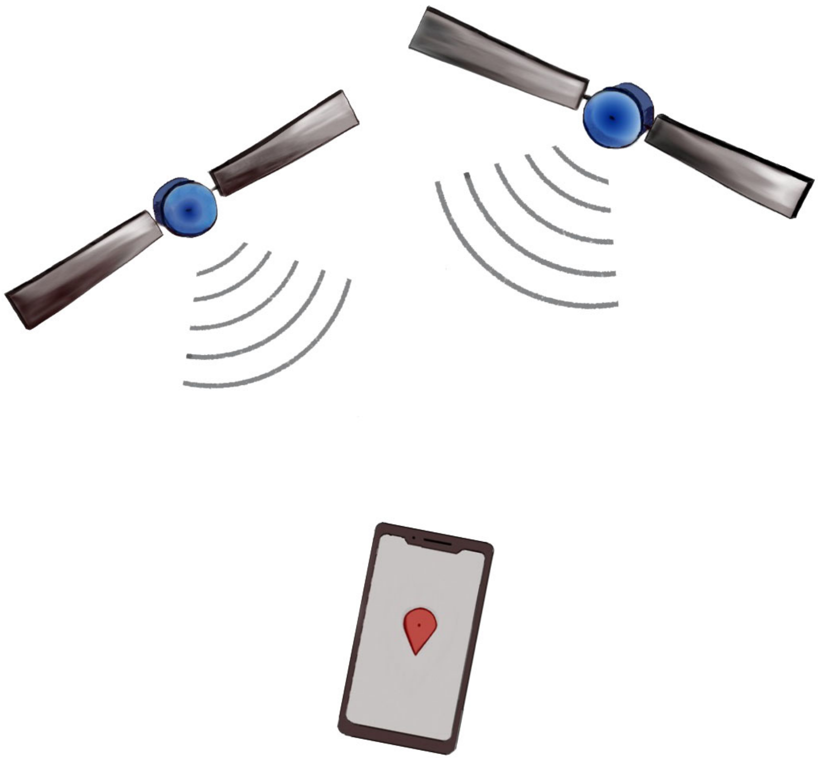
Single vs Dual-frequency signal of GNSS on Mobile Phone.

4th semester Thesis project at MSc Surveying and Mapping
Group 4, Aalborg



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Topic:

The dual frequency on mobile phones is expected to provide higher accuracy to Geo-positioning. With Galileo constellation already completed and the implement of dual-frequency chips on mobile phones it will be possible to evaluate the accuracy of the position that can be reached by a mobile phone.

Project:

4rd semester project

Project period:

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Project group:

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TECH 30 ECTS:
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Appendix: See attached folder
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Synopsis:

The aim of the project is to perform an analysis that provide us information enough to know how accurate could be Geo-positioning using a dual frequency mobile phone. To be able to achieve the target of this project it is necessary to know first how dual frequency positioning works on the mobile phone and its expected precision and accuracy for the computed position. On the other side it will be needed have access to the raw data from the mobile phone and its post processing. Once that this is known it will be performed some measurement under different scenarios. The results of theses measurements will provide information to analyze the quality of the positioning obtained with the mobile phone.

Summary

The aim of this project is to evaluate the quality of the positioning of a dual frequency mobile phone, the *Xiaomi Mi8*. This mobile phone is the first dual frequency mobile pone that hits the market. It has a huge expectation because it expected that it will provide an accuracy of decimeters. For test the mobile phone, an analysis has been made of what factors affect to obtain of the position, how it can be tested and what it is the actual quality of the data that the mobile provide.

The *Xiaomi Mi8* is equipped with a *Broadcom BCM47755* chip capable of processing dual frequency GNSS signals. In the same way, it is capable of processing the signals of different GNSS, *GPS, Galileo, Beidou and Glonass* constellations. This project focuses only on the study of dual frequency positioning in *GPS* and *Galileo* since they are the only constellations with dual frequency that the mobile receives. For this study, tests are designed in both static and kinematic in order to obtain the necessary information to evaluate its precision and accuracy. These tests will be carried out both, in an open sky environment and in an urban environment where the signal suffers distortions that affect the quality of the final positioning. Thus, the quality of the positioning of this mobile phone can be evaluated in a favorable situation and in another that is not.

Once the measurements have been carried out in the field, the statistical evaluation of the results was carried out, thus being able to quantify the pricing and accuracy in each case. For the evaluation of the quality of the results, the calculation of the standard deviation with respect to the midpoint of each measurement was carried out in the case of static tests. On the other hand, kinematics tests were performed, to evaluate the quality of these it was necessary to obtain the regression line of each data set to later calculate how much the points vary with respect to this line by obtaining the orthogonal distance and its standard deviation.

Una vez efectuados las mediciones sobre el terreno, se procede a la evaluacion estadistica de los resultados pudiendo asi cuantificar la preciosion y exactitud en cada caso. para la evaliacion de la calidad de los resultados se efectuará el calculo de la desviacion estandar respecto del punto medio de cada medicion en el caso de los test en estatico. Por otro lado se realizan tesd en cinematico, para evaluar la calidad de estos es necesario obtener la linea de regresion de cada set de datos para posteriormente caluclar cuando varian los puntos respecto a esta linea mediante la obtencion de la desviacion estandar.

Corona

The objective of this project that started in early February 2020 was to evaluate the precision and accuracy of a dual frequency mobile phone, *Xiaomi Mi8*. To do this, it began to write this project designing some tests in which the precision and accuracy would be evaluated by comparing measurements made simultaneously with a *Leica 1230+ gnss* device provided by the Jaén University

Once the tests were designed, the coronavirus crisis began. In Spain a state of alarm was decreed from March 14^{th} in which free movement of people was prohibited and Universities and any public places were closed. This directly affected the implementation of the project since have access to the *Leica 1230+ gnss* equipment that the University of Jaén was going to lend me was impossible. Also, the fact of not being able to circulate freely made it impossible to carry out the measurements properly. It was not until May 2^{nd} when we began to be able to go outside but with time limitations. From then on, measurements were performed only with the mobile phone but always conditioned by the time we could be on the street. That is why the time used in the measurements is so short, when the ideal would have been to make much longer measurements.

All of the above has conditioned and limited the completion of this project as originally planned, as well as having to face and redo a part of the project has reduced the time taken for its completion.

Foreword

This is the thesis project on the MSc program for Surveying and Mapping at Aalborg University. The project period stretches from 1^{st} February 2020 to 12^{th} June 2020. During the period that the world suffered the corona virus situation that affect directly the process of this thesis.

This project is concerned with the quality of the positioning provided by a mobile phone, collect the raw data from satellites signals and the evaluation of the precision and accuracy of the positioning obtained. The Thesis is based on the data collected with mobile phone Xiaomi Mi8. The reason to use this mobile is because it is the first mobile equipped with dual frequency positioning.

A big thanks is given to the thesis supervisor Jens Peter Cederholm for guiding the thesis towards the right direction. Also I want to thanks to Vojtech Jindra for his patience and help when I started to learn Python.

Readers Guide:

Appendix

There is an appendix for this project containing the following items:

- Python code.
- Results of python calculations and plots.
- Results of the post-process with GnssAnalisysApp.
- Information about the geodetic point.

List of abbreviations

APP: Application

ADR: Accumulated Delta Range

ETRS89: **E**uropean **T**errestrial **R**eference **S**ystem 1989

GNSS: **G**lobal **N**avigation **S**atellite **S**ystem

GPS: **G**lobal **P**osition **S**ystem

GSA: European **G**N**S**S **A**gency

GLONASS: **GLO**bal **NA**vigation **S**atellite **S**ystem

IGN: **I**nstituto **G**eográfico **N**acional; *Geographic National Institute*

ns: nanosecond

RL: **R**egression **L**ine

ROI: **R**ed de **O**rden **I**nferior; *second level geodetic network*

RNSS: **R**adio **N**avigation **S**atellite **S**ystem

SD: **S**tandard **D**eviation

UTM: **U**niversal **T**ransversal **M**ercator

WGS: **W**orld **G**eodetic **S**ystem

WLS: **W**eighted **L**east **S**quares

Contents

1 Introduction

The continuous development of GNSS technology looking for a higher accuracy and precision lead us to the dual-frequency on mobile phones. Dual-frequency signal allows to compute a better positioning and navigation in urban environments.

The investment from the European **G**N**S**S **A**gency, GSA, on dual-frequency trying to get the non-professional costumer about the benefits of use this technology was huge. With this statement I want to know how the use of dual frequency will implement the quality of positioning from a qualitative point of view. Knowing the expected accuracy that that it could be reached I will performed measurement to get my own result and compare with the expected.

Dual-frequency signal GNSS in combination with the Broadcom BCM47755 chip, a chip able read dual-frequency on mobile phones, is expected to compute the position with an accuracy of up to a few decimetres. This *"will create opportunities for new applications in areas such augmented reality, vehicle navigation and mapping."*[1]

Dual-frequency has led to improvements in the different uses of positioning. For example, vehicle navigation, one of the most common used of positioning, will increase substantially it quality in urban environment due to dual-frequency on mobile phones. Also when positioning is used for safety purposes, dual-frequency implies a decreased in the area of search, therefore find the location of an accident in the country side will be faster. These are just a couple examples for non survey uses but there are wide options that apply to other business large benefits from agriculture to geo-marketing.

2 Problem statement

This section will establish the problem statement which will be developed on following sections and try to answered the questions exposed below on this section.

Performing measurement and collecting raw data to evaluate the increment of accuracy expected using dual-frequency positioning will be the aim of this project. Once that the data will be processed it should be possible to answered the following question:

- Which positional accuracy and precision does a dual-frequency GNSS mobile phone deliver?

To find out the answer to this main question it will be necessary to look for how to evaluate the different factors that affect the position. Finding the sources of errors and how each one affect to the position, how the position is computed and any other parameter that could interfere substantially on the final position. Answering these factor allows us to get to answer the previous question. The way in which it is intended to achieve this is to answer the sub-questions exposed below:

- Which are the parameters that affect to compute the position and how do the errors affect to compute the position?

- How it will be tested the accuracy and the precision?

Finally, the conclusion will provide information based on the previous results about the possible used of this technology.

3 Method

To address the project it is necessary to set up a path that will be followed to be able to find an answer to the questions exposed on *section 2*, Problem Statement.

Firstly on *section 4, Theory,* is divided in different subsections. Where the first one, *section 4.3,* exposed what frequencies are used on this project, to which constellations belongs and how many satellites have available the frequencies needed for the project. Also it is explained the benefits of positioning based on dual-frequency. And also, talks about the errors that can affect the signal and the distance range error of each one. Next *subsection, section 4.4,* explain the mobile phone used on this project, the chip that is capable to get positioning using dual-frequency. Then *section 4.5* explain how it is possible to get raw measurement data from the mobile phone, the data provided and the app use for this purpose and how the app works. Finally is the *section 4.6,* this subsection is the most relevant for the project, it is on this subsection where is explained how the position is computed and data will be tested. An explanation of the software used, how the data will be computed and the output data will be found. Also the statistics used in order to evaluate the accuracy and precision.

Then on *section 5, Test Design,* we will found two main subsections, *Static* and *Kinematic*. Both of them explain the location of each measurement. The case of the *Static* is divided in two, one uses a point of known coordinates so it will be possible to evaluate precision and accuracy and a second one in an urban scenario. The *Kinematic* it is only performed on a urban scenario. On this section it is explained the design to test the position, what constellation and frequencies will be used to compute the position. Finally the analysis of all the computed position mentioned before. The statistic used on each case and how it will be evaluated.

On *section 6, Test execution*, describes all the work performed on the site and the post-process of the data as was explained on the previous sections. Here will be found a representative example of each situation, the rest of the tests are found on the attached appendix.

The *section 7, Results* will expose the results of the evaluation of all tests. How precise the position is at each case and also how accurate it is on the case of the static test over the point of known coordinates. It will include too the explanation of why there are some measurements provide some errors and where it comes from.

section 8, Conclusion will expose the finding from this project also there are the answer to the questions exposed on *section 2*.

The following chapters will explain deeply the whole process exposed above.

4 Theory

4.1 Introduction

The section is divided on subsections where it is explained the signals frequencies that it use for this project, the sources of distortions and how much affects them to compute the distance to get the position. Also exposed the hardware and software need to get the data and compute the position. A final section will explain how the computed position will be evaluated and tested.

The information obtained on this section will provide the knowledge to design and execute the test.

4.2 Coordinate system

Positioning based on GNSS technology uses the reference frame World Geodetic System 1984 [2] (WGS84), that is a 3-dimensional coordinate reference frame for establishing latitude, longitude and heights. The coordinates obtained for the project will be based on this system expressed on degrees, latitude and longitude, the heights will be expressed on meter from the ellipsoid. These coordinates will be used to compute distances and evaluate the precision and accuracy so it is decided to transform them to ETRS89 [3] UTM zone 30 and works with X, Y, h coordinates in meters.

The ERTS89 system is high precision system for GNSS georeferencing in Europe, this is based on the ellipsoid GRS80 that is slightly different from the one of WGS84. The differences are minimum for the area located on the projection UTM zone 30 where this project take place. Then the project will be performed with coordinates ETRS89 UTM zone 30.

4.3 Dual-frequency

Originally it was needed an expensive device and to get access to more than one signal from the satellites. The new cheaper and smaller chips for mobile allow us to compute get benefits to compute the position using dual-signal.

Positioning based on dual-frequency or multi-frequency was not possible for mobile phones until *Xiaomi Mi 8* mobile phone was launched to the market. Then positioning using dual-frequency based on signal *L1+L5* in the case of *GPS* constellation and *E1+E5a* from *Galileo* constellation was possible. Based on this two constellation and signals the project will be developed.

To obtain the position based on dual-frequency it is needed that both, space segment and user segment works with the same signals. On the space segment side, satellites emit different signals but the ones that concerned this project are the ones mentioned above from *GPS* and *Galileo*. *Galileo* is the constellation that has more satellites available that works on *E5a* signal. On the other side the user segment must be able to collect and process those multi-frequency signals in order to compute the position. For this purpose the dual-frequency mobile phone *Xiaomi Mi8* is used to perform the tests that will be exposed on *section 5*.

The addition to use of the frequencies signals like *L5 and E5a* to compute dial-frequency positioning have benefits like: [4] [5]

- Improved ionospheric correction.
- Signal redundancy.
- Improved signal accuracy.
- Reduced multipath problems.
- Improved accuracy of positioning and timing..

Following it will be explain briefly about the signals that are used by *GPS* and *Galileo* constellations that can be collected for the mobile phone.

GPS Signal Characteristics [6]

GPS constellation consists of 31 satellites in orbit, dated 20 February 2020, of which only 12 of them emit L5 signal. Therefore, the global positioning based on dual frequency L1 + L5 is limited. These are the *GPS* frequencies used for the civil use and belong to RNSS (Radio Navigation Satellite System) from what it would be called the lower band and upper band, for L1 its frequency is 1575.42 MHz and in the case of L5 is 1176.45 MHz.

Galileo Signal Characteristics [7]

Galileo constellation consists of 28 satellites, of which 26 are operational in orbit. It is estimated that the constellation will be completed throughout this year 2020 with the launch of four new satellites to have a backup in case of any fail. Unlike *GPS*, all satellites emit both signals E1 and E5 frequencies. Consequently, this supposes a better global coverage and a significant positioning improvement from *GPS*. *Galileo* constellation also works in upper and lower band with the same frequencies as *GPS*.

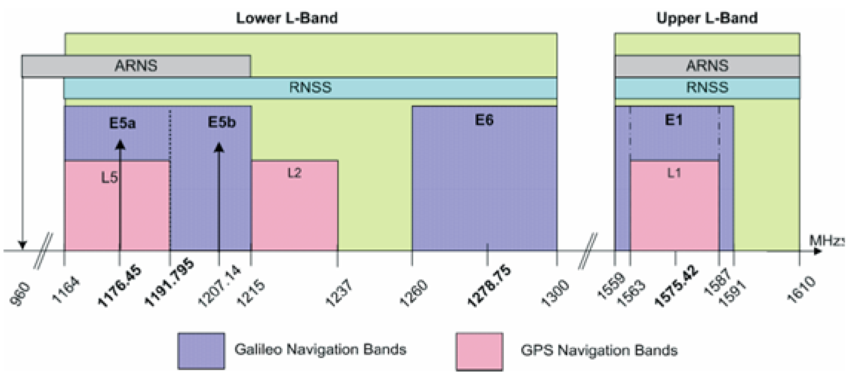


Figure 1: RNSS *Galileo* and *GPS* Frequencies Plans.

Errors[8]

Different factors affect directly the computed position. Some come from external parameters and other come from the receiver or the satellites. The quantity of how the parameters affect the position is different as it is shown on the following *table 1* this is refereed to a Single Frequency Absolute Positioning. This section will be explain where this parameters come from and in which amount they affect.

Contributing Source	Error Range
Satellite Clocks	±2 m
Orbit Errors	±2.5 m
Ionospheric Delays	±5 m
Tropospheric Delays	±0.5 m
Receiver Noise	±0.3 m

Table 1: Errors sources for Standard measurement.[8] [9]

Satellite Clocks.

The satellite clock is highly accurate but they still drift a small amount that has a significantly consequence in the position obtained by the receiver. This drift use to be around a few nanoseconds (ns) but a drift from 8 to 17 ns become in an error around ±2.5 to ±5 meters.

