The case for geographical diversification in renewable energy portfolios

Does the application of volatility forecasting and Mean Variance Optimization provide more value than equally weighted portfolios?

Aalborg University Business School

Master Thesis

MSc in Finance

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May 31, 2023

Abstract

This thesis delves into the evolving realm of renewable energy, exploring the potential of leveraging portfolio optimization models traditionally utilized in financial markets, with a key emphasis on the Dynamic Conditional Correlation GARCH (DCC GARCH) model in conjunction with the time-tested Mean Variance Optimization (MVO) method. Amid a global transition towards green energy, and spurred by projections of renewable and nuclear technologies contributing over 50% of global electricity production by 2030, the study investigates the efficacy of geographical portfolio diversification within the renewable energy sector, specifically across Denmark, Finland, Italy, Belgium, and Germany. The thesis concludes that the implementation of a DCC GARCH based Mean Variance Optimization does in fact yield higher value for the respective portfolios than the simple equally weighted method of portfolio diversification. It does however also suggest that more complex methods of calculating portfolio risk than simply the portfolio variance would be required to more adequately account for the risk/reward profile of each portfolio, especially with more extreme price scenarios occurring in the electricity markets.

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1 Introduction

In today's world where the effects of global warming are become more apparent every year with more extreme heat waves, forest fires, flash floods, hurricanes and many more weather phenomenon, the case for increased production of renewable energy becomes better and better. At the current rate, the World Economic Forum expects that by 2030 over 50% of the worlds electricity production will be from renewable technology and nuclear technology. The WEF cites a collection of 47 peer reviewed research papers investigating the possibilities for countries to completely switch to renewable energy by 2050 and how that could keep the planet below the 1.5°global warming target set by the Paris agreement. (WEF, 2022 (b))

This significant increase in global renewable energy capacity also opens up doors for many companies to facilitate this transition to green energy. A relatively new standard of contracts within the renewable sector is the Power Purchase Agreement (PPA) which is an agreement that can be between a producer and consumer or as is often the case, between a producer and a Balance Responsible Party (BRP) who then makes another PPA with a consumer. This can often make it easier on the producer as they only need to focus on producing while the BRP takes care of delivering the physical electricity to the end consumer or sells it off on the Day-Ahead electricity exchange.(Next-Kraftwerke, n.d. (b))

These PPA contracts also allow large corporations that want to live up to certain climate standards, where companies such as Google have signed multiple long term PPA contracts in order to meet their climate neutrality goals. This allows them at the same time to secure and fix their cost of electricity by the amount contracted. (McKinsey, 2018)

This thesis looks at how a BRP can diversify it's portfolio geographically by using known portfolio optimization methods from the more common financial markets. The countries used for the optimization will be Denmark, Finland, Italy, Belgium and Germany, they were chosen based on a correlation study made by Eurostat (2023), however the choice of countries was also limited by access to data through Centrica Energy Trading's database. The methods used will be a combination of volatility modelling by the use of a Dynamic Conditional Correlation GARCH (DCC GARCH) proposed by Engle in 1982 in combination with the well known Mean Variance Optimization (MVO) method introduced and popularized by Markowitz in 1952.

The DCC GARCH model will be used to forecast for three different time horizons within a year and the resulting covariance matrixes will be used for the Mean Variance Optimization. That will result in different portfolio weights of the five respective countries with each country having its own volume weighted average price (VWAP) and the portfolio its own VWAP based on the combination of weights. These portfolios will be compared against a portfolio optimized on the sample covariance of each training period as well as a portfolio diversified equally with each country having a 20% weights, referred to in this thesis as the naive equally weighted portfolio.

These different portfolios will then be compared against each other and their respective performance will predominantly be observed by the portfolios VWAP as it reflects the average price for each MW produced, where the higher the VWAP, the better. Each portfolios variance will also be used as a metric for their respective risk, making the VWAP more relative to portfolio risk.

2 Introduction to the Power Markets

This chapter serves as a foundational introduction to power markets, enabling readers to better understand the various concepts explored throughout this thesis. Although many of the financial derivatives available to market participants resemble those used in the broader equity markets, there are subtle differences.

The European Power Market is a vast and intricate system involving many countries, each with their own electricity production and consumption. At present, 19 countries in Europe, which account for 85% of the region's total electricity consumption, participate in the Multi-Regional Coupling (MRC). The MRC facilitates the seamless flow of electricity between participating countries, as long as transmission capacity is available. The MRC's primary goal is to maximize social welfare across all member countries by importing power where prices are high and exporting power from countries with lower prices. Consequently, this results in smaller price differences between member nations (Next Kraftwerke, n.d. (a))

In the power market, there are several types of market participants, each with distinct roles and responsibilities. These participants include utilities, independent power producers (IPPs), large energy consumers, and energy traders. Utilities are companies responsible for generating, transmitting, and distributing electricity to end-users, such as households and businesses. IPPs are non-utility entities that generate and sell electricity to utilities, grid operators, or large energy consumers. Large energy consumers include industrial and commercial facilities that consume significant amounts of electricity and may engage directly in the power market to manage their electricity costs and hedge against price volatility. Energy traders are specialized market participants that buy and sell electricity and other energy-related financial products, such as futures, options, and swaps. They play a crucial role in managing price risks, providing liquidity, and promoting price discovery in the power market. (EIA, n.d.)

To ensure that supply and demand discrepancies in the power market do not escalate, Balance Responsible Parties (BRPs) are employed. BRPs trade electricity through various methods, including forward contracts, futures contracts, Over the Counter (OTC) trades, Day-Ahead Auctions, and the Intra-Day market. Among these, the Day-Ahead and Intra-Day markets are particularly significant.

The Day-Ahead Auction helps maintain adequate balance within each country's power grid for the following day, preventing excessive or insufficient power that could lead to unstable grid conditions, such as flickering lights or blackouts. In the auction, BRPs submit market orders indicating the amount of electricity they wish to buy and sell and the desired price. The auction is blind, ensuring that no one knows who is buying or selling, thereby maintaining a level playing field. Once the auction concludes, an algorithm matches all buy and sell bids to determine an equilibrium price for each hour of the day. This is the market-setting price at which BRPs buy or sell their electricity. (Energifakta Norge, 2022)

The Intra-Day market opens daily, allowing for the sale or purchase of electricity to balance discrepancies between expected and actual production in each price area. The concept of a balanced power grid is essential, and all market participants recognize its importance. BRPs can only profit from a position in balancing by helping the grid. For example, if the system is short and requires more power, it must upregulate and activate more expensive power reserves than the Day-Ahead price. Consequently, market participants must hold a long position in that market to profit, which also aids the system if they can direct power from one area to the one in need. (Energifakta Norge, 2022)

The forward and futures markets cater more to longer-term hedging or speculation of financial derivatives related to electricity production. Market participants can trade in various forward products, such as baseload products (a product that represents a fixed amount of MW per hour, every hour for the specified period) over a month, quarter, half-year, etc. Many participants use these products to hedge production or consumption, similar to how equities are hedged for risk. (Energifakta Norge, 2022)

3 Problem Formulation

As described in the introduction, in the current geopolitical climate where much focus and resources are being put to use in increasing the production capacity of renewable energy technologies like wind mills, photovoltaic panels and hydroelectric reserves it has introduced a phenomenon called price cannibalisation. This phenomenon can lead to depressed electricity prices during times of high renewable penetration which in turn reduces revenue of electricity producers.

3.1 Research question

Can the use of financial forecasting models be effective in constructing diversified renewable energy portfolios that aim to minimize production variance of wind parks, and does the resulting optimisation result in higher financial value than a naive equally weighted portfolio?

The aim of this thesis is to investigate the effects of geographical diversification within the renewable electricity markets as the the effects of price cannibalisation become more prevalent within the industry and require risk and diversification methods to reduce exposure during times of negative prices. The methods used have been extrapolated from methods that are widely accepted within the broader financial markets and the efficacy will be tested within the energy markets. The focus will be on geographical diversification of wind based renewable assets and how it can be utilized to increase value of a portfolio consistent of exclusively wind assets.

4 Literature Review

4.1 Electricity price cannibalisation

The need for hedging renewable energy production is just as important for renewable energy portfolio optimisation as hedging of the more common equity based assets such as stocks and bonds. There are many types of risks that need to be accounted for by energy producers as well as the off-takers of the energy, which in many cases are intermediaries who then deliver the power to the end consumer.

One of the most common risks for energy producers is revenue risk or merchant risk which refers to the uncertainty of revenue received for the power produced. An example would be the revenue risk of a wind mill owner that only receives revenue for the actual power produced by the wind mill, which is dependent on the amount of wind blowing at each point in time. This type of renewable energy production is called intermittent energy production as it is dependent on external variables such as wind speed and wind direction. For solar parks the dependent variable would be cloud cover and solar radiation determining how much electricity can be produced by the solar panels. However with the current increase in renewable energy investments the general price level of energy during windy and sunny days has decreased due to the increased capacity of these renewable energy sources as the cost of producing renewable energy is less than conventional carbon based energy that rely on input costs such as oil, gas or coal in order to produce energy. This means that renewable power producers get less revenue per MW produced than only a few years ago and therefore their initial profit expectations might be significantly higher than the realised profit. This phenomenon is referred to as price cannibalisation and has been observed on both sides of the Atlantic Ocean. (McKinsey, 2018)

Prol, Steininger and Zilberman (2020) observed a clear cannibalisation effect of electricity prices in California between 2013 and 2017 with increasing capacity of solar and wind production. They did observe that the increase in wind production had a negative impact on the value factor of both wind and solar generation while an increase in solar production had a negative effect on the value factor of solar but an increase in the value factor of wind power. The reason for this inverse effect of solar penetration on wind value factor is that solar penetration is at its peak around noon which has a steeper downwards effect on electricity prices and a spike upwards when solar penetration tapers off and wind generation picks up in the evening. The absolute cannibalisation effect is the strongest in high wind and high solar scenarios with low consumption.

De Lagarde and Lantz (2018) Observed a price cannibalisation effect in the German electricity market and conclude that the increase in wind production does lead to longer and more frequent periods of low prices. They also observed that with the increased capacity of renewable production in the German grid there was more volatility during periods of high prices and low renewable production due to the merit-order effect. It is also important to note that in the German market there are many consumers that have solar panels on their roofs which leads to a double effect of solar production on electricity prices. This is because consumers with solar panels do not need to rely on the power grid to supply them with electricity when the sun is shining, or at least to a lesser degree, decreasing the demand load on the grid on top of the increase in renewable production in the grid.

Clò, Cataldi and Zoppoli (2015) investigated the effects of increased renewable production in the Italian market and found results similar to the two above mentioned papers. They observed a direct correlation between level of renewable production and price level as well as a relationship between renewable production and price volatility, similar to De Lagarde and Lantz (2018).

4.2 Risk associated with renewable production and hedging

As the previous sub-chapter explained, there is a significant negative correlation between renewable production and the price level of electricity. This is an issue for future and current investments in renewable energy as the decrease in electricity prices might outpace the decrease in building costs of renewable power plants, such as wind mill parks or solar parks. This has and will continue to decrease the expected profits of investments in renewable energy without proper diversification across energy technologies and geographical locations.

In addition to the effects of price cannibalisation, there are other risks that need to be evaluated, Mack (2014) lists a few that include but are not limited to: Credit Risk, Liquidity Risk, Market Risk, Operational Risk and Political Risk. These risk factors are very similar to the common risk factors when discussing equity based portfolio risk. In order to lessen or minimize the risk associated with an investment, hedging is usually required.

Hedging refers to the act of investing in an asset that has a non-perfect positive or a negative correlation to a main investment in order to reduce risks associated with the main investment. In most cases, if a hedge is utilized in order to lower possible risk, the possible returns are usually also reduced as the investment and the hedged asset do not have a perfect positive correlation. This means that when the investment increases in value, the hedge most often decreases in value, if the hedge is performed properly. It is important to note that hedging does not always refer to offsetting a certain financial position with the opposite hedge. It can also refer to the act of minimizing of a specific attribute of a financial position, such as minimizing the volatility of a portfolio or minimizing the correlation of a portfolio. The hedging of these two attributes does not necessarily require a ”long-short” approach like many portfolio optimisation methods suggest. (Mack, 2014)

Many electricity producers are also exposed to the cost of input materials such as coal or gas in order to produce power. During late 2021 and all of 2022, the cost of electricity rose exponentially with the increased cost of gas and oil due to the war in Ukraine, resulting in increased price levels as well as increased volatility for those producers that depended on gas in order to produce electricity (WEF, 2022a). This also spilled over into the general electricity market with the forward markets pricing some longer term contracts drastically higher than ever experienced before. This price risk can be very important to hedge for producers of conventional carbon based electricity in order to prevent large decreases in profitability or even result in losses.(Mack, 2014)

This type of input price risk can be mitigated by entering into long forward contracts on the required input if the owner of the production unit suspects an increase in the the price of the input in the future. This type of hedging is not required when discussing renewable energy production as the input cost is considered zero as the input is either wind, solar or hydro. (Mack, 2014)

For renewable energy producers, volumetric risk, the risk of not producing the expected amount of electricity which was sold in the market on the Day-Ahead Auction, be it more or less than expected can result in unwanted outcomes. An example could be that a supplier sold a baseload profile of 10MW per hour for the following day but on the day they only produce 8MW per hour. This means that they have sold too much power on the Day-Ahead Auction and need to purchase the difference in the Intra-Day market. Depending on the market conditions on the day, it can more or less expensive for the producer to live up to its obligation of delivering 10MW of electricity per hour. The opposite can also happen with the producer producing more than expected and having to sell it off in the Intra-Day market. (Mack, 2014)

Masala, Micocci and Rizk (2022) observe the effects of using weather derivatives to hedge against volumetric risk and found a significant increase in the worst value of a renewable portfolio that was properly hedged against volumetric risk. They did see a small decrease in the Earnings at Risk while implementing a hedge but it was deemed less significant than the absolute increase in the value of the portfolio in a worst-case scenario.

4.3 Volatility of Energy Prices

Electricity price volatility is an issue that needs to be properly addressed due to is impact on renewable investment decisions, risk management of renewable portfolios and the adoption of renewable energy sources. Weron (2014) conducted a comprehensive review of different methods of forecasting electricity prices. In the review, Weron (2014) highlights some of the main drivers of price volatility, with the more impactful drivers include demand fluctuations, production plant outages, changes in fuel prices for traditional carbon based production and interventions by regulatory authorities. Weron (2014) emphasizes the need for accurate electricity price forecasting for market participants, such as producers, consumers, and traders for effective risk management. The author also identified numerous forecasting methods that have been used in order to address the specific challenges presented by the unique characteristics of electricity markets, such as high volatility, mean-reversion and extreme values. The forecasting techniques include statistical models, such as autoregressive integrated moving average (ARIMA) models, as well as machine learning approaches and hybrid models that combine multiple methodologies.

Fanone, Gamba, and Prokopczuk (2013) observed the existence of negative electricity prices during Day-Ahead auctions which were attributed to elevated levels of renewable energy penetration while the flexibility of conventional power plants is limited. Negative prices occur when supply exceeds demand during a certain period which leads to producers paying consumers to absorb excess power in the system. This scenario is most common in markets where a significant portion of it's electricity generation is wind and solar based, as these sources are characterized by volatility and intermittency. The authors find that negative prices are more likely to occur during periods of low demand and high renewable energy generation, leading to increased volatility in electricity prices as deviations in the renewable production can not be offset instantly by carbon based production due to ramping and igniting of those plants. This finding underscores the need for market participants to develop effective risk management strategies and forecasting tools that can account for the inherent volatility and unique features of electricity markets, especially as renewable energy sources continue to grow in importance.

4.4 Multivariate GARCH models in energy markets

Multivariate GARCH models have become increasingly popular in energy market research due to their ability to capture the dynamic nature of volatility and the relationships between multiple time series. Song, F., Cui, J. and Yu, Y. (2022) observe the dynamic volatility spillover effects between solar generation and wind generation in order to construct an optimal portfolio with weights in both wind and solar generation. The authors use a DCC GARCH model in order to capture the time varying correlation between the two technologies and the risk complementarities in order to minimize the risk of power generation.

Efimova, O. and Serletis, A. (2014) apply a periodic GARCH-M model to improve the understanding of wholesale electricity price volatility. In the study the authors break down the price risk into two components, its pure price component and its skeweness price component. The papers result indicate the the growth rate of electricity prices behave differently to those of financial rates with past positive shocks having a more significant impact on volatility than past negative shocks of the same size. This research highlights the importance of understanding price volatility for large consumers and producers of electricity in order to properly hedge their exposure.

The use of multivariate GARCH models in energy markets contributes to a better understanding of the complex interactions between different energy commodities and financial assets. These models help market participants, such as investors, traders, and policy-makers, make more informed decisions by providing a deeper understanding of the sources of risk and the potential for diversification. Additionally, multivariate GARCH models can be used to analyze the impact of renewable energy sources on energy markets, as they can account for the unique characteristics of renewable energy generation, such as variability, intermittency and complementarity of different renewable sources.

5 Methodology

In this chapter, an introduction to the methodology for implementing a Dynamic Conditional Correlation (DCC) GARCH model, a multivariate extension of the univariate GARCH model, to analyze the time-varying correlations and volatilities in renewable energy production. The DCC GARCH model, proposed by Engle (2002), allows for capturing of the dynamic relationship between wind production in different countries, which is crucial for understanding the potential benefits of geographical portfolio diversification and risk management in the energy sector. This methodology follows a similar structure to that of Song, F., Cui, J. and Yu, Y. (2022) in order to keep in line with previously accepted methodology. All model estimations and data processing will be conducted in R and the code will be attached in an external file.

Before estimating the DCC GARCH model, a series of preliminary tests will be performed, including tests for stationarity of the data set, to ensure that the data is suitable for the analysis. The Phillips Perron will be used to check for the presence of unit roots in the time series data. If needed, appropriate transformations will be applied, such as differencing, to achieve stationarity.

After the data has been confirmed to be suitable for the analysis, either as is or after some transformation, the dataset will be split up into three different training sets where each training set includes the previous testing set and therefore is a rolling training set. The testing set will always be fixed at a length of one year from the end of the respective training set. From the training sets different mean models will be estimated on the data as the DCC GARCH model needs a mean model with a variance to have been estimated as the DCC GARCH is estimated on the variance of the mean model. The mean models will be a version of an Autoregressive (AR) or Autoregressive Moving Average (ARMA) process depending on what attributes the data shows. As renewable energy production usually shows signs of seasonality, a variation of the ARMA process called Seasonal Autoregressive Integrated Moving Average (SARIMA) could be used. Seasonality in the data will be confirmed before estimating the model in order for the appropriate model type to be estimated.

Once the mean model process has been chosen, it will be fitted to all 5 sets of production data and the appropriate lag structures will be decided by using information criteria such as Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC). After fitting the mean models, the standardized residuals are calculated by dividing the raw residuals by their estimated conditional standard deviations. These standardized residuals will be used as inputs for the DCC GARCH model.

In order to assess the performance of the DCC GARCH model, a naive mean portfolio with 20% weights in each country will be made as well as portfolios based on the different sample covariances for the different periods. This will allow for a comparison of a very basic hedging strategy with the equally weighted portfolio as well as a more complex sample covariance portfolio. This will provide grounds for an analysis of the different methods within the renewable energy sector and lead to a discussion of the different methods.

Finally, the results from the DCC GARCH model will be used to construct a Mean Variance Portfolio (MVP) for the renewable production portfolio. The MVP aims to minimize the overall portfolio variance profile by selecting the optimal combination of assets based on their dynamic conditional covariance. This portfolio will be periodically adjusted based on the DCC GARCH variance forecast in order to better adjust the weights of the portfolio dynamically.

The following subsections will describe each step in the methodology in further detail in order for the reader to get a better understanding of the underlying theory.

