# Master Thesis

- Advanced Analysis of Steel Structures -


Project Report by

Anders Anthonippilai



Aalborg University
M.Sc. in Structural and Civil Engineering

$4^{th}$ Semester
May $24^{th}$ 2019

**AALBORG UNIVERSITY**
STUDENT REPORT

**The School of Engineering and Science**
Study Board of Civil Engineering
Thomas Manns Vej 23, 9220 Aalborg Ø
http://www.ses.aau.dk/

**Title:**
Advanced Analysis of Steel Structures

**Theme:**
Design of Steel Structures

**Project Period:**
Spring Semester 2019

**Participant:**
Anders Anthonippilai

**Supervisor:**
Johan Christian Clausen

**Copies:** 1

**Page Numbers:** 84

**Date of Completion:**
May 24th 2019

**Abstract:**

In this report 2 methods from the design guide Eurocode (EC) 1993-1-1 has been employed to highlight and quantify load carrying capacity of steel structures.
Method 1 being an analytical method in EC clause 6.3.3 where only hand calculation is needed. The hand calculation was made with the help of calculation program MathCAD.
Method 2 in EC clause 6.3.4 described as the general method is a numerical approach where the FEA software tool Abaqus has been used in this project. General method has its advantages of being applicable for material non-linearities and large deformation.
A literature study provided description for the theoretical background of the two methods which has been presented in this report.
Finally, a comparison between the two methods has been made and discussed.

# Preface

This report is written by 4th semester student as a part of the Master's programme in Structural and Civil Engineering at Aalborg University.

Prerequisites for reading the report is knowledge regarding the AAU PBL method, steel structures in a technical perspective, methods from Eurocode, and basic knowledge of numerical solutions using Abaqus/CAE.

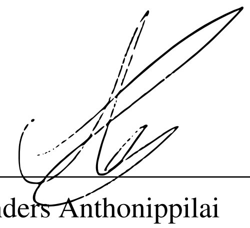
Great gratitude is addressed to the supervisor of the project, Assoc. prof. Johan C. Clausen.

**Reading guide**
References to sources are in the form of the Harvard method, and a complete source list is stated in the bibliography. References are made to sources with either "[Surname/organisation, Year]" or "Surname/organisation [Year]" and, when relevant, specific pages, tables or figures may be stated. Websites are specified by author, title, URL and date. Books are specified by author, title, publisher and edition, where available. Papers are furthermore specified with journal, conference papers with time and venue, when available.

The report contains figures and tables, which are enumerated according to the respective chapter. E.g. the first figure in Chapter 5 has number 5.1, the second number 5.2 and so on.

References are made to folders on the enclosures-CD attached to the report, which contains digital files of various kinds. The reference are in the form: "[Enclosures-CD, Folder name]".

Aalborg University, May 24th 2019



Anders Anthonippilai
<aantho14@student.aau.dk>

# Symbols

| | |
|---|---|
| $\alpha$ | Imperfection factor |
| $\alpha_{LT}$ | Imperfection factor for lateral-torsional buckling |
| $\alpha_{cr,op}$ | Minimum load amplifier out-of-plane |
| $\alpha_{ult,k}$ | Minimum load amplifier in-plane |
| $\beta$ | Factor to use of determining critical elastic flexural-torsional buckling force, $N_{cr,TF}$ |
| $\beta_A$ | Ratio between $A$ and $A_{eff}$ |
| $\gamma_{M0}$ | Partial coefficients |
| $\gamma_{M1}$ | Partial coefficients |
| $\gamma_{M2}$ | Partial coefficients |
| $\varepsilon$ | Factor to use of determining cross-section classification |
| $\overline{\lambda}$ | Non-dimensional slenderness |
| $\lambda_1$ | Factor to use of determining non-dimensional slenderness |
| $\overline{\lambda}_{op}$ | Global non-dimensional slenderness |
| $\overline{\lambda}_{LT}$ | Non-dimensional slenderness for lateral-torsional buckling |
| $\overline{\lambda}_T$ | Non-dimensional slenderness for torsional and torsional-flexural buckling |
| $\rho$ | Density |
| $\tau_{Ed}$ | Shear stress |
| $\upsilon$ | Poisson's ratio |
| $\phi$ | Factor to use of determining reduction factor, $\chi$ |
| $\phi_{LT}$ | Factor to use of determining reduction factor for lateral-torsional buckling, $\chi_{LT}$ |
| $\chi$ | Reduction factor for lateral buckling |
| $\chi_{op}$ | Minimum reduction factor of $\chi$ and $\chi_{LT}$ |
| $\chi_{LT}$ | Reduction factor for lateral-torsional buckling |
| $\chi_y$, $\chi_z$ | Reduction factors due to flexural buckling for respectively strong and weak axis |

| | |
|---|---|
| $f_y$ | Nominal values of yield strength of structural steel |
| $f_u$ | Ultimate tensile strength of structural steel |
| $h$ | Height |
| $i$ | Radius of gyration about the relevant axis |
| $i_c$ | Radius of polar gyration |
| $k_z$, $k_w$ | Effective length factors that depend on the support conditions at the end sections |
| $k_{ii}$ | Interaction factor |
| $q$ | Design load |
| $q_{applied}$ | Applied load |
| $q_{ult}$ | Ultimate load |
| $t$ | Thickness of the examined point |
| $y_c$ | Distance along the y axis between the shear centre and the centroid of the section |
| $z_a$, $z_s$ | Coordinates of the point of application of the load and of the shear centre, relative to the centroid of the cross-section |
| $z_g$ | $z_g = (z_a - z_s)$ |
| $z_j$ | Parameter that reflects the degree of asymmetry of the cross-section in relation to the strong axis |
| $t$ | Time-step |

| | |
|---|---|
| $A$ | Area of cross-section |
| $A_{eff}$ | Effective area of cross-section |
| $A_v$ | Shear area of the cross-section |
| $C_1, C_2, C_3$ | Coefficients depending on the shape of the bending moment diagram and on support conditions |
| $E$ | Young's Modulus |
| $G$ | Shear Modulus |
| $I$ | Second moment of area |
| $I_T$ | Torsion constant |
| $I_z$ | Moment of inertia about the weak axis |
| $I_w$ | Warping constant |
| $L$ | Geometrical length |
| $L_{cr}$ | Buckling length |
| $L_E$ | Buckling length |
| $L_{ET}$ | Equivalent length |
| $M_{b,Rd}$ | Design buckling resistance moment |
| $M_{Ed}$ | Design value of the moment |
| $M_{c,Rd}$ | Design resistance for bending moment |
| $M_{cr}$ | Elastic critical moment for lateral-torsional buckling |
| $M_{y,Ed}$ | Design value of the moment about the strong axis |
| $M_{y,Rk}$ | Characteristic value of the moment about the strong axis |
| $M_{z,Ed}$ | Design value of the moment about the weak axis |
| $M_{z,Rk}$ | Characteristic value the moment about the weak axis |
| $\Delta M_{y,Ed}$, $\Delta M_{z,Ed}$ | Moments due to the shift of the centroidal axis |
| $N_{Ed}$ | Design value of the compression force |
| $N_{b,Rd}$ | Design buckling resistance |
| $N_{c,Rd}$ | Design resistance of the compression member |
| $N_{cr}$ | Elastic critical force |
| $N_{cr,T}$ | Elastic torsional buckling force |
| $N_{cr,TF}$ | Elastic torsional-flexural buckling force |
| $N_{cr,y}$ | Critical load for flexural buckling about the strong axis |
| $N_{Rk}$ | Characteristic value of the compression force |
| $S$ | First moment of area |
| $UR$ | Utilization ratio |
| $V_{c,Rd}$ | Design elastic shear resistance |
| $V_{Ed}$ | Design value of shear force |
| $V_{pl,Rd}$ | Plastic shear resistance |
| $W_{pl}$ | Plastic section modulus |
| $W_{el,min}$ | Elastic section modulus |
| $W_{eff,min}$ | Effective minimum section modulus |
| $W_y$ | Section modulus of the compression flange |

# Table of Contents


**Preface** **III**

**Symbols** **V**

**Table of Contents** **IX**

**Chapter 1 Introduction** **1**
- 1.1 Aim of the Project . . . . . . . . 1
- 1.2 Instability Modes Regarding Steel Structures . . . . . . . . 2
- 1.3 Methods . . . . . . . . 5
- 1.4 Limitations . . . . . . . . 6

**Chapter 2 Steel Structures** **7**
- 2.1 Static system . . . . . . . . 7
- 2.2 Profiles . . . . . . . . 8
- 2.3 Material Properties . . . . . . . . 8
- 2.4 Beam . . . . . . . . 9
- 2.5 Frame . . . . . . . . 9

**Chapter 3 Analytical** **11**
- 3.1 Method 6.3.3 . . . . . . . . 11
- 3.2 Results . . . . . . . . 23

**Chapter 4 Numerical** **27**
- 4.1 General method - Method 6.3.4 . . . . . . . . 27
- 4.2 Abaqus . . . . . . . . 27
- 4.3 Determining of the Minimum Load Amplifier, $\alpha_{ult,k}$ . . . . . . . . 33
- 4.4 Determining of the Minimum Load Amplifier, $\alpha_{cr,op}$ . . . . . . . . 37
- 4.5 Results . . . . . . . . 39

**Chapter 5 Comparison** **40**

**Chapter 6 Conclusion** **42**

**Bibliography** **44**

**Appendix A Analytical Calculation** **45**

**Appendix B Numerical Approach** **80**

# Introduction 1

*In this chapter the projects relevance is discussed, some theoretical background is explained, and material properties are presented.*

Steel structures are favourable due to numerous reasons: the ability of prefabricating which reduces errors on the construction sites, labour hours at construction sites, and cost of the material compared with the space needed of cross-section.

A number of failures have been recorded regarding steel frames and therefore a closer examination and insight has become essential. As an example of an accident of steel frame structure can be seen in Figure 1.1 which took place in Rønbæk in 2016.

***Figure 1.1.*** https://stiften.dk/article_gallery/420020

There are several approaches based on EC standards for a design of steel structures, which have been used over the years. In [Standard, 2005] there is introduced a new methodology to be used for steel structure design. The main purpose of this project is to compare the existing method with the new proposed method and to make parameter and sensitivity studies.

## 1.1 Aim of the Project

In ultimate limit state (ULS) analyses of steel frames; compression forces and bending moments are of concern, as they may lead to global instability manifested in either buckling or lateral torsion buckling failure. The design guide Eurocode (EC) sets up procedures for evaluating the ULS and in EC different designs approaches are suggested. Some EC-approaches are more simplifying than others resulting that the final evaluation of the ULS depends on the method chosen for the evaluation. The aim of the project is to highlight and quantify load carrying capacity of steel frames employing different methods, ranging from basic methods to more advanced methods, and with different steel configurations. In all methods a comparison will be made with FE-analyses to various degree of complexity. Furthermore the method 6.3.4 from EC 3-1-1 (which is applicable for material non-linearities and large deformations) will be investigated which requires non-linear analysis.

### 1.1.1 Linear and Non-linear material behaviour

The global analysis of a steel structure provides with sufficient accuracy the internal forces, moments, and the corresponding displacements. The internal forces and displacements may be determined using either an elastic or plastic analysis. Elastic analysis is based on the assumption of a linear stress-strain relation for steel (see Figure 1.2). Plastic analysis, assumes progressive yielding of some cross-sections of the structures, normally leading to plastic hinges and a redistribution of forces as explained in [da Silva et al., 2010]. For design purposes, steel is idealized as an elastic-perfectly plastic material as seen in Figure 1.2 b.

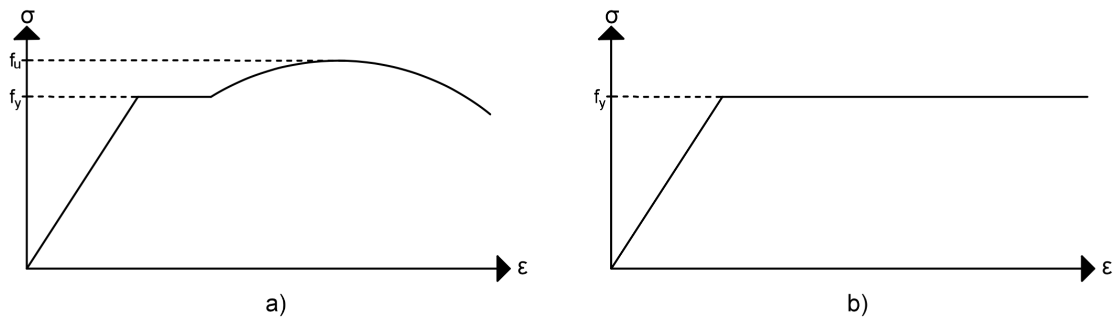



***Figure 1.2.*** Stress-strain relation of steel a) Real behaviour, b) Perfect elastic-plastic behaviour.

Sometimes it is also necessary to model a non-linear geometry analysis, referring to the second order analysis. In the first order analysis the internal forces and displacements are obtained with reference to the undeformed structure (small displacements assumption). In the second order analysis the influence of the deformation of the structure is taken into account (large displacements), the procedure and the methodology used is explained in Chapter 4. The different deformation shapes influencing are explained and described furthermore in Section 1.2.

## 1.2 Instability Modes Regarding Steel Structures

As explained in [da Silva et al., 2010], the resistance of a steel member subjected to axial compression depends on the cross-section resistance or the occurrence of instability phenomena. As steel

members usually have high slenderness the design for compression is governed by the instability phenomena such as:
• Flexural buckling
• Torsional buckling
• Flexural torsional buckling
• Lateral torsional buckling

The buckling resistance should be evaluated according to the relevant buckling mode and relevant imperfections of real members, as described in the following sections.

### 1.2.1 Flexural Buckling

Flexural buckling is a phenomenon that occurs about the axis of the highest slenderness ratio and the smallest radius of gyration. It can happen in any member subjected to compression, which in the end will lead to deflection of the member. An illustration of the flexural buckling can be seen in Figure 1.3.

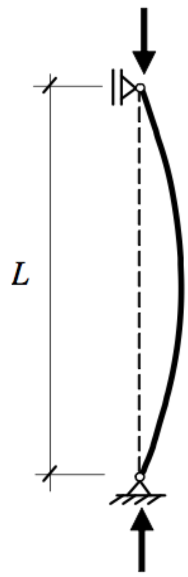



***Figure 1.3.*** Flexural buckling of a column, [da Silva et al., 2010].

### 1.2.2 Torsional Buckling

Torsional buckling is a form of buckling occurring about the longitudinal axis of a member, where the center of the member remains straight while the rest of the section rotates. An illustration of torsional buckling can be seen in Figure 1.4.

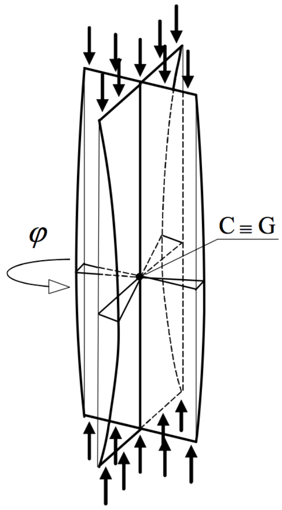



***Figure 1.4.*** Torsional buckling, [da Silva et al., 2010].

### 1.2.3 Flexural Torsional Buckling

According to [da Silva et al., 2010], flexural torsional buckling consists of the simultaneous occurrence of torsional and bending deformations along the axis of the member. An illustration of this can be seen in Figure 1.5.

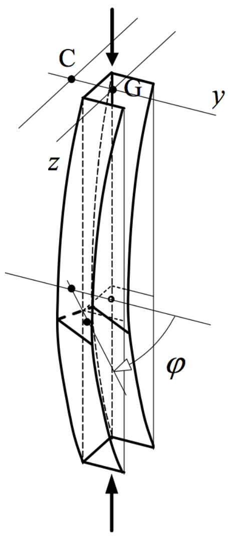



***Figure 1.5.*** Flexural torsional buckling, [da Silva et al., 2010].

### 1.2.4 Lateral Torsional Buckling

Lateral torsional buckling is as stated in [da Silva et al., 2010], characterized by lateral deformation of the compressed part of the cross-section. In an I-profile, the compressed part will be one of the flanges. As a part of the member will behave under compression, it will also simultaneously have one continuously restrained by the part of the section in tension. This will result in a deformation of the cross-section where both lateral and torsion buckling is included. There is a difference between constrained and unconstrained lateral torsional buckling as they will behave differently under the buckling process. It is understood that with constrained lateral torsional buckling means that a point of the member is restrained against deformations across the length of the member. This means that the axis of rotation is made fixed, which is where the member buckles around (see Figure 1.6). With unconstrained lateral torsional buckling, the axis of rotation is not given in

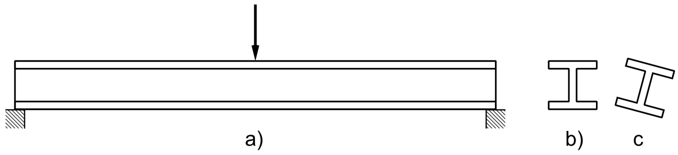



***Figure 1.6.*** Lateral torsional buckling a) Longitudinal view, b) Cross-section near support, c) Cross-section in center with lateral-torsional buckling.

advance, and it is therefore more complicated to determine the capacity, as it is dependent of the members internal balance at buckling.

The point of application in respect to the load will influence the elastic critical moment of a member. As stated in [da Silva et al., 2010], a gravity load applied below the shear centre C (that coincides with the centroid, in case of doubly symmetric I or H sections) has a stabilizing effect (Mcr,1 > Mcr), whereas the same load applied above this point has a destabilizing effect (Mcr,2 < Mcr). This is illustrated in Figure 1.7.

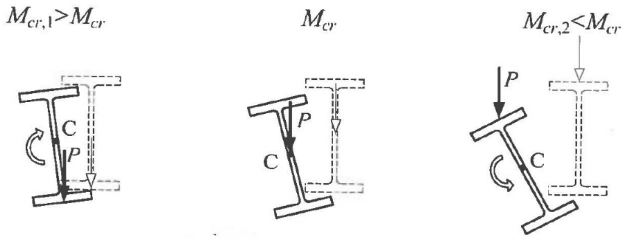



***Figure 1.7.*** Displacement influenced by elastic critical moment, [da Silva et al., 2010].

## 1.3 Methods

In order to achieve the aim of the project and be able to understand the behaviour of a steel frame, a literature study is made to understand the behaviour of a steel frame and the parameters influencing this. The focus is on literature explaining the different mechanisms of a frame, but also on EC 3 part 1-1, where detailed suggestions on how to calculate a steel frame are presented. In addition, the different compositions of a steel structure is compared to investigate optimised solution. In order to make a reasonable comparison between the analytical solution based on the equations in the [Standard, 2005] and the models made in Abaqus software, a further understanding of Abaqus is also a necessity. In this thesis Abaqus is used to analyse a frame numerically by the Finite

Element Method (FEA). In addition, a parameter study is also conducted in order to elicit the behaviour of a steel frame.

## 1.4 Limitations

The load applied in the project is a design load uniformly distributed and not the most critical load combination of permanent, variable nor accidental loads. It seems fulfilling because the aim of the project is to compare the two methods in EC and not to find an exact solution of a final design of the structure.

Usually, there will be placed bracing along the steel frames to prevent the before mentioned instability modes but in this project the structures are assumed not to have these kind of supports.

Because of the complexity of frame some parameters and geometries which are well suited for an analysis of a frame are not included. Only some parameters has been chosen to be further investigated.

# Steel Structures 2

*In this chapter the type of steel profile, static system, material properties of the steel structures that will be investigated and analysed are presented.*

## 2.1 Static system

In this project, two type of structures are examined. Firstly, a beam with simple support, as seen in Figure 2.1, will be investigated to make a comparison between the different methods.

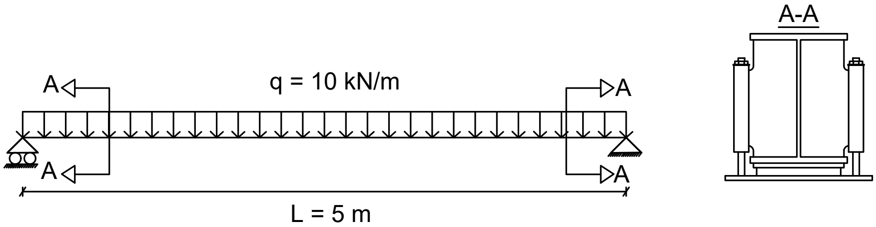



***Figure 2.1.*** Static model of a simply supported beam.

Secondly, a steel frame with pinned supports is analysed as seen in Figure 2.2.

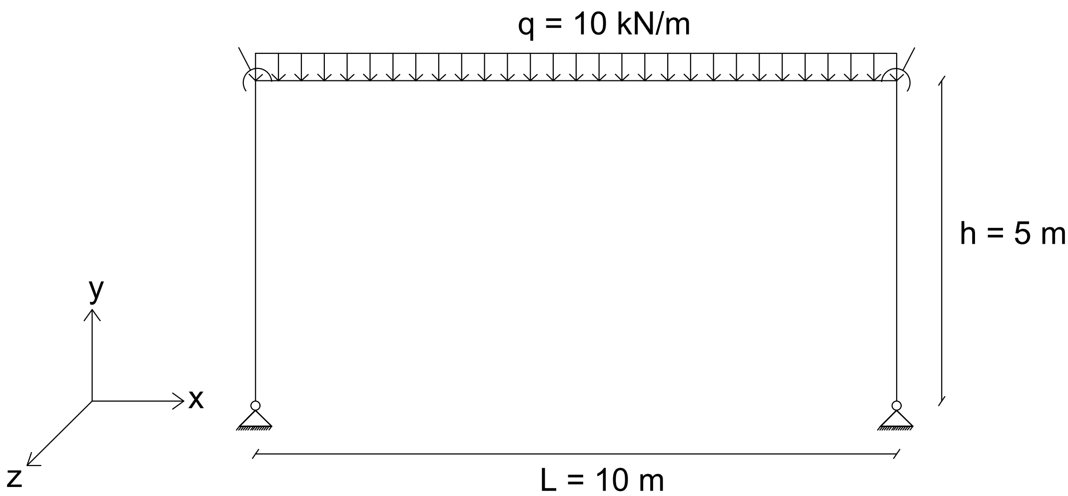



***Figure 2.2.*** Static model of a frame with pinned supports and fork supports in the corner.

## 2.2 Profiles

The steel profile which will be used throughout the project is HE400A. The cross-section of HE400A as it will be in reality can be seen in Figure 2.3 b). Because of the limitations of Abaqus the fillet radius is being neglected as seen in Figure 2.3 a). This error can be ignored as the same cross-section will be used in the analytical analysis.

