IMPROVING LED EFFICIENCY THROUGH THERMOELECTRIC COOLERS

Master Thesis at Thermal Energy Engineering

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AALBORG UNIVERSITY
STUDENT REPORT

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SYNOPSIS:

Recent developments in power Light Emitting Diode (LED) industry made LEDs suitable for being efficiently used in high intensity lighting fixtures. The applied power can reach up to 60 W/mm2 in the high output LEDs, while 80% of this power dissipated as heat loss. The heat loss increases the LED temperature, so that its efficiency and lifetime reduce. An effective cooling can provide suitable temperature distribution on the LED. Each LED needs specific design and cooling properties of the heat sink system. An interesting cooling technology that can be integrated with the LEDs is thermoelectric cooler (TEC). Thermoelectric cooler is able to move the hot zone, due to the heat loss, in the LEDs to the heat sink. Therefore, the conversion efficiency of LEDs enhances at lower operation temperature.

Pages, total:	47
Appendices:	1
Supplements:	1 zip file with code and model

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Summary

Chapter 1 - Introduction

This chapter provides a general overview about LED systems, their main performance problems associated with temperature. Moreover, a State of the Art of the existing cooling technologies is presented.

Chapter 2 - Problem statement

The problem statement considered on this project is defined in this chapter. The assumptions made during the project are also listed here as well as the software used through this project.

Chapter 3 - LED Systems

In this chapter a short introduction to LED technology is provided to derive to thermal management of LED systems. Next, a description of the main components involved in standard LED systems is discussed. It is shown the implementation of the model in Icepak. Lastly, an analysis of the LED efficiency and lifetime of LED is made for different T_j through the use of a Simulink model which obtains the electrical and optical properties of the LED by look up tables extracted from manufacturers information.

Chapter 4 - Thermoelectrics

This chapter presents the main principles and equations of Thermoelectric (TEC) devices. Moreover the conduction model of the TEC is presented and implemented in Icepak. By last, an evaluation of the performance of the TEC with different boundary conditions is made to find the more suitable situations for TEC technology.

Chapter 5 - Results

The results obtained by the different models are presented in this chapter. Firstly, an individual study of each model is presented. Next a comparison between the models is done at particular situations. By last, the COP of both systems is studied and compared.

Chapter 6 - Conclusion

This chapter summarizes the results and facts found during the the development of the project for the different models.

Chapter 7 - Further Work

This chapter presents the ideas and knowledge gained through this project that due to the limitations of the project, it has not been possible to elaborate further.

Preface

This Master Thesis is written by a Semester MSc Student on Energy Engineering with Thermal Energy and Process Engineering specialization. The focus of this semester is modelling and optimization of energy systems in thermal energy engineering . The purpose of this project is an study on how to improve the efficiency of a LED system by the use of Thermoelectrics coolers.

Prerequisites

It is expected from the reader to have basic knowledge about mathematical modelling and thermodynamics.

Reading Guide

The equations, figures and tables presented in this project are numbered according to the chapter they belong to. As an example, Figure 2.3 indicates that it is the 3^{rd} figure of chapter 2. All objects include a short description of its meaning.

The variables presented along the project are defined together with its unit and symbol and are placed on a list called 'Nomenclature' where abbreviations and subscripts used are also included. The definition of the abbreviations is described when they first appear for the first time.

On this projects, citations are mentioned by brackets and drive to the reader to the corresponding bibliography at the end of this project. The citations give the information about the sources of the information as author, website, title or dates, year of publication, etc. The sources can be books, articles, interviews or websites.

Appendix is named A and is placed at the end of this project, to support it.

The main softwares used for this report are Icepak ANSYS®, MATLAB® and Simulink®.

Nomenclature

Special Symbols and Denotations

Symbol	Description	Unit
P	Electrical Power	[W]
Q	Thermal Power	[W]
T	Temperature	[$°C$]
η_e	LED Efficiency	[-]
S	Effective Seebeck coefficient	[-]
R	Serial Thermal resistance	[$°C/W$]
K	Parallel Thermal Conductance	[W/K]
I	Current	[A]
V	Voltage	[V]
ϕ	Radiometric Power	[W]
G	Geometric ratio	[m]
L_e	Length thermoelement	[m]
ϕ_v	Luminous flux	[lm]
η_v	Efficacy	[lm/W]
j	Current density	[A/mm^2]
α	Seebeck Coefficient or thermopower	[V/K]
k	Thermal conductivity	[W/mK]
ρ	Electrical resistivity	[Ωm]
S_h	Volumetric heat source	[W/m^3]
N	Number of thermocouples	[-]
Z	Figure of Merit	[K]
ZT	A-dimensional Figure of Merit	[-]
f	Filling factor	[-]
L	Total length TEC device	[m]
A	Total area TEC	[m^2]
C	Electrical capacitance	[J/K]

Subscripts

Subscript	Description
j	junction
jc	junction case
jb	junction-board
jhs	junction-heat sink
jref	junction-reference
amb	ambient
opt	optical power
h	hot side
c	cold side
th	thermal
tec	thermoelectric
elec	electrical Power
htc	heat transfer coefficient
rtyp	Radiometric power datasheet
vtyp	Luminous Flux datasheet
ref	Reference
f	forward
fmax	forward max
case	referent to the LED case
heat	referent to heating power
v,ref	luminous flux reference
n	n type thermoelement
p	p type thermoelement
e	Referent to thermoelement
cop	coefficient of performance
o	optimum

Acronyms

Acronym	Description
AAU	Aalborg University
LED	Light Emiting Diode
TIM	Thermal Material Interface
WPE	Wall Plug Efficiency
HS	Heat Sink
COP	Coefficient of performance
TEC	Thermo Electric Cooler
TEG	Thermo Electric Generator
MCPCB	Metal Circuit Printed circuit Boards
TIM	Thermal Material Interface
MLT	Median Lifetime

Table of contents

1 Introduction

A Light Emitting Diode(LED) is a light source formed by a p–n junction diode which emits light when it is activated. With proper voltage applied to its terminals, electrons can recombine with electron holes within the device. This recombination releases energy as photons which is known by electroluminescence.

These devices are highly reliable and efficient when compared with other light sources, however, there is an inherent problem associated with these systems: the raise of temperature at the junction of the LED. Due to this, thermal management of LEDs applications has become an important issue as LED sizes have grown up to 8 times what was available ten years ago, while the packaging is required to be tighter for some applications.

The problem of temperature raise becomes more accentuated when speaking about red LEDs, which voltage decreases considerably (with respect to orange or blue LEDs) with the rise of temperature at the junction (see figure 1.2). A model of red LED is presented on the left, in the figure 1.1.

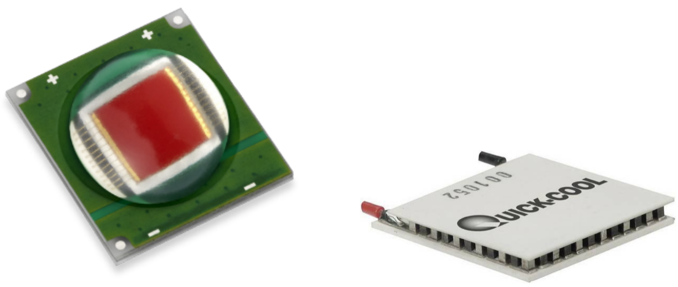


Figure 1.1: *LED and TEC technologies*

High junction temperatures decrease the optical output, vary the color of light and reduce lifetime considerably (lifetime measured in terms of relative luminous flux, e.g 70% of the initial light output) [Wang, 2014].

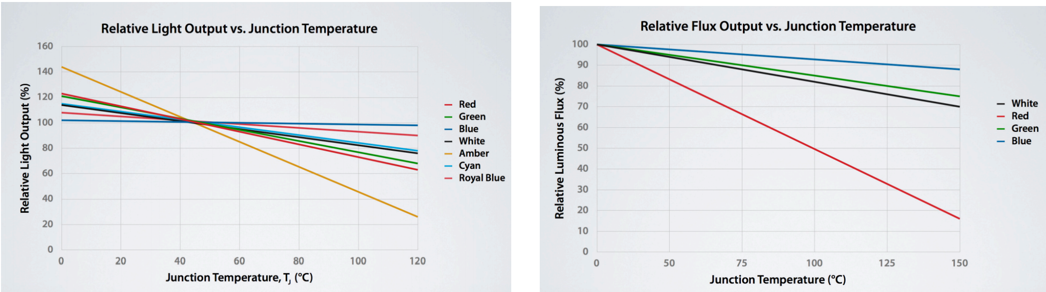


Figure 1.2: *Performance dependancy of temperature of different LED colours [Gala, 2017]*

1.1 State of the art

To address the thermal inconveniences explained above, different alternatives are being taken. The most common over the past years has been the optimization in design of Heat Sinks (HS) for which there are different studies about the geometric shape of the fins, the disposition of them (in line, staggered) or the surface characteristics to favor thermal radiation.
The authors on the paper [Shirazy, 2014] postulated that through numerical and analytical models the thermal performance of the LED module can be predicted when all the component dimensions and materials properties are available.
Furthermore, It was observed that the bigger thermal resistance in the systems comes from the Heat Sink (more than 50% of the total thermal resistance) due to the the low air natural convection coefficient.
On the paper [Royston Marlon Mendonca, Sai Sharan Yalamarty, 2011] different fin heat sinks geometries are compared concluding that circular geometries were more efficient.
On the paper [Bing, 2012] ANSYS software is used to carry out the thermal analysis of 60W LED illumination lamps by optimizing the heat sink structure. The results showed that fin lengths follow a linear relationship with the maximum junction temperature, T_j, with longer fins implying lower junction temperature. Furthermore, cross fin increases convective heat transfer requiring 20% less in material than when comparing with rectangular fin.

Another approach to reduce or control T_j is by the use of Thermoelectric Coolers (TEC). TECs (see figure 1.1) are devices that, using the Peltier effect, are able to produce a heat flux between its surfaces when a current flows through the junctions of the different materials forming the device. Despite of their low conversion efficiency, their application for cooling electronics is growing as they have the advantages of no need of maintenance, do not have moving parts neither leaks as no liquid circulation is needed. There are different techniques to tackle the analysis of TEC systems. In different papers[Chávez et al., 2000], [Hasan and Toh, 2007]the electrical approach is used to represent the power flux in the thermal network due to the similarities of thermal systems with electrical systems in terms of resistances, sources and temperatures. The equivalence between the different components is presented at the next table:

Table 1.1: *Similarities between electrical and thermal systems*

Thermal variable	Electrical variable
Heat Flow, Q[W]	Current flow, I [A]
Temperature, T[K]	Voltage, V [V]
Thermal resistance, R_{th} [W/K]	Electrical resistance, R [Ω]
Thermal mass, C_{th} [J/K]	Electrical capacitance, C [F]

Some authors are using these similarities to model thermal systems in Spice as [Chávez et al., 2000], Lineykin and Ben-Yaakov [2005]. The schematic look of this systems can be seen at the next figure:

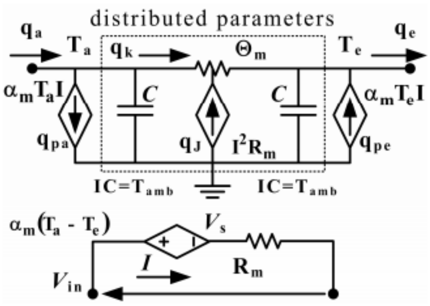


Figure 1.3: *Distributed TEC system in Spice[Lineykin and Ben-Yaakov, 2005]*

On the thesis [Version and Technology, 2012], the simplified thermal model of Luminus Rebel diodes is elaborated through thermal elements ("cauer" and "foster" at Simulink) with thermal resistances and capacitances associated to each element. This model is illustrated at the figure 1.4.