5.1 Autoregressive Moving Average Model

The Autoregressive Moving Average (ARMA) Model is a combination of two components: the autoregressive (AR) and moving average (MA) models. It is used in time series analysis to model stationary data that do not exhibit trends or cyclical patterns, such as seasonality. The AR component accounts for the persistence of the series, while the MA component captures the random noise present in the data. An ARMA(1,1) model would be as follows

$$r_t = \phi_0 + \phi_1 r_{t-1} + \varepsilon_t + \theta_1 \varepsilon_{t-1}, \quad \varepsilon_t \sim WN(0, \sigma^2) \tag{1}$$

Where:
r_t: Return at time t.
ϕ_0: The constant term.
ϕ_1: The autoregressive (AR) coefficient of order 1.
r_{t-1}: The return at time $t-1$.
θ_1: The moving average (MA) coefficient of order 1.
ε_t: The error term at time t.
ε_{t-1}: The error term at time t-1.

The data set will first be linearly differenced in order to remove any trend it exhibits and if it still is non stationary it will either be linearly differenced again or quadratically differenced if it shows signs of seasonality. These transformations would mean that an ARIMA or SARIMA model would not be necessary, therefore the ARMA model will be used in order to calibrate the optimal mean model for the GARCH process. (Tsay, 2010)

5.2 The GARCH Model

As this thesis will be incorporating a variation of the GARCH model in it's multivariate form, a brief introduction to the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model is deemed necessary in order for the reader to better understand the mechanics and purpose of the model.

The GARCH model is a generalization of the ARCH model, which is a model used for modelling volatility and was first introduced by Engle in 1982. One of the main purposes of the ARCH is to model a time series' volatility over time and capture the time dependency of the volatility. This model is often used for capturing volatility clustering and allows for the conditional variance to be adjusted over time as a function of residual errors of a mean process. (Engle, 1982)

The GARCH model introduces a moving average component in addition to the autoregressive component in the ARCH model. Specifically, it introduces a lag term for the variance which allows for the conditional variance to change over time based on previous changes in conditional variances. This model can be considered more accurate in predicting a time series' variance when there can be observed longer term persistence in variance.

A simple GARCH(1,1) model is as follows:

$$
\begin{aligned}
&r_t = \mu_t + \varepsilon_t, \quad \varepsilon_t = v_t \sqrt{\sigma_t^2} \quad v_t \sim N(0,1) \\
&\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2
\end{aligned}
$$

Where:

r_t : Observed time series value at time t

μ : Constant mean of the time series

ε_t : Error term at time t

v_t : shock in the variance at time t

σ_t^2 : Variance of the time series at time t

α_1 : ARCH parameter

β_1 : GARCH parameter

ε_{t-1}^2 : Squared error term at lag one (represents shocks in volatility)

σ_{t-1}^2 : Variance at lag one (represents persistence in volatility)

5.3 DCC - GARCH Model

The Dynamic Conditional Correlation (DCC) GARCH model, developed by Engle (2002), is an extension of the multivariate GARCH model designed to estimate time-varying conditional correlations between multiple time series. The DCC GARCH model is particularly useful for modeling and forecasting correlations in multivariate financial time series data, such as stock returns, exchange rates, and commodity prices. By capturing the dynamic nature of correlations, the DCC GARCH model provides valuable insights into the relationships between different assets and can be used for portfolio optimization, risk management, and other financial applications. In this thesis it will be used as a basis for a mean variance portfolio optimisation problem which will use the DCC model output and build a portfolio of five different wind parks in five different countries with the aim of minimizing the variance between the parks and therefore the internal variance of the portfolio.

Bauwens, Laurent and Rombouts (2006) explain the process in the following way: "$x_t\{t = 1,2,...T\}$ is a stochastic vector process of financial returns with Nx1 dimension and mean vector μ_t, given the information set I_{t-1}:

$$x_t|I_{t_1} = \mu_t + \varepsilon_t, \tag{2}$$

where the residuals of the process are modelled as:

$$\varepsilon_t = H_t^{1/2} z_t \tag{3}$$

and $H_t^{1/2}$ is and NxN positive definite matrix such that H_t is the conditional covariance matrix of x_t, and z_t and Nx1 i.i.d. random vector, with centered and scaled first 2 moments:

$$\begin{aligned} E[z_t] &= 0, \\ Var[z_t] &= I_N, \end{aligned} \tag{4}$$

with I_N denoting the identity matrix of order N. The conditional covariance matrix H_t of x_t may be defined as":

$$\begin{aligned} Var(x_t|I_{t-1}) = Var_{t-1}(x_t) &= Var_{t-1}(\varepsilon_t) \\ &= H_t^{1/2} Var_{t-1}(z_t)(H_t^{1/2})' \\ &= H_t \end{aligned} \tag{5}$$

For the purpose of the Dynamic Conditional Correlation model, the covariance matrix H_t can be decomposed into:

$$H_t = D_t R_t D_t \tag{6}$$

where D_t is a diagonal matrix of conditional covariances $D_t = diag(\sqrt{h_{11,t}}, ..., \sqrt{h_{nn,t}})$ and R_t is a time varying conditional correlation matrix. (Engle, 2002)

In this thesis the mean vector from equation (2) is estimated with an ARMA estimation process.

5.4 Phillips Perron Test

The Phillips-Perron (PP) test is a widely used method to test for stationarity in a time series data. A time series is said to be stationary if its statistical properties, such as mean, variance, and autocovariance, do not change over time. There is however, an important distinction to be made between a time series that is weak-stationary and a time series that is strictly-stationary. Time series is considered strictly-stationary when the joint distribution of $(r_{t_1}, ...rt_{1+k})$ is identical to that of $(r_{t_{1+t}}, ...r_{t_{1k+t}})$ for all t where k is an arbitrary positive integer and $(t_1, ..., t_k)$ is a collection of k positive integers. In other words, the joint distribution does not change under time shift. A time series is considered weak-stationary where the mean of r_t and the covariance between r_t and r_{t-m} are time invariant, where m is an arbitrary integer. (Tsay, 2010)

Testing for stationarity is important because many time series models assume that the underlying data is stationary in order to be predictable as it exhibits the trait of mean reversion as well as the constant mean and covariance described above. Mean reversion refers to the process where a time series' mean and variance always revert to the conditional mean and conditional variance given enough time. If the data is non-stationary, it may need to be transformed before being used in such models.

The Phillips-Perron test can be seen as a sophisticated evolution of the Dickey-Fuller test, a well-established method for assessing stationarity. The inherent challenge this test aims to address is the potential problem of autocorrelation and heteroskedasticity within the error term of the Dickey-Fuller test, which might lead to distorted results.

The essence of the Phillips-Perron test is captured in this model:

$$\Delta Y_t = \rho Y_{t-1} + \varepsilon_t \tag{7}$$

Where $\Delta Y_t = Y_t - Y_{t-1}$ signifies the first difference in the time series Y, ρ represents the coefficient to be estimated, and ε_t is the error term. The null hypothesis (H_0) and alternative hypothesis (H_1) in this context are:

$$H_0 : \rho = 0 \quad \text{(The time series possesses a unit root, indicating non-stationarity)}$$
$$H_1 : \rho \neq 0 \quad \text{(The time series exhibits stationarity)}$$

Calculating the test statistic starts with the estimation of the coefficient ρ using the ordinary least squares (OLS) regression. Subsequently, determining the test statistic is as follows:

$$Z(\rho) = \frac{(\hat{\rho} - 0)}{\mathrm{SE}(\hat{\rho})} \tag{8}$$

In this equation, $\hat{\rho}$ is the OLS estimate of ρ, and $\mathrm{SE}(\hat{\rho})$ is the standard error of $\hat{\rho}$. According to the null hypothesis, the test statistic $Z(\rho)$ complies with a non-standard distribution, known as the Dickey-Fuller distribution.

However, a challenge arises if the error term ε_t showcases autocorrelation or heteroskedasticity. These characteristics can skew the test statistic. The Phillips-Perron test resolves this by modifying the test statistic:

$$Z(\rho)_{\text{PP}} = \frac{(\hat{\rho} - 0)}{\text{SE}(\hat{\rho})_{\text{PP}}} \tag{9}$$

In this scenario, $\text{SE}(\hat{\rho})_{\text{PP}}$ is the Phillips-Perron adjusted standard error. This adjustment incorporates non-parametric corrections for the OLS standard error, which account for the autocorrelation and heteroskedasticity of the error term.

The execution of the test involves comparing the calculated $Z(\rho)_{\text{PP}}$ statistic with critical values derived from the Dickey-Fuller distribution. If the absolute value of the test statistic exceeds the critical value at a designated significance level, we reject the null hypothesis, concluding that the time series is stationary. (Phillips, P.C.B. and Perron, P., 1988)

5.5 Ljung-Box Test

An statistical method for testing for autocorrelation in a set of time series data is the Ljung-Box Test (LB test). The LB test, created by Ljung and Box in 1978, is a modification of the Box-Pierce test, another statistical Q test, and offers a finite-sample correction.

The Ljung-Box test typically posits the following hypotheses:

$$H_0 : \textit{The autocorrelations are all zero}$$

$$H_1 : \textit{At least one autocorrelation is non-zero}$$

The test statistic, used to assess these hypotheses, is given by:

$$Q = n(n+2)\sum_{k=1}^{h} \frac{{p_k}^2}{n-k} \tag{10}$$

In this equation, n represents the sample size, p_k denotes the autocorrelation at lag k. The test statistic follows a chi-squared (χ^2) distribution with h degrees of freedom.

The LB test will be utilized in the analysis to determine whether the models estimated exhibit autocorrelation in their residuals. If the residuals show signs of autocorrelation, it could mean that the models do not adequately capture the correlation within the data and might need to be re-estimated with different lags. A lag value of 10 will be used in the test for the models during the different time periods and this decision is based on Tsay (2010) where they observed that simulation studies suggested that the number of lags be roughly calculated as the log() of the number of observations. For the three different periods the $log(n) \approx 10$ therefore it was kept constant for all models. (Ljung, G.M. and Box, G.E.P., 1978)

5.6 ARCH-LM Test

The Autoregressive Conditional Heteroskedasticity Lagrange Multiplier test (ARCH-LM test) is a statistical test for volatility clustering within a time series and is used to detect if the variance of a series changes over time and was developed by Engle in 1982. The test takes the residuals of a mean model and tests the squared residuals of the model for ARCH effects, where the null hypothesis is that there are no ARCH effects present in the squared residuals and the alternative hypothesis is that there are some ARCH effects. The following regression is used for the test:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + ... + \alpha_p \varepsilon_{t-p}^2 + e_t \tag{11}$$

Where e_t is a White noise error term. The above mentioned hypothesis is then written as follows:

$$H_0 : \alpha_1 = ... = \alpha_p = 0$$
$$H_1 : \alpha_1 = ... = \alpha_p \neq 0$$

In order to test the null hypothesis the Lagrange Multiplier test is used, it is where the ”LM” part of the ARCH-LM name comes from and the multiplier test is defined as follows:

$$LM = (T-p)R^2 \; \chi^2(p) \tag{12}$$

Under the null hypothesis we have no ARCH effects present while under the alternative at least one of the parameters is different from 0. If the alternative hypothesis cannot be rejected it would indicate that a models such as the ARCH or GARCH would be required to appropriately account for and model the time varying conditional variance. (Engle, 1982)

5.7 Mean Variance Portfolio

The Mean Variance Portfolio theory is a popular popular portfolio optimization method where it is assumed that investors have perfect information and make decisions based on that to construct a portfolio with the best risk/reward profile, where the best portfolio for a given risk level gets plotted and is known as the efficient frontier. This theory was developed by Harry Markowitz in the early 1950s and has been used in finance ever since. The optimization is usually computed using quadratic programming where the goal is to minimize the portfolio variance with a given expected return. The most commonly used version of this optimization is defined as follows:

$$\begin{aligned} \text{Minimize:} \quad & \frac{1}{2} w^T \Sigma w \\ \text{Subject to:} \quad & w^T \mu = \mu_p \\ & w^T \mathbf{1} = 1 \end{aligned}$$

Where:
w denotes the weights vector.
Σ is the covariance matrix of asset returns.
μ is the growth rate vector.
μ_p is the target portfolio growth rate.
$\mathbf{1}$ is a vector of ones.

This quadratic programming problem will be used on the covariance matrix's estimated by the DCC GARCH forecasts as well as the different Sample Covariance matrix's.

When it comes to electricity markets and the attributes of physical electricity production, the theory is not quite as useful as there is no certain risk and reward with renewable production except for the uncertainty of what price will be paid for the electricity produced. The theory also assumes a target portfolio return which in the case of a renewable energy portfolio would be the growth rate of the portfolio, which is not necessarily a parameter to be optimized on, unless the purpose of the portfolio optimization is production growth. This makes the minimum variance portfolio appealing as it can be calibrated to minimize the variance of electricity production within a portfolio giving it the highest chance of receiving most value for the electricity as the overall production profile should stay more consistent and therefore have more production during higher priced period than if it was not diversified. This is due to the negative correlation between renewable electricity production and electricity prices, explained earlier in this thesis. (Markowitz, 1952)

5.8 Out-of-Sample forecasting

This project will make use of the Out-of-Sample(OOS) forecasting method for estimating and subsequently testing forecasting models. OOS and also enables evaluation of forecasting studies with a time series dataset.

Out-of-Sample forecast splits the data into two parts, where the first part is used for estimating a predictive model and the second part is used for testing the model. Since there is no data available for the future, a part of data will be used for testing the developed model in order to evaluate its effectiveness. Meaning that out of the available data, around 2/3 of the portion of the data will be used to estimate a model while the rest will subsequently be used for testing. This provides an opportunity to test a model on a sample that was not used on estimating it therefore providing insight into how the model works with unknown data. In this project, the first training period is from the 1st to the 17,517th observation, and subsequent testing periods add 8760 observations one before it while the first testing period is from the 17,518th observations to the 26,278th period. The testing period will then be shifted by 8760 observations each time. (Wooldridge, 2015)

5.9 Forecasting Evaluation

In order to determine which forecasting model is the most accurate of the ones tested the volume weighted average price of portfolios constructed based on a minimum variance optimisation with the different correlation forecasts as well as the sample covariance. The model with the the highest volume weighted average price would be the most desirable as it gives more tangible and financially relevant results.

6 Data

This thesis aims to construct a portfolio of renewable production assets diversified geographically. The goal is to smooth out the portfolio's overall production profile by minimizing it's variance and, consequently, achieve the highest volume-weighted average price possible.

The study uses historical production data from wind farms located in Denmark, Belgium, Germany, Finland, and Italy, spanning from January 1, 2018, to December 31, 2022. This data presents the most accurate depiction of renewable production as it comprises a time series of actual hourly production instead of forecasts.

Country selection was influenced by a study conducted by Eurostat, examining correlation between countries during heating and cooling days. Furthermore, access to production data through Centrica Energy Trading's actual physical asset portfolio limited the choice of countries. (Eurostat, 2023)

In addition to the physical production hourly values during the period, hourly spot prices will be used to evaluate the constructed portfolio's performance and discern potential modifications to improve it.

The data from Germany and Belgium underwent transformation as it initially represented quarter-hour intervals. To align with the hourly data from Finland, Denmark, and Italy, the four quarterly values of each hour were averaged to yield the hourly value.

To ensure stationarity of the production data, which is a prerequisite for effectively estimating the empirical model, the logarithmic function was applied and the difference was calculated, yielding the $Log(Return)$. Although termed as Log Return, it, in reality, signifies the rate of growth of production. Stationary data boasts constant mean, variance, and lacks seasonality and trend, facilitating more precise and straightforward forecasting. The computation of $Log(return)$ is as follows:

$$Log(growthrate_t) = Log(production_t) - Log(production_{t-1}) \tag{13}$$

This calculation will be repeated from $t = 1, ..., t = k$.

Finally, the data for each country is split into distinct training and testing periods for model estimation over different time periods. The initial training period starts from January 1, 2018, and concludes on December 31, 2019, furnishing the model with two years of hourly data. The corresponding testing period spans the following year.

As the testing progresses, the training period extends by one year, integrating the previous testing period. The testing period, however, remains a constant duration of one year but moves forward in time. This pattern continues until the final training period, which runs from January 1, 2018, to December 31, 2021, with the final testing period from January 1, 2022, to December 31, 2022.

7 Empirical Analysis

The objective of this analysis is to compare and evaluate the performance of three different portfolio construction methods, specifically: (1) a Dynamic Conditional Correlation (DCC) GARCH portfolio, (2) a portfolio based on sample covariances, and (3) an equally weighted portfolio.

Modern portfolio theory suggests that the optimal allocation of assets within a portfolio depends not only on the expected returns of individual assets, but also on the correlations between these assets. Traditionally, correlations are assumed to be constant over time or are estimated using a simple sample covariance matrix. As stated earlier the expected return of a renewable electricity producer is not the same as an expected return of an equity based asset such as stocks. Due to this difference there will be a slight adjustment made to the minimum variance optimization

In this analysis, we will construct portfolios based on these four methods and compare their performances over the period from 2020 to 2023. The main criterion for comparison will be the portfolio value, which reflects both the return and risk characteristics of the portfolio. By comparing the portfolio values obtained by the different methods, the aim will be to gain insights into the practical implications of the different portfolio construction methods and the potential benefits of using advanced models such as DCC for portfolio allocation.

As explained in the methodology section, before doing any model estimation the data needed to inspected and transformed in order for the data to be usable. The data for the five countries came in one excel sheet that had to be split up into five different time series. The different time series were then logged and differenced as they showed signs of seasonality and trend which would make estimating and forecasting more cumbersome. Below is an example of the raw data from Italy. It can be easily observed that there are signs of seasonality and trend in the data.

After logging and taking the first difference of the raw production data, the data looks to become stationary if plotted. Of course this was also confirmed by the use of Phillips-Perron test for stationarity where each test gave a p-value of ¡= 0.01. From Figure 2 it can be observed that the raw production data does indeed appear stationary.

After transforming each of the time series, they were then combined into a growth rate matrix for the DCC GARCH estimation later in the analysis. Before the GARCH estimation process, a mean model estimation for each time series was performed as the GARCH estimation and specification will be based on the residuals of the mean models.

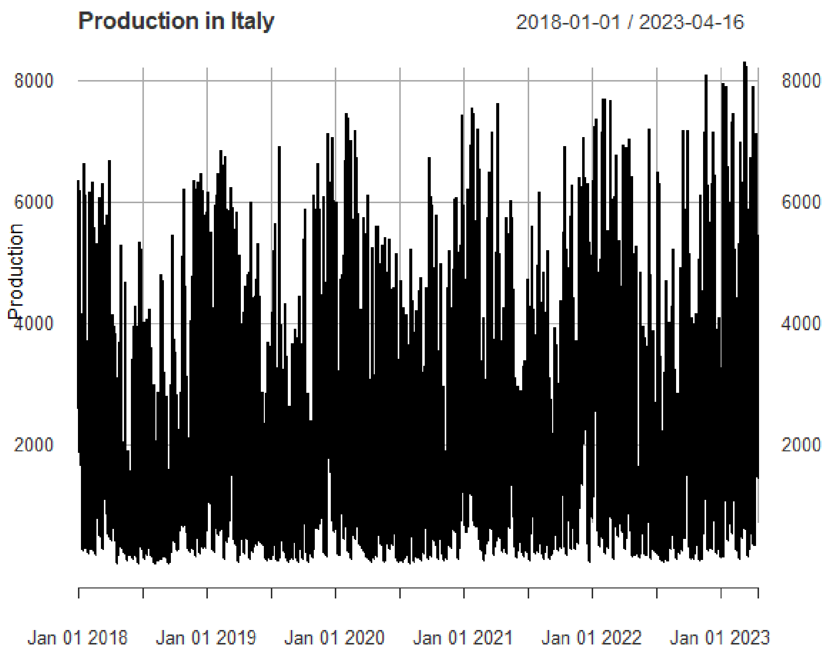


Figure 1: Raw Italy Production Data; (Appendix)

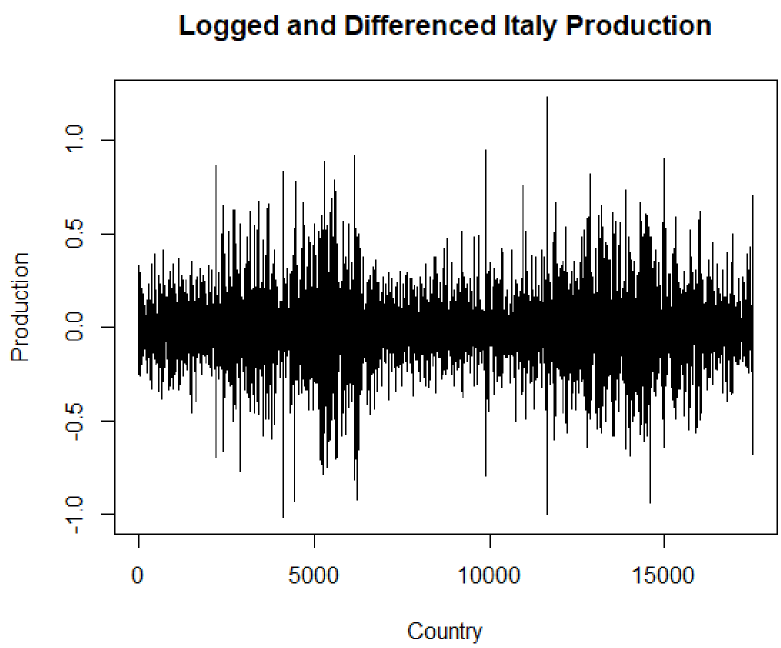


Figure 2: Stationary Italy Production Data; (Appendix)

7.1 Mean model estimation

As the time series' were logged and differenced and were indeed deemed stationary by the Phillips-Perron test, there was no reason for a SARIMA model estimation as there was no seasonality or trend in the stationary time series, the only attributes the data exhibited was mean reversion and volatility clustering, which is ideal for model forecasting.