Orbit errors.

Satellites are travelling in known orbit so the position of the satellite is know when the signal is sent. However the orbit can vary a small amount that affect to the result of compute the distance from the satellite to the receiver. The orbits of the satellites are corrected by the ground segment but anyway there are still a small error that could up to ±2.5 meters.

Ionospheric Delays.

The ionosphere is a layer of the atmosphere that contains ions. These ions cause delay on the signal that could cause error of ±5 meters. The ionospheric delay is the largest error that affects positioning.

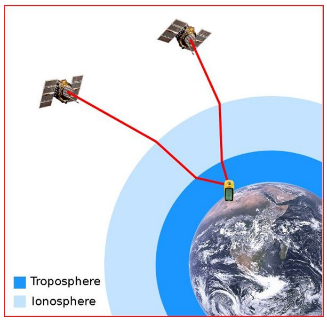


Figure 2: Ionospheric and Tropospheric Delays

Tropospheric Delays.

The troposphere is a layer of the atmosphere that is find closest to the Earth's surface. This error is caused by the humidity, temperature and atmospheric pressure.

There exits correction model for these atmospheric errors that can be apply in order to reduce them.

Receiver noise.

This error varies in relation with the hardware of the receiver. Lower cost receivers should have more noise. As the case of the project is to evaluate a mobile phone the the receiver noise is expected to be high.

Multipath.

Multipath occurs when the signal is reflected by any object before reach the receiver so the signal takes more time to be read by the receiver. That imply that the computed distance from the satellite to the receiver is longer.

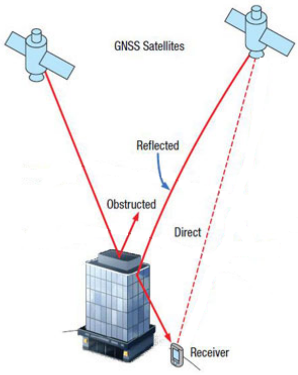


Figure 3: Multipath

4.4 Receiver

For this project it will be used a *Xiaomi Mi8* mobile phone to perform the measurements.

Xiaomi MI8 [10]

Xiaomi Mi8 mobile phone is the first one to hit the market that is able to compute the position using dual-frequency technology. This is thanks to the *Broadcom BCM47755* [11]. The chip is able to read dual-frequency from the signal L1 + L5 from *GPS* and E1 + E5a from *Galileo* among other constellations but only these two will be used on this project.

4.5 Data collection

Another issue that need to be handle is get access to the raw data from the mobile phone. Collect raw data is important to be able to compute the position using the data that we I want. Been able to perform the test only with the data that matters to achieve the aim of this project.

The data collection from the mobile phone is performed with mobile app *GnssLogger*, this App was developed by *Android Developers* [12] from *Google* and the app collected and created a .txt file with the raw data measured from all the satellites available at the moment. The file is automatically named with the date and the time when start to collect data. *fig. 4*.

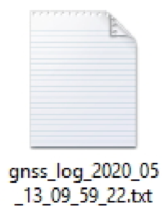
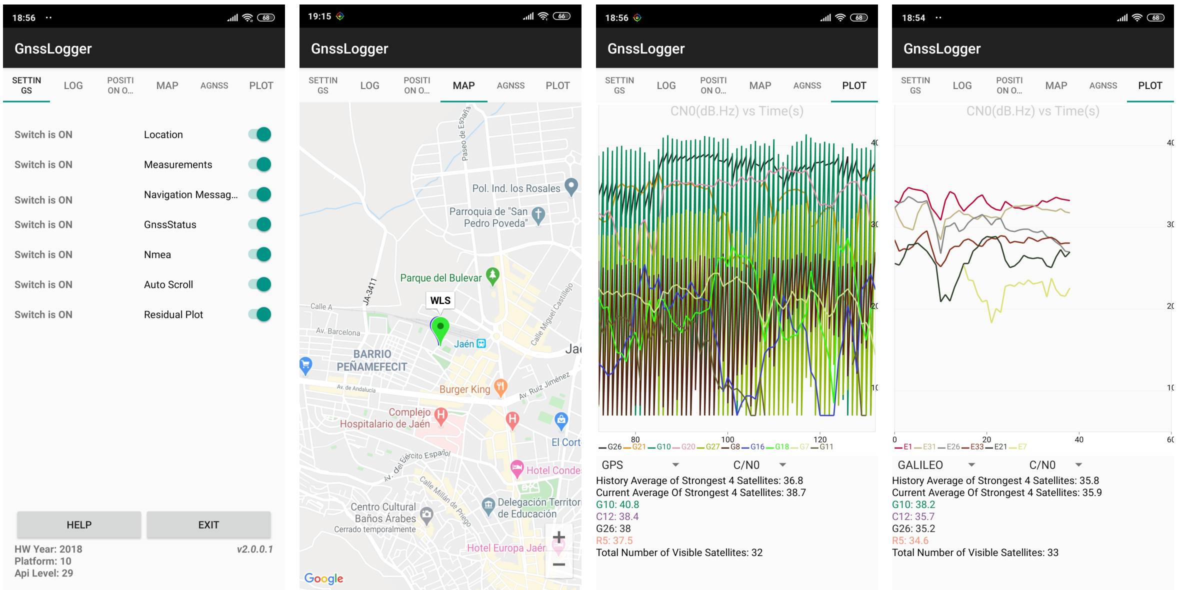


Figure 4: TXT file derived from *GnssLoger*

Also the App will provide some information during the measurement as it show below. There are four screenshots from the App *GnssLogger* where it can set up some parameter that will be added to the measurements *fig. 5a*. Also a comparison between the position computed at each epoch and the mean WSL position computed *fig. 5b*. The last two images provide an example of the signal strength from each satellite. On this case the *fig. 5c* shows the satellites from *GPS* constellation and *fig. 5d* shows the satellites from *Galileo* constellation. It can be observed that beside the *GPS* has more satellites on the sky at that moment, the instability is so high in comparison with *Galileo* that are only 6 satellites but much more stable.



(a) Options. (b) Position (c) *GPS* signal strength. (d) *Galileo* signal strength.

Figure 5: GnssLogger Screenshots

4.6 Post-process/Positioning

Post-processing the data will be handle with *GnssAnalysisApp*. This desktop app was developed also from *Android Developers* [12] from *Google*. It is a *Matlab* script that allow to adjust parameters like the elevation mask, the tropospheric and ionospheric errors before compute the position. Also it analyzes the signal strength, the clock errors and the position.

The figure below, *fig. 6*, shows the main screen of *GnssAnalysisApp* where it can be possible to select satellites, apply or not atmospheric corrections or change the angle of the elevation mask. *GnssAnalysisApp* is provided with extra scripts that can be added in order to adjust some parameters or apply some filters that it is not possible to do it directly from the main screen.

After running the software it will bring graphic information about the signal strength, clock errors and positioning. Also it will deliver files with the position computed raw applying Kalman filter and ADR (Acumulated Delta Range).

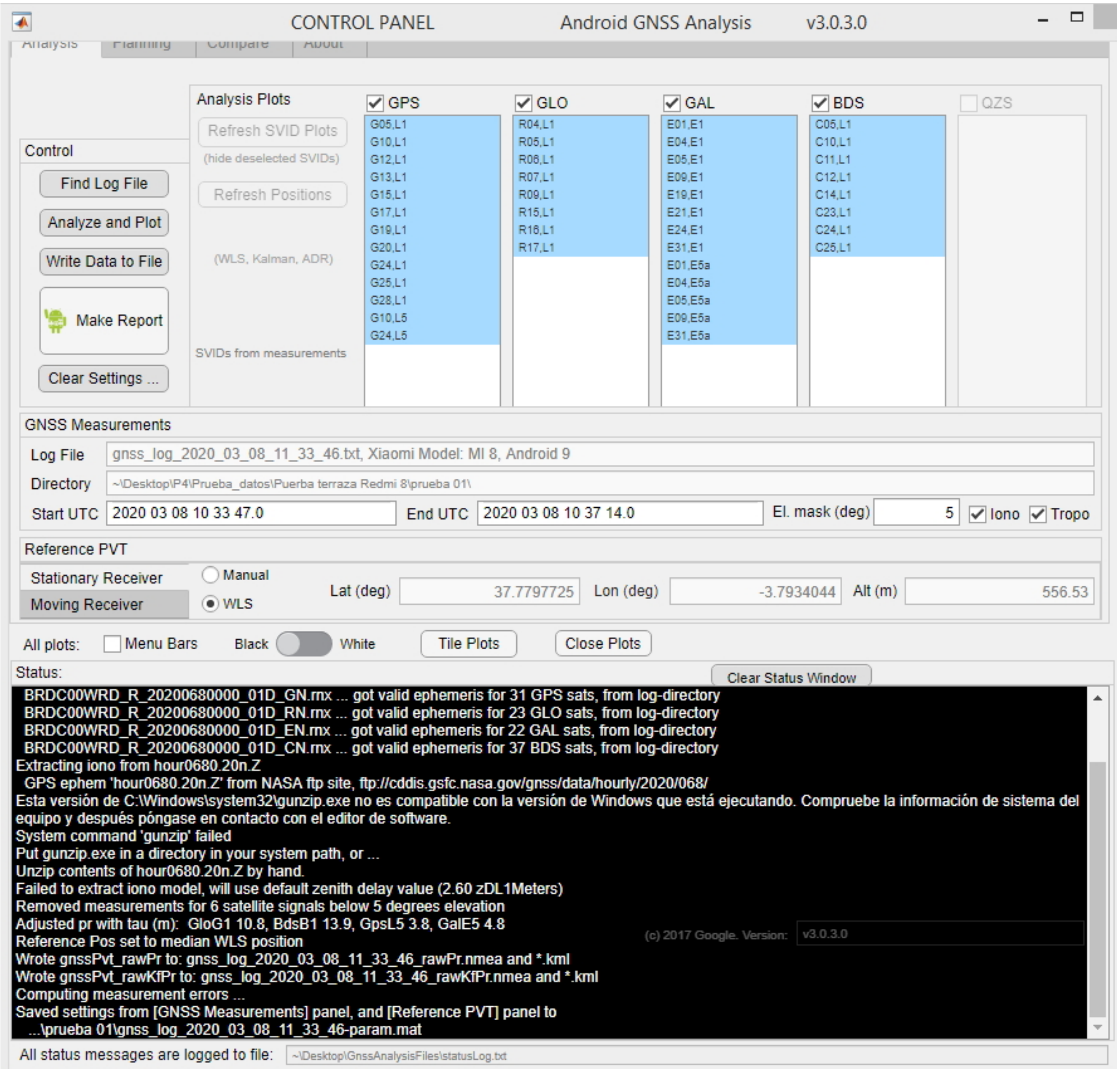


Figure 6: GnssAnalysisApp Control Panel.

The analysis to study the quality of each test will be done by evaluate the precision and the accuracy. Depend of each test, if there are know coordinates to compare or not, it will be possible to calculate both or only the precision. But each situation will be evaluated by the analysis of the raw position delivered by *GnssAnalysis*. This data is the position computed by WLS (Weighted Least Squares) using the raw pseudoranges.

Once that the data is process with *GnssAnalysis*, it will print out the plots shows on *fig. 7 and fig. 8*. These plots provide information about the signal *fig. 7 - 1 and 2*, and the skyplot, *fig. 8 - 8*, of all the satellites at the moment of the measurement. Also shows the plots about the clocks, the *fig. 7 - 3* shows the pseudoranges. Then *fig. 7 - 4* plot shows the receiver clock offset and frequency. Finally it shows plots for pseudorange and ADR (Accumulated Delta Range) measurements. The *fig. 8 - 5* plot shows the position from weighted-least-squares thus it is a way to visualize the aggregate measurement errors. The *fig. 8 - 6 and 7* plots show individual measurement residual errors. The residual error is the difference between the measurement and the expected value.

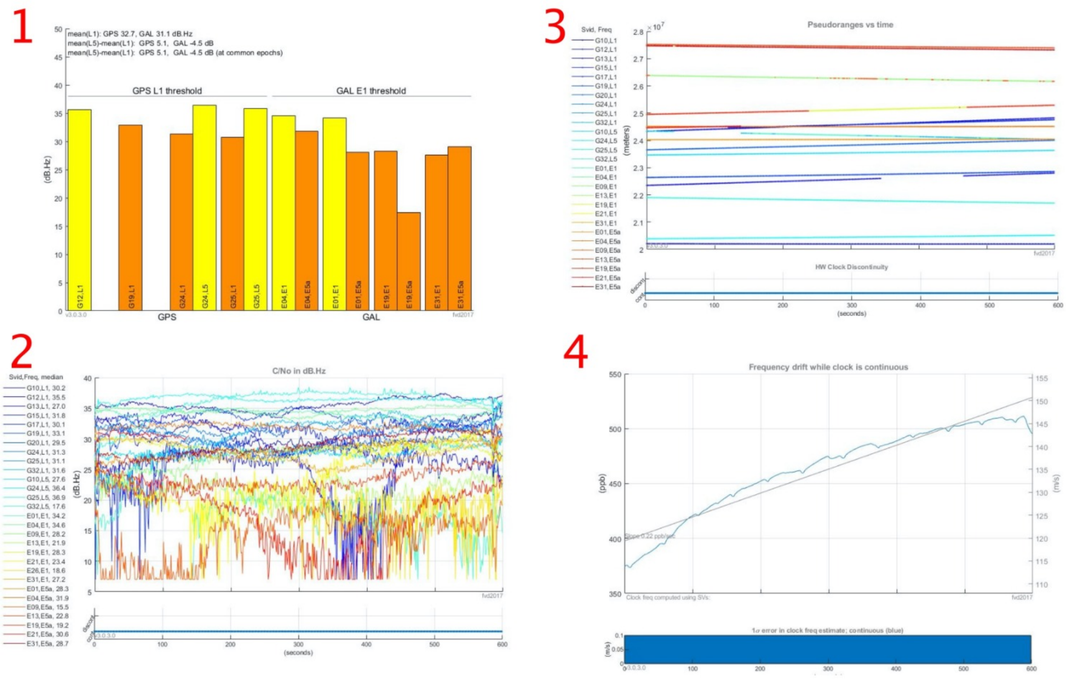


Figure 7: GnssAnalysisApp Plots, I.

The most important for this project are the *fig. 8 - 5 and 6.* Taking a look on the second one*fig. 8 - 6* will give information about if there are any signal with a frequency error too big, so this satellite will be avoid to compute the position. Then the *fig. 8 - 5* will provide the raw position that will be evaluated.

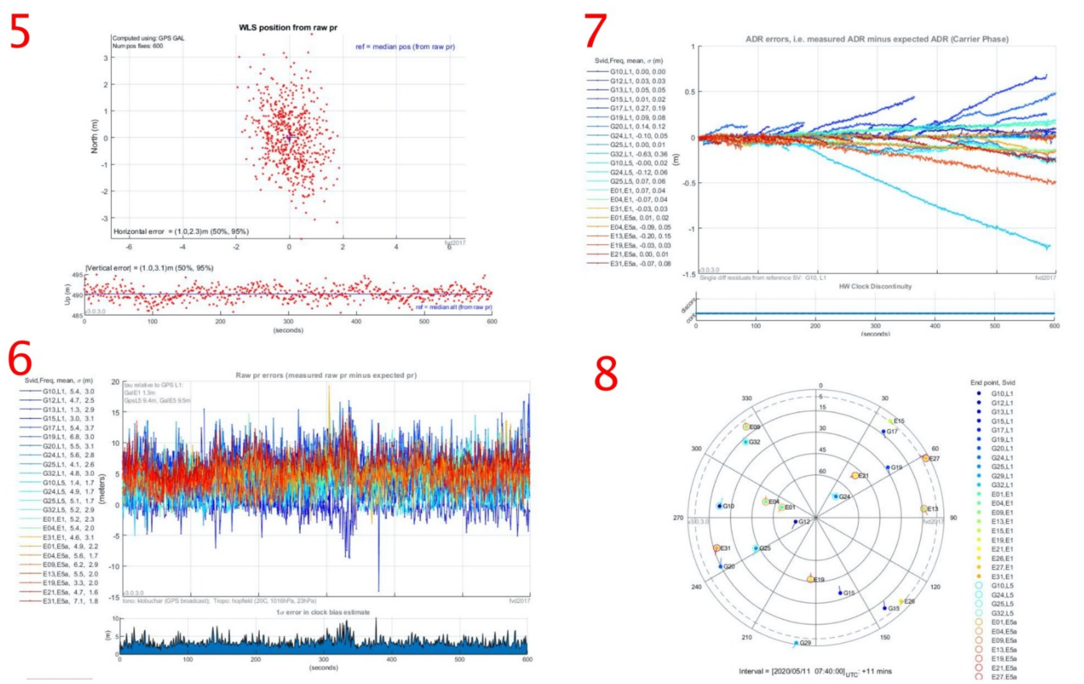


Figure 8: GnssAnalysisApp Plots, II.

After the program finish all the computations, automatically deliver 12 files as show the image below, fig. 9. There are rinex files for each constellation, the KMl files and nmea from the computed position under the three situations, raw, Kalman and ADR, The correction model for the atmospheric corrections and a copy of the raw data. The one that will be used to evaluated the position is the *rawPr.kml* file. This file provide coordinates from the WGS84 system computed position using only the raw pseudorange data. These coordinates will be used for the statistical analysis of the position as is exposed below.