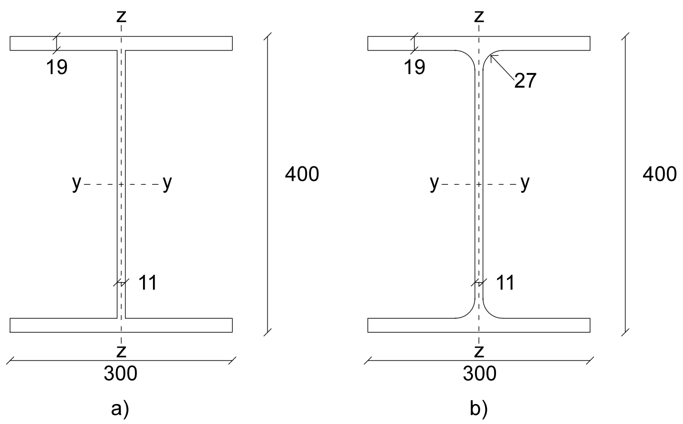



***Figure 2.3.*** HE400A steel profile a) Assumed HE400A profile b) Real HE400A profile

## 2.3 Material Properties

The material properties of the steel profile can be seen in Tabel 2.1 and found in [Jensen et al., 2011].

***Table 2.1.*** Material properties

| Material Properties | Values | Units |
|---|---|---|
| Young's Modulus, $E$ | $2.1 \times 10^5$ | MPa |
| Shear Modulus, $G$ | $8.1 \times 10^4$ | MPa |
| Yield strength of Steel, $f_y$ | 235 | MPa |
| Ultimate strength of Steel, $f_u$ | 360 | MPa |
| Poisson's Ratio, $\nu$ | 0.3 | - |
| Density, $\rho$ | 7850 | $\frac{kg}{m^3}$ |

These values are used to calculate design values from the characteristic values divided by appropriate partial factor $\gamma_M$:
• $\gamma_{M0}$, resistance of cross-sections to excessive yielding including local buckling (depending on $f_y$)
• $\gamma_{M1}$, resistance of members to member buckling
• $\gamma_{M2}$, resistance of cross-sections in tension to fracture (depending on $f_u$)
where the recommended values of the partial factors are:
• $\gamma_{M0} = 1.00$

- $\gamma_{M1} = 1.10$
- $\gamma_{M2} = 1.25$

## 2.4 Beam

The normal force-, shear force-, and bending moment diagram for the simply supported beam can be seen in Figure 2.4, 2.5, and 2.6. FEM Design software tool has been used.

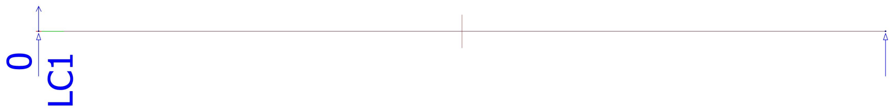



***Figure 2.4.*** Normal force diagram

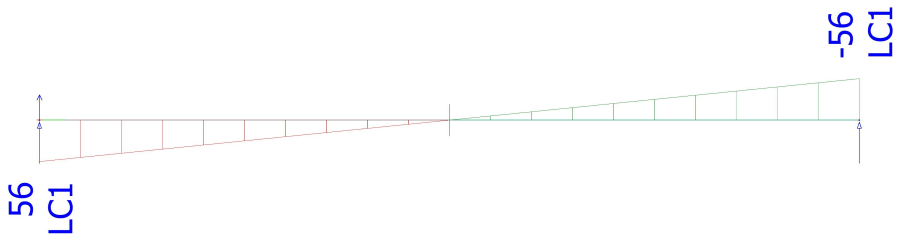



***Figure 2.5.*** Shear force diagram

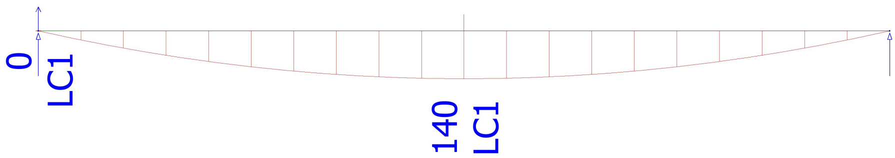



***Figure 2.6.*** Bending moment diagram

## 2.5 Frame

The normal force-, shear force-, and bending moment diagram for the frame model with pinned supports at the bottom and fork supports in the corners can be seen in Figure 2.7, 2.8, and 2.9.

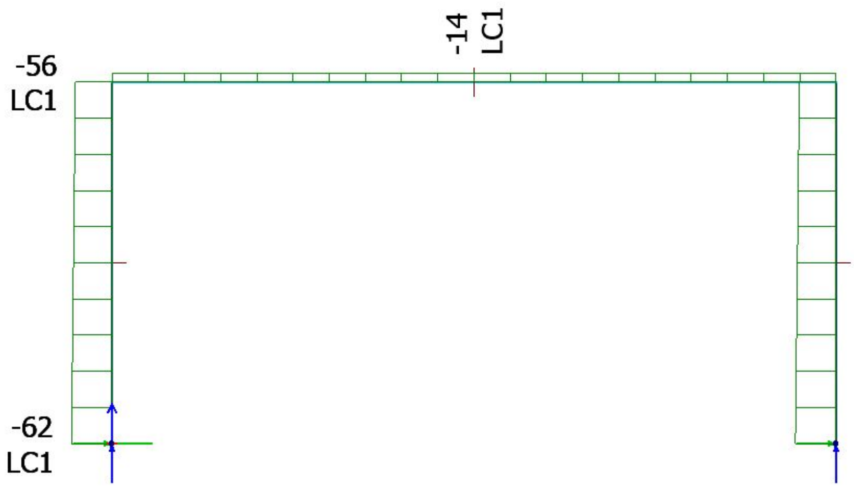



*Figure 2.7.* Normal force diagram

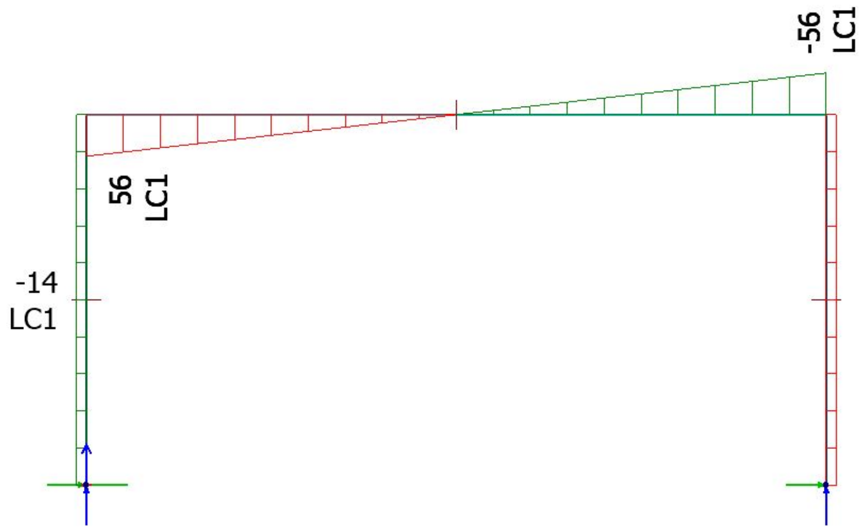



*Figure 2.8.* Shear force diagram

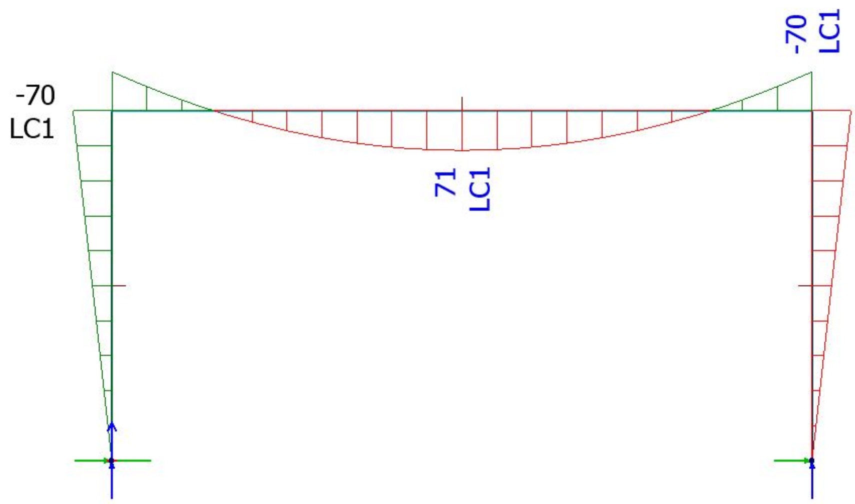



*Figure 2.9.* Bending moment diagram

# Analytical 3

*In this chapter the theoretical background of the analytical analysis is presented. The method is based on [Standard, 2005] and [da Silva et al., 2010]*

## 3.1 Method 6.3.3

Firstly the analytical analysis is made for the simply supported beam and secondly the pinned supported steel frame is analysed.

Before determining the occurrence of instability phenomena the cross-section resistance to axial compression should be verified where the classification of cross-section is needed.

### 3.1.1 Classification of Cross-section

According to clause 5.5.2(1) in [Standard, 2005], four classes of cross-sections are defined, depending on their rotation capacity and ability to form rotational plastic hinges:
• **Class 1**: Cross-sections are those which can form a plastic hinge with the rotation capacity required from plastic analysis without reduction of the resistance.
• **Class 2**: Cross-sections are those which can develop their plastic resistance moment, but have limited rotation capacity because of local buckling.
• **Class 3**: Cross-sections are those in which the stress in the extreme compression fibre of the steel member, assuming an elastic distribution of stresses, can reach the yield strength. However, local buckling is liable to prevent development of the plastic resistance moment.
• **Class 4**: Cross-sections are those in which local buckling will occur before the attainment of yield stress in one or more parts of the cross section.

### 3.1.2 Compression Verification

According to clause 6.2.4(1) in EC, the cross-section resistance of axially compressed members should be verified by the following condition:

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1 \tag{3.1}$$

where

| | |
|---|---|
| $N_{Ed}$ | Design value of the compression force |
| $N_{c,Rd}$ | Design resistance of the compression member |

For members with cross-section classification 1, 2, and 3 $N_{c,Rd}$ is determined by:

$$N_{c,Rd} = A\frac{f_y}{\gamma_{M0}} \tag{3.2}$$

where

| | |
|---|---|
| $A$ | Area of cross-section |
| $\gamma_{M0}$ | Partial coefficient |

For classification 4 it is determined by:

$$N_{c,Rd} = A_{eff}\frac{f_y}{\gamma_{M0}} \tag{3.3}$$

where

| | |
|---|---|
| $A_{eff}$ | Effective area of cross-section |

### 3.1.3 Shear Verification

The design value of the shear force should satisfy at each cross-section:

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1 \tag{3.4}$$

where

| | |
|---|---|
| $V_{Ed}$ | Design value of shear force |
| $V_{c,Rd}$ | Design elastic shear resistance determined by the Equation (3.5) |

$$V_{c,Rd} = \frac{\tau_{Ed}}{\frac{f_y}{\sqrt{3}\gamma_{M0}}} \tag{3.5}$$

where

| | |
|---|---|
| $\tau_{Ed}$ | Shear stress determined by the Equation (3.6) |

$$\tau_{Ed} = \frac{V_{Ed}S}{It} \tag{3.6}$$

where

| | |
|---|---|
| $S$ | First moment of area |
| $I$ | Second moment of area |
| $t$ | Thickness of the examined point |

In ultimate designing condition the plastic shear resistance will be used:

$$V_{c,Rd} = V_{pl,Rd} = \frac{A_v \left( \frac{f_y}{\sqrt{3}} \right)}{\gamma_{M0}} \tag{3.7}$$

where

| | |
|---|---|
| $V_{pl,Rd}$ | Plastic shear resistance |
| $A_v$ | Shear area of the cross-section |

### 3.1.4 Bending Moment Verification

According to clause 6.2.5 in [Standard, 2005] the design value of the bending moment $M_{Ed}$ at each cross-section shall satisfy Equation (3.8).

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1 \tag{3.8}$$

where

| | |
|---|---|
| $M_{Ed}$ | Design value of the moment |
| $M_{c,Rd}$ | Design resistance moment |

The design resistance for bending moment is depended on the cross-section classification which can be seen in the following Equations (3.9) to (3.11).

Class 1 or 2 cross-sections:

$$M_{c,Rd} = \frac{W_{pl} f_y}{\gamma_{M0}} \tag{3.9}$$

Class 3 cross-sections:

$$M_{c,Rd} = \frac{W_{el,min} f_y}{\gamma_{M0}} \tag{3.10}$$

Class 4 cross-sections:

$$M_{c,Rd} = \frac{W_{eff,min} f_y}{\gamma_{M0}} \tag{3.11}$$

where

| | |
|---|---|
| $W_{pl}$ | Plastic section modulus |
| $W_{el,min}$ | Elastic section modulus |
| $W_{eff,min}$ | Effective minimum section modulus |

### 3.1.5 Buckling Resistance of Compression

After verifying the cross-section resistance of axial compression and bending moment the next step is to verify the buckling resistance. According to clause 6.3.1 in [Standard, 2005] the buckling resistance of compression shall fulfil Equation (3.12).

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1 \tag{3.12}$$

where

| | |
|---|---|
| $N_{b,Rd}$ | Design buckling compression resistance |

For class 1, 2, and 3 cross-section:

$$N_{b,Rd} = \chi A \frac{f_y}{\gamma_{M1}} \tag{3.13}$$

For class 4 cross-section:

$$N_{b,Rd} = \chi A_{eff} \frac{f_y}{\gamma_{M1}} \tag{3.14}$$

where

| | |
|---|---|
| $\chi$ | Reduction factor |

The reduction factor is determined from Equation (3.15).

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \overline{\lambda}^2}} \tag{3.15}$$

where

| | |
|---|---|
| $\phi$ | Factor to use of determining $\chi$ |
| $\overline{\lambda}$ | Non-dimensional slenderness |

To determine reduction factor, $\phi$ must be determined first. This lead to determining Equation (3.17) and (3.18) where Figure 3.2 is used to determine the imperfection factor.

$$\phi = 0.5 \left[1 + \alpha \left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right] \tag{3.16}$$

where

| | |
|---|---|
| $\alpha$ | Imperfection factor |

For class 1, 2, and 3 cross-section:

$$\overline{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} \tag{3.17}$$

For class 4 cross-section:

$$\overline{\lambda} = \sqrt{\frac{A_{eff}f_y}{N_{cr}}} \tag{3.18}$$

where

| | |
|---|---|
| $N_{cr}$ | Elastic critical force for the relevant buckling mode based on the gross cross sectional properties |

Elastic critical load is determined by Equation (3.19).

$$N_{cr} = \frac{\pi^2 EI}{L_E^2} \tag{3.19}$$

where

| | |
|---|---|
| $L_E$ | Buckling length |

Buckling length can be determined by looking at the supports of a simple static model like a simply support beam. But in the case of frame model the supports, loading, equality between beam element and column element cross-section, and the geometry has influence on the buckling length, [Ehlers, 2009].

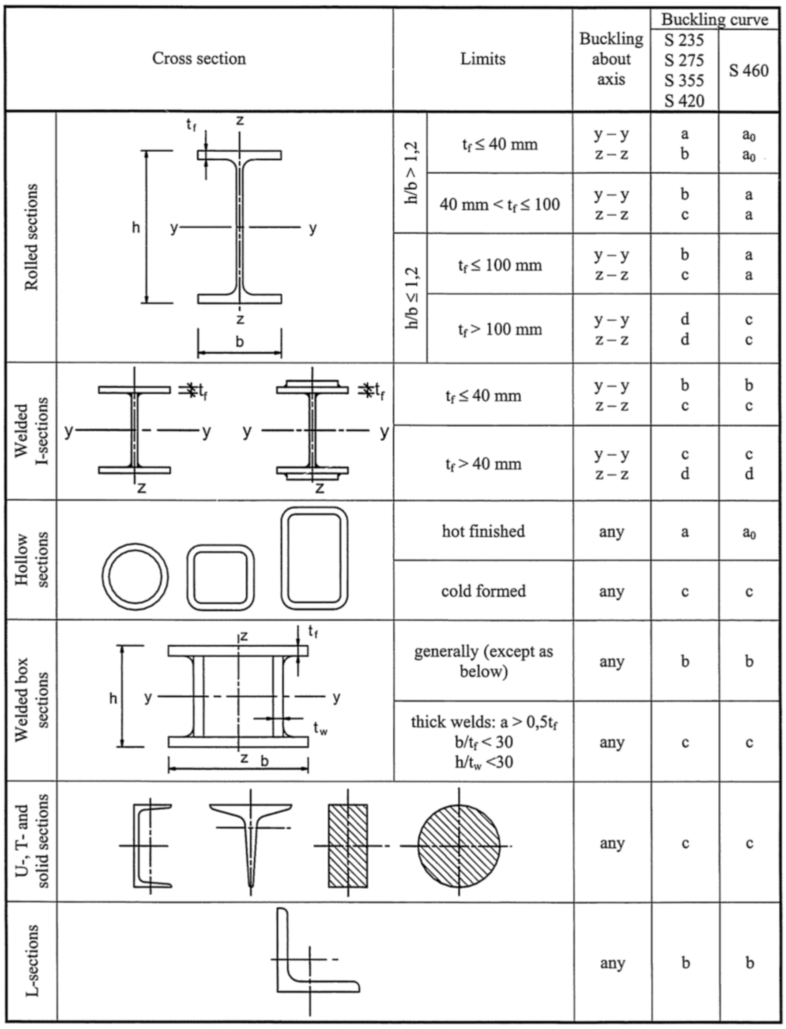
Selection of buckling curve for a cross-section is decided by the type of cross-section, which axis buckling is about, yield strength, and in the case of H-profile the flange thickness compared to the width/height relation of the cross-section is important. This can also be seen in Figure 3.1.

The imperfection factor takes into account the effect of the imperfections and the values corresponding to the appropriate buckling curve can be obtained from Table 3.1 but more precise imperfection factors can be obtained by use of Table 3.1 orthe graph seen in Figure 3.2 by knowing $\chi$ and $\overline{\lambda}$.

***Table 3.1.*** Imperfection factors for buckling curves.

| Buckling curve | Imperfection factor, $\alpha$ |
|---|---|
| $a_0$ | 0.13 |
| $a$ | 0.21 |
| $b$ | 0.34 |
| $c$ | 0.49 |
| $d$ | 0.76 |

A graphical representation of Equation (3.15) can be seen in Figure **??**.



| Cross section | Limits | | Buckling about axis | Buckling curve S 235, S 275, S 355, S 420 | Buckling curve S 460 |
|---|---|---|---|---|---|
| Rolled sections | h/b > 1,2 | $t_f \le 40$ mm | y – y<br>z – z | a<br>b | $a_0$<br>$a_0$ |
| | | 40 mm < $t_f \le 100$ | y – y<br>z – z | b<br>c | a<br>a |
| | h/b ≤ 1,2 | $t_f \le 100$ mm | y – y<br>z – z | b<br>c | a<br>a |
| | | $t_f > 100$ mm | y – y<br>z – z | d<br>d | c<br>c |
| Welded I-sections | $t_f \le 40$ mm | | y – y<br>z – z | b<br>c | b<br>c |
| | $t_f > 40$ mm | | y – y<br>z – z | c<br>d | c<br>d |
| Hollow sections | hot finished | | any | a | $a_0$ |
| | cold formed | | any | c | c |
| Welded box sections | generally (except as below) | | any | b | b |
| | thick welds: $a > 0{,}5t_f$<br>$b/t_f < 30$<br>$h/t_w < 30$ | | any | c | c |
| U-, T- and solid sections | | | any | c | c |
| L-sections | | | any | b | b |

***Figure 3.1.*** Selction of buckling curves, [Standard, 2005]

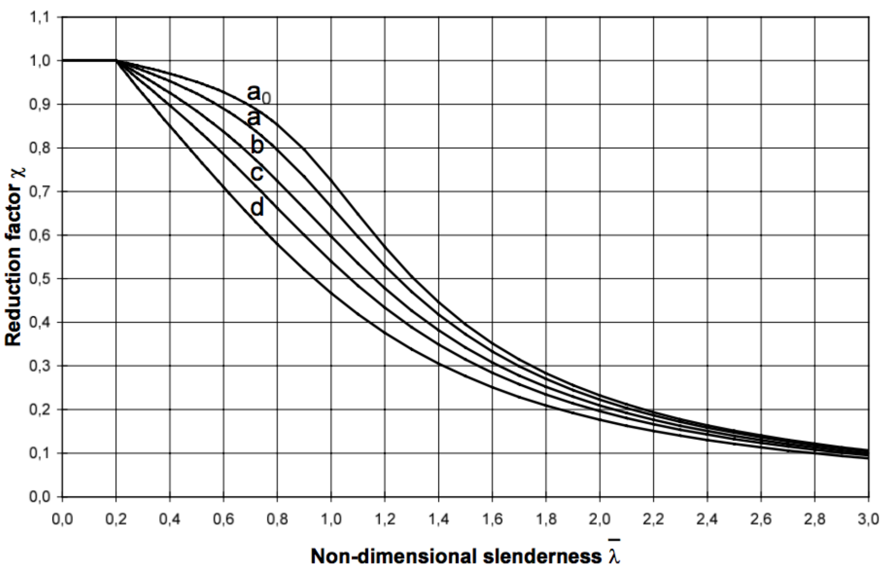



***Figure 3.2.*** Buckling curves, [Standard, 2005].

### Flexural Buckling

The non-dimensional slenderness, $\overline{\lambda}$, for flexural buckling can be determined by buckling lenght, $L_{cr}$, instead of the elastic critical force, $N_{cr}$, as seen in Equation (3.20) and (3.21).

