Probabilistic Assessment of Existing Structures

Reliability Updating Through Testing

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Master's Thesis, Structural and Civil Engineering

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Supervisors:

Jannie Sønderkær Nielsen
John Dalsgaard Sørensen

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Abstract:

The purpose of this project is to study how various of testing methods can be used to access and/or update the reliability level of existing structures. This is investigated both on a component and system level and is carried out considering the various methods with respect to proposed decision models. The analyses are all carried out for a representative linear limit state equation where only one variable load is present at a time. Relevant stochastic models are established, where especially the stochastic models related to the resistance are of interest in regards to the obtained results.

The first part of the project mainly revolves around how proof load testing of components in a system can be used to update the reliability level based on whether prior information is available or not. The total expected costs are then estimated in regards to a decision model.

The last part of the project assesses how cylinder and CAPO-tests can be used to assess the reliability level and which method is the most cost-beneficial in regards to average total expected costs. The total expected costs are also in this case estimated in regards to a decision model.

Preface

This report and its appendices are the product of the undersigned master's thesis at the 4th semester of the Master's programme in Structural and Civil Engineering at the Department of Civil Engineering. The report is prepared and made in accordance and compliance to the current curriculum at the School of Engineering and Science from the 1st of February 2018 to the 8th of June 2018 under the supervision of John Dalsgaard Sørensen and Jannie Sønderkær Nielsen.

The theme of the project is *Assessment of the structural reliability for existing structures*, where the purpose of this project is to assess an approach to reliability update a structure through different testing methods.

In regards to the project, many appreciations goes towards above mentioned supervisors for their supervision, expertise and flexibility throughout the project period.

Meick J. Jørgensen
Simon Bak
Aalborg, 8th of June 2018

Reading guide

The project consists of two parts: A main report and its appendices, which are found at the back of the report. The main report refers to the appendix where the appertaining calculations and extensional documents are to be found. The files used for software, e.g MATLAB, are attached in a digital folder. The main report is numbered numerically, so that the first chapter is called Chapter 1 and sections here-under 1.1, 1.2 etc. The appendices are analogously referred to alphabetically e.g. A, B etc. Figures and tables are numbered with respect to the their respective chapter, meaning that the first figure in Chapter 2 is referred to as Figure 2.1 etc. Equations are referred to analogously to this with the exception of having ( ) in-closing the reference, e.g. Equation (2.1). The digital folder contains literature, experimental data, results and MATLAB scripts.

This report uses the Harvard method of bibliography with the name of the author and year of publication into brackets after the text, e.g [Ayyub and McCuen, 2002]. If the source reference is positioned before a punctuation it only refers to the sentence whereas if it is placed behind the punctuation it refers back to the whole text section. A list of all the source references is given in the bibliography list at the end of the main report.

Resume

Dette kandidatspeciale er udarbejdet i perioden d. 1. februar 2018 - d. 8. juni 2018 og er produktet af kandidatudannelsen Structural and Civil Engineering på Aalborg Universitet.

Titlen på projektet er: *Probabilistic Assessment of Existing Structures* med den tilhørende undertitel *Reliability Updating Through Testing*.

For en eksisterende konstruktion kan et ønske om at ændre anvendelsen eller forlænge levetiden føre til en ombygning eller renovering. Hvis en ombygning, renovering eller ændret anvendelse indebærer udskiftning eller ændring af eksisterende, bærende konstruktionsdele, statisk virkemåde eller lasters størrelse og omfordelingen af disse, så skal konstruktionens bæreevne naturligvis revurderes, hvis de ikke umiddelbart erstattes med nye komponenter. Forstærkning eller udskiftning af bærende konstruktionsdele i en eksisterende konstruktion er en dyr løsning sammenlignet med samme tiltag for en ny konstruktion i designfasen. Herved kan det være af stor interesse at vide, hvorvidt det er muligt gennem forsøg at bevise, om pålideligheden er tilstrækkelig, eller hvor meget forstærkning der i så fald skal vendes for at opnå et bestemt ønsket sikkerhedsindeks.

Formålet med dette projekt er at undersøge, hvordan forskellige testmetoder kan bruges til at vurdere og/eller opdatere sikkerhedsniveauet for eksisterende konstruktioner. Dette undersøges både på komponent- og systemniveau med hensyn til forskellige testmetoder i relation til foreslåede beslutningsmodeller. Analyserne udføres alle for en repræsentativ lineær svigtfunktion, hvor kun en variabel last er til stede af gangen. Relevante stokastiske modeller etableres, hvor især de stokastiske modeller relateret til styrken er af interesse med hensyn til de opnåede resultater.

Den første del af projektet omhandler primært, hvordan prøvebelastninger komponenter i et system kan anvendes til at opdatere sikkerhedsniveauet, alt efter om der foreligger forhåndsviden eller ej. De samlede forventede omkostninger beregnes derefter i forbindelse med en beslutningsmodel.

Den sidste del af projektet vurderer, hvordan cylinder- og CAPO-test kan bruges til at vurdere sikkerhedsniveauet af en eksisterende konstruktion, samt hvilken metode som er den mest omkostningseffektive med hensyn til gennemsnitlige samlede forventede omkostninger. De samlede forventede omkostninger beregnes også i dette tilfælde i forbindelse med en beslutningsmodel.

Table of Contents

Chapter

1 Introduction

An assessment or reevaluation of the reliability level of an already existing structure is in some cases necessary due to a change in circumstances regarding the structure. For instance, this could be caused by a change in loading or usage, discovery of damage on the structure, extension of service life or in general doubts about the reliability level and structural safety of the given structure.

The documentation of the load-bearing capacity of components in a structure was previously based on a different basis than today. As a result, the safety of an already existing structure based on previous codes cannot be used as documentation in regards to the current codes. Therefore, the structural safety of an existing structure has to be updated either by reinforcement of the load-bearing components of concern in case the load-bearing capacity is insufficient, or by proving that the components have sufficient strength and safety despite the change in circumstances by gathering additional information through tests or more advanced calculations. In general, reinforcement of components in already existing structures is expensive in comparison to new structures that are still in the design phase. Instead a less expensive method is to conduct tests on the existing structure in order to potentially improve the certainty of material strength proving that the reliability level is sufficient. Different methods for testing exist, but they are only briefly accounted for in regards to the assessment of structural safety of existing structures in the codes. In general the codes for assessment of structural safety for existing structures are not well developed, which is problematic, when a reevaluation of the structural safety of an existing structure has to be carried out. However, new codes are currently under development that seek to better establish a set of guidelines or rules for different problems to assess structural safety for existing structures. [SBI 251, 2015]

1.1 Safety - then and now

The basis for design of structures in Denmark has changed over the past years. In the following a distinction between three periods is presented [SBI 251, 2015]:

1856-1908 - Design requirements

In the structural codes of Copenhagen from 1856 [Bygningslov, 1856], 1871 [Bygningslov, 1871] and 1889 [Bygningslov, 1889] a detailed description of materials, dimensions and interior design is described. The basis of these descriptions are from artisan-related traditions.

1908-1965 - Permissible stresses

The Danish Engineering Association's codes for reinforced concrete structures operates with the calculation of permissible actions in the form of permissible stresses. [Ingeniørforening, 1908] After this period, structures are designed in regards to functional requirements, i.e. the desire for a given load-bearing capacity instead of specified design descriptions. The permissible stresses are expressed as a given percentage of the mean strength of the material. In 1945 the first edition of DS 410 [1945] is published. Until then, the magnitude of the loads could be found in Norm 12 by Ingeniørforening [1916]. Load combinations and the magnitude of the permissible stresses were still found in the structural codes.

1965 - Partial safety method

After 1965 the structural codes began to assign partial safety factors on the load- and strength side as well as describe methods for estimation of the characteristic values of the load and strength properties. The value of the characteristic strength was typically expressed as the 5-10%-quantile in a probability distribution. The value of the characteristic variable load was typically expressed as the 98%-quantile in a distribution function corresponding to a 50 year return period.

In the beginning, the partial safety factors were based on experience such that the obtained dimensions are similar to the ones obtained from the permissible stresses. Later on, the reliability index method was used as basis for the calibration of the partial safety factors in the safety codes. [NKB, 1978] This method was also used for verifying the load-bearing capacity of larger structures.

In 1982 the first edition of the Code of Practice for the Safety of Structures, DS 409, was released. [Ingeniørforening, 1982] This code governs the rest of the structural codes by establishing a set of rules for load combinations and rules for estimation of the partial safety factors in these codes.

In 2001 the Joint Committee of Structural Dafety publication on assessment of existing structures was published, which recommends procedures and tools for assessment of existing structures' safety level. [Diamantidis, 2001]

From 2009 Eurocodes are applied together with the Danish National Annexes for design of new strutures, where the safety system is accounted for in Eurocode 0, DS/EN 1990 [2007] and loads in Eurocode 1, DS/EN 1991-1 [2007]. In DS/INF 172 [2009] the basis for the partial safety factors in the National Annex and Eurocodes that result in an acceptable reliability level has been described. The partial safety factors are estimated by the use of the reliability index method and a set of prerequisites are related to this, which involves probability distribution types and quantile-values for load- and strength parameters.

1.2 Problem statement

This project aim is to study how to assess the reliability level of existing structures, both on a component and system level with focus on performing tests as basis for assessment. This is to be carried out considering various testing methods, and relevant decision models are to be proposed.

1.3 Prerequisites and delimiters

Only linear limit state equations are considered. Furthermore, only one variable load is present at a time, which means that presented results are not valid for combination of several loads.

Analyses are not carried out for any specific materials, apart from in Chapter 5 where concrete subjected to compression is assessed.

Due to limited simulation time, almost all results are obtained for specific defined values that in reality can vary a lot.

1.4 Structure of the report

In this section a short description of the report's overall structure is presented.

As a basis, reasons and methods for assessing the structural reliability of an existing structure are described.

Some of the various of test methods are analyzed with respect to assessing and/or updating the structural reliability of existing structures. In relation to this, assessment- and decision models are constructed to clarify the method of approach for the case studies related to the different methods.

A cost-benefit analysis is performed between two testing methods, cylinder compression test and CAPO-test. The stochastic models for cylinder compression tests are based on codes, while the stochastic modelling of the CAPO-test in this project is based on experimental measurements.

Lastly, an overall assessment of the results obtained in the different analyses are evaluated and discussed, so that a proper conclusion to the problem statement can be formulated.

The above-described is summarized in Figure 1.1, which shows a structure diagram of the content of the report.

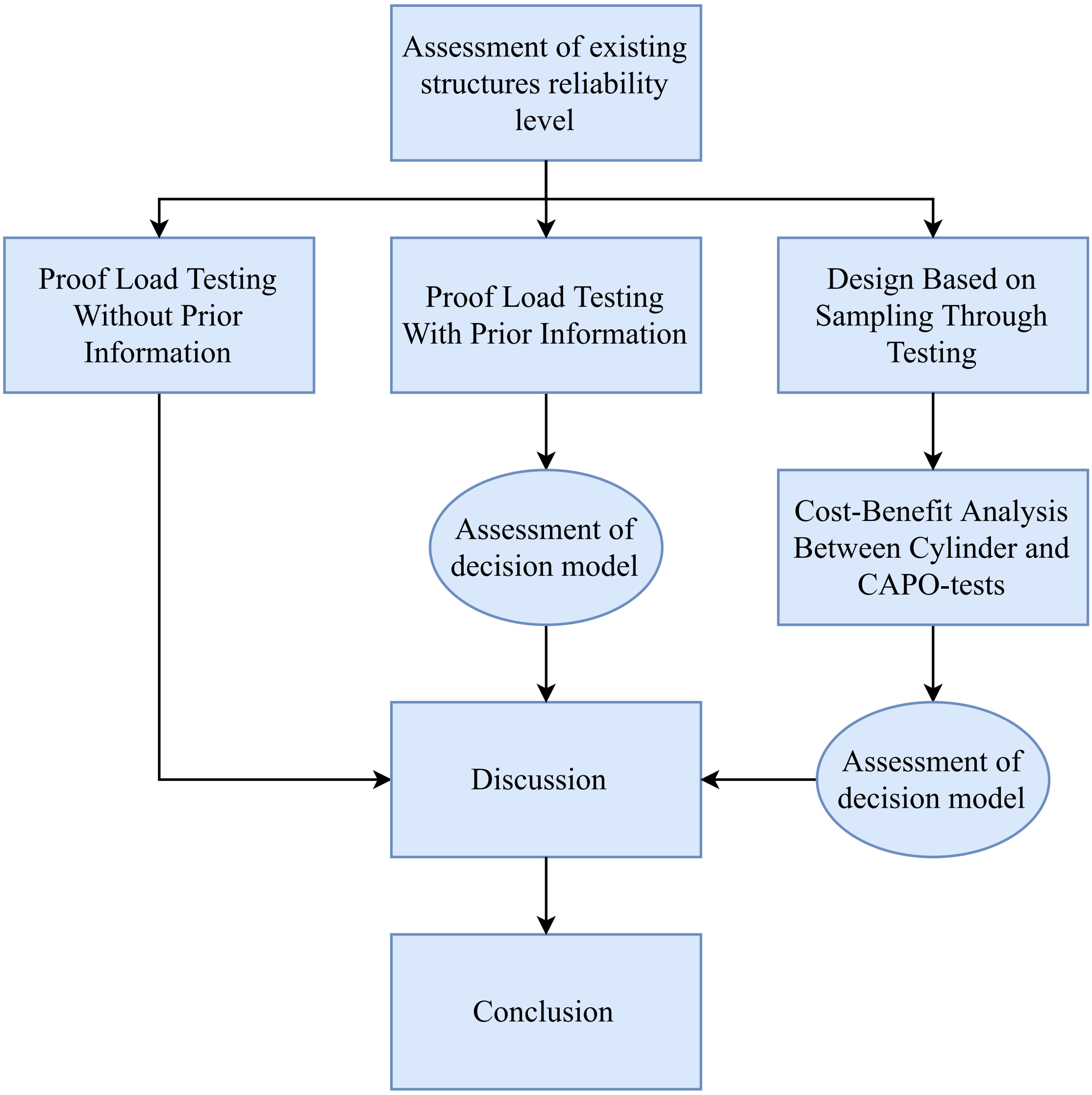

Figure 1.1: Structure diagram of the content of the report.

Chapter

2 Assessment of Existing Structures Reliability Level

In this chapter the purpose or reassessing the reliability level of an existing structures and when to do so is described along with the various of methods to evaluate and/or update the reliability level of an existing structure. Theory and methods described in this chapter are found in SBI 251 [2015].

2.1 Reasons for assessment of existing structures

An existing structure designed on the basis of previous codes can in most cases not be documented in accordance to the codes currently valid for new structures. This is a result of the change of codes from then to now, as described in Chapter 1, and the fact that the structure naturally decays over its lifetime, meaning that the remaining lifetime will usually be shorter than what is required in the current codes for new structures. For instance, an existing structure with a remaining lifetime of 20 years will most likely not have an acceptable reliability level in accordance with a required lifetime of 50 years in the current codes, if fatigue or other damage accumulation is of importance. However, it is generally accepted that existing structures do not have to comply with the current codes given that they comply with codes at the time of construction, unless changes has occurred or will occur. In this case, the reliability level of the existing structure has to be assessed, which among others might be caused by the following:

- Change of application of the structure.
- Change in static system of the load-bearing structure.
- Change in load conditions or surroundings, e.g. accumulation of snow, increase in wind load or change in conditions of foundation.
- Change of lifetime for a structure.
- Deterioration or damage of the bearing structure, e.g. collision, overloading, settlements or unstable cracks.
- Discovery of errors in relation to design and construction.

The assessment of an existing structure should initially be based on the current codes, since they are assumed to be created on the basis of greater knowledge than the previous codes. The assessment requires that information about the strength characteristics of the structure is known, either from historical documentation or by conducting proof load tests. This information is then used to update the reliability level of the structure in accordance to the new codes through Bayesian updating. In cases, for which the current codes do not cover the existing structures, they have to be considered from an engineering point of view in regards to functionality and capacity. However, circumstances related to uncertainty of strength characteristics and determination of loads still have to be considered in accordance with the current codes.

2.2 Methods for obtaining new information

When reevaluating the structural reliability of an existing structure, it is necessary to obtain new information regarding the resistance of the structure. This information is generally obtained through either measurements and/or observations of the stochastic variable i.e. the resistance or through observation of events. Methods used for measuring and observing the stochastic variables can be either direct or indirect and examples of these are as follows:

- Direct measurements of the resistance, e.g. from bored out cylinders of a concrete structure.
- Indirect measurements of the resistance, e.g. by performing CAPO-test on a concrete structure.

Observation of events include events caused by actively performing a test on the structure or by natural occurrences. This includes the following examples:

- Proof load testing a structure and observe if failure occurs.
- Observation of non-failure during a natural extreme event, e.g. a storm.

Reevaluation of the structural reliability through cylinder test, CAPO-test and proof load testing will be assessed in this project. These test methods are intended to be either destructive, non-destructive or partly destructive and will be described in the following.

2.2.1 Proof load tests

Proof load testing is a common method for obtaining new information on a structural component or a whole system of components. The method can be performed, when no prior information associated to the existing structure is available or when the available information does not document the reliability level of the structure. It is an experimental method, where a component or system is subjected to a load close to the extreme exposures throughout the service life. This test establishes a minimum load level that the tested specimen can sustain. Proof load testing is considered or is at least intended to be a non-destructive testing method, meaning that a probability of not successfully carrying out the test exists. Proof load testing is expensive and bears the risk of damage to the tested structure and its surroundings. Thus, the decision to carry out proof load tests should depend on a decision model that includes the cost and risks involved in comparison to alternative methods.

In regards to Bayesian statistics, the resistance of the structural component is updated by knowing that the component survived the proof load and is therefore described as an observation of an event. There are two types of observations in context to this, which are an equality event and an inequality event, see Appendix A.4. If no prior information about the resistance is available when proof load testing, then the updating of the probability of failure, should the component survive the proof load test, is modelled by an equality event given the correct prerequisites. When no prior information is available and the proof load is not increased to failure, then it only validates the reliability level of the particular structural member that was tested. This means that the whole population in a structural system has to be tested in order to have a total validation for the whole structure. Proof load testing without prior information is studied in Chapter 3. If the contrary scenario is considered e.g. having prior information about the resistance, then the updating of the probability of failure is modelled by an inequality event, see Appendix A.4. In this case, it is sufficient to perform proof load tests on a portion of the structural components. Likewise, if the proof load is increased to failure, only a portion of the population has to be tested in order to assess the reliability level. Proof load testing with prior information is studied in Chapter 4.

Proof load testing is not always a possibility when having to assess the reliability level of an existing structure. Reasons for this could be that the proof load test is too difficult to perform, e.g. on offshore structures, the component cannot properly be accessed or perhaps the risk of the proof load test is too expensive. Nonetheless, there are alternative methods in relation to obtaining new information of existing structures.

2.2.2 Resistance sampling tests

Resistance sampling tests are other methods that can be used for obtaining new information about a structure. The principle of these testing methods is to directly measure the strength of the material, i.e. the stochastic variable, through test samples from the given structure. The information obtained through these type of tests can be used to update the stochastic variables, if prior information is available, or otherwise give an estimate of the stochastic variable. This is described in Appendix A.4. Three common methods to measure the resistance of a structural component is; cylinder compression test, CAPO-test and LOK-test.

The cylinder compression test is carried out by boring out a cylindrical test specimen from the component. The specimen is then compression tested in a laboratory and the resistance is measured, which is why this is considered a destructive testing method. This method is only used for concrete structures. For CAPO-test and LOK-test the compressive strength of the material is directly measured on-site by the use of a pull-out test. The pull-out force (tensile force) is converted to a strength resistance. These testing methods are considered partly destructive. A more detailed description of the various of sampling test methods is found in Chapter 5, while Chapter 6 describes how they are used to assess the reliability level of an existing structure.

Even though these resistance sampling tests are considered either destructive or partly destructive, it is intended that these tests are carried out so that the component is not damaged. The decision to perform a cylinder compression test or a CAPO/LOK-test can be evaluated based on a cost-benefit analysis of the tests, which will be studied in Chapter 6.

2.3 Estimation of reliability level

2.3.1 General

When a suitable combination of loads and load-bearing properties e.g. geometry and strengths causes the structure to have exhausted its load-bearing capacity, then the structure will go into failure-mode. The probability of failure is normally assessed in terms of calculation models related to the load-bearing capacity. The calculation model has to be provided with specific values representing the loads and the load-bearing properties. These values are uncertain (stochastic) and are expressed by the use of distribution functions. By using these distribution functions together with the calculation model it is possible to estimate the probability of obtaining a combination of loads and load-bearing capacities that yields failure. The stochastic values are included in the calculation model for the probability of failure with different weights. This is why it is appropriate to be able to describe the most influential stochastic variables as accurate as possible.

2.3.2 Estimation of the probability of failure

In Figure 2.1 distribution functions are illustrated in a simple case, e.g. a single stochastic load and a single stochastic strength property.

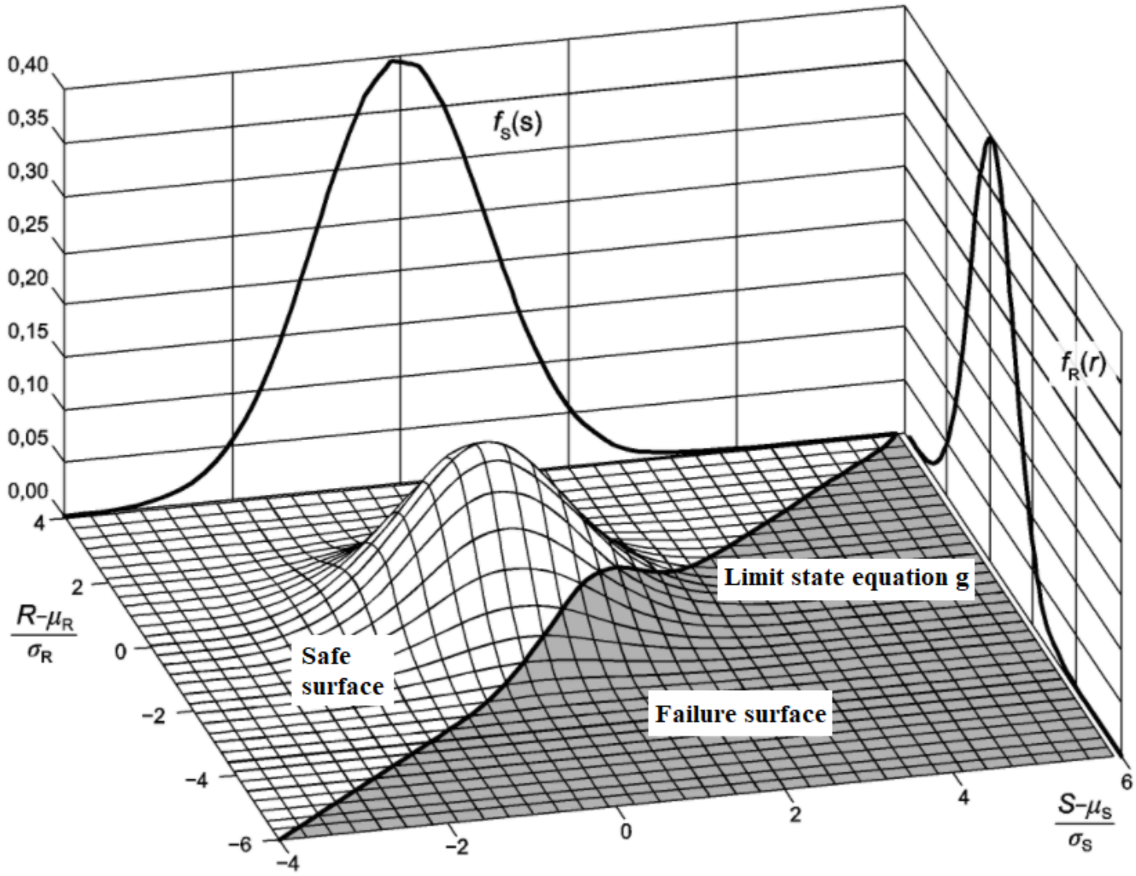

Figure 2.1: Illustration of a joint probability distribution function for two uncertain parameters. [SBI 251, 2015]

The shown surface is the joint probability distribution function of the two separate distribution functions of the load and strength property. The joint probability distribution function showcases the frequency of the combined events. The joint probability function is standardized such that the area below equals 1, e.g. having 100% probability of having any event. Standardizing the surface means that the variables are transformed into standard normal distributed variables with a mean value, $\mu = 0$, and standard deviation, $\sigma = 1$.

Furthermore, in Figure 2.1 a limit state equation, $g = R - S$, is shown for which R is the resistance and S is the load. The probability of failure is found at the most probable combination of events that will cause failure. In case of failure, the combined events yield a resistance less than the load.

The probability of failure is equal to the volume beneath the failure surface, which is shown on Figure 2.1. The failure surface is the area in which the strength is less than the load. The minimum acceptance criteria for the probability of failure is of the magnitude 10^{-5} and with a reference period of 1 year (annual probability of failure). This is why the upper tail of load-related stochastic variables are of importance and the contrary applies for the strength-related stochastic variables.

When dealing with normal distributed stochastic variables and linear limit state equations, then the probability of failure is estimated as $\Phi(-\beta)$, where β is called the reliability index. The reliability index is defined as the shortest distance between the origin of the standardized stochastic variables and the failure surface in the standardized space, while $\Phi(\bullet)$ is the standardized normal distribution function. This explanation is illustrated in Figure 2.2.

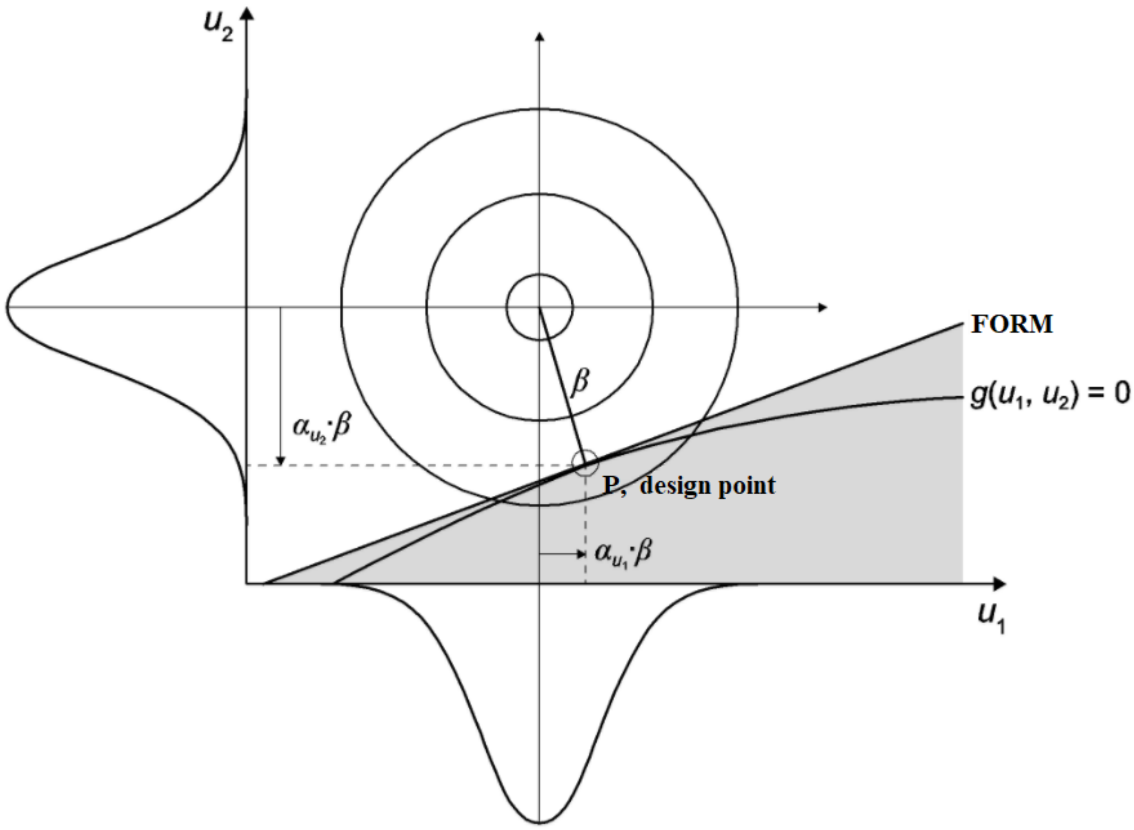

Figure 2.2: Illustration of probability distribution functions in regards to estimating the reliability index, β, for the design point, P. SBI 251 [2015]

Non-linear limit state equations can be approximated with a tangent in the point, which has the lowest β-value. This is an iterative process to estimate the point, P, which yields the smallest value for β. This point is defined as the design point and represents the combination of multiple stochastic events that will most likely cause failure. The components in the vector from the origin to P express the reliability index's sensitivity towards changes in the various of stochastic variables' distribution functions.