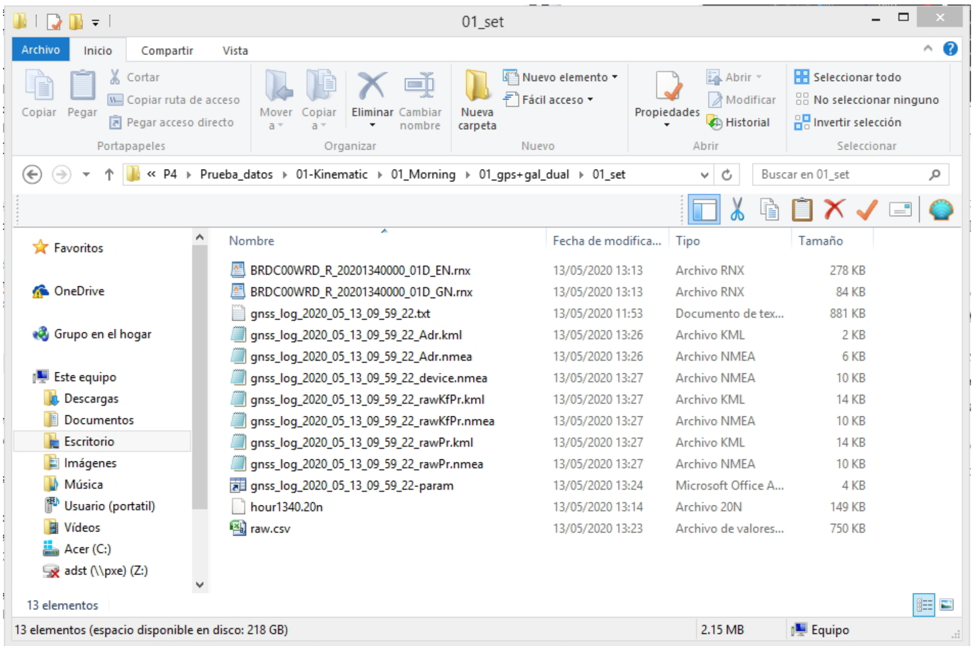


Figure 9: Output files from GnssAnalysis.

Finally, to analyse the quality of the position the standard deviation for the static tests will be computed and the regression line in the case of the kinematic. For theses statistic analysis was written a *Python* script for each case that provide the statistical results and a graphic to show how the points are spared and the distances to the line. Scripts are found at appendix *section 9.2 and section 9.3.*

"The standard deviation (SD) is a measure of the amount of variation or dispersion of a set of values. A low standard deviation indicates that the values tend to be close to the mean or the expected value of the set, while a high standard deviation indicates that the values are spread out over a wider range." [13]

The *SD* will be applied to the static tests. To obtain the *SD* of the coordinates points normally is computed the mean of the coordinates and the *SD* will give dispersion of the points respect of the mean. On this project exits two situations, the test over the geodetic point, where the coordinates are known and the tennis court test where the coordinates are not known. So over the geodetic point instead of use the mean to compute the *SD* it will be used the known coordinates of the point. In this way it will be possible to evaluate the precision and accuracy. The case of the tennis court has not the true coordinates of the point so *SD* will be calculated with the mean of the data set, this only provide information about precision. Both cases will be evaluated also by computing the error ellipse of each set.

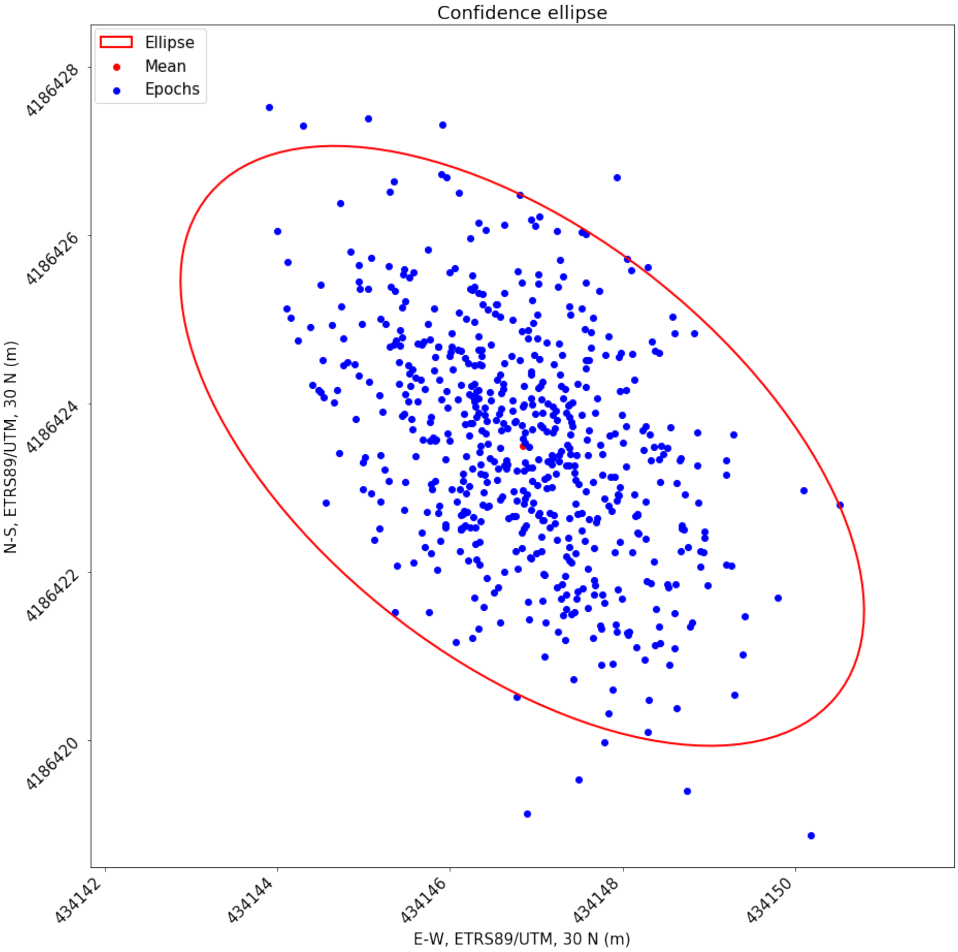


Figure 10: Example for computing the standard deviation of a static test.

"Linear regression (LR) is a linear approach to modeling the relationship between a scalar response (or dependent variable) and one or more explanatory variables (or independent variables). The case of one explanatory variable is called simple linear regression. [14]

Same that happened at the tennis court, the kinematic test has not know coordinates to compare and evaluate the measurement so for the kinematic test will be computed the *LR*. By calculated the *LR* we are going to obtain a line from the measured points. This line represent the best adjust to the data, once that this line is obtained we can set this one as reference to compute the precision of the measurements by computing the orthogonal distance from each point to the line. The distances will be used to obtain the precision of the measurements.

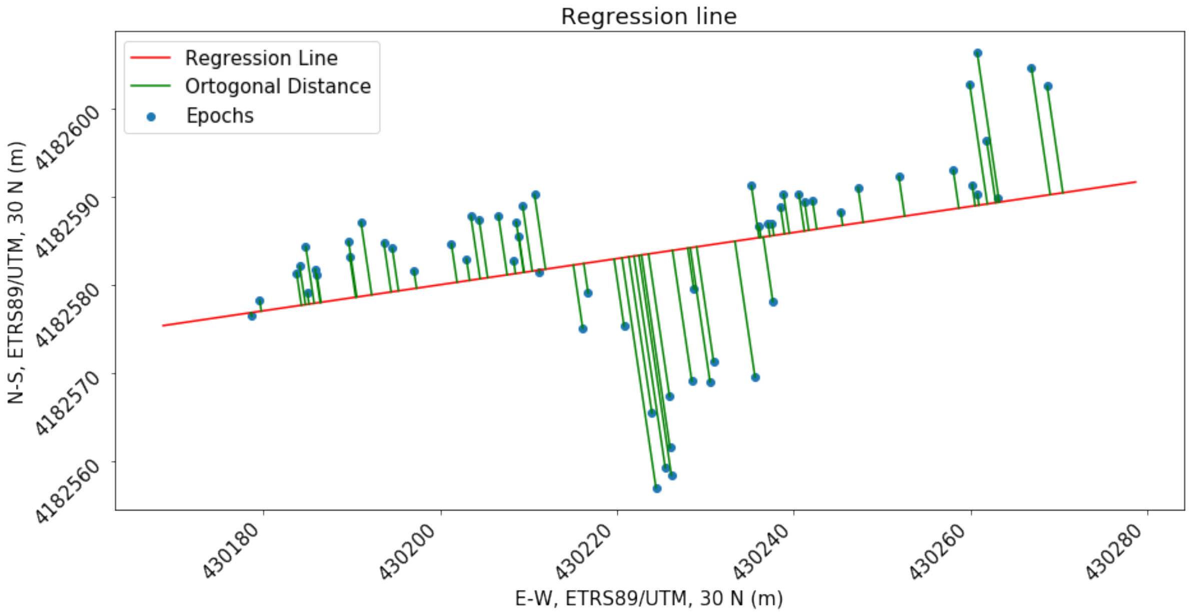


Figure 11: Example of computing the regression line for a kinematic test.

Performing these calculations it will be obtained how much the position vary from the true position in the case of the geodetic point and how much drift from the mean position in the others tests. This results should provide the information required to evaluate if dual frequency increased the precision and accuracy of positioning.

5 Tests Design

On this section it will be exposed how the tests are designed in order to get information enough to evaluate the quality of the positioning used a dual-frequency mobile phone. These tests will evaluated the precision and accuracy. For this purpose the section is structured in the following subsections, *Static* and *Kinematic*. On them it be explained how each tests was performed.

Due to Corona situation, this project altered the initial design of the tests and consequently how to deal with the measurement on the site. Originally, the tests were designed to evaluate the position obtained with the mobile phone by comparing the results with the measurement performed with a *Leica 1230+ Gnss* borrowed by the *University of Jaén*. Because of the lockdown that was not possible. Another issue derived from the Corona situation was the schedule restriction to go out. It was possible to go out only in two windows of time each day but only once per day. This schedule conditioned the duration and when the measurement could be performed. This is the reason why the tests are designed as it is explained in the following sections.

Figure 12: Jaén, Spain

Tests were performed in Jaén, capital of the province with the same name, *fig. 12*, the province is located in Andalusia, South of Spain. The location belong to the UTM zone 30 N.

5.1 Static

5.1.1 Data collection

The static test it designed to evaluate the precision and accuracy of the computed position by the mobile phone over a point where the coordinates are known. The measurement was performed over a geodetic point, *San Juan de Díos* sited at 6 kilometers to the North the city of Jaén (Spain). This geodetic point belong to the ROI, Red de Orden Inferior, a second level of geodetic points with a baseline around 7 kilometers between points. The coordinates of those geodetic points have freely access at the IGN, Instituto Geográfico Nacional [15] (Geographic National Institute). The coordinates are referred to the ETRS89 and ED50 system and the elevation is ellipsoidal and ASL at Alicante (Spain). The geographic coordinates refereed to the ETRS89 system are the following:

- X: 434148,756 m
- Y: 4186420,323 m
- Alt. Ellipsoidal: 489.366 m

This coordinates were compensated on November 1st 2009 and have a 95% confidence ellipse. All the info referred to the geodetic point is found in *section section 9.1 at page 49*.

Because of the restrictions derived from the Corona situation where people was not allow to go out freely and most of public areas where closed the data was collected on two set of 10 minutes each, once in the morning another in the evening, with the APP *GnssLogger*. The measurement were performed on the 11^{th} of May, 2020. The mobile phone were placed on top of the geodetic point. Despite the app collect data for the four constellations, *GPS, Galileo, Glonass and Beidu* for the aim of the project only *GPS and Galileo* will be used because these two are the only one that provide a second frequency, L5/E5a respectively, from which the mobile phone is able to collect data.

Figure 13: Geodetic Point Location.

Then to evaluate the quality of the positioning under a close sky scenario a static measurement will be performed in an urban area *fig. 14* where the signal will be affected by multipath. The analysis and comparison between the geodetic point and the urban scenario results will evaluate how much vary the computed position from one to another scenario. This comparison must be done by evaluating the results in a relative way because the impossibility of use the *Leica GNSS*. So it was not possible to compare with know coordinate point in the urban scenario.

The data for the static urban test was collected on the 15th of May, 2020 in the same way of the one measured on the geodetic point. Also the same restrictions that happened with the geodetic point test so the data was collected also on two set of 10 minutes, one set in the morning and another in the evening. For the urban scenario the place chosen was the central corner of the service line of a tennis court as show the *fig. 14*. This point was chosen because at the surrounding there are building about 20 meters high and about 35 meters between them so the signal will suffer from multipath. Also is a point easily recognisable on *Google Earth* therefore it will provide a visual information about how much vary the computed position.

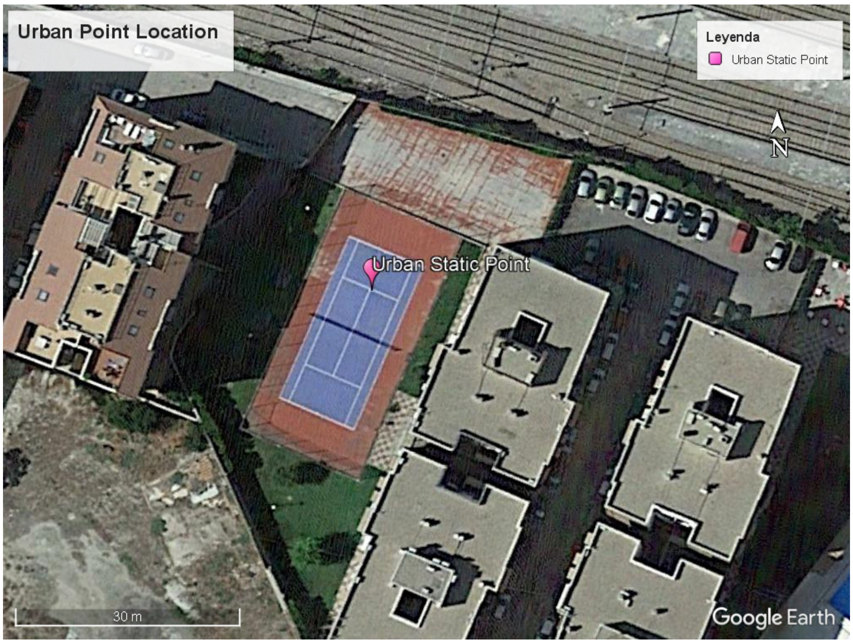


Figure 14: Urban Point Location.

5.1.2 Post-process

The data that concern this project belong to the *GPS and Galileo* constellations. Thus data must be filtered by constellations before proceed to compute the position. Also because of the computed position will be evaluated by signals (dual or single-frequency) these must be filtered too.

To filter the data it is needed a one of the *Matlab* scripts provided with *GnssAnalysis*. Select the data that we want to filter from the header of the file delivered from the app and place the script on the same folder of the file before running *GnssAnalysis* and the filter will be applied.

Once that the filters were applied, the position can be computed for each situation that will be evaluated. The situations are combinations of the both constellations and each one apart also, dual vs single-frequency. The *fig. 15* show what constellations and signals will be computed to obtain the position.

In order to compute the position the software *GnssAnalysis* will be used as was mentioned on *section 4.6*. Because the aim project is to evaluate dual vs single frequency under the same circumstances it will be applied for each test the atmospheric corrections and the elevation mask will be set at 5 degrees on each situation. *GnssAnalysis* compute the position in three different ways as was exposed on *section 4.6* but this project only use the raw data positioning obtained from the raw pseudorange uncertainty without apply any smooth. Not use the smooth data mill make possible to compare the differences between dual and single-frequency.

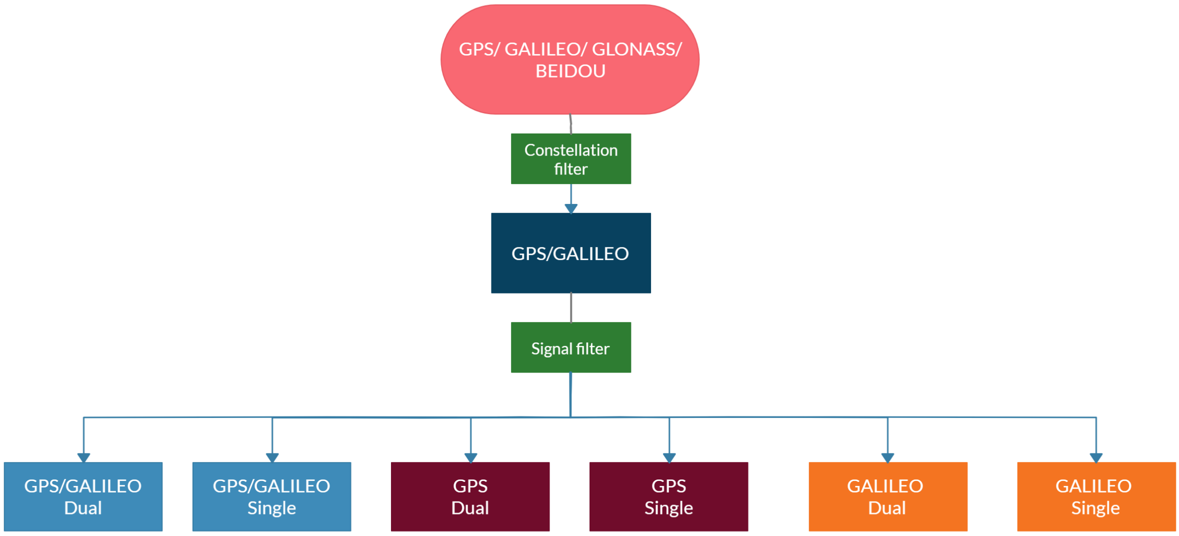


Figure 15: How the position will be computed

Finally after all the positions were computed the analysis will be done as is explain on next section.