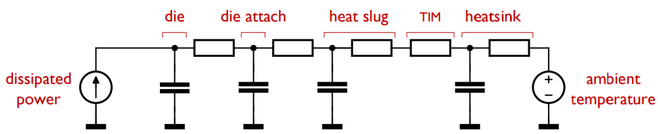


Figure 1.4: *Cauer LED Thermal Model [Version and Technology, 2012]*

Drawbacks and misinformation

When designing LED-based systems, the most important parameters to consider are ϕ_v, the luminous flux [lm], and η_v, efficacy [lm/W] of the LED. However, these parameters are not linearly dependant on T_j and the driving current,I_f, so for the analysis of the efficiency of the LED system it is necessary extra-tools as fitting tools or to look up tables to obtain more realistic results.

The data available in datasheets is usually given for a $T_j = 25°C$ which is not real as these temperatures are economically too costly to reach by standard LED systems. Over this optimum temperature, the performance of the LED can decrease around 50% at the maximum rated power. Furthermore, mostly due to this misleading, it is not possible to compare directly datasheets from different manufacturers.

Temperature dependence with luminous flux is given by graphs instead of exact equations which can make the designer more prone to commit errors.

Problem Statement 2

2.1 Methodology

The objective of this project is to analyze whether the efficiency of a high power red LED can be improved by reducing the T_j with the use of TEC.
To compare both systems, an study of a SST 90R LED model[1] have been chosen and its datasheet has been studied for the implementation of the LED Thermal Model.
Furthermore, a model is developed to evaluate the variation of optical power or luminous flux with T_j.
At last, the TEC technology has been investigated and modelled for its integration within LED systems.
The results obtained with both models have been compared for different situations of ambient temperature (T_{amb}), HS surfaces and thermocouples lenghts.

To develop this analysis, various assumptions are taken:

- Steady state conditions
- Thermal modelling considering only conduction
- Simplification of the LED model with a two resistor model [Thermal and Guideline]
- Metal Core Printed Board (MC PCB) has been considered only in the STD case
- The Thermal Material Interface (TIM) have the same material and dimension properties

2.2 Software

In this project, thermal modelling has been made with the finite volume Icepak ANSYS® software application while the data representation has been analyzed and presented in MATLAB®. Simulink MATLAB® has been used for modelling the efficiency of the LED system while Latex® is used to present this project.

[1] Shipping2015

LED systems 3

3.1 Brief introduction to LEDs

The first crude solid-state lamp was operated in 19007 by Henry Reund, an electrical engineer, who found a crystal of silicon carbide connected to a battery. He reported that by applying a potential difference between the two terminals, a yellow light was emitted. A simple and effective way to produce light is by building a pn junction diode. The wavelenght (λ) and frequency of the photon to be emitted (v) is determined by:

$$v = W_g/h \tag{3.1}$$

$$\lambda = c/v \tag{3.2}$$

where $W_g = W_c - W_v$, is the bandgap energy, h is here the Planck constant and c represents the speed of light.

The light appears when an excited electron reach equilibrium through a direct conduction band, by the recombination of the electron with a hole in the valence band.

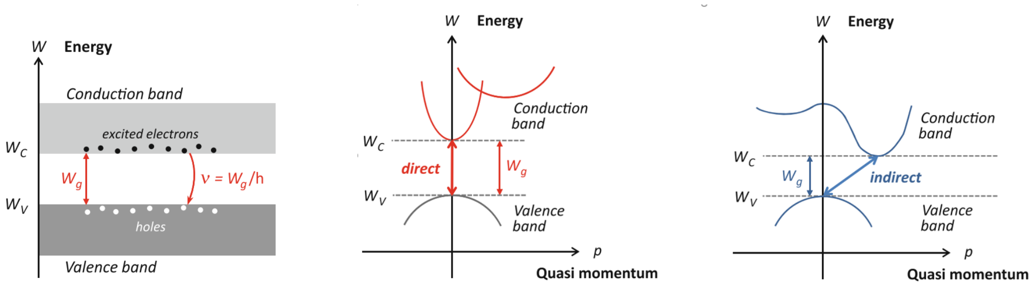


Figure 3.1: *Band structure [Editors and Thermal]*

This energy band is the most important parameter to define the light output properties. There are two types of semiconductor materials, direct bandgap and indirect bandgap. Direct bandgap is when the minimum and maximum energy level of conduction are at the same quasi momentum or wavelength number (see figure 3.1). The electrical current has to cross a potential barrier, the pn junction, but for doing that the energy of electrons has to be raised from the valence band to the conduction band [Editors and Thermal].
When the current has crossed, the electrones can recombine with holes resulting in light emision, what is called radiative recombination. In opposite, with the non radiative recombination, the energy is transferred to vibration energy of the semiconductor lattice resulting in heat, the main source of degradation and inefficiency in LEDs.

3.2 Thermal management of LED systems

The heat produced by the non-radiative recombination needs to be evacuated through the base of the LED, which makes thermal management of the heat power, P_h, essential to ensure an expected lifetime and adequate light output.
LED packages are usually mounted on a MC PCB (figure 3.2) generally formed by a first metal conductive layer (usually Cu) for the electrical connections of the LED, a second layer, a dielectric, which allows the thermal conduction but disables the electrical conduction; and a third thicker layer (generally Al) which behaves as a HS. The design of this component highly affects the thermal behaviour of the entire system. The connection of the device to the board is made through a specific solder that functions as thermal interface.

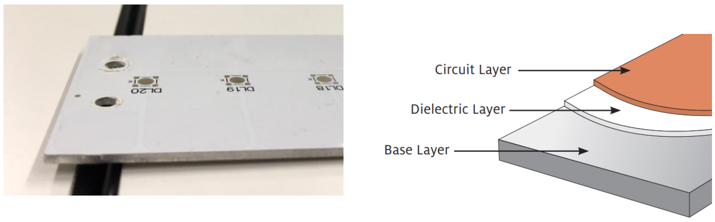


Figure 3.2: *MC PCB [Guide]*

To favor heat spreading, HS with different configurations are also applied, presenting large convection areas. This could be a part of the led itself, depending on the application. In some cases radiation is also an important factor for heat dissipation and is determined by the surface characteristics as texture and emissivity. Powder coating and painted surfaces favour radiation better than bare aluminium or shinny chrome surfaces.
TIM are the materials used to fix the PCB to the HS and the quality of this material will also play an important role in the thermal pad.
The resistance of this thermal pad (junction-case-board-heat sink-ambient) is given in the datasheet of the LED device as R_{jc} or R_{ja}, depending on if it refers to the case temperature (T_{case}), or to ambient temperature (T_{amb}).

3.3 LED model description

The LED chosen for the analysis is a red LED SST-90R. The following parameters for thermal and optical analysis have been considered for the modelling and are provided by the manufacturers datasheets:

Table 3.1: *Datasheet electrical properties for SST 90R LED*

Drive condition		**3.15A**	**9A**	
Parameter	**Symbol**	**Values**		**Units**
Current density	j	0.35	1	A/mm2
Forward voltage	V_{fmin}	1.8	-	V
	V_F	2.1	2.5	V
	V_{fmax}	2.8	-	V
Radiometric power	ϕ_{rtyp}	2.4	6.4	W
Luminous Flux	ϕ_{vtyp}	540	1500	lm

Table 3.2: *Datasheet thermal properties for SST 90R LED*

Common Characteristics			
Emiting Area		9	mm^2
Thermal Resistances[1]	R_{jc}	0.5	C/W
	R_{jb}	1.2	C/W
	R_{jhs}	1.4	C/W
Absolute max Junction Temperature	T_{jmax}	110	C
Thermal Coefficient of Junction Voltage		-1.3	mV/C
Thermal Coefficient of Photometric Flux		-0.96	%/V
Thermal Coeficient of Radiometric Flux		-0.52	%/V
Forward Current		0.2-18	A

The reference points for the given values are illustrated at the figure below:

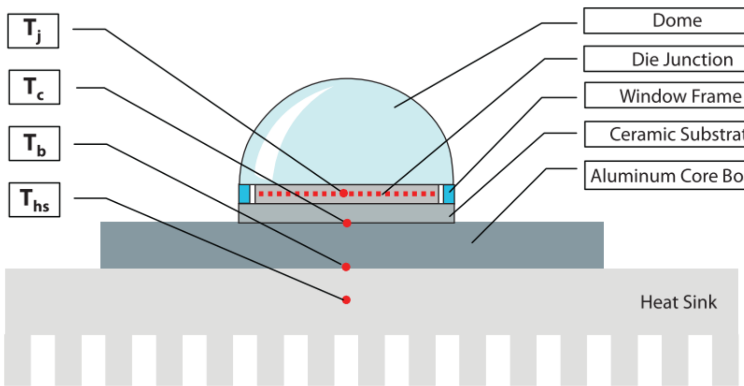


Figure 3.3: *Typical thermal resistances [Shipping et al., 2015]*

3.3.1 Thermal conduction model in Icepak

Due to the information given on datasheets, the LED system has been modelled using a two resistor compact model [Thermal and Guideline].

[1]The table provide above are given for a $T_j = 25C$. The given thermal resistances were measured using SAC305 solder, a Berquist Al-Clad MCPCB and eGraff 1205 TIM.

[1]T_{hs}definition=3mm from core board

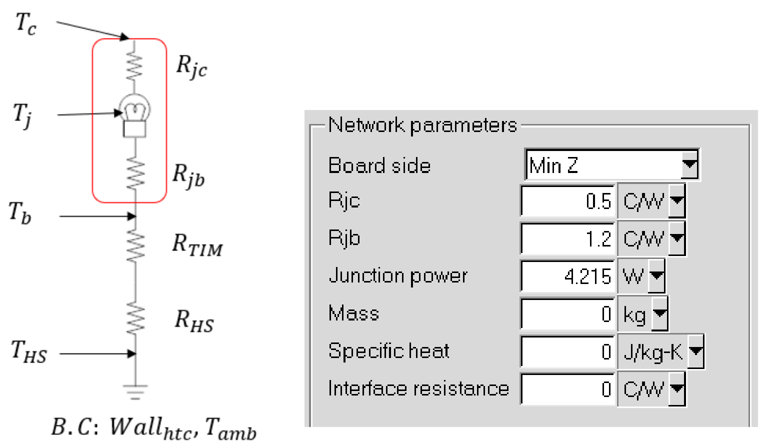
This model is the most intuitive and simplest method to analyze T_j, however when high accuracy is needed, it is suggested to compare this model with other methods. In the case of this project, the accuracy provided by the method is considered enough. For a more detailed study, the properties of the layers of the LED must be defined.

The two resistor model defines the LED package thermal behaviour and its interconnections with the adjacent system. It considers the heat flux leaving the node through the case node and through the board.

Under this assumption, the LED system can be simplified by using the given resistances R_{jc} and R_{jb} or R_{jb} given at the table above 3.2. This model is based on the following equation [Thermal and Guideline] and illustrated at the figure **??**:

$$\theta_{J-ref} = (T_j - T_{ref})/P_h \tag{3.3}$$

Being θ_{J-ref} the resistance of the LED between the junction and a reference point in K/W, T_{ref} is the temperature of the point of reference and P_h, the heating power generated at the junction. The network model is illustrated at the figure 3.4a and the corresponding values for the modelling of the network block at Icepak are presented at the table 3.4b:



(a) *Network block represented by the red box*

(b) *R_{jc},R_{jb} are the values provided by datasheet and P_h is the heat power P_h (see equation 3.3.1)*

Figure 3.4: *Compact two resistor model LED: T_j is the junction temperature, T_c is the case temperature,T_b represents the temperature at the bottom of the MC PCB and T_{HS} defines the temperature at the base of the HS. Boundary conditions (B.C): $Wall_{htc}$ and T_{amb}*

The complete network for the model is shown below. As shown in figure 3.4a, the defined thermal resistances correspond to junction-board and junction-case. Doing so, there is no need to model the PCB, its thermal resistance is already included in the given value R_{jb} as $R_{jb} = R_{jc} + R_{cb}$. The physical representation of this model in Icepak can be seen at the figure below where the values given for the network block can be seen.

