Reliability updating of existing bridges based on proof loading

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Synopsis:

This masters thesis investigates how proof loading could be used to reassess the reliability level of existing bridges. When the resistance of the bridge is greater than the applied proof load, the bridge is considered to be safe and the test successful.

The first part of this thesis presents modern state-of-the-art assessments of previous work done by different researchers. This is to give an overview of the effort done to see how, when using reliability analysis, the bridges can be safely designed so as to utilise their maximum capacity.

The stochastic model is established in Chapter 3, with which the reliability analysis is carried out for the generic limit state equation. The results are presented for EN 1990 standard CC2 and CC3 consequence classes. Based on the sensitivity measures it was possible to determine which stochastic variables could be updated in order to update the reliability level of existing bridge.

Bayesian decision theory, introduced in Chapter 4, is used to update the reliability level of existing concrete bridge based on proof loading. The updated failure probabilities and corresponding reliability indices indicated which proof load level is required to obtain the sufficient reliability level of structure.

Future aspects were mentioned in Conclusion.

Front page image: Taken by Arquivo O Globo [1974].

Preface

Reading guide
References to figures and tables are indicated as “figure x.y” and “table x.y” where x refers to the chapter in which the reference is placed, and y refers to the figure/table number within the chapter. References to equations are indicated as “equation (x.y)”, using the same system as presented for figures and tables.

The bibliography is according to the Harvard method where an active reference in the text is shown as “author [year of publishing]”. When using a passive reference the source is put in the end of the paragraph as “[author, year of publishing]”. The bibliography is placed at the end of the main report, and is sorted alphabetical.

Jelena Perisa
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Spring 2021

Contents

Introduction 1

Most of the bridges in Denmark have been built in the 20th century. Due to paced expansion of cities and need for faster commuting between them, bridges were built to span a valley, road, body of water or other physical obstacles.

Figure 1.1: The Oresund bridge (2000), Mauritz Antin [2020]

Bridges offer faster transport of not only people, but also goods. Sometimes, the routes to deliver the goods might be longer because bridges on the way are not classified to support such a heavy load. This does not only increase time of delivery, but makes deliveries more expensive. Because bridges have a significant role in everyday life, it is necessary to perform a reliability analysis for those that could be of a better use. If analysis determines that they would be able to withstand a greater load, they may be repurposed.

Existing bridges have been built based on the codes for new structures that existed at the time of the building that are in some cases no longer valid. The advantage of the existing bridges is that geometrical measurements, characteristics of the materials, loads and structural response can be measured directly on the bridge.

Current codes (Eurocodes) for the calculation of new bridges are based on conservative assumptions. Those could be regarding the magnitude of the action and their design response to these actions. When calculating a new bridge, one should check that the structure is capable of a particular intended use during the estimated structure life.

These requirements in particular (designed actions on structure) cannot be changed for the existing bridges. That is why considering currently used codes, existing bridges are proven to have insufficient strength to withstand the updated loads that they are exposed to daily or; quite on contrary, that some of those bridges could be up-classified.

Since the direct tests conducted on these kind of bridges (such as proof load or material tests) might be very expensive, probabilistic models are set to develop a framework which could determine if the bridge could be up-classifies. This in particular is the topic; or better said, matter in question of this report. How can one, based on the assessed current state of the bridge build 10, 20 or more years ago, update its reliability without making any physical tests. Conducting first a reliability analysis of an existing bridge and checking its consequence class, the bridge will be subjected to proof loading test in form of simulations conducted in Matlab. Based on those results, it will be concluded whether the existing bridge should be up- or down-classified. Two cases for the bridge will be examined: once when the concrete compressive strength will be considered as dominant material strength and once when reinforcement will be considered as dominant material strength, given that the bridge in question is concrete bridge.

1.1 Classification of structures and vehicles

Bridge classification is carried out for two situations:

- Normal passage
  - The standard vehicles pass the bridge without any limitation on other traffic

- Conditional passage
  - The standard vehicles are the only traffic load on the bridge. They travel on a specified lane at a reduced speed (V = 10 km/h)

Classification is an iterative process. It first starts with selecting the required class for which it is desired to verify that the bridge has the required capacity. The procedure is depicted below (see figure 1.2).

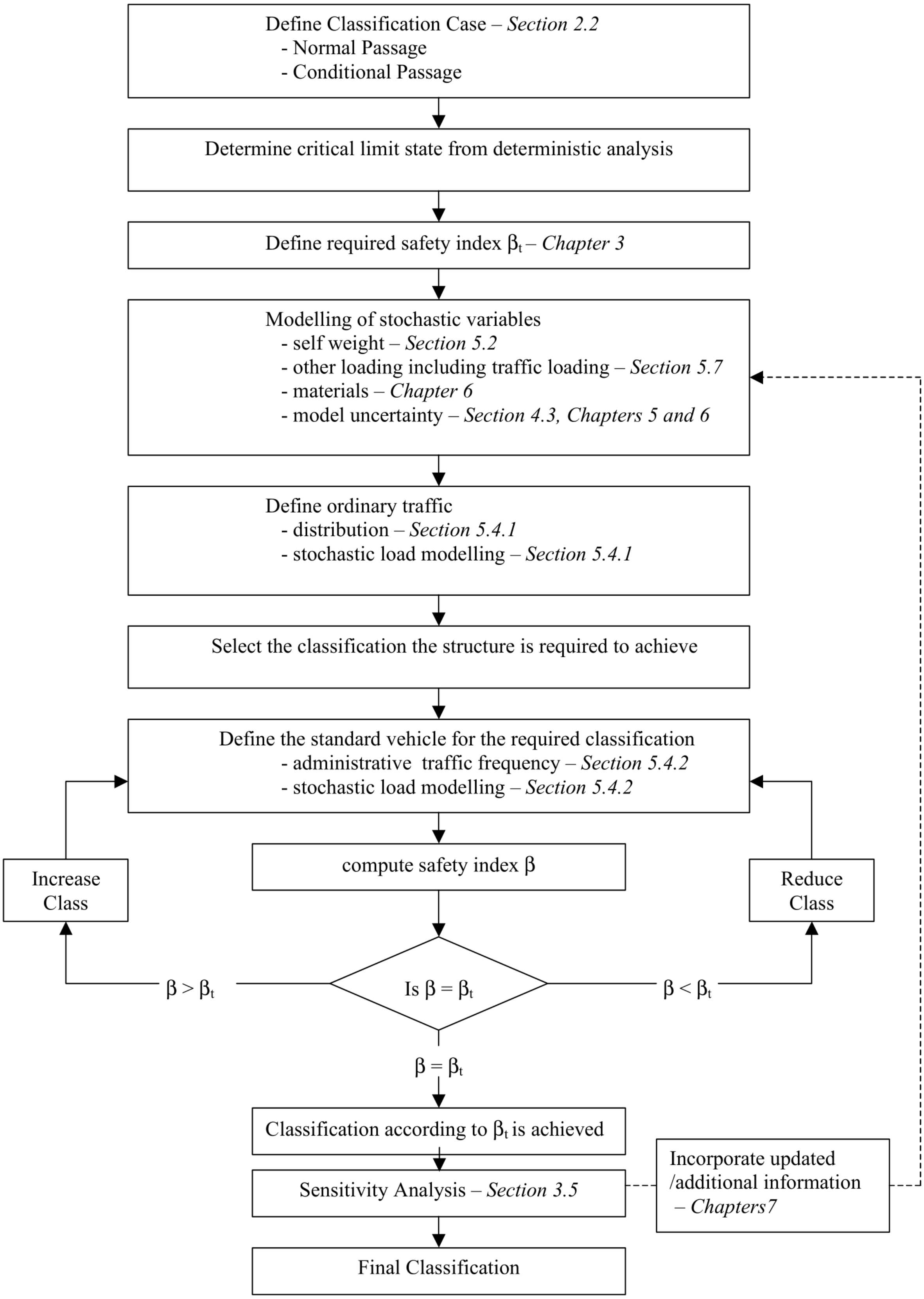

Figure 1.2: Probability based classification, Von Scholten et al. [2004]

For the purposes of this paper, conditional passage will be investigated.

In Denmark, the vehicle modelling is divided into two categories:

- Modelling of ordinary transports in conformity with the traffic regulations
- Modelling of standard vehicles for bridge classification

In this report, the modelling of the standard vehicles for bridge classification will be considered.

Standard vehicles in Denmark are sorted into classes based on their weight. The axle load is taken as distributed on two 600 mm wide wheels. The contact length in the direction of travel is taken as 200 mm.
The weight of standard vehicles, W, is assumed to be normal distributed. The parameters for each class are given in figure .

Standard vehicle Weight W_i	Mean weight [tons]	Standard Deviation [tons]
Class 50	53.1	5.0
Class 60	63.4	5.0
Class 70	72.2	5.0
Class 80	82.5	5.0
Class 90	95.4	5.0
Class 100	109.2	5.0
Class 125	131.4	5.0
Class 150	157.6	5.0
Class 175	170.2	5.0
Class 200	201.0	5.0

Figure 1.3: Standard vehicles, Von Scholten et al. [2004]

The corresponding axle configuration for each class is shown in figure 1.4.

Class	Axle Configuration Axle Weight in tonnes and Axle Spacing in m	Width m
10	2,0 4,4 4,4 3,2 1,4	2,6
20	3,0 5,0 6,8 6,8 3,2 3,2 1,4	2,6
30	5,0 8,0 9,3 9,3 3,2 3,5 1,4	2,6
40	5,5 5,5 5,5 4,2 10,8 10,8 3,2 1,4 3,5 1,4 1,4	2,6
50	6,5 6,5 6,5 10,9 11,8 10,9 3,2 1,4 4,5 1,4 1,4	2,6
60	6,5 7,0 7,0 9,3 12,8 11,5 9,3 3,2 1,4 6,0 1,4 1,4 1,4	2,6
70	6,5 9,0 9,0 11,6 11,0 13,5 11,6 3,2 1,4 6,0 1,4 1,4 1,4	2,6
80	8,0 9,0 9,0 9,0 9,0 14,0 14,0 10,5 3,2 1,4 6,0 1,4 1,4 1,4 1,4	2,6
90	7,0 7,0 9,5 9,5 11,0 11,0 14,7 14,7 11,0 1,4 3,2 1,4 6,0 1,4 1,4 1,4 1,4	2,6
100	7,0 7,0 9,5 9,5 11,5 11,5 11,5 15,1 15,1 11,5 1,4 3,2 1,4 6,0 1,4 1,4 1,4 1,4 1,4	2,6
125	7,0 7,0 9,5 9,5 6 × 16,4 1,4 3,2 1,4 6,0 1,4 1,4 1,4 1,4 1,4	2,8
150	7,0 7,0 9,5 9,5 7 × 17,8 1,4 3,2 1,4 6,0 1,4 1,4 1,4 1,4 1,4 1,4	2,8
175	7,0 7,0 9,5 9,5 7 × 19,6 1,4 3,2 1,4 6,0 1,4 1,4 1,4 1,4 1,4 1,4	2,8
200	7,0 7,0 9,5 9,5 8 × 21 1,4 3,2 1,4 6,0 1,4 1,4 1,4 1,4 1,4 1,4 1,4	2,8

Figure 1.4: Axle configuration for standard vehicles, Von Scholten et al. [2004]

The number of the standard vehicles in each class is determined separately. It is a function of the bridge's position in the road network and the envisaged use of the bridge for heavy transports.

Class/ Road type	50	60	70	80	90	100	125	150	175	200
Motorways	200	200	200	150	150	100	50	50	50	50
Main roads	100	100	100	80	80	50	20	20	20	20
Other	50	50	50	40	40	20	10	10	10	10

Figure 1.5: Administratively determined annual number of standard vehicles, N, Von Scholten et al. [2004]

1.2 Evaluation of bridges

The information that will be presented in this section will be based on EN 1990 [2002] for the design of the new structures.

EN1990 Basis of structural design establishes principles and requirements for the safety, serviceability and durability of new structures. It describes the basis for their design and verification and gives guidelines for related aspects of structural reliability.
It is intended to be used in conjunction with EN 1991 to EN 1999 for the structural design of buildings and civil engineering works, including geotechnical aspects, structural fire design, situations involving earthquakes, execution and temporary structures.

The draft version of prEN1990-2 for assessment of existing structures states that every structure shall be designed to have adequate:

- structural resistance
- serviceability
- durability

Requirements regarding structural modelling

- Calculations shall be carried out using appropriate structural models involving relevant variables
- The structural models selected should be those appropriate for predicting structural behaviour with an acceptable level of accuracy. The structural models should also be appropriate to the limit states considered.
- Structural models shall be based on established engineering theory and practice. If necessary, they shall be verified experimentally.

The traditional way of bridge safety design is by using deterministic analysis (for both ULS and SLS). Deterministic analysis does not take into account the uncertainties and variation. These simplifications provide less accurate and, in many cases, conservative design outcomes. This often leads to more expensive upgrades since a lot can be overlooked during the analysing the structure (which is included in semi-probabilistic or probabilistic analysis like uncertainties or reliability). Unless the models are simple by nature and reflect the real case, it should be advised to use semi-probabilistic or probabilistic analysis instead. This will expectedly lead to more accurate representations of real life structures.

The semi-probabilistic approach can be seen as the safe design correction to this and is founded on limit state principles where uncertainties of design parameters are taken into account by means of safety factors (for example the partial safety factors in the Eurocode). Partial safety factors represent safety measures. Without the doubt, it is a better representation of model of real life bridge. However, this approach is still recognised as conservative for more complex structures. According to DS/ISO 2394 [2015], this approach is appropriate for the basis of design and assessment for the structures for which the damages and consequences of failure are well understood. Standards ensure the quality of analysis, design, maintenance etc.

The third approach is risk-informed and reliability-based approach.
In risk-informed design and/or assessment, decisions are optimised considering the total risk. Assessment of total risk is represented by scenarios and by probabilistic models of the failure events, exposures, direct or indirect consequences. Meaning, the decision should be optimized on basis of the maximization of the expected value of benefits as illustration in figure 1.6 shows.

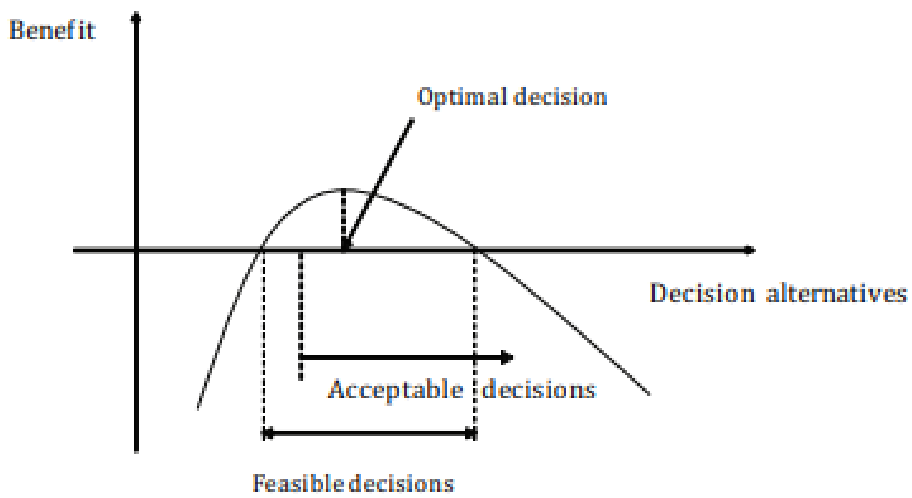

Figure 1.6: The optimization principle, DS/ISO 2394 [2015]

Reliability-based approach is an alternative to risk-based design and assessment. It is based on limit state principle. This approach assesses minimization of costs and/or minimization of committed resource usage subject to given reliability requirements for the structure (DS/ISO 2394 [2015]). The difference between semi-probabilistic and reliability-based approach is that the safety measure is represented by reliability index and probability of failure. Both are directly related and the reliability index is obtained by approximation techniques. All uncertainties are quantified or estimated. The structure is deemed safe the moment it reaches a required minimum structural reliability, known as target reliability index (see figure 3.1).

Certainly, reliability assessment of existing bridges is different from the one for new bridges.
Increasing the safety level is more expensive for existing than the new bridges.
The remaining working life of existing bridges often differs from the standard design working life. The standard design working life is 100 years for new bridges. Some of the ways the remaining working life of existing bridges influence reliability requirements are in terms of fatigue or durability.
Inspections, tests, measurements etc. which provide actual structural conditions are different for

existing and new bridges.

1.3 Problem statement

"How can a reliability level of existing bridge be updated based on the proof loading? "

1.4 Objectives

A framework for reliability-based up-classification will be investigated. Reliability analysis will be updated using proof loading. The deterministic approach is simple to use, but may give inadequate results since it will often lead to conservative design. Reliability analysis determine the probability of failure of the structure under various loading scenarios, and therefore provide more reliable and prosperous results.

The model of the bridge in question will be simplified. Only conditional passage will be considered. It will be assumed that only one truck at the time will be crossing the bridge.

2 Up-classification of existing bridges – State of the art

In this chapter, previous work dealing with similar objective will be presented and commented. Each paper will be presented in short and will present different approaches investigated in each paper respectively. It will provide better understanding of the matter in question.

2.1 Reliability-based assessment procedures for existing concrete structures

In, Jeppsson [2003], the aim was to show a transparent assessment procedure for the residual service life assessment of concrete structures using reliability theory and statistical tools.
The focus was to show applicability and usefulness of reliability theory as a tool for assessment. Later on the comparison between deterministic and probabilistic results was conducted.
Jeppsson [2003] highlights that one of the most important factors for using reliability analysis to assess the existing structures is large economical benefit. Not only would the up-classification of existing structures be a sustainable solution, but also economical one. It is understood that not every reliability update will lead to up-classification, but rather indicate that the reliability index is lower that suggested and will lead to deterioration. Either way, using reliability analysis, it will be possible to look into the current state of the existing bridges and apply the necessary care or up-classify the bridge.
For the purposes of this paper, the railway bridge will be used in the assessment as an example (The Swedish National Railroad Administration (SNRA) is the owner of a bridge that was built with cast-in-place concrete in 1955. It is a two span trough bridge with continuous girder designed as a frame, assuming interaction between the girder and the supports) and can be seen in figure 2.1. The assessment of an existing railway bridge should be performed in safety class 3, which may be interpreted such that the safety index should be 4.8. The parameters are updated describing a random variable. The main idea is to update prior probabilistic information concerning the parameters, with information from testing. Often, this is referred to as Bayesian updating. Bayesian updating results in posterior probabilistic model.
The strength of the concrete has a significant influence on the load carrying capacity. Therefore, assessment of the material parameters investigated is crucial for the load carrying capacity of the bridge. Model uncertainties that are related to different failure modes and failure mechanisms are also very important parameters in reliability analysis. It is also noted that in used standards, sufficient information on model uncertainties was not provided (in Regulations for Structural Design, NKB 55 (1987)). In this research, the sensitivity analysis suggested that the dynamic amplification factor is of considerable importance for the safety of the bridge. The size of the amplification factor is evaluated in a crude manner giving safe values on the safe side.
In general, assessing the existing structures is a demanding task. Moreover, economical consequences are significant. This way, performing reliability analyses it is possible to utilise the structure to its maximum capacity.
Reliability does not always result in an increase in the permissible load, but the procedure of doing the assessment has the same positive influence as a risk analysis. It becomes evident which variables have the greatest influence on the safety, indicating where money is best spent in order to increase the

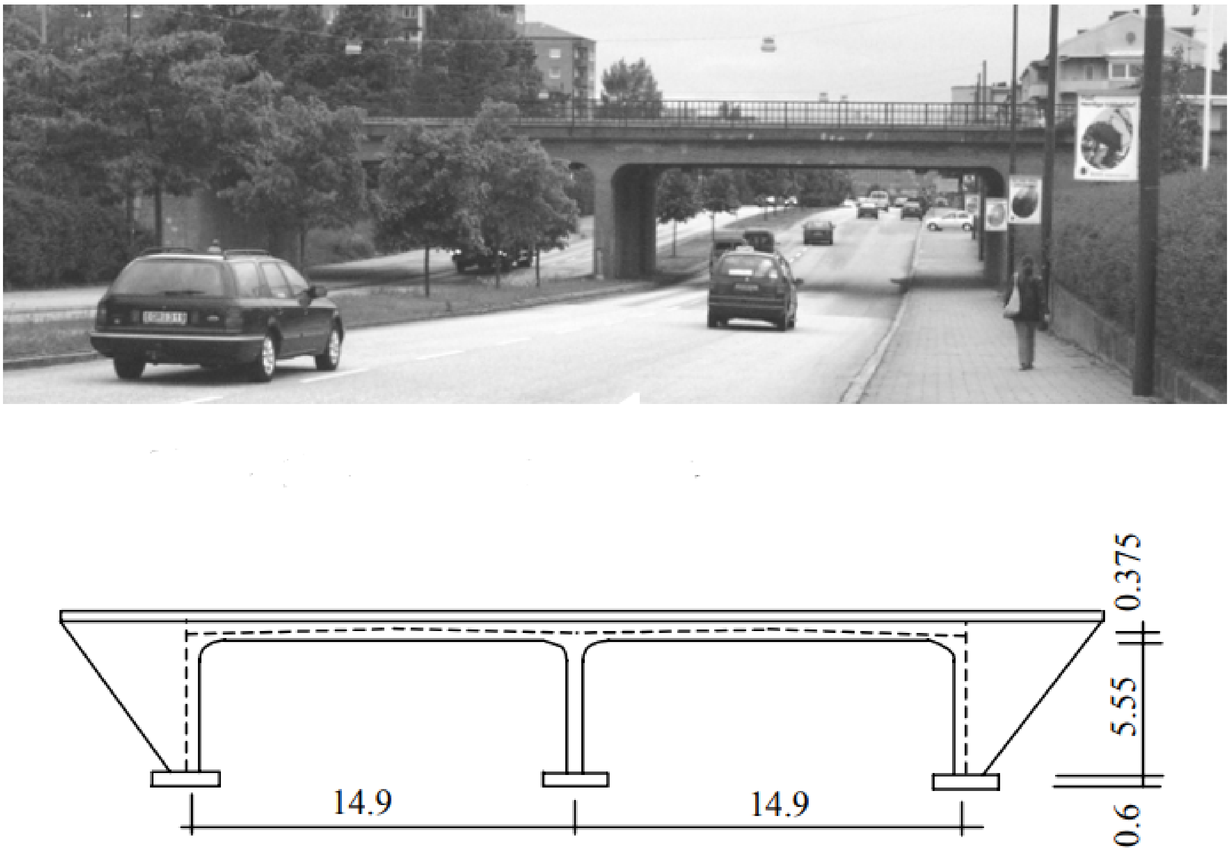

Figure 2.1: Photograph of the investigated bridge, Jeppsson [2003]

permissible load.

