA or ARMA models AIC and visual analysis of the correlograms of ACF and PACF is used. The latter is an indispensable part of the model identification and diagnostics part (Box et al., 2015).

For estimation of the parameters the Maximum-Likelihood method is applied, thereby the estimated parameters are obtained by maximizing the likelihood function to obtain the best fit between the identified model and the observed data. The Maximum Likelihood approach aims to find the parameter estimate that make the observed data most likely to occur based on the identified model structure (Tsay, 2005).

3.6.4 Forecasting

There are multiple methods for forecasting returns for an ARIMA model. In this thesis the method applied is the one-step a head forecast with an expanding window. This is chosen contrary to the multiple steps ahead forecast, because of the quality of the predictions increases when allowing the model to forecast using the latest observations in the period, and reestimate the parameters as estimation sample increases. The equation for the one-step ahead forecast for an ARMA (1,1) is defined in equation 17.

$$\hat{R}_{t+1} = \phi_0 + \phi_{1_t} r_t + \theta_1 \hat{\varepsilon}_t \tag{17}$$

(Tsay, 2005)

3.7 Akaike information criterion

As part of identifying the best fitting ARIMA model to a time series Akaike Information criterion (AIC) is applied. For model identification AIC is often referred to as a complementary use as described in chapter two regarding the Box-Jenkins methodology. AIC is defined as:

$$AIC = \frac{-2}{n} \ln(likelihood) + \frac{2}{n} * (Number\ of\ parameters) \tag{18}$$

In equation 18 n is the sample size of the estimation period, and ln(*likelihood)* is the log likelihood of the given model (Tsay, 2005). In equation 20 it's presented there is an allowance for insignificant parameters, and a penalization of additional model complexity. The selection rule for AIC is to select the model with the lowest value, since the estimate of the equation is the relative information lost by a model (Wooldridge, 2015). As described in the literature review AIC is recommend as tool for model identification in time series analysis for financial data (Mondal et al., 2014; Tsay, 2005; and Dong et al., 2015). AIC as a selection criterion allows for estimating multiple models efficiently, though with constraints regarding a maximum lag order of five for both AR and MA lags with, since over specifying is to be avoided (Box et al., 2015).

3.8 Ljung-Box test

To test for jointly significant autocorrelation in a timeseries the Ljung-Box test is applied, because of its usage in prior research articles with similar objectives and methodology. The test is defined in equation 19.

$$Q(m) = N(N+2) \sum_{\ell=1}^{m} \frac{\hat{\rho}_\ell^{\,2}}{N-\ell} \tag{19}$$

The Ljung-Box test applies a chi-squared distribution, and the hypothesis for the test is defined as:

$$H_0 : \rho_1 = \cdots = \rho_m = 0$$
$$H_a : \rho_i \neq 0$$

In the Ljung-Box test the null hypothesis is rejected if Q(m) > X_a^2 (Tsay, 2005). Though with the application of statistical software the p-value for Q(m) is provided and is rejected if below the chosen significance level. In equation 21 m is the degrees of freedom, *N* the number of observations, and the test squares the autocorrelation for a given number of lags ℓ (Tsay, 2005). The Ljung-Box test is applied for diagnostics in the Box-Jenkins methodology. In the literature review the study by Shaker (2013) tested the efficiency of the Swedish market using the Ljung-Box on a daily time series of the OMXS30. Generally, the test is applied in time series to the residuals of a model for evaluating misspecification, since if autocorrelation is identified in the residuals the model is not capturing all past information.

3.9 Goodness of fit

AIC is applied for both model identification and as a measure for goodness of fit. Besides this visual analysis of the ACF and PACF correlogram's of the residuals is necessary when evaluating model fit to a timeseries. Other than that, the following measurements shall be used to evaluate the models on behalf of the forecasted results (Box et al., 2015).

The Root Mean Squared Error (RMSE) is the standard deviation of the prediction errors, which is the forecasted log return subtracted by the actual log return for that period. RMSE is defined in equation 20:

$$RMSE = \sqrt{\sum_{i=1}^{n} \frac{(\widehat{r_i} - r_i)^2}{n}} \tag{20}$$

In equation 22 $\widehat{r_i}$ represents the predicted value of *r*, and *n* is the total number of observations. The squared values turn negative values into positive, which ultimately makes the method great for forecasts with outliers (Wooldridge, 2015).

The last method for evaluating the goodness of fit, is the Mean Absolute Error (MAE) which estimates the actual average difference between the predicted values from the models and the actual values observed values. The method is defined as:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|r_i - \widehat{r_i}\,| \tag{21}$$

As above $\widehat{r_i}$ presents the predicted value at time. MAE indicates the average magnitude of errors in the predictions, and a low MAE indicates the predicted values are closed to the actual data (Wooldridge, 2015).

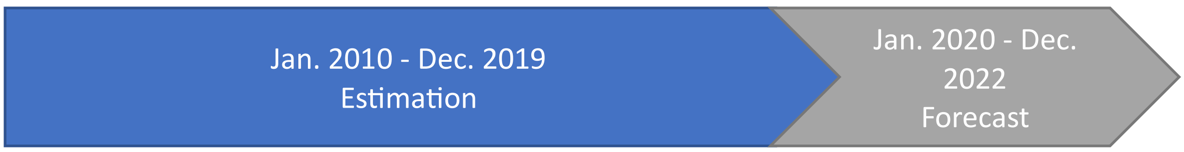
4. Data collection

This section explains the data used for this study with regard to the specific industries and the method applied to create the Swedish industry portfolios.

4.1 Stock selection

The companies for the industry portfolios have been found using FactSet's universal stock screener function where the following criterions was sat: 1) The stocks had to be listed on the Swedish exchange, 2) Be common stock like the criteria of Wu et al. (2016), 3) Have a closing price in the time frame of the 1st of January 2010 until 31st December 2022. The reasoning for stocks had to be listed during this timeframe is due to the avoidance of survivorship bias. This is valid because of the allowance to be listed or delisted during the period, and by randomly picking stocks from the investment universe.

Figure 1: Figure of forecasting- and estimation period



Setting these criteria's, the total investment universe was constituted of 1.068 stocks before industry selection. The price data retrieved is closing prices on the last day of the month and closing prices on Fridays for weekly data. Using monthly and weekly data is similar to Mondal et al. (2014), Kim and Shamsuddin (2015), and Lo and MacKinlay (1988). The data is corrected for dividends and stock splits and represents the total returns. Five FactSet industries was chosen for this study after setting the above constraints. These was chosen due to their industry idiosyncratic risk is different.

Biotechnology

Biotechnology is an industry that involves the use of biological organism to create new products and technologies. This industry is primarily driven by research and development efforts for the specific company. The biotech industry includes companies that are engaged in the development of drugs, diagnostics, and medical devices (Investopedia.com). Biotech companies typically operate in a high-risk, high-reward environment due to the long development timelines and regulatory hurdles associated with their products, this implies a high level of company specific risk. The number of stock available was 48 during the whole period.

Information Technology Services

The information technology services industry includes companies that provide hardware, software, and services related to computing and telecommunications etc. Information technology companies operate in a variety of sectors, including healthcare, finance, and consumer goods (Statista.com). The differentiation of the companies can therefore vary. The number of stocks available was 62 during the whole period.

Packaged Software

The Packaged Software industry involves the development and sale of software packages. These packages can include accounting-, data structuring-, and gaming software therefore the homogeneity of the companies vary with regard to end customers and general market. The number of stocks available for the whole period was 154.

Real Estate Development

Real estate development involves the construction and sale of residential-, commercial-, and industrial properties. This industry is cyclical and is heavily influenced by macroeconomic factors such as interest rates (CFAinstitue.com). Real estate development is often considered a stable and profitable industry with steady cashflows. The number of stocks available was 143 for the whole period.

Industrial Machinery

The Industrial Machinery industry involves the manufacturing and sale of equipment and machinery used in a variety of sectors such as agriculture, construction, and manufacturing. The industrial machinery industry is influenced by the general economy, though the differentiation of the companies also is high. The number of stocks available for the whole period was 64.

Stocks were randomly picked after filtering for each industry at a time. The stock for each portfolio is listed in table 1 in appendix A. The sample consist of a total of 780 monthly and 3.385 weekly observations.

4.2 Portfolio construction

After the collection of price data, the industry portfolios were constructed as equally-weighted and rebalanced at the beginning of every year. For calculation of the portfolio value the following formula was used:

$$p_v = n_{i_t} \cdot p_{i_t} + n_{k_t} \cdot p_{k_t}, \dots, n_{p_t} \cdot p_{p_t} \tag{22}$$

The value of a portfolio p_v is defined by the stock prices $p_{i,k,p_{,t}}$ and number of shares held n_{i,k,p_t} for each stock in the portfolios. For rebalancing the weights is defined as $w = \frac{1}{n}$, where n is the number of investable stocks for the industry for the given year. The logarithmic price return is then calculated as in equation 6 for the portfolios. Going forward the industry portfolios will interchangeably be referred to as Biotechnology (B), Information Technology Services (IT), Real Estate Development (RED), Industrial Machinery (IM), and Packaged Software (PS).

4.3 Descriptive statistics

The Box-Jenkins framework for model estimation and selection has been applied to the period from January 2010 to December 2019 with a total of 119 observations for monthly data and 519 for weekly data per industry. As mentioned in the introduction this study investigates the log difference of closing prices for the industry portfolios as calculated in equation 6.

In table 2 the descriptive statistics is presented for each industry portfolio using monthly data. The five industry portfolios have similar characteristics regarding their average value which for all is slightly positive. Moreover, is the skewness of their distributions positive indicating a right-skewed distribution meaning a higher frequency of positive values. These instances are common when working with stock returns because stock prices generally will increase through time. The Swedish

GDP had an CAGR of 0,82% during the estimation period, therefore the Swedish economy in general was moving forward (Statista.com, 2022). The kurtosis varies slightly throughout the industry portfolios, while all is below three. Generally, a normal distribution is expected to have a kurtosis value equal to three, indicating that all industry portfolios can be regarded as platykurtic (Wooldridge, 2015). This indicates the observations has a low number of extreme values, which is due to the observations consists of monthly log returns and diversified portfolios aren't expected to experience a high degree of large price changes. Applying the Jarque-Bera test, where the null hypothesis states the data are normally distributed (Wooldridge, 2015). The null hypothesis is rejected for RED and PS. The histograms can be seen in Appendix A from figure 1-5. The relatively large maximum and minimum values is due to the observations is gathered monthly. *Avg. Holdings* represents the average number of stocks in the portfolio for each year.

Table 2: Descriptive statistics of monthly data

In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Monthly data	B	RED	IT	IM	PS
n	119	119	119	119	119
Avg. Holdings	8	9	9	8	8
Avg. Log return	0,002	0,011	0,006	0,010	0,015
Std.	0,084	0,045	0,050	0,064	0,059
Maximum	0,257	0,180	0,127	0,181	0,233
Minimum	-0,203	-0,093	-0,111	-0,138	-0,143
Kurtosis	0,799	1,470	-0,432	0,043	1,311
Skewness	0,407	0,705	0,083	0,318	0,668
Jarque-Bera	0.0621	0.0001***	0.5235	0.3836	0.0005***

Notes: The sample period is from Jan. 2010 to December 2019.
*** Corresponds to 1% signifiance level

A similar table of the weekly data is provided in Appendix A Table 2. For the weekly data the kurtosis is similar throughout the portfolios with values ranging from -0.432-1,470. All distributions can be characterized as platykurtic meaning the tails is shorter than a normal distribution (Wooldridge, 2015).

The correlation coefficients for the five industry portfolios is shown in table 3 using monthly observations. The highest correlation is between RED and IT. Biotechnology has in general the lowest correlation coefficients with the other industries, which can be due price movements for this industry is due to idiosyncratics event rather than systemtic factors. Though none of the correlation coefficients rises any concerns.

Table 3: Correlation matrix for monthly log returns from Jan. 2010 to Dec. 2019

In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Correlation	B	RED	IT	IM	PS
B	1	0.172	0.286	0.121	0.097
RED	0.172	1	0.432	0.351	0.382
IT	0.286	0.432	1	0.422	0.390
IM	0.121	0.351	0.422	1	0.309
PS	0.097	0.382	0.390	0.309	1

The dataset applied in this study differentiates from prior studies in Sweden since the portfolios sampled act as proxies for their given industries. Therefore, the portfolios don't represent the whole market of Sweden. Further this study differentiates by using self-constructed equal-weighted portfolios to avoid potential biases from the value weighted main index used by Frennberg and Hansson (1993) and Shaker (2013), and eliminates idiosyncratic behaviour from single stocks, which was used by Jennergren and Korsvold (1974).

5. Analysis of results

In this chapter the results from applying the Box-Jenkins methodology to the log returns of the Swedish industry portfolios are presented. The results are presented in separate sections in the chronological order of identification, estimation, diagnostics, and ending with the results from goodness of fit measurements.

5.1 Identification

As mentioned in section 2.4 a time series must be stationary prior to identification of lags. Therefore, the ADF-test was conducted to assess stationarity and if differencing should be applied to smooth the time series. Subsequently, the Ljung-Box test is employed to investigate the presence of significant joint autocorrelation. The identification of potential models is then performed using the ACF and PACF correlograms, followed by AIC selection.

5.1.1 Augmented Dickey-Fuller test

The null hypothesis of the ADF test states the series is non-stationary and follows a random walk as described in 2.2.1. Table 4 presents the obtained p-values using the ADF test to log returns of the industry portfolios. The results shows that all time series are stationary at a significance level of 1% meaning they exhibit mean-reverting behaviour, and the random walk model is rejected. Ultimately the number of differencing's *d* in the coming models is equal to ARIMA(*p,0,d*).

Table 4: ADF Test p-values for estimation period Jan. 2010 to Dec. 2019

In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

ADF test	B	RED	IT	IM	PS
Monthly	2.48e-11***	1.43e-09***	7.27e-09***	2.5e-07***	6.13e-09***
Weekly	2e-16***	2e-16***	2e-16***	2e-16***	2e-16***

*** Corresponds to 0.001 signifiance level [6]

The results presented in Table 4 are supported by a visual inspection. The figures from 6-11 in Appendix A shows the time series plots, and indicates all processes are centred around zero and

[6] The ADF test was also applied for the whole sample period from Jan. 2010 to Dec. 2022 with similar rejection results.

doesn't have indication of trend. Notably, there are large spikes around 2012 and 2019 for all portfolios, which could be due to the European banking crisis in 2012, the trade war between USA and China in 2019, and the Covid-19 pandemic in 2019 and 2020.

These findings support the earlier work of Shaker (2013), who rejected the null hypothesis of the ADF-test using daily log returns for the main index in Sweden OMXS30. The differences between the collected data sample used by Shaker (2013) and this study, are the frequency of observations and the nature of the sample as shown in table 1 of prior literature.

5.1.2 Ljung-Box test

The Ljung-Box test is commonly used to assess potential model misspecifications by examining the residuals. In this thesis, the Ljung-Box test is performed on the series of log returns and the residuals of the estimated models. The results presented in table 5 is of the log returns and shows that only weekly Packages Software exhibits significant evidence of joint autocorrelation.

Table 5: Ljung-Box Test p-values for estimation period Jan. 2010 to Dec. 2019

In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Ljung-Box test	B	RED	IT	IM	PS
Monthly	0.9726	0.6281	0.1104	0.9335	0.4869
Weekly	0.3105	0.186	0.656	0.6095	0.0066**

** Corresponds to 0.001 significance level [7]

These findings differ from those reported by Shaker (2013), who discovered significant autocorrelation up to 10 lags at a 1% significance level. An explanation for this discrepancy is the use of daily observations, which are expected to exhibit a higher level of autocorrelation relative to weekly and monthly observations, due to for example clustering effects on a daily basis.

[7] The Ljung-Box test was also applied for the whole sample period from Jan. 2010 to Dec. 2022 with similar rejection results.

5.1.3 Visual analysis of ACF & PACF

The ACF and PACF correlograms for the industry portfolios at both weekly and monthly observations is shown in Appendix B, figure 11-21. As mentioned in section 3.5 the objective is to identify potential models for estimation by interpreting the significance of the correlation coefficients at different lags. The general framework for interpreting ACF and PACF correlograms is shown in the table below 6.

Table 6: Framework for interpretation of ACF and PACF correlograms

Framework	*AR(p)*	*MA(q)*	*ARIMA(p,d,q)*
ACF	Tails off/Geometric decay	Cutoff after lag *q*	Tails off/Geometric decay
PACF	Cutoff after lag *p*	Tails off/Geometric decay	Tails off/Geometric decay

(Box et al. 2015)

If the correlograms of the industry portfolios do not exhibit any significant lags of interest, the time series is considered to follow a white noise process, indicating unpredictable behaviour using ARIMA models. However, when dealing with a time series that includes both AR and MA lags, the framework presented in Table 6 has limitations.

5.1.3.1 Monthly returns

The plots of monthly correlograms are shown in Appendix B figure 11-15. For Biotechnology, Real Estate Development, and Industrial Machinery, the ACF and PACF correlograms showed no significant lags. This indicates that these time series follow a white noise process, and thereby the absence of Autoregressive (AR) and Moving Average (MA) processes. Therefore, the visual analysis suggest predictability using ARIMA isn't found, which is supported by the Ljung-Box test results.

For Information Technology Services, the ACF exhibits significance at lags 10 and 12, but this should not be interpreted as evidence for a well-defined model. Significance at later stages can indicate trends or seasonality in the time series. However, considering the lack of clear seasonality in the monthly plot (Appendix A, figure 7) and the results from the Ljung-Box test, these significant lags are random effects in the time series. The PACF shows similar significant lags with the addition of lag 20, further supporting the interpretation of the time series as a white noise process.

For Packaged Software the ACF shows significance at lag 6 followed by a cutoff, while the same lag is significant in the PACF followed by a cufoff. Due to a lag restriction of five for both AR and MA parameters, this lag won't be captured directly, but negative autocorrelation in the sixth lag could be an indication of a semi-annual mean reverting behaviour. Comparing this interpretation with the monthly plot (Appendix A, figure 8) and the results from the Ljung-Box test, the significant sixth lag should not be overemphasized. Ultimately this inspection suggests the time series to be a white noise process.

The visual analysis of the correlograms aligns with the results of the Ljung-Box test, as no significant autocorrelation is found at meaningful lags for most of the industry portfolios.

5.1.3.2 Weekly returns

In the analysis of weekly log returns, similar interpretations of the correlograms are made as for the monthly time series, the correlograms are shown in Appendix B figure 16-20. For Biotechnology and Information Technology, both the ACF and PACF is significant at the first lag followed by a cutoff. This indicates that an AR(1) or MA(1) could be an appropriate model to fit for these time series.

For Packaged Software the correlograms indicate the appropriate model is more complex than the previous simple models. The first lag is insignificant, but two significant lags with geometric decay are observed in both the ACF and PACF. Additionally, three other lags (lags 6, 16, and 24) are significant with direct decays. Although the latter three lags are not of great interest, this inspection suggests the need for multiple models and parameter combinations. The following models could be considered: AR(2), MA(2), ARMA(2,2), ARMA(2,1), and ARMA(1,2).

For Real Estate Development the correlograms indicate a white noise behaviour. The only significant lags are observed at 19 and 20 for both the ACF and PACF. While a geometric decay is observed in the ACF and PACF for the first three lags, though not significant. The late significant lags are random behaviour, since withholding this inspection with the results from the Ljung-Box test, there isn't indications of other explanations.

For Industrial Machinery the correlograms suggest a white noise process, as significant lags are found at 10 and 14 in both the ACF and PACF. However, these lags do not provide substantial evidence for model identification, therefore these two is interpreted as random behaviour, supported by the Ljung-Box results.

The interpretations of the ACF and PACF correlograms are contrary to the results of the Ljung-Box test for 3 out of 5 industry portfolios using weekly log returns. On the other hand, the correlograms were consistent with the Ljung-Box test for the monthly log returns, indicating white noise processes. These results highlight the importance of visual inspection, as the Ljung-Box test assesses overall joint autocorrelation, while the correlograms help isolate individual lags.

5.1.4 Akaike information criterion

In accordance with the literature review in section 2.4, AIC is used for model identification. The maximum constraints of five lags for both the AR and MA regressors were applied to test models ranging from ARIMA(0,0,0) to ARIMA(5,0,5). The selection rule for AIC involves selecting the model with the lowest value.

5.1.4.1 Monthly returns

In Table 7, the best-fitted model for each industry portfolio is highlighted in bold font. The complete table of AIC values for all models can be found in Appendix B, from Table 1 to Table 5, for the industry portfolios.

ARIMA(0,0,0) with a zero-mean was identified as the best-fitted models according to AIC for Biotechnology and Information Technology, which is equivalent to a white noise process. Similarly, the best-fitted models for Real Estate Development and Industrial Machinery were ARIMA(0,0,0), but with a non-zero mean indicating a drift in the time series, therefore the series should be interpreted as random noise, since white noise processes is conditioned to a zero-mean. The best fitting model for Packaged Software according to AIC was an MA(1) with a drift indicating dependence on past residuals.

Table 7: Best fitted models using AIC for monthly data

In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Model	B	IT	PS	RED	IM
ARIMA(0,0,0)*	**-252,911**	**-373,654**	-328,312	-393,383	-312,618
ARIMA(0,0,0)	-250,973	-373,206	-333,773	**-395,183**	**-313,648**
ARIMA(0,0,1)	-248,974	-371,206	**-334,601**	-390,800	-312,552

Notes: * indicates a zero-mean

The models identified as the best-fitted models using AIC align with the results obtained from the Ljung-Box test, which found no significant evidence of autocorrelation. Additionally, the interpretations of the ACF and PACF for all industry portfolios supported the models for white noise behaviour, which indicates ARIMA models isn't appropriate for prediction of the returns. An exception is made for Packaged Software, where an MA(1) model was identified as the best model indicating dependence on past residuals.

5.1.4.2 Weekly returns

In Table 8, the best-fitted models for the industry portfolios using AIC are highlighted with bold font. Notably, Industrial Machinery stands out as the only industry portfolio where the best-fitted model is an ARIMA(0,0,0) with a non-zero mean, meaning the series is best explained as random noise. This finding is consistent with the interpretation of the correlograms and the results of the Ljung-Box test.

Table 8: Best fitted models using AIC for weekly data

In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Model	B	IT	PS	RED	IM
ARIMA(0,0,2)*	**-1.806,140**	- 2.340,073	-2.118,253	-2.509,700	-2.048,732
ARIMA(2,0,2)*	-1.802,283	**-2.343,425**	-2.122,483	Inf	Inf
ARIMA(1,0,4)	-1.799,096	-2.339,476	**-2.129,906**	-2.512,971	-2.051,969
ARIMA(1,0,0)	-1.803,649	-2.342,557	-2.121,438	**-2.517,274**	-2.054,123
ARIMA(0,0,0)	-1.795,439	-2.340,264	-2.123,090	-2.516,576	**-2.055,845**

Notes: * indicates a zero-mean

For Biotechnology, AIC identified an additional Moving Average lag than that interpreted by the correlograms. For Information Technology AIC identified a combined ARMA(2,2). The most complex model was identified for Packaged Software, as it included the highest number of lags, according to both AIC and the correlograms. Additionally, Packaged Software was the only industry portfolio where the null hypothesis was rejected for the Ljung-Box test, providing significant evidence for autocorrelation in the time series. For Real Estate Development, the ACF and PACF correlograms did not provide any indications of potential models. However, according to AIC, the best-fitted model for the series was an AR(1).

5.1.5 Identified models

In Table 9 a summary is presented of the identified models for the monthly data. This analysis suggests that white- or random noise process is present for most industry portfolios, while AIC identified an MA(1) model found for Packaged Software.

Table 9: Identified ARIMA models for monthly data

In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Model	B	RED	IT	IM	PS
ARMA(0,0,0)	ACF & PACF	ACF & PACF	ACF & PACF	ACF & PACF	ACF & PACF
MA(1)					AIC
AR(1)					AIC

A greater number of models was identified for weekly observations using both visual analysis of correlograms and the results from AIC, as summarized in Table 10. This suggests the weekly time series are more characterised by Autoregressive and Moving average processes compared to monthly.

Table 10: Identified ARIMA models for weekly data

In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Model	B	RED	IT	IM	PS
ARMA(0,0,0)		ACF & PACF		ACF & PACF + AIC	
AR(1)	ACF & PACF	AIC	ACF & PACF		
AR(2)					ACF & PACF
AR(3)					ACF & PACF
MA(1)	ACF & PACF		ACF & PACF		
MA(2)	AIC				ACF & PACF
MA(3)					ACF & PACF
ARMA(1,2)					ACF & PACF
ARMA(2,1)					ACF & PACF
ARMA(2,2)			AIC		ACF & PACF
ARMA(1,4)					AIC

5.2 Model estimation

In this stage of the Box-Jenkins methodology, the identified models from table 9 and 10 have been fitted to their given time series for the estimation period from January 2010 to December 2019. An inspection of the results is provided beginning with the use of monthly data.

5.2.1 Monthly returns

In table 11 a summary of the estimated models is provided. For Biotechnology and Information Technology the inspection of correlograms did not identify any potential lags, and AIC indicated that the best fitting models as ARIMA(0,0,0). This is evidence for the series to be white noise processes and thereby unpredictable using ARIMA.

For Packaged Software the MA(1) model is significant at 10% level relative to an insignificant AR(1). Besides this the MA lag has a parameter estimate of 0.1604, meaning that with 90% certainty the returns of today as a 16,04% dependence of the previous lagged residual. The constant of the models is equivalent to the drift of the series and significant within a 5% level.

For Real Estate Development and Industrial Machinery, the visual analysis of correlograms did not identify any potential lags since the time series displayed a white noise behaviour. Applying AIC for model identification, the proposed models were ARIMA(0,0,0) with non-zero means equivalent to the models being random noise with a drift. The drift for Real Estate Development is significant within a 1% level, whereas the drift for Industrial Machinery is insignificant within 5%.

Table 11: Estimated models for monthly data

Industry portfolio	Model	Parameter	Estimate	Standard Error	Z-value	p-value	AIC
Biotechnology	ARIMA(0,0,0)	-	0.000	-	-	-	-252.91
Information Technology	ARIMA(0,0,0)	-	0.000	-	-	-	-373.65
Packaged Software	ARIMA(0,0,0)	Constant	0.0150	0.0054	2.7751	0.0055 **	-334.43
	MA(1)	Constant	0.0149	0.0062	2.4059	0.0161 *	-334.6
		MA(1)	0.1604	0.0924	1.7338	0.0829 .	
	AR(1)	Constant	0.0149	0.0062	2.3840	0.0171 *	-334.28
		AR(1)	0.1441	0.0905	1.5929	0.1111	
Real Estate Development	ARIMA(0,0,0)	Constant	0.0106	0.0041	2.6017	0.0092**	-400.76
Industrial Machinery	ARIMA(0,0,0)	Constant	0.0103	0.0059	1.7516	0.0799 .	-313.65

Notes: *** Corresponds to 0.001 signifiance level, ** Corresponds to 0.01 signifiance level,
* Corresponds to 0.05 signifiance level '"." Corresponds to 0.1

5.2.2 Weekly returns

For the weekly observations more potential AR and MA lags was identified in the time series for all industry portfolios except for Industrial Machinery. The results of the estimated models are shown in table 12.

For Biotechnology the correlograms indicated the potential of AR(1) or MA(1), and AIC proposed an MA(2), where all estimated models have a zero mean. The AR(1) indicates a negative autocorrelation of the first lag of -0.1402, which is the highest estimated parameter estimate for Biotechnology and significant at 1% level. The MA(1) has a parameter estimate of -0.1226 and significant at 1% level. According to AIC the best fitted model is an MA(2), for which the first lag is estimated to -0.1366 and the second lag 0.0865, where latter is insignificant at the 5% level. Ultimately the results indicate with 96% certainty linear dependence in the time series for all three models.

For Information Technology the visual analysis of correlograms identified the models of AR(1) and MA(1), which is estimated to have a zero mean. The magnitude of the estimated parameters in the AR(1) and MA(1) is low meaning a weak level of autocorrelation is in the time series, though both are significant at the 5% level. Therefore, significant evidence for dependence of past returns and forecasting error, but due to the magnitude of the parameters is low, actual economic value is not expected for forecasting. The ARMA(2,2) identified by AIC has a higher level of parameter estimates and significant at 5% level. The model is mean reverting since the absolute values AR of MA lags doesn't exceed 1 in absolute value, though high parameter estimate for the AR and MA lags has an offsetting effect.

For Packaged Software multiple models was identified as potential, and all estimated with a significant drift at 1% level. A general evaluation of the models indicates that the simple models AR(2), AR(3), MA(2) and MA(3) doesn't capture the same information as the combined models of ARMA(1,2) and ARMA(1,4), when comparing AIC values. The models of ARMA(1,2) and ARMA(2,1) is significant through all parameters at the 5% level, and the first having the lowest AIC of -2128,29 relative to -2127,05. Regarding the most complex models namely ARMA(2,2) and ARMA(1,4), where the latter was identified using AIC, all the estimated parameters isn't significant, which in general is allowed if the last estimated lag is significant within 5% level. Besides this the magnitude of the parameter estimates could be expected to have an economic value add when forecasting.

For Real Estate Development, the identified model by AIC is the AR(1), where the Autoregressive lag is significant at 1% level and estimated at 0.0721. This level of autocorrelation is considered weak and therefore, the autocorrelation in the series is not expected to add significant economic value for predictions. By interpreting the correlograms, an ARIMA(0,0,0) was proposed and estimated with a significant constant equivalent to the mean of the process.

For Industrial Machinery, both the correlograms and AIC suggested an ARIMA(0,0,0) model, as the series exhibited a white noise behaviour. The model was estimated with a drift, indicating that the time series has a positive constant, and the series is random noise with a drift.