The described method is called First Order Reliability Method (FORM). The error related to the probability of failure, $\Phi(-\beta)$, is directly related to the failure surface's deviation from the linear surface, and hence the part of the failure surface that is not included. In these cases another method can be used instead. This method is called Second Order Reliability Method (SORM), which approximates the failure surface with 2. order derivatives and thus gives more precise results. Using FORM yields sufficient results as stated in DS/EN 1990 [2007]. The tangent line develops into a tangent plane if three stochastic variables are introduced and into a hyper plane if further are added.

The stochastic variables are not always given in terms of normal distributions. When dealing with various of distribution functions it is essential to transform these into normal distributed uncertainties instead. This is carried out by substituting the stochastic variables in the limit state equation with normal distributed stochastic variables, see Sørensen [2011] for further explanation.

Alternatively, Monte-Carlo simulation of a large amount of combined events can be carried out and the amount of obtained failures compared to the non-failure events is counted, see Appendix A. Hereby, an estimate of the probability of failure is found. The amount of samples that has to be simulated has to be sufficiently large, so that an even larger amount of samples will yield the same results. In this project, it is chosen to use Monte-Carlo simulation for all analyses for estimating the probability of failure.

2.3.3 Distribution functions

The distribution functions that form a basis for the safety assessment includes both physical uncertainties, statistical uncertainties and model uncertainties. The physical uncertainties on the load side is related to the variations that are expected to happen, e.g. wind load during a storm. On the resistance side the physical uncertainty is related to e.g. the construction, geometry and natural variations in the construction materials.

The statistical uncertainty is governed by the extent of observations and test results, which the physical uncertainty is based on.

A model uncertainty is associated to the load-bearing model. This model uncertainty indicates how good a relation there is between the resistance estimated by a model and the strength estimated through tests, see DS/EN 1990 [2007], Annex D.

For the load model the uncertainty expresses the uncertainty related to the conversion between the load and the load effect.

2.3.4 Reliability level

When the involved distribution functions related to the load- and resistance side describe the probability of having a maximum load within a given reference period, then this means that the reliability level is the probability of having failure within this reference period. The reference period that is most commonly used is an annual one, i.e. one year.

For structures under normal circumstances this annual probability of failure has to have a magnitude less than 10^{-5}. This corresponds to a reliability index with a value of approximately $\beta = 4.3$. Since the required probability of failure is annual, it means that the remaining life time on the structure is of no importance in the assessment of the structures safety given that no degradation is present, e.g. fatigue or corrosion.

As a minimum the desired reliability level has to be obtained. In a specific existing structure some circumstances might cause this to be unreasonably expensive or impossible to carry out. In these cases the government governs deviation from the codes if necessary.

The requirement for the reliability level depends on the structure's consequence class and failure type, see Table 2.1. A description related to the failure types are found in Table 2.2. Values for failure type II are found from DS/EN 1990 [2007]. Values for the other failure types I and III are found by respectively increasing and reducing the annual probability of failure with a factor 10, as indicated in DS/INF 172 [2009].

Table 2.1: Tentative target values for the annual reliability indices for different reliability classes and failure types cf. DS/INF 172 [2009].

Reliability Class	Failure Type I	Failure Type II	Failure Type III
CC1 (less serious)	3.2	3.8	4.3
CC2 (serious)	3.8	4.3	4.8
CC3 (very serious)	4.2	4.7	5.2

Table 2.2: Classification and description of failure types cf. DS/INF 172 [2009]

Failure type	Description
Type I	Alerted failure with extra load-bearing capacity reserves from e.g. hardening of steel
Type II	Alerted failure without extra-load bearing capacity
Type III	Failure caused by e.g. loss of stability

Alternative tentative reliability indices have also been proposed by the Joint Committee of Structural Safety, which are shown in Figure 2.3. These values are based on a cost-benefit analysis and the choice of target reliability index depends on an assessment of consequence class and cost of safety measures.

Table 2.3: Tentative target values for the annual reliability indices for different reliability classes and cost of failures cf. Joint Committee on Structural Safety [2001].

Consequence Class	Large cost of safety measures (A)	Normal cost of safety measures (B)	Low cost of safety measures (C)
CC1 (less serious)	3.1	3.7	4.2
CC2 (serious)	3.3	4.2	4.4
CC3 (very serious)	3.7	4.4	4.7

2.4 Overall decision model

An overall decision model over the different test methods that can be used for reevaluating the reliability level of existing structures will be established in this section. The decision model takes basis in a realization that one of the circumstances listed in Section 2.1 leads to the conclusion that the reliability level of an existing structure has to be reevaluated. The possible test methods used in the reevaluation are illustrated in the decision model in Figure 2.3.

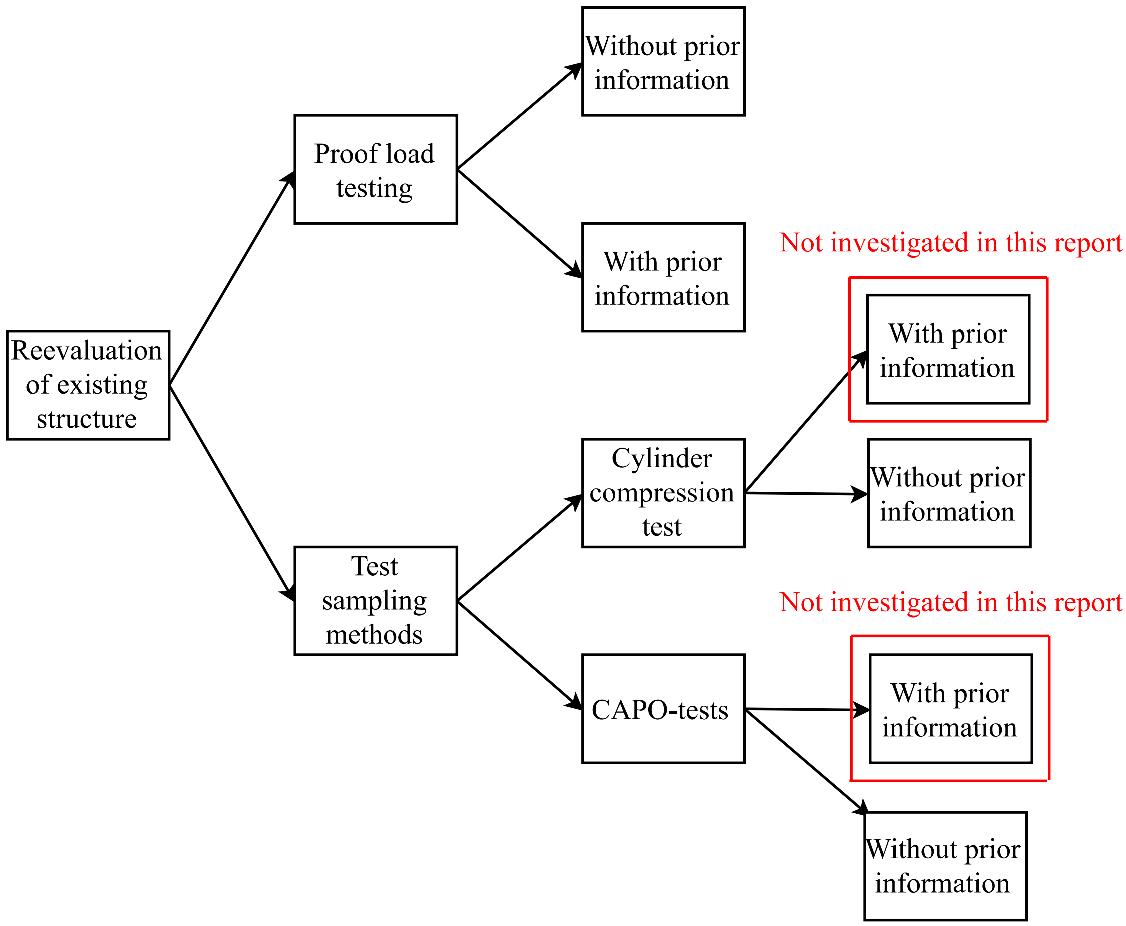

Figure 2.3: Overall decision model.

An initial decision is made between performing proof load tests or test sampling methods like cylinder compression tests or CAPO-tests. This decision depends on parameters such as the type of material used in the structure and total expected costs. If proof load tests are performed, then it has to be assessed whether prior information is available of not. The case without prior information in regards to reevaluating the reliability level of the structure is evaluated in Chapter 3. The case with prior information is studied in Chapter 4 in which a more detailed decision model is established for the different decisions and actions that can be taken in this case. If test sampling methods are decided as choice of action, then it has to be decided whether to perform cylinder or CAPO-tests as well as assess whether prior information is available of not. The cases for which prior information is available are not investigated in this report. The other cases for which no prior information is available are evaluated in Chapter 5 and 6. The decision between performing cylinder or CAPO-tests is based on a cost-benefit analysis and is used as part of a more detailed decision model in Chapter 6.

Chapter

3 Proof Load Testing Without Prior Information

This chapter will describe how proof load testing can be used to assess the reliability level of an existing structure, when no prior information about the resistance is available.

When no prior information associated to the existing structure is available, then the reliability level has to be assessed solely from the information obtained through proof load testing. In this case information about a system can be obtained by either proof load testing a portion of the components to failure or by proof load testing all components in a system with a proof load corresponding to a target reliability level. This chapter will only study the latter in which an analysis of how proof load testing can be used to update the reliability level of an existing structure. In relation to this, the chapter will study how different stochastic models for the variable load influence the reliability level, when performing proof load tests.

3.1 Reliability level

The analysis for assessing how the reliability level can be updated by performing proof load testing takes basis in the reliability levels for new structures. The target reliability level is chosen for different cases listed in Table 2.1. This includes the following component reliability indices with a reference period of one year in accordance to Table 3.1:

Table 3.1: The target reliability indices that are used for assessing how the reliability level can be updated by performing proof load testing without prior information.

Target reliability index, β^t_{comp}				
$\beta = 3.2$	$\beta = 3.8$	$\beta = 4.3$	$\beta = 4.7$	$\beta = 5.2$

3.2 Limit state equation

A limit state equation has to be established that includes both the proof load and the permanent and variable loads multiplied with model uncertainties related to the loads. It is assumed that only a single variable load is present at a time, e.g. wind or snow load. This is assessed to be a reasonable assumption, since e.g. no snow load is present when wind is assumed to be the dominating load.

Thus, a generic representative linear limit state equation can be expressed as following:

$$g = P_{\mathrm{pl}} - \left(X_{G,i}G(1-\alpha) + X_{Q,i}Q\alpha\right) \tag{3.1}$$

where

P_{pl}	Proof load.
G	Permanent load.
Q	Variable load.
$X_{G,i}$	Model uncertainties for the permanent load.
$X_{Q,i}$	Model uncertainties for the variable load.
α	$\alpha = 0$ corresponds to full permanent load and $\alpha = 1$ corresponds to full variable load.

The ratio for the permanent and variable load can be varied in the interval between 0 and 1, i.e. the interval between full permanent and full variable load respectively for the characteristic load requirement. The loads and model uncertainties depend on the chosen stochastic models listed in the following. The proof load is a deterministic load that is determined by:

$$P_{\mathrm{pl}} = P_{\mathrm{test}} \cdot k_{\mathrm{test}} \tag{3.2}$$

where

k_{test}	Multiplication factor dependent on the percentage permanent load of the characteristic load and the target reliability index, β^t.
P_{test}	Characteristic load requirement.

The characteristic load requirement is defined as the additional load that the structure has to carry in addition to the permanent load already applied to the structure before subjecting it to the proof load. The purpose is to estimate the k_{test}-factors that corresponds to the target reliability indices. This can be done by assuming that the resistance of the tested component in the structure is emptied at a value just above the proof load, since no knowledge of the resistance above the proof load is available, given that the conditions for the proof load is equivalent to the real load size and duration. Thus, the assessment of the reliability level is based on an equality event, which is described in Appendix A.4, i.e. the resistance is assumed equal to the proof load. The probability of failure can be estimated as the probability that a load greater than the proof load subjects the structure.

3.3 Stochastic models

Stochastic models are established for all stochastic variables in the limit state equation. The models include distribution type, mean value,μ, coefficient of variation, V, and quantiles, p, for the characteristic values. The stochastic models of interest are listed in Table 3.2 for which model 2 and 3 for the wind and snow load respectively are determined from Equation (3.3) - (3.6).

$$Q_{w,2} = C_{pe}C_rC_gQ_{ref,2} \quad \text{for wind model 2} \tag{3.3}$$

$$Q_{w,3} = X_{Q_{ref}}C_{pe}C_rC_gQ_{ref,3} \quad \text{for wind model 3} \tag{3.4}$$

$$Q_{s,2} = X_{Q_{sg,2}}Q_{sg,2} \quad \text{for snow model 2} \tag{3.5}$$

$$Q_{s,3} = X_{Q_{s,3}}Q_{sg,3} \quad \text{for snow model 3} \tag{3.6}$$

Table 3.2: Stochastic models for loads and model uncertainties.

Load type	Model	Random variable	Symbol	Dist. type	Mean	V	Quantile
Permanent	Model 1	Permanent load	G_P	Normal	1.00	0.10	0.50
Wind	Model 1	Wind load	$Q_{w,1}$	Gumbel	1.00	0.40	0.98
	Model 2	Pressure coeff.	C_{pe}	Gumbel	1.00	0.15	0.78
		Roughness coeff.	C_r	Lognorm.	1.00	0.15	1.00
		Gust coeff.	C_g	Lognorm.	1.00	0.10	1.00
		Wind pressure[1]	$Q_{ref,2}$	Gumbel	1.00	0.25	0.98
	Model 3	Model uncertainty	$X_{Q_{ref}}$	Lognorm.	0.80	0.20	1.00
		Pressure coeff.	C_{pe}	Gumbel	1.00	0.10	0.78
		Roughness coeff.	C_r	Lognorm.	0.80	0.10	1.00
		Gust coeff.	C_g	Lognorm.	1.00	0.10	1.00
		Wind pressure[1]	$Q_{ref,3}$	Gumbel	1.00	0.25	0.98
Snow	Model 1	Snow load	$Q_{s,1}$	Gumbel	1.00	0.40	0.98
	Model 2	Model uncertainty	$X_{Q_{s,2}}$	Lognorm.	1.00	0.30	$(\mu + \sigma)$
		Snow load[2]	$Q_{sg,2}$	Gumbel	1.00	0.40	0.98
	Model 3	Model uncertainty	$X_{Q_{s,3}}$	Lognorm.	1.00	0.35	0.50
		Snow load[2]	$Q_{sg,3}$	Gumbel	1.00	0.40	0.98

[1] Annual reference wind pressure.
[2] Snow load at ground level.

Currently, there is no clear-cut stochastic model for the variable loads that should be used. Thus, it is of interest to assess the difference between the models in regards to assessing the reliability level of an existing structure through proof load testing.

The stochastic models for the permanent load as well as model 1 for wind and snow load are used as basis for the loads in DS/EN 1991-1 [2007] and often used for design. Physical and model uncertainties are already included in model 1 for the wind and snow load. Model 1 for wind and snow load are identical and normally referred to under the same category of 'variable load' in the current codes. However, it might be fair to assume that wind and snow loads do not follow the same stochastic model, which is why it is interesting to also evaluate alternative proposed stochastic models for the wind and snow load. This includes model 2 and 3 for the wind and snow load respectively in Table 3.2. Model 3 for the wind load and model 2 for the snow load are described in JCSS [2001], while model 2 for the wind load and model 3 for the snow load are described in S A K O [1999] and Sanpolesi [1997] respectively.

For model 2 and 3 of the wind load in comparison to model 1, it can be seen that the coefficient of variation is lower for the annual reference wind pressure compared to the wind load, which is caused by the models not yet accounting for a number of different time-invariant factors related to pressure, roughness and gust. An uncertainty and thus a stochastic model is related to each of these factors as listed in Table 3.2. Furthermore, a model uncertainty is added in model 3.

The stochastic model for the snow load at ground level in model 2 and 3 is identical to model 1 of the snow load. However, model 2 and 3 have been added a model uncertainty with a rather high coefficient of variation to take the snow time-invariant part into account. The high uncertainty is caused by the snow load being dependent on the shape of the structure and shelter from the surroundings.

3.4 Updated reliability level

The reliability level of a system can be updated by proof load testing components in the system. Since no prior information is available, all components have to be proof load tested, given they are not tested to failure. Firstly, the stochastic models for model 1 described in Section 3.3 will be used for estimating the necessary k_{test}-factors for obtaining the target reliability levels listed in Section 3.1. These k_{test}-factors are estimated for a varying weight factor, α. This study has previously been carried out in SBI 251 [2015], and thus the initial purpose is to recreate these results before different alternative proposed stochastic models are used in the analysis. The results are listed in Table 3.3.

Table 3.3: k_{test}-factors for the 1. stochastic model of permanent and wind/snow load in SBI 251 [2015].

Characteristic permanent load percentage of characteristic load requirement [%]	Factor: k_{test}				
	$\beta = 3.2$	$\beta = 3.8$	$\beta = 4.3$	$\beta = 4.8$	$\beta = 5.2$
0	1.52	1.85	2.19	2.48	2.87
25	1.39	1.65	1.89	2.11	2.40
50	1.28	1.45	1.61	1.75	1.95
60	1.24	1.38	1.51	1.62	1.78
65	1.22	1.34	1.46	1.56	1.69
70	1.20	1.31	1.41	1.50	1.61
75	1.20	1.29	1.37	1.44	1.54
80	1.20	1.27	1.34	1.40	1.48
85	1.21	1.28	1.33	1.37	1.43
90	1.24	1.30	1.34	1.38	1.43
95	1.28	1.34	1.38	1.42	1.47
100	1.32	1.38	1.43	1.47	1.52

Instead of estimating the k_{test}-factors in the entire interval of α, an average value is usually suggested to be used. The weighting between the permanent and variable load usually lies in the interval $\alpha = [0.3; 0.8]$ and thus an average value of the k_{test}-factors can be estimated for this interval. The results are listed in Table 3.4.

Table 3.4: Average k_{test}-factors in the interval $\alpha = [0.3; 0.8]$ for model 1.

k_{test}-factor				
$\beta = 3.2$	$\beta = 3.8$	$\beta = 4.3$	$\beta = 4.7$	$\beta = 5.2$
1.39	1.60	1.79	1.96	2.19

Similarly, the k_{test}-factors for the four alternative stochastic models of the variable loads can be estimated. If the target reliability indices listed in 3.1 are used again, the estimated k_{test}-factors will become unreasonable large due to all the additional variables in the models that add up to a larger coefficient of variation. Thus, the probability of the components actually surviving the proof load is really low and therefore not feasible. Instead, it will be assessed how the reliability level differs between the different stochastic models of the variable load when using the same magnitude of proof load. These reliability indices can then be used as new target reliability indices for which k_{test}-factors can be estimated. These target reliability indices can be estimated by inserting k_{test}-factors in the interval $\alpha = [0.3; 0.8]$ from model 1 into the limit state equation in Equation (3.1) for the stochastic models 2 and 3 and calculating an average reliability index for these stochastic models. This is done for a reliability index of $\beta = 4.3$. The following values are found for the different stochastic models:

Table 3.5: Average annual targeted reliability indices for the alternative stochastic models in the interval $\alpha = [0.3; 0.8]$ in comparison to $\beta = 4.3$ for model 1.

Target reliability index, β^t_{comp}			
Wind model 2	Wind model 3	Snow model 2	Snow model 3
2.6	3.5	1.9	1.8

The results clearly shows that the reliability level decreases significantly when using the alternative stochastic models of the variable load, especially for the snow load. Average values of the k_{test}-factor in the interval $\alpha = [0.3; 0.8]$ can now be estimated for the new target reliability indices and compared in order to assess the sensitivity of the k_{test}-factors, when using different stochastic models. The k_{test}-factors are estimated for the target reliability indices in Table 3.5 as well as $\beta^t_{\text{comp}} \pm 0.5$ and 1.0 that are assessed to represent two reliability classes above and below the reliability indices in Table 3.5 similar to the case for model 1. The results are seen in Tables 3.6 - 3.9, while more detailed tables for varying α-values can be found in Appendix B, if accurate information about the percentage permanent load of the characteristic load requirement is known.

Table 3.6: Average k_{test}-factors in the interval $\alpha = [0.3; 0.8]$ for wind model 2.

k_{test}-factor				
$\beta = 1.6$	$\beta = 2.1$	$\beta = 2.6$	$\beta = 3.1$	$\beta = 3.6$
1.32	1.48	1.68	1.92	2.21

Table 3.7: Average k_{test}-factors in the interval $\alpha = [0.3; 0.8]$ for wind model 3.

k_{test}-factor				
$\beta = 2.5$	$\beta = 3.0$	$\beta = 3.5$	$\beta = 4.0$	$\beta = 4.5$
1.28	1.46	1.68	1.94	2.27

Table 3.8: Average k_{test}-factors in the interval $\alpha = [0.3; 0.8]$ for snow model 2.

k_{test}-factor				
$\beta = 0.9$	$\beta = 1.4$	$\beta = 1.9$	$\beta = 2.4$	$\beta = 2.9$
1.21	1.42	1.68	2.01	2.42

Table 3.9: Average k_{test}-factors in the interval $\alpha = [0.3; 0.8]$ for snow model 3.

k_{test}-factor				
$\beta = 0.8$	$\beta = 1.3$	$\beta = 1.8$	$\beta = 2.3$	$\beta = 2.8$
1.19	1.40	1.68	2.03	2.48

By comparing the average k_{test}-factors between the models, it can be seen that the results only differ slightly from each other. This indicates that the k_{test}-factors have a low sensitivity in regards to which model that is used, given that the resulting reliability indices are accepted.

Chapter

4 Proof Load Testing With Prior Information

This chapter will make an assessment of how the reliability level of an existing structure can be updated based on proof load testing of components in a system, when prior information about the resistance is available.

When prior information is available for a structural system consisting of a number of components with resistances that can be considered as realizations from a homogeneous population in the existing structure, then the reliability level can be updated by conducting proof load tests on a portion of the components. The prior information includes knowledge about the mean value, μ_R, and coefficient of variation, V_R, of the resistance, R, of the components in a system. An example of such a system is illustrated in Figure 4.1 for a reinforced concrete flooring structure, where the steel reinforcement bars in the upper and lower part respectively are considered components of a homogeneous population in a system. If changes to the structure lead to an increase in the loads, which are carried by the reinforcement bars, then proof load testing can be performed to ensure an acceptable reliability level of the structure.

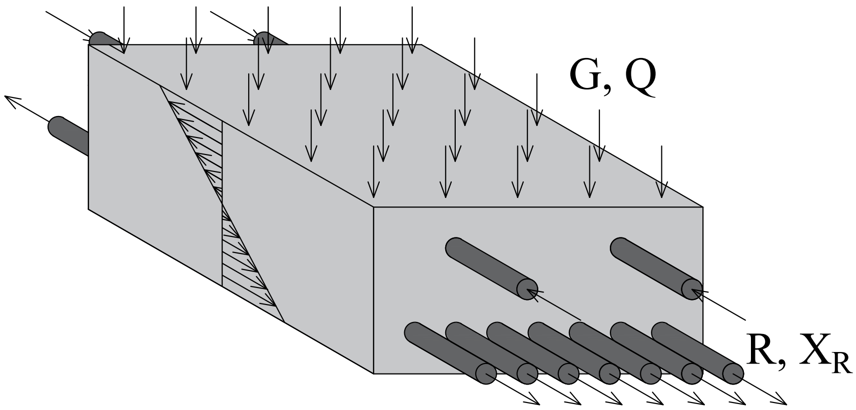

Figure 4.1: A concrete component consisting of a number of identical reinforcement bars with the resistance parameters, R and X_R, and subjected to a permanent and variable load, G and Q.

For an existing structure, the stochastic models for the loads and model uncertainties can be considered known, but there will always be an uncertainty related to the resistance of the components, despite prior information being available. Based on the prior information available and an assessment of the uncertainty related to this information, different actions can be taken to ensure that the structure has an acceptable reliability level, and these are described by a decision model that is based on a preliminary inspection and rating of the resistance.

4.1 Assessment model: Preliminary inspection and rating

An assessment model that takes basis in a preliminary inspection and rating of the resistance will be established. This assessment model constitutes a scenario in which the reliability level of an existing structure has to be reevaluated. For instance, an office building has to be changed into a fitness center, meaning that the loads are increased on the load-bearing structures such as a concrete flooring structure as previously illustrated in Figure 4.1. The flooring structure can be considered a system consisting of a number of reinforcement bars from a homogeneous population. In this case, the reinforcement bars in the lower part of the flooring structure will be subjected to increased tension forces, and thus the reliability level has to be reevaluated. The assessment model in Figure 4.2 has been established to illustrate the process behind a reevaluation.

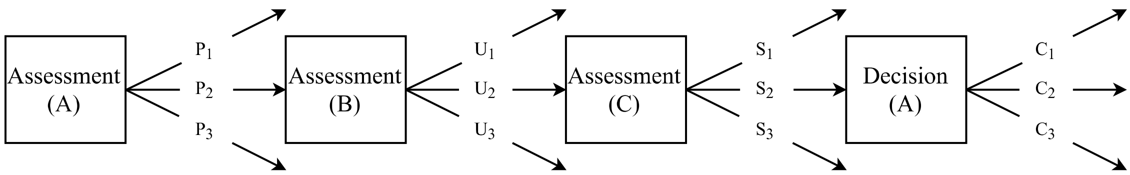

Figure 4.2: Assessment model that takes basis in a preliminary inspection and rating of the resistance and describes the different assessment that have to be made when the reliability level of an existing structure has to be reevaluated.

Assessment (A) comprises an assessment of the prior information, P_i, available for the resistance of the existing structure. The prior information might be believed to still hold true or having changed due to time having passed. For example, cracks might be discovered, which leads to a belief that the resistance is less than documented. Likewise, the resistance of e.g. a concrete structure might be believed to be greater than documented, since the compressive strength of concrete increases with time.

In assessment (B), the level of uncertainty, $V(\mu_R)$ and (V_R), related to the prior information, μ_R and V_R, have to be assessed, since the resistance is an uncertain variable. This is modelled by the symbol U_i in the figure. The third assessment (C) is to identify what type of system that has to be reevaluated and the number of components in the system, denoted S_i in the figure. This includes whether the system is a series or parallel system as well as the static mode of action of the system, i.e. is the failure alerted or instantaneous.

Finally, decision (A) is whether to perform proof load testing on a number of components in the system or to reinforce the components in the system to ensure an acceptable reliability level. This decision is based on an analysis of cost, C_i, for performing the respective actions. This cost depends on the number of proof load tested components and the probability of failure occurring during the proof load testing. The optimal decision can then be made based on the action with the least cost.

4.2 Prerequisites

The aim of this chapter is to assess the influence of proof load testing on the reliability level of a structural system consisting of a number of components and in order to do so, it is necessary to establish some prerequisites for the resistance. This includes both the stochastic model of the resistance, μ_R and V_R, as well as the uncertainties, $V(\mu_R)$ and $V(V_R)$, related to μ_R and V_R. The prerequisites used for μ_R and V_R are described in Section 4.5.2 for the stochastic model. The uncertainty on the statistical parameters, μ_R and V_R, is a result of the structure having already existed in a period of time, which generates an uncertainty related to the resistance of the components. Therefore, when assessing the reliability of an existing structure, the level of uncertainty, $V(\mu_R)$ and $V(V_R)$, is important to take into consideration. In this analysis, the uncertainty is considered to be classified in the following levels assuming that all parameters are lognormal distributed: [Sørensen, 2016]

- Small for $V(\mu_R) \leq 15\%$ and $V(V_R) \leq 7.5\%$
- Large for $15\% < V(\mu_R) \leq 20\%$ and $7.5\% < V(\mu_R) \leq 10\%$
- Very large for $20\% < V(\mu_R) \leq 30\%$ and $10\% < V(\mu_R) \leq 15\%$

The influence of proof load testing will be assessed for when the uncertainty for estimating μ_R and V_R is either small, large or very large respectively. This analysis will be carried out for both series and parallel systems.