2.2 Structural evaluation updating based on quality control and proof loads

In their study, Abbadi and Lamdouar [2018], argue that the reliability of structures based on probabilistic theories provides an accurate estimate of the current strength and the remaining life time of the structure.
Since design codes provide rather conservative safety margins, it is concluded that some characteristics can be defined with more accurately.
The highlight of this study is the new information obtained on material properties and proof loads. Proof load may indicate a greater resistance or load-bearing capacity of the structure. Quality control of material properties during the construction process can be used as a first update of the partial factors used in the design. This should be carried out within the framework of a probabilistic reliability theory, in accordance with the margin of safety intended in the Eurocode.
This paper was intended to provide a semi-probabilistic format in verification of existing reinforced concrete Bridges (in accordance with the target reliability suggested by Eurocode). Therefore, this paper had 2 goals:

- For a given reliability level, derive new partial factors
- For a given partial factors, estimate the real reliability level

The characteristic value, R_k, of a resistance variable, R is defined as its 5% fractile. If R is normally distributed, then the characteristic values is given as

$$R_k = \mu_R - 1{,}645\sigma_R \tag{2.1}$$

where μ_R is mean value and σ_R is standard deviation of performance function.
Design value, R_d, of R can be estimated as

$$R_d = \mu_R - \alpha_R\beta\sigma_R \tag{2.2}$$

where α_R is sensitivity factor (taken from Eurocode). Based on characteristic and design value of R as normal distributed variable, partial factor can be assessed. Considering R as lognormal distributed variable, characteristic value of R is described as

$$R_k = \mu_R \cdot exp(-1{,}645\sigma_R) \tag{2.3}$$

and design value as

$$R_d = \mu_R \cdot exp(-\alpha_R\beta\sigma_R) \tag{2.4}$$

and the partial factor for lognormal distributed R can be expressed as

$$\gamma_R = \frac{R_k}{R_d} = \frac{\mu_R \cdot exp(-1{,}645\sigma_R)}{\mu_R \cdot exp(-\alpha_R\beta\sigma_R)} \tag{2.5}$$

The truncated method was used for updating by proof load. It consists of updating the probability density function (PDF) of resistance, after experiencing a load proof with a convincing results in term of deflection and safety. First, the PDF before experiencing proof load is defined as

$$\int_{-\infty}^{+\infty} f_R(x)dx = F_R(R_{test}) + \int_{R_{test}}^{+\infty} f_R(x)dx \tag{2.6}$$

The truncated distribution function after successful test under a proof load is

$$\int_{R_{test}}^{+\infty} f_{R,up}(x)dx = 1 \tag{2.7}$$

The updated PDF, $f_{R,up}(x)$ is described based on equations 2.6 and 2.7.

$$f_{R,up}(x) = \begin{cases} = \frac{f_R(x)}{1-F_R(R_{test})}, & x \geq R_{test} \\ = 0, & x < R_{test} \end{cases}$$

Following are the updated mean value and variance:

$$\mu_{R,up} = \int_{-\infty}^{+\infty} x f_{R,up}(x)dx = \frac{\mu_R - \int_{-\infty}^{R_{test}} x f_R(x)dx}{1 - F_R(R_{test})} \tag{2.8}$$

$$V_{R,up} = \frac{V_R - (\mu_{R,up} - \mu_R)^2 - \int_{-\infty}^{R_{test}} (x - \mu_{R,up})^2 f_R(x)dx}{1 - F_R(R_{test})} \tag{2.9}$$

The results are presented in figure 2.2. The load may be a service load or an exceptional load.

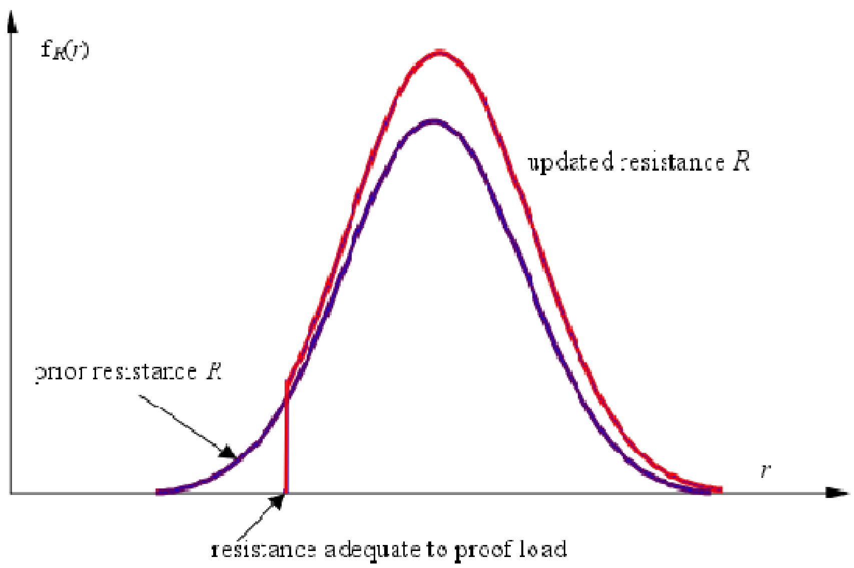

Figure 2.2: Probability density function of resistance prior to proof loading and after surviving the proof loading, Abbadi and Lamdouar [2018]

Based on target reliability index, β, the reliability index related to probability of failure during the proof load of proof load is expressed as

$$\beta = \frac{\mu_R - Q_{test}}{\sigma_R} \tag{2.10}$$

This paper suggests that the proof load can be fixed with accuracy (even though it is considered that some uncertainty will be present), its value was taken as deterministic in this case. After updating resistance, and preserving the same reliability level

$$\beta = \frac{\mu_{R,up} - Q_{test,up}}{\sigma_{R,up}} \tag{2.11}$$

From that equation and the updated proof level it is calculated as it follows

$$\frac{Q_{test,u}}{Q_{test}} = \frac{\mu_{R,up} - \beta \cdot \sigma_{R,up}}{\mu_R - \beta \cdot \sigma_R} = \frac{\mu_{R,up}}{\mu_R}\left(\frac{1 - \beta \cdot V_{R,up}}{1 - \beta \cdot V_R}\right) \tag{2.12}$$

In this study, a slab was used in experimental work. A mid-cross section of simply supported slab with the span of 12 m.
A sample of n=120 concrete compressive strength measurement, obtained during the construction, was used to assess the characteristic value of the concrete strength, f_{ck}.
In figure 2.3, the resume of the statistical results of concrete strength and reinforcement strength (yield) characteristics can be found. A sample of n=40 yield tensile measurements, which were obtained during the construction.

	Concrete	yield
Mean (MPa)	34.63	521.06
Standard deviation (MPa)	4.10	10.64
Coefficient of variation	0.12	0.02
Characteristic value (MPa)	27.88	503.92
Design value (MPa)	22.15	489.38
Updated Partial factor	1.26	1.03
Initial partial factor	1.50	1.15

Figure 2.3: Statistical results of concrete, Abbadi and Lamdouar [2018]

Now, if the same reliability level is maintained, the proof load can be updated as shown in the equation 2.13. In this case this means that the proof load level could be raised for around 23% within staying in the same safety margin.

$$\frac{\sigma_{test,up}}{\sigma_{test}} = 1{,}23 \tag{2.13}$$

From the test-based updating it was proven that the resistance of a bridge is greater than the proof load. This increased the bridge reliability and reduced the uncertainty in bridge resistance. The truncated method constituted a great approach. Combined with the update by control quality, it gives better results.
As a main results, partial factors of concrete and yield were decreased about 16% and 10%. The reliability index was greatly improved after update by both quality control and proof load.

2.3 Reliability analysis of a prestressed bridge beam

This paper will be based on Brazilian standards (NBR6118 and NBR7188).
Economic losses are one of the main factors that show how valuable reliability assessments are. As reliability analysis being one of the most used methods to estimate the safety of the structure.
The studied bridge has 33.5 meters of span, is simply supported, constituted by five precast concrete beams with U section (see figure 2.4).

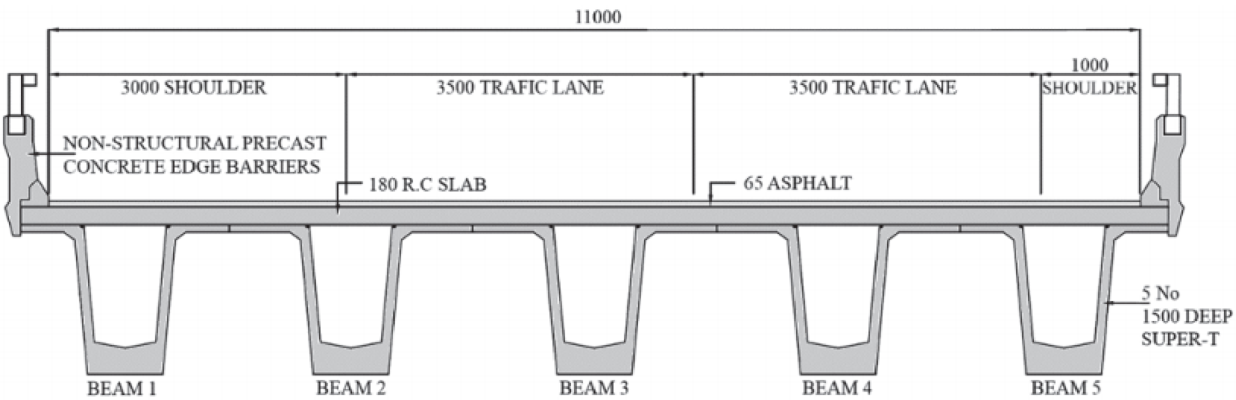

Figure 2.4: Bridge deck layout, cite

In the following figure (see figure 2.5), the methodology of this paper is described.

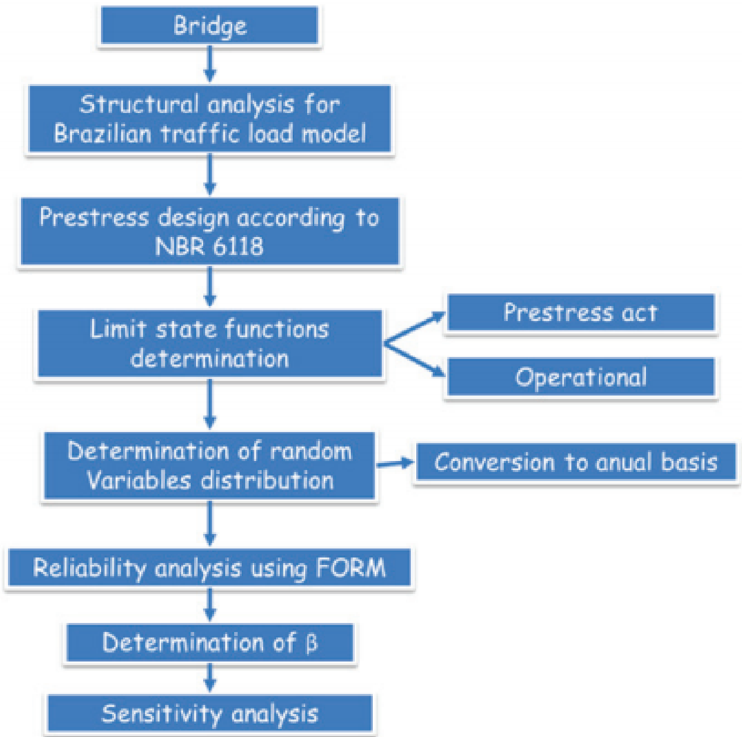

Figure 2.5: Methodology of work, cite

The reliability analysis was carried out using two methods for the four limit state equations:

- First Order Mean Value (FOMV)
- First Order Reliability Method (FORM)

Sensitivity analyzes were performed to consider both the relative contribution of these variables and the effect of their distributions on the annual reliability indexes for SLS.
It was verified that the effect of load trains and the allowable stress significantly reduce the reliability index obtained for Brazilian standard. The service limit state equations are particularly sensitive to

load trains, allowable stress and prestress losses, as well as their respective distributions.
There are four (4) limit state equations

- 2 for prestressing
- 2 for operation

The first equation is for the tensile stress at the upper fiber and the second equation is for the compressive stress at the lower fiber. Both equations are used during prestressing. For these two situations, the immediate losses of prestressing force are considered to have already occurred. The third and fourth equation are for the tensile stress at the lower fiber and the compressive stress at the upper fiber, respectively, in the operation situation. Considering the total losses of prestressing force for both equations.
In table 2.6, it can be seen the results of the annual reliability index using the First-Order Mean Value (FOMV) method, First-Order Reliability Method (FORM) and the probability of failure (p_f), for each limit state equation.

	$g_1(x)$	$g_2(x)$	$g_3(x)$	$g_4(x)$
β_{FOMV}	5.57	2.84	3.70	2.29
β by FORM	5.53	2.84	3.71	2.26
p_f by FORM	1.6 x 10-8	2.26 x 10-3	5.59 x 10-4	1.19 x 10-2

Figure 2.6: Annual serviceability reliability indices for the four limit state functions, cite

The difference between FOMV and FORM exists because the FOMV method consists of the first-order approximation around the mean point, and FORM consists of building a joint probability distribution function and its transformation into the standard normal space. Comparison of the annual reliability indexes can be made since the same geometric characteristics and materials of the bridge were used.
In table 2.7 it can be seen the comparison of the annual reliability indexes, obtained by FORM, for the Australian (AS), European (EN) and the Brazilian (NBR) standards.

	AS[1]	EN[1]	NBR
$g_1(x)$	4.81	5.55	5.53
$g_2(x)$	2.12	2.82	2.84
$g_3(x)$	3.36	3.74	3.72
$g_4(x)$	1.54	2.44	2.26
Number of strands	48	40	40

[1] Reliability indices from Caprani, Mayer and Siamphukdee

Figure 2.7: Annual serviceability reliability indices for the four limit state functions from Australian, Europe and Brazilian Standard, cite

The reliability index obtained by using equation $g_4(x)$ for the three standards did not reach the target reliability index of 2,9 provided by the European standard (see table 2.8).

Reference	β_{alvo}
EN1990:2002 – basis of structural design	2.9
JCSS 2000b (low relative cost of safety measure)	1.3
JCSS 2000b (moderate relative cost of safety measure)	1.7
JCSS 2000b (high relative cost of safety measure)	2.3

Figure 2.8: Literature annual target reliability indices for the different service limit state functions, cite

The reason for the results for 4 different limit state equations presented in tables 2.6 and 2.7 might be that these four limit state equations studied were assumed to be perfect; that is, the model error was assumed as unitary since there is no study assessing the model error for the serviceability limit state of prestressed bridge girders.
The analysis of reliability allows design standards to establish a minimum safety level, and analysts to assess the safety performance of structures designed according to these standards.

3 Reliability analysis

The following chapter is based on Sørensen et al. [2009] and Von Scholten et al. [2004].

The general requirements for the reliability of a structure shall ensure that:

- The structure has sufficient safety against failure in its lifetime
- The structure functions satisfactorily with normal use
- The structure has satisfactory durability and robustness

Limit states determine weather a structure (or a structural component) functions satisfactorily. Limit states determine at which point does the structure (or a structural component) start to fail.

Generally, there are 2 principal groups of limit state:

- Ultimate limit sate
- Serviceability limit state

Ultimate limit state represent more critical state and some examples of the failure of a structure or component are: buckling, tilting, folding and overturning. Serviceability limit state represent the failure in normal use and some examples of these kinds of failures are: deformations, crack formation and unacceptable oscillation.

A structure's reliability can be determined at two levels:

- System safety, in which all components of the structure and all forms of failure are taken into account.
- Safety at the component level, in which the safety of a component or, when a mechanism is considered, several components, with respect to a single form of failure is taken into account.

The safety requirement for the ultimate limit state depends on the type of failure that occurs.

- Failure with warning and with load-bearing capacity reserve, which includes ductile failure, for which a capacity reserve in addition to the defined capacity is required, for example in the form of deformation tempering.
- Failure with warning but without load-bearing capacity reserve, which includes ductile failure without extra load-bearing capacity.
- Failure without warning, which includes brittle failure and stability failure..

The safety index, β, is given as the reliability requirement:

$$\beta = -\Phi^{(-1)}(P_f) \tag{3.1}$$

Φ	Distribution function of the standardized normal distribution
P_f	Probability of the limit state under consideration being exceeded

The table 3.1 from Von Scholten et al. [2004] shows the required safety reliability index, β_t, and corresponding probability of failure, P_f. The reliability requirements correspond to a formal annual probability of failure. The required safety index is for the ultimate limit state with the high safety class because according to the Road Directorate's regulations, all road bridges shall be in the high safety class (CC3).

Failure type	Failure with warning and bearing capacity reserve	Failure with warning but without capacity reserve	Failure without warning
β_t	4,26	4,75	5,20
P_f	10^{-5}	10^{-6}	10^{-7}

Table 3.1: Required safety index for ultimate limit states (corresponds to annual values)

Target reliability index is introduced depending on the consequence class:

	CC2	CC3
β_t	4,8	5,2

Table 3.2: Target reliability index,β_t

3.1 Limit state equation

The following generic limit state equation, g, can be expresses as

$$g = zX_mR - [(1-\kappa)(G + X_g) + \kappa QX_q] \tag{3.2}$$

z	Design parameter (according to Sørensen et al. [2009])
X_m	Material model uncertainty
R	Material strength
κ *	1 – no unfavorable permanent load; 0 – no variable load
G	Dead load
X_g	Dead load model uncertainty
Q	Traffic load
X_q	Traffic load model uncertainty

*For concrete bridges with vehicle load, κ is typically in range [0,2-0,5].
*Load effect calculation model uncertainty is included in load variables.

3.1.1 Design parameter

The design parameter is determined based on load combinations 6.10a and 6.10b from EN 1990 [2002].

Load combination corresponding to EN 1990 [2002]: STR / GEO (6.10a).

$$z_A \frac{R_k}{\gamma_M} - ((1-\kappa)\gamma_{G,sup} G_k) \geq 0 \tag{3.3}$$

Load combination corresponding to EN 1990 [2002]: STR / GEO (6.10b).

$$z_B \frac{R_k}{\gamma_M} - ((1-\kappa)\gamma_{G,sup} G_k + \kappa\gamma_Q Q_k) \geq 0 \tag{3.4}$$

Design parameter, z, that will be used in limit state equation is determined as $z = max\{z_A, z_B\}$.
NOTE: Characteristic value of load bearing capacity, R_k, is determined as the 5% fraction of the total bearing capacity, $X_M R$.

To determine the design parameter, the maximum annual vehicle load, P, need to be determined. Distribution function for maximum annual load is given in Sørensen et al. [2009] as

$$F_P(x) = exp(-[1 - F_W(x)]N) \tag{3.5}$$

$F_W(x)$	Distribution function for individual loads from a single vehicle / axle number
N	Number of vehicle passages per year

If distribution function for individual loads from a single vehicle / axle number is considered to be normally distributed (this can be used for standard vehicles according to Sørensen et al. [2009])

$$F_W(x) = \Phi\left(\frac{x - \mu_W}{\sigma_W}\right) \tag{3.6}$$

then realization, P, of $F_p(x)$ (eq. 3.5) is

$$P = \mu_W + \sigma_W \Phi^{-1}\left(1 - \frac{1}{N} ln(\Phi(u))\right) \tag{3.7}$$

μ_W	Mean weight of the vehicles
σ_W	Standard deviation of the vehicles
u	Standard normal stochastic variable associated with the annual maximum traffic load

As described in Sørensen et al. [2009], characteristic value of maximum annual load represents 98% quantile for standard vehicles (for normal vehicles, Eurocodes assume a 1000 years return period value as characteristic value) and this value will be used to calculate the design parameter, z.

Subsequently, maximum annual load, Q, is determined as

$$Q = e_Q X_Q (1 + \rho) P \tag{3.8}$$

e_Q	Influence coefficient
X_Q	Model uncertainty
ρ	Dynamic factor

From any chosen deterministic valued κ=[0,2-0,5], dead load, G, will be determined accordingly.

$$\kappa = \frac{Q}{G+Q} \tag{3.9}$$

Partial coefficients for permanent load and variable load are given in Table 2. in Sørensen et al. [2009]. CC3 (high consequence class) is the recommended class from Von Scholten et al. [2004] (K_{FI} = 1,1), but because of the computation time, CC2 will be considered.

In table 3.3 parameters used to estimate design parameters when **concrete compression strength** is considered as **dominant material strength** and table 3.5 highlights the parameters used to estimate design parameters when **reinforcement** represents **dominant material strength**.

Parameter	Value
$f_{ck}[MPa]$	30
γ_M	1,45
$\gamma_{G,a}$	1,2K_{fi}
$\gamma_{G,b}$	1,0K_{fi}
γ_Q	1,4K_{fi}
κ	[0,2-0,5]
$G_k[N]$	1.5575e+06
$Q_k[N]$	1.5575e+06

Table 3.3: Parameters for determining design parameters, concrete compressive strength as dominant material strength

NOTE: G_k and Q_k in this table are expressed in [N] as stated in the table. Since single vehicle load used in eq. 3.6 was given in tons [t] as it can be seen in figure 1.3, final value of characteristic dead and traffic load was converted from tons to Newtons. The same has been done in table 3.5.

Factor K_{FI} depend on the consequence class.

