Aalborg University

Department of Electronic Systems

Long Master Thesis

Information-Theoretic Modeling and Performance Analysis of Femtocells

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Abstract:

A superposition coding transmission scheme is evaluated in a representative femtocell network scenario. In order to extract conclusions on the performance of the scheme, two state-of-the-art reference scenarios are also defined. For the comparison, some optimization parameters are selected to run in a Montecarlo method simulation. The problem is formulated in information-theoretic terms.

Preface

This report constitutes the Master Thesis Project of Carlos Yáñez Roche for the Telecommunications Engineering degree at the Miguel Hernández University from Elche, Spain. It has been written using LaTeX and consists of the following chapters: C1 introduction; C2 scenario definition; C3 use of information-theoretic techniques in femtocells; C4 rate analysis and multiple access channel constraints; C5 comparison between superposition coding and reference scenarios; C6 conclusions. Bibliography and Appendixes are included in the end of the report.

MATLAB has been used to give support to the different calculations performed.

The author would like to acknowledge Petar Popovski, Troels B. Sørensen and Troels Kolding for their assistance during the whole project work time.

Carlos Yáñez Roche
Aalborg University, 27th May, 2011

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In a personal way, I also thank all my friends from the Miguel Hernández University for the great five years I spent with them studying my degree.

I would like to give my special thanks to my friend in Aalborg, Sara, Fran, Marta, Mario, and Pablo, and my friends and partners in RATE Lucas, Ignacio and Javier for making my stay here such an enjoyable experience and becoming my family in my days in Denmark.

I will continue in Spanish for my last acknowledgements to my family.

Por encima de lo demás, quisiera dar las gracias a mi familia, especialmente a mis padres José y María Jesús y hermana Nuria, por la educación y el apoyo que me han dado durante todos estos años, permitiéndome alcanzar los objetivos que me haya marcado y

ser todo lo que ahora mismo sea.

"I do not think that the wireless waves I have discovered will have any practical application".

Heinrich Rudolf Hertz

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To José and María Jesús

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Chapter 1

Introduction

1.1 Femtocell networks

1.1.1 Generalities

The needs for bandwidth in wireless networks have experienced an exponential growth in the past years. Many new standards have been developed in order to deal with this necessity, such as WiMAX, High Speed Packet Access (HSPA) and LTE standards. In indoor scenarios, Wi-Fi networks are widely deployed, but they will not be able in all cases to support the mobility and coverage requirements of cellular users [1].

A typical way of achieving the goal of increasing the bandwidth is to bring the transmitter closer to the receiver. Given the inherent mobility of the cellular system users, this can only be achieved deploying more infrastructure, usually in the form of microcells, relays or distributed antennas. However, the main drawback of the cell-size reduction of the cellular network is that the network infrastructure becomes too expensive.

An alternative are femtocells, or home base stations (HBS). HBS are low-power base stations installed at the user premises and connected to the rest of the network via digital subscriber line (DSL), cable modem, or an orthogonal radio frequency channel (backhaul link). HBS's main feature is that they are much cheaper than microcell base station and sometimes can be acquired by mobile network users, which reduces drastically the expenses in the deployment for the mobile operator [1].

1.1.2 Advantages and challenges

The key arguments in favor to femtocells [1] are summarized in the following:

- **Better coverage and capacity.** Since femtocells are close to the users, they make it possible to use lower transmit power. Then, higher signal-to-interference-plus-noise ratio (SINR) values are achievable, because the noise created by interfering stations is lower but the useful signal level is still sufficiently high.
- **Improved macrocell reliability.** If the traffic generated indoor can be absorbed by femtocells, macrocells can dedicate their resources to outdoor users, providing a more reliable service.
- **Cost benefits.** For the Mobile Network Operators, the deployment of femtocells reduces the capital expenditure. The cost of a macrocell base station includes the site lease, electricity and backhaul. By deploying a femtocell network, the network infrastructure decreases and, with it, the expenses of the operator. A recent study [2] shows that the operating expenses scale from $60,000/year/macrocell to just $200/year/femtocell.
- **Reduced subscriber turnover.** Poor indoor coverage can originate costumer dissatisfaction, making them consider switching operators. With a wide femtocell deployment attending the necessities of indoor users, they will less likely be unhappy with their carriers.

Some of the greater challenges that a successful femtocell deployment will have to face are:

- **Inter-tier interference.** Due to aggressive spectral reuse, the system throughput in two-tier networks is limited by inter-tier interference, caused by femto and macrocells sharing the spectrum. This calls for efficient solutions like distributed power allocation or interference avoidance techniques.
- **Reliability of the backhaul link between the HBS and the services provider.** As it was mentioned above, HBSs connect to the rest of the network infrastructure via an external backhaul link provided by an Internet Services Provider (ISP) financed by customers. The backhaul link is not usually designed to serve to this matter, so it may not always offer a performance as good as required, unlike other structures such as microcells. Particularly, studies [1] have shown that in backhaul links implemented with DSL connections shared with Wi-Fi, the femtocell performance is severely affected.

- **Techniques for efficient use of the overall infrastructure.** Currently, there is a well developed infrastructure after years of mobile services being offered, that will be enriched when femtocells begin to be also spread in the cities. A new challenge is to look for techniques that allow the operators to make the most of the deployed equipment (both old and new) by making an efficient use of the resources. This project will concentrate in this point, studying innovative techniques that can boost the system capacities.

1.1.3 Typical femtocell network configuration

Usually, a typical network configuration consists of macrocells with embedded femtocell "hot-spots". This can be seen as a two-tier network structure. This setup seems particularly appropriate if one considers that 50% of voice traffic and 70% of data traffic are originated indoor [3]. In addition, users would experience seamless connectivity employing the same handset both indoors and outdoors.

An example of a femtocell network setup can be found in Figure 1.1. In it, we can see a macrocell base station giving service to some outdoor users. Surrounding it, several femtocell hot spots serving indoor (or nearby) users are found.

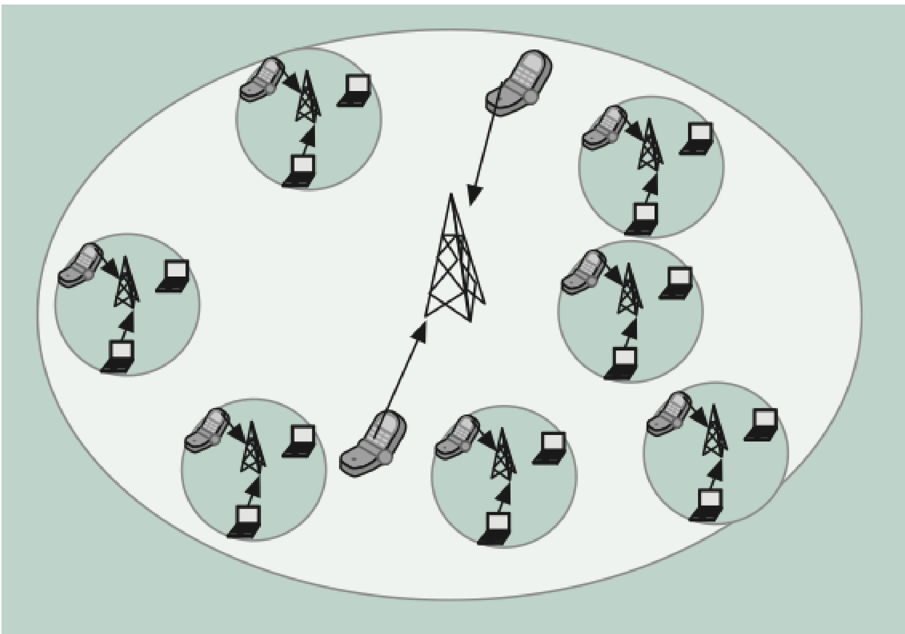

Figure 1.1: Typical femtocell network scenario [1]

1.2 Motivation for an information-theoretic study

Information-theory is a branch of the applied mathematics that deals with the quantification of information. It was developed by Claude E. Shannon [9] to find the theoretic limits on signal processing operations such as data compression, storing and reliability of communications. It has application in a wide range of areas such as cryptography, statistical inference and other forms of data analysis.

One of the most important applications of it is the coding theory, which can be divided in source coding (data compression) and channel coding (error-correction codes). Using a statistical description for data, information theory quantifies the number of bits needed to describe the data, which is the information entropy of the source. In this text, this application of the information-theory will be used in an innovative manner to study a representative case that involves the use of femtocells for a first approximation to know the potential benefits of the deployment of this new kind of infrastructure.

In particular, the aim of performing a theoretic approach is to know the fundamental boundaries of some coding scheme. Knowing these boundaries will allow us to decide if it is worth it to continue evaluating the scheme. In this kind of studies, many practical issues are not considered and many assumptions are made (e.g. assuming synchronization, normalizing noise powers, etc.) in order to make the problem easier to solve from a mathematical point of view. Consequently, it can be said that the *real* performance will always be worse than the *theoretic* boundaries indicate. Therefore, if no benefit (or not remarkable) is achieved in theory, the coding scheme can be roled out without having to go deeper into the problem (considering complicated practical issues). If, on contrary, a potential benefit is detected, it will encourage further investigation to get more realistic results.

Another motivation to use information-theory is to test new ways of sending the information that imply cooperation between base stations. This is motivated by the characteristics of the network infrastructure. Basically, having many base stations close to the user creates a situation favorable to break the traditional schemes, where each user connects to the best serving base stations only. Cooperative transmissions may be reliably implemented thanks to the fact that one user is likely to have more than one good serving base stations at the same time. This kind of transmissions schemes consider the possibility of treating the network as a whole. Base stations can cooperate to send or process the signals to/from users in a coordinated way.

1.3 Problem preview

1.3.1 Steps of the problem and project organization

This project formulates and solves an information theoretic problem relevant in the femtocell networks context. It is done in the following way:

- A scenario that is regarded to be representative of a typical femtocell situation consisting of outdoor/indoor users and base stations, like the one shown in Figure 1.1, is chosen. For this, the common characteristics of this situation are taken into account to be adapted to an mathematically tractable scenario.
- Along with the previous scenario, other reference scenarios, that represent state-of-the-art systems, are defined in order to benchmark the benefits of using the innovative techniques.
- Several coding schemes are proposed and the possible mapping and utilization of them in the representative scenario is discussed. One of them is chosen to be tested.
- Having decided *what* to test *where*, some evaluation parameters are defined for the comparison to be made between the proposed scheme and the ones currently being used.
- Simulations are designed and carried out to get the evaluation parameters for all the situation. Numerical and graphic results are presented.
- Conclusions are drawn using the results obtained from the simulation. Way forward is suggested.

This steps also coincide with the organization of the report itself.

1.3.2 Uplink vs Downlink

For the study a problem, a *downlink* scheme was chosen to be tested. The reason for this selection was that there were already some uplink information-theoretic studies, such as [3] and [4]. Some interesting downlink schemes were possible to test, and the possible advantages of having HBSs interconnected via backhaul links wanted to be explored.

1.4 State-of-the-art

In this section, some of the systems and coding schemes currently being used are briefly described. Some of them served as inspiration for the proposed scheme and some served as reference in order to test the improvement that brings using the innovative techniques.

1.4.1 Base station selection

In the traditional mobile communication systems, such as GSM or UMTS, the user equipment survey the incoming signals from the control channels of each base station to get the received signal power from them. With this information, the selection of the serving base station is made based on the Signal to Interference and Noise Ratio (SINR), being chosen the base station providing the highest value of this parameter. Given the mobility inherent to the users, when one user is moving from the coverage area of a base stations to to area of another, the connection switches to the new base station (handover).

Based on this transmission scheme, one of the reference scenarios is defined, as it will be explained in following chapters, concretely, in Section 2.3.1.

1.4.2 Beamforming with timesharing

In the information-theoretic context, beamforming is a transmission technique in which a group of base stations send signals weighted with phase-correcting coefficients over the same channel to one user during a certain period of time. Periodically, the system shifts and the group of base stations give service to another user for a while. This process is called timesharing.

This scheme was used to define the other reference case to compare with the solution being studied. Further details will be given in Section 2.3.2.

1.4.3 Wyner model

The Wyner model [5] is a basic and simple model used as a start point for many studies. It obtains the theoretic limits for a very simple cellular multiple-access system. In this fundamental information-theoretic model, the received signal at a given base station is the sum of the signals transmitted from within that cell plus a factor α times the sum of the signals transmitted from the adjacent cells plus some Gaussian noise. Although this simple model is not realistic, it provides results considerable good results to bring insight into the real systems. Its simplicity makes it ideal for getting first approximations in theoretic studies.

The scenario used in this model is shown with more details in Section 2.1.1.

1.4.4 O. Simeone's work

As a start point for this project, the papers from O. Simeone [3], [4], were used. In [3] the concept of multicell processing (MCP) is presented. This new transmission technique, also known as network MIMO, proposes a joint encoding and decoding across the different cells. The decoding of the joint signals takes places in a network element called central processor (CP), to which the signals that each base station gathers are relayed via the corresponding backhaul links. This situation is represented in Figure 1.2. The paper shows conclusions regarding how using different coding schemes (decode & forward, compress & forwards...) and access methods (open or closed access) affect in the performance of the system.

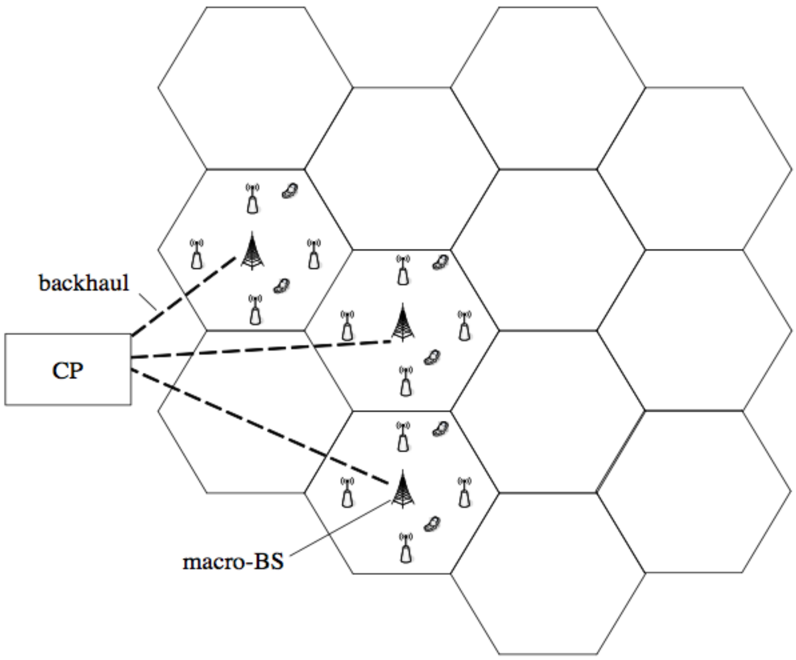

Figure 1.2: Multicell processing network infrastructure [3]

This paper was used mainly as a reference in Section 2.1.1 for choosing the scenario in which the coding schemes are tested, following the assumptions presented in it.

In [4], a single macrocell serving a number of outdoor users, overlaid with a femtocell which includes several home users, is studied. The home users in the femtocell are served by a home base station (HBS) that is connected to the macrocell base station (BS) via an unreliable connection (e.g., DSL). Robust communications strategies for the home users are investigated. The problem is formulated in information-theoretic terms and rate bounds are given to achievable sum-rates for outdoor and home users. The scenario used in this paper is shown in Figure 1.3.

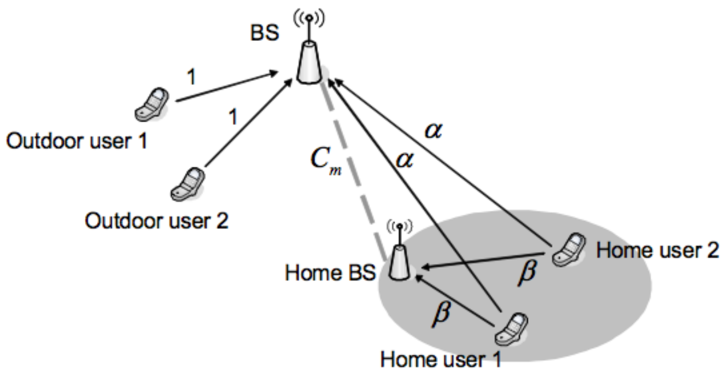

Figure 1.3: Scenario for robust communications in femtocells [4]

Work in [4] was used in our investigation as a reference in how to define mathematically the problem in Section 3.2.

Chapter 2

Definition of the scenarios

In this chapter, the evolution of the process of choosing the scenario in which the study is concentrated is described. This scenario has to be representative for the situation that wants to be studied, i.e. a typical femtocell environment. After having discussed the basic structure of the scenario, a more detailed description is shown, and all the notation that will be used throughout the report is introduced. Finally, a description of the reference scenarios with which the scenario under study will be compared are presented.