4.3 Reliability level

The analysis for assessing the influence of proof load testing on the system reliability will take its basis in a target reliability level for new structures. The target reliability level is chosen for a reliability class CC2 and failure type II, which corresponds to the following component reliability index with a reference period of one year in accordance to Table 2.1:

- $\beta^t_{\text{comp}} = 4.3$

However, it should be noted that the reliability level for existing structures is normally accepted to be lower than that of new structures, since the cost of safety measure is larger for an existing structure compared to a new structure.

4.4 Limit state equation

In order to estimate the reliability level of a system, the following generic, representative, linear limit state equation is used for each of the components for calibrating the mean value of the resistance, μ_R, to result in a targeted reliability index of $\beta^t_{\text{comp}} = 4.3$:

$$g = RX_R - (G(1-\alpha) + Q\alpha) \tag{4.1}$$

where

G	Normalized permanent load.
Q	Normalized variable load.
X_R	Model uncertainty of the resistance, R.

In Equation (4.1), $g < 0$ will lead to failure. It is assumed that only a single variable load is applied to the structure at a time, meaning Equation (4.1) is not valid, when multiple variable loads are present simultaneously, e.g. both wind and snow load. Furthermore, the weighting factor is initially assumed to be $\alpha = 0.5$ for all analyses, since it is assessed to be a representative average value, but will in the later analyses be varied to assess the sensitivity of the weighting factor.

4.5 Stochastic models

The stochastic models used for the variables in Equation (4.1) will be established in the following, including the mean value, μ, coefficient of variation, V, and characteristic value for the stochastic variables, R_k. Firstly, the load models will be described, since the loads are the known stochastic variables, which can then be used to give an estimate of the resistance that is the unknown variable.

4.5.1 Loads

The following assumptions are chosen for the normalized characteristic value of the permanent and variable load applied to the components in the system:

- $G_k = 1$
- $Q_k = 1$

The permanent and variable load are assumed to be distributed in accordance to the stochastic models described in DS/INF 172 [2009] and are listed in Table 4.1. The statistical parameters, i.e. μ and V are estimated directly from the characteristic values of the 50%-quantile for the permanent load and 98%-quantile for the variable load. The statistical parameters for the permanent load are estimated by:

$$\mu_G = G_k \tag{4.2}$$

$$V_G = \frac{\sigma_G}{\mu_G} = 0.10 \tag{4.3}$$

Similarly to Equation (4.2) and (4.3), the statistical parameters for the variable load are estimated by the following, assuming $V = 40\%$ and the characteristic value is equal to 1:

$$\mu_Q = 0.490\, Q_k \tag{4.4}$$

$$V_Q = \frac{\sigma_Q}{\mu_Q} = 0.40 \tag{4.5}$$

The distribution of the variable load is an expression for the probability that a particular load is the maximum load to occur in a reference period of one year, i.e the variable load models are the maximum annual load.

Table 4.1: Statistical parameters for the stochastic models of the permanent and variable loads.

Parameter	Symbol	Distribution type	μ	V	Characteristic value
Permanent load	G	Normal	1.00	0.10	1.00
Variable load	Q	Gumbel	0.49	0.40	1.00

4.5.2 Resistance and model uncertainties

The resistance is the uncertain and unknown variable, meaning that it is necessary to set up prerequisites for establishing a stochastic model that can be used for the analysis. The resistance will be evaluated in two cases for which the following assumptions listed in Table 4.2 are made for the distribution type, V, and quantile for R_k corresponding to the total resistance, RX_R.

Table 4.2: Stochastic models of the resistances and model uncertainty.

Parameter	Symbol	Distribution type	V	R_k
Resistance 1	R_1	Lognormal	0.10	5 %-quantile
Resistance 2	R_2	Lognormal	0.20	5 %-quantile
Model uncertainty	X_R	Lognormal	0.05	

The first stochastic model of the resistance corresponds to a material in between steel and concrete in regards to $V_R = 10\%$ while the second stochastic model represents laminated wood with $V_R = 20\%$. The same model uncertainty related to the resistance has been chosen for both stochastic models with V_{X_R}. The mean, μ_R, and characteristic value, R_k, for the two stochastic models will be estimated corresponding to a component reliability index, $\beta^t_{\text{comp}} = 4.3$. The mean values can be estimated through iteration by using Monte-Carlo simulation for which the inverse method, $x = F_x^{-1}(F_U(U))$, is used, where $F_U(U)$ is the distribution function for a uniform distributed stochastic variable and $F_U(U) = U$ for $0 \leq U < 1$ and $F_U(U) = 1$ for $u > 1$, see Appendix A. The following simulation algorithm is used for estimating these mean values and is also illustrated in the flowchart in Figure 4.3 to better visualize the procedure.

1. A mean value of the resistance, μ_R, is guessed.
2. $N = 10^7$ random realizations of the resistance, R_i, model uncertainty, $X_{R,i}$, and the loads G_i and Q_i using the appropriate stochastic parameters and distribution functions are simulated.
3. The simulated variables in step 2 are placed into Equation 4.1 and g is calculated. $g < 0$ leads to failure.
4. The probability of failure, P_f, for a component is estimated as the ratio between the amount of failed components and number of total simulations, N.
5. The reliability index, β, is calculated by $\beta_{\text{comp}} = -\Phi^{-1}(P_f)$.
6. If $|\beta^t - \beta| \leq 0.1$, the correct mean value, μ_R, has been guessed, otherwise step 1-6 is repeated until convergence is met.

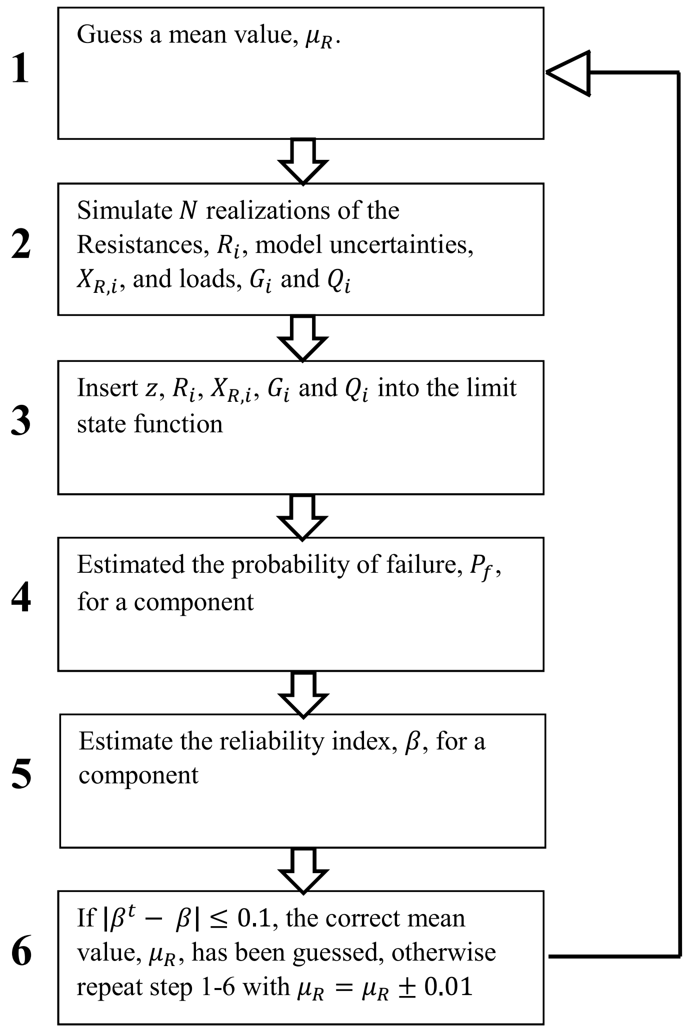

Figure 4.3: Algorithm for estimating μ_R corresponding to $\beta^t_{\text{comp}} = 4.3$ for a component, when using different $\mu_{R,i}$ and $V_{R,i}$.

The results for the mean values are listed in Table 4.3.

Table 4.3: Mean values of the resistances.

Parameter	Symbol	$\mu_{R,i}$
Resistance 1	R_1	1.82
Resistance 2	R_2	2.34

When $\mu_{R,i}$ for the stochastic models have been estimated, then $R_{k,i}$ can be determined as the 5%-quantile of the total resistance, RX_R. Both R and X_R are lognormal distributed, meaning that their product is likewise lognormal distributed.

Firstly, the lognormal statistical parameters, σ_{L,X_R} and μ_{L,X_R}, of X_R can be determined by:

$$\sigma_{L,X_R} = \sqrt{\ln(V_{X_R}^2 + 1)} \tag{4.6}$$

$$\mu_{L,X_R} = \ln(\mu_{X_R}) - \frac{1}{2}\sigma_{L,X_R}^2 \tag{4.7}$$

The only parameters in Equation (4.6) and (4.7) are V_{X_R} and X_R. The assumed value of X_R and V_{X_R} can be found in Table 4.2. Analogously, the lognormal statistical parameters, $\sigma_{L,R}$ and $\mu_{L,R}$, of R are determined from:

$$\sigma_{L,R} = \sqrt{\ln(V_R^2 + 1)} \tag{4.8}$$

$$\mu_{L,R} = \ln(\mu_R) - \frac{1}{2}\sigma_{L,R}^2 \tag{4.9}$$

The characteristic value, R_k, of the 5%-quantile of the resistance can be determined by the following:

$$\ln(x_{0.05}) = \mu_L - 1.645\sigma_L \tag{4.10}$$

The value -1.645 corresponds to the 5% quantile of the standard normal distribution. The mean value and standard deviation of the product of R and X_R are determined by:

$$\mu_L = \mu_{L,X_R} + \mu_{L,R} \tag{4.11}$$

$$\sigma_L = \sqrt{\sigma_{L,X_R}^2 + \sigma_{L,R}^2} \tag{4.12}$$

Through Equation (4.6) - (4.12) it is possible to determine R_k if μ and V of R and X_R are known. The results for R_k as well as the final stochastic models for R and X_R are shown in Table 4.4.

Table 4.4: Statistical parameters for the stochastic models of the resistances and model uncertainty.

Parameter	Symbol	Distribution type	μ	V	R_k
Resistance 1	R_1	Lognormal	1.82	0.10	1.49
Resistance 2	R_2	Lognormal	2.34	0.20	1.55
Model uncertainty	X_R	Lognormal	1.00	0.05	

4.6 Estimation of system reliability

Now that the appropriate stochastic models and limit state equation have been described, the procedure used to simulate and estimate the reliability index of a system, β_{sys}, will be explained.

As previously mentioned β^t is set to 4.3 and relates to a single component's safety. If more than one component engages with other components, it can be considered either a series, parallel system or a combination of both. For a series system, the corresponding $\beta_{\text{sys}}^{\text{s}}$ will be less than the components' β_{comp}^t, since the typical prerequisite of a series system is that if a single component fails, then failure will occur in the entire system. The opposite would most likely be the case for a parallel system, where the system reliability index, $\beta_{\text{sys}}^{\text{p}}$, usually is higher than that of the components due to mechanical behaviour and the requirement that all components fail.

The first step in this analysis is to get an idea of the current reliability level of the systems, both series and parallel system, before they are proof load tested. This is done with a varying amount of components and with varying uncertainties on the stochastic models related to the resistance, see the beginning of Chapter 4. A system consisting of 10, 50 and 100 components will be evaluated for the following cases:

Table 4.5: The cases that will be evaluated including the statistical parameters and uncertainties.

Cases	μ_R	V_R	$V(\mu_R)$	$V(V_R)$
1a	1.82	10 %	0 %	0 %
2a	1.82	10 %	15 %	7.5 %
3a	1.82	10 %	20 %	10 %
4a	1.82	10 %	30 %	15 %
1b	2.34	20 %	0 %	0 %
2b	2.34	20 %	15 %	7.5 %
3b	2.34	20 %	20 %	10 %
4b	2.34	20 %	30 %	15 %

These cases are denoted 1a-4b, where *a* indicates the first stochastic model for the resistance and *b* the other stochastic model, while 1-4 indicates the different uncertainties related to the statistical parameters.

The established stochastic models are used to estimate the corresponding system reliability level for the eight cases. For this analysis, the following prerequisites are used in regards to correlation:

- The statistical parameters, $\mu_{R,i}$ and $\mu_{R,i}$, are fully correlated, i.e. all components in a system have the same statistical parameters.
- Three different cases of correlation between resistances, R_i, of components in a system will be assessed. This includes an uncorrelated case and a case for which the correlation coefficient, ρ, is set equal to $\rho = 0.5$ and $\rho = 0.8$ respectively.
- The loads, G_i and Q_i, are assumed to be fully correlated in the systems, i.e. the load is the same for all components in the system.

The procedure for estimating the system reliability levels is described in the following three subsections for series systems and parallel systems respectively.

4.6.1 Series system

Firstly, series systems are considered. The prerequisite for this procedure is that a series system fails if any of its components fails. The simulation algorithm used to estimate $\beta^{\text{s}}_{\text{sys}}$ is similar to the procedure above, but with a few modifications.

1. $N = 10^7$ set of the stochastic parameters, $\mu_{R,i}$ and $V_{R,i}$, related to a batch of components are Monte-Carlo simulated, including the uncertainties $V(\mu_R)$ and $V(V_R)$ for each case.
2. $N \cdot n$ random realizations of the resistance, R_i, model uncertainty, $X_{R,i}$, and loads, G_i and Q_i, are simulated depending on the assumption for the correlation. n is the number of components in each batch/system. The theory for adding correlation between component resistances in a system is described in Appendix A.
3. The random simulated variables are inserted into Equation 4.1 and g is calculated for n components in N batches/systems. Failure in any component leads to failure of the system.
4. The probability of failure, P_f^{sys}, is estimated as the ratio between the amount of failed batches/systems and the number of simulations, N.
5. The reliability index, $\beta^{\text{s}}_{\text{sys}}$, is calculated by $\beta^{\text{s}}_{\text{sys}} = -\Phi^{-1}\left(P_f^{\text{sys}}\right)$.

By following the above-mentioned procedure, $\beta_{\text{sys}}^{\text{s}}$ corresponding to $\beta_{\text{comp}}^{t} = 4.3$ for no uncertainty can then be estimated. The results are listed in Tables 4.6 - 4.8 for a correlation coefficient of $\rho = 0$, $\rho = 0.5$ and $\rho = 0.8$ respectively.

Table 4.6: The reliability indices of the series system for the various of cases with varying n and $\rho = 0$.

Cases	$\beta_{\text{sys}}^{n=1}$	$\beta_{\text{sys}}^{n=10}$	$\beta_{\text{sys}}^{n=50}$	$\beta_{\text{sys}}^{n=100}$
1a	4.30	3.83	3.55	3.43
2a	3.57	3.10	2.82	2.71
3a	3.17	2.70	2.44	2.33
4a	2.45	2.05	1.81	1.72
1b	4.30	3.75	3.36	3.18
2b	3.72	3.13	2.72	2.55
3b	3.40	2.79	2.38	2.21
4b	2.78	2.17	1.80	1.65

Table 4.7: The reliability indices of the series system for the various of cases with varying n and $\rho = 0.5$.

Cases	$\beta_{\text{sys}}^{n=1}$	$\beta_{\text{sys}}^{n=10}$	$\beta_{\text{sys}}^{n=50}$	$\beta_{\text{sys}}^{n=100}$
1a	4.30	3.94	3.72	3.63
2a	3.57	3.20	3.00	2.93
3a	3.17	2.81	2.63	2.55
4a	2.45	2.14	1.98	1.92
1b	4.30	3.81	3.50	3.38
2b	3.72	3.23	2.92	2.80
3b	3.40	2.91	2.61	2.50
4b	2.78	2.31	2.06	1.96

Table 4.8: The reliability indices of the series system for the various of cases with varying n and $\rho = 0.8$.

Cases	$\beta_{\text{sys}}^{n=1}$	$\beta_{\text{sys}}^{n=10}$	$\beta_{\text{sys}}^{n=50}$	$\beta_{\text{sys}}^{n=100}$
1a	4.30	4.05	3.90	3.83
2a	3.57	3.32	3.19	3.14
3a	3.17	2.92	2.81	2.76
4a	2.45	2.24	2.15	2.11
1b	4.30	3.94	3.72	3.63
2b	3.72	3.36	3.16	3.09
3b	3.40	3.05	2.86	2.79
4b	2.78	2.47	2.31	2.24

The tables show a significant decrease in $\beta_{\text{sys}}^{\text{s}}$ when adding an uncertainty, $V(\mu_R)$ and $V(V_R)$. This is caused by an increase in the deviation of the resistances, meaning that more values in the lower tail of the distribution are obtained in the system. Because of the prerequisite for the series system that failure in a single component leads to total failure of the system, this means that there is a higher probability of failure. Moreover, this prerequisite is also a cause for the decrease in β^{s}_{sys} for an increasing n in the system. Furthermore, the results show that an increase in correlation between the resistance of components in a system consisting of more than one component result in larger β^{sys}, especially for $n = 100$. This agrees well with what is to be expected of a series system.

The influence of the correlation is also illustrated in Figure 4.4 for case 1a and 1b with $V(\mu_R) = 0$ and $V(V_R) = 0$. In the figure, it should be noted that case 1a for $\rho = 0.5$ and case 1b for $\rho = 0.8$ have the same curve by chance.

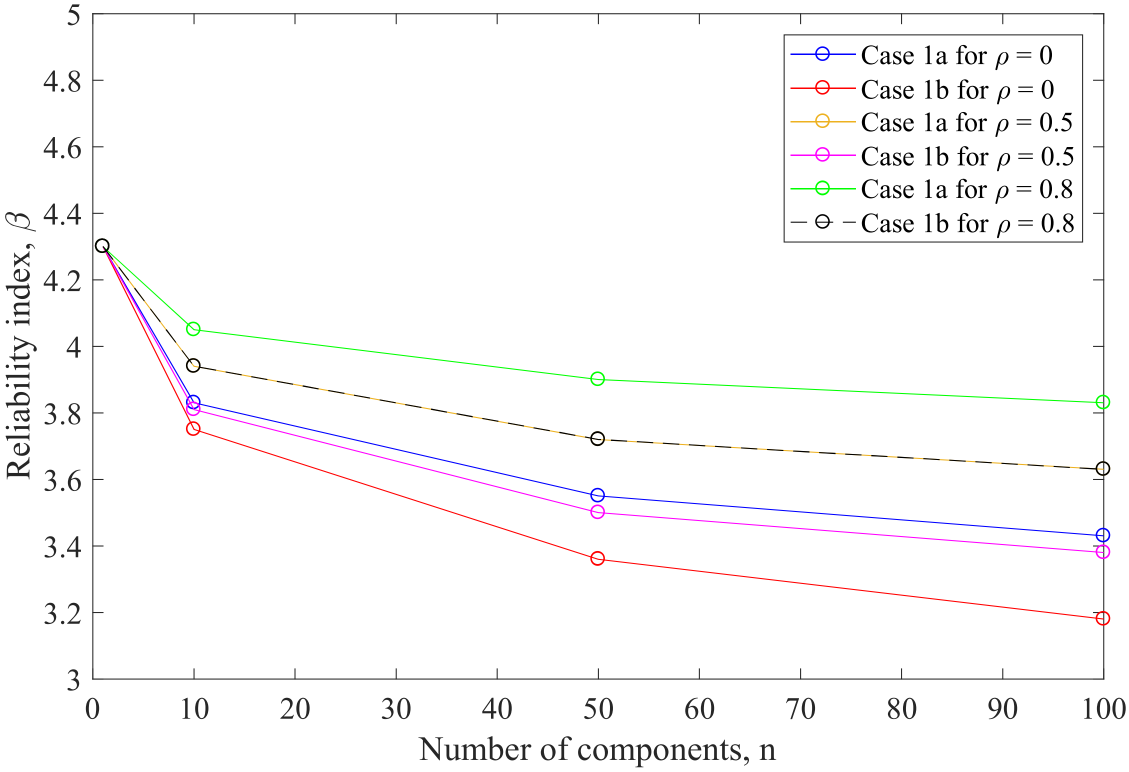

Figure 4.4: Illustration of β^{s}_{sys} for $V(\mu_R)$ and $V(V_R)$ as function of n components in a series system.

4.6.2 Parallel system - brittle

Secondly, parallel systems are considered. For this case, the parallel system is assumed brittle, meaning that each component in the system does not have any extra bearing capacity and fails when the load reaches the load-bearing capacity, i.e. resistance. This is illustrated by the stress-strain curve in Figure 4.5.

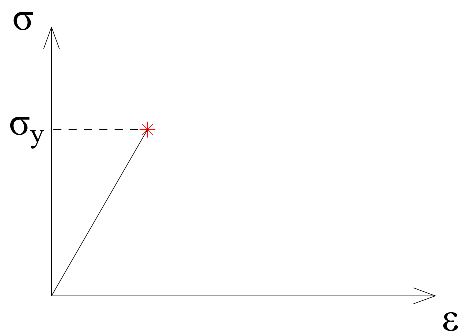

Figure 4.5: Stress-strain curve for a brittle material.

When a component fails, the load applied to the failed component is assumed to be uniformly distributed to the rest of the system. The simulation algorithm used for estimating $\beta_{\text{sys}}^{\text{p}}$ for a parallel brittle system is as follows:

1. $N = 10^7$ set of the stochastic parameters, $\mu_{R,i}$ and $V_{R,i}$, related to a batch of components are Monte-Carlo simulated, including the uncertainties $V(\mu_R)$ and $V(V_R)$ for each case.
2. $N \cdot n$ random realizations of the resistance, R_i, model uncertainty, $X_{R,i}$, and loads, G_i and Q_i, are simulated depending on the assumption for the correlation. n is the number of components in each batch/system. Correlation between the resistance of components in each system can be varied.
3. The random simulated variables are inserted into Equation 4.1 and g is calculated for n components in N batches/systems. If any component fails in a system, the applied load is uniformly distributed to the rest of the components. This process is repeated until no failure occurs in a component or the system fails.
4. The probability of failure, P_f^{sys}, is estimated as the ratio between the amount of failed batches/systems and the number of simulations, N.
5. The reliability index, $\beta_{\text{sys}}^{\text{p}}$, is calculated by $\beta_{\text{sys}}^{\text{p}} = -\Phi^{-1}\left(P_f^{\text{sys}}\right)$.

The results for $\beta_{\text{sys}}^{\text{p}}$ for the same cases as the series system are listed in Tables 4.9 - 4.11

Table 4.9: The reliability indices of the brittle parallel system for the various of cases with varying amount of components in the system and $\rho = 0$.

Cases	$\beta_{\text{sys}}^{n=1}$	$\beta_{\text{sys}}^{n=10}$	$\beta_{\text{sys}}^{n=50}$	$\beta_{\text{sys}}^{n=100}$
1a	4.30	3.93	3.85	3.81
2a	3.57	3.17	3.06	3.03
3a	3.17	2.76	2.64	2.61
4a	2.45	2.06	1.95	1.93
1b	4.30	4.37	4.32	4.33
2b	3.72	3.56	3.48	3.45
3b	3.40	3.14	3.04	3.00
4b	2.78	2.40	2.27	2.25

Table 4.10: The reliability indices of the brittle parallel system for the various of cases with varying amount of components in the system and $\rho = 0.5$.

Cases	$\beta_{\text{sys}}^{n=1}$	$\beta_{\text{sys}}^{n=10}$	$\beta_{\text{sys}}^{n=50}$	$\beta_{\text{sys}}^{n=100}$
1a	4.30	3.96	3.86	3.80
2a	3.57	3.22	3.12	3.09
3a	3.17	2.83	2.72	2.70
4a	2.45	2.15	2.05	2.03
1b	4.30	3.99	3.90	3.87
2b	3.72	3.37	3.26	3.24
3b	3.40	3.02	2.92	2.89
4b	2.78	2.49	2.40	2.38

Table 4.11: The reliability indices of the brittle parallel system for the various of cases with varying amount of components in the system and $\rho = 0.8$.

Cases	$\beta_{\text{sys}}^{n=1}$	$\beta_{\text{sys}}^{n=10}$	$\beta_{\text{sys}}^{n=50}$	$\beta_{\text{sys}}^{n=100}$
1a	4.30	4.04	3.93	3.91
2a	3.57	3.33	3.23	3.21
3a	3.17	2.93	2.85	2.82
4a	2.45	2.25	2.17	2.16
1b	4.30	3.95	3.83	3.84
2b	3.72	3.39	3.29	3.27
3b	3.40	3.08	2.98	2.96
4b	2.78	2.49	2.40	2.38

The results for $\beta_{\text{sys}}^{\text{p}}$ are slightly higher than for the series system, but the same tendencies are seen, when varying $V(\mu_R)$, $V(V_R)$ and changing n in a system. The similarities are due to the parallel system being considered brittle, which means that even though all components in the system have to fail for total failure of the system, the redistribution of the full load from the failed components often leads to a chain reaction of component failures. Thus, system failure might have a high probability to occur anyway due to an initial failure in a component. The opposite will most likely be the case for a ductile parallel system, which is assessed in the following. The influence ρ is also illustrated in Figure 4.6 for the cases with $V(\mu_R) = 0$ and $V(V_R) = 0$.

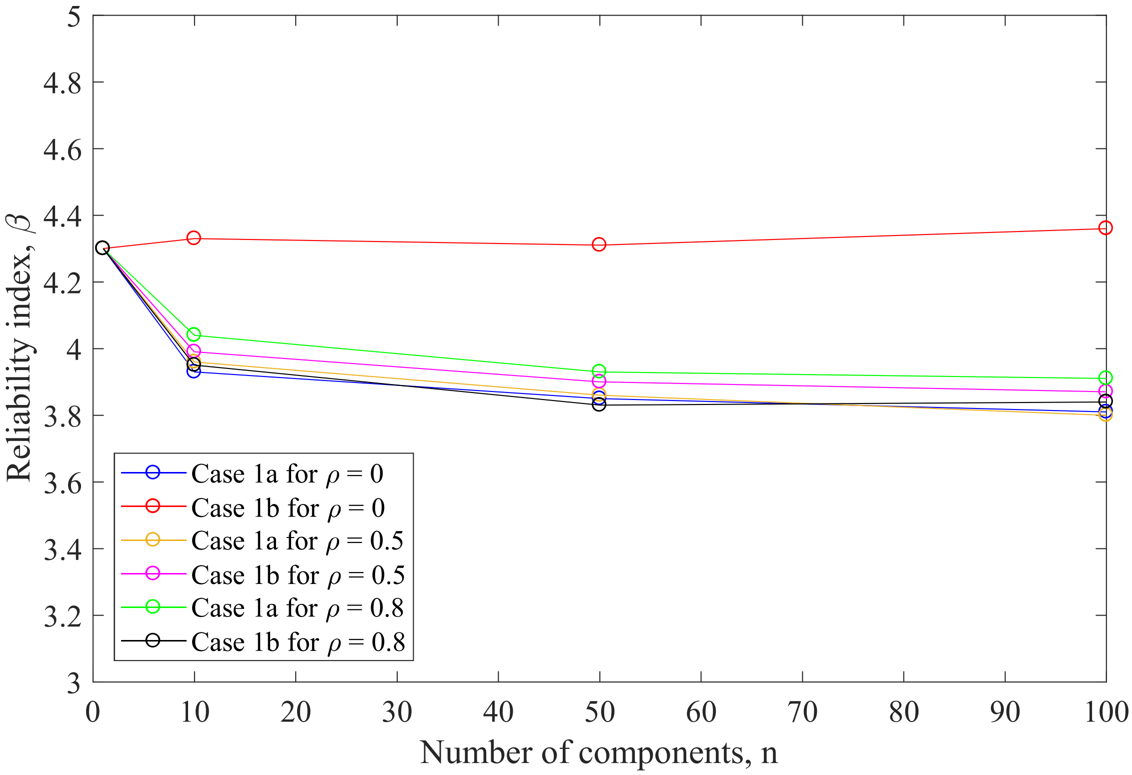

Figure 4.6: Illustration of $\beta_{\text{sys}}^{\text{p}}$ for $V(\mu_R) = 0$ and $V(V_R) = 0$ as function of n in a brittle parallel system.