After confirming the stationarity of the data, the partial autocorrelation function for all countries was plotted in order to understand how many AR parameters would be adequate for an AR or ARIMA model estimated on the data. Below in Figure 3 is the PACF plot of Danish production in the first training period, it can be observed that there is some correlation of residuals from the first lag to the sixth but the seventh lag has almost none. All ACF and PACF figures can be found in the Appendix. For the models that pass the LB test of autocorrelation, they will then be compared by their respective Akaike Information Criterion (AIC) and the model with the lowest AIC will be used as the respective country's mean model.

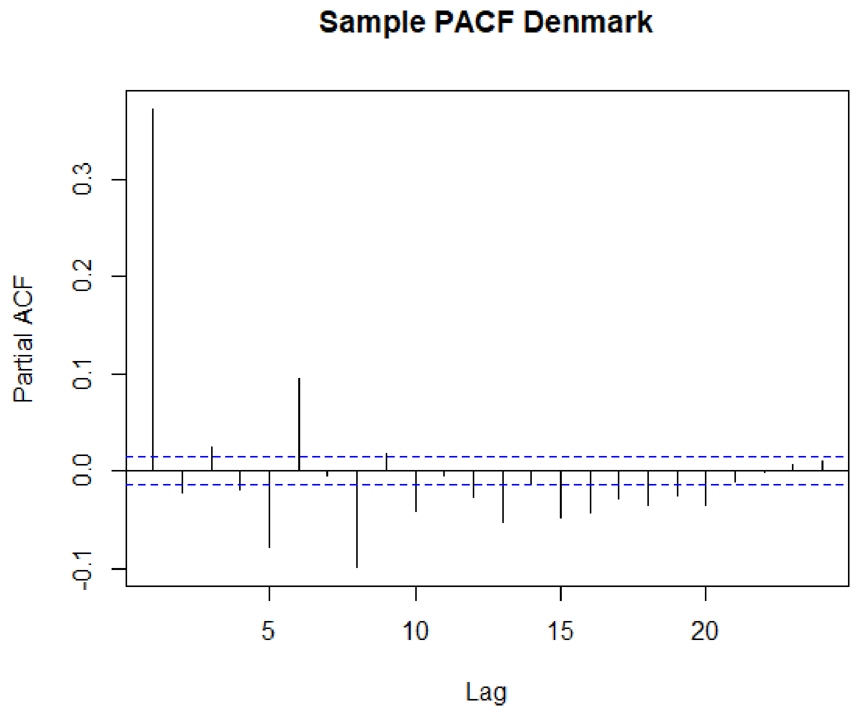


Figure 3: PACF plot of Danish Production; (Appendix)

After estimating many different models with different ARIMA parameters for each country in each training period, the following model specifications resulted in the least information loss as implied by the AIC.

As can be seen in Table 1, many of the ARIMA model parameters stay consistent for two out of the three different training periods indicating that the dynamics in the data do not change significantly enough over the years to warrant different models. From these ARIMA models the residuals were then extracted and tested for autocorrelation by the use of the Ljung Box Test. If the residuals show signs of any significant correlation, it would indicate that the mean models were not adequately fitted to the data and therefore would need to be re-estimated. As confirmed with the Ljung Box Test, the residuals of the ARIMA models estimated did not show

Model and Forecasting year	AR (P)	I (D)	MA (Q)
DK Model 2020	12	0	2
DK Model 2021	10	0	0
DK Model 2022	10	0	0
FI Model 2020	10	0	1
FI Model 2021	10	0	1
FI Model 2022	14	0	4
IT Model 2020	7	0	1
IT Model 2021	8	0	1
IT Model 2022	8	0	1
BE Model 2020	15	0	0
BE Model 2021	15	0	0
BE Model 2022	15	0	0
DE Model 2020	15	0	0
DE Model 2021	15	0	0
DE Model 2022	20	0	0

Table 1: Different ARIMA Model Parameters

any signs of significant autocorrelation, meaning that the models were adequately fitted as all tests resulted in a p-value above 0.05, therefore the residuals could be used for the GARCH estimation.

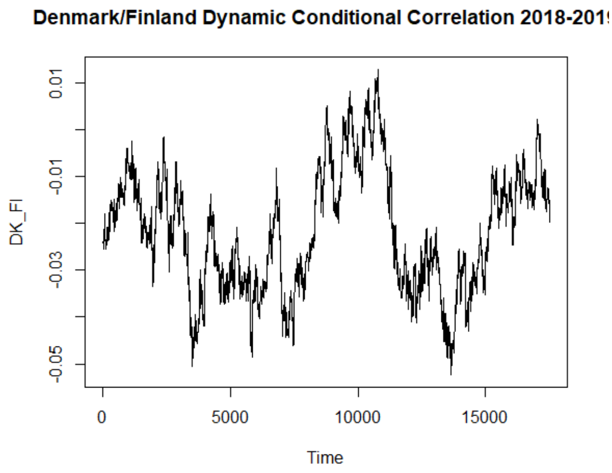
The ARCH-LM test was also utilized to test the squared residuals of the ARIMA models for Autoregressive Heteroskedasticity (volatility clustering) as if the residuals would not exhibit any volatility clustering, an ARCH/ GARCH model would not be required to forecast the time series. The ARCH-LM test confirmed what could be observed by the logged and differenced production data above in Figure 2 that there is indeed volatility clustering with p-values of each models squared residuals being well below 0.05 or 2.2e-16 to be specific, confirming that a GARCH model could be adequate at forecasting the volatility of the time series'.

7.2 DCC GARCH Model

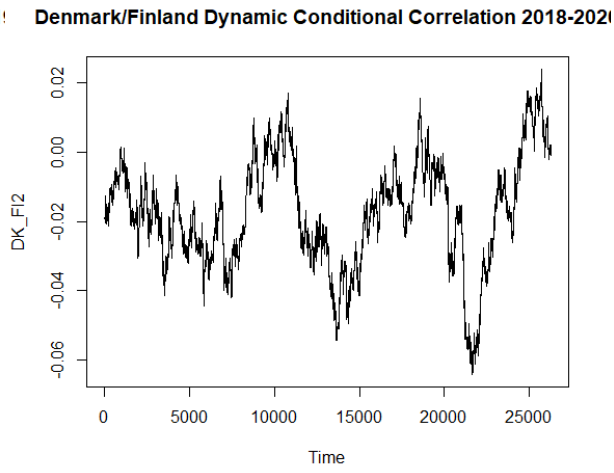
After estimating the mean models and extracting the residuals from them, univariate GARCH models with an ARMA(0,0) mean were specified and combined into a multivariate GARCH specification. First the GARCH models were constructed based on the PACF plots of each country and used as a starting point for the number of ARCH parameters. They were then fitted to the data and tested for autocorrelation of standardized residuals as well as heteroskedasticity. All GARCH models reject the null hypothesis of the Ljung Box test indicating autocorrelation in the standardized residuals. This does not necessarily mean that the models are not useful as the GARCH model inherently accounts for autocorrelation with the lagged variance terms. All models except for the Germany model failed to reject the null hypothesis of the ARCH-LM test during the first training period, indicating that they capture the significant volatility clustering in the data. During the second and third training periods the Italy model also rejected the null hypothesis of the ARCH-LM test no matter the specification, thus the specification that was closest to not rejecting was chosen. The same was observed with the Finland specification during the third period and the same process was used for it as Italy. Multiple attempts were made to specify a GARCH model for Germany that would pass the ARCH-LM test but to no avail, therefore the commonly used GARCH(1,1) was used for that country in all periods as all the specifications resulted in the same p-value.

After the estimation and fit of the univariate GARCH models, a DCC GARCH(1,1) model was estimated from multivariate specification of the univariate GARCH models and fit to the data. From the output of the DCC(1,1) model all lagged parameters further from t than $t-1$ were observed as insignificant with their p-values well above 0,05 indicating that though the GARCH models appear to have captured most of the volatility clustering in the time series', the model itself can be deemed quite insignificant. Due to this insignificance of parameters it was decided that a model with GARCH (1,1) specifications would also be constructed as the DCC GARCH(1,1) model output resulted in all parameters being significant at the 0,05 level. These models will then be compared against each other in order to observe if the insignificant model that captures the volatility leads to higher value than a significant model that does not. Figures for the six fitted DCC GARCH model outputs can be found in the Appendix (Figures 16-21)

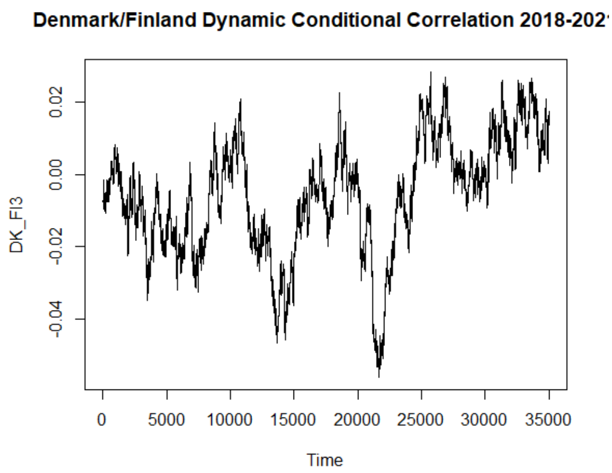
The reason for the ARMA (0,0) specification in the GARCH specification is due to the fact that the DCC GARCH will be fit to the already constructed residual matrix of the different ARIMA models, therefore there is no need to include ARMA models with any lag parameters in the univariate GARCH specification. This multivariate specification was then used for the DCC (1,1) specification as the DCC is a multivariate model. After the DCC (1,1) model had been specified it was then fit to the combined residual matrix of the ARIMA models. This was done for all periods and below are figures showing how the dynamic conditional correlation between Denmark production and Finland production changes over the different periods:



(a) DK/FI Correlation between 2018-2019



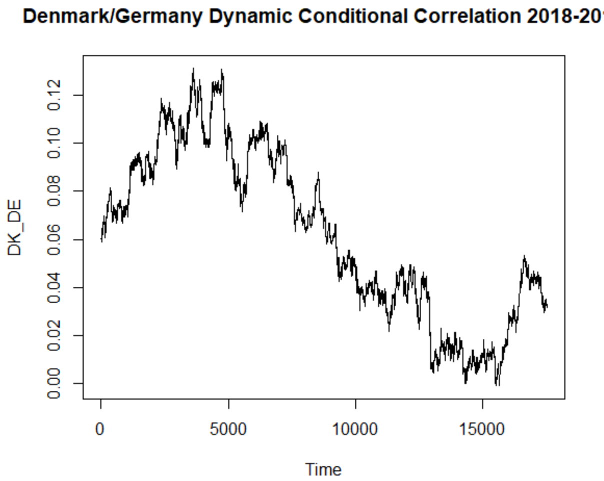
(b) DK/FI Correlation between 2018-2020



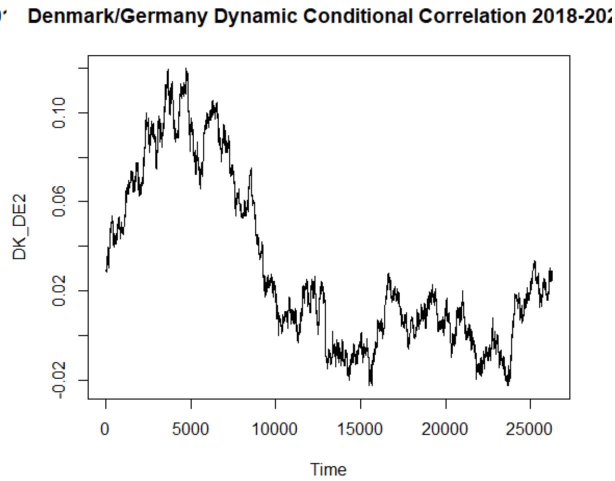
(c) DK/FI Correlation between 2018-2021

Figure 4: Denmark/Finland Correlation Development

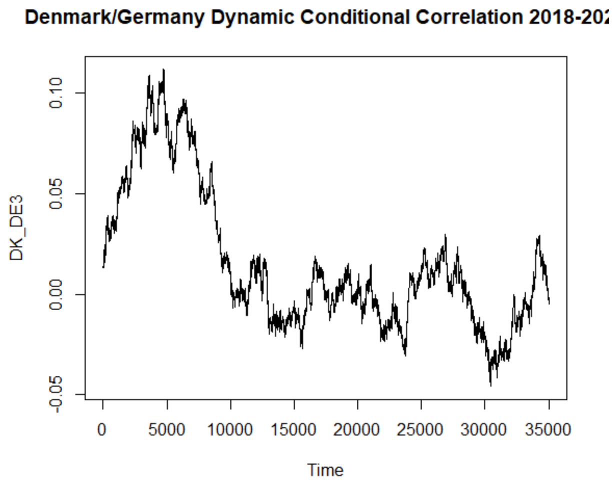
These Figures show that the correlation dynamics between the two countries fluctuate quite a bit where in the period from 2018-2020 there is consistent negative correlation between the countries but between 2020-2021 the dynamics change and the countries are mostly positively correlated during that period. This change in the correlations would intuitively necessitate a change in portfolio weights of the two countries. This is also reflected in the model forecasts and minimum variance optimization, which will be elaborated on later in this chapter. Another correlation pair that is interesting to look at is Denmark and Germany production as they exhibit the opposite of the Denmark/Finland correlation being positively correlated in the first period and then dynamically flipping between being positively and negatively correlated.



(a) DK/DE Correlation between 2018-2019



(b) DK/DE Correlation between 2018-2020



(c) DK/DE Correlation between 2018-2021

Figure 5: Denmark/Germany Correlation Development

This change in the correlation structures of the different countries could indicate that a model such as the DCC GARCH would be more optimal for portfolio optimization than a model that assumes a fixed correlation structure over the whole period. All correlation Figures can be found in the Appendix.

For the minimum variance optimization of the DCC GARCH models three forecasts are made, the first forecast is one day ahead, the second is one month ahead and the third is one year ahead. These forecasts are in hourly granularity which results in 24, 720 and 8760 correlation and covariance matrix forecasts respectively. As these are very large numbers of matrix forecasts the last days forecast of each is used as that is the end point of the forecast. From the 24 hourly matrix's the 13th hour matrix is used for the minimum variance optimization. The reason for the choice of the 13th hour forecast is that it is the hour during the day where the most extreme prices most often occur, which is due to the highest level of photovoltaic production of the day and the highest level of consumption during the day. This combination can lead to some interesting price dynamics in the electricity markets and was therefore deemed the most significant hour of the day.

After the DCC GARCH forecasts were made from the different training periods, quadratic programming was used in order to optimize the weights of each country for minimizing the portfolio variance. This is the same procedure as Markowitz' Mean Variance Portfolio Theory (1952) popularized for equity based portfolios. The standard MVP assumes a target return of the portfolio, which in this case would be a target growth rate of the different countries. As the growth rate of each country is not a variable that has a target value which can be associated with a higher return, the average growth rate for the training period is used as it reflects the average over the period.

In order to stay coherent with common risk management and diversification practises there are constraints on the lower and upper bound of the portfolio weights, these bounds are 10% and 40% respectively. This is done in order to ensure a well diversified portfolio as it is possible that a solution to the minimization problem would be significantly skewed to one country and omit others.

With this information in mind the quadratic program is expressed as follows:

$$\begin{aligned}
\text{Minimize:} \quad & \frac{1}{2} w^T \Sigma w \\
\text{Subject to:} \quad & w^T \mu = \mu_p \\
& w^T \mathbf{1} = 1 \\
& 0.10 \leq w_i \leq 0.40 \quad \text{for all } i = 1,..,5
\end{aligned}$$

Where:
w denotes the weights vector.
Σ is the covariance matrix of asset returns.
μ is the growth rate vector.
μ_p is the target portfolio growth rate.
1 is a vector of ones.
The constraint $0.10 \leq w_i \leq 0.40$ enforces that each weight should be between 10% and 40%.
The constraint is applied to all assets, with i ranging from 1 to 5 to denote each of the 5 assets.

This optimization was done for all three forecasts and for all three training periods and resulted in the following portfolio weights:

Forecast for 2020					
Forecast	Denmark	Finland	Italy	Belgium	Germany
1 day	13,44%	18,42%	28,10%	30,05%	10,00%
1 day*	10,68%	18,68%	30,92%	29,72%	10,00%
1 month	15,95%	12,21%	30,07%	31,77%	10,00%
1 month*	12,57%	10,00%	35,47%	31,97%	10,00%
1 year	15,88%	11,98%	30,32%	31,81%	10,00%
1 year*	16,08%	10,50%	31,24%	32,18%	10,00%

Table 2: DCC GARCH Portfolio Weights for 2020, * signifies the insignificant models

Forecast for 2021					
Forecast	Denmark	Finland	Italy	Belgium	Germany
1 day	23,16%	10,00%	31,18%	25,66%	10,00%
1 day*	29,75%	10,00%	34,28%	15,98%	10,00%
1 month	12,76%	12,59%	24,65%	40,00%	10,00%
1 month*	13,42%	10,00%	26,58%	40,00%	10,00%
1 year	12,51%	12,74%	24,28%	40,00%	10,00%
1 year*	13,28%	10,55%	26,17%	40,00%	10,00%

Table 3: DCC GARCH Portfolio Weights for 2021, * signifies the insignificant models

Forecast for 2022					
Forecast	Denmark	Finland	Italy	Belgium	Germany
1 day	24,93%	12,71%	24,13%	28,22%	10,00%
1 day*	25,84%	10,92%	22,69%	30,55%	10,00%
1 month	13,29%	20,99%	15,71%	40,00%	10,00%
1 month*	17,35%	16,67%	15,98%	40,00%	10,00%
1 year	10,00%	24,51%	15,49%	40,00%	10,00%
1 year*	18,81%	15,11%	16,08%	40,00%	10,00%

Table 4: DCC GARCH Portfolio Weights for 2022, * signifies the insignificant models

Looking at the three tables it is clear that for most countries the weights change based on the forecast horizon ie. one day, one month or one year. One exception is Germany which is allocated at the lower bound of 10% for all forecasts. This might indicate that the internal variance of the production is so high that it warrants the lowest allocation in all periods. Interestingly, the insignificant models do not vary that much form the significant ones but it will be interesting to see how they perform in comparison to the significant ones. The performance of these different portfolios will be elaborated on once the sample covariance portfolio has been explained.