Table 12: Estimated models for weekly data

Industry portfolio	Model	Parameter	Estimate	Standard Error	Z-value	p-value	AIC
Biotechnology	AR(1)	Zero mean	-	-	-	-	-1805.64
		AR(1)	-0.1402	0.0437	-3.2107	0.0013**	
	MA(1)	Zero mean		-	-	-	-1804.38
		MA(1)	-0.1226	0.0404	-3.0361	0.0023 **	
	MA(2)	Zero mean	-	-	-	-	-1806.14
		MA(1)	-0.1366	0.0439	-3.1094	0.0019 **	
		MA(2)	0.0872	0.0446	1.9558	0.0504 .	
Information Technology	AR(1)	Zero mean	-	-	-	-	-2342.02
		AR(1)	-0.0865	0.0437	-1.977	0.0481 *	
	MA(1)	Zero mean	-	-	-	-	-2342
		MA(1)	-0.0855	0.0430	-1.9909	0.0465 *	
	ARMA(2,2)	Zero mean	-	-	-	-	-2343.42
		AR(1)	1.3884	0.1566	8.8643	2.2e-16***	
		AR(2)	-0.6233	0.1617	-3.8548	0.0001 ***	
		MA(1)	-1.4772	0.1352	-10.9262	2.2e-16 ***	
		MA(2)	0.7379	0.1397	5.2824	1.275e-07 ***	
Packaged Software	AR(2)	Constant	0.0045	0.0015	3.0281	0.0025 **	-2125.46
		AR(1)	-0.0228	0.0436	-0.5232	0.6008	
		AR(2)	0.1076	0.0438	2.4650	0.0138 *	
	AR(3)	Constant	0.0045	0.0016	2.7486	0.0063 **	-2127.76
		AR(1)	-0.0325	0.0437	-0.7431	0.4574	
		AR(2)	0.1105	0.0436	2.5336	0.0112 *	
		AR(3)	0.0912	0.0439	2.0784	0.0376 *	
	MA(2)	Constant	0.0045	0.0014	3.1087	0.0020 **	-2125.59
		MA(1)	-0.0422	0.0447	-0.9443	0.3450	
		MA(2)	0.1076	0.0431	2.4969	0.0125 *	
	MA(3)	Constant	0.0045	0.0016	2.8701	0.0041 **	-2127.97
		MA(1)	-0.0416	0.0437	-0.9527	0.3407	
		MA(2)	0.1073	0.0414	2.5928	0.0095 **	
		MA(3)	0.0888	0.0420	2.1136	0.0345 *	
	ARMA(1,2)	Constant	0.0045	0.0017	2.6859	0.0081 **	-2128.29
		AR(1)	0.5381	0.1622	3.3182	0.0009 ***	
		MA(1)	-0.5782	0.1622	-3.5649	0.0004 ***	
		MA(2)	0.1483	0.0459	3.2351	0.0012 **	
	ARMA(2,1)	Constant	0.0045	0.0017	2.6572	0.0072**	-2127.05
		AR(1)	0.4224	0.1724	2.4506	0.0143 *	
		AR(2)	0.1331	0.0442	3.0104	0.0026 **	
		MA(1)	-0.4532	0.1703	-2.6606	0.0078 **	
	ARMA(2,2)	Constant	0.0045	0.0016	2.7212	0.0065 **	-2127.45
		AR(1)	0.7962	0.2692	2.9579	0.0031 **	
		AR(2)	-0.2857	0.2494	-1.1456	0.2519	
		MA(1)	-0.8323	0.2555	-3.2568	0.0011 **	
		MA(2)	0.4219	0.2320	1.8184	0.0690 .	
	ARMA(1,4)	Constant	0.0045	0.0016	2.7805	0.0055 **	-2129.91
		AR(1)	-0.8656	0.0942	-9.1871	2.2e-16 ***	
		MA(1)	0.8394	0.1020	8.2305	2.2e-16 ***	
		MA(2)	0.0744	0.0569	1.3084	0.1907	
		MA(3)	0.1804	0.0556	3.2441	0.0011 **	
		MA(4)	0.1406	0.0444	3.1673	0.0015 **	
Real Estate Development	ARIMA(0,0,0)	Constant	0.0030	0.0009	3.2255	0.0012 **	-2516.58
	AR(1)	Constant	0.0030	0.001	3.0049	0.0999 .	-2517.27
		AR(1)	0.0721	0.0438	1.6449	0.0026 **	
Industrial Machinery	ARIMA(0,0,0)	Constant	0.0037	0.0015	2.5043	0.0123 *	-2055.84

Notes: *** Corresponds to 0.001 signifiance level, ** Corresponds to 0.01 signifiance level,
* Corresponds to 0.05 signifiance level '"." Corresponds to 0.1

The estimation period begins two years after the 2008 financial crisis and ends with the effect of the COVID-19 pandemic. For this period positive drifts are found significant for the monthly and weekly models of Packaged Software, Real Estate Development, and for Industrial Machinery only for the weekly data.

5.3 Diagnostics

To ensure the consistency and right specification of the estimated models, it is essential to conduct appropriate diagnostic tests. In accordance with the Box-Jenkins methodology, this study examines the residuals of the models to evaluate their behaviour, which should resemble white noise. Specifically, this study employs the Ljung-Box test and analyse the ACF and PACF correlograms of the residuals to assess their behaviour.

5.3.1 Ljung-Box test results

The Ljung-Box test is regarded obligatory when doing diagnostics of ARIMA models, since all autocorrelation in the time series should be captured, otherwise the time series isn't properly specified. Although the Ljung-Box test didn't provide evidence for autocorrelation in the log returns prior to estimation except for weekly Packaged Software, the residuals of the models still must be tested.

The ARIMA(0,0,0) models have the same significance levels as the time series, for Packaged Software there wasn't significant evidence for autocorrelation prior to model fit. Ultimately the MA(1) model was properly fitted with a parameter estimate of 0.16, but only significant within the 10% level.

Table 13: Ljung-Box test p-values of model residuals using monthly data
In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Ljung-Box test	B	RED	IT	IM	PS
ARIMA(0,0,0)	(0.9257)		(0.1406)		
ARIMA(0,0,0)		0.6843		0.9382	0.538
MA(1)					0.6534
AR(1)					0.6406

Note: () Indicates a zero-mean

Prior to model estimation, the weekly data for Packaged Software showed significant evidence of autocorrelation. Table 14 presents the p-values from the Ljung-Box test for the weekly data. The results indicate that for all models besides the simple models of AR(2) and MA(2), the null hypothesis fails to reject at a 5% significance level. Therefore, no evidence for autocorrelation in the residuals is found for the weekly models.

Table 14: Ljung-Box test p-values of model residuals using weekly data
In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Ljung-Box	B	RED	IT	IM	PS
ARIMA(0,0,0)		0.186		0.6095	
AR(1)	(0.8808)	0.3783	(0.8224)		
AR(2)					0.0805 .
AR(3)					0.2272
MA(1)	(0.9116)		(0.8183)		
MA(2)	(0.9376)				0.0823 .
MA(3)					0.2258
ARMA(1,2)					0.2661
ARMA(2,1)					0.1941
ARMA(2,2)			(0.9882)		0.3091
ARMA(1,4)					0.478

Notes: "." Corresponds to 0.1 significance level and () indicates a zero mean

5.3.2 ACF and PACF of residuals

To visually diagnose the residuals of the estimated models, the ACF and PACF are applied to evaluate whether the residuals of the models follow a white noise process. Only models with a total number of lags above zero are inspected since otherwise, the correlograms is similar to the time series of log returns.

5.3.2.1 Monthly returns

For the monthly data only Packaged Software models are inspected. The correlograms in Appendix B figure 21-22 shows the ACF and PACF for the MA(1) and AR(1) respectively. Comparing these to the correlograms of the raw series, the residuals correlation coefficient becomes smaller and has a clearer expression of being white noise. Though the model can't capture the significant correlation at lag 6 for both models. This ultimately can be handled by increasing the number of lags to six for both models. The results from the Ljung-Box test indicates there isn't evidence for autocorrelation in the series, and combining these results the most appropriate model is the MA(1). The strong significant lag 6 could be an indication of semi-annual mean reversion.

5.3.2.2 Weekly returns

For the weekly models the residuals of AR(1), MA(1) and MA(2) for Biotechnology is presentenced in appendix B figure 23-25. Comparing these to the time series of log returns, where the first lag was significant, indicates that all identified models capture this lag. This is complemented by the failure to reject the null hypothesis for the Ljung-Box test for all instances, therefore the residuals have a white noise behaviour.

For Real Estate Development the proposed model by AIC was an AR(1) model. The correlograms of the log returns had white noise behaviour, though significant lags at 18 and 19. The low magnitude of the estimated AR parameter indicates a low dependency of past returns in the time series. Combining these interpretations of the correlograms with the results from the Ljung-Box test the best model is an ARIMA(0,0,0) with a drift equivalent to random noise with a drift.

For Information Technology the proposed model using AIC as selection criteria was the ARMA(2,2), whereof the correlograms of the residuals can be seen in Appendix B figure 27. The model captures the autocorrelation in the time series, since the behaviour of the error terms is a clear white noise process. The correlograms for the MA(1) and AR(1) is shown in Appendix B in figure 28 and 29 respectively. Through a visual analysis it can be seen, that both models capture the first significant lag, though the white noise process of the residuals isn't as clear as for the ARMA(2,2).

In the case of Packaged Software multiple models has been estimated. The simple models of AR(3) and MA(3) was able to capture the significant lags until the 17, the correlograms hereof is shown in Appendix B figure 30 and 32 respectively. When decreasing the number of lags to an AR(2) and MA(2) model, for which the correlograms are shown in Appendix B figure 31 and 33 respectively. For these the significant lag 6 is not captured. The more complex models of ARMA(1,2) and ARMA(2,1) is shown in appendix B figure 34 and 35 respectively. The correlograms indicates for ARMA(1,2) that all autocorrelation is captured until 17, though for the ARMA(2,1) the seventh lag is slightly significant for the PACF. The correlograms of ARMA(2,2) and ARMA(1,4) is shown in Appendix B figure 36 and 37 respectively. The residuals of the latter two models have the clearest resemblance of white noise.

5.4. Forecast and model evaluation

To assess the predictive power of the estimated models and determine the best performing model for each industry, an out-of-sample one-step ahead forecast was conducted with an expanding window. The forecasting period ranged from January 2020 to December 2022, with a total of 36 monthly observations and 155 weekly observations. The accuracy of the forecasts was evaluated using two metrics: the Root Mean Square Error (RMSE), which measures the standard deviation of the residuals, and the Mean Absolute Error (MAE), which measures the mean of the differences between the predicted values and the actual observations. The lowest values for these measurements, as well for AIC are presented in bold font in table 15 and 16 for monthly and weekly data respectively.

5.4.1 Monthly returns

For the predictive performance of the monthly models, only Packaged Software was forecasted and evaluated, as no alternative model specifications were considered for the other industry portfolios besides an ARIMA(0,0,0) model. Table 15 presents the performance values for the monthly models of Packaged Software. The model with the best performance according to the selection criteria's is the MA(1) model. It has the best overall performance with an AIC of -334,60, RMSE of 0.08002, and MAE of 0.06465. While the performance differences among the models are small, the MA(1) model exhibits the strongest predictive strength for the time series. The models for Packaged Software has a standard deviation of prediction errors of approximately 8,0% and the absolute deviation between the forecasts and actual values ranges from 6,37%-6,47%. These values are relatively high, which is due to the observation frequency being monthly, and the period for estimation is volatile with regard to the covid-19 pandemic.

Table 15: Evaluation of models with AIC, RMSE and MAE

Industry	Model	*AIC*	*RMSE*	*MAE*
Packaged Software	ARIMA(0,0,0)	-334,40	0.08077	**0.06374**
	MA(1)	**-334,60**	**0.08002**	0.06465
	AR(1)	-334,28	0.08006	0.06478

[8]

5.4.2 Weekly returns

In table 16 the forecasting results is shown for the estimated models for the weekly industry portfolios together with their AIC value. For Biotechnology the best model according to AIC was the MA(2), which has superior performance in the prediction period compared to the AR(1) and MA(1) model, though the second lag of the MA(2) models is not significant within the 5% level, but within the 10% level. This means if constraining the significance level for all parameters to the normally applied level of 5% the MA(1) model is best, since it performs better in the forecasting period compared to the AR(1), even though the AIC value is higher for the AR(1) model. The average deviation for the forecasts are approximately 3,6%, while the absolute deviation is approximately 2,7%.

For Information Technology the best model using AIC as selection criteria is the ARMA(2,2) but the simple regression indicates a bias due to a significant constant. Comparing the forecast performance for AR(1) and MA(1), they have a minor outperformance in RMSE and MAE respectively, ultimately the best performing model is AR(1). The standard deviation of the error forecast is highest for this industry at approximately 4,2%, with an average prediction error of 3,1%.

The Packaged Software time series proved to be the most challenging to interpret based on its correlograms, leading to the identification of multiple potential models with a drift. Upon evaluating the results in table 16, it was found that the ARMA(1,4) model, which was selected using the AIC, demonstrated the best performance across the goodness of fit measurements. The difference in RMSE and MAE is generally low for all models. For RMSE the lowest value is 0.03444

[8] A simple regression using the actual observations in the forecasting period as the dependent variable and forecasted observations as independent variable showed insignificant results within 5% level for both the estimated constant and beta.

for ARMA(1,4) and the highest is 0.03483 for ARMA(2,2). Taken the results for the forecasting period into account, the best model for Packaged Software is the ARMA (1,4), since the measurements indicates the greatest model fit.

For Real Estate Development the identified model by AIC was an AR(1), while the correlograms of the log returns series didn't give incentive to model estimation, due to a white noise behaviour. In Table 16 the results indicate that the best performing model in the forecasting period is the ARMA(0,0) with a drift compared to the AR(1) with a drift. The ARMA(0,0) with a drift states the return of today is defined by a random noise process with a drift.

Table 16: Evaluation of models with AIC, RMSE and MAE for weekly models

Industry	Model	*AIC*	*RMSE*	*MAE*
Biotechnology	AR(1)*	-1805,64	0.03694	0.02782
	MA(1)*	-1804,38	0.03694	0.02769
	MA(2)*	**-1806,14**	**0.03614**	**0.02763**
Information Technology	AR(1)*	-2342,02	**0.04219**	0.03114
	MA(1)*	-2342,00	0.04219	**0.03113**
	ARMA(2,2)*	**-2343,42**	0.04294	0.03157
Packaged Software	AR(2)	-2125,46	0.03460	**0.02632**
	AR(3)	-2127,76	0.03477	0.02664
	MA(2)	-2125,59	0.03465	0.02643
	MA(3)	-2127,97	0.03478	0.02669
	ARMA(1,2)	-2128,29	0.03478	0.02660
	ARMA(2,1)	-2127,05	0.03478	0.02649
	ARMA(2,2)	-2127,45	0.03483	0.02674
	ARMA(1,4)	**-2129,91**	**0.03444**	0.02635
Real Estate Development	ARMA(0,0)	-2516,58	**0.03423**	**0.02500**
	AR(1)	**-2517,27**	0.03435	0.02511
Industrial Machinery	ARMA(0,0)	-2055,84	0.03716	0.02615

Notes: * indicates a zero mean [9]

5.5 Summary of results

The result from this study provides evidence for the time series for all industry portfolios to be stationary using both monthly and weekly observations. The rejection of the null hypothesis of the ADF test provides significant evidence against the random walk model and for stationarity shown

[9] A simple regression using the actual observations in the forecasting period as the dependent variable and forecasted observations as independent variable gave insignificant results withing 5% level for both the constant and beta for all models, except for ARMA(2,2) for IT, where the constant and beta was significant indicating a poor model fit.

in Table 4. These results support prior research in Sweden namely the ones by Shaker (2013). The results from the Ljung-Box test shown Table 5 was contrary to prior results for the OMXS30, except for the weekly observations for Packaged Software, where significant evidence for autocorrelation in the time series was provided. This could be an effect of the observation frequency differs since Shaker (2013) used daily log returns, and the nature of the applied series. Ultimately the results indicate a rejection of the weak-form of efficiency using the ADF test, while the results from the Ljung-Box test indicates the majority of the time series should be interpreted as white noise. The difference in results is explained by the objectives of the tests are different.

Using visuals analysis of the correlograms and AIC for model identification gave incentive to estimate ARIMA models. The estimated models hereof can be seen in Table 11 for monthly observations and Table 12 for weekly observations. Using monthly data an MA(1) model with a was estimated with a 0,16 parameter estimate, significant within 10% level, and with predictive power compared to a random noise process. Generally, the industry portfolios using monthly observations had a white- and random noise behaviour with estimation of ARIMA(0,0,0) for Biotechnology and Information Technology, and ARIMA(0,0,0) with a drift for Real Estate Development and Industrial Machinery.

For the weekly model's significant lags within the 5% level was found for all industry portfolios except for Industrial Machinery, which can be due to autocorrelation in general is higher for weekly observations. In general, the estimated parameters are low, since the parameters with both AR and MA lags can have an offsetting effect. The significant positive drifts for Packaged Software, Real Estate Development, and Industrial Machinery can be explained by stock returns generally will increase through time. In the estimation period the main index of Sweden increased from 936 SEK to 1771 SEK equivalent to a CAGR of 6,28% (Nasdaqomxnordic.com). Given the contrarian effect of equal-weighting and exposure to small- and midcap stocks positive drifts is regarded as accurate presentations of these time series.

An explanation for the difference in the results with regard to parameter magnitude and significance level for the estimated models is, that the industries have different characteristics meaning that the homogeneity of the firms wary across industries. Companies such as Real Estate Developers has a high level of homogeneity, since business models and products are highly similar, and the complexity of the companies is low, while returns are expected to be driven by steady cash flows and macroeconomic factors such as interest rates and general economic development in the country. In an industry such as Biotechnology, the companies are highly specialized in their given research where the companies have a higher level of differentiation. The returns of Biotechnology companies are therefore more driven by company specific factors, such as achieving governmental approvals or successful research studies. The complexity in Biotechnology is therefore much higher relative to Real Estate Development. This could explain the higher level of parameter magnitude for weekly Biotechnology relative to other industries, since information is priced at a less efficient level compared to other industries.

The distributions of all observations for the industry portfolios, was platykurtic meaning the data has a lower probability of extreme values compared to a normal distribution. This has an effect when estimating parameters, since an underestimation of extreme events is implied in the models. This implies platykurtic distributions isn't optimal for forecasting through a volatile period. A simple regression was tested, where the observed values acted as dependent variable and predicted values as independent variable, the results hereof was insignificant constant and beta, except for model ARMA(2,2) for weekly IT. These results generally indicate the predicted values are not statistically significant in predicting the actual values, and the time series for the given period can be explained by other factors than autocorrelation.

6. Reflection

The objective of this chapter is to reflect upon the methodology and results of this thesis and provide recommendations as to how these can be applied in future research.

6.1 Implementation of volatility

The period for estimation and forecasting includes significant events of systematic risk, such as the trade war between the United States and China in 2019 and the COVID-19 pandemic, which caused high levels of uncertainty and volatility in the financial markets (Chaudhary, Bakhshi, & Gupta, 2020; Nishimura, Dong & Sun, 2021). Estimating ARIMA models with relatively low parameter estimations and forecasting through a period with high levels of volatility, there is a possibility for enhancing the predictability by implementing volatility adjustments.

Volatility of financial assets is commonly estimated as the variance or standard deviation of the given time series. Generally, the usage of volatility models implies the asset return of today depends on the past returns in a non-linear relationship, which can be captured using volatility models. A characteristic of volatility of financial assets is its dependence of time, since if returns followed a random walk the variance should be independent of time, but studies have found that the volatility of yesterday can help explain the volatility today (Tsay, 2005). The common applied models for forecasting volatility are the Autoregressive Conditional Heteroskedastic (ARCH) and Generalized Autoregressive Conditional Heteroskedastic (GARCH). The structure of these models is similar to an ARIMA model, since past values of the same time series is applied to estimate the conditional variance, which equals the variance of the next period (Tsay, 2005).

An ARCH model has *p* number of lags for the regressor α_p. The regressors captures the volatility at time *t-1* and *t-p,* which is conditioned to have an effect at time *t.* Though ARCH models have limitations, because it doesn't differentiate negative and positive shocks in a time series, and it don't capture the persistence of the shocks (Tsay, 2005). The most applied volatility model is GARCH (*p,q*), where *p* is the number of ARCH terms and *q* the number of GARCH terms in the model, which is seen in equation 23. Whereas the return equation is presented in equation 24 (Tsay, 2005).

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_p \varepsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_p \sigma_{t-p}^2 \tag{23}$$

$$r_t = \mu_t + \varepsilon_t, \qquad \varepsilon_t = v_t \sqrt{\sigma^2}_t \qquad v_t \sim N(0,1) \tag{24}$$

The rationale for using GARCH models relative to ARCH models is that ARCH models capture volatility spikes but do not adequately capture the persistence of volatility and the mean-reverting behaviour (Tsay, 2005). In contrast, GARCH models incorporate GARCH regressors β_p that capture the persistence of the volatility. This is important because of the pace in which volatility mean reverts (Tsay, 2005). There has been developed various variants of ARCH and GARCH models to enhance their performance for specific modelling purposes. For example, the ARCH and GARCH models with t-innovations allows for higher kurtosis, accommodating greater shocks in the data (Tsay, 2005). This allowance for greater shocks can be suitable for industries with a higher frequency of large shocks, such as the Biotechnology industry. By implementing ARCH and GARCH models, one can capture time-varying volatility, and thereby incorporate past information that is not captured by ARIMA models and improve the accuracy of forecasting (Tsay, 2005).

A previous study by Dritsakis and Savvas (2017) had the objective of studying the volatility effects in the four main Nordic indexes for Norway, Denmark, Finland, and Sweden. The data for Sweden was daily log returns from 30thSeptember 1986 to May 11th 2016. In the article the squared log returns are applied for the detection of an MA(3) for the Swedish index. A low magnitude of estimated parameters is found significant at 5% level, and a constant with a p-value of 0.0593. In the article Dritsakis and Savvas (2017) found the best model for describing the volatility in Sweden to be an ARMA(0,3)-GARCH-M(1,1). Where GARCH-M models implements a constant in the model. This is due since log returns of stocks or indexes is expected to depend upon the volatility. Therefore, a constant effect is applied in the return equation such as: $r_t = \mu_t + c\ \sigma_t^2 + \varepsilon_t$. In the equation the constant effect if positive can be attributed as a risk premium factor, meaning if the volatility increases the return will increase as well (Tsay, 2005). Therefore, the GARCH-M can be favourable for stock returns because stock returns are known to have significantly different reactions to negative- and positive news (Zhang, 2006).

In an article by Hyytinen (1999) a similar study was made for Finland, Norway, and Sweden for the conditional volatility, though applying weekly data for value-weighted market indexes for the whole markets in the period 1983 until 1997. The results showed that the most appropriate model for the Swedish market was an EGARCH, where the E stands for Exponential. This model is appropriate for asymmetric volatility, meaning having a greater power of explaining distributions with fat tails (Hyytinen, 1999; Tsay, 2005).

6.2 Equal- vs value-weighted

The constructed industry portfolios for this study applied the equal-weighted method to determine the size for each holding at the start of every year, which was described in chapter 4. Data collection. Another method commonly used by financial institutions to construct indexes and benchmark portfolios is the value-weighted method. Using this method, the weight of each asset is determined by its market capitalization and the total market capitalization of the portfolio or index, meaning stocks with large market capitalizations has a greater weight in the portfolio or index, and the opposite for companies with small market capitalizations. Besides this the value-weighted method have a momentum characteristic, since increasing capitalization of a stock naturally will increase its weight in the portfolio or index, whereas an equal-weighted portfolio with rebalancing can be said to have a contrarian approach, because of rebalancing the portfolio will sell companies with increasing prices and buy companies with decreasing share prices (Bolognesi, Torluccio and Zucherri, 2013).

In an article by Bolognesi et al. (2013) evidence is found for equal-weighted indexes outperforms value-weighted indexes in the European markets using stocks from the DJ Euro Stoxx index. Besides this an equal-weighted index or portfolio allows for a greater diversification and exposure to small- and midcap stocks. These results are important findings since a common method for benchmark construction is value-weighing for financial asset management divisions and mutual funds. Though when deciding whether to apply equal- or value-weighted the objective for the portfolio construction should be the decider, since the methods is appropriate for different purposes (Bolognesi et al., 2013).

For industry and sector indexes Kittsley (2006) specifies why equal-weighting is the preferred method. He states these indexes are specified to segments that are smaller compared to large indexes, and therefore should focus on bearing the specific sector or industry risk. By applying equal-weighting the diversification increases, and idiosyncratic company risk minimizes implying the exposure to the sector or industry is more clearly reflected. When the objective for portfolio construction is test the market efficiency of an industry, and whether the time series of returns is predictable, the equal-weighted method is preferred. Another reason for choosing the equal-weighted index is that the effect of rebalancing avoids momentum effects. A negative effect of an equal-weighted portfolio is it won't capture the aggregate market of the industry compared to a value-weighted index (Kittsley, 2006). Future research could therefore create value-weighted portfolios containing the total number of stocks for the given industry. This could help to verify the results of this study and provide new evidence for broader industry portfolios or -indexes.

6.3 Industry complexity

As mentioned in the literature review the latest study about market efficiency in Sweden was for the main index by Shaker (2013), where evidence was found for stationarity and autocorrelation. The result from this thesis provides evidence for stationary time series for all industry portfolios independent of observation frequency. As mentioned in section 5.5 Summary of Results*,* the results for autocorrelation vary. This could be due to the homogeneity of the industries can have an effect on the predictability. From this study models with impactful magnitude have been found like an MA(1) for Packaged Software using monthly observations and ARMA(1,4) using weekly. This raises the question are some industries more predictable than others?

Ultimately it could be due to variations in industry complexity, due to variation in homogeneity of companies and steadiness of cash flows. A study by Jim Liew and Ryan Roberts (2013) tested a statistical arbitrage strategy relying on mean reversion for nine different US industries such as materials, energy, financials, and technology. Their findings are that mean reversion strategies aren't suited for all industries and sectors, since it according to their study depends upon the level of variables needed to explain the variation in returns for each industry. Therefore, the greater the level of variables needed to explain the variation in returns, the more appropriate is the industry for mean reverting investment strategies (Liew & Roberts, 2013). Their findings were that sectors such as materials, energy and utilities wasn't appropriate for a statistical arbitrage strategy relying on mean reversion, since the industry complexity is relatively low (Liew & Roberts, 2013).

6.4 Limitations and further research

Data limitations: The results of this study is constrained to the data applied for the chosen time period (Jan. 2010 to Dec. 2022), since it might not capture all relevant market conditions or economic events that could impact the industry portfolios. Besides this the whole time period including forecasting suffers from abnormal events such as the trade war between America and China, and the covid-19 pandemic. This study forecasted from Jan. 2020 to Dec. 2022 to capture the long-term performance of ARIMA predictions. Besides this the collected data doesn't account for transaction costs and liquidity since the objective of this study has been to test the weak-form of market efficiency and predictability of returns. Future studies could apply similar methodology to other geographical markets and industries, use daily observations and test the forecasting performance through another period.

Generalizability: This study focuses specifically on the Swedish market and the selected industries (Biotechnology, Information Technology, Packaged Software, Real Estate Development, and Industrial Machinery) for the given time period. Therefore, the findings and conclusions may not be directly applicable to other markets or industries. Sweden was chosen due to prior studies had found evidence for stationarity and autocorrelation in daily log returns, and the number of listed companies makes it appropriate for statistical analysis. In future research the geographic reach could be broadened, due to industries could be expected to have cross-border effects, such as companies in Packaged Software and Information Technology competing in a broader market of Scandinavia, Europe, or globally. This would complement the results of this study and provide new results about market efficiency and predictability for industry returns.

Model selection: While ARIMA models are widely used in time series analysis, they are just one of many possible models used for forecasting, though the objective for this study was regarding the relevance of ARIMA for predictability equity returns. In general, the results indicate the economic value added for ARIMA prediction is not robust across industry portfolios. Therefore, other statistical models could enhance the predictability. Alternative models, such as ARCH and GARCH, could yield different results, therefore future research could implement ARCH and GARCH models. This would provide new information about volatility of industry portfolios and verify a rejection of the weak-form of market efficiency and potentially enhance the predictability.

7. Conclusion

This study had the objective to investigate the validity of the weak-form of the efficient market hypothesis, and test if ARIMA models still are relevant in predicting the returns of self-constructed industry portfolios in Sweden with a sample period from Jan. 2010 to Dec. 2022.

The results from the ADF test gave significant evidence for rejecting the null hypothesis stating the series follows a random walk for all industry portfolios independent of observation frequency. Therefore, evidence was also found for stationarity. For the Ljung-Box test the null hypothesis was solely rejected for weekly observations of Packaged Software, indicating the majority of the time series follows a white- or random noise process, which is contrary to the results of Shaker (2013). This can be explained by the frequency of observations, since autocorrelation can be expected to be more frequent at daily observations, and general differences between OMXS30 and self-constructed portfolios. Ultimately the test results are contradicting, regarding whether to reject the weak-form of efficiency, since the tests are applied for different properties of the time series. These results contribute with new results for weak-form of market efficiency in Sweden for the given industry portfolios. Future studies could apply models such as ARCH and GARCH to verify a rejection of the weak-form of efficiency.

Prior to forecasting the Box-Jenkins methodology was applied for fitting ARIMA models to the time series. For 4 out of 5 industry portfolios using monthly data there wasn't evidence for AR and MA processes. Using weekly observations AR and MA lags was identified and estimated significant within a 5% level for 4 out of 5 industries. For evaluating the predictive power of the estimated models a simple regression of the actual- and predicted values, AIC, RMSE and MAE was applied. The results from this study shows ARIMA models isn't robust for creating economic value-added predictions for Swedish industry portfolios in the given period, despite evidence for significant dependence on past returns and forecasting errors. This is due to log return processes are complex and external factors other than autocorrelation in the time series has great impact. However, it's worth noting that the forecasting period has a high level of volatility due to the systematic event of covid-19.