The deviation between the curves for $\beta_{\text{sys}}^{\text{p}}$ is small compared to the series system. Some of the curves crosses each other, which might indicate that there is a minor instability in regards to the number of simulations. Furthermore, it is seen that case 1b for $\rho = 0$ deviates from the tendency of the other cases with $\beta^{\mathrm{p}}_{\mathrm{sys}}$ being almost constant regardless of n. This indicates that a high V_R is beneficial for a brittle parallel system, when there is no correlation between the resistance of components. In general, it is seen that $\beta^{\mathrm{p}}_{\mathrm{sys}}$ increases for an increasing ρ for $V_R = 10\%$, while the opposite is the case for $V_R = 20\%$.

4.6.3 Parallel system - ductile

The second type of parallel system that is considered is a ductile system. It is assumed that the components in this ductile parallel system behave as ideal plastic materials as illustrated by the stress-strain curve in Figure 4.7.

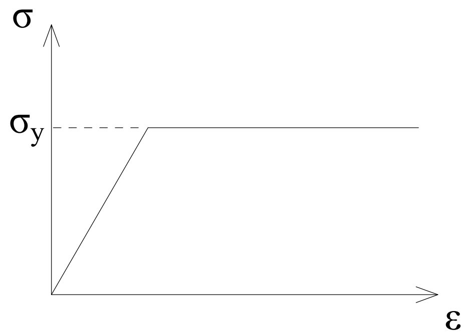

Figure 4.7: Stress-strain curve for an ideal plastic material.

When a component in the ductile parallel system is subjected to a load larger than the resistance, then the component will not fail. Instead the component will carry the load up to the resistance, while the residual load will be uniformly redistributed to the rest of the components in the parallel system. This means that the system only fails, if the load surpasses the resistance of all the components in the system. The simulation algorithm is similar to the one for brittle material with the exception of step 3 for which only the load surpassing the resistance of a component is redistributed to the remaining components. This analysis is only carried out for the cases with no uncertainty, i.e. 1a and 1b for the various ρ. The reason for this is that the ductile system is expected to have a very high $\beta^{\mathrm{p}}_{\mathrm{sys}}$, thus it is only necessary to confirm this hypothesis. The results are listed in Table 4.12, 4.13 and 4.14. It should be noted that $\beta^{\mathrm{p}}_{\mathrm{sys}} > 5.20$ means that no components fail in $N = 10^7$ simulations.

Table 4.12: The reliability indices of the ductile parallel system for the various of cases with varying n in the system and $\rho = 0$.

Cases	$\beta^{\mathrm{sys}}_{n=1}$	$\beta^{\mathrm{sys}}_{n=10}$	$\beta^{\mathrm{sys}}_{n=50}$	$\beta^{\mathrm{sys}}_{n=100}$
1a	4.30	4.83	>5.20	4.72
1b	4.30	>5.20	>5.20	>5.20

Table 4.13: The reliability indices of the ductile parallel system for the various of cases with varying n in the system and $\rho = 0.5$.

Cases	$\beta^{sys}_{n=1}$	$\beta^{sys}_{n=10}$	$\beta^{sys}_{n=50}$	$\beta^{sys}_{n=100}$
1a	4.30	4.55	4.65	4.58
1b	4.30	4.99	4.86	4.94

Table 4.14: The reliability indices of the ductile parallel system for the various of cases with varying n in the system and $\rho = 0.8$.

Cases	$\beta^{sys}_{n=1}$	$\beta^{sys}_{n=10}$	$\beta^{sys}_{n=50}$	$\beta^{sys}_{n=100}$
1a	4.30	4.43	4.37	4.46
1b	4.30	4.53	4.56	4.51

The results show that the ductile parallel system in general has a very high reliability level, especially when having no correlation between the resistance of the components as would have been expected. The basis reliability levels indicate that a ductile parallel system will have a satisfying safety and it is likely that a change of function or loading on an existing structure will not lead to failure. Thus, it will only be necessary to proof load test a single component in this ductile parallel system with a magnitude equal to the characteristic load requirement to validate the safety level. This means that the ductile parallel system will not be analyzed in the following Section 4.7.

4.7 Updated system reliability

The reliability level of a system can be updated by using information gained through proof load tests on a number of components in a system, see Appendix A.4. The general idea behind proof load testing is to apply a proof load on e.g. 5 components in a system of 25 identical components and observe if failure occurs in any component in the system. An observation of no failure means that β_{sys} can then be updated depending on the magnitude of the proof load. The limit state function used for proof load testing is expressed as the following:

$$g_{\text{test}} = RX_R - P_{pl} \qquad \text{where} \qquad P_{pl} = k_{\text{test}} P_k \tag{4.13}$$

where

P_{pl}	Proof load.
k_{test}	Multiplication factor to the characteristic load requirement.
P_k	Characteristic load requirement.

The characteristic load requirement is determined by a weighting between the permanent and variable load as expressed by:

$$P_k = G_k(1 - \alpha) + Q_k \alpha \tag{4.14}$$

The proof load is applied to a portion of the components in each system. The portion sizes that are used are 10%, 20%, 30%, 40% and 50% of the total numbers of components rounded to the following whole number respectively. The procedure for assessing the influence of proof load testing is as following and can be seen visualized in the flowchart in Figure 4.8.

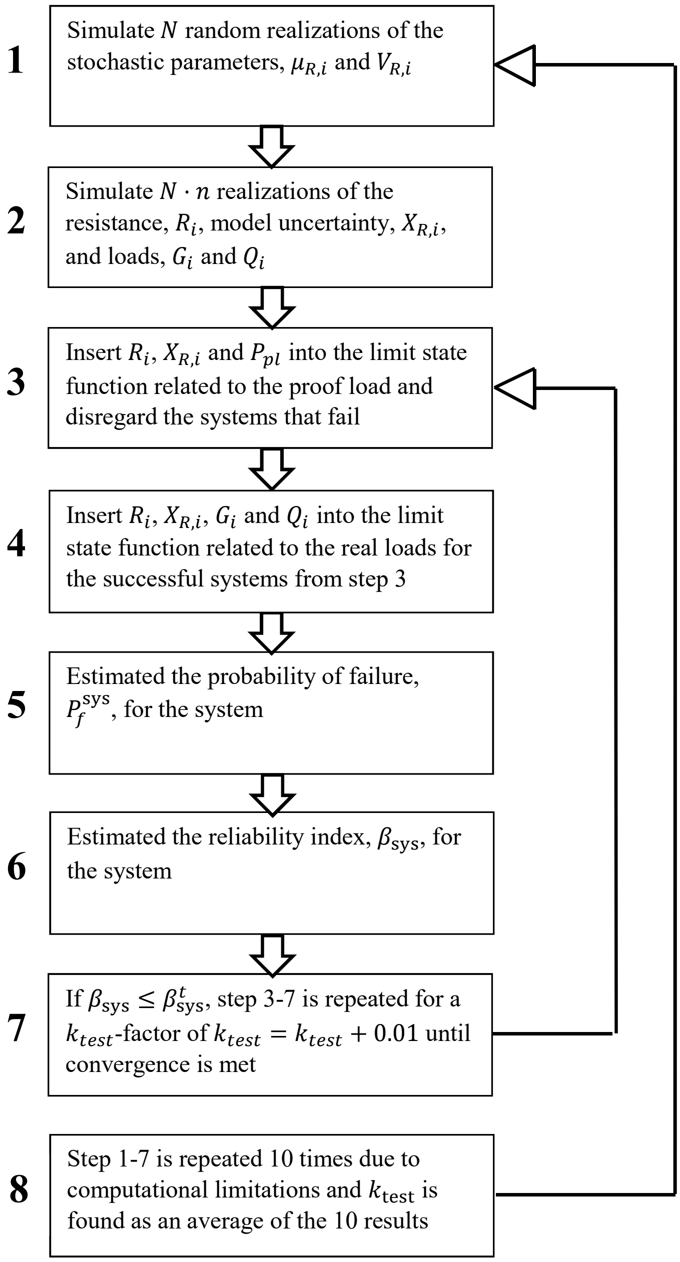

Figure 4.8: Algorithm for estimating the k_{test} corresponding to a targeted system reliability index, β_{sys}^t, when using different $V(\mu_R)$ and $V(V_R)$.

1. $N = 10^6$ random realizations of the stochastic parameters, $\mu_{R,i}$ and $V_{R,i}$ related to a system are Monte-Carlo simulated, including the uncertainties $V(\mu_R)$ and $V(V_R)$ for each case.
2. $N \cdot n$ random realizations of the resistance, R_i, model uncertainty, $X_{R,i}$, and loads, G_i and Q_i, are simulated for which n is the number of components in each batch/system. Correlation between the resistance of components in each system can be varied.
3. The simulated variables related to the resistance in step 2 and the proof load, P_{pl}, are placed into Equation 4.13 and g_{test} is calculated. $g_{\text{test}} < 0$ leads to failure and the failed systems are disregarded for the remaining procedure.

4. The real loads, G_i and Q_i, are now applied to the remaining systems from step 3 and Equation 4.1 is evaluated. The failure criterion is $g < 0$.
5. The probability of failure, P_f^{sys}, for a system is estimated as the ratio between the amount of failed systems and number of total systems that survived the proof load test.
6. The reliability index, β, is calculated by $\beta_{\text{sys}} = -\Phi^{-1}(P_f^{\text{sys}})$.
7. If $\beta_{sys} < \beta_{\text{sys}}^t$, step 3-7 is repeated for a k_{test}-factor of $k_{\text{test}} = k_{\text{test}} + 0.01$ until convergence is met.
8. Step 1-7 is repeat 10 times due to computational limitations in regards to memory when saving matrices in step 2 of more than $10^6 \cdot n$ in size. The k_{test}-factor is then found as an average of the 10 simulations.

4.8 Analyses

In this section, a number of analyses will be carried out for assessing the influence of proof load testing on the reliability level. This includes assessment of k_{test}-factors for the series system and brittle parallel system as well as sensitivity analyses on selected results.

4.8.1 Proof load test on series system

For the first analysis, the series system will be evaluated in order to determine the k_{test}-factors for the respective cases. The factors will be estimated for a varying amount of components and varying portion size of proof load tested components in the system. The number of components and test portions are described in Section 4.6 and 4.7 respectively.

The analysis will be carried out for all cases with an uncertainty related to the statistical parameters, μ_R and V_R, i.e. case 2a - 4b listed in Table 4.5. The reason for this is that proof load testing a portion of components in a system, when $V(\mu_R) = 0$ and $V(V_R) = 0$, does not provide information about the rest of the components. The k_{test}-factors will be calibrated to have a β_{sys}^t corresponding to $V(\mu_R) = 0$ and $V(V_R) = 0$, i.e. case 1a and 1b in Tables 4.6, 4.7 and 4.8 depending on the correlation between components in the system. For instance, $\beta_{\text{sys}}^t = 3.83$ is used for a series system consisting of 10 components in case 2a, 3a and 4a, when estimating the k_{test}-factors.

The procedure for estimating the k_{test}-factors are explained in Section 4.7 and the stochastic models are shown in Section 4.5. The results are listed in Table 4.15, 4.16 and 4.17 for $\rho = 0$, $\rho = 0.5$ and $\rho = 0.8$ respectively. The tables are generalized on the basis of the results obtained in Appendix C for the sake of clarity. Because of the generalization, the results have been made conservative, but more detailed results can be found in Tables C.1 - C.18.

Table 4.15: Test portions and k_{test}-factors for a various number of components, n, for a correlation of $\rho = 0$. Values inside the parenthesis () indicate the amount of series systems that succeeded the proof load testing.

Uncertainty related to μ_R and V_R	$V_R \leq 10\%$ (a-cases)	$10\% < V_R \leq 20\%$ (b-cases)
Small	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 43\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 58\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 67\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 67\%$)	10% and $k_{\text{test}} = 2.9$ if $n \geq 10$ ($\geq 18\%$) 10% and $k_{\text{test}} = 1.7$ if $n \geq 100$ ($\geq 48\%$) 50% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 59\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 71\%$)
Large	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 45\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 57\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 60\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 62\%$)	10% and $k_{\text{test}} = 3.0$ if $n \geq 10$ ($\geq 16\%$) 10% and $k_{\text{test}} = 1.7$ if $n \geq 100$ ($\geq 48\%$) 50% and $k_{\text{test}} = 1.8$ if $n \geq 10$ ($\geq 51\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 65\%$)
Very large	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 38\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 55\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 60\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 56\%$)	10% and $k_{\text{test}} = 3.3$ if $n \geq 10$ ($\geq 13\%$) 10% and $k_{\text{test}} = 1.7$ if $n \geq 100$ ($\geq 46\%$) 50% and $k_{\text{test}} = 1.8$ if $n \geq 10$ ($\geq 47\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 58\%$)

Table 4.16: Test portions and k_{test}-factors for a various number of components, n, for a correlation of $\rho = 0.5$. Values inside the parenthesis () indicate the amount of series systems that succeeded the proof load testing.

Uncertainty related to μ_R and V_R	$V_R \leq 10\%$ (a-cases)	$10\% < V_R \leq 20\%$ (b-cases)
Small	10% and $k_{\text{test}} = 1.6$ if $n \geq 10$ ($\geq 71\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 76\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq 79\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 79\%$)	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 81\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 88\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq 92\%$) 50% and $k_{\text{test}} = 1.2$ if $n \geq 100$ ($\geq 92\%$)
Large	10% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 64\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 71\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 72\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 73\%$)	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 72\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 83\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 86\%$) 50% and $k_{\text{test}} = 1.2$ if $n \geq 100$ ($\geq 88\%$)
Very large	10% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 58\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 63\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq 64\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 65\%$)	10% and $k_{\text{test}} = 2.1$ if $n \geq 10$ ($\geq 57\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 72\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 75\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 76\%$)

Table 4.17: Test portions and k_{test}-factors for a various number of components, n, for a correlation of $\rho = 0.8$. Values inside the parenthesis () indicate the amount of series systems that succeeded the proof load testing.

Uncertainty related to μ_R and V_R	$V_R \leq 10\%$ (a-cases)	$10\% < V_R \leq 20\%$ (b-cases)
Small	10% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq$ 82%) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq$ 85%) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq$ 85%) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq$ 85%)	10% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq$ 95%) 10% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq$ 96%) 50% and $k_{\text{test}} = 1.3$ if $n \geq 10$ ($\geq$ 97%) 50% and $k_{\text{test}} = 1.2$ if $n \geq 100$ ($\geq$ 97%)
Large	10% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq$ 76%) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq$ 79%) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq$ 78%) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq$ 79%)	10% and $k_{\text{test}} = 1.6$ if $n \geq 10$ ($\geq$ 90%) 10% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq$ 93%) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq$ 94%) 50% and $k_{\text{test}} = 1.2$ if $n \geq 100$ ($\geq$ 94%)
Very large	10% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq$ 67%) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq$ 69%) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq$ 68%) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq$ 69%)	10% and $k_{\text{test}} = 1.6$ if $n \geq 10$ ($\geq$ 81%) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq$ 84%) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq$ 85%) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq$ 86%)

From the results in Table 4.15, 4.16 and 4.17 it can be seen that the k_{test}-factor does not differ much for different $V(\mu_R)$ and $V(V_R)$, i.e. from case 2a to 4a and from case 2b to 4b respectively. It could have been expected that the k_{test}-factors should increase with an increasing $V(\mu_R)$ and $V(V_R)$, since Table 4.6 shows a decrease in β^{s}_{sys} pre-proof load testing with an increasing uncertainty. However, this indicates that more information is gained through proof load testing of a system with a larger uncertainty.

Furthermore, by comparing a-cases and b-cases it can be seen that a larger V_R generally results in larger k_{test}-factors, especially for a low value of ρ when proof load testing a smaller test portion of components in a system or when having a system consisting of few components. Nevertheless, the tendencies between a-cases and b-cases are similar. A large test portion and/or a large amount of components in a system results in a lower k_{test}-factor. The survival rate of the proof load testing has the opposite tendencies in comparison to the k_{test}-factors, i.e. increasing survival rate for larger test portions and amount of components in a system. This is the opposite of what would be expected, if the same proof load is applied, thus indicating that the decrease in k_{test}-factors out-weights the negative effect on the survival rate when increasing test portion and number of components. Furthermore, by comparing the results for different ρ, it can also be seen that an increase in ρ leads to a decrease in k_{test}-factors, which is to be expected for a series system.

The survival rate of proof load testing is an important factor to take into consideration, when deciding whether to perform the proof load testing or just reinforce the components instead. This aspect will be treated in relation to a decision model based on a cost-benefit analysis in Section 4.10.

4.8.2 Proof load test on brittle parallel system

The brittle parallel system is now considered in regards to estimating the k_{test}-factors for the respective cases. The procedure is similar to that for the series system with the exception of how failure of the system is modelled. Failure in a brittle parallel system occurs, if all the components fail, under the assumption that the full load applied to a component that fails is evenly redistributed to the remaining components.

The analysis is similarly to the series system carried out for all cases with an uncertainty $V(\mu_R) \neq 0$ and $V(V_R) \neq 0$. The k_{test}-factors are calibrated to have β^t_{sys} for case 1a and 1b listed in Table 4.9, 4.10 and 4.11 depending on ρ. The results are seen in Table 4.18, 4.19 and 4.20 for $\rho = 0$, $\rho = 0.5$ and $\rho = 0.8$ respectively.

Table 4.18: Test portions and k_{test}-factors for a varying n and $\rho = 0$. Values inside the parenthesis () indicate the amount of brittle parallel systems that succeeded the proof load testing.

Uncertainty related to μ_R and V_R	$V_R \leq 10\%$ (a-cases)	$10\% < V_R \leq 20\%$ (b-cases)
Small	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 44\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 59\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 66\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 61\%$)	10% and $k_{\text{test}} = 2.8$ if $n \geq 10$ ($\geq 24\%$) 10% and $k_{\text{test}} = 1.8$ if $n \geq 100$ ($\geq 43\%$) 50% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 64\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 56\%$)
Large	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 45\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 57\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 63\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 59\%$)	10% and $k_{\text{test}} = 3.0$ if $n \geq 10$ ($\geq 17\%$) 10% and $k_{\text{test}} = 1.7$ if $n \geq 100$ ($\geq 45\%$) 50% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 56\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 49\%$)
Very large	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 41\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 54\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 56\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 55\%$)	10% and $k_{\text{test}} = 3.0$ if $n \geq 10$ ($\geq 18\%$) 10% and $k_{\text{test}} = 1.7$ if $n \geq 100$ ($\geq 46\%$) 50% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 54\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 49\%$)

Table 4.19: Test portions and k_{test}-factors for a varying n and $\rho = 0.5$. Values inside the parenthesis () indicate the amount of brittle parallel systems that succeeded the proof load testing.

Uncertainty related to μ_R and V_R	$V_R \leq 10\%$ (a-cases)	$10\% < V_R \leq 20\%$ (b-cases)
Small	10% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 70\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 78\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 79\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 79\%$)	10% and $k_{\text{test}} = 1.8$ if $n \geq 10$ ($\geq 83\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 90\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq 93\%$) 50% and $k_{\text{test}} = 1.2$ if $n \geq 100$ ($\geq 92\%$)
Large	10% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 65\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 72\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 72\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 73\%$)	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 75\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 85\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq 87\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 87\%$)
Very large	10% and $k_{\text{test}} = 1.6$ if $n \geq 10$ ($\geq 59\%$) 10% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 70\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq 65\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 69\%$)	10% and $k_{\text{test}} = 2.0$ if $n \geq 10$ ($\geq 61\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 74\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 78\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 76\%$)

Table 4.20: Test portions and k_{test}-factors for a varying n and $\rho = 0.8$. Values inside the parenthesis () indicate the amount of brittle parallel systems that succeeded the proof load testing.

Uncertainty related to μ_R and V_R	$V_R \leq 10\%$ (a-cases)	$10\% < V_R \leq 20\%$ (b-cases)
Small	10% and $k_{test} = 1.5$ if $n \geq 10$ ($\geq$ 82%) 10% and $k_{test} = 1.4$ if $n \geq 100$ ($\geq$ 88%) 50% and $k_{test} = 1.4$ if $n \geq 10$ ($\geq$ 84%) 50% and $k_{test} = 1.3$ if $n \geq 100$ ($\geq$ 88%)	10% and $k_{test} = 1.5$ if $n \geq 10$ ($\geq$ 96%) 10% and $k_{test} = 1.2$ if $n \geq 100$ ($\geq$ 99%) 50% and $k_{test} = 1.3$ if $n \geq 10$ ($\geq$ 97%) 50% and $k_{test} = 1.1$ if $n \geq 100$ ($\geq$ 99%)
Large	10% and $k_{test} = 1.6$ if $n \geq 10$ ($\geq$ 74%) 10% and $k_{test} = 1.4$ if $n \geq 100$ ($\geq$ 80%) 50% and $k_{test} = 1.4$ if $n \geq 10$ ($\geq$ 78%) 50% and $k_{test} = 1.3$ if $n \geq 100$ ($\geq$ 81%)	10% and $k_{test} = 1.6$ if $n \geq 10$ ($\geq$ 91%) 10% and $k_{test} = 1.2$ if $n \geq 100$ ($\geq$ 96%) 50% and $k_{test} = 1.4$ if $n \geq 10$ ($\geq$ 94%) 50% and $k_{test} = 1.2$ if $n \geq 100$ ($\geq$ 96%)
Very large	10% and $k_{test} = 1.6$ if $n \geq 10$ ($\geq$ 66%) 10% and $k_{test} = 1.4$ if $n \geq 100$ ($\geq$ 72%) 50% and $k_{test} = 1.4$ if $n \geq 10$ ($\geq$ 69%) 50% and $k_{test} = 1.3$ if $n \geq 100$ ($\geq$ 71%)	10% and $k_{test} = 1.6$ if $n \geq 10$ ($\geq$ 82%) 10% and $k_{test} = 1.3$ if $n \geq 100$ ($\geq$ 88%) 50% and $k_{test} = 1.4$ if $n \geq 10$ ($\geq$ 86%) 50% and $k_{test} = 1.2$ if $n \geq 100$ ($\geq$ 90%)

The same tendencies as for the series system are seen for the brittle parallel system. However, the brittle parallel system results generally in slightly lower k_{test}-factors compared to the series system due to the assumption that all components in a system have to fail for total failure to occur. The survival rate is almost identical. Thus, it can be concluded that there is only a small difference between the brittle parallel system and the series system. The reason for this could be that the failure in a component in the parallel system might start a chain-reaction of components that fail due to redistribution of the load.

4.9 Sensitivity analysis

This section will evaluate the sensitivity of various important parameters used for obtaining the results in Section 4.8. This includes a sensitivity analysis of the mean value, μ_R, and the weighting factor, α. The sensitivity analyses will only be performed for case 2a and 2b in the uncorrelated cases, $\rho = 0$, since analyzing a selection of the results in regards to sensitivity is sufficient for assessing the influence of the parameters.

4.9.1 Influence of μ_R

Firstly, the influence of μ_R is evaluated. The reason for this is that μ_R is the uncertain parameter, when dealing with an existing structure, and thus a parameter of importance. The results in Section 4.8 are obtained on the basis of the assumption that μ_R in the stochastic models can be estimated by having it correspond to $\beta^t_{comp} = 4.3$. This implies that the structure initially has a satisfying resistance, which might not have been the case. Therefore, it is of interest to evaluate the influence of the results for k_{test}-factors, when decreasing and increasing μ_R. This sensitivity analysis will be performed for a 10% decrease and increase of μ_R. The results are shown in Table 4.21 for which the k_{test}-factors corresponds to β_{sys} in Table 4.6 for case 1a and 1b respectively, i.e. the k_{test}-factors for 0.9μ, 1.0μ and 1.1μ correspond to the same β_{sys}.

Table 4.21: Comparison between k_{test}-factors and survival rate when having different mean values, μ_R.

Scaling of the mean value, μ_R	$V_R \leq 10\%$ (a-cases)	$10\% < V_R \leq 20\%$ (b-cases)
$0.9 \cdot \mu_R$	10% and $k_{\text{test}} = 2.1$ if $n \geq 10$ ($\geq 11\%$) 10% and $k_{\text{test}} = 1.6$ if $n \geq 100$ ($\geq 25\%$) 50% and $k_{\text{test}} = 1.6$ if $n \geq 10$ ($\geq 31\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 32\%$)	10% and $k_{\text{test}} = 3.5$ if $n \geq 10$ ($\geq 2\%$) 10% and $k_{\text{test}} = 1.8$ if $n \geq 100$ ($\geq 16\%$) 50% and $k_{\text{test}} = 2.0$ if $n \geq 10$ ($\geq 19\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 33\%$)
$1.0 \cdot \mu_R$	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 43\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 58\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 67\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 67\%$)	10 % and $k_{\text{test}} = 2.9$ if $n \geq 10$ ($\geq 18\%$) 10 % and $k_{\text{test}} = 1.7$ if $n \geq 100$ ($\geq 48\%$) 50% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 59\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 71\%$)
$1.1 \cdot \mu_R$	10% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 85\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 89\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq 93\%$) 50% and $k_{\text{test}} = 1.2$ if $n \geq 100$ ($\geq 94\%$)	10% and $k_{\text{test}} = 2.2$ if $n \geq 10$ ($\geq 72\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 86\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 92\%$) 50% and $k_{\text{test}} = 1.2$ if $n \geq 100$ ($\geq 95\%$)

The sensitivity analysis shows a small decrease in k_{test}-factors for $0.9\mu_R$ and opposite a small increase k_{test}-factors for $1.1\mu_R$ for obtaining the same reliability level, when having $V_R \leq 10\%$. For $10\% < V_R \leq 20\%$ a larger decrease and increase is seen when testing a small portion of components in a system consisting of few components. As a result, the survival rate when having 0.9μ is very low, especially for a large V_R. The opposite is seen for 1.1μ, where the survival rate is very high and satisfying. Thus, the results are sensitive to a variation of μ_R.

Instead of estimating the k_{test}-factors corresponding to the same reliability level when having different μ_R it might also be interesting to evaluate a case in which β^{s}_{sys} are estimated for different μ_R when using the k_{test}-factors for $1.0\mu_R$. This corresponds to a situation, where μ_R is thought to be $1.0\mu_R$, but in reality is either lower or higher, meaning that the system will have a different reliability level than β^{t}_{sys}. These β^{s}_{sys} for different μ_R are shown in Table 4.22 and 4.24.

Table 4.22: Reliability indices for $0.9\mu_R$.

Cases	$\beta^{n=10}_{\text{sys}}$	$\beta^{n=50}_{\text{sys}}$	$\beta^{n=100}_{\text{sys}}$
2a	3.70	3.48	3.38
2b	3.53	3.21	3.05

Table 4.23: Reliability indices for $1.0\mu_R$.

Cases	$\beta^{n=10}_{\text{sys}}$	$\beta^{n=50}_{\text{sys}}$	$\beta^{n=100}_{\text{sys}}$
2a	3.83	3.55	3.43
2b	3.75	3.36	3.18

Table 4.24: Reliability indices for $1.1\mu_R$.

Cases	$\beta_{\text{sys}}^{n=10}$	$\beta_{\text{sys}}^{n=50}$	$\beta_{\text{sys}}^{n=100}$
2a	4.03	3.77	3.65
2b	3.97	3.57	3.40

The results show that the reliability level becomes lower for a smaller μ_R and opposite, higher for a larger μ_R, which is what would have been expected. Furthermore, it can be seen that an increase in μ_R has a bigger influence on the reliability level than a decrease in μ_R, especially for case 2a with the smaller V_R.

4.9.2 Influence of α

The weighting factor, α, is another parameter that might have a significant influence on the k_{test}-factors for obtaining β_{sys}^t in Table 4.6. Therefore it is interesting to analyze the sensitivity of α, since it describes the weight between the two load variables that have very different stochastic models. All previous analyses have been performed based on the assumption that $\alpha = 0.5$. However, this is a rough assumption that is not always the case as $\alpha = 0.5$ was initially used as an average assessed value. This sensitivity analysis will be performed for $\alpha = 0.3$ and $\alpha = 0.8$, i.e. a high weighting on the permanent and variable load respectively. The results are listed in Table 4.25.

Table 4.25: Comparison between k_{test}-factors and survival rate when having different weighting factors, α.

