
Optimal Control in Water Distribution

- Multivariable Control System -

Project Report

20gr1031



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Abstract:

This paper investigates MPC control for a water distribution network. A laboratory setup was used to emulate a real water distribution network. The laboratory includes a complete network with a pump, consumers, pipes and a water tower. The cost function for the MPC was designed to reduce operation cost and pressure variance. The results of the implementation were evaluated by its objective of reducing operation cost or pressure variance. However, this was first tested in a simulated environment to examine the behaviour of the MPC controller. The MPC was compared to an on/off controller with the ability to turn the pump on or off. The MPC proved to produce better results than the on/off controller for each objective.

Nomenclature

Abbreviation

Abbreviation	Meaning
Eq	Equation
Fig	Figure
MPC	Model predictive control
OD	Opening degree
PI	Proportional–Integral

When reading this report, there will be repeating notations such as, subscript k which will refer to the k^{th} component in the network, subscript C refers to the chords where subscript T refers to the tree. When referring to the water tank, τ is used, e.g. p_τ would be the pressure at the water tank node.

General symbols

Symbol	Meaning	Unit
Δp	Difference in pressure	[bar]
\dot{p}	The rate of change in pressure	[bar/h]
p	Pressure	[bar]
\dot{q}	The rate of change in flow	[m ³ /h ²]
q	Flow rate	[m ³ /h]
λ	Pressure lost due to resistance in a pipe	[bar]
h	Elevation of water level in the water tank	[m]
J	Mass inertia of water	[kgm ²]
Z	Pressure change due to geodesic level difference	[bar]
D	Diameter of a pipe	[m]
h_f	Surface resistance	[Pa]
h_m	Form resistance	[Pa]
L	Length of given pipe	[m]
f	Coefficient of surface resistance	[]
ϵ	Height of roughness projection	[m]
R	Reynolds number	[]
k_f	Form-loss coefficient	[]
Ω	The full resistance of the pipe	[bar]
P	Power consumption	[W]
η	Pump efficiency	[]
τ	Cross sectional area	[bar/m ³]
p_h	Pressure head	[m]
d_p	Flow rate from the pump	[m ³ /h]
d_c	Flow rate from the consumers	[m ³ /h]
d_τ	Flow rate at the water tank node	[m ³ /h]
p_p	Pressure at the pump	[bar]
P_{rice}	Flow rate at the water tank node	[DKK]

Graph theory

Symbol	Meaning
v	Nodes
N	Set of nodes
e	Edges
E	Set of edges
A	Incidence matrix
G	Directed and connected graph
B	Loop matrix
M	Transformation vector
d	Disturbance vector

Constants

Symbol	Meaning	Unit
ρ	Water density	$[kg/m^3]$
g	Gravitational acceleration	$[m/s^2]$

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Chapter 1

Introduction

Water distribution is an essential factor in human society, and with an increase in population and industry, the need for water increases. Therefore is the groundwater a valuable resource, and with increasing water demand, the water supply system needs to be more efficient. Having an efficient water supply system can be achieved by having a pipe layout which can be easily maintained, repaired or/and having control which uses the future electricity prices, water consumption and leakage predictions to control the system. Improving the system can also be seen as reducing a by-product like CO_2 , as shown in [1].

The need for water is increasing due to population growth which will result in growth in the network, and therefore, it is essential to optimize the water network [2]. In 2016 an average Dane would use 104 litres of water daily. However, this consumption has been reduced by 15% from 2006 [3]. The expanding water networks also increase the importance of handling pressure variation for the consumers, and the economic burden increases as well since the network affects more people. Energy prices vary throughout the day, due to the change in prices and therefore is it essential to plan the usage of the pump [4]. The energy prices vary, but it has a pattern whereas the price is lowest in the morning and night and most expensive in the day, this is because the prices follow the demand for electricity and other factors [5]. When improving a water distribution network, there are two areas to optimize, and these two are consumer comforts and supplier benefits. The consumer comfort can be a reduction in pressure variation, so the pressure does not change during the day. Whereas the supplier wants to reduce the cost of operating the network by storing more water when the price is the lowest.

Different control approaches have been investigated like the use of model predictive control (MPC) seen in [6] and [7]. The benefit of MPC compared to other controllers is that it can use future estimates to provide an input which uses knowledge about the future. The change in demand from the consumers and prices can be taken into account for and used to achieve the desired goal. This project will investigate how a

water tower can be used to reduce energy consumption while having an acceptable pressure at the consumers. An optimal controller will be used, to accomplish an optimal control of a water distribution network, and therefore will MPC be investigated. The focus and problem statement of this report is:

Problem statement: *How can a water distribution network be optimized, in terms of pressure variation and economics with the use of MPC.*

Chapter 2

Network Analysis

This chapter will explore each part of a water distribution network and explain the functionalities. First, a general understanding of the parts in a water distribution network is needed. After that will the dynamics be explored, which leads into a complete dynamic model of the whole system.

2.1 Water distribution networks

The function of a water distribution network is to deliver high-quality water while being able to satisfy the consumer's water demand. Water distribution networks can vary in size and complexity, an example of a water distribution network is shown in Fig. 2.1 and the different colours indicate the different sizes of pipes. Before the water is distributed to the consumers in the network, it has to follow regulations.

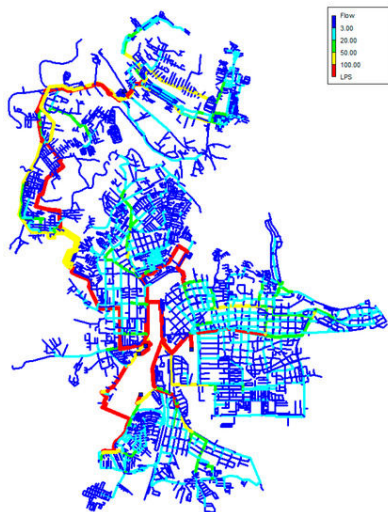


Figure 2.1: Water distribution network with different pipe diameters [8].

After the groundwater has been cleaned, then the quality of the water depends on the distribution network, so the water is safe to drink and use for households and industry consumption. A method for keeping the water quality in the desired range is to keep the water flowing in an acceptable rate in the pipes which would result in less clustering of salt and metal on the pipe walls, thus less troublesome microorganisms [9]. A simplified water distribution network is shown in Fig. 2.2. In the simplified system, the pump is the main element giving water to the system and when the water consumption is greater than the input the water tower applies the difference and when the water consumption is less the water in the water tower increases.

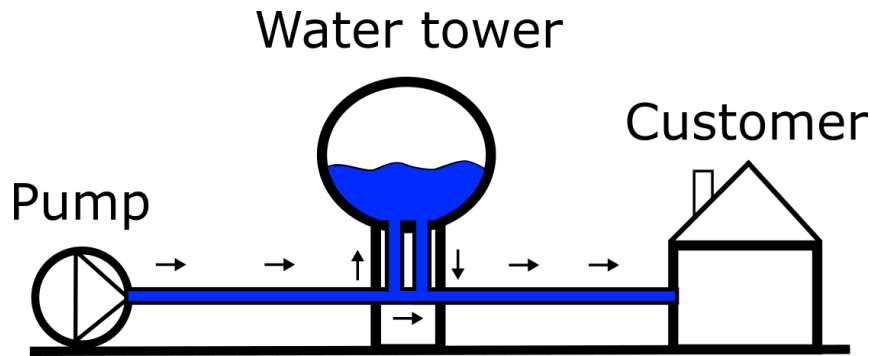


Figure 2.2: Simplified water distribution network.

A more extensive network and different altitudes make water transportation more complex and change the pressure, a way to handle this problem is to use a water tower. A water tower is elevated storage of water which therefore gives another source of pressure. Without the water tower, the network would lose the additional supply of water and pressure, the consequences of not having a water tower would be to have the pump producing the demanded amount of water at any time. While using a water tower, the controller would be able to use water from the storage if the price of water is too high or if the pumps cannot produce enough flow. Thus, the water tower is a useful tool for optimizing water distribution networks.

Pipe network

Water distribution networks have different layouts, each bringing different pros and cons. The layout will be chosen depending on what is needed for the network to perform. The three layouts which will be looked into are the branch, grid and loop configuration. The three layouts all have three different types of pipes; transmission line, arterial main, and distribution main, this is shown in figure 2.3 and are sized respectively.

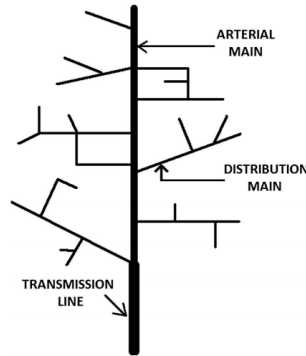


Figure 2.3: Simplified branch layout[10]

The first layout is the branched configuration and is shown in Fig. 2.3, as the name applies it branches out in smaller pipes. It is to be noted that there is not drawn any cut-off valves, but in this layout, they would be placed at the beginning of a distribution line and therefore be able to cut parts off. The advantage of this layout is its simplicity. However, since it is mainly one pipeline, it can be a problem if leaks appear in it, and due to the tree-like design, remote areas can increase unwanted difference in pressure [10].

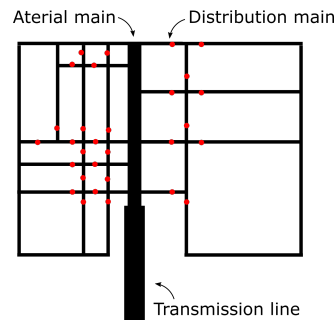


Figure 2.4: Simplified grind layout, with cut-off valves shown as red dots.

The second layout is the grid configuration which is shown in Fig. 2.4. The cut-off valves would be placed so cutting off a specific distribution line would be possible, which means that many more valves would be needed for this configuration compared to branched. The advantages are the ability to cut off small parts of the network if it needs to be repaired and due to the intersections, a smaller amount of pressure is lost. The disadvantages are that the network requires more pipes and cut-off valves than the breach, which makes it more expensive to make [10].

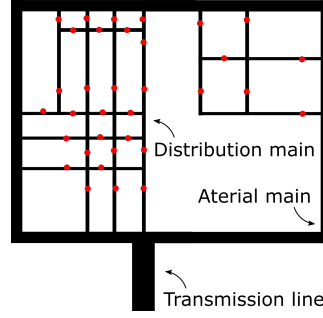


Figure 2.5: Simplified loop layout, with cut-off valves shown as red dots.

The third layout is the loop configuration, and as the name applies, there is a loop through the pipes, this is shown in figure 2.5. It is to be noted that the loop does not require arterial lines as the outer ring also can be distribution lines. The advantages and disadvantages are similar to the grid. However, it is more suited for well-planned cities, and the arterial lines are much longer, which means a higher quantity of water will be available when needed [10].