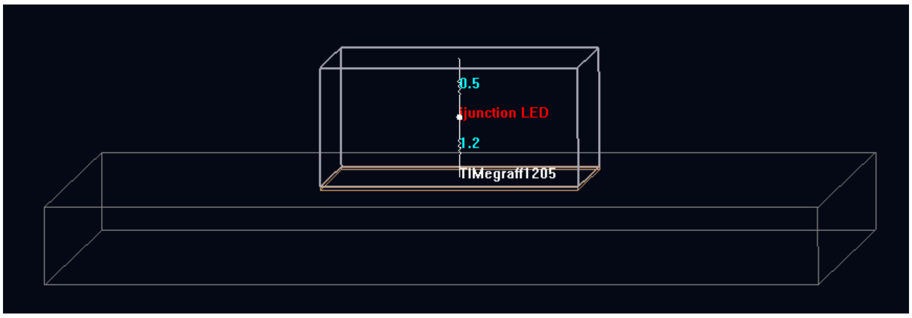


Figure 3.5: *Complete STD LED model at Icepak. Network block (defining the* $R_{jc} = 0.5$*,* R_{jb}*=1.2,* P_h*=4.215), conducting plate as TIM, HS of* $40 \cdot 40mm^2$ *and the wall (*$Wall_{htc}$*), not visible here, to define the heat transfer coefficient on the bottom surface of the HS respect to* T_{amb}

The wall, under the HS, is defined with a constant heat transfer coefficient for a defined T_{amb}. Both systems, Standard and TEC, are modelled in Icepak using this network approach, assuming only heat conduction. It is also assumed that the heat generated at the junction will flow down through the case and not to the sides or top of the case.

The next table 3.3 gather up the components used in Icepak, one network block for the LED-case-PCB (see figure 3.4) and two conducting plates for the TIM and the HS:

Table 3.3: *Established dimensions and materials for the Standard Model at Icepak*

Standard LED Model dimensions		$[mm^3]$
Block two resistor		$10 \cdot 11 \cdot 5.4$
TIM		$10 \cdot 11 \cdot 0.127$
Heat Sink Block[2]		$40 \cdot 40 \cdot 3$
Standard LED Model materials		k $[W/mK]$
TIM	egraff 1205	7.5
HS	Aluminium	205

With the given standard values by the datasheet (T_j=25° C) it is possible to obtain the electrical power, P_{elec} and thermal power, P_h, with the following equations [Clemens Lasance, 2014]:

$$P_{elec} = I_f \cdot V_f \tag{3.4}$$

where I_f and V_f are the forward current and voltage respectively

$$P_h = P_{elec} - P_{opt} \tag{3.5}$$

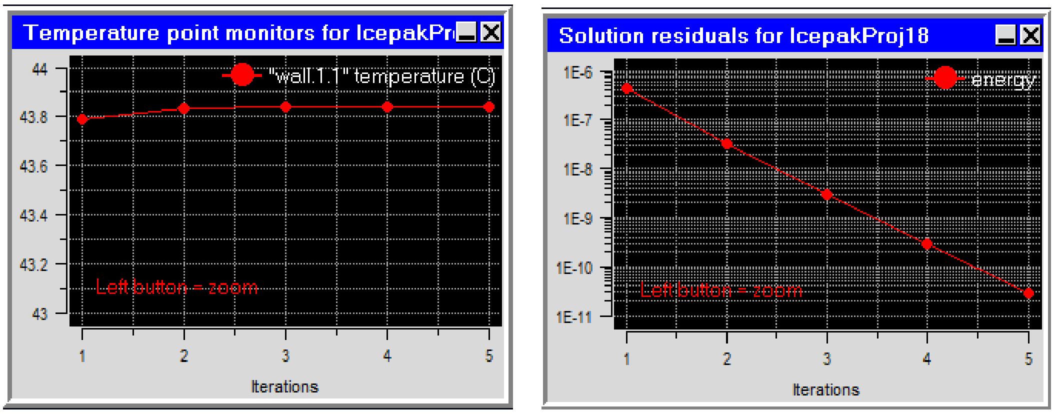
The radiant efficiency, Wall Plug Efficiency (WPE) or power efficiency is defined as:

$$\eta_{LED} = P_{opt}/P_{elec} \tag{3.6}$$

[2]Some manufactures recommend a heat sink surface of $228.6mm^2/W$ for LEDs over 350mA[Supply]

Solver set up

The system has been analyzed as steady state, $T_{amb} = 40°C$. The variable set at the solver in Icepak is the temperature, no radiation either convection has been considered. The discretization schema applied for temperature calculations is set to first order and the residual monitors of Flow, Energy and Joule Heating are fixed to $1E-07$. The number of iterations has been set to 20 as the system convergence is relatively fast (when the solution presents a stable value and the convergence criterion has been reached for flow and Joule heating).



(a) *Temperature point monitor scope* **(b)** *Solution residuals scope*

Figure 3.6: *Monitoring solution process in Icepak*

Independency study and mesh quality

Meshing is an important step in the analysis as it forms the basics for the solution procedure. A good mesh should have proper resolution, smoothness, low skewness and the appropite number of elements. Icepak automates the mesh generation but enables the user to redefine the meshing procedure when it is required.

The meshing procedure at Icepak follows different rules, depending on the object being modelled.

The quality of the mesh is essential to ensure a good solution. It requires to be enough fine nea the objects where gradients of physical properties vary abruptly. The expansion ratio is suggested to be kept between 2 and 5 although it can be lower for complex geometries. For simple geometries, Icepak recommends the use of Mesher-HD (Hexadominant). With this method, firstly, a coarse mesh is generated with a minimum number of elements required to define the geometry and to satisfy the default meshing rules for each object in the model.

The solution is computed for this coarse mesh. Then a new thinner mesh is generated and the new solution is computed. By displaying and checking the mesh, it can be seen the properties of the mesh which needs to satisfy the following rule: the minimum number of elements between two solid faces must be at least 2. When the solution does not vary with the number of elements, the solution can be said to be mesh independent. Four different resolution for the mesh have been tested, providing the following results (figure 3.4):

Table 3.4: *Mesh independancy study for the LED model*

Mesh resolution	T_j
59 K	45.52
115 K	45.46
161 K	45.38
225 K	45.5

From the table 3.4, the T_j does not vary significantly from 59 K, however, 115 K is the chosen resolution for investigation to be on safe side and to ensure higher accuracy for the TEC model.

3.4 Efficiency analysis

In order to study the efficiency of the system, it is necessary to establish a mathematical relation between optical and thermal properties. The conversion efficiencies are not given in datasheets so some designers prefer to take a conservative approach considering that all the power drawn by the LED is dissipated as heat.

In this project a constant conversion efficiency, (η_e) has been adopted, obtained as follows:

$$\eta_e = \frac{P_{opt}}{P_{elec}} \tag{3.7}$$

For the mathematical relations, three graphs from data-sheet are used:

1. Relative Output Flux vs. Forward Current
2. Forward Current vs. Forward Voltage
3. Relative Luminous Flux vs.Heat Sink Temperature

From the graph 1, and with the given value of thermal coefficient of junction voltage, the first look up table is obtained which outputs the P_{elec} depending on this T_j. The second look up table is obtained through the graph 2, and the thermal coefficient of photometric flux.
From the graph 3, a linear relation between luminous flux and T_j is obtained, which for a specified T_j give the corresponding light output. Since the light output depends on two variables, I_f and T_j, and there is no given relationship in data-sheet, it is reasonable to assume that the light output is the product of the ratios $\frac{\phi(T_{HS})}{\phi_{v,ref}}$ and $\frac{\phi(I_f)}{\phi_{v,ref}}$ [Bider, 2009].

$$\phi_v = \phi_{v,ref} \cdot \frac{\phi(T)}{\phi_{v,ref}} \cdot \frac{\phi(I_f)}{\phi_{v,ref}} \tag{3.8}$$

In order to find these ratios, a simulink model has been developed. This process is shown on the following schema 3.7:

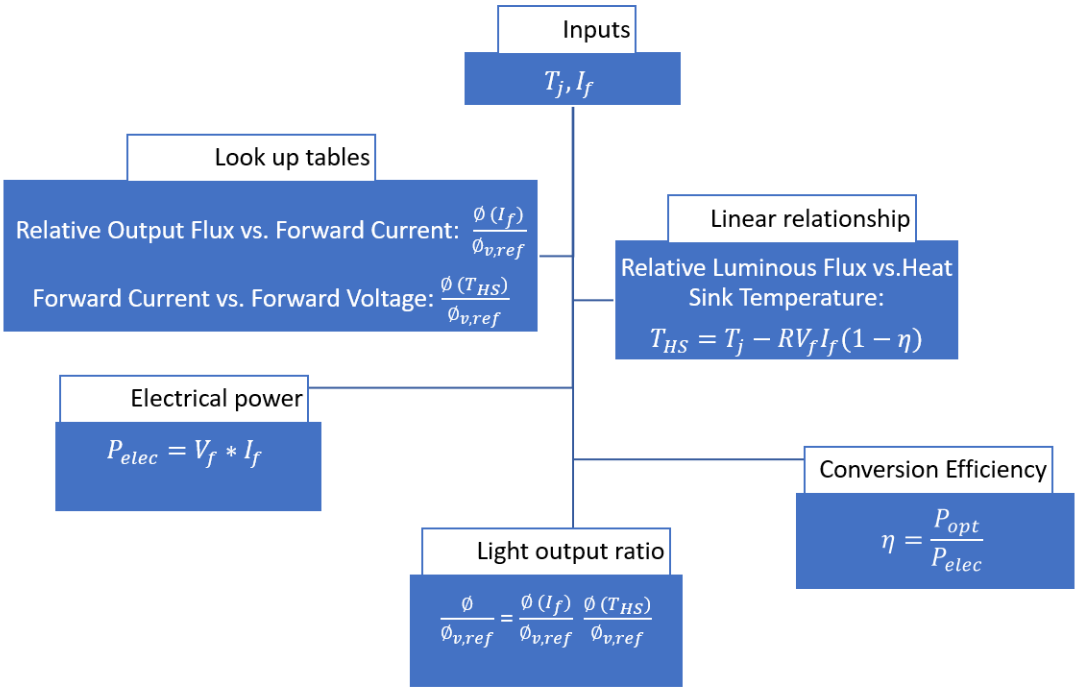


Figure 3.7: *Schema describing the process to obtain Light output*

As shown in the Simulink model (see Appendix A) the inputs for the model are I_f and T_j.

1. With I_f and T_j the P_{elec} is obtained through the first look up table 1.
2. Entering at the second look up tables 2 With I_f and T_j, the percentage of real lumens can be obtained, $\frac{\phi(I_f)}{\phi_{v,ref}}$.
3. The third graph is described with a linear relationship as follows:
   Relative luminous flux $= -0.9091 * T_{HS} + 137.33$
4. Having the already mentioned relation:

$$T_{HS} = T_j - R_{jhs} V_f I_f (1 - \eta) \tag{3.9}$$

   together with R_{jhs} and η_e, which is considered constant with a value of 0.34, the T_{HS} can be obtained.