2.1 Selection of the representative scenario

2.1.1 Wyner-based scenarios

As a starting point, the scenario proposed in [3] was taken into consideration. This paper analyzes uplink transmission schemes using multicell processing (MCP). Even though our investigation concentrates on downlink, this paper was taken as a first reference since the modeling is made from an information-theoretic points of view, like in our case. Therefore, most of the assumptions in it are also valid for this study.

In [3], a basic information-theoretic reference model was used: the Wyner model, [5]. This model is depicted in Figure 2.1. It represents a unidimensional scenario in which it is considered that all the base stations (BS) are located in a straight line, with a constant separation of $2R$ between them. The mobile station (MS) is situated in a fixed position too. The BS giving service this MS has a channel such that there is a power gain of 1 between this BS and the MS. There are only two interfering BSs, the ones just in the left and right of the serving BS, i.e., if the MS is being served by BS n, the only interfering BSs are the neighboring BSs $n-1$ and $n+1$. These two interfering BSs have the same power gain towards the MS, α, satisfying $0 \leq \alpha \leq 1$.

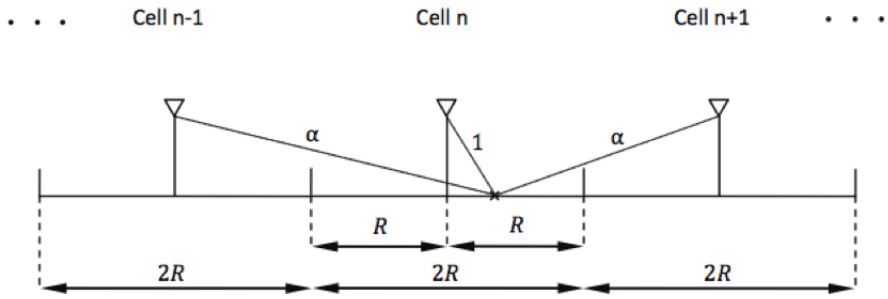

Figure 2.1: The Wyner model [5]

Even though this model 2.1 seems very basic, it has been widely used in many information-theoretic studies since it is regarded to give acceptable results and it is mathematically tractable.

However, some studies have been made in order to test the accuracy of the Wyner model, like [6]. This paper comes to the conclusion that in systems with a small number of users the Wyner model is not accurate, because the interference power parameter α cannot statistically represent all the users, given the randomness of their possible locations. On the other hand, in cases in which there are many users, it is possible to set a value for α that can be statistically representative, and therefore the results of the model can be acceptable.

Back to the Simeone's paper [3], the model used in this study is based on the work in [5]. The uplink adaptation of this scenario used in it for the case of Multicell Processing (MCP) transmission schemes is shown in Figure 2.2.

In this scenario, some modification with respect to the Wyner model can be seen. This modifications aim to adapt the model to a femtocell environment. We can distinguish outdoor base station, home base station (HBSs) and both indoor and outdoor users. The HBS are connected to their respective BS via a backhaul link with C bits/s/Hz. About the pathlosses, outdoor users have a α power loss variable towards their main BS, and β_H towards their serving HBS. Outdoor user l presents a gain of 1 to its BSs (like

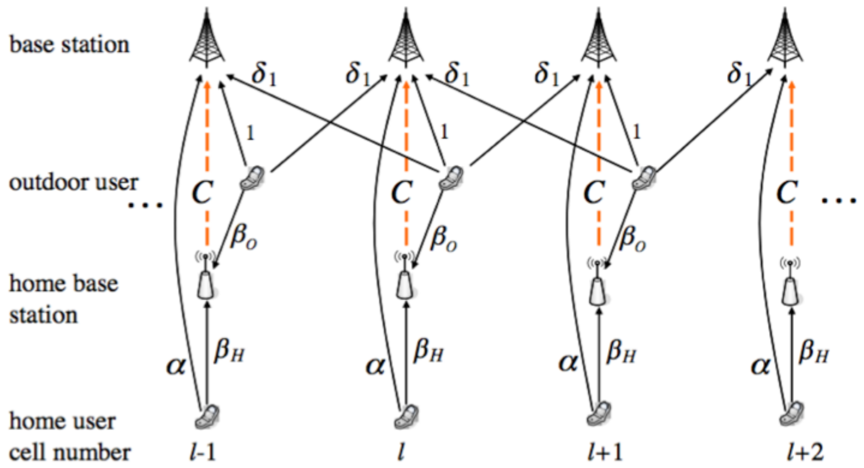

Figure 2.2: Simeone's MCP scenario [3]

in Wyner model), δ_1 to the BSs $l-1$ and $l+1$ and a gain of β_O to their closest HBS. Here, $\beta_O > 1$, because the variables are normalized with respect to the link between the home user and the BS.

This model presents some issues that were discussed. The first of the concerns the power variables δ_1, β_O, β_H and α. As it was previously stated in relation to the work in [6], the accuracy of the Wyner model (and consequently to the model being discussed here) depends highly on the concentration of users in a particular scenario. Since this study is concentrating on a femtocell case, in which very varying circumstances can be found, it is hard to find valid values for these variables so that the results are representative. In addition, as it is also pointed out in [6], the accuracy of the model depends on the multiple access technique used, too. Therefore, it cannot be generalized that for every case the scenario provides acceptable results.

Another problem with the model from [3] is that it is not considered realistic to model that one outdoor user is able to connect to several BSs and only one HBS. The reason for this is that it is expected that in a common femtocell network deployment, the density of HBSs will be much higher than the density of BSs. This is because HBSs are supposed to be in many user premises, while macrocell BSs are usually separated distances ranging 1km. Under this assumption, it would be more common for a user to connect to one macrocell BS and more than one femtocell HBS, unlike it is modeled in this scenario.

2.1.2 Modification of Simeone's scenario

As a first approximation to the final scenario, a different model that intends to fix this problems was proposed. This model can be found in Figure 2.3.

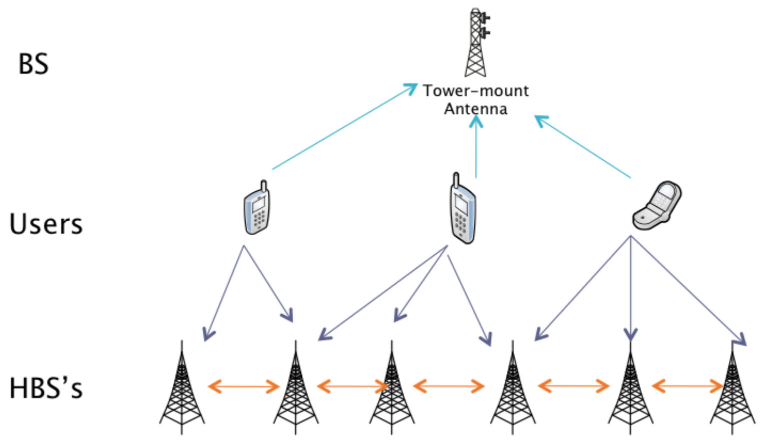

Figure 2.3: Modification of Simeone's MCP in a HBS environment scenario

The first modification that can be observed in Figure 2.3 is that the considered links change in order to fix the defect that was explained in the end of the previous section. Now, it is allowed that users can connect to more than one HBS but only one macrocell BS. This is because in a normal urban deployment many more HBSs can be found near a typical user than macrocell BSs. To perform the MCP, all the received signals in the BS and HBSs would be decoded in a central processor (CP), that is connected to all of them using a backhaul link with C bits/s/Hz, like in the previous case.

However, it should be noted that the restriction of only connecting to one macrocell BS is not so important, because since there is a well developed backbone communication in the currently deployed mobile networks, exchanging information between BSs is easy and effective. Therefore, even though it is not considered in the model, it should not be discarded that the users can connect to more than one BS at the same time.

A new option that can be considered is the possibility to create a inner-tier link between HBSs. This low-capacity link would allow HBS to exchange small quantities of information that could be very useful for many different purposes, e.g. make the coordination of the spectrum allocation easier for techniques such as [7]. A disadvantage that would imply using this link between HBSs is that it would make them more expensive and that there are no current studies that assure the reliability that can be attained for it.

2.1.3 Simple versions of the new model

A new simplified version of the model described above was proposed in order to make the study easier to deal with, in the sense that a more restricted number of architecture elements are considered. This new version is shown in Figure 2.4.

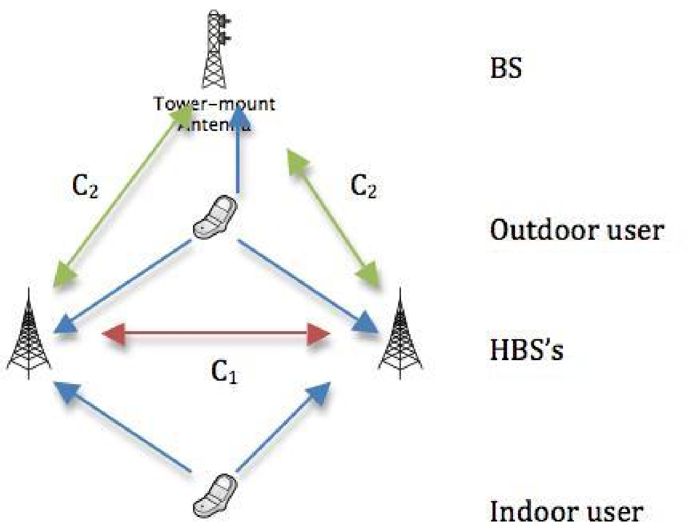

Figure 2.4: Simpler version of Figure 2.3

In this version, 3 base stations are considered: two HBS and one macrocell BS. Only two users are taken into account. This users, in principle, are regarded to be one outdoor and one indoor. The indoor users only is able to connect to two close HBS, while the outdoor user get service from the BS and both HBSs.

In this case, the BS acts as the central processor (where the decoding process is done), so all the information gathered by the HBSs is relayed to it, via the backhaul links C_2, considered equal for both HBS-BS connections. In this approach, the inner-tier link between HBSs is considered, having a capacity of C_1.

2.2 Description of the chosen scenario

After having discussed the evolution of the process of election the scenario in which the information-theoretic techniques will be tested in this investigation, the final scenario is presented here, along with the notation regarding the model.

The practical scenario chosen to be tested is represented in Figure 2.5. As it can be seen, it consists of 3 HBSs and 2 users (UEs). In the rest of the report, the subscript k will refer to the UEs ($k = 1, 2$) and the subscript i to the HBSs ($i = 1, 2, 3$). The HBSs are connected to a gateway (GW) from the operator via their respective backhaul links C_{B1}, C_{B2} and C_{B3} (shown in Figure 2.5 with a dotted line). Connectivity between HBS's also exists via inner-tier links $C_{k,j}$, with $(k, j) = (1, 2), (1, 3), (2, 3)$ (discontinuous line). In the scenario a downlink transmission takes place from the GW towards the two UEs. In the rest of the report, when talking in general terms, it will be said that the scenario consists of B HBSs giving service to U users.

The reason for choosing this amount of HBSs and UEs is because it was thought that it could be representative of a real practical situation, so that the results obtained for this could be extrapolated to a more generic scenario.

In this case, no distinction has been made between indoor and outdoor users, since it has been considered that, operating under open access conditions, for the transmission scheme, there is no difference in the treatment of the different kinds of users.

2.2.1 Transmission description

For the scenario described above, depicted in Figure 2.5, we consider a downlink transmission from the GW to the two UEs. Through the links C_{B1}, C_{B2} and C_{B3}, the information to be transmitted to the UEs is received in all three HBSs. These backhaul links are considered to have enough capacity to bring the information to the HBSs correctly before the transmissions have to be made. The basic idea of the investigations is to use some information-theoretic coding technique to send the information to the

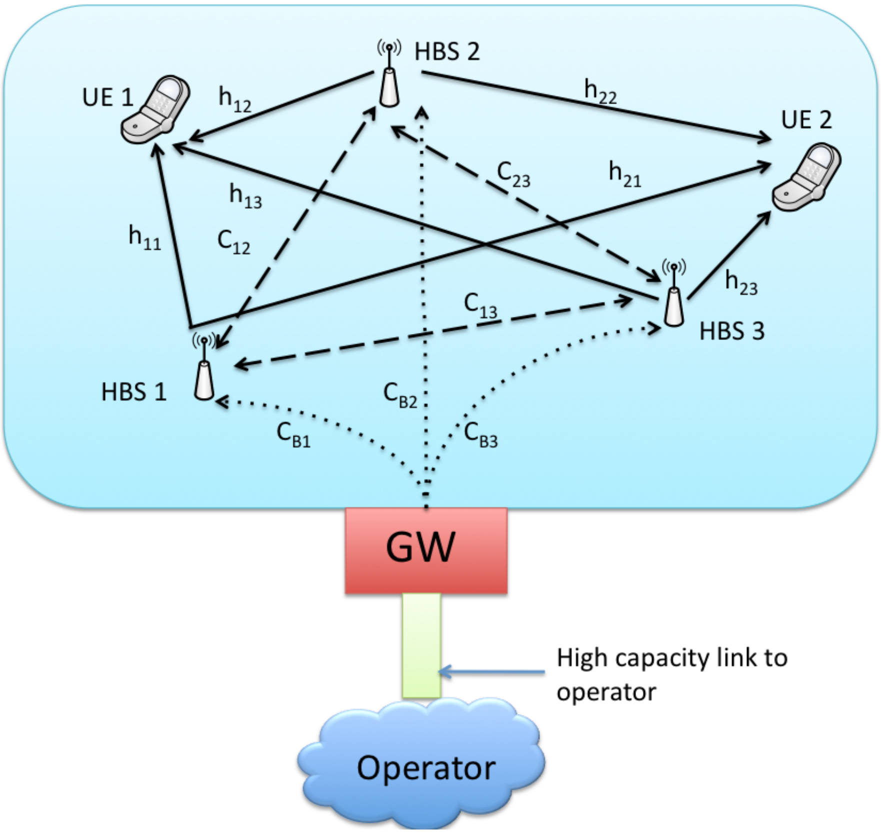

Figure 2.5: Scenario for the problem

two UEs using the three HBSs, which send the corresponding signals to the two users containing their messages. These transmissions are represented in the figure with a continuous line, where the notation $h_{k,i}$ is used to represent the channel coefficients from the base station i to the UE k. It is assumed that the length of the transmitted symbols and the information packets are sufficiently short for $h_{k,i}$ to be considered constant for a given symbol, i.e. block-fading channel.

It will be considered that the physical resource used during the transmission is shared by all the B HBS's. This means that the HBSs send their sequences at the same time using the same bandwidth.

2.2.2 Channel coefficients

The channel coefficients $h_{k,i}$ measure the voltage variation of the signal strength in their way from the transmitting HBS i to the receiving UE k. They are modeled as the product of a Rayleigh-distributed random variable and a pathloss variable. Mathematically, their magnitude can be expressed as

$$|h_{k,i}|^2 = s\ d^{-n_{pl}}|h_{ff}|^2 \tag{2.1}$$

where s is a pathloss constant, d is the distance UE-HBS ($d > 0$), n_{pl} is the pathloss exponent and h_{ff} is a Rayleigh-distributed variable representing fast fading. The value of n_{pl} ranges typically between 3 and 4 for indoor scenarios.

2.3 Reference scenarios

In order to test the effectivity of the proposed coding schemes in the scenario just described, it is necessary to define some reference scenarios. These scenarios are general cases that represent some state-of-the-art systems, and they will allow us to benchmark how far the innovative techniques can go in comparison to technologies currently in use.

2.3.1 Reference scenario 1 - One dedicated HBS serving each UE

The first reference case consists of the two UEs and only two serving HBSs (i.e., the third one is supposed to be transmitting to some other user, affecting only creating interference in the considered users). The serving HBSs will be chosen by each user depending on the signal levels received from them, selecting the one that provides best SINR levels. Like in the main scenario of the investigation, in this reference scenario a downlink transmission takes place from the operator to the two UEs, which previously are transmitted to the two serving HBSs via C_{B1}, C_{B2} or C_{B3}. In this case, as an example, it is supposed that HBS1 will be assigned to UE1, and HBS2 to UE2, being HBS3 an interfering station for both users.

For these two transmissions, the same physical resources are used. Consequently, the received signal in one user coming from the HBS not giving service to it, is considered to be interference. In such conditions, $h_{1,1}$ and $h_{2,2}$ represent the channel coefficients of the useful signal's link for users 1 and 2 respectively, while $h_{1,2}$ and $h_{2,1}$ correspond to the interfering links.