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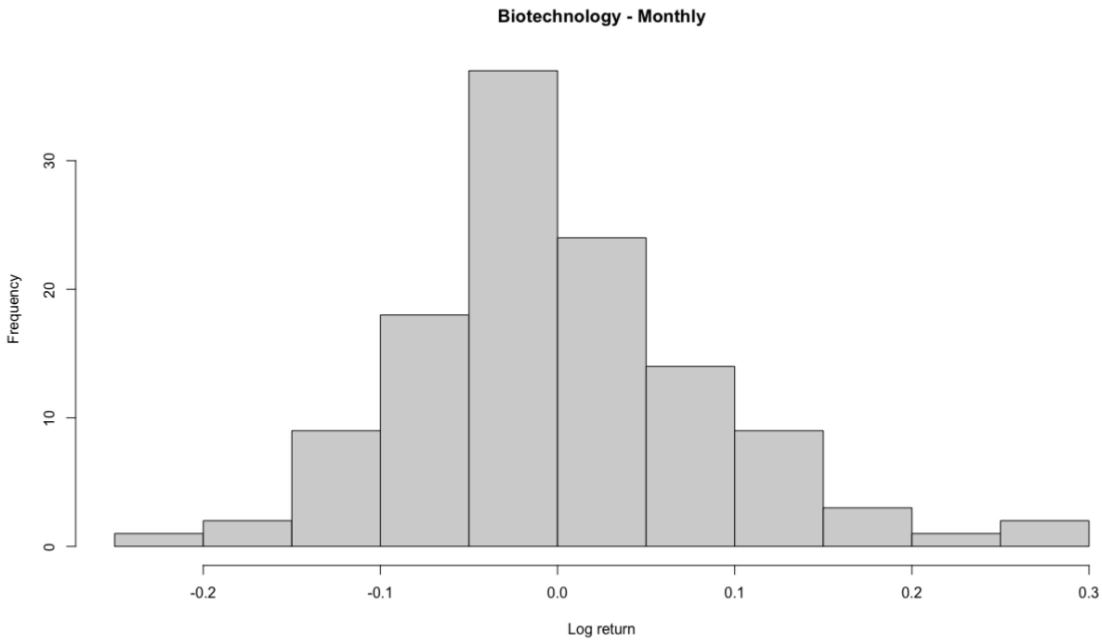
Appendix

Appendix A

Histograms of monthly- and weekly log returns

Figure 1: Histogram of the log returns Biotechnology from Jan. 2010 to Dec. 2019



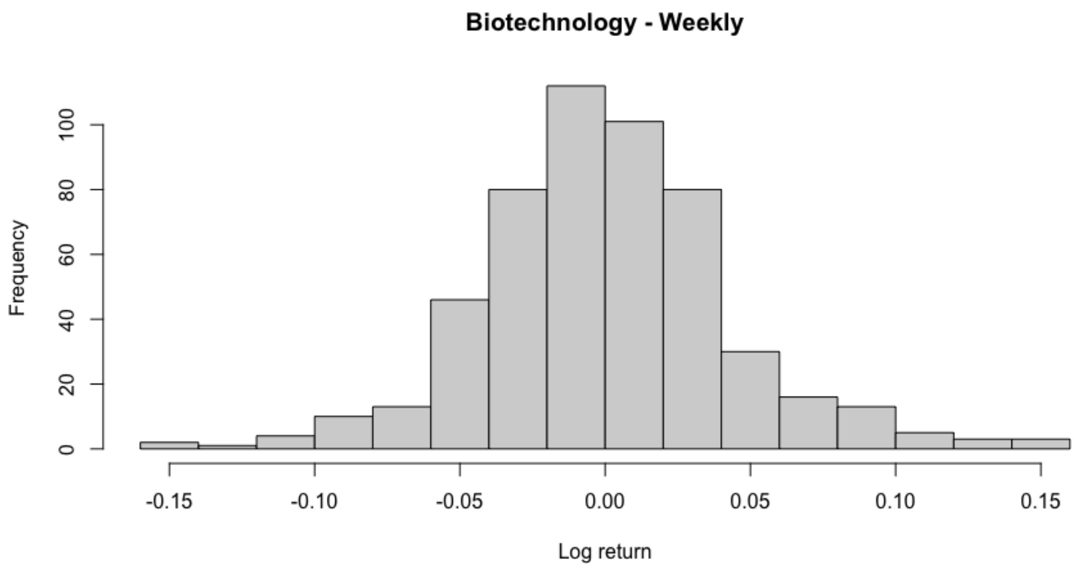
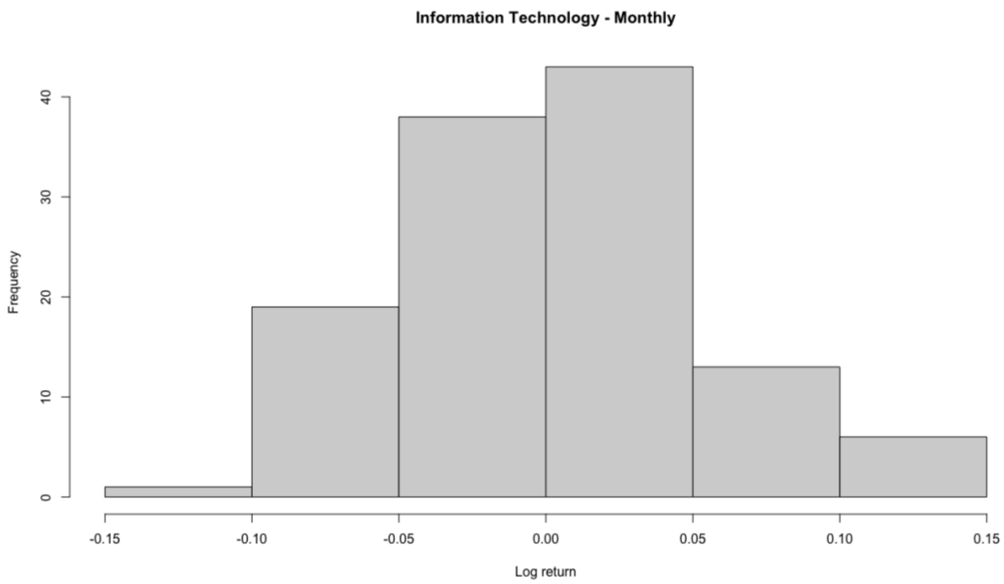


Figure 2: Histogram of the log returns for Information Technology from Jan. 2010 to Dec. 2019



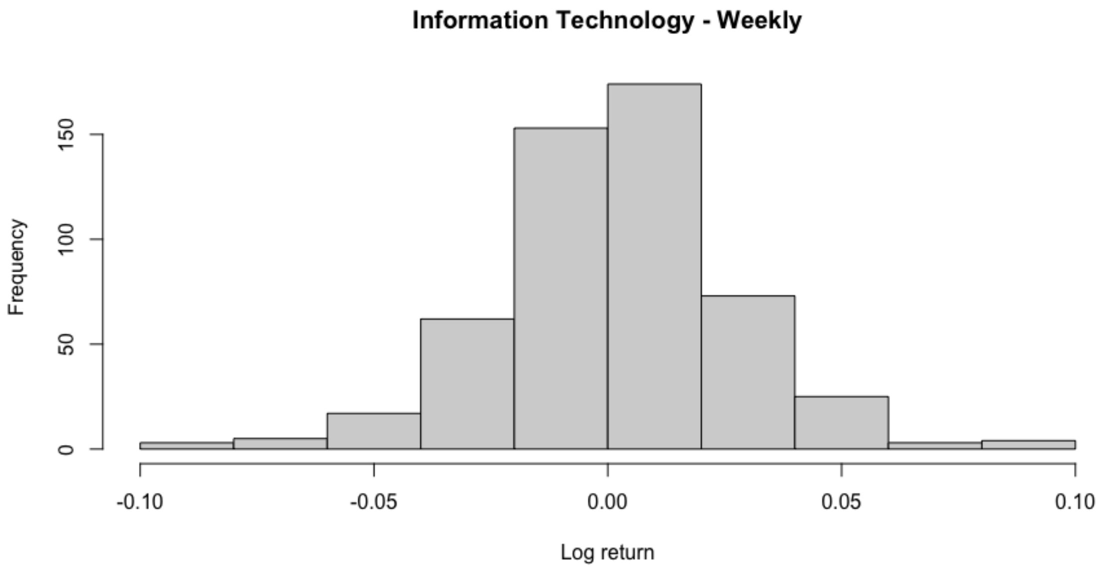
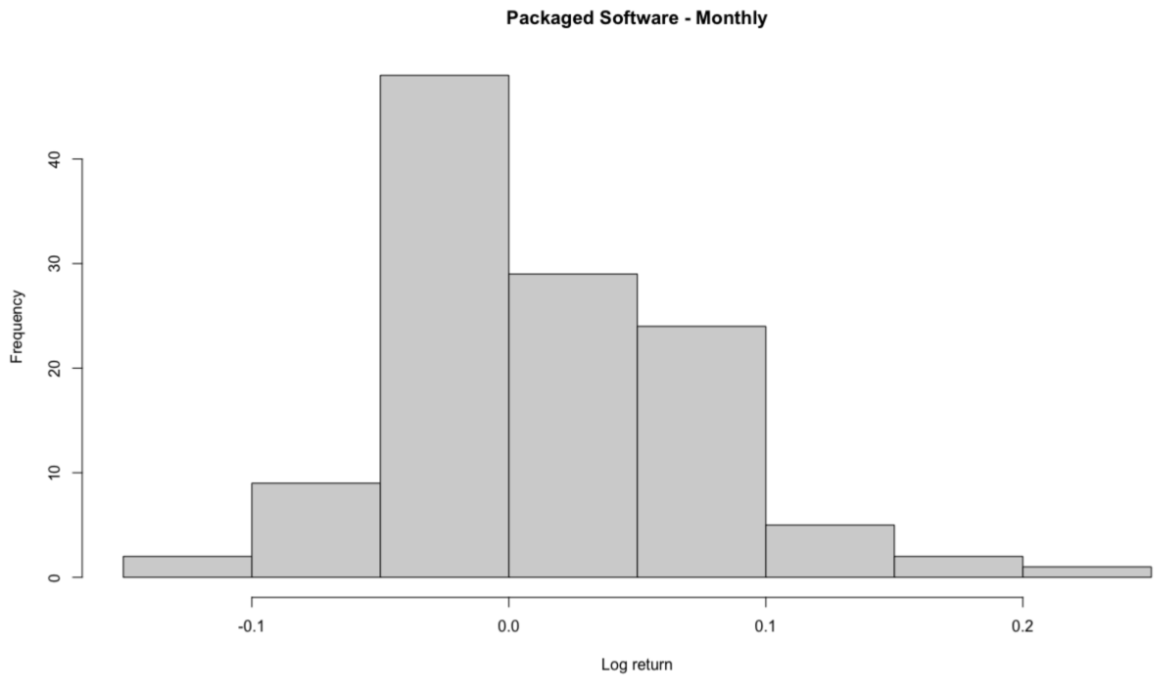


Figure 3: Histogram of the log returns for Packaged Software from Jan. 2010 to Dec. 2019



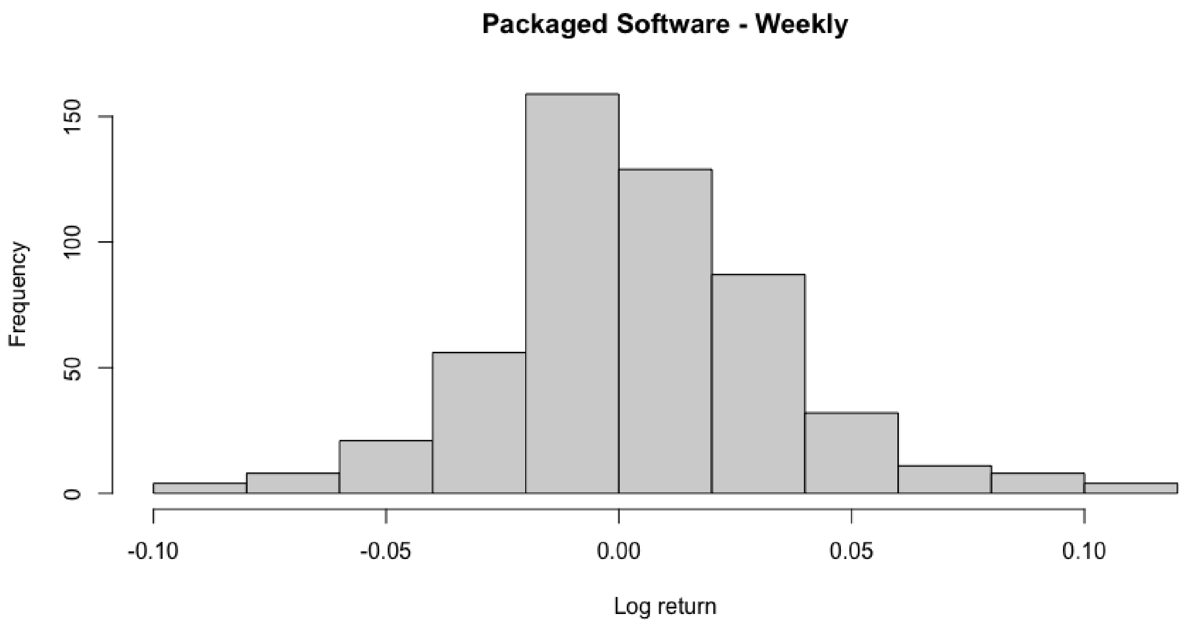
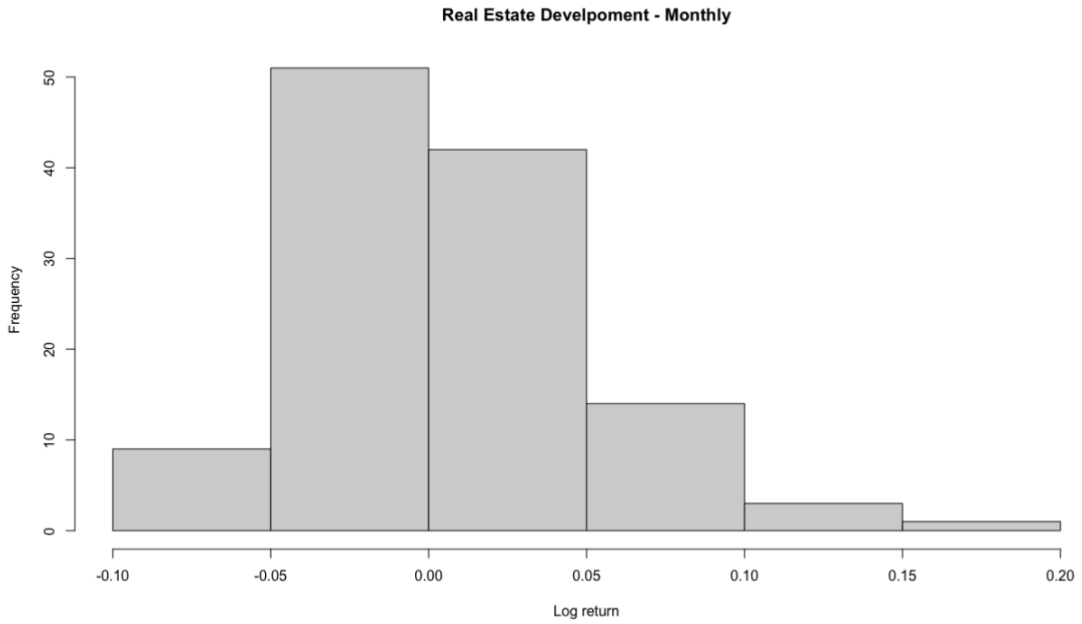


Figure 4: Histogram of the log returns for Real Estate Development from Jan. 2010 to Dec. 2019



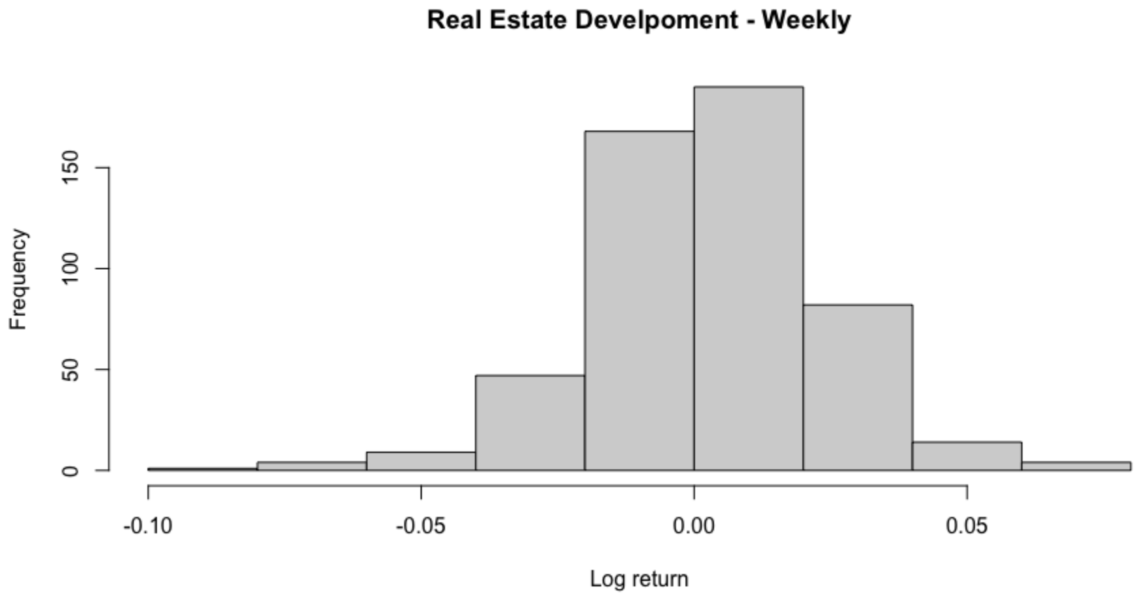
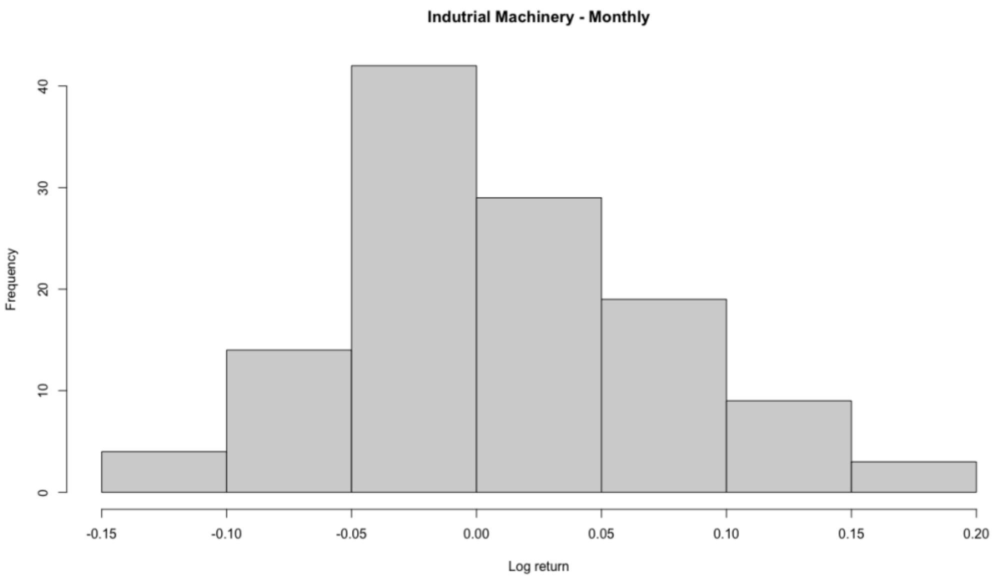
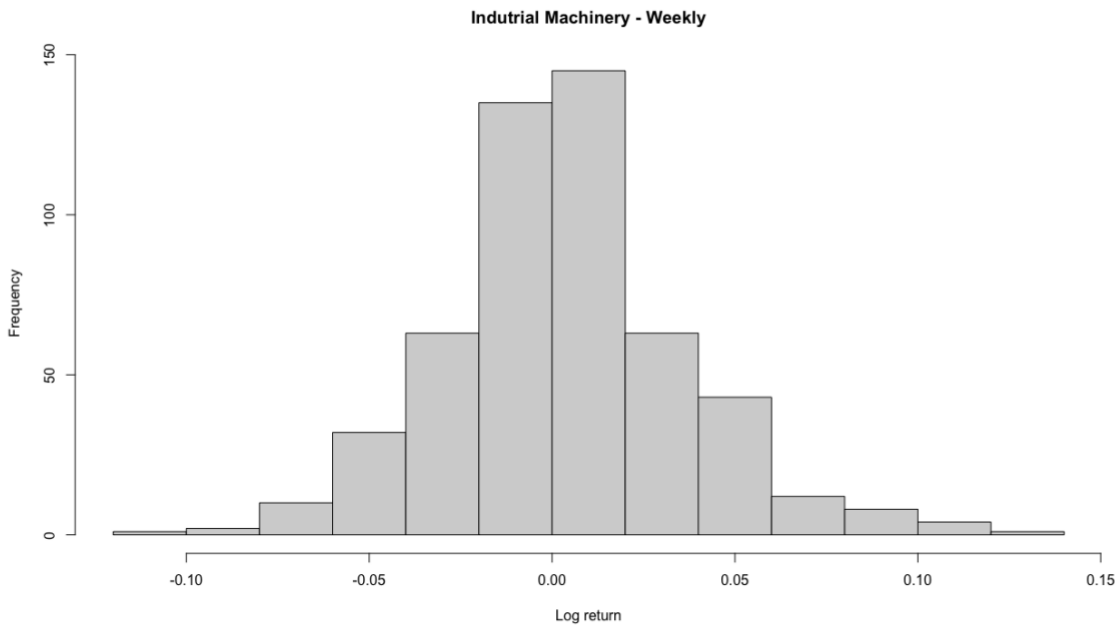


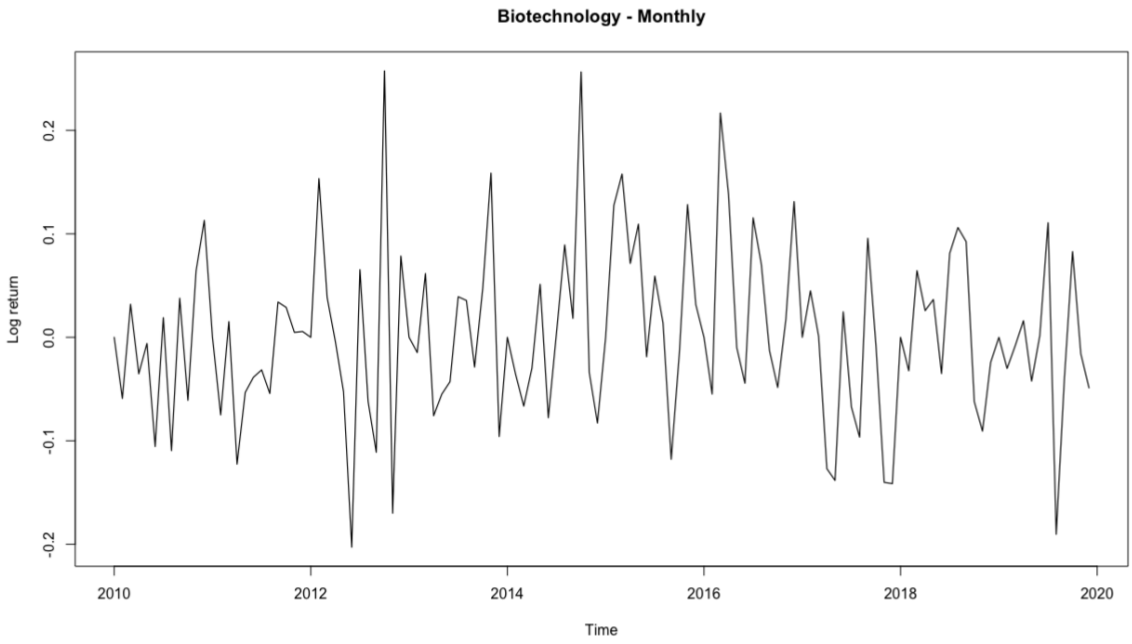
Figure 5: Histogram of the log returns for Industrial Machinery from Jan. 2010 to Dec. 2019





Time series plots of monthly- and weekly returns

Figure 6: Plot of the log returns for Industrial Machinery from Jan. 2010 to Dec. 2019



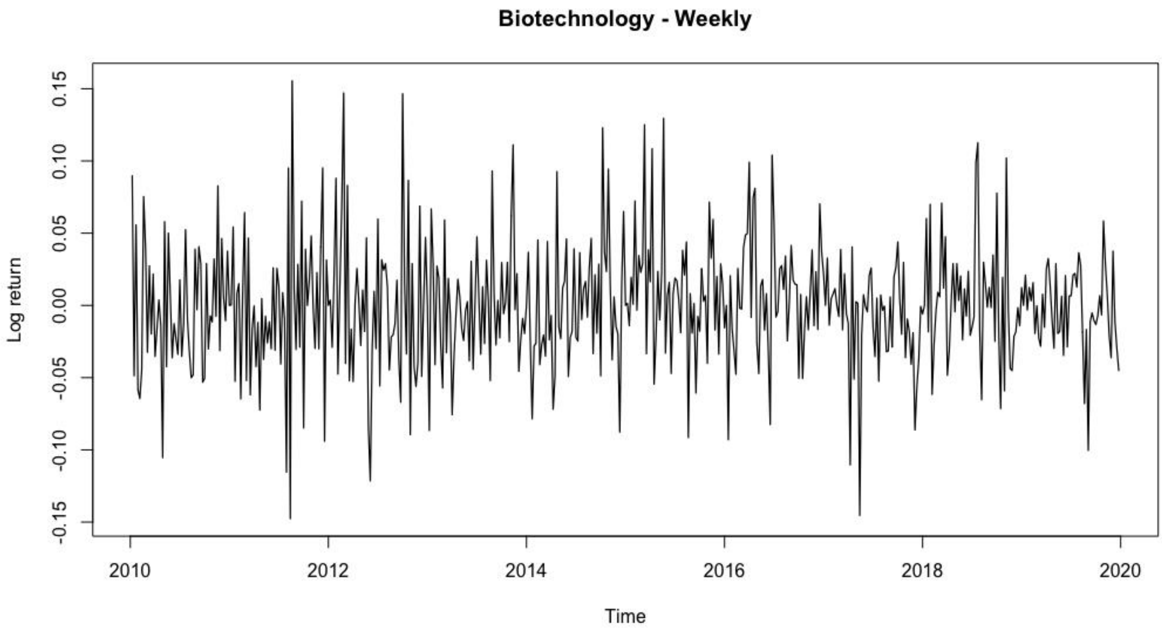
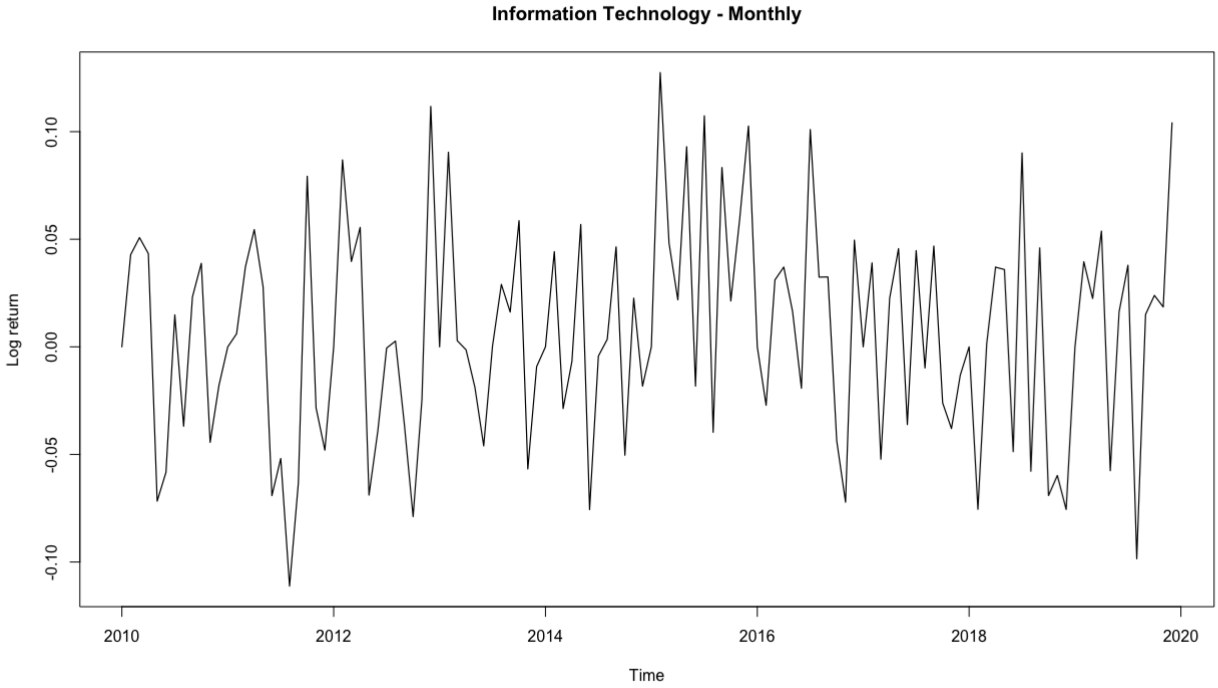


Figure 7: Plot of the log returns for Information Technology from Jan. 2010 to Dec. 2019