7.3 Sample Covariance Portfolio

In order to observe how well the DCC GARCH portfolio performs in comparison to other methods of establishing portfolio weights, a portfolio based on the sample covariance of the different training periods is constructed. This portfolio has the same conditions as the DCC GARCH portfolios where the lower and upper bounds are set for the portfolio weights. The largest difference is that the sample covariance can not be forecasted as it is not a model and therefore only one portfolio is constructed for each of the periods. The quadratic problem is the same as for the DCC GARCH with the difference being the covariance matrix used here is the sample covariance matrix while it was the forecasted covariance matrix for the DCC GARCH.

The resulting portfolio weights are listed in the table below:

Sample Covariance					
Forecast	Denmark	Finland	Italy	Belgium	Germany
2020	12,96%	24,60%	32,83%	10,84%	18,77%
2021	16,28%	19,90%	32,72%	10,56%	20,54%
2022	16,72%	20,11%	32,70%	10,00%	20,47%

Table 5: Sample Covariance Portfolio Weights

Interestingly the difference in weight allocations from the second to the third period is marginal, indicating that the sample covariance does not display significant change with the addition of 2020 in the sample mix.

7.4 Equally Weighted Portfolio

The third portfolio that was constructed for the DCC portfolio performance evaluation is the naive equally weighted portfolio. It is naive in the sense that it does not do any analysis or estimations and only weights the countries equally so the portfolio weight equals 100%. This is not something that would be realistic in practice as risk and portfolio management can be highly complex. Nevertheless it was chosen as a benchmark to evaluate if the more complicated methods of the DCC GARCH and sample covariance portfolios would pay dividends and result in better value.

The portfolio weights are listed in the table below for coherence:

Equally Weighted					
Forecast	Denmark	Finland	Italy	Belgium	Germany
2020	20%	20%	20%	20%	20%
2021	20%	20%	20%	20%	20%
2022	20%	20%	20%	20%	20%

Table 6: Equally weighted portfolios

7.5 Portfolio Performance

Now that all the different portfolios have been estimated their financial value is required in order to evaluate their effectiveness in providing the most value for the electricity produced. The method of calculating each portfolios respective value will be the volume weighted average price of each portfolio. The volume weighted average price is calculated as the sum of each hours production multiplied by the respective hours spot price, this is done for each hour within a year where the sum of all hourly weighted prices are then divided by the total volume produced within the respective year, this gives more weight to the prices where the production is highest and gives a more accurate view the price within a given year. These volume weighted average prices are then multiplied by the respective country weights estimated by the different models resulting in the portfolio VWAP. As the portfolios were constructed using Mean Variance Optimisation, each portfolio's internal variance is also calculated in order to present the value with respect to its risk. Below are tables presenting the performance of each portfolio in a given year:

Portfolio Performance 2020		
Portfolio	VWAP	Portfolio Variance
DCC day	29,27 EUR/MW	1,08%
DCC day*	29,69 EUR/MW	1,09%
DCC month	29,46 EUR/MW	1,12%
DCC month*	30,24 EUR/MW	1,17%
DCC year	29,49 EUR/MW	1,12%
DCC year*	29,60 EUR/MW	1,14%
Sample Covariance	29,21 EUR/MW	0,78%
Equally Weighted	27,73 EUR/MW	0,80%
Average	29,03 EUR/MW	0,98%

Table 7: Portfolio Performance in 2020, * signifies the insignificant models

Portfolio Performance 2021		
Portfolio	VWAP	Portfolio Variance
DCC day	98,55 EUR/MW	0,88%
DCC day*	99,57 EUR/MW	0,93%
DCC month	95,74 EUR/MW	1,43%
DCC month*	97,03 EUR/MW	1,43%
DCC year	95,15 EUR/MW	1,43%
DCC year*	96,75 EUR/MW	1,43%
Sample Covariance	96,55 EUR/MW	0,56%
Equally Weighted	91,70 EUR/MW	0,72%
Average	95,54 EUR/MW	1,00%

Table 8: Portfolio Performance in 2021, * signifies the insignificant models

Portfolio Performance 2022		
Portfolio	VWAP	Portfolio Variance
DCC day	201,93 EUR/MW	0,93%
DCC day*	201,93 EUR/MW	1,01%
DCC month	186,08 EUR/MW	1,46%
DCC month*	190,03 EUR/MW	1,42%
DCC year	182,87 EUR/MW	1,51%
DCC year*	191,45 EUR/MW	1,41%
Sample Covariance	202,83 EUR/MW	0,59%
Equally Weighted	190,50 EUR/MW	0,72%
Average	192,84 EUR/MW	1,04%

Table 9: Portfolio Performance 2022, * signifies the insignificant models

By looking at the tables it can be seen that in all periods, a portfolio optimized on the DCC GARCH forecasts brings the most value, which is reflected in the volume weighted average price of the portfolios. Interestingly during the year 2020, six out of five portfolios delivered a VWAP within half a euro of each other, indicating that when prices are more stable it might be harder to squeeze out much extra value. It is however clear that during 2022 there is much value to be created by establishing a portfolio with a non equal weights and that is where the Sample Covariance portfolio comes out on top, significantly beating out five of the eight portfolios with only the DCC GARCH 1 day portfolios delivering within one EUR/MW in value. Comparing the insignificant models whose univariate GARCH models adequately captured most of the volatility clustering effects to the significant models whose univariate GARCH models did not capture the volatility clustering. It becomes clear that a model that more accurately captures the volatility results in more value as the insignificant models beat the significant ones in every instance except for the one day model in 2022 where they bring equal value.

In relation to portfolio performance, looking only at the volume weighted average price does not always tell the whole story. It is also important to look at each portfolios variance as it measures the risk of each portfolio and compare each portfolios variance like was done for the VWAP. During the three periods the portfolio optimized on the sample covariance had the lowest variance of all the portfolios estimated, meaning that it produced the most consistent amount of electricity. It then becomes a trade off between nominal financial value of the DCC portfolios and the production stability of the sample covariance portfolio as the DCC* 1 day and 1 month portfolios provided a higher VWAP than the Sample Covariance in 2020 and 2021.

As the tables above show, the portfolio which was based on minimizing the covariance from the 13th hour of the last day from any of the DCC GARCH forecasting horizons produced more value than the equally weighted portfolio in all three years. Comparing it to the equally weighted portfolio it does indeed indicate that a more elaborate method of estimating portfolio weights does provide additional value to a portfolio of renewable energy assets. The tables also show that a DCC GARCH model based on a multivariate specification where most of the heteroskedasticity is captured, despite the DCC GARCH model itself being deemed insignificant, outperforms a DCC GARCH model that does not capture the heteroskedastic properties of the data but is significant.

When comparing the portfolios to each individual country's volume weighted average price for each period it is clear that the diversified portfolios do perform better than four out of the five individual countries, with only Italy providing the highest average price in each period. However, looking only at the average price of the portfolios and countries does not paint the whole picture as the standard deviation of prices for each country describe how wide of a span the prices are on. Below is a table documenting the average price of each country as well as one standard deviation of their price for each year.

Volume Weighted Average Prices and Standard Deviation					
	Denmark	Finland	Italy	Belgium	Germany
2020					
VWAP	21,54 EUR	25,43 EUR	37,30 EUR	28,77 EUR	25,58 EUR
Std.Dev	17,44 EUR	21,11 EUR	14,53 EUR	16,54 EUR	17,21 EUR
2021					
VWAP	87,93 EUR	67,65 EUR	127,59 EUR	90,06 EUR	85,26 EUR
Std.Dev	64,75 EUR	65,99 EUR	80,49 EUR	79,45 EUR	73,70 EUR
2022					
VWAP	195,40 EUR	109,81 EUR	286,92 EUR	186,44 EUR	173,91 EUR
Std.Dev	145,41 EUR	132,37 EUR	132,28 EUR	134,66 EUR	141,60 EUR

Table 10: Average Prices and Standard Deviations

The table shows that for all of the countries that 68% (one standard deviation) of the time, the received price of electricity is positive but once the price falls two standard deviations from the mean, the price of electricity can become negative in all countries except Italy in 2022. This shows how wide the spectrum of prices is and is important to take into consideration when constructing renewable energy portfolios. The table also shows that it would be most profitable to construct a portfolio that only includes Italy. That however, does not follow common practises of risk management and diversification and as is commonly said with financial performance of equities: past performance is not indicative of future performance.

This section has showed that utilizing a diversification method that is more intricate than the naive equally weighted portfolio does indeed result in higher financial value of a renewable energy portfolio consisting solely of wind producing assets. In this analysis, mean variance optimisation has been utilized to construct multiple portfolios based on Dynamic Conditional GARCH forecasts and sample covaraince estimations. The DCC GARCH forecasts estimate multiple covariance matrixes and those are then used for the mean variance optimization through the use of quadratic programming. The same was done for the sample covariances. Interestingly enough not all DCC GARCH portfolios beat the equally weighted portfolio, specifically the statistically significant 1 month and 1 year portfolios in 2022 indicating that complexity is not always better. However, the right level of complexity might be better than none as both of the 1 day DCC portfolios and the sample covariance portfolio beat the equally weighted in all periods. This does indeed indicate that geographical diversification within an renewable energy portfolio based on estimation methods more complex than the naive equally weighted model does result in higher financial value of the portfolios.

8 Discussion

As this thesis shows, the use of empirical models commonly used in finance can translate well to the renewable energy market with the positive effects of diversification also present in these markets, just like in the broader financial markets. There are however different risk factors that need to be investigated as well such as the costs associated with producing more or less electricity than expected when prices are very volatile and extreme. The latest example of extreme prices took place on the 28th of May 2023 where for some hours around noon in most countries in Western Europe, electricity prices were negative.(EPEX SPOT, 2023)

Most countries exhibited negative prices between the 12th and the 15th hour of the day with prices reaching as low as -400€ in the Netherlands, meaning that it costs 400€/MW for a producer of electricity to supply the grid with electricity. This is very costly for any electricity producer whether they are producing renewable electricity or not, now imagine if a wind park or solar park produces more than expected during the Day-Ahead auction where they sold off all the electricity they expected to produce, it would further increase their losses. Due to this increased negative price risk it could be appropriate to estimate Value at Risk models alongside the Mean Variance Portfolios in order to assess the tail risk when these significant price events occur as they can have drastic implications on the profitability of establishing a portfolio of renewable energy assets.

One of the limitations of this thesis is that it estimated the portfolios based on market wind data, meaning that the data is an aggregate of many different wind parks all over the individual countries, except for Germany where the data comes from wind parks located in the eastern part of Germany. Due to this limitation it is hard to estimate the volumetric risk of any portfolio (how much electricity is produced versus how much was forecasted). It could be beneficial for future research if the volumetric risk was also profiled and therefore quantifiable as it could then be used in a Value at Risk model, especially if there are clear patterns in the difference between forecasted and actual production. This wou This would only be possible with wind park specific data where both realized historical production and historical expected production was available.

Another limitation to the analysis is the fact that the GARCH models that passed the test for autocorrelation of the residuals were insignificant when modelled with the DCC GARCH process, therefore resulting in the model itself being insignificant. Due to that it was decided for the purposes of the thesis to compare the financial performance of the insignificant models and the significant ones. The difference in the superior financial performance of the insignificant models over the significant ones does raise questions regarding the power of the significant models as the analysis does indicate that despite the DCC GARCH models being insignificant, they still perform better due to their ability to better capture the volatility structure of the data which in practice is more appealing than statistical significance.

Another expansion on this thesis could be the inclusion of photovoltaic assets in the portfolio and how the volatility spillover and complementary of each technology affects the portfolio weights in addition to a Value at Risk model, similar to the work of Song, F., Cui, J. and Yu, Y. (2022) and Jurasz, J. et al. (2020) as there has been observed a relationship between the two technologies. An interesting research area could be how the surface heating of the earth by the sun can affect the amount of wind energy in a given geographical area and if that could lead to higher volumetric risk within a portfolio that is concentrated in that area with both solar and wind production. As this weather environment would lead to depressed prices in those periods and therefore lead to lower revenue for the portfolio. (Libretexts, 2022)

9 Conclusion

The purpose of this thesis was to investigate whether models commonly used in financial markets for portfolio optimisation would be effective in creating value in the renewable energy markets. The Dynamic Conditional Correlation GARCH model was the main model of focus where it forecasted dynamic conditional covariances between renewable production in Denmark, Finland, Italy, Belgium and Germany for three different years. Those covariance matrix's were then used for the commonly used Mean Variance Optimisation process popularized by Markowitz in 1952 and the portfolios constructed from the optimization were compared to a naive equally weighted portfolio. The DCC portfolios were also compared against a less complicated Mean Variance optimization with the Sample Covariance portfolios. A significant difference between the value of a renewable energy portfolio and an more classic equity portfolio is that for each stock in a portfolio there is one price, the market price at each given point in time. For the renewable energy portfolio the value of the portfolio is usually calculated as the spot price multiplied by the amount of energy produced divided by the sum of the production, resulting in the volume weighted average price where more weight is given to prices where production is higher, therefore being more important than the average price.

This thesis showed that the six estimated DCC GARCH portfolios provided more value than the equally weighted portfolio in all instances except for the significant 1 month and 1 year forecast portfolios in 2022, indicating that the portfolio optimization method indeed brings extra value over the naive method of equal weights. It does that in periods of more stable electricity prices and in periods where there is more price volatility. The DCC one day portfolios was the superior of the DCC models as it outperformed the other two in two out of three years. However, as mentioned in the discussion for further research it might be valuable to expand on this topic by combining the DCC GARCH Mean Variance Optimisation with more risk centered models such as Value at Risk in order to quantify tail risk due to extreme price events in the energy markets.