	CC1	CC2	CC3
K_{FI}	0,9	1,0	1,1

Table 3.4: Factor K_{FI}

Parameter	Value
$f_y[MPa]$	275
γ_M	1,2
$\gamma_{G,a}$	$1{,}2K_{fi}$
$\gamma_{G,b}$	$1{,}0K_{fi}$
γ_Q	$1{,}4K_{fi}$
κ	[0,2-0,5]
$G_k[N]$	1.5575e+06
$Q_k[N]$	1.5575e+06

Table 3.5: Parameters for determining design parameters, reinforcement as dominant material strength

Table below (3.6) shows the design parameters obtained for both aforementioned cases.

Design parameter $[mm^2]$				
	$f_{ck} = dominant$		$f_y = dominant$	
κ	CC2	CC3	CC2	CC3
0.2	90 390	99 430	6 740	7 415
0.3	93 730	103 120	6 989	7 690
0.4	97 080	106 800	7 239	7 941
0.5	100 430	110 480	7 489	8 239

Table 3.6: Design parameters

3.1.2 Material model uncertainty, X_m

Material model uncertainty is Lognormal distributed and is modelled based on Sørensen et al. [2009]. Mean value, μ_m, is given as 1,0 and coefficient of variation, V_{I_m}, for material parameter is given as 0,11 when concrete compressive strength is the dominant material strength and 0,05 when reinforcement is.

3.1.3 Material strength, R

For the characteristic strength of the concrete, the mean value of the compressive strength $E[f_c]$ and the variation coefficient V_{f_c}, as given in table 6.1 in Von Scholten et al. [2004].
Based on variation coefficient and mean value, standard deviation was obtained and material strength, R, was modelled as stochastic variable with Lognormal distribution.

3.1.4 Dead load, G

Mean value of dead load is equal to characteristic value. Coefficient of variation is given as 10%. After determining the standard deviation of the dead load, it is described as Normal distributed stochastic variable.

3.1.5 Dead load uncertainty, X_g

Dead load uncertainty is Normal distributed variable with mean values 0 and a standard deviation of 5% of the mean value of the permanent load (characteristic value of G) (Von Scholten et al. [2004]).

3.1.6 Traffic load, Q

Traffic load is a stochastic variable calculated from 3.8 where dynamic factor, ρ, and traffic load uncertainty, X_Q, are modelled as stochastic variables as described in Sørensen et al. [2009].

3.1.7 Traffic load uncertainty, X_q

Following the recommendations from EN 1990 [2002], traffic load uncertainty is described with mean value equal to 1 and coefficient of variation equal to 0,1.

3.2 Crude Monte Carlo simulation

In this project work, Monte Carlo simulation was performed to obtain probability of failure and reliability index based on probability of failure.
The inputs in Monte Carlo simulation are random variables with known probability distributions. The model is represented by the mathematical function, limit state equation = g.

Monte Carlo simulation is often used to model exceedingly complicated systems such as those governed by a large number of (sometimes indeterminable) variables, or when analytical solutions simply cannot be obtained. In the case of future development in the system, simulation methods are more appropriate since future developments may be more tractable.

Probability of failure, $\hat{P}_f$, is estimated as

$$\hat{P}_f \approx \frac{1}{N} \sum_{i=1}^{N} I[g(\hat{u}_j)] \tag{3.10}$$

N	Number of simulations
$\hat{u}_j$	Sample no. j of a standard normally distributed stochastic vector U

The indicator function, $I[g(\hat{u}_j)]$, is defined as

$$I[g(\hat{u}_j)] = \begin{cases} = 1\, if & g(\hat{u}_j) \leq 0 \quad failure \\ = 0\, if & g(\hat{u}_j) > 0 \quad safe \end{cases}$$

The standard error is estimated as

$$s = \sqrt{\frac{\hat{P}_f(1 - \hat{P}_f)}{N}} \tag{3.11}$$

The number of simulations, n_{sim}, is described as a vector that is generated for each variable. The limit state function is evaluated by elementwise operations. Every time the limit state equation has a result less than 0, it counts it as a failure. So, probability of failure is the ration between number of failures and number of simulations. Thus, for a greater number of simulations, the greater the accuracy.

$$P_f = \frac{n_{fail}}{n_{sim}} \tag{3.12}$$

Each time the simulation results in failure, u-values are extracted. The u represents a matrix with n_{sim} number of rows and a number of columns corresponding to the number of independent u-variables. Then, the variables are transformed from the u-space to the specific distributions.
The design point $u*$ is estimated based on mean value of each extracted u-values. U-value was extracted for each variable respectively. It is how the design point is approximated. Design point is not on failure surface, but within the failure domain. The α-vector can be estimated as normalization of the average design point (for each u-value respectively), and sensitivity measures can be estimated based on this.

$$\alpha - vector = \frac{u*}{norm(u*)} \tag{3.13}$$

The components of the α-vector are representation of sensitivity factors giving the relative importance of the individual random variables (for the reliability index β).

3.3 Results

3.3.1 Standard vehicle class 100

Both Matlab and Comrel were used to perform a reliability analysis. Table 3.8 present the results for 1 million of simulation in Comrel, that is the maximum limit of simulations, and 17 million simulations in Matlab (in 10 blocks; meaning, 170 million of simulations all together).
The results presented in this section are referring to standard vehicle class 100.

Case 1: Concrete compressive strength as dominant material strength

Variable	Comment	Distribution	μ	σ
ρ [N]	Dynamic factor	Normal	41500/W	41500/W
W [N]	Individual loads from a single vehicle	Normal	1072344	49100
G [N]		Normal	1.5574e+6	155820
X_g	Dead load uncertainty	Normal	0	77877
R [MPa]	Material resistance stochastic variable	Lognormal	36,2	5,068
X_q	Traffic load uncertainty	Normal	1	0,1
X_m	Material model uncertainty	Lognormal	1	0,11
U	Standard normal stochastic variable	Normal	0	1

Table 3.7: Variables used in Comrel to perform reliability analysis

Table 3.7 describes variables used to execute the reliability analysis using Comrel. In table 3.8 the corresponding reliability index, β, and probability of failure, P_f, are expressed.
First, CC2 consequence class was considered. in Matlab, the limit state equation was run 17 million times in 10 blocks and the results are recorded in table 3.8. It can be seen also that target reliability index for class CC2 (β_t=4,8) has not been reached unless for κ=0,5, but only for Monte Carlo simulation carried out in Comrel.

	Matlab		Comrel			
	MC		MC		FORM	
κ	P_f	β	P_f	β	P_f	β
0,2	$5{,}4 \cdot 10^{-5}$	3,9	$5{,}4 \cdot 10^{-5}$	3,9	$5{,}3 \cdot 10^{-5}$	3,9
0,3	$1{,}5 \cdot 10^{-5}$	4,2	$1{,}4 \cdot 10^{-5}$	4,2	$1{,}4 \cdot 10^{-5}$	4,2
0,4	$5{,}0 \cdot 10^{-6}$	4,4	$5{,}0 \cdot 10^{-6}$	4,4	$4{,}0 \cdot 10^{-6}$	4,5
0,5	$2{,}0 \cdot 10^{-6}$	4,7	$1{,}0 \cdot 10^{-6}$	4,8	$1{,}5 \cdot 10^{-6}$	4,7

Table 3.8: Comparison of reliability index and probability of failure using Matlab and Comrel for CC2 (f_{ck}=dominant strength material)

Then, CC3 consequence class was considered and it was run in 25 million of simulations in 10 blocks. Results are recorded in table below.

	Matlab		Comrel			
	MC		MC		FORM	
κ	P_f	β	P_f	β	P_f	β
0,2	$6{,}1 \cdot 10^{-6}$	4,4	$4{,}0 \cdot 10^{-6}$	4,5	$6{,}4 \cdot 10^{-6}$	4,4
0,3	$1{,}2 \cdot 10^{-6}$	4,7	$2{,}0 \cdot 10^{-6}$	4,6	$1{,}4 \cdot 10^{-6}$	4,7
0,4	$3{,}0 \cdot 10^{-7}$	5,0	$1{,}0 \cdot 10^{-6}$	5,0	$3{,}8 \cdot 10^{-7}$	5,0
0,5	$1{,}0 \cdot 10^{-7}$	5,2	$1{,}0 \cdot 10^{-6}$	5,2	$1{,}3 \cdot 10^{-7}$	5,2

Table 3.9: Comparison of reliability index and probability of failure using Matlab and Comrel for CC3 (f_{ck}=dominant strength material)

*NOTE: MC= Monte Carlo and FORM= First Order Reliability Method.

As seen in table 3.8, reliability indices estimated by performing Monte Carlo simulation in both Matlab and Comrel are quite similar and follow the same trend of only increasing with increasing the value of assigned κ and design value, z. The comparison was made between two classes to see which one is more suitable. CC2 class demands less simulations and therefore less computation work. The estimated values for consequence class CC2 are more accurate as more failures are recorded for less simulations. The reliability indices are higher for CC3 class as that corresponds to higher $K_{fi} = 1{,}1$ than for CC2 class when $K_{fi} = 1{,}0$. Target reliability index for class CC3 (β_t=5,2) has been reached for κ=0,5.

As the scope of a sensitivity analysis is to ensure assurance in the results of the reliability analysis and also to authenticate that the results obtained are adequately robust, the reliability analysis always include sensitivity analysis. α-vector is a sensitivity measure that indicates which parameters are the most influential.
In Monte Carlo simulation in Matlab, α-vector is estimated by first extracting all the failures from u-vector (for each variable and alpha-value respectively). Then design point (point on failure surface), u* is estimated. By normalizing u* vector, α-vector is estimated and used to indicate the influence of each variable for different alpha-values.
In table 3.10 comparison between α-vectors obtained in Comrel using FORM method and Matlab by performing Monte Carlo simulation will be presented.

α-vector																
	P_s		W		φ		G		R		X_g		X_q		X_m	
κ	Ml	C	Ml	C	Ml	C	Ml	C	Ml	C	Ml	C	Ml	C	Ml	C
0,2	0.011		0.004		0.011	0.0	0.370	0.44	-0.706	-0.71	0.194	0.22	0.145	0.30	-0.553	-0.67
0,3	0,006		0.002		0.027	0,0	0.309	0,32	-0.716	-0,71	0.162	0.16	0.218	0.23	-0.563	-0.56
0,4	0.007		0.007		0.007	0,0	0.275	0.28	-0.721	-0,70	0.154	0.14	0.276	0.32	-0.552	-0,55
0,5	0.017		0.036		0.027	0,0	0.231	0.22	-0.695	-0.69	0.125	0.11	0.342	0.41	-0.572	-0.54

Table 3.10: Comparison between different α-vector values, for CC2

*NOTE: Ml= Matlab and C=Comrel.

If α-value is positive that means the associated random variable is capacitive type. In other words, the reliability increases if the mean of the random variable is increased. If α is negative, the variable is of resistance type. Consequently, reliability decreases if mean of random variable is increased which can be seen form the elasticity coefficients (3.3).

Here, in table 3.10, it is seen that the biggest influence on reliability has material resistance variable, R. It is followed by the influence of material mode uncertainty, X_m.
It is noticed that influence of dead load , G, on reliability is decreased as the assigned κ is increased, as expected. On the contrary, traffic load uncertainty, X_q increases with assigned κ.

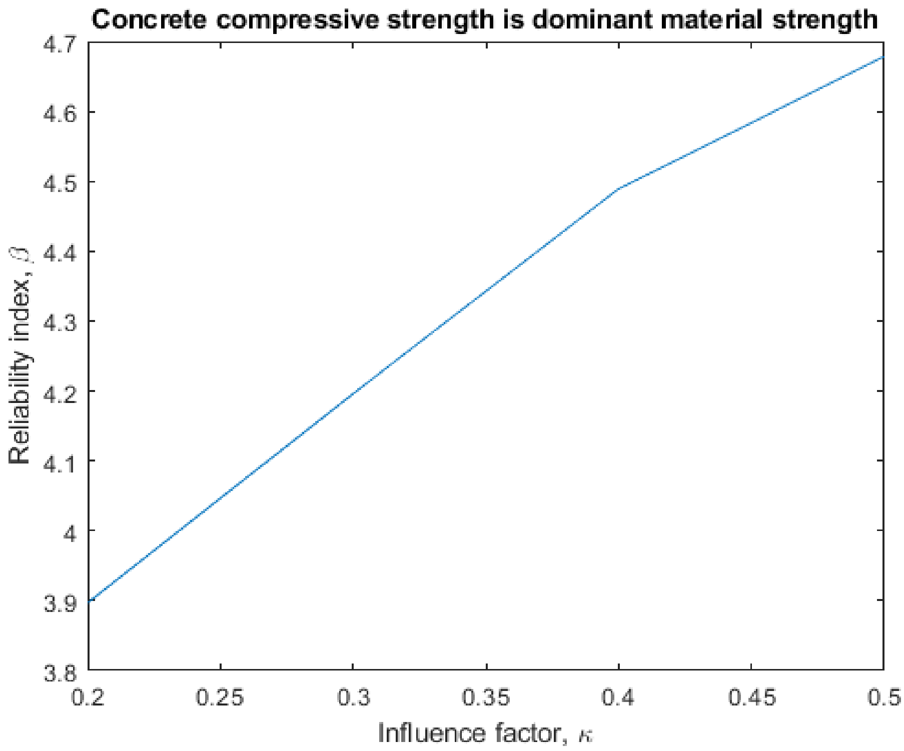

Figure 3.1: Concrete compressive strength as dominant material strength

Below is the figure 3.6 graphically representing the α-values when κ=0,2. For other value, see Appendix B.

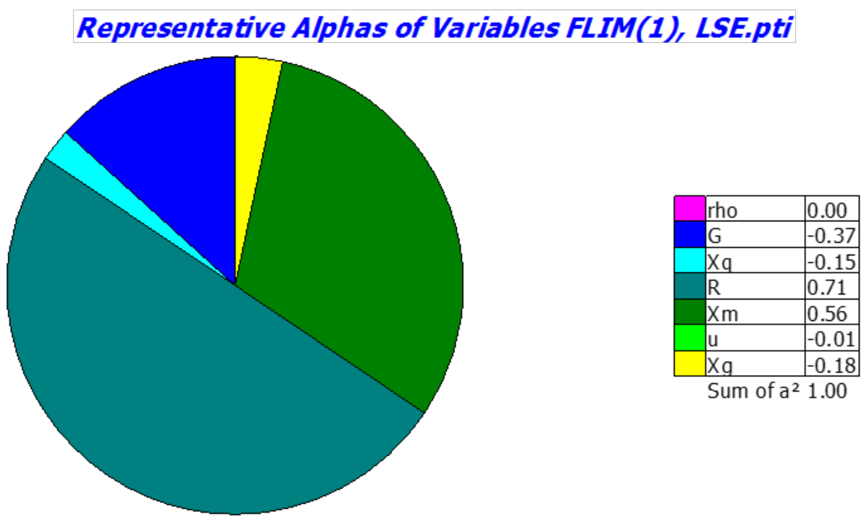

Figure 3.2: Comrel, α- vector value for κ=0,2

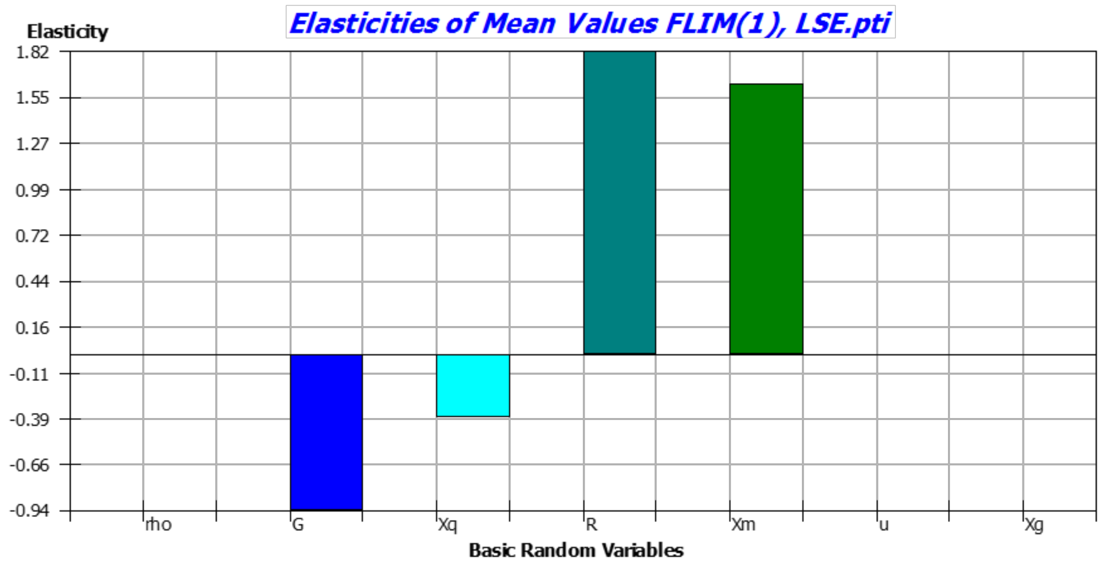

Figure 3.3: Comrel, elasticities of mean values for κ=0,2

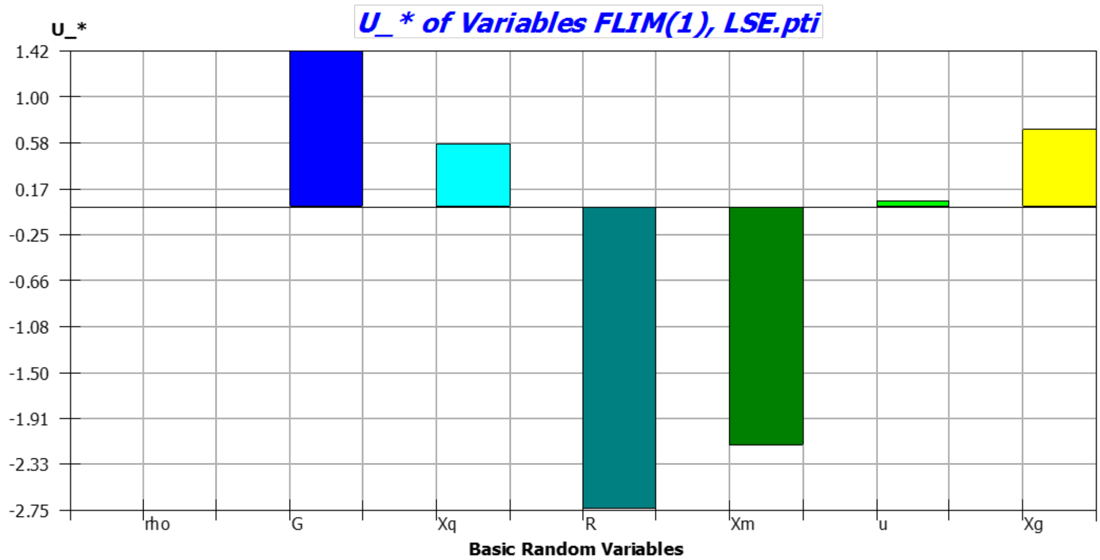

Figure 3.4: Comrel, $u*$- vector value for κ=0,2

Case 2:Reinforcement as dominant material strength

Variable	Comment	Distribution	μ	σ
ρ [N]	Dynamic factor	Normal	41500/W	41500/W
W [N]	Individual loads from a single vehicle	Normal	1072344	49100
G [N]	Dead load	Normal	1.5574e+6	155820
X_g	Dead load uncertainty	Normal	0	77877
R [MPa]	Material resistance	Lognormal	345	25
X_q	Traffic load uncertainty	Normal	1	0,15
X_m	Material model uncertainty	Lognormal	1	0,1039
U	Standard normal stochastic variable	Normal	0	1

Table 3.11: Variables used in Comrel to perform reliability analysis

Another case was taken into account, if the reinforcement was the dominant material strength.
Table 3.11 describes variables used to execute the reliability analysis using Comrel. In table 3.13 the corresponding reliability index, β, and probability of failure, P_f, are expressed. Limit state equation was simulated 17 million times in 10 blocks for CC2 consequence class and 17 million in 10 blocks for CC3 consequence class.

	Matlab		Comrel			
	MC		MC		FORM	
κ	P_f	β	P_f	β	P_f	β
0,2	$1{,}7 \cdot 10^{-4}$	3,6	$1{,}44 \cdot 10^{-4}$	3,6	$1{,}6 \cdot 10^{-4}$	3,6
0,3	$2{,}1 \cdot 10^{-5}$	4,1	$1{,}5 \cdot 10^{-5}$	4,2	$1{,}9 \cdot 10^{-5}$	4,1
0,4	$4{,}1 \cdot 10^{-6}$	4,5	$1{,}8 \cdot 10^{-6}$	4,5	$3{,}1 \cdot 10^{-6}$	4,5
0,5	$1{,}3 \cdot 10^{-6}$	4,7	$1{,}0 \cdot 10^{-6}$	4,8	$9{,}1 \cdot 10^{-7}$	4,8

Table 3.12: Comparison of reliability index and probability of failure using Matlab and Comrel CC2 (f_y=dominant material strength)

	Matlab		Comrel			
	MC		MC		FORM	
κ	P_f	β	P_f	β	P_f	β
0,2	$5{,}1 \cdot 10^{-6}$	4,4	$5{,}0 \cdot 10^{-6}$	4,4	$5{,}0 \cdot 10^{-6}$	4,4
0,3	$1{,}2 \cdot 10^{-6}$	4,9	$4{,}8 \cdot 10^{-6}$	4,8	$3{,}9 \cdot 10^{-7}$	4,9
0,4	$5{,}1 \cdot 10^{-7}$	5,2	/	/	$5{,}2 \cdot 10^{-8}$	5,3
0,5	$1{,}3 \cdot 10^{-7}$	5,3	/	/	$1{,}6 \cdot 10^{-8}$	5,4

Table 3.13: Comparison of reliability index and probability of failure using Matlab and Comrel CC2 (f_y=dominant material strength)

*NOTE: MC= Monte Carlo and FORM= First Order Reliability Method.
*NOTE: "/" as a result in table means that the result of the analysis was inconclusive; therefore, will not be taken into account.