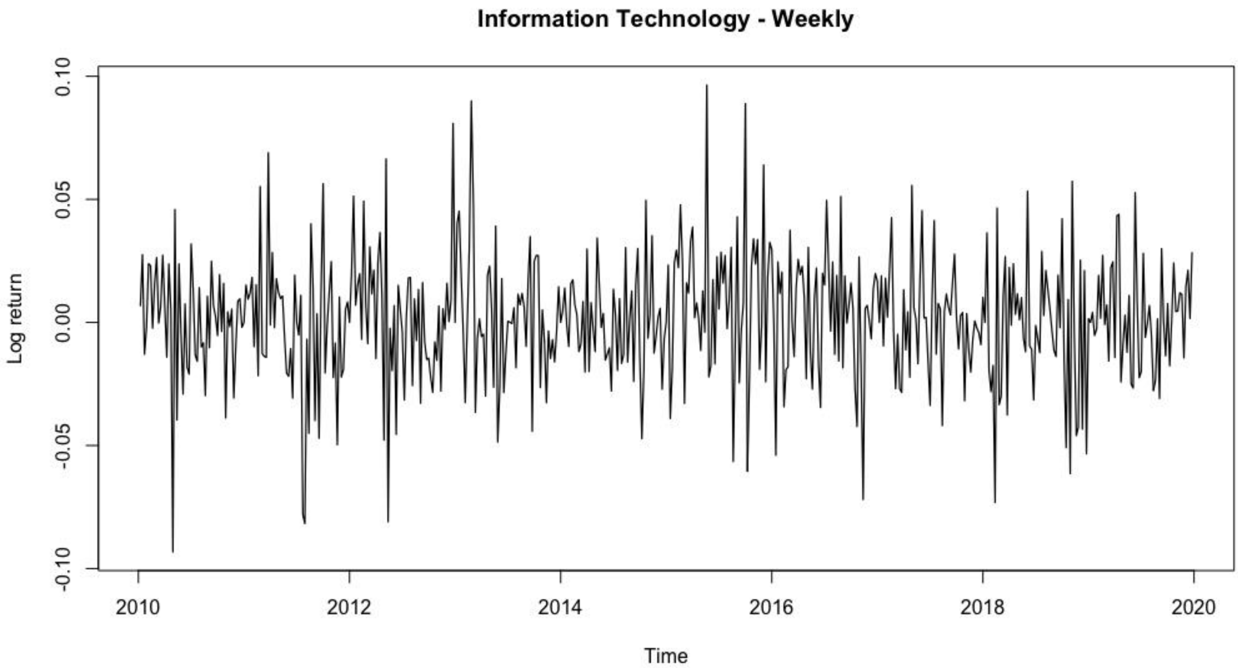
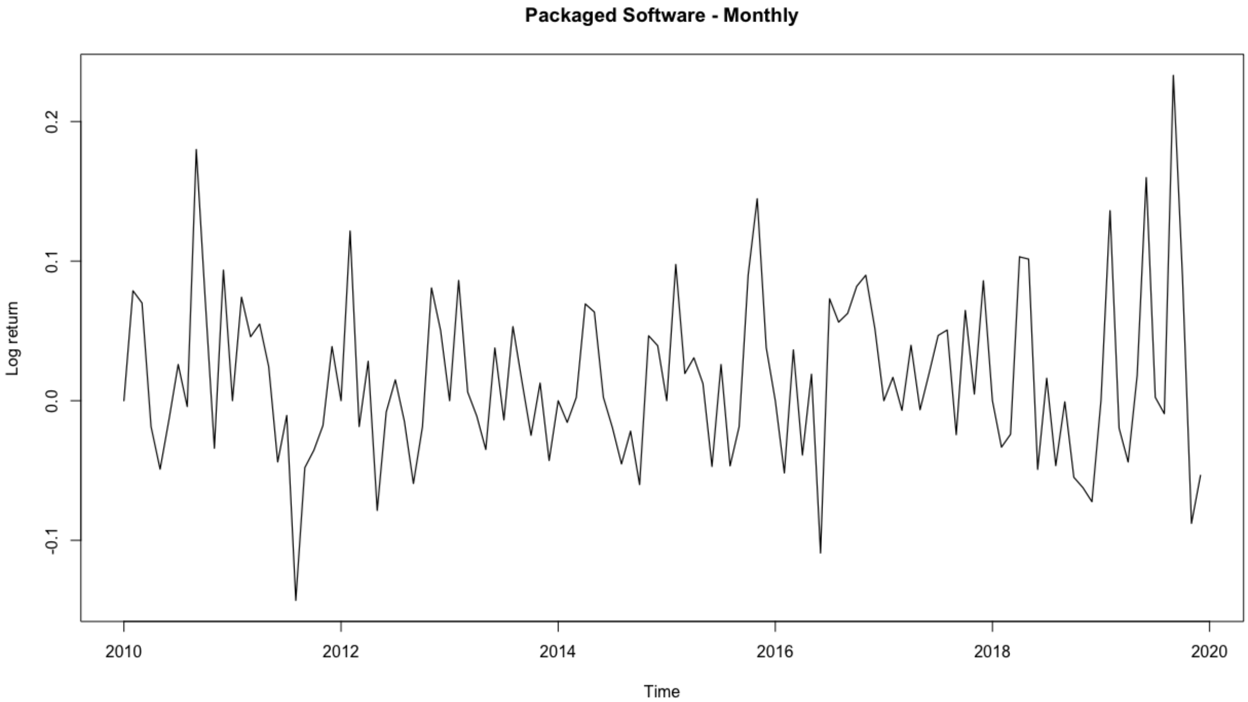


Figure 8: Plot of the log returns Packaged Software from Jan. 2010 to Dec. 2019



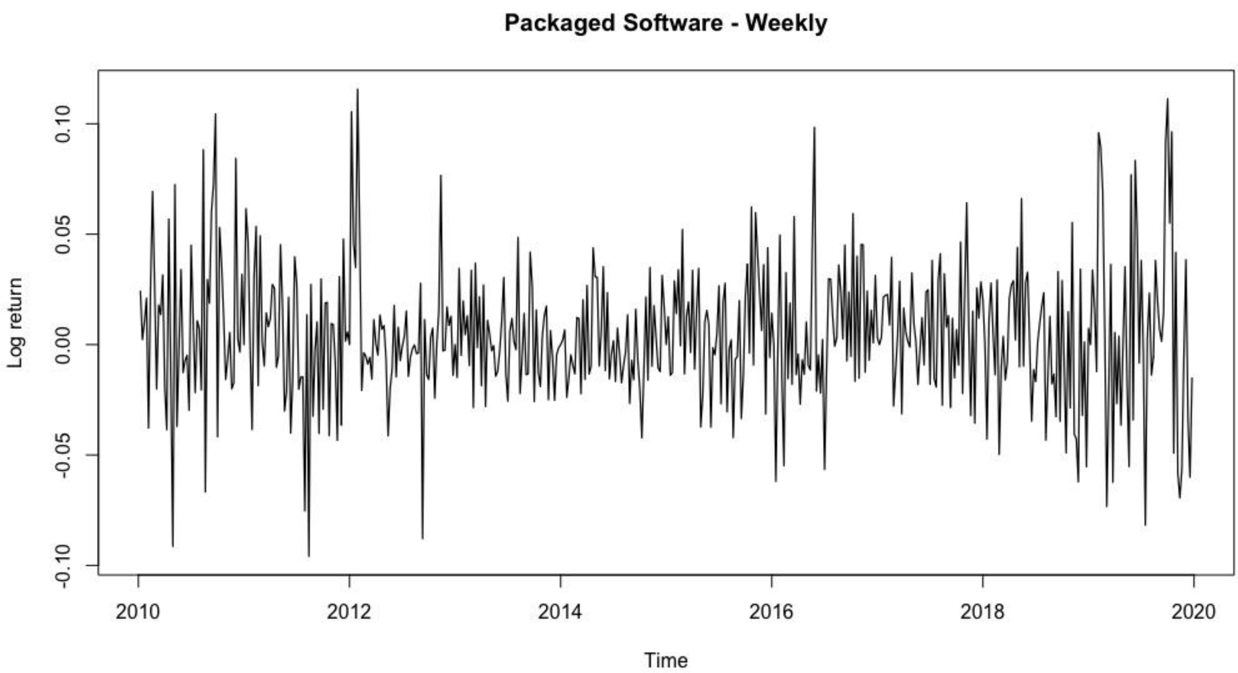
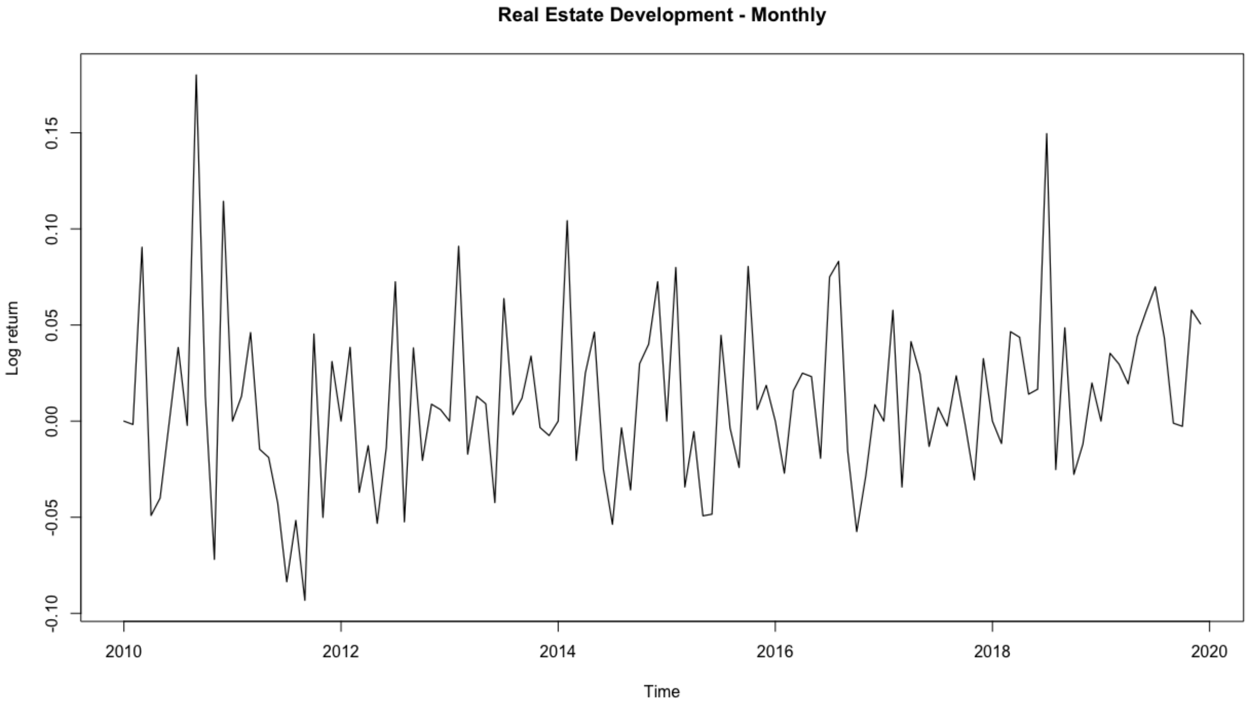


Figure 9: Plot of the log returns Real Estate Development from Jan. 2010 to Dec. 2019



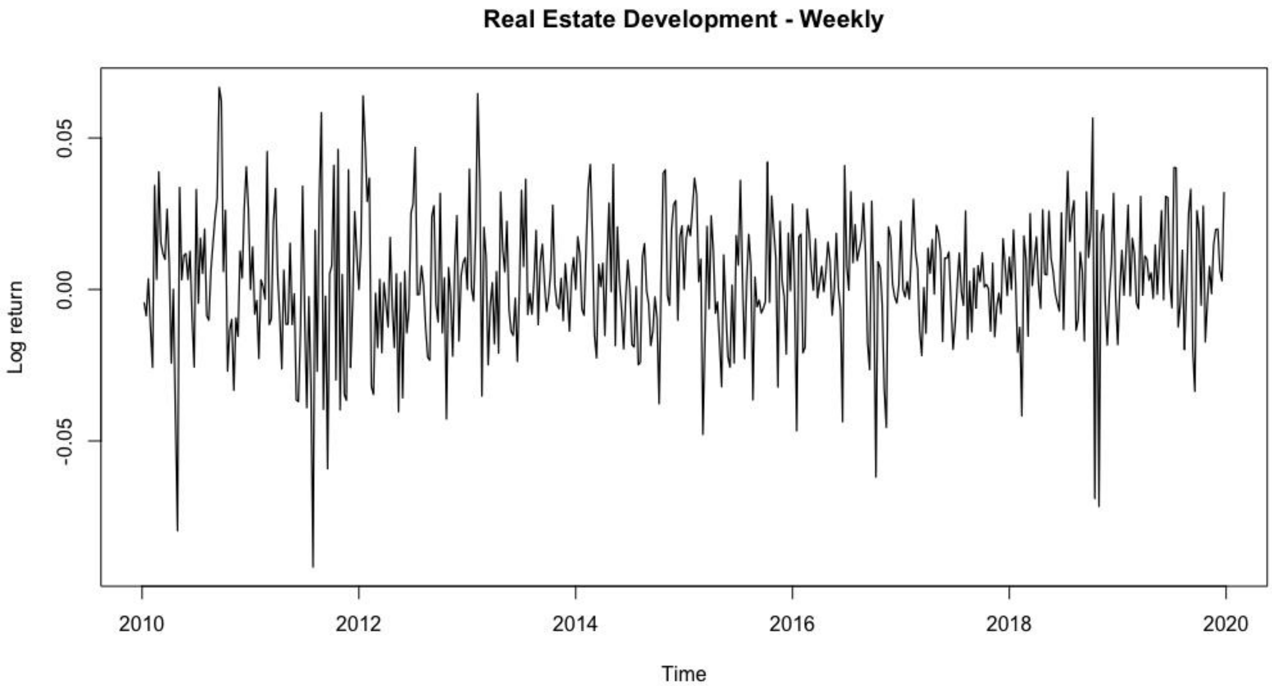
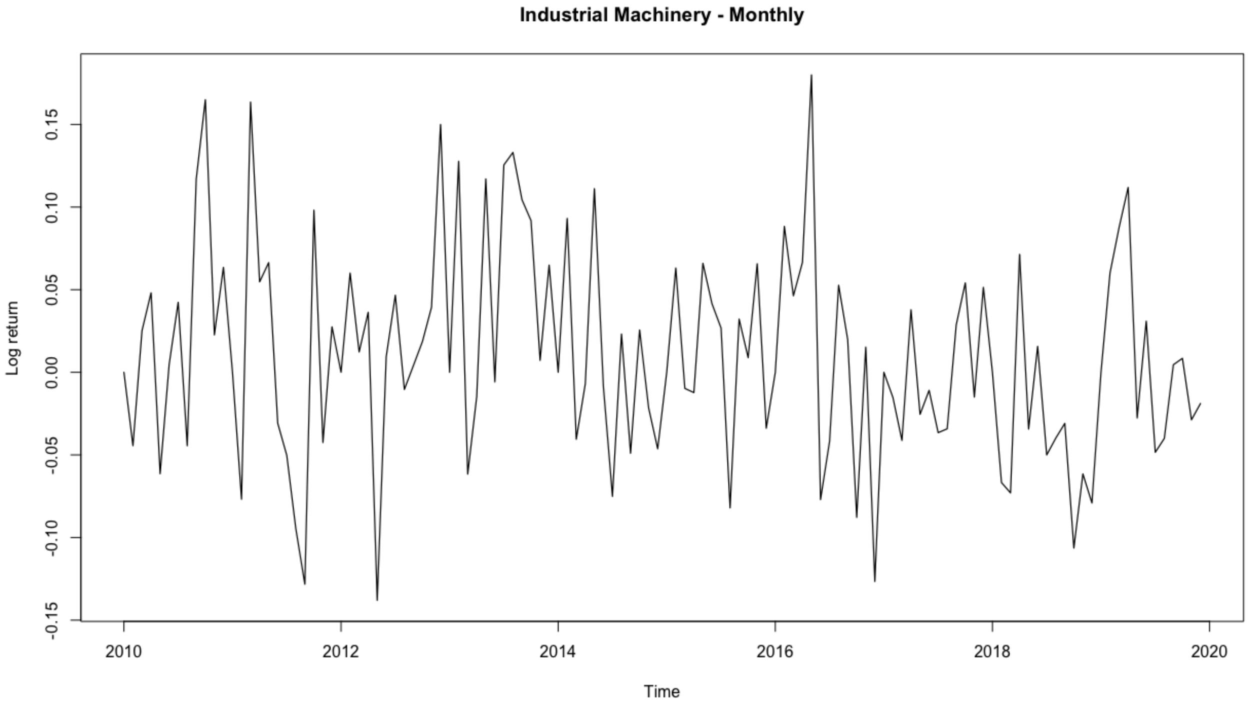


Figure 10: Plot of the log returns Industrial Machinery from Jan. 2010 to Dec. 2019



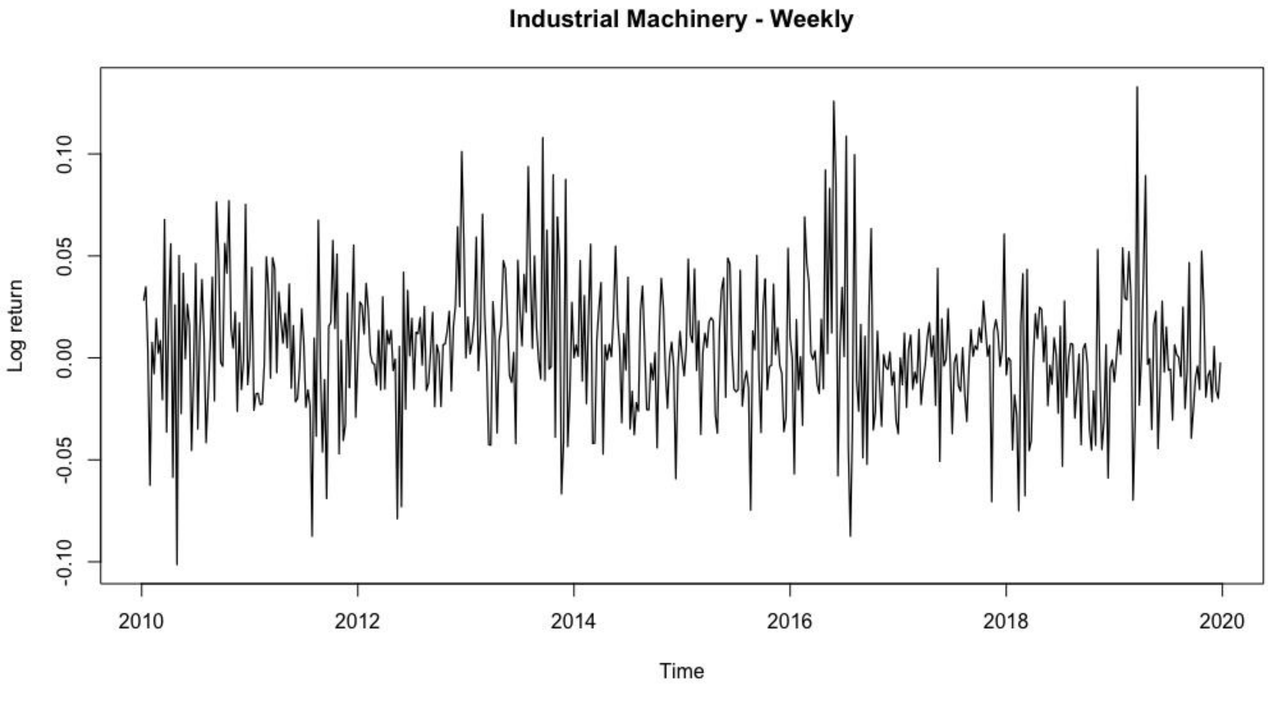


Table 1: Stocks in industry portfolios

In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

B	IT	PS	RED	IM
Spago Nanomedical AB	EXINI Diagnostics AB	FormPipe Software AB	Kungsleden AB	HEXPOL AB Class B
RLS Global AB	Servage AB	Aspiro AB	Logistea AB Class B	Lindab International AB
BioPhausia AB	Hoylu AB	Hoodin AB	FastPartner AB	Munters Group AB
BioInvent International AB	Transmode AB	Enea AB	Neobo Fastigheter AB	Josab Water Solutions AB
QuiaPEG Pharmaceuticals Holding AB	Terranet AB Class B	Plejd AB	Dagon AB	ITAB Shop Concept AB
Toleranzia AB	Acando AB	G5 Entertainment AB	Mofast AB	Rentunder Holding AB
Xintela AB	3L System AB	Ranplan Group AB	Samtrygg Group AB Class B	Finepart Sweden AB
Lipum AB	HMS Networks AB	LYYN AB	NP3 Fastigheter AB	EasyFill AB Class B
SenzaGen AB	Exalt AB	Edgeware AB	HEBA Fastighets AB Class B	Arcam AB
Saniona AB	Vitec Software Group AB Class B	Diadrom Holding AB	Wihlborgs Fastigheter AB	Alfa Laval AB
Active Biotech AB	Cybercom Group AB	Eurocon Consulting AB	Castellum AB	Hedson Technologies International AB
Fluicell AB	Empir Group AB Class B	Spiffbet AB	JM AB	
Mendus AB	Addnode Group AB Class B	Fortnox AB	AB Sagax	
Probi AB	Pricer AB Class B		Catena AB	

Table 2: Descriptive statistics for weekly data
In the table the letters are an abbreviation for the given industries: B (Biotechnology), RED (Real Estate Development), IT (Information Technology Services), IM (Industrial Machinery), PS (Packaged Software).

Weekly data	B	RED	IT	IM	PS
n	519	519	519	519	519
Avg. Holdings	8	9	9	8	8
Avg. Log return	0,000	0,003	0,002	0,004	0,005
Std.	0,043	0,021	0,025	0,033	0,031
Maximum	0,155	0,067	0,096	0,133	0,116
Minimum	-0,148	-0,092	-0,093	-0,101	-0,096
Kurtosis	1,303	1,439	1,415	1,179	1,266
Skewness	0,273	-0,374	-0,043	0,380	0,328
Jarque-Bera	8.071e-10 ***	2.402e-07 ***	7.761e-10 ***	1.322e-09 ***	3.947e-10 ***

Notes: The sample period is from Jan. 2010 to December 2019.
*** Corresponds to 0.000 signifiance level

Appendix B

ACF and PACF correlograms of monthly- and weekly log returns

Figure 11: ACF & PACF of monthly log returns for Biotechnology from Jan. 2010 to Dec. 2019

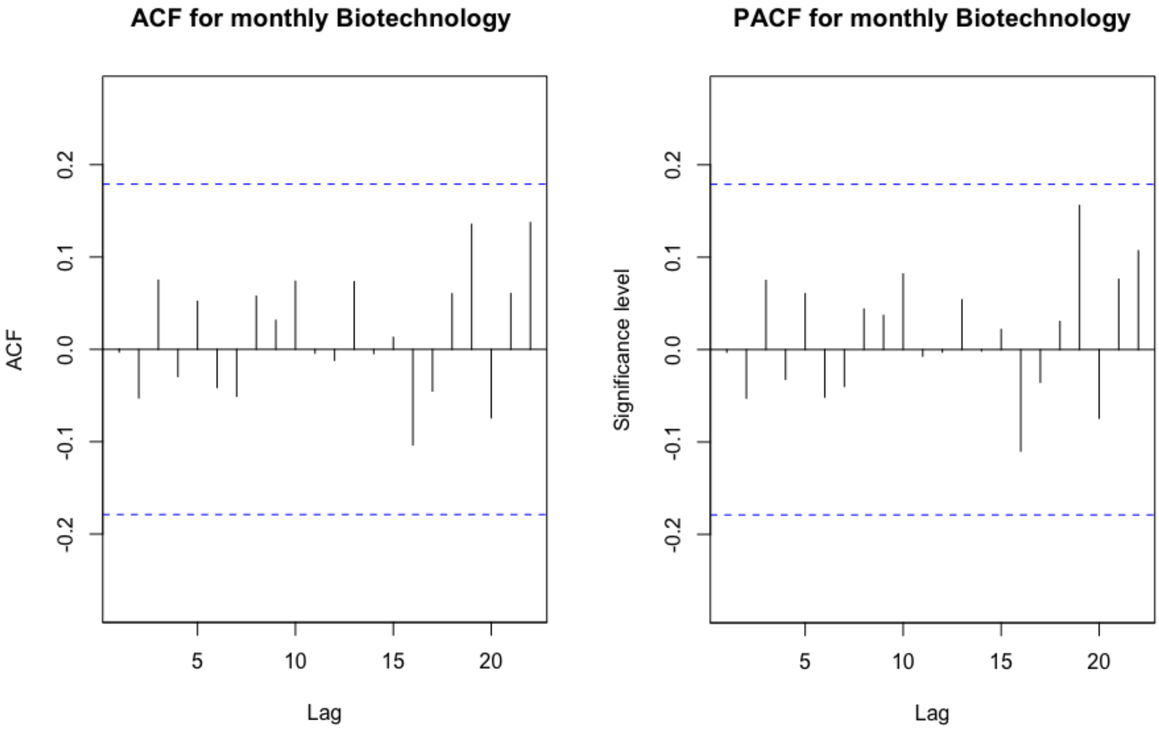


Figure 12: ACF & PACF of monthly log returns for Information Technology from Jan. 2010 to Dec. 2019

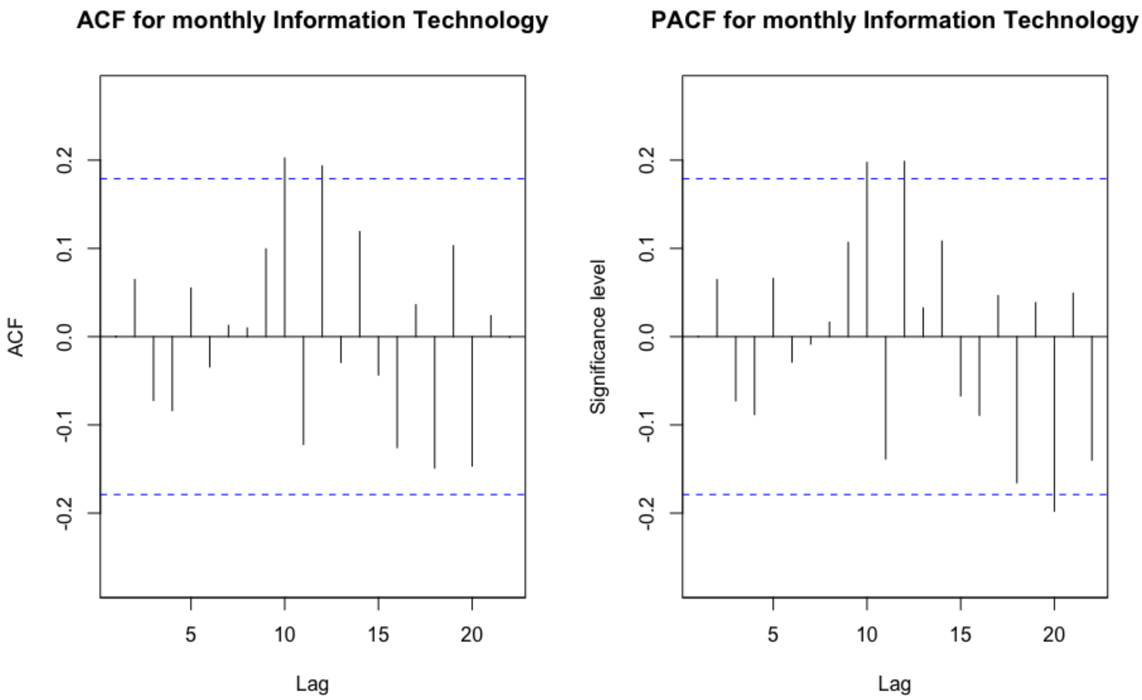


Figure 13: ACF & PACF of monthly log returns for Packaged Software from Jan. 2010 to Dec. 2019

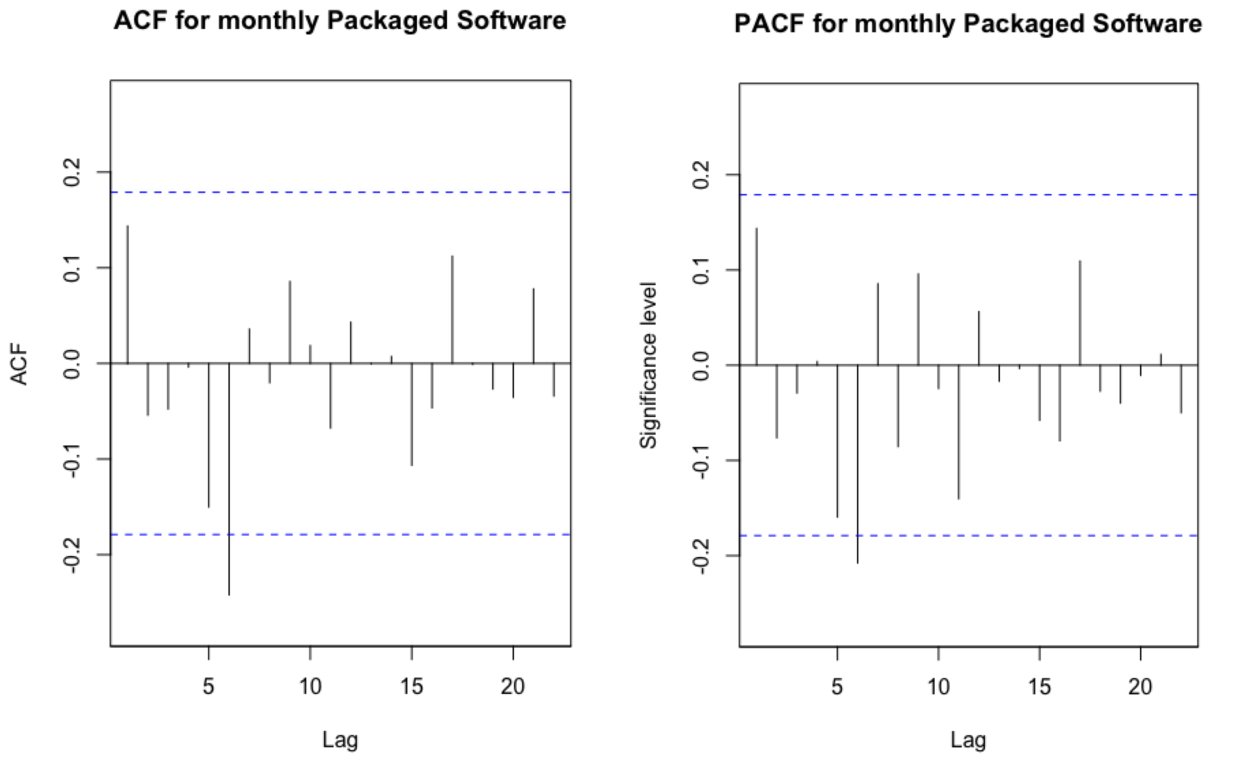


Figure 14: ACF & PACF of monthly log returns for returns Real Estate Development from Jan. 2010 to Dec. 2019