2.2 pipes

The pressure difference in all components can be described as:

$$\Delta p_k = p_{in,k} - p_{out,k} \quad (2.1)$$

Where Δp_k is the pressure difference, and subscript k refers to the k^{th} component. For the pipe model, it is assumed that all the pipes have the same diameter and the forces acting on the pipe is shown in figure 2.6. The change in pressure is given as:

$$\Delta p_k = J_k \dot{q}_k + \lambda_k(q_k) + Z_k \quad (2.2)$$

Where, J_k is mass inertia of water in the given pipe, $\lambda_k(q_k)$ change in pressure because of resistance in k^{th} pipe and Z_k is a pressure change due to geodesic level difference. The mass inertia of water J_k can be neglected, and this is due to the water distribution network in this work will have a water tower. The dynamic of the water tower has a more significant influence than the pipes.

The λ can be split up into two parts, surface resistance and form resistance, h_f and h_m , respectively. Surface resistance can be found by the use of the Darcy-Weisbach equation [11]:

$$h_f = f \frac{8Lq^2}{\pi^2 g D^5} \quad (2.3)$$

Where, D is the diameter of the given pipe, $[m]$, g is gravitational acceleration, $[m/s^2]$, L is the length of given pipe, $[m]$, f is the coefficient of surface resistance and h_f surface resistance which results in a head loss. Head loss is measured in meter and is the height the water can climb in the network [11]. The coefficient of surface resistance can be calculated by:

$$f = 1.325 \left(\log \left(\frac{\epsilon}{3.7D} + \frac{5.74}{R^{0.9}} \right) \right)^{-2} \quad (2.4)$$

Where ϵ is the height of roughness projection, and \mathbf{R} is the Reynolds number. Eq. (2.4) is chosen due to the flow being turbulent, i.e. $\mathbf{R} \geq 4000$. The form resistance is losses which appear due to the changes in the pipe, like bends and intersections. Therefore will any imperfections to the pipe will result in a form loss. The form resistance is found by Eq. (2.5).

$$h_m = k_f \frac{8q^2}{\pi^2 g D^4} \quad (2.5)$$

Where k_f is the form-loss coefficient and can be challenging to estimate and depends on the pipe layout [11]. Both resistances shows the loss in pressure head and therefore needs to be changed, so it is the wanted units $[Pa]$. The converting is done by $p = \rho g p_h$, where ρ is water density and p_h is the pressure head. It is to be noted that Z can be rewritten by the correlation between pressure and pressure head.

With the two resistances showing pressure drop Eq. (2.2) can be rewritten as:

$$\Delta p_k = \left(f \frac{8L\rho}{\pi^2 D^5} + k_f \frac{8\rho}{\pi^2 D^4} \right) \|q_k\| q_k - pg\Delta h \quad (2.6)$$

This equation uses standard SI for pressure, $[Pa]$, and for flow, $[m^3/s]$. However, this will be changed into $[bar]$ and $[m^3/h]$, respectively. The transformation is performed to make the system more practical for later implementation. The unit transformation changes the equation into:

$$\Delta p_k = \left(f \frac{8L\rho}{\pi^2 D^5 10^5 3600^2} + k_f \frac{8\rho}{\pi^2 D^4 10^5 3600^2} \right) \|q_k\| q_k - \frac{pg\Delta h}{10^5} \quad (2.7)$$

In this report, the h_m is assumed to be equal to h_f onwards. The resistance due to the pipes will be described as:

$$\lambda(q_k) = \Omega_k q_k |q_k| \quad (2.8)$$

where, Ω is the summed value of h_m and h_f .

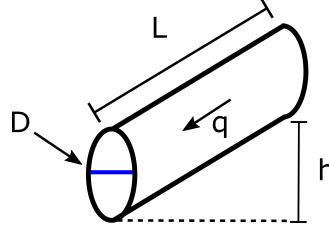


Figure 2.6: Water pipe units

2.3 Valve

A valve is used to control the flow of a fluid in a pipe network. The function of a valve is to control the fluid flow rate going through by choosing the opening degree (OD). There are different types of valves with different functions and some valves for specific applications. Flow control valves make it possible to choose a specific flow rate, and check valves function is to let the fluid flow one way only. Some valves like the ball valve can change the network structure while it is operating, valves with an on/off control, these valves are also called cut-off valves. The cut-off valves can change the structure by cutting off pipelines. Since valves have no energy consumption while not changing they act like passive actuators. The dynamics of a valve can be seen as a pressure drop from one point to another, and this is shown in Eq. (2.9).

$$\Delta p_k = p_{in,k} - p_{out,k} = \mu_k(q_k, OD_k). \quad (2.9)$$

Where,

$$\mu_k(q_k) = \frac{1}{K_{v,k}(OD_k)^2} |q_k| q_k \quad (2.10)$$

q_k is the flow rate, $K_{v,k}$ is the hydraulic resistance in the valve, OD_k is the opening degree and μ_k is a smooth function which is strictly monotone increasing. When there is no flow through the valve, then $u_k = 0$ [12].

2.4 Pump

Pumps are used for moving fluids in water networks, and pumps can move fluids by changing the pressure. There are three different approaches to transporting the fluids, direct lift, displacement, and gravity pumps. Pumps generate pressure with mechanical work which can be rotary or reciprocating. Since a pump generates mechanical work, it needs a power source which can be controlled, therefore acts as an active actuator in a water network. The relation between power consumption and

pressure is formulated as:

$$\Delta p = \frac{P\eta}{q} \quad (2.11)$$

Where P is the power consumption $[W]$, Δp is the change in pressure $[Pa]$, q is the volume flow rate $[m^3/s]$ and η is the pump efficiency.

2.5 Water tower

The function of a water tower is to have another method to generate pressure and also have a water tank to use for supplying water if needed. The water tower can generate pressure due to the difference in altitude, and it can also provide water to the network if needed. This means the water towers are an essential part of the network since it can be filled up when the price is low or when the consumption of water is low, and it can be used when the price is high or supply extra water into the system when needed. For the model, it is assumed that the tank is a cylinder.

$$\dot{p}_\tau = -\tau q_\tau \quad (2.12)$$

Where \dot{p}_τ is the rate of change in pressure, q_τ is the flow to or from the water tower, and this means the flow can be negative whenever the pump is producing more than the consumers or positive when supplying water to the network. τ is a parameter made from the cross-sectional area:

$$\tau = \rho g \frac{1}{\pi r^2} \quad (2.13)$$

With the equation for pressure head, the units can be changed to be describing the height instead of the pressure:

$$p = p_h \rho g \quad (2.14)$$

where p_h is the pressure head of the fluid on the water tower and therefore Eq. (2.12) can be rewritten as:

$$\dot{p}_h = -\tau q \quad (2.15)$$

where,

$$\tau = \frac{1}{\pi r^2} \quad (2.16)$$

2.6 Graph-based network modelling

This section will go through the formulation of the model and explain the different steps and assumptions. Fig. 2.7 shows a graphical hydraulic system where the edges are referred to as e_k and nodes as v_k and have two customers, a pump and a water tower. The customers are shown as edge 8 and 9, the pump is edge 11, the water tower is edge 10, and the remaining edges are pipes. This is a network in is simplified in size and complexity compared to a bigger network, a network can have multiple pumps, customers and water towers.

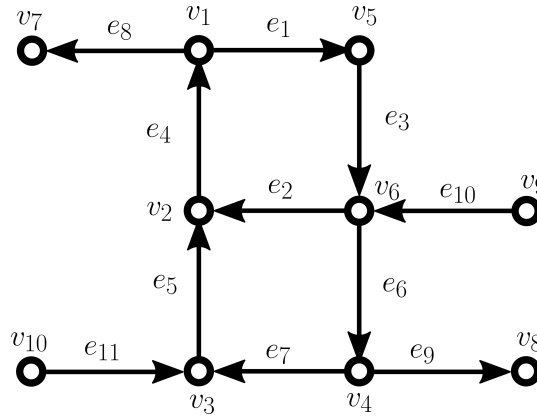


Figure 2.7: Graph-based network

The graphical network can be simplified by having the inputs and outputs of the network to be seen as disturbances and therefore, the last four nodes will be removed, and the last four edges will be disturbances. This can be seen in Fig. 2.8 where the customers, pump and water tower is replaced by d_1 and d_2 , d_p and d_τ , respectively. The system which will be used for this project and the nodes and edges will stay the same way as Fig. 2.8.

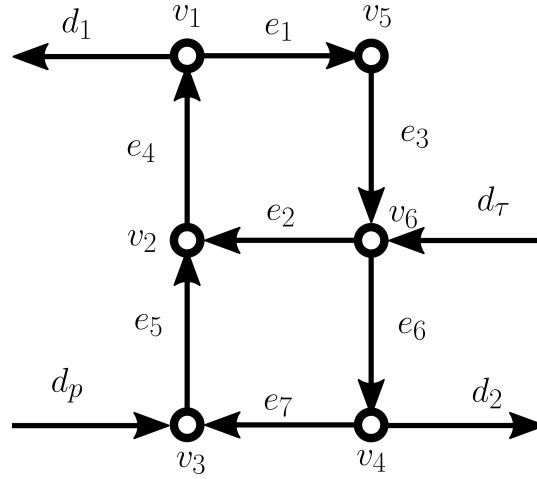


Figure 2.8: Simplified graph-based network

Fig. 2.8 can be described by a set of nodes and edges. The advantage of this is to have a mathematical representation of the network, and this is expressed as G .

$$G = (N, E) \quad (2.17)$$

Where N is a set of nodes $N = v_1, \dots, v_n$ and E is a set of edges $E = e_1, \dots, e_n$

$$A_{ij} = \begin{cases} 1, & e_j \text{ is leaving } v_i \\ -1, & e_j \text{ is entering } v_i \\ 0, & \text{otherwise} \end{cases} \quad (2.18)$$

The incidence matrix A of the network shows the connections of the pipes in the system. The edges can be divided into chords, and the remaining are the tree edges. The chords have to be selected so the fewest amount of edges is removed and no loops occur in the system. For this network, e_1 and e_2 is chosen and will be the first two columns.

$$A = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (2.19)$$

A node has to be removed to get the reduced incidence matrix, and the node will be used as a reference. The selected reference node is node 6, which is the water tower.

$$\bar{A} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.20)$$

However, \bar{A} can be split up into \bar{A}_C and \bar{A}_T which are representing the chords and tree part, respectively. This notation will be used for other variables as well when referring to the chords or three edges.

$$B_{ij} = \begin{cases} 1, & e_j \text{ is following the loop direction of chord } i \\ -1, & e_j \text{ is the opposite direction of the chord } i \\ 0, & \text{if } e_j \text{ is not in the loop} \end{cases} \quad (2.21)$$

B represent all the loops of the network and is construed with Eq. (2.21). B_s is created by the network loops that go through only one of the selected chords, e_1 or e_2 :

$$B_s = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \quad (2.22)$$

Hydraulic network model formulation

A hydraulic network can be described with the use of Kirchoff's node law:

$$Aq = 0 \quad (2.23)$$

Where q is a vector with the flow of each edge, however, the network has been reduced and by doing that introduced an extra element d , disturbance.