Again, it can be seen in the table 3.13 that the discrepancy between the reliability index, β, and corresponding probability of failure, P_f, is very low. This proposition is further validated, as the results obtained when using independent software suites yield similar results.
For higher κ, the reliability index increases as expected, meaning the probability of failure is lower.
As mentioned before, CC3 will need significantly higher number of simulations and that might be limited by personal computer used to carry out the analysis. Therefore, it is why estimated reliability indices and probability of failures yield more accurate results for CC2

α-values are compared between both software suites, and it is concluded that both yield similar results and indicated that the resistance had the biggest influence like for case 1. The uncertainty of material strength and dead load are second and third most influential variables in this reliability analysis when reinforcement is the dominant material strength.

α-vector																
	P_s		W		φ		G		R		X_g		X_q		X_m	
κ	Ml	C	Ml	C	Ml	C	Ml	C	Ml	C	Ml	C	Ml	C	Ml	C
0,2	0.02		0.00		0.00	0,0	0.57	0,57	-0.59	-0,59	0.29	0,29	0.24	0,24	-0.42	-0,42
0,3	0.04		0,00		0,00	0,0	0.50	0,50	-0.59	-0,60	0.25	0,25	0.39	0,38	-0.43	-0,43
0,4	0.06		0.00		0.01	0,0	0.42	0,41	-0.59	-0,58	0.20	0,21	0.52	0,52	-0.41	-0,42
0,5	0.07		0.02		0.02	0,0	0.33	0,32	-0.57	-0,55	0.15	0,16	0.63	0,64	-0.38	-0,40

Table 3.14: Comparison between different α-vector values, CC2

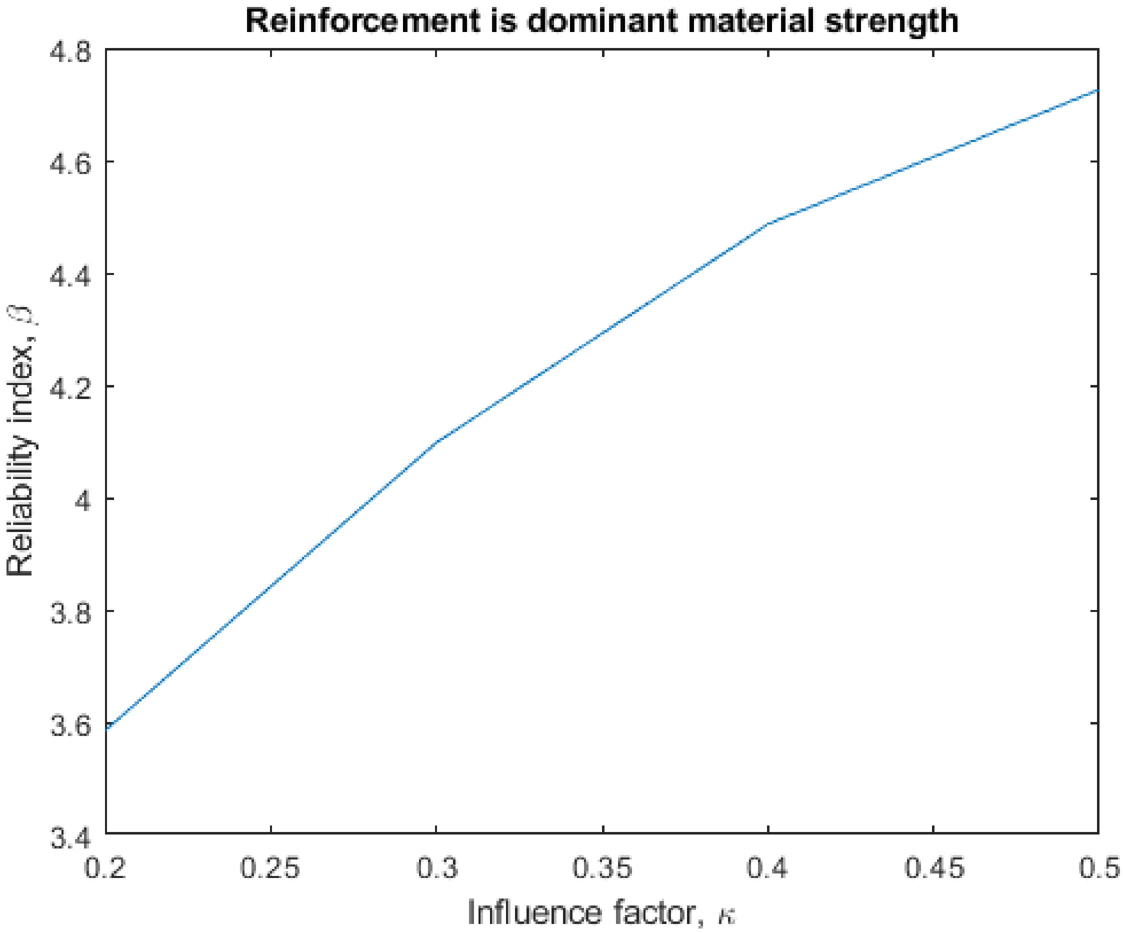

Figure 3.5: Reinforcement as dominant material strength, CC2

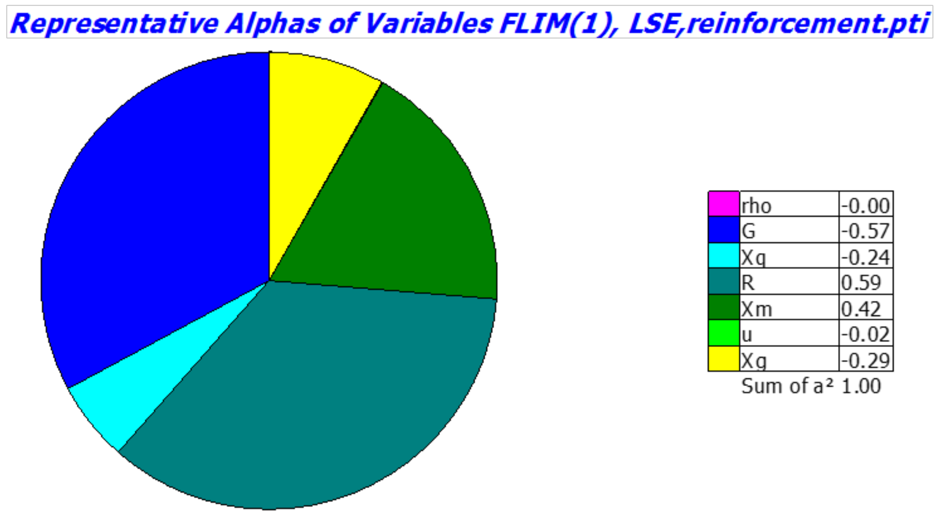

Figure 3.6: Comrel, α- vector value for κ=0,2

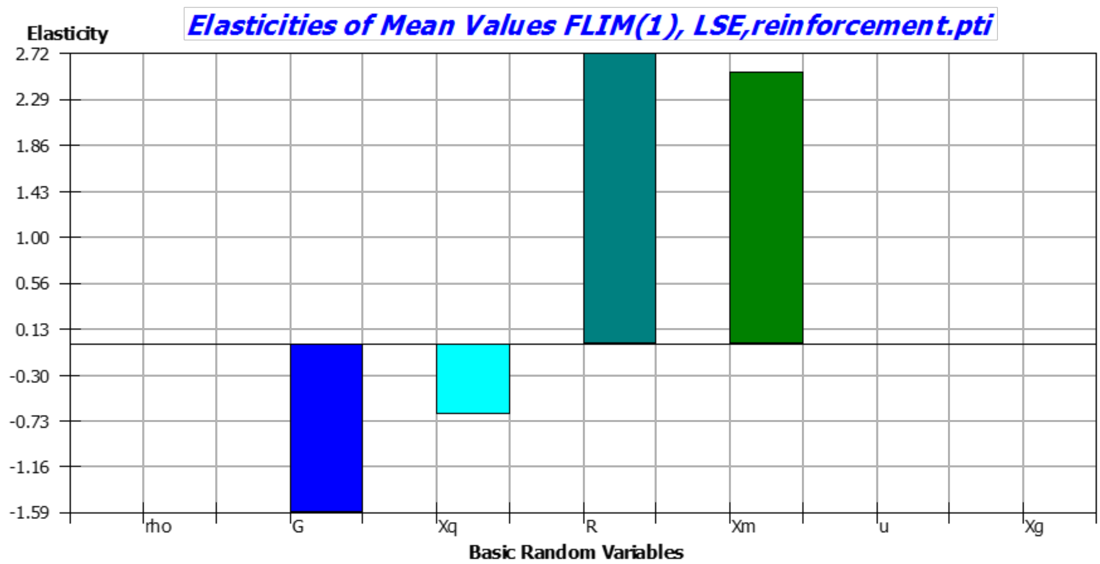

Figure 3.7: Comrel, elasticities of mean values for κ=0,2

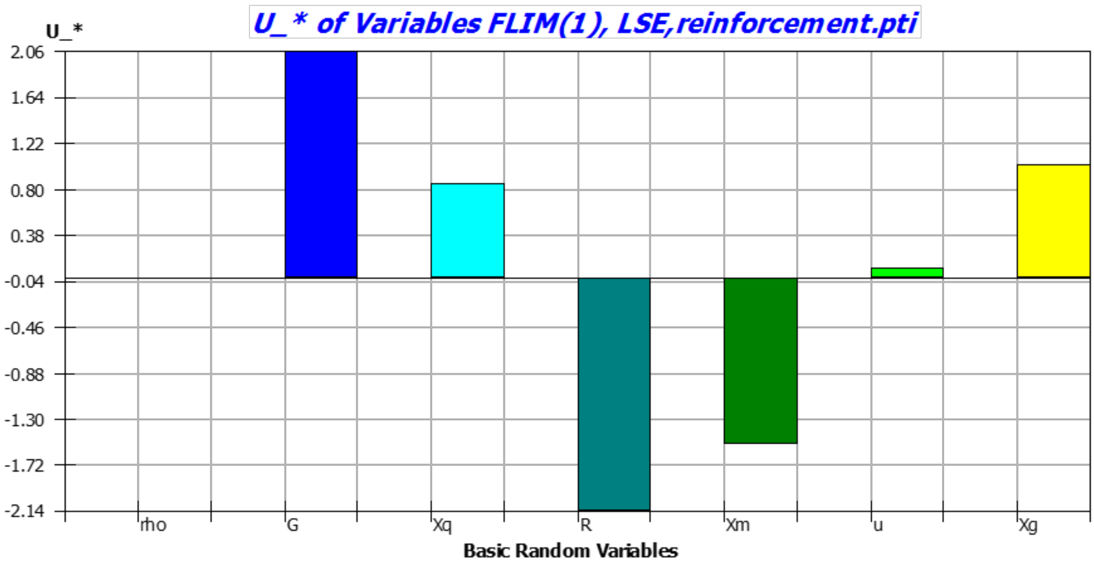

Figure 3.8: Comrel, $u*$- vector value for κ=0,2

3.3.2 Standard vehicle class 125

Additionally, the results for standard vehicles of class 125 will be presented. As previously stated, estimates from both Matlab and Comrel will be presented int his chapter. In Matlab, for consequence class CC2, 10 million simulations in 10 blocks were performed and for CC3 class, 17 million simulations in 10 blocks.

Case 1: Concrete compressive strength as dominant material strength

Variable	Comment	Distribution	μ	σ
ρ [N]	Dynamic factor	Normal	41500/W	41500/W
W [N]	Individual loads from a single vehicle	Normal	1290348	49100
G [N]		Normal	1.8185e+6	181850
X_g	Dead load uncertainty	Normal	0	90927
R [MPa]	Material resistance stochastic variable	Lognormal	36,2	5,068
X_q	Traffic load uncertainty	Normal	1	0,1
X_m	Material model uncertainty	Lognormal	1	0,11
U	Standard normal stochastic variable	Normal	0	1

Table 3.15: Variables used in Comrel to perform reliability analysis for vehicle class 125, CC2

Table 3.15 describes the parameters used to carry out the reliability analysis in Comrel.

	Matlab		Comrel			
	MC		MC		FORM	
κ	P_f	β	P_f	β	P_f	β
0,2	$8{,}9 \cdot 10^{-5}$	3,8	$5{,}5 \cdot 10^{-5}$	3,9	$5{,}3 \cdot 10^{-5}$	3,9
0,3	$2{,}4 \cdot 10^{-5}$	4,1	$1{,}5 \cdot 10^{-5}$	4,1	$1{,}4 \cdot 10^{-5}$	4,2
0,4	$7{,}3 \cdot 10^{-6}$	4,4	$6{,}0 \cdot 10^{-6}$	4,4	$4{,}1 \cdot 10^{-6}$	4,6
0,5	$2{,}7 \cdot 10^{-6}$	4,6	$1{,}0 \cdot 10^{-6}$	4,8	$1{,}5 \cdot 10^{-6}$	4,7

Table 3.16: Comparison of reliability index and probability of failure using Matlab and Comrel for CC2, vehicle class 125, (f_{ck}=dominant strength material)

The results of the analysis between two softwares are similar especially for smaller values of κ. Based on these results, it would mean that only for κ=0,5 would the target reliability index be reached, but performing FORM analysis in Comrel and Monte Carlo simulations in Matlab, it is seen how results differ; therefore, it is not possible with certainty to conclude that the target reliability index (β_t=4,8) is reached.

Case 2: Reinforcement as dominant material strength

Variable	Comment	Distribution	μ	σ
ρ [N]	Dynamic factor	Normal	41500/W	41500/W
W [N]	Individual loads from a single vehicle	Normal	1290348	49100
G [N]		Normal	1.8185e+6	181850
X_g	Dead load uncertainty	Normal	0	90927
R [MPa]	Material resistance stochastic variable	Lognormal	345	24,15
X_q	Traffic load uncertainty	Normal	1	0,1
X_m	Material model uncertainty	Lognormal	1	0,05
U	Standard normal stochastic variable	Normal	0	1

Table 3.17: Variables used in Comrel to perform reliability analysis for vehicle class 125, CC2

Table 3.17 describes variables used in Comrel to carry out the reliability analysis.

	Matlab		Comrel			
	MC		MC		FORM	
κ	P_f	β	P_f	β	P_f	β
0,2	$1{,}6 \cdot 10^{-4}$	3,6	$1{,}5 \cdot 10^{-4}$	3,6	$1{,}6 \cdot 10^{-4}$	3,6
0,3	$2{,}0 \cdot 10^{-5}$	4,1	$2{,}1 \cdot 10^{-5}$	4,1	$1{,}9 \cdot 10^{-5}$	4,1
0,4	$3{,}3 \cdot 10^{-6}$	4,5	$5{,}0 \cdot 10^{-6}$	4,4	$3{,}2 \cdot 10^{-6}$	4,5
0,5	$1{,}1 \cdot 10^{-6}$	4,8	$3{,}0 \cdot 10^{-6}$	4,8	$9{,}6 \cdot 10^{-7}$	4,8

Table 3.18: Comparison of reliability index and probability of failure using Matlab and Comrel for CC2, vehicle class 125, (f_y=dominant strength material)

Results presented in table 3.18 are for consequence class CC2; therefore, the target reliability index β_t=4,8 has been reached when κ=0,5. Estimates between the softwares and different methods are quite similar, as expected.

Reliability updating 4

The two common reasons to perform a reliability analysis on an existing structure/bridge are

- to confirm the existing load rating (meaning, the bridge had shown the signs deterioration)
- to possibly increase the load rating

The conservative nature of the deterministic approach will sometimes lead the actual load carrying capacity of a bridge being considerably larger than the predicted capacity.
Therefore, a diagnostic test may be used to verify or refine analytical or predictive structural models. Also, a proof load test is used to assess the actual load carrying capacity of a bridge. A successful proof load test demonstrates immediately that the resistance of the bridge is greater than the proof load. It should be recognised also that there is a risk that the bridge will be damaged or not survive a proof load test and so proof load testing may not always be cost-effective (refer to ; M.H. Faber et al. / Engineering Structures 22 (2000) 1677–1689).

Having said that, this master thesis will focus on a reliability-based method to determine the target proof load where reliability is used as the measure of structural performance.

Two main types of updating of the probability of failure estimates are in general considered in bridge management systems:

- Updating of stochastic variables based on measured samples of the stochastic variables, e.g. measurements of the yield strength of the reinforcement.

- Updating based on general information, e.g. the observation that the structure has not failed or that a corrosion degree less than a certain value is measured.

(refer to CHAPTER 78; BRIDGE RELIABILITY IN DENMARK)

4.1 Estimating and updating structural reliability

In the observation of a load variable the distribution parameters for the variable such as mean value and standard deviation can be estimated and updated. The distribution parameters will be estimated on the basis of Bayesian statistics. Using Bayesian statistics it is likewise possible to quantify the uncertainty of the estimated distribution parameters. It is possible to update the reliability on the basis of a given event. An event in question will be modelled on the basis of a limit state function.

Using the Bayesian theorem, it is possible to update the structural reliability directly.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{4.1}$$

A	Local or global structural failure
B	Information gathered by proof loading
$\cap$	Intersection of two events
$\vert$	Conditional upon

If g1(x) is an initial limit state equation and x is the vector of basic variables, then A is the failure of initial limit state equation (g1(x)<0). B on the other hand is failure of an updated limit state equation (g2(x)<0). Now it is rewritten as

$$P(g1(x) < 0|g2(x) < 0) = \frac{P(g1(x) < 0 \cap g2(x) < 0)}{P(g2(x) < 0)} \tag{4.2}$$

4.2 Updating on proof load test

The traffic load and deterioration cause by environmental effects and structure itself, raise concern regarding bridge reliability. For that reason, proof load tests are conducted on existing bridges to inspect the current state of one. Performing such test, it is possible to instantly observe if the resistance of the bridge is greater then the proof load. Downside of performing a physical testing is that it is highly expensive and can cost up to 6% of the bridge replacement, Faber et al. [2000]. Also, if the test is unsuccessfully it can damage the bridge.
The intensities of proof loads considered in this paper are calibrated based on a target reliability index. Both CC2 and CC3 consequence class will be mentioned and discussed. Reliability updating for proof load test will be done in Matlab and vehicle class 100 will be considered in further analysis.

The corresponding generic limit state equation is:

$$g1 = zX_mR - [(1-\alpha)(G + X_g) + \alpha Q X_q] \tag{4.3}$$

and conditional equation is:

$$g2 = zX_mR - [(1-\alpha)(G + X_g) + \alpha Pl] \tag{4.4}$$

where Pl the load used to simulate the proof load.

Now, Bayesian theorem will be applied for this case:

$$P(g1(x) < 0|g2(x) > 0) = \frac{P(g1(x) < 0 \cap g2(x) > 0)}{P(g2(x) > 0)} \tag{4.5}$$

Two cases will be examined as before. First when concrete compressive strength is the dominant material strength and then when reinforcement is the dominant material strength.

4.2.1 Standard vehicle class 100

Case 1: Concrete compressive strength as dominant material strength

In the case when concrete compressive strength is the dominant material strength, the proof load applied to this bridge of interest was considered in range of 50% to 140% of characteristic value of

maximum annual traffic load for vehicle class 100, P_c.
The results of this analysis for CC2 consequence class are shown in table 4.1.

Pl [N] \ κ	0,2		0,3		0,4		0,5	
	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$
50% P_c	3,9	4,1 e^{-5}	4,2	1,3e^{-5}	4,5	4,2e^{-6}	4,7	1,4e^{-6}
60% P_c	4,0	3,6 e^{-5}	4,2	1,3e^{-5}	4,5	4,1e^{-6}	4,7	1,4e^{-6}
70% P_c	4,0	3,1 e^{-5}	4,2	1,2e^{-5}	4,5	4,0e^{-6}	4,7	1,4e^{-6}
80% P_c	4,1	2,4e^{-5}	4,3	1,1e^{-5}	4,5	3,6e^{-6}	4,7	1,4e^{-6}
90% P_c	4,1	1,8e^{-5}	4,3	8,1e^{-6}	4,5	3,3e^{-6}	4,7	1,3e^{-6}
100% P_c	4,2	1,5e^{-5}	4,4	6,2e^{-6}	4,5	2,9e^{-6}	4,7	1,2e^{-6}
110% P_c	4,4	5,7e^{-6}	4,4	4,4e^{-6}	4,6	2,4e^{-6}	4,8	9,0e^{-7}
120% P_c	4,5	3,2e^{-6}	4,6	2,5e^{-6}	4,7	1,7e^{-6}	4,8	7,0e^{-7}
130% P_c	4,7	1,4e^{-6}	4,7	1,1e^{-6}	4,7	1,1e^{-6}	4,9	5,0e^{-7}
140% P_c	4,9	4,0e^{-6}	4,9	4,5e^{-7}	4,9	5,5 e^{-7}	4,9	4,0 e^{-7}

Table 4.1: CC2, Reliability updating for standard vehicle class 100

In the figure 4.1, four lines represent four values of κ as indicated by legend. The graph in figure explains that for the greater the applied load the greater the reliability index. Finally, at 140% the reliability index succeeds the target reliability index (see table 4.1), which is 4,8, and it deems the bridge a safe. The analysis was done in 20 million of simulations ran in Matlab.