5. Lastly, the Relative luminous flux, $\frac{\phi(T_{HS})}{\phi_{v,ref}}$, can be calculated using T_{HS}.

The results obtained with the model are shown on the graphs below 3.8:

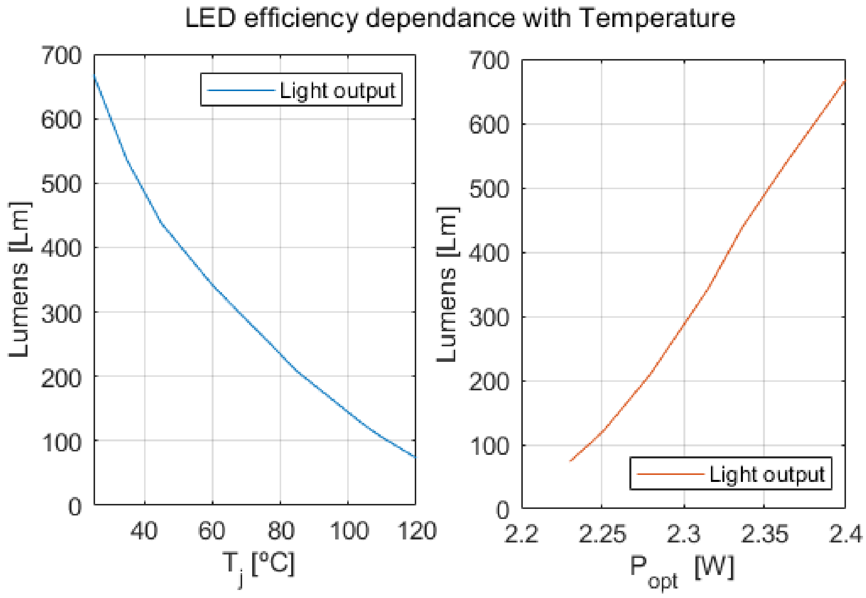


Figure 3.8: *Luminous flux vs. T_j (Left). Luminous flux vs. P_{elec} (right). Both graphs represent the relation between the Lumens, T_j and P_{opt} for $I_f = 3.15A$ and $\eta_e = 0.34$.*

These results have been obtained for $I_f = 3.15A$ and $\eta_e = 0.34$. When the junction temperature increases, the forward voltage decreases and consequently the P_{opt} available will also decrease.

3.5 Lifetime analysis

Lastly, the Median Lifetime is analyzed. Theoretically, by keeping a T_j of $25°C$, the Lifetime could be increased til 800,000 h.
Lifetime in LEDs is measured with respect the output flux which decreases with time and the raise of T_j. Furthermore, changes in the solder or TIM properties can also affect the operation of the LED as the thermal pad for the heat disipation is modified.
To use the Lifetime data for the study of efficiency, the increment in the Median Lifetime, MLT, provoked by the reduction of T_j to $25°C$, could be translated to an increment of P_{elec} gained, what being traduced to P_{opt} can increase the efficiency of the later proposed TEC system.

The contribution to the optical power gained by increasing the MLT can be written as follows:

$$P_{opt,extra} = \eta_e(P_{elec}(25°C) - P_{elec}(T_j)) \tag{3.10}$$

With this, the COP of both systems is calculated:

$$COP_{std} = \frac{P_{opt}}{P_e} = \frac{\eta_e \cdot P_{elec}(T_j)}{P_{elec}(25°C)} \tag{3.11}$$

$$COP_{tec} = \frac{P_{opt}}{P_{total}} = \frac{P_{opt}(25°C) + \eta_e(P_{elec}(25°C) - P_{elec}(T_j))}{P_{total}} \tag{3.12}$$

$$P_{total} = P_{tec}(25°C) + P_{elec}(25°C) \tag{3.13}$$

where $P_{opt}(25°C) = 2.4W$, $P_{elec}(25°C)$ is the P_{elec} corresponding to $T_j = 25°C$, the $P_{tec}(25°C)$ represents the power consumed by the TEC to keep the T_j at $25°C$, $P_{elec}(T_j)$ is the value of P_{elec} depending of the T_j and η_e, being η_e considered constant.

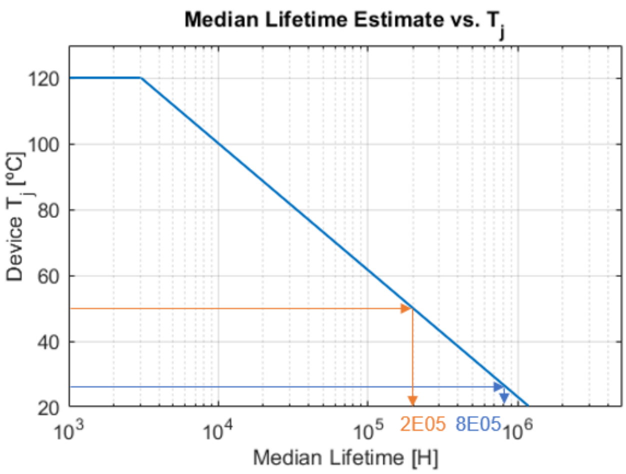


Figure 3.9: *Median Lifetime Estimate. Lifetime defined as time to 70% of initial intensity [Shipping et al., 2015]*

Thermoelectrics 4

4.1 Thermoelectrics effects

Thermoelectricity was discovered in the nineteen century being directly associated with thermal and electrical phenomena. When considering thermoelectrics, there are three effects interacting:

- **Seebeck effect.** Effect known as the conversion of a temperature difference into electric current. When two wires A and B are joined together and a temperature difference is imposed between two junctions, a voltage will appear on the voltmeter placed in wire B, being this potential proportional to the temperature difference.

  $$V = \alpha \Delta T$$

  where α is the seebeck coeficient or thermopower, measured on $[V/K]$ and ΔT is the temperature difference between the hot and cold junction, [K].
- **Peltier Effect.** When current flows through a junction between two different wires, heat must be continuously added or subtracted at the junction to keep the temperature constant at the junction.
- **Thomson effect.** When a current flows through a wire with a temperature gradient, heat is absorbed or released depending on the material and the direction of the current. This is the only effect measurable directly on individual materials.

Thomson Relationships

The relationship between these thermoelectrics effects was explained by Thomson by applying the first and second law of thermodynamics assuming reversible and irreversible processes are separable [Figure 4.1].

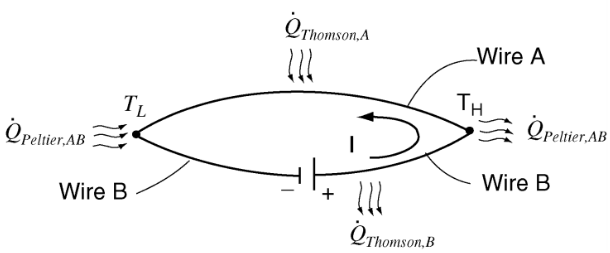


Figure 4.1: *Thomson relatioship [Peltier et al., 1960]*

The Thomson effect is very small when compared with Peltier effect but it is necessary to deduce the Thomson relationships which lead to the Peltier equation for cooling:

$$\dot{Q}_{Peltier} = \alpha T I$$

being T the temperature at the juntion between the two wires, [K], I the current circulating by the wires, [A], and $\dot{Q}$ the amount of heat per unit time, [W].

4.2 TEC model description.

From the effects explained above, two similar technologies have emerged, the TEC and the TEG (Thermo Electric Generator) which perform in opposite way.
Both technologies use the thermoelectric effect. This effect is reversible so both of the systems can work as TEC or TEG.

However both technologies are optimized to work in different temperature ranges so it is recommended to use them specifically for the application they will be integrated in.
At the figure 4.2, a TEC is illustrated. Its basic unit is the thermoelectric element or thermoelement, which consists of n-type and p-type of semiconductors joined by conductive plates or metal joints. This thermocouple is connected electrically in series to other thermocouples in the TEC module and all the thermoelements are thermally connected in parallel by two ceramic plates or substrates.

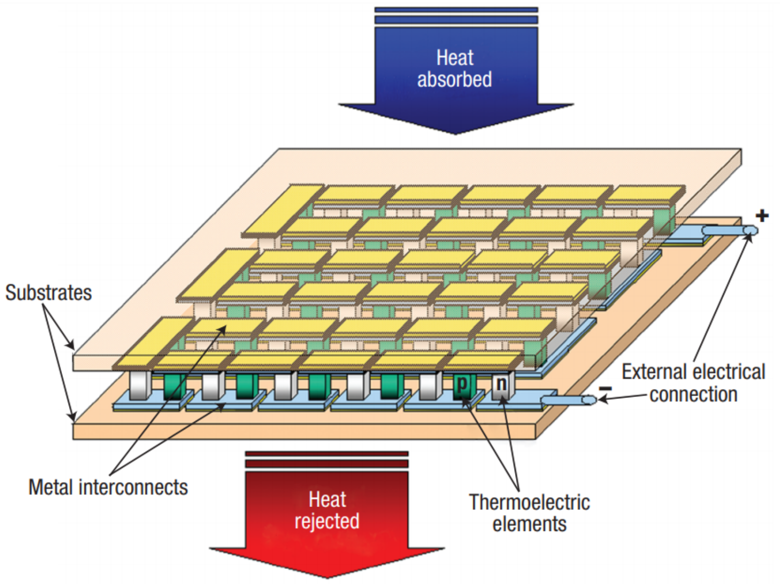


Figure 4.2: *TEC module Snyder et al. [2003].*

The heat produced by the LED chip will be absorbed by the top ceramic plate and rejected by the bottom ceramic plate to the ambient temperature through the HS and wall, in the case of this project.

On the figure below, it is illustrated the temperature profile inside a TEC. It can be seen the temperature extremes corresponds to the electrical connections in both sides, cold and hot.

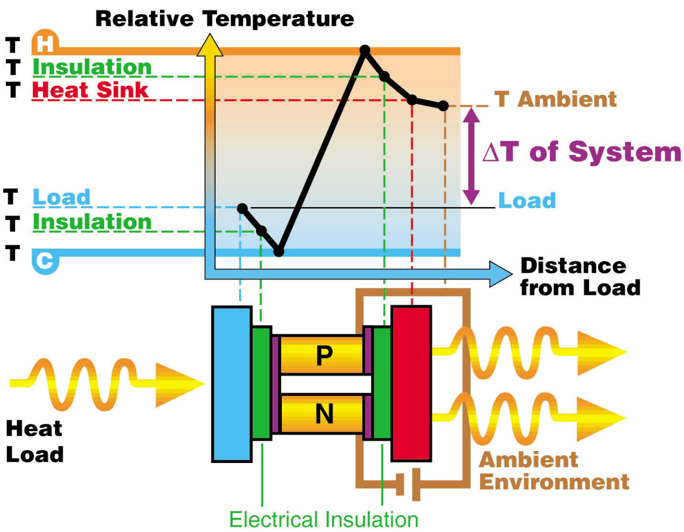


Figure 4.3: *TEC temperature distribution [Briefs]*

Icepak models the TEC by a compact model composed by two ceramic blocks representing the ceramic plates, two sources as the electrical connections and one block to represent the equivalent properties for all the thermoelements in the TEC (see figure 4.4).