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11 Appendix

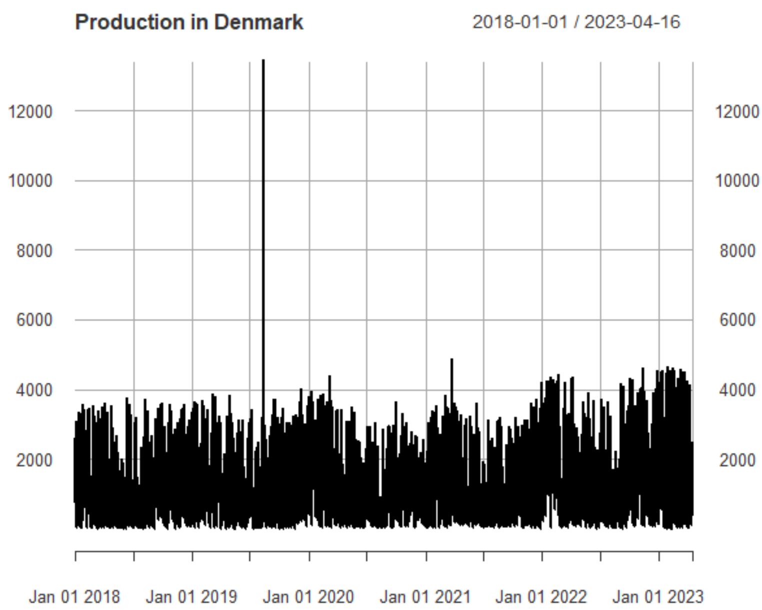

Figure 6: Denmark Production

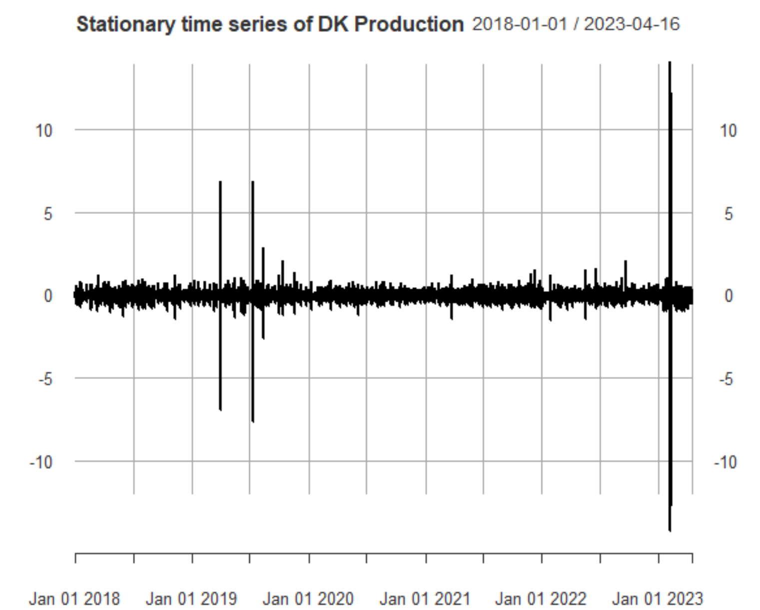

Figure 7: Denmark Stationary Production

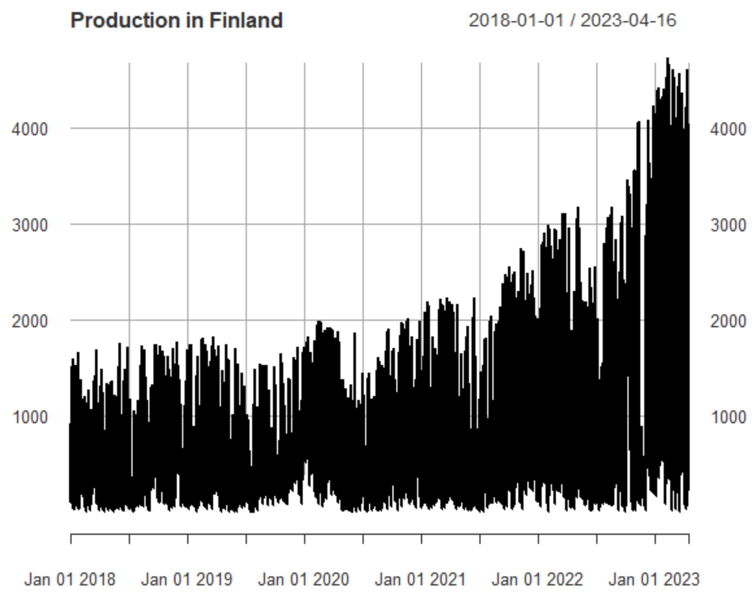

Figure 8: Finland Production

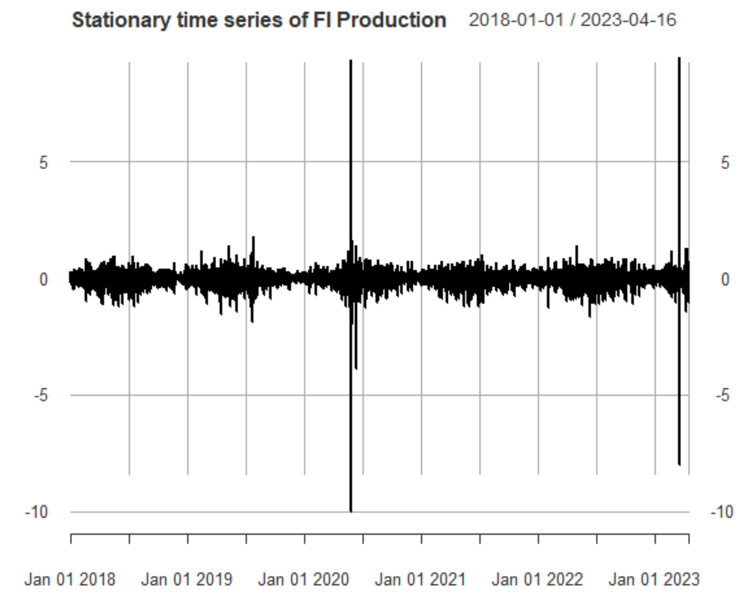

Figure 9: Finland Stationary Production

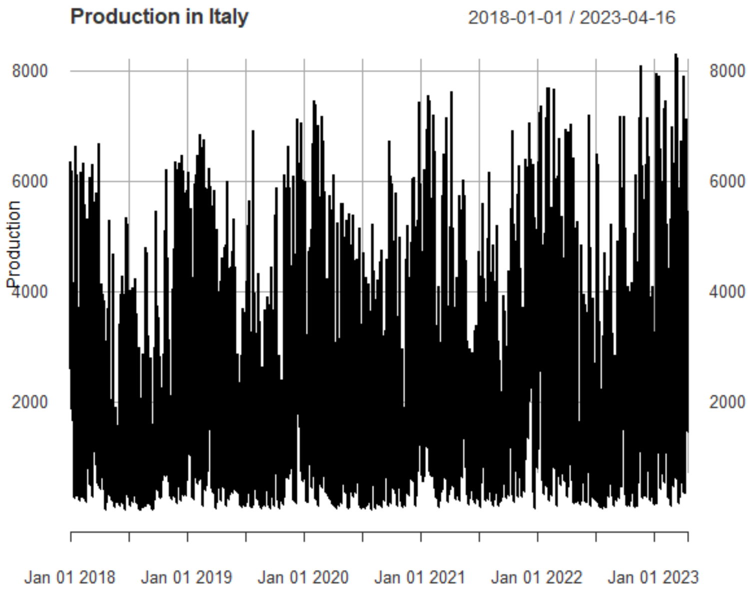

Figure 10: Italy Production

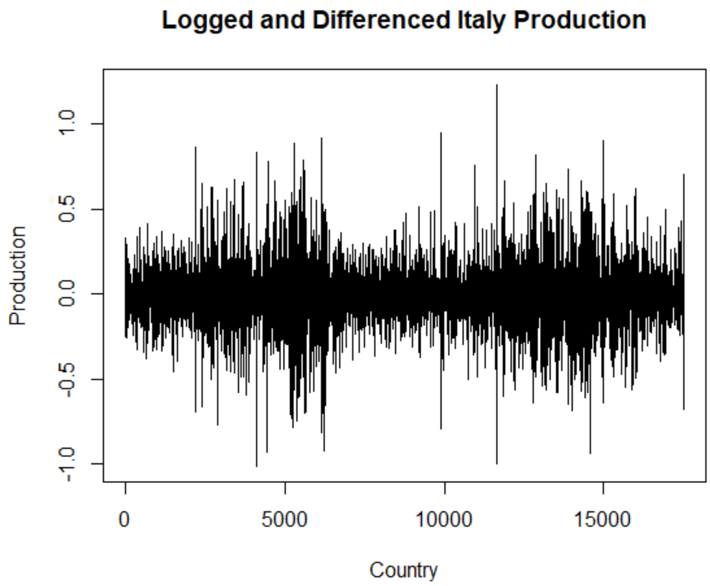

Figure 11: Italy Stationary Production

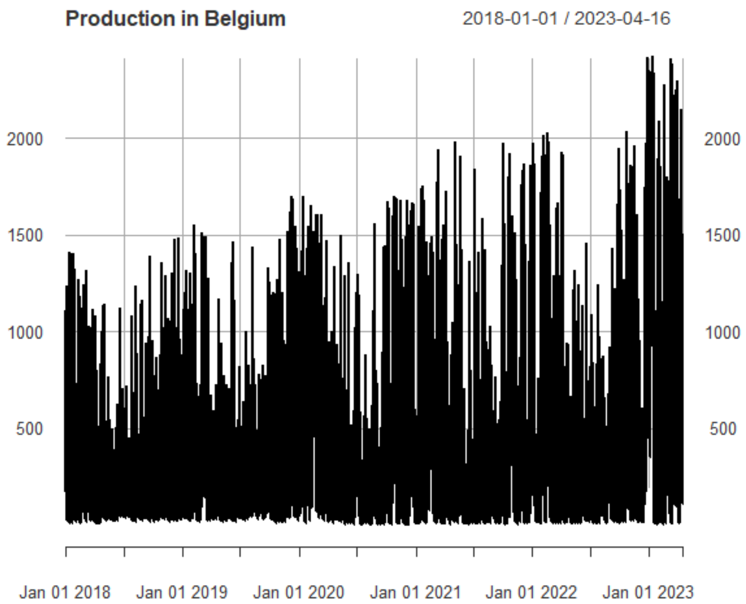

Figure 12: Belgium Production

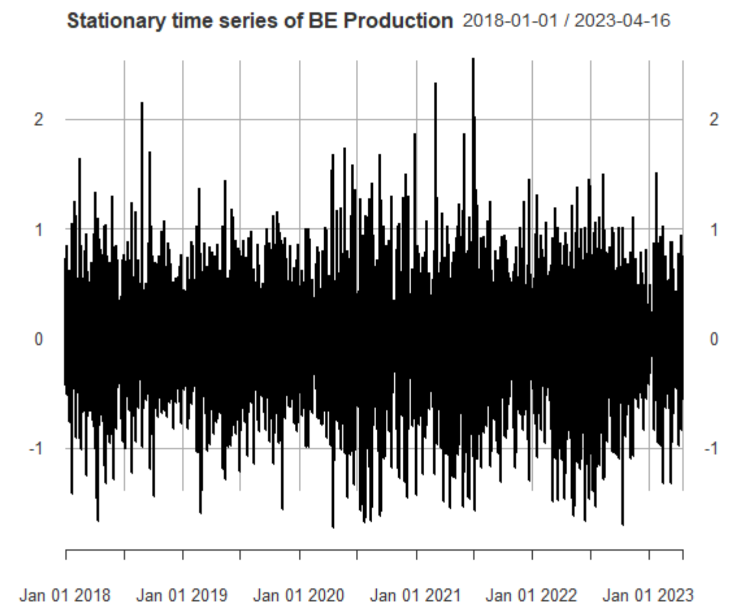

Figure 13: Belgium Stationary Production

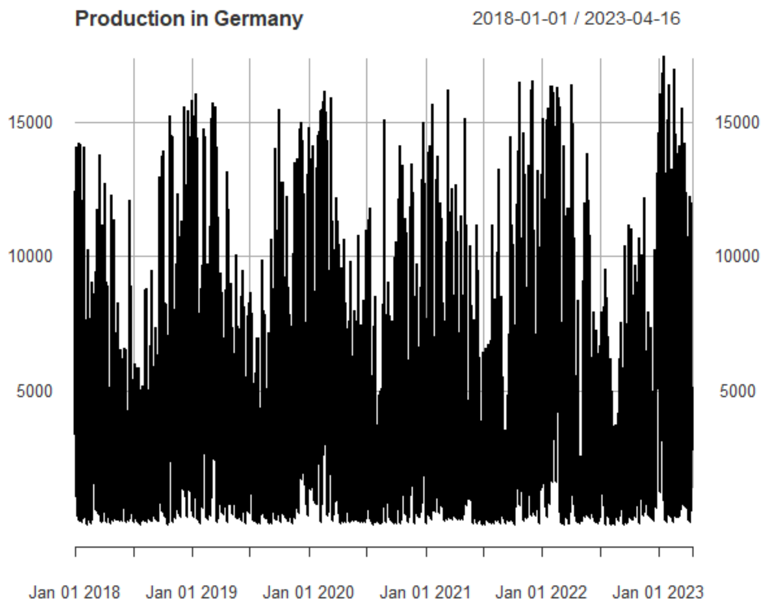

Figure 14: Germany Production

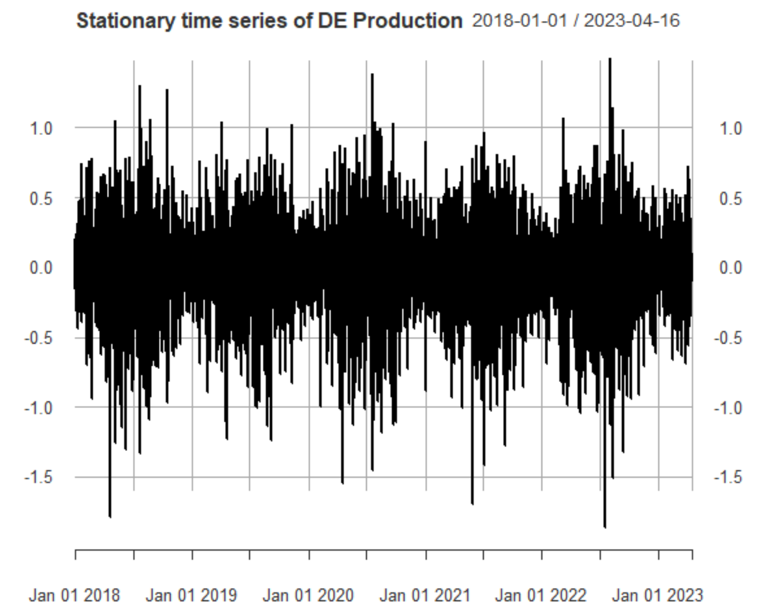

Figure 15: Germany Stationary Production

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  32
[VAR GARCH DCC UncQ] : [0+20+2+10]
No. Series           :  5
No. Obs.             :  17517
Log-Likelihood       :  62771.73
Av.Log-Likelihood    :  3.58

Optimal Parameters
-----------------------------------
                                          Estimate  Std. Error    t value Pr(>|t|)
[manual_arima_modelDK_residuals13].mu     0.004339    0.001184    3.66638 0.000246
[manual_arima_modelDK_residuals13].omega  0.001117    0.000111   10.02323 0.000000
[manual_arima_modelDK_residuals13].alpha1 0.359859    0.023198   15.51264 0.000000
[manual_arima_modelDK_residuals13].beta1  0.639140    0.015463   41.33385 0.000000
[manual_arima_modelFI_residuals13].mu     0.000318    0.000741    0.42902 0.667905
[manual_arima_modelFI_residuals13].omega  0.000609    0.000299    2.03568 0.041783
[manual_arima_modelFI_residuals13].alpha1 0.312883    0.021937   14.26274 0.000000
[manual_arima_modelFI_residuals13].beta1  0.686117    0.040762   16.83218 0.000000
[manual_arima_modelIT_residuals1].mu      0.000716    0.000714    1.00228 0.316209
[manual_arima_modelIT_residuals1].omega   0.000604    0.000079    7.65617 0.000000
[manual_arima_modelIT_residuals1].alpha1  0.211647    0.015962   13.25944 0.000000
[manual_arima_modelIT_residuals1].beta1   0.757654    0.017939   42.23459 0.000000
[manual_arima_modelBE_residuals12].mu     0.003124    0.001416    2.20663 0.027340
[manual_arima_modelBE_residuals12].omega  0.004051    0.000690    5.87176 0.000000
[manual_arima_modelBE_residuals12].alpha1 0.314308    0.023150   13.57726 0.000000
[manual_arima_modelBE_residuals12].beta1  0.632869    0.031241   20.25788 0.000000
[manual_arima_modelDE_residuals12].mu     0.000387    0.000873    0.44381 0.657183
[manual_arima_modelDE_residuals12].omega  0.001181    0.000402    2.93327 0.003354
[manual_arima_modelDE_residuals12].alpha1 0.471776    0.038503   12.25283 0.000000
[manual_arima_modelDE_residuals12].beta1  0.527223    0.064943    8.11823 0.000000
[Joint]dcca1                              0.000612    0.000127    4.84043 0.000001
[Joint]dccb1                              0.998941    0.000258 3875.35430 0.000000

Information Criteria
---------------------

Akaike       -7.1633
Bayes        -7.1491
Shibata      -7.1633
Hannan-Quinn -7.1586
```

Figure 16: DCC Model Output for 2020

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  32
[VAR GARCH DCC UncQ] : [0+20+2+10]
No. Series           :  5
No. Obs.             :  26276
Log-Likelihood       :  92076.3
Av.Log-Likelihood    :  3.5

Optimal Parameters
-----------------------------------
                                          Estimate  Std. Error    t value Pr(>|t|)
[manual_arima_modelDK_residuals2].mu      0.000109    0.000777 1.4024e-01  0.88847
[manual_arima_modelDK_residuals2].omega   0.001109    0.000103 1.0780e+01  0.00000
[manual_arima_modelDK_residuals2].alpha1  0.343105    0.019327 1.7752e+01  0.00000
[manual_arima_modelDK_residuals2].beta1   0.655895    0.014529 4.5145e+01  0.00000
[manual_arima_modelFI_residuals23].mu     0.000001    0.000667 1.8690e-03  0.99851
[manual_arima_modelFI_residuals23].omega  0.000584    0.000283 2.0634e+00  0.03907
[manual_arima_modelFI_residuals23].alpha1 0.329080    0.024993 1.3167e+01  0.00000
[manual_arima_modelFI_residuals23].beta1  0.669920    0.042641 1.5711e+01  0.00000
[manual_arima_modelIT_residuals22].mu     0.000194    0.000736 2.6386e-01  0.79189
[manual_arima_modelIT_residuals22].omega  0.000840    0.000160 5.2367e+00  0.00000
[manual_arima_modelIT_residuals22].alpha1 0.246037    0.025003 9.8405e+00  0.00000
[manual_arima_modelIT_residuals22].beta1  0.716110    0.029022 2.4675e+01  0.00000
[manual_arima_modelBE_residuals22].mu     0.000434    0.001161 3.7387e-01  0.70851
[manual_arima_modelBE_residuals22].omega  0.002572    0.000408 6.2967e+00  0.00000
[manual_arima_modelBE_residuals22].alpha1 0.322934    0.020921 1.5436e+01  0.00000
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[manual_arima_modelDE_residuals22].mu     0.000193    0.000714 2.6963e-01  0.78745
[manual_arima_modelDE_residuals22].omega  0.001148    0.000311 3.6943e+00  0.00022
[manual_arima_modelDE_residuals22].alpha1 0.463997    0.030033 1.5450e+01  0.00000
[manual_arima_modelDE_residuals22].beta1  0.535003    0.050318 1.0632e+01  0.00000
[Joint]dcca1                              0.000594    0.000106 5.6225e+00  0.00000
[Joint]dccb1                              0.998996    0.000212 4.7169e+03  0.00000

Information Criteria
---------------------

Akaike       -7.0060
Bayes        -6.9960
Shibata      -7.0060
Hannan-Quinn -7.0027
```

Figure 17: DCC Model Output for 2021

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  32
[VAR GARCH DCC UncQ] : [0+20+2+10]
No. Series           :  5
No. Obs.             :  35034
Log-Likelihood       :  123357.3
Av.Log-Likelihood    :  3.52

Optimal Parameters
-----------------------------------
                                           Estimate  Std. Error    t value Pr(>|t|)
[manual_arima_modelDK_residuals3].mu       0.000296    0.000700 4.2341e-01 0.671997
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[manual_arima_modelDK_residuals3].alpha1   0.329065    0.016823 1.9560e+01 0.000000
[manual_arima_modelDK_residuals3].beta1    0.667977    0.013630 4.9007e+01 0.000000
[manual_arima_modelFI_residuals33].mu      0.011917    0.000980 1.2163e+01 0.000000
[manual_arima_modelFI_residuals33].omega   0.000521    0.000174 2.9955e+00 0.002740
[manual_arima_modelFI_residuals33].alpha1  0.325643    0.018418 1.7681e+01 0.000000
[manual_arima_modelFI_residuals33].beta1   0.673357    0.029105 2.3136e+01 0.000000
[manual_arima_modelIT_residuals32].mu      0.000261    0.000594 4.3956e-01 0.660252
[manual_arima_modelIT_residuals32].omega   0.000622    0.000094 6.6197e+00 0.000000
[manual_arima_modelIT_residuals32].alpha1  0.216110    0.017685 1.2220e+01 0.000000
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[manual_arima_modelBE_residuals32].mu      0.000013    0.001009 1.3049e-02 0.989589
[manual_arima_modelBE_residuals32].omega   0.002562    0.000345 7.4207e+00 0.000000
[manual_arima_modelBE_residuals32].alpha1  0.326700    0.017139 1.9062e+01 0.000000
[manual_arima_modelBE_residuals32].beta1   0.672300    0.018986 3.5411e+01 0.000000
[manual_arima_modelDE_residuals3].mu       0.000242    0.000653 3.7003e-01 0.711362
[manual_arima_modelDE_residuals3].omega    0.001067    0.000224 4.7691e+00 0.000002
[manual_arima_modelDE_residuals3].alpha1   0.441169    0.024571 1.7955e+01 0.000000
[manual_arima_modelDE_residuals3].beta1    0.557831    0.037671 1.4808e+01 0.000000
[Joint]dcca1                               0.000570    0.000089 6.4038e+00 0.000000
[Joint]dccb1                               0.999021    0.000180 5.5392e+03 0.000000

Information Criteria
---------------------

Akaike       -7.0403
Bayes        -7.0326
Shibata      -7.0403
Hannan-Quinn -7.0379
```

Figure 18: DCC Model Output for 2022

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  72
[VAR GARCH DCC UncQ] : [0+60+2+10]
No. Series           :  5
No. Obs.             :  17517
Log-Likelihood       :  63106.91
Av.Log-Likelihood    :  3.6

Optimal Parameters
-----------------------------------
                                           Estimate  Std. Error    t value Pr(>|t|)
[manual_arima_modelDK_residuals13].mu      0.004339    0.001197 3.6262e+00 0.000288
[manual_arima_modelDK_residuals13].omega   0.001275    0.000161 7.8937e+00 0.000000
[manual_arima_modelDK_residuals13].alpha1  0.426438    0.039037 1.0924e+01 0.000000
[manual_arima_modelDK_residuals13].beta1   0.396886    0.072298 5.4896e+00 0.000000
[manual_arima_modelDK_residuals13].beta2   0.147831    0.114015 1.2966e+00 0.194773
[manual_arima_modelDK_residuals13].beta3   0.000009    0.102148 8.4000e-05 0.999933
[manual_arima_modelDK_residuals13].beta4   0.027832    0.059418 4.6841e-01 0.639493
[manual_arima_modelDK_residuals13].beta5   0.000001    0.051485 2.2000e-05 0.999982
[manual_arima_modelDK_residuals13].beta6   0.000001    0.086584 1.2000e-05 0.999991
[manual_arima_modelDK_residuals13].beta7   0.000000    0.082438 5.0000e-06 0.999996
[manual_arima_modelDK_residuals13].beta8   0.000000    0.039620 8.0000e-06 0.999993
[manual_arima_modelDK_residuals13].beta9   0.000000    0.028036 1.1000e-05 0.999991
[manual_arima_modelDK_residuals13].beta10  0.000000    0.048374 5.0000e-06 0.999996
[manual_arima_modelFI_residuals13].mu      0.000318    0.000731 4.3474e-01 0.663751
[manual_arima_modelFI_residuals13].omega   0.000599    0.000262 2.2859e+00 0.022259
[manual_arima_modelFI_residuals13].alpha1  0.381995    0.028105 1.3592e+01 0.000000
[manual_arima_modelFI_residuals13].beta1   0.486837    0.074063 6.5733e+00 0.000000
[manual_arima_modelFI_residuals13].beta2   0.037130    0.081859 4.5358e-01 0.650129
[manual_arima_modelFI_residuals13].beta3   0.000001    0.052734 1.7000e-05 0.999986
[manual_arima_modelFI_residuals13].beta4   0.000003    0.043000 6.6000e-05 0.999948
[manual_arima_modelFI_residuals13].beta5   0.000003    0.053688 6.0000e-05 0.999952
[manual_arima_modelFI_residuals13].beta6   0.000001    0.041077 1.3000e-05 0.999990
[manual_arima_modelFI_residuals13].beta7   0.000002    0.033190 5.1000e-05 0.999959
[manual_arima_modelFI_residuals13].beta8   0.093029    0.028312 3.2858e+00 0.001017
[manual_arima_modelIT_residuals1].mu       0.000760    0.000719 1.0567e+00 0.290635
[manual_arima_modelIT_residuals1].omega    0.000649    0.000092 7.0467e+00 0.000000
[manual_arima_modelIT_residuals1].alpha1   0.265403    0.025508 1.0405e+01 0.000000
[manual_arima_modelIT_residuals1].beta1    0.558024    0.092455 6.0357e+00 0.000000
[manual_arima_modelIT_residuals1].beta2    0.062740    0.135850 4.6183e-01 0.644202
[manual_arima_modelIT_residuals1].beta3    0.002442    0.125386 1.9479e-02 0.984459
[manual_arima_modelIT_residuals1].beta4    0.000002    0.083697 2.1000e-05 0.999983
[manual_arima_modelIT_residuals1].beta5    0.000001    0.094780 1.0000e-05 0.999992
[manual_arima_modelIT_residuals1].beta6    0.000001    0.420124 3.0000e-06 0.999998
[manual_arima_modelIT_residuals1].beta7    0.000063    0.538934 1.1700e-04 0.999907
[manual_arima_modelIT_residuals1].beta8    0.064012    0.182821 3.5013e-01 0.726239
[manual_arima_modelIT_residuals1].beta9    0.014006    0.099305 1.4104e-01 0.887839
[manual_arima_modelIT_residuals1].beta10   0.000003    0.062649 4.0000e-05 0.999968
[manual_arima_modelIT_residuals1].beta11   0.000002    0.054662 3.9000e-05 0.999969
[manual_arima_modelBE_residuals12].mu      0.003124    0.001660 1.8818e+00 0.059867
[manual_arima_modelBE_residuals12].omega   0.002727    0.000561 4.8631e+00 0.000001
[manual_arima_modelBE_residuals12].alpha1  0.346621    0.031734 1.0923e+01 0.000000
[manual_arima_modelBE_residuals12].beta1   0.513221    0.081801 6.2740e+00 0.000000
[manual_arima_modelBE_residuals12].beta2   0.017327    0.088415 1.9598e-01 0.844628
[manual_arima_modelBE_residuals12].beta3   0.008781    0.082232 1.0678e-01 0.914966
[manual_arima_modelBE_residuals12].beta4   0.037085    0.088454 4.1926e-01 0.675027
[manual_arima_modelBE_residuals12].beta5   0.000001    0.067128 8.0000e-06 0.999993
[manual_arima_modelBE_residuals12].beta6   0.000000    0.053724 5.0000e-06 0.999996
[manual_arima_modelBE_residuals12].beta7   0.000000    0.062084 7.0000e-06 0.999995
[manual_arima_modelBE_residuals12].beta8   0.014932    0.054362 2.7467e-01 0.783569
[manual_arima_modelBE_residuals12].beta9   0.000002    0.079615 2.6000e-05 0.999980
[manual_arima_modelBE_residuals12].beta10  0.000000    0.112621 3.0000e-06 0.999998
[manual_arima_modelBE_residuals12].beta11  0.022495    0.097179 2.3148e-01 0.816944
[manual_arima_modelBE_residuals12].beta12  0.000000    0.028465 9.0000e-06 0.999993
[manual_arima_modelBE_residuals12].beta13  0.000000    0.100977 1.0000e-06 0.999999
[manual_arima_modelBE_residuals12].beta14  0.000000    0.116490 1.0000e-06 0.999999
[manual_arima_modelBE_residuals12].beta15  0.018512    0.067050 2.7610e-01 0.782472
[manual_arima_modelDE_residuals12].mu      0.000387    0.000873 4.4380e-01 0.657185
[manual_arima_modelDE_residuals12].omega   0.001181    0.000402 2.9333e+00 0.003354
[manual_arima_modelDE_residuals12].alpha1  0.471776    0.038503 1.2253e+01 0.000000
[manual_arima_modelDE_residuals12].beta1   0.527223    0.064942 8.1184e+00 0.000000
[Joint]dcca1                               0.000604    0.000122 4.9685e+00 0.000001
[Joint]dccb1                               0.998964    0.000241 4.1497e+03 0.000000

Information Criteria
---------------------

Akaike       -7.1970
Bayes        -7.1651
Shibata      -7.1970
Hannan-Quinn -7.1865
```