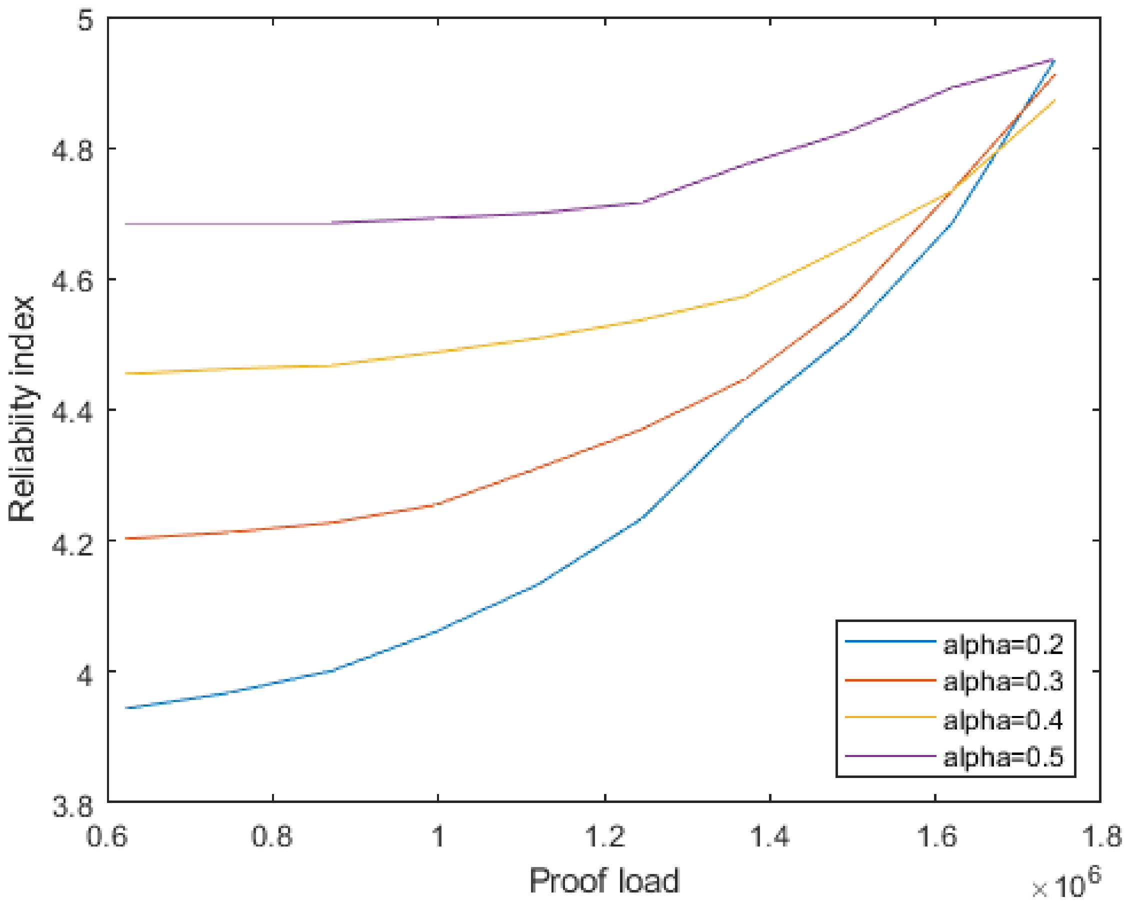

Figure 4.1: Proof loading updating when concrete compressive strength is the dominant material strength, vehicle class 100

In figure 4.1, it is seen that all reliability indices, what can be observed converge, to the same reliability level. Now, this might not be the real representation and might be the cause of the number of simulations. For smaller values of κ, there is always significantly higher number of failures which makes the result more accurate. The higher the reliability index, the smaller number of failures observed and therefore less accurate result. If it is supposed that, because of the aforementioned reason, κ=0,2 represents the curve in this figure of how the reliability index rises with proof load, then it could be that if the sufficient number of simulations was used, other κ values would now overlap or "converge" to the same reliability level.

Case 2: Reinforcement as dominant material strength

Now, the second case will be inspected, the case when reinforcement is the dominant material strength. The proof load applied to this bridge of interest was considered in range of 50% to 170% of characteristic value of maximum annual traffic load for vehicle class 100, P_c.
The results of this analysis for CC2 consequence class are shown in table 4.2.
It is seen from the table that the by increasing the proof load all up to 170% P_c, it is possible to meet the required reliability level. This represent the sufficient load level to consider the bridge safe.

Pl [N] \ κ	0,2		0,3		0,4		0,5	
	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$
50% P_c	3,5	2,3 e^{-4}	4,0	2,7e^{-5}	4,4	4,4e^{-6}	4,7	1,5e^{-6}
60% P_c	3,5	2,2 e^{-4}	4,0	2,7e^{-5}	4,4	4,4e^{-6}	4,7	1,5e^{-6}
70% P_c	3,5	2,0 e^{-4}	4,0	2,7e^{-5}	4,4	4,4e^{-6}	4,7	1,5e^{-6}
80% P_c	3,6	1,8 e^{-4}	4,0	2,6e^{-5}	4,4	4,3e^{-6}	4,7	1,5 e^{-6}
90% P_c	3,6	1,4 e^{-4}	4,1	2,2e^{-5}	4,4	4,3e^{-6}	4,7	1,5e^{-6}
100% P_c	3,7	1,2 e^{-4}	4,1	1,8e^{-5}	4,4	4,1e^{-6}	4,7	1,5e^{-6}
110% P_c	3,8	7,5e^{-5}	4,1	1,4e^{-5}	4,5	3,4e^{-6}	4,7	1,5e^{-6}
120% P_c	3,9	4,6e^{-5}	4,2	9,4e^{-6}	4,5	2,5e^{-6}	4,7	1,5e^{-6}
130% P_c	4,1	2,4e^{-5}	4,3	5,7e^{-6}	4,6	1,8e^{-6}	4,7	1,5e^{-6}
140% P_c	4,2	1,1 e^{-5}	4,4	2,4e^{-6}	4,6	1,1 e^{-6}	4,7	1,3 e^{-6}
150% P_c	4,4	5,2e^{-6}	4,6	1,4e^{-6}	4,7	8,0e^{-7}	4,8	1,0e^{-6}
160% P_c	4,6	2,2e^{-6}	4,7	7,0e^{-7}	4,8	6,0e^{-7}	4,8	8,0e^{-7}
170% P_c	4,8	8,0 e^{-7}	4,8	3,0e^{-7}	4,9	2,0 e^{-7}	4,9	5,0 e^{-7}

Table 4.2: CC2, Reliability updating for standard vehicle class 100

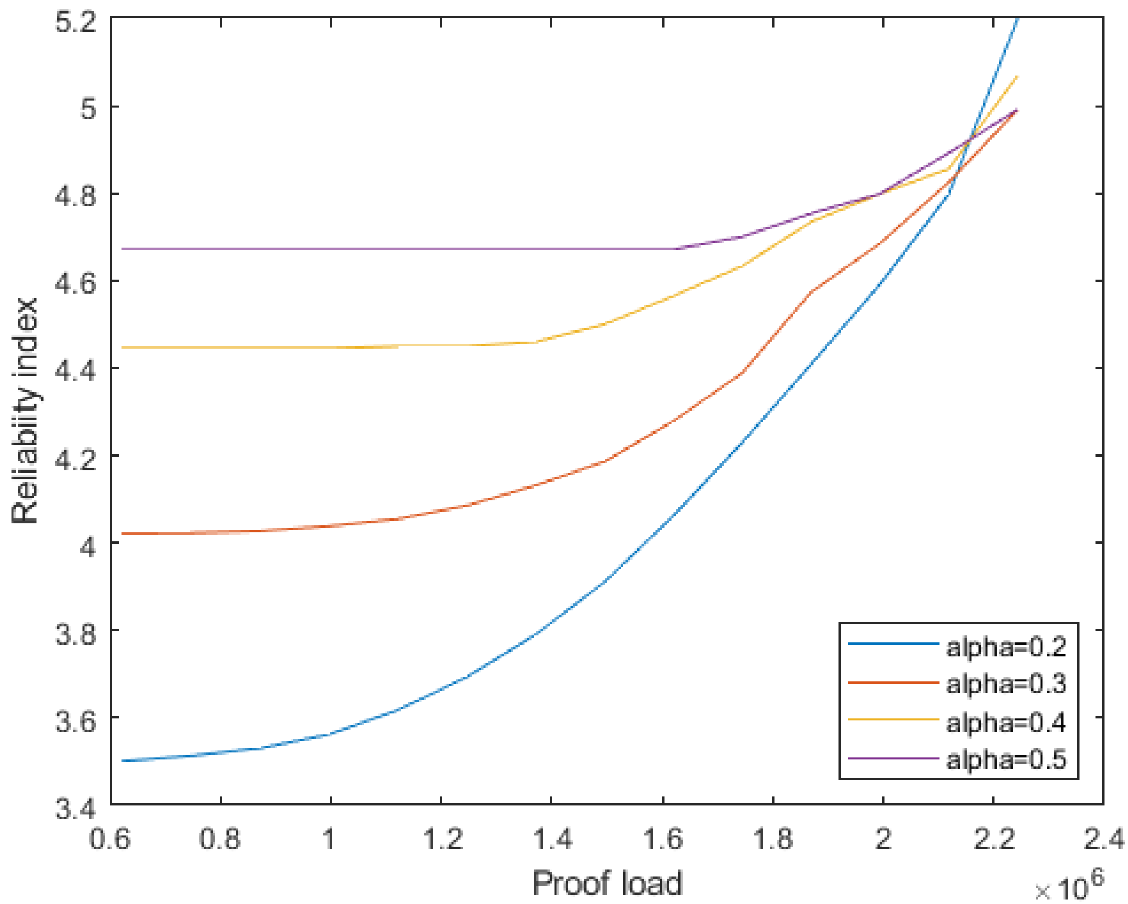

Figure 4.2: Proof loading updating when reinforcement is the dominant material strength, vehicle class 100

In the figure 4.2, four lines represent four values of κ as indicated by legend. As it can be seen in figure 4.2, the greater the applied proof load, the greater the reliability index. For consequence class CC2, the target reliability index is 4,8. For higher values of κ, it is noticed that results are not as accurate as for lower values. This could be the influence of the number of simulations that are needed. In this case, it was done in 25 million of simulations. For higher consequence class, the results are even less accurate. The reason for seemingly converging κ values is due to accuracy caused by insufficient number of simulations.

4.2.2 Standard vehicle class 125

Case 1: Concrete compressive strength as dominant material strength

In the table below, it can be seen that for standard vehicle class 125 and consequence class CC2, target reliability index is reached when proof loading level reaches 140% P_c.The bridge is deemed same.

Pl [N] \ κ	0,2		0,3		0,4		0,5	
	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$
50% P_c	3,9	4,3 e^{-5}	4,2	1,4e^{-5}	4,5	3,9e^{-6}	4,7	1,5e^{-6}
60% P_c	4,0	3,7 e^{-5}	4,2	1,3e^{-5}	4,5	3,9e^{-6}	4,7	1,5e^{-6}
70% P_c	4,0	3,2 e^{-5}	4,2	1,2e^{-5}	4,5	3,8e^{-6}	4,7	1,5e^{-6}
80% P_c	4,1	2,5 e^{-5}	4,3	1,1e^{-5}	4,5	3,7e^{-6}	4,7	1,5e^{-6}
90% P_c	4,1	1,9 e^{-5}	4,4	1,0e^{-5}	4,5	3,5e^{-6}	4,7	1,3e^{-6}
100% P_c	4,2	1,4 e^{-5}	4,4	6,6e^{-6}	4,5	2,9e^{-6}	4,7	1,2e^{-6}
110% P_c	4,4	7,6e^{-6}	4,4	4,8e^{-6}	4,6	2,1e^{-6}	4,7	1,3e^{-6}
120% P_c	4,5	4,3e^{-6}	4,6	2,8e^{-6}	4,7	1,4e^{-6}	4,8	1,3e^{-6}
130% P_c	4,6	2,2e^{-6}	4,7	1,3e^{-6}	4,8	8,0e^{-7}	4,9	1,1e^{-6}
140% P_c	4,9	6,7e^{-7}	4,9	5,3e^{-7}	5,0	3,3 e^{-7}	5,0	3,3 e^{-7}

Table 4.3: CC2, Reliability updating for standard vehicle class 125

Figure 4.3 shows the increase of reliability level as the proof load level increases.

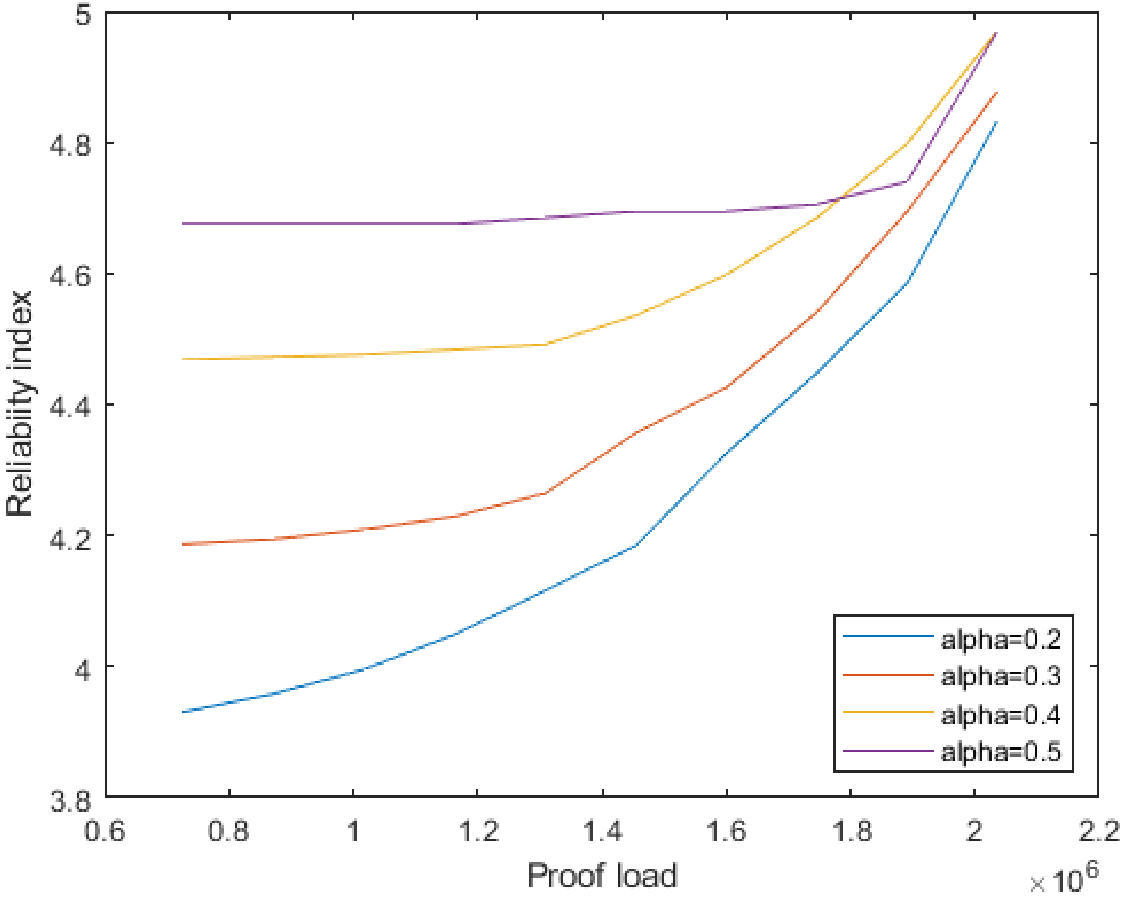

Figure 4.3: Proof loading updating when concrete compressive strength is the dominant material strength, vehicle class 125

Case 2: Reinforcement as dominant material strength

Table 4.4 indicates that 160% P_c is sufficient proof load level to achieve the target reliability index.

Pl [N] \ κ	0,2		0,3		0,4		0,5	
	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$	β_{pl}	$P_{f,pl}$
50% P_c	3,6	1,4 e^{-4}	4,1	2,1e^{-5}	4,5	3,2e^{-6}	4,8	9,0e^{-7}
60% P_c	3,6	1,4 e^{-4}	4,1	2,1e^{-5}	4,5	3,2e^{-6}	4,8	9,0e^{-7}
70% P_c	3,7	1,3 e^{-4}	4,1	2,1e^{-5}	4,5	3,2e^{-6}	4,8	9,0e^{-7}
80% P_c	3,7	1,1 e^{-4}	4,1	2,0e^{-5}	4,5	3,2e^{-6}	4,8	9,0e^{-7}
90% P_c	3,8	9,3 e^{-5}	4,1	1,9e^{-5}	4,5	3,2e^{-6}	4,8	9,0e^{-7}
100% P_c	4,0	6,9 e^{-5}	4,2	1,7e^{-5}	4,5	3,1e^{-6}	4,8	9,0e^{-7}
110% P_c	4,0	4,6e^{-5}	4,2	1,4e^{-5}	4,5	2,9e^{-6}	4,8	9,0e^{-7}
120% P_c	4,2	2,6e^{-5}	4,2	1,1e^{-5}	4,6	2,7e^{-6}	4,8	9,0e^{-7}
130% P_c	4,4	1,4e^{-5}	4,3	8,1e^{-6}	4,6	1,9e^{-6}	4,8	9,0e^{-7}
140% P_c	4,4	6,8 e^{-6}	4,4	4,4e^{-6}	4,7	1,5 e^{-6}	4,8	9,0 e^{-7}
150% P_c	4,6	2,6e^{-6}	4,6	1,8e^{-6}	4,8	1,0 e^{-6}	4,8	8,0e^{-7}
160% P_c	4,8	1,0 e^{-6}	4,8	7,0e^{-7}	4,9	4,0 e^{-7}	4,9	2,0 e^{-7}

Table 4.4: CC2, Reliability updating for standard vehicle class 125

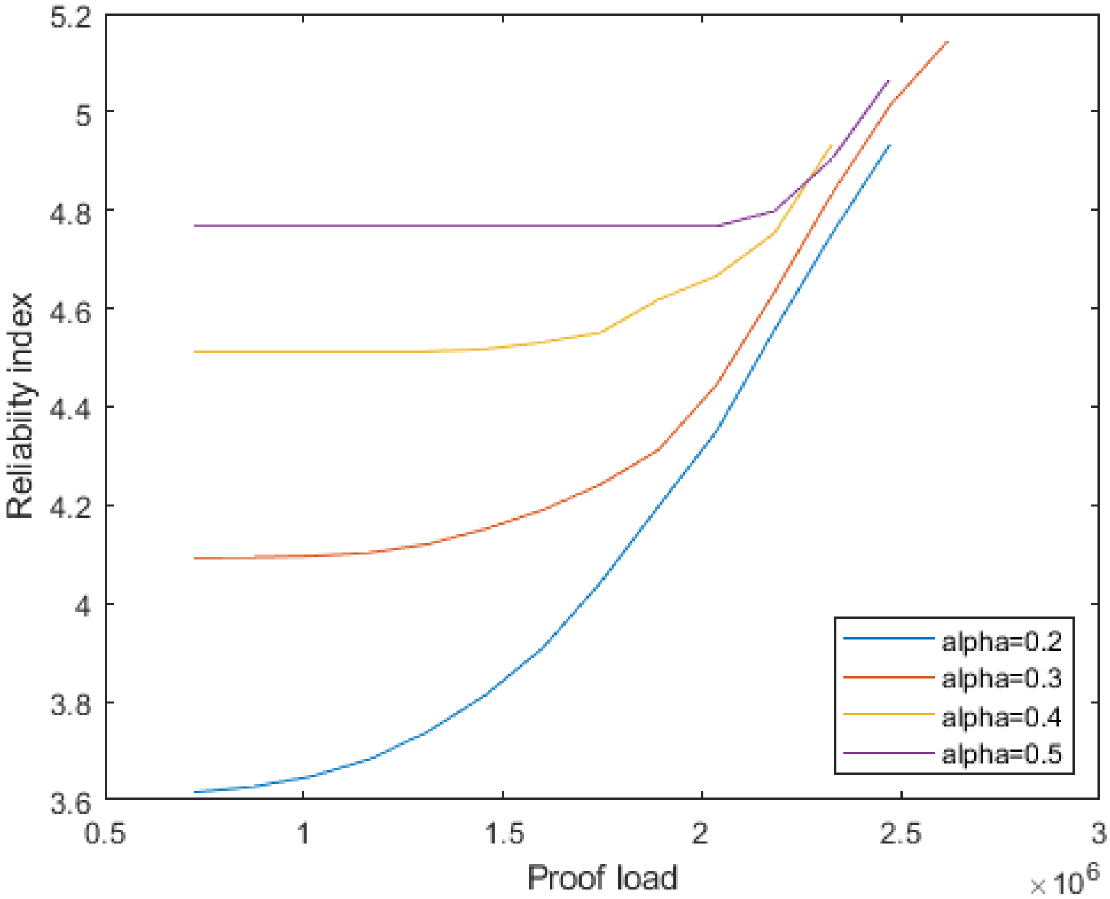

Figure 4.4: Proof loading updating when reinforcement is the dominant material strength, vehicle class 125

Conclusion 5

The aim of this project was to inspect how can a reliability level of an existing bridge be re-evaluated based on the proof loading tests.
In reality, these tests are not so common. Reason for that lies in the cost to conduct one and possible damage on the structure it can cause. This only stresses out the importance of this topic.

In the initial part of the report, more general introduction of this matter was given in order give better understanding of topic. Introduced were the Eurocodes and other documents which served as guidelines on how to establish a stochastic model which was later used in reliability analysis.

Variables were modelled based on the recommendations from Sørensen et al. [2009] and Von Scholten et al. [2004]. Then, the reliability analysis was carried out to check if the reliability requirements are fulfilled for 2 cases of interest:

- Concrete compression strength is the dominant material strength
- Reinforcement is the dominant material strength

Two consequence classes were considered as well:

- CC2 consequence class with β_t=4,8
- CC3 consequence class with β_t=5,2

Also, two standard vehicle classes were considered.

- Standard vehicle class 100
- Standard vehicle class 125

Standard vehicle class 100 was checked for both CC2 and CC3 consequence class while for class 125 only CC2 was checked. Reason for that is that it was concluded that CC3 class required significantly higher number of simulations which lead to longer computation time and insufficient computer capacity to conduct such an analysis.
Both classes 100 and 125 only satisfied the reliability level for case when κ=0,5.

Based on the sensitivity measures estimated from reliability analysis, it was determined which is the most influential factor. For both analysis that was the resistance. To check the resistance of the bridge, proof load was applied to the bridge (or in this case, stochastic model). Proof load intensities were calibrated based on the target reliability index and in this case it was β_t=4,8.

After updating the reliability level of the two vehicle classes it is possible to compare intensities at which the required reliability level was reached.