For class 1, 2, and 3 cross-section:

$$\overline{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{1}{\lambda_1} \tag{3.20}$$

For class 4 cross-section:

$$\overline{\lambda} = \sqrt{\frac{A_{eff}f_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{\sqrt{\beta_A}}{\lambda_1} \tag{3.21}$$

where

| | |
|---|---|
| $L_{cr}$ | Buckling length |
| $i$ | Radius of gyration about the relevant axis, determined using the properties of the gross cross-section |
| $\beta_A$ | Ratio between $A$ and $A_{eff}$ |
| $\lambda_1$ | Determined in Equation (3.23) |

$$\beta_A = \frac{A_{eff}}{A} \tag{3.22}$$

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93.9\varepsilon \tag{3.23}$$

$$\varepsilon = \sqrt{\frac{235}{f_y}} \tag{3.24}$$

### Torsional and Torsional-flexural Buckling

The non-dimensional slenderness, $\overline{\lambda}_T$, for torsional or torsional-flexural buckling can be determined by Equation (3.25) and (3.26).

For class 1, 2, and 3 cross-section:

$$\overline{\lambda}_T = \sqrt{\frac{Af_y}{N_{cr}}} \tag{3.25}$$

For class 4 cross-section:

$$\bar{\lambda}_T = \sqrt{\frac{A_{eff} f_y}{N_{cr}}} \tag{3.26}$$

where, $N_{cr} = N_{cr,TF}$ but should satisfy $N_{cr} < N_{cr,T}$

| | |
|---|---|
| $N_{cr,T}$ | Elastic torsional buckling force |
| $N_{cr,TF}$ | Elastic torsional-flexural buckling force |

The critical elastic torsional buckling force, $N_{cr,T}$, can be determined by Equation (3.27):

$$N_{cr,T} = \frac{1}{i_c^2}\left(GI_T + \frac{\pi^2 EI_w}{L_{ET}^2}\right) \tag{3.27}$$

where

| | |
|---|---|
| $i_c$ | Radius of polar gyration determined by Equation (3.28) |
| $I_T$ | Torsion constant |
| $I_w$ | Warping constant |
| $L_{ET}$ | Equivalent length that depends on the restrictions to torsion and warping at the end sections |

$$i_c^2 = y_c^2 + \frac{I_y + I_z}{A} \tag{3.28}$$

where

| | |
|---|---|
| $y_c$ | Distance along the y axis between the shear centre and the centroid of the section |

The critical elastic flexural-torsional buckling force, $N_{cr,TF}$, can be determined by Equation (3.29):

$$N_{cr,TF} = \frac{1}{2\beta}\left[(N_{cr,y} + N_{cr,T}) - \sqrt{(N_{cr,y} + N_{cr,T})^2 - 4\beta N_{cr,y} N_{cr,T}}\right] \tag{3.29}$$

where

| | |
|---|---|
| $N_{cr,y}$ | Critical load for flexural buckling about the strong axis |
| $\beta$ | Factor given by $\beta = 1 - (y_c/i_c)^2$ |

### 3.1.6 Lateral-torsional Buckling Resistance

According to clause 6.3.2 in [Standard, 2005] the lateral-torsional buckling resistance is determined by Equation (3.30).

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1 \tag{3.30}$$

where

| | |
|---|---|
| $M_{b,Rd}$ | Design buckling resistance moment |

The design buckling resistance moment is determined by Equation (3.31).

$$M_{b,Rd} = \chi_{LT} W_y \frac{f_y}{\gamma_{M1}} \tag{3.31}$$

where

| | |
|---|---|
| $\chi_{LT}$ | Reduction factor for lateral-torsional buckling |
| $W_y$ | Section modulus of the compression flange |

The value of $W_y$ is determined from Table 3.2.

***Table 3.2.*** The value of $W_y$ categorised by the cross-section classification.

| | |
|---|---|
| Class 1 or 2 cross-section | $W_y = W_{pl,y}$ |
| Class 3 cross-section | $W_y = W_{el,y}$ |
| Class 4 cross-section | $W_y = W_{eff,y}$ |

Reduction factor for lateral-torsional buckling is determined by Equation (3.32).

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \overline{\lambda}_{LT}^2}} \quad \text{but} \quad \chi_{LT} \leq 1.0 \tag{3.32}$$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} \left(\overline{\lambda}_{LT} - 0.2\right) + \overline{\lambda}_{LT}^2\right] \tag{3.33}$$

where

| | |
|---|---|
| $\phi_{LT}$ | Factor to use of determining reduction factor for lateral-torsional buckling |
| $\alpha_{LT}$ | Imperfection factor for lateral-torsional buckling |
| $\overline{\lambda}_{LT}$ | Non-dimensional slenderness for lateral-torsional buckling |

Imperfection factor for lateral-torsional buckling, $\alpha_{LT}$, is determined by the conditions seen in Figure 3.3.

| Section | Limits | Buckling curve (EC3-1-1) |
|---|---|---|
| I or H sections rolled | $h/b \leq 2$ | $b$ |
| | $h/b > 2$ | $c$ |
| I or H sections welded | $h/b \leq 2$ | $c$ |
| | $h/b > 2$ | $d$ |

***Figure 3.3.*** Buckling curves, [Standard, 2005].

Non-dimensional slenderness lateral-torsional buckling is determined by the Equation (3.34).

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} \tag{3.34}$$

where

| | |
|---|---|
| $M_{cr}$ | Elastic critical moment for lateral-torsional buckling |

The elastic critical moment, $M_{cr}$, can be determined according to [da Silva et al., 2010] by Equation (3.35).

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{(k_z L)^2} \left\{ \left[ \left( \frac{k_z}{k_w} \right)^2 \frac{I_w}{I_z} + \frac{(k_z L)^2 G I_T}{\pi^2 E I_z} + (C_2 z_g - C_3 z_j)^2 \right]^{0.5} - (C_2 z_g - C_3 z_j) \right\} \tag{3.35}$$

where

| | |
|---|---|
| $L$ | Beam length |
| $I_z$ | Moment of inertia about the weak axis |
| $C_1$, $C_2$, and $C_3$ | Coefficients depending on the shape of the bending moment diagram and on support conditions |
| $k_z$ and $k_w$ | Effective length factors that depend on the support conditions at the end sections |
| $z_g = (z_a - z_s)$ | $z_a$ and $z_s$ are the coordinates of the point of application of the load and of the shear centre, relative to the centroid of the cross-section |
| $z_j$ | Parameter that reflects the degree of asymmetry of the cross-section in relation to the y axis |

Parameter $z_j$ can be determined by Equation (3.36).

$$z_j = z_s - \left( 0.5 \int_A (y^2 + z^2) \left( \frac{z}{I_y} \right) dA \right) \tag{3.36}$$

The conservative values will be $k_z = 1$ and $k_w = 1$ which will be used further on in the analytical solution.

The coefficients of the parameters $C_1$, $C_2$, and $C_3$ for the Equation (3.35) can be determined by the Figures 3.4, 3.5, and 3.6 depending on how the beam is loaded.

### 3.1.7 Bending and Axial Compression

According to [da Silva et al., 2010] and clause 6.3.3(1) two distinct situations should be considered:

<table>
<tr><th rowspan="2">Loading and support conditions</th><th rowspan="2">Diagram of moments</th><th rowspan="2">$k_z$</th><th rowspan="2">$C_1$</th><th colspan="2">$C_3$</th></tr>
<tr><th>$\psi_f \leq 0$</th><th>$\psi_f > 0$</th></tr>
<tr><td rowspan="9">M ΨM</td><td>Ψ = +1</td><td>1.0<br>0.5</td><td>1.00<br>1.05</td><td colspan="2">1.000<br>1.019</td></tr>
<tr><td>Ψ = +3/4</td><td>1.0<br>0.5</td><td>1.14<br>1.19</td><td colspan="2">1.000<br>1.017</td></tr>
<tr><td>Ψ = +1/2</td><td>1.0<br>0.5</td><td>1.31<br>1.37</td><td colspan="2">1.000<br>1.000</td></tr>
<tr><td>Ψ = +1/4</td><td>1.0<br>0.5</td><td>1.52<br>1.60</td><td colspan="2">1.000<br>1.000</td></tr>
<tr><td>Ψ = 0</td><td>1.0<br>0.5</td><td>1.77<br>1.86</td><td colspan="2">1.000<br>1.000</td></tr>
<tr><td>Ψ = −1/4</td><td>1.0<br>0.5</td><td>2.06<br>2.15</td><td>1.000<br>1.000</td><td>0.850<br>0.650</td></tr>
<tr><td>Ψ = −1/2</td><td>1.0<br>0.5</td><td>2.35<br>2.42</td><td>1.000<br>0.950</td><td>$1.3-1.2\psi_f$<br>$0.77-\psi_f$</td></tr>
<tr><td>Ψ = −3/4</td><td>1.0<br>0.5</td><td>2.60<br>2.45</td><td>1.000<br>0.850</td><td>$0.55-\psi_f$<br>$0.35-\psi_f$</td></tr>
<tr><td>Ψ = −1</td><td>1.0<br>0.5</td><td>2.60<br>2.45</td><td>$-\psi_f$<br>$-0.125-0.7\psi_f$</td><td>$-\psi_f$<br>$-0.125-0.7\psi_f$</td></tr>
<tr><td colspan="6">▪ In beams subject to end moments, by definition $C_2 z_g = 0$.<br>▪ $\psi_f = \frac{I_{fc} - I_{ft}}{I_{fc} + I_{ft}}$, where $I_{fc}$ and $I_{ft}$ are the second moments of area of the compression and tension flanges respectively, relative to the weak axis of the section (z axis);<br>▪ $C_1$ must be divided by 1.05 when $\frac{\pi}{k_w L}\sqrt{\frac{E I_W}{G I_T}} \leq 1.0$, but $C_1 \geq 1.0$.</td></tr>
</table>

***Figure 3.4.*** Coefficients $C_1$, $C_3$, and $k_z$ factor for beams with end moments, [da Silva et al., 2010].

| Loading and support conditions | Diagram of moments | $k_z$ | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|---|---|
| p | | 1.0<br>0.5 | 1.12<br>0.97 | 0.45<br>0.36 | 0.525<br>0.478 |
| P | | 1.0<br>0.5 | 1.35<br>1.05 | 0.59<br>0.48 | 0.411<br>0.338 |
| P P<br>d d d d | | 1.0<br>0.5 | 1.04<br>0.95 | 0.42<br>0.31 | 0.562<br>0.539 |

***Figure 3.5.*** Coefficients $C_1$, $C_2$, $C_3$, and $k_z$ factor for beams with transverse loads, [da Silva et al., 2010].

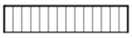
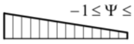
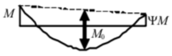
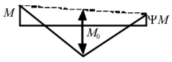
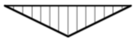
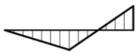
| Diagram of bending moments | $k_c$ |
|---|---|
| $\Psi = +1$  | 1.0 |
| $-1 \leq \Psi \leq 1$  | $\frac{1}{1.33 - 0.33\Psi}$ |
|  | |
|  | 0.94 |
|  | 0.90 |
|  | 0.91 |
|  | |
|  | 0.86 |
|  | 0.77 |
|  | 0.82 |
| $\Psi$ - ratio between end moments, with $-1 \leq \Psi \leq 1$. | |

***Figure 3.6.*** Correction factors, $k_c$, to determining $C_1$ coefficient, [da Silva et al., 2010].

*- Members not susceptible to torsional deformation, such as members of circular hollow section or other sections restrained from torsion. Here, flexural buckling is the relevant instability mode.*
*- Members that are susceptible to torsional deformations, such as members of open section (I or H sections) that are not restrained from torsion. Here, lateral torsional buckling tends to be the relevant instability mode.*[da Silva et al., 2010]

A single span member of doubly symmetric section is subjected to bending moment and axial compression should satisfy Equation (3.37) and (3.38).

$$\frac{N_{Ed}}{\chi_y \frac{N_{Rk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1.0 \quad (3.37)$$

$$\frac{N_{Ed}}{\chi_z \frac{N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1.0 \quad (3.38)$$

where

Interaction factors can be determined by formulas seen in Figure 3.7.

To be able to determine the interactions factors auxiliary terms are needed which can be seen in Figure 3.8 and 3.9.

Equivalent factors of uniform moment, $C_{mi,0}$, is determined by the corresponding bending moment diagrams which can be seen in Figure 3.10.

| | |
|---|---|
| $N_{Ed}$ | Design value of the compression force |
| $N_{Rk}$ | Characteristic value of the compression force |
| $M_{y,Ed}$ | Design value of the moment about the strong axis |
| $M_{y,Rk}$ | Characteristic value of the moment about the strong axis |
| $M_{z,Ed}$ | Design value of the moment about the weak axis |
| $M_{z,Rk}$ | Characteristic value the moment about the weak axis |
| $\Delta M_{y,Ed}$ and $\Delta M_{z,Ed}$ | Moments due to the shift of the centroidal axis |
| $\chi_y$ and $\chi_z$ | Reduction factors due to flexural buckling for respectively strong and weak axis |
| $k_{yy}$, $k_{yz}$, $k_{zy}$, and $k_{zz}$ | Interaction factors |

| Interaction factors | Elastic sectional properties (Class 3 or 4 sections) | Plastic sectional properties (Class 1 or 2 sections) |
|---|---|---|
| $k_{yy}$ | $C_{my}\,C_{mLT}\dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,y}}}$ | $C_{my}\,C_{mLT}\dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,y}}}\dfrac{1}{C_{yy}}$ |
| $k_{yz}$ | $C_{mz}\dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,z}}}$ | $C_{mz}\dfrac{\mu_y}{1-\dfrac{N_{Ed}}{N_{cr,z}}}\dfrac{1}{C_{yz}}0.6\sqrt{\dfrac{w_z}{w_y}}$ |
| $k_{zy}$ | $C_{my}\,C_{mLT}\dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,y}}}$ | $C_{my}\,C_{mLT}\dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,y}}}\dfrac{1}{C_{zy}}0.6\sqrt{\dfrac{w_y}{w_z}}$ |
| $k_{zz}$ | $C_{mz}\dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,z}}}$ | $C_{mz}\dfrac{\mu_z}{1-\dfrac{N_{Ed}}{N_{cr,z}}}\dfrac{1}{C_{zz}}$ |

***Figure 3.7.*** Interaction factors according to [da Silva et al., 2010].

Equations (3.37) and (3.38) can now be checked for instability .

According to EC3-1-1 two methods are given for the calculation of the interaction factors. Method 1 and Method 2 (beskriv forskellen og hvorfor jeg går videre med Method 2 da IT er mindre end Iy. Men selvom It er mindre end Iy så kan metode 1 anvendes hvis en formel eftervises.

The HE400B profile has been verified in MathCAD calculation by Method 2 and can be seen in Appendix A.

## 3.2 Results

The utilization ratio determined by the analytical approach can be seen from Table 3.3.

The results will be discussed and compared to the results determined by the numerical approach in Chapter 5.

**Auxiliary terms:**

$$\mu_y = \frac{1 - \frac{N_{Ed}}{N_{cr,y}}}{1 - \chi_y \frac{N_{Ed}}{N_{cr,y}}}; \quad \mu_z = \frac{1 - \frac{N_{Ed}}{N_{cr,z}}}{1 - \chi_z \frac{N_{Ed}}{N_{cr,z}}}; \quad w_y = \frac{W_{pl,y}}{W_{el,y}} \leq 1.5; \quad w_z = \frac{W_{pl,z}}{W_{el,z}} \leq 1.5.$$

$n_{pl} = \frac{N_{Ed}}{N_{Rk}/\gamma_{M1}}$; $a_{LT} = 1 - \frac{I_T}{I_y} \geq 0$; $C_{my}$ and $C_{mz}$ are factors of equivalent uniform moment, determined by Table 3.15.
For class 3 or 4, consider $w_y = w_z = 1.0$.

$$C_{yy} = 1 + (w_y - 1)\left[\left(2 - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max} - \frac{1.6}{w_y} C_{my}^2 \bar{\lambda}_{max}^2\right) n_{pl} - b_{LT}\right] \geq \frac{W_{el,y}}{W_{pl,y}},$$

where $b_{LT} = 0.5\, a_{LT}\, \bar{\lambda}_0^2 \frac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \frac{M_{z,Ed}}{M_{pl,z,Rd}}$.

$$C_{yz} = 1 + (w_z - 1)\left[\left(2 - 14 \frac{C_{mz}^2 \bar{\lambda}_{max}^2}{w_z^5}\right) n_{pl} - c_{LT}\right] \geq 0.6 \sqrt{\frac{w_z}{w_y}} \frac{W_{el,z}}{W_{pl,z}},$$

where $c_{LT} = 10\, a_{LT} \frac{\bar{\lambda}_0^2}{5 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}}$.

$$C_{zy} = 1 + (w_y - 1)\left[\left(2 - 14 \frac{C_{my}^2 \bar{\lambda}_{max}^2}{w_y^5}\right) n_{pl} - d_{LT}\right] \geq 0.6 \sqrt{\frac{w_y}{w_z}} \frac{W_{el,y}}{W_{pl,y}},$$

where $d_{LT} = 2\, a_{LT} \frac{\bar{\lambda}_0}{0.1 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}} \frac{M_{z,Ed}}{C_{mz} M_{pl,z,Rd}}$.

$$C_{zz} = 1 + (w_z - 1)\left[\left(2 - \frac{1.6}{w_z} C_{mz}^2 \bar{\lambda}_{max} - \frac{1.6}{w_z} C_{mz}^2 \bar{\lambda}_{max}^2\right) - e_{LT}\right] n_{pl} \geq \frac{W_{el,z}}{W_{pl,z}}, \quad 4$$

where $e_{LT} = 1.7\, a_{LT} \frac{\bar{\lambda}_0}{0.1 + \bar{\lambda}_z^4} \frac{M_{y,Ed}}{C_{my} \chi_{LT} M_{pl,y,Rd}}$.

***Figure 3.8.*** Auxiliary terms for the calculation of the interaction factors, $k_{ij}$, [da Silva et al., 2010].

**Auxiliary terms (continuation):**

$\bar{\lambda}_{\max} = \max\left(\bar{\lambda}_y, \bar{\lambda}_z\right)$;

$\bar{\lambda}_0 =$ non dimensional slenderness for lateral torsional buckling due to uniform bending moment, that is, taking $\Psi_y = 1.0$ in Table 3.15;

$\bar{\lambda}_{LT} =$ non dimensional slenderness for lateral torsional buckling;

If $\bar{\lambda}_0 \leq 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}$: $C_{my} = C_{my,0}$; $C_{mz} = C_{mz,0}$; $C_{mLT} = 1.0$;

If $\bar{\lambda}_0 > 0.2\sqrt{C_1}\sqrt[4]{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}$: $C_{my} = C_{my,0} + \left(1 - C_{my,0}\right)\frac{\sqrt{\varepsilon_y}\, a_{LT}}{1+\sqrt{\varepsilon_y}\, a_{LT}}$;

$$C_{mz} = C_{mz,0};\; C_{mLT} = C_{my}^2 \frac{a_{LT}}{\sqrt{\left(1-\frac{N_{Ed}}{N_{cr,z}}\right)\left(1-\frac{N_{Ed}}{N_{cr,T}}\right)}} \geq 1;$$

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A}{W_{el,y}}$ for class 1, 2 or 3 cross sections;

$\varepsilon_y = \frac{M_{y,Ed}}{N_{Ed}}\frac{A_{eff}}{W_{eff,y}}$ for class 4 cross sections;

$N_{cr,y}$ is the elastic critical load for flexural buckling about $y$;

$N_{cr,z}$ is the elastic critical load for flexural buckling about $z$;

$N_{cr,T}$ is the critical load for torsional buckling;

$I_T$ is the constant of uniform torsion or St. Venant's torsion;

$I_y$ is the second moment of area about $y$;

$C_1 = \left(\frac{1}{k_c}\right)^2$ where $k_c$ is taken from Table 3.10.