With the signal and interference links shown, the expressions of the Signal to Interference and Noise Ratio (SINR) can be easily obtained for this example as the following:

$$\bar{\gamma}_{1,ref1} = \frac{E\left[\left|\sqrt{P_1}h_{11}\right|^2\right]}{E\left[\left|\sqrt{P_2}h_{12}+\sqrt{P_3}h_{13}\right|^2\right]+\sigma_1^2} = \frac{\left|\sqrt{P_1}h_{11}\right|^2}{\left|\sqrt{P_2}h_{12}+\sqrt{P_3}h_{13}\right|^2+\sigma_1^2} \quad (2.2)$$

$$\bar{\gamma}_{2,ref1} = \frac{E\left[\left|\sqrt{P_2}h_{22}\right|^2\right]}{E\left[\left|\sqrt{P_1}h_{21}+\sqrt{P_3}h_{23}\right|^2\right]+\sigma_2^2} = \frac{\left|\sqrt{P_2}h_{22}\right|^2}{\left|\sqrt{P_1}h_{21}+\sqrt{P_3}h_{23}\right|^2+\sigma_2^2} \quad (2.3)$$

being $\bar{\gamma}_k$ the SINR for user k (the bar on top of γ denotes that we are talking about the average SINR, not the instantaneous); P_i is the power of the signal transmitted by HBS i; and σ_k^2 is the variance (power) of the noise in receiver k, which will be normalized to 1 for simplicity in the simulations presented here. The expectation operator is used to get the average of the instantaneous values of $h_{k,i}$. However, since block-fading channel is being assumed (as explained in section 2.2.1), it can be removed, because the channel coefficient will remain constant in that interval.

For the calculation of the maximum channel capacity that can be achieved in each user, we will use the Shannon-Hartley's theorem, [9]. According to this theorem, one can estimate the maximum rate usable for a message to (theoretically) be able to get an error-free transmission. This theorem, in a normalized-per-bandwidth version, can be expressed as

$$R_k = \log_2(1+\bar{\gamma}_k) = C(\bar{\gamma}_k). \quad (2.4)$$

In this text, the notation $C(\bar{\gamma}_k)$ will be used when referring to calculations using the Shannon's theorem.

Once the SINR expressions have been derived, simply combining equations (2.2), (2.3) and (2.4) one can obtain the final capacity of the channels for both users in this reference case as

$$R_{1,ref1} = C(\bar{\gamma}_{1,ref1}) = \log_2\left(1+\frac{\left|\sqrt{P_1}h_{11}\right|^2}{\left|\sqrt{P_2}h_{12}+\sqrt{P_3}h_{13}\right|^2+\sigma_1^2}\right) \quad (2.5)$$

$$R_{2,ref1} = C(\bar{\gamma}_{2,ref1}) = \log_2\left(1+\frac{\left|\sqrt{P_2}h_{22}\right|^2}{\left|\sqrt{P_1}h_{21}+\sqrt{P_3}h_{23}\right|^2+\sigma_2^2}\right). \quad (2.6)$$

Other considerations on this reference scenario are given in Section 5.3.2.

2.3.2 Reference scenario 2 - Single-user beamforming

The second reference scenario is represented in Figure 2.6 at a particular instant of time.

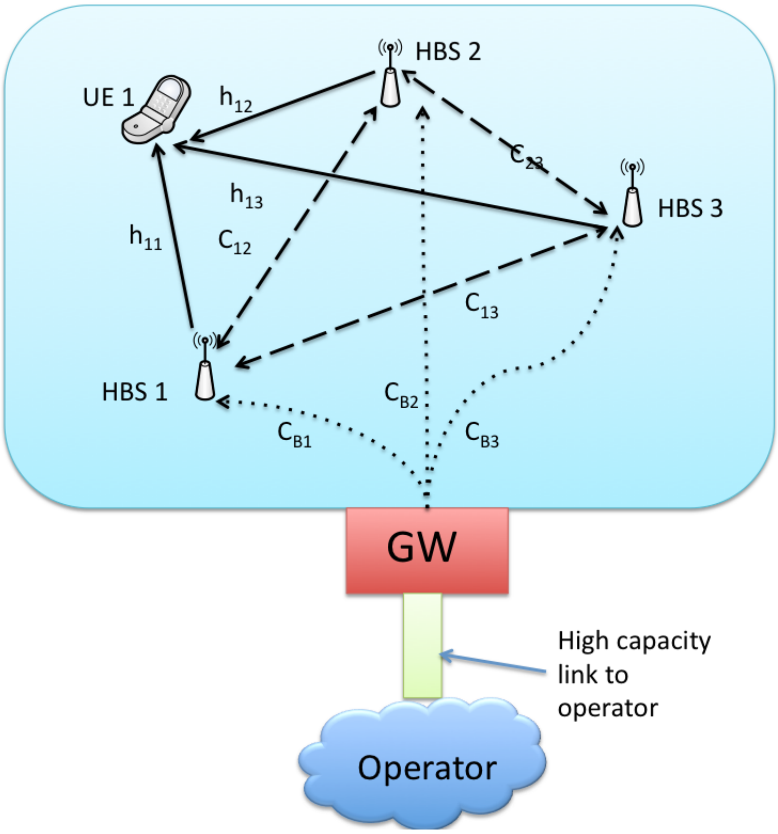

Figure 2.6: Reference scenario 2 in a concrete instant

In this case, as it can be seen, there is only one UE being served at a time and three operating HBSs. The system switches periodically between the situation shown in Figure 2.6 and an analog one where the user being served is UE2. This process is called *time-sharing*. Like in the previous scenario, the information to be sent to UE1 arrives to the three HBSs through their backhaul links C_{B1}, C_{B2} and C_{B3} and then the three HBSs send the same message to the user using the channels represented by $h_{1,1}$, $h_{1,2}$ and $h_{1,3}$. Synchronization between the three signals is supposed. Also, co-phasing in the users is achieved by setting suitable coefficients in the transmitter, so that the receiver in UE1 can add them up coherently. Under this assumptions, the SINR in the receiver will be

The rate expressions and other notation and results regarding this reference scenario will be introduced in Section 5.3.1.

Chapter 3

Use of information-theoretic techniques in femtocells

After having decided the scenario in which the investigation is situated, the purpose of this chapter is to present some information-theoretic coding schemes which were candidates to be studied in the scenario described in Section 2.2.

First, a general description of the main techniques is presented to finally describe in more detail the chosen technique (superposition coding), including the mapping of the technique in our scenario, the problem formulation and some guidelines of how this coding scheme would be implemented in practice.

3.1 Proposed information-theoretic coding schemes

Among the existing coding schemes, three were considered as candidates to be more deeply examined and tested in the femtocell scenario under study: multiple description, Slepian-Wolf coding and superposition coding, [10].

For a simpler explanation of the techniques, the scenario that will be used to present them considers only one user and two HBSs, which act as relays forwarding the information from a BS to a UE (downlink transmission). Specifically, this scenario was presented in Figure 2.3 with the only modification of removing the outdoor user (Section 2.1.3).

3.1.1 Multiple Description

The Multiple Description problem is described in chapter 2 from [10]. The block diagram of this coding technique in the selected scenario is shown in Figure 3.1.

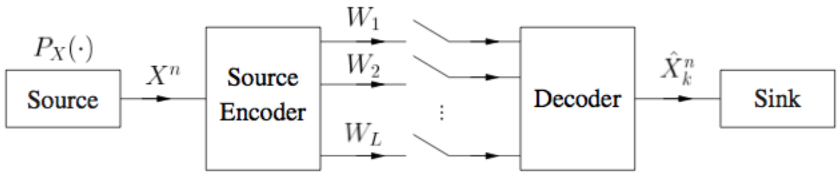

Figure 3.1: Block diagram of Multiple Description [10]

Basically, it is a source coding scheme in which the source sends a message using more than one index, i.e., different representations of it (multiple descriptions). The multiple descriptions are sent through more than one path at the same time, so spatial diversity is obtained. This fits specially well in the scenario being considered here, since the user could send the same message using the path that every HBS offer. Some of these paths may not always be reliable (e.g. C_2 is implemented with technologies such as DSL) so having this kind of redundancy provides several benefits. Some of these benefits can be, for example, having a second version of the message in the receiver in case that the first is not received correctly; or allowing two levels of quality in a multimedia signal, having a basic version of the message if only one of the descriptions is correctly decoded and a higher quality version when both are received correctly (for users in better coverage conditions).

In Figure 3.2, W_1 and W_2 are two different descriptions received via the backhaul links in both HBSs that will be then forwarded to the UE using their respective radio links. W_3 is an alternative path that could be used in the case that the backhaul link of the HBS on the bottom was better than the other one and the link between femtocells could handle it.

3.1.2 Slepian-Wolf problem - Distributed source coding

The Slepian-Wolf problem is explained in chapter 4 from [10]. The solution of this problem is a well-known coding scheme called binning. A short theoretic explanation of this

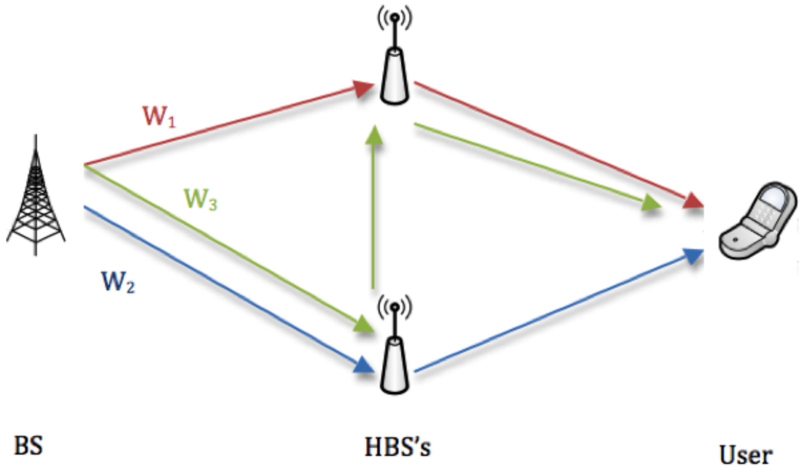

Figure 3.2: Possible implementation of Multiple Description

techniques is given in the following.

A discrete memoryless source (DMS) that emits two sequences of symbols x^n and y^n (where n is the length of the sequence) as depicted in Figure 3.3, is considered. Each encoder maps the sequences x^n and y^n to indexes w_1 and w_2 that will be sent through the channel to their destination. This indexes code the source for a more efficient transmission (source coding). In the receiver, the decoder produces $\hat{x}^n(w_1, w_2)$ and $\hat{y}^n(w_1, w_2)$ using these indexes. The codebooks have to be defined in such way that it can be assured that the error probability satisfies

$$P_e = P\{(\hat{x}^n, \hat{y}^n) \neq (x^n, y^n)\} \to 0 \tag{3.1}$$

when n is sufficiently large.

In information-theory, a known fact is that in source coding, for an input sequence x^n, the rate of the coded sequence must be at least equal to the entropy of the sequence, that is, $R_x > H(X)$, in order to be able to satisfy 3.1. Remember that the entropy of a signal is defined as

$$H_b(X) = -\sum_{i=1}^{N} p(x_i^n) \log_b p(x_i^n) \tag{3.2}$$

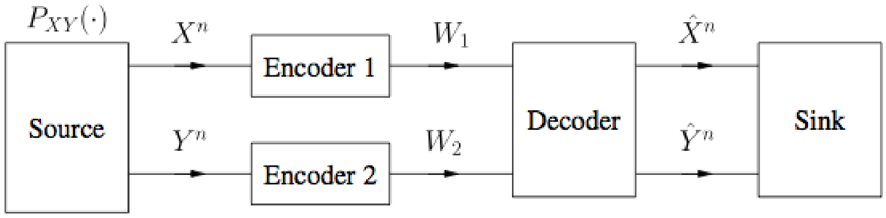

Figure 3.3: The Distributed source coding problem [10]

where x_i^n are the N possible values of x^n, and commonly $b = 2$ so the entropy is measured in *bits*.

Back in the distributed source coding problem, we can say that a rate larger that the joint entropy of the two variables, $R > H(X,Y)$, would be sufficient for the encoding if both of the sequences are jointly coded. Note that in source coding it is interesting to have the lowest possible rates, since they allow the information of the source to be represented with the minimum number of symbols. If both sequences were to be decoded separately (as it is the case), a rate $R = R_1 + R_2 > H(X) + H(Y)$ would obviously work. It should be also noted that $H(X) + H(Y) > H(X,Y)$.

A surprising result that was shown in [8] is that using a rate $R = H(X,Y) < H(X) + H(Y)$ is also sufficient to decode both sequences separately. These techniques are widely known as *binning*.

It is then the objective to find a couple of rates (R_1, R_2) for which, with sufficiently large n, one can satisfy equation (3.1).

The coding procedure consists in assigning to each pair of sequences x^n and y^n a pair of indexes which only occur at the same time when those two sequences have been sent. The key concept that allows the messages to be sent using a sum-rate of only $R_1 + R_2 = H(X,Y) < H(X) + H(Y)$ is the *bin*. A bin is a group of indexes, ω_1 or ω_2, that have the same corresponding sequence $x^n(\omega_1)$ or $y^n(\omega_2)$. The decoder will only be able to reconstruct a sequence if it knows also the index of the other sequence that has been sent along. The benefit, of course, is that a reduction of the rate used is achieved. It can be shown that the theoretic achievable region of rates of the Slepian-Wolf problem is defined by the following Figure 3.4. The points beyond the border (top right side)

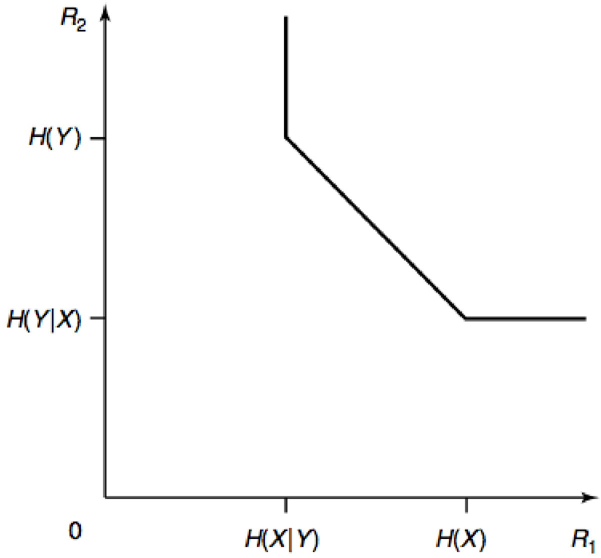

Figure 3.4: Rate region for Slepian-Wolf encoding [10]

represent couples of rates that can be used with this codification.

Figure 3.5 shows a simple example of how binning can be applied in an scenario similar to the one being considered here.

The user sends its message x to both HBSs. The signals that are received in them, x^n and y^n, which consist of the information sent by the user x, plus the noise in each of the receiver z_1 and z_2, are obviously correlated, because they share the information term. This facts permits coding the construction of binning-coded indexes, that require that certain correlation between the sequences of the source exists.

There are some drawbacks in using this technique in the way it was proposed. The main of them is that for the coding process it is mandatory that the two HBSs agree on the rates that they will use to code the messages. This implies that there should be an efficient and reliable coordination between them. Since the communication link between HBSs is not fully developed and tested at the present time, it would be too risky to depend so strongly on it for the coding to be possible.

Another disadvantage is that for the message to be recovered in the receiver, it is

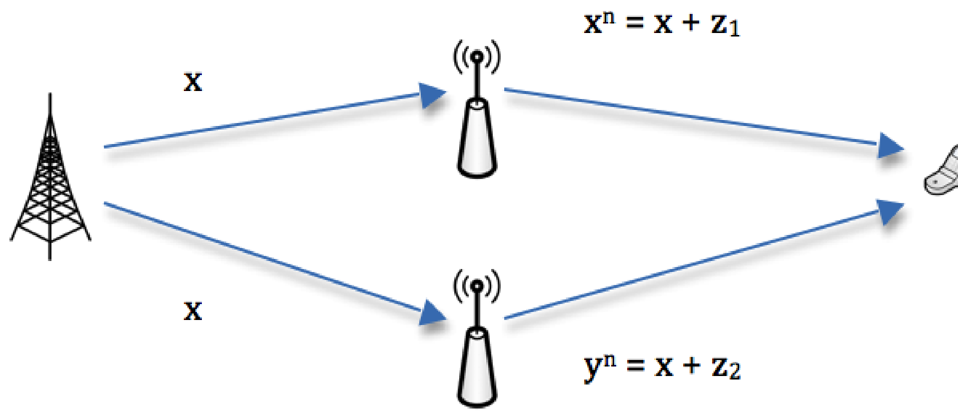

Figure 3.5: Mapping of the Slepian-Wolf problem in scenario

necessary that the signals from both HBSs in the UE arrive correctly to their destination. If one of them fails to do so, the transmission would not succeed. This kind of problems caused that this coding scheme was ruled out to be further studied.

3.1.3 Superposition coding

The last coding scheme that was suggested is superposition coding, presented in Chapter 7 from [10]. Superposition coding is a broadcast channel method to stack codebooks. It has several applications in many systems, such as in high-definition TV (HDTV), where it allows different layers of information to be transmitted, so that depending on the capacity of the receiver, more refined versions of the message can be obtain when there are favorable channel conditions [11]. It is always ensured though, that bad a receiver will get at least a basic version of the transmitted message.

Superposition coding can be applied in many different ways. Here, a general case will be explained. In the following section, a more detailed description will be given when it is mapped to the final scenario chosen for this investigation, described in Section 2.2.