	Standard vehicle class 100		Standard vehicle class 125	
	$f_{ck} = dom.$	f_y=dom	f_{ck}=dom	f_y=dom
Proof load level	140% P_c	170% P_c	140% P_c	160% P_c

Table 5.1: Required proof load level to reach sufficient reliability level

Even thought that both for class 100 and 125 the reliability requirements estimated after reliability analysis were not fulfilled for this class of standard vehicles, during reliability updating it was proven that reliability of existing bridges can be updated based on the proof load. It is concluded that by increasing the level of proof load it is possible to increase the reliability level of the structure (table 5.1, for each case respectively).

5.1 Future aspects

One of the things that was not investigated in this report is updating the reliability analysis based on material parameters. Also, it could be interesting to see the economical aspect of both methods to update a reliability level. Which method would give better results at smaller cost?
Talking about proof loading, it is important to mention that before conducting physical test, stop criteria is determined so that it doesn't lead to collapse or damage of the bridge.
More complex model of the bridge could be taken into account with normal passage situation instead of conditional for example.

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Appendices

A Matlab codes

Reliability analysis files, when concrete compressive strength is the dominant material strength, compose of function file and script to make iterations:
-function:

```
function [Pf, betas, nfail, ufail, ustar, alphavector]=f_mYsimulation(nsim)

%% PARAMETERS FOR CLASS 100 VEHICLE
mu_w= 131.4*1000*9.82;  %[t]-->[N]
sigma_w=5*1000*9.82;    %[t]-->[N]
N=50;

%% Characteristic values- design equation (z)

%calcualtion of MAXIMUN ANNUAL TRAFFIC LOAD FOR CLASS 100 (VEHICLE),  P_c
%characteristic value, 98% quantile

P_c= mu_w + sigma_w*((norminv(1+(1/N)*(log(0.98)))));



%traffic load uncertaainty,                                          Xq
X_Q=1;

e_q= 1;
rho=0.25;

%trafic load- characteristic value:
Q_c=e_q*X_Q*(1+rho)*P_c;

%chosen alpha
alpha_c=0.5;

%dead load- characteristic value:
G_c=((Q_c)/(alpha_c)) -Q_c;



%% Copied values from below:

nsim_f=10000000;
u=normrnd(0,1,nsim_f,8);

%material model uncertainty: Lognormal distribution
mu_m=1;                                 %mean value
V_Im=0.11;                              %COV
std_m=V_Im*mu_m;                        %standard deviation
std_mm=sqrt(log(((std_m/mu_m)^2)+1));
mu_mm=log(mu_m)- (0.5*(std_mm)^2);

Xm=(exp(mu_mm+ u(:,8)*std_mm));

%Material resistance stohastic variable- Lognormal distibution    [N/mm^2] (R)
V_m=0.14;                             % COV
mu_R=36.2;                            % mean value
std_R=V_m*mu_R;                       % standard deviation
std_RR=sqrt(log(((std_R/mu_R)^2)+1));
mu_RR=log(mu_R)- (0.5*(std_RR)^2);
```

Figure A.1

```
R= (exp(mu_RR + u(:,5)*std_RR));


%% DESIGN PARAMETER
%for CC3, K_fi=1.1:
%for CC2, K_fi=1.0:

gamma_MC=1.45;
gamma_Ga=1.2;           % 1,2*1,1 =1.32
gamma_Gb=1;             % 1,0*1,1 =1.1
gamma_Qa=0;
gamma_Qb=1.4;           % 1,4*1,1 =1.54

R_c=Xm.*R;              % R= 5 percent quantile of Xm * R
std_Rc=sqrt(log(((std(R_c)/mean(R_c))^2)+1));
mu_Rc=log(mean(R_c))- (0.5*(std_Rc)^2);

%for small COV:
Rk=mean(R_c)*exp(-1.645*(std(R_c)/mean(R_c)));

alpha=[ 0.2; 0.3; 0.4; 0.5];
Gk=G_c;               %[N]
Qk=Q_c;               %[N]

% design variables z_a and z_b are calculated in [mm^2]

z_a = (gamma_MC*((1-alpha)*gamma_Ga*Gk))/Rk;

z_b = (gamma_MC*((1-alpha)*gamma_Gb*Gk+alpha*gamma_Qb*Qk))/Rk;

 %% Stohastic variables -limit state equation (g)

P_s= mu_w + sigma_w*((norminv(1+(1/N)*(log(normcdf(u(:,1)))))));


%Q- traffic load:

%trafic load uncertainty:
muq=1;
COV=0.1;                   %!!CHANGED: from 0.15 to 0.1 ! !
stdq=muq*COV;

X_q=muq+stdq*u(:,7);


W= (sigma_w)*(u(:,2))+ (mu_w);


%dynamic factor:
mu_rho=41500/W ;                  %[N]
std_rho=41500/W ;                 %[N]
rho_s=std_rho*u(:,3)+ mu_rho ;
```

Figure A.2

```
Q_s=e_q.*X_q.*(1+rho_s').*P_s;


%% LIMIT STATE EQUATION:


z_list=z_b;                                     % z=[mm^2]


alpha_list=[0.2 0.3 0.4 0.5];


%Dead load stohastic variable- Normal distribution=       [N]              (G)
mu_G=G_c;
COV_G=0.1;
std_G=COV_G*mu_G;

G=(u(:,4).*std_G+mu_G);


%Traffic load stohastic variable-Normal distribution      [N]              (Q)

Q=Q_s;

%Material resistance stohastic variable- Lognormal distibution    [N/mm^2] (R)
V_m=0.14;        % COV of R
mu_R=36.2;
std_R=V_m*mu_R;
std_RR=sqrt(log(((std_R/mu_R)^2)+1));
mu_RR=log(mu_R)- (0.5*(std_RR)^2);

R= (exp(mu_RR + u(:,5)*std_RR));




%dead load uncertainty: Normal distribution
mug=0;          % given mean
stdg=0.05*G_c;  % given std

Xg=mug+stdg*u(:,6);

%traffic load uncertainty: Normal distribution
muq=1;
COV=0.1;
stdq=muq*COV;

X_q=muq+stdq*u(:,7);
```

Figure A.3

```
%material model uncertainty: Lognormal distribution
mu_m=1;
V_Im=0.11;
std_m=V_Im*mu_m;
std_mm=sqrt(log(((std_m/mu_m)^2)+1));
mu_mm=log(mu_m)- (0.5*(std_mm)^2);

Xm=(exp(mu_mm+ u(:,8)*std_mm));




    alpha=alpha_list;
    z=z_list;


    g=z'.*Xm.*R-((1-alpha).*(G+Xg)+alpha.*X_q.*Q);
    fail=g<=0;
%indices for realizations in the failure domain
    nfail=sum(fail);
%number of failures
    Pf=nfail./nsim;
%probability of failure


    for i=1:length(alpha_list)
        ufail(i).list=u(fail(:,i),:);        % extract u values where had
failure (size 1x8 for each alpha)
        ustar(i,:)=mean(ufail(i).list,1);    % columnwise means for each u,
rows for each alpha
        alphavector(i,:)=ustar(i,:)./norm(ustar(i,:)); % normalization
    end



Pf;
betas=-norminv(Pf);

rng shuffle % reset random number generator (seed)

end
```

Figure A.4

-script to run iteration process (simulation in blocks):

```
clc;clearvars
nsim=10000000;
ustar_all=[];alphavector_all=[];ufail_iter=struct;
alpha=[0.2; 0.3; 0.4; 0.5];

for i=1:10
[Pf(i,:), betas(i,:),nfail(i,:), ufail, ustar,
alphavector]=f_mYsimulation(nsim);

ustar_all=vertcat(ustar_all(:,:),ustar(1:4,:));
alphavector_all=vertcat(alphavector_all(:,:),alphavector(1:4,:));

ufail_iter(i).ufail.list=ufail.list;
s=sqrt((Pf.*(1-Pf)/nsim));

end


ustar_a1=ustar_all(1:4:end,:); % for alpha=0.2, 10sims
ustar_a2=ustar_all(2:4:end,:); % for alpha=0.3, 10sims
ustar_a3=ustar_all(3:4:end,:); % for alpha=0.4, 10sims
ustar_a4=ustar_all(4:4:end,:); % for alpha=0.5, 10sims

ustar_a1_weighted=sum((ustar_a1).*nfail(:,1),1)/sum(nfail(:,1)); %weighted
values of u* (in respect to number fo failures) for alpha=0.2
ustar_a2_weighted=sum((ustar_a2).*nfail(:,2),1)/sum(nfail(:,2));
ustar_a3_weighted=sum((ustar_a3).*nfail(:,3),1)/sum(nfail(:,3));
ustar_a4_weighted=sum((ustar_a4).*nfail(:,4),1)/sum(nfail(:,4));


alphavector_a1_w=ustar_a1_weighted./norm(ustar_a1_weighted);     %alphavector
based on wighted u* for alpha=0.2
alphavector_a2_w=ustar_a2_weighted./norm(ustar_a2_weighted);
alphavector_a3_w=ustar_a3_weighted./norm(ustar_a3_weighted);
alphavector_a4_w=ustar_a4_weighted./norm(ustar_a4_weighted);

% plot(alpha,mean(betas,1))
% title('Concrete compressive strength is dominant material strength')
% ylabel('Reliability index, {\beta}')
% xlabel('Influence factor, {\kappa}')

% for first 10x simulations: ufail_iter(1).ufail.list
% for 2nd 10x simulations:   ufail_iter(2).ufail.list
```

Figure A.5

Reliability analysis files, when reinforcement is the dominant material strength, compose of function file and script to make iterations:
-function:

```
function [Pfr, betasr, nfailr, ufailr, ustarr,
alphavectorr]=f_mYsimulation_R(nsim)

%% PARAMETERS FOR CLASS 100 VEHICLE
mu_wr= 109.2*1000*9.82;  %[t]-->[N]
sigma_wr=5*1000*9.82;    %[t]-->[N]
Nr=100;

%% Characteristic values- design equation (z)

%calcualtion of MAXIMUN ANNUAL TRAFFIC LOAD FOR CLASS 100 (VEHICLE),  P_c
%characteristiv value, 98% quantile

P_cr= mu_wr + sigma_wr*((norminv(1+(1/Nr)*(log(0.98)))));


%traffic load uncertaainty,                                              Xq
X_Qr=1;

e_qr= 1;
rhor=0.25;

%trafic load- characteristic value:
Q_cr=e_qr*X_Qr*(1+rhor)*P_cr;

%chosen alpha
alpha_cr=0.5;

%dead load- characteristic value:
G_cr=((Q_cr)/(alpha_cr)) -Q_cr;


%% Copied values from below:

nsim_fr=10000000;
ur=normrnd(0,1,nsim_fr,8);

%Material resistance stohastic variable- Lognormal distibution    [N/mm^2]
%(R) for fyk=275 [MPa]

COVr=0.07;
mu_Rr=345;
std_Rr=COVr*mu_Rr;
std_RRr=sqrt(log(((std_Rr/mu_Rr)^2)+1));
mu_RRr=log(mu_Rr)- (0.5*(std_RRr)^2);

Rr= (exp(mu_RRr + ur(:,5)*std_RRr));

%material model uncertainty: Lognormal distribution
mu_mr=1;
V_Imr=0.05;
std_mr=V_Imr*mu_mr;
std_mmr=sqrt(log(((std_mr/mu_mr)^2)+1));
mu_mmr=log(mu_mr)- (0.5*(std_mmr)^2);
```

Figure A.6

```
Xmr=(exp(mu_mmr+ ur(:,8)*std_mmr));

%% DESIGN PARAMETER
%for CC3, K_fi=1.1:

gamma_MC_r=1.2;
gamma_Gar=1.2;
gamma_Gbr=1;
gamma_Qar=0;
gamma_Qbr=1.4;

R_cr=Xmr.*Rr;            % R= 5 percent quantile of Xm * R
std_Rcr=sqrt(log(((std(R_cr)/mean(R_cr))^2)+1));
mu_Rcr=log(mean(R_cr))- (0.5*(std_Rcr)^2);

%for small COV:
Rkr=mean(R_cr)*exp(-1.645*(std(R_cr)/mean(R_cr)));


alphar=[ 0.2; 0.3; 0.4; 0.5];
Gkr=G_cr;            %[N]
Qkr=Q_cr;            %[N]

% design variables z_a and z_b are calculated in [mm^2]

z_aR = (gamma_MC_r*((1-alphar)*gamma_Gar*Gkr))/Rkr;

z_bR = (gamma_MC_r*((1-alphar)*gamma_Gbr*Gkr+alphar*gamma_Qbr*Qkr))/Rkr;



 %% Stohastic variables -limit state equation (g)

P_sr= mu_wr + sigma_wr*((norminv(1+(1/Nr)*(log(normcdf(ur(:,1)))))));



%Q- traffic load:

%trafic load uncertainty:
muqr=1;
COVr=0.10;
stdqr=muqr*COVr;

X_qr=muqr+stdqr*ur(:,7);


Wr= (sigma_wr)*(ur(:,2))+ (mu_wr);


%dynamic factor:
```

Figure A.7

```
mu_rhor=41500/Wr ;                  %[N]
std_rhor=41500/Wr ;                 %[N]
rho_sr=std_rhor*ur(:,3)+ mu_rhor ;


Q_sr=e_qr.*X_qr.*(1+rho_sr').*P_sr;



%% LIMIT STATE EQUATION:


z_listr=z_bR;                                      % z=[mm^2]


alpha_listr=[0.2 0.3 0.4 0.5];



%Dead load stohastic variable- Normal distribution=       [N]              (G)
mu_Gr=G_cr;
COV_Gr=0.1;
std_Gr=COV_Gr*mu_Gr;

Gr=(ur(:,4).*std_Gr+mu_Gr);


%Traffic load stohastic variable-Normal distribution      [N]              (Q)

Qr=Q_sr;

%Material resistance stohastic variable- Lognormal distibution    [N/mm^2] (R)
(fyk=275 [MPa])

COVr=0.07;
mu_Rr=345;
std_Rr=COVr*mu_Rr;
std_RRr=sqrt(log(((std_Rr/mu_Rr)^2)+1));
mu_RRr=log(mu_Rr)- (0.5*(std_RRr)^2);

Rr= (exp(mu_RRr + ur(:,5)*std_RRr));



%dead load uncertainty: Normal distribution
mugr=0;           % given mean
stdgr=0.05*G_cr;  % given std

Xgr=mugr+stdgr*ur(:,6);

%traffic load uncertainty: Normal distribution
muqr=1;
```

Figure A.8

```
COVr=0.1;
stdqr=muqr*COVr;

X_qr=muqr+stdqr*ur(:,7);


%material model uncertainty: Lognormal distribution
mu_mr=1;
V_Imr=0.05;
std_mr=V_Imr*mu_mr;
std_mmr=sqrt(log(((std_mr/mu_mr)^2)+1));
mu_mmr=log(mu_mr)- (0.5*(std_mmr)^2);

Xmr=(exp(mu_mmr+ ur(:,8)*std_mmr));



    alphar=alpha_listr;
    zr=z_listr;


    gr=zr'.*Xmr.*Rr-((1-alphar).*(Gr+Xgr)+alphar.*X_qr.*Qr);
    failr=gr<=0;
%indices for realizations in the failure domain
    nfailr=sum(failr);
%number of failures
    Pfr=nfailr./nsim_fr;
%probability of failure


    for i=1:length(alpha_listr)
        ufailr(i).list=ur(failr(:,i),:); % extract u values where had failure
(size 1x8 for each alpha)
        ustarr(i,:)=mean(ufailr(i).list,1); % columnwise means for each u,
rows for each alpha
        alphavectorr(i,:)=ustarr(i,:)./norm(ustarr(i,:)); % normalization
    end


Pfr;
betasr=-norminv(Pfr);

rng shuffle % reset random number generator (seed)

end
```

Figure A.9

-script to run iteration process (simulation in blocks):

```
clc;clearvars
nsimr=10000000;
ustarr_all=[];alphavectorr_all=[];ufailr_iter=struct;
alphar=[ 0.2; 0.3; 0.4; 0.5];

for i=1:10
[Pfr(i,:), betasr(i,:),nfailr(i,:), ufailr, ustarr,
alphavectorr]=f_mYsimulation_R(nsimr);

ustarr_all=vertcat(ustarr_all(:,:),ustarr(1:4,:));
alphavectorr_all=vertcat(alphavectorr_all(:,:),alphavectorr(1:4,:));

ufailr_iter(i).ufailr.list=ufailr.list;
sr=sqrt((Pfr.*(1-Pfr)/nsimr));

end


ustar_a1r=ustarr_all(1:4:end,:); % for alpha=0.2, 10sims
ustar_a2r=ustarr_all(2:4:end,:); % for alpha=0.3, 10sims
ustar_a3r=ustarr_all(3:4:end,:); % for alpha=0.4, 10sims
ustar_a4r=ustarr_all(4:4:end,:); % for alpha=0.5, 10sims

ustar_a1_weightedr=sum((ustar_a1r).*nfailr(:,1),1)/sum(nfailr(:,1)); %weighted
values of u* (in respect to number fo failures) for alpha=0.2
ustar_a2_weightedr=sum((ustar_a2r).*nfailr(:,2),1)/sum(nfailr(:,2));
ustar_a3_weightedr=sum((ustar_a3r).*nfailr(:,3),1)/sum(nfailr(:,3));
ustar_a4_weightedr=sum((ustar_a4r).*nfailr(:,4),1)/sum(nfailr(:,4));


alphavector_a1_wr=ustar_a1_weightedr./norm(ustar_a1_weightedr);
%alphavector based on wighted u* for alpha=0.2
alphavector_a2_wr=ustar_a2_weightedr./norm(ustar_a2_weightedr);
alphavector_a3_wr=ustar_a3_weightedr./norm(ustar_a3_weightedr);
alphavector_a4_wr=ustar_a4_weightedr./norm(ustar_a4_weightedr);


plot(alphar,mean(betasr,1))
title('Reinforcement is dominant material strength')
ylabel('Reliability index, {\beta}')
xlabel('Influence factor, {\kappa}')

% for first 10x simulations: ufail_iter(1).ufail.list
% for 2nd 10x simulations:   ufail_iter(2).ufail.list
```

Figure A.10

Script for updating reliability of existing bridges for proof load test. When concrete compressive strength is the dominant material strength.

```
clc;clearvars

%% PARAMETERS FOR CLASS 100 VEHICLE
mu_w= 131.4*1000*9.82;  %[t]-->[N]
sigma_w=5*1000*9.82;    %[t]-->[N]
N=50;

%% Characteristic values- design equation (z)

%calcualtion of MAXIMUN ANNUAL TRAFFIC LOAD FOR CLASS 100 (VEHICLE),  P_c
%characteristiv value, 98% quantile

P_c= mu_w + sigma_w*((norminv(1+(1/N)*(log(0.98)))));



%traffic load uncertaainty,                                            Xq
X_Q=1;

e_q= 1;
rho=0.25;

%trafic load- characteristic value:
Q_c=e_q*X_Q*(1+rho)*P_c;

%chosen alpha
alpha_c=0.5;

%dead load- characteristic value:
G_c=((Q_c)/(alpha_c)) -Q_c;

%% Copied values

nsim=15e6;
u=normrnd(0,1,nsim,8);

%material model uncertainty: Lognormal distribution
mu_m=1;
V_Im=0.11;
std_m=V_Im*mu_m;
std_mm=sqrt(log(((std_m/mu_m)^2)+1));
mu_mm=log(mu_m)- (0.5*(std_mm)^2);

Xm=(exp(mu_mm+ u(:,8)*std_mm));


%Material resistance stohastic variable- Lognormal distibution    [N/mm^2] (R)
V_m=0.14; % COV of R
mu_R=36.2;
std_R=V_m*mu_R;
std_RR=sqrt(log(((std_R/mu_R)^2)+1));
mu_RR=log(mu_R)- (0.5*(std_RR)^2);
```

Figure A.11

```
R= (exp(mu_RR + u(:,5)*std_RR));

%% DESIGN PARAMETER
%for CC3, K_fi=1.1:

gamma_MC=1.45;
gamma_Ga=1.2;
gamma_Gb=1;
gamma_Qa=0;
gamma_Qb=1.4;

R_c=Xm.*R;            % R= 5 percent quantile of Xm * R
std_Rc=sqrt(log(((std(R_c)/mean(R_c))^2)+1));
mu_Rc=log(mean(R_c))- (0.5*(std_Rc)^2);

%for small COV:
Rk=mean(R_c)*exp(-1.645*(std(R_c)/mean(R_c)));

alpha=[ 0.2; 0.3; 0.4; 0.5];
Gk=G_c;             %[N]
Qk=Q_c;             %[N]

% design variables z_a and z_b are calculated in [mm^2]

z_a = (gamma_MC*((1-alpha)*gamma_Ga*Gk))/Rk;

z_b = (gamma_MC*((1-alpha)*gamma_Gb*Gk+alpha*gamma_Qb*Qk))/Rk;



 %% Stohastic variables -limit state equation (g)

P_s= mu_w + sigma_w*((norminv(1+(1/N)*(log(normcdf(u(:,1)))))));


%Q- traffic load:

%trafic load uncertainty:
muq=1;
COV=0.1;
stdq=muq*COV;

X_q=muq+stdq*u(:,7);


W= (sigma_w)*(u(:,2))+ (mu_w);


%dynamic factor:
mu_rho=41500/W ;                  %[N]
std_rho=41500/W ;                 %[N]
rho_s=std_rho*u(:,3)+ mu_rho ;
```

Figure A.12

```
Q_s=e_q.*X_q.*(1+rho_s').*P_s;


%% LIMIT STATE EQUATION:

z_list=z_b;
alpha_list=[0.2 0.3 0.4 0.5];

%Dead load stohastic variable- Normal distribution=       [N]              (G)
mu_G=G_c;
COV_G=0.1;
std_G=COV_G*mu_G;

G=(u(:,4).*std_G+mu_G);


%Traffic load stohastic variable-Normal distribution      [N]              (Q)

Q=Q_s;

%Material resistance stohastic variable- Lognormal distibution    [N/mm^2] (R)
V_m=0.14; % COV of R
mu_R=36.2;
std_R=V_m*mu_R;
std_RR=sqrt(log(((std_R/mu_R)^2)+1));
mu_RR=log(mu_R)- (0.5*(std_RR)^2);

R= (exp(mu_RR + u(:,5)*std_RR));




%dead load uncertainty: Normal distribution
mug=0;  % given mean
stdg=0.05*G_c;  % given std

Xg=mug+stdg*u(:,6);

%traffic load uncertainty: Normal distribution
muq=1;
COV=0.1;
stdq=muq*COV;

X_q=muq+stdq*u(:,7);

%material model uncertainty: Lognormal distribution
mu_m=1;
V_Im=0.11;
std_m=V_Im*mu_m;
std_mm=sqrt(log(((std_m/mu_m)^2)+1));
mu_mm=log(mu_m)- (0.5*(std_mm)^2);
```