Weighting factor, α	$V_R \leq 10\%$ (a-cases)	$10\% < V_R \leq 20\%$ (b-cases)
0.3	10% and $k_{\text{test}} = 1.8$ if $n \geq 10$ ($\geq 56\%$) 10% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 67\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 10$ ($\geq 79\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 75\%$)	10% and $k_{\text{test}} = 3.2$ if $n \geq 10$ ($\geq 10\%$) 10% and $k_{\text{test}} = 1.8$ if $n \geq 100$ ($\geq 39\%$) 50% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 47\%$) 50% and $k_{\text{test}} = 1.4$ if $n \geq 100$ ($\geq 62\%$)
0.5	10% and $k_{\text{test}} = 1.9$ if $n \geq 10$ ($\geq 43\%$) 10% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 58\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 10$ ($\geq 67\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 67\%$)	10% and $k_{\text{test}} = 2.9$ if $n \geq 10$ ($\geq 18\%$) 10% and $k_{\text{test}} = 1.7$ if $n \geq 100$ ($\geq 48\%$) 50% and $k_{\text{test}} = 1.7$ if $n \geq 10$ ($\geq 59\%$) 50% and $k_{\text{test}} = 1.3$ if $n \geq 100$ ($\geq 71\%$)
0.8	10% and $k_{\text{test}} = 2.4$ if $n \geq 10$ ($\geq 8\%$) 10% and $k_{\text{test}} = 1.7$ if $n \geq 100$ ($\geq 25\%$) 50% and $k_{\text{test}} = 1.8$ if $n \geq 10$ ($\geq 22\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 32\%$)	10% and $k_{\text{test}} = 3.4$ if $n \geq 10$ ($\geq 6\%$) 10% and $k_{\text{test}} = 1.9$ if $n \geq 100$ ($\geq 30\%$) 50% and $k_{\text{test}} = 2.0$ if $n \geq 10$ ($\geq 32\%$) 50% and $k_{\text{test}} = 1.5$ if $n \geq 100$ ($\geq 49\%$)

Table 4.25 show that for the k_{test}-factors becomes smaller for $\alpha = 0.3$ in the a-cases and opposite larger for $\alpha = 0.8$. A different tendency is seen for the b-cases for which both $\alpha = 0.3$ and $\alpha = 0.8$ result in larger k_{test}-factors. This indicates that the relation between α and the k_{test}-factors is non-linear with an optimum/minimum point between the two α-values. Though, in general it is seen that $\alpha = 0.8$ results in larger k_{test}-factors, which means that a large weight on the variable load is more critical. This is caused by the large V on the variable load despite the permanent load having a higher μ in the stochastic models described in Section 4.5. Thus, V_R of the stochastic load has a significant influence on the results.

4.10 Decision model

The decision for whether to perform proof load testing or not usually comes down to an economical point of view, although the practical aspect also is of importance, since it might not always be possible to perform proof load tests. A decision model can be established to provide an overview of different decisions and actions that can be taken. The decision model is illustrated in Figure 4.9 and is based on the prerequisite that the endpoint of each branch in the model corresponds to a fixed target reliability level for a component. In the analyses in this report, $\beta^t_{\text{comp}} = 4.3$ has been used. Furthermore, the cost related to collapse of the structure for $\beta^t_{\text{comp}} = 4.3$ is assumed to be very small and thus negligible, and the proof load tests are assumed to be carried out such that failure in a component does not lead to collapse of structure.

The decision model constitutes of an initial decision to replace all components in the system with new components that corresponds to a desired β^t or proof load test a number of components in the system. Replacement of components is usually an expensive solution, why the decision to proof load test might be the preferable choice of action. The decision to perform proof load testing leads to an event, where failure is observed or not. If not, then the system has the required β^t_{comp} given the prerequisites about the resistance hold true. If failure is observed, then the structure does not have the required β^t_{comp}. The question then becomes whether to replace the components or to continue performing tests. If it is decided to continue testing, then the failed component firstly has to be replaced. An option is to repeat the proof load testing, but this requires that the analysis in this chapter is repeated with the prerequisite that the simulation in Section 4.7 only disregards system for which more than one component fail. If failure is observed again, this process can be repeated again and so on. This is not investigated in this report and therefore it is assumed that failure in one component results in all components having to be replaced. This is a strict rule, but the analyses in this report can not account for these situations. Another option is to proof load test a few components to failure in order to obtain information about the resistance. This can be used in addition to the information already obtained from the previously failed component and hereby estimate how much the components must be reinforced. If the components are made of concrete, other test methods like cylinder and CAPO-tests can be used for this estimation. The analysis

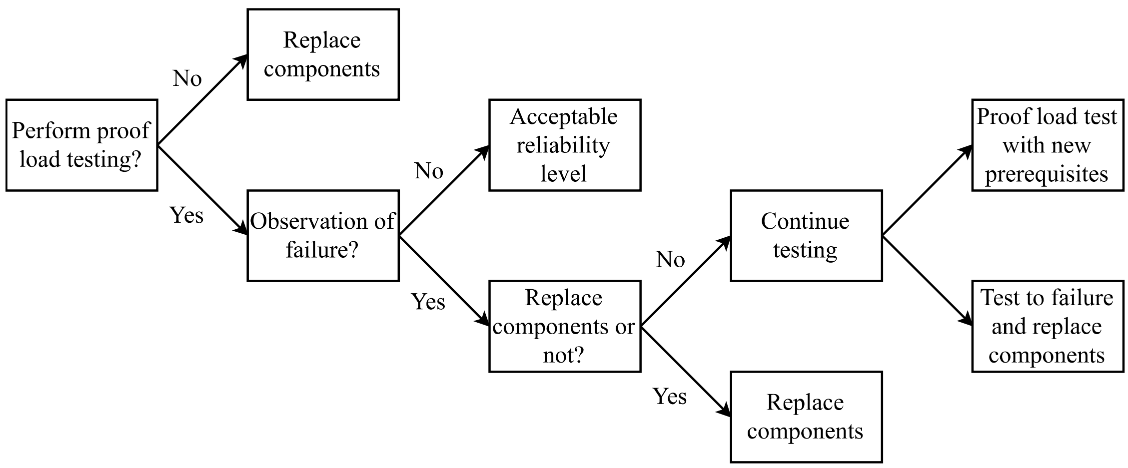

Figure 4.9: Decision model for reassessment of the reliability level for an existing structure through proof load testing with prior information. The endpoint of a branch corresponds to a fixed target reliability level, $\beta^t_{\text{comp}} = 4.3$.

then becomes similar to that described in Chapter 6 that studies how the reliability level can be updated by observation of the stochastic variable that is the resistance. However, this is not further investigated in this report.

An analysis of the cost related to each decision will be performed to provide a basis for choosing between the different solutions for updating the reliability level of an existing structure. The comparison will be between the decision to replace the components in a system immediately or to proof load test a portion of the components based on the average cost of these. The analysis in this report will not take basis in the correct costs of the different options for updating the reliability level, since they are difficult to obtain given that they depend on a lot of parameters such as the present market values, the man hours used for performing the solutions etc. Instead a generic analysis will be performed for which a ratio is specified between the cost of proof load testing and replacement of components in order to illustrate the principle behind the decision problem. The cost of collapse for the fixed $\beta^t_{\text{comp}} = 4.3$ is assumed to be negligible and thus not listed. The following costs are chosen for the analysis, where E indicates a unit cost and each cost is for testing and replacing of a single component.

- Proof load testing, C_{pl}: 1 E per component
- Replacement of components, C_{re}: 10 E per component

Proof load testing a single component in a system corresponds to a unit cost, E, while replacement of components is 10 times the unit cost per component. The replacement of components has been chosen to have a significantly larger cost than proof load testing, since that is to be expected in reality. This assumption will probably result in the decision to perform proof load test usually being less expensive.

Example

The specified cost and results for the survival rate of systems obtained in Section 4.8 can be used in the decision model in Figure 4.9 to determine the optimal. An example of this is shown in Figure 4.10 for a series system consisting of 100 components for which 10% of the components are proof load tested. The survival rates of the test is taken for a case with a small uncertainty related to μ_R and V_R for $V_R = 10\%$ and $\rho = 0$ as seen in Table 4.15, which is 58%. The last part of the decision model is not included in the analysis of cost, since the analysis required for the continuation of testing has not been investigated in this report. Thus, observation of failure during the initial proof load testing results in all the components having to be replaced.

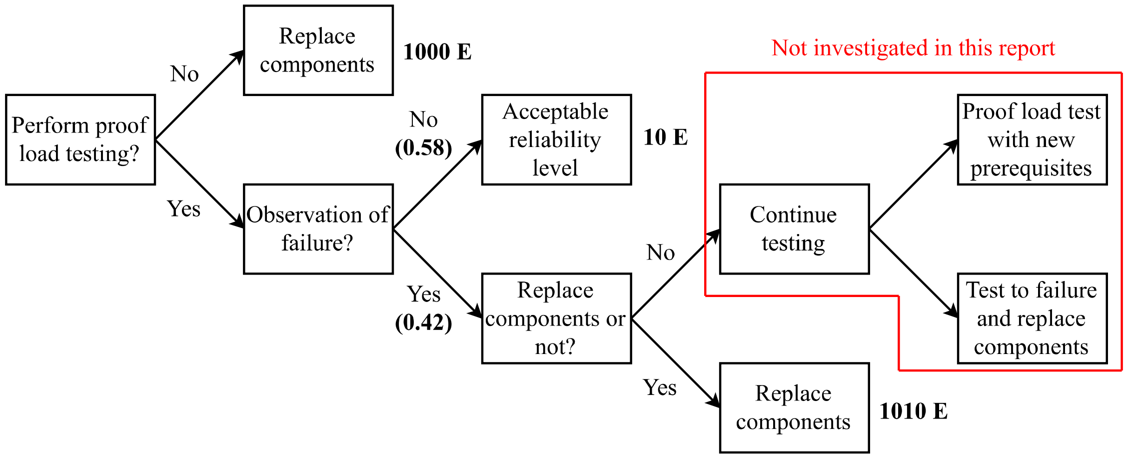

Figure 4.10: Example of a decision tree for reevaluating the reliability level of an existing structure.

The decision between replacing the components in the system or performing proof load testing from an economical point of view depends on the average cost of the options. These are calculated in the following:

Replacing the components: 1000 E

Proof load testing: $0.58 \cdot 10\text{ E} + 0.42 \cdot 1010\text{ E} = 426\text{ E}$

For this particular case, proof load testing has a significantly less average cost for updating the reliability level. Based on the survival rates, the ratio between the cost of the two options for when the average cost is equal to each other can be estimated to $C_{\mathrm{pl}}/C_{\mathrm{re}} = 5.8$. This means that the proof load tests have to be significantly more expensive than replacement of the components.
This analysis should be repeated for different test portions of component and compared to determine the optimal solution. The survival rates for a test portion of 20%, 30%, 40% and 50% are listed in Table C.1 in Appendix C for case 2a with $\rho = 0$. By repeating the above procedure, the following costs in Table 4.26 are estimated.

Table 4.26: Comparison between cost of option A and B for different test portions for case 2a with a coefficient correlation of $\rho = 0$.

Test portion	k_{test}	Survival rate	Replacing components	Proof load testing (replacement)
10%	1.46	58%	1000 E	426 E
20%	1.39	61%	1000 E	402 E
30%	1.35	64%	1000 E	379 E
40%	1.32	65%	1000 E	376 E
50%	1.30	67%	1000 E	364 E

In this case, the optimal solution in regards to the cost is to proof load test 50% of the 100 components. The results show that performing proof load tests in general becomes less expensive, if more components are tested, which is due to the survival rate increasing with the size of test portion and the fact that the specified cost for proof load tests in comparison to replacement of components is small. The decreasing cost could also mean that it is beneficial to proof load test more than 50% of the components, but this has not been investigated. The increase in survival rate is caused by the decrease in k_{test}-factors that are used for the proof load. If ρ is increased, the survival rate will tend to become more constant for all test portions, meaning that a lower test portion will probably be the optimal solution. Thus, it is important to assess different test portions of components.

In reality, an assessment of the resistance has to be made through a preliminary inspection and rating, and if this assessment fits the prerequisites that are assumed about the resistance in this chapter, then the optimal solution for updating the reliability level can be estimated by using the results obtained in this chapter and Appendix C, given that the correct costs are known.

Chapter

5 Design Based on Sampling Through Testing

This chapter describes how to perform cylinder compression tests and CAPO-tests. Furthermore, the model uncertainty related to CAPO-tests is assessed in regards to various of calculation models, which will be used in a cost-benefit analysis between the two test methods in Chapter 6.

When having to reevaluate the reliability level of existing structures, it may be very important to obtain new or additional information about the resistance of the structure. This information can be obtained through testing of components in regards to measuring the strength and updating the stochastic variables through Bayesian updating of variables, see Appendix A.4. In general, two types of tests are distinguished between, which are destructive and non-destructive tests. The cylinder compression test is a destructive test method, while CAPO and LOK-tests are examples of non-destructive test methods, despite being partly destructive methods. A more detailed description of the test methods will be described in the following sections. LOK-testing, which is very similar to CAPO-testing, will not be investigated in this study.

5.1 Cylinder test

A cylinder test is a commonly used destructive method for concrete structures and lightweight concrete structures for directly measuring the compressive strength of the structure. This test method is used for both existing structures and new structures still in the design phase. When performing destructive tests, they have to be performed in accordance with the relevant codes or other relevant documented instructions. Moreover, when extracting test samples, it has to be ensured that it does not reduce the functionality of the structure or if so that the functionality can be reestablished. Similarly, the consequences of potential damage on the structure as well as possible structural failures during testing have to be assessed, before performing a destructive test. [SBI 251, 2015] For existing structures, cores are bored out directly from the structure with a typical dimension of 150 $\times$ 300 mm for diameter and height respectively. It is recommended to bore a large number of core samples in different locations on the structure in order to evaluate the compressive strength of the concrete with a high precision. When boring out the cores as well as transporting them to the laboratory, it has to be ensured that the structures and cores receive minor damage. [FPrimeC Solutions Inc., 2016] The extracted cylinder of the structure is then tested in a laboratory for estimating the characteristic values of the resistance in accordance with Annex D7.2 in DS/EN 1990 [2007].

5.2 CAPO-test

The CAPO-test is a partly destructive testing method that can be performed on existing structures without having pre-installed inserts in the specimen. The test is regarded as a non-destructive test, since the test is intended to be carried out with almost no consequence in terms of damage and cost. The test makes use of a pullout system as illustrated in Figure 5.1 for indirectly estimating the compressive strength of concrete. It is a fast and cheap alternative to core testing and does not require much space, but yields a larger uncertainty.

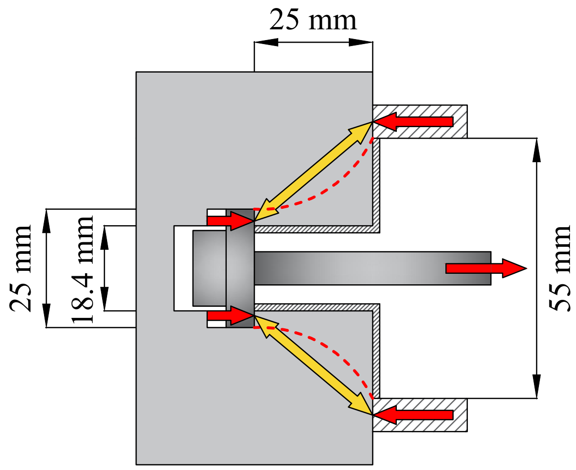

Figure 5.1: Cross-sectional view of a CAPO-test. [Germann Instruments, 2018]

The procedure for performing a CAPO-test is as follows [Germann Instruments, 2018]:

- The pullout system is placed on the specimen with an inner diameter of 55 mm between the reinforcement bars at the desired location. When doing this, it has to be ensured that the reinforcement bars are not within the failure region of the concrete.
- A 65 mm deep hole is cored with a diameter of 18.4 mm perpendicular to the surface of the specimen.
- The surface is levelled to a depth of approximately 3 mm, since an irregular surface will provide invalid results.
- A recess with a diameter of 25 mm is drilled at a depth of 25 mm.
- A split ring is expanded and placed in the recess, while a counter pressure ring is placed between the reinforcement bars on the surface.
- The expanded ring is pulled out by applying a tension force. When performing the test, the concrete in between the expanded ring and counter pressure ring will be in compression. Thus, the pullout force is directly related to the compressive strength of the concrete.

5.2.1 Theoretical models

It is important to note that the CAPO-test result does not equal the compressive strength of concrete, meaning that a correlation between the pullout force and compressive strength of the concrete has to be established for the CAPO-test to be applicable. Various of models for correlating these have been developed during recent decades through a number of studies. These models will be presented in the following with the purpose of carrying out a statistical assessment of the models in relation to experimental data.

A commonly used method takes its basis in Equation (5.1), which relates the pullout force to the compressive strength of 150 × 300 mm cylinders. [Germann Instruments, 2018]

$$f_{\mathrm{cyl}} = 0.69F^{1.12} \tag{5.1}$$

where

f_{cyl}	Compressive strength of cylinder [MPa]
F	Maximum pullout force [kN]

Another common model is the one found in BS EN 13791 [2007] where the compressive strength, $f_{c,EN}$, is expressed as following:

$$f_{c,EN} = 1.33(F - 10) \qquad 10\text{kN} \leq F \leq 60\text{kN} \tag{5.2}$$

The last model that will be presented is the best linear fit to the experimental data, which will have the following appearance:

$$f_c = aF + b \tag{5.3}$$

where

a	Parameter estimated by linear least squares method, $a = 1.18$	[MPa/kN]
b	Parameter estimated by linear least squares method, $b = -3.40$	[MPa]

5.2.2 Experimental data

The experimental data which will be used in regards to the statistical assessment of the theoretical models is based on the results in Appendix E and is shown in Figure 5.2. Each point in the data is an average of 2-9 cylinders and 4-24 CAPO-test performed on the vertical faces of accompanying 200 mm cubes.

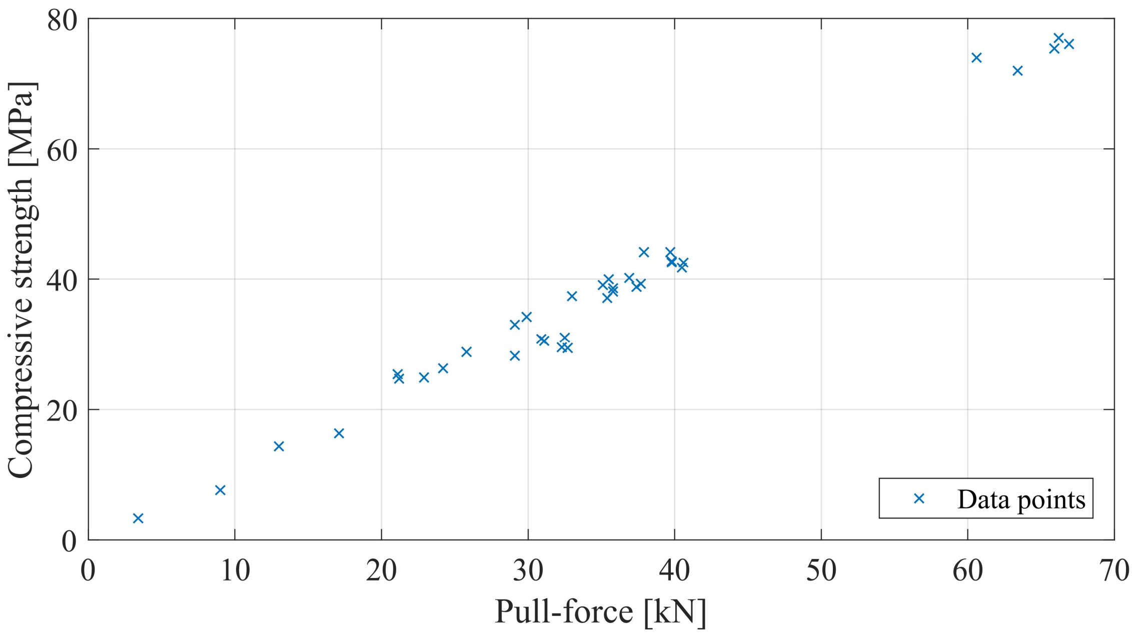

Figure 5.2: Experimental data

5.2.3 Statistical assessment

The procedure used to perform the statistical assessment is based on the procedure found in DS/EN 1990 [2007] Annex D - Design assisted by testing to estimate the mean, μ, and the coefficient of variation, V, of the model uncertainty. The procedure will be presented in the following.

The model is rewritten as:

$$Y = f(\mathbf{X}) \cong b\Delta h(\mathbf{X}) \tag{5.4}$$

where

b	Bias corresponding to the mean value of F_c
Δ	Δ modelled by a lognormal distributed stochastic variable with mean 1 and standard deviation σ_Δ
$h(\mathbf{X}$	Theoretical model, see Equations (5.1) - (5.3)

It is assumed that the parameters $F_i, ..., F_n$ are included directly in the model $h(X)$.

Using the experimental results in Appendix E and assuming that the test results are statistically independent, then the bias, b, is estimated using a linear least squares method:

$$b = \frac{\sum_{i=1}^{N} y_i h(\mathbf{x}_i)}{\sum_{i=1}^{N} h(\mathbf{x}_i)} \tag{5.5}$$

where

y_i	Experimental value
$h(\mathbf{x}_i)$	Theoretical value

Realizations of the lognormally distributed variable F_c for each test is obtained from:

$$\Delta_i = \ln\left(\frac{y_i}{bh(\mathbf{x}_i)}\right) \tag{5.6}$$

The standard deviation, σ_Δ, is estimated as:

$$s_\Delta = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\Delta_i - \bar{\Delta}\right)} \tag{5.7}$$

where $\bar{\Delta}$ is the mean:

$$\bar{\Delta} = \frac{1}{N}\sum_{i=1}^{N}\Delta_i \tag{5.8}$$

The corresponding coefficient of variation of the model uncertainty is found from:

$$V_\Delta = \sqrt{\exp\left(s_\Delta^2\right) - 1} \tag{5.9}$$

By following this procedure the model uncertainty related to the various of theoretical models can be estimated. In Figure 5.3 the different calculation models are plotted with respect to the data.

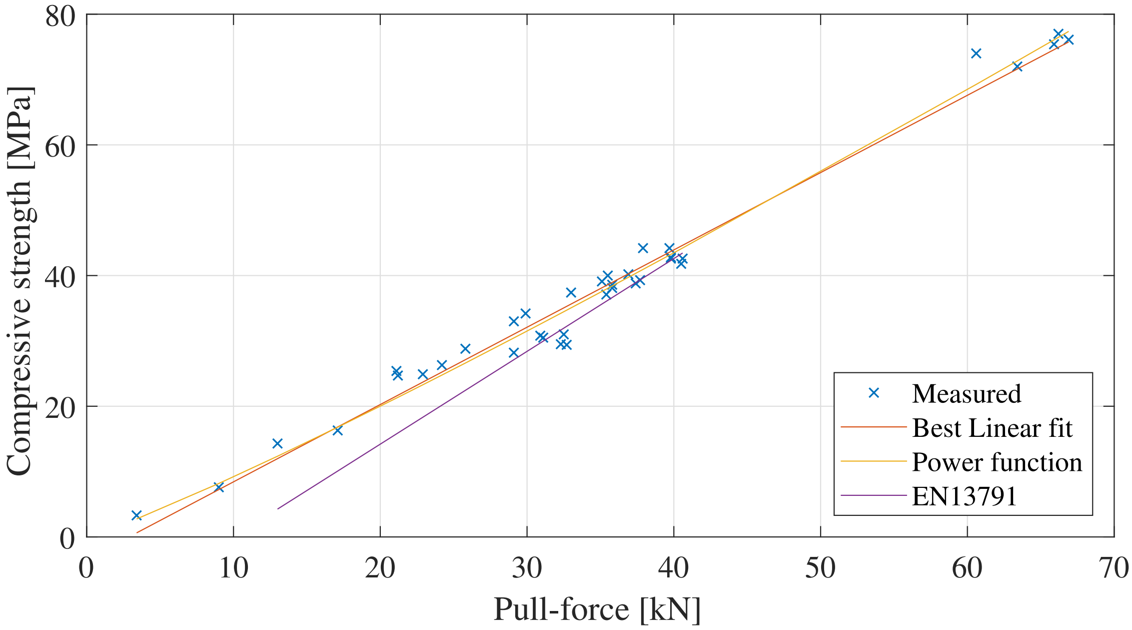

Figure 5.3: Calculation models versus experimental data

5.2.4 Results

By using the procedure described above for each of the three theoretical models, see Equation 5.1-5.3, the results shown in Table 5.1 are obtained.

Table 5.1: Statistical parameters of the model uncertainty for the various of theoretical models.

Model	Bias (Mean value), μ	Coefficient of variation, V
Best linear fit	1.00	0.29
Power function	1.01	0.08
EN13791:2007	1.07	0.26

What can be said about the bias in the table above for the various of calculation models, except the best linear fit, is that they generally produce a conservative result, which can also be seen on Figure 5.3. The best linear fit's bias is expected to be 1, since the model is calibrated from a least squares method of the data. In relation to the cost-benefit analysis that will be carried out in the next section, the model with the smallest V is chosen, which in this case is the power function.

Chapter

6 Cost-Benefit Analysis Between Cylinder and CAPO-tests

In this chapter the purpose is to perform a cost-benefit analysis of cylinder and CAPO-tests and compare these in different situations. The cost-benefit is part of a decision model for choosing the optimal solution in regards to cost when having to assess the reliability level of an existing structure. This chapter makes use of results obtained in Chapter 5.

For existing structures, cylinder and CAPO tests can be performed to obtain information about the resistance of the components in the structure. This information can be used in combination with prior or without prior information to estimate the stochastic variables. Based on the information obtained, the reliability level of the structure can then be assessed and updated if necessary by reinforcing the component to an acceptable reliability level. The decision whether to perform one test or the other can be chosen based on an economical point of view, where a cost-benefit analysis is carried out for the different tests. A cost-benefit analysis is a tool used for calculating the total expected cost of carrying out a chosen strategy. The cost-benefit analysis will depend on the type of test, amount of performed tests and the information gained from performing the individual test. The results can then be used in a decision model to determine the optimal solution in regards to cost.

The planning of tests should follow the general guidelines/requirements described in Annex D, DS/EN 1990 [2007]. This includes prerequisites, that requires the data to be representative, statistically homogeneously and statistically independent.

6.1 Assessment model: Preliminary inspection and rating

An assessment model related to a preliminary inspection and rating of the resistance of an existing structure is established. The reason behind this assessment model is to give a qualitative guess about the resistance, since the decision whether to perform cylinder or CAPO tests based on a cost-benefit analysis will depend on the actual resistance of the structure. When prior information is available, this rating of the resistance will be less uncertain than a case for which no prior information is available. Depending on the rating, different actions can be taken and these are illustrated in the assessment model in Figure 6.1 similar to that in Chapter 4.

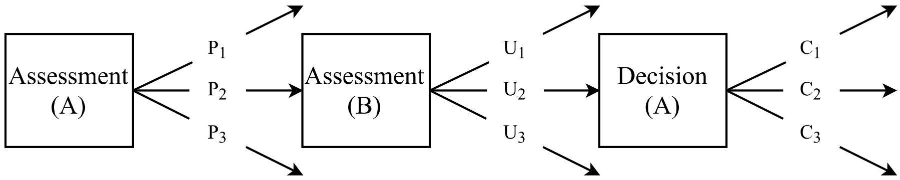

Figure 6.1: Decision model that takes basis in a preliminary inspection and rating of the resistance and describes the different decisions that have to be made, when the reliability level of an existing structure has to be reevaluated.

Assessment (A) is a rating of the resistance of the structure, either based on prior information or no prior information. Cylinder tests might be better for a given resistance and CAPO tests for another, but without giving a qualitative rating of the resistance, the decision then becomes completely random. However, the cost-benefit analysis that will be carried out in this chapter might prove that one of the test types in general is less expensive, since the analysis is made for several different cases. Assessment (B) is an assessment of an uncertainty related to the rating of the resistance. Based on the rating of resistance and uncertainty, decision (A) is whether to perform cylinder or CAPO tests based on a cost-benefit analysis.

The prerequisites about the resistance and uncertainty that are used for the cost-benefit analysis in this chapter are described in Section 6.2.

Case study
The following case study can be considered for when cylinder and CAPO tests can be used for assessment of the reliability level of an existing structure. The reliability level of a concrete structure consisting of a number of components from a homogeneous population has to be reevaluated due to an increase in loads. This could be due to a change of function of the structure, for instance an office building has to be changed into a fitness center, meaning new, heavier interior and perhaps more screed is added.