***Figure 3.9.*** Auxiliary terms for the calculation of the interaction factors, $k_{ij}$, [da Silva et al., 2010].

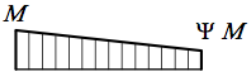
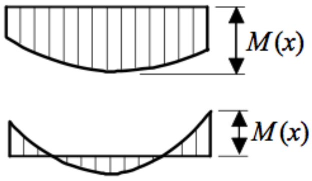
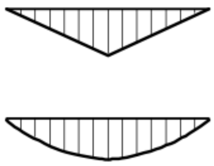
| Diagram of moments | $C_{mi,0}$ |
|---|---|
|  | $C_{mi,0} = 0.79 + 0.21\Psi_i + 0.36(\Psi_i - 0.33)\frac{N_{Ed}}{N_{cr,i}}$ |
|  | $C_{mi,0} = 1 + \left(\frac{\pi^2 EI_i \lvert\delta_x\rvert}{L^2 \lvert M_{i,Ed}(x)\rvert} - 1\right)\frac{N_{Ed}}{N_{cr,i}}$<br>$M_{i,Ed}(x)$ is the maximum moment $M_{y,Ed}$ or $M_{z,Ed}$ according to the first order analyses<br>$\lvert\delta_x\rvert$ is the maximum lateral deflection $\delta_z$ (due to $M_{y,Ed}$) or $\delta_y$ (due to $M_{z,Ed}$) along the member |
|  | $C_{mi,0} = 1 - 0.18\frac{N_{Ed}}{N_{cr,i}}$<br>$C_{mi,0} = 1 + 0.03\frac{N_{Ed}}{N_{cr,i}}$ |

***Figure 3.10.*** Equivalent factors of uniform moment, $C_{mi,0}$, [da Silva et al., 2010].

***Table 3.3.*** Utilization Ratio, $UR$, determined by analytical approach.

| | | |
|---|---|---|
| Simply supported beam | Strong axis | 0.428 |
| Simply supported beam | Weak axis | 0.318 |
| Beam element of frame structure | Strong axis | 0.202 |
| Beam element of frame structure | Weak axis | 0.157 |
| Column element of frame structure | Strong axis | 0.140 |
| Column element of frame structure | Weak axis | 0.136 |

# Numerical 4

*In this chapter the general method proposed by [Standard, 2005] clause 6.3.4 is applied and performed with Abaqus software as FEA tool and presented.*

## 4.1 General method - Method 6.3.4

According to [Standard, 2005] clause 6.3.4 frames composed of beams or columns or beam-columns subject to mono-axial bending and compression, the assessment for lateral torsional buckling out of the plane of the frame may be performed in the following way:

• For the distribution of action effects on the frame resulting from the analysis of the frame for the design loads the multiplier $\alpha_{cr,op}$ of these design loads to reach the elastic critical resistance of the frame with regard to lateral deformations should be determined
• For the same distribution of action effects the minimum multiplier $\alpha_{ult,k}$ of the design loads to reach the characteristic resistance of the frame without taking lateral torsional buckling into account should be determined.

## 4.2 Abaqus

Abaqus FEA software includes several kinds of finite element programs. In this project Abaqus/CAE has been used. Abaqus/CAE is described by [Simulia] as an interactive environment used to create finite element models, submit Abaqus analyses, monitor and diagnose jobs, and evaluate results.

The unit values used in Abaqus in this project are $N$, $m$, and $kg$.

### 4.2.1 Theory

Five aspects of an element characterize its behaviour, Simulia:
• Family
• Degrees of freedom (directly related to the element family)
• Number of nodes
• Formulation

• Integration

**Element type**

Figure 4.1, shows the element families that are used most commonly in a stress analysis and used in this project, [Simulia].

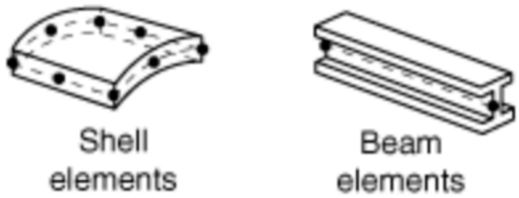



***Figure 4.1.*** Element types used in this project, [Simulia]

**Degrees of Freedom**

The degrees of freedom are the fundamental variables calculated during the analysis. For a stress/displacement simulation the degrees of freedom are the translations and, for shell and beam elements, the rotations at each node, [Simulia].

**Number of Nodes and Order of Interpolation**

Displacements or other degrees of freedom are calculated at the nodes of the element. At any other point in the element, the displacements are obtained by interpolating from the nodal displacements. Usually the interpolation order is determined by the number of nodes used in the element. Elements that have nodes only at their corners, such as the 8-node brick shown in Figure 4.2, use linear interpolation in each direction and are often called linear elements or first-order elements. In Abaqus/Standard elements with midside nodes, such as the 20-node brick shown in Figure 4.2, use quadratic interpolation and are often called quadratic elements or second-order elements. Modified triangular or tetrahedral elements with midside nodes, such as the 10-node tetrahedron shown in Figure 4.2, use a modified second-order interpolation and are often called modified or modified second-order elements, [Simulia].

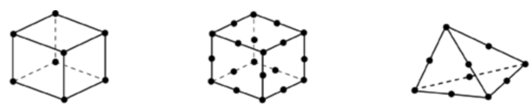


***Figure 4.2.*** Node element types in Abaqus, [Simulia]

In this project the quadratic elements have been used.

**Integration**

Abaqus uses numerical techniques to integrate various quantities over the volume of each element, thus allowing complete generality in material behavior. Using Gaussian quadrature for most elements, Abaqus evaluates the material response at each integration point in each element. Some continuum elements in Abaqus can use full or reduced integration, a choice that can have a significant effect on the accuracy of the element for a given problem. Shell, pipe, and beam element properties can be defined as general section behaviors; or each cross-section of the element can be integrated numerically, so that nonlinear response associated with nonlinear material behavior can be tracked accurately when needed. In addition, a composite layered section can be specified for shells and, in Abaqus/Standard, three-dimensional bricks, with different materials for each layer through the section, [Simulia].

### 4.2.2 Boundary Conditions

Figure 4.3, 4.4, and 4.5 shows the boundary conditions of the beam model.

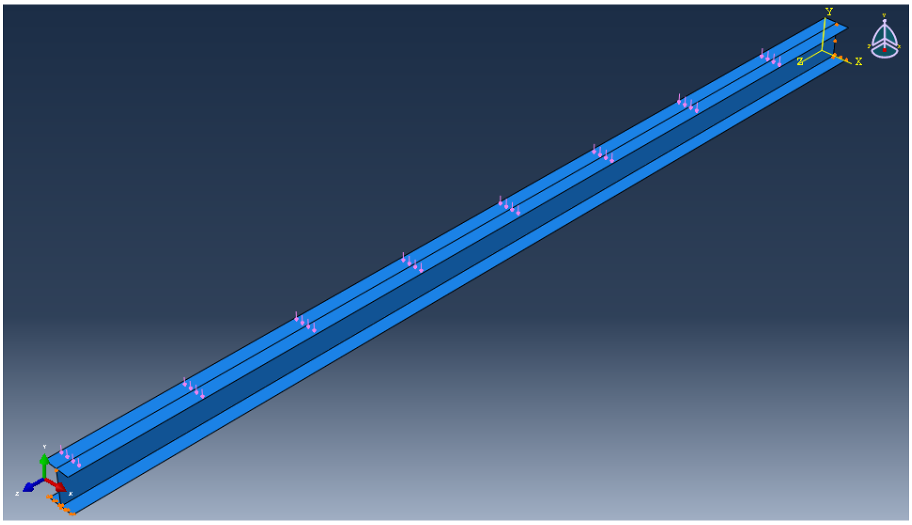


***Figure 4.3.*** Boundary conditions of beam model.

The boundary condition in z-direction of beam model is set only at one end of the beam at one point which is the crossing point of the web and bottom flange.

Boundary condtion of the frame model can be seen in Figure 4.6 and 4.7.

### 4.2.3 Loads

Figure 4.8, 4.9, and 4.10 shows the load applied of the beam model and frame model. Load has been converted to pressure in beam model so it will be evenly distributed on top flange which contains shell elements. On the frame model a line load has been applied.

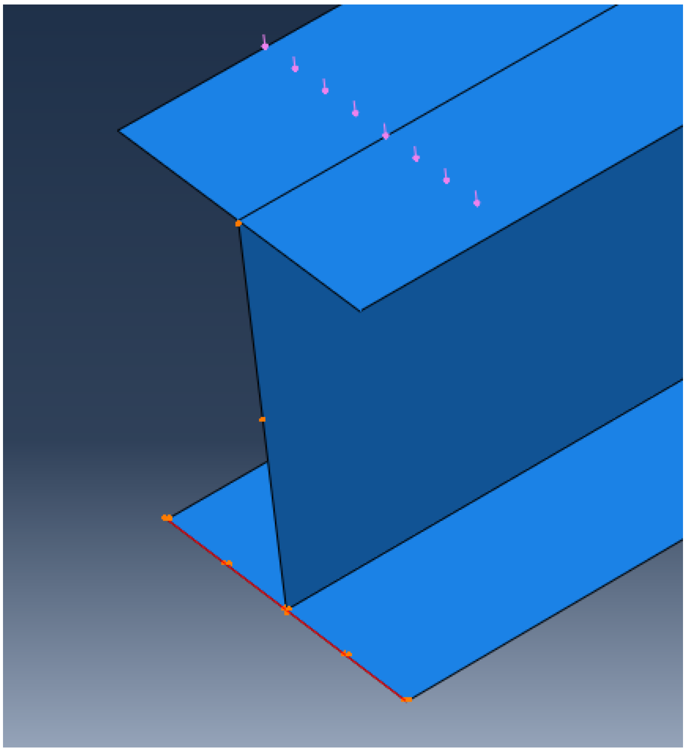


*Figure 4.4.* Boundary condition of beam model in x-direction.

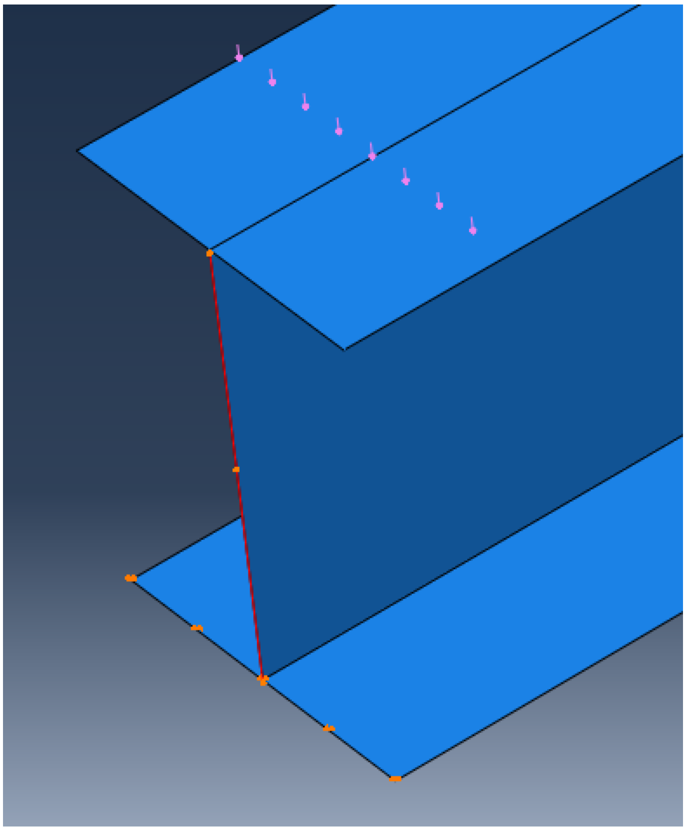


*Figure 4.5.* Boundary condition of beam model in y-direction.

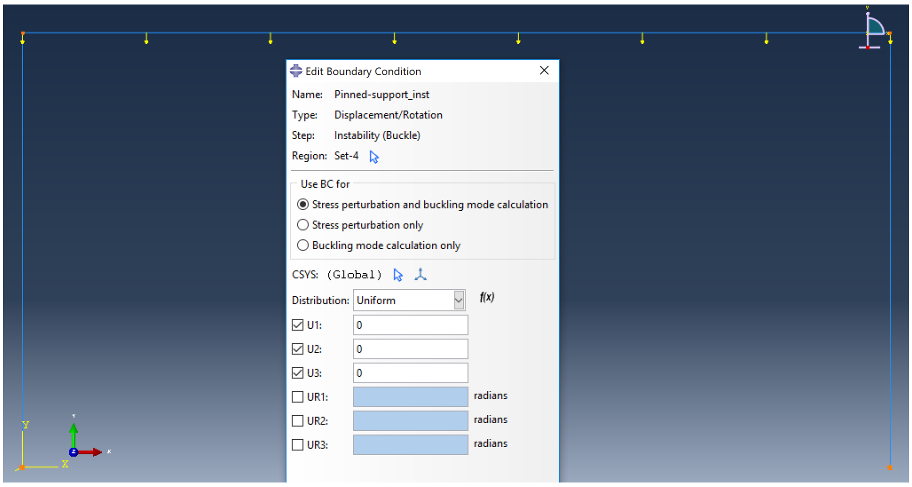



***Figure 4.6.*** Boundary condition of frame model at the bottom supports.

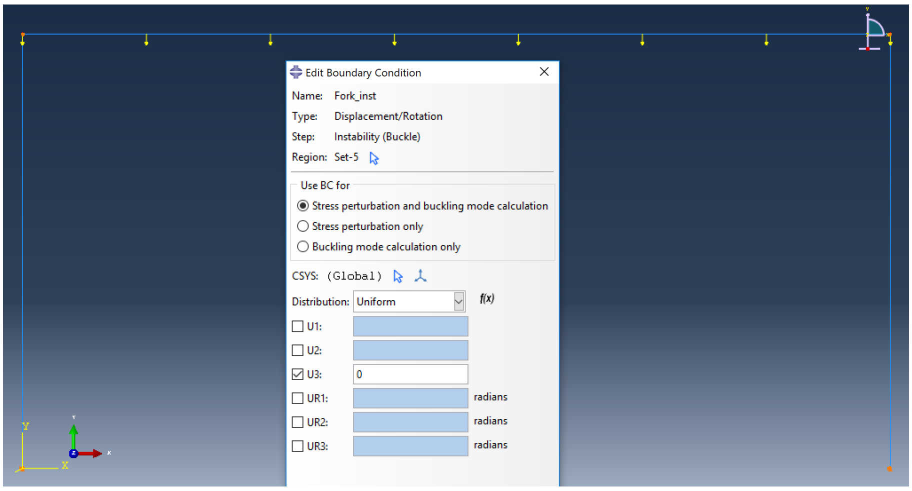



***Figure 4.7.*** Boundary condition of frame model at the corner supports.

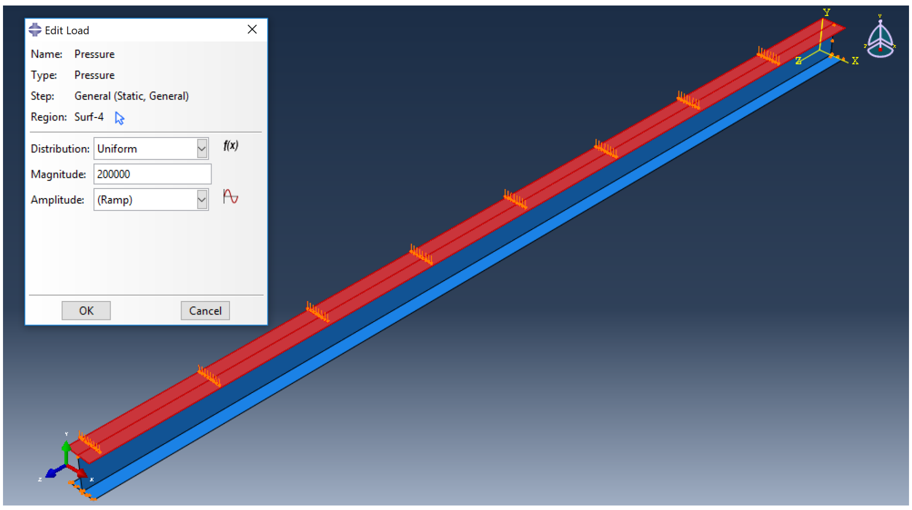



***Figure 4.8.*** Load applied on beam model for bifurcation analysis.

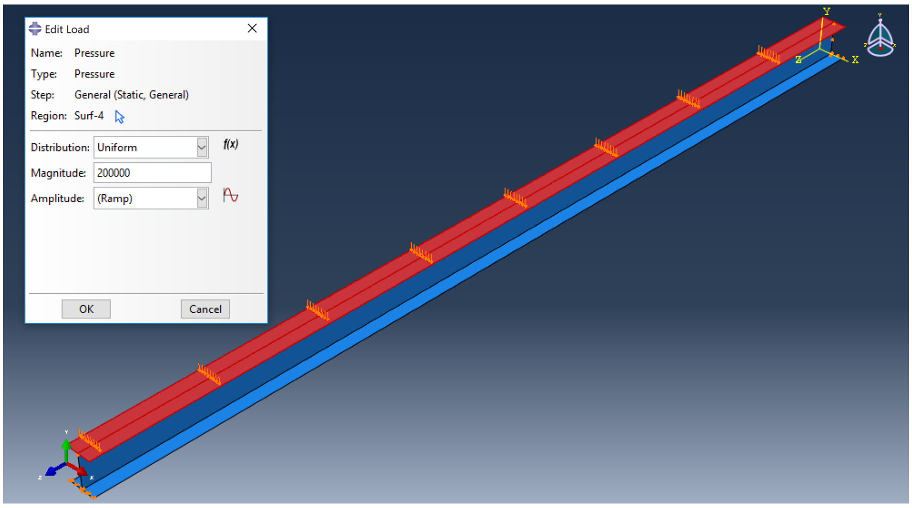



*Figure 4.9.* Load applied on beam model for large displacement analysis.

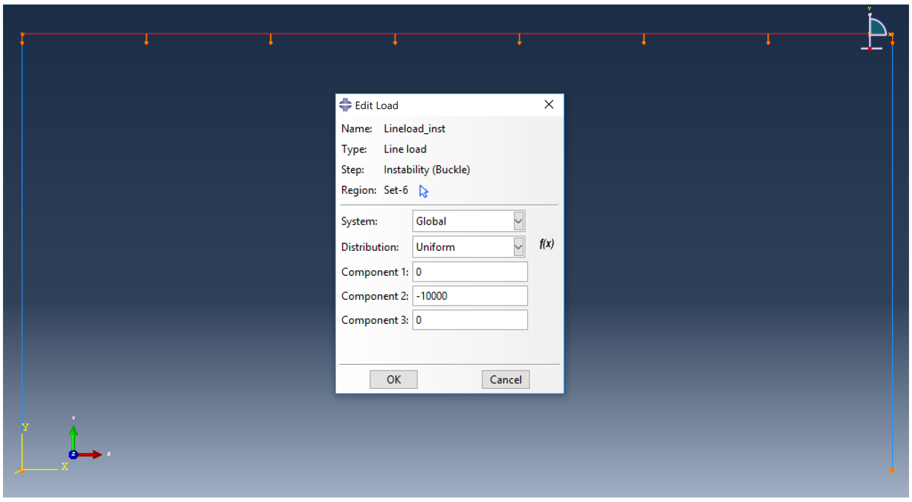



*Figure 4.10.* Load applied on frame model.

### 4.2.4 Mesh

The mesh of beam model in both bifurcation analysis and large displacement analysis can be seen in Figure 4.11 and 4.12.

### 4.2.5 Simulation

There has to be made a bifurcation and large displacement analyses to get the minimum load amplifier outputs. The bifurcation analysis provides the time-step before reaching yielding and large displacement analysis provides the eigenvalue.

In Figure 4.13, 4.14,4.15, and 4.16 shows the choices made to achieve the wanted outputs.

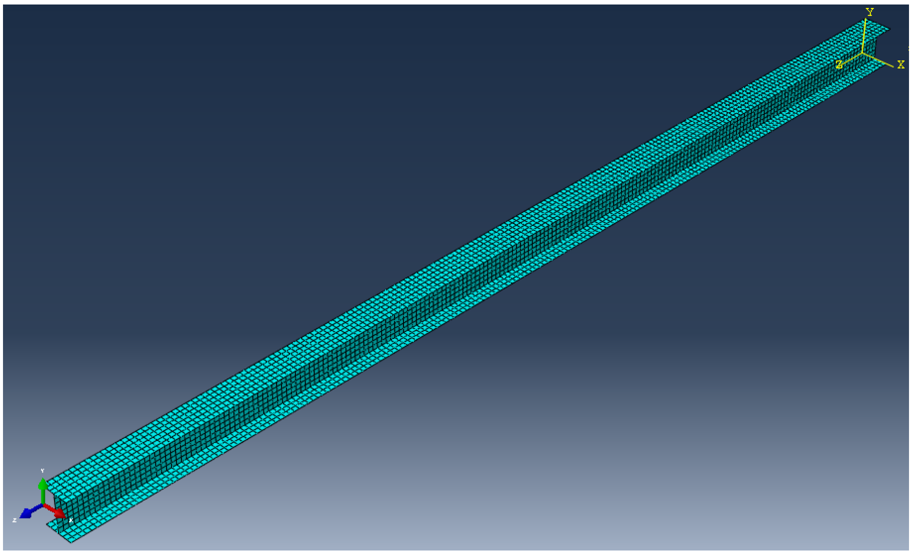


***Figure 4.11.*** Beam element partitioning of bifurcation analysis.

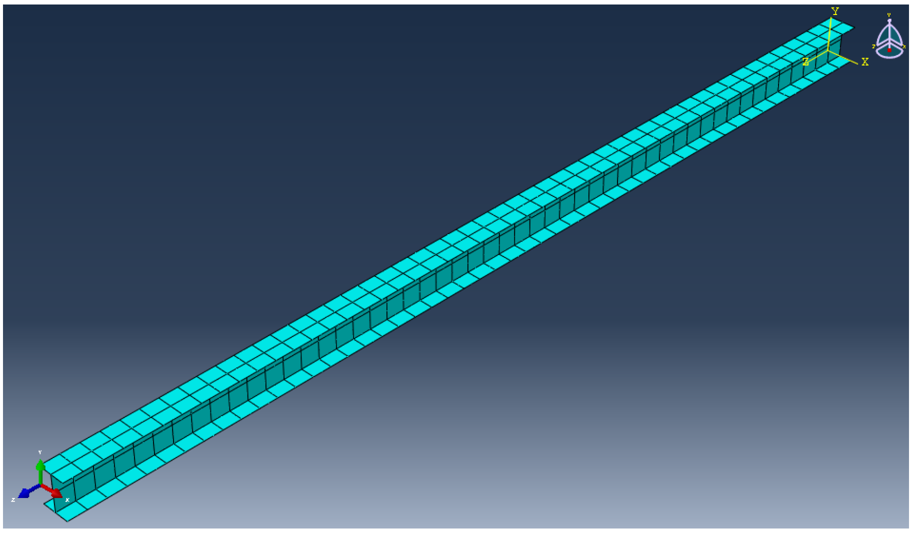


***Figure 4.12.*** Beam element partitioning of large displacement analysis.

### 4.2.6 Output

The outputs achieved by the simulations of both bifurcation and large displacement analyses can be seen in Figure 4.17, 4.18, 4.19, 4.20, and 4.21.