5.1.3 Analysis

Be able to adjust the parameter mentioned above will allow us to get positioning under different scenarios that will provide information about the quality of the final position. The quality of this position will be calculated by compute the standard deviation [16] *eq.* (1) of the results for each situation.

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{X})^2}{n}}$$
$$\sigma_y = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \overline{Y})^2}{n}} \quad (1)$$

Where x_i and x_i are the ETRS89 UTM 30 coordinates for feature i, $\{\overline{X}, \overline{Y}\}$ represents the coordinates of the geodetic point or the mean center for the features, depend if the data is from the geodetic point or the urban one respectively, and n is equal to the total number of features.

By combine both formulas from *eq.* (1) we will obtained the covariance, *eq.* (2). The result of the covariance will indicates the correlation of all the measurement from the mean.

$$cov(x,y) = \sigma_{xy} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{X})(y_i - \overline{Y}) \tag{2}$$

Finally the uncertainty ellipse will be computed with three degrees levels of confidence.[17]

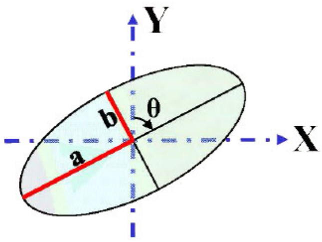


Figure 16: Confidence ellipse.

$$a = \sqrt{\frac{\sigma_x^2 + \sigma_y^2}{2} + \sqrt{\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2}} \tag{3}$$

$$b = \sqrt{\frac{\sigma_x^2 + \sigma_y^2}{2} - \sqrt{\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2}} \tag{4}$$

$$tan2\theta = \frac{2\sigma_{xy}}{\sigma_y^2 - \sigma_x^2} \tag{5}$$

Where 'a' and 'b' are the large and sort axis of the ellipse respectively; σ_x and σ_y are the standard deviation of X and Y coordinates σ_{xy} is the covariance of XY; θ represents the rotation angle of the ellipse.

5.2 Kinematic

Kinematic test is design to evaluate the position while the mobile phone is moving. Same as happened in the *section 5.1*, the not access to a *Leica 1230+ Gnss* will conditioned the way perform the evaluation of the positioning. Because of that the test is design to collect raw data along a line, this line would be an architectural element, large enough to be able to recognize it on *Goolge Earth*. And evaluate how the measured track with the mobile phone varies from the real one.

5.2.1 Data collection

The aim of this test is to collect data in an urban environment where the signal will suffer distortions. Normally, people use GNSS navigation apps on the mobile phone in urban scenarios. This is the reason to perform the measurements for this test at an urban scenario. The location chosen for the kinematic test is street divider located at the street *Antonio Romero Maroto* sited on *Jaén, Spain*, *fig. 17.* It has a straight line shape that will be follow in order to obtain a straight line with the measurements. The street divider is approximately 90 meters long and the building around the are about 20 meters high with a street width of 33 meters. With this situation is expected that some satellites signals will suffer multipath.

The mobile phone was collecting data from one point per second. The measurement are expected to describe a straight line so to evaluate the data it will be calculate the *regression line* then, from each point, the distance to the line will be calculated. Finally the distance will be analysed to obtain how much vary the position of the measurement from the expected line.

Figure 17: Location for Kinematic Test.

As said above this test pretend to emulate the way that a user will navigate with the mobile phone. For this reason the mobile phone is holding as show the *fig. 20a* while walking along the wall. There will be performed two set of 10 measurements one in the morning and the other in the evening. Also same as with the static test the data of our interest is from *GPS and Galileo* constellation. But different from the static test, the analysis and evaluation will be done computing the regression line and evaluate how much each measurement vary from it. Also by comparison between the path measured with the mobile phone and the street divider on *Google Earth, fig. 20a* it will provide a visual information about how the measurement vary from the real position.

5.2.2 Post-process

Same as was exposed on *section 5.1.2* the post-process will start by apply a filter to get only the data from *GPS and Galileo*. Also, after that it will be needed to compute the position different times to obtain the position filtered by signals (dual or single-frequency).

Once that the filters were applied, the position can be computed and evaluated. For this test we have two set of 10 measurement that will be evaluated to compare dual and single-frequency on multiconstellation GPS+Galileo as it is show below:

- Multiconstellation GPS+Galileo dual-frequency.

- Multiconstellation GPS+Galileo single-frequency.

In order to compute the position the software *GnssAnalysis* will be used as was mentioned on *section 4.6*. Setting it up the option for a moving receiver. It will be applied the atmospheric corrections and the elevation mask will be set at 5 degrees on each situation. *GnssAnalysis* compute the position in three different ways as was exposed on *section 4.6* but this project only use the positioning obtained from the raw pseudorange uncertainty without apply any smooth.

Finally after all the positions were computed the analysis will be done as is explain on next section.

5.2.3 Analysis

Be able to adjust the parameter mentioned above will allow us to get positioning under different scenarios that will provide information about the quality of the final position. The quality of this position will be calculated by compute the linear regression of the points measures and calculate how much is the distance from the line.

The equation for a linear regression if just the equation for any linear function:[18]

$$y = a \cdot x + b \tag{6}$$

In order to find the two constants for the equation a and b two formulas are needed.

$$a = \frac{n \sum(xy) - \sum x \sum y}{n \sum x^2 - (\sum x)^2} \tag{7}$$

$$b = \frac{\sum y - a \sum x}{n} \tag{8}$$

Where n is the number of observations, and x and y are the observed coordinates.

The results obtained with this test will allow us just to evaluate the relative positioning because is not possible to get a true reference positioning for this measurement. Anyway theses results will be compared with *Google Earth* as reference but, of course the value of this comparison can not be evaluated properly. It will provide only a visual idea of the quality of the measurement because of it will not be possible to get the exact reference data from *Google Earth*.

To evaluate the precision of those measurements it will be computed the standard deviation of the distances from each point to the regression line. The formula of the standard deviation eq. (9) normally ν is referred to the mean of the measurements. On this case the standard deviation is computed to calculate how much vary the points measured from the regression line, so the result will be evaluated how close to 0 will be.

$$\sigma^2 = \frac{\sum(\mu - d)^2}{n} \tag{9}$$

6 Tests execution

This section will explain the whole process of the execution of the tests. From collecting the data on the site until the post-process to compute the position and the calculations to evaluate the quality.

The section is structured in two main subsections: *Measurement* and *Post-process*. The first one will expose how and where the data were collected. On the second how the data will be computed and the position obtained on each case will be exposed.

6.1 Measurement

It was performed three measurements for the execution of the tests, two statics and one kinematic. The three of them consist in two measurement along the day, one at morning time and the second at evening time. For the static tests the two measurement consists on 10 minutes collecting data over the chosen point. In the case of the kinematic test it consist in walk along a straight line while collecting data 10 times for each set, morning and evening set.

6.1.1 Static

The data was collected from the geodetic point was on 11^{th} May 2020 and for the urban point was collected on 15^{th} May 2020.

Geodetic Point

This test was performed over the geodetic point *San Juan de Dios* as was exposed on *section 5.1.1 fig. 13*. The mobile was placed over the pillar, *fig. 18* and *GnssLogger* was collecting raw measurement during 10 minutes. This measurement was performed two times a long the day, once at morning time and the second at the evening.

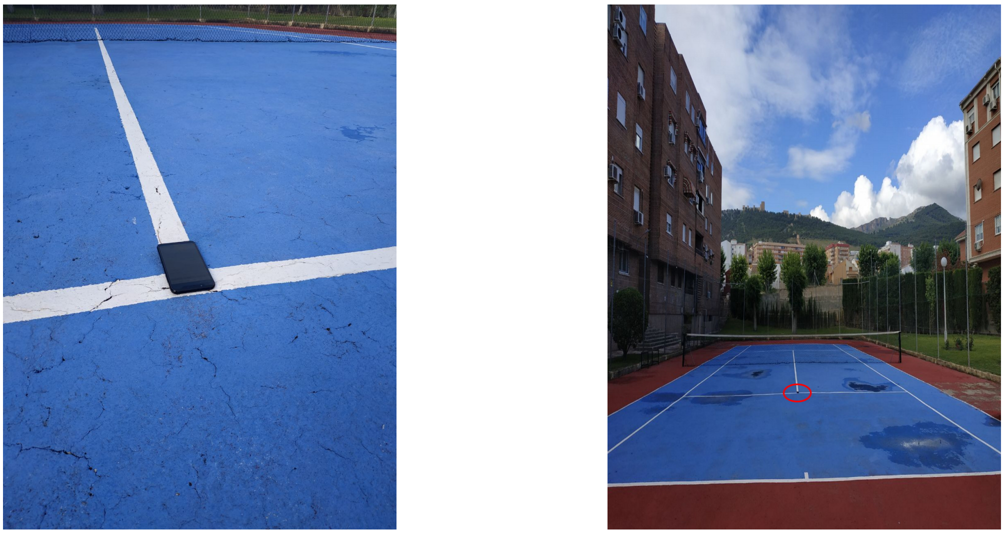
(a) Mobile phone over geodetic point (b) Geodetic point pillar

Figure 18: Geodetic point measurement.

This point was chosen to know in advance the true coordinates of a point so the results could be compared. Also because of the geodetic point is placed on top of a hill in the country side so there distortions from multipath that the signal could suffer are fewer. The execution and analysis of this test will provide information about the accuracy and the precision of the measurement.

Urban point

This test was performed over the point defined by the serve lines at a tennis court as was exposed on *section 5.1.1 fig. 14*. The mobile was placed over the cross serve line of the tennis court as shows *fig. 19a* and *GnssLogger* was collecting raw measurement during 10 minutes. This measurement was performed two times a long the day, once at morning time and the second at the evening.



(a) Mobile phone over the urban point

(b) Urban point location. Red circle is the mobile phone

Figure 19: Urban point measurement

The data collected at this location is expected to suffer distortion because of multipath effect. On this point it was not possible to know its true coordinates due to don't have access to the *Leica 1230+ Gnss*. Do not have the true coordinates will limit the evaluation of the measurement to analysed only the precision.

The analysis of this measurement will give information about the precision of the measurement in an urban environment. By the comparison between the results of the geodetic point and this one it can be quantify how much affected the the multipath to the compute position.

6.1.2 Kinematic

This test was performed over the straight line defined by the central divider of the street *Antonio Romero Maroto* sited on *Jaén, Spain*, as was exposed on *section 5.2.1*, *fig. 17*. This is a 90 meters length line and the data was collect holding by hand the mobile phone and walk following the line, *fig. 20a*. The reason to perform the measurement on this way because it was tried to replicate the way that people use the mobile phone when they are following an address. Also the data for this test was collected with

GnssLogger twenty times, one set of ten at morning time and another set at evening time.

The date of collect the data was originally on 13th May 2020 but the evening measurements were not recorded so it was need to go again on 1st June 2020 to complete the evening measurements.

(a) Kinematic data collection

(b) Kinematic location

Figure 20: Kinematic measurement.

The location for this test was chosen because the intent to replicate a common use of a mobile phone navigator. Normally used in an urban environment and walking holding the phone with the hand. Also to be able to make a study of the quality of the measurement and, because of it was not possible to use the*Leica 1230+ Gnss* it was need to choose a straight line to follow. Having this line to compare it will be possible to evaluate the distance from the measurements to the straight line. Also, because this line is recognizable on *Google Earth* it will provide a visual idea of how the data drift from where it was measured.

6.2 Post-process

This section will expose how the data was processed in order to compute the position and to analysed the quality of the position. It es possible to split into tow step the whole process. The first step is to compute the position, that will be performed with *GnssAnalysis*. As was exposed on *section 4.6* the software will provide among other information, the data for the raw positioning that is the one used for this project.

Right after compute the position it will obtained the kml file with the longitude and latitude coordinates referred to WGS84 system. Before start the statistical analysis it was decided to work with coordinates referred to the ETRS89 UTM 30 so instead of working with geocentric coordinates measured in degrees the analysis was done with Cartesian coordinates in meters. To transform the coordinates was used the software *QGIS*. Once that all the coordinates are transformed to Cartesian the analysis starts. To analysed the data it was written two *Python* scripts that can be found on *section 9.2* for the regression line computed at *Kinematic* test and *section 9.3* for the standard deviation computed at *Static* tests.

6.2.1 Static

Static tests are structured in two sections, the test over the Geodetic point and the test at the urban scenario. Both points was measured two times, morning and evening during 10 minutes as was exposed on *section 6.1*, Then, the data was processed with *GnssAnalisys* to obtain the coordinates in six different situations as shows *table 2*. It represents which constellations frequencies are used for the test and if exists any error with the data that make not possible to compute the position. There are a total of 24 data processed, from all this data there are four situations that could not be obtained the position because of the receiver get weak signal.

	Static			
	Geodetic point		**Urban point**	
	Morning	Evening	Morning	Evening
GPS+Gal. dual	✓	✓	✓	✓
GPS+Gal. single	✓	✓	✓	Weak Gal signal
Galileo dual	✓	✓	Weak signal	✓
Galileo single	✓	Weak signal	Weak signal	Weak signal
GPS dual	✓	✓	✓	✓
GPS single	✓	✓	✓	✓

Table 2: Static measurements

As we can see in the table above the four times that the position could not be computed belongs to *Galileo* and three of them to the urban point that was affected for the distortion of the buildings that were around.

Geodetic Point

The graphic bellow, *fig. 21* shows that the receiver could get signal for 8 satellites but only two of them has a frequency mean above 30 dB.Hz. That mean this two are the only ones that can be possible to use to compute the position and because it is needed have at least 4 satellites to compute the position this test could not be performed.

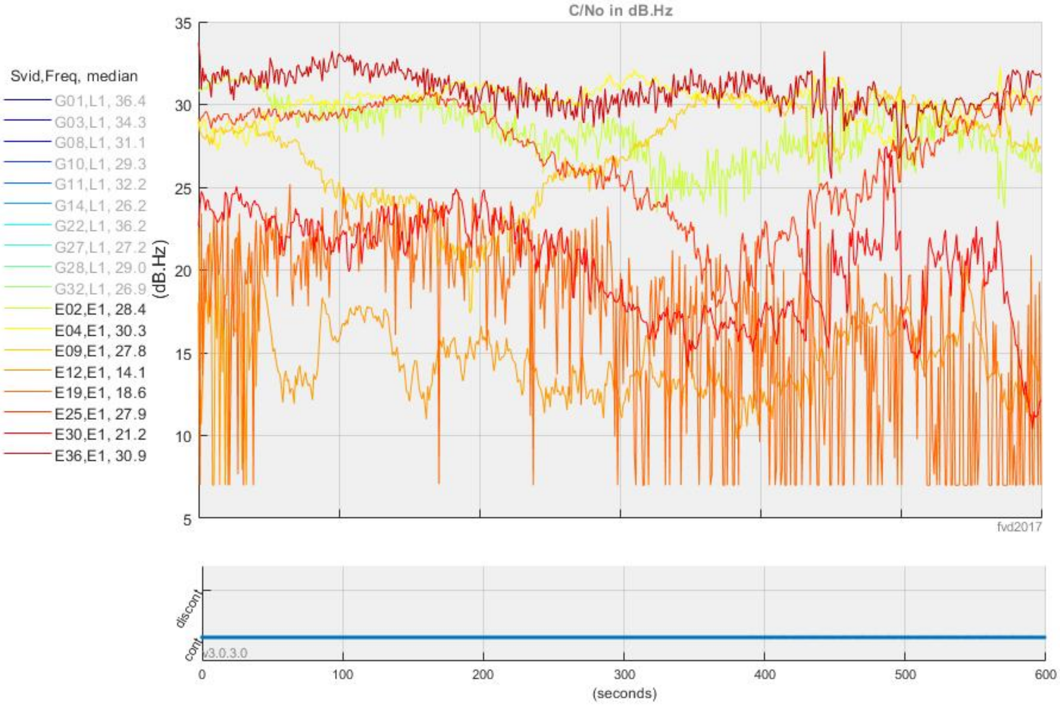


Figure 21: C/No during the test of *Galileo* Single evening

Also the following graphic, *fig. 22*, shows that the only two satellites that the signal is strong enough to be used for compute the position has a huge pseudorange error. Both situations, the lack of satellites with strong signal and the pseudo range error make not possible to compute the signal

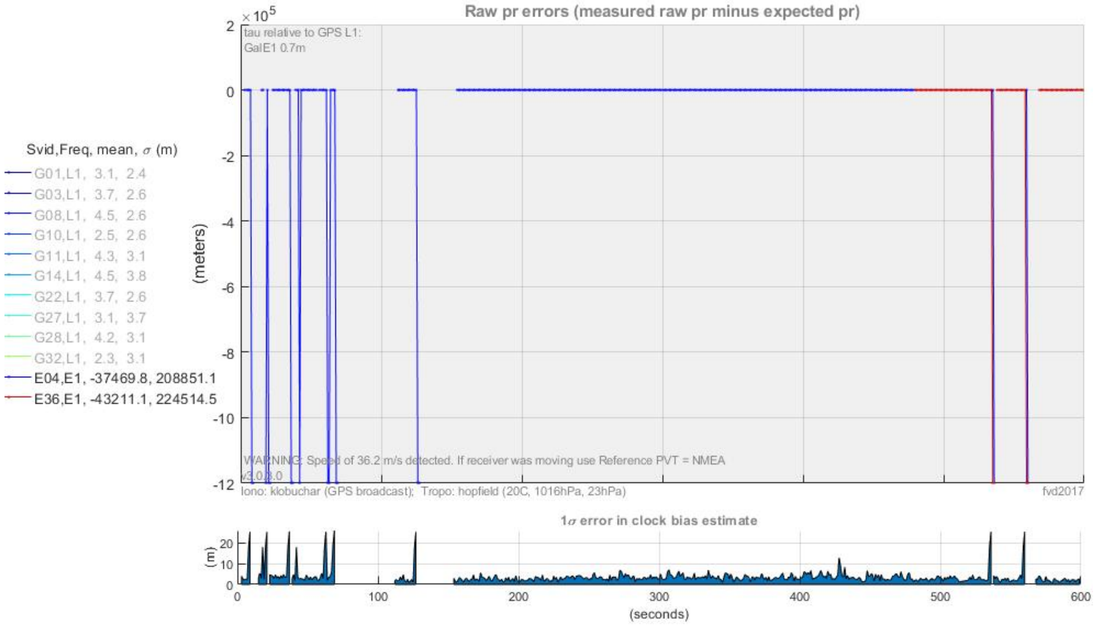


Figure 22: Pseudorange error for the test of *Galileo* Single evening

Both situations, the lack of satellites with strong signal and the pseudorange error for the ones that have strong signal make not possible to compute the position for this test.