For this general explanation, we will suppose that there are only two levels of information. In this case, two different messages x_1 and x_2 are to be transmitted with the two layers of information. These messages are sent jointly in the same signal with a coefficient to split the power that the sender uses for each message, α. The way the

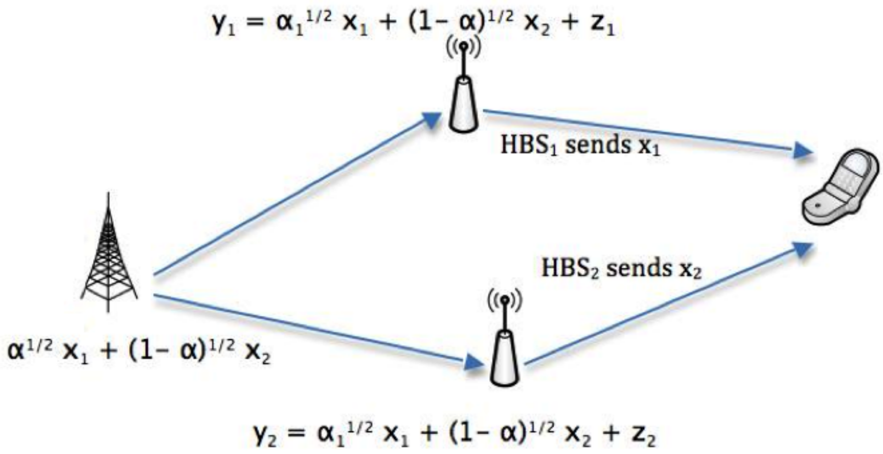

Figure 3.6: Possible implementation of superposition coding

information flows is show in the diagram in Figure 3.6.

In this possible application of superposition coding, the macrocell BS sends the same message containing both layers of information to the two HBSs. The received signals in the HBSs are

$$y_1 = \sqrt{\alpha}\, x_1 + \sqrt{1-\alpha}\, x_2 + z_1 \tag{3.3}$$

$$y_2 = \sqrt{\alpha}\, x_1 + \sqrt{1-\alpha}\, x_2 + z_2 \tag{3.4}$$

where z_k are variables representing the noise, the power sent by the macrocell BS has been normalized to 1 in this case and $0 \leq \alpha \leq 1$. If we assume for the noise variances that $\sigma_1^2 > \sigma_2^2$, it can be considered that HBS1 has a weaker link that HBS2. Under this assumption, HBS1 can concentrate in decoding only message x_1 and considering the term that goes with x_2 as noise. This way, the SINR required for it in the decoding process decreases, so it is more likely that the receiver succeeds in it. Then, the requited rate is

$$R_1 \leq C\left(\frac{\alpha}{(1-\alpha)+\sigma_1^2}\right). \tag{3.5}$$

HBS2 has a better link quality, so it can decode all the information that HBS1 can decode. Therefore, HBS2 can decode message x_1 and subtract it from the joint signal. This allows that, in the end, the noise term in the SINR expression is lower, so the rate that that can be decoded by HBS2 is higher. This rate can be

$$R_2 \leq C\left(\frac{1-\alpha}{\sigma_2^2}\right). \tag{3.6}$$

Globally, it can be said that superposition coding suits the purpose for investigating femtocell cooperation. Some advantages of this kind of implementation are that since the signals are decoded in the HBSs before forwarding them the UE the noise of the backhaul links that connect the BS with the HBSs is eliminated. Also, it was considered that studying the performance of superposition coding would be more approachable than other information-theoretic techniques. From a practical point of view, the combination of signals in the modulation level is the simplest for the considered techniques. These are some of the reasons why it was chosen as the coding scheme to be considered in this investigation.

Even though this is not the exact way that superposition coding will be applied in the scenario considered in our study, it can be seen that this kind of coding can be good for such kind of scenarios with many base stations sending to more that one user. In the following section, it is presented how exactly we apply it in our case.

3.2 Superposition coding in the selected scenario

Once the superposition coding has been chosen as the technique to be used, in this section it will be presented in detail how the messages are sent, how the signals are constructed, etc.

Back on the scenario presented in Figure 2.5, we already described the set up of the base stations and users in this scenario. Now that we know that the superposition coding technique is to be tested in it, we can formulate the problem with the corresponding notation and definitions.

We clarified in Section 3.1.3 that with this kind of coding more than one messages can be sent at the same time in combined signals to various destinations. In the scenario in Figure 2.5, apart from having more than one destination, we have more than one source - the three HBSs. As a consequence, now we will have three HBSs sending the two messages addressed to the users x_1 and x_2 in a joint way.

3.2.1 Properties of the signals and messages

Once all the HBS's have received the user messages $x_1, x_2, ..., x_U$, being U the total number of users in a generic description, via their backhaul links $C_{B,i}$, the *i-th* HBS uses superposition coding to produce the signal

$$\begin{aligned} b_i &= \sqrt{P_i}(\alpha_{1,i}x_1 + \alpha_{2,i}x_2 + ... + \alpha_{U,i}x_U) \\ &= \sqrt{P_i}\sum_{k=1}^{U}\alpha_{k,i}x_k \end{aligned} \tag{3.7}$$

where $\alpha_{k,i}$ is the voltage division coefficient used in the superposition coding scheme, indicating the amount of power that HBS i dedicates for the message to user k. This variables can take complex values if it is required; U represents the total number of users in a more generic scenario ($U = 2$ in this problem); and P_i scales the power of the downlink signal sent by HBS i. In general, we will say that there are B HBS's working cooperatively (as stated before, $B = 3$ here). The voltage division coefficients must satisfy

$$\sum_{k=1}^{U}\alpha_{k,i}^2 = 1, \forall i \in \{1, ..., B\}. \tag{3.8}$$

and the power associated to the k user message is $P_k = 1, k = 1, ..., U$ for all HBSs. Under these conditions it is ensured that the transmitted power by HBS i is P_i.

When the B superpositioned messages from each HBS have been transmitted, the *k-th* user receives

$$\begin{aligned} y_k &= \sum_{i=1}^{B} h_{k,i}b_i + z_k \\ &= \sum_{i=1}^{B} h_{k,i}\sqrt{P_i}\sum_{k=1}^{U}\alpha_{k,i}x_k + z_k. \end{aligned} \tag{3.9}$$

where z_k is an additive white Gaussian noise for user k and $h_{k,i}$ is the channel coefficient for user k for the signal sent by HBS i. Users can reconstruct the messages sent to them from y_k using the corresponding decoding techniques.

At this point, we will introduce a notation that will be used in simulations and explanations along the report. A voltage gain variable $g_{k,i}$ has been defined as

$$g_{k,i} = \sqrt{P_i} h_{k,i} \tag{3.10}$$

to represent the channel from user k to HBS i. It will also be useful to concentrate the amount of power that a certain user receives from one of the U user messages after joining all the signal received from the different HBSs in a single variable. For that, let us define

$$\beta_{k,j} = \left| \sum_{i=1}^{U} g_{k,i} \alpha_{j,i} \right|^2 \tag{3.11}$$

for $j, k = 1, ..., U$. This $\beta_{k,j}$ parameter represents the amount power that user k receives of message x_j after adding up all the signals that are received from the B base stations. In a more restricted case like in ours where $B = 3$ and $U = 2$, we can express (3.11) as

$$\beta_{k,j} = |g_{k,1}\alpha_{j,1} + g_{k,2}\alpha_{j,2} + g_{k,3}\alpha_{j,3}|^2 \tag{3.12}$$

with j and k only having as possible values 1 and 2, and where there is one term corresponding to each HBS. As an example, $\beta_{1,2}$ represents the total power that UE1 has in its receiver from message x_2.

3.2.2 Optimization problem

The objective of the investigation is to study the benefit in the global rate of the system from using superposition coding. Let R_k be the rate for user k. Using Shannon's formula, as we know, we can say that

$$R_k = \log_2(1 + \mathrm{SINR}_k) = C(\gamma_k) \tag{3.13}$$

The main optimization problem to be solved is to maximize the system sum-rate R giving the power division coefficients $\alpha_{k,i}$ suitable values, i.e.,

$$\max_{\alpha_{k,i}} R = \sum_{k=1}^{U} R_k. \tag{3.14}$$

This optimization problem is to be solved by one of the HBSs to distribute the optimal coefficient values to the HBSs. It is required that the HBS knows all user messages, that will arrive to them through the backhaul links to the GW. The voltage division coefficients $\alpha_{k,i}$ will have to be adapted so maximize the rate of the system. Therefore, apart from the user messages, the channel state information for each wireless link and the AWGN at every receiver also have to be known by the HBS solving the problem. For sharing these channel state coefficients, the HBSs can use their inner-tier links $C_{k,j}$.

Depending on the particular scenario, it is possible that in some cases the solution of the optimization problem formulated in (3.14) is not practical, e.g. if giving all the resources to one user is the optimal solution. Therefore, some other optimization parameters are introduced. One of them is the best achievable rate for both users at the same time, that is,

$$\begin{aligned} &\max_{\alpha_{k,j}} R_1 \\ &\text{s.t. } R_1 = R_2. \end{aligned} \tag{3.15}$$

With this optimization parameter, it is assured that total fairness between users exists.

Another optimization parameter used in this study is the best sum-rate region area, i.e. the biggest set of achievable pairs of rates possible for a set up.

3.2.3 Preliminary results

For a preliminary study, we can start by calculating the SINR values for each user for this coding scheme when no interference cancellation method is used.

For user 1, using equation (3.9), the received signal is

$$y_1 = \sqrt{P_1}h_{11}(\alpha_{11}x_1 + \alpha_{21}x_2) + \sqrt{P_2}h_{12}(\alpha_{12}x_1 + \alpha_{22}x_2) + \sqrt{P_3}h_{13}(\alpha_{13}x_1 + \alpha_{23}x_2) + z_1 \tag{3.16}$$

With this received signal, the average SINR can be calculated considering that the messages addressed to user 1 are the useful signal and the ones to user 2 plus noise are unwanted. Being σ_1^2 the variance of the Gaussian noise, we can say that

$$\bar{\gamma}_1 = \frac{|g_{11}\alpha_{11} + g_{12}\alpha_{12} + g_{13}\alpha_{13}|^2}{|g_{11}\alpha_{21} + g_{12}\alpha_{22} + g_{13}\alpha_{23}|^2 + \sigma_1^2}. \tag{3.17}$$

Similarly, for user 2 we have

$$\bar{\gamma}_2 = \frac{|g_{21}\alpha_{21} + g_{22}\alpha_{22} + g_{23}\alpha_{23}|^2}{|g_{21}\alpha_{11} + g_{22}\alpha_{12} + g_{23}\alpha_{13}|^2 + \sigma_2^2}. \tag{3.18}$$

For the two users, considering that the channel coefficients $h_{k,i}$ and the noise powers σ_k^2 are known, the aim of the problem would be to choose $\alpha_{k,i}$ such that the sum-rate $R_1 + R_2 = C(\bar{\gamma}_1) + C(\bar{\gamma}_2)$ is maximized.

Chapter 4

Rate analysis for broadcasting with superposition coding. Multiple Access Channel constraints.

In this chapter, a first step to attempt solving the problem formulated in Section 3.2 is to determine how the achievable rate regions look like. An outer bound for the rate regions is shown in Appendix A. Some theoretic considerations that have to be accounted for the rate analysis are presented with examples that aimed to bring some insight to the problem.

4.1 Aim of the simulations

In the simulation for our scenario shown in Appendix A, some constraints were not considered for simplicity. This constraints are related to the Multiple Access Channel (MAC). The theoretic fundamentals of the MAC rate-regions and new simulation results and conclusions are presented here.

4.2 Theoretic fundamentals of Multiple Access Channel

An interesting finding from this investigation is that there are two different for analyzing the MAC constraints that give exactly the same result. Both of them are presented in the following.

4.2.1 a) Taking the most restrictive rates

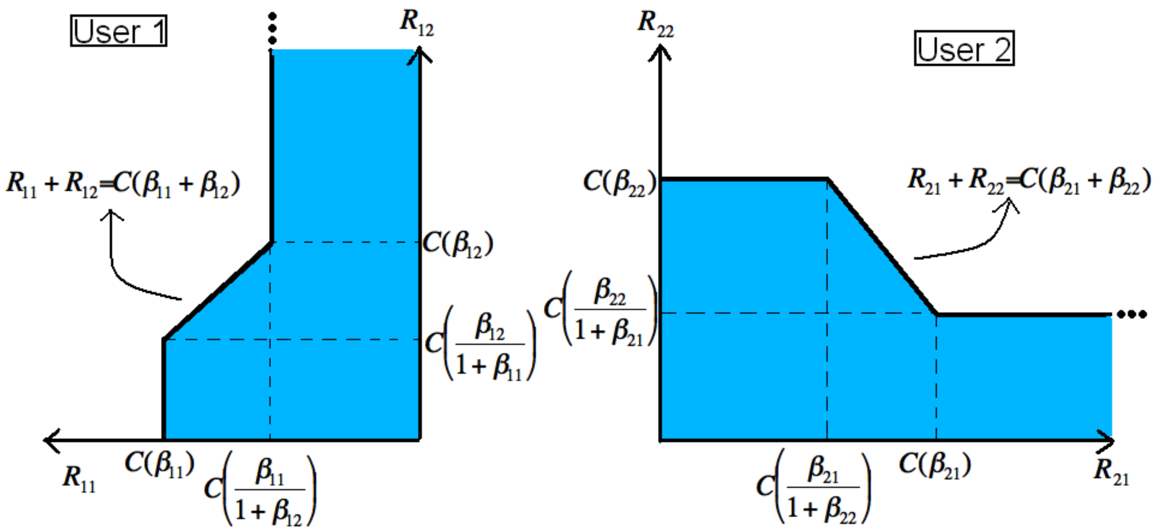

Figure 4.1: Theoretic achievable rate region for a Multiple Access Channel with two users separately considered

Let us assume that there are two users sharing a channel, like is the case in the scenario being studied. In this situation, we have two user messages, x_1 and x_2, to be distributed to users 1 and 2, respectively. As we know, these messages are sent by all the base stations with combined signals. Therefore, the two messages will be received in the two users. We can then define $R_{k,j}$ as the potential rate of message j in receiver k that user k would be able to achieve in the case that it was interested in decoding message j. Despite this, each user's final goal is to decode only their message x_k, and consequently, they are only interested in decoding other user's message if it will help them to decode their own with a higher rate (e.g. by interference cancellation). The rates $R_{k,j}$ represent a situation in which the users are working independently. From the set of $R_{k,j}$, for each message, one will be chosen to be the definitive rate R_k with which message k will be sent.

A possible rate region for each of the users is represented in Figure 4.1. In the left of it, we can see the achievable rate-region for user 1, in which $R_{1,1}$ (potential rate for user 1 to message x_1) is plotted versus $R_{1,2}$ (rate for user 1 to message x_2). To understand the boundaries of the region, let us first recall the definition introduced in equation (3.12)

$$\beta_{k,j} = |g_{k,1}\alpha_{j,1} + g_{k,2}\alpha_{j,2} + g_{k,3}\alpha_{j,3}|^2$$

for $j, k = 1, 2$.

This $\beta_{k,j}$ parameter represents the amount power that user k receives of message x_j after adding up all the signals that are received from the 3 base stations. In (3.12), $g_{k,i}$ is the channel voltage coefficient from base station i to user k (including transmit power and path losses) and $\alpha_{k,i}$ is the voltage division coefficient. Also remember that $C(x) = log_2(1 + x)$.

After defining the parameters, let us examine Figure 4.1 for user 1. It can be seen the *required* $R_{1,2}$ for receiver 1 to be able to decode x_1 with a certain *desired* $R_{1,1}$ (viceversa for user 2). It should be pointed out that the value of $R_{1,2}$ only matters if the desired rate $R_{1,1}$ will be achievable only from a concrete value of R_2 and below. This happens when when x_2 has to be (partially) decoded by user 1.

Looking at the shape of the graphic (again concentrating on user 1), two cases can be distinguished:

1. When $R_{1,1} \leq C\left(\frac{\beta_{1,1}}{1+\beta_{1,2}}\right)$, $R_{1,1}$ is sufficiently low for receiver 1 to successfully decode x_1 by considering x_2 as noise. Therefore, it is not useful to try to decode x_2, and consequently $R_{1,2}$ can take any value (it is irrelevant, it will not be used), meaning that user 1 will not force any constraint in the possible values for R_2. That is why the rate-region is infinitely high under this condition. Seeing it the other way around, $R_{1,2}$ would limit the maximum $R_{1,1}$ that user 1 can handle according to the interference cancellation needs.

2. When $C(\beta_{1,1}) \geq R_{1,1} \geq C\left(\frac{\beta_{1,1}}{1+\beta_{1,2}}\right)$, it is required that receiver 1 decodes at least partially message x_2, because this will allow that some interference can be removed, thus being able to achieve a higher SINR level and rate.

The upper bound for $R_{1,1}$ is $C(\beta_{1,1})$, since that is the case in which the interference that creates x_2, represented by the power term $\beta_{1,2}$, is completely removed. In the subregion in which x_2 is (partially) decoded, the condition

$$R_{1,1} + R_{1,2} = C(\beta_{1,1} + \beta_{1,2}) \tag{4.1}$$

applies. Using (4.1), one can estimate the required rate of the message of the other user so that a certain rate for his own is achievable.