Figure A.13

```
Xm=(exp(mu_mm+ u(:,8)*std_mm));


%Proof load: x% P_c:

PI=(0.5:0.1:1.4).*P_c;



    alpha=alpha_list;
    z=z_list;


    g1=z'.*Xm.*R-((1-alpha).*(G+Xg)+alpha.*X_q.*Q);
    fail_g1=g1<0;                                           %indices for
realizations in the failure domain
    nfail_g1=sum(fail_g1);                                  %number of
failures
    Pf_g1=nfail_g1./nsim;                                   %probability of
failure

for i=1:length(PI)
    clearvars g2 fail_g2 nfail_g2 Pf_g2 Pf_proof betas
    g2=z'.*Xm.*R-((1-alpha).*(G+Xg)+alpha.*PI(i));
    fail_g2=g2>0;
    nfail_g2=sum(fail_g2);
    Pf_g2=nfail_g2./nsim;

Pf_proof=((sum(fail_g1==1 & fail_g2==1))/nsim)./Pf_g2;

betas=-norminv(Pf_proof);

Pf_g2_row4eachPIvalue(i,:)=Pf_g2;       % row for each PI value and columns by
each alpha
Pf_proof_row4eachPIvalue(i,:)=Pf_proof; % row for each PI value and columns by
each alpha
betas_row4eachPIvalue(i,:)=betas;       % row for each PI value and columns by
each alpha


end
%%
% Betas over PI for each alpha
figure(1)
clf
plot(PI,betas_row4eachPIvalue(:,:))
xlabel('Proof load')
ylabel('Reliabiity index')
legend('alpha='+string(alpha),'Location','southeast')
```

Figure A.14

Script for updating reliability of existing bridges for proof load test. When reinforcement is the dominant material strength.

```
clc;clearvars -except nsim

%% PARAMETERS FOR CLASS 100 VEHICLE
mu_wr= 109.2*1000*9.82;  %[t]-->[N]
sigma_wr=5*1000*9.82;    %[t]-->[N]
Nr=100;

%% Characteristic values- design equation (z)

%calcualtion of MAXIMUN ANNUAL TRAFFIC LOAD FOR CLASS 100 (VEHICLE),  P_c
%characteristiv value, 98% quantile

P_cr= mu_wr + sigma_wr*((norminv(1+(1/Nr)*(log(0.98)))));


%traffic load uncertaainty,                                          Xq
X_Qr=1;

e_qr= 1;
rhor=0.25;

%trafic load- characteristic value:
Q_cr=e_qr*X_Qr*(1+rhor)*P_cr;

%chosen alpha
alpha_cr=0.5;

%dead load- characteristic value:
G_cr=((Q_cr)/(alpha_cr)) -Q_cr;



%% Copied values from below:

nsim_fr=15000000;
ur=normrnd(0,1,nsim_fr,8);

%Material resistance stohastic variable- Lognormal distibution    [N/mm^2]
%(R) for fyk=275 [MPa]

COVr=0.07;
mu_Rr=345;
std_Rr=COVr*mu_Rr;
std_RRr=sqrt(log(((std_Rr/mu_Rr)^2)+1));
mu_RRr=log(mu_Rr)- (0.5*(std_RRr)^2);

Rr= (exp(mu_RRr + ur(:,5)*std_RRr));

%material model uncertainty: Lognormal distribution
mu_mr=1;
V_Imr=0.05;
std_mr=V_Imr*mu_mr;
std_mmr=sqrt(log(((std_mr/mu_mr)^2)+1));
```

Figure A.15

```
mu_mmr=log(mu_mr)- (0.5*(std_mmr)^2);

Xmr=(exp(mu_mmr+ ur(:,8)*std_mmr));

%% DESIGN PARAMETER
%for CC3, K_fi=1.1:

gamma_MC_r=1.2;
gamma_Gar=1.2;
gamma_Gbr=1;
gamma_Qar=0;
gamma_Qbr=1.4;

R_cr=Xmr.*Rr;            % R= 5 percent quantile of Xm * R
std_Rcr=sqrt(log(((std(R_cr)/mean(R_cr))^2)+1));
mu_Rcr=log(mean(R_cr))- (0.5*(std_Rcr)^2);

%for small COV:
Rkr=mean(R_cr)*exp(-1.645*(std(R_cr)/mean(R_cr)));

alphar=[ 0.2; 0.3; 0.4; 0.5];
Gkr=G_cr;             %[N]
Qkr=Q_cr;             %[N]

% design variables z_a and z_b are calculated in [mm^2]

z_aR = (gamma_MC_r*((1-alphar)*gamma_Gar*Gkr))/Rkr;

z_bR = (gamma_MC_r*((1-alphar)*gamma_Gbr*Gkr+alphar*gamma_Qbr*Qkr))/Rkr;



 %% Stohastic variables -limit state equation (g)

P_sr= mu_wr + sigma_wr*((norminv(1+(1/Nr)*(log(normcdf(ur(:,1)))))));


%Q- traffic load:

%trafic load uncertainty:
muqr=1;
COVr=0.10;
stdqr=muqr*COVr;

X_qr=muqr+stdqr*ur(:,7);


Wr= (sigma_wr)*(ur(:,2))+ (mu_wr);


%dynamic factor:
mu_rhor=41500/Wr ;                  %[N]
std_rhor=41500/Wr ;                 %[N]
```

Figure A.16

```
rho_sr=std_rhor*ur(:,3)+ mu_rhor ;


Q_sr=e_qr.*X_qr.*(1+rho_sr').*P_sr;



%% LIMIT STATE EQUATION:


z_listr=z_bR;                                       % z=[mm^2]


alpha_listr=[0.2 0.3 0.4 0.5];


%Dead load stohastic variable- Normal distribution=       [N]              (G)
mu_Gr=G_cr;
COV_Gr=0.1;
std_Gr=COV_Gr*mu_Gr;

Gr=(ur(:,4).*std_Gr+mu_Gr);


%Traffic load stohastic variable-Normal distribution      [N]              (Q)

Qr=Q_sr;

%Material resistance stohastic variable- Lognormal distibution    [N/mm^2] (R)

COVr=0.07;
mu_Rr=345;
std_Rr=COVr*mu_Rr;
std_RRr=sqrt(log(((std_Rr/mu_Rr)^2)+1));
mu_RRr=log(mu_Rr)- (0.5*(std_RRr)^2);

Rr= (exp(mu_RRr + ur(:,5)*std_RRr));





%dead load uncertainty: Normal distribution
mugr=0;  % given mean
stdgr=0.05*G_cr;  % given std

Xgr=mugr+stdgr*ur(:,6);

%traffic load uncertainty: Normal distribution
muqr=1;
COVr=0.1;
stdqr=muqr*COVr;
```

Figure A.17

```
X_qr=muqr+stdqr*ur(:,7);

%material model uncertainty: Lognormal distribution
mu_mr=1;
V_Imr=0.05;
std_mr=V_Imr*mu_mr;
std_mmr=sqrt(log(((std_mr/mu_mr)^2)+1));
mu_mmr=log(mu_mr)- (0.5*(std_mmr)^2);

Xmr=(exp(mu_mmr+ ur(:,8)*std_mmr));




%Proof load: x% P_c:
PI=(0.5:0.1:1.8).*P_cr;



    alphar=alpha_listr;
    zr=z_listr;


    g1=zr'.*Xmr.*Rr-((1-alphar).*(Gr+Xgr)+alphar.*X_qr.*Qr);
    fail_g1=g1<0;                                               %indices for
realizations in the failure domain
    nfail_g1=sum(fail_g1);                                      %number of
failures
    Pf_g1=nfail_g1./nsim_fr;                                    %probability of
failure

for i=1:length(PI)
    clearvars g2 fail_g2 nfail_g2 Pf_g2 Pf_proof betas
    g2=zr'.*Xmr.*Rr-((1-alphar).*(Gr+Xgr)+alphar.*PI(i));
    fail_g2=g2>0;
    nfail_g2=sum(fail_g2);
    Pf_g2=nfail_g2./nsim_fr;

Pf_proof=((sum(fail_g1==1 & fail_g2==1))/nsim)./Pf_g2;

betas=-norminv(Pf_proof);

Pf_g2_row4eachPIvalue(i,:)=Pf_g2;       % row for each PI value and columns by
each alpha
Pf_proof_row4eachPIvalue(i,:)=Pf_proof; % row for each PI value and columns by
each alpha
betas_row4eachPIvalue(i,:)=betas;       % row for each PI value and columns by
each alpha


end
%%
```

Figure A.18

```
% Betas over PI for each alpha
figure(1)
clf
plot(PI,betas_row4eachPIvalue(:,:))
xlabel('Proof load')
ylabel('Reliabiity index')
legend('alpha='+string(alphar),'Location','southeast')
```