Figure 19: Insignificant DCC Model Output for 2020

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  77
[VAR GARCH DCC UncQ] : [0+65+2+10]
No. Series           :  5
No. Obs.             :  26276
Log-Likelihood       :  92706.12
Av.Log-Likelihood    :  3.53

Optimal Parameters
-----------------------------------
                                          Estimate  Std. Error    t value Pr(>|t|)
[manual_arima_modelDK_residuals2].mu       0.000109    0.000771 1.4141e-01 0.887545
[manual_arima_modelDK_residuals2].omega    0.001232    0.000150 8.2369e+00 0.000000
[manual_arima_modelDK_residuals2].alpha1   0.403591    0.023959 1.6845e+01 0.000000
[manual_arima_modelDK_residuals2].beta1    0.432437    0.042879 1.0085e+01 0.000000
[manual_arima_modelDK_residuals2].beta2    0.116224    0.061092 1.9024e+00 0.057113
[manual_arima_modelDK_residuals2].beta3    0.000000    0.050533 9.0000e-06 0.999993
[manual_arima_modelDK_residuals2].beta4    0.046746    0.041220 1.1341e+00 0.256770
[manual_arima_modelDK_residuals2].beta5    0.000000    0.044442 2.0000e-06 0.999998
[manual_arima_modelDK_residuals2].beta6    0.000000    0.055043 2.0000e-06 0.999999
[manual_arima_modelDK_residuals2].beta7    0.000000    0.064115 1.0000e-06 0.999999
[manual_arima_modelDK_residuals2].beta8    0.000000    0.046023 3.0000e-06 0.999998
[manual_arima_modelDK_residuals2].beta9    0.000000    0.021810 6.0000e-06 0.999995
[manual_arima_modelDK_residuals2].beta10   0.000000    0.039873 2.0000e-06 0.999998
[manual_arima_modelFI_residuals23].mu      0.000001    0.000678 1.8370e-03 0.998534
[manual_arima_modelFI_residuals23].omega   0.000596    0.000231 2.5803e+00 0.009872
[manual_arima_modelFI_residuals23].alpha1  0.408868    0.023155 1.7658e+01 0.000000
[manual_arima_modelFI_residuals23].beta1   0.461118    0.055178 8.3569e+00 0.000000
[manual_arima_modelFI_residuals23].beta2   0.019279    0.048228 3.9975e-01 0.689342
[manual_arima_modelFI_residuals23].beta3   0.011217    0.043308 2.5900e-01 0.795632
[manual_arima_modelFI_residuals23].beta4   0.000016    0.039922 4.0000e-04 0.999681
[manual_arima_modelFI_residuals23].beta5   0.012867    0.027758 4.6353e-01 0.642986
[manual_arima_modelFI_residuals23].beta6   0.000008    0.029530 2.6600e-04 0.999788
[manual_arima_modelFI_residuals23].beta7   0.085628    0.017779 4.8163e+00 0.000001
[manual_arima_modelIT_residuals22].mu      0.000042    0.000705 5.8905e-02 0.953028
[manual_arima_modelIT_residuals22].omega   0.000741    0.000229 3.2316e+00 0.001231
[manual_arima_modelIT_residuals22].alpha1  0.302288    0.030022 1.0069e+01 0.000000
[manual_arima_modelIT_residuals22].alpha2  0.000000    0.105334 2.0000e-06 0.999998
[manual_arima_modelIT_residuals22].beta1   0.494668    0.287251 1.7221e+00 0.085056
[manual_arima_modelIT_residuals22].beta2   0.000001    0.149291 9.0000e-06 0.999993
[manual_arima_modelIT_residuals22].beta3   0.105696    0.056029 1.8865e+00 0.059233
[manual_arima_modelIT_residuals22].beta4   0.012458    0.080446 1.5486e-01 0.876934
[manual_arima_modelIT_residuals22].beta5   0.000000    0.079969 1.0000e-06 1.000000
[manual_arima_modelIT_residuals22].beta6   0.000000    0.092240 0.0000e+00 1.000000
[manual_arima_modelIT_residuals22].beta7   0.000000    0.098637 2.0000e-06 0.999999
[manual_arima_modelIT_residuals22].beta8   0.015597    0.131400 1.1870e-01 0.905511
[manual_arima_modelIT_residuals22].beta9   0.000000    0.098338 2.0000e-06 0.999999
[manual_arima_modelIT_residuals22].beta10  0.000000    0.057247 0.0000e+00 1.000000
[manual_arima_modelIT_residuals22].beta11  0.000000    0.057276 0.0000e+00 1.000000
[manual_arima_modelIT_residuals22].beta12  0.000000    0.034562 0.0000e+00 1.000000
[manual_arima_modelIT_residuals22].beta13  0.000000    0.039261 0.0000e+00 1.000000
[manual_arima_modelIT_residuals22].beta14  0.000000    0.055511 1.0000e-06 1.000000
[manual_arima_modelIT_residuals22].beta15  0.000000    0.094923 0.0000e+00 1.000000
[manual_arima_modelIT_residuals22].beta16  0.039425    0.064759 6.0880e-01 0.542660
[manual_arima_modelBE_residuals22].mu      0.000434    0.001341 3.2365e-01 0.746202
[manual_arima_modelBE_residuals22].omega   0.002216    0.000353 6.2853e+00 0.000000
[manual_arima_modelBE_residuals22].alpha1  0.367456    0.024030 1.5291e+01 0.000000
[manual_arima_modelBE_residuals22].beta1   0.510207    0.087253 5.8474e+00 0.000000
[manual_arima_modelBE_residuals22].beta2   0.033915    0.123344 2.7496e-01 0.783345
[manual_arima_modelBE_residuals22].beta3   0.000001    0.097216 1.3000e-05 0.999990
[manual_arima_modelBE_residuals22].beta4   0.008685    0.068127 1.2748e-01 0.898560
[manual_arima_modelBE_residuals22].beta5   0.000001    0.037145 2.6000e-05 0.999979
[manual_arima_modelBE_residuals22].beta6   0.000006    0.042128 1.3400e-04 0.999893
[manual_arima_modelBE_residuals22].beta7   0.000368    0.043708 8.4240e-03 0.993279
[manual_arima_modelBE_residuals22].beta8   0.020351    0.036769 5.5347e-01 0.579941
[manual_arima_modelBE_residuals22].beta9   0.040757    0.046668 8.7333e-01 0.382481
[manual_arima_modelBE_residuals22].beta10  0.000001    0.060785 1.0000e-05 0.999992
[manual_arima_modelBE_residuals22].beta11  0.017250    0.068157 2.5309e-01 0.800198
[manual_arima_modelBE_residuals22].beta12  0.000001    0.030598 2.8000e-05 0.999978
[manual_arima_modelBE_residuals22].beta13  0.000000    0.072503 5.0000e-06 0.999996
[manual_arima_modelBE_residuals22].beta14  0.000000    0.060036 4.0000e-06 0.999996
[manual_arima_modelBE_residuals22].beta15  0.000002    0.052031 3.1000e-05 0.999975
[manual_arima_modelDE_residuals22].mu      0.000193    0.000714 2.6962e-01 0.787453
[manual_arima_modelDE_residuals22].omega   0.001148    0.000311 3.6944e+00 0.000220
[manual_arima_modelDE_residuals22].alpha1  0.463997    0.030032 1.5450e+01 0.000000
[manual_arima_modelDE_residuals22].beta1   0.535003    0.050318 1.0632e+01 0.000000
[Joint]dcca1                               0.000589    0.000101 5.8036e+00 0.000000
[Joint]dccb1                               0.999008    0.000200 5.0066e+03 0.000000

Information Criteria
---------------------

Akaike       -7.0505
Bayes        -7.0265
Shibata      -7.0505
Hannan-Quinn -7.0427
```

Figure 20: Insignificant DCC Model Output for 2021

```
*---------------------------------*
*          DCC GARCH Fit          *
*---------------------------------*

Distribution         :  mvnorm
Model                :  DCC(1,1)
No. Parameters       :  77
[VAR GARCH DCC UncQ] : [0+65+2+10]
No. Series           :  5
No. Obs.             :  35034
Log-Likelihood       :  124177.2
Av.Log-Likelihood    :  3.54

Optimal Parameters
-----------------------------------
                                         Estimate  Std. Error   t value Pr(>|t|)
[manual_arima_modelDK_residuals3].mu     0.000296    0.000702 4.2189e-01 0.673105
[manual_arima_modelDK_residuals3].omega  0.001292    0.000114 1.1362e+01 0.000000
[manual_arima_modelDK_residuals3].alpha1 0.399592    0.020444 1.9545e+01 0.000000
[manual_arima_modelDK_residuals3].beta1  0.420554    0.044510 9.4486e+00 0.000000
[manual_arima_modelDK_residuals3].beta2  0.123768    0.063382 1.9527e+00 0.050852
[manual_arima_modelDK_residuals3].beta3  0.000000    0.053402 2.0000e-06 0.999999
[manual_arima_modelDK_residuals3].beta4  0.049784    0.040794 1.2204e+00 0.222330
[manual_arima_modelDK_residuals3].beta5  0.000000    0.038839 1.0000e-06 1.000000
[manual_arima_modelDK_residuals3].beta6  0.000000    0.039478 1.0000e-06 0.999999
[manual_arima_modelDK_residuals3].beta7  0.000000    0.045494 0.0000e+00 1.000000
[manual_arima_modelDK_residuals3].beta8  0.000000    0.036057 2.0000e-06 0.999998
[manual_arima_modelDK_residuals3].beta9  0.000000    0.020224 1.2000e-05 0.999991
[manual_arima_modelDK_residuals3].beta10 0.005301    0.017689 2.9968e-01 0.764419
[manual_arima_modelFI_residuals33].mu    0.011917    0.000820 1.4533e+01 0.000000
[manual_arima_modelFI_residuals33].omega 0.000552    0.000122 4.5118e+00 0.000006
[manual_arima_modelFI_residuals33].alpha1 0.414111   0.017296 2.3943e+01 0.000000
[manual_arima_modelFI_residuals33].beta1 0.472314    0.042610 1.1085e+01 0.000000
[manual_arima_modelFI_residuals33].beta2 0.014660    0.040295 3.6383e-01 0.715989
[manual_arima_modelFI_residuals33].beta3 0.000149    0.036278 4.1140e-03 0.996717
[manual_arima_modelFI_residuals33].beta4 0.010874    0.026975 4.0313e-01 0.686852
[manual_arima_modelFI_residuals33].beta5 0.001188    0.024624 4.8235e-02 0.961529
[manual_arima_modelFI_residuals33].beta6 0.026217    0.024365 1.0760e+00 0.281922
[manual_arima_modelFI_residuals33].beta7 0.000011    0.025463 4.4200e-04 0.999648
[manual_arima_modelFI_residuals33].beta8 0.059475    0.016938 3.5114e+00 0.000446
[manual_arima_modelIT_residuals32].mu    0.000138    0.000567 2.4313e-01 0.807908
[manual_arima_modelIT_residuals32].omega 0.000615    0.000070 8.7593e+00 0.000000
[manual_arima_modelIT_residuals32].alpha1 0.281503   0.026728 1.0532e+01 0.000000
[manual_arima_modelIT_residuals32].beta1 0.521122    0.090290 5.7717e+00 0.000000
[manual_arima_modelIT_residuals32].beta2 0.026257    0.082397 3.1866e-01 0.749983
[manual_arima_modelIT_residuals32].beta3 0.086861    0.047292 1.8367e+00 0.066258
[manual_arima_modelIT_residuals32].beta4 0.000897    0.068351 1.3117e-02 0.989534
[manual_arima_modelIT_residuals32].beta5 0.000001    0.086788 8.0000e-06 0.999993
[manual_arima_modelIT_residuals32].beta6 0.000000    0.085936 3.0000e-06 0.999997
[manual_arima_modelIT_residuals32].beta7 0.000001    0.077831 9.0000e-06 0.999992
[manual_arima_modelIT_residuals32].beta8 0.024482    0.095678 2.5588e-01 0.798041
[manual_arima_modelIT_residuals32].beta9 0.000001    0.088553 8.0000e-06 0.999993
[manual_arima_modelIT_residuals32].beta10 0.000000   0.062706 5.0000e-06 0.999996
[manual_arima_modelIT_residuals32].beta11 0.000000   0.055001 4.0000e-06 0.999997
[manual_arima_modelIT_residuals32].beta12 0.000000   0.027755 6.0000e-06 0.999995
[manual_arima_modelIT_residuals32].beta13 0.000000   0.042972 2.0000e-06 0.999998
[manual_arima_modelIT_residuals32].beta14 0.000000   0.071980 2.0000e-06 0.999998
[manual_arima_modelIT_residuals32].beta15 0.000000   0.083467 2.0000e-06 0.999998
[manual_arima_modelIT_residuals32].beta16 0.034269   0.055404 6.1854e-01 0.536221
[manual_arima_modelBE_residuals32].mu    0.000013    0.001215 1.0840e-02 0.991351
[manual_arima_modelBE_residuals32].omega 0.002065    0.000369 5.5901e+00 0.000000
[manual_arima_modelBE_residuals32].alpha1 0.355786   0.026025 1.3671e+01 0.000000
[manual_arima_modelBE_residuals32].beta1 0.531131    0.074237 7.1545e+00 0.000000
[manual_arima_modelBE_residuals32].beta2 0.041405    0.088951 4.6547e-01 0.641592
[manual_arima_modelBE_residuals32].beta3 0.000001    0.055695 1.6000e-05 0.999987
[manual_arima_modelBE_residuals32].beta4 0.000018    0.011945 1.5210e-03 0.998786
[manual_arima_modelBE_residuals32].beta5 0.000002    0.037089 4.5000e-05 0.999964
[manual_arima_modelBE_residuals32].beta6 0.010227    0.038628 2.6476e-01 0.791194
[manual_arima_modelBE_residuals32].beta7 0.005688    0.038092 1.4932e-01 0.881303
[manual_arima_modelBE_residuals32].beta8 0.022901    0.033923 6.7510e-01 0.499614
[manual_arima_modelBE_residuals32].beta9 0.008510    0.064204 1.3255e-01 0.894552
[manual_arima_modelBE_residuals32].beta10 0.000002   0.060753 3.2000e-05 0.999974
[manual_arima_modelBE_residuals32].beta11 0.002025   0.056796 3.5661e-02 0.971553
[manual_arima_modelBE_residuals32].beta12 0.000001   0.021542 3.1000e-05 0.999975
[manual_arima_modelBE_residuals32].beta13 0.000000   0.052682 6.0000e-06 0.999995
[manual_arima_modelBE_residuals32].beta14 0.000000   0.051964 6.0000e-06 0.999995
[manual_arima_modelBE_residuals32].beta15 0.021302   0.053965 3.9474e-01 0.693035
[manual_arima_modelDE_residuals3].mu     0.000242    0.000653 3.7002e-01 0.711364
[manual_arima_modelDE_residuals3].omega  0.001067    0.000224 4.7691e+00 0.000002
[manual_arima_modelDE_residuals3].alpha1 0.441169    0.024570 1.7955e+01 0.000000
[manual_arima_modelDE_residuals3].beta1  0.557831    0.037670 1.4808e+01 0.000000
[Joint]dcca1                             0.000556    0.000085 6.5390e+00 0.000000
[Joint]dccb1                             0.999045    0.000170 5.8937e+03 0.000000