For user 2, an analog description can be made, obtaining the required $R_{2,1}$ so that some desired $R_{2,2}$ is achievable.

Having considered the users separately and knowing that in some cases, when interference cancellation is required, receivers may have to decode messages not addressed to them, now we consider the two users jointly. The main constraint that considering the MAC implies is that the rates used to send messages x_1 and x_2, R_1 and R_2, can be limited by both users according to their desired rates $R_{1,1}$ and $R_{2,2}$, and the conditions that the other user's rates, $R_{1,2}$ and $R_{2,1}$, have to satisfy for those desired rates to be achievable. The conditions to be satisfied by the selected rates with respect to the desired and required rates are

$$\begin{aligned} R_1 &\leq R_{k,1} \\ R_2 &\leq R_{k,2} \end{aligned} \tag{4.2}$$

for all $k = 1, 2$. These conditions are equivalent to setting R_1 and R_2 as the minimum possible value. Consequently, the rates used to send the messages can be obtained as

$$\begin{aligned} R_1 &= min(R_{1,1}, R_{2,1}) \\ R_2 &= min(R_{1,2}, R_{2,2}) \end{aligned} \tag{4.3}$$

so that the chosen rates are the most limiting ones.

As a result of all these considerations, the theoretic achievable rate-region for the MAC can be estimated by intersecting both graphs in Figure 4.1. For the final region to be generated, a last modification is to be made. This modification is the convexification of the region. This process will be explained later, in Section 4.3. The shape of the joint rate region for the selected R_1 and R_2 changes depending on the resulting $\beta_{k,j}$ values obtained with (3.12). Plotting the joint graph for different values of $\beta_{k,j}$, one can see which are the cases in which some benefit can be obtain. Some examples will be shown in next sections.

4.2.2 b) Limiting maximum rates by other user's power constraints

A second approach to get the theoretic rate regions for our problem considering the MAC constraints equally valid is now presented. For it, a particular example is given. Figure 1, 2 and 3 show a particular case for some concrete values or the power parameters. Particularly, we used $\beta_{1,1} = \beta_{2,1} = \beta_{2,2} = 10$ and $\beta_{1,2} = 1$.

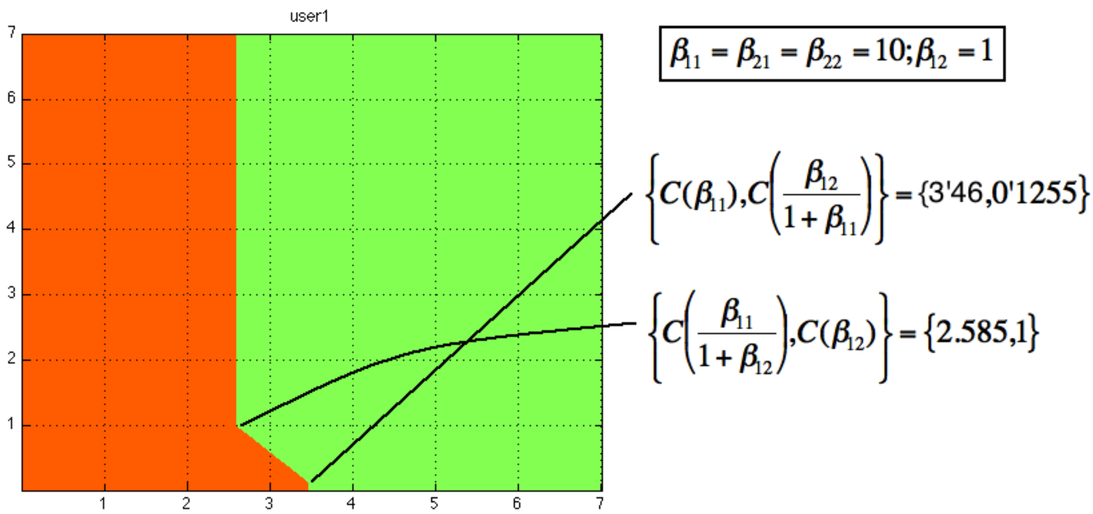

Figure 4.2: Achievable rates for user 1 working independently

Figure 4.2 and 4.3 represent the achievable rates for each user regardless the other user (as we were explaining in the previous section). In these figures, the orange inner part represent the achievable rates, while the green area is not achievable. This is why they go up to infinity if they are below their "treat as noise" threshold, as we were discussing before. A little modification is made prior to joining both regions: limiting the unrestricted axis (y for user 1, x for user 2) by the maximum rate that can be set for the other user, that is, limiting $R_{2,1} \leq C(\beta_{1,1}) = 3,46$ and $R_{1,2} \leq C(\beta_{2,2}) = 3,46$. We can see the resulting regions in Figure 4.4. In particular, user 1's constrained region can be obtained by joining the subregions numbered by 1 and 2 in the figure; and user 2's regions, joining subregions 1 and 3.

The next step for generating the final region is to make the intersection of these two regions, to fulfill both users' requirements. This achievable region corresponds to

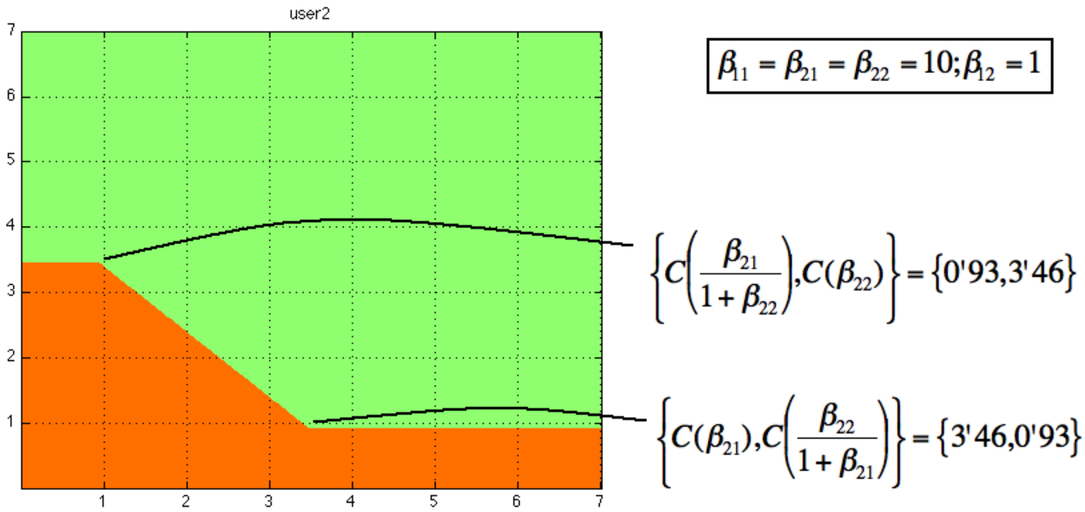

Figure 4.3: Achievable rates for user 2 working independently

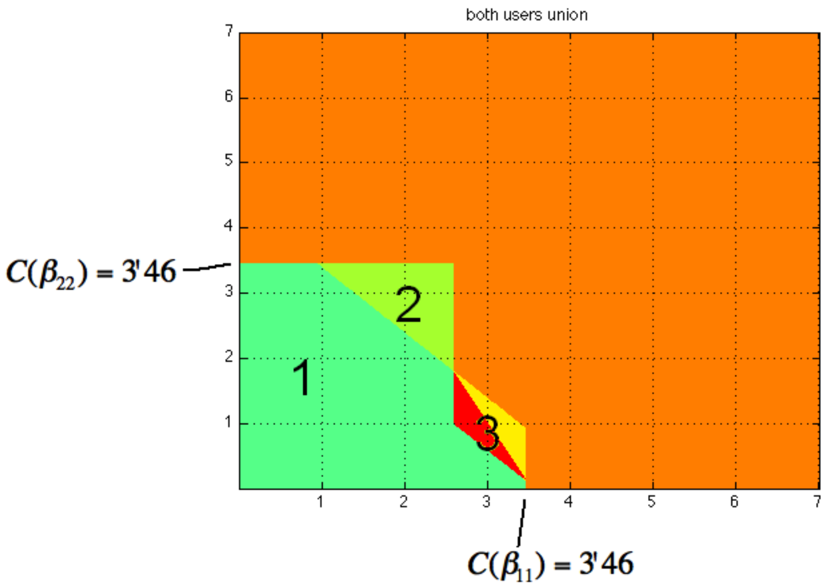

Figure 4.4: Intersection of the resulting regions shown in subregion 1. Subregions 2 and 3 correspond to subregions achievable only by user 1 and 2, respectively

subregion number 1 from Figure 4.4, resulting from putting user1's region on top of user 2's. One again, the last step to get the final region is the convexification of the resulting region, explained just above.

As a conclusion, both methods that have been presented are equally valid to generate the regions. In method b) previous step is introduced for a more intuitive derivation of the regions, forcing the obvious constraint of limiting each rate by the corresponding total interference cancellation case. In the examples shown in the next section, both methods can be used for the generation independently.

4.3 Convexification of superposition coding region

A more advanced transmission scheme can mix both superposition coding and beamforming. The way to do it is the following. Taking any rate pair that is a result of time-sharing between two points with fixed beamforming coefficients is also achievable. Therefore we need to do a convexification con the rate region. This means that we can take the vertexes of the superposition coding region and, when they do not have a boundary with a convex shape, the we apply time-sharing between two of them. This will allow that all the points in/above the union line are also achievable.

An example of this situation can be observed in Figure 4.4. We said that the final achievable region uses *pure* superposition coding corresponded to the subregions number 1. It can be seen that this subregion does not have a convex shape. Therefore, applying time-sharing between the two vertexes a convex version is achieved. The additional region added to the rate region is colored in red in Figure 4.4.

This improvement of the scheme, even though it enlarges the region, does not improve the maximum achievable sum-rate. This is because the optimum point will be found at a vertex of the region, and thus will not vary if two vertexes are joined by a line. Clearly, on the other hand, doing this convexification process does increase the area of the region, and may affect also the other optimization parameter, the best equal rate.

4.4 Finding the optimum rate point

After knowing how to generate the achievable rate regions respecting the MAC constraints, an interesting parameter to find is the optimum pair of rates to be used that maximize the sum-rate of the system, using the maximization problem introduced in equation (3.14) from the previous chapter.

An important note should be made in this point. It can be demonstrated that the optimum in the rate region will always be found in a vertex of the rate region. To prove this, say we take a random point $(R_1, R_2) = (a, b)$ from the achievable set of rates. It is easy to see that if it is possible to "move" forward in any of the axis, i.e., increase the value of R_1 or R_2, being in the achievable region yet, the new point $(R_1', R_2') = (a + \Delta R_1, b + \Delta R_2)$ will always offer a higher sum-rate. This leads to the conclusion that the best sum-rate will always be found in the boundary of the achievable region.

On the other hand, we should take into account that the boundary is in all cases defined by straight lines with 0 or ∞ slope (horizontal or vertical lines, respectively), or lines with a -1 slope ($R_{k,1} + R_{k,2} = C(\beta_{k,1} + \beta_{k,2})$ = constant, see Figure 4.1). With this facts, we can say that in the 0 or ∞ slope lines, the best point that can be achieved will always be found in the extreme of the line with highest value of the non-constant variable. In the -1 slope line, the sum of the value of both axis is constant. As a conclusion, the optimum point will always be in a vertex of the region, as we wanted to prove. Note that this does not imply that the optimum points have to be located only there, since they may also be contained in a whole segment of a straight line (like, e.g., a complete -1 slope line).

This proven fact will be used in simulations yet to be presented to find the optimum points easier.

4.5 MAC region examples

In this section, some examples of rate regions generated using both procedures explained in Section 4.2 are presented.

4.5.1 Example 1 - one user better served

For this first example, it was represented that user 1 gets very good signal levels compared to user 2. For this, the selected $\beta_{k,j}$ values were $\beta_{1,1} = \beta_{1,2} = 10$ (user 1 can get adding up all the incoming signals from all base stations a 10 power value for both messages x_1 and x_2) and $\beta_{2,1} = \beta_{2,2} = 1$.

Using method a), the individual rate regions for each users can be seen in Figure 4.5. Green area represents the achievable rates and blue the not achievable rates, as before.

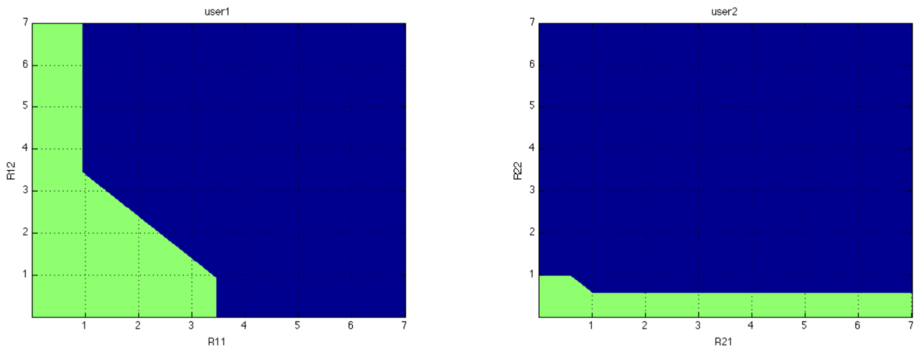

Figure 4.5: Independent regions for example 1

It can be observed for each of the users how the unconstrained axis ($R_{1,2}$ for user 1 and $R_{1,2}$ for user 2) go up to infinity. Also, note how the rate region for user 1 is larger than user 2's, as it was expected. Now, joining both regions by taking the common points results in the region shown in Figure 4.6. Like before, an additional area (marked in red) is achievable via time-sharing convexificating the region.

Regarding the optimum sum-rate for this case, it is located in the vertex point in the right side of the graph. It takes advantage of the fact that user 1 has better signal conditions. In particular, the optimum pair of rates for this case is $R_1 + R_2 = 3.45 + 0.58 = 4.03$.

Now using method 2, the resulting constrained graphs and final rate region is shown in Figure 4.7. The final rate region corresponds to the blue inner area. It can be seen that this subregion is equal to the one shown in Figure 4.6. In this particular case, taking the union of both the blue and green subregions get the constrained region of user 1 (limited in the R_2 axis by $1 = C(\beta_{2,2}) = C(1)$. User 2's constrained region, in this case, coincides with the intersection of both users' constrained regions (blue area plus convexification area, in red).

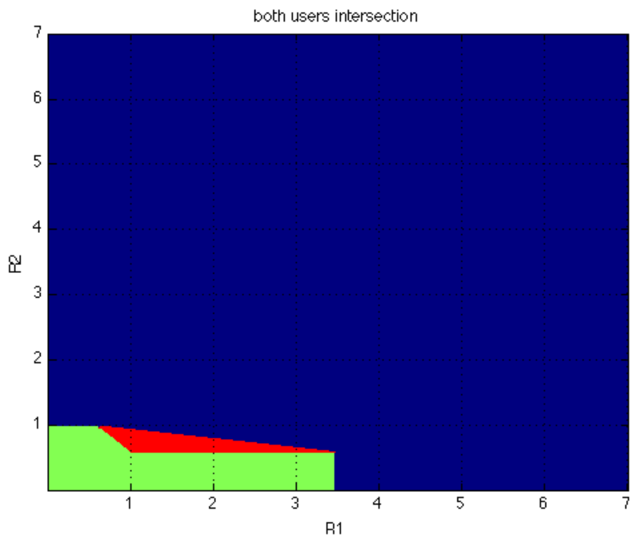

Figure 4.6: Final achievable region for both users in example 1 using method 1

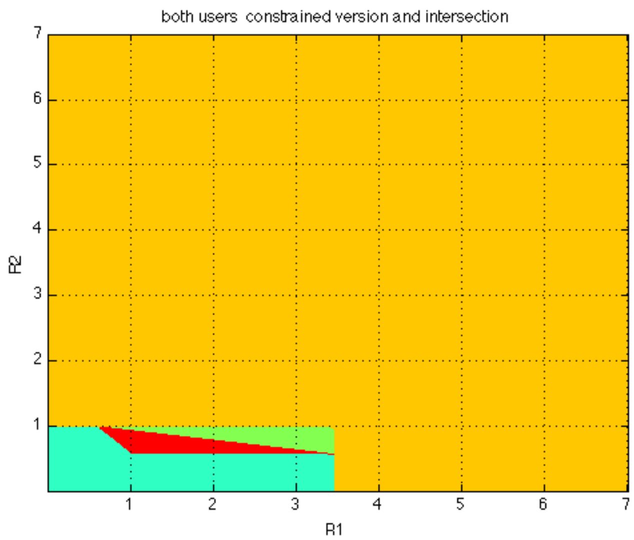

Figure 4.7: Final achievable region for both users in example 1 using method 2

4.5.2 Example 2 - Equal service for both users

For this second example, it is supposed that the strength of the received signal is the same for both users and messages. In particular, $\beta_{k,j} = 5$ was chosen for all k, j. For this second case, the individual results for each user can be found in Figure 4.8, and for the complete system in Figure 4.9.