There are two different scenarios to consider if it is not immediately chosen to replace the load-bearing components, which is an expensive solution and can be difficult to perform, if the components act as a series system. Either the tests are performed and it is discovered that the characteristic strength of the concrete, R_k is satisfying or it is discovered that the components have to be reinforced or replaced. If R_k is satisfying, it means no further action has to be taken.

6.2 Prerequisites

The aim of this chapter is to perform a cost-benefit analysis considering the two testing methods that can be used in relation to a decision model, when assuming no prior information is available, hence having a diffuse prior. In regards to the uncertainty related to the resistance, the uncertainty is assumed classified in the following levels assuming that all parameters are lognormally distributed:

- Small for $V(\mu_R) \leq 15\%$ and $V(V_R) \leq 7.5\%$
- Large for $15\% < V(\mu_R) \leq 20\%$ and $7.5\% < V(\mu_R) \leq 10\%$
- Very large for $20\% < V(\mu_R) \leq 30\%$ and $10\% < V(\mu_R) \leq 15\%$

Furthermore, it is assumed that there cannot be performed more tests than the amount of components in a system. The mechanical function of the system (series or parallel system) is not included in this study. A limit state equation with concrete compression strength is considered, and the stochastic models are described in Section 6.5.

6.3 Reliability level

The target reliability level used for this analysis is the same as described in Section 4.3, i.e. $\beta^t = 4.3$.

6.4 Limit state equation

The limit state equation used for this analysis is the same as the one described in Section 4.4.

6.5 Stochastic models

Loads

The stochastic load model is the same as the one described in Section 4.5.

Resistance and model uncertainties

The stochastic resistance model is specifically for in-situ concrete in compression and is shown in Table 6.1 Sørensen [2009]. The characteristic value, R_k is estimated for the 5%-quantile of the total resistance, RX_R Sørensen [2009]. The model uncertainty related to the CAPO-test is the one chosen from Section 5.2.4 for the power function because of its small V. This model uncertainty is assumed representative for all CAPO tests in this analysis. The additional model uncertainty related to the conversion to concrete strength is taken into account when simulating realizations from a CAPO-test.

Table 6.1: Stochastic models of the resistances and model uncertainties for in-situ concrete in compression.

Parameter	Symbol	Distribution type	V	R_k
Resistance	R	Lognormal	0.14	5 %-quantile
Model uncertainty, cylinder	X_R	Lognormal	0.11	
Model uncertainty, CAPO	$X_{R,\mathrm{CAPO}}$	Lognormal	0.08	

The mean, μ_R, and R_k of R will be estimated corresponding to $\beta^t_{\mathrm{comp}} = 4.3$. The mean values are estimated by a Monte-Carlo simulation for which the theory can be found in Appendix A. The result for μ_R is listed in Table 6.2.

Table 6.2: Mean values of the resistance.

Parameter	Symbol	Mean value, μ_R
Resistance	R	2.15

When μ_R for the stochastic model have been estimated, R_k can then be estimated. The resistance and model uncertainty are lognormally distributed, meaning that their product is likewise lognormally distributed. Therefore, the same procedure as described in Section 4.5.2 is applied.

The results for R_k as well as the final stochastic model for the resistance and model uncertainties are shown in Table 6.3.

Table 6.3: Statistical parameters for the stochastic model of the resistance and model uncertainties.

Parameter	Symbol	Distribution type	μ	V	Characteristic value
Resistance	R	Lognormal	2.15	0.14	1.52
Model uncertainty, cylinder	X_R	Lognormal	1.00	0.11	
Model uncertainty, CAPO	$X_{R,CAPO}$	Lognormal	1.01	0.08	

It is important to keep in mind that this obtained characteristic strength is the minimum value needed to obtain a satisfying reliability level of the component of $\beta^t = 4.3$.

6.6 Updating of the characteristic strength

When no prior information is available and the characteristic strength, R_k, of a component has to be estimated through tests, then R_k can be estimated from Equation (6.1), which corresponds to having an unknown coefficient of variation of the resistance, V_R. [DS/EN 1990, 2007]

$$R_k = \bar{X} - k_n S \tag{6.1}$$

where

$\bar{X}$	Mean value of test results
k_n	Factor that accounts for statistical uncertainty.
S	Standard deviation of test results

The mean value and standard deviation of the resistance are estimated from realizations, $\hat{x}$ on a number of components by Equation (6.2) and (6.3):

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n}\hat{x}_i \tag{6.2}$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\hat{x} - \bar{X})^2 \tag{6.3}$$

The realization, $\hat{x}$, are from the total distribution RX_R or RX_RX_{CAPO} depending on the test type that is being evaluated. The k_n-factors are found in Table 6.4 and takes into account the statistical uncertainty due to a limited number of tests.

Table 6.4: Values of k_n for the 5% characteristic value. [DS/EN 1990, 2007]

n	1	2	3	4	5	6	8	10	20	30	∞
V_{X} unknown	-	-	3.37	2.63	2.33	2.18	2.00	1.92	1.76	1.73	1.64

6.7 Simulation algorithm

A simulation algorithm that takes basis in the stochastic models and equation for estimating the characteristic value from test results will now be explained. The simulation algorithm is used to produce results of the characteristic value of components in a system based on a number of cylinder or CAPO-tests. These results will then be used in a cost-benefit analysis to determine the optimal number of tests to perform in regards to total cost in different cases. The procedure is as following:

1. A sensitivity analysis with respect to the assumed value of μ_R is investigated, why the following values of μ_R will be considered: $\mu_R = 0.6\mu_R$, $\mu_R = 0.8\mu_R$ and $\mu_R = 1.0\mu_R$ respectively.
2. Simulate N random realizations of the stochastic parameters, $\mu_{R,i}$ and V_{R_i}, for each value of $\mu_{R,i}$ depending on the chosen level of uncertainty, $V(\mu_R)$ and $V(V_R)$.
3. Simulate $N \cdot n$ realizations from the total distribution RX_R or RX_RX_{CAPO} depending on the test type that is being evaluated. The realizations in each system are correlated by having the same distribution function.
4. Estimate the mean, $\bar{X}$ and standard deviation, S, of each test series
5. Each of the test series characteristic strengths are calculated from Equation (6.1).
6. The estimated characteristic strengths are compared to the necessary one estimated earlier to be $R_{k,\text{necessary}} = 1.52$
7. The possibility of obtaining an characteristic value smaller than the necessary is estimated for every value of $\mu_{R,i}$ for various amount of tests and chosen uncertainty levels. This possibility is denoted P_{R}.
8. The average amount of reinforcement needed is expressed as the average, ratio between $R_{k,\text{necessary}}/\bar{R}_k$. This is the value, z, used in regards to the cost-benefit analysis.
9. The average amount of reinforcement needed when having to reinforce is expressed as the average ratio between $R_{k,\text{necessary}}/\bar{R}_k$ for $R_{k,\text{necessary}} > R_k$. This is the value, z_{T}, used in regards to calculating the expected cost for the alternatives in a decision model, see Figure 6.2. The 'T' indicates that the values are truncated.

The results for the characteristic values and average amount of reinforcement needed to obtained the β^t_{comp} is used in a cost-benefit analysis. This cost-benefit analysis is part of the decision model illustrated in Figure 6.2. The decision model is based on the prerequisites that the endpoint of each branch corresponds to a fixed β^t_{comp} and that cylinder and CAPO-tests are always performed without damaging the structure, meaning that collapse is not considered a risk when performing the tests.

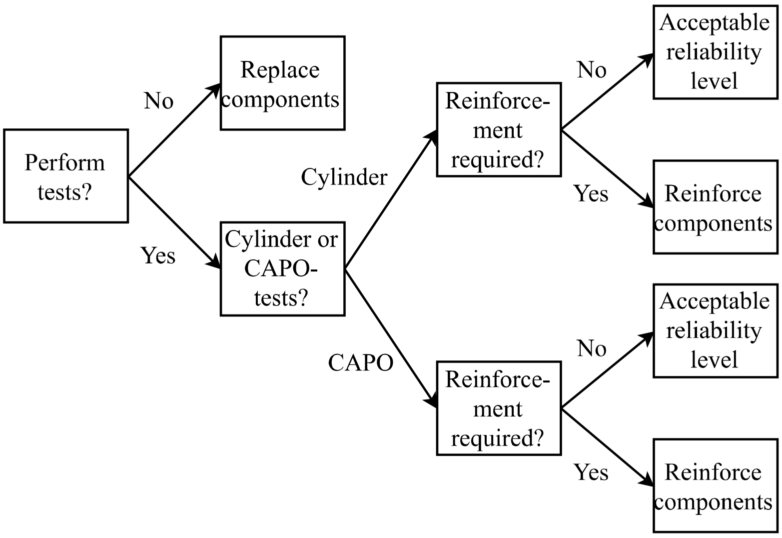

Figure 6.2: Decision model for reassessment of the reliability level for an existing structure by performing cylinder and CAPO-tests. The branches corresponds to a fixed β^t_{comp}.

The decision model constitutes of an initial decision to immediately replace components in a system with new components that corresponds to β^t_{comp} or perform cylinder or CAPO-tests with the objective to update the reliability level to β^t_{comp} if necessary. Based on a cost-benefit analysis between cylinder and CAPO-tests, the optimal method can be used for assessing the current reliability level. If $\beta_{\text{comp}} \geq \beta^t_{\text{comp}}$, then no further action has to be performed. If $\beta_{\text{comp}} < \beta^t_{\text{comp}}$, then the components must be reinforced to correspond to β^t_{comp}, given that the reinforcement is less expensive than replacing the components. A cost-benefit analysis will be carried out for cylinder and CAPO-tests in the following.

6.8 Cost-benefit analysis

A cost-benefit analysis is performed as part of the decision model for the cylinder and CAPO-tests in order to assess the optimal method in regards to total expected cost. The expected cost that is used for each solution in the decision model will depend on the choice of μ_R, level of uncertainty, test method, amount of tests and amount of components in the system considered. The correct costs have not been obtained for the analysis in this report. Instead the analysis is carried out for the following unit costs, E:

- CAPO-test, C_{CAPO}: 1 E per test
- Cylinder test, C_{cyl}: 5 E per test
- Reinforcement, C_z: 50 E per component for $z > 1$
- Replacement, C_{re}: 500 E per component

Based on the expected costs listed above, the total expected cost for cylinder and CAPO-tests can be estimated by Equation (6.4) and (6.5) respectively.

$$C_{\text{tot,cyl}} = n(C_{\text{cyl}} + (z-1)C_z) \tag{6.4}$$

$$C_{\text{tot,CAPO}} = n(C_{\text{CAPO}} + (z-1)C_z) \tag{6.5}$$

This analysis is based on average values for the expected cost of reinforcement needed, meaning that the probability of having to reinforce the components is not considered in the cost-benefit analysis. However, this will later be taken into account, when giving an example of expected cost in the decision model.

The cost-benefit analysis will be carried out for a system with varying mean value, μ_R, uncertainties, $V(\mu_R)$ and $V(V_R)$, number of components, n, and correlation coefficient, ρ. The following cases are analyzed:

- Mean value: $\mu_R = 0.6\mu_R$, $\mu_R = 0.8\mu_R$ and $\mu_R = 1.0\mu_R$
- Uncertainty: Small, large and very large, see Section 6.2
- Number of components: $n = 10$, $n = 50$ and $n = 100$
- Coefficient correlation: $\rho = 0$, $\rho = 0.5$ and $\rho = 0.8$

All results obtained for the cost-benefit analysis is graphically shown in Appendix D. In the following, a few selected cases are presented to show the influence of the different parameters. The case for non-correlated simulated realizations, a small level of uncertainty and a system consisting of 100 components will be used as reference for assessing the influence of the different parameters. Relevant results obtained for the analysis of this case are shown in Figure 6.3 and Table 6.5 and 6.6.

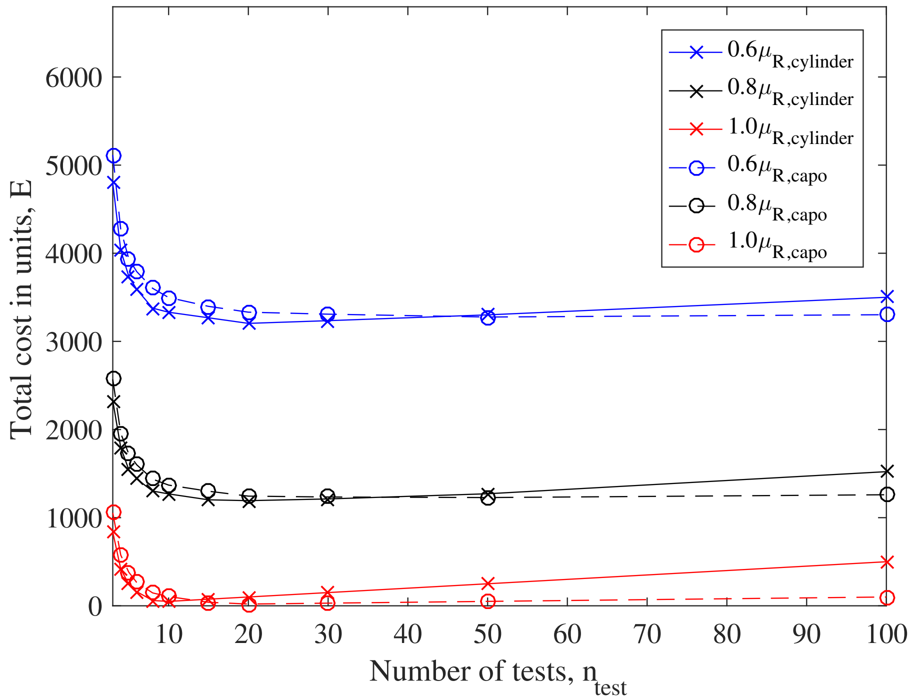

Figure 6.3: Cost-benefit analysis of cylinder vs CAPO-tests for total average expected cost.

Table 6.5: Optimal amount of tests and minimal average expected cost for the two test types.

Test type	Optimal amount of tests		Average minimum expected cost	
	Cylinder	CAPO	Cylinder	CAPO
$0.6\mu_R$	20	50	3205	3277
$0.8\mu_R$	20	50	1193	1228
$1.0\mu_R$	10	20	50	20

Table 6.6: Corresponding average reinforcement values and probability of having to reinforce.

	z		z_T		P_R	
Type type	Cylinder	CAPO	Cylinder	CAPO	Cylinder	CAPO
$0.6\mu_R$	0.63	0.66	0.63	0.66	99.9	99.8
$0.8\mu_R$	0.22	0.24	0.26	0.27	90.8	92.5
$1.0\mu_R$	0	0	0.16	0.14	50.6	54

It is seen from Figure 6.3 and Table 6.5 that the cylinder test is the most cost-beneficial for small values of μ_R. For an increasing resistance the CAPO test tends towards becoming more cost-beneficial than cylinder tests based on the specified costs. The curves tend to become linear after a certain amount of tests performed, which is due to the cost of performing the tests starting to heavily out-weight the benefits from performing additional tests. Furthermore, the results show that the cylinder test reaches its optimum at a lower amount of performed tests than CAPO-tests, which indicates that more information is obtained per test as would be expected due to the additional model uncertainty for CAPO-tests. Additionally, Figure 6.3 indicates that a lot of information in comparison to cost is obtained when increasing the amount of components tested from a low amount. The difference in expected cost between $0.6\mu_R$ and $1.0\mu_R$ is significant, but this is caused by no reinforcement being needed for $1.0\mu_R$ on average and reinforcement of existing structure being very costly.

6.8.1 Variation of components in a system

In this analysis, the same analysis is carried out with the exception of the amount of components in a system being changed to 10 components, i.e. small uncertainty, $n = 10$ and $\rho = 0$. Results are shown in Figure 6.4 and Table 6.7 and 6.8.

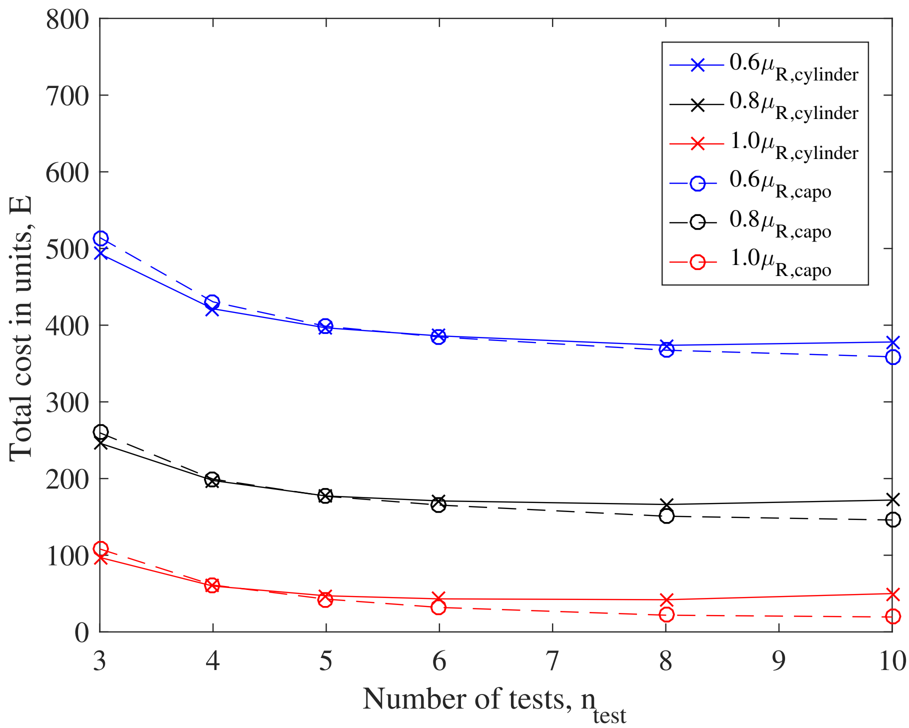

Figure 6.4: Cost-benefit analysis of cylinder vs CAPO-tests for total average expected cost.

Table 6.7: Optimal amount of tests and minimal average expected cost for the two test types.

Test type	Optimal amount of tests		Average minimum expected cost	
	Cylinder	CAPO	Cylinder	CAPO
$0.6\mu_R$	8	10	374	359
$0.8\mu_R$	8	10	166	146
$1.0\mu_R$	8	10	42	19.5

Table 6.8: Corresponding average reinforcement values and probability of having to reinforce.

	z		z_T		P_R	
Type type	Cylinder	CAPO	Cylinder	CAPO	Cylinder	CAPO
$0.6\mu_R$	0.70	0.70	0.70	0.71	99.8	99.9
$0.8\mu_R$	0.24	0.27	0.32	0.29	90.57	91.8
$1.0\mu_R$	0	0	0.17	0.17	54.0	57.1

Figure 6.4 and Table 6.8 show that CAPO-tests tend to become more cost beneficial than cylinder tests for the all three cases, when the amount of components in the system is decreased from 100 to 10. The cylinder test reaches its optimum for less tests than the amount of components, while the minimum cost for CAPO-tests is obtained by testing all components. This indicates that there is a possibility for the real optimum to be located for $n_{\text{test}} > 10$. Due to the assumption made in this analysis that only one test per component can be performed, this optimum will not be estimated.

6.8.2 Variation of uncertainty

This analysis is carried out for a very large uncertainty, $n = 100$ and $\rho = 0$. The results are shown in Figure 6.5 and Table 6.9 and 6.10.

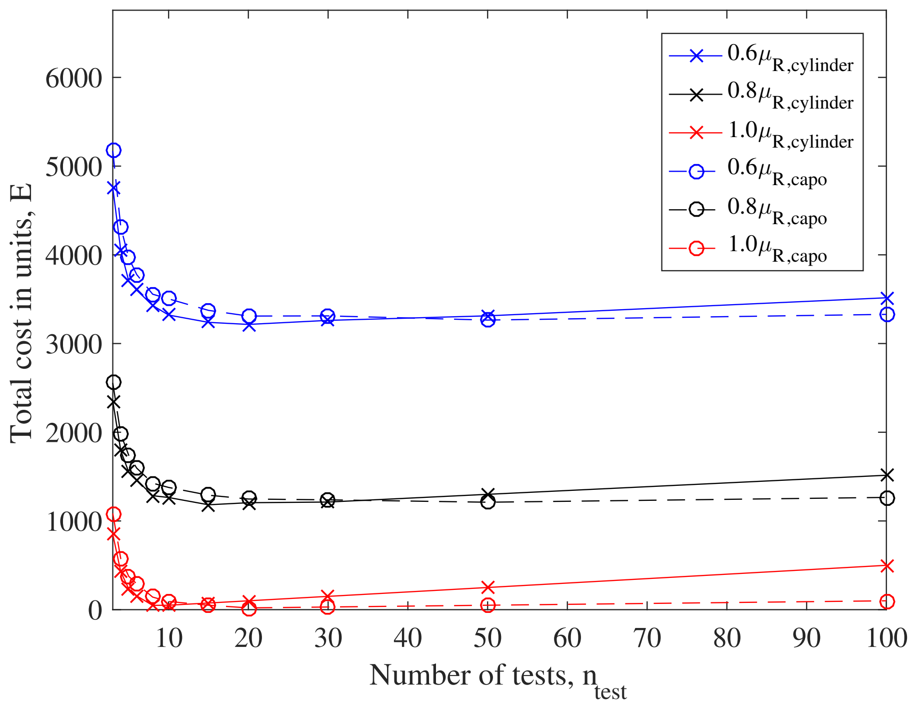

Figure 6.5: Cost-benefit analysis of cylinder vs CAPO-tests for total average expected cost.

Table 6.9: Optimal amount of tests and minimal average expected cost for the two test types.

Test type	Optimal amount of tests		Average minimum expected cost	
	Cylinder	CAPO	Cylinder	CAPO
$0.6\mu_R$	20	50	3208	3298
$0.8\mu_R$	20	50	1194	1202
$1.0\mu_R$	10	20	50	20

Table 6.10: Corresponding average reinforcement values and probability of having to reinforce.

	z		z_T		P_R	
Type type	Cylinder	CAPO	Cylinder	CAPO	Cylinder	CAPO
$0.6\mu_R$	0.65	0.66	0.72	0.72	96.0	96.6
$0.8\mu_R$	0.19	0.23	0.36	0.40	78.8	80.6
$1.0\mu_R$	0	0	0.29	0.26	55.9	55.2

By increasing the level of uncertainty from small to very large uncertainty, the tendencies are similar to that of small uncertainty for which the cylinder test is now more cost-effective for a small μ_R, while CAPO-tests is less expensive for $1.0\mu_R$. This indicates that the results for the cost is not very sensitive towards changes in the uncertainty. Though, it should be noted that the probability of having to reinforce the components is lower for a very large uncertainty for $0.6\mu_R$ and $0.8\mu_R$. The reason for this is that the very large $V(\mu_R)$ and $V(V_R)$ results in more characteristic values in the upper-tail of the distribution and thus being beneficial for low values of μ_R.

6.8.3 Variation of correlation coefficient in a system

The following analysis is now carried out for $n = 100$, small uncertainty and a correlation coefficient of $\rho = 0.8$. The results are illustrated in Figure 6.6 and listed in Table 6.11 and 6.12.

Table 6.11: Optimal amount of tests and minimal average expected cost for the two test types.

Test type	Optimal amount of tests		Average minimum expected cost	
	Cylinder	CAPO	Cylinder	CAPO
$0.6\mu_R$	10	30	1867	1803
$0.8\mu_R$	15	30	163	119
$1.0\mu_R$	3	3	15	3

Table 6.12: Corresponding average reinforcement values and probability of having to reinforce.

	z		z_T		P_R	
Type type	Cylinder	CAPO	Cylinder	CAPO	Cylinder	CAPO
$0.6\mu_R$	0.37	0.38	0.41	0.42	93.3	92.4
$0.8\mu_R$	0.01	0.01	0.20	0.20	57.3	57.6
$1.0\mu_R$	0	0	0.19	0.20	37.9	39.7

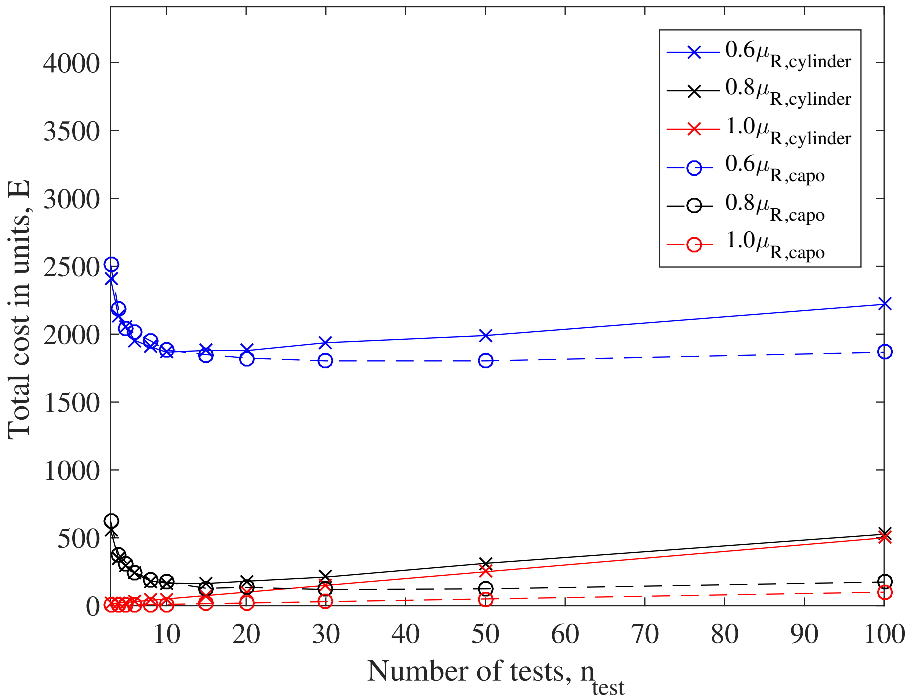

Figure 6.6: Cost-benefit analysis of cylinder vs CAPO-tests for total average expected cost.

By comparing the results above with those obtained in the first analysis for $\rho = 0$ it is clearly seen that the cost significantly decreases by introducing a large ρ. As a result, the optimums are located for a smaller amount of tests and CAPO-tests are the most cost-beneficial solution for all the evaluated values of μ_R. In reality, it is fair to assume that their is a correlation between components in a system, if they are from the batch. Hence, a lot of cost is saved by using a correlation coefficient in the cost-benefit analysis.

6.8.4 Cost related to decision model

The cost-benefit analysis can be used in addition to the specified cost in the decision model in Figure 6.2 to decide whether to replace components in a system or perform cylinder or CAPO-tests based on the average cost. An example of this will now be evaluated, which is shown in Figure 6.7 for $0.8\mu_R$, small uncertainty, $n = 100$ and $\rho = 0$. The cost related to collapse of the structure for $\beta^t_{\text{comp}} = 4.3$ is assumed to be negligible.

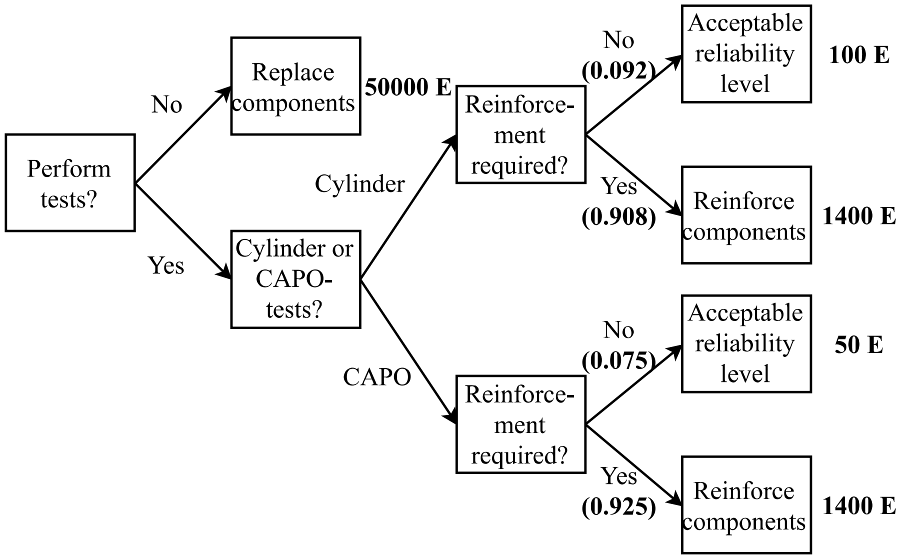

Figure 6.7: Cost related to decision model for reassessment of the reliability level for an existing structure by performing cylinder and CAPO-tests. The branches corresponds to a fixed β^t_{comp}.