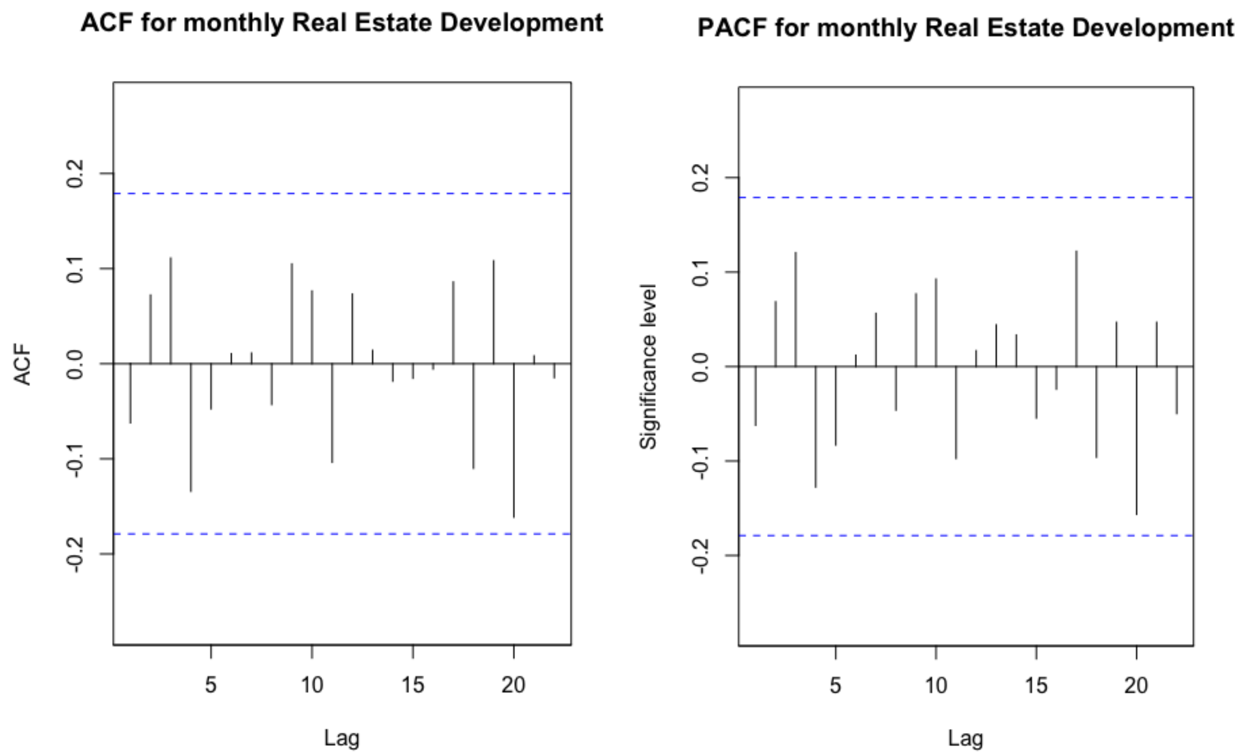


Figure 15: ACF & PACF of monthly log returns for Industrial Machinery from Jan. 2010 to Dec. 2019

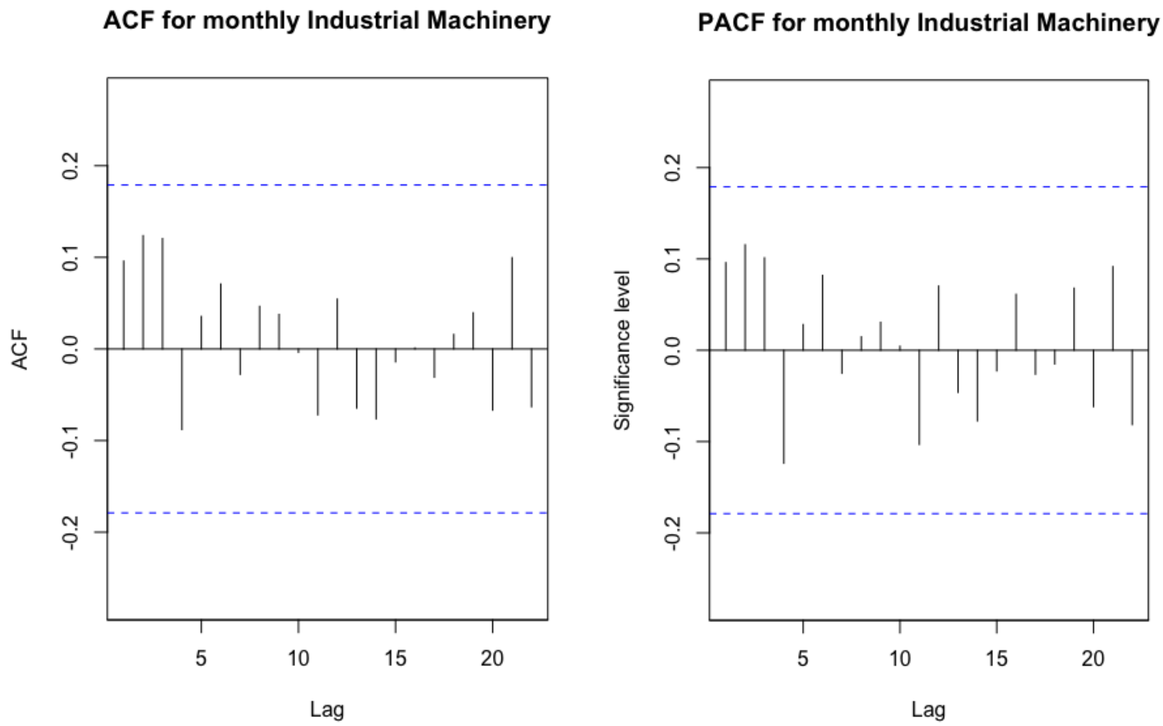


Figure 16: ACF & PACF of weekly log returns for Biotechnology from Jan. 2010 to Dec. 2019

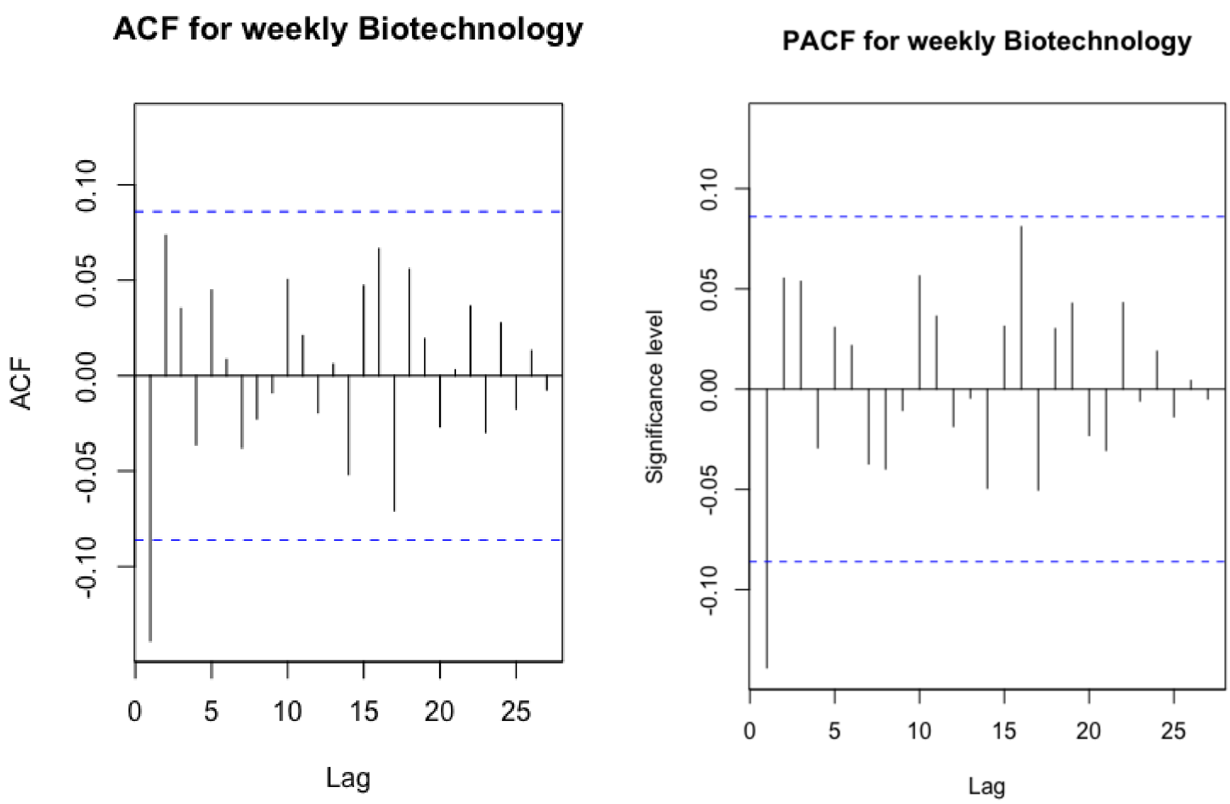


Figure 17: ACF & PACF of weekly log returns for Information Technology from Jan. 2010 to Dec. 2019

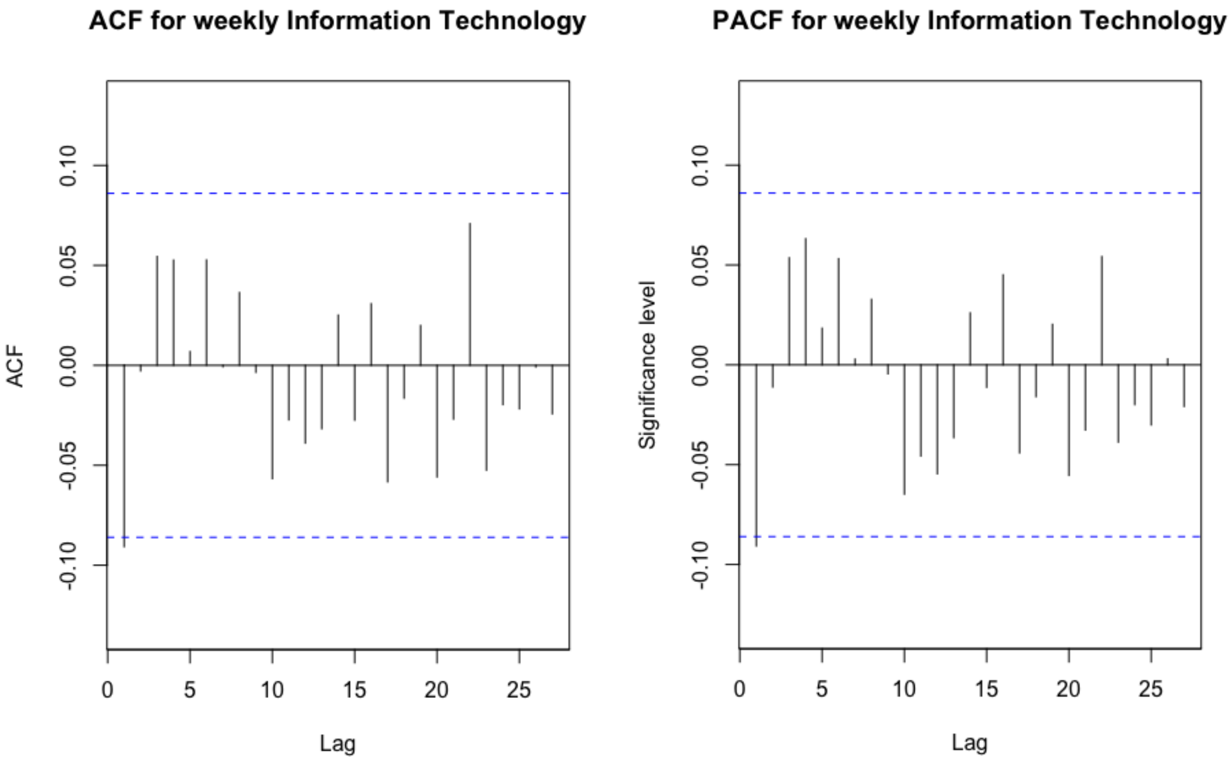


Figure 18: ACF & PACF of weekly log returns for Packaged Software from Jan. 2010 to Dec. 2019

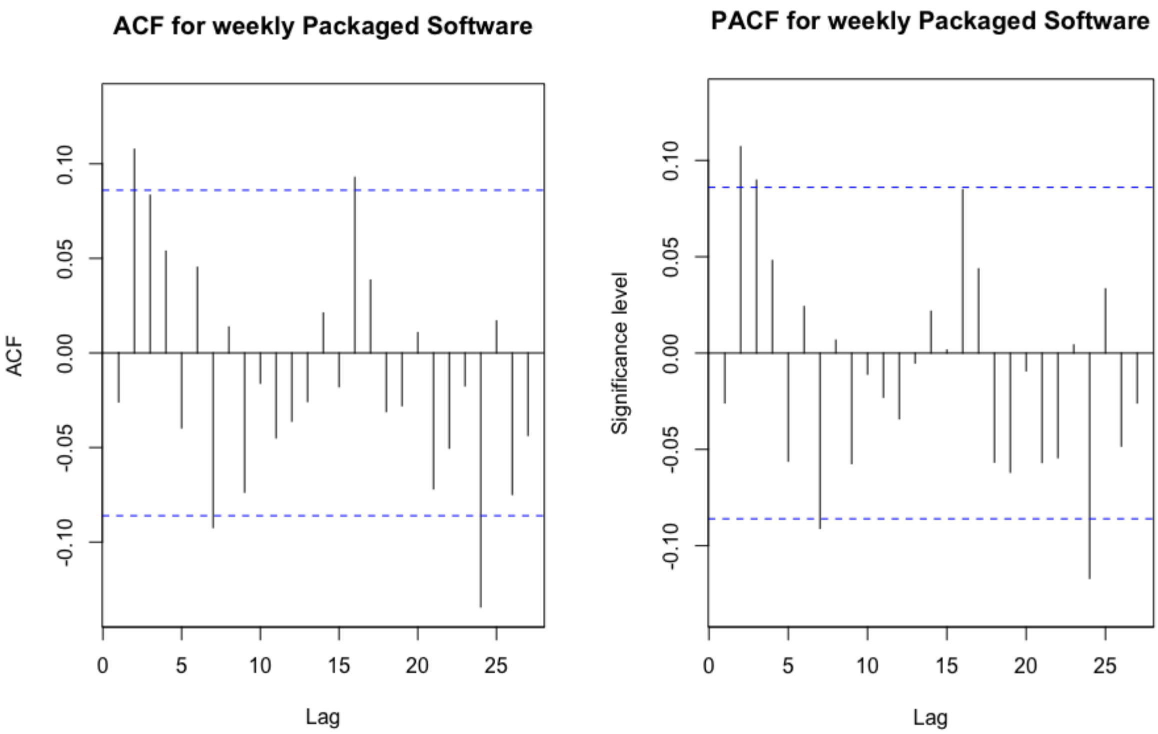


Figure 19: ACF & PACF of weekly log returns for Real Estate Development from Jan. 2010 to Dec. 2019

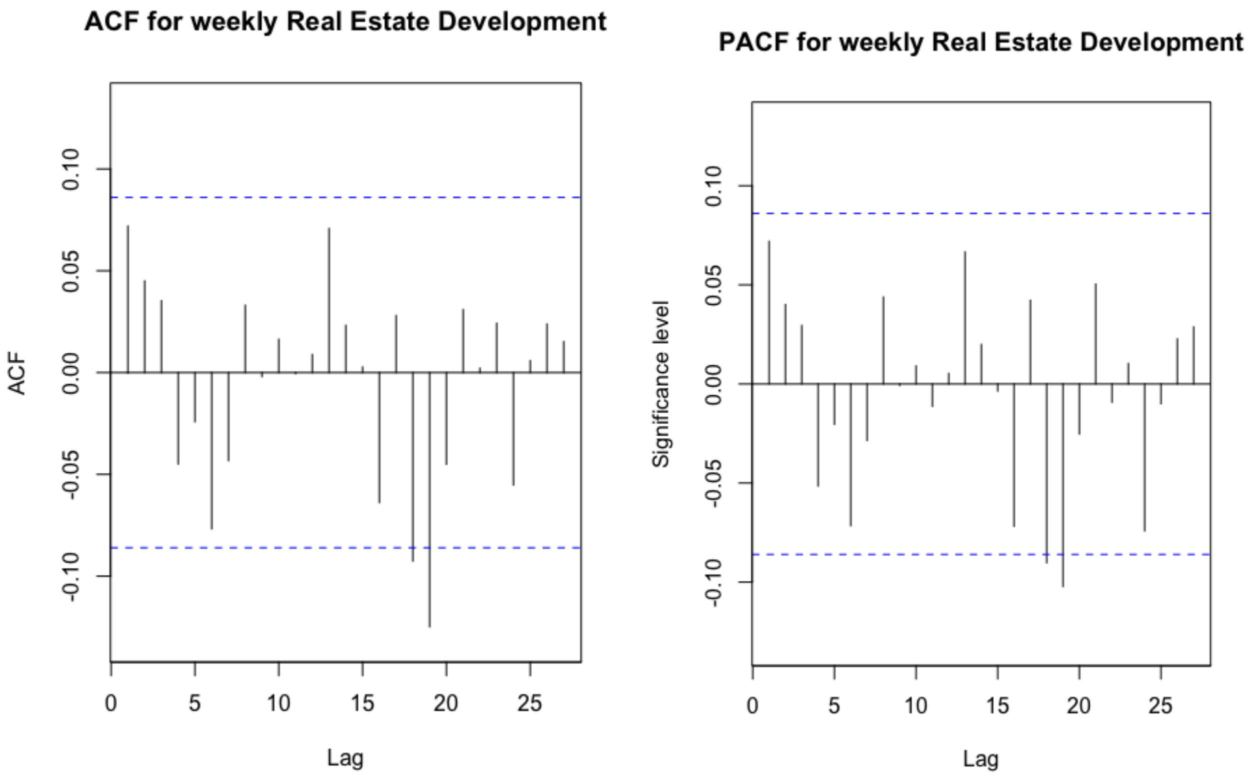
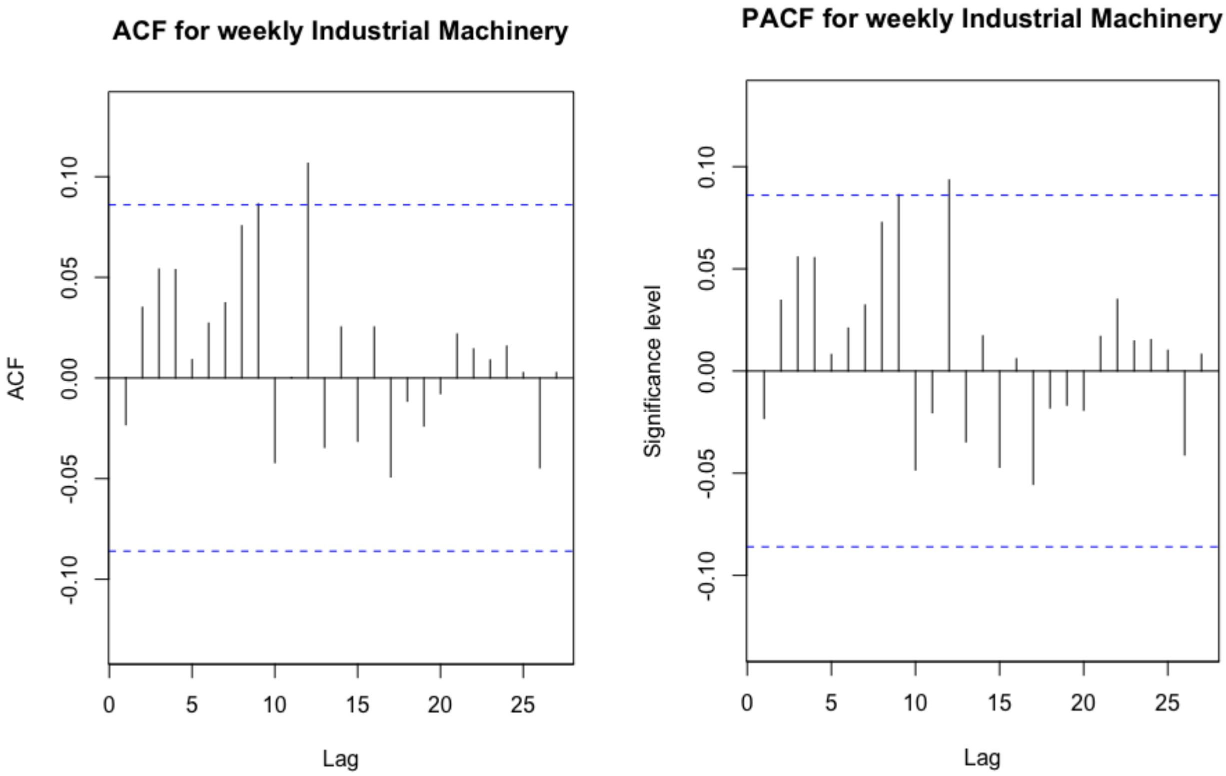


Figure 20: ACF & PACF of weekly log returns for Industrial Machinery from Jan. 2010 to Dec. 2019