$$Aq = d \quad (2.24)$$

Where d is the vector of the extra inputs which is applied to the network, and each entry can be positive or negative. If an entry of d is negative, then water is being removed from the system, and if d is positive, then water is running into the system. Vector d will be referred to as a disturbance even though the controlled input is part of it. It is to be noted that the disturbances are known, and due to mass conservation they add up to 0:

$$0 = d_\tau + d_p + d_{c1} + d_{c2} \quad (2.25)$$

Where, d_τ is the water tower flow rate, d_{c1} and d_{c2} are the consumers disturbance flow rate, d_p is the pump flow rate and controlled input. The remaining nodes which are not connected to any pump, customer or water tower do not have any disturbance, and therefore they are seen as 0. All of the disturbances can then be described as:

$$d = M_p d_p + M_c d_c + M_\tau d_\tau \quad (2.26)$$

Where, d_c is a vector of both customers disturbances, M_p is a vector representing where the pump is connected to the node and transforms d_p into the space of d , M_c and M_τ is the same for d_c and d_τ , respectively.

$$M_p = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad M_c = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_\tau = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (2.27)$$

By removing the reference node, M will be reduced to \bar{M} :

$$\bar{M}_p = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \bar{M}_c = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \bar{M}_\tau = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.28)$$

thus,

$$\bar{d} = \bar{M}_p d_p + \bar{M}_c d_c + \bar{M}_\tau d_\tau \quad (2.29)$$

The flow rates of the network can be divided into the flow rate in the chords q_C , and the tree edges' flow rate q_T . However, the disturbances also need to be accounted for, and q can be derived as:

$$q = \begin{pmatrix} q_C \\ q_T \end{pmatrix} = B_s^T q_C + \begin{bmatrix} 0_{1 \times n-1} \\ A_T^{-1} \end{bmatrix} \bar{d} \quad (2.30)$$

Eq. (2.30) is formulated with the use of Kirchhoff's first law, which states that the total flow that goes into a node must also be the total flow out, therefore by looking at each node other than the reference 6 equations can be derived.

$$n_1 : 0 = -q_1 + q_4 + d_{c1} \quad (2.31a)$$

$$n_2 : 0 = q_2 - q_4 + q_5 \quad (2.31b)$$

$$n_3 : 0 = -q_5 + q_7 + d_p \quad (2.31c)$$

$$n_4 : 0 = q_6 - q_7 + d_{c2} \quad (2.31d)$$

$$n_5 : 0 = -q_3 + q_1 \quad (2.31e)$$

With Eq. (2.31) all tree edge flows can be isolated for:

$$q_3 = q_1 \quad (2.32a)$$

$$q_4 = q_1 - d_{c1} \quad (2.32b)$$

$$q_5 = q_1 - q_2 - d_{c1} \quad (2.32c)$$

$$q_6 = q_1 - q_2 - d_{c1} + d_{c2} + d_p \quad (2.32d)$$

$$q_7 = q_1 - q_2 - d_{c1} + d_p \quad (2.32e)$$

Kirchhoff's second law states that the pressure drop for a closed loop in a hydraulic network should end up having no pressure difference. The pressure drop due to the pipes is formulated in Eq. (2.8) and since the network is only made of pipes, Kirchhoff's second law can be written as:

$$0 = B_s \lambda(q) \quad (2.33)$$

where $\lambda(q)$ is the vector with pipe resistance found in Eq. (2.8). Eq.(2.33) can be used as two equations with two unknown being the two flow chords q_1 and q_2 , and it is possible due to Eq. (2.32) where all the tree flows are written with respect to the chords. Thus,

$$0 = \lambda(q_1) + \lambda(q_3) + \lambda(q_6) + \lambda(q_7) + \lambda(q_5) + \lambda(q_4) \quad (2.34)$$

$$0 = \lambda(q_2) - \lambda(q_5) - \lambda(q_7) - \lambda(q_6) \quad (2.35)$$

This method obtains q_C with the entries q_1 and q_2 . Now, with q_C the next step is to calculate q_T . This is done by replacing B_s and \bar{d} in Eq. 2.30 so q_T is the output, thus q_T can be calculated:

$$q_T = -\bar{A}_T^{-1} \bar{A}_C q_C + \bar{A}_T^{-1} M_p d_p + \bar{A}_T^{-1} M_c d_c + \bar{A}_T^{-1} M_\tau d_\tau \quad (2.36)$$

The difference in pressure and height can be described as:

$$\Delta p = A^T p \quad (2.37)$$

$$\Delta z = A^T z \quad (2.38)$$

Eq.(2.37) can be rewritten into:

$$\begin{bmatrix} \Delta p_C \\ \Delta p_T \end{bmatrix} = \begin{bmatrix} A_C^T \\ A_T^T \end{bmatrix} p \quad (2.39)$$

thus,

$$A_T^T p = \Delta p_T = \lambda_T(q_T) - \Delta z_T \quad (2.40)$$

From Eq. (2.38) it can be further rewritten into:

$$A_T^T p = \Delta p_T = \lambda_T(q_T) - A_T^T z \quad (2.41)$$

Lemma 1 *Let T be a directed tree with the incidence matrix A_T and reduced incidence matrix \bar{A}_T (without loss of generality we assume that we delete the last row of A_T to obtain \bar{A}_T). The reduced incidence matrix is invertible since a tree is a connected graph with $n - 1$ edges [13]. Then the following holds*

$$A_T \bar{A}_T^{-1} = \begin{bmatrix} I_{n-1} \\ -\mathbb{1}^T \end{bmatrix} \quad (2.42)$$

where $\mathbb{1}$ is a vector of ones, and I_{n-1} is the $n - 1$ identity matrix [14].

Using Eq. (2.41) and multiplying \bar{A}_T^{-T} to each side and thereafter using Lemma 1, Eq. (2.41) can be expanded into:

$$\bar{A}_T^{-T} A_T^T p = \bar{p} - \mathbb{1} p_\tau = \bar{A}_T^{-T} \lambda_T(q_T) - \bar{A}_T^{-T} A_T^T z \quad (2.43a)$$

$$\bar{p} - \mathbb{1} p_\tau = \bar{A}_T^{-T} \lambda_T(q_T) - (\bar{z} - \mathbb{1} z_r) \quad (2.43b)$$

$$\bar{p} = \bar{A}_T^{-T} \lambda_T(q_T) - (\bar{z} - \mathbb{1} z_r) + \mathbb{1} p_\tau \quad (2.43c)$$

Where \bar{z} is the height of the non-reference node, z_r being the height of the reference node and p_τ being the pressure of the reference node. Thus, the model is ready to be implemented.

Implementation of model

The specification of the pipes can be seen in table 2.1 and it is to be noted that pipe 1 is edge 1, and etc. The reason for choosing these numbers is because they fit the test setup, which will be explained later in this report.

Table 2.1: specification of pipes

Pipe	Length [m]	Diameter [m]	Height of roughness [m]
1	20	0.025	0.00005
2	10	0.015	0.00005
3	10	0.020	0.00005
4	5	0.025	0.00005
5	25	0.025	0.00005
6	20	0.020	0.00005
7	10	0.025	0.00005

The different elevations in the network with the first entry being the first node:

Table 2.2: Elevations of the nodes

Node	Height [m]
v_1	0.9
v_2	0
v_3	0
v_4	0.9
v_5	0
v_6	3

Table 2.3: Specification of the water tower located at node 2

Tank	Radius [m]	Height [m]	Cross sextional area [m ²]	Volume [m ³]
n6	0.3	0.7	0.28	0.28

Simulation of non-linear model

The simulation of the model is made in MATLAB/Simulink, and the model is simulated for 48 minutes in simulation time. However, this will be seen as 48 hours to emulate two days. This method of representing a day as 24 minutes will be used throughout the report. The algorithm for the simulation is implemented in a MATLAB function, and the approach can be seen below:

Start

Read input signals; d_{c1} , d_{c2} , d_p and p_6

Initialize:

Reduced incidence matrix (2.20)

Transformation vectors M_p and M_c (2.28)

Flow vector q with respect to the chords flow q_1 and q_2 (2.32)

Elevation of each node, shown on table 2.2

Calculate resistance in all pipes due to flow q (2.8)

Solve Eq. (2.33) for chord flows q_1 and q_2

Calculate tree flow q_T (2.36)

Calculate pressure of non-reference node \bar{p} (2.43)

Output pressure vector \bar{p} and d_τ

End

Algorithm 1: Implementation in MATLAB/Simulink

To control the network, a basic control designed to be an on/off controller has been used, the controller works by using the water level in the tank as an indicator to turn on/off. The chosen value for turning off is $0.35m$, and it turns on again at $0.2m$, this

can be seen in Fig. 2.9. The figure also shows how the level of water changes over time. The two consumer disturbances are shown in Fig. 2.10 and the flow supplied by the pump is shown in Fig. 2.11. Fig. 2.11 also shows the flow rate of d_τ , which represent the in and outtake of the water tower.

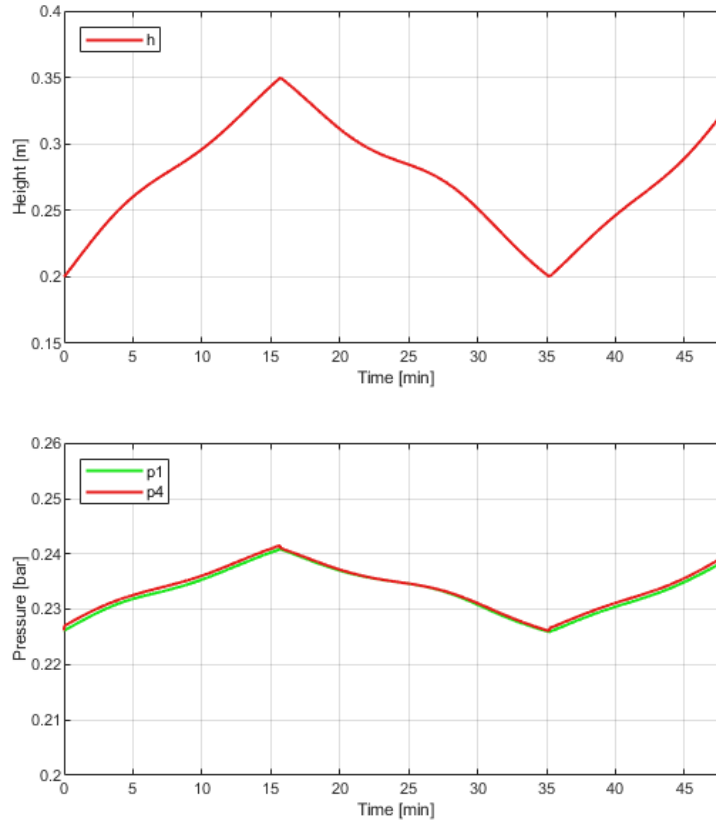


Figure 2.9: On/off controller controlling water level in tank and pressure at node 6 and the pressure for at the consumer nodes, p_1 and p_4

Fig. 2.9 shows the pressures at the customer node. Where, p_1 and p_4 are the customers. The pump is turned off whenever the water level reaches its maximum capacity. The change in input can be seen by the pressure drop. The reason why the pressure does not drop to 0 when the pump is turned off is due to the water tower. Furthermore, the effect of the pump turning off is also the reason for the slope downwards for p_1 and p_4 in Fig. 2.9.