Urban Point

At the urban point the test that could not be computed are from the four scenarios that belong only to *Galileo* constellation too. The reason of this problem is because of the weak signal that the receiver gets. The graphic bellow *fig. 24* shows the signals from the 6 satellites that the receiver collects, 5 of them are E1 and E5a frequencies the last one it is only E1 frequency.

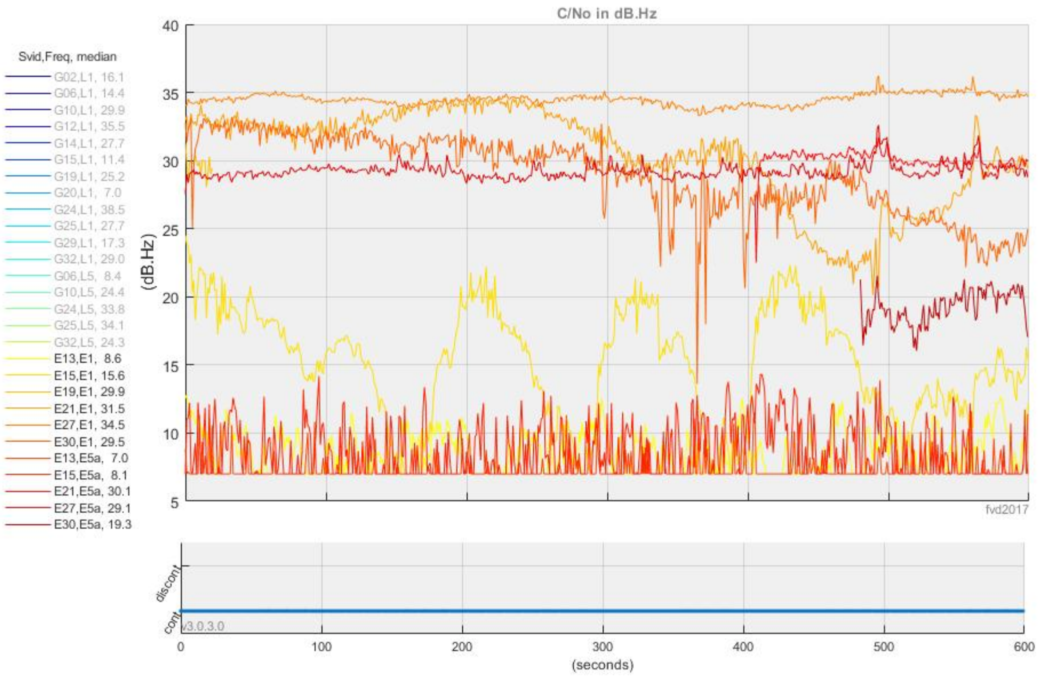


Figure 23: C/No during the test of *Galileo* dual morning

From these satellites there are only three that have the signal strong enough to be used for compute the position as shows *fig. 24*. Once again only three satellites are not enough to compute the position so the test can not be performed.

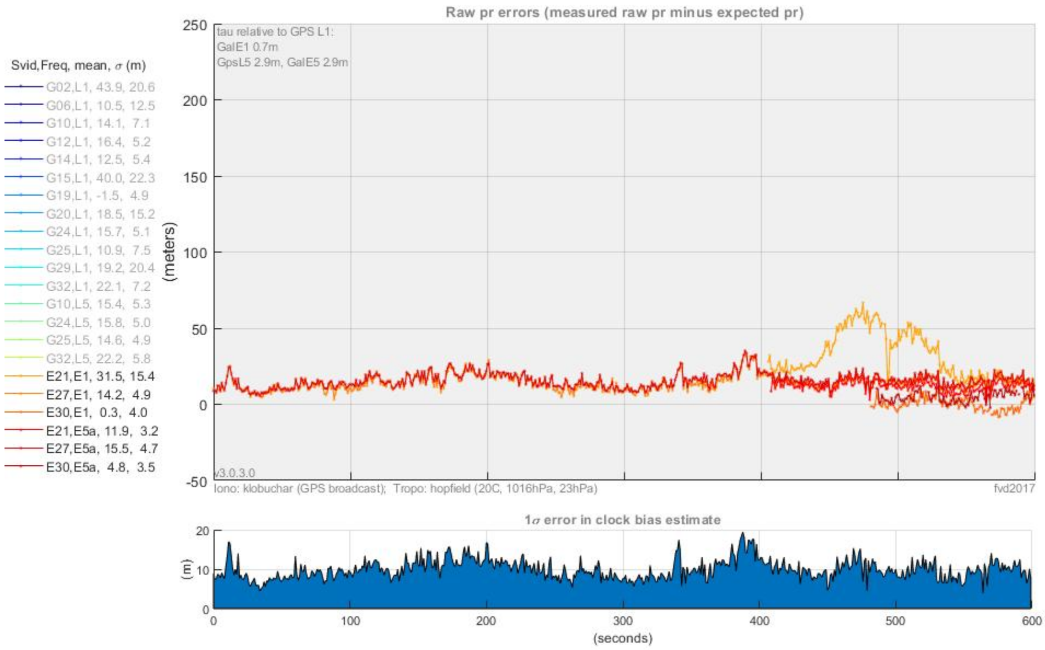


Figure 24: Pseudorange error for the test of *Galileo* dual morning

These graphic above, *fig. 24* and *fig. 23* belong to the Galileo dual test at morning time but the base data are the same for the Galileo single test at morning time so when compute the position for the single test it appear the same problems. Weak signal collected by the receiver and from less than four satellites that are the minimum to compute the position. That means the single frequency test can not be performed as well.

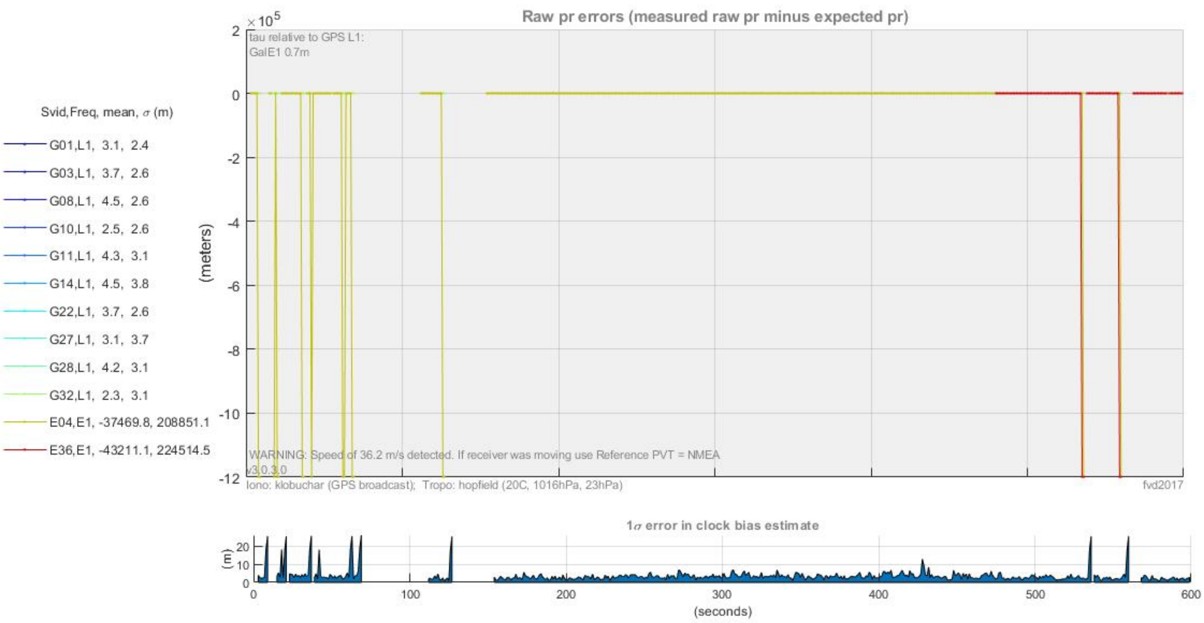


Figure 25: Pseudorange error for the test of *GPS+Galileo* single evening

Then, for the urban static test during the evening, the test could not be performed for Galileo single frequency. The reason was that only two satellites with single frequency have the strength enough but the pseudorange error for those two is to high to compute the position. So the lack of satellites and the pseudorange error make not possible to perform the test for Galileo single frequency and multiconstellation single frequency as show the *fig. 25*. This last one has the same result as GPS single test because only the GPS satellites were used.

6.2.2 Kinematic

	Kinematic		
	Set	**Morning**	**Evening**
GPS+Gal dual	1	✓	✓
	2	✓	✓
	3	✓	✓
	4	✓	✓
	5	✓	✓
	6	Data error	✓
	7	✓	✓
	8	✓	✓
	9	✓	✓
	10	✓	✓
GPS+Gal single	1	✓	✓
	2	✓	✓
	3	✓	✓
	4	✓	✓
	5	✓	✓
	6	Data error	✓
	7	✓	✓
	8	✓	✓
	9	✓	✓
	10	✓	✓

Table 3: Kinematic points measured status

The table above, *table 3*, shows the tests performed and if there was any error or situation that made not possible to compute the position and evaluate it. There are one set that it was kept out of the analysis, set number 6. This set delivered data that is out of the area, a huge distortions should happened because the data is far away from where it suppose to be. For these reason the sets were keep out of the analysis.

Because of the data it is the same for the dual an single frequency tests the problem to compute the position affect both scenarios.

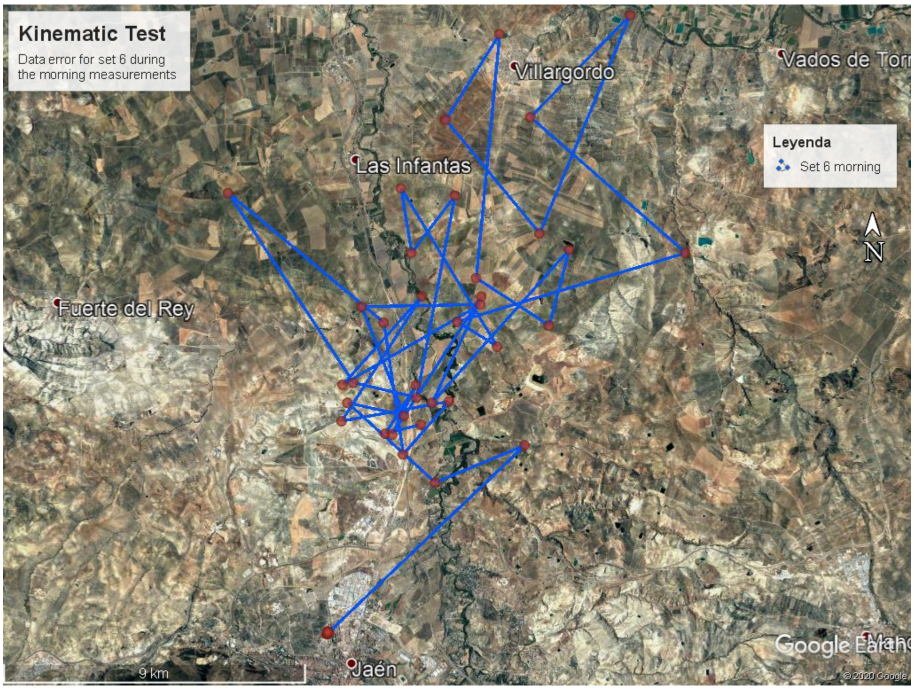


Figure 26: Measurements for set 6

The figure above, *fig. 26*, correspond to the measurements of the set 6. This measurement started from West to East, at the beginning the data collected is around the area that should be, then the distortion started and it is show on *fig. 26*, the measurement are spared more than 20 km to the North from the original point. This distortion problem only affect set 6, the rest of the measurement were performed without that problem.

7 Results

The section is divided in two main sections the *Static* and *Kinematic* where it is exposed the statistical results obtained from the test executed on the previous section *section 6*. The results try to expose the quality of the measurement under each scenario.

7.1 Static

Static test have two subsections, one for the test performed over the geodetic point where the precision was evaluated with the analyse of the standard deviation of the measurements and then the accuracy by comparison between the mean position and the coordinates of the geodetic point. The second subsection belong to the urban point where only the precision was evaluated.

Geodetic Point

As was said before the precision for the test performed over the geodetic point was computed the SD and the results are found at *table 4*. On this table we can observe the SD and covariance of all measurements performed.

	Geodetic point					
	Morning			**Evening**		
	σ_x	σ_y	σ_{xy}^2	σ_x	σ_y	σ_{xy}^2
GPS+Gal dual	1,2570	1,8964	-0,6674	0,2640	0,3494	-0,0532
GPS+Gal single	1,7171	2,9247	-0,2778	2,3853	3,8093	0,3472
Gal dual	0,7796	13,0908	-1,9927	1,2106	1,8007	-0,8305
Gal single	2,1044	3,5045	-0,5450	-	-	-
GPS dual	1,2185	1,8346	-0,6502	1,2877	1,6633	-0,7857
GPS single	2,1150	3,4917	-0,5528	2,3853	3,8093	0,3472

Table 4: Standard Deviation and covariance for Geodetic point measurements.

At evening time it need to be mentioned that the empty cells is because of the position could not be computed as was exposed on *table 2*. Also we can see that the SD values for *GPS+Gal single* are the same as *GPS single* during the evening measurements. That is related with having weak signal for *Gal single*, this problem with the signal make impossible to use *Galileo* satellites at any test. So *GPS+Gal single* only use *GPS single* signal.

We can see from this results that the precision is always better when the position is computed with dual frequency instead of single frequency. During the measurement performed at morning time we can observe that SD for Y coordinate when the position was computed with *Gal dual* it extremely high. After analyze the satellite signals and pseudorange errors there are not any remarkable error. Following we can observe at the *fig. 27 and fig. 28* that there are not any reason to think that the signal is the problem to have this results for Y coordinate when the SD for X coordinate is really small.