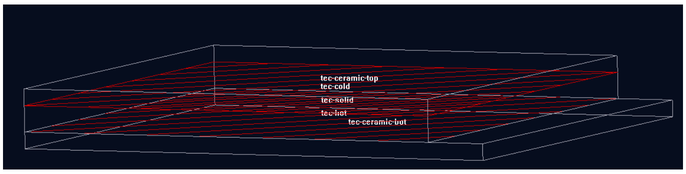


Figure 4.4: *TEC model in Icepak represented by two ceramic plates at top and bottom, two sources to simulate the conduction plates and a solid block representing the thermoelectric material of the TEC*

The governing equations to analyze the TEC behaviour are the energy conservation equation and the transport equation. For conduction in solids, Icepak solves a simple conduction equation which includes the heat flux due to conduction and the volumetric heat sources inside the solid [Canonsburg, 2018]. The simplified equation for energy conservation is expressed below 4.2.

$$\frac{\nabla}{\nabla t}(\rho h) = \nabla(k\nabla T) + S_h \tag{4.1}$$

where ρ is density, k is conductivity, T is temperature, and S_h is the volumetric heat source. Solving for a steady state condition, these equations can be simplified to the following forms:

$$Q_c = IST_c - \frac{1}{2}I^2R - K(T_h - T_c) \tag{4.2}$$

$$Q_h = IST_c + \frac{1}{2}I^2R - K(T_h - T_c) \tag{4.3}$$

$$P_{tec} = Q_h - Q_c = I^2 R + IS(T_h - T_c) \tag{4.4}$$

$$V_{tec} = S(T_h - T_c) + RI \tag{4.5}$$

$$Z_{tec} = \frac{\alpha^2}{RK} \tag{4.6}$$

These equations are present in several papers [Chávez et al., 2000],[Mitrani et al., 2007], [Seebeck et al., 1854]. The heat absorbed at the cold side of the TEC, Q_c, the heat released at the hot surface, Q_h, the power applied to the TEC, P_{tec} and the voltage, V_{tec} can be easily calculated having the material properties.
Z_{tec} is the well known figure of merit which determines the performance of the TEC module. On the paper Chen and Snyder [2013], a compact model of a TEC is developed where the material properties of the thermoelectric are obtained as follows:

$$\alpha = \frac{Q_{max}(T_h - \Delta T_{max})}{NT_h^2 I_{max}} \tag{4.7}$$

$$\rho = \frac{Af(T_h - \Delta T_{max})^2}{2T_h^2 Le} \cdot \frac{Q_{max}}{N^2 I_{max}^2} \tag{4.8}$$

$$k = \frac{Le(T_h - \Delta T_{max})^2}{AfT_h^2} \frac{Q_{max}}{\Delta T_{max}} \tag{4.9}$$

This shape allows the analyst or designer to obtain the materials properties by using the datasheet information.
Total Seebeck coefficient:

$$S = 2N\alpha \ [V/K] \tag{4.10}$$

Serial electrical resistance:

$$R = \frac{N^2 \cdot L_e}{Af} 4\rho \ [\Omega] \tag{4.11}$$

Parallel thermal conductance:

$$K = \frac{Af}{L_e} k \ [W/K] \tag{4.12}$$

where α is the Seebeck coefficient, [V/K], k the thermal conductivity, [W/mK], ρ the electrical resistivity, $[\Omega/m]$ and f is the filling factor defined as $f = 2A_e N/A$.

This procedure has been developed for the chosen TEC by a small program in Matlab (see Appendix A) through which the needed current, I_{tec}, for the cooling requirement is easily obtained.

4.2.1 Model TEC Marlow RC 12-2.5

In order to choose an adequate TEC module for the analysis, it is necessary to define the hole system, that is to say, defining the thermal resistances and temperatures in both sides of the TEC. The assumptions made for the choice of TEC are as follows:

- $T_{amb} = (273.15 + 40)\ K$
- $T_j = (273.15 + 25)\ K$
- $Wall_{htc} = 1500\ W/m^2K$
- $Q_c = 4.215\ W$
- $R_{hs} = 0.2\ K/W$
- $R_{jc} = 0.5\ K/W$
- No TIM is considered in this case

And the schematic representation of the system is shown at the figure 4.5:

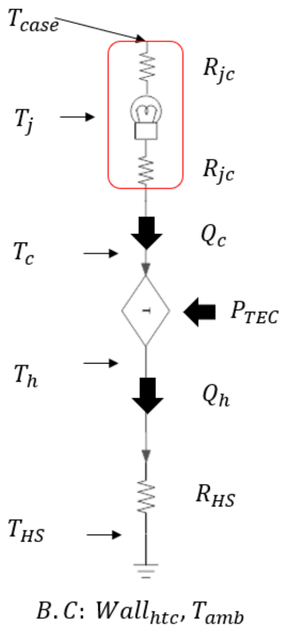


Figure 4.5: *Network thermal TEC model representing the thermal resistances, temperatures and power fluxes occurring in the TEC system*

Under this circumstances considered, the chosen TEC is a Marlow RC12-2.5 [Vi and Dallas]. The module specifications in the datasheet are presented at the table below 4.1:

Table 4.1: *Nominal performance in Nitrogen of RC12-2.5*

Nominal performance in Nitrogen of RC12-2.5		
Hot side temperature [$°C$]	27	50
ΔT_{max}	66	74
$Q_{max}[W]$	23	26
$I_{max}[A]$	2.5	2.5
$V_{max}[V]$	14.7	16.4
Device ZT	0.77	–

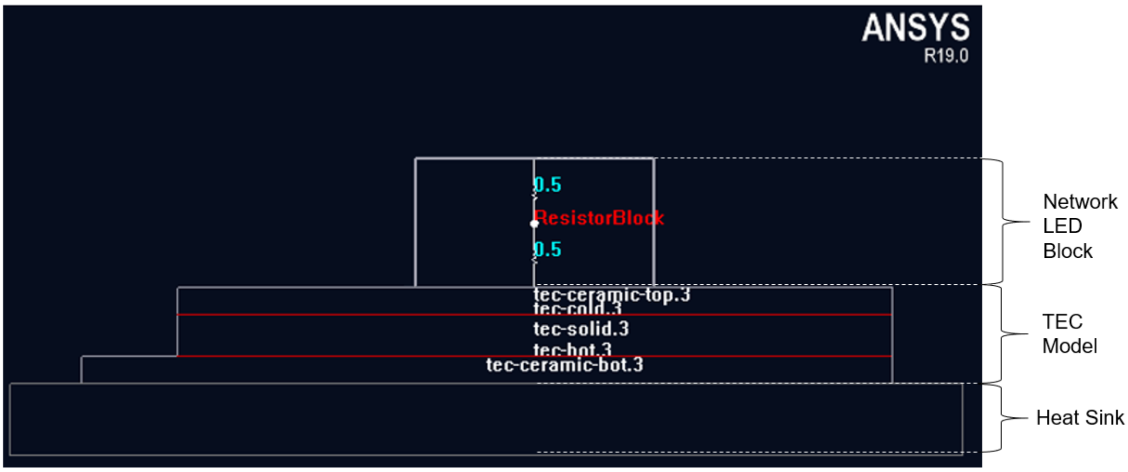
The material used for the thermoelements is Bismuth-Telluride, Bi_2Te_3, whose properties are temperature dependant and calculated by Icepak with the equation 4.13. The coefficients for this equation are presented at the table 4.2. Furthermore, all the properties given for the TEC model at Icepak are summarized at the same table.

$$f(T) = a0 + a1 \cdot T + a2 \cdot T^2 \tag{4.13}$$

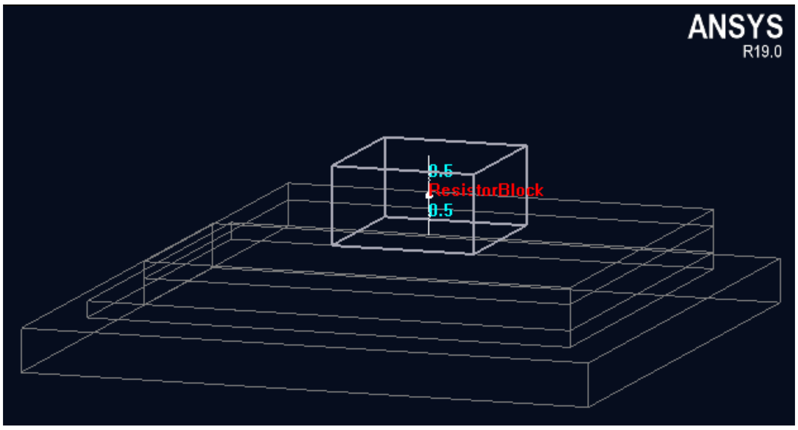
Table 4.2: *Material properties and dimensions of the TEC module*

Material properties (temperature dependant)			
	a0	a1	a2
Seebeck coefficient [V/K]	$-2.0185E-5$	$1.1425E-6$	$-1.2919E-9$
Electrical resistivity [Ohm-cm]	$2.5162E-5$	$1.2175E-6$	$7.0104E-9$
Thermal conductivity [W/cmK]	$4.0986E-2$	$-1.5085E-4$	$2.0708E-7$
Ceramic material: Alumina typical [W/mK]			27
Conduction plates	Orthotropic [W/mK]		k=[0.026,0.026,0.405]
TEC module	Top Ceramic [m^3]		$30 \cdot 30 \cdot 1.13$
	Bottom Ceramic [m^3]		$34 \cdot 30 \cdot 1.13$
n=127	$L_e[m]$		1.778E-3
$I_p = 2.5[A]$	G [m]		0.000551906

The complete 2D LED/TEC model implemented in Icepak can be seen at the figure 4.6a while in figure 4.6b a 3D representation is illustrated. As commented before, the TEC model comprises the Network block representing the junction case, the TEC device, the HS and the wall, not visible in the model illustrated here.



(a) *2D model of the TEC system for* $HS = 40 \cdot 40mm^2$



(b) *3D model of the TEC system. The values seen on the figure represents the given* $R_{jc} = 0.5\ K/W$ *available at datasheet.*

Figure 4.6: *LED model in Icepak formed by a network block, a TIM, a HS and a wall,* $Wall_{htc}$ *to represent the heat transfer coefficient of the HS base for a temperature reference,* T_{amb}*.*

Results and discussion 5

5.1 Standard LED model results

To illustrate how the choice of HS surface influences the results on the STD LED model, two different HS dimensions has been tested.

HS surface of $34 \cdot 30 mm^2$

The temperature evolution of the Standard LED system is shown at the figure 5.1 for different $Wall_{htc}$ and T_{amb}.
As it can be seen, for T_{amb} over $20°C$ the T_j will increase considerably.
Besides, the ideal temperature of $25°C$ (as mentioned on data-sheet) would never be economically reachable for the junction as the $Wall_{htc}$ would be excessively high.