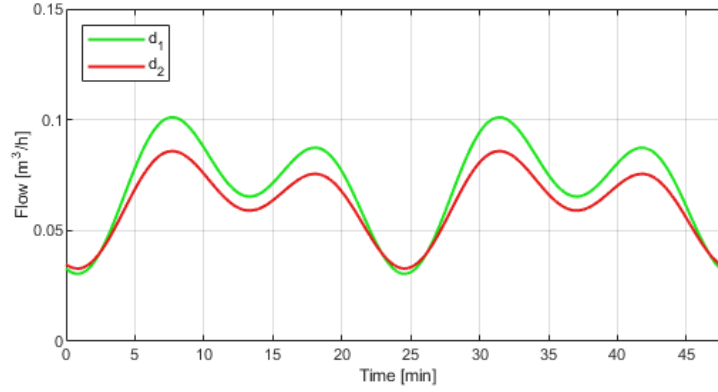


Figure 2.10: The customer's daily water usage flow pattern.

The demand from the customers shown in Fig. 2.10 is made to replica an average household consumption, and it is to be noted that the consumption is shown as positive values to make it more reader-friendly. The reasoning for the pattern is to simulate an average household by having a higher water usage in the morning and evening and lower by night.

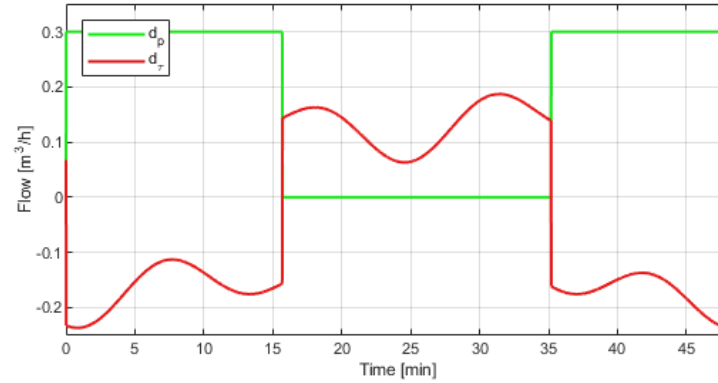


Figure 2.11: The flow from the pump and the flow from the node connected to the water tower.

The usage of the water tower and flow rate produced from the pump is shown in Fig. 2.11, where it can be seen when d_τ is negative the water tank takes water out of the system and stores it for when it is needed. Whenever d_τ is positive, it supplies the stored water into the system. While the pump is active, it is producing a constant flow rate of $0.3\text{m}^3/\text{h}$.

In this chapter, the model has been investigated and derived with the use of graph

theory and tested with an on/off controller. In the next chapter, an optimal controller will be designed to archive optimal inputs for the objective.

Chapter 3

Controller design

This chapter will focus on formulating the control problem and develop a control solution. To solve the problem Model Predictive Control (MPC) will be investigated and explained. The constraints and cost function will be developed, and the non-linear model is linearised to be able to design the MPC.

3.1 MPC and control problem

This project works towards achieving an optimal controller which controls the flow in a water distribution network and uses a water tower to minimize the cost function. The purpose of the controller is to minimize the cost function and thereby reducing operational cost and fluctuation in pressure for the consumers. First, a simple cost function is made to reduce operational cost and after that will pressure variation be added to the cost function. The cost function will be used by the MPC to find the preferred input. A simplified illustration of a MPC is shown in Fig. 3.1. MPC is a complex control method based on the model, and by using the model, it produces an optimal control input. A cost function will be described to represent the objective, and the purpose of the controller is to minimize the cost function by adjusting the input. MPC figures out the most optimal input by estimating a number of steps into the future, the number of steps is the prediction horizon. A system can become more effective by increasing the length of the prediction horizon. However, small model errors can propagate and lead to worse estimates. Weights in the cost function are used to provoke wanted behaviour, e.g. fewer changes to the input. The controller repeats itself at the next time step and calculates another input which is valued to be the most optimal at a given time.

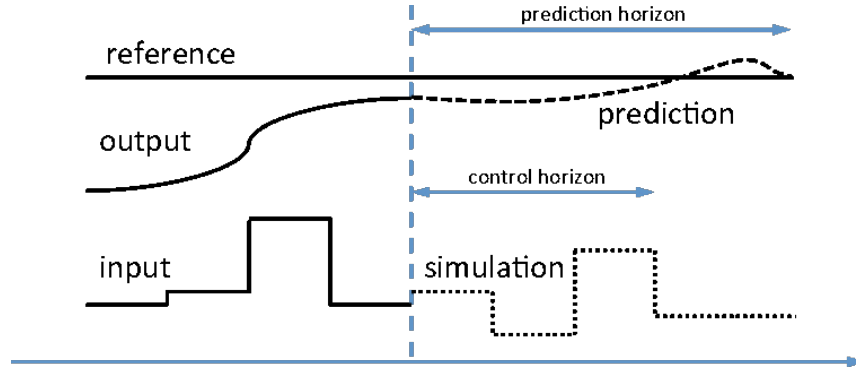


Figure 3.1: Illustration of MPC [15]

MPC can also satisfy multiple constraints and optimize the current moment while taking future predictions into account. To summarize, the objectives of this project are:

- Minimize the operational cost
- Minimize the pressure variation at the consumer's end

Cost function

The first objective is to minimize the operational cost, and this is accomplished by using the pump more efficiently. The power consumption is derived by the pressure at the pump, the flow produced and the efficiency of the pump.

$$P = \frac{d_p p_p}{\eta 10^3} \quad (3.1)$$

Where P is the power consumption, d_p is the flow rate from the pump, p_p is the pressure at the node connected to the pump and η is the efficiency of the pump which is assumed to be 0.8. However, the units have to be changed into $[bar]$ and $[m^3/h]$ and therefore.

$$P = \frac{u p_p}{\eta 10^3 10^5 3600} \quad (3.2)$$

To standardise, d_p will be rewritten into u following this. The price for electricity at a given time incident is needed to get the cost of the power consumption, and the price can then be calculated at every time sample as:

$$\epsilon = P P_{rice} T_s \quad (3.3)$$

Where ϵ is the price of using the pump, P_e is the price of electricity and T_s is the sample time. This can be rewritten into:

$$J = u\hat{P}P_{rice}p_pT_s \quad (3.4)$$

Where J is the cost and \hat{P} is:

$$\hat{P} = \frac{1}{\eta 10^3 10^5 3600} \quad (3.5)$$

The second objective is to minimize the difference in pressure at the nodes connected to the costumers, and this can be derived as the variance of pressures:

$$J = (p_c - p_{avg,c})^T (p_c - p_{avg,c}) \quad (3.6)$$

where p_c is a vector with the pressure of the nodes connected to costumers and $p_{avg,c}$ is a vector of the mean of p_c . The mean is accumulated with a moving window, and this means that it only uses current and previous of p_c . This type of mean calculation makes it more forgiving to change the pressure first, but the later estimates have to be similar to the first calculation. The reason for using $p_c - p_{avg,c}$ twice in the cost function is to ensure a positive cost. The two cost functions can be summed up as:

$$J = u\hat{P}P_{rice}p_pT_sQ + (p_c - p_{avg,c})^T R(p_c - p_{avg,c}) \quad (3.7)$$

The optimization problem can then be rewritten into.

$$v(k) = \min_u \sum_{i=0}^{N-1} \left(u_i \hat{P}P_{rice,i}p_{p,i}T_sQ + (p_{c,i} - p_{avg,c})^T R(p_{c,i} - p_{avg,c}) \right) \quad (3.8)$$

Where N is the prediction horizon, Q and R are both weights to adjust for desired behaviour.

Constraints

The constraints for this system are:

- Network model
- Tank model
- Limitations of the pump

The network model has been derived in section 2.6 and will be used to design the MPC controller. However, the non-linearities have to be linearized for it to be usable

for a standard MPC. The system equations are:

$$0 = \lambda_C(q_C) - B_{s,T} \lambda_T(q_T) \quad (3.9a)$$

$$\bar{p} = \bar{A}_T^{-T} \lambda_T(q_T) - (\bar{z} - \mathbf{1} z_r) + \mathbf{1} p_\tau \quad (3.9b)$$

$$p_\tau(k+1) = p_\tau(k) + \tau d_\tau(k) T_s \quad (3.9c)$$

where q_C is the flow in the chords and q_T is the flow in the tree and described as:

$$q_T = -\bar{A}_T^{-1} \bar{A}_C q_C + \bar{A}_T^{-1} M_p d_p + \bar{A}_T^{-1} M_c d_c + \bar{A}_T^{-1} M_\tau d_\tau \quad (3.10)$$

Recall, Eq. (3.9a) is obtained with the use of the law of conservation of mass, which means the flows in the loops sum up to 0. Eq. (3.9b) formulates the pressure of the nodes which are not connected to the water tower and Eq.(3.9c) is the discretized model of the pressure for the node connected to the water tower.