Information Criteria
---------------------

Akaike       -7.0846
Bayes        -7.0660
Shibata      -7.0846
Hannan-Quinn -7.0786
```

Figure 21: Insignificant DCC Model Output for 2022

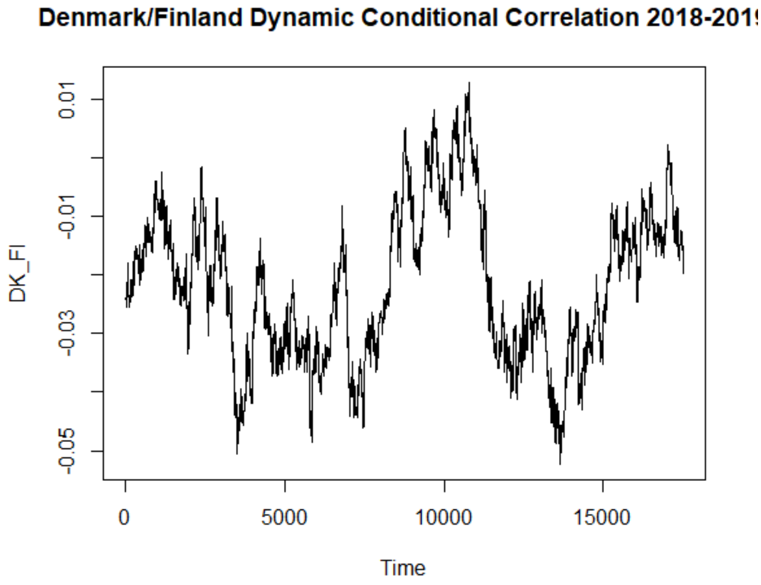

Figure 22: DK/FI Correlation from 2018-2019

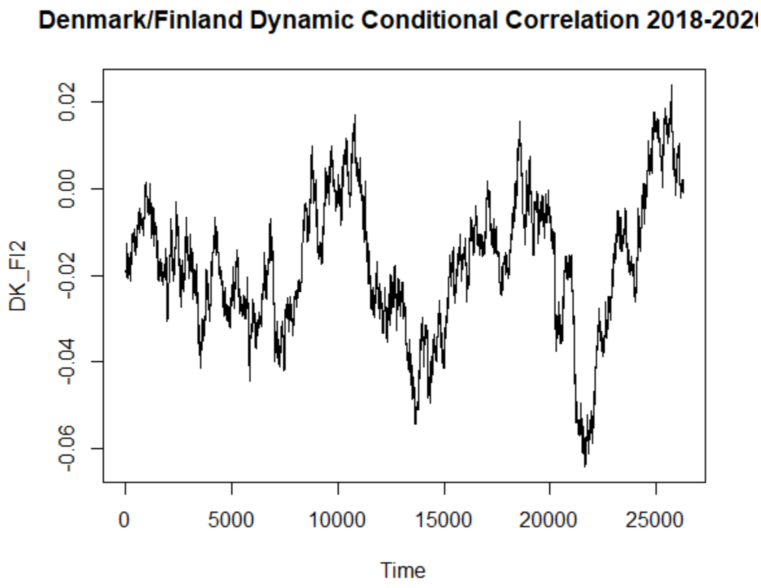

Figure 23: DK/FI Correlation from 2018-2020

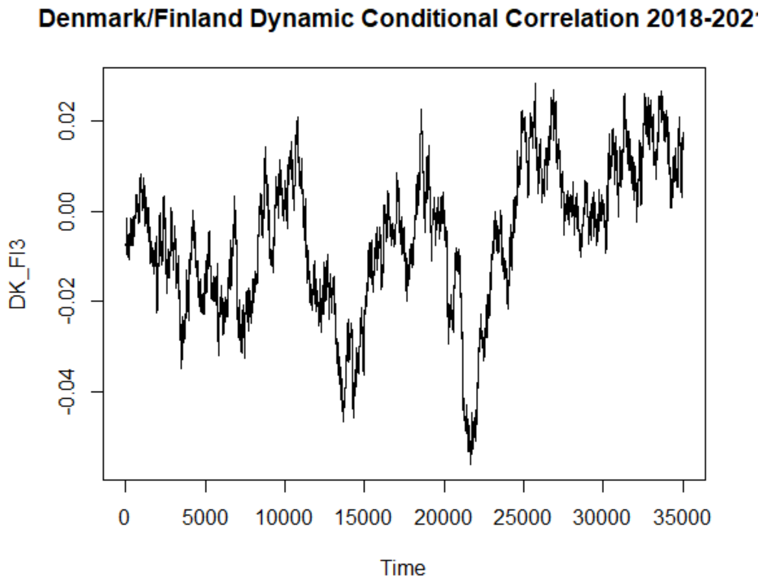

Figure 24: DK/FI Correlation from 2018-2021

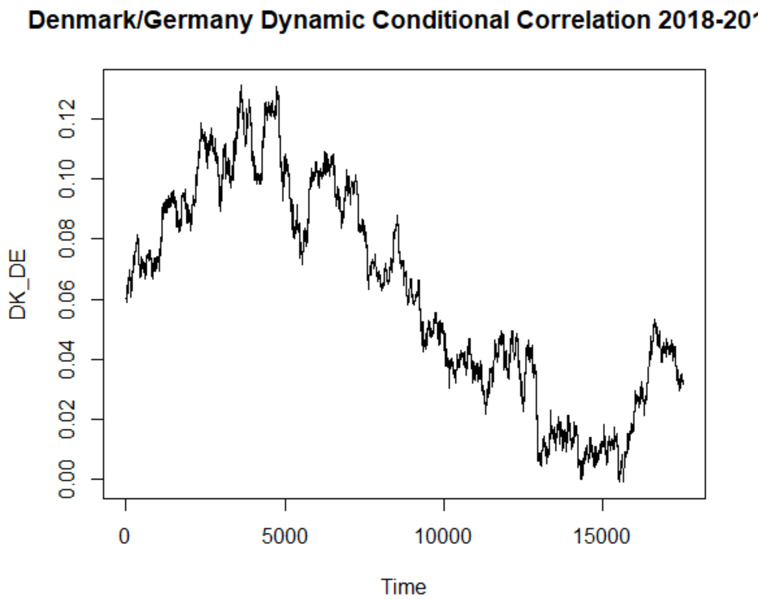

Figure 25: DK/DE Correlation from 2018-2019

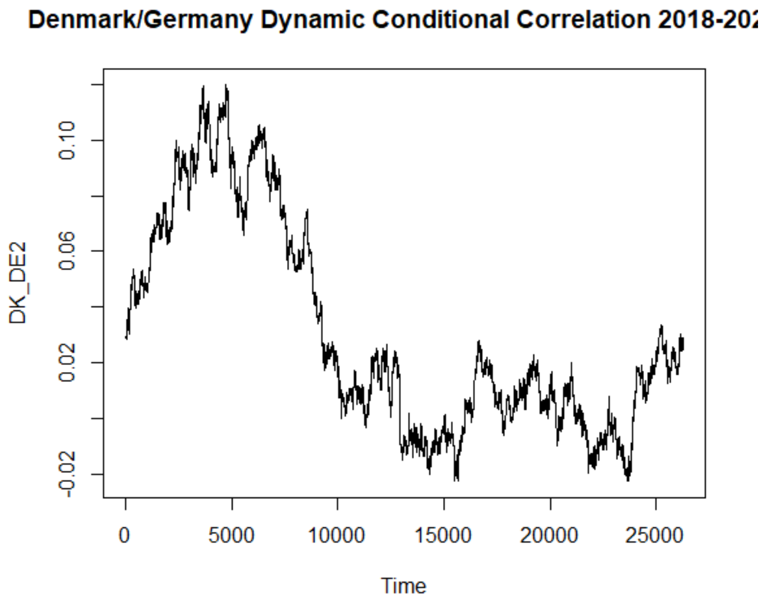

Figure 26: DK/DE Correlation from 2018-2020

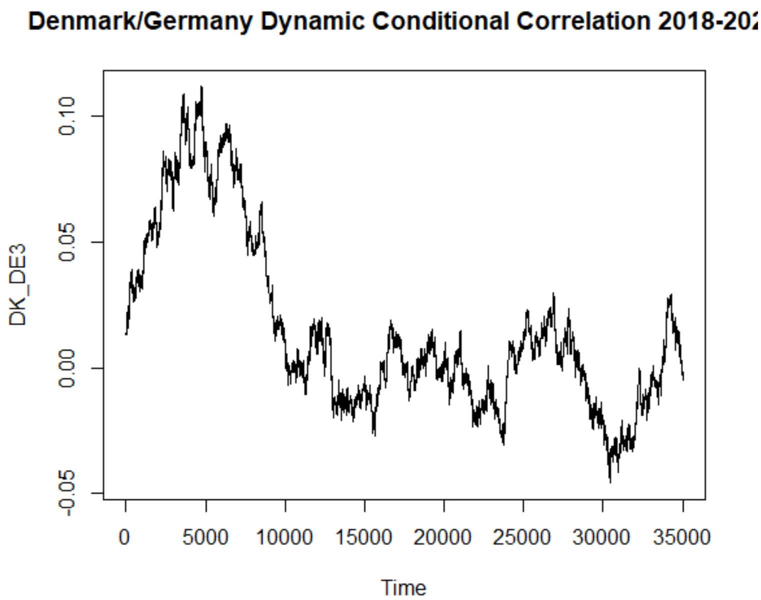

Figure 27: DK/DE Correlation from 2018-2021

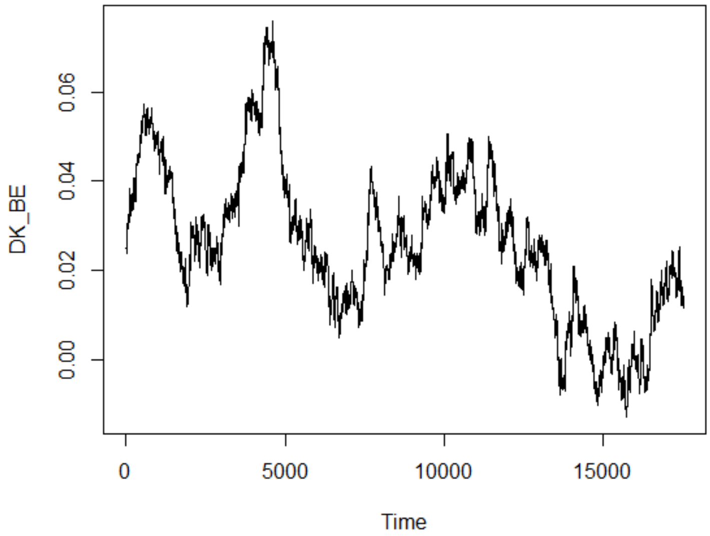

Figure 28: DK/BE Correlation from 2018-2019

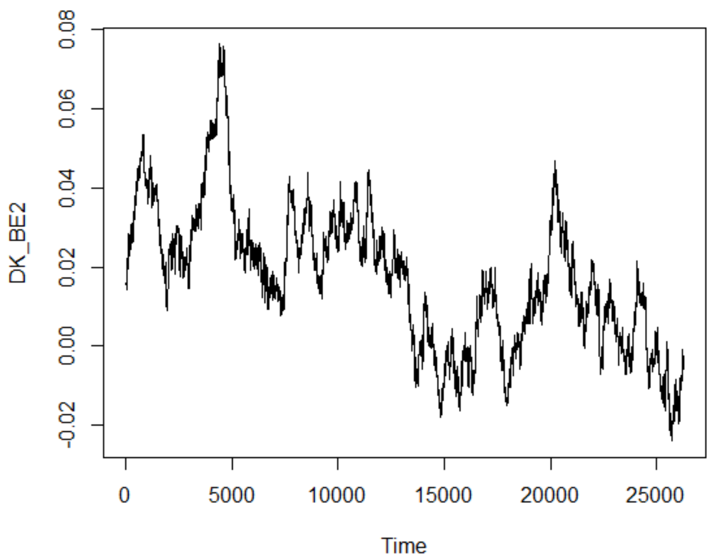

Figure 29: DK/BE Correlation from 2018-2020

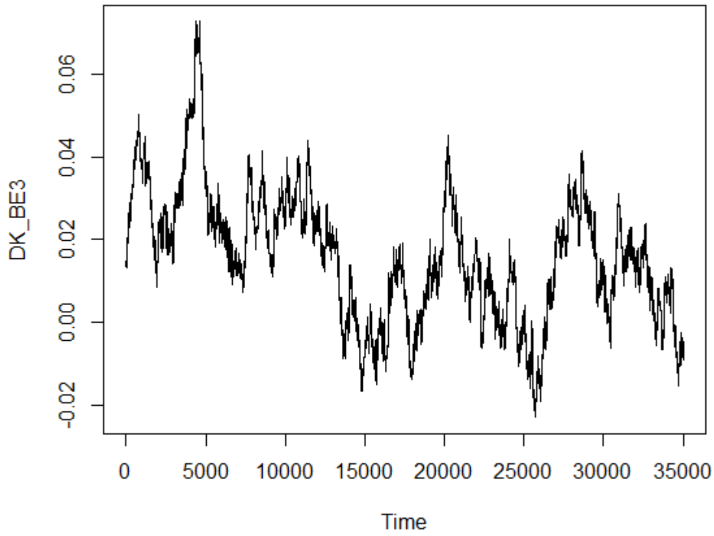

Figure 30: DK/BE Correlation from 2018-2021

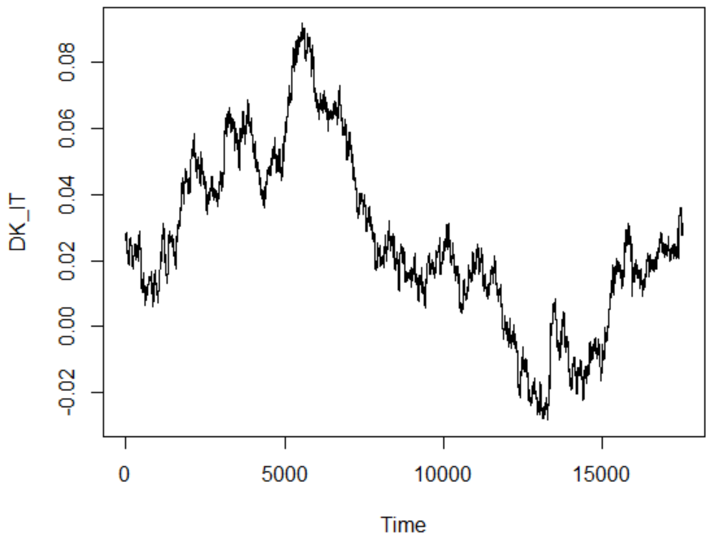

Figure 31: DK/IT Correlation from 2018-2019

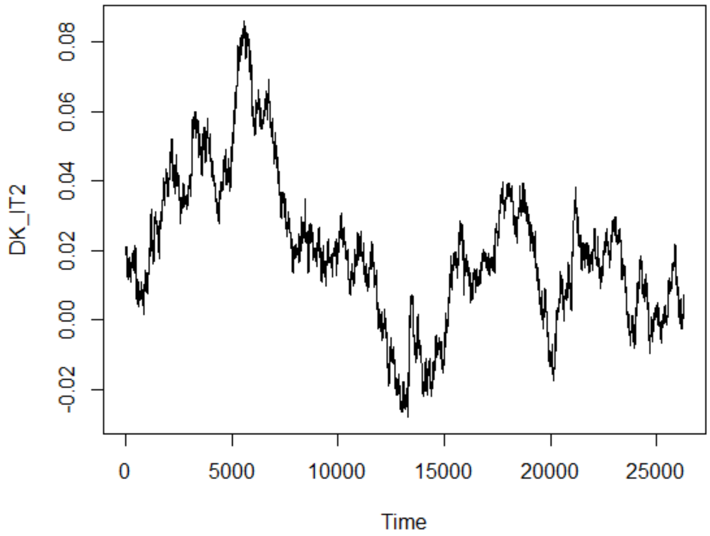

Figure 32: DK/IT Correlation from 2018-2020

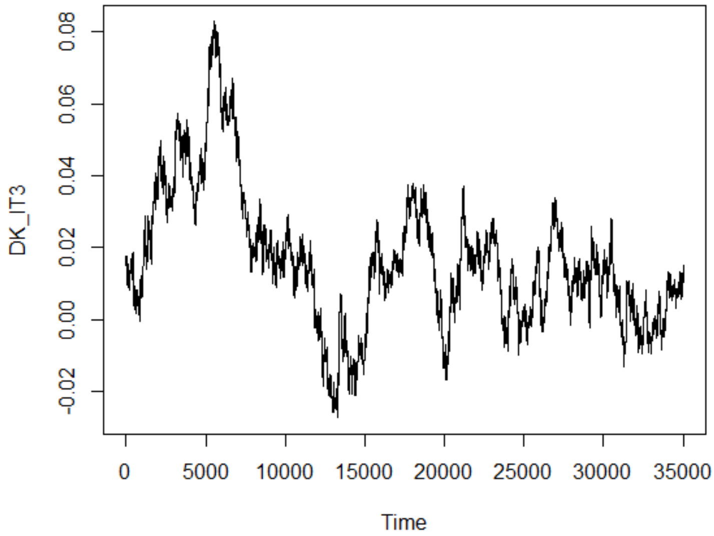

Figure 33: DK/IT Correlation from 2018-2021

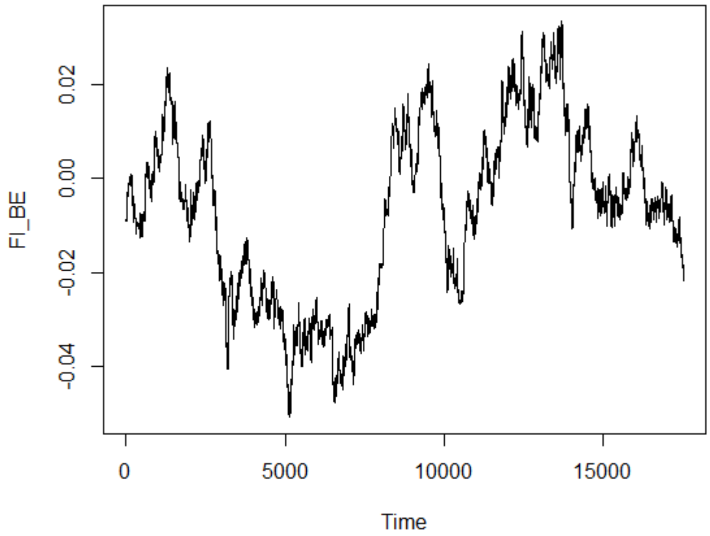

Figure 34: FI/BE Correlation from 2018-2019

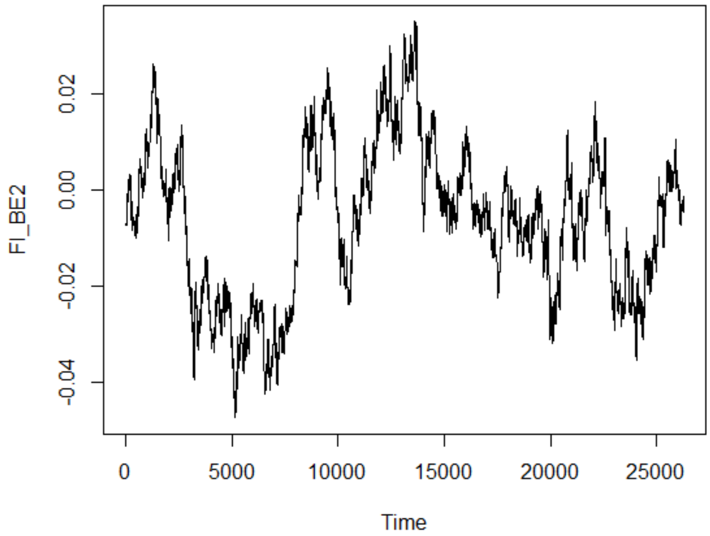

Figure 35: FI/BE Correlation from 2018-2020