ACF and PACF correlogram of monthly- and weekly residuals

Figure 21: ACF and PACF for monthly residuals for MA(1) Packaged software

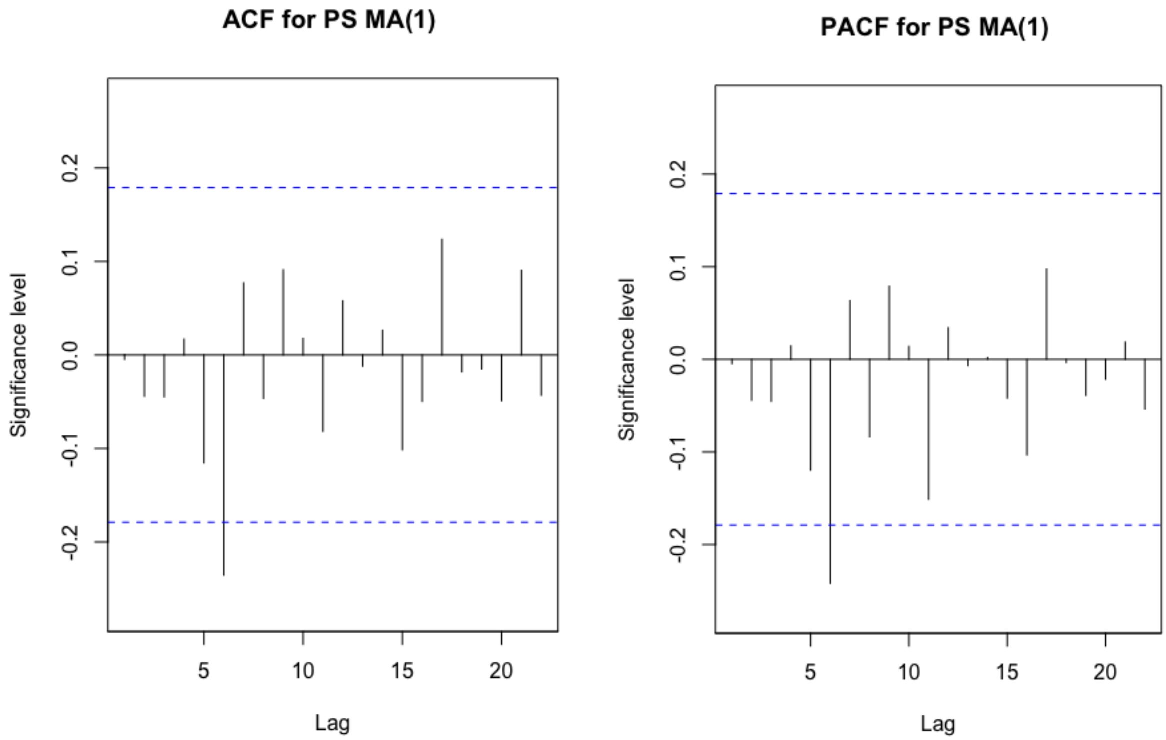


Figure 22: ACF and PACF for monthly residuals for AR(1) Packaged software

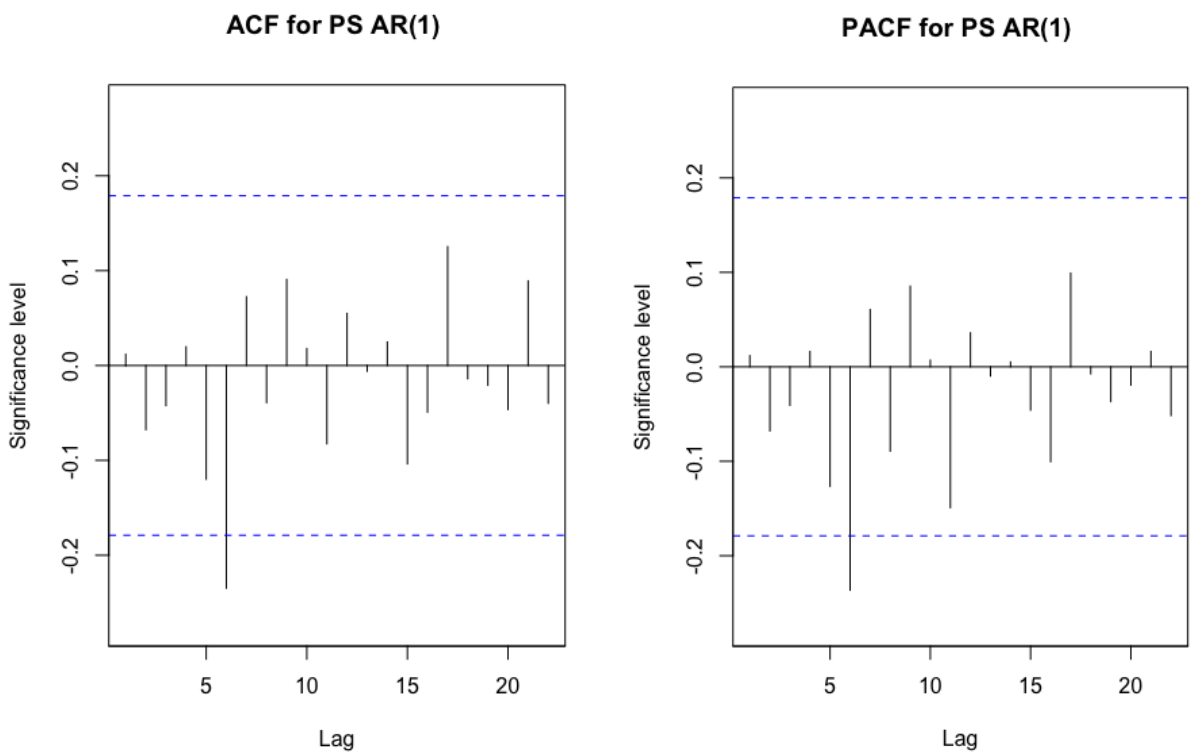


Figure 23 ACF and PACF for weekly residuals for MA(1) Biotechnology

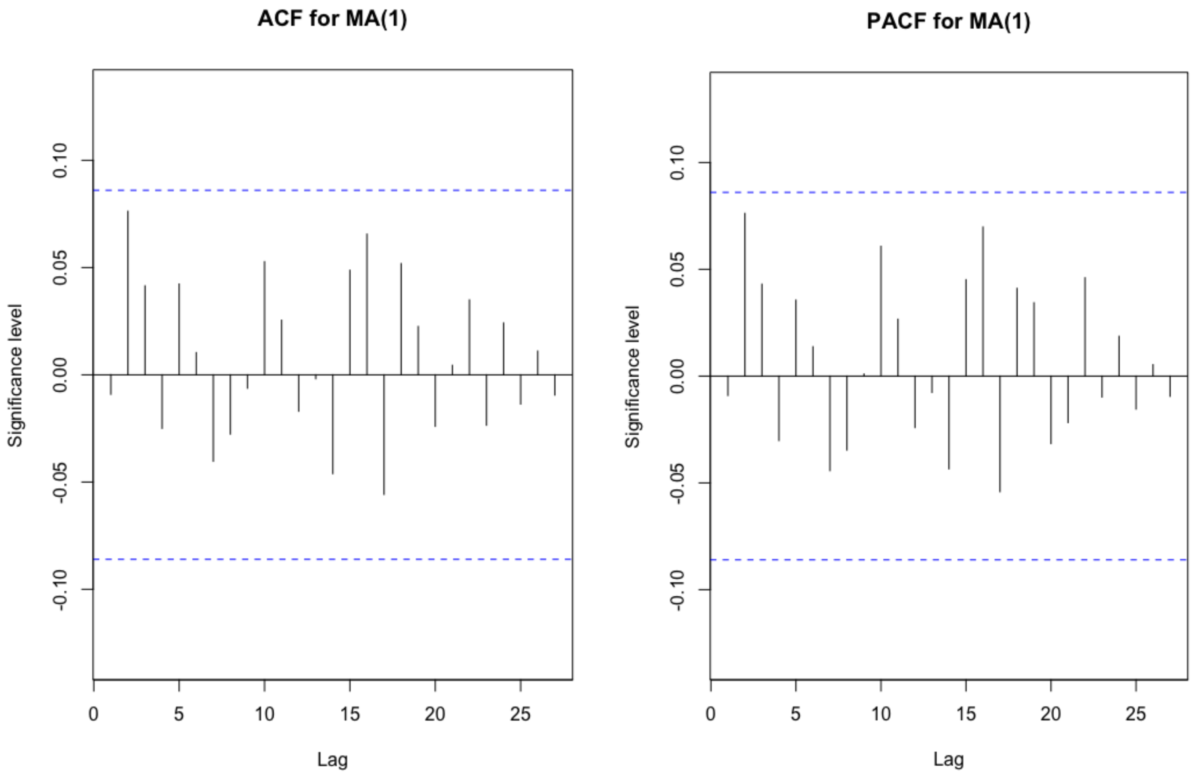


Figure 24: ACF and PACF for weekly residuals for AR(1) Biotechnology

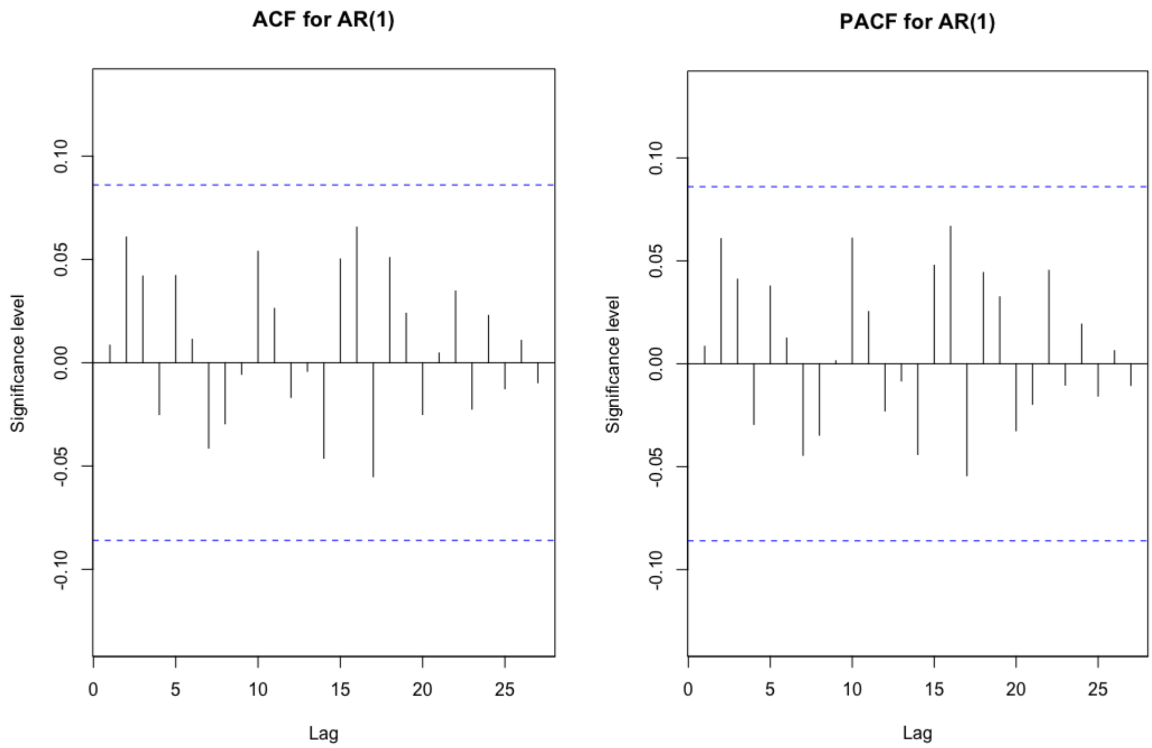


Figure 25: ACF and PACF for weekly residuals for MA(2) Biotechnology

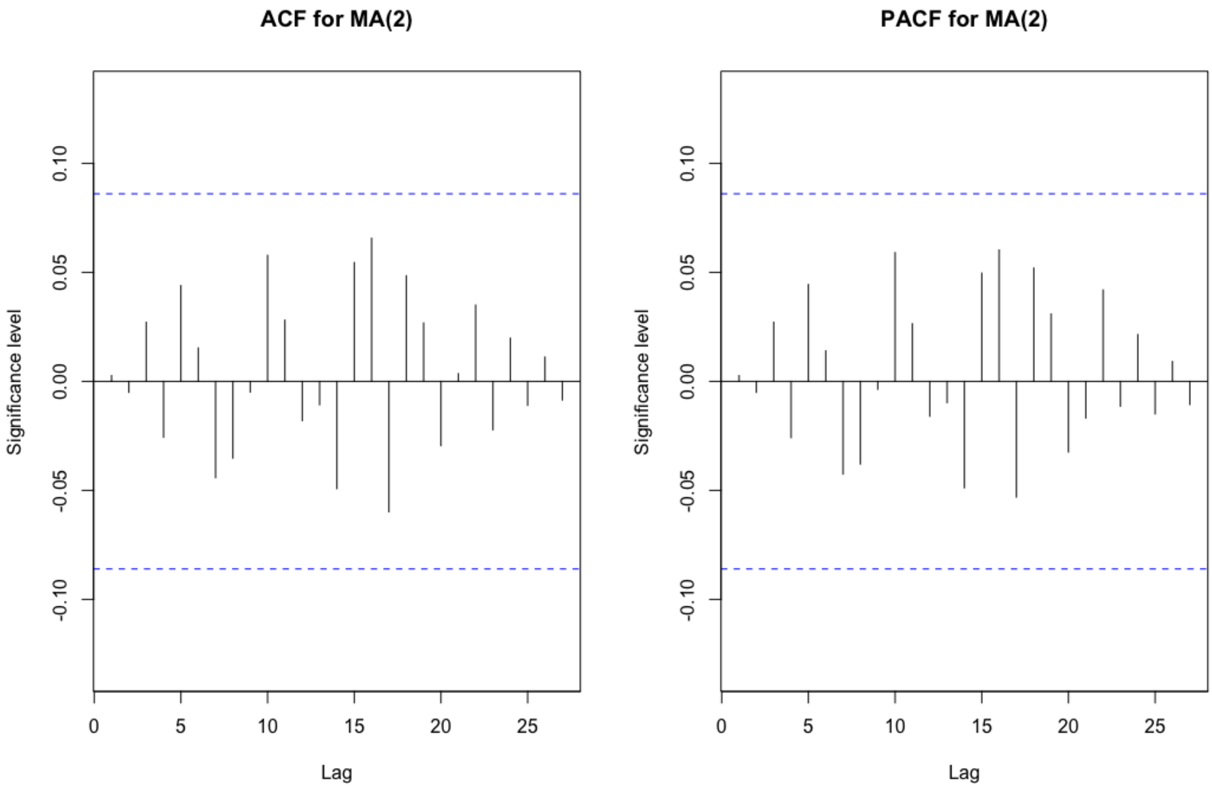


Figure 26: ACF and PACF for weekly residuals for AR(1) Real Estate Development

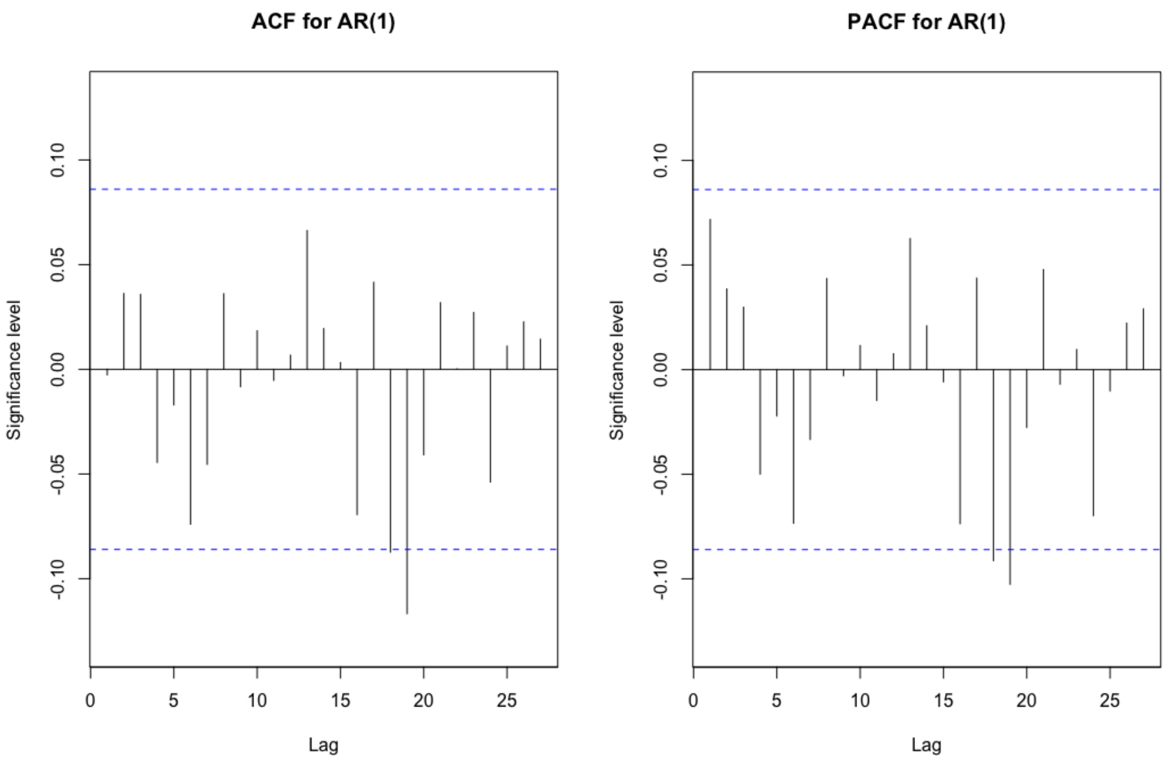
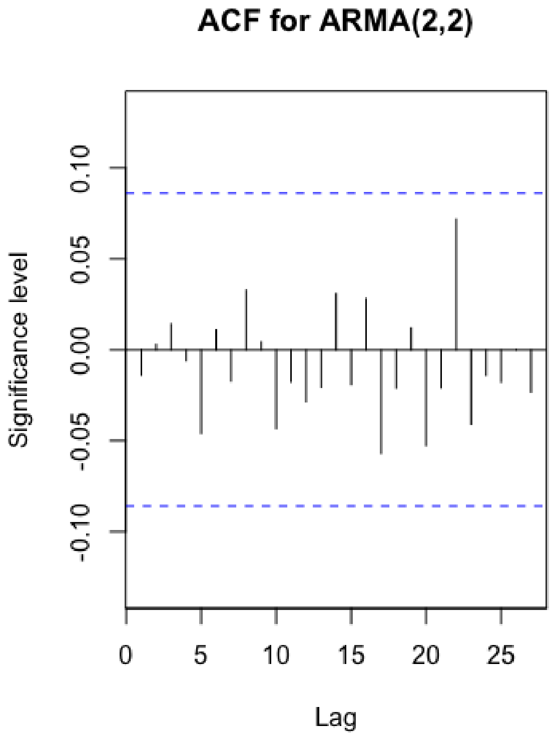


Figure 27: ACF and PACF for weekly residuals for Information Technology ARMA(2,2)



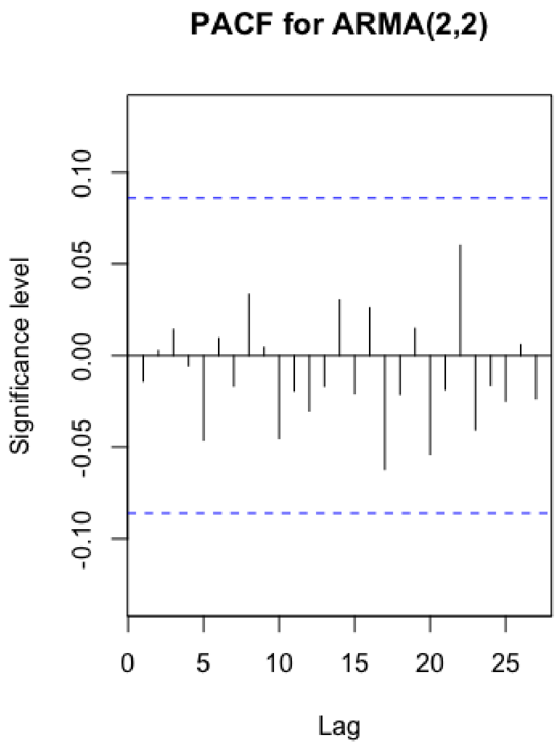
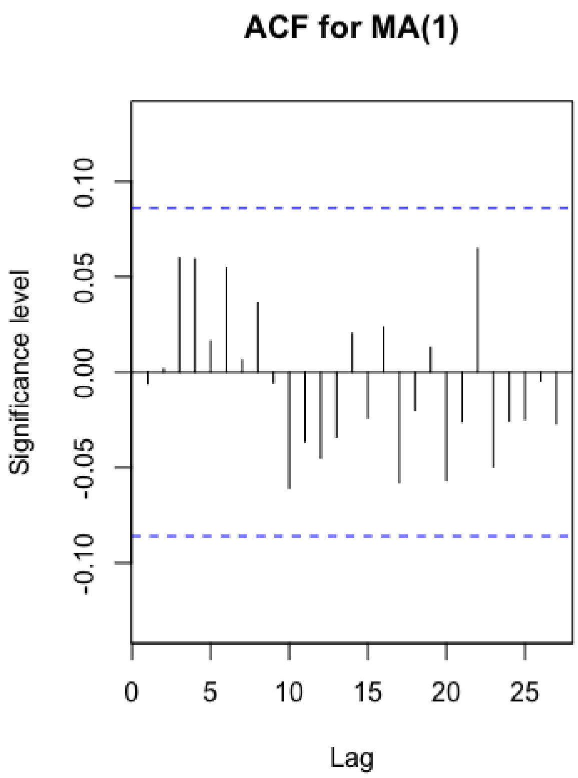


Figure 28: ACF and PACF for weekly residuals for Information Technology MA(1)



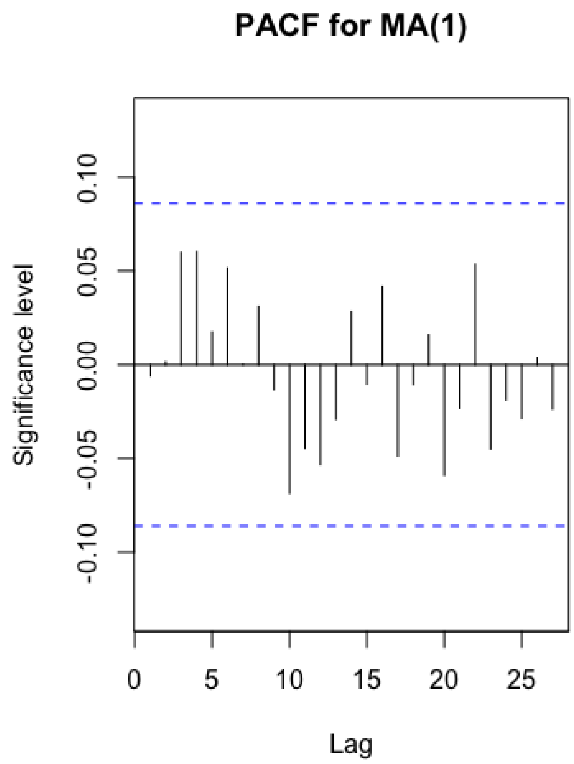


Figure 29: ACF and PACF for weekly residuals for Information Technology AR(1)

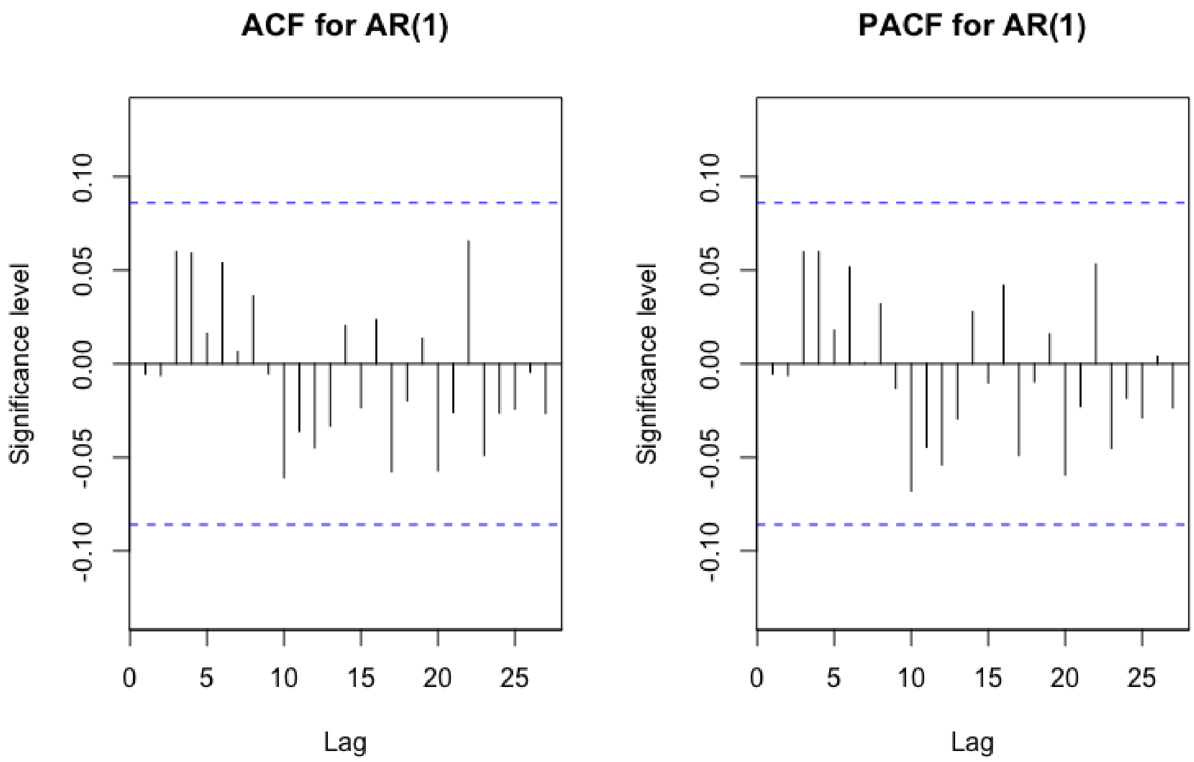


Figure 30: ACF and PACF for weekly residuals for Packaged Software AR(3)

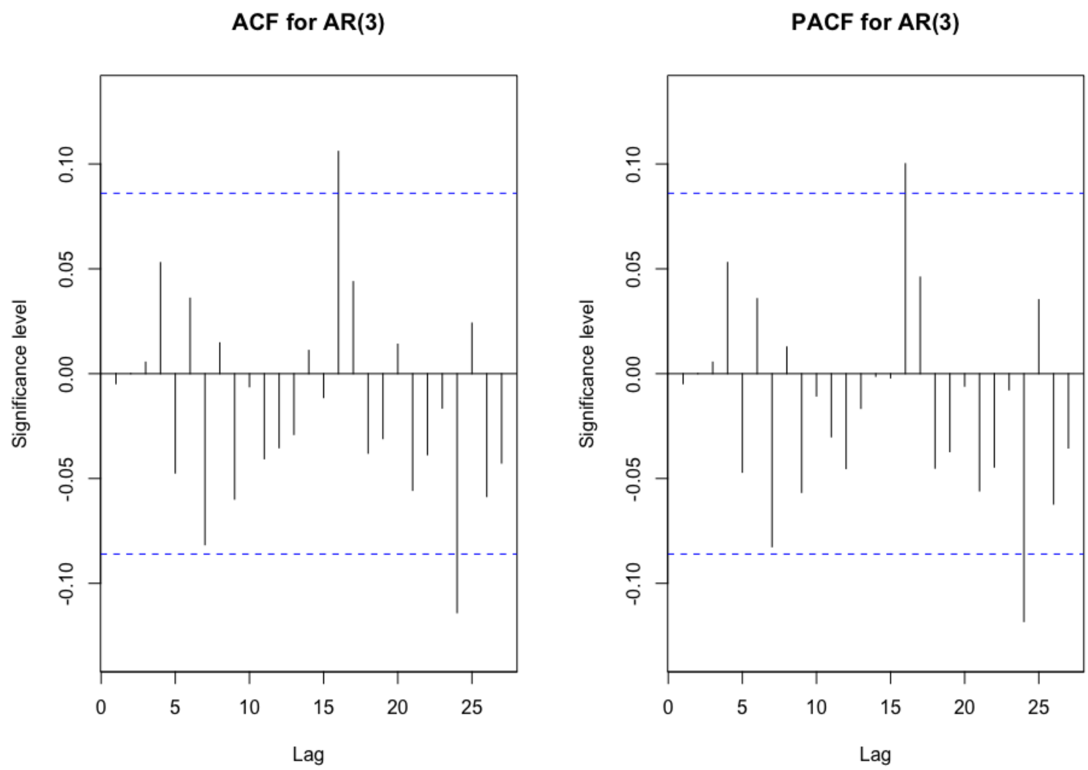


Figure 31: ACF and PACF for weekly residuals for Packaged Software AR(2)

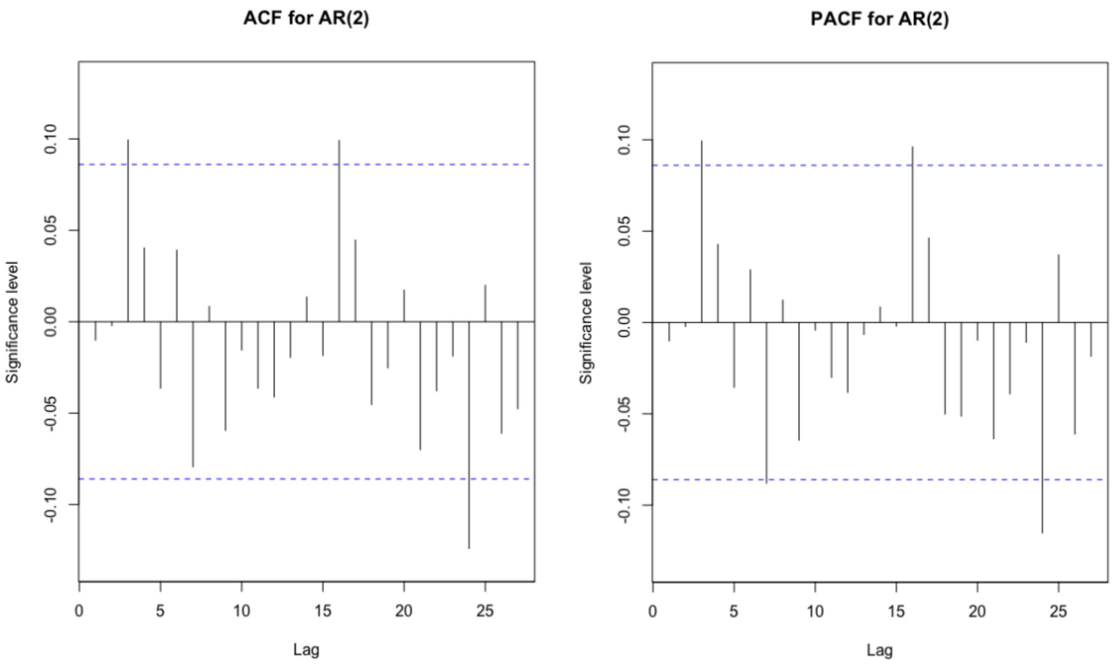


Figure 32: ACF and PACF for weekly residuals for Packaged Software MA(3)

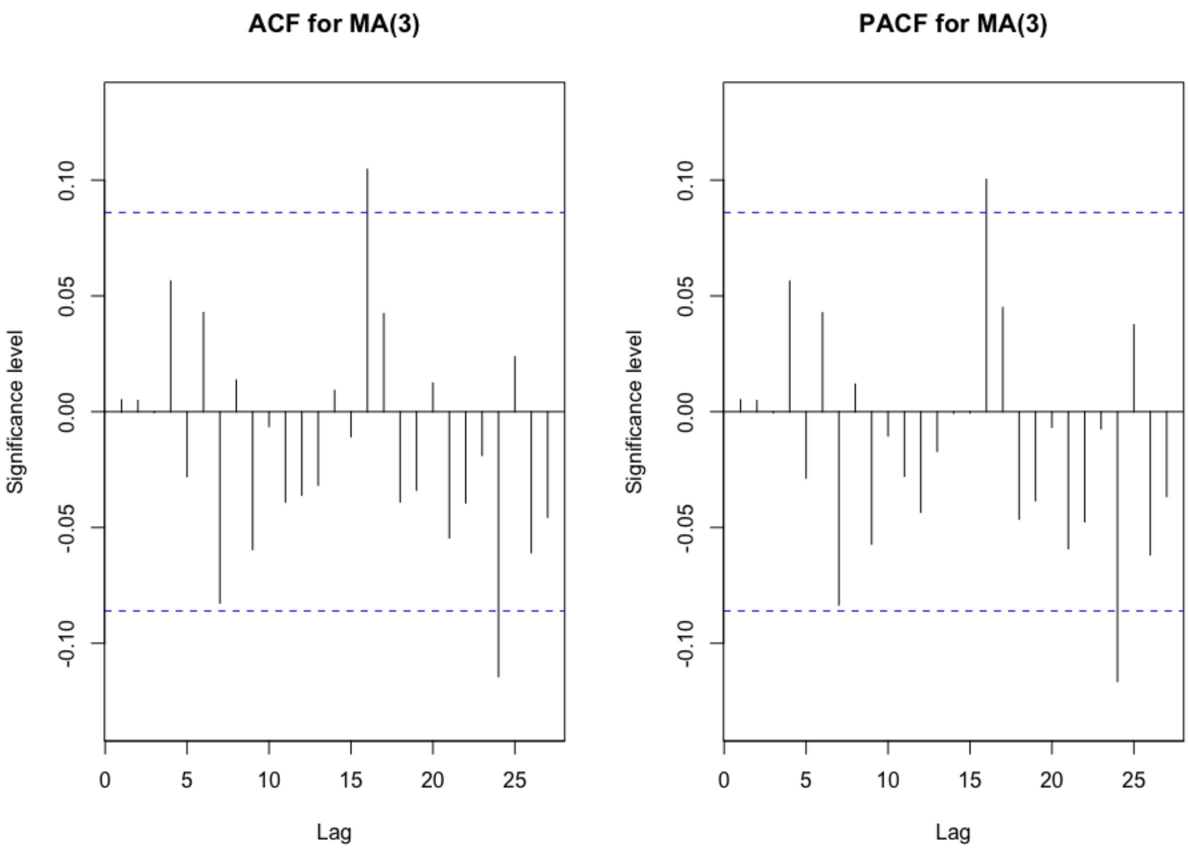


Figure 33: ACF and PACF for weekly residuals for Packaged Software MA(2)

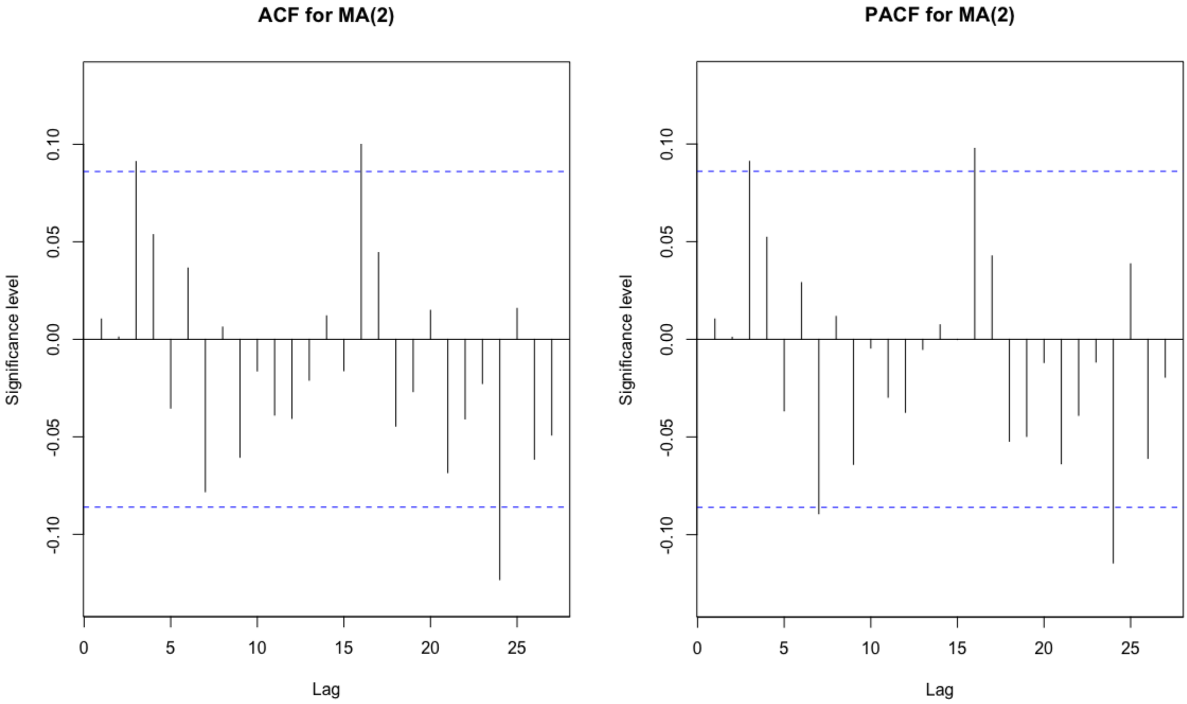


Figure 34: ACF and PACF for weekly residuals for Packaged Software ARMA(1,2)