3.2 Model linearization

To linearize the non-linearity in Eq. (3.9a) and Eq. (3.9b) Taylor series approximation has been chosen. Operation points have to be chosen, to start with p_τ is chosen from experiments. d_{c1} and d_{c2} is chosen from their averages over a day. The flow d_τ has been chosen as 0 and u is therefore the negative sum of d_{c1} and d_{c2} because of:

$$0 = d_\tau + d_p + d_{c1} + d_{c2} \quad (3.11)$$

q_C , on the other hand, is calculated with Eq. (2.33) from section 2.6. The operation points are therefore:

$$\tilde{q}_C = \begin{bmatrix} -0.157 \\ 0.048 \end{bmatrix} \quad (3.12a)$$

$$\tilde{u} = 0.122 \quad (3.12b)$$

$$\tilde{d}_{c1} = -0.064 \quad (3.12c)$$

$$\tilde{d}_{c2} = -0.058 \quad (3.12d)$$

$$\tilde{d}_\tau = 0 \quad (3.12e)$$

$$\tilde{p}_\tau = 0.01 \quad (3.12f)$$

The two equations are:

$$f_1 \rightarrow \lambda_C(q_C) - B_{s,T} \lambda_T(q_T) = 0 \quad (3.13)$$

$$f_2 \rightarrow \bar{p} = \bar{A}_T^{-T} \lambda_T(q_T) - (\bar{z} - \mathbf{1}z_r) + \mathbf{1}p_r \quad (3.14)$$

The method for Taylor series approximation is

$$f_1 \approx \tilde{f}_1 + \left. \frac{\partial f_1}{\partial q_C} \right|_o (q_C - \tilde{q}_C) + \left. \frac{\partial f_1}{\partial u} \right|_o (u - \tilde{u}) + \left. \frac{\partial f_1}{\partial d_{c1}} \right|_o (d_{c1} - \tilde{d}_{c1}) + \left. \frac{\partial f_1}{\partial d_{c2}} \right|_o (d_{c2} - \tilde{d}_{c2}) + \left. \frac{\partial f_1}{\partial d_\tau} \right|_o (d_\tau - \tilde{d}_\tau) \quad (3.15)$$

Where \tilde{f}_1 is the system at the given operation point, and o indicates the given operating point.

$$q_C = \begin{bmatrix} 0.0195 & 0.0357 & 0.0113 \\ -0.0195 & -0.0325 & -0.0113 \end{bmatrix} \begin{bmatrix} u \\ d_{c1} \\ d_{c2} \end{bmatrix} - \begin{bmatrix} 1.03e-6 \\ -4.45e-6 \end{bmatrix} \quad (3.16)$$

The same process is done for f_2 , which leads to:

$$\bar{p} = \begin{bmatrix} -0.0357 & 0.0325 & 0.0195 & 0.0357 & 0.0113 & 1 \\ -0.0325 & 0.0325 & 0.0195 & 0.0325 & 0.0113 & 1 \\ -0.0195 & 0.0195 & 0.0195 & 0.0195 & 0.0113 & 1 \\ -0.0113 & 0.0113 & 0.0113 & 0.0113 & 0.0113 & 1 \\ 0.00183 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_C \\ u \\ d_{c1} \\ d_{c2} \\ p_\tau \end{bmatrix} - \begin{bmatrix} 0.216 \\ 0.308 \\ 0.309 \\ 0.220 \\ 0.307 \end{bmatrix} \quad (3.17)$$

These two equations, Eq. (3.16) and (3.17) can be rewritten into:

$$q_C = A_{f1} \begin{bmatrix} u \\ d_{c1} \\ d_{c2} \end{bmatrix} - b_{f1} \quad (3.18a)$$

$$\bar{p} = A_{f2} \begin{bmatrix} q_C \\ u \\ d_{c1} \\ d_{c2} \\ p_\tau \end{bmatrix} - b_{f2} \quad (3.18b)$$

Validation of Linear model

It is essential to know that the linear and non-linear models are coherent and therefore, usable for the controller. Both models will be tested with an on/off controller and have a simulation time of 48 minutes. The linear model is essential because it will be used by the MPC to estimate the next states. The difference in consumer pressure p_1 and p_4 will be investigated to validate the linear model.

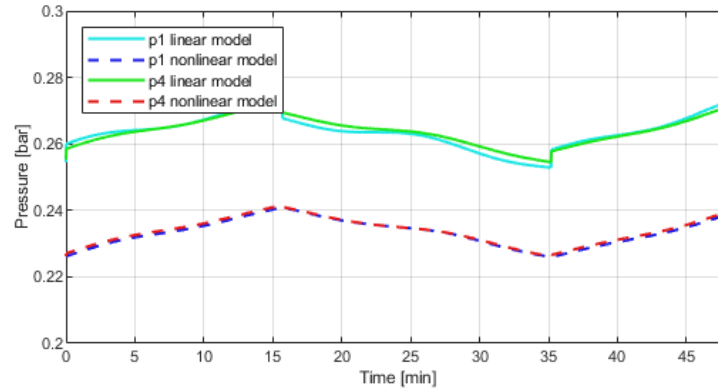


Figure 3.2: Pressure of node 1 and 4 with nonlinear model and Linear model

Figure 3.2 shows the pressures at the node connected to the customers. The input changes from $0.3m^3/h$ to 0 at the same time as the pressure goes straight down. The immediate change in pressure happens when the water level in the tank reaches a height of $0.35m$, and the opposite happens whenever the water level reaches $0.2m$. The difference in the outcome can be due to the point of linearization, which for the input is assumed to be the negative sum of the disturbance wherein reality the input is 0 or 0.3.

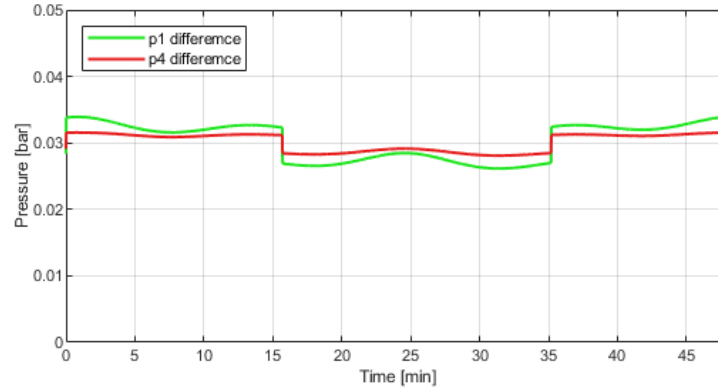


Figure 3.3: Difference non-linear model and Linear model

Figure 3.3 shows the difference more clearly, and the difference between the models are on the y-axis. Both pressures are close to an offset of 0.03, which is an acceptable accuracy for an approximation.

3.3 MPC implementation and results

This project uses YALMIP as a solver to the optimization problem in MATLAB, and the results will be displayed in 48 minutes to emulate two days. When implementing the MPC as a controller, constraints have to be formulated to fit the system's component specifications. Firstly, the pump can only add water to the network, and therefore it needs to have a flow larger than 0, and the limit of the pump determines the maximum flow. This pump-related constraint can be written as:

$$0 \leq u \leq u_{max} \quad (3.19)$$

Where u_{max} is the maximum flow from the pump, and 0 is the minimum since it cannot remove water from the network.

The water tank has some physical limits to it, which is, how much water can be stored into the tank.

$$h_{\tau,min} \leq h \leq h_{\tau,max} \quad (3.20)$$

Where $h_{\tau,min}$ is the minimum height in the water tank and $h_{\tau,max}$ is the maximum height at the water tank. $h_{\tau,min}$ and $h_{\tau,max}$ will be chosen to make sure the water level is in between the water tank safety levels

The MPC optimization problem can now be derived with the linearized model equations.

$$v(k) = \min_u \sum_{i=0}^{N-1} \left(u \hat{P} P_e p_p T_s Q + (p_c - p_{avg,c})^T R (p_c - p_{avg,c}) \right) \quad (3.21)$$

subject to

$$q_C = A_{f1} \begin{bmatrix} u \\ d_{c1} \\ d_{c2} \end{bmatrix} - b_{f1} \quad (3.22a)$$

$$\bar{p} = A_{f2} \begin{bmatrix} q_C \\ u \\ d_{c1} \\ d_{c2} \\ p_\tau \end{bmatrix} - b_{f2} \quad (3.22b)$$

$$p_\tau(k+1) = p_\tau(k) + \tau d_\tau(k) T_s \quad (3.22c)$$

$$0 \leq u \leq 0.3 \quad (3.22d)$$

$$0.2 \leq h \leq 0.35 \quad (3.22e)$$

Soft constraints

When applying MPC constraints to a system, whether it is a simulation or an implementation, problems can arise whenever the disturbances are prominent. The disturbances can make it, so the MPC is unable to follow the constraints which will make the problem infeasible. However, a slack variable can be used to increase the already established constraint at a high cost. A slack variable needs a high weight for it only to be used whenever the system is not able to stay inside a constraint [16]. The slack variable is e will be used to soften the constraint on the height of the water, this means the system will be able to go below $0.2m$ if needed to or above $0.35m$. The weight on the slack variable is D and is chosen to be large enough to limit any use of e . The use of a slack variable changes the cost function and makes it, so e has to be minimized as well, and thus the optimization will be derived as:

$$v(k) = \min_{u,e} \sum_{i=0}^{N-1} \left(u \hat{P} P_{rice} p_p T_s Q + (p_c - p_{avg,c})^T R (p_c - p_{avg,c}) + e D \right) \quad (3.23)$$

subject to

$$q_C = A_{f1} \begin{bmatrix} u \\ d_{c1} \\ d_{c2} \end{bmatrix} - b_{f1} \quad (3.24a)$$

$$\bar{p} = A_{f2} \begin{bmatrix} q_C \\ u \\ d_{c1} \\ d_{c2} \\ p_\tau \end{bmatrix} - b_{f2} \quad (3.24b)$$

$$p_\tau(k+1) = p_\tau(k) + \tau d_\tau(k) T_s \quad (3.24c)$$

$$0 \leq u \leq 0.3 \quad (3.24d)$$

$$0.2 - e \leq h \leq 0.35 + e \quad (3.24e)$$

$$0 \leq e \quad (3.24f)$$

Results

To be able to simulate the network with a MPC the water tower needs an initial condition of at least $0.2m$ water, This is due to the safety of the tank and therefore a constraint in the MPC. Therefore will the pressures start above 0, which will be shown in the results. First off, some price curve has to be chosen for the test, the purpose of these results is to test the MPC, so the price does not have to fit a realistic evolution or be realistic prices. Therefore has a square wave been chosen to represent

the price for electricity over a day, this is shown in Fig. 3.4. The weights have to be selected to satisfy the desired behaviour of the MPC and to achieve low energy consumption Q has to be weighted higher than R . The prediction horizon for this test is 6 steps.

$$Q = 1e9$$

$$R = 1$$

$$D = 1e7$$

$$N = 6$$

Where Q is the weight for energy, R is the weight for pressure differences, D is the weight for the slack variable, and N is the prediction horizon.

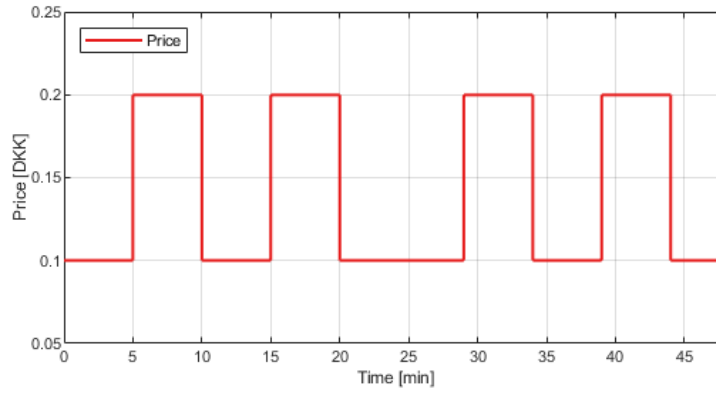


Figure 3.4: Price of electricity during one day

With the prices shown in Fig. 3.4, the larger usage of the pump can be estimated to be between 0-5, 10-15 and 20-24 for the MPC to work as expected. Consumption is shown in Fig. 3.5 with two daily peaks at 7 and 17.