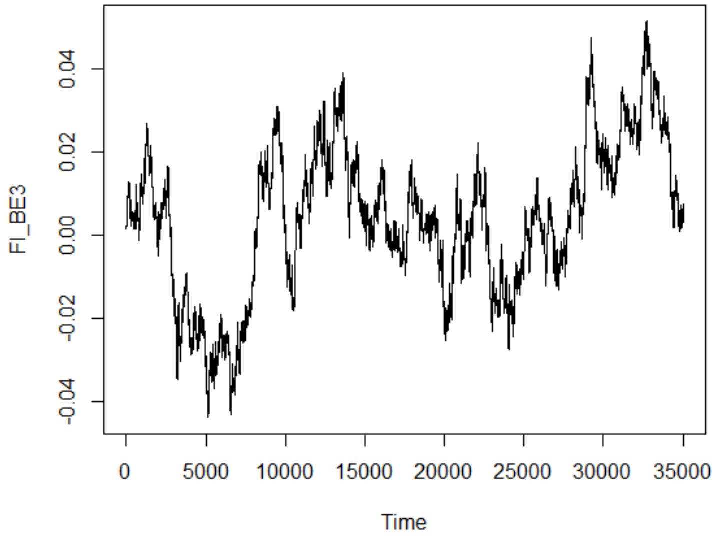

Figure 36: FI/BE Correlation from 2018-2021

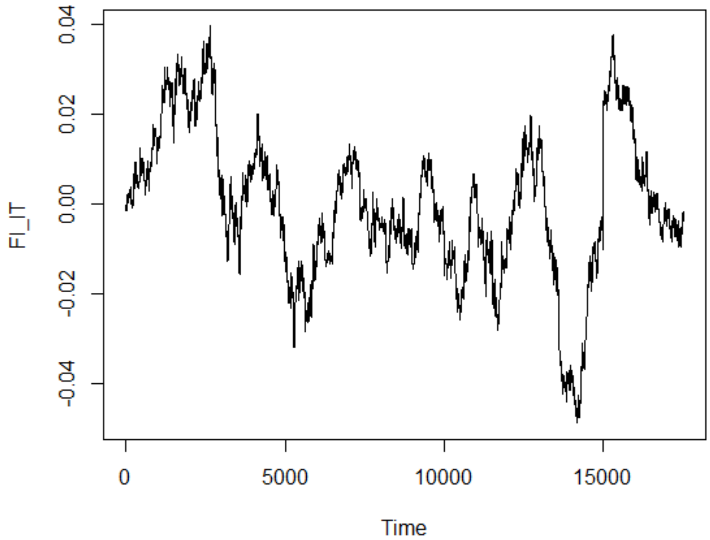

Figure 37: FI/IT Correlation from 2018-2019

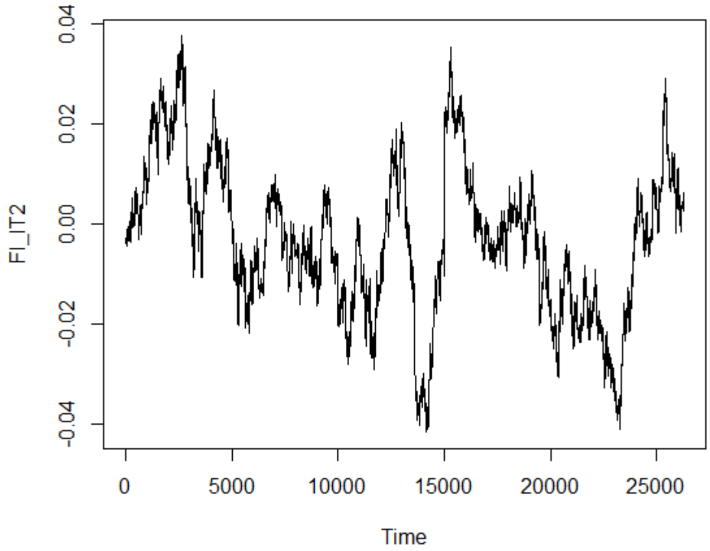

Figure 38: FI/IT Correlation from 2018-2020

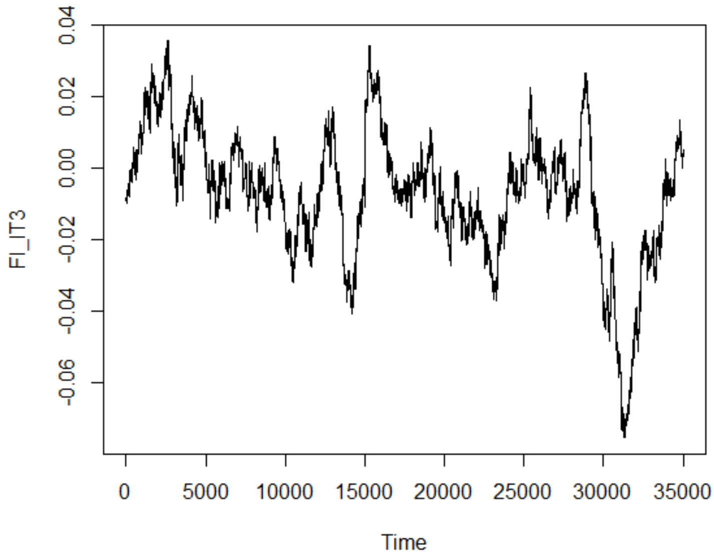

Figure 39: FI/IT Correlation from 2018-2021

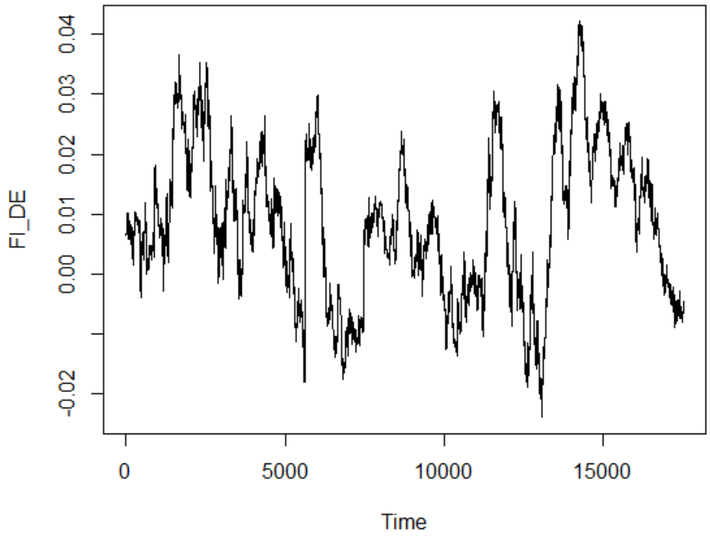

Figure 40: FI/DE Correlation from 2018-2019

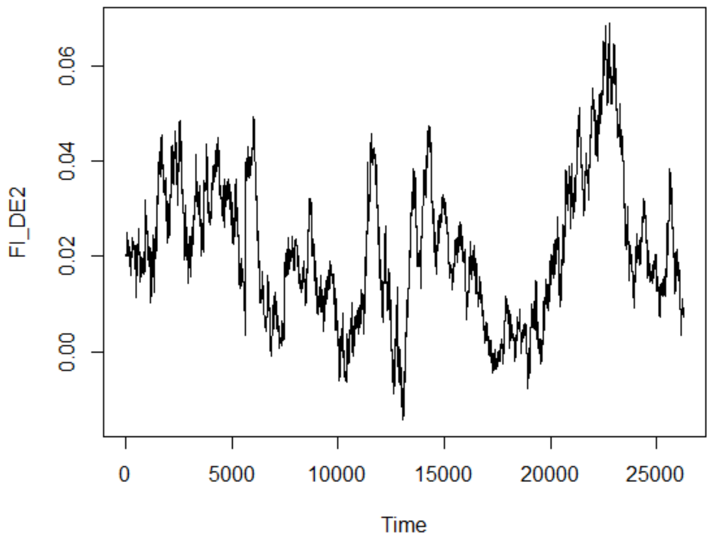

Figure 41: FI/DE Correlation from 2018-2020

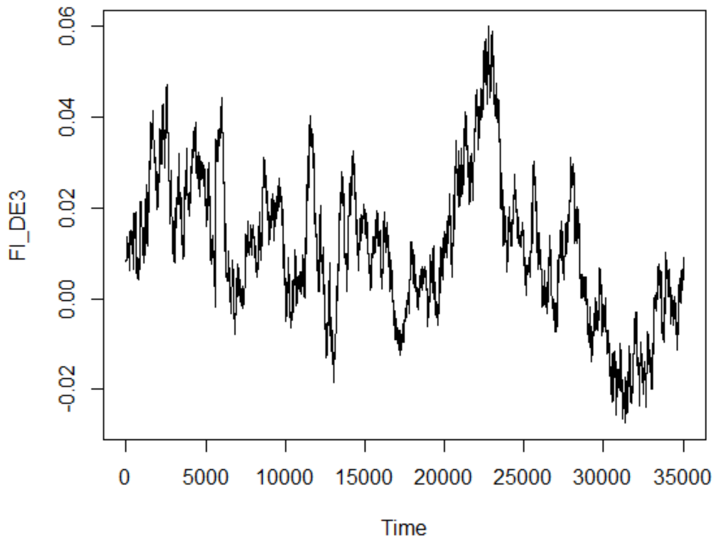

Figure 42: FI/DE Correlation from 2018-2021

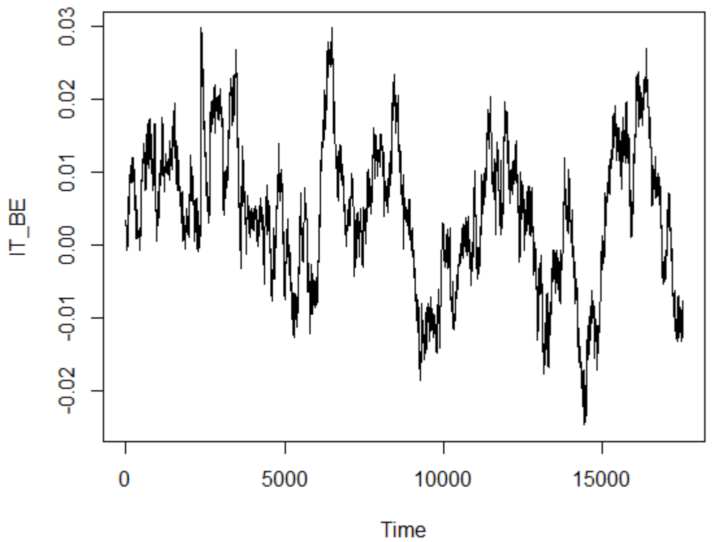

Figure 43: IT/BE Correlation from 2018-2019

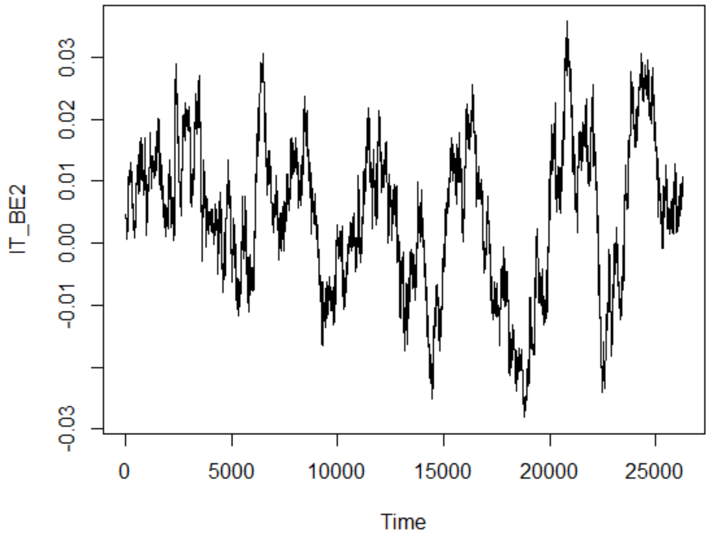

Figure 44: IT/BE Correlation from 2018-2020

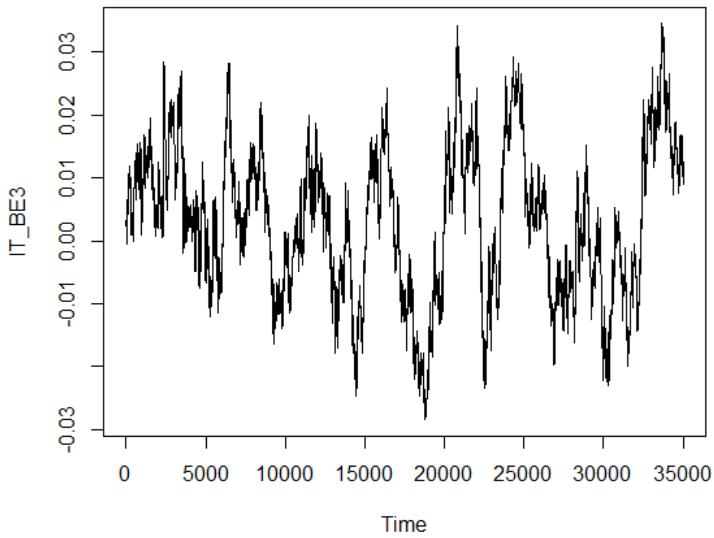

Figure 45: IT/BE Correlation from 2018-2021

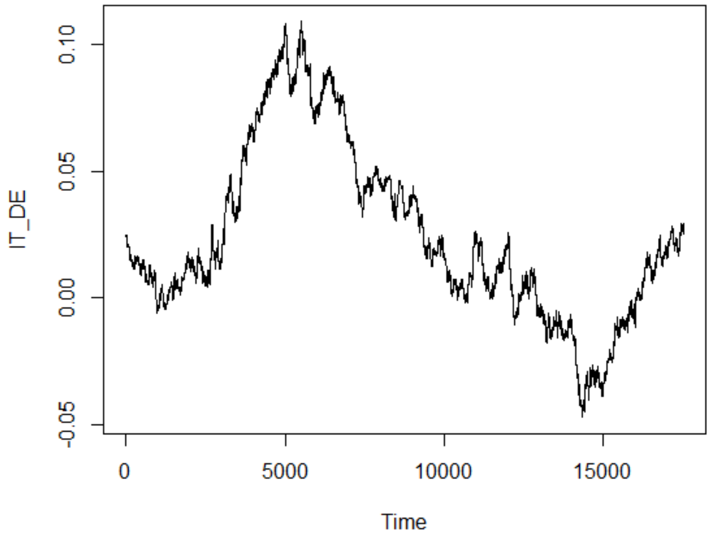

Figure 46: IT/DE Correlation from 2018-2019

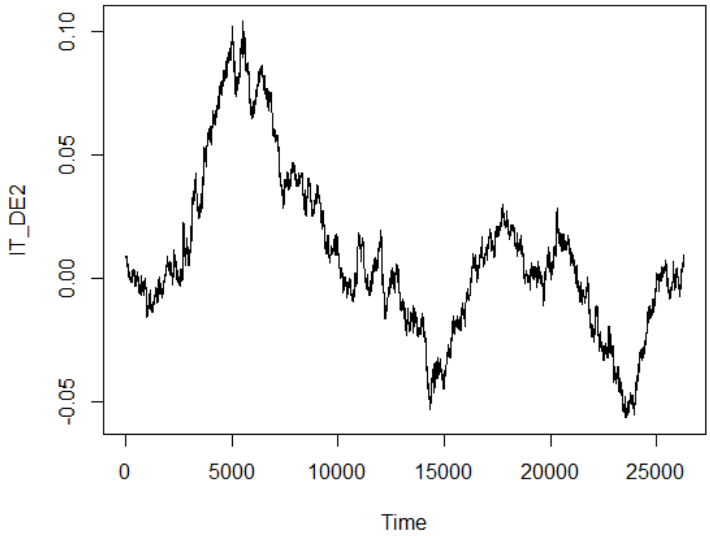

Figure 47: IT/BE Correlation from 2018-2020

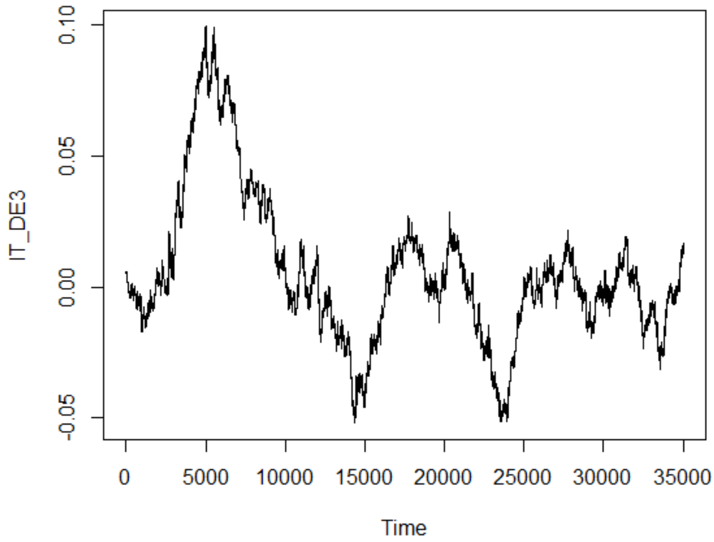

Figure 48: IT/BE Correlation from 2018-2021

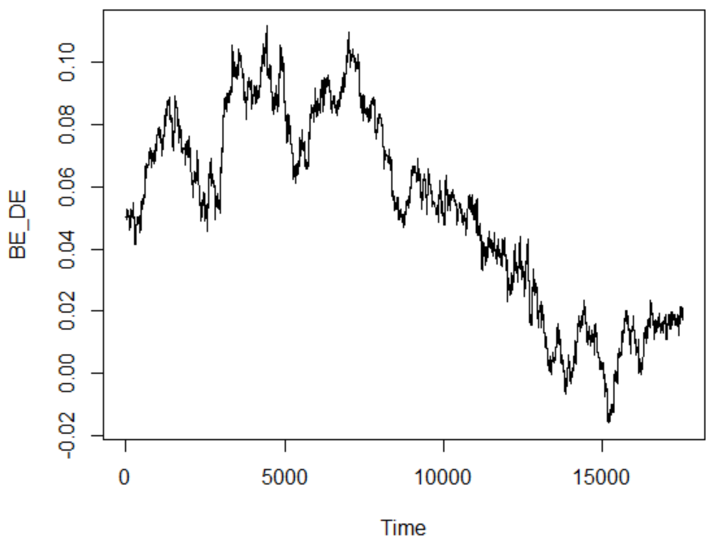

Figure 49: BE/DE Correlation from 2018-2019

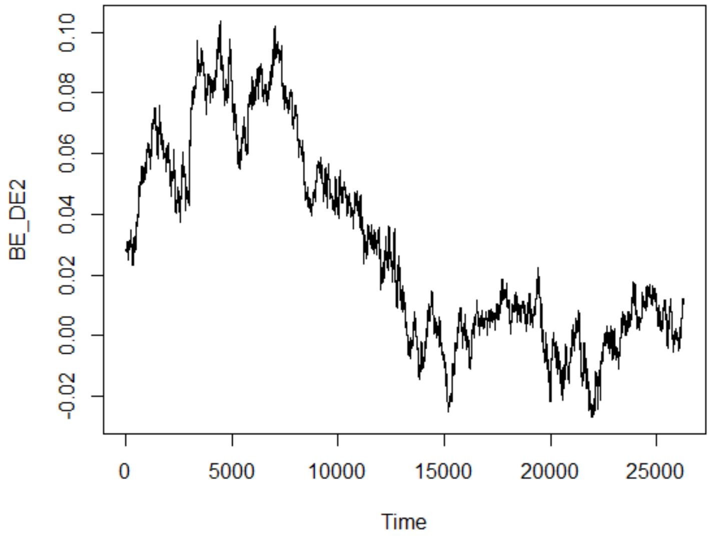

Figure 50: BE/DE Correlation from 2018-2020

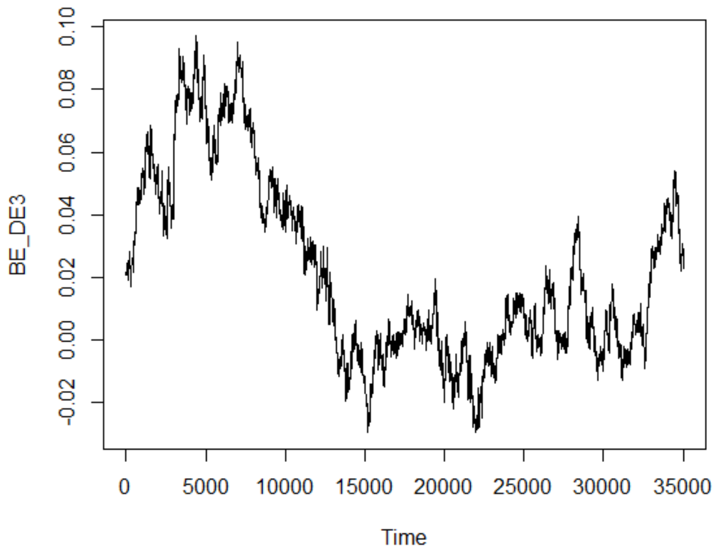

Figure 51: BE/DE Correlation from 2018-2021