The outputs have been used in Appendix B with the formulas presented in the following sections.

## 4.3 Determining of the Minimum Load Amplifier, $\alpha_{ult,k}$

Firstly the ultimate load, $q_{ult}$, will be determined by Equation (4.1).

$$q_{ult} = q_{applied} t \tag{4.1}$$

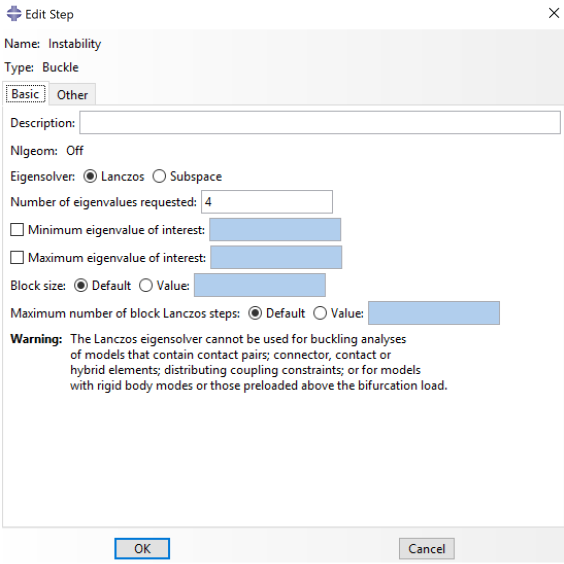



***Figure 4.13.*** Bifurcation analysis.

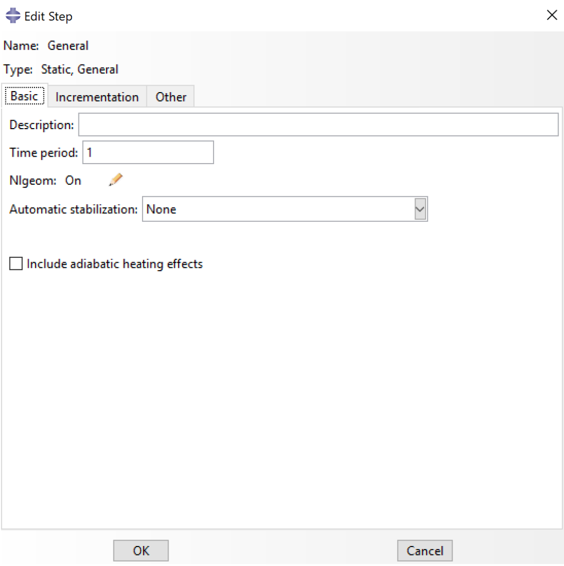



***Figure 4.14.*** Large displacement analysis - Basic.

where

| | |
|---|---|
| $q_{applied}$ | Applied load in Abaqus |
| $t$ | Time-step before reaching the yielding in Abaqus |

The minimum load amplifier, $\alpha_{ult,k}$, can be determined by Equation (4.2).

$$\alpha_{ult,k} = \frac{q_{ult}}{q} \tag{4.2}$$

where

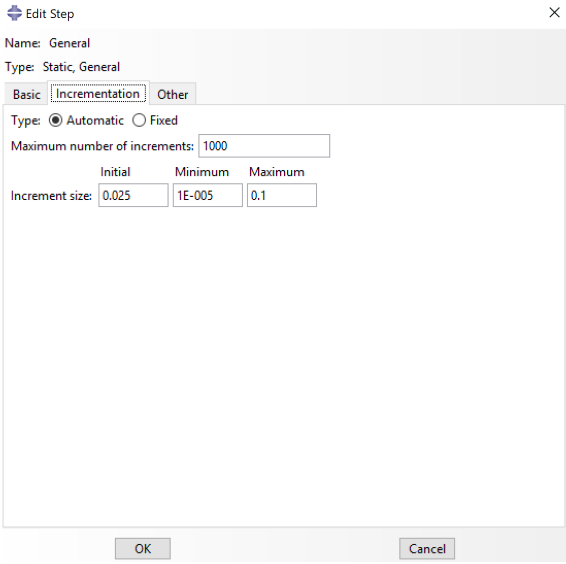



***Figure 4.15.*** Large displacement analysis - Incrementation.

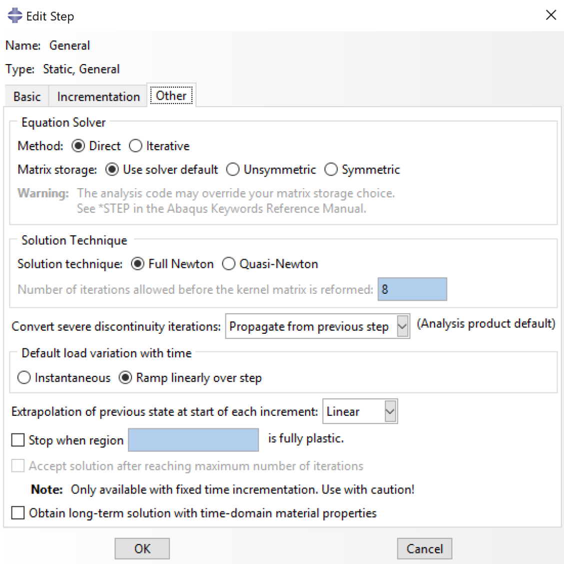



***Figure 4.16.*** Large displacement analysis - Iteration method.

| | |
|---|---|
| $\alpha_{ult,k}$ | Minimum load amplifier of the design loads to reach the characteristic resistance of the frame without taking lateral torsional buckling into account |
| $q$ | Design load |

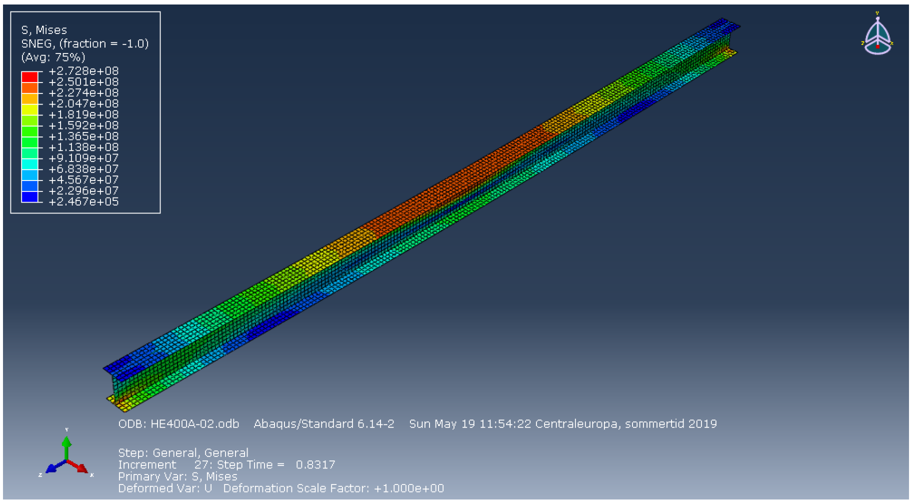



***Figure 4.17.*** Time-step output from beam model analysis.

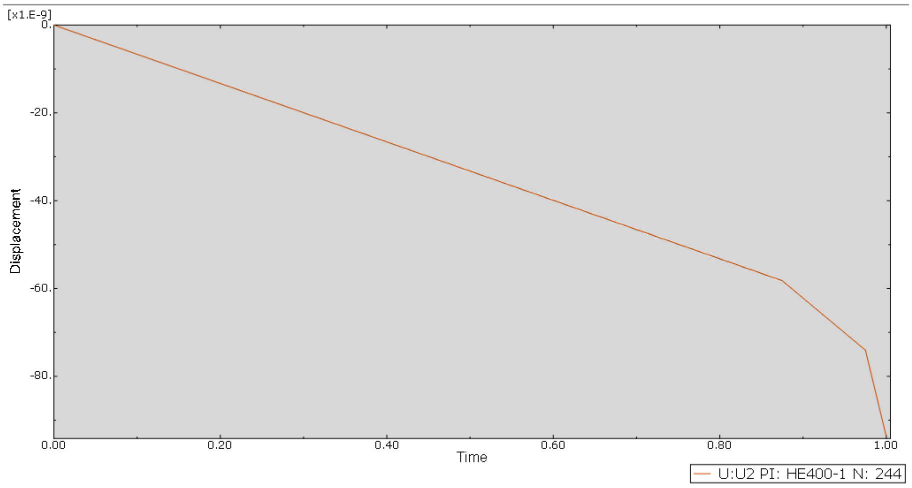



***Figure 4.18.*** Time-step and displacement plot of beam model.

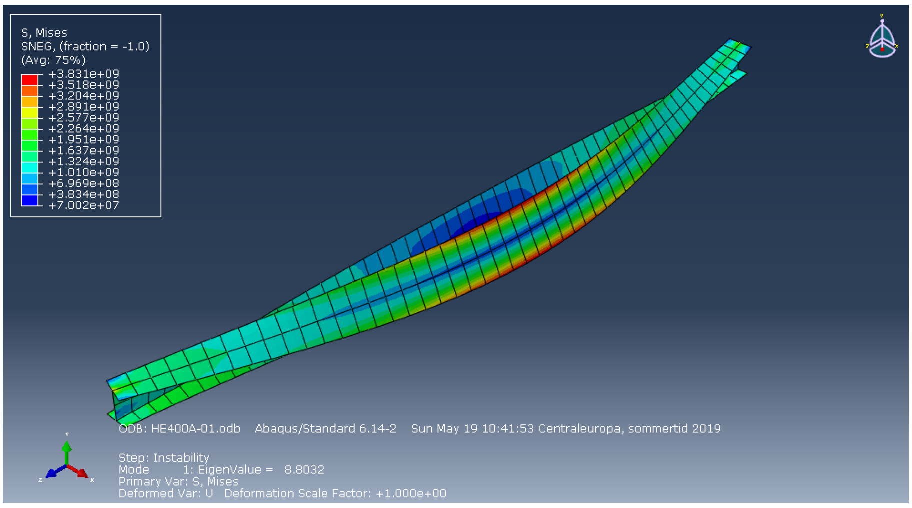



***Figure 4.19.*** 1st eigenvalue of beam model.

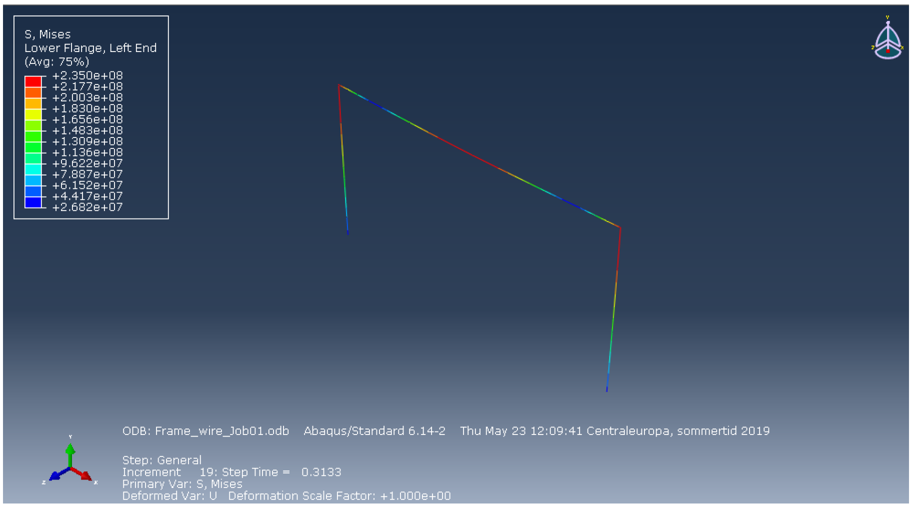



***Figure 4.20.*** Time-step output from frame model analysis.

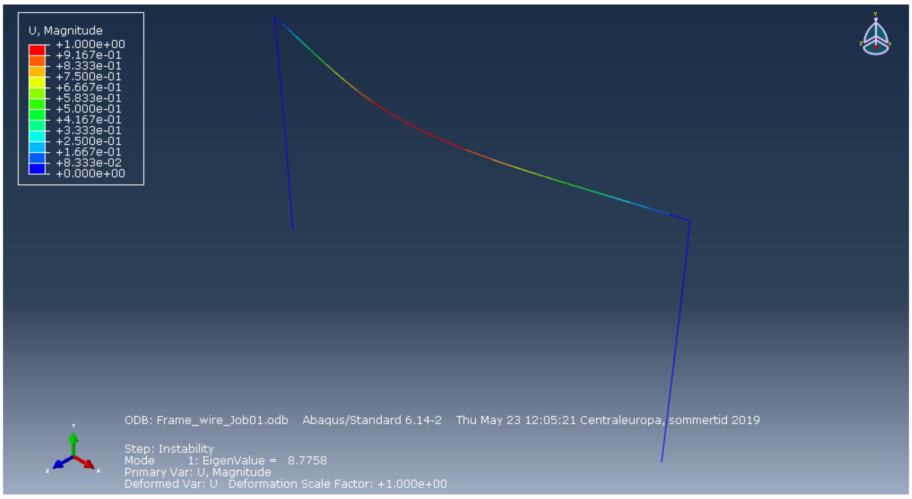



***Figure 4.21.*** 1st eigenvalue of frame model.

## 4.4 Determining of the Minimum Load Amplifier, $\alpha_{cr,op}$

The minimum load amplifier, $\alpha_{cr,op}$, is obtained by making a buckle analysis in Abaqus with employing the design load.

### 4.4.1 Determining of the Utilization Ratio

When the minimum load amplifier for both in-plane and out-of-plane behaviour is obtained the utilization ratio of the structure can be determined.

Firstly the the global non-dimensional slenderness, $\overline{\lambda}_{op}$, of the structure is determined by the

Equation (4.3).

$$\overline{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \tag{4.3}$$

where

| | |
|---|---|
| $\alpha_{cr,op}$ | Minimum load amplifier of the design loads to reach the elastic critical resistance of the frame with regard to lateral torsional deformation |

Secondly the reduction factor for lateral torsional buckling, $\chi_{LT}$, is determined by the Equation (4.4) where the factor $\phi_{LT}$ is determined by Equation (4.5).

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta\overline{\lambda}_{LT}^2}} \quad \text{but} \quad \chi_{LT} \leq 1.0 \tag{4.4}$$

where

| | |
|---|---|
| $\beta$ | Correction factor for the lateral torsional buckling curves |
| $\overline{\lambda}_{LT}$ | Plateau length of the lateral torsional buckling curves |

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\overline{\lambda}_{op} - \overline{\lambda}_{LT}\right) + \beta\overline{\lambda}_{LT}^2\right] \tag{4.5}$$

where

| | |
|---|---|
| $\alpha_{LT}$ | Imperfection factor for lateral torsional buckling |

Reduction factor for lateral torsional buckling, $\chi_{LT}$, is determined but it is a necessity for the lateral buckling, $\chi$, to be determined. Because the minimum value of $\chi_{LT}$ and $\chi$ will result in the maximum utilization ratio. Determining of reduction factor for lateral buckling will be similar to the analytical calculation Equation (3.15) and (3.16) where the non-dimensional slenderness is replaced by $\overline{\lambda}_{op}$ determined in Equation (4.3).

Verification of the structural element is made by the Equation (4.6).

$$\frac{\alpha_{ult,k}\chi_{op}}{\gamma_{M1}} \geq 1.0 \tag{4.6}$$

where

| | |
|---|---|
| $\chi_{op}$ | The minimum value of $\chi_{LT}$ and $\chi$ |

The utilization ratio, $UR$, is determined by the Equation (4.7).

$$UR = \frac{1}{\frac{\alpha_{ult,k}\chi_{op}}{\gamma_{M1}}} \quad \text{but} \quad UR \leq 1.0 \tag{4.7}$$

## 4.5 Results

The utilization ratio determined by the numerical approach in Appendix B can be seen from Table 4.1.

***Table 4.1.*** Utilization Ratio, $UR$, determined by numerical approach.

| | |
|---|---|
| Simply supported beam | 0.293 |
| Beam element of frame structure | 0.204 |

The results will be discussed and compared to the results determined by the analytical approach in Chapter 5.

# 5 Comparison

*In this chapter results from the analytical method 6.3.3 and numerical approach method 6.3.4 will be compared and discussed.*

The results from the analytical and numerical approach is shown in Table 5.1.

***Table 5.1.*** Utilization ratio, $UR$, determined by analytical and numerical approach.

| Structural element | Axis | Analytical | Numerical |
|---|---|---|---|
| ① Simply supported beam | Strong axis | 0.428 | 0.293 |
| Simply supported beam | Weak axis | 0.318 | - |
| ② Beam element of frame structure | Strong axis | 0.202 | 0.204 |
| Beam element of frame structure | Weak axis | 0.157 | - |
| Column element of frame structure | Strong axis | 0.140 | - |
| Column element of frame structure | Weak axis | 0.136 | - |

From Table 5.1 it can be seen that $UR$ only for the simply support beam about the strong axis and beam element of frame structure about the strong axis can be compared.

Deviation between analytical and numerical approach for the two structural elements are given as:
- ① Simply supported beam $UR$ about strong axis deviates 46.1%
- ② Beam element of frame structure $UR$ about strong axis deviates 1.0%

The tendency clearly shows with ① simply supported beam the analytical approach provides the most conservative $UR$ with deviation of 46.1%.

But in the case of ② beam element of frame structure in both analytical and numerical for the $UR$ shows similar results with numerical solution slightly being conservative by 1.0%.
One of the reason why the $UR$ for ② frame model from Abaqus is being more conservative than the analytical result could be that a convergence analysis is missing. There should have been made a convergence analysis of the discretization of the structure. Too few elements would result in incorrect and inapplicable $UR$ and too many would make the computational time of simulations too time consuming.

In the following problems and uncertainties of the analytical approach 6.3.3 method is showcased. During the calculation some parameters and determining of these, e.g. buckling length of frame structure was dealt with some uncertainties which could have led to some wrong results in the end.

There is a discrepancy in the analytical determination of buckling curve for lateral and torsional bending between [Standard, 2005] and [da Silva et al., 2010]. The impact of difference in the results should be investigated.

In the following problems and uncertainties of the numerical approach 6.3.4 general method is showcased.
When using Abaqus a result is obtained faster compared to undergo many calculations and formulas in the analytical method. But during analytical calculation there will be conditions that should be satisfied and thereby an ongoing control will be made.
Even though numerical method is faster to achieve result with this could lead to incorrect results and without any conditions nor equations to confirm the result can easily be misleading without having knowledge of this.

# Conclusion 6

*In this chapter the conclusion of the results from the analytical method 6.3.3 and numerical approach method 6.3.4 will be discussed. Further investigations intended or provided in an extension of this project will be described.*

As the project and report was intended an investigation of method proposed in [Standard, 2005] clause 6.3.3 and clause 6.3.4 has been performed and compared by the final outputs. The theoretical background of both methods was examined during the calculations and procedures.

It should be mentioned that the steel structures in this project with beam and frame model and its simplicity made the analytical approach accommodating. If more complexity was introduced to the structures; e.g. the profile was tapered, the applied load was unevenly distributed or added more supports and placed irregularly obtaining results analytically would be difficult. The deviation between the clause 6.3.3 and 6.3.4 in [Standard, 2005] would have been more significant.

Furthermore investigations could be made to compare the analytical and numerical approach:
• A structure or element that can not be categorised as standard profile
• Different supports and the effects of these
• Uneven distributed loading

There should have been made a convergence analysis of the amount of elements needed before reaching a converging result. Which could lead to a applicable comparison between the accuracy of results and time consumption of hand calculation and computational time of the simulations.

A parameter and sensitivity studies could have been made for the following:
• Shell and beam - Structural model analysed with both shell and beam elements and convergence analysis for both cases
• Supports - the impact of different kind of supports of the results and deviation between the two methods
• $C_1$, $C_2$, and $C_3$ coefficients from the analytical approach should be examined because the values are in an interval, which will cause a range in the end results

It can not be denied the visualization of the model in FEA software also being favourable when choosing methods. Whether it is deformation, stress/strain or $UR$ the impact of the values can be seen. For inexperienced engineering student without any practical experience the visualization gives an insight.

Both methods has some advantages and disadvantages but in the end both methods should be

handled with proper understanding and knowledge of the material behaviour and using software.
This leads to the final mark:
In extension of this project an experimental execution can be a recommendable addition. Both the analytical and numerical method can be compared to the reality and see how much the results deviate.

# Bibliography

**da Silva et al., 2010**. Luis Simoes da Silva, Rui Simoes and Helena Gervasio. *Design of Steel Structures*. ECCS European Convention for Constructional Steelwork, 2010.