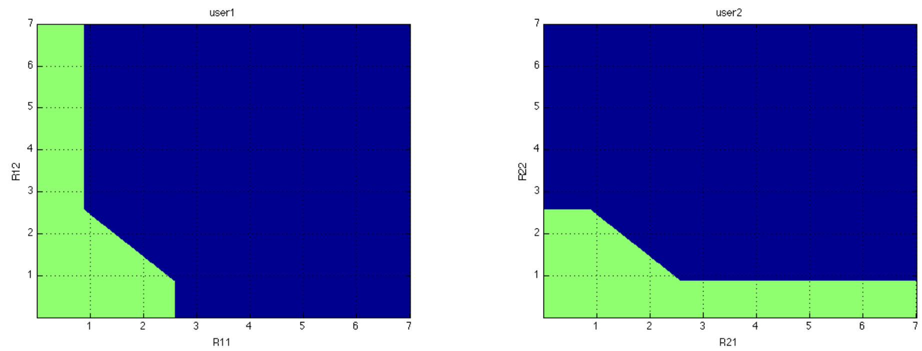

Figure 4.8: Independent regions for example 2

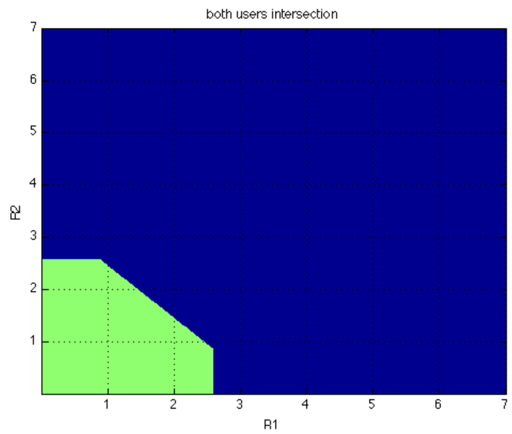

Figure 4.9: Final achievable region for both users in example 2 using method 1

It can be noticed the symmetry for the two users in this case given the equal parameters that are set. Here, an optimum sum-rate is found in $R_1 + R_2 = 0.87 + 2.58 = 3.45$. It should be said that in this case user 2 has priority over user 1. It has been given to it the highest possible rate for the set of rates that are optimum. It can be observed that in this case all the points in the -1 slope line are equally optimum.

Now talking about the second method for generating the region, the result for this case is shown in Figure 4.10. Again, the results coincide in both methods. Notice that this time only one achievable area is drawn (blue) because the constrained versions of both users are the same (symmetric parameters).

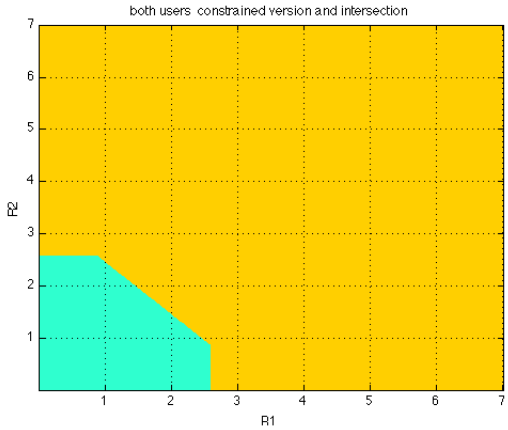

Figure 4.10: Final achievable region for both users in example 2 using method 2

Chapter 5

Comparing superposition coding scheme with reference scenarios

In this chapter, the complete optimization problem including the MAC constraints is set out. Some discussion regarding this optimization problem are carried out. Then, a Montecarlo method (MCM) simulation that was designed to attempt to give an approximate solution to the formulated problem is described, along with a comparison with the reference scenarios already defined. In this simulation some theoretic results regarding the reference are used. They are also derived in this chapter. Finally, some results for particular cases are given with the conclusions that can be extracted from the whole simulation process.

5.1 Presentation of the optimization problem

5.1.1 Mathematical description of the optimization problem

Here we consider the optimization problem for the application of superposition coding in our scenario. The objective of the problem (5.1) is to obtain the optimal $\alpha_{k,i}$ parameters that maximize the system sum-rate. The mathematical description of the whole problem can be stated as follows:

$$\max R_1 + R_2 \tag{5.1}$$

s.t.

$$\left\{\left[R_1 < C\left(\frac{\beta_{11}}{1+\beta_{12}}\right)\right]\bigcup\left[(R_1 + R_2 < C\left(\beta_{11}+\beta_{12}\right))\bigcap(R_1 < C(\beta_{11}))\right]\right\}\bigcap$$
$$\left\{\left[R_2 < C\left(\frac{\beta_{22}}{1+\beta_{21}}\right)\right]\bigcup\left[(R_1 + R_2 < C\left(\beta_{21}+\beta_{22}\right))\bigcap(R_2 < C(\beta_{22}))\right]\right\} \tag{5.2}$$

$$R_1, R_2 \geq 0 \tag{5.3}$$

$$\sum_k |\alpha_{k,i}|^2 = 1 \tag{5.4}$$

$$\Re\{\alpha_{k,i}\} \geq 0 \tag{5.5}$$

with

$$C(x) = \log_2(1+x) \tag{5.6}$$

$$\beta_{k,j} = \left|\sum_i g_{k,i}\alpha_{j,i}\right|^2. \tag{5.7}$$

In it, the maximization of the sum-rate (adding up both users' rates) is done respecting the constraints of the Multiple Access Channel, explained in the Section 4. Equation (5.2) defines the resulting region of achievable rates that arises from this consideration, graphically achieved by intersecting both plots from Figure 4.1 (method 1 for generating the regions is used, Section 4.2.1). As it can be seen, it consists in the intersection of the each user's rate limitations forced by the received signal levels for each message, represented by the $\beta_{k,j}$ terms. Inside each user's region, there is the union of two subregions. On one side, the achievable rates by considering the unwanted message as noise, constrained by the term $C\left(\frac{\beta_{k,k}}{1+\beta_{k,j}}\right), k \neq j$. On the other, the additional area achievable thanks to the partial decoding of the other message (limited by a -1 slope straight line and the total subtraction of the interference term, $C(\beta_{k,k})$).

About the other constraints, an obvious one is that rates have to have positive values, (5.3) . The ones applying to $\alpha_{k,i}$, (5.4), (5.5), were already discussed in the presentation of the notation, in Chapter 2.

5.1.2 Problems solving the optimization problem

Attaining the goal of solving the optimization problem is quite tricky given the logarithmic dependence of the optimization variables that are being treated here. Therefore, advanced optimization tools are required in order to tackle this problem. That is why a simpler approach was selected in this study. This approach is a new Montecarlo Integration simulation.

5.2 Montecarlo method simulation

A Montecarlo method (MCM) simulation is a numerical method that uses random numbers to evaluate a function. This kind of simulation was chosen because it can provide preliminary results and gives a valid approximation while it remains simple to set up.

The simulation will compare the superposition coding scenario with the two reference cases that are used to benchmark its performance: beamforming scenario (that uses time-sharing) and dedicated HBSs' scenario, in which the traditional scheme where UEs choose the best serving station is simulated.

5.2.1 Optimization parameters

For the comparison between the scenarios being considered, three optimization parameters were selected. A reminder of which they are follows:

- **Sum-rate.** The first (and more obvious) optimization parameter is the best sumrate $R_1 + R_2$ that is achievable using a particular technique, i.e., (5.1).
- **Achievable rate area.** The second parameter is basically the area under the boundary of the rate region in each scenario. This gives an idea of the amount of rate pairs that are usable in each case.
- **Best equal rate.** The last test parameter is the best equal rate. In other words, the highest rate that can be given to both users at the same time in each of the cases.

5.2.2 Simulation description

A general description of the MATLAB script that performs the MCM simulation and calculates the optimization parameters for each of the scenarios in presented in the following lines.

Setting channel coefficients. First, the channel gain values $g_{k,i} = \sqrt{P_i} h_{k,i}$ (taken into account both fading and transmit power from base stations) are fixed as input parameters in the beginning of the simulation. The chosen values have to be representative for an interesting situation that wants to be examined. Of course, these variables can take complex values to represent the different phases with which the signals arrive to the receivers depending on the paths they follow.

Calculation of the reference scenario optimization parameters. As we will see in the following section, the optimization parameters are fixed when the channel values are also fixed. Prior to the calculation of the optimization parameters, in the dedicated HBSs scenario, the best serving HBS is chosen for each user, following the criteria of selecting the highest $g_{k,i}$. Immediately after having selected the HBSs, the three optimization parameters are calculated for both scenarios for being compared with the superposition coding ones after they are computed with the MCM simulation.

Generation of the (complex) $\alpha_{k,i}$ values. As a part of the MCM, the $\alpha_{k,i}$ are randomly generated. The alphas are the input to the MC method, generated over the domain in (A.9) for $\alpha_{k,i}$ bounded between 0 and 1. They will be used later to calculate the corresponding $\beta_{k,i}$ parameters in the superposition coding scheme. The $\alpha_{k,i}$ coefficients are generated with the MATLAB function *rand* and using the condition

$$\sum_{k=1}^{U} |\alpha_{k,i}|^2 = 1. \tag{5.8}$$

In the particular case being studied here, with 2 users, first $\alpha_{1,i}$ is generated randomly and with its value is forced that $\alpha_{2,i} = \sqrt{1 - \alpha_{1,i}^2}$. One half of the points are generated this way. After that, the generation for the next half is done the other way around, first generating $\alpha_{2,i}$. It is done this way so the average values of both variables is equalized, for a fair distribution of the random value. With all the obtained points, the Montecarlo integration is performed. For considering the complex part, another uniformly distributed random variable between 0 and 1 is generated to decide the sign of the complex part of $\alpha_{k,i}$: if it is higher than 0.5, then the complex part is positive, and vice-versa in the complementary case.

Generation of β-regions. After having generated vectors containing the sets of $\alpha_{k,i}$, for each of these sets the corresponding received power variables $\beta_{k,j}$ are calculated simply using expression (5.7). With them, applying the MAC contraints mathematically expressed as 5.2 as we commented before, the set of β-regions are computed.

Looking for the optimum in all the β-regions. Following the criteria of looking for the highest sum-rate in the superposition coding scenario, we look for the best couple of points achievable in all the regions generated. For this, first of all, the best point of each region is obtained. This is done by adding up both axis' value for all the boundary of the region (in which, evidently, the optimum will be found) and selecting the best of them. It should be noted at this point that since one of the limiting functions is a straight line with a -1 slope, if the optimum point is contained in this line, all the points in it are equally optimal, because the sum of all of them is constant. For these cases, it was chosen that rate R_2 for user 2 has priority over R_1. Another observation is that in every case the optimum point will be in a vertex of the region. This is because the region is limited by straight lines, so depending on the value of the slope of each of them, the best point will be in one extreme or another, but always in a vertex.

Once the optimum points of all the regions are gathered in a vector, the best of all of them is obtained. The addition of the 2 coordinates of the points will be the optimum sum-rate of the system using superposition coding, and the region containing it is selected as the optimal β-regions which are used to calculate the rest of the optimization parameters to be compared with the ones from the reference cases.

Calculation of comparison parameters and presentation of results The last step in the simulation consists in calculating some comparative parameters between scenarios, like ratios between optimum sum-rates. After that, the results are presented with graphics with the achievable rate regions and the numeric results of the comparison parameters and the values of the set of $\alpha_{k,i}$.

5.2.3 Other considerations

The main limitation of using a simulation like MCM in our case is the reduced number of iterations to get different β-regions that can be done. Also, in order to avoid memory problems in MATLAB in the computers used to run the simulation, there should be a moderate number of points in the regions, although this problem is not as important as the first.

To fight this problem, several runs of the simulation are carried out, taking the best result of all of them. It is considered that after a sufficient number of runs the final result approaches to the optimum solution.

5.3 Theoretic analysis of the reference scenarios

As we explained previously, a theoretic study of the two reference scenarios can be done in order to derive the expressions of the optimization parameters for each case. These results are applied in the simulation to calculate the optimization parameters which depend directly on the channel values.

5.3.1 Beamforming scenario

Rate region and parameter definition

The beamforming scenario uses certain coefficients that allow co-phasing the signals in the receiver depending on the channel conditions. Co-phasing allows the signals coming from different HBSs to be added in phase, so no cancellation between each other exists. It also uses time-sharing to consider two rate points: in one of them the system gives all the resources to one user and no service to the other user; and in the other point it is done the other way around. The system periodically switches from working in one point to the other. This way, a range of different pairs of rates can be achieved depending on the amount of time that the system is working in one extreme point of another. The rate region for this kind of situation is shown in Figure 5.1.

For this kind of transmission scheme to be implemented, certain cooperation between HBSs must exist. Particularly, HBSs must be synchronized when transmitting the signals for each user. Also, for the co-phasing process, the transmitters must estimate the received phase in each receiver in order to compensate it with the corresponding coefficient, as will be shown just above.

Actually, beamforming could be seen as a special case of superposition coding scenario in which two extreme points are used: giving all the resources to each user, respectively, and co-phasing the signals. Using the superposition coding notation, we can say

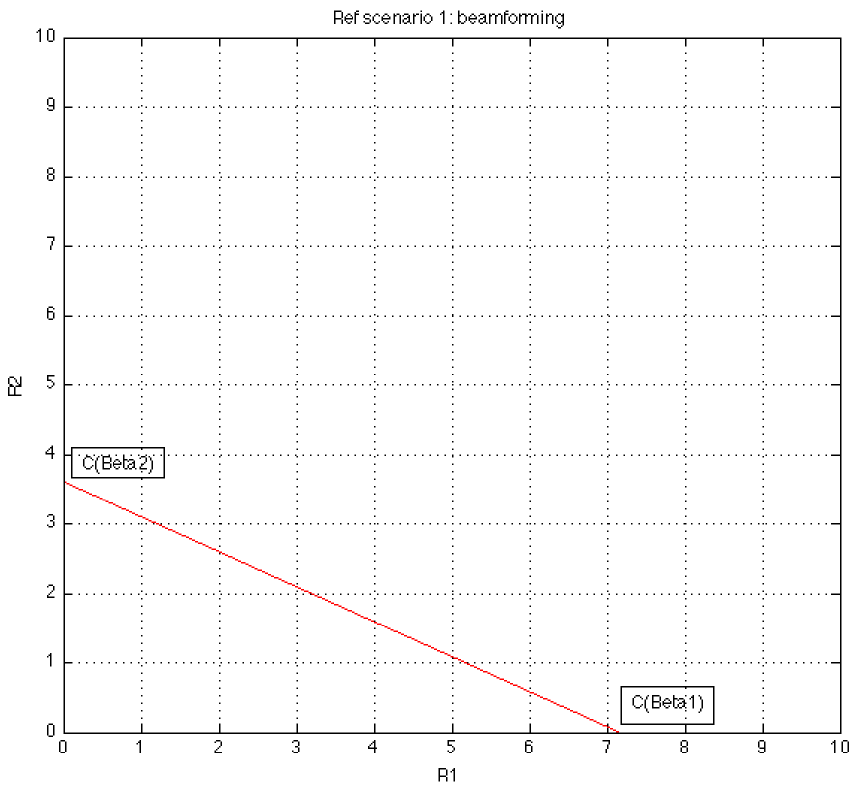

Figure 5.1: Example of rate-region achievable through beamforming

that for the point of work serving user k, in the following called $C(\beta_k^{BF})$, the following condition applies:

$$\alpha_{n,i} = e^{-j\angle g_{n,i}} \qquad \text{if } n = k \tag{5.9}$$

$$\alpha_{n,i} = 0 \qquad \text{if } n \neq k \tag{5.10}$$

for $n = 1, ..., U$, and where $\angle g_{n,i}$ is the phase of the channel gain coefficient and j is the imaginary unit.

Basically, this condition means that when the system is working serving user k, the coefficients are tuned so that the signals arrive to the receiver are co-phased, while all the power is dedicated to serve user k, and thus $|\alpha_{n,i}| = 1$. With this remark, let us now define

$$\beta_k^{BF} = \left| \sum_{i=1}^{U} g_{k,i} e^{-j\angle g_{k,i}} \right|^2 . \tag{5.11}$$

With this parameter, the points of work are directly the capacity of these power variables, i.e., $C(\beta_k^{BF})$. An important observation regarding (5.11) is that, in general, when comparing beamforming with superposition coding, it is true that $\beta_k^{BF} \geq \beta_{k,k}^{SC}$. The reason for this is that if we go back to expression (3.12) we can see that all the $g_{k,i}$ parameters are multiplied by their corresponding $\alpha_{k,i}$, for which $|\alpha_{k,i}| \leq 1$. The implications of this fact are that the top values in the axis of both users' rate will be higher in the case of beamforming than in superposition coding.

Optimization parameters

Sum-rate. Given the characteristics of the shape of the rate region, shown in Figure 5.1, one can easily see that the optimum sum-rate of the system will be found at one extreme point or another depending on the value of the slope joining both of them. Being m the value of the slope of the straight line, it can be said that

- if $m < -1$, then the optimum sum-rate is $C(\beta_2^{BF})$;
- if $0 > m > -1$, then the optimum sum-rate is $C(\beta_1^{BF})$;
- if $m = -1$, then the optimum sum-rate are all the points contained in the straight line, since $C(\beta_1^{BF}) = C(\beta_2^{BF})$.