Figure A.19

Sensitivity measures B

B.0.1 Case 1: Concrete compressive strength as dominant material strength

α-values

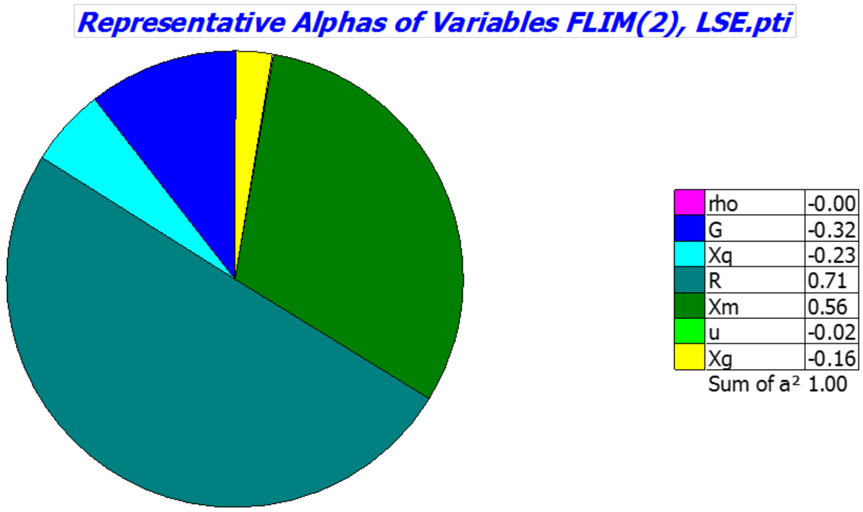

Figure B.1: Comrel, α- vector value for α=0,3

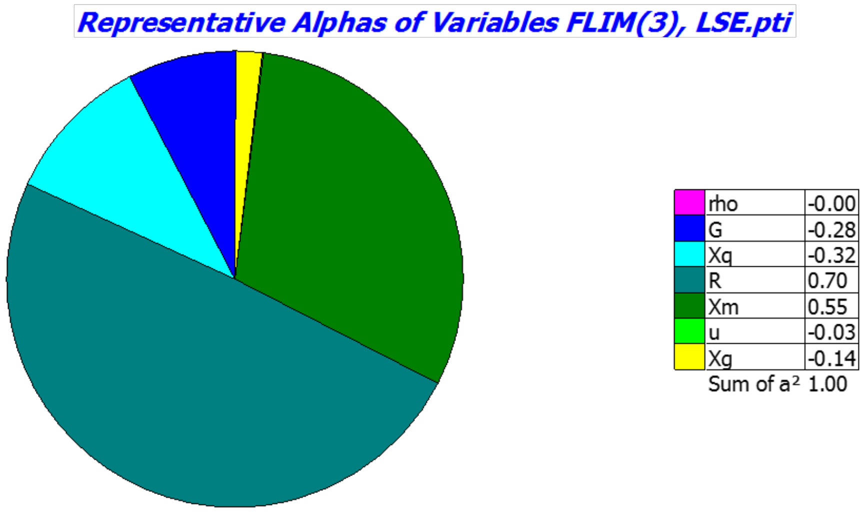

Figure B.2: Comrel, α- vector value for α=0,4

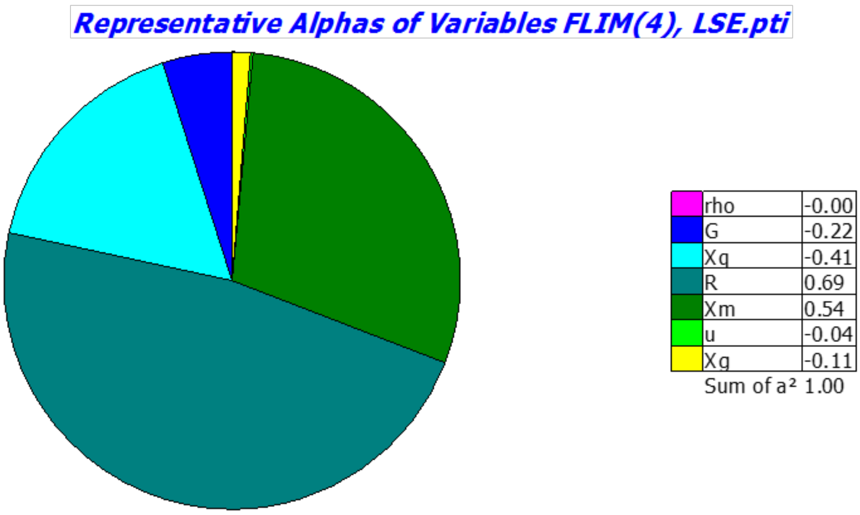

Figure B.3: Comrel, α- vector value for α=0,5

u^*- values

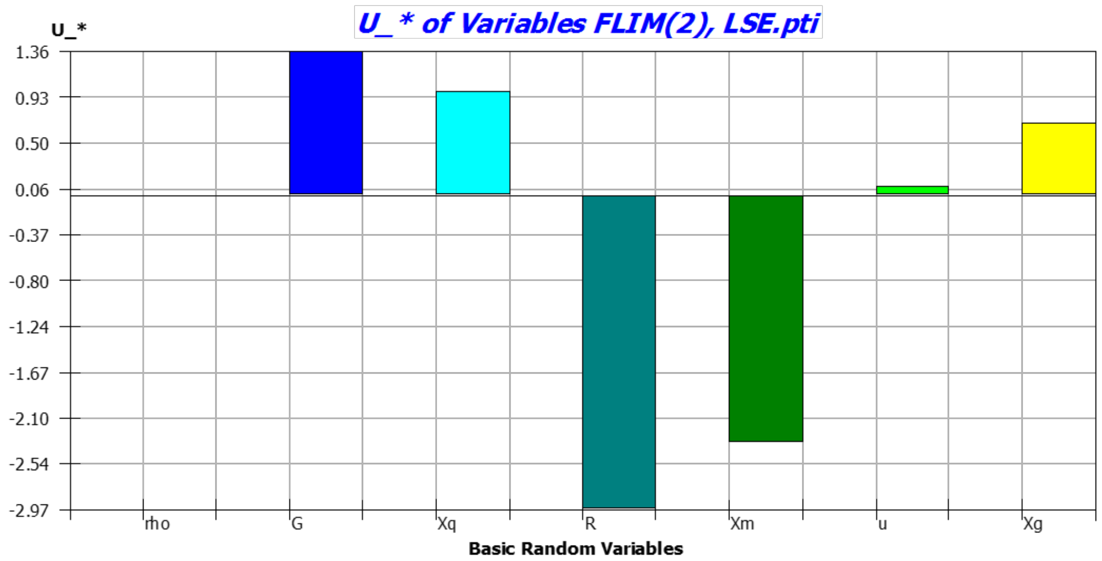

Figure B.4: Comrel, $u*$- vector value for α=0,3

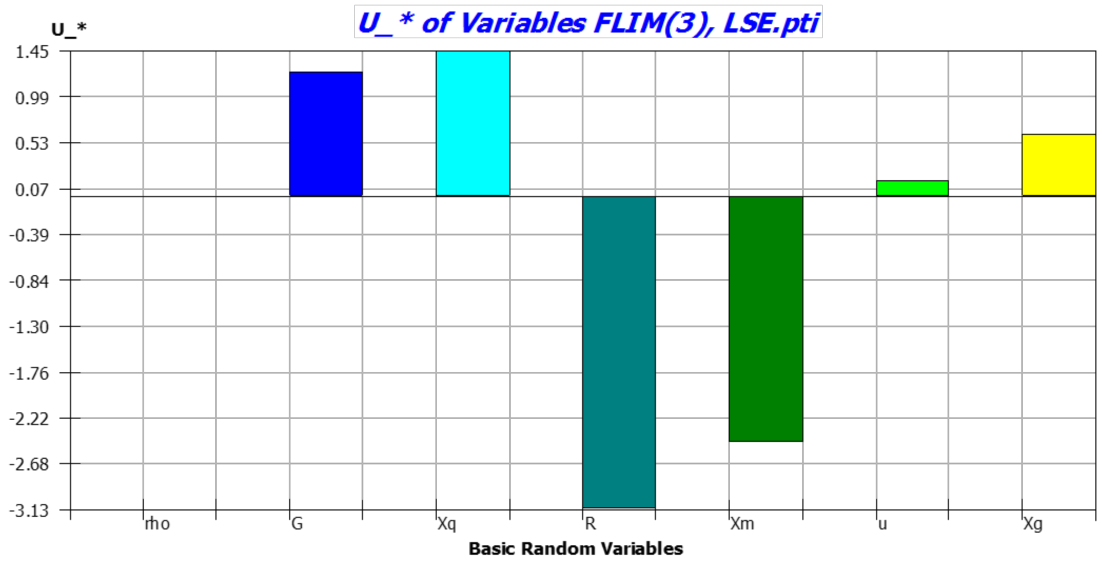

Figure B.5: Comrel, $u*$- vector value for α=0,4

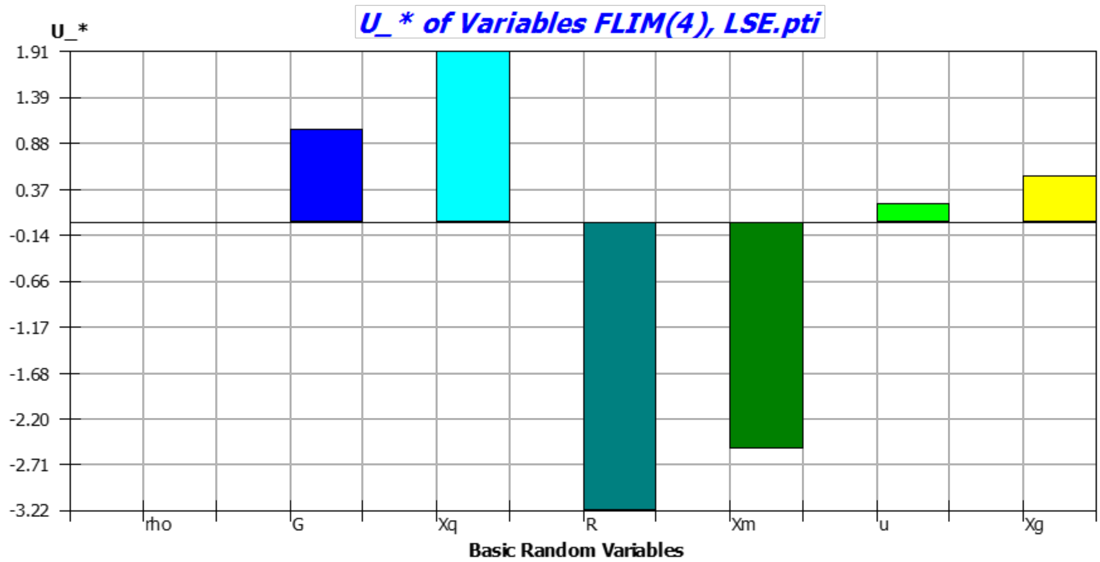

Figure B.6: Comrel, $u*$- vector value for α=0,5

Elasticities of mean values

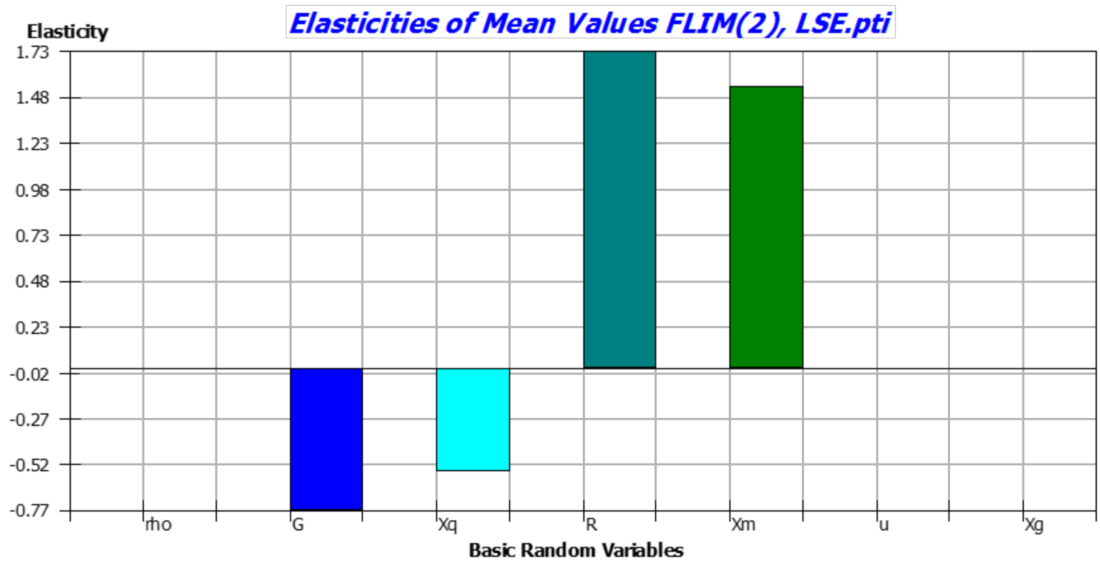

Figure B.7: Comrel, elasticities of mean values for α=0,3

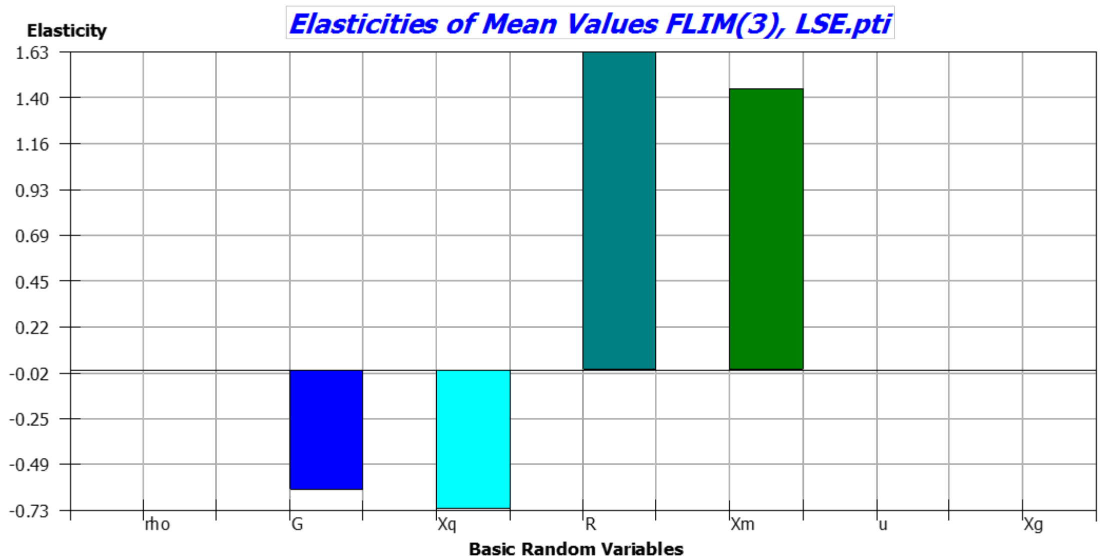

Figure B.8: Comrel, elasticities of mean values for α=0,4

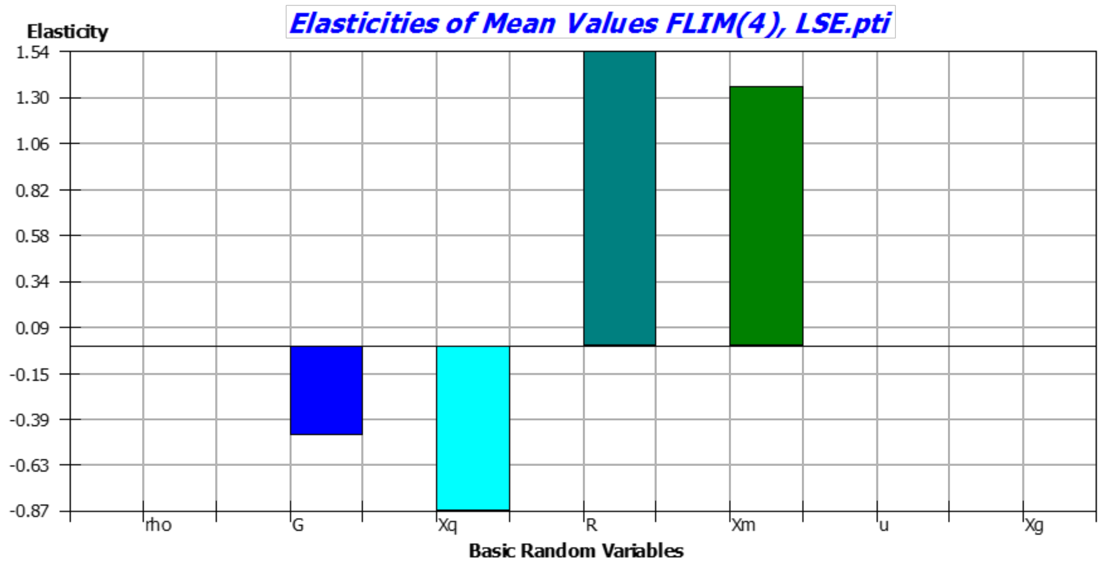

Figure B.9: Comrel, elasticities of mean values for α=0,5

B.0.2 Case 2:Reinforcement as dominant material strength

α-values

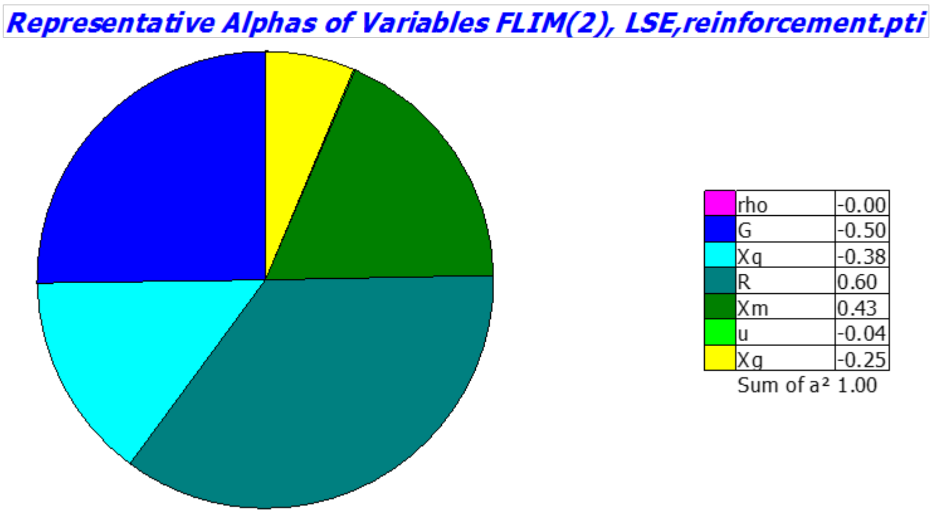

Figure B.10: Comrel, α- vector value for α=0,3

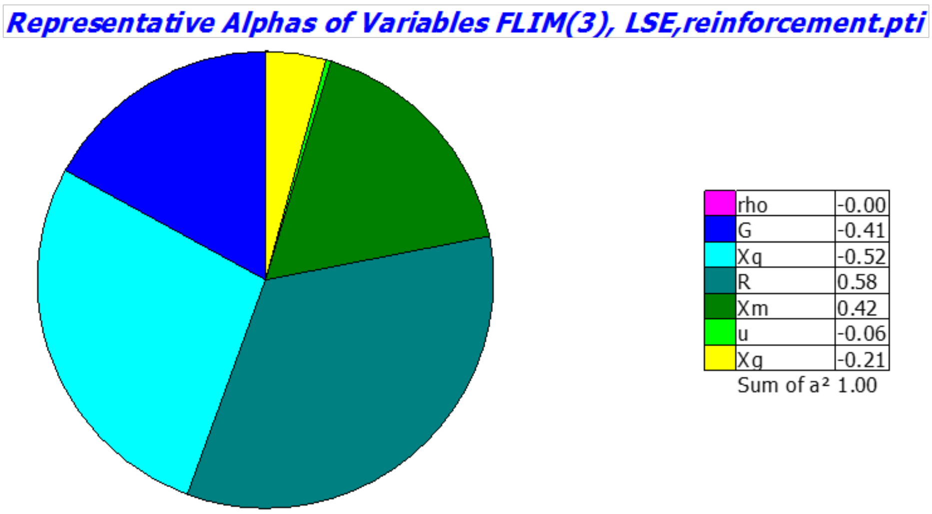

Figure B.11: Comrel, α- vector value for α=0,4

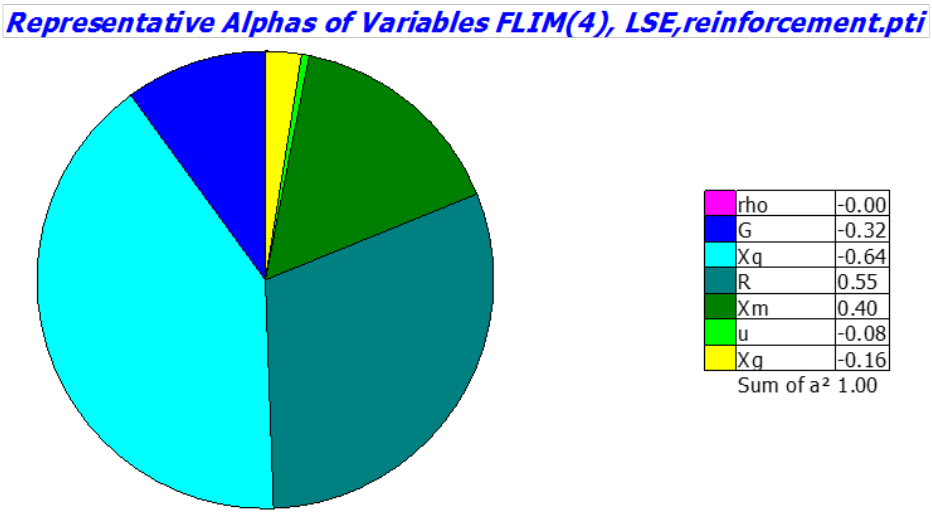

Figure B.12: Comrel, α- vector value for α=0,5

u^*- values

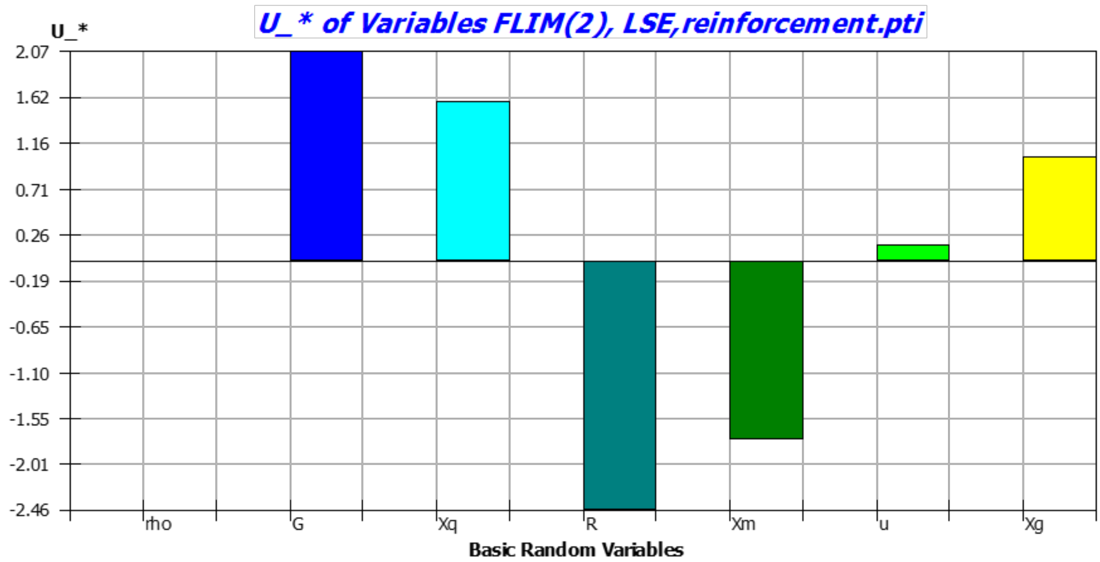

Figure B.13: Comrel, $u*$- vector value for α=0,3

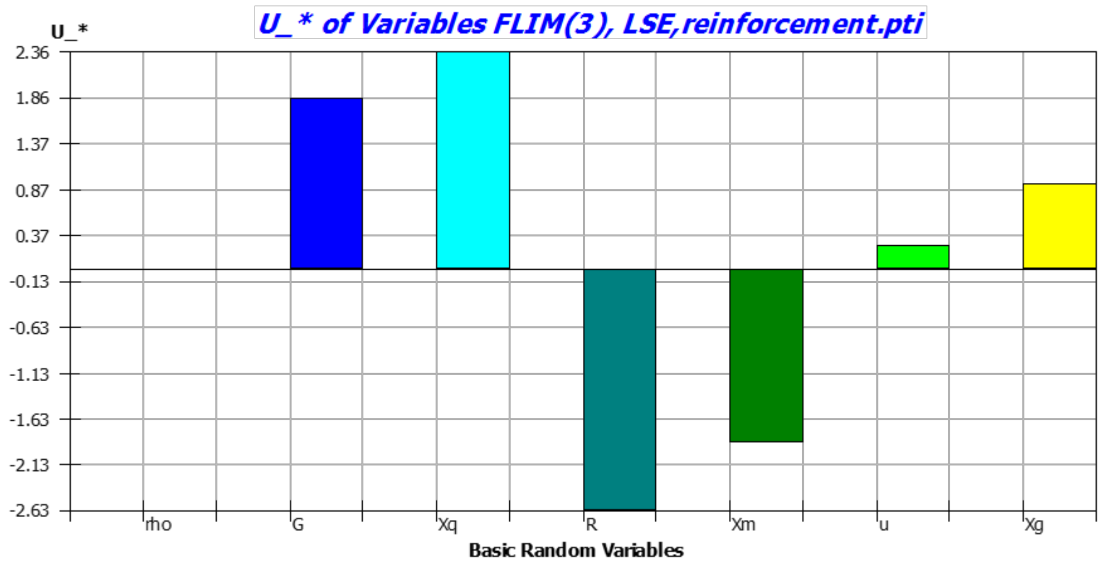

Figure B.14: Comrel, $u*$- vector value for α=0,4

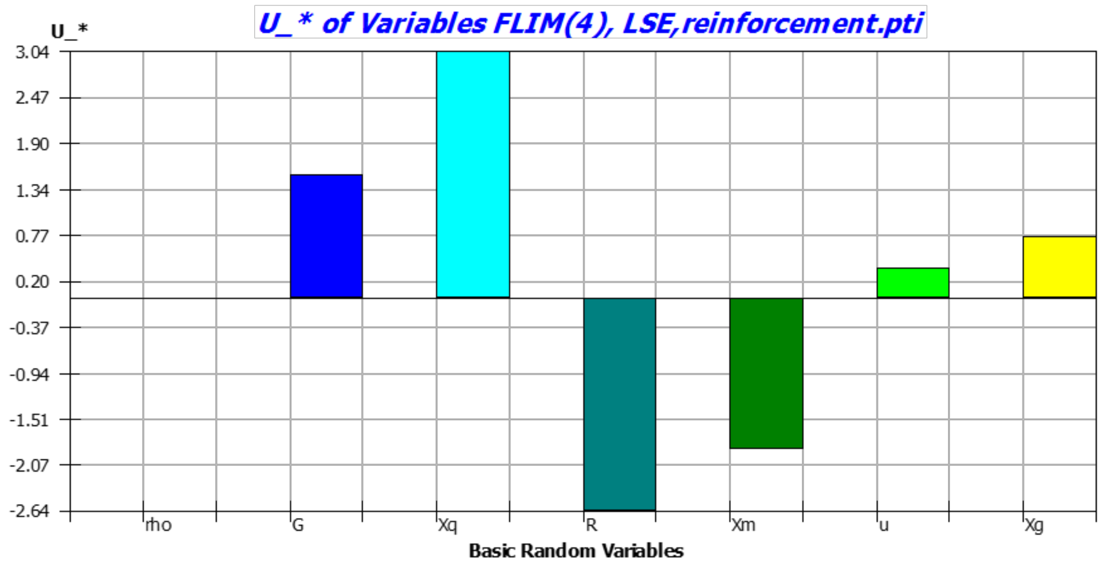

Figure B.15: Comrel, $u*$- vector value for α=0,5

Elasticities of mean values

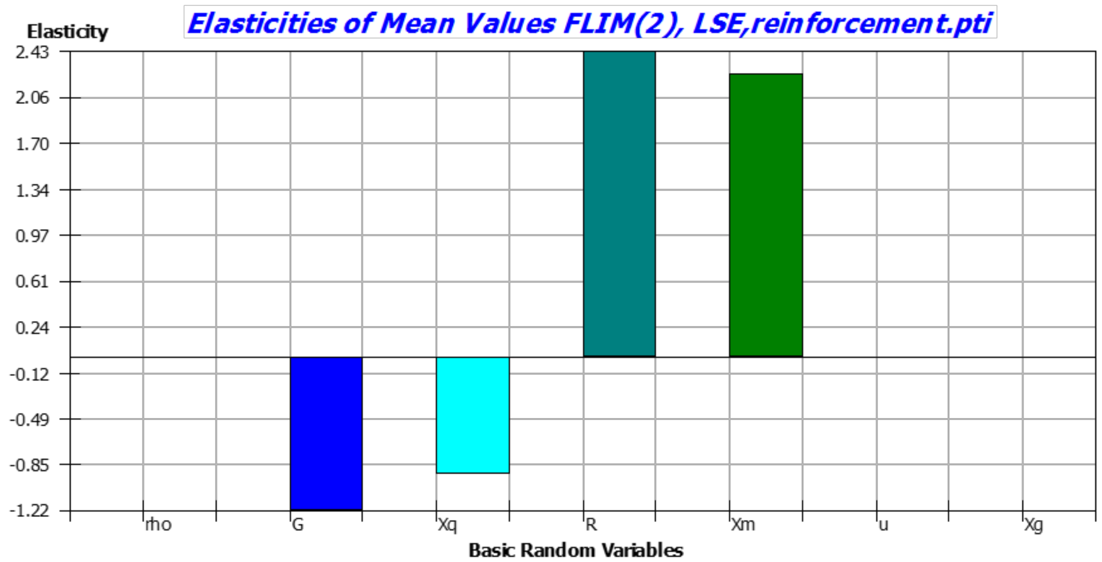

Figure B.16: Comrel, elasticities of mean values for α=0,3

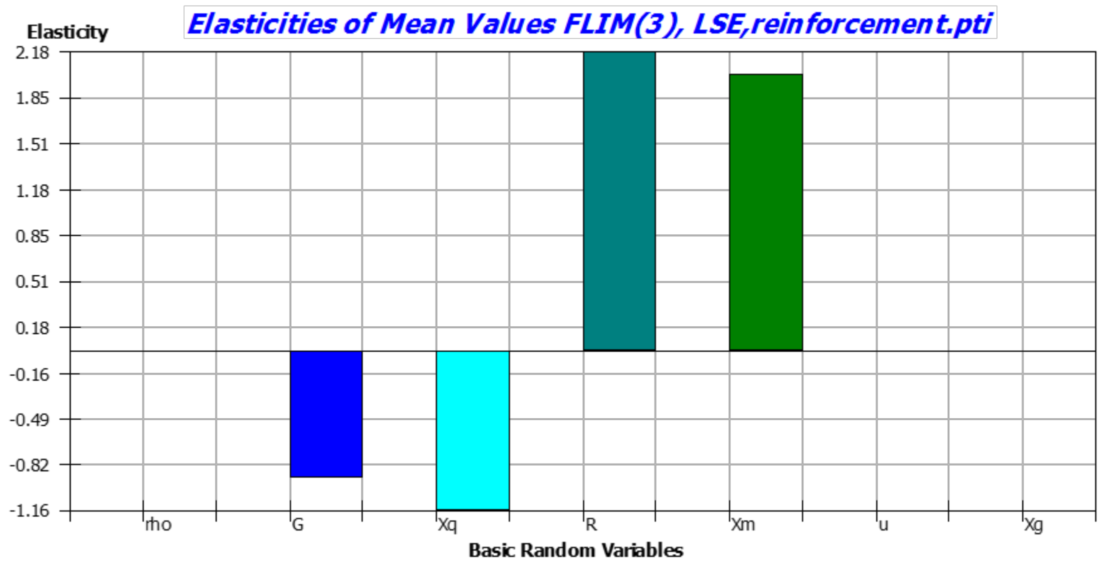

Figure B.17: Comrel, elasticities of mean values for α=0,4

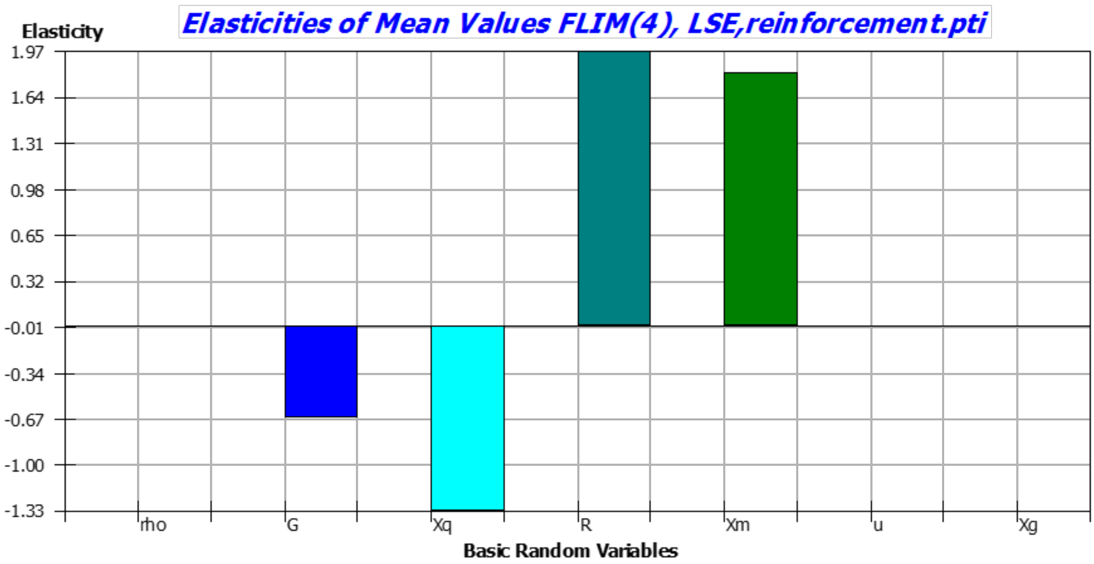

Figure B.18: Comrel, elasticities of mean values for α=0,5