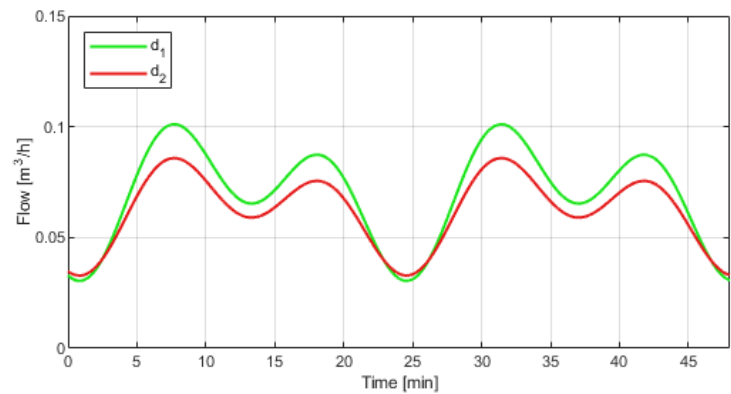


Figure 3.5: Consumption of water by costumers

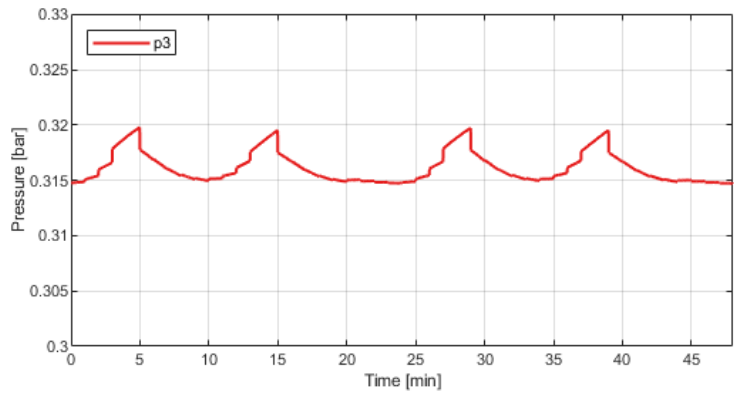


Figure 3.6: Pressure at nodes connected to the pump

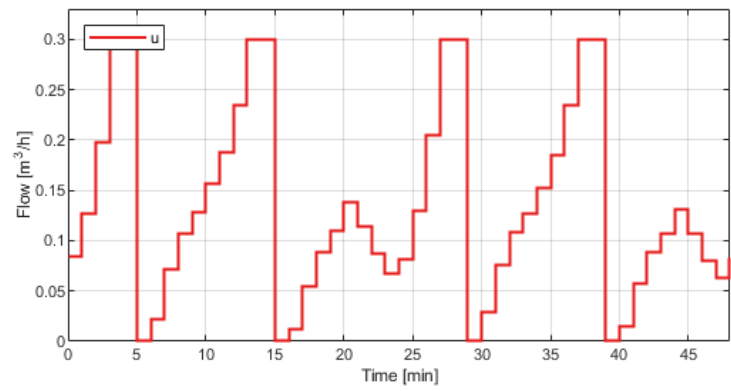


Figure 3.7: Flow rate generated by the pump

Fig. 3.6 shows the pressure of node 3, which is connected to the pump. Fig. 3.7 shows the control input generated by the MPC. With these two figures, it can be seen that the pump produces the most whenever the price is the lowest. However, it is also important that the water tower stores water whenever the price is low. Fig. 3.8 shows how the water level increases when the price is low. The spikes in the water level and pump pressure results in spikes in the pressure at the consumers, and this can be seen in Fig. 3.8

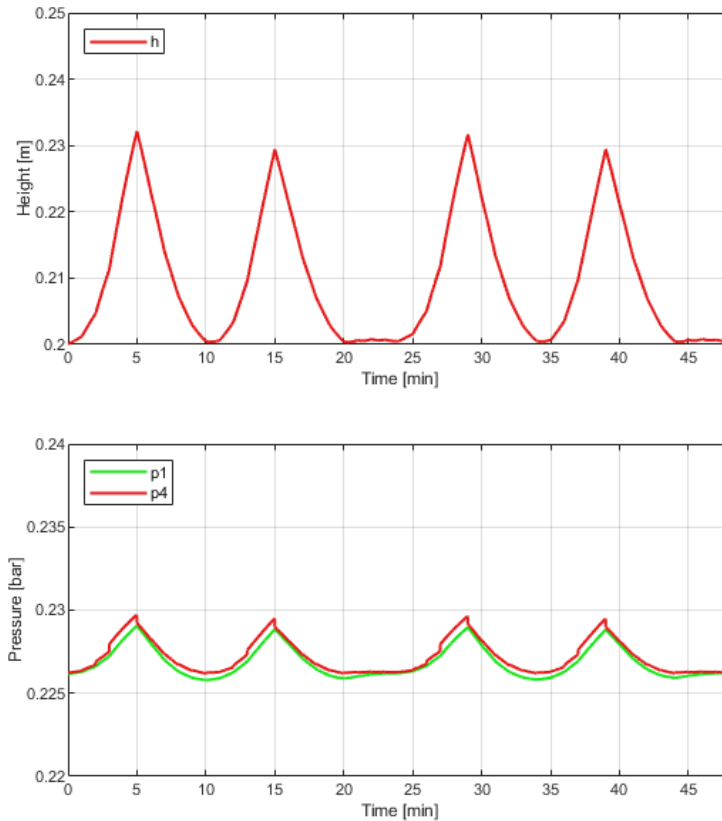


Figure 3.8: Pressure at node 1 and 4 with the level in the water tower for two days

With a prediction horizon of 6 steps and a cost function weighted towards reducing energy usage, and the MPC controller is not violating the constraints.

Turning variables

This MPC has two weights for tuning the importance of energy and consumer pressure stabilization, Q and R , respectively. However, another critical factor is the prediction horizon, and the MPC will be tested for three different lengths, 6, 12, and 24 steps. Fig. 3.9 shows how the total cost evolves, and it can be seen how similar

they are, it is to be pointed out that they are scaled with the same constant. The similarity can be due to the simple price pattern. Because the longest expensive stretch is only 5 steps, however, the longest stretch of cheap electricity is 9 steps, which is where the small prediction horizon has its shortcoming. Therefore if only looking at one day, a prediction horizon of 6 steps will be sufficient for detecting the next change in price for that day. However, this is not sufficient for multiple days due to the prediction horizon not being long enough.

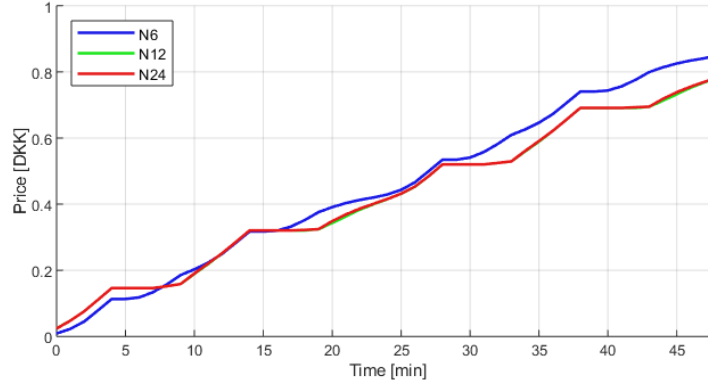


Figure 3.9: Commutative sum of cost over time with different prediction horizons

Table 3.1: Cost for different prediction horizons

Prediction horizon	Total cost
N6	0.85
N12	0.78
N24	0.78

Table 3.1 shows the total cost of running for 48 minutes where $N12$ and $N24$ performs the best. Since there is no real change in cost from $N12$ to $N24$, benefit gaining for choosing a larger than 24 will not outdo the extra computational time. Thus longer prediction horizons will not be investigated any further.

Pressure variation minimization

Two tests will be performed with a prediction horizon of 24, to examine how the relation of the weights changes the behaviour of the system. The first test was to have a greater weight on energy which will punish usage of the pump whenever the price is high, and this test is what can be seen above. The second test will be with a greater weight on pressure change, and both combinations have its use for each

objective. Fig. 3.10 shows the pressure which has been flattened out as a result of the objective. The water level in the tank can be seen in Fig. 3.10 and since the price has not much weight to it fills up regardless of the price. Table 3.2 shows the total cost of the two approaches with the MPC, where priority Q indicates a high weight on Q, and likewise for R. The highly weighted Q shows the lowest cost of operation. However, it is also the objective whereas a priority on R minimizes the pressure change instead. Therefore, is the pressure of the two consumer nodes what is essential, the results of this controller is shown on table 3.3. The variance of the controller with a high weighted R has a smaller variance, thus proves to lower pressure change for consumers.

$$\begin{aligned} Q &= 1 \\ R &= 1e9 \\ D &= 1e7 \\ N &= 24 \end{aligned}$$

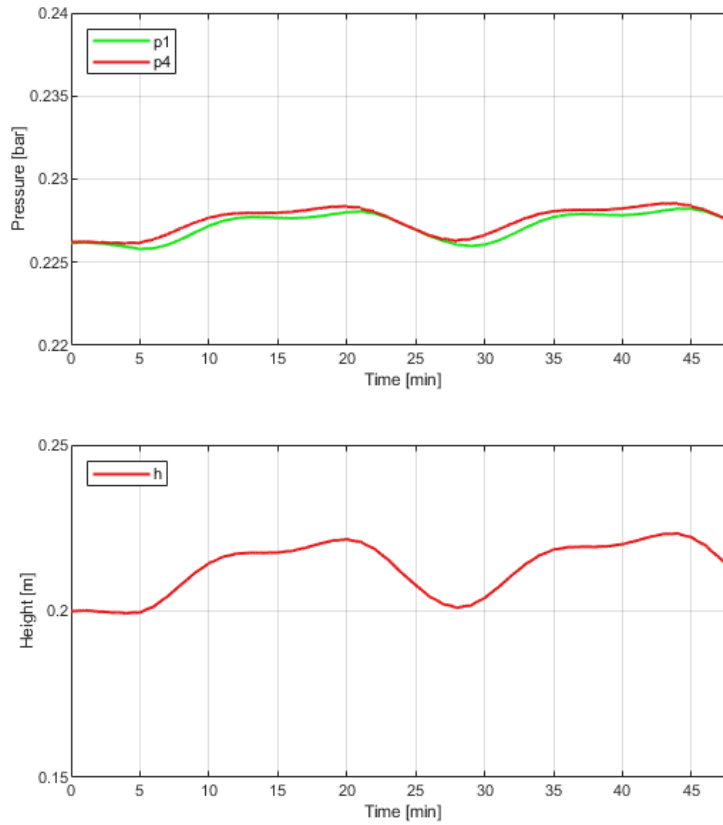


Figure 3.10: Pressure at node 1 and 4 with the level in the water tower for two days

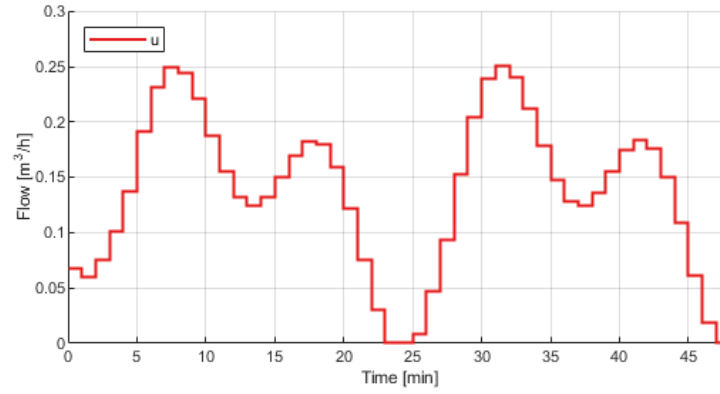


Figure 3.11: Flow rate generated by the pump

Table 3.2: Cost for different objectives

Priority	Total cost
Q	0.7840
R	1.1816

Table 3.3: Total variance of MPC with a priority of Q and R

Q	Variance	R	Variance
p1	1.74e-6	p1	6.23e-7
p4	1.87e-6	p4	6.20e-7

The MPC has been derived and tested in Simulink with the non-linear model in this chapter and will be implemented in a laboratory in the next.

Chapter 4

Laboratory Implementation

This chapter will explain the laboratory, how it is used and the changes to the control, so it matches the laboratory. The results in the laboratory will be compared to an on/off controller to verify the advantages with a MPC.

4.1 Laboratory setup

The laboratory consists of different modules, each containing a part of the network, and the layout and combination of the modules can be seen in Fig. 4.2 and the real laboratory can be seen in Fig. 4.1. The modules are a pumping module, a consumer module, a tank module and two pipe modules. The consumer module has two independent valves which are shown as d_1 and d_2 and the water tower has a pressure measurement. The different components are shown in Fig. 4.2, where a is a pipe, b is the pump, c is a tank and d is the connection between components or different pipes.



Figure 4.1: Water laboratory setup

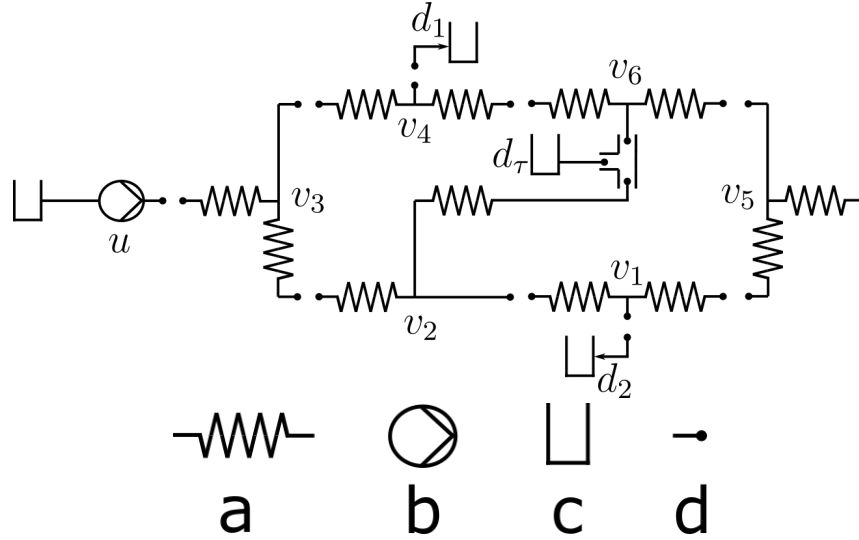


Figure 4.2: The top figure is the laboratory setup with nodes v_1 to v_6 and below can the different components be seen. The components are: a is a pipe, b is a pump, c is a tank and d is a connection point between the components.

In the simulation framework, the flow reference for the pump was calculated by the controllers and used on an ideal controller to generate the flow rate. Thus the flow rate reference from the MPC was used directly as the output of the pump. However, for the laboratory experiments, the pump has to be controlled, and a PI controller will accomplish this. Since there now are two controllers, it becomes cascaded control, the structure can be seen in Fig. 4.3. Now the PI controllers job is to make the pump produce the right amount of flow rate, which is determined by the MPC. The PI controller will have a sampling time of one second, and the MPC will have a

sampling time of 60 seconds. It can be seen in Fig. 4.3 that the MPC has three inputs, price, d and h . The price is the price curve, d is the consumer's disturbance curve, and h is the water level in the tank. The PI controller has u as an input which is the wanted flow rate calculated from the MPC and ω is the power given to the pump.

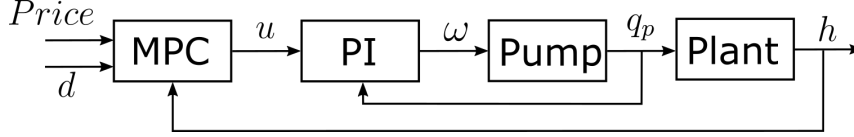


Figure 4.3: Control of pump with a PI

The consumer module controls both water consumptions d_1 and d_2 independently by adjusting valves. The control is achieved by using a PI controller to regulate the opening degree of the valves and using the flow sensors to calculate the error, and this can be seen in Fig. 4.4. The method of control produces oscillating behaviour since it is non-linear. However, this can be reduced to be acceptable by tuning the PI controller. It is to be noted that this laboratory uses two different kinds of sensors to measure the flow, and they vary in accuracy, this difference can be seen later on Fig. 4.5 and Fig. 4.6. The pump has a more accurate sensor, whereas the consumers use a noisy sensor compared to the other, and this results in a more noisy flow rate for the consumers. A moving average filter has been used to counteract the noisy sensor measurement of the disturbance.

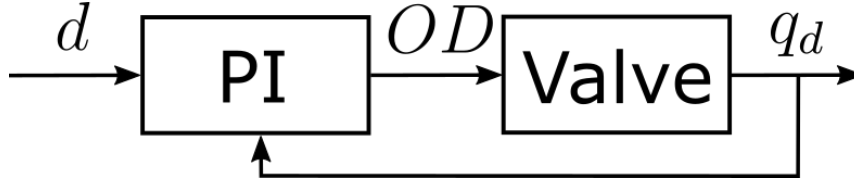


Figure 4.4: Control of valve with a PI

Controller benchmark

The on/off controller is used as a form of a benchmark to compare the MPC to. The pump and valves are controlled by PI controllers which have been manually tuned. This test also showcases the behaviour of the consumers, which will have a similar pattern for testing the MPC. The oscillation of the pump flow can be seen in Fig 4.5, the reason for the oscillation in the flow rate is due to sensor noise and the PI controller acting as a non-linear controller. The non-linearity is due to the PI using the flow rate as an input. However, it is actually controller the power given to the pump.

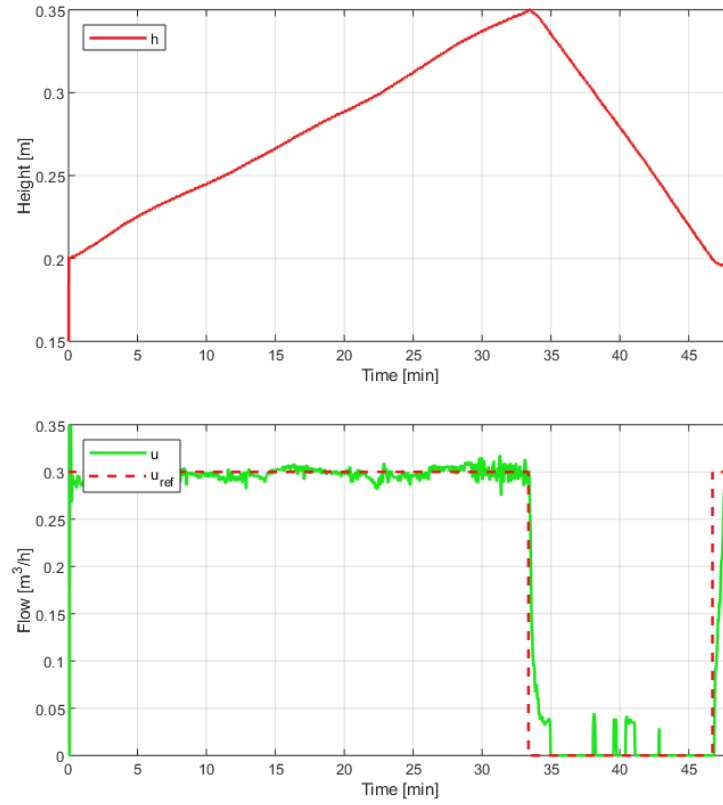


Figure 4.5: On/off controller behaviour with a maximum flow rate of $0.3\text{m}^3/\text{h}$. The top figure is the height in the water tank, and the bottom figure is the control input.

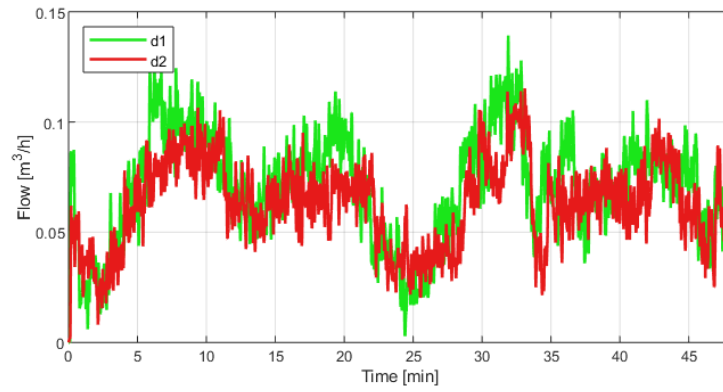


Figure 4.6: Flow at consumer nodes, d_1 and d_2 , controlled by a PI

4.2 Laboratory MPC controller

The MPC controller in the laboratory uses the same PI configurations for controlling the flow rate of the pump and consumers as the on/off controller. The MPC uses the price and disturbances to calculate the most optimal input for the current time step. Fig. 4.7 shows the price for usage where it has two price increases a day, from 5 to 10 and from 15 to 20. The MPC weights and prediction horizon are:

$$Q = 1e9$$

$$R = 1$$

$$D = 1e7$$

$$N = 24$$

For this test, the considerable difference in the weights is to focus on reducing the energy consumption and therefore is Q larger than R . Thus the pressure variation of the consumers is not of concern for the MPC controller.

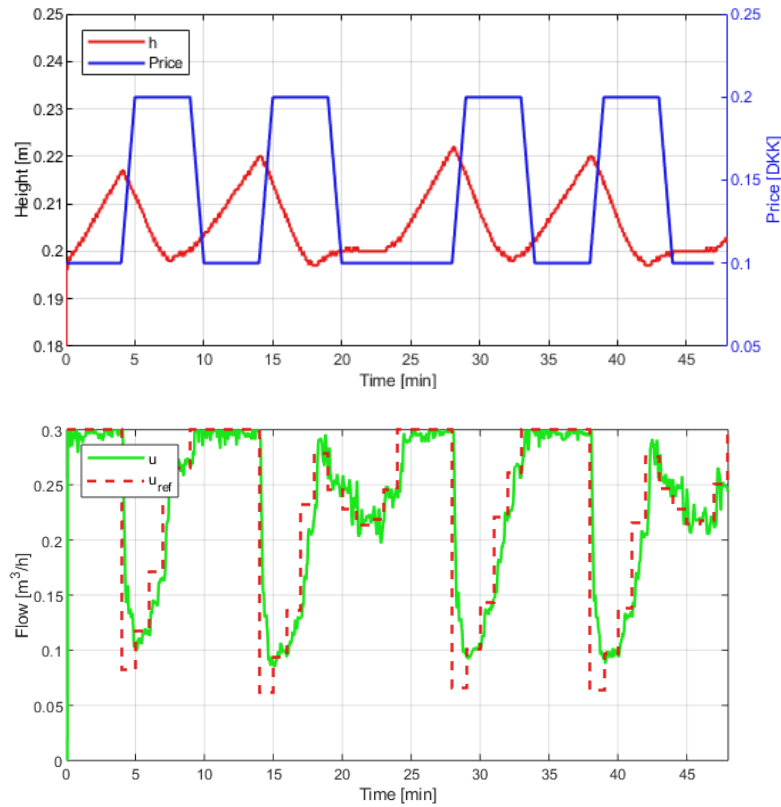


Figure 4.7: The first figure is the price curve and its influence on the water level. The second is the reference from the MPC and flow rate input from the pump.

Fig. 4.7 shows how the MPC wants to fill up the water tower whenever the price is low. Thus, the MPC is achieving its desired behaviour by using the water tower to its advantage to lower the operation cost. It can be seen in Fig. 4.7 that the water level starts to decrease at 4 whereas in the simulation it starts at 5. The change is because the MPC in the laboratory starts its time step at 1 whereas the simulation starts in 0. However, the MPC shows expected behaviour by filling the tank whenever the price is low and using it when the price is high. The best way to validate the controller is to look at the objective of the designed MPC, which is to reduce the operating cost. Fig. 4.8 shows how the behaviour of the cumulative cost over time, and the MPC can be seen as the lower one. Firstly looking at the on/off it can be seen whenever the price changes the slope does as well and when it is turned off the cost flattens out. In contrast, the MPC has a lower slope which results in a lower overall cost. The total cost for operating 48 minutes is 1.8783 and 1.5965 [DKK] for the on/off and MPC respectively.

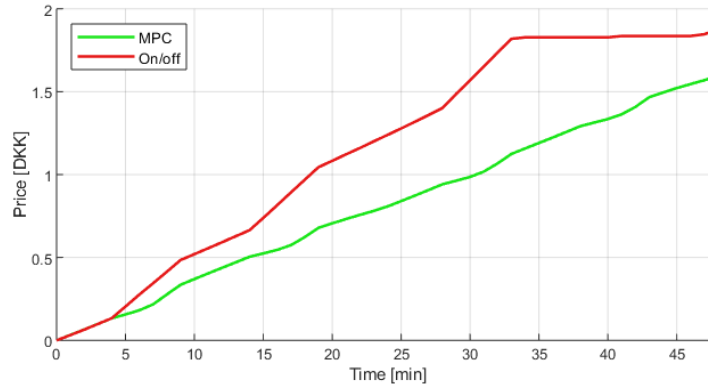


Figure 4.8: The cost of operating on/off and MPC over time

Table 4.1: Cost for different prediction horizons

Controller	Total cost [DKK]
On/off	1.8783
MPC	1.5965

Reduction of pressure variation

For the MPC to reduce the pressure variation R has to be larger than Q , this would result in the MPC punishing changes in pressure more than changes in price. The objective will be to minimise pressure variation by using the pump and water tower to reduce spikes and have a constant pressure which also would mean to have fewer changes to the height in the water tower. For testing this, the variables are:

$$\begin{aligned}
 Q &= 1 \\
 R &= 1e9 \\
 D &= 1e7 \\
 N &= 24
 \end{aligned}$$

Fig. 4.9 shows the water level inside the tank and the flow rate from the pump and its reference. Water is not being stored whenever the price is low but kept at a minimum. It means the pump is trying to produce whenever amount the consumers are taking from the network. Thus reducing the pressure variation by not gaining additional pressure by having more water in the tank.

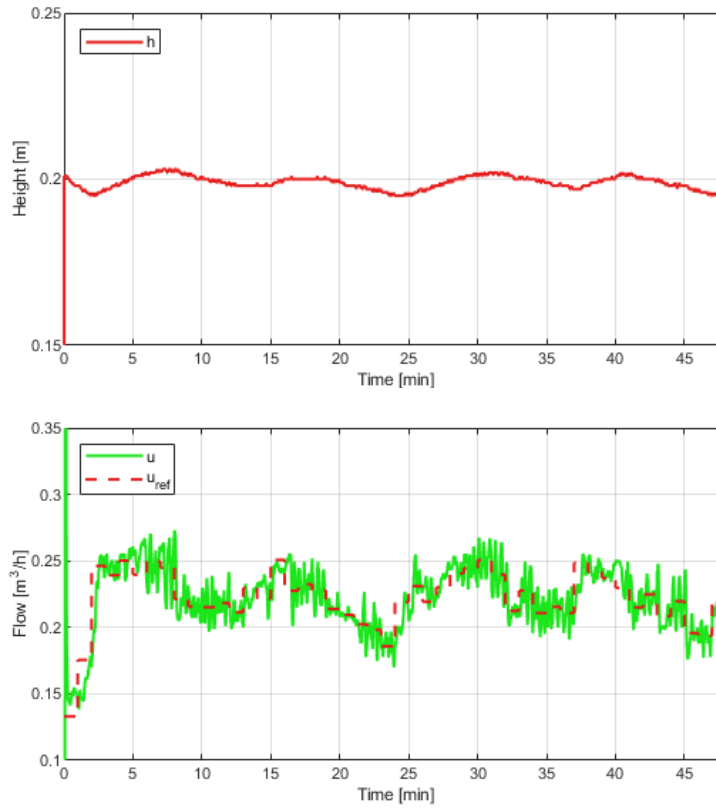


Figure 4.9: Height of water in the water tank, and the flow rate generated from the pump.

Chapter 5

Discussion

When comparing the results from the simulation to the laboratory results, it is clear that there is a difference in cost. However, the behaviour of the controllers are similar. The controller produces an input which either minimises the input and slack variable or pressure variation and slack variable. Some of the differences in the cost from the simulation to the real lab could be due to some pipes having the wrong length or incorrect water tank variables. These small changes could change the pressure and therefore be the reason why the simulation and the laboratory have different costs.

For controlling the flow rate of the pump and consumers, PI controllers were added, this made the system into a cascaded control which meant the flow rate did not change into the control input immediately and therefore can a settling time be seen. Furthermore, better control of the pump's flow rate could reduce the oscillation, and therefore being able to have a more aggressive PI control to reduce the settling time in the laboratory experiments.

Determining which MPC explored in the paper is the better one depends on the objective due to the multiple objective cost function. Therefore, having a larger R than Q would result in better consumer quality due to constant pressure. Whereas having a more energy cost-efficient control reduces the price of operating the water distribution, which benefits the supplier. Furthermore, different relations between the two weights would produce different outcomes: however, in this project, the two iterations of MPC focused on achieving the best goals for each objective. Therefore could a third design be made which found a desired balance between the two weights. Additional constraints could be added to increase the consumer quality, and one constraint could be a range for the consumer's pressure. Furthermore, by choosing a small range, spikes in the pressure would also be limited, and a specific desired pressure could be obtained.

The MPC was implemented in a laboratory setup which required the use of PI controllers to control the valves and pump, which made the MPC and PI into a cascaded control. The MPC was evaluated with the same conditions as the simulation, and both objectives were investigated. The result from the MPC with a focus on economics showed a reduction in operation cost of 15% for two days compared to an on/off controller. Furthermore, The MPC controller, with the focus on reducing the pressure variation showed a stabilisation in the level in the water tank and a more constant flow rate from the pump.

Since the MPC prove to a suitable controller for water distribution networks, the next step would be to achieve better results. An improvement to the system could be the additional pump to provide a greater flow rate when the price is low. The addition of adding an extra constraint with a range for the consumer's pressure to in would mean that the supplier could guarantee a specific pressure for the consumers at all time.

Chapter 6

Conclusion

Water distribution networks are everywhere and are an essential part of the infrastructure. A network can have different designs depending on the consumers and geographical location. Water distribution network with a water tower has an advantage by being able to store water and therefore pressure as well. By using future prices, the water tower can be used as a buffer to handle whenever the prices seem to be too expensive. Furthermore, the water tower is also a way to guarantee that there will be water in the system if the consumer demand becomes larger than the pump's capacity. This leads to the problem statement:

How can a water distribution network be optimized, in terms of pressure variation and economics with the use of MPC.

The project investigated an approach to optimising a water distribution network by MPC control. The dynamics of each component of the system has been derived, and after that, with the use of graph theory, the network could be reduced to only a link of pipes. By removing the pump, tank and consumer nodes, and therefore, having them as disturbances, the network became simpler. With graph theory, a non-linear model was obtained, which was used to simulate the water distribution network.

To validate the behaviour of the network, an on/off controller was derived, and it also acted as a benchmark controller for later comparisons. The MPC was designed with two objectives in mind, reducing the operation cost and pressure variation, which made the cost function multi-objective. Before implementing the MPC, the system equations had to be linearised, and this is due to the MPC needing a linear system to be able to predict the future states. The constraints were developed to fit the dynamics of the system and the limitations of the components. The optimisation problem was solved in MATLAB with the use of YALMIP. Comparing the different prediction horizons for the MPC shows a decrease in 8% from a prediction horizon

of 6 to 24 steps. After that, the effects of changing weights were examined. Showing the behaviour of both objectives and also proving the usefulness of the linear model used to predict future states.

The MPC was implemented in a laboratory setup which required the use of PI controllers to control the valves and pump, which made the MPC and PI into a cascaded control. The MPC was evaluated with the same conditions as the simulation, and both objectives were investigated. The result from the MPC with a focus on economics showed a reduction in operation cost of 15% for two days compared to an on/off controller. Furthermore, the MPC with a focus on pressure variation showed a stabilisation for the input and water level and therefore reducing the spikes in the consumer's pressure.

Since the MPC prove to a suitable controller for water distribution networks, the next step would be to achieve better results. An improvement to the system could be the additional pump to provide a greater flow rate when the price is low. The addition of adding an extra constraint with a range for the consumer's pressure to in would mean that the supplier could guarantee a specific pressure for the consumers at all time.

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