**Ehlers, 2009**. Peter Ehlers. *Stålkonstruktioner BK301*. Ingeniørhøjskolen i Århus, 2009.

**Jensen et al., 2011**. Bjarne Chr. Jensen, Gunnar Mohr, Erik Brandt, Bo Mortensen, Lars Pilegaard Hansen, Carsten Munk Plum, Eilif Svensson, Svend Ole Hansen, Olaf Precht Sørensen, Dirch H. Bager, Henning Lausten, Ejnar Søndergaard, Martin Uhre Mandrup, Jørgen Munch-Andersen, Thomas Cornelius, Lars Zenke Hansen, Per Goltermann, Jørgen S. Steenfelt, Carsten S. Sørensen and Egil Borchersen. *Teknisk Ståbi 21. udgave*. ISBN: 978-87-571-2729-4, Book. Nyt Teknisk Forlag, 2011.

**Simulia**. Simulia. *Abaqus 6.13 Online Documentation*. URL `http://dsk.ippt.pan.pl/docs/abaqus/v6.13/`.

**Standard, 2005**. European Standard. *Eurocode 3: Design of Steel Structures - Part 1-1: General Rules and Rules for Buildings*. European Standard, 2005.

# Analytical Calculation A

*In Appendix A the analytical calculation has been made by use of MathCAD software.*

## Simply Supported Beam, HE400A

### *Inputs*

Line load : $q := 10\frac{kN}{m}$

Length of beam : $l := 10m$

### *Internal Forces*

Axial force : $N_{Ed} := 0kN$

Shear force : $V_{Ed} := \frac{1}{2}\cdot q\cdot l = 50\cdot kN$

Bending moment : $M_{max} := \frac{1}{8}\cdot q\cdot l^2 = 125\cdot kN\cdot m$

### *Material Properties*

Yielding strength : $f_y := 235MPa$

Ultimate strength : $f_u := 360MPa$

Young modulus of elasticity : $E := 210\cdot 10^3 MPa$

Shear modulus : $G := 810\cdot 10^2 MPa$

Poisson's ratio : $\upsilon := 0.3$

Partial safety factors : $\gamma_{M0} := 1$ $\gamma_{M1} := 1.1$

Constant to determine cross-section classification : $\varepsilon := \sqrt{\frac{235MPa}{f_y}} = 1$

### *Cross-section Properties for HE400A*

Height : $h := 390mm$

Width : $b_f := 300mm$

Web thickness : $t_w := 11mm$

Flange thickness : $t_f := 19mm$

Web height: $h_w := h - 2\cdot t_f = 352\cdot mm$

Radius : $r := 27mm$

Cross sectional area : $A := 15.9 \cdot 10^3 mm^2$

Moment of inertia, y-axis : $I_y := 450.7 \cdot 10^6 mm^4$

Elastic section modulus, y-axis : $W_{el.y} := 2310 \cdot 10^3 mm^3$

Plastic section modulus, y-axis : $W_{pl.y} := 2560 \cdot 10^3 mm^3$

Radius of giration, y-axis : $i_y := 168mm$

Cross sectional shear area : $A_V := A - \left(2 \cdot b_f \cdot t_f\right) + \left(t_w + 2 \cdot r\right) \cdot t_f = 5735 \cdot mm^2$

Moment of inertia, z-axis : $I_z := 85.6 \cdot 10^6 mm^4$

Elastic section modulus, z-axis : $W_{el.z} := 571 \cdot 10^3 mm^3$

Plastic section modulus, z-axis : $W_{pl.z} := 873 \cdot 10^3 mm^3$

Radius of giration, z-axis : $i_z := 73mm$

Warping Constant : $I_w := 2940 \cdot 10^9 mm^6$

St. Venant torsional constant : $I_t := 1900 \cdot 10^3 mm^4$

***Cross-section Classification***

Internal Web in Compression

Effected length : $c_w := h - 2 \cdot t_f - 2.r = 352 \cdot mm$

Factor to determine cross-section classification : $\frac{c_w}{t_w} = 32$

Cross-section class :

$$Web_{class} := \begin{vmatrix} 1 \text{ if } \frac{c_w}{t_w} \le 33\varepsilon \\ 2 \text{ if } 33 \cdot \varepsilon \le \frac{c_w}{t_w} \le 38\varepsilon \\ 3 \text{ if } 38 \cdot \varepsilon \le \frac{c_w}{t_w} \le 42\varepsilon \\ 4 \text{ if } 42 \cdot \varepsilon < \frac{c_w}{t_w} \end{vmatrix} = 1$$

Outer Flange in Compression

Effected length : $c_f := \frac{b_f - t_w - 2 \cdot r}{2} = 144.5 \cdot mm$

Factor to determine cross-section classification : $\frac{c_f}{t_f} = 7.6$

Cross-section class :

$$Flange_{class} := \begin{vmatrix} 1 \text{ if } \frac{c_f}{t_f} \le 33\varepsilon \\ 2 \text{ if } 33 \cdot \varepsilon \le \frac{c_f}{t_f} \le 38\varepsilon \\ 3 \text{ if } 38 \cdot \varepsilon \le \frac{c_f}{t_f} \le 42\varepsilon \\ 4 \text{ if } 42 \cdot \varepsilon < \frac{c_f}{t_f} \end{vmatrix} = 1$$

### *Compression resistance*

Axial force : $N_{Ed} := 0kN$

Resistance axial force : $N_{c.Rd} := A \cdot \frac{f_y}{\gamma_{M0}} = 3736.5 \cdot kN$ For cross-section class 1, 2 and 3!

$\frac{N_{Ed}}{N_{c.Rd}} = 0$

Validation of resistance of compression : $\text{if}\left(\frac{N_{Ed}}{N_{c.Rd}} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

### *Shear Resistance*

Shear force : $V_{Ed} = 50 \cdot kN$

Resistance shear force : $V_{c.Rd} := \frac{A_v}{\sqrt{3}} \cdot \frac{f_y}{\gamma_{M0}} = 778.1 \cdot kN$

$\frac{V_{Ed}}{V_{c.Rd}} = 0.1$

Validation of resistance of shear force : $\text{if}\left(\frac{V_{Ed}}{V_{c.Rd}} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

### *Bending Moment Resistance*

Bending moment : $M_{Ed} := M_{max} = 125 \cdot kN \cdot m$

Resistance of bending, y-axis : $M_{c.Rd} := W_{pl.y} \cdot \frac{f_y}{\gamma_{M0}} = 601.6 \cdot kN \cdot m$

$\frac{M_{Ed}}{M_{c.Rd}} = 0.2$

Validation of resistance of bending moment : $\text{if}\left(\frac{M_{Ed}}{M_{c.Rd}} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

### *Elastic Critical Load for Torsional Buckling, $N_{cr,T}$*

$\frac{h - t_f}{t_w} = 33.7$ ▮ $> 10$ Thin-walled section

$\frac{b_f}{t_f} = 15.8$ ▮ $> 10$ Thin-walled section

Determination of torsion constant : $I_T := \frac{1}{3}\left[\left(h - t_w\right)\cdot t_w^{\ 3} + 2\cdot b_f\cdot t_f^{\ 3}\right] = 1.54 \times 10^6\cdot mm^4$

Determination of warping constant : $I_W := \frac{t_f\cdot\left(h - t_w\right)^2\cdot b_f^{\ 3}}{24} = 3.1 \times 10^{12}\cdot mm^6$

Distance along the y-axis between the shear centre and the centroid of the section :

$y_c := 0$

Radius of polar gyration : $i_c := \sqrt{y_c^{\ 2} + \frac{I_y + I_z}{A}} = 183.7\cdot mm$

Buckling lenght for the torsional bucling mode : $L_{ET} := 1.0l = 10\cdot m$

Critical axial load for torsional buckling : $N_{cr.T} := \frac{1}{i_c^{\ 2}}\cdot\left(G\cdot I_T + \frac{\pi^2\cdot E\cdot I_W}{L_{ET}^{\ 2}}\right) = 5584.8\cdot kN$

### *Buckling Resistance of Compression*

Uniform members in compression

Compression force : $N_{Ed} := 0kN$

Determination of Slenderness for Flexural Buckling

$\lambda_1 := 93.9\cdot\varepsilon = 93.9$

Buckling length for simple supported beam in both y and z axis:

$L_{cr.y} := 10m$

$L_{cr.z} := 10m$

Slenderness, y-axis : $\lambda\_y := \frac{L_{cr.y}}{i_y}\cdot\frac{1}{\lambda_1} = 0.63$

24th May 2019 **Simply Supported Beam, HE400A** 6/11

Slenderness, y-axis : $\lambda\_z := \frac{L_{cr.z}}{i_z} \cdot \frac{1}{\lambda_1} = 1.46$

Slenderness, maximum : $\lambda\_ := \max(\lambda\_y, \lambda\_z) = 1.46$

Determination of buckling curve, α:

$\frac{h}{b_f} = 1.3$ if $\frac{h}{b_f} \le 1.2$

$t_f = 19 \cdot mm$ if $t_f \le 40mm$

for S 235, buckling curve:

y-y => a => $\alpha_y := 0.21$

z-z => b => $\alpha_z := 0.34$

Determination of reduction factor, χ:

Factor to determine reduction factor : $\phi_y := 0.5 \cdot \left[1 + \alpha_y \cdot (\lambda\_y - 0.2) + \lambda\_y^2\right] = 0.746$

$\phi_z := 0.5 \cdot \left[1 + \alpha_z \cdot (\lambda\_z - 0.2) + \lambda\_z^2\right] = 1.778$

$\phi := \max(\phi_y, \phi_z) = 1.778$

Reduction factor, y-axis: $\chi_y := \frac{1}{\phi_y + \sqrt{\phi_y^2 - \lambda\_y^2}} = 0.877$

Reduction factor, z-axis: $\chi_z := \frac{1}{\phi_z + \sqrt{\phi_z^2 - \lambda\_z^2}} = 0.358$

Reduction factor, maximum : $\chi := \min(\chi_y, \chi_z) = 0.358$

Verification of buckling resistance of compression:

$N_{b.Rd} := \chi \cdot A \cdot \frac{f_y}{\gamma_{M1}} = 1215.4 \cdot kN$

$\frac{N_{Ed}}{N_{b.Rd}} = 0$

$\text{if}\left(\frac{N_{Ed}}{N_{b.Rd}} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

24th May 2019 **Simply Supported Beam, HE400A** 7/11

***Buckling Resistance of Bending***

Uniform members in bending

Coefficent from figure 3.4 : $C_1 := 1.12$

Coefficent from figure 3.4 : $C_2 := 0.45$

Coefficent from figure 3.4 : $C_3 := 0.525$

Factor depending on the supports : $k_z := 1$

Factor depending on the supports : $k_w := 1$

Coordinate of the applied load : $z_a := h$

Coordinate of the applied load : $z_s := \frac{h}{2}$

$z_g := z_a - z_s = 195 \cdot mm$

Parameter, assymetry of the cross-section : $z_j := 0mm$ for doubly symmetric cross-section

Determination of elastic critical moment for lateral-torsional buckling:

$$M_{cr} := C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{(k_z \cdot L_{cr.z})^2} \cdot \left[ \left[ \left( \frac{k_z}{k_w} \right)^2 \cdot \frac{I_w}{I_z} + \frac{(k_z \cdot L_{cr.z})^2 \cdot G \cdot I_T}{\pi^2 \cdot E \cdot I_z} + (C_2 \cdot z_g - C_3 \cdot z_j)^2 \right]^{0.5} - (C_2 \cdot z_g - C_3 \cdot z_j) \right] = 491.7 \cdot kN \cdot$$

Section modulus : $W_y := W_{pl.y}$ for Class 1 cross-section

Determination of imperfection factor for for Rolled I-section for lateral-torsional buckling:

Dertermining of buckling curve from Figure 3.1 : $\frac{h}{b_f} = 1.3$ if $\frac{h}{b} \le 2$ => $\alpha_{LT} := 0.34$ Buckling curve b

Non-dimensional slenderness : $\lambda_{\_LT} := \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} = 1.106$

Factor to dertimne reduction factor : $\phi_{LT} := 0.5 \cdot \left[ 1 + \alpha_{LT} \cdot (\lambda_{\_LT} - 0.2) + \lambda_{\_LT}^2 \right] = 1.266$

Reduction factor : $\chi_{LT} := \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{\_LT}^2}} = 0.532$

Buckling resistance of bending : $M_{b.Rd} := \chi_{LT} \cdot W_y \cdot \frac{f_y}{\gamma_{M1}} = 290.7 \cdot kN \cdot m$

$$\frac{M_{Ed}}{M_{b.Rd}} = 0.4$$

Verification of buckling resistance of bending: $\text{if}\left(\frac{M_{Ed}}{M_{b.Rd}} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

***Buckling Resistance of Bending and axial compression***

Uniform members in bending and axial compression

Characteristic compression force : $N_{Rk} := A \cdot f_y = 3736.5 \cdot kN$

Characteristic moment, y-axis : $M_{y.Rk} := W_{pl.y} \cdot f_y = 601.6 \cdot kN \cdot m$

Reduction factor, y-axis : $\chi_y = 0.877$

Reduction factor, z-axis : $\chi_z = 0.358$

Elastic critical load for flexural bucklingm, y-axis : $N_{cr.y} := \frac{\pi^2 \cdot E \cdot I_y}{L_{cr.y}^2} = 9341.3 \cdot kN$

Elastic critical load for flexural buckling, z-axis : $N_{cr.z} := \frac{\pi^2 \cdot E \cdot I_z}{L_{cr.z}^2} = 1774.2 \cdot kN$

Auxiliary term from Figure 3.6 : $\mu_y := \frac{1 - \frac{N_{Ed}}{N_{cr.y}}}{1 - \chi_y \cdot \frac{N_{Ed}}{N_{cr.y}}} = 1$

Auxiliary term from Figure 3.6 : $\mu_z := \frac{1 - \frac{N_{Ed}}{N_{cr.z}}}{1 - \chi_z \cdot \frac{N_{Ed}}{N_{cr.z}}} = 1$

Auxiliary term from Figure 3.6 : $w_y := \frac{W_{pl.y}}{W_{el.y}} = 1.1$

$\text{if}\left(w_y \le 1.5, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$

Auxiliary term from Figure 3.6 : $w_z := \frac{W_{pl.z}}{W_{el.z}} = 1.53$

$\text{if}\left(w_z \le 1.5, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Redimension"}$

Factor to determine $C_{ij}$ : $n_{pl} := \frac{N_{Ed}}{\frac{N_{Rk}}{\gamma_{M1}}} = 0$

Factor to determine $C_{ij}$ : $a_{LT} := 1 - \frac{I_t}{I_y} = 0.996$ where $a_{LT} \ge 0$

Factor to determine $N_{cr,TF}$ : $\beta := 1 - \left(\frac{y_c}{i_c}\right)^2 = 1$

Elastic critical axial load for flexural-torsional buckling, y-axis :

$$N_{cr.TF} := \frac{1}{2\cdot\beta}\cdot\left[\left(N_{cr.y} + N_{cr.T}\right) - \sqrt{\left(N_{cr.y} + N_{cr.T}\right)^2 - 4\cdot\beta\cdot N_{cr.y}\cdot N_{cr.T}}\right] = 5584.8\cdot kN$$

Non dimensional slenderness for lateral torsional buckling :

$\lambda_{\_0} := \lambda_{\_LT} = 1.1$

Because $\lambda_{\_0} > 0.2\cdot\sqrt{C_1}\cdot\sqrt[4]{\left(1 - \frac{N_{Ed}}{N_{cr.z}}\right)\cdot\left(1 - \frac{N_{Ed}}{N_{cr.TF}}\right)} = 0.212$ from figure 3.7

Becase $\lambda_{\_0} > 0.2$:

Equivalent moment factor : $C_{my.0} := 1 + 0.03\cdot\frac{N_{Ed}}{N_{cr.y}} = 1$ from figure 3.8

$\varepsilon_y = \frac{M_{Ed}}{N_{Ed}}\cdot\frac{A}{W_{el.y}}$ for class 1 => $\varepsilon_y := 0$ because $N_{Ed} = 0$

Equivalent moment factor : $C_{my} := C_{my.0} + \left(1 - C_{my.0}\right)\cdot\frac{\sqrt{\varepsilon_y} a_{LT}}{1 + \sqrt{\varepsilon_y} a_{LT}} = 1$

Equivalent moment factor : $C_{mz} := 0$ The moment around the z-axiz is 0

Equivalent moment factor : $C_{mLT} := C_{my}^2 \cdot \frac{a_{LT}}{\sqrt{\left(1 - \frac{N_{Ed}}{N_{cr.z}}\right)\cdot\left(1 - \frac{N_{Ed}}{N_{cr.T}}\right)}} = 1$ where $C_{mLT} \geq 1$

Determination of interaction factors:

$M_{y.Ed} := M_{Ed} = 125 \cdot kN \cdot m$

$M_{pl.y.Rd} := M_{c.Rd} = 601.6 \cdot kN \cdot m$

$M_{pl.z.Rd} := W_{pl.z} \cdot \frac{f_y}{\gamma_{M0}} = 205.2 \cdot kN \cdot m$

$M_{z.Ed} := 0kN \cdot m$

Factor to determine $C_{yy}$ : $b_{LT} := 0.5 \cdot a_{LT} \cdot \lambda\_0^2 \cdot \frac{M_{y.Ed}}{\chi_{LT} \cdot M_{pl.y.Rd}} \cdot \frac{M_{z.Ed}}{M_{pl.z.Rd}} = 0$

Slenderness, maximum : $\lambda_{max} := \lambda\_$

Auxiliary term from Figure 3.6 : $C_{yy} := 1 + (w_y - 1) \cdot \left[\left(2 - \frac{1.6}{w_y} \cdot C_{my}^2 \cdot \lambda_{max} - \frac{1.6}{w_y} \cdot C_{my}^2 \cdot \lambda_{max}\right) \cdot n_{pl} - b_{LT}\right] = 1$

$\text{if}\left(C_{yy} \geq \frac{W_{el.y}}{W_{pl.y}}, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$

Factor to determine $C_{zy}$ : $d_{LT} = 2 \cdot a_{LT} \cdot \frac{\lambda\_0}{0.1 + \lambda\_z^4} + \frac{M_{y.Ed}}{C_{my} \cdot \chi_{LT} \cdot M_{pl.y.Rd}} \cdot \frac{M_{z.Ed}}{C_{mz} \cdot M_{pl.z.Rd}}$

$d_{LT} := 2 \cdot a_{LT} \cdot \frac{\lambda\_0}{0.1 + \lambda\_z^4} = 0.476$

Auxiliary term from Figure 3.6 : $C_{zy} := 1 + (w_y - 1) \cdot \left[\left(2 - 14 \cdot \frac{C_{my}^2 \cdot \lambda_{max}}{w_y^5}\right) \cdot n_{pl} - d_{LT}\right] = 0.949$

$\text{if}\left(C_{zy} \geq 0.6 \cdot \sqrt{\frac{w_y}{w_z}} \cdot \frac{W_{el.y}}{W_{pl.y}}, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$

Interaction factor : $k_{yy} := C_{my} \cdot C_{mLT} \cdot \frac{\mu_y}{1 - \frac{N_{Ed}}{N_{cr.y}}} \cdot \frac{1}{C_{yy}} = 1$

Interaction factor : $k_{zy} := C_{my} \cdot C_{mLT} \cdot \frac{\mu_z}{1 - \frac{N_{Ed}}{N_{cr.y}}} \cdot \frac{1}{C_{zy}} \cdot 0.6 \cdot \sqrt{\frac{w_z}{w_y}} = 0.7$

Equation (3.30) - Strong axis : $\frac{N_{Ed}}{\frac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}} + k_{yy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} = 0.428$

$$\text{if}\left( \frac{N_{Ed}}{\frac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}} + k_{yy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} \le 1, \text{"Ok"}, \text{"Redimension"} \right) = \text{"Ok"}$$

Equation (3.30) - Weak axis : $\frac{N_{Ed}}{\frac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}} + k_{zy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} = 0.318$

$$\text{if}\left( \frac{N_{Ed}}{\frac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}} + k_{zy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} \le 1, \text{"Ok"}, \text{"Redimension"} \right) = \text{"Ok"}$$

## Beam Element of the Frame, HE400A

### *Inputs*

Line load : $q := 10\frac{kN}{m}$

Length of beam : $l := 10m$

### *Internal Forces - Obtained from FEM Design Software*

Axial force : $N_{Ed} := 14kN$

Shear force : $V_{Ed} := 56kN$

Bending moment : $M_{Ed} := 70kN\cdot m$ $M_{max} := 71kN\cdot m$

### *Material Properties*

Yielding strength : $f_y := 235MPa$

Ultimate strength : $f_u := 360MPa$

Young modulus of elasticity : $E := 210\cdot 10^3 MPa$

Shear modulus : $G := 810\cdot 10^2 MPa$

Poisson's ratio : $\upsilon := 0.3$

Partial safety factors : $\gamma_{M0} := 1$ $\gamma_{M1} := 1.1$

Constant to determine cross-section classification : $\varepsilon := \sqrt{\frac{235MPa}{f_y}} = 1$

### *Cross-section Properties for HE400A*

Height : $h := 390mm$

Width : $b_f := 300mm$

Web thickness : $t_w := 11mm$

Flange thickness : $t_f := 19mm$

Web height: $h_w := h - 2\cdot t_f = 352\cdot mm$

| | |
|---|---|
| Radius : | $r := 27mm$ |
| Cross sectional area : | $A := 15.9 \cdot 10^3 mm^2$ |
| Moment of inertia, y-axis : | $I_y := 450.7 \cdot 10^6 mm^4$ |
| Elastic section modulus, y-axis : | $W_{el.y} := 2310 \cdot 10^3 mm^3$ |
| Plastic section modulus, y-axis : | $W_{pl.y} := 2560 \cdot 10^3 mm^3$ |
| Radius of giration, y-axis : | $i_y := 168mm$ |
| Cross sectional shear area : | $A_v := A - \left(2 \cdot b_f \cdot t_f\right) + \left(t_w + 2 \cdot r\right) \cdot t_f = 5735 \cdot mm^2$ |
| Moment of inertia, z-axis : | $I_z := 85.6 \cdot 10^6 mm^4$ |
| Elastic section modulus, z-axis : | $W_{el.z} := 571 \cdot 10^3 mm^3$ |
| Plastic section modulus, z-axis : | $W_{pl.z} := 873 \cdot 10^3 mm^3$ |
| Radius of giration, z-axis : | $i_z := 73mm$ |
| Warping Constant : | $I_w := 2940 \cdot 10^9 mm^6$ |
| St. Venant torsional constant : | $I_t := 1900 \cdot 10^3 mm^4$ |

### *Cross-section Classification*

Internal Web in Compression

Effected length : $c_w := h - 2 \cdot t_f - 2.r = 352 \cdot mm$

Factor to determine cross-section classification : $\frac{c_w}{t_w} = 32$

Cross-section class :

$$Web_{class} := \begin{vmatrix} 1 \text{ if } \frac{c_w}{t_w} \le 33\varepsilon \\ 2 \text{ if } 33 \cdot \varepsilon \le \frac{c_w}{t_w} \le 38\varepsilon \\ 3 \text{ if } 38 \cdot \varepsilon \le \frac{c_w}{t_w} \le 42\varepsilon \\ 4 \text{ if } 42 \cdot \varepsilon < \frac{c_w}{t_w} \end{vmatrix} = 1$$

Outer Flange in Compression

Effected length : $c_f := \frac{b_f - t_w - 2 \cdot r}{2} = 144.5 \cdot mm$

Factor to determine cross-section classification : $\frac{c_f}{t_f} = 7.6$

Cross-section class :

$$Flange_{class} := \begin{vmatrix} 1 \text{ if } \frac{c_f}{t_f} \le 33\varepsilon \\ 2 \text{ if } 33 \cdot \varepsilon \le \frac{c_f}{t_f} \le 38\varepsilon \\ 3 \text{ if } 38 \cdot \varepsilon \le \frac{c_f}{t_f} \le 42\varepsilon \\ 4 \text{ if } 42 \cdot \varepsilon < \frac{c_f}{t_f} \end{vmatrix} = 1$$

### *Compression resistance*

Axial force : $N_{Ed} = 14\,kN$

Resistance axial force : $N_{c.Rd} := A \cdot \frac{f_y}{\gamma_{M0}} = 3736.5 \cdot kN$ For cross-section class 1, 2 and 3!