Equivalently, expressed in a mathematical way,

$$R^*_{BF} = \max(R_1 + R_2) = \max\{C(\beta_1^{BF}), C(\beta_2^{BF})\}. \tag{5.12}$$

having defined R^*_{BF} as the optimum rate for beamforming.

Rate region area. It is easy to see that being the shape of the rate region a triangle, directly it can be said that

$$A_{BF} = \frac{C(\beta_1^{BF})C(\beta_2^{BF})}{2}. \tag{5.13}$$

Best equal rate. A way of solving the problem of the best equal rate for both users is to see where the boundary of the region intersects with a 45? straight line from the origin of the graph. Mathematically, it is equivalent to solving the following system of equation:

$$R_2 = -\frac{C(\beta_2^{BF})}{C(\beta_1^{BF})}R_1 + C(\beta_2^{BF}) \tag{5.14}$$

$$R_1 = R_2. \tag{5.15}$$

Equation (5.14) defines the boundary of the region, while (5.15) is the equal rate straight line. Solving this easy system of equation, it can be derived that the best equal rate is

$$R^{eq}_{BF} = \frac{C(\beta_1^{BF})C(\beta_2^{BF})}{C(\beta_1^{BF}) + C(\beta_2^{BF})}. \tag{5.16}$$

5.3.2 Dedicated HBSs scenario

Rate region and parameter definition

As it was already explained, in the dedicated HBS scenario, each user selects the best serving HBS and connects to it. Since all the users and base station share the same physical resources (frequency and time), the signals received from HBSs not giving service to a user in its receiver are considered interference which can be added to the noise term. For a formal definition, let us define

$$g_k^s = \max\{g_{k,i}\}, \text{ for } i = 1, ..., U \tag{5.17}$$

$$g_k^{int} = \sum_{i=1}^{U} g_{k,i} - \max\{g_{k,i}\}. \tag{5.18}$$

In other words, g_k^s represent the signal term coming from the selected HBS, and g_k^{int} is the interfering term resulting from adding the rest of the signals. Using this definitions, we can now define the following general SINR expression for user k:

$$\beta_k^D = \frac{|g_k^s|^2}{1 + |g_k^{int}|^2} \tag{5.19}$$

Clearly, in this case, $\beta_k^D < \beta_k^{BF}$, because in the dedicated HBSs' term there ir interference and less terms in the numerator.

Now that we have defined the parameters related to this scenario, we can examine the properties of the rate region. As we know, in this scheme there is no coordination between base station (that is why, e.g., there is no power adaptation for the sent signals). Consequently, the achievable rates for each user are completely independent from each other. This implies that the rate-region is a rectangle constrained by the capacities of each user. An example of this is shown in Figure 5.2.

Optimization parameters

Sum-rate. In this case, since users are operating completely independently, the best achievable sum-rate will be directly the sum of the capacities of each user:

$$R_D^* = \max(R_1 + R_2) = C(\beta_1^D) + C(\beta_2^D). \tag{5.20}$$

Rate region area. Again, it is very simple to calculate the area of the rate region using the expression for the area of a rectangle:

$$A_D = C(\beta_1^D)C(\beta_2^D). \tag{5.21}$$

Best equal rate. For the best rate achievable for the two users at the same time, the best is constrained by the lower of the rates, because this will be the maximum achievable

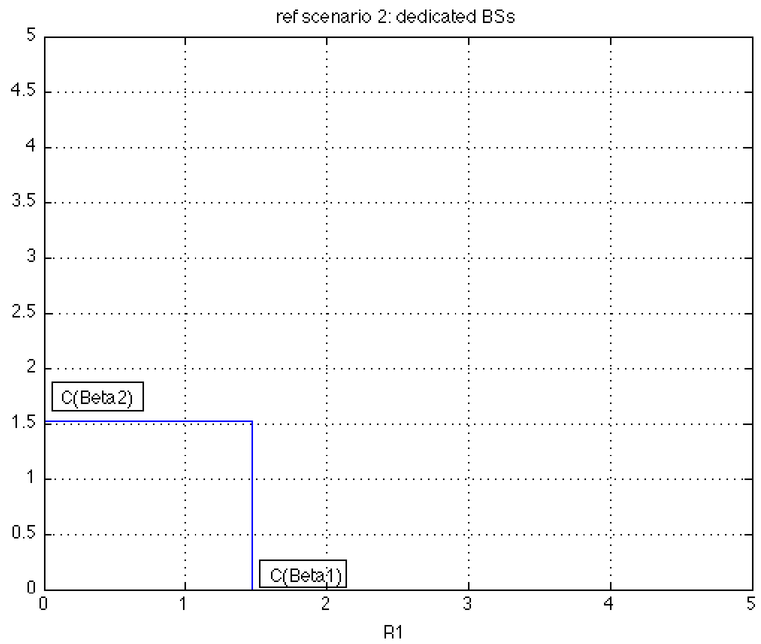

Figure 5.2: Example of rate-region achievable through dedicated HBSs

rate for one user and it will be contained in the set of achievable rates of the other, with a higher SINR. Therefore,

$$R_D^{eq} = \min\{C(\beta_1^D), C(\beta_2^D)\}. \tag{5.22}$$

5.4 Cases

5.4.1 Case 1: very asymmetric channel gains distribution

For a first practical case to simulate the superposition coding scenario versus the reference scenarios, it was chosen to set channel gains $g_{k,i}$ that represented a very asymmetric case, that is, that one user has very good signal levels while the other has very poor. The concrete chosen parameters were $g_{1,1} = 6$; $g_{1,2} = 6 + 0.5j$; $g_{1,3} = 6 - 0.2j$; $g_{2,1} = 0.4$; $g_{2,2} = 0.3 + 0.2j$ and $g_{2,3} = 0.6 - 0.1j$. It can be seen that all the $g_{k,i}$ with $k = 1$ have absolute values around 6, while the ones having $k = 2$ range around 0.5.

The numerical results for the optimum parameters of superposition coding scheme

User k	$(\alpha_{k,1}, \alpha_{k,2}, \alpha_{k,3})$	R_k
User 1	(0.19-0.71j, 0.11-0.91j, 0.73-0.58j)	7.78
User 2	(0.6-0.31j, 0.3-0.27j, 0.35-0.08j)	0.22

Table 5.1: Superposition coding parameters in case 1

Scheme	Sum-rate	Equal rate	Area
Superposition coding	8	0.48	2
Dedicated HBSs	0.45	0.17	0.05
Beamforming	8.35	1.29	6.36

Table 5.2: Comparison parameters in case 1

calculated for this case can be found in Table 5.1. Table 5.2 contains the values for the optimization parameters in both the superposition coding and the reference scenarios, where the rates are expressed in bits/s/Hz since the bandwidth is normalized in the capacity formula being used.

Regarding the rate areas, they can be compared in Figure 5.3. In green we can see the superposition coding achievable area (after convexification); the red line limits the area achievable by beamforming; and the yellow box shows the boundary in the dedicated HBSs case.

For analyzing these results, let us first calculate the ratios between the best sum-rates for superposition coding compared to each scenario. The ratio for superposition coding versus beamforming's best sum-rate gives a result of 0.96, while against the dedicated scenario the superposition coding gets an approximate ratio of 17.7.

Observing both the regions and the numerical results, we can say that beamforming scenario gets the highest results in the three optimization parameters. In terms of sum-rate, the difference is not very remarkable - superposition coding is only approximately 4 % lower than beamforming. However, talking about rate area and equal rate, in this case the improvement got from using beamforming compared to superposition coding is considerable. Specially, it should be highlighted that user 2 (the one with poor coverage conditions) is very benefitted from using beamforming schemes in terms of achievable rates, as can be seen in Figure 5.3. An advantage of using superposition coding is that

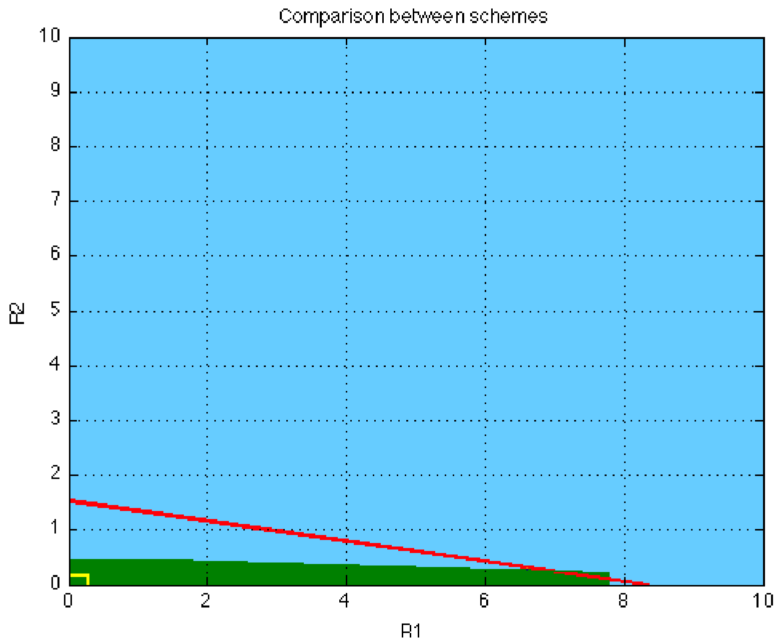

Figure 5.3: Rate regions for case 1. Green = SC; red = BF; yellow = Dedicated

some rates that are not achievable using beamforming become achievable now by using this scheme.

Regarding the comparison with the dedicated HBSs scenario, we can say that superposition coding gets very good results. In terms of sum-rate, superposition coding gets a sum-rate 17.7 times higher, and also gets about 3 times higher best equal rate and 40 times higher area. Therefore, in every situation for these channel gains using superposition coding over having a dedicated HBSs is preferable.

However, it should be noted that this particular distribution of channel gains is very disadvantageous for the dedicated case. This is because the user has one serving base station and two interfering base stations with a very similar signal levels. This creates very high interference in the user, and thus the capacity decreases dramatically.

5.4.2 Case 2: very symmetric channel gains distribution

Now for the second example of the simulation, it was chosen a fairer situation in this users get identical channel conditions. In particular, a strong link, a weak link and a

User k	$(\alpha_{k,1}, \alpha_{k,2}, \alpha_{k,3})$	R_k
User 1	(0.07-0.72j, 0.14-0.65j, 0.22-0.76j)	0.96
User 2	(0.5+0.47j, 0.09+0.74j, 0.5+0.35j)	5.64

Table 5.3: Superposition coding parameters in case 2

Scheme	Sum-rate	Equal rate	Area
Superposition coding	6.6	3.24	20.21
Dedicated HBSs	8.47	4.23	17.97
Beamforming	6.68	3.32	22.01

Table 5.4: Comparison parameters in case 2

medium link for each of the users. The chosen values are $g_{1,1} = 8+2.3j$, $g_{1,2} = 1.4-0.6j$; $g_{1,3} = 0.1-0.2j$; $g_{2,1} = 7+3.5j$; $g_{2,2} = 1.3-0.2j$; and $g_{2,3} = 0.2+0.6j$. The simulations results are shown in Tables 5.3 and 5.4 and Figure 5.4. Remember that for the distribution of rates, priority is given to user 2 in the case of having the same sum-rate values for different sets of user rates. On the other hand, since the region for superposition coding (green in the figure) in this case was already convex, no convexification was needed to be done.

Regarding the rate areas, they can be compared in Figure 5.3. In green we can see the superposition coding achievable area (after convexification); the red line limits the area achievable by beamforming; and the yellow box shows the boundary in the dedicated HBSs case.

About this second case, it can be said that about the in terms of best sum-rate, dedicated HBS gives some advantage. The ratios of superposition coding's best sum-rate against beamforming and dedicated HBSs are 0.99 and 0.78, respectively, so it can be concluded that there is some benefit in this optimization parameter in the dedicated case. Observing the best equal rate parameter, we can see that the dedicated HBSs scenario outperforms the other two scenarios, but the improvement is not very important.

The greater differences in this case are found in the area optimization parameter. Beamforming and superposition coding give similar performances. The only difference are the extreme areas close to the vertexes of the triangle defining the beamforming area

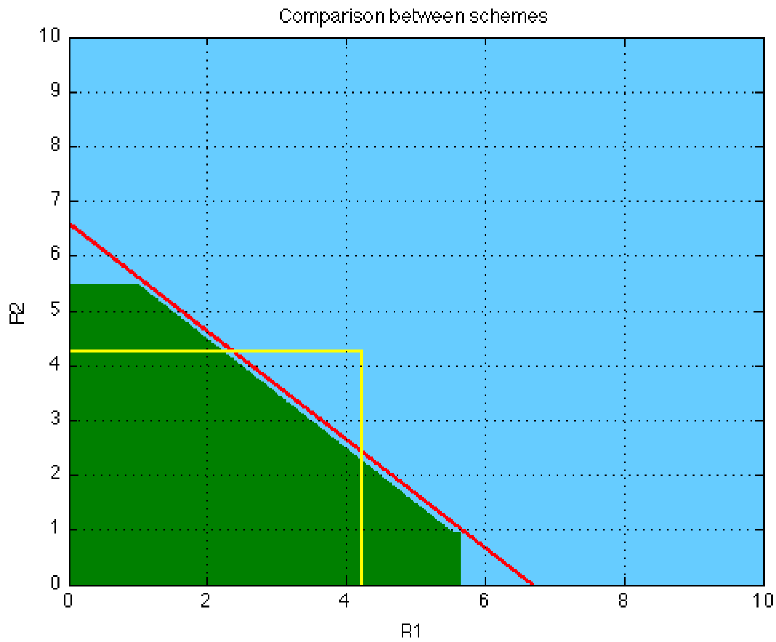

Figure 5.4: Rate regions for case 2. Green = SC; red = BF; yellow = Dedicated

in Figure 5.4. Since normally it is desirable that both users get a fairly good service level, it is not very common to work on these areas. Therefore, we can conclude that using superposition coding or beamforming for these channel conditions makes no great differences in the performance.

Regarding the comparison with the dedicated HBSs scenario, as we said, best sum rate and equal rate are outperformed by the reference scenario in this case. The reason is that for these given channel gains, the *signal* term in the SINR is considerably higher than the interference term, because the base station with highest signal level is chosen as the serving one. However, in the quantity of achievable rate pair, related to the area optimization parameter) seeing the rate regions we detect an improvement provided by using superposition coding. Basically, this improvement consist in that a wider range of possible rates can be used keeping a very similar sum-rate in the system. This is not possible using the dedicated HBSs scheme, which is constrained by the fixed SINR levels.

It might be surprising to see that the dedicated scenario can outperform both superposition coding and beamforming scenarios in terms of sum-rate (those are more advanced techniques). This is due to the shape of the rate regions. In the dedicated case, the only connection between the rates at each receiver is the transmitted power by each base station. It is possible though that the channel conditions are such that the final received power is low for the interfering signals. Case 2 would be an example for this. Therefore, in some conditions it is possible that adding both users' rate gets a higher value than the best possible sum-rate in beamforming, which corresponds to one of the vertexes of the triangle defining the rate region in beamforming (recall Figure 5.1 and equation (5.12)).

5.5 General conclusions of the simulation

Some conclusions that can be extracted from this cases are summarized in the following points:

- The best achievable rates are similar in the cases of beamforming and superposition coding in all cases. The main benefit from using superposition coding compared to using beamforming is that normally in this optimum point both users are assigned acceptable rates. Furthermore, there is often a wider range of rate values that work in the optimum point of the system.

- The performance of the dedicated HBSs scenario in term of sum-rate is constrained by the distribution of the channel gains. Since all users and HBSs share the same bandwidth, if all the channel gains have similar absolute values the interference created in the receivers is very high, so poor results are achieved. If on contrary there is a dominant HBS, the dedicated HBS scenario offers good sum-rate results.

- In the equal rate parameters, normally the beamforming scenario slightly outperforms the superposition coding scenario, and this offers normally results as good as the dedicated HBS scenario.

- The biggest benefit of using beamforming lies in the area optimization parameter. In all cases beamforming offers achievable rates that superposition cannot attain. However, sometimes superposition coding achieves rate pairs that are not achievable via beamforming. Regarding the dedicated HBSs scenario, it normally get poorer results in this parameter compared to the other two schemes.

Given all these conclusions, it should be remarked again that these simulations represent only a first approach in attempting to solve the optimization problem in (5.1).

With a better computer to perform a larger MCM simulation or with more suitable optimization tools to solve the problem, the results obtained for superposition coding may improve.

Chapter 6

Conclusion of the investigation

In this project, we have explored the potential benefits that using cooperative transmission schemes can bring in the context of femtocell networks using the information theoretic framework. For this purpose, a representative scenario for a femtocell deployment was selected, consisting of 3 HBSs and two users. Among several possible coding techniques, superposition coding was selected to be applied in this scenario because of its practicality.

To be aware of the possible improvements that the proposed coding scheme can bring, it was compared to two state-of-the-art reference scenarios: beamforming using timesharing and choosing a dedicated HBS for each UE. Also, convenient optimization parameters were defined to get numerical results for the comparison to be made - best sum-rate and equal rate, and biggest rate-region area.