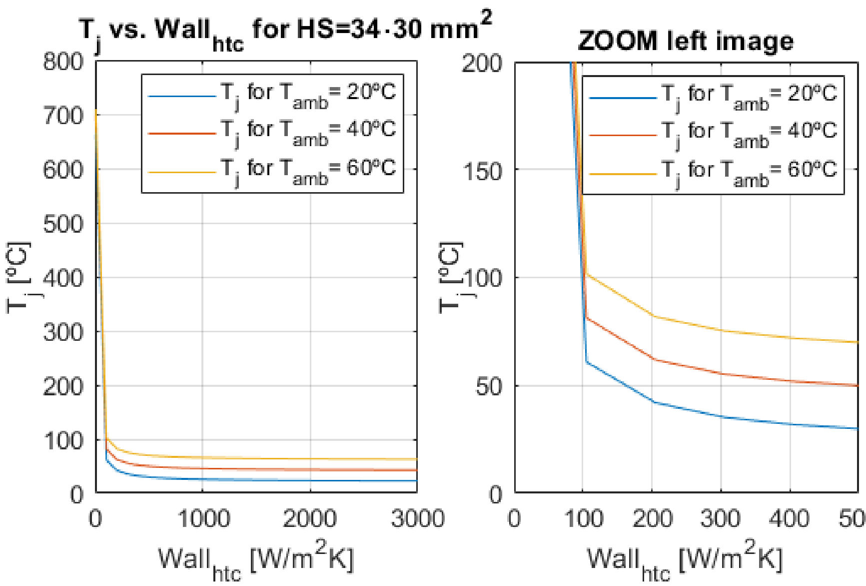


Figure 5.1: *Temperature distribution for different* $Wall_{HTC}$ *and different* T_{amb} *for a HS surface of* $40 \cdot 40\ mm^2$

As illustrated at the figure above, the STD LED system needs at least a $Wall_{htc} = 150\ W/m^2K$ to reduce the temperature of the junction below $100°C$.
Furthermore, from 500 W/m^2K the STD system seems to reach a constant T_j over the interval of $Wall_{htc}$ tested, what meansthat for high T_{amb} (over 20 $°C$) the ideal T_j of 25 $°C$ mentioned on datasheet would never be reachable with the STD systems.

HS surface of $40 \cdot 40\ mm^2$

The analysis of the STD system with $HS = 40 \cdot 40\ mm^2$ is shown below (see figure 5.2). The difference of T_j for the STD system with $HS = 30 \cdot 34\ mm^2$ and $HS = 40 \cdot 40\ mm^2$ is lying between 1 and 3 degrees being. The most remarkable differences between the two systems is seen for low values of HTC, under $500 W/m^2K$ the difference in T_j is around $5°C$.

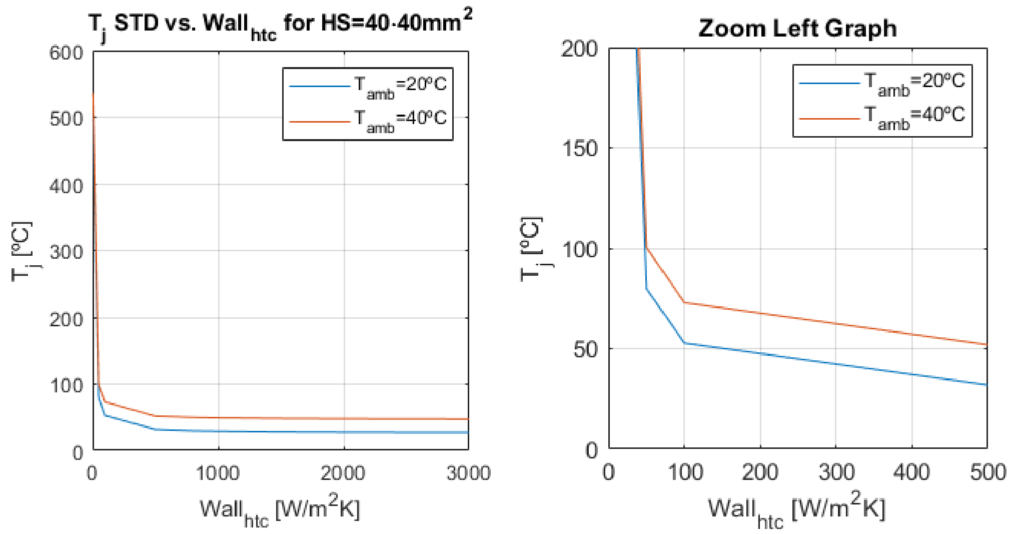


Figure 5.2: *Temperature distribution for different* $Wall_{HTC}$ *and different* T_{amb} *and a HS surface of* $34 \cdot 30\ mm^2$

On the figure below it is illustrated the results of the modelling for the Standard LED case in Icepak, for one the case with $Wall_{htc} = 1500\ W/Km^2$ and $T_{amb} = 60\ °\ C$ and $HS = 40 \cdot 40\ mm^2$.

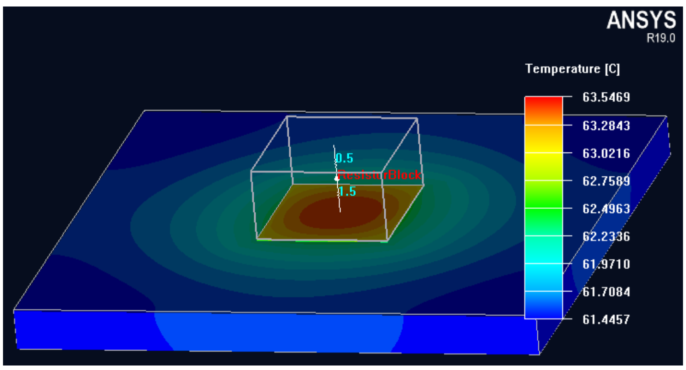


Figure 5.3: *Distribution temperature for LED model*

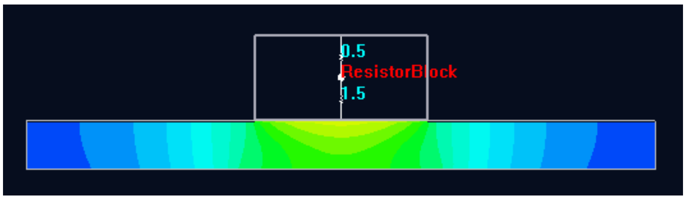


Figure 5.4: *Section of the STD LED model*

The COP of the STD system depending on T_j is shown below.

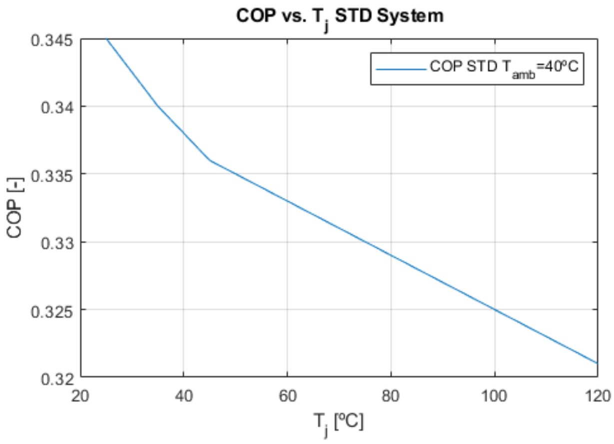


Figure 5.5: *COP of the Standard LED system as function of T_j*

5.2 TEC Model results

As already mentioned, the performance of the TEC is dependent on the system definition as, for example, T_{amb}, $Wall_{HTC}$ and HS surface, so in order to know which is the best combination for the better performance of the TEC model, different options has been considered.

T_{amb} influence on TEC performance

At the graphs below (figure 5.6), it is shown the results of the TEC model for the different T_{amb} considered. As at Standard LED Systems, high T_{amb}, have a negative impact on the performance of the system as the power needed in the TEC to keep a T_j of 25 $°C$ is approximately 65% bigger for 40$°C$ than for 20$°C$ ambient temperature.

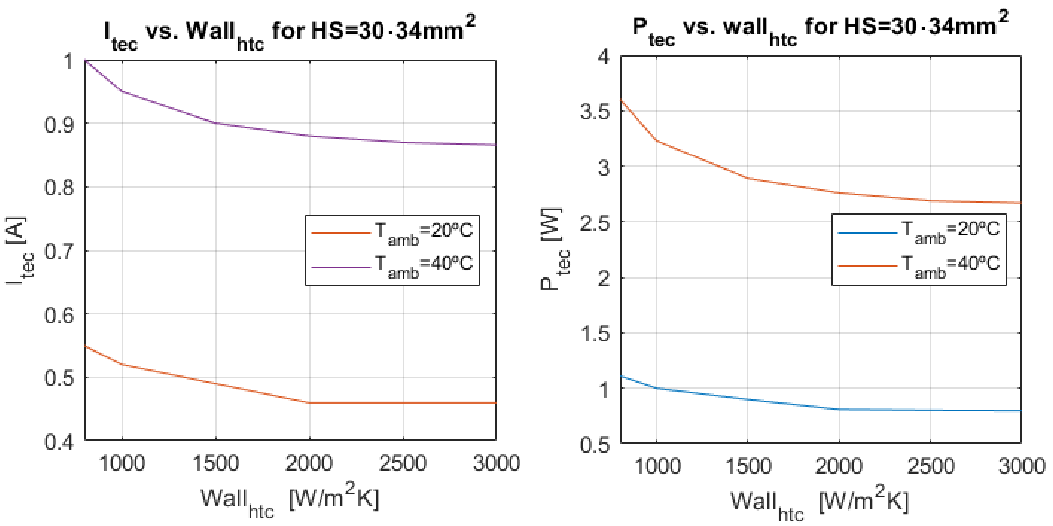
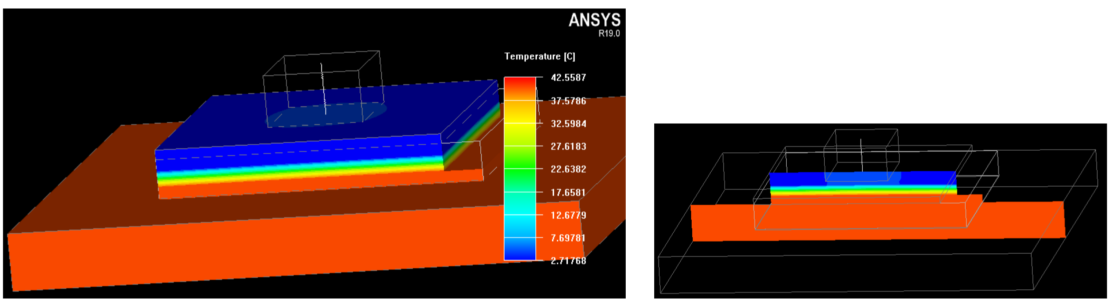


Figure 5.6: *Current and power against different $Wall_{htc}$ with a HS surface of* $30 \cdot 34mm^2$

The next figure illustrates the temperature distribution of the TEC/LED model for a $T_{amb} = 40°C$, $Wall_{HTC} = 1500$ and I= 1A.



(a) *Temperature distribution in the TEC* **(b)** *Section of TEC and HS*

Figure 5.7: *Results of temperature distribution on the TEC system in Icepak*

Heat Sink surface influence

On the next figure, it is seen that for a HS surface with the same size as the TEC, the required power is much higher as there is less $Wall_{htc}$ available for heat rejection.