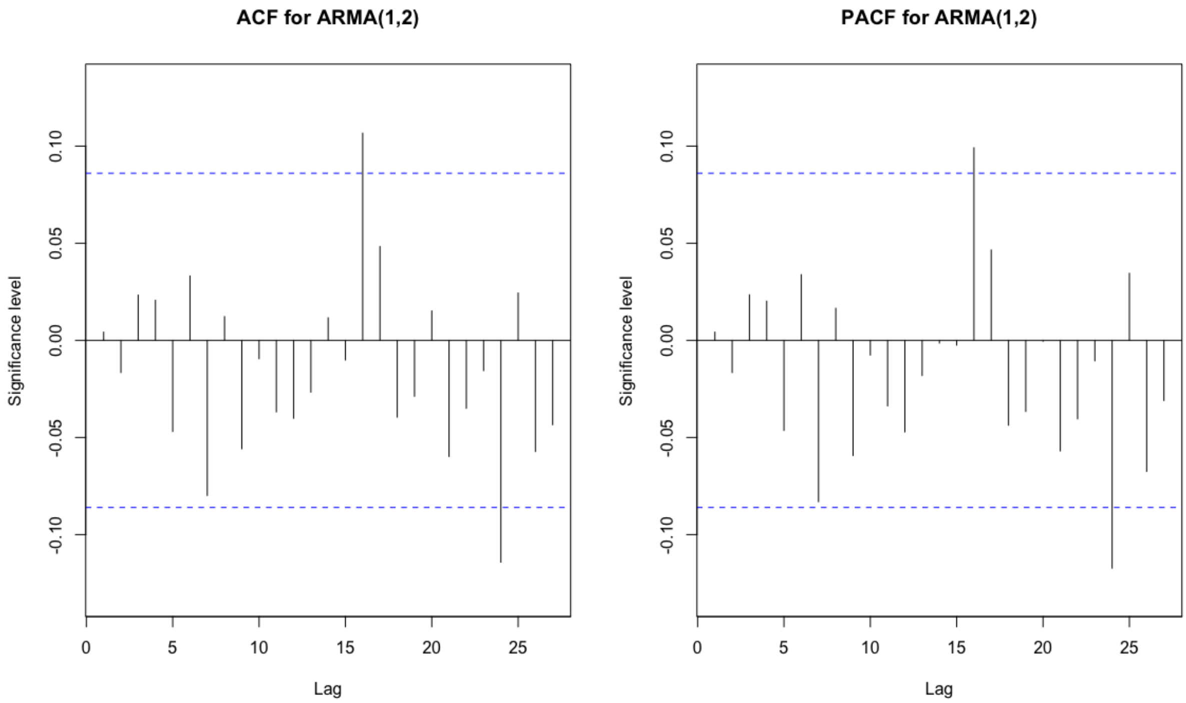
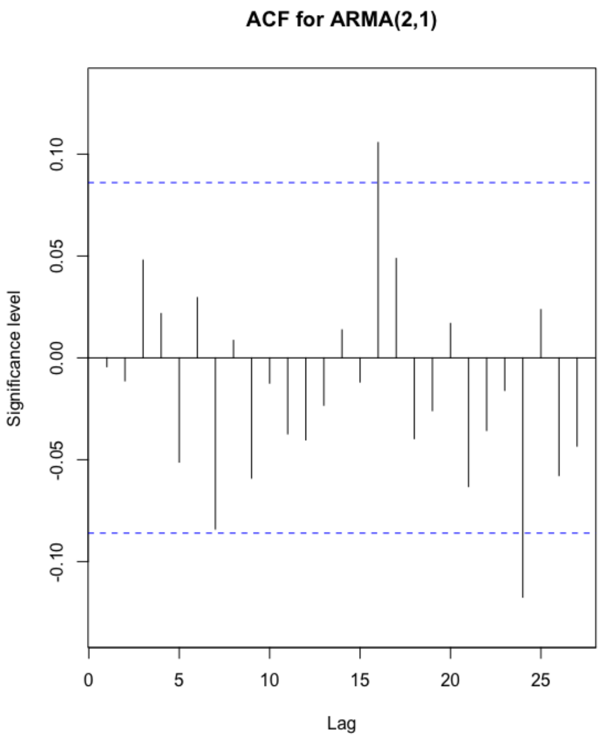


Figure 35: ACF and PACF for weekly residuals for Packaged Software ARMA(2,1)



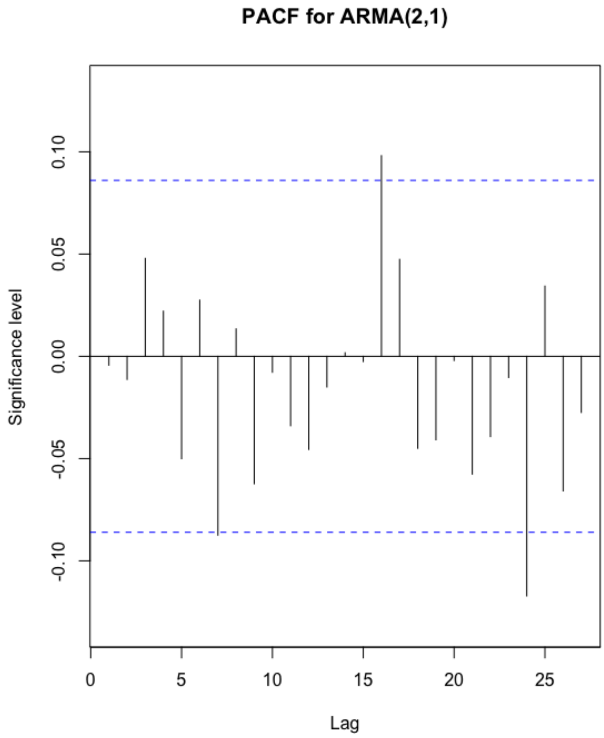
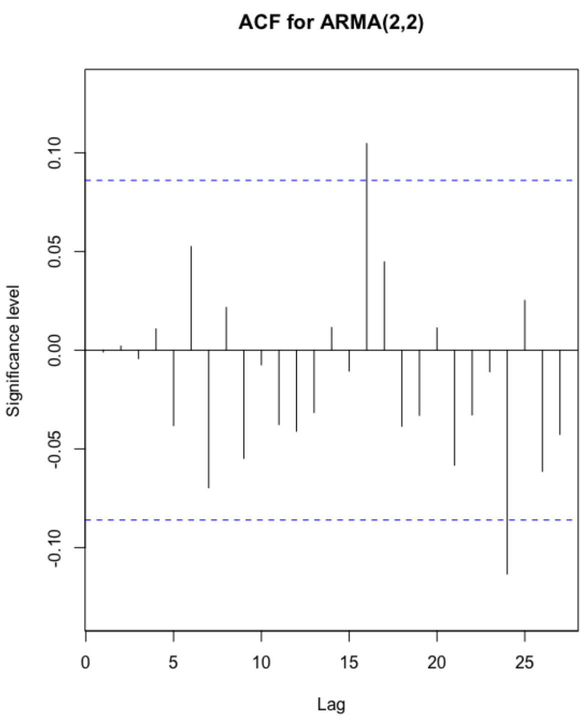


Figure 36: ACF and PACF for weekly residuals for Packaged Software ARMA(2,2)



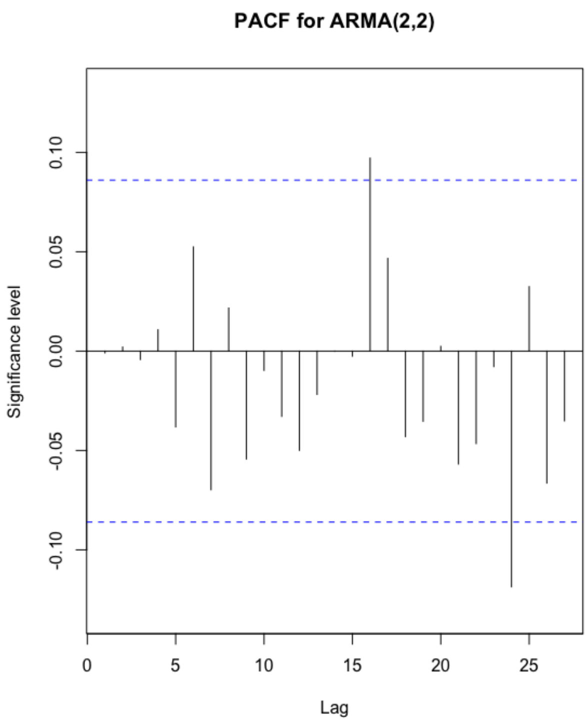
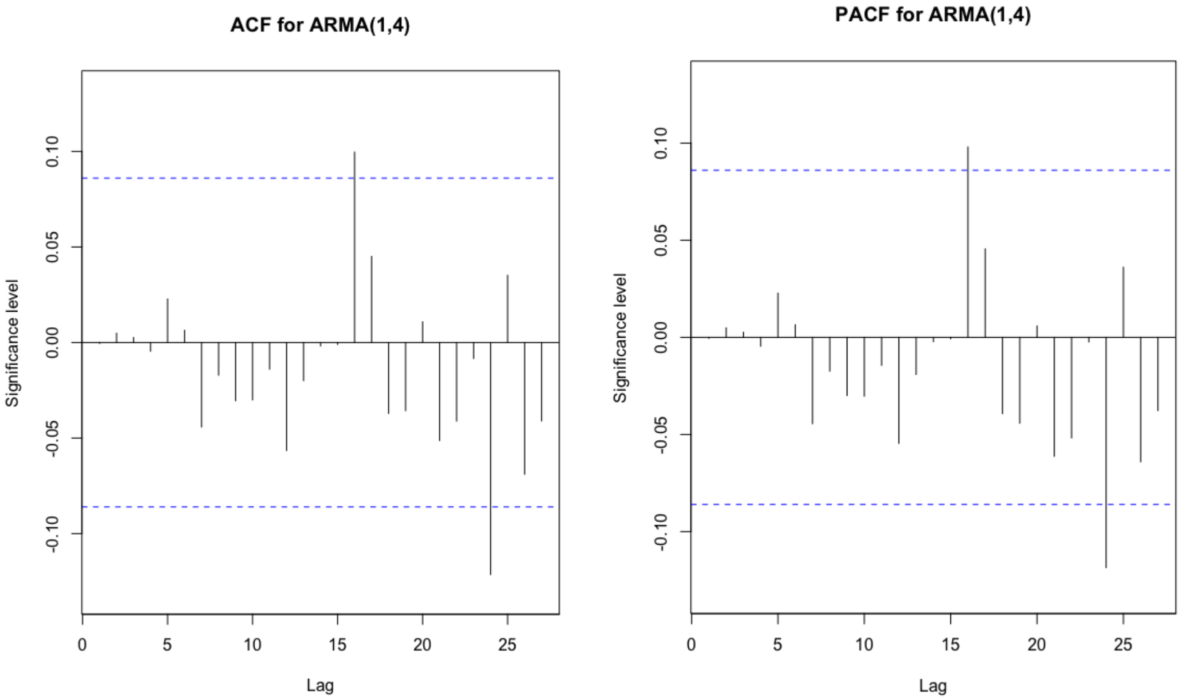


Figure 37: ACF and PACF for weekly residuals for Packaged Software ARMA(1,4)



AIC selection tables for monthly- and weekly models

Table 1: Biotechnology results from auto.arima function in R

Monthly				Weekly			
Model	**Mean**		**AIC**	**Model**	**Mean**	**0**	**AIC**
ARIMA(0,0,0)	**with zero mean**	**:**	**-252,911**	ARIMA(0,0,0)	with zero mean	:	-1.797,434
ARIMA(0,0,0)	with non-zero mean	:	-250,973	ARIMA(0,0,0)	with non-zero mean	:	-1.795,439
ARIMA(0,0,1)	with zero mean	:	-250,912	ARIMA(0,0,1)	with zero mean	:	-1.804,383
ARIMA(0,0,1)	with non-zero mean	:	-248,974	ARIMA(0,0,1)	with non-zero mean	:	-1.802,388
ARIMA(0,0,2)	with zero mean	:	-249,257	**ARIMA(0,0,2)**	**with zero mean**	**:**	**-1.806,140**
ARIMA(0,0,2)	with non-zero mean	:	-247,330	ARIMA(0,0,2)	with non-zero mean	:	-1.804,144
ARIMA(0,0,3)	with zero mean	:	-248,120	ARIMA(0,0,3)	with zero mean	:	-1.804,333
ARIMA(0,0,3)	with non-zero mean	:	-246,182	ARIMA(0,0,3)	with non-zero mean	:	-1.802,338
ARIMA(0,0,4)	with zero mean	:	-246,189	ARIMA(0,0,4)	with zero mean	:	-1.802,564
ARIMA(0,0,4)	with non-zero mean	:	-244,255	ARIMA(0,0,4)	with non-zero mean	:	-1.800,568
ARIMA(0,0,5)	with zero mean	:	-244,357	ARIMA(0,0,5)	with zero mean	:	-1.802,113
ARIMA(0,0,5)	with non-zero mean	:	-242,413	ARIMA(0,0,5)	with non-zero mean	:	-1.800,117
ARIMA(1,0,0)	with zero mean	:	-250,912	ARIMA(1,0,0)	with zero mean	:	-1.805,643
ARIMA(1,0,0)	with non-zero mean	:	-248,974	ARIMA(1,0,0)	with non-zero mean	:	-1.803,649
ARIMA(1,0,1)	with zero mean	:	Inf	ARIMA(1,0,1)	with zero mean	:	-1.804,497
ARIMA(1,0,1)	with non-zero mean	:	-246,990	ARIMA(1,0,1)	with non-zero mean	:	-1.802,502
ARIMA(1,0,2)	with zero mean	:	-248,114	ARIMA(1,0,2)	with zero mean	:	-1.804,290
ARIMA(1,0,2)	with non-zero mean	:	-246,186	ARIMA(1,0,2)	with non-zero mean	:	-1.802,291
ARIMA(1,0,3)	with zero mean	:	-246,286	ARIMA(1,0,3)	with zero mean	:	-1.802,867
ARIMA(1,0,3)	with non-zero mean	:	-244,352	ARIMA(1,0,3)	with non-zero mean	:	-1.800,871
ARIMA(1,0,4)	with zero mean	:	-244,287	ARIMA(1,0,4)	with zero mean	:	-1.801,087
ARIMA(1,0,4)	with non-zero mean	:	-242,353	ARIMA(1,0,4)	with non-zero mean	:	-1.799,096
ARIMA(1,0,5)	with zero mean	:	-242,403	ARIMA(1,0,5)	with zero mean	:	-1.800,114
ARIMA(1,0,5)	with non-zero mean	:	-240,461	ARIMA(1,0,5)	with non-zero mean	:	-1.798,117
ARIMA(2,0,0)	with zero mean	:	-249,231	ARIMA(2,0,0)	with zero mean	:	-1.805,263
ARIMA(2,0,0)	with non-zero mean	:	-247,303	ARIMA(2,0,0)	with non-zero mean	:	-1.803,267
ARIMA(2,0,1)	with zero mean	:	-248,154	ARIMA(2,0,1)	with zero mean	:	-1.804,089
ARIMA(2,0,1)	with non-zero mean	:	-246,226	ARIMA(2,0,1)	with non-zero mean	:	-1.802,092
ARIMA(2,0,2)	with zero mean	:	Inf	ARIMA(2,0,2)	with zero mean	:	-1.802,283
ARIMA(2,0,2)	with non-zero mean	:	Inf	ARIMA(2,0,2)	with non-zero mean	:	-1.800,287
ARIMA(2,0,3)	with zero mean	:	Inf	ARIMA(2,0,3)	with zero mean	:	Inf
ARIMA(2,0,3)	with non-zero mean	:	Inf	ARIMA(2,0,3)	with non-zero mean	:	-1.799,557
ARIMA(2,0,4)	with zero mean	:	Inf	ARIMA(2,0,4)	with zero mean	:	Inf
ARIMA(2,0,4)	with non-zero mean	:	Inf	ARIMA(2,0,4)	with non-zero mean	:	Inf
ARIMA(2,0,5)	with zero mean	:	-240,324	ARIMA(2,0,5)	with zero mean	:	Inf
ARIMA(2,0,5)	with non-zero mean	:	Inf	ARIMA(2,0,5)	with non-zero mean	:	Inf
ARIMA(3,0,0)	with zero mean	:	-247,912	ARIMA(3,0,0)	with zero mean	:	-1.804,731
ARIMA(3,0,0)	with non-zero mean	:	-245,974	ARIMA(3,0,0)	with non-zero mean	:	-1.802,735
ARIMA(3,0,1)	with zero mean	:	-246,318	ARIMA(3,0,1)	with zero mean	:	-1.803,258
ARIMA(3,0,1)	with non-zero mean	:	-244,384	ARIMA(3,0,1)	with non-zero mean	:	-1.801,262
ARIMA(3,0,2)	with zero mean	:	-244,418	ARIMA(3,0,2)	with zero mean	:	Inf
ARIMA(3,0,2)	with non-zero mean	:	Inf	ARIMA(3,0,2)	with non-zero mean	:	-1.799,552
ARIMA(3,0,3)	with zero mean	:	Inf	ARIMA(3,0,3)	with zero mean	:	Inf
ARIMA(3,0,3)	with non-zero mean	:	Inf	ARIMA(3,0,3)	with non-zero mean	:	Inf
ARIMA(3,0,4)	with zero mean	:	Inf	ARIMA(3,0,4)	with zero mean	:	Inf
ARIMA(3,0,4)	with non-zero mean	:	Inf	ARIMA(3,0,4)	with non-zero mean	:	Inf
ARIMA(3,0,5)	with zero mean	:	Inf	ARIMA(3,0,5)	with zero mean	:	Inf
ARIMA(3,0,5)	with non-zero mean	:	Inf	ARIMA(3,0,5)	with non-zero mean	:	Inf
ARIMA(4,0,0)	with zero mean	:	-246,033	ARIMA(4,0,0)	with zero mean	:	-1.803,152
ARIMA(4,0,0)	with non-zero mean	:	-244,101	ARIMA(4,0,0)	with non-zero mean	:	-1.801,156
ARIMA(4,0,1)	with zero mean	:	-244,244	ARIMA(4,0,1)	with zero mean	:	Inf
ARIMA(4,0,1)	with non-zero mean	:	-242,309	ARIMA(4,0,1)	with non-zero mean	:	Inf
ARIMA(4,0,2)	with zero mean	:	-242,450	ARIMA(4,0,2)	with zero mean	:	Inf
ARIMA(4,0,2)	with non-zero mean	:	-240,483	ARIMA(4,0,2)	with non-zero mean	:	Inf
ARIMA(4,0,3)	with zero mean	:	Inf	ARIMA(4,0,3)	with zero mean	:	Inf
ARIMA(4,0,3)	with non-zero mean	:	Inf	ARIMA(4,0,3)	with non-zero mean	:	Inf
ARIMA(4,0,4)	with zero mean	:	Inf	ARIMA(4,0,4)	with zero mean	:	Inf
ARIMA(4,0,4)	with non-zero mean	:	Inf	ARIMA(4,0,4)	with non-zero mean	:	Inf
ARIMA(4,0,5)	with zero mean	:	Inf	ARIMA(4,0,5)	with zero mean	:	-1.798,582
ARIMA(4,0,5)	with non-zero mean	:	Inf	ARIMA(4,0,5)	with non-zero mean	:	-1.796,585
ARIMA(5,0,0)	with zero mean	:	-244,474	ARIMA(5,0,0)	with zero mean	:	-1.801,679
ARIMA(5,0,0)	with non-zero mean	:	-242,525	ARIMA(5,0,0)	with non-zero mean	:	-1.799,683
ARIMA(5,0,1)	with zero mean	:	-242,595	ARIMA(5,0,1)	with zero mean	:	-1.799,741
ARIMA(5,0,1)	with non-zero mean	:	-240,649	ARIMA(5,0,1)	with non-zero mean	:	-1.797,744
ARIMA(5,0,2)	with zero mean	:	Inf	ARIMA(5,0,2)	with zero mean	:	-1.801,334
ARIMA(5,0,2)	with non-zero mean	:	Inf	ARIMA(5,0,2)	with non-zero mean	:	-1.799,337
ARIMA(5,0,3)	with zero mean	:	Inf	ARIMA(5,0,3)	with zero mean	:	-1.799,747
ARIMA(5,0,3)	with non-zero mean	:	Inf	ARIMA(5,0,3)	with non-zero mean	:	-1.797,754
ARIMA(5,0,4)	with zero mean	:	Inf	ARIMA(5,0,4)	with zero mean	:	Inf
ARIMA(5,0,4)	with non-zero mean	:	Inf	ARIMA(5,0,4)	with non-zero mean	:	Inf
ARIMA(5,0,5)	with zero mean	:	Inf	ARIMA(5,0,5)	with zero mean	:	Inf
ARIMA(5,0,5)	with non-zero mean	:	Inf	ARIMA(5,0,5)	with non-zero mean	:	Inf
Best model: ARIMA(0,0,0) with zero mean				Best model: ARIMA(0,0,2) with zero mean			

Table 2: Information Technology results from auto.arima function in R

Monthly				Weekly			
Model	Mean		AIC	Model	Mean	0	AIC
ARIMA(0,0,0)	**with zero mean**	**:**	**-373,654**	ARIMA(0,0,0)	with zero mean	:	-2.340,128
ARIMA(0,0,0)	with non-zero mean	:	-373,206	ARIMA(0,0,0)	with non-zero mean	:	-2.340,264
ARIMA(0,0,1)	with zero mean	:	-371,668	ARIMA(0,0,1)	with zero mean	:	-2.342,002
ARIMA(0,0,1)	with non-zero mean	:	-371,206	ARIMA(0,0,1)	with non-zero mean	:	-2.342,571
ARIMA(0,0,2)	with zero mean	:	-370,516	ARIMA(0,0,2)	with zero mean	:	-2.340,073
ARIMA(0,0,2)	with non-zero mean	:	-369,844	ARIMA(0,0,2)	with non-zero mean	:	-2.340,604
ARIMA(0,0,3)	with zero mean	:	-369,174	ARIMA(0,0,3)	with zero mean	:	-2.340,275
ARIMA(0,0,3)	with non-zero mean	:	-368,685	ARIMA(0,0,3)	with non-zero mean	:	-2.340,593
ARIMA(0,0,4)	with zero mean	:	-367,886	ARIMA(0,0,4)	with zero mean	:	-2.339,872
ARIMA(0,0,4)	with non-zero mean	:	-367,533	ARIMA(0,0,4)	with non-zero mean	:	-2.340,026
ARIMA(0,0,5)	with zero mean	:	-366,565	ARIMA(0,0,5)	with zero mean	:	-2.338,072
ARIMA(0,0,5)	with non-zero mean	:	-366,098	ARIMA(0,0,5)	with non-zero mean	:	-2.338,169
ARIMA(1,0,0)	with zero mean	:	-371,670	ARIMA(1,0,0)	with zero mean	:	-2.342,022
ARIMA(1,0,0)	with non-zero mean	:	-371,206	ARIMA(1,0,0)	with non-zero mean	:	-2.342,557
ARIMA(1,0,1)	with zero mean	:	-369,733	ARIMA(1,0,1)	with zero mean	:	-2.340,030
ARIMA(1,0,1)	with non-zero mean	:	-369,314	ARIMA(1,0,1)	with non-zero mean	:	-2.340,585
ARIMA(1,0,2)	with zero mean	:	-368,734	ARIMA(1,0,2)	with zero mean	:	-2.341,221
ARIMA(1,0,2)	with non-zero mean	:	-368,085	ARIMA(1,0,2)	with non-zero mean	:	-2.341,082
ARIMA(1,0,3)	with zero mean	:	-367,391	ARIMA(1,0,3)	with zero mean	:	-2.341,420
ARIMA(1,0,3)	with non-zero mean	:	-366,988	ARIMA(1,0,3)	with non-zero mean	:	-2.341,207
ARIMA(1,0,4)	with zero mean	:	-366,259	ARIMA(1,0,4)	with zero mean	:	-2.339,709
ARIMA(1,0,4)	with non-zero mean	:	-365,869	ARIMA(1,0,4)	with non-zero mean	:	-2.339,476
ARIMA(1,0,5)	with zero mean	:	-368,087	ARIMA(1,0,5)	with zero mean	:	-2.337,821
ARIMA(1,0,5)	with non-zero mean	:	-366,823	ARIMA(1,0,5)	with non-zero mean	:	-2.337,568
ARIMA(2,0,0)	with zero mean	:	-370,342	ARIMA(2,0,0)	with zero mean	:	-2.340,045
ARIMA(2,0,0)	with non-zero mean	:	-369,719	ARIMA(2,0,0)	with non-zero mean	:	-2.340,622
ARIMA(2,0,1)	with zero mean	:	-368,480	ARIMA(2,0,1)	with zero mean	:	-2.338,170
ARIMA(2,0,1)	with non-zero mean	:	-367,872	ARIMA(2,0,1)	with non-zero mean	:	-2.338,774
ARIMA(2,0,2)	with zero mean	:	Inf	**ARIMA(2,0,2)**	**with zero mean**	**:**	**-2.343,425**
ARIMA(2,0,2)	with non-zero mean	:	Inf	ARIMA(2,0,2)	with non-zero mean	:	-2.343,262
ARIMA(2,0,3)	with zero mean	:	Inf	ARIMA(2,0,3)	with zero mean	:	-2.341,662
ARIMA(2,0,3)	with non-zero mean	:	Inf	ARIMA(2,0,3)	with non-zero mean	:	-2.341,568
ARIMA(2,0,4)	with zero mean	:	Inf	ARIMA(2,0,4)	with zero mean	:	-2.338,056
ARIMA(2,0,4)	with non-zero mean	:	Inf	ARIMA(2,0,4)	with non-zero mean	:	-2.337,837
ARIMA(2,0,5)	with zero mean	:	-368,044	ARIMA(2,0,5)	with zero mean	:	-2.336,234
ARIMA(2,0,5)	with non-zero mean	:	-367,140	ARIMA(2,0,5)	with non-zero mean	:	-2.335,975
ARIMA(3,0,0)	with zero mean	:	-368,841	ARIMA(3,0,0)	with zero mean	:	-2.339,786
ARIMA(3,0,0)	with non-zero mean	:	-368,356	ARIMA(3,0,0)	with non-zero mean	:	-2.340,111
ARIMA(3,0,1)	with zero mean	:	-367,079	ARIMA(3,0,1)	with zero mean	:	-2.340,767
ARIMA(3,0,1)	with non-zero mean	:	-366,717	ARIMA(3,0,1)	with non-zero mean	:	-2.340,595
ARIMA(3,0,2)	with zero mean	:	Inf	ARIMA(3,0,2)	with zero mean	:	-2.341,651
ARIMA(3,0,2)	with non-zero mean	:	Inf	ARIMA(3,0,2)	with non-zero mean	:	-2.341,554
ARIMA(3,0,3)	with zero mean	:	Inf	ARIMA(3,0,3)	with zero mean	:	Inf
ARIMA(3,0,3)	with non-zero mean	:	Inf	ARIMA(3,0,3)	with non-zero mean	:	Inf
ARIMA(3,0,4)	with zero mean	:	Inf	ARIMA(3,0,4)	with zero mean	:	Inf
ARIMA(3,0,4)	with non-zero mean	:	Inf	ARIMA(3,0,4)	with non-zero mean	:	-2.338,034
ARIMA(3,0,5)	with zero mean	:	Inf	ARIMA(3,0,5)	with zero mean	:	Inf
ARIMA(3,0,5)	with non-zero mean	:	Inf	ARIMA(3,0,5)	with non-zero mean	:	Inf
ARIMA(4,0,0)	with zero mean	:	-367,629	ARIMA(4,0,0)	with zero mean	:	-2.340,114
ARIMA(4,0,0)	with non-zero mean	:	-367,327	ARIMA(4,0,0)	with non-zero mean	:	-2.340,182
ARIMA(4,0,1)	with zero mean	:	-366,191	ARIMA(4,0,1)	with zero mean	:	-2.339,214
ARIMA(4,0,1)	with non-zero mean	:	-365,832	ARIMA(4,0,1)	with non-zero mean	:	-2.339,024
ARIMA(4,0,2)	with zero mean	:	Inf	ARIMA(4,0,2)	with zero mean	:	Inf
ARIMA(4,0,2)	with non-zero mean	:	Inf	ARIMA(4,0,2)	with non-zero mean	:	Inf
ARIMA(4,0,3)	with zero mean	:	Inf	ARIMA(4,0,3)	with zero mean	:	-2.338,057
ARIMA(4,0,3)	with non-zero mean	:	Inf	ARIMA(4,0,3)	with non-zero mean	:	Inf
ARIMA(4,0,4)	with zero mean	:	Inf	ARIMA(4,0,4)	with zero mean	:	Inf
ARIMA(4,0,4)	with non-zero mean	:	Inf	ARIMA(4,0,4)	with non-zero mean	:	Inf
ARIMA(4,0,5)	with zero mean	:	Inf	ARIMA(4,0,5)	with zero mean	:	Inf
ARIMA(4,0,5)	with non-zero mean	:	Inf	ARIMA(4,0,5)	with non-zero mean	:	Inf
ARIMA(5,0,0)	with zero mean	:	-366,389	ARIMA(5,0,0)	with zero mean	:	-2.338,362
ARIMA(5,0,0)	with non-zero mean	:	-365,881	ARIMA(5,0,0)	with non-zero mean	:	-2.338,357
ARIMA(5,0,1)	with zero mean	:	-365,701	ARIMA(5,0,1)	with zero mean	:	-2.337,216
ARIMA(5,0,1)	with non-zero mean	:	-364,489	ARIMA(5,0,1)	with non-zero mean	:	-2.337,025
ARIMA(5,0,2)	with zero mean	:	-363,816	ARIMA(5,0,2)	with zero mean	:	-2.337,955
ARIMA(5,0,2)	with non-zero mean	:	-362,629	ARIMA(5,0,2)	with non-zero mean	:	-2.337,836
ARIMA(5,0,3)	with zero mean	:	Inf	ARIMA(5,0,3)	with zero mean	:	-2.336,274
ARIMA(5,0,3)	with non-zero mean	:	Inf	ARIMA(5,0,3)	with non-zero mean	:	-2.336,281
ARIMA(5,0,4)	with zero mean	:	Inf	ARIMA(5,0,4)	with zero mean	:	Inf
ARIMA(5,0,4)	with non-zero mean	:	Inf	ARIMA(5,0,4)	with non-zero mean	:	Inf
ARIMA(5,0,5)	with zero mean	:	Inf	ARIMA(5,0,5)	with zero mean	:	Inf
ARIMA(5,0,5)	with non-zero mean	:	Inf	ARIMA(5,0,5)	with non-zero mean	:	Inf
Best model: ARIMA (0,0,0) with zero mean				Best model: ARIMA (2,0,2) with zero mean			