As a first approach, the outer bound for this scenarios was calculated (Appendix A) without considering the Multiple Access Channel constraints. Then, a Montecarlo method simulations was designed considering these restrictions to tackle the complex optimization problem and give an approximate solution to be aware of the possible gain that using this technique can attain.

In general, the results that are obtained using MCM simulation show that some benefit can be achieved in certain situations. Basically, the biggest improvements are achieved in comparison to the dedicated HBSs scenario, one of the most extended transmission schemes.

Given the limitations of the computers used to run the implemented simulations and the lack of computational efficiency of the designed algorithm, the results that were presented here can only be taken as a first approximation to the final solution. However, some of the results obtained (specially those related to the comparison to the dedicated HBSs) are good enough to encourage further investigation within this area.

For future work directly related to superposition coding, a more efficient optimization method should be used to solved the problem set out in Chapter 5 in order to get more accurate results and conclusions on the applicability of the coding scheme. On the other hand, other information-theoretic may also be studied to investigate the possible implementation in a femtocell network deployment.

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Appendix A

Outer bound of rate regions using unconstrained Montecarlo method

For a first approximation to see how the rate regions look like, a simple simulation was designed. The method used for this simulation was a Montecarlo Integration. Montecarlo method (MCM) simulation is a numerical method that uses a large set of random numbers to evaluate a function and get statistical results with them. This kind of simulation was chosen because it can provide preliminary fairly good results and gives a valid approximation while it remains simple to set up.

A.1 Aim of the simulations

To have a first idea of the potential benefits that using a codification scheme such as the one proposed in the problem stated (superposition coding in a downlink 2-UE/3-HBS scenario), this simulation is presented. For simplicity, in this simulation the constraints that are imposed by the Multiple Access Channel (explained later this chapter) are not considered yet. It should be noted that this implies that the real results will be upper bounded by the ones obtained via this method. However, it helped us to get an idea of how good the scheme can me compared to some reference case. For comparison, the chosen reference scenario consists of 2 HBSs serving 2 users, one connected to each of them and sharing the channel.

A.2 Simulation description

A.2.1 General comments

In this simulation, the Successive Interference Cancellation (SIC) decoding method is considered. SIC is a decoding method that permits partial (or total) cancellation of the signals in a receiver that are considered to be interference. In the superposition coding scheme set out here, this signals are the ones addressed for the other user.

The simulations get the achievable rate-regions for 4 different cases: the reference scenario and the superposition coding scenario without Successive Interference Cancellation (SIC), with SIC in one user and SIC in both users. In the SIC case, it is chosen that the interference cancellation has 100% efficiency. Again, we note that a 100% efficient SIC in both users is seldom achievable in a real case if the MAC constraints are considered, but this will serve as a first approach. The channel coefficients $h_{k,i}$, the transmit power of each base station P_i and the noise variances N are fixed. The noise variances have been normalized to 1 for simplicity. The voltage gain variable $g_{k,i}$ defined in equation (3.10) is used to represent the amount of power received in user k from HBS i, being considered both pathloss and transmitted power. With this definition, for each of the cases, the SINR can be expressed as

Ref. scenario $\rightarrow$
$$\gamma_1 = \frac{|g_{11}|^2}{1 + |g_{12}|^2} \tag{A.1}$$
$$\gamma_2 = \frac{|g_{22}|^2}{1 + |g_{21}|^2} \tag{A.2}$$

SC without SIC $\rightarrow$
$$\gamma_1 = \frac{|\alpha_{11}g_{11} + \alpha_{12}g_{12} + \alpha_{13}g_{13}|^2}{1 + |\alpha_{21}g_{11} + \alpha_{22}g_{12} + \alpha_{23}g_{13}|^2} \tag{A.3}$$
$$\gamma_2 = \frac{|\alpha_{21}g_{21} + \alpha_{22}g_{22} + \alpha_{23}g_{23}|^2}{1 + |\alpha_{11}g_{21} + \alpha_{12}g_{22} + \alpha_{13}g_{23}|^2} \tag{A.4}$$

SC + SIC, 1user $\rightarrow$
$$\gamma_1 = \frac{|\alpha_{11}g_{11} + \alpha_{12}g_{12} + \alpha_{13}g_{13}|^2}{1 + |\alpha_{21}g_{11} + \alpha_{22}g_{12} + \alpha_{23}g_{13}|^2} \tag{A.5}$$
$$\gamma_2 = |\alpha_{21}g_{21} + \alpha_{22}g_{22} + \alpha_{23}g_{23}|^2 \tag{A.6}$$

SC + SIC, 2 users $\rightarrow$
$$\gamma_1 = |\alpha_{11}g_{11} + \alpha_{12}g_{12} + \alpha_{13}g_{13}|^2 \tag{A.7}$$
$$\gamma_2 = |\alpha_{21}g_{21} + \alpha_{22}g_{22} + \alpha_{23}g_{23}|^2. \tag{A.8}$$

With these SINR values, the capacity in each case can be calculated using (3.13).

In the case of the reference scenario, since every parameter affecting (A.1) is fixed, the achievable rates will be contained in a rectangle when representing rates R_1 against R_2.

A.2.2 Generation of the power division coefficient values

The $\alpha_{k,i}$ parameters are generated randomly to perform the MCM study evaluating expressions (A.3) to (A.8). Two different situations have been tested. In the first simulation, it was considered that $\alpha_{k,i}$ can only take real and positive values. In the second, it is allowed that there is a complex part too, which can be either positive or negative.

- Real power division coefficients

  In this case, the $\alpha_{k,i}$ coefficients are generated with the MATLAB function *rand* and using the condition

$$\sum_{k=1}^{U} \alpha_{k,i}^2 = 1. \tag{A.9}$$

  In the particular case being studied here, with 2 users, first $\alpha_{1,i}$ is generated randomly and with its value it is forced that $\alpha_{2,i} = \sqrt{1 - \alpha_{1,i}^2}$. After that, the generation for the next points is done the other way around. It is done this way so the average values of both vectors is equalized (the average value of both vector containing $\alpha_{1,i}$ and $\alpha_{2,i}$ is not the same if this process is not done). With all the obtained points, the Montecarlo method is performed.

- Complex power division coefficients

  In the complex case, the condition equivalent to (A.9) is

$$\sum_{k=1}^{U} |\alpha_{k,i}|^2 = 1. \tag{A.10}$$

  Another random variable to decide the sign of the complex part of $\alpha_{k,i}$ is introduced. After computing all the values, the same process as in the real case is carried out.

A.3 Results

The chosen state for the links to show as example is one trying to represent that HBS1 gives a good service to user 1 and not so good for user 2; viceversa for HBS 2; and

HBS3 is equally good for both of them. The concrete values are $g_{11} = \sqrt{10 + 0.3j}$; $g_{22} = \sqrt{10 - 0.2j}$; $g_{12} = \sqrt{1 - 0.5j}$; $g_{21} = \sqrt{1 + 0.6j}$; $g_{13} = \sqrt{3}$; and $g_{23} = \sqrt{3 + 0.1j}$.

A.3.1 Real $\alpha_{k,i}$

With this set of values for the power division coefficients, the rate regions are represented in Figure A.1. The red inner rectangle corresponds to the reference scenario; the green area to the superposition coding scenario without SIC; and in the blue one SIC is included for user 2.

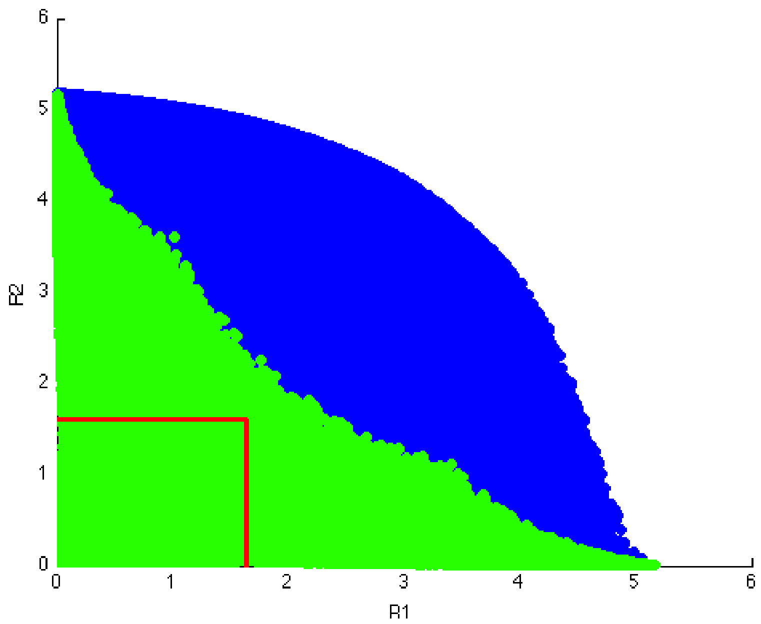


Figure A.1: Achievable rate region for real values of $\alpha_{k,i}$

The superposition coding (SC) + SIC area has a convex shape, so the optimum point is somewhere in the middle of the curve (because the values in the axis are similar). However, this is not the case for the green function, that has a concave-like shape, which implies that the optimum sum-rate corresponds to set all the $\alpha_{k,i}$ of one user to 1 and 0 to the other.

For the chosen channel characteristics, the best sum-rate in the SC + SIC in 1 user scenario is $R_1 + R_2 = 3.3113 + 3.9654 = 7.2768$, and it is obtained with the values

$(\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{21}, \alpha_{22}, \alpha_{23})$ = (1,0.26039, 0.89587,0.00066,0.9655,0.44432).

Meanwhile, the reference scenario's best achievable sum-rate is $R_1 + R_2 = 1.6336 + 1.5971 = 3.2307$. This means that by using SIC in one user, an improvement of $\frac{7.2768}{3.2307} = 2.2524 = 125\%$ can be obtained.

Now regarding the SIC in 2 users' case, the results are presented in Figure A.2. This time, the curve in green (in the right) corresponds to the SIC in 2 users's case. As it can be seen, a better performance is achieved. Giving concrete numerical results, the best sum-rate for this new case in the same channel conditions is $R_1 + R_2 = 4.445 + 4.4376 = 8.8827$, achieved using $(\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{21}, \alpha_{22}, \alpha_{23})$ = (0.95413, 0.31406, 0.70311, 0.2994, 0.9494, 0.71108).

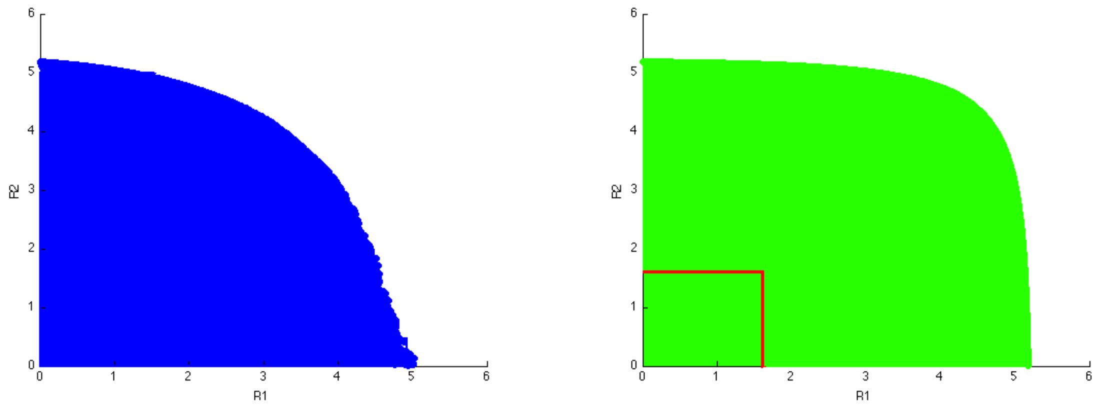


Figure A.2: Comparison of rate regions with SIC done in 1 and 2 users

This results imply that using SIC in two users implies an improvement of $\frac{8.8827}{7.2768} = 1.2207 = 22\%$ in the system sum-rate with respect to using it only in one, and $\frac{8.8827}{3.2307} = 2.7501 = 175\%$ against the reference scenario.

A.3.2 Complex $\alpha_{k,i}$

This time it is allowed that $\alpha_{k,i}$ can be complex with an either positive of negative imaginary part, forcing condition (A.10). Results for this case are shown in Figure A.3 for the SC + SIC for one user.

After running several simulations (the needed number of points for the Montecarlo

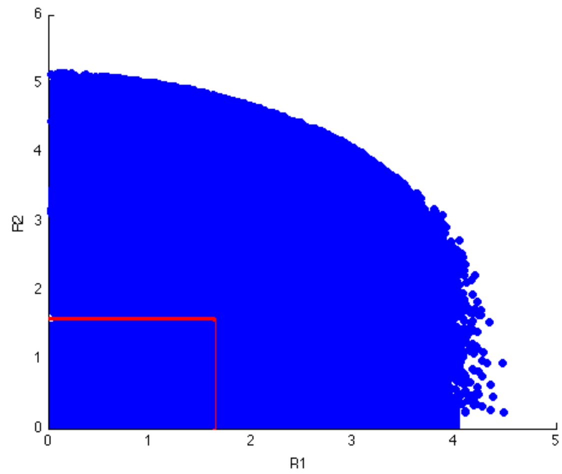


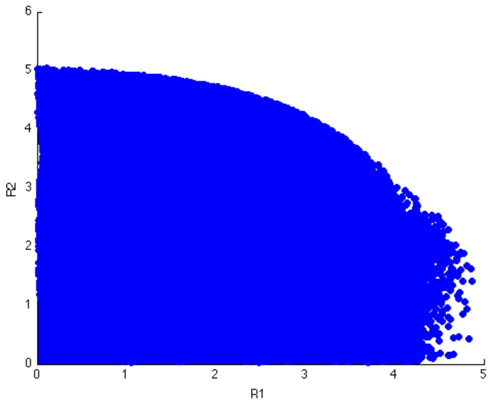
Figure A.3: Achievable rate region for complex values of $\alpha_{k,i}$

method is too large to perform only one simulation with complex numbers in this computer), the best solution of the optimization problem for this case gives an aggregate rate of $R_1 + R_2 = 4.072 + 3.29 = 7.3623$, using the set $(\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{21}, \alpha_{22}, \alpha_{23}) = (0.049796-0.96321i, 0.12033-0.33862i, 0.27985-0.94718i, 0.092228+0.24746i, 0.30545-0.88179i, 0.054861-0.14674i)$. This means an improvement of only $\frac{7.3623}{7.2768} = 1.0117 = 1.1\%$ with respect to the real case.

A comparison between SIC in 1 and 2 users with $\alpha_{k,i} \in \Im$ is shown in Figure A.4. Note that the scattered points in the right part of the plots give us an intuition of how the shape of the region looks like. Some points are less probable, so that is why the whole region is not shown.

In this case, the best sum-rate obtained with several simulations is $R_1 + R_2 = 4.4526 + 4.4381 = 8.8907$, which is only $\frac{8.8907}{8.8827} = 1.0009$ times better than with real values, and $\frac{8.8907}{7.3623} = 1.2076 = 20\%$ better than SIC in only 1 user with complex power division values. For this solution, the optimal $\alpha_{k,i}$ are $(\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{21}, \alpha_{22}, \alpha_{23}) = (0.038196-0.947i, 0.054124-0.31404i, 0.0085662-0.7153i, 0.14468+0.28424i, 0.17694+0.93121i, 0.071205+0.69512i)$.

As it can be seen again, adding a complex term to $\alpha_{k,i}$ does not bring an important benefit. Obviously, since using complex numbers correspond to a more general case, all *real* $\alpha_{k,i}$ cases are included in the *complex* ones. That is why complex values are



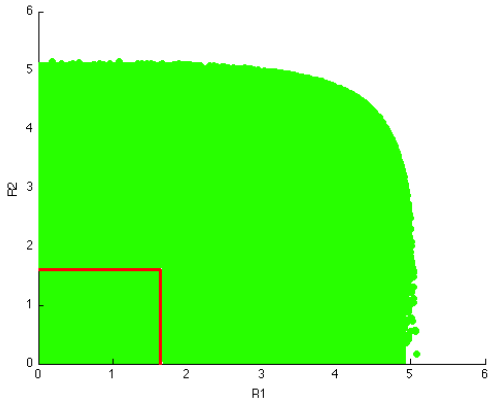


Figure A.4: Comparison of rate regions with SIC done in 1 and 2 users

evaluated for not losing generality.

A.4 Conclusions

The main conclusions that can be extracted from these preliminary simulations are:

- The improvement achieved by using superposition coding with total interference cancellation compared to the reference scenario is remarkable (above 125% in all cases). Again, we insist that this would be only an upper bound because no MAC constraints are considered.
- If it is accomplished to cancel the interference completely in 2 users, a benefit of around 20% with respect to doing it in one user is brought.
- Allowing that $\alpha_{k,i}$ can take complex values brings a slight benefit in comparison to considering only real values. Complex values will be used in new simulations for not losing generality.