$\frac{N_{Ed}}{N_{c.Rd}} = 3.7 \times 10^{-3}$

Validation of resistance of compression : $\text{if}\left(\frac{N_{Ed}}{N_{c.Rd}} \leq 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

### *Shear Resistance*

Shear force : $V_{Ed} = 56 \cdot kN$

Resistance shear force : $V_{c.Rd} := \frac{A_v}{\sqrt{3}} \cdot \frac{f_y}{\gamma_{M0}} = 778.1 \cdot kN$

$\frac{V_{Ed}}{V_{c.Rd}} = 0.1$

Validation of resistance of shear force : $\text{if}\left(\frac{V_{Ed}}{V_{c.Rd}} \leq 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

### *Bending Moment Resistance*

Bending moment : $M_{max} = 71\,kN \cdot m$

Resistance of bending, y-axis : $M_{c.Rd} := W_{pl.y} \cdot \frac{f_y}{\gamma_{M0}} = 601.6 \cdot kN \cdot m$

$\frac{M_{Ed}}{M_{c.Rd}} = 0.1$

Validation of resistance of bending moment : $\text{if}\left(\frac{M_{Ed}}{M_{c.Rd}} \leq 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

### *Elastic Critical Load for Torsional Buckling, $N_{cr,T}$*

$\frac{h - t_f}{t_w} = 33.7$ ▮ $> 10$ Thin-walled section

$\frac{b_f}{t_f} = 15.8$ ▮ $> 10$ Thin-walled section

Determination of torsion constant : $I_T := \frac{1}{3}\left[\left(h - t_w\right)\cdot t_w^{\,3} + 2\cdot b_f\cdot t_f^{\,3}\right] = 1.54 \times 10^6\cdot mm^4$

Determination of warping constant : $I_W := \frac{t_f\cdot\left(h - t_w\right)^2\cdot b_f^{\,3}}{24} = 3.1 \times 10^{12}\cdot mm^6$

Distance along the y-axis between the shear centre and the centroid of the section :

$y_c := 0$

Radius of polar gyration : $i_c := \sqrt{y_c^{\,2} + \frac{I_y + I_z}{A}} = 183.7\cdot mm$

Buckling lenght for the torsional buckling mode : $L_{ET} := 0.5l = 5\cdot m$

Critical axial load for torsional buckling : $N_{cr.T} := \frac{1}{i_c^{\,2}}\cdot\left(G\cdot I_T + \frac{\pi^2\cdot E\cdot I_W}{L_{ET}^{\,2}}\right) = 11244.7\cdot kN$

### *Buckling Resistance of Compression*

Uniform members in compression

Compression force : $N_{Ed} = 14\,kN$

Determination of Slenderness for Flexural Buckling

$\lambda_1 := 93.9\cdot\varepsilon = 93.9$

Buckling length for simple supported beam in both y and z axis:

$L_{cr.y} := L_{ET} = 5\,m$

$L_{cr.z} := 1.0\cdot l = 10\,m$

24th May 2019 **Beam Element of the Frame, HE400A** 6/11

Slenderness, y-axis : $\lambda\_y := \frac{L_{cr.y}}{i_y} \cdot \frac{1}{\lambda_1} = 0.32$

Slenderness, z-axis : $\lambda\_z := \frac{L_{cr.z}}{i_z} \cdot \frac{1}{\lambda_1} = 1.46$

Slenderness, maximum : $\lambda\_ := \max(\lambda\_y, \lambda\_z) = 1.46$

Determination of buckling curve, α:

$\frac{h}{b_f} = 1.3$ if $\frac{h}{b_f} \le 1.2$

$t_f = 19 \cdot mm$ if $t_f \le 40mm$

for S 235, buckling curve:
y-y => a => $\alpha_y := 0.21$

z-z => b => $\alpha_z := 0.34$

Determination of reduction factor, χ:

Factor to determine reduction factor :

$\phi_y := 0.5 \cdot \left[1 + \alpha_y \cdot (\lambda\_y - 0.2) + \lambda\_y^2\right] = 0.563$

$\phi_z := 0.5 \cdot \left[1 + \alpha_z \cdot (\lambda\_z - 0.2) + \lambda\_z^2\right] = 1.778$

$\phi := \max(\phi_y, \phi_z) = 1.778$

Reduction factor, y-axis: $\chi_y := \frac{1}{\phi_y + \sqrt{\phi_y^2 - \lambda\_y^2}} = 0.973$

Reduction factor, z-axis: $\chi_z := \frac{1}{\phi_z + \sqrt{\phi_z^2 - \lambda\_z^2}} = 0.358$

Reduction factor, maximum : $\chi := \min(\chi_y, \chi_z) = 0.358$

Verification of buckling resistance of compression:

$N_{b.Rd} := \chi \cdot A \cdot \frac{f_y}{\gamma_{M1}} = 1215.4 \cdot kN$

$\frac{N_{Ed}}{N_{b.Rd}} = 0$

$$\text{if}\left(\frac{N_{Ed}}{N_{b.Rd}} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$$

***Buckling Resistance of Bending***

Uniform members in bending

Coefficent from figure 3.6 : $k_c := 0.90$

Coefficent from figure 3.4 : $C_1 := \left(\frac{1}{k_c}\right)^2 = 1.2$

Coefficent from figure 3.3 : $C_2 := 0$ When Subjected end moment

Coefficent from figure 3.3 : $C_3 := 0$

Factor depending on the supports : $k_z := 1$

Factor depending on the supports : $k_w := 1$

Coordinate of the applied load : $z_a := h$

Coordinate of the applied load : $z_s := \frac{h}{2}$

$z_g := z_a - z_s = 195 \cdot mm$

Parameter, assymetry of the cross-section : $z_j := 0mm$ for doubly symmetric cross-section

Determination of elastic critical moment for lateral-torsional buckling:

$$M_{cr} := C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{(k_z \cdot L_{cr.z})^2} \cdot \left[\left[\left(\frac{k_z}{k_w}\right)^2 \cdot \frac{I_w}{I_z} + \frac{(k_z \cdot L_{cr.z})^2 \cdot G \cdot I_T}{\pi^2 \cdot E \cdot I_z} + (C_2 \cdot z_g - C_3 \cdot z_j)^2\right]^{0.5} - (C_2 \cdot z_g - C_3 \cdot z_j)\right] = 708.6 \cdot kN \cdot m$$

Section modulus : $W_y := W_{pl.y}$ for Class 1 cross-section

Determination of imperfection factor for for Rolled I-section for lateral-torsional buckling:

Dertermining of buckling curve from Figure 3.1 : $\frac{h}{b_f} = 1.3$ if $\frac{h}{b} \le 2$ => $\alpha_{LT} := 0.34$ Buckling curve b

24th May 2019 **Beam Element of the Frame, HE400A** 8/11

Non-dimensional slenderness : $\lambda_{\_LT} := \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} = 0.921$

Factor to dertimne reduction factor : $\phi_{LT} := 0.5 \cdot \left[1 + \alpha_{LT} \cdot \left(\lambda_{\_LT} - 0.2\right) + \lambda_{\_LT}^2\right] = 1.047$

Reduction factor : $\chi_{LT} := \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{\_LT}^2}} = 0.647$

Buckling resistance of bending : $M_{b.Rd} := \chi_{LT} \cdot W_y \cdot \frac{f_y}{\gamma_{M1}} = 354.1 \cdot kN \cdot m$

$\frac{M_{Ed}}{M_{b.Rd}} = 0.2$

Verification of buckling resistance of bending: $\text{if}\left(\frac{M_{Ed}}{M_{b.Rd}} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

### *Buckling Resistance of Bending and axial compression*

Uniform members in bending and axial compression

Characteristic compression force : $N_{Rk} := A \cdot f_y = 3736.5 \cdot kN$

Characteristic moment, y-axis : $M_{y.Rk} := W_{pl.y} \cdot f_y = 601.6 \cdot kN \cdot m$

Reduction factor, y-axis : $\chi_y = 0.973$

Reduction factor, z-axis : $\chi_z = 0.358$

Elastic critical load for flexural buckling, y-axis : $N_{cr.y} := \frac{\pi^2 \cdot E \cdot I_y}{L_{cr.y}^2} = 37365.1 \cdot kN$

Elastic critical load for flexural buckling, z-axis : $N_{cr.z} := \frac{\pi^2 \cdot E \cdot I_z}{L_{cr.z}^2} = 1774.2 \cdot kN$

Auxiliary term from Figure 3.6 : $\mu_y := \frac{1 - \frac{N_{Ed}}{N_{cr.y}}}{1 - \chi_y \cdot \frac{N_{Ed}}{N_{cr.y}}} = 1$

Auxiliary term from Figure 3.6 : $\mu_z := \dfrac{1 - \dfrac{N_{Ed}}{N_{cr.z}}}{1 - \chi_z \cdot \dfrac{N_{Ed}}{N_{cr.z}}} = 0.995$

Auxiliary term from Figure 3.6 : $w_y := \dfrac{W_{pl.y}}{W_{el.y}} = 1.1$

$\text{if}\left(w_y \le 1.5, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$

Auxiliary term from Figure 3.6 : $w_z := \dfrac{W_{pl.z}}{W_{el.z}} = 1.53$

$\text{if}\left(w_z \le 1.5, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Redimension"}$ The value is so close to 1,5 that it will be neglected!

Factor to determine $C_{ij}$ : $n_{pl} := \dfrac{N_{Ed}}{\dfrac{N_{Rk}}{\gamma_{M1}}} = 0.0041$

Factor to determine $C_{ij}$ : $a_{LT} := 1 - \dfrac{I_t}{I_y} = 0.996$ where $a_{LT} \ge 0$

Factor to determine $N_{cr,TF}$ : $\beta := 1 - \left(\dfrac{y_c}{i_c}\right)^2 = 1$

Elastic critical axial load for flexural-torsional buckling, y-axis :

$$N_{cr.TF} := \frac{1}{2\cdot\beta}\cdot\left[\left(N_{cr.y} + N_{cr.T}\right) - \sqrt{\left(N_{cr.y} + N_{cr.T}\right)^2 - 4\cdot\beta\cdot N_{cr.y}\cdot N_{cr.T}}\right] = 11244.7\cdot kN$$

Non dimensional slenderness for lateral torsional buckling :

$\lambda\_0 := \lambda\_LT = 0.9$

Because $\lambda\_0 > 0.2\cdot\sqrt{C_1}\cdot\sqrt[4]{\left(1 - \dfrac{N_{Ed}}{N_{cr.z}}\right)\cdot\left(1 - \dfrac{N_{Ed}}{N_{cr.TF}}\right)} = 0.222$ from figure 3.9

Becase $\lambda\_0 > 0.2$ :

Equivalent moment factor : $C_{my.0} := 1 + \left(\dfrac{\pi^2\cdot E\cdot I_y\cdot 7mm}{l^2\cdot M_{max}} - 1\right)\cdot\dfrac{N_{Ed}}{N_{cr.y}} = 1$ from figure 3.10 and deflection obtained from FEM Design

24th May 2019 **Beam Element of the Frame, HE400A** 10/11

$$\varepsilon_y := \frac{M_{max}}{N_{Ed}} \cdot \frac{A}{W_{el.y}} = 34.9$$ for class 1 => $\varepsilon_y := 0$

Equivalent moment factor : $$C_{my} := C_{my.0} + \left(1 - C_{my.0}\right) \cdot \frac{\sqrt{\varepsilon_y} a_{LT}}{1 + \sqrt{\varepsilon_y} a_{LT}} = 1$$

Equivalent moment factor : $C_{mz} := 0$ The moment around the z-axiz is 0

Equivalent moment factor : $$C_{mLT} := C_{my}^2 \cdot \frac{a_{LT}}{\sqrt{\left(1 - \frac{N_{Ed}}{N_{cr.z}}\right) \cdot \left(1 - \frac{N_{Ed}}{N_{cr.T}}\right)}} = 1$$ where $C_{mLT} \geq 1$

Determination of interaction factors:

$M_{y.Ed} := M_{max} = 71 \cdot kN \cdot m$

$M_{pl.y.Rd} := M_{c.Rd} = 601.6 \cdot kN \cdot m$

$M_{pl.z.Rd} := W_{pl.z} \cdot \frac{f_y}{\gamma_{M0}} = 205.2 \cdot kN \cdot m$

$M_{z.Ed} := 0 kN \cdot m$

Factor to determine $C_{yy}$ : $$b_{LT} := 0.5 \cdot a_{LT} \cdot \lambda\_0^2 \cdot \frac{M_{y.Ed}}{\chi_{LT} \cdot M_{pl.y.Rd}} \cdot \frac{M_{z.Ed}}{M_{pl.z.Rd}} = 0$$

Slenderness, maximum : $\lambda_{max} := \lambda\_$

Auxiliary term from Figure 3.6 : $$C_{yy} := 1 + \left(w_y - 1\right) \cdot \left[\left(2 - \frac{1.6}{w_y} \cdot C_{my}^2 \cdot \lambda_{max} - \frac{1.6}{w_y} \cdot C_{my}^2 \cdot \lambda_{max}\right) \cdot n_{pl} - b_{LT}\right] = 1$$

$$\text{if}\left(C_{yy} \geq \frac{W_{el.y}}{W_{pl.y}}, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$$

Factor to determine $C_{zy}$ : $$d_{LT} = 2 \cdot a_{LT} \cdot \frac{\lambda\_0}{0.1 + \lambda\_z^4} + \frac{M_{y.Ed}}{C_{my} \cdot \chi_{LT} \cdot M_{pl.y.Rd}} \cdot \frac{M_{z.Ed}}{C_{mz} \cdot M_{pl.z.Rd}}$$

$$d_{LT} := 2 \cdot a_{LT} \cdot \frac{\lambda\_0}{0.1 + \lambda\_z^4} = 0.396$$

Auxiliary term from Figure 3.6 : $$C_{zy} := 1 + \left(w_y - 1\right) \cdot \left[\left(2 - 14 \cdot \frac{C_{my}^2 \cdot \lambda_{max}}{w_y^5}\right) \cdot n_{pl} - d_{LT}\right] = 0.953$$

$$\text{if}\left(C_{zy} \geq 0.6 \cdot \sqrt{\frac{w_y}{w_z}} \cdot \frac{W_{el.y}}{W_{pl.y}}, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$$

Interaction factor :
$$k_{yy} := C_{my} \cdot C_{mLT} \cdot \frac{\mu_y}{1 - \frac{N_{Ed}}{N_{cr.y}}} \cdot \frac{1}{C_{yy}} = 1$$

Interaction factor :
$$k_{zy} := C_{my} \cdot C_{mLT} \cdot \frac{\mu_z}{1 - \frac{N_{Ed}}{N_{cr.y}}} \cdot \frac{1}{C_{zy}} \cdot 0.6 \cdot \sqrt{\frac{w_z}{w_y}} = 0.7$$

Equation (3.30) - Strong axis :
$$\frac{N_{Ed}}{\frac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}} + k_{yy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} = 0.202$$

$$\text{if}\left(\frac{N_{Ed}}{\frac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}} + k_{yy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} \leq 1, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$$

Equation (3.30) - Weak axis :
$$\frac{N_{Ed}}{\frac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}} + k_{zy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} = 0.157$$

$$\text{if}\left(\frac{N_{Ed}}{\frac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}} + k_{zy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} \leq 1, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$$

## Column Element of the Frame, HE400A

### *Inputs*

Line load : $q := 10\frac{kN}{m}$

Length of column : $l := 5m$

### *Internal Forces - Optained from FEM Design Software*

Axial force : $N_{Ed} := 62kN$

Shear force : $V_{Ed} := 14kN$

Bending moment : $M_{Ed} := 70kN \cdot m$

### *Material Properties*

Yielding strength : $f_y := 235MPa$

Ultimate strength : $f_u := 360MPa$

Young's modulus of elasticity : $E := 210 \cdot 10^3 MPa$

Shear modulus : $G := 810 \cdot 10^2 MPa$

Poisson's ratio : $\upsilon := 0.3$

Partial safety factors : $\gamma_{M0} := 1$ $\gamma_{M1} := 1.1$

Constant to determine cross-section classification : $\varepsilon := \sqrt{\frac{235MPa}{f_y}} = 1$

### *Cross-section Properties for HE400A*

Height : $h := 390mm$

Width : $b_f := 300mm$

Web thickness : $t_w := 11mm$

Flange thickness : $t_f := 19mm$

Web height: $h_w := h - 2 \cdot t_f = 352 \cdot mm$

Radius : $r := 27mm$

24th May 2019 **Colmn Element of the Frame, HE400A** 2/12

| | |
|---|---|
| Cross sectional area : | $A := 15.9 \cdot 10^3 mm^2$ |
| Moment of inertia, y-axis : | $I_y := 450.7 \cdot 10^6 mm^4$ |
| Elastic section modulus, y-axis : | $W_{el.y} := 2310 \cdot 10^3 mm^3$ |
| Plastic section modulus, y-axis : | $W_{pl.y} := 2560 \cdot 10^3 mm^3$ |
| Radius of giration, y-axis : | $i_y := 168mm$ |
| Cross sectional shear area : | $A_v := A - (2 \cdot b_f \cdot t_f) + (t_w + 2 \cdot r) \cdot t_f = 5735 \cdot mm^2$ |
| Moment of inertia, z-axis : | $I_z := 85.6 \cdot 10^6 mm^4$ |
| Elastic section modulus, z-axis : | $W_{el.z} := 571 \cdot 10^3 mm^3$ |
| Plastic section modulus, z-axis : | $W_{pl.z} := 873 \cdot 10^3 mm^3$ |
| Radius of giration, z-axis : | $i_z := 73mm$ |
| Warping Constant : | $I_w := 2940 \cdot 10^9 mm^6$ |
| St. Venant torsional constant : | $I_t := 1900 \cdot 10^3 mm^4$ |

***Cross-section Classification***

Internal Web in Compression

$$c_w := h - 2 \cdot t_f - 2.r = 298 \cdot mm$$

$$\frac{c_w}{t_w} = 27.1$$

$$Web_{class} := \begin{cases} 1 & \text{if } \frac{c_w}{t_w} \le 33\varepsilon \\ 2 & \text{if } 33 \cdot \varepsilon \le \frac{c_w}{t_w} \le 38\varepsilon \\ 3 & \text{if } 38 \cdot \varepsilon \le \frac{c_w}{t_w} \le 42\varepsilon \\ 4 & \text{if } 42 \cdot \varepsilon < \frac{c_w}{t_w} \end{cases} = 1$$

Outer Flange in Compression

$$c_f := \frac{b_f - t_w - 2 \cdot r}{2} = 117.5 \cdot mm$$

$$\frac{c_f}{t_f} = 6.2$$

$$Flange_{class} := \begin{cases} 1 & \text{if } \frac{c_f}{t_f} \le 33\varepsilon \\ 2 & \text{if } 33 \cdot \varepsilon \le \frac{c_f}{t_f} \le 38\varepsilon \\ 3 & \text{if } 38 \cdot \varepsilon \le \frac{c_f}{t_f} \le 42\varepsilon \\ 4 & \text{if } 42 \cdot \varepsilon < \frac{c_f}{t_f} \end{cases} = 1$$

***Compression resistance***

$N_{Ed} = 62 \cdot kN$

$N_{c.Rd} := A \cdot \frac{f_y}{\gamma_{M0}} = 3736.5 \cdot kN$

$\frac{N_{Ed}}{N_{c.Rd}} = 0.017$

$if\left(\frac{N_{Ed}}{N_{c.Rd}} \le 1, "OK", "Redimension"\right) = "OK"$

***Shear Resistance***

$V_{Ed} = 14 \cdot kN$

$V_{c.Rd} := \frac{A_v}{\sqrt{3}} \cdot \frac{f_y}{\gamma_{M0}} = 778.1 \cdot kN$