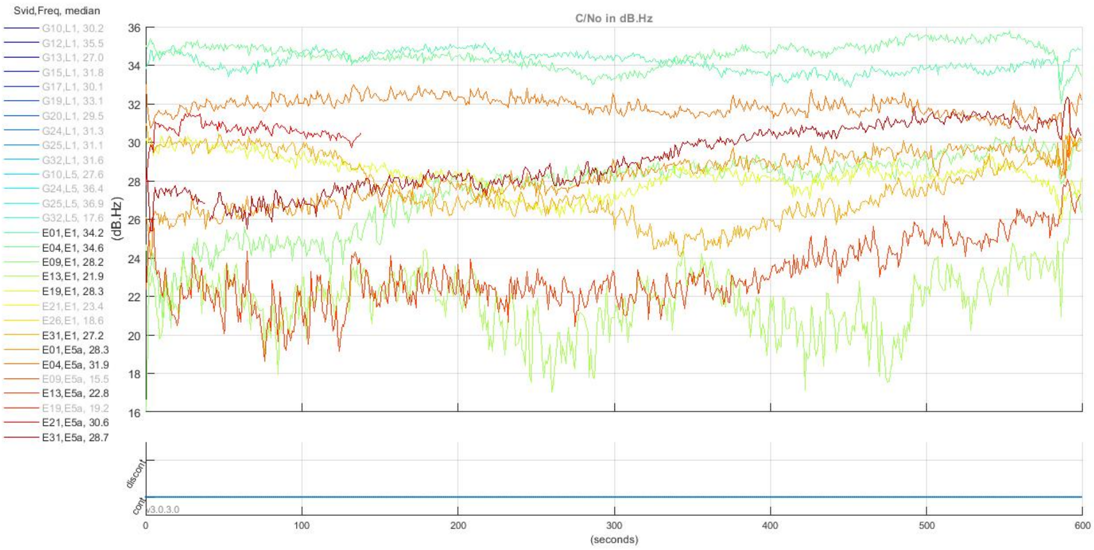


Figure 27: C/No for the test of *Galileo dual* morning

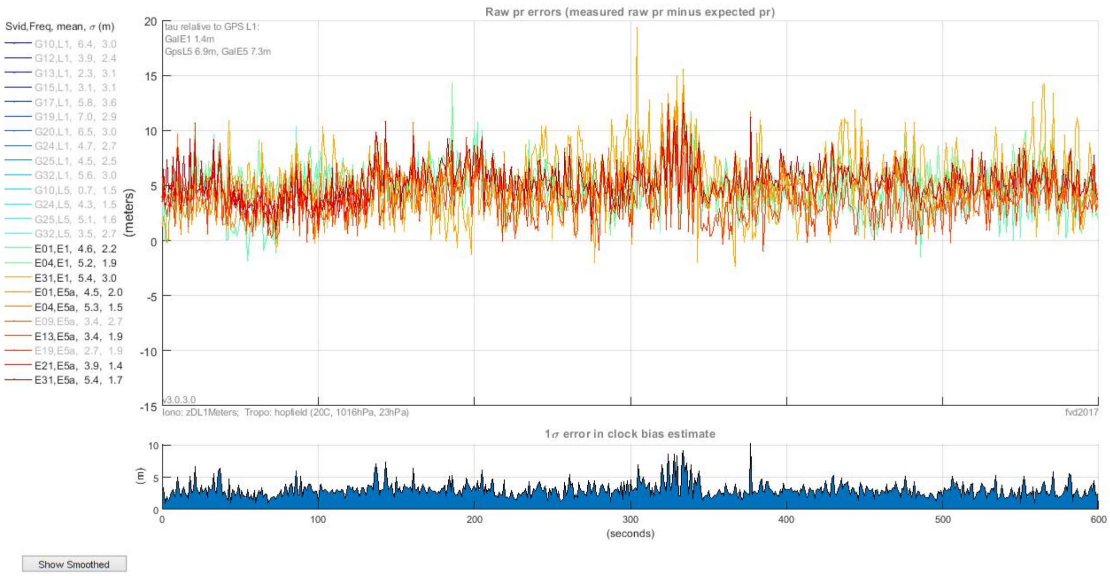


Figure 28: Pseudorange error for the test of *Galileo dual* morning

On the other side we can see that at the evening measurement. When the position is computed with *GPS+Gal dual* satellites we obtained a really good precision, less than half meter for both coordinates. This result is what was expected from the article *'World's first dual-frequency'* [1] published at the GSA web site. But have only one result with this precision is not enough to consider it as a reference when the others results are about 1.5 meters for X and Y coordinates.

The position obtained by compute the mean of at the values collected at each test will be compared with the coordinates of the Geodetic point that provide the *IGN* [15], at *section 9.1* are found the information about the Geodetic point.

The coordinates used for the comparison are referred to the ETRS89 UTM30 system:

- X: 434148,756 m
- Y: 4186420,323 m

	Geodetic point measured coordinates			
	Morning		Evening	
	X_m	Y_m	X_m	Y_m
GPS+Gal dual	434146,8369	4186423,4945	434149,0973	4186423,5695
GPS+Gal single	434148,0021	4186421,3825	434148,5429	4186421,6626
Gal dual	434149,1057	4186422,9125	434148,8342	4186424,2984
Gal single	434148,3360	4186421,1476	-	-
GPS dual	434147,6234	4186423,0553	434149,4408	4186424,9721
GPS single	434148,0560	4186421,3176	434148,5434	4186421,6648

Table 5: Coordinates obtained during the test over the Geodetic point, ETRS89 UTM30

The table above, *table 5* shows the coordinates obtained by compute the mean of the 600 epochs that were measured during the tests.

To understand better the variations of the computed position measured from the Geodetic point was computed the distances as show the table below, *table 6*.

	Geodetic point distances					
	Morning			Evening		
	ΔX_m	ΔY_m	d_m	ΔX_m	ΔY_m	d_m
GPS+Gal dual	1,919	-3,171	3,707	-0,341	-3,247	3,264
GPS+Gal single	0,754	-1,059	1,300	0,213	-1,340	1,356
Gal dual	-0,350	-2,590	2,613	-0,078	-3,975	3,976
Gal single	0,420	-0,825	0,925	-	-	-
GPS dual	1,133	-2,732	2,958	-0,685	-4,649	4,699
GPS single	0,700	-0,995	1,216	0,213	-1,342	1,359

Table 6: Distances from the measurements to the Geodetic point

It can be observed that the X coordinate always have less difference with the real point than the Y coordinates. But the most noticeable information that provide this comparison is that when the position was computed with dual frequency the accuracy get worst than with single frequency. This is something unexpected because dual frequency must increase the accuracy.

Urban Point

The Urban point test was designed to test the precision of the measurement under a scenario where the signal suffer multipath distortion. Unlike the previous test, this test only evaluate the precision of the measurements because it was not possible to get the true coordinates of the point. Same as with the Geodetic point test, the precision was evaluated by compute the SD and the results are show on the next *table 7*.

	Urban point					
	Morning			Evening		
	σ_x	σ_y	σ_{xy}^2	σ_x	σ_y	σ_{xy}^2
GPS+Gal dual	5,1032	12,8271	-5,9763	1,6643	1,1740	-0,5225
GPS+Gal single	1,4053	6,8593	22,9732	6,3916	5,5028	-2,0349
Gal dual	4.5287	12.5175	-5.0508	-	-	-
Gal single	-	-	-	-	-	-
GPS dual	4,3031	26,0912	-6,9337	2,4162	3,5641	0,2208
GPS single	9,8969	38,8969	-1,4100	9,0022	9,7294	-2,2826

Table 7: Standard Deviation and covariance for Urban point

Again we need to mention some situation where the position could not be computed. These situations are one in the morning measurements that belong to *Galileo single* and two during the evening measurements that belongs to *Galileo single and dual* at some tests, in this case it is because there are only three satellites *fig. 29* that the mobile phone was recorded signal stronger enough to compute the position and the minimum need to be four.

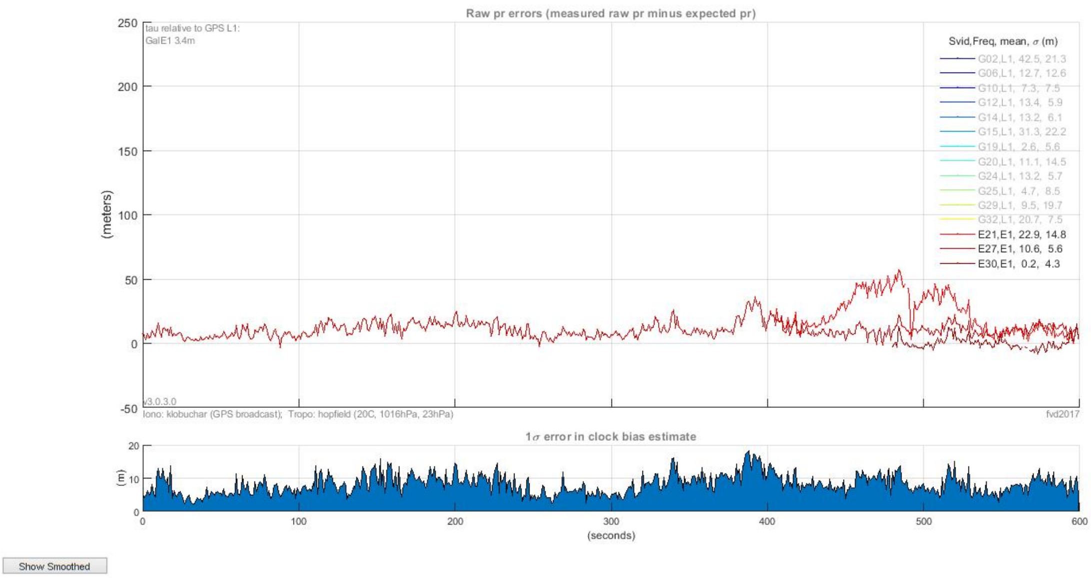


Figure 29: Pseudorange error for the test Galileo single morning

Then for the evening measurement we have the situation that the position was not computed for *Galileo* constellation, both single and dual frequencies. This time the mobile phone collected data from five satellites but two of them the signal was too weak so can not be used to compute the position and once again there are only three satellites left to compute the position that is not possible. The *fig. 30* shows the carrier phase of the *Galileo* satellites dual and singe frequency collected by the mobile phone, as it shows satellites E29 and E25 has too weak signal in both frequencies.

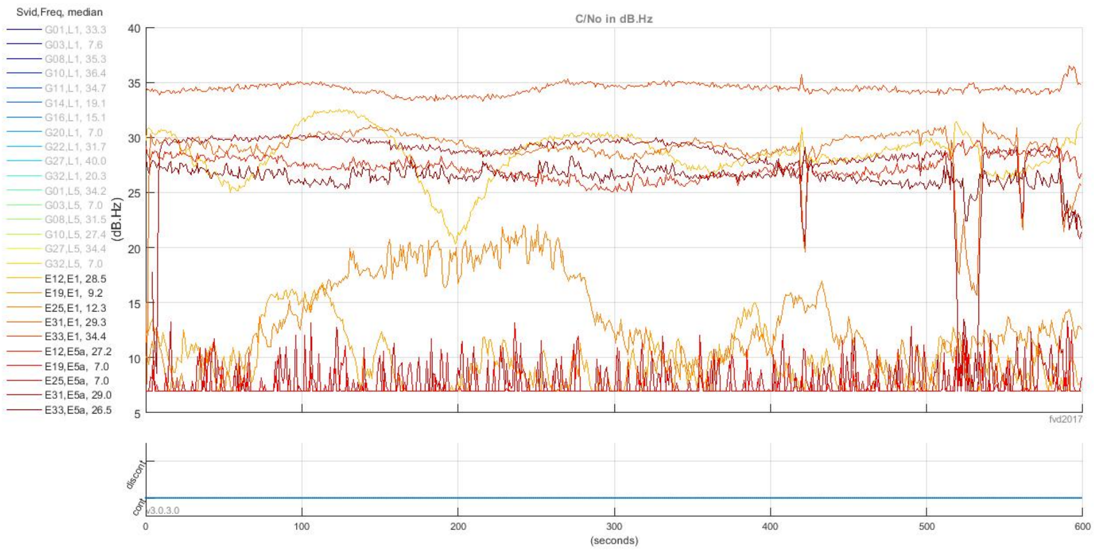


Figure 30: C/No for the test Galileo dual and single morning

This test also show same as happened with the geodetic point test the SD for the Y coordinate are higher than the SD for the X coordinates but different than Geodetic point test at the evening the are more similar between X and Y SD.

As expected the distortion of multipath make this results worst than the ones of the Geodetic point test. Also it is noticeable that during morning test something affect the dual frequency tests probably the L5/E5 frequencies because the test where the position was computed using multi-constellation dual frequency the SD is bigger than the single something that shouldn't be like that.

Finally the coordinates of the Urban point refereed to the ETRS89 UTM30 taked as an approximation reference were taken from *Google Earth*:

- X: 430043.76 m
- Y: 4181754.35 m

These coordinates are just a reference to be used as information but can not be used to test accuracy because of the way that was obtained.

The following table, *table 8*, shows the coordinates of the computed position of each test. From this point was calculate the distance to the point obtained from *Google Earth* as shows *table 9*

	Urban point measured coordinates			
	Morning		Evening	
	X_m	Y_m	X_m	Y_m
GPS+Gal dual	430044,7009	4181746,9893	430045,8534	4181757,2188
GPS+Gal single	430050,1330	4181750,8999	430046,4766	4181754,2110
Gal dual	430044,4934	4181747,6343	-	-
Gal single	-	-	-	-
GPS dual	430044,4929	4181746,9839	430045,8965	4181757,0178
GPS single	430049,1543	4181750,1127	430046,0097	4181754,8762

Table 8: Urban point X,Y UTM30 mean coordinates

During the tests performed with the morning data, dual tests have higher differences on the Y coordinates while the X keep values bellow the meter. On the other side the single tests have higher distances at X coordinate than at Y. Then at the evening test, besides the *Galileo* constellation could not be used itself because of the lack of satellites it provide support to compute the position by multi-constellation and the results are much better than in the morning but slightly worst than when the position is computed only using *GPs* constellation.

	Urban point distances					
	Morning			Evening		
	ΔX_m	ΔY_m	d_m	ΔX_m	ΔY_m	d_m
GPS+Gal dual	-0,941	7,361	7,421	-2,093	-2,869	3,551
GPS+Gal single	-6,373	3,450	7,247	-2,717	0,139	2,720
Gal dual	-0,733	6,716	6,756	-	-	-
Gal single	-	-	-	-	-	-
GPS dual	-0,733	7,366	7,402	-2,137	-2,668	3,418
GPS single	-5,394	4,237	6,860	-2,250	-0,526	2,310

Table 9: Distances from the measurements to the Urban point

7.2 Kinematic

Kinematic test was design to evaluate only the precision. This time the precision was evaluated also by computing the SD of the distances from the measured points to the LR. Same as happen with the Urban point test the reason to do that is because the absence of a reference where the coordinates where known.

At the *table 10* it is exposed the result of the standard deviation computed from the distances that exits between the point and the regression line.

		Kinematic	
		Morning	**Evening**
	Set	σ_d	σ_d
GPS+Gal dual	1	10,9841	5,9289
	2	9,7086	6,7730
	3	14,6667	6,7825
	4	9,9420	6,8267
	5	11,2848	6,6316
	6	-	6,8018
	7	11,6320	6,7640
	8	10,1679	6,2739
	9	10,2493	7,0726
	10	8,2580	7,1438
GPS+Gal single	1	11,0781	8,3978
	2	10,8003	7,1085
	3	9,5614	8,6716
	4	10,9868	7,1860
	5	9,6875	8,0987
	6	-	6,9178
	7	10,0935	8,0903
	8	10,9687	7,1203
	9	9,7625	7,6955
	10	9,7750	7,9727

Table 10: Standard Deviation Kinematic

It can be observed that the measurement performed during morning time have worst results than the ones at the evening. It is something to mention because that happened also to the static test that have better results at the evening besides the measurement were performed on different days.

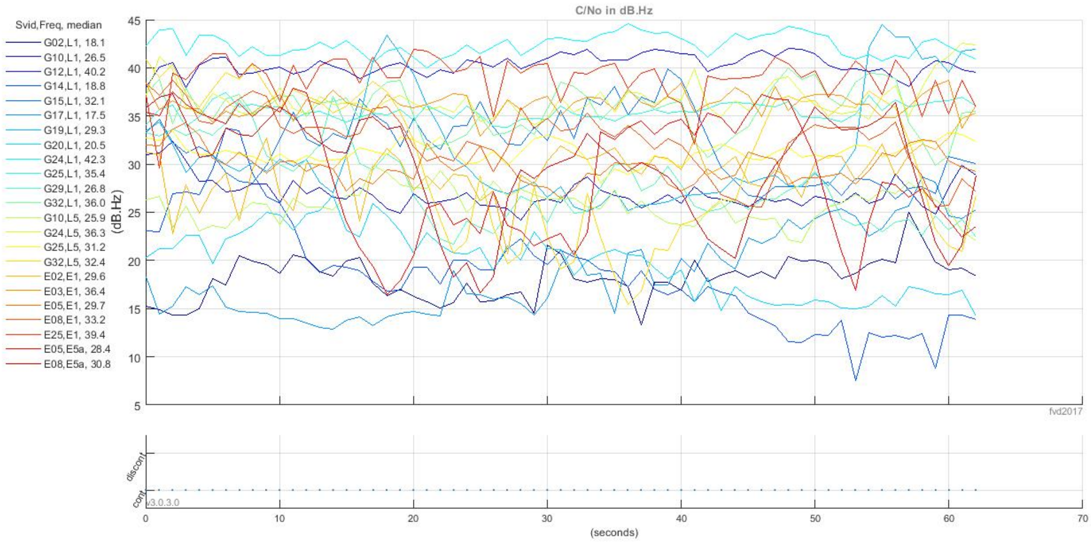


Figure 31: C/No for the test of the set 6 during morning

During the morning measurements there was a problem with the data for the set number 6. As was exposed on *section 6.2.2* the data was not used to evaluate the precision of the measurements because the it had a huge error on the positioning results, more than 20 km. This time the signal was good as show the *fig. 31* but the problem appear with the clock. There raw data recorder was missing two field: *Drift Nanos Per Second* and *Drift Uncertainty Nanos Per Second*. The lack of this data make impossible to compute the position properly and as show *fig. 32* the results are too bad to include it on the calculation of the precision.