Table 3: Packaged software results from auto.arima function in R

Monthly				Weekly			
Model	**Mean**		**AIC**	**Model**	**Mean**		**AIC**
ARIMA(0,0,0)	with zero mean	:	-328,312	ARIMA(0,0,0)	with zero mean	:	-2.114,316
ARIMA(0,0,0)	with non-zero mean	:	-333,773	ARIMA(0,0,0)	with non-zero mean	:	-2.123,090
ARIMA(0,0,1)	with zero mean	:	-331,137	ARIMA(0,0,1)	with zero mean	:	-2.112,326
ARIMA(0,0,1)	**with non-zero mean**	**:**	**-334,601**	ARIMA(0,0,1)	with non-zero mean	:	-2.121,376
ARIMA(0,0,2)	with zero mean	:	-329,165	ARIMA(0,0,2)	with zero mean	:	-2.118,253
ARIMA(0,0,2)	with non-zero mean	:	-332,765	ARIMA(0,0,2)	with non-zero mean	:	-2.125,585
ARIMA(0,0,3)	with zero mean	:	-327,198	ARIMA(0,0,3)	with zero mean	:	-2.122,046
ARIMA(0,0,3)	with non-zero mean	:	-331,721	ARIMA(0,0,3)	with non-zero mean	:	-2.127,968
ARIMA(0,0,4)	with zero mean	:	-326,110	ARIMA(0,0,4)	with zero mean	:	-2.122,597
ARIMA(0,0,4)	with non-zero mean	:	-329,736	ARIMA(0,0,4)	with non-zero mean	:	-2.127,622
ARIMA(0,0,5)	with zero mean	:	-324,479	ARIMA(0,0,5)	with zero mean	:	-2.120,610
ARIMA(0,0,5)	with non-zero mean	:	-329,341	ARIMA(0,0,5)	with non-zero mean	:	-2.125,813
ARIMA(1,0,0)	with zero mean	:	-331,078	ARIMA(1,0,0)	with zero mean	:	-2.112,328
ARIMA(1,0,0)	with non-zero mean	:	-334,282	ARIMA(1,0,0)	with non-zero mean	:	-2.121,438
ARIMA(1,0,1)	with zero mean	:	-329,163	ARIMA(1,0,1)	with zero mean	:	-2.114,272
ARIMA(1,0,1)	with non-zero mean	:	-332,693	ARIMA(1,0,1)	with non-zero mean	:	-2.123,532
ARIMA(1,0,2)	with zero mean	:	Inf	ARIMA(1,0,2)	with zero mean	:	-2.123,721
ARIMA(1,0,2)	with non-zero mean	:	Inf	ARIMA(1,0,2)	with non-zero mean	:	-2.128,286
ARIMA(1,0,3)	with zero mean	:	-325,317	ARIMA(1,0,3)	with zero mean	:	-2.122,403
ARIMA(1,0,3)	with non-zero mean	:	-329,760	ARIMA(1,0,3)	with non-zero mean	:	-2.127,132
ARIMA(1,0,4)	with zero mean	:	-324,147	ARIMA(1,0,4)	with zero mean	:	-2.124,459
ARIMA(1,0,4)	with non-zero mean	:	-327,762	**ARIMA(1,0,4)**	**with non-zero mean**	**:**	**-2.129,906**
ARIMA(1,0,5)	with zero mean	:	-324,327	ARIMA(1,0,5)	with zero mean	:	-2.122,535
ARIMA(1,0,5)	with non-zero mean	:	-331,760	ARIMA(1,0,5)	with non-zero mean	:	-2.127,907
ARIMA(2,0,0)	with zero mean	:	-329,158	ARIMA(2,0,0)	with zero mean	:	-2.118,743
ARIMA(2,0,0)	with non-zero mean	:	-332,966	ARIMA(2,0,0)	with non-zero mean	:	-2.125,456
ARIMA(2,0,1)	with zero mean	:	Inf	ARIMA(2,0,1)	with zero mean	:	-2.122,522
ARIMA(2,0,1)	with non-zero mean	:	-333,919	ARIMA(2,0,1)	with non-zero mean	:	-2.127,047
ARIMA(2,0,2)	with zero mean	:	Inf	ARIMA(2,0,2)	with zero mean	:	-2.122,483
ARIMA(2,0,2)	with non-zero mean	:	Inf	ARIMA(2,0,2)	with non-zero mean	:	-2.127,451
ARIMA(2,0,3)	with zero mean	:	Inf	ARIMA(2,0,3)	with zero mean	:	-2.120,481
ARIMA(2,0,3)	with non-zero mean	:	Inf	ARIMA(2,0,3)	with non-zero mean	:	Inf
ARIMA(2,0,4)	with zero mean	:	-326,213	ARIMA(2,0,4)	with zero mean	:	-2.122,671
ARIMA(2,0,4)	with non-zero mean	:	Inf	ARIMA(2,0,4)	with non-zero mean	:	-2.127,907
ARIMA(2,0,5)	with zero mean	:	-325,707	ARIMA(2,0,5)	with zero mean	:	-2.121,018
ARIMA(2,0,5)	with non-zero mean	:	-331,942	ARIMA(2,0,5)	with non-zero mean	:	Inf
ARIMA(3,0,0)	with zero mean	:	-327,185	ARIMA(3,0,0)	with zero mean	:	-2.122,699
ARIMA(3,0,0)	with non-zero mean	:	-331,090	ARIMA(3,0,0)	with non-zero mean	:	-2.127,756
ARIMA(3,0,1)	with zero mean	:	Inf	ARIMA(3,0,1)	with zero mean	:	-2.121,791
ARIMA(3,0,1)	with non-zero mean	:	-329,150	ARIMA(3,0,1)	with non-zero mean	:	-2.126,292
ARIMA(3,0,2)	with zero mean	:	Inf	ARIMA(3,0,2)	with zero mean	:	-2.120,483
ARIMA(3,0,2)	with non-zero mean	:	Inf	ARIMA(3,0,2)	with non-zero mean	:	-2.125,480
ARIMA(3,0,3)	with zero mean	:	Inf	ARIMA(3,0,3)	with zero mean	:	-2.120,844
ARIMA(3,0,3)	with non-zero mean	:	Inf	ARIMA(3,0,3)	with non-zero mean	:	Inf
ARIMA(3,0,4)	with zero mean	:	Inf	ARIMA(3,0,4)	with zero mean	:	-2.121,589
ARIMA(3,0,4)	with non-zero mean	:	Inf	ARIMA(3,0,4)	with non-zero mean	:	-2.126,361
ARIMA(3,0,5)	with zero mean	:	-323,971	ARIMA(3,0,5)	with zero mean	:	Inf
ARIMA(3,0,5)	with non-zero mean	:	-331,052	ARIMA(3,0,5)	with non-zero mean	:	Inf
ARIMA(4,0,0)	with zero mean	:	-325,425	ARIMA(4,0,0)	with zero mean	:	-2.122,666
ARIMA(4,0,0)	with non-zero mean	:	-329,092	ARIMA(4,0,0)	with non-zero mean	:	-2.126,969
ARIMA(4,0,1)	with zero mean	:	-323,609	ARIMA(4,0,1)	with zero mean	:	-2.125,130
ARIMA(4,0,1)	with non-zero mean	:	-330,204	ARIMA(4,0,1)	with non-zero mean	:	-2.129,660
ARIMA(4,0,2)	with zero mean	:	Inf	ARIMA(4,0,2)	with zero mean	:	-2.123,193
ARIMA(4,0,2)	with non-zero mean	:	-329,909	ARIMA(4,0,2)	with non-zero mean	:	-2.127,996
ARIMA(4,0,3)	with zero mean	:	Inf	ARIMA(4,0,3)	with zero mean	:	-2.121,526
ARIMA(4,0,3)	with non-zero mean	:	Inf	ARIMA(4,0,3)	with non-zero mean	:	-2.126,121
ARIMA(4,0,4)	with zero mean	:	Inf	ARIMA(4,0,4)	with zero mean	:	-2.119,612
ARIMA(4,0,4)	with non-zero mean	:	Inf	ARIMA(4,0,4)	with non-zero mean	:	-2.126,377
ARIMA(4,0,5)	with zero mean	:	Inf	ARIMA(4,0,5)	with zero mean	:	Inf
ARIMA(4,0,5)	with non-zero mean	:	Inf	ARIMA(4,0,5)	with non-zero mean	:	Inf
ARIMA(5,0,0)	with zero mean	:	-325,077	ARIMA(5,0,0)	with zero mean	:	-2.121,647
ARIMA(5,0,0)	with non-zero mean	:	-330,430	ARIMA(5,0,0)	with non-zero mean	:	-2.126,645
ARIMA(5,0,1)	with zero mean	:	-323,845	ARIMA(5,0,1)	with zero mean	:	-2.123,171
ARIMA(5,0,1)	with non-zero mean	:	-331,263	ARIMA(5,0,1)	with non-zero mean	:	-2.127,913
ARIMA(5,0,2)	with zero mean	:	-325,242	ARIMA(5,0,2)	with zero mean	:	Inf
ARIMA(5,0,2)	with non-zero mean	:	-331,312	ARIMA(5,0,2)	with non-zero mean	:	-2.129,029
ARIMA(5,0,3)	with zero mean	:	-323,262	ARIMA(5,0,3)	with zero mean	:	-2.119,539
ARIMA(5,0,3)	with non-zero mean	:	Inf	ARIMA(5,0,3)	with non-zero mean	:	-2.127,046
ARIMA(5,0,4)	with zero mean	:	Inf	ARIMA(5,0,4)	with zero mean	:	Inf
ARIMA(5,0,4)	with non-zero mean	:	-327,582	ARIMA(5,0,4)	with non-zero mean	:	Inf
ARIMA(5,0,5)	with zero mean	:	Inf	ARIMA(5,0,5)	with zero mean	:	Inf
ARIMA(5,0,5)	with non-zero mean	:	Inf	ARIMA(5,0,5)	with non-zero mean	:	Inf
Best model: ARIMA (0,0,1) with non-zero mean				Best model: ARIMA(1,0,4) with non-zero mean			

Table 4: Real Estate Development results from auto.arima function in R

Monthly				Weekly			
Model	**Mean**		**AIC**	**Model**	**Mean**		**AIC**
ARIMA(0,0,0)	with zero mean	:	-393,383	ARIMA(0,0,0)	with zero mean	:	-2.508,252
ARIMA(0,0,0)	**with non-zero mean**	**:**	**-395,183**	ARIMA(0,0,0)	with non-zero mean	:	-2.516,576
ARIMA(0,0,1)	with zero mean	:	-388,600	ARIMA(0,0,1)	with zero mean	:	-2.510,051
ARIMA(0,0,1)	with non-zero mean	:	-390,800	ARIMA(0,0,1)	with non-zero mean	:	-2.517,073
ARIMA(0,0,2)	with zero mean	:	-386,235	ARIMA(0,0,2)	with zero mean	:	-2.509,700
ARIMA(0,0,2)	with non-zero mean	:	-387,270	ARIMA(0,0,2)	with non-zero mean	:	-2.515,976
ARIMA(0,0,3)	with zero mean	:	-384,235	ARIMA(0,0,3)	with zero mean	:	-2.509,631
ARIMA(0,0,3)	with non-zero mean	:	-383,940	ARIMA(0,0,3)	with non-zero mean	:	-2.515,047
ARIMA(0,0,4)	with zero mean	:	-380,562	ARIMA(0,0,4)	with zero mean	:	-2.507,722
ARIMA(0,0,4)	with non-zero mean	:	-381,638	ARIMA(0,0,4)	with non-zero mean	:	-2.513,545
ARIMA(0,0,5)	with zero mean	:	-375,847	ARIMA(0,0,5)	with zero mean	:	-2.505,731
ARIMA(0,0,5)	with non-zero mean	:	-376,856	ARIMA(0,0,5)	with non-zero mean	:	-2.511,632
ARIMA(1,0,0)	with zero mean	:	-388,601	ARIMA(1,0,0)	with zero mean	:	-2.510,475
ARIMA(1,0,0)	with non-zero mean	:	-390,861	**ARIMA(1,0,0)**	**with non-zero mean**	**:**	**-2.517,274**
ARIMA(1,0,1)	with zero mean	:	-383,885	ARIMA(1,0,1)	with zero mean	:	-2.510,216
ARIMA(1,0,1)	with non-zero mean	:	-386,176	ARIMA(1,0,1)	with non-zero mean	:	-2.516,009
ARIMA(1,0,2)	with zero mean	:	-382,613	ARIMA(1,0,2)	with zero mean	:	-2.508,580
ARIMA(1,0,2)	with non-zero mean	:	-382,863	ARIMA(1,0,2)	with non-zero mean	:	-2.514,289
ARIMA(1,0,3)	with zero mean	:	-380,362	ARIMA(1,0,3)	with zero mean	:	-2.507,790
ARIMA(1,0,3)	with non-zero mean	:	-380,444	ARIMA(1,0,3)	with non-zero mean	:	-2.513,515
ARIMA(1,0,4)	with zero mean	:	-375,817	ARIMA(1,0,4)	with zero mean	:	Inf
ARIMA(1,0,4)	with non-zero mean	:	-376,856	ARIMA(1,0,4)	with non-zero mean	:	-2.512,971
ARIMA(1,0,5)	with zero mean	:	inf	ARIMA(1,0,5)	with zero mean	:	Inf
ARIMA(1,0,5)	with non-zero mean	:	Inf	ARIMA(1,0,5)	with non-zero mean	:	-2.510,987
ARIMA(2,0,0)	with zero mean	:	-385,511	ARIMA(2,0,0)	with zero mean	:	-2.510,109
ARIMA(2,0,0)	with non-zero mean	:	-386,632	ARIMA(2,0,0)	with non-zero mean	:	-2.516,109
ARIMA(2,0,1)	with zero mean	:	-381,901	ARIMA(2,0,1)	with zero mean	:	-2.508,507
ARIMA(2,0,1)	with non-zero mean	:	-382,195	ARIMA(2,0,1)	with non-zero mean	:	-2.514,227
ARIMA(2,0,2)	with zero mean	:	Inf	ARIMA(2,0,2)	with zero mean	:	Inf
ARIMA(2,0,2)	with non-zero mean	:	Inf	ARIMA(2,0,2)	with non-zero mean	:	Inf
ARIMA(2,0,3)	with zero mean	:	Inf	ARIMA(2,0,3)	with zero mean	:	-2.505,831
ARIMA(2,0,3)	with non-zero mean	:	Inf	ARIMA(2,0,3)	with non-zero mean	:	Inf
ARIMA(2,0,4)	with zero mean	:	Inf	ARIMA(2,0,4)	with zero mean	:	Inf
ARIMA(2,0,4)	with non-zero mean	:	Inf	ARIMA(2,0,4)	with non-zero mean	:	-2.510,953
ARIMA(2,0,5)	with zero mean	:	Inf	ARIMA(2,0,5)	with zero mean	:	-2.501,949
ARIMA(2,0,5)	with non-zero mean	:	Inf	ARIMA(2,0,5)	with non-zero mean	:	-2.509,071
ARIMA(3,0,0)	with zero mean	:	-383,807	ARIMA(3,0,0)	with zero mean	:	-2.509,119
ARIMA(3,0,0)	with non-zero mean	:	-383,657	ARIMA(3,0,0)	with non-zero mean	:	-2.514,561
ARIMA(3,0,1)	with zero mean	:	-379,636	ARIMA(3,0,1)	with zero mean	:	-2.507,796
ARIMA(3,0,1)	with non-zero mean	:	-379,735	ARIMA(3,0,1)	with non-zero mean	:	-2.513,359
ARIMA(3,0,2)	with zero mean	:	Inf	ARIMA(3,0,2)	with zero mean	:	-2.505,893
ARIMA(3,0,2)	with non-zero mean	:	Inf	ARIMA(3,0,2)	with non-zero mean	:	-2.512,023
ARIMA(3,0,3)	with zero mean	:	Inf	ARIMA(3,0,3)	with zero mean	:	Inf
ARIMA(3,0,3)	with non-zero mean	:	Inf	ARIMA(3,0,3)	with non-zero mean	:	Inf
ARIMA(3,0,4)	with zero mean	:	Inf	ARIMA(3,0,4)	with zero mean	:	Inf
ARIMA(3,0,4)	with non-zero mean	:	Inf	ARIMA(3,0,4)	with non-zero mean	:	Inf
ARIMA(3,0,5)	with zero mean	:	Inf	ARIMA(3,0,5)	with zero mean	:	Inf
ARIMA(3,0,5)	with non-zero mean	:	Inf	ARIMA(3,0,5)	with non-zero mean	:	Inf
ARIMA(4,0,0)	with zero mean	:	-380,084	ARIMA(4,0,0)	with zero mean	:	-2.507,814
ARIMA(4,0,0)	with non-zero mean	:	-380,971	ARIMA(4,0,0)	with non-zero mean	:	-2.513,928
ARIMA(4,0,1)	with zero mean	:	-375,371	ARIMA(4,0,1)	with zero mean	:	-2.506,059
ARIMA(4,0,1)	with non-zero mean	:	-376,725	ARIMA(4,0,1)	with non-zero mean	:	-2.513,196
ARIMA(4,0,2)	with zero mean	:	Inf	ARIMA(4,0,2)	with zero mean	:	Inf
ARIMA(4,0,2)	with non-zero mean	:	-372,459	ARIMA(4,0,2)	with non-zero mean	:	Inf
ARIMA(4,0,3)	with zero mean	:	Inf	ARIMA(4,0,3)	with zero mean	:	Inf
ARIMA(4,0,3)	with non-zero mean	:	Inf	ARIMA(4,0,3)	with non-zero mean	:	Inf
ARIMA(4,0,4)	with zero mean	:	Inf	ARIMA(4,0,4)	with zero mean	:	Inf
ARIMA(4,0,4)	with non-zero mean	:	Inf	ARIMA(4,0,4)	with non-zero mean	:	Inf
ARIMA(4,0,5)	with zero mean	:	Inf	ARIMA(4,0,5)	with zero mean	:	Inf
ARIMA(4,0,5)	with non-zero mean	:	Inf	ARIMA(4,0,5)	with non-zero mean	:	Inf
ARIMA(5,0,0)	with zero mean	:	-375,499	ARIMA(5,0,0)	with zero mean	:	-2.505,824
ARIMA(5,0,0)	with non-zero mean	:	-377,119	ARIMA(5,0,0)	with non-zero mean	:	-2.512,140
ARIMA(5,0,1)	with zero mean	:	-370,790	ARIMA(5,0,1)	with zero mean	:	-2.505,614
ARIMA(5,0,1)	with non-zero mean	:	-372,344	ARIMA(5,0,1)	with non-zero mean	:	-2.511,602
ARIMA(5,0,2)	with zero mean	:	Inf	ARIMA(5,0,2)	with zero mean	:	-2.503,953
ARIMA(5,0,2)	with non-zero mean	:	Inf	ARIMA(5,0,2)	with non-zero mean	:	-2.511,033
ARIMA(5,0,3)	with zero mean	:	Inf	ARIMA(5,0,3)	with zero mean	:	Inf
ARIMA(5,0,3)	with non-zero mean	:	Inf	ARIMA(5,0,3)	with non-zero mean	:	Inf
ARIMA(5,0,4)	with zero mean	:	Inf	ARIMA(5,0,4)	with zero mean	:	Inf
ARIMA(5,0,4)	with non-zero mean	:	Inf	ARIMA(5,0,4)	with non-zero mean	:	Inf
ARIMA(5,0,5)	with zero mean	:	Inf	ARIMA(5,0,5)	with zero mean	:	Inf
ARIMA(5,0,5)	with non-zero mean	:	Inf	ARIMA(5,0,5)	with non-zero mean	:	Inf
Best model: ARIMA (0,0,0) with non-zero mean				Best model: ARIMA(1,0,0) with non-zero mean			

Table 5: Industrial Machinery results from auto.arima function in R

Monthly				Weekly			
Model	**Mean**		**AIC**	**Model**	**Mean**		**AIC**
ARIMA(0,0,0)	with zero mean	:	-312,618	ARIMA(0,0,0)	with zero mean	:	-2.051,605
ARIMA(0,0,0)	**with non-zero mean**	**:**	**-313,648**	**ARIMA(0,0,0)**	**with non-zero mean**	**:**	**-2.055,845**
ARIMA(0,0,1)	with zero mean	:	-311,972	ARIMA(0,0,1)	with zero mean	:	-2.049,663
ARIMA(0,0,1)	with non-zero mean	:	-312,552	ARIMA(0,0,1)	with non-zero mean	:	-2.054,105
ARIMA(0,0,2)	with zero mean	:	-312,288	ARIMA(0,0,2)	with zero mean	:	-2.048,732
ARIMA(0,0,2)	with non-zero mean	:	-312,338	ARIMA(0,0,2)	with non-zero mean	:	-2.052,774
ARIMA(0,0,3)	with zero mean	:	-312,560	ARIMA(0,0,3)	with zero mean	:	-2.048,918
ARIMA(0,0,3)	with non-zero mean	:	-312,144	ARIMA(0,0,3)	with non-zero mean	:	-2.052,392
ARIMA(0,0,4)	with zero mean	:	-311,404	ARIMA(0,0,4)	with zero mean	:	-2.048,408
ARIMA(0,0,4)	with non-zero mean	:	-311,298	ARIMA(0,0,4)	with non-zero mean	:	-2.051,459
ARIMA(0,0,5)	with zero mean	:	-309,696	ARIMA(0,0,5)	with zero mean	:	-2.046,408
ARIMA(0,0,5)	with non-zero mean	:	-309,454	ARIMA(0,0,5)	with non-zero mean	:	-2.049,487
ARIMA(1,0,0)	with zero mean	:	-312,318	ARIMA(1,0,0)	with zero mean	:	-2.049,669
ARIMA(1,0,0)	with non-zero mean	:	-312,750	ARIMA(1,0,0)	with non-zero mean	:	-2.054,123
ARIMA(1,0,1)	with zero mean	:	-312,833	ARIMA(1,0,1)	with zero mean	:	-2.054,784
ARIMA(1,0,1)	with non-zero mean	:	-312,250	ARIMA(1,0,1)	with non-zero mean	:	-2.052,255
ARIMA(1,0,2)	with zero mean	:	-311,414	ARIMA(1,0,2)	with zero mean	:	-2.055,063
ARIMA(1,0,2)	with non-zero mean	:	-311,080	ARIMA(1,0,2)	with non-zero mean	:	-2.055,723
ARIMA(1,0,3)	with zero mean	:	-311,268	ARIMA(1,0,3)	with zero mean	:	-2.053,224
ARIMA(1,0,3)	with non-zero mean	:	-310,945	ARIMA(1,0,3)	with non-zero mean	:	-2.053,968
ARIMA(1,0,4)	with zero mean	:	-309,490	ARIMA(1,0,4)	with zero mean	:	-2.051,229
ARIMA(1,0,4)	with non-zero mean	:	-309,355	ARIMA(1,0,4)	with non-zero mean	:	-2.051,969
ARIMA(1,0,5)	with zero mean	:	-308,299	ARIMA(1,0,5)	with zero mean	:	-2.049,240
ARIMA(1,0,5)	with non-zero mean	:	-307,655	ARIMA(1,0,5)	with non-zero mean	:	-2.049,970
ARIMA(2,0,0)	with zero mean	:	-312,523	ARIMA(2,0,0)	with zero mean	:	-2.048,783
ARIMA(2,0,0)	with non-zero mean	:	-312,362	ARIMA(2,0,0)	with non-zero mean	:	-2.052,743
ARIMA(2,0,1)	with zero mean	:	-311,226	ARIMA(2,0,1)	with zero mean	:	-2.054,954
ARIMA(2,0,1)	with non-zero mean	:	-310,783	ARIMA(2,0,1)	with non-zero mean	:	-2.055,572
ARIMA(2,0,2)	with zero mean	:	-309,756	ARIMA(2,0,2)	with zero mean	:	Inf
ARIMA(2,0,2)	with non-zero mean	:	-309,721	ARIMA(2,0,2)	with non-zero mean	:	Inf
ARIMA(2,0,3)	with zero mean	:	-311,420	ARIMA(2,0,3)	with zero mean	:	-2.047,016
ARIMA(2,0,3)	with non-zero mean	:	Inf	ARIMA(2,0,3)	with non-zero mean	:	-2.050,910
ARIMA(2,0,4)	with zero mean	:	Inf	ARIMA(2,0,4)	with zero mean	:	-2.046,299
ARIMA(2,0,4)	with non-zero mean	:	Inf	ARIMA(2,0,4)	with non-zero mean	:	-2.049,792
ARIMA(2,0,5)	with zero mean	:	-308,448	ARIMA(2,0,5)	with zero mean	:	-2.046,034
ARIMA(2,0,5)	with non-zero mean	:	Inf	ARIMA(2,0,5)	with non-zero mean	:	Inf
ARIMA(3,0,0)	with zero mean	:	-312,130	ARIMA(3,0,0)	with zero mean	:	-2.049,086
ARIMA(3,0,0)	with non-zero mean	:	-311,583	ARIMA(3,0,0)	with non-zero mean	:	-2.052,364
ARIMA(3,0,1)	with zero mean	:	-311,103	ARIMA(3,0,1)	with zero mean	:	-2.053,232
ARIMA(3,0,1)	with non-zero mean	:	-310,607	ARIMA(3,0,1)	with non-zero mean	:	-2.053,960
ARIMA(3,0,2)	with zero mean	:	-312,222	ARIMA(3,0,2)	with zero mean	:	Inf
ARIMA(3,0,2)	with non-zero mean	:	Inf	ARIMA(3,0,2)	with non-zero mean	:	-2.050,953
ARIMA(3,0,3)	with zero mean	:	Inf	ARIMA(3,0,3)	with zero mean	:	-2.052,347
ARIMA(3,0,3)	with non-zero mean	:	Inf	ARIMA(3,0,3)	with non-zero mean	:	-2.052,748
ARIMA(3,0,4)	with zero mean	:	Inf	ARIMA(3,0,4)	with zero mean	:	Inf
ARIMA(3,0,4)	with non-zero mean	:	Inf	ARIMA(3,0,4)	with non-zero mean	:	Inf
ARIMA(3,0,5)	with zero mean	:	Inf	ARIMA(3,0,5)	with zero mean	:	Inf
ARIMA(3,0,5)	with non-zero mean	:	Inf	ARIMA(3,0,5)	with non-zero mean	:	Inf
ARIMA(4,0,0)	with zero mean	:	-311,571	ARIMA(4,0,0)	with zero mean	:	-2.049,313
ARIMA(4,0,0)	with non-zero mean	:	-311,417	ARIMA(4,0,0)	with non-zero mean	:	-2.051,975
ARIMA(4,0,1)	with zero mean	:	-309,660	ARIMA(4,0,1)	with zero mean	:	-2.051,235
ARIMA(4,0,1)	with non-zero mean	:	-309,459	ARIMA(4,0,1)	with non-zero mean	:	-2.051,968
ARIMA(4,0,2)	with zero mean	:	Inf	ARIMA(4,0,2)	with zero mean	:	Inf
ARIMA(4,0,2)	with non-zero mean	:	Inf	ARIMA(4,0,2)	with non-zero mean	:	-2.050,492
ARIMA(4,0,3)	with zero mean	:	Inf	ARIMA(4,0,3)	with zero mean	:	-2.049,594
ARIMA(4,0,3)	with non-zero mean	:	Inf	ARIMA(4,0,3)	with non-zero mean	:	Inf
ARIMA(4,0,4)	with zero mean	:	Inf	ARIMA(4,0,4)	with zero mean	:	-2.049,065
ARIMA(4,0,4)	with non-zero mean	:	Inf	ARIMA(4,0,4)	with non-zero mean	:	-2.049,518
ARIMA(4,0,5)	with zero mean	:	Inf	ARIMA(4,0,5)	with zero mean	:	-2.047,258
ARIMA(4,0,5)	with non-zero mean	:	Inf	ARIMA(4,0,5)	with non-zero mean	:	-2.047,827
ARIMA(5,0,0)	with zero mean	:	-309,829	ARIMA(5,0,0)	with zero mean	:	-2.047,467
ARIMA(5,0,0)	with non-zero mean	:	-309,520	ARIMA(5,0,0)	with non-zero mean	:	-2.050,008
ARIMA(5,0,1)	with zero mean	:	-308,505	ARIMA(5,0,1)	with zero mean	:	-2.049,326
ARIMA(5,0,1)	with non-zero mean	:	-307,832	ARIMA(5,0,1)	with non-zero mean	:	-2.050,011
ARIMA(5,0,2)	with zero mean	:	Inf	ARIMA(5,0,2)	with zero mean	:	Inf
ARIMA(5,0,2)	with non-zero mean	:	Inf	ARIMA(5,0,2)	with non-zero mean	:	-2.048,544
ARIMA(5,0,3)	with zero mean	:	Inf	ARIMA(5,0,3)	with zero mean	:	-2.047,671
ARIMA(5,0,3)	with non-zero mean	:	Inf	ARIMA(5,0,3)	with non-zero mean	:	-2.047,746
ARIMA(5,0,4)	with zero mean	:	Inf	ARIMA(5,0,4)	with zero mean	:	-2.047,247
ARIMA(5,0,4)	with non-zero mean	:	Inf	ARIMA(5,0,4)	with non-zero mean	:	-2.047,804
ARIMA(5,0,5)	with zero mean	:	Inf	ARIMA(5,0,5)	with zero mean	:	Inf
ARIMA(5,0,5)	with non-zero mean	:	Inf	ARIMA(5,0,5)	with non-zero mean	:	Inf
Best model: ARIMA (0,0,0) with non-zero mean				Best model: ARIMA(0,0,0) with non-zero mean			

Appendix C

Excel files

1) *Portfolio_return_monthly.xlsx*

   a. A file containing the monthly log returns for all industry portfolios.

2) *Portfolio_return_weekly.xlsx*

   b. A file containing the weekly log returns for all industry portfolios.

RStudio code

1) Code_monthly.R

   (a) A code containing all code applied in this study for the monthly data

2) *Code_weekly.R*

   (a) A code containing all code used for this study for the weekly data

3) *exp.R*

   (a) Contains a function for the out of sample 1-step ahead with expanding window forecast for a vector of error prediction.

4) *exp1.R*

   (a) Contains a function for the out of sample 1-step ahead with expanding window forecast for log returns.