$\frac{V_{Ed}}{V_{c.Rd}} = 0.018$

$if\left(\frac{V_{Ed}}{V_{c.Rd}} \le 1, "OK", "Redimension"\right) = "OK"$

***Bending Moment Resistance***

$M_{Ed} = 70 \cdot kN \cdot m$

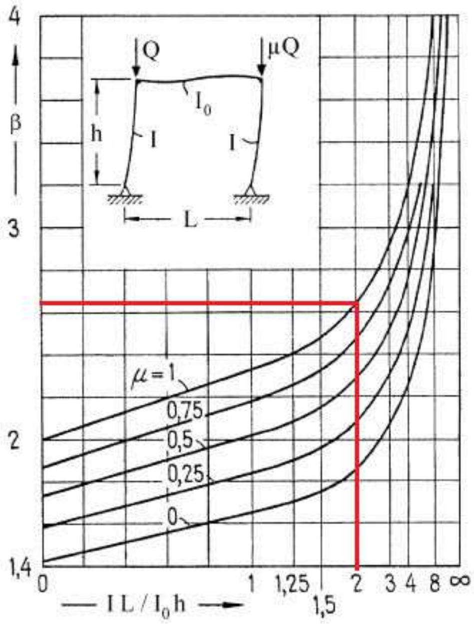
$M_{c.Rd} := W_{pl.y} \cdot \frac{f_y}{\gamma_{M0}} = 601.6 \cdot kN \cdot m$

$\frac{M_{Ed}}{M_{c.Rd}} = 0.116$

$if\left(\frac{M_{Ed}}{M_{c.Rd}} \le 1, "OK", "Redimension"\right) = "OK"$

***Determening of critical load***

$\frac{I \cdot L_{beam}}{I_0 \cdot h_{column}} = \frac{L_{beam}}{h_{colum}}$ $\frac{10m}{5m} = 2$




The value of beta is determined by the knowlegde of the moment of inertia of the beam and columns of the frame structure and also the support conditions.

beta := 2.62

Buckling lenght, y-axis : $L_{cr.y} := \text{beta}\cdot 5\text{m} = 13.1\ \text{m}$

Critical load, y-axis : $N_{cr.y} := \dfrac{\pi^2 \cdot E \cdot I_y}{L_{cr.y}^2} = 5.4 \times 10^3 \cdot \text{kN}$

Buckling lenght, z-axis : $L_{cr.z} := l = 5\ \text{m}$ The column is simply support in the z-axis

Critical load, z-axis : $N_{cr.z} := \dfrac{\pi^2 \cdot E \cdot I_z}{L_{cr.z}^2} = 7.1 \times 10^3 \cdot \text{kN}$

***Elastic Critical Load for Torsional Buckling, $N_{cr,T}$***

$\dfrac{h - t_f}{t_w} = 33.7$ > 10 Thin-walled section

$\frac{b_f}{t_f} = 15.8$ $> 10$ Thin-walled section

Determination of torsion constant : $I_T := \frac{1}{3}\left[\left(h - t_w\right) \cdot t_w^3 + 2 \cdot b_f \cdot t_f^3\right] = 1.54 \times 10^6 \cdot mm^4$

Determination of warping constant : $I_W := \frac{t_f \cdot \left(h - t_w\right)^2 \cdot b_f^3}{24} = 3.1 \times 10^{12} \cdot mm^6$

Distance along the y-axis between the shear centre and the centroid of the section :

$y_c := 0$

Radius of polar gyration : $i_c := \sqrt{y_c^2 + \frac{I_y + I_z}{A}} = 183.7 \cdot mm$

Buckling lenght for the torsional bucling mode : $L_{ET} := L_{cr.y} = 13.1 \cdot m$

Critical axial load for torsional buckling : $N_{cr.T} := \frac{1}{i_c^2} \cdot \left(G \cdot I_T + \frac{\pi^2 \cdot E \cdot I_W}{L_{ET}^2}\right) = 4797.5 \cdot kN$

***Buckling Resistance of Compression***

Uniform members in compression

Determination of Slenderness for Flexural Buckling

$\lambda_1 := 93.9 \cdot \varepsilon = 93.9$

Buckling length for the column in the frame structure in both y and z axis:

$\lambda\_y := \frac{L_{cr.y}}{i_y} \cdot \frac{1}{\lambda_1} = 0.83$

$\lambda\_z := \frac{L_{cr.z}}{i_z} \cdot \frac{1}{\lambda_1} = 0.73$

$\lambda\_ := \max\left(\lambda\_y, \lambda\_z\right) = 0.83$

Determination of buckling curve, α:

24th May 2019 **Colmn Element of the Frame, HE400A** 7/12

$\frac{h}{b_f} = 1.3 \qquad \text{if} \qquad \frac{h}{b_f} \le 1.2$

$t_f = 19 \cdot mm \qquad \text{if} \qquad t_f \le 40mm$

for S 235, buckling curve:
y-y => a => $\alpha_y := 0.21$

z-z => b => $\alpha_z := 0.34$

Determination of reduction factor, χ:

$$\phi_y := 0.5 \cdot \left[1 + \alpha_y \cdot \left(\lambda\_y - 0.2\right) + \lambda\_y^2\right] = 0.911$$

$$\phi_z := 0.5 \cdot \left[1 + \alpha_z \cdot \left(\lambda\_z - 0.2\right) + \lambda\_z^2\right] = 0.856$$

$$\phi := \max\left(\phi_y, \phi_z\right) = 0.911$$

$$\chi_y := \frac{1}{\phi_y + \sqrt{\phi_y^2 - \lambda\_y^2}} = 0.778$$

$$\chi_z := \frac{1}{\phi_z + \sqrt{\phi_z^2 - \lambda\_z^2}} = 0.767$$

$$\chi := \min\left(\chi_y, \chi_z\right) = 0.767$$

Verification of buckling resistance of compression:

$$N_{b.Rd} := \chi \cdot A \cdot \frac{f_y}{\gamma_{M1}} = 2604.8 \cdot kN$$

$$\frac{N_{Ed}}{N_{b.Rd}} = 0$$

$$\text{if}\left(\frac{N_{Ed}}{N_{b.Rd}} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$$

## Uniform members in bending

Determination of elastic critical moment:

$k_z := 1$

$k_w := 1$

$\psi := 0$ The moment is zero at end point of the support of the columns

$C_1 := 1.77$

$C_2 := 0$ When subjected end moment

$C_3 := 1.0$

$z_g := 0mm$

$z_j := 0mm$ for doubly symmetric cross-section

$$M_{cr} := C_1 \cdot \frac{\pi^2 \cdot E \cdot I_z}{\left(k_z \cdot L_{cr.z}\right)^2} \cdot \left[ \left[ \left(\frac{k_z}{k_w}\right)^2 \cdot \frac{I_w}{I_z} + \frac{\left(k_z \cdot L_{cr.z}\right)^2 \cdot G \cdot I_T}{\pi^2 \cdot E \cdot I_z} + \left(C_2 \cdot z_g - C_3 \cdot z_j\right)^2 \right]^{0.5} - \left(C_2 \cdot z_g - C_3 \cdot z_j\right) \right] = 2.9 \times 10^3 \cdot kN$$

$W_y := W_{pl.y}$ for Class 1 cross-section

Determination of imperfection factor for for Rolled I-section for lateral-torsional buckling:

$\frac{h}{b_f} = 1.3$ if $\frac{h}{b} \le 2$ => $\alpha_{LT} := 0.21$ Buckling curve a

Determination of the coefficient of non-dimensional slenderness:

$$\lambda_{\_LT} := \sqrt{\frac{W_y \cdot f_y}{M_{cr}}} = 0.458$$

$$\phi_{LT} := 0.5 \cdot \left[1 + \alpha_{LT} \cdot \left(\lambda_{\_LT} - 0.2\right) + \lambda_{\_LT}^2\right] = 0.632$$

$$\chi_{LT} := \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \lambda_{\_LT}^2}} = 0.937$$

Verification of buckling resistance of bending:

$$M_{b.Rd} := \chi_{LT} \cdot W_y \cdot \frac{f_y}{\gamma_{M1}} = 512.3 \cdot kN \cdot m$$

$$\frac{M_{Ed}}{M_{b.Rd}} = 0.1$$

$$\text{if}\left(\frac{M_{Ed}}{M_{b.Rd}} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$$

Uniform members in bending and axial compression

$N_{Rk} := A \cdot f_y = 3736.5 \cdot kN$

$M_{y.Rk} := W_{pl.y} \cdot f_y = 601.6 \cdot kN \cdot m$

$\chi_y = 0.778$

$\chi_z = 0.767$

$N_{cr.y} = 5443.3 \cdot kN$

$N_{cr.z} = 7096.6 \cdot kN$

$$\mu_y := \frac{1 - \frac{N_{Ed}}{N_{cr.y}}}{1 - \chi_y \cdot \frac{N_{Ed}}{N_{cr.y}}} = 0.997$$

$$\mu_z := \frac{1 - \frac{N_{Ed}}{N_{cr.z}}}{1 - \chi_z \cdot \frac{N_{Ed}}{N_{cr.z}}} = 0.998$$

$w_y := \frac{W_{pl.y}}{W_{el.y}} = 1.1$ where $w_y \leq 1.5$

$w_z := \frac{W_{pl.z}}{W_{el.z}} = 1.5$ where $w_z \leq 1.5$

$$n_{pl} := \frac{N_{Ed}}{\frac{N_{Rk}}{\gamma_{M1}}} = 0.0183$$

$a_{LT} := 1 - \frac{I_t}{I_y} = 0.996$ where $a_{LT} \geq 0$

Factor to determine $N_{cr,TF}$ : $\beta := 1 - \left(\frac{y_c}{i_c}\right)^2 = 1$

Elastic critical axial load for flexural-torsional buckling, y-axis :

$$N_{cr.TF} := \frac{1}{2 \cdot \beta} \cdot \left[\left(N_{cr.y} + N_{cr.T}\right) - \sqrt{\left(N_{cr.y} + N_{cr.T}\right)^2 - 4 \cdot \beta \cdot N_{cr.y} \cdot N_{cr.T}}\right] = 4797.5 \cdot kN$$

Non dimensional slenderness for lateral torsional buckling :

$\lambda\_0 := \lambda\_LT = 0.5$

Because $\lambda\_0 > 0.2 \cdot \sqrt{C_1} \cdot \sqrt[4]{\left(1 - \frac{N_{Ed}}{N_{cr.z}}\right) \cdot \left(1 - \frac{N_{Ed}}{N_{cr.TF}}\right)} = 0.265$ from figure 3.7

Becase $\lambda\_0 > 0.2$ :

Equivalent moment factor : $C_{my.0} := 0.79 + 0.21 \cdot \psi + 0.36 \cdot (\psi - 0.33) \cdot \left(\frac{N_{Ed}}{N_{cr.y}}\right) = 0.79$ from figure 3.8

$\varepsilon_y := \frac{M_{Ed}}{N_{Ed}} \cdot \frac{A}{W_{el.y}} = 7.8$ for class 1

Equivalent moment factor : $C_{my} := C_{my.0} + \left(1 - C_{my.0}\right) \cdot \frac{\sqrt{\varepsilon_y} a_{LT}}{1 + \sqrt{\varepsilon_y} a_{LT}} = 0.9$

Equivalent moment factor : $C_{mz} := 0$ The moment around the z-axiz is 0

Equivalent moment factor : $C_{mLT} := C_{my}{}^2 \cdot \frac{a_{LT}}{\sqrt{\left(1 - \frac{N_{Ed}}{N_{cr.z}}\right) \cdot \left(1 - \frac{N_{Ed}}{N_{cr.T}}\right)}} = 0.9$ where $C_{mLT} \geq 1$

Determination of interaction factors:

$M_{y.Ed} := M_{Ed}$

$M_{pl.y.Rd} := M_{c.Rd}$

$M_{pl.z.Rd} := M_{c.Rd}$

$M_{z.Ed} := 0kN \cdot m$

Factor to determine $C_{yy}$ : $b_{LT} := 0.5 \cdot a_{LT} \cdot \lambda\_{0}^{2} \cdot \frac{M_{y.Ed}}{\chi_{LT} \cdot M_{pl.y.Rd}} \cdot \frac{M_{z.Ed}}{M_{pl.z.Rd}} = 0$

Slenderness, maximum : $\lambda_{max} := \lambda\_$

Auxiliary term from Figure 3.6 : $C_{yy} := 1 + (w_y - 1) \cdot \left[\left(2 - \frac{1.6}{w_y} \cdot C_{my}^{2} \cdot \lambda_{max} - \frac{1.6}{w_y} \cdot C_{my}^{2} \cdot \lambda_{max}\right) \cdot n_{pl} - b_{LT}\right] = 1$

where $C_{yy} \geq 1$

Factor to determine $C_{zy}$ : $d_{LT} = 2 \cdot a_{LT} \cdot \frac{\lambda\_{0}}{0.1 + \lambda\_{z}^{4}} + \frac{M_{y.Ed}}{C_{my} \cdot \chi_{LT} \cdot M_{pl.y.Rd}} \cdot \frac{M_{z.Ed}}{C_{mz} \cdot M_{pl.z.Rd}}$

$d_{LT} := 2 \cdot a_{LT} \cdot \frac{\lambda\_{0}}{0.1 + \lambda\_{z}^{4}} = 2.383$

Auxiliary term from Figure 3.6 : $C_{zy} := 1 + (w_y - 1) \cdot \left[\left(2 - 14 \cdot \frac{C_{my}^{2} \cdot \lambda_{max}}{w_y^{5}}\right) \cdot n_{pl} - d_{LT}\right] = 0.734$

where $C_{zy} \geq 0.6 \cdot \sqrt{\frac{w_y}{w_z}} \cdot \frac{W_{el.y}}{W_{pl.y}} = 0.5$

Interaction factor : $k_{yy} := C_{my} \cdot C_{mLT} \cdot \frac{\mu_y}{1 - \frac{N_{Ed}}{N_{cr.y}}} \cdot \frac{1}{C_{yy}} = 0.9$

Interaction factor : $k_{zy} := C_{my} \cdot C_{mLT} \cdot \frac{\mu_z}{1 - \frac{N_{Ed}}{N_{cr.y}}} \cdot \frac{1}{C_{zy}} \cdot 0.6 \cdot \sqrt{\frac{w_z}{w_y}} = 0.8$

Equation (3.30) - Strong axis : $\frac{N_{Ed}}{\frac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}} + k_{yy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} = 0.14$

$\text{if}\left(\frac{N_{Ed}}{\frac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}} + k_{yy} \cdot \frac{M_{Ed}}{\chi_{LT} \cdot \frac{M_{y.Rk}}{\gamma_{M1}}} \leq 1, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$

Equation (3.30) - Weak axis : $\dfrac{N_{Ed}}{\dfrac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}} + k_{zy} \cdot \dfrac{M_{Ed}}{\chi_{LT} \cdot \dfrac{M_{y.Rk}}{\gamma_{M1}}} = 0.136$

$$\text{if}\left(\dfrac{N_{Ed}}{\dfrac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}} + k_{zy} \cdot \dfrac{M_{Ed}}{\chi_{LT} \cdot \dfrac{M_{y.Rk}}{\gamma_{M1}}} \le 1, \text{"Ok"}, \text{"Redimension"}\right) = \text{"Ok"}$$

# Numerical Approach B

*In Appendix B the numerical approach by use of Abaqus software is shown. Abaqus is used to obtain $\alpha_{ult,k}$ and $\alpha_{cr,op}$ which are the minimum multiplier of the design loads. $\alpha_{ult,k}$ takes the in-plane and $\alpha_{cr,op}$ takes the out-of-plane behaviour into account.*

## Simply Supported Beam in Abaqus, HE400A

### *Input & Output from Abaqus*

The inputs and parameters to optain and determine minimum load amplifier:

Width of the beam : $b := 300\text{mm}$

Design load on the beam : $q_{design} := \dfrac{10\dfrac{kN}{m}}{b} = 3.3 \times 10^4 \cdot \dfrac{N}{m^2}$

Applied load in Abaqus : $q_{applied} := 200000\dfrac{N}{m^2}$

Timestep in Abaqus : $t := 0.8317$

Ultimate load of the beam : $q_{ult} := q_{applied} \cdot t = 1.7 \times 10^5 \cdot \dfrac{N}{m^2}$

Determing of $\alpha_{ult,k}$ : $\alpha_{ult.k} := \dfrac{q_{ult}}{q_{design}} = 4.99$

Determing of $\alpha_{cr,op}$ : $\alpha_{cr.op} := 8.8032$

### *Determining of Utilization Ratio*

$$\lambda_{\_op} := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr.op}}} = 0.753$$

Imperfection factor : $\alpha_{LT} := 0.34$ For lateral torsional, rolled I-section buckling curve b

$\lambda_{LT.0} := 0.4$

$\beta := 0.75$

$$\Phi_{LT} := 0.5\left[1 + \alpha_{LT} \cdot \left(\lambda_{\_op} - \lambda_{LT.0}\right) + \beta \cdot \lambda_{\_op}^{2}\right] = 0.773$$

$$\chi_{LT} := \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^{2} - \beta \cdot \lambda_{\_op}^{2}}} = 0.842$$

Verification of reduction factor: $\text{if}\left(\chi_{LT} \leq 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

and

$$\text{if}\left(\chi_{LT} \leq \frac{1}{\lambda_{\_op}^{2}}, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$$

24th May 2019 **Simply Supported Beam in Abaqus, HE400A** 2/2

Imperfection factor : $\alpha := 0.34$ buckling curve b!

$$\Phi := 0.5\left[1 + \alpha\cdot\left(\lambda_{\_op} - 0.2\right) + \lambda_{\_op}^{2}\right] = 0.877$$

Reduction factor : $$\chi := \frac{1}{\Phi + \sqrt{\Phi^{2} - \lambda_{\_op}^{2}}} = 0.753$$

Minimum value of reductions factor : $$\chi_{op} := \min\left(\chi_{LT}, \chi\right) = 0.753$$

Partial factor : $\gamma_{M1} := 1.1$

Verification of the element : $$\frac{\alpha_{ult.k}\cdot\chi_{op}}{\gamma_{M1}} = 3.416 \qquad \text{if}\left(\frac{\alpha_{ult.k}\cdot\chi_{op}}{\gamma_{M1}} \geq 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$$

Utilization ratio : $$UR := \frac{1}{\frac{\alpha_{ult.k}\cdot\chi_{op}}{\gamma_{M1}}} = 0.293 \quad \text{if}\left(\frac{1}{\frac{\alpha_{ult.k}\cdot\chi_{op}}{\gamma_{M1}}} \leq 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$$

## Frame in Abaqus, HE400A

### *Input & Output from Abaqus*

The inputs and parameters to optain and determine minimum load amplifier:

Design load on the beam : $q_{design} := 10000 \frac{N}{m}$

Applied load in Abaqus : $q_{applied} := 300000 \frac{N}{m}$

Timestep in Abaqus : $t := 0.3133$

Ultimate load of the beam : $q_{ult} := q_{applied} \cdot t = 9.4 \times 10^4 \cdot \frac{N}{m}$

Determing of $\alpha_{ult,k}$ : $\alpha_{ult.k} := \frac{q_{ult}}{q_{design}} = 9.399$

Determing of $\alpha_{cr,op}$ : $\alpha_{cr.op} := 8.7758$

### *Determining of Utilization Ratio*

$$\lambda_{\_op} := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr.op}}} = 1.035$$

Imperfection factor : $\alpha_{LT} := 0.34$ For lateral torsional, rolled I-section buckling curve b

$\lambda_{LT.0} := 0.4$

$\beta := 0.75$

$$\Phi_{LT} := 0.5\left[1 + \alpha_{LT} \cdot \left(\lambda_{\_op} - \lambda_{LT.0}\right) + \beta \cdot \lambda_{\_op}^{\ 2}\right] = 1.01$$

$$\chi_{LT} := \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^{\ 2} - \beta \cdot \lambda_{\_op}^{\ 2}}} = 0.678$$

Verification of reduction factor: $\text{if}\left(\chi_{LT} \le 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

and

$$\text{if}\left(\chi_{LT} \le \frac{1}{\lambda_{\_op}^{\ 2}}, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$$

Imperfection factor : $\alpha := 0.34$ buckling curve b!

24th May 2019 **Frame in Abaqus, HE400A** 2/2

$$\Phi := 0.5\left[1 + \alpha\cdot\left(\lambda_{-op} - 0.2\right) + \lambda_{-op}^{2}\right] = 1.177$$

Reduction factor : $\chi := \dfrac{1}{\Phi + \sqrt{\Phi^{2} - \lambda_{-op}^{2}}} = 0.575$

Minimum value of reductions factor : $\chi_{op} := \min\left(\chi_{LT}, \chi\right) = 0.575$

Partial factor : $\gamma_{M1} := 1.1$

Verification of the element : $\dfrac{\alpha_{ult.k}\cdot\chi_{op}}{\gamma_{M1}} = 4.913$ $\quad \text{if}\left(\dfrac{\alpha_{ult.k}\cdot\chi_{op}}{\gamma_{M1}} \geq 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$

Utilization ratio : $UR := \dfrac{1}{\dfrac{\alpha_{ult.k}\cdot\chi_{op}}{\gamma_{M1}}} = 0.204$ $\quad \text{if}\left(\dfrac{1}{\dfrac{\alpha_{ult.k}\cdot\chi_{op}}{\gamma_{M1}}} \leq 1, \text{"OK"}, \text{"Redimension"}\right) = \text{"OK"}$