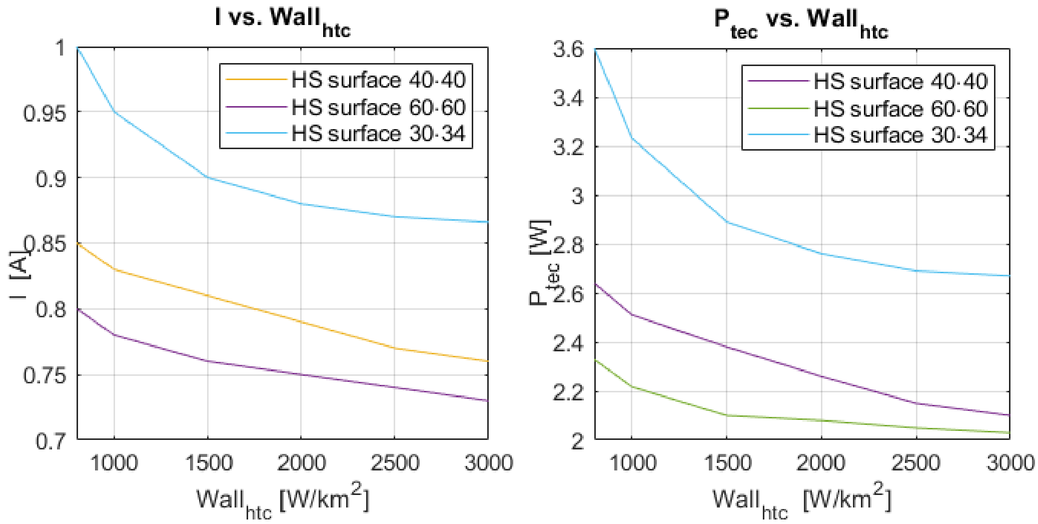


Figure 5.8: *Current and power against different* $Wall_{htc}$ *for* $T_{amb} = 40°C$

Thermocouple length influence

The other parameter studied is the length of the thermoelement. The results are illustrated on the following graphs (see figure 5.9):

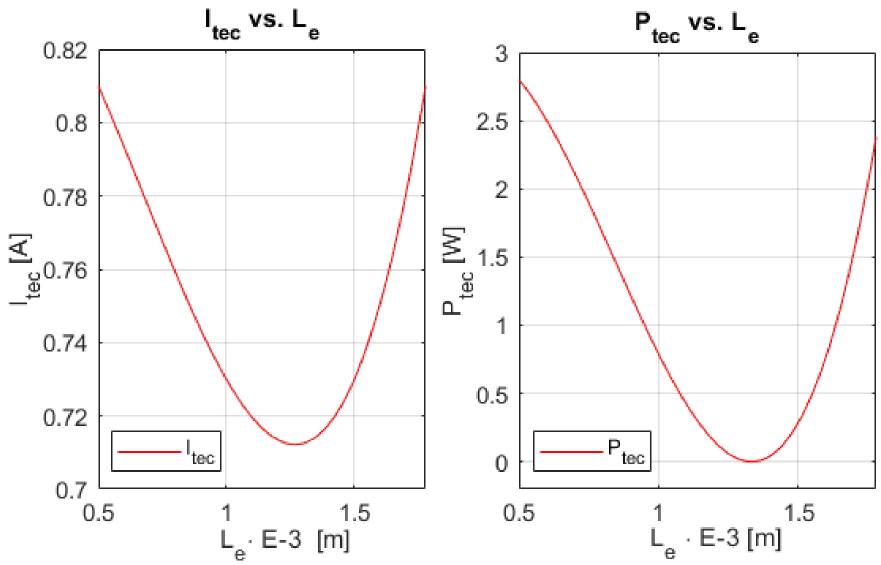


Figure 5.9: *Current and power against different thermoelement longitudes for* $Wall_{htc} = 1500\ W/Km^2$ *for* $T_{amb} = 40\ °\ C$

From these results, the optimum length for the TEC analyzed is found to be around 1.3E-3 m.

5.3 Comparison Standard LED Model vs. TEC Model

After the analysis of the results provided by the different models, it has been chosen to compare both systems for two different T_{amb}, 20 and 40 $°$ C and for HS surfaces of $34 \cdot 30$ mm^2 and 40 $\cdot$ 40 mm^2. At the graphs presented below, for an easy comparison between the two models, the results from both models, for the same conditions, are presented at the same line. On the left side it is shown the T_j evolution of the STD system for different $Wall_{htc}$ while on the right side the graph represents the electrical power consumed by the TEC system for different $Wall_{htc}$.

HS with $34 \cdot 30mm^2$ surface

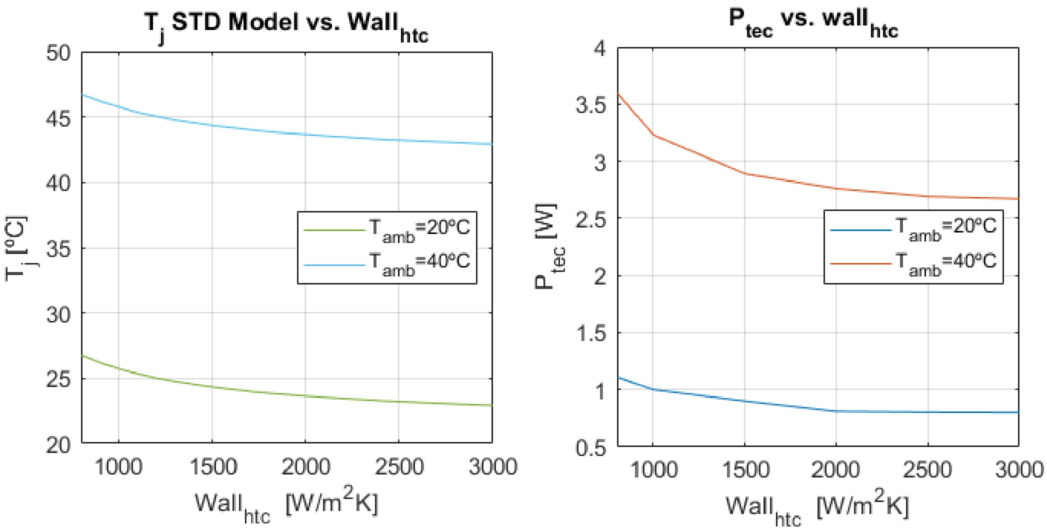


Figure 5.10: *T_j at STD model (left). P_{tec} consumed by the TEC to keep a T_j of $25°C$ (right). Heat sink surface: $34 \cdot 30mm^2$*

HS with $40 \cdot 40mm^2$ surface

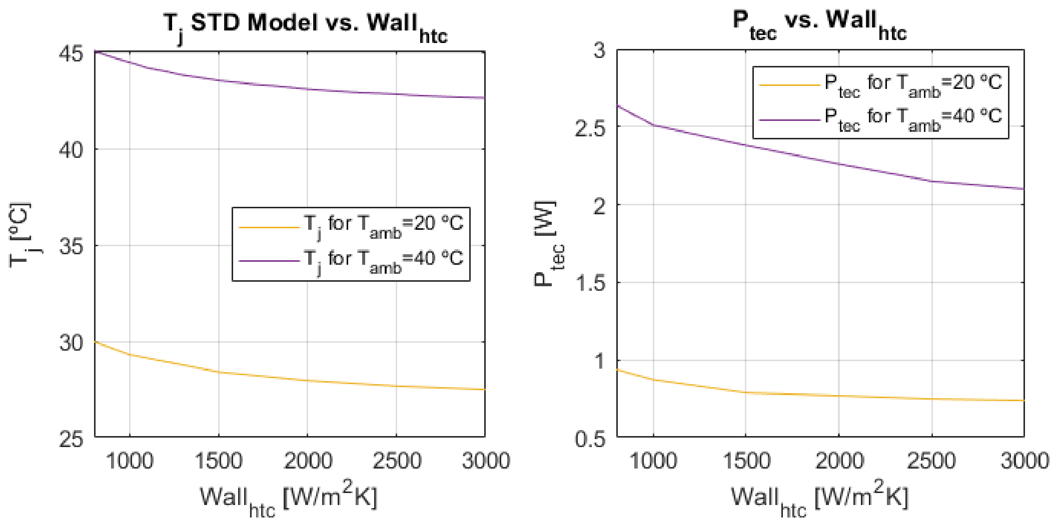


Figure 5.11: *T_j at STD model (left). P_{tec} consumed by the TEC to keep a T_j of $25°C$ (right). Heat sink surface:* $40 \cdot 40mm^2$

From the results shown, it seems more interesting the use of TEC for ambient temperatures over $20°CC$. The reason is that for a STD model, even for high $Wall_{htc}$, the T_j can never been reduced under certain values. However, TEC models improve in efficiency when increasing the $Wall_{htc}$, requiring less electrical power, P_{tec} for the same cooling capacity.

5.3.1 Study cases

Considering a $T_{amb} = 40\ °C$, the proposed systems have been compared in terms of COP for different $Wall_{htc}$. With the equations presented at section 3.5, the COP is calculated as follows:.

$$COP_{std} = \frac{P_{opt}}{P_e} = \frac{\eta_e \cdot P_{elec}(T_j)}{P_{elec}(25°C)} \tag{5.1}$$

$$COP_{tec} = \frac{P_{opt}}{P_{total}} = \frac{P_{opt}(25°C) + \eta_e(P_{elec}(25°C) - P_{elec}(T_j))}{P_{total}} \tag{5.2}$$

$$P_{total} = P_{tec}(25°C) + P_{elec}(25°C) \tag{5.3}$$

$P_{elec}(25°C) = 6.95W$
$P_{opt}(25°C) = 2.4W$
$\eta_e = 0.34$

COP for $Wall_{htc} = 800W/m^2K$

Proceding in the same manner as in the above case, the results in this case are:

- $P_{elec}(T_j) = 6.74W$
- $T_j, std = 49.95°C$
- $P_{tec} = 2.63W$

$COP_{std} = 0.335$
$COP_{tec} = 0.263$

COP for $Wall_{htc} = 1000W/m^2K$

- $P_{elec}(T_j) = 6.74W$
- $T_j, std = 49.32°C$
- $P_{tec} = 2.62W$

$COP_{std} = 0.34$
$COP_{tec} = 0.26$

COP for $Wall_{htc} = 2000W/m^2K$

- $P_{elec}(T_j) = 6.74W$
- $T_j, std = 47.94°C$
- $P_{tec} = 2.21W$

$COP_{std} = 0.34$
$COP_{tec} = 0.28$

COP for $Wall_{htc} = 3000W/m^2K$

- $P_{elec}(T_j) = 6.73W$
- $T_j, std = 47.49°C$
- $P_{tec} = 2.1W$

$COP_{std} = 0.34$
$COP_{tec} = 0.26$

Looking at the results, there is a approximately constant difference in the COP of both systems. The STD LED system shows better COP in terms of P_{elec} and P_{opt}.

6 Conclusion

The aim of this project has been the analysis of the efficiency improvement in LEDs by means of TEC technology.

First, the LED standard system has been implemented in Icepak according with the values provided in datasheet.
Then an analysis of the T_j has studied over a range of $Wall_{htc}$ from 50 to 3000 W/m^2K for different $T_{ambient}$.
From these results, for high T_{amb} (over 20 $°C$), the T_j increase considerably what can be detrimental for the LED lifetime as well as for the efficiency of the system. The light output has been evaluated for the different T_j through a model which integrates the graphs provided at the datasheet for the different working conditions of the LED.

Second, a TEC has been modelled in Icepak which function is to keep the T_j to 25 $°C$ which gives the maximum light output and lifetime according to manufacturer datasheet.
In the process of thermal modelling of the TEC it has been found to be essential the exact definition of the system, as thermal resistances of each component, the cooling required, based on the P_h of the LED and the value of $Wall_{htc}$.
With all these characteristics defined, the choice of the TEC is more accurate as different modules can work on the same application but each one will provide different efficiency to the complete system, depending on the application and the TEC module chosen.

By last, a comparison between the two models is done in terms of COP. The LED system still shows better performance as the electrical power of the TEC is high. However, it is seen, the efficiency of the light and the lifetime of the LED is improved by keeping the T_j at 25$°C$.
So if the application requirements in light quality or lifetime are high, TEC modules would be a good choice as it seems the only technology at the market able to provide such low temperatures at the junction for different boundary conditions.

Future work 7

In this project, some aspects have not being thoroughly studied due to the multidisciplinary nature of the project however some of the ideas gained are presented here.

The integration of the LED system with the TEC has been left out which could be an interesting future work. By implementing an algorithm, the necessary TEC parameters for a particular system could be easily extracted doing the choice of the TEC device more adequate to the application requirements.