The cost for cylinder and CAPO-tests at the end of the branches are estimated for a truncated distribution of the reinforcement needed, z_T, expressed as an average value. The average cost of the different options in Figure 6.7 is calculated by:

$$C = C_\text{re}\, n \tag{6.6}$$

$$C = P_\text{R}\,(C_\text{z}\, z_\text{T}\, n + C_{\text{cyl/CAPO}}\, n_\text{test}) + (1 - P_\text{R})\, C_{\text{cyl/CAPO}}\, n_\text{test} \tag{6.7}$$

The values for z_T and the probability of having to reinforce, P_R, is found in Table 6.6. The results for the average cost of replacing the components in a system or performing cylinder or CAPO-tests are as following:

Replace comp.: $C = 500\text{ E} \cdot 100 = 50000\text{ E}$

Cylinder test: $C = 0.908 \cdot (50\text{ E} \cdot 0.26 \cdot 100 + 5\text{ E} \cdot 20) + 0.092 \cdot 5\text{ E} \cdot 20 = 1280\text{ E}$

CAPO-test: $C = 0.925 \cdot (50\text{ E} \cdot 0.27 \cdot 100 + 1\text{ E} \cdot 50) + 0.075 \cdot 1\text{ E} \cdot 100 = 1299\text{ E}$

The results show that performing cylinder test on average is the optimal solution in terms of cost in this case for the specified costs. Similarly, this analysis can be carried out for the other case studies evaluated in this chapter, but this will not be the case, since the purpose of this study is to describe the process behind choosing the optimal solution based on cost in a decision model.

In reality, the cost-benefit analysis in addition to a decision model has to be carried out for the correct costs based on an assessment of the resistance through a preliminary inspection and rating. The optimal decision can then be chosen based on the average cost.

Chapter

7 Discussion

The purpose of this chapter is to discuss the importance and choice of different prerequisites that has been used for obtaining the results in this report. Furthermore, a perspectivation will be carried out discussing different analyses that can be investigated for future studies in addition to this project.

The analyses in this report, except for some analyses in Chapter 3, use a target reliability level of $\beta^t_{\text{comp}} = 4.3$ corresponding to a probability of failure, P_F, of a magnitude 10^{-5}. This is not necessarily an unrealistic target reliability level, as it is commonly used for new structures, cf. Table 2.1. However, for existing structures it is sometimes proposed and accepted to use a reliability level lower than that of new structures, see Sørensen [2016]. The reason for accepting a lower reliability level of existing structures is that the cost of safety measures is very large, e.g. reinforcement or replacement of components, in comparison to new structures still in the design phase. This means that if the analyses in this report are carried out again for $\beta^t_{\text{comp}} = 3.8$, then smaller k_{test}-factors are necessary for obtaining β^t_{comp} and thus having an increased survival rate, given the same prerequisites are kept. This is beneficial in regards to the decision model for choosing to perform proof load tests instead of replacing the components, still neglecting the cost for collapse of the structure. In the cost-benefit analysis for cylinder and CAPO-tests, the lower reliability level will result in less total expected costs, if the same resistance models are kept. This is due to the necessary characteristic value of the resistance, $R_{\text{k,necessary}}$, being smaller, hence less average reinforcement is needed, if any. Therefore, the choice of the target reliability level is important when deciding how to update the reliability level of an existing structure.

Another aspect to be discussed is the prerequisites used for the stochastic models of the resistance in relation to the reevaluation of the reliability level by performing proof load tests with prior information. The coefficient of variation for the resistance, V_R, in the evaluated cases is assumed to be 10% and 20%, which does not correspond to commonly used materials. In addition to this, the model uncertainty, X_R, related to the resistance has a fixed value for both cases, despite X_R normally being calibrated for specific materials. Thus, these prerequisites make it difficult to relate the results to a real situation, but by keeping the same model uncertainty, it is possible to evaluate the influence of V_R. Instead, recommended stochastic resistance models from the Joint Committee on Structural Safety [2001] for e.g. concrete in compression or steel could have been specifically evaluated.

In addition to not using recommended stochastic resistance models, a fixed value of the weighting factor, α, between permanent and variable load has been used. The weighting factor normally depends on the material, e.g. high weighting on permanent load for concrete structures as described in Sørensen [2011]. A value of $\alpha = 0.5$ has been used for all analyses, meaning that the results solely corresponds to this weighting. The sensitivity of α was investigated, see Table 4.25, for which the results for a high weighting on the variable load showed significantly larger k_{test}-factors due to the large V. The large k_{test}-factors indicates that the reliability level pre-proof load testing is lower than for $\alpha = 0.5$. Instead of using a fixed value for α, the average of a representative interval for α could have been used similar to Chapter 3, so that the results on an average satisfies β^t_{comp}.

The decision model associated to updating of the reliability level of an existing structure through proof load testing with prior information, see Figure 4.2, has been given a strict rule that all components have to be replaced, if failure is observed during proof load testing. This is most likely not the optimal choice of action, but is what have been analyzed within the scope of this project. The reason for ending the decision model a this point is that there exist a large number of alternative actions that require in-depth analyses analogously to the the ones evaluated in this project. For instance, new k_{test}-factors can be estimated for which the simulation in Section 4.7 is performed such that the systems with failure in more than one component is disregarded in the analysis. Another example of a choice of action is to proof load test a few components to failure and use the information about the resistance to update the stochastic variables similar to test sampling methods evaluated in Chapter 6. More choice of actions are in other words available and these can be included in future extensional case studies.

Chapter

8 Conclusion

An assessment of how to reevaluate the reliability level of existing structures on component and system level has been carried out for proof load testing with and without prior information as well as cylinder and CAPO-tests without prior information.

The reliability level can be reevaluated through proof load testing without prior information by using multiplication factors found in Table 3.4 on the characteristic load requirement. Multiplication factors have also been estimated by using alternative proposed stochastic models of the variable load, but the corresponding reliability levels are low, meaning that the results are only valid, if the these reliability levels are accepted.

If prior information is available, then the reliability level can be reevaluated through proof load testing of a number of components in a series or parallel system from a statistical homogeneous population. This is done by using the multiplication factors on the characteristic load requirement presented in Chapter 4. The results are based on a preliminary assessment of the uncertainty related to the resistance. A decision model including this analysis has been established to estimate the optimal choice of action based on total expected costs.

Test sampling methods like cylinder compression tests and CAPO-tests can be used to reevaluate the reliability level of an existing structure. A cost-benefit analysis between the two tests has been performed. The analyses show that CAPO-tests tend to become more cost-beneficial on average for larger values of the resistance, smaller amount of components in a system and a larger correlation coefficient. The cost-benefit analysis is used as part in a decision model to estimate the optimal decision in regards to total expected cost.

Bibliography

Anindita, 2017. Anindita. *The Challenge Of Converting Existing Buildings Into Energy-Efficient Structures*, 2017. URL `https://www.makaan.com/iq/living/the-challenge-of-converting-existing-buildings-into-energy-efficient-structures`.

BS EN 13791, 2007. Eurocode BS EN 13791. *Assessment of in-situ compressive strength in structures and pre-cast concrete components (BS EN 13791:2007)*. Dansk Standard, 2007.

Bygningslov, 1856. Bygningslov. *Bygningslov for Staden Kjøbenhavn og dens forstæder*. 1856.

Bygningslov, 1871. Bygningslov. *Bygningslov for Staden Kjøbenhavn, Lovtidende nr. 53, lov nr. 164*. 1871.

Bygningslov, 1889. Bygningslov. *Bygningslov for Staden Kjøbenhavn, Lovtidende nr. 15*, 1889.

Diamantidis, 2001. D. Diamantidis. *Probabilistic Assessment of Existing Structures - JCSS Report*. JCSS, 2001.

DS 410, 1945. Dansk Standard DS 410. *Normer for bygningskonstruktioner, 1. Belastningsforskrifter*. Dansk Standard, 1945.

DS/EN 1990, 2007. Dansk Standard DS/EN 1990. *Projekteringsgrundlag for bærende konstruktioner (DS/EN 1990:2007)*. Dansk Standard, 2007.

DS/EN 1991-1, 2007. Dansk Standard DS/EN 1991-1. *Last på bærende konstruktioner (DS/EN 1991-1:2007)*. Dansk Standard, 2007.

DS/INF 172, 2009. Dansk Standard DS/INF 172. *Baggrundsundersøgelser i forbindelse med udarbejdelse af Nationale Annekser til EN 1990 og EN 1991 - Sikkerhedsformat, lastkombinationer, partialkoefficienter, udmattelse, snelast, vindlast, m.m. (DS/INF172)*. Dansk Standard, 2009.

FPrimeC Solutions Inc., 2016. FPrimeC Solutions Inc. *How to Evaluate Compressive Strength of Concrete*, 2016. URL `http://www.fprimec.com/how-to-evaluate-compressive-strength-of-concrete/`.

Germann Instruments, 2018. Germann Instruments. *CAPO-TEST catalog*, 2018. URL `http://germann.org/products-by-application/category-1/capo-test`.

Ingeniørforening, **1908**. Dansk Ingeniørforening. *Normer for jærnbeton-konstruktioner (Norm 7)*. Dansk Ingeniørforening, 1908.

Ingeniørforening, **1916**. Dansk Ingeniørforening. *Norm for beregning af husbygningskonstruktioner (Norm 12)*. Dansk Ingeniørforening, 1916.

Ingeniørforening, **1982**. Dansk Ingeniørforening. *Sikkerhedsbestemmelser for konstruktioner (DS 409)*. Dansk Ingeniørforening, 1982.

JCSS, **2001**. JCSS. *JCSS probabilistic model code part 2: Load models*. Joint Committee on Structural Safety, 2001.

Joint Committee on Structural Safety, **2001**. Joint Committee on Structural Safety. *Probabilistic Assessment of Existing Structures*. RILEM Publications S.A.R.L., 2001.

NKB, **1978**. NKB. *Recommendations for loading and safety regulations for structural design, Report 36*. NKB, 1978.

S A K O, **1999**. S A K O. *Basis of design of structures - Proposals for Modification of Partial Safety Factors in Eurocodes*. Nordic Committee on Building Regulations, 1999.

Sanpolesi, **1997**. Luca Sanpolesi. *Scientific support activity in the field of structural stability of civil engineering works snow loads*. 1997.

SBI 251, **2015**. Erik Steen Pedersen SBI 251. *Vurdering af eksisterende konstruktioners bæreevne*. Statens Byggeforskningsinstitut, 2015.

Sørensen, **2016**. John Dalsgaard Sørensen. *Bæreevnevurdering af eksisterende konstruktioner*. 2016.

Sørensen, **2011**. John Dalsgaard Sørensen. *Structural Reliability Theory And Risk Analysis*. John Dalsgaard Sørensen, 2011.

Sørensen, **2009**. John Dalsgaard Sørensen. *Baggrundsundersøgelse ifm. udarbejdelse af brospecifikke Nationale Annekser til Eurocodes EN1990 og EN1991*, Aalborg Universitet, 2009.

Appendix

A Simulation Techniques

In this appendix the necessary tools used to estimate the probability of failure will be described. This includes simulation of outcomes of stochastic variables with an arbitrary distribution and with/ without correlation between the variables. It also includes the simulation technique used to estimate the probability of failure and the error related to this. The theory accounted for in this appendix is based on Sørensen [2011].

A.1 Simulation of stochastic variables

Consider a stochastic variable, X, with a corresponding distribution function, $F_X(x)$. In the following, the inverse method is used to simulate random realizations from the distribution function. There are two steps that are needed to generate an outcome $\hat{x}$ of X:

1. Generate an outcome $\hat{v}$ of V which is uniformly distributed between 0 and 1.
2. Determine the outcome of $\hat{x}$ from Equation (A.1).

$$\hat{x} = F_X^{-1}\left(F_V\left(\hat{v}\right)\right) = F_X^{-1}\left(\hat{v}\right) \tag{A.1}$$

The method is illustrated in Figure A.1 and it is seen that the distribution function for $\hat{X}$ with outcomes simulated by this procedure is:

$$F_{\hat{X}}\left(x\right) = P\left(\hat{X} \leq x\right) = P\left(F_X^{-1}\left(V\right) \leq x\right) = P\left(V \leq F_X\left(x\right)\right) = F_x\left(x\right) \tag{A.2}$$

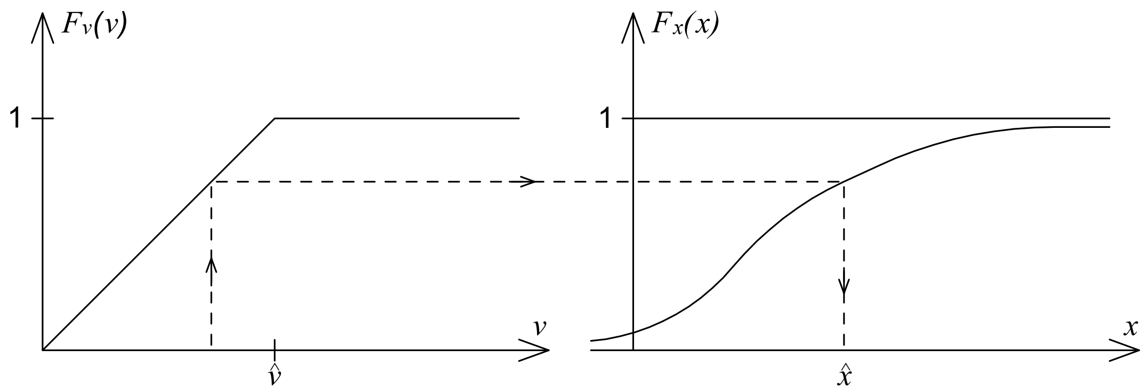

Figure A.1: The concept of the inverse method

In the following, the derived equations, which will be obtained by applying the inverse method to a normal distribution, lognormal distribution and gumbel distribution will be presented. These are the various of distribution functions that have been used throughout this project.

Simulation of random realization of normally distributed random variable

The distribution function for a normally distributed variable with expected value, μ, and standard deviation, σ is defined by:

$$F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right) \tag{A.3}$$

where $\Phi(u)$ is the standardized distribution function for a normal distributed stochastic variable with $\mu = 0$ and $\sigma = 1$.

By applying Equation (A.1) following expression to simulate realizations of a normal distributed random variable is obtained:

$$\hat{x} = \mu + \Phi(\hat{v})\,\sigma \tag{A.4}$$

Simulation of random realization of lognormally distributed random variable

The distribution function for a lognormally distributed stochastic random variable with expected value, μ, and standard deviation, σ is defined by:

$$F_X(x) = \Phi\left(\frac{\ln x - \mu_Y}{\sigma_Y}\right) \tag{A.5}$$

where

μ_Y	Lognormal parameter
σ_Y	Lognormal parameter

The lognormal parameters μ_Y and σ_Y is calculated from Equations (A.6) and (A.7), respectively.

$$\sigma_Y = \sqrt{\ln\left(\left(\frac{\sigma}{\mu}\right)^2 + 1\right)} = \sqrt{\ln\left(V^2 + 1\right)} \tag{A.6}$$

$$\mu_Y = \ln\mu - \frac{1}{2}\sigma_Y^2 \tag{A.7}$$

The lognormally distributed random variable, Y, has the following relationship to the normal distributed random variable:

$$Y = \ln X \tag{A.8}$$

The Equation that is obtained by applying Equation (A.1) and (A.8) is:

$$\hat{x} = \exp\hat{y} = \exp\left(\mu_Y + \Phi(\hat{v})\,\sigma_Y\right) \tag{A.9}$$

Simulation of random realization of Gumbel distributed random variable

The distribution function for a stochastic variable with expected value, μ, and standard deviation, σ is defined by:

$$F_X(x) = \exp(-\exp(-\alpha(x-\beta))) \tag{A.10}$$

where

α	Shape parameter
β	Scale parameter

The shape and scale parameter is related to the mean and standard deviation:

$$\mu = \beta + \frac{0.5772}{\alpha} \tag{A.11}$$

$$\sigma = \frac{\pi}{\alpha\sqrt{6}} \tag{A.12}$$

By applying Equation (A.1) following expression is obtained:

$$\hat{x} = \hat{v} - \frac{1}{\alpha}\ln(-\ln(\hat{v})) \tag{A.13}$$

A.2 Simulation of correlated lognormally distributed numbers

In the previous section the inverse method used to simulate stochastic variables was explained. In this section a technique to simulate correlated log- and normally distributed variables will be described.

Let the stochastic variables $X_i, \quad i = 1, \ldots, n$ be normally distributed with expected value $\mu_{X_1}, \ldots, \mu_{X_n}$, standard deviations $\sigma_{X_1}, \ldots, \sigma_{X_n}$ and correlation coefficients $\rho_{ij}, \quad i, j = 1, \ldots, n$. Firstly, a transformation from correlated to uncorrelated stochastic variables is necessary. This transformation can be carried out in various of ways, but here the Choleski triangulation is used. The procedure described in the following requires the correlation matrix ρ to be positive definite.

The first step is to determine the normalized variables $Y_i, i = 1, \ldots, n$ with $\mu = 0$ and $\sigma = 1$:

$$Y_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}, \quad i = 1, \ldots, n \tag{A.14}$$

It should be noted that $\mathbf{Y}$ will have a covariance matrix and correlation coefficient matrix equal to ρ. Now a transformation from $\mathbf{Y}$ to uncorrelated and normalized variables $\mathbf{U}$ is defiend as:

$$\mathbf{Y} = \mathbf{TU} \tag{A.15}$$

where $\mathbf{T}$ is a lower triangular matrix meaning. The covariance matrix $\mathbf{C_Y}$ for $\mathbf{Y}$ can be expressd as:

$$\mathbf{C_Y} = E\left[\mathbf{YY}^T\right] = E[\mathbf{TUU}^T\mathbf{T}^T] = \mathbf{T}E\left[\mathbf{UU}^T\right]\mathbf{T}^T = \mathbf{TT}^T = \rho \tag{A.16}$$

The transformation from **X** to **U** is written as:

$$\hat{\mathbf{X}} = \mu_{\mathbf{x}} + \mathbf{DT}\hat{\mathbf{U}} \tag{A.17}$$

where **D** is a diagonal matrix with standard deviations in the diagonal.

Equation (A.17) is used to simulate random correlated normal distributed variables. If lognormally distributed correlated random variables are to be simulated, then the following equation is used:

$$\hat{\mathbf{X}} = \exp\left(\hat{\mathbf{Y}}\right) = \exp\left(\mu_{\mathbf{x},\mathbf{Y}} + \mathbf{DT}\hat{\mathbf{U}}\right) \tag{A.18}$$

where **D** is a diagonal matrix with the lognormal standard deviations in the diagonal.

In the following section a simulation method to estimate the probability of failure is described:

$$P_f = P\left(g\left(\mathbf{X}\right)\right) \leq 0 \tag{A.19}$$

Here the presented failure function is assumed to be modelled in the physical space.

A.3 Crude Monte Carlo simulation

A crude Monte Carlo simulation is a method for estimating the probability of failure, P_f, which is estimated by:

$$\hat{P}_f = \frac{1}{N}\sum_{j=1}^{N} I\left[g(\hat{\mathbf{x}}_j)\right] \tag{A.20}$$

where

N	Number of simulations
$\hat{\mathbf{x}}_j$	Sample no. j of a randomly sampled stochastic vector **U**.
$I\left[g(\mathbf{u})\right]$	The indicator function

The indicator function is defined by the following:

$$I\left[g(\mathbf{u})\right] = \begin{cases} 0 & \text{if} \quad g(\mathbf{x}) > 0 \quad \text{(safe)} \\ 1 & \text{if} \quad g(\mathbf{x}) < 0 \quad \text{(failure)} \end{cases} \tag{A.21}$$

Thus, if evaluation of the limit state function results in a value above 0, the component in the sample is denoted 0 corresponding to no failure in the component. Likewise the component is denoted 1, if the limit state function results in a value below 0.
The standard error of the estimated probability of failure, $\hat{P}_f$, is estimated by the following:

$$\sqrt{\frac{\hat{P}_f(1-\hat{P}_f)}{N}} \tag{A.22}$$

Confidence intervals can also be established for the estimate of the probability of failure by using that $\hat{P}_f$ becomes normally distributed for the number of samples, N, going towards infinite, i.e. $N \longrightarrow \infty$.

A.4 Reliability updating

Reliability updating is a method for which new available information is used to update the stochastic models and probability of failure for a given component or structure. Thus, reliability updating is useful for assessing the reliability of existing structures. When performing the updating, it depends on the type of information available. In general, two types of new information are distinguished between, which are:

- Observation of events that are described by a single or multiple stochastic variables. If a non-failure event is observed, it is modelled by an event margin, while a failure event is modelled by a safety margin.
- Test samples or measurements of a stochastic variable, e.g. concrete resistance for which the updating can be performed by using Bayesian statistics.

The methods for performing the updating depending on the new available information are described in the following part of the section.

A.4.1 Bayesian updating of failure events

When modelling the observed events, firstly an event function has to be established for the stochastic variable:

$$H = h(\boldsymbol{X}) \tag{A.23}$$

The event function, h, corresponds to the limit state function, while the observations are considered as samples of the stochastic variable H. An example of an observed event can be the damage level of an existing structure that is subjected to a well defined proof load. Usually no damage is observed in this case. These observations are assumed to be modelled by either:

- Inequality events $\{\boldsymbol{H \leq 0}\}$, i.e. the observed quantity is less than or equal to a limit.
- Equality events $\{\boldsymbol{H = 0}\}$, i.e. the observed quantity is equal to a limit.

When modelling the observations by inequality events, the updated probability of failure is estimated by the following, cf. Sørensen [2011]:

$$P_f^U = P\left(g(\boldsymbol{X}) \leq 0 | h(\boldsymbol{X}) \leq 0\right) = \frac{P\left(g(\boldsymbol{X}) \leq 0 \cap h(\boldsymbol{X}) \leq 0\right)}{P\left(h(\boldsymbol{X}) \leq 0\right)} \tag{A.24}$$

In equation (A.24), $M = g(\boldsymbol{X})$ is the safety margin related to the limit state function, $g(\boldsymbol{X})$, while $\boldsymbol{X}$ represents the stochastic variables. The equation takes into account that the probability of an event A given an event B, normally denoted by $P(A|B)$, can be expressed as $\dfrac{P(A \cap B)}{P(B)}$. Furthermore, it shoud be noted that the expression $P\left(g(\boldsymbol{X}) \leq 0 \cap h(\boldsymbol{X}) \leq 0\right)$ corresponds to a parallel system consisting of two elements. When evaluating equation (A.24), FORM or SORM methods can be used.

If the observations are modelled by equality events instead for which the observed quantity is equal to a limit, then the probability of failure is estimated by, cf. Sørensen [2011]:

$$P_f^U = P\left(g(\boldsymbol{X}) \leq 0 | h(\boldsymbol{X}) = 0\right) = \frac{P\left(g(\boldsymbol{X}) \leq 0 \cap h(\boldsymbol{X}) = 0\right)}{P\left(h(\boldsymbol{X}) = 0\right)} = \frac{\dfrac{\partial}{\partial z} P\left(g(\boldsymbol{X}) \leq 0 \cap h(\boldsymbol{X}) \leq z\right)}{\dfrac{\partial}{\partial z} P\left(h(\boldsymbol{X}) \leq z\right)} \tag{A.25}$$

Equation (A.25) can similarly to equation (A.24) be evaluated by FORM or SORM methods and the equation is simple to generalize in case that more than one event is observed.

A.4.2 Bayesian updating of stochastic variables

For parameter estimations, it is recommended to use Bayesian techniques whenever possible, since the statistical uncertainty related to the estimated parameters can be determined through Bayesian estimation. Furthermore, Bayesian techniques makes it easy to update a model, when new information becomes available.

When one or more observations are available of the stochastic variables, $\boldsymbol{X}$, it is possible to update the probabilistic model related to the stochastic variables and thus the probability of failure. The density function for a stochastic variable, X, is now considered:

$$f_X(x, \boldsymbol{q}) \tag{A.26}$$

In equation (A.26), $\boldsymbol{q}$ denotes a vector of parameters governing the distribution of the stochastic variable, X, e.g. mean value and standard deviation of X, if X is normally distributed.

If there is an uncertainty related to the parameters, $\boldsymbol{q}$, then the density function, $f_X(x, \boldsymbol{q})$, can instead be considered as a conditional density function, $f_X(x|\boldsymbol{Q})$, for which q denotes a realization of $\boldsymbol{Q}$. The prior density function of $\boldsymbol{Q}$ is denoted $f'_Q(\boldsymbol{q})$ and contains information from e.g. historical knowledge or previous documentation about the parameter, $\boldsymbol{Q}$.

For updating the density function, it is assumed that n observations/realizations of the stochastic variable, X, are available, i.e. $\hat{\boldsymbol{x}} = (\hat{x}_1, \hat{x}_2, ..., \hat{x}_n)$, and these realizations are independent. The posterior density function of the uncertain parameters, $\boldsymbol{Q}$, given the realizations can then be expressed as:

$$f''_Q(\boldsymbol{q}|\hat{\boldsymbol{x}}) = \frac{f_X(\hat{\boldsymbol{x}}|\boldsymbol{q}) f'_Q(\boldsymbol{q})}{\int f_X(\hat{\boldsymbol{x}}|\boldsymbol{q}) f'_Q(\boldsymbol{q}) d\boldsymbol{q}} \tag{A.27}$$

In equation (A.27), $f_X(\hat{\boldsymbol{x}}|\boldsymbol{q}) = \prod_{i=1}^{N} f_X(\hat{x}_i|\boldsymbol{q})$ is the probability density for the observations based on the assumption that the distribution parameter are $\boldsymbol{q}$. The distribution type of the prior distribution for a stochastic variable, X, is typically chosen identical to the posterior distribution.

Finally the predictive density function of the stochastic variable, X, given the realization, $\hat{\boldsymbol{x}}$, can be expressed by:

$$f_X(x|\hat{\boldsymbol{x}}) = \int f_X(x|\boldsymbol{q}) f''_Q(\boldsymbol{q}|\hat{\boldsymbol{x}}) d\boldsymbol{q} \tag{A.28}$$

It is possible to quantify both the physical and statistical uncertainty related to the given variable and the model parameters respectively by using the Bayesian methods described above. However, it should be noted that the measurement and model uncertainties have to be taken into account when formulating the probabilistic model.

Example When updating stochastic variables through Bayesian methods, the distribution type of the density function as well as the parameters of interest have to be identified, since the updating depends on these. As an example, consider a normally distributed stochastic variable, X, with a known variance, σ^2. The normal distribution is expressed as the following for which the parameter of interest is the unknown mean value of the stochastic variable:

$$f_X(x|\mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \tag{A.29}$$

The prior distribution of the unknown mean value, μ, is normally distributed with a mean value, μ', and variance, σ'^2. New objective information has been obtained by performing tests, resulting in a test sample, n, with a mean value, $\overline{X}$. A likelihood function can be computed based on the normal distribution in equation (A.29) and inserted in equation (A.27). The posterior mean value, μ'', and variance, σ'', is then determined by:

$$\mu'' = \frac{n\overline{X}\sigma'^2 + \mu'\sigma^2}{n\sigma'^2 + \sigma^2} \tag{A.30}$$

$$\sigma''^2 = \frac{\sigma'^2\sigma^2}{n\sigma'^2 + \sigma^2} \tag{A.31}$$

Similarly, the mean value, μ'', and standard deviation, σ''', for the predictive distribution can be expressed, which also becomes normal distributed:

$$\mu'' = \frac{n\overline{X}\sigma'^2 + \mu'\sigma^2}{n\sigma'^2 + \sigma^2} \tag{A.32}$$

$$\sigma''' = \sqrt{\sigma''^2 + \sigma^2} \tag{A.33}$$

When the stochastic variable, X, has a different distribution type, the same procedure is followed, i.e. the likelihood function of the given distribution type can be inserted in equation (A.27) for which the parameters of interest are determined.

B Factors for Proof Load Testing Without Prior Information

Table B.1: k_{test}-factors for the 2. stochastic model of wind load.

Characteristic permanent load percentage of characteristic load requirement [%]	Factor: k_{test}				
	$\beta = 1.6$	$\beta = 2.1$	$\beta = 2.6$	$\beta = 3.1$	$\beta = 3.6$
0	1.56	1.85	2.21	2.65	3.18
25	1.42	1.64	1.91	2.24	2.64
50	1.29	1.44	1.62	1.83	2.10
60	1.24	1.36	1.50	1.68	1.89
65	1.22	1.32	1.45	1.60	1.78
70	1.19	1.29	1.39	1.52	1.68
75	1.18	1.25	1.34	1.45	1.58
80	1.16	1.22	1.30	1.39	1.49
85	1.15	1.21	1.26	1.33	1.41
90	1.15	1.20	1.25	1.30	1.35
95	1.15	1.20	1.25	1.30	1.35
100	1.16	1.21	1.26	1.31	1.36

Table B.2: k_{test}-factors for the 3. stochastic model of wind load.

Characteristic permanent load percentage of characteristic load requirement [%]	Factor: k_{test}				
	$\beta = 2.5$	$\beta = 3.0$	$\beta = 3.5$	$\beta = 4.0$	$\beta = 4.5$
0	1.49	1.81	2.21	2.70	3.30
25	1.37	1.61	1.91	2.27	2.72
50	1.26	1.42	1.62	1.86	2.16
60	1.22	1.35	1.50	1.70	1.94
65	1.20	1.31	1.45	1.62	1.83
70	1.18	1.28	1.40	1.54	1.72
75	1.17	1.25	1.35	1.47	1.62
80	1.17	1.23	1.31	1.40	1.52
85	1.18	1.23	1.29	1.35	1.43
90	1.20	1.24	1.29	1.34	1.39
95	1.22	1.27	1.32	1.37	1.42
100	1.26	1.31	1.36	1.41	1.46

Table B.3: k_{test}-factors for the 2. stochastic model of snow load.

Characteristic permanent load percentage of characteristic load requirement [%]	Factor: k_{test}				
	$\beta = 0.9$	$\beta = 1.4$	$\beta = 1.9$	$\beta = 2.4$	$\beta = 2.9$
0	1.38	1.75	2.22	2.82	3.57
25	1.28	1.56	1.92	2.36	2.93
50	1.19	1.38	1.62	1.92	2.29
60	1.16	1.31	1.50	1.74	2.04
65	1.14	1.27	1.44	1.65	1.91
70	1.13	1.24	1.38	1.56	1.79
75	1.11	1.21	1.33	1.48	1.67
80	1.10	1.18	1.28	1.40	1.55
85	1.09	1.15	1.23	1.32	1.43
90	1.08	1.14	1.20	1.26	1.33
95	1.09	1.14	1.18	1.23	1.28
100	1.09	1.14	1.19	1.24	1.29

Table B.4: k_{test}-factors for the 3. stochastic model of snow load.

Characteristic permanent load percentage of characteristic load requirement [%]	Factor: k_{test}				
	$\beta = 0.8$	$\beta = 1.3$	$\beta = 1.8$	$\beta = 2.3$	$\beta = 2.8$
0	1.34	1.72	2.22	2.86	3.69
25	1.25	1.54	1.92	2.40	3.02
50	1.17	1.37	1.62	1.94	2.35
60	1.14	1.30	1.50	1.75	2.08
65	1.13	1.26	1.44	1.66	1.95
70	1.11	1.23	1.38	1.58	1.82
75	1.10	1.20	1.33	1.49	1.69
80	1.09	1.17	1.27	1.40	1.57
85	1.08	1.15	1.23	1.32	1.44
90	1.08	1.13	1.19	1.26	1.34
95	1.08	1.13	1.18	1.23	1.28
100	1.09	1.14	1.19	1.24	1.29

Appendix

C Factors for Proof Load Testing With Prior Information

C.1 Series systems

C.1.1 Results for correlation $\rho = 0$

Table C.1: k_{test}-factors for $\rho = 0$ in case 2a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.85 (43%)	1.53 (57%)	1.46 (58%)
20%	1.66 (53%)	1.45 (60%)	1.39 (61%)
30%	1.56 (60%)	1.40 (63%)	1.35 (64%)
40%	1.51 (63%)	1.37 (65%)	1.32 (65%)
50%	1.48 (67%)	1.35 (65%)	1.30 (67%)

Table C.2: k_{test}-factors for $\rho = 0$ in case 3a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.83 (45%)	1.52 (55%)	1.44 (57%)
20%	1.65 (52%)	1.44 (58%)	1.36 (61%)
30%	1.56 (56%)	1.39 (60%)	1.35 (59%)
40%	1.52 (57%)	1.36 (62%)	1.30 (63%)
50%	1.48 (60%)	1.33 (63%)	1.30 (62%)

Table C.3: k_{test}-factors for $\rho = 0$ in case 4a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.90 (38%)	1.50 (52%)	1.40 (55%)
20%	1.65 (48%)	1.40 (55%)	1.35 (55%)
30%	1.50 (56%)	1.35 (57%)	1.30 (58%)
40%	1.50 (54%)	1.35 (55%)	1.30 (56%)
50%	1.45 (60%)	1.30 (59%)	1.25 (56%)

Table C.4: k_{test}-factors for $\rho = 0$ in case 2b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	2.88 (18%)	1.87 (42%)	1.68 (48%)
20%	2.24 (33%)	1.66 (50%)	1.51 (57%)
30%	1.98 (44%)	1.55 (56%)	1.43 (61%)
40%	1.83 (51%)	1.47 (62%)	1.37 (65%)
50%	1.71 (59%)	1.41 (67%)	1.30 (71%)

Table C.5: k_{test}-factors for $\rho = 0$ in case 3b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	2.98 (16%)	1.87 (42%)	1.66 (48%)
20%	2.28 (31%)	1.66 (49%)	1.50 (54%)
30%	2.03 (40%)	1.55 (53%)	1.42 (58%)
40%	1.89 (45%)	1.47 (58%)	1.35 (62%)
50%	1.77 (51%)	1.40 (63%)	1.31 (65%)

Table C.6: k_{test}-factors for $\rho = 0$ in case 4b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	3.30 (13%)	1.90 (39%)	1.66 (46%)
20%	2.41 (27%)	1.65 (47%)	1.50 (50%)
30%	2.10 (35%)	1.55 (50%)	1.40 (54%)
40%	1.90 (43%)	1.46 (54%)	1.35 (56%)
50%	1.78 (47%)	1.40 (56%)	1.30 (58%)

C.1.2 Results for correlation $\rho = 0.5$

Table C.7: k_{test}-factors for $\rho = 0.5$ in case 2a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.61 (71%)	1.45 (75%)	1.40 (76%)
20%	1.51 (74%)	1.39 (77%)	1.35 (78%)
30%	1.48 (75%)	1.38 (76%)	1.34 (77%)
40%	1.45 (77%)	1.34 (79%)	1.32 (78%)
50%	1.41 (79%)	1.32 (80%)	1.30 (79%)

Table C.8: k_{test}-factors for $\rho = 0.5$ in case 3a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.64 (64%)	1.45 (70%)	1.40 (71%)
20%	1.53 (68%)	1.40 (71%)	1.35 (72%)
30%	1.49 (69%)	1.36 (72%)	1.33 (73%)
40%	1.44 (72%)	1.35 (72%)	1.31 (73%)
50%	1.42 (72%)	1.34 (72%)	1.30 (73%)

Table C.9: k_{test}-factors for $\rho = 0.5$ in case 4a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.63 (58%)	1.44 (62%)	1.39 (63%)
20%	1.52 (61%)	1.38 (63%)	1.34 (64%)
30%	1.46 (63%)	1.35 (64%)	1.32 (64%)
40%	1.43 (64%)	1.35 (63%)	1.30 (65%)
50%	1.41 (64%)	1.31 (66%)	1.29 (65%)

Table C.10: k_{test}-factors for $\rho = 0.5$ in case 2b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.82 (81%)	1.46 (88%)	1.40 (88%)
20%	1.60 (87%)	1.36 (90%)	1.30 (90%)
30%	1.50 (89%)	1.30 (91%)	1.25 (91%)
40%	1.43 (91%)	1.26 (92%)	1.23 (91%)
50%	1.39 (92%)	1.25 (92%)	1.20 (92%)

Table C.11: k_{test}-factors for $\rho = 0.5$ in case 3b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.91 (72%)	1.50 (81%)	1.40 (83%)
20%	1.67 (79%)	1.40 (83%)	1.34 (83%)
30%	1.55 (82%)	1.35 (84%)	1.28 (85%)
40%	1.48 (84%)	1.30 (86%)	1.25 (86%)
50%	1.43 (86%)	1.27 (87%)	1.20 (88%)

Table C.12: k_{test}-factors for $\rho = 0.5$ in case 4b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	2.06 (57%)	1.55 (70%)	1.45 (72%)
20%	1.74 (67%)	1.41 (74%)	1.35 (74%)
30%	1.61 (71%)	1.35 (76%)	1.30 (75%)
40%	1.53 (74%)	1.31 (77%)	1.25 (77%)
50%	1.48 (75%)	1.30 (76%)	1.25 (76%)

C.1.3 Results for correlation $\rho = 0.8$

Table C.13: k_{test}-factors for $\rho = 0.8$ in case 2a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.51 (82%)	1.42 (83%)	1.38 (85%)
20%	1.47 (82%)	1.38 (84%)	1.36 (84%)
30%	1.43 (84%)	1.36 (85%)	1.34 (85%)
40%	1.42 (84%)	1.36 (84%)	1.33 (85%)
50%	1.40 (85%)	1.35 (85%)	1.32 (85%)

Table C.14: k_{test}-factors for $\rho = 0.8$ in case 3a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.51 (76%)	1.42 (77%)	1.38 (79%)
20%	1.46 (77%)	1.38 (79%)	1.35 (79%)
30%	1.42 (79%)	1.38 (77%)	1.35 (78%)
40%	1.42 (78%)	1.36 (78%)	1.34 (78%)
50%	1.41 (78%)	1.35 (78%)	1.32 (79%)

Table C.15: k_{test}-factors for $\rho = 0.8$ in case 4a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.51 (67%)	1.40 (69%)	1.37 (69%)
20%	1.47 (67%)	1.38 (69%)	1.35 (69%)
30%	1.43 (68%)	1.37 (69%)	1.35 (69%)
40%	1.41 (69%)	1.35 (70%)	1.33 (69%)
50%	1.42 (68%)	1.35 (69%)	1.33 (69%)

Table C.16: k_{test}-factors for $\rho = 0.8$ in case 2b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.49 (95%)	1.31 (96%)	1.28 (96%)
20%	1.40 (96%)	1.27 (97%)	1.24 (96%)
30%	1.35 (96%)	1.25 (96%)	1.20 (97%)
40%	1.32 (96%)	1.23 (97%)	1.20 (97%)
50%	1.29 (97%)	1.22 (97%)	1.18 (97%)

Table C.17: k_{test}-factors for $\rho = 0.8$ in case 3b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.55 (90%)	1.37 (92%)	1.30 (93%)
20%	1.45 (92%)	1.31 (93%)	1.25 (93%)
30%	1.39 (93%)	1.27 (93%)	1.25 (93%)
40%	1.36 (93%)	1.25 (93%)	1.22 (93%)
50%	1.33 (94%)	1.25 (93%)	1.20 (94%)

Table C.18: k_{test}-factors for $\rho = 0.8$ in case 4b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.60 (81%)	1.40 (84%)	1.35 (84%)
20%	1.49 (83%)	1.33 (85%)	1.30 (84%)
30%	1.43 (84%)	1.30 (85%)	1.27 (85%)
40%	1.38 (85%)	1.29 (85%)	1.25 (85%)
50%	1.38 (85%)	1.26 (86%)	1.23 (86%)

C.2 Parallel systems

C.2.1 Results for correlation $\rho = 0$

Table C.19: k_{test}-factors for $\rho = 0$ in case 2a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.84 (44%)	1.56 (53%)	1.46 (59%)
20%	1.65 (55%)	1.46 (59%)	1.40 (60%)
30%	1.59 (56%)	1.43 (58%)	1.36 (62%)
40%	1.52 (62%)	1.40 (59%)	1.35 (61%)
50%	1.47 (66%)	1.37 (62%)	1.33 (61%)

Table C.20: k_{test}-factors for $\rho = 0$ in case 3a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.82 (45%)	1.52 (55%)	1.44 (57%)
20%	1.62 (55%)	1.45 (56%)	1.38 (59%)
30%	1.57 (56%)	1.40 (58%)	1.35 (59%)
40%	1.49 (61%)	1.37 (60%)	1.33 (59%)
50%	1.45 (63%)	1.36 (59%)	1.32 (59%)

Table C.21: k_{test}-factors for $\rho = 0$ in case 4a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.86 (41%)	1.52 (51%)	1.42 (54%)
20%	1.64 (49%)	1.43 (53%)	1.36 (54%)
30%	1.54 (53%)	1.40 (53%)	1.33 (55%)
40%	1.49 (55%)	1.35 (55%)	1.31 (55%)
50%	1.46 (56%)	1.33 (56%)	1.30 (55%)

Table C.22: k_{test}-factors for $\rho = 0$ in case 2b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	2.72 (24%)	1.90 (39%)	1.72 (43%)
20%	2.20 (37%)	1.65 (51%)	1.56 (49%)
30%	1.94 (48%)	1.56 (55%)	1.51 (48%)
40%	1.79 (56%)	1.53 (54%)	1.45 (52%)
50%	1.67 (64%)	1.49 (56%)	1.40 (56%)

Table C.23: k_{test}-factors for $\rho = 0$ in case 3b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	2.96 (17%)	1.89 (40%)	1.69 (45%)
20%	2.26 (33%)	1.67 (48%)	1.54 (50%)
30%	1.96 (45%)	1.58 (50%)	1.45 (54%)
40%	1.79 (53%)	1.50 (54%)	1.46 (49%)
50%	1.71 (56%)	1.47 (55%)	1.44 (49%)

Table C.24: k_{test}-factors for $\rho = 0$ in case 4b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	3.00 (18%)	1.90 (39%)	1.66 (46%)
20%	2.34 (30%)	1.65 (47%)	1.53 (48%)
30%	2.00 (40%)	1.54 (51%)	1.49 (47%)
40%	1.82 (47%)	1.52 (49%)	1.42 (50%)
50%	1.68 (54%)	1.44 (53%)	1.41 (49%)

C.2.2 Results for correlation $\rho = 0.5$

Table C.25: k_{test}-factors for $\rho = 0.5$ in case 2a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.63 (70%)	1.45 (75%)	1.38 (78%)
20%	1.53 (74%)	1.40 (77%)	1.35 (78%)
30%	1.48 (75%)	1.38 (76%)	1.32 (79%)
40%	1.44 (78%)	1.36 (77%)	1.31 (80%)
50%	1.42 (79%)	1.34 (78%)	1.31 (79%)

Table C.26: k_{test}-factors for $\rho = 0.5$ in case 3a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.62 (65%)	1.45 (70%)	1.39 (72%)
20%	1.51 (70%)	1.39 (72%)	1.35 (72%)
30%	1.47 (71%)	1.36 (72%)	1.32 (74%)
40%	1.44 (72%)	1.35 (72%)	1.30 (74%)
50%	1.42 (72%)	1.35 (71%)	1.30 (73%)

Table C.27: k_{test}-factors for $\rho = 0.5$ in case 4a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.61 (59%)	1.38 (67%)	1.30 (70%)
20%	1.52 (61%)	1.33 (68%)	1.27 (70%)
30%	1.46 (63%)	1.30 (69%)	1.25 (70%)
40%	1.43 (64%)	1.30 (68%)	1.24 (70%)
50%	1.41 (65%)	1.29 (68%)	1.25 (69%)

Table C.28: k_{test}-factors for $\rho = 0.5$ in case 2b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.78 (83%)	1.44 (89%)	1.36 (90%)
20%	1.57 (88%)	1.37 (90%)	1.28 (91%)
30%	1.47 (91%)	1.30 (91%)	1.25 (91%)
40%	1.41 (92%)	1.27 (92%)	1.24 (91%)
50%	1.36 (93%)	1.25 (92%)	1.20 (92%)

Table C.29: k_{test}-factors for $\rho = 0.5$ in case 3b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.86 (75%)	1.47 (83%)	1.38 (85%)
20%	1.64 (80%)	1.36 (86%)	1.31 (85%)
30%	1.51 (85%)	1.32 (86%)	1.25 (87%)
40%	1.45 (86%)	1.31 (85%)	1.25 (86%)
50%	1.41 (87%)	1.28 (86%)	1.22 (87%)

Table C.30: k_{test}-factors for $\rho = 0.5$ in case 4b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.99 (61%)	1.52 (72%)	1.41 (74%)
20%	1.70 (69%)	1.40 (75%)	1.32 (76%)
30%	1.56 (74%)	1.35 (76%)	1.29 (76%)
40%	1.49 (76%)	1.31 (77%)	1.25 (77%)
50%	1.43 (78%)	1.29 (77%)	1.25 (76%)

C.2.3 Results for correlation $\rho = 0.8$

Table C.31: k_{test}-factors for $\rho = 0.8$ in case 2a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.51 (82%)	1.39 (86%)	1.34 (88%)
20%	1.47 (82%)	1.36 (86%)	1.32 (88%)
30%	1.42 (85%)	1.36 (85%)	1.30 (88%)
40%	1.41 (85%)	1.34 (86%)	1.30 (88%)
50%	1.41 (84%)	1.33 (86%)	1.30 (88%)

Table C.32: k_{test}-factors for $\rho = 0.8$ in case 3a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.53 (74%)	1.41 (78%)	1.36 (80%)
20%	1.46 (77%)	1.38 (79%)	1.32 (82%)
30%	1.43 (78%)	1.35 (80%)	1.32 (81%)
40%	1.42 (78%)	1.35 (79%)	1.31 (81%)
50%	1.41 (78%)	1.34 (80%)	1.30 (81%)

Table C.33: k_{test}-factors for $\rho = 0.8$ in case 4a with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.52 (66%)	1.39 (70%)	1.34 (72%)
20%	1.47 (67%)	1.36 (71%)	1.33 (71%)
30%	1.43 (69%)	1.35 (70%)	1.30 (73%)
40%	1.42 (69%)	1.35 (69%)	1.30 (72%)
50%	1.40 (69%)	1.34 (70%)	1.30 (71%)

Table C.34: k_{test}-factors for $\rho = 0.8$ in case 2b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.45 (96%)	1.22 (98%)	1.15 (99%)
20%	1.37 (96%)	1.19 (98%)	1.11 (99%)
30%	1.33 (97%)	1.16 (98%)	1.09 (99%)
40%	1.29 (97%)	1.15 (98%)	1.09 (99%)
50%	1.27 (97%)	1.14 (98%)	1.07 (99%)

Table C.35: k_{test}-factors for $\rho = 0.8$ in case 3b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.52 (91%)	1.28 (95%)	1.20 (96%)
20%	1.40 (94%)	1.22 (95%)	1.15 (96%)
30%	1.36 (94%)	1.20 (95%)	1.15 (96%)
40%	1.32 (94%)	1.19 (95%)	1.12 (96%)
50%	1.32 (94%)	1.18 (95%)	1.12 (96%)

Table C.36: k_{test}-factors for $\rho = 0.8$ in case 4b with a various n and test portions. Values inside parenthesis () indicate the amount of systems that succeeded the proof load testing.

Test portion	$k_{\text{test}}^{n=10}$	$k_{\text{test}}^{n=50}$	$k_{\text{test}}^{n=100}$
10%	1.58 (82%)	1.31 (88%)	1.25 (88%)
20%	1.46 (84%)	1.26 (88%)	1.20 (89%)
30%	1.40 (85%)	1.25 (88%)	1.19 (89%)
40%	1.36 (86%)	1.22 (88%)	1.17 (89%)
50%	1.34 (86%)	1.21 (88%)	1.15 (90%)

Appendix

D Cost-Benefit Analysis of Sampling Methods

D.0.1 Korrelation = 0

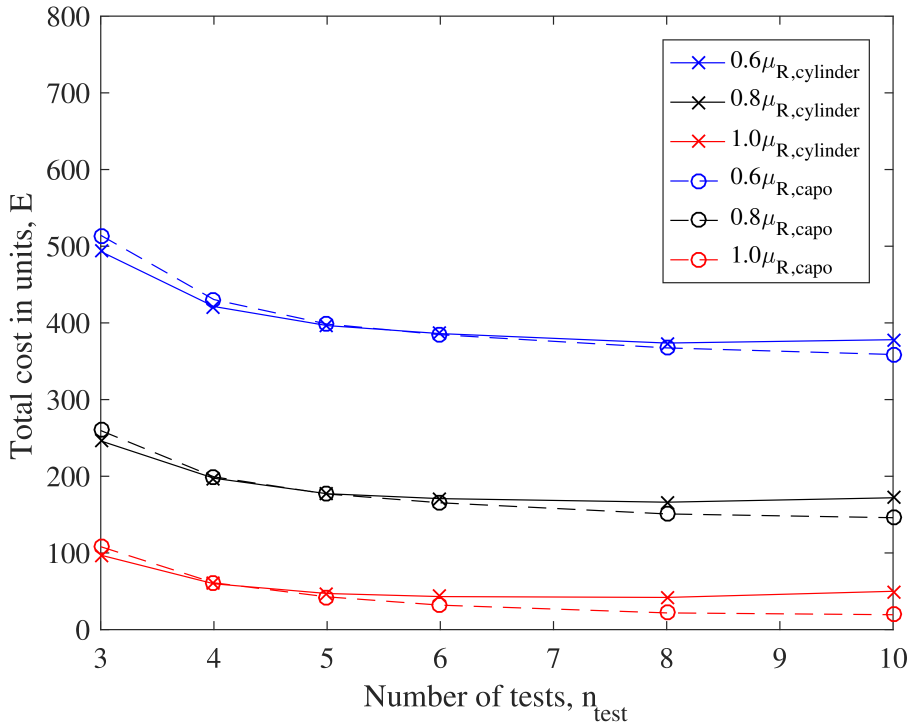

Figure D.1: Cost-benefit analysis of cylinder tests. Total cost for performing the test and reinforcing the component if necessary

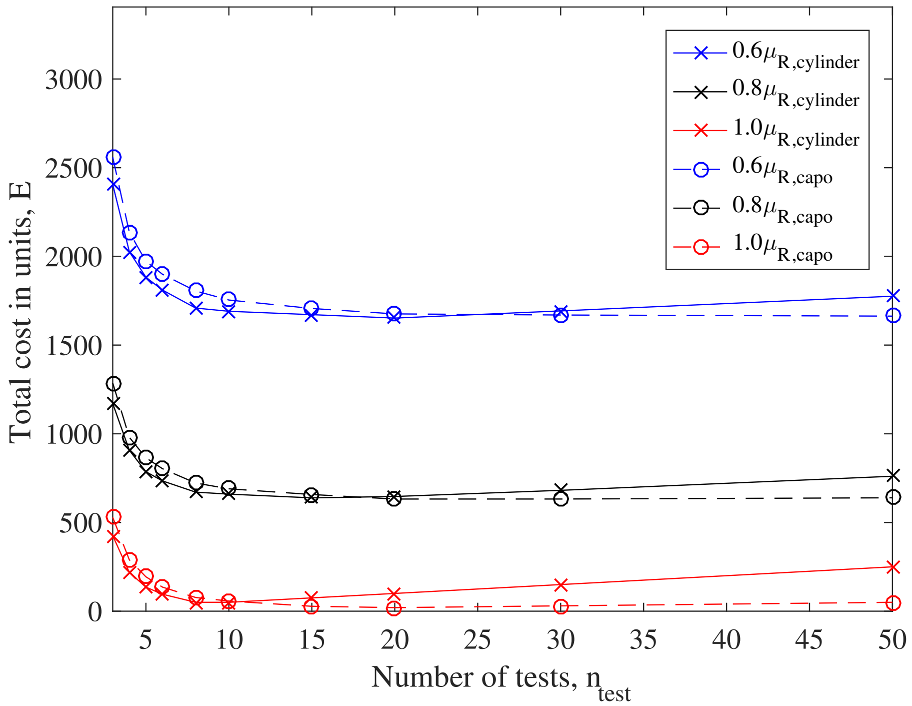

Figure D.2: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

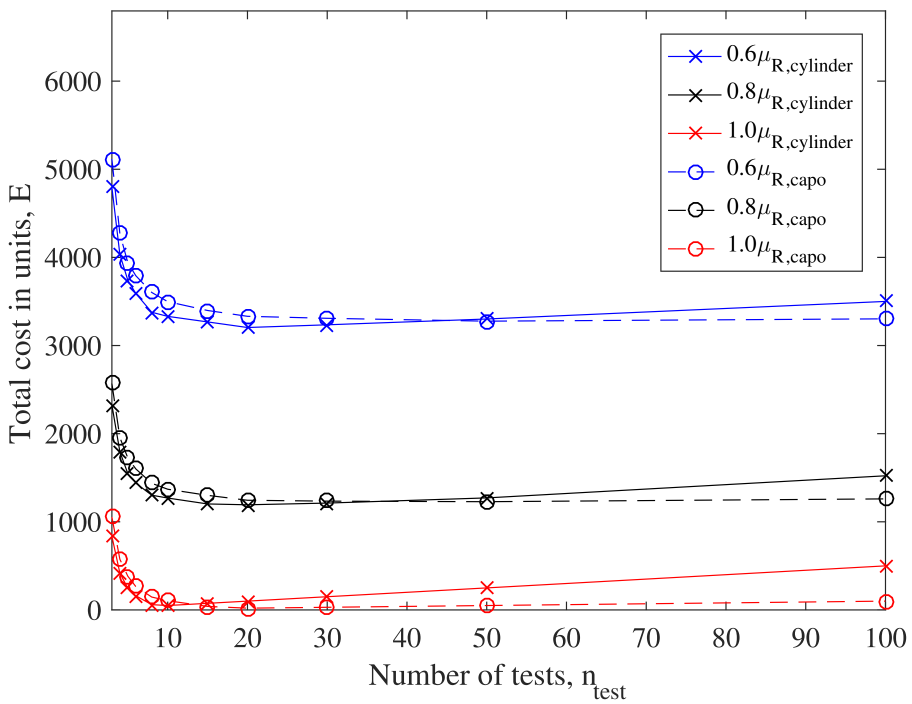

Figure D.3: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

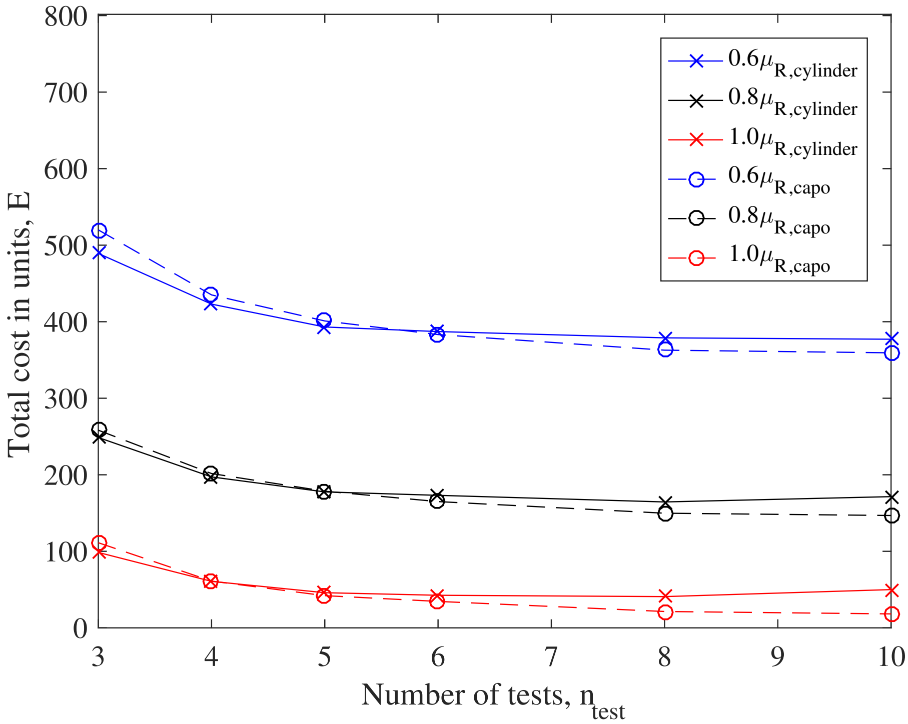

Figure D.4: Cost-benefit analysis of cylinder tests. Total cost for performing the test and reinforcing the component if necessary