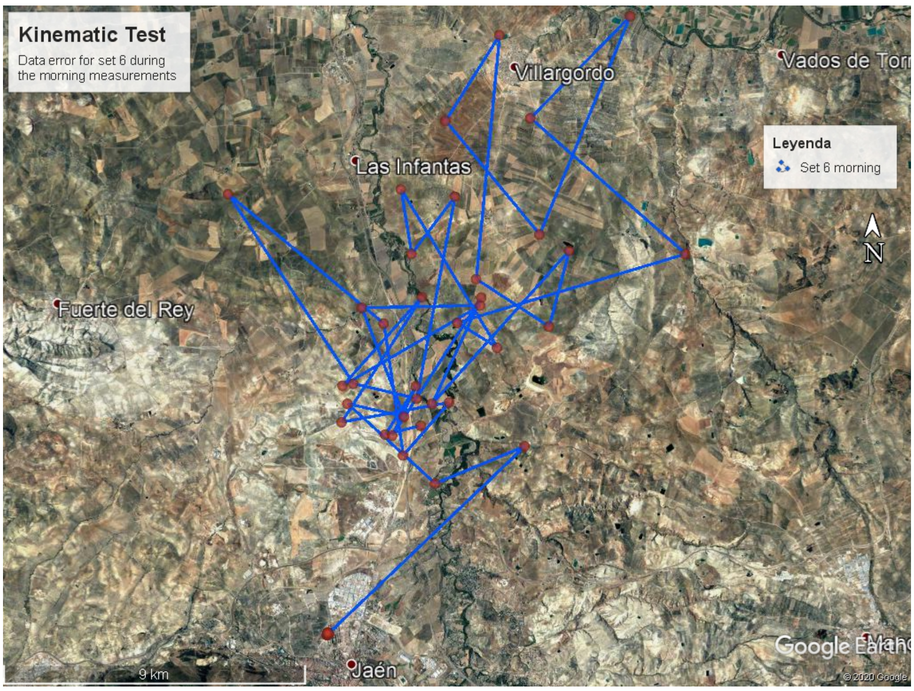


Figure 32: Measurements for set 6

To simplify the results of the SD exposed on *table 10* was computed the mean and it is exposed on the next table:

	Kinematic mean of SD	
	Morning (m)	**Evening** (m)
GPS+Gal dual	10,766	6,700
GPS+Gal single	10,302	7,726

Table 11: Mean of the Standard Deviation

Focusing on the evening test that are the ones that suffered less distortion on the static tests. We can observe that the results are better when the position it is computed with dual frequency but only one meter. If we compare this results with the statics tests was expected better results.

8 Conclusion

Considering that the aim of this project was to study the quality of dual frequency positioning on a mobile phone versus single frequency, after having done the tests designed for that case, and taking into account the difficulties suffered during the project's development due to the situation arising from corona virus, I must begin saying that the conclusions drawn from this work cannot be conclusive but if it serves as guidance

In order to answer the questions established in the problem statement, *section 2*, first of all we will start with the subquestions.

Regarding how precision and accuracy have been tested, three different tests were carried out, in all of them the precision was tested and in one only accuracy could be tested. The tests performed were one in kinematic and two in static. Of the two statics, in one the accuracy could be tested, since we knew the coordinates of the point previously.

The standard deviation on both the X-axis and the Y-axis was calculated for static tests, obtaining the results of the differences with the mean and with it the precision of each measure. On the other hand, to calculate the accuracy of one of them, the difference between the mean position and the known coordinate point was calculated.

The case of the dynamic test, only the precision was calculated. This was done by calculating the regression line for each set and the distance from each point measured to that line. The standard deviation of these distances was then calculated, thus obtaining the precision of each set.

Answering question, *'which parameters affect to compute the position?'*, in section *section 4.3* was exposed which parameters affect to compute positioning and it magnitude. Once that these parameters were exposed and after read from the GSA [19] that dual frequency positioning reduce the errors cause by multipath this situation wanted to be evaluated.To do this, an open sky test was performed and two others were carried out in the city, where it was ensured that the signal suffered from multipath.

Finally, to answer the main question, and considering that the GSA when talk about dual frequency mobile and in particular this mobile, says that the accuracy achieved will be of decimeters.

After analyzing the results obtained in this project it is seen that; In the case of accuracy, the minimum values found are around 1.30 meters, being 0.923 meters the lowest and this belongs to the position computed with Galileo single frequency. In fact, all the results obtained using double frequency are much worse than those obtained with simple frequency. Getting differences with the known coordinate point of more than the double that the results obtained with single frequency.

Regarding precision, we observe that the improvement in positioning is much better for cases where positioning has been carried out using double frequency, obtaining values from 1.2 meters to 3.8m, with some exceptions. Obtaining similar results regardless of the constellation used. We only obtain a result of decimeters in the case where the position was calculated in the evening, whose standard deviations are for the X-axis 0.264 meters and, for the Y-axis 0.3494 meters.

When the precision of the tests was evaluated in an urban environment where the signal suffers from multipath, the results are seen to worsen markedly. It is also observed that the results are not as stable as they were in the case of the open sky test. Here a minimum error of 1.40 meters was found, obtaining errors of up to 38 meters.

Finally, when the precision of the kinematics measurements was tested, the results obtained show a similarity in the errors. Very similar in the test carried out during the morning, only 40 centimeters difference between single and dual frequency, but better in single. However in the evening the results are better in the case of dual frequency. In this case the results obtained are 6.7 meters for single and 7.7 for dual frequency.

As a final conclusion we can say that it is true that the improvement in positioning using double frequency is notable despite being far from the expected improvement of decimeters. But being fair, it must be mentioned that the conditions under which the mobile phone has been tested have not been the best in order to obtain a conclusive result. For which its should have done at least a measurement for a longer time and thus be able to obtain a more objective result.

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9 Appendixes

9.1 Appendix A

Geodetic Point Information

Reseña Vértice Geodésico

1-abr-2020

Número.....: 94739
Nombre.....: San Juan de Dios
Municipios: Jaén
Provincias: Jaén
Fecha de Construcción........: 12 de noviembre de 1990
Pilar sin centrado forzado..: 1,20 m **de alto,** 0,30 m **de diámetro.**
Último cuerpo......................: 0,50 m **de alto,** 1,00 m **de ancho.**
Total cuerpos.........: 1 **de** 0,50 m **de alto.**

Coordenadas Geográficas:

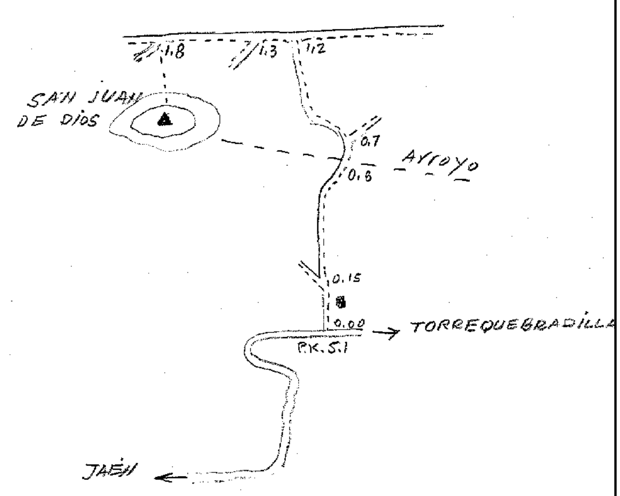
Sistema de Ref.:	ED 50	ETRS89
Longitud...........:	- 3° 44' 49,0058"	- 3° 44' 53,64602" ±0.073 m
Latitud..............:	37° 49' 26,6142"	37° 49' 22,14595" ±0.077 m
Alt. Elipsoidal...:		489,336 m ±0.076 (BP)
Compensación.:	04 de febrero de 1994	01 de noviembre de 2009

Elipse de error al 95% de confianza.

Coordenadas UTM. Huso 30 :

Sistema de Ref.:	ED 50	ETRS89
X.......................:	434260,36 m	434148,756 m
Y.......................:	4186626,36 m	4186420,323 m
Factor escala....:	0,999653224	0,999653409
Convergencia...:	- 0° 27' 29"	- 0° 27' 32"

Altitud sobre el nivel medio del mar: 439,757 m. (BP)



Situación:

Situado en lo más alto del paraje conocido por San Juan de Dios o La Cantarera.

Acceso:

Desde Jaén, por la carretera de Torrequebradilla, en el P.K. 5,100 se toma un camino que sale a la izquierda; a los 100 m. se pasa junto a una caseta, a los 50 m. se encuentra un complejo arqueológico y se deja un camino que sale a la izquierda, a los 450 m. se cruza un arroyo, a los 100 m. se deja un camino a la derecha, a los 500 m. se gira a la izquierda, a los 100 m. se deja un camino a la izquierda (con cadena), a los 500 m., donde sale otro camino, se deja el vehículo y en 5 minutos, andando, se llega al vértice.

Observaciones:

Horizonte GPS:

Despejado

CF: Centrado Forzado. CP: Cabeza Pilar. BP: Base Pilar. CN: Clavo Nivelado. CS: Clavo Suelo.
@ anulado, © no pertenece a bloque de compensación, ® recrecido, reparado o reconstruido.

Informe del estado del Vértice: ftp://ftp.geodesia.ign.es/utilidades/InfoRG.pdf

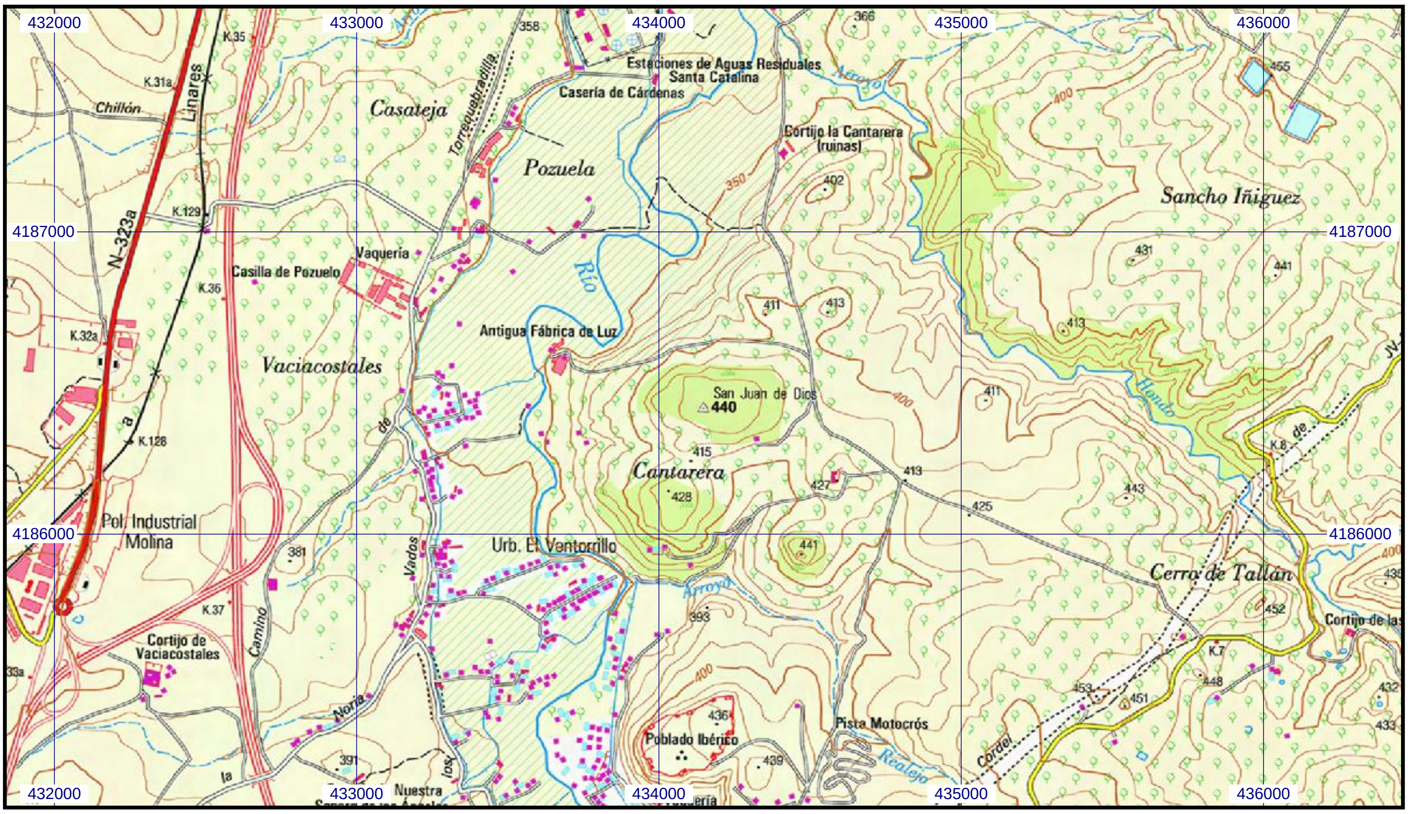
GOBIERNO DE ESPAÑA
MINISTERIO DE FOMENTO
INSTITUTO GEOGRÁFICO NACIONAL

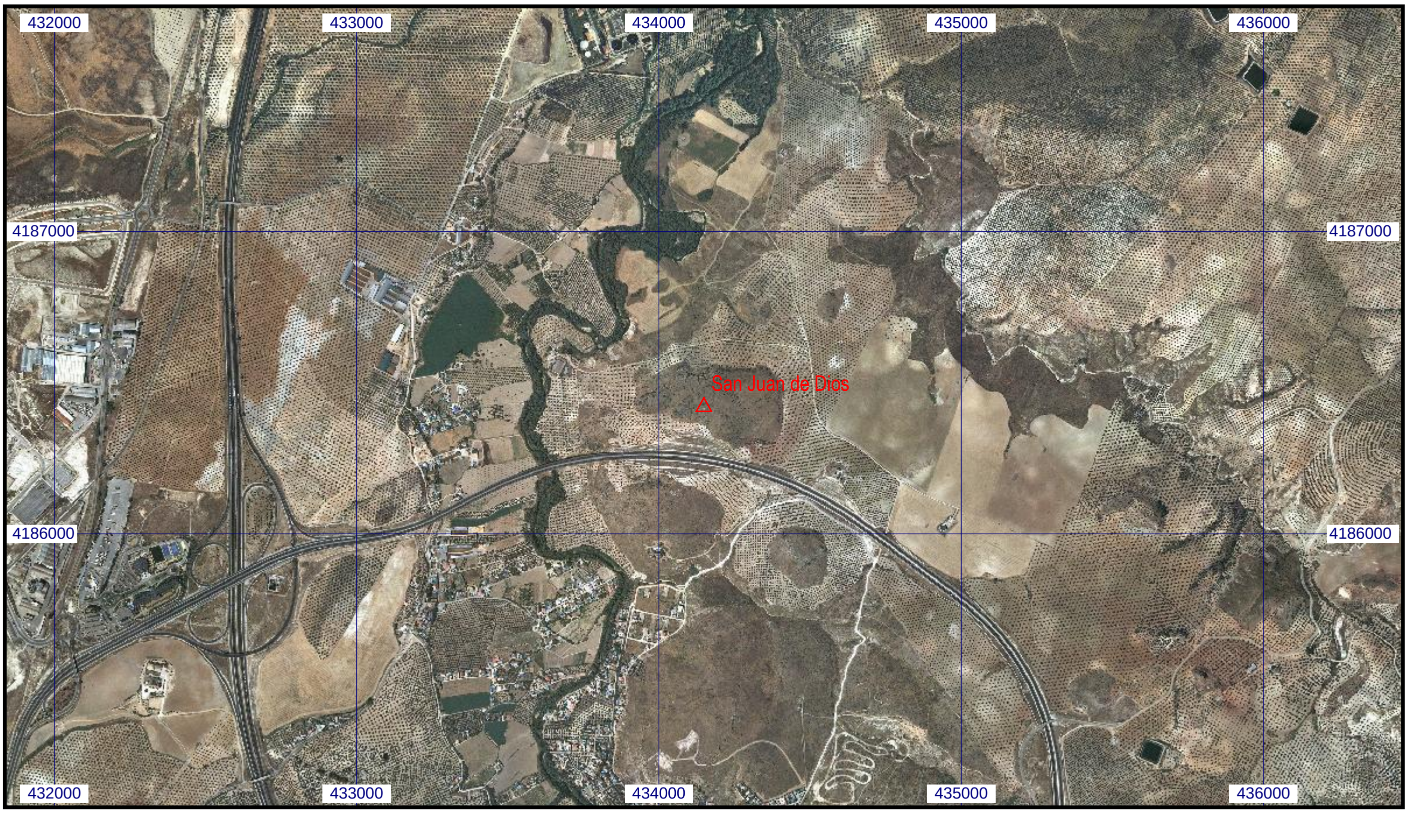
Área de Geodesia
Subdirección General de Geodesia y Cartografía

Cartografía de situación

1-abr-2020

Escala 1:25.000 | **094739 San Juan de Dios** | **Coordenadas ETRS89. Huso 30**





9.2 Appendix B

Python script for the compute of the regression line and the standard deviation for the Kinematic tests

In [1]:

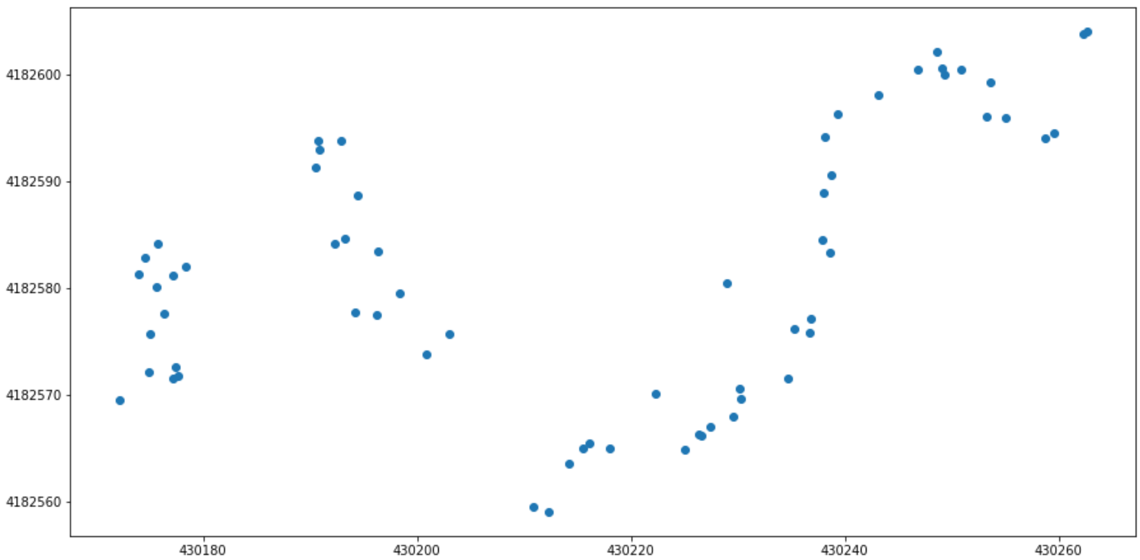
```
import pandas as pd #for bigdata
import numpy as np #for basic math etc
import matplotlib.pyplot as plt # to plot graphics
from sklearn.linear_model import LinearRegression #science and engineering
from scipy.odr import *
import math
```

In [2]:

```
df = pd.read_csv("set_1UTMs.csv", #import the data
                sep=',', #is separete by space
                header=None, #the file doesnt have header
                names=['X', 'Y', 'Z'], #write the headers names of each colum
                usecols=['X', 'Y', 'Z'], #specifying the colum that I want to use
                dtype={'X': np.float64, 'Y': np.float64} #this change the format of the
number to float number para ver los numeros sin exponencial
                #dtype={'X': np.str, 'Y': np.str} #this change the format of the number
to string para ver los numeros sin exponencial
                )
```

In [4]:

```
plt.figure(figsize=(15,15))

plt.ticklabel_format(useOffset=False)

plt.scatter(df['X'], df['Y']) #to plot the graphic of the point cloud
plt.gca().set_aspect('equal', adjustable='box')
```



In [5]:

```
x = np.array(df['X']).reshape(-1, 1) # we need to reshape the x variable because otherw
ise the LinearRegression is not able to work with it
y = np.array(df['Y'])  # no need to be reshaping because once that the X is reshaped th
e Y follow it
```

In [6]:

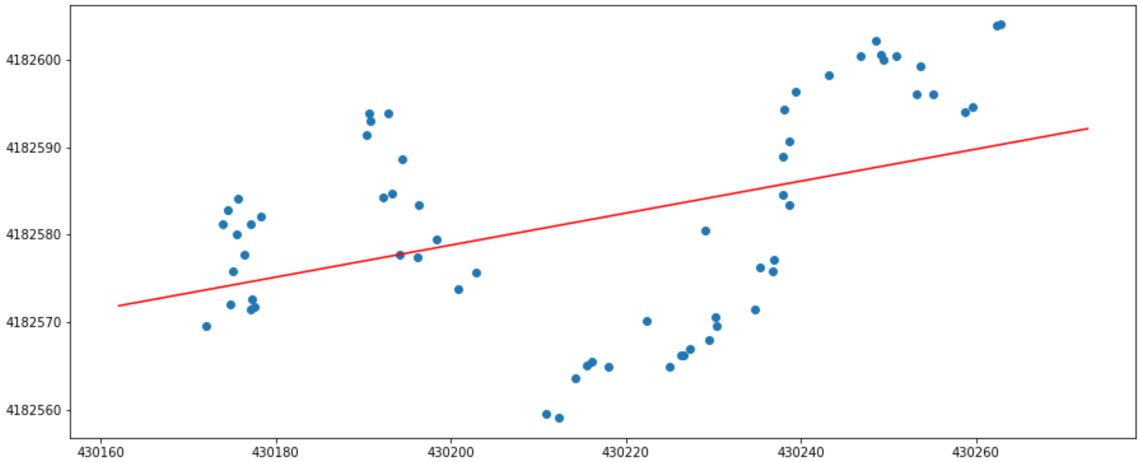
```
mmcc = LinearRegression() #initialize the linear regression
mmcc.fit(x, y) #calculate the two parametres (slove and intersection)
```

Out[6]:

```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
         normalize=False)
```

In [7]:

```
plt.figure(figsize=(15,15))

plt.ticklabel_format(useOffset=False)

plt.scatter(x, y) #to print
plt.plot(
    [x.min()-10, x.max()+10],mmcc.predict(np.array([x.min()-10,x.max()+10]).reshape(-1,
1)),
    c='r')#print the line and give it colour red

plt.gca().set_aspect('equal', adjustable='box')
```



Parameters of the regession line

In [8]:

```
a = mmcc.coef_[0] #slope
```

In [10]:

```
b = mmcc.intercept_ #intersection with Y axis
```

Orthogonal distance

In [12]:

```
n = abs(a * x - y.reshape(-1, 1) + b) # numerador
d = math.sqrt(a**2+1) # denominador
```

In [13]:

```
dist = n/d #distance from the point to the line
```

In [16]:

```
sdd = math.sqrt(sum((0-dist)**2)/len(dist))
```

In [19]:

```
with open('set_1UTMs_sd.csv', 'w') as f:
    f.write(f'Sd: {sdd}\n')
```

In [21]:

```
pd.DataFrame(dist).to_csv('set_1UTMs_dist_perp.csv', index = None, header = None)
```

In [22]:

```
binter = y.reshape(-1, 1)+(1/a)*x # interseccion con eje y de cada punto
```

In [23]:

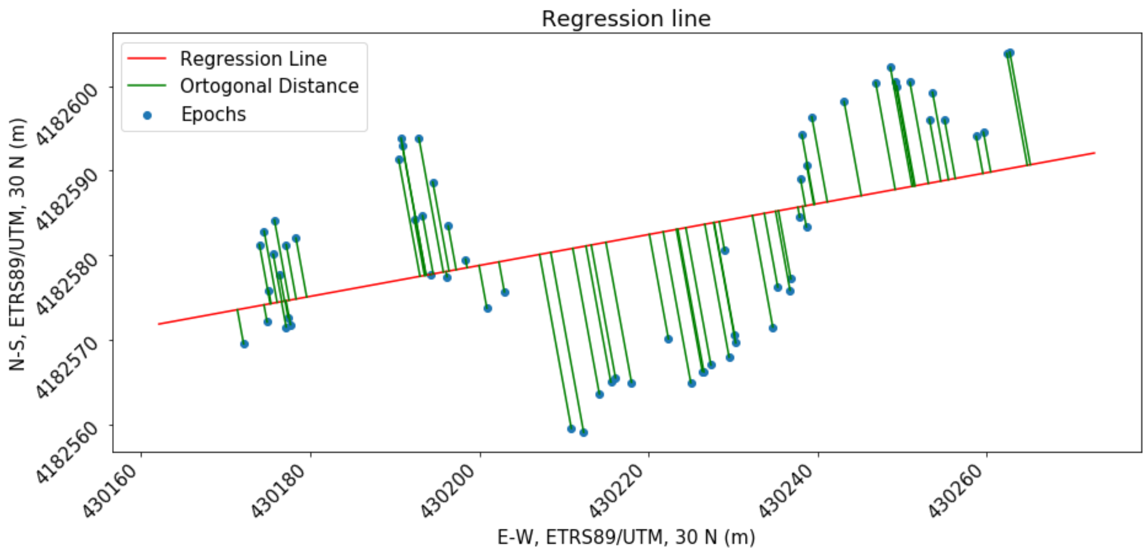
```
n1 = binter-b # numerador
d1 = (a**2+1)/a # denominador
xptos = n1/d1 #calculo de x
yptos = -(1/a)*xptos+binter #calculo de y
```

In [27]:

```
plt.rcParams.update({'font.size': 15})
plt.figure(figsize=(15,15))

plt.ticklabel_format(useOffset=False)
plt.xticks(rotation=45, horizontalalignment='right')
plt.yticks(rotation=45, horizontalalignment='right')

plt.title('Regression line')

plt.scatter(x, y, label='Epochs') #to print
#plt.scatter(xptos, yptos) #puntos sobre la linea
plt.plot(
    [x.min()-10, x.max()+10],mmcc.predict(np.array([x.min()-10,x.max()+10]).reshape(-1,
1)),
    c='r', label='Regression Line') #print the line and give it colour red
for i in range(len(x)):
    plt.plot([x[i], xptos[i]], [y[i], yptos[i]], c='g', label='Ortogonal Distance')

plt.ylabel('N-S, ETRS89/UTM, 30 N (m)')
plt.xlabel('E-W, ETRS89/UTM, 30 N (m)')


plt.legend(bbox_to_anchor=(1.05, 1), loc='upper left', borderaxespad=0.)


plt.gca().set_aspect('equal', adjustable='box')


handles, labels = plt.gca().get_legend_handles_labels()
i = 1
while i < len(labels):
    if labels[i] in labels[:i]:
        del(labels[i])
        del(handles[i])
    else:
        i += 1

plt.legend(handles, labels)
```

Out[27]:

<matplotlib.legend.Legend at 0x875b43e7f0>



9.3 Appendix C

Python script for the compute of standard deviation for the static tests

In [1]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Ellipse
from sklearn.linear_model import LinearRegression
from scipy.odr import *
import math
```

In [2]:

```
df = pd.read_csv("SJD_03_t.csv",
                sep=',',
                header=None,
                names=['X', 'Y', 'Z'],
                usecols=['X', 'Y', 'Z'],
                dtype={'X': np.float64, 'Y': np.float64}
                #dtype={'X': np.str, 'Y': np.str}
                )
```

In [3]:

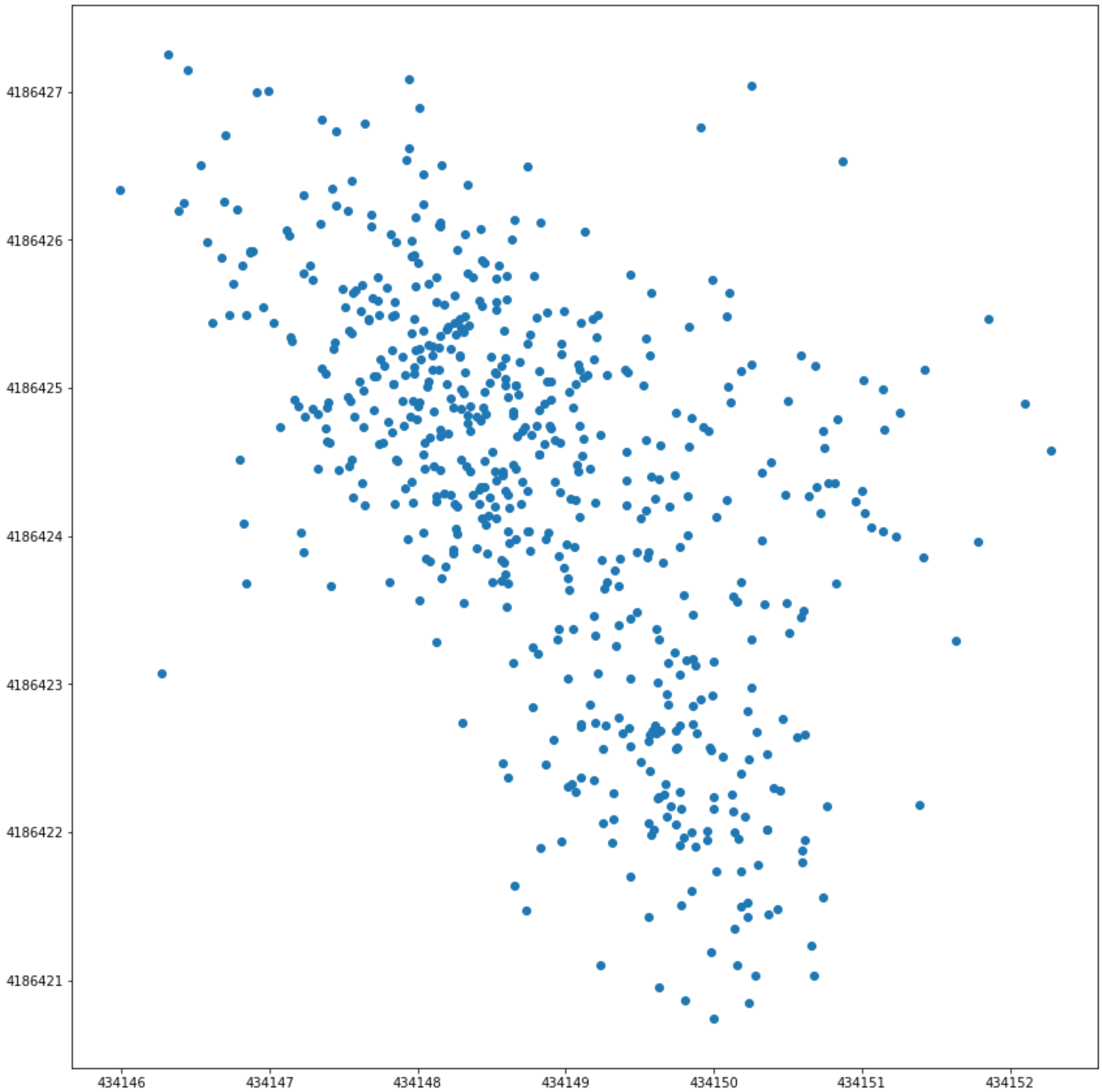
```
#CENTER OF THE ELLIPSE
#Geodetic point coordinates, ETRS89 UTM 30 N
x_sjd = 434148.756
y_sjd = 4186420.323
```

In [4]:

```
center = [x_sjd,y_sjd]
```

In [5]:

```
plt.figure(figsize=(15,15))

plt.ticklabel_format(useOffset=False)

plt.scatter(df['X'], df['Y'])
plt.gca().set_aspect('equal', adjustable='box')
```



In [6]:

```
x = np.array(df['X']).reshape(-1, 1)
y = np.array(df['Y']).reshape(-1, 1)
```

Standard deviation, cov y RMS

In [7]:

```
rex = np.mean(x) #mean x
rey = np.mean(y) #mean y
```

In [8]:

```
cov = np.cov(np.hstack([x,y]).T)
print(cov)
```

```
[[ 1.21054788 -0.83046489]
 [-0.83046489  1.80072747]]
```

In [9]:

```
sigx = cov [0,0]
sigy = cov [1,1]
covxy = cov [0,1]
```

In [10]:

```
with open('SJD_03_t_param.csv', 'w') as f:
    f.write(f'cov: {covxy}\n')
    f.write(f'sigx: {sigx}\n')
    f.write(f'sigy: {sigy}\n')
```

Confidence ellipse

In [11]:

```
coel1 = (sigx+sigy)/2
coel2 = (((sigx-sigy)**2)/4)+covxy**2
a = math.sqrt(coel1+math.sqrt(coel2))
b = math.sqrt(coel1-math.sqrt(coel2))
print(a)
print(b)
```

```
1.5449827978146686
0.7901287860420235
```

In [12]:

```
numer = 2*covxy
denom = (sigy-sigx)
print(numer)
print(denom)
```

```
-1.6609297858261214
0.5901795897098472
```

In [13]:

```
t1 = (2*covxy)/(sigy-sigx)
theta = 90-(math.atan(t1)/2)*180/math.pi
print(theta)
```

125.21915240197745

In [15]:

```
plt.rcParams.update({'font.size': 15})
plt.figure(figsize=(15,15))

plt.ticklabel_format(useOffset=False)
plt.xticks(rotation=45, horizontalalignment='right')
plt.yticks(rotation=45, horizontalalignment='right')

plt.title('Confidence ellipse')

#plt.figure()
ax = plt.gca()#.set_aspect('equal', adjustable='box')
plt.scatter(rex,rey, c='r', label='Mean')
plt.scatter(x, y, c='b', label='Epochs')

prueba = Ellipse(xy=(rex, rey), width=3.035*a*2, height=3.035*b*2, angle=theta,
                        edgecolor='r', fc='None', lw=2, label='Ellipse')

plt.ylabel('N-S, ETRS89/UTM, 30 N (m)')
plt.xlabel('E-W, ETRS89/UTM, 30 N (m)')


ax.set_xlim(rex-5, rex+5)
ax.set_ylim(rey-5, rey+5)

#plt.gca().set_aspect('equal', adjustable='box')

ax.add_patch(prueba)

plt.legend(bbox_to_anchor=(0.005, 0.995), loc='upper left', borderaxespad=0.)
```

Out[15]:

<matplotlib.legend.Legend at 0x7c8ab25128>