The algorithm could be integrated on a network by the definition of variables describing the process or network. A possible configuration of a LED/TEC built in Simscape is shown at the figure 7.1. This model could be used to couple the LED efficiency model with this TEC model, getting a complete integrated solution for LED/TEC systems.

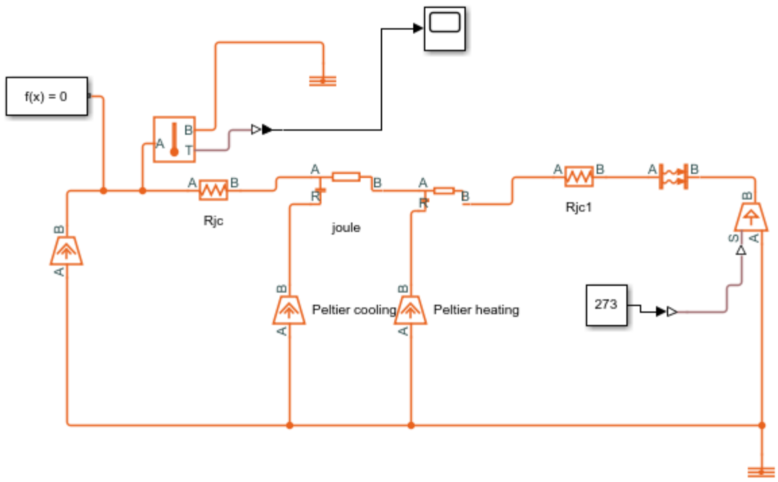


Figure 7.1: *Distributed TEC model with Simscape objects*

The relation between cooling technologies and W/m^2K is also an interesting point of study since by optimizing the TEC system, the requirements in W/m^2K could be decreased.

Lastly, the implementation of TEC in cascade is left for further study and also the combination of TEG-TEC which could bring up interesting results.

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Appendix A

A.1 Physics of Thermoelectrics modules

A thermocouple is illustrated below at the figure A.1. Typically a thermocouple consists on two semiconductors of n-type, p-type. These are conected electrically in series and thermally in parallel forming a thermoelectric module. Assuming neglegible the electrical and thermal contact resistances and constant properties for the materials, the performance of theses devices is obtained by the figure of merit, defined as:

$Z = \alpha^2/\rho k$ $[1/C]$

The material more widely used for thermoelectric coolers is Bithmuth Telluride (Bi_2Te_3) for which the figure of merit take the value of $Z = 2.5 \cdot 10^{-3} K^{-1}$ As shown on figure A.1, these devices are composed of p-type, n-type semiconductors. The bar over the terms denotes the mean values between the hot and cold junction temperature.
$\alpha = \overline{\alpha}_p - \overline{\alpha}_n$
$\rho = \overline{\rho}_p + \overline{\rho}_n$
$k = \overline{k}_p + \overline{k}_n$

For similar materials, the p, n properties take the following shape:
$\alpha_p \approx -\alpha_n$
$\rho_p \approx \rho_n$

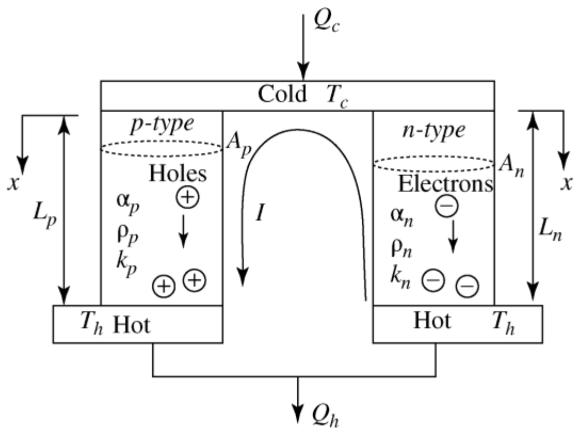

Figure A.1: *Thermoelectric couple with two dissimilar materials [Peltier et al., 1960].*

$k_p \approx k_n$

As the more common cases make use of dissimilar materials, the figure of merit for an optimum geometry can be obtained as:

$$Z = \frac{(\alpha_p - \alpha_n)^2}{[(k_p\rho_p)^{1/2} + (k_n\rho_n)^{1/2}]^2} = \frac{(\alpha_p - \alpha_n)^2}{KR} \tag{A.1}$$

Maximizing Z can be done by minimizing KR leading to the next useful relation for the design of thermoelectrics in terms of materials and geometry.

$$\frac{L_n/A_n}{L_p/A_p} = \left[\frac{\rho_p k_n}{\rho_n k_p}\right]^{1/2} \tag{A.2}$$

For the design of Peltier refrigerators, the amount oh heat absorbed at the cold junction is a combination of Peltier cooling, joule heating and thermal conduction effects. The heat removed at the cold junction can be obtained as:

$$\dot{Q}_c = \alpha T_c I - \frac{1}{2}I^2R - K\Delta T \tag{A.3}$$

and the amount of heat removed at the hot junction is:

$$\dot{Q}_h = \alpha T_h I + \frac{1}{2}I^2R - K\Delta T \tag{A.4}$$

Aplying the first law of thermodynamics to the thermoelectric couple, the electrical power is obtained by:

$$\dot{W}_c = \dot{Q}_h - \dot{Q}_c = \alpha I(T_h - T_c) + I^2R \tag{A.5}$$

being $V = \dot{W}/I$

The coeficient of performance is the defined as follows:

$$COP = \frac{\dot{Q}_c}{\dot{W}} \tag{A.6}$$

Optimum current for maximum cooling rate

For a given geometry, the maximum cooling rate is obtained as follows:

$$\frac{d\dot{Q}_c}{dI} = 0 \qquad I_0 = \frac{\alpha T_c}{R} \tag{A.7}$$

Maximizing performance parameters

The maximum temperature difference will occur for the following conditions:

$I = I_{max}$ and $\dot{Q}_c = 0$

$$\Delta T_{max} = \frac{\alpha^2 T_c^2}{2KR} \tag{A.8}$$

where $KR = k\rho$ or $KR = [(k_p\rho_p)^{1/2} + (k_n\rho_n)^{1/2}]^2$ for similar or dissimilar materials respectively.

The maximum cooling power is obtained by substituting I_{max} into A.7. By simplifying, the current-optimized cooling rate is presented as:
$(\dot{Q})_{I,0} = K(\Delta T_{max} - \Delta T)$

while the maximum possible cooling rate for the a given material and geometry is given when $\Delta T = 0$.

$(\dot{Q}_c)_{max} = K(\Delta T_{max} - \Delta T)$

Optimum current for the maximum COP

The maximum COP is obtained by differentiating the equation A.6:

$$\frac{d(COP)}{dI} = 0 \qquad I_{COP} = \frac{\alpha(T_h - T_c)}{R[(1 + Z\overline{T})^{1/2} - 1]} \tag{A.9}$$

By substituting A.9 into equation A.6 the maximum COP is obtained as:

$$COP_{max} = \frac{T_c}{T_h - T_c} \frac{(1 + Z\overline{T})^{1/2} - \frac{T_h}{T_c}}{(1 + Z\overline{T})^{1/2} + 1} \tag{A.10}$$

Optimum geometry for the maximum cooling in similar materials

Defining G as the geometric ratio, $G = A/L$ the resistance become $R = \frac{\rho L}{A} = \frac{\rho}{G}$ and the cooling rate become:

$$\dot{Q}_c = \alpha T_c I - \frac{1}{2} I^2 \frac{\rho}{G} - KG\Delta T \tag{A.11}$$

to maximize the cooling rate, the equation above is differentiated respect the geometric ratio obtaining the next relation:

$$G_0 = I \left(\frac{\rho}{2k\Delta T} \right)^{1/2} \tag{A.12}$$

A.2 Obtaining TEC material properties from datasheet values

3/06/19 23:04 C:\Users\User\Desktop\...\TECModelScript.m 1 of 1

```
%%datasheet parameters___input. Fixed for each module
DeltaTmax= 74  %(K)
Imax=2.5  %(A)
vmax=15.9 % (V)
Qmax=28   %(W)
Th=273.15+50  %K
G=0.000551906  %(m)

Le=0.001778 %(m)
L=0.00403 %(m)
A=0.03*0.03 %(m^2)
N=127
Ae=G*Le  %(m^2)
f=(2*Ae*N)/A

%%%%Thermoelectric material properties
alpha=Qmax*(Th-DeltaTmax)/(N*Th^2*Imax)   %(V/K)
rho=(A*f*(Th-DeltaTmax)^2/(2*Le*Th^2))*(Qmax/(N^2*Imax^2))  %(\Ohmios*m)
kon=Le*(Th-DeltaTmax)^2/(A*f*Th^2)*(Qmax/DeltaTmax)  %(W/Km)

%%%%Defining the real situation in terms of temperature
Threal=273.15+40;
Tcreal=273.15+25;
deltaT=Threal-Tcreal;
Tave=(Threal+Tcreal)/2

%%%Main parameters of Qc, Qh, V
K=A*f*kon/Le   %(W/K)
R=(N^2*le*4*rho)/(A*f)   %(Ohmios)
S=2*N*alpha   %(V/K)

%%Having Rm, Km and alpham we can calculate now the real situation for
%Th and deltaT

qc=4.21;
syms I
eq1=I*S*Tcreal -0.5*I^2*R-K*deltaT==qc
solveeq1 = solve(eq1,I)
 I=vpa(solveeq1)

 %%calculating the power TEC
qh=I(1)*S*Tcreal +0.5*I(1)^2*R-K*deltaT
 %%Qh
 P=qh-qc
 V=P/I
```

A.3 Simulink Model

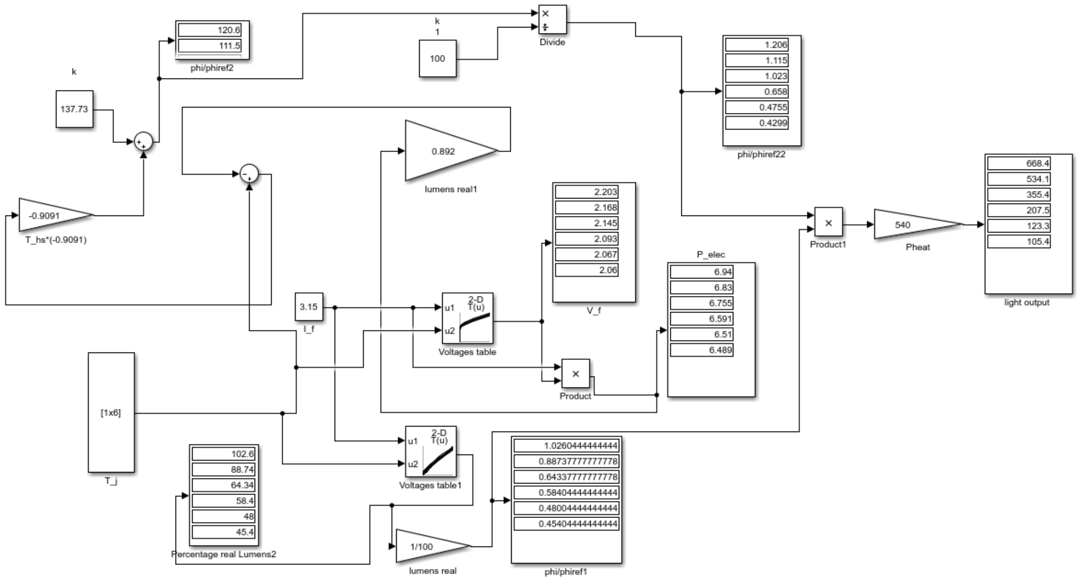

Figure A.2: *Simulink model for the analysis of T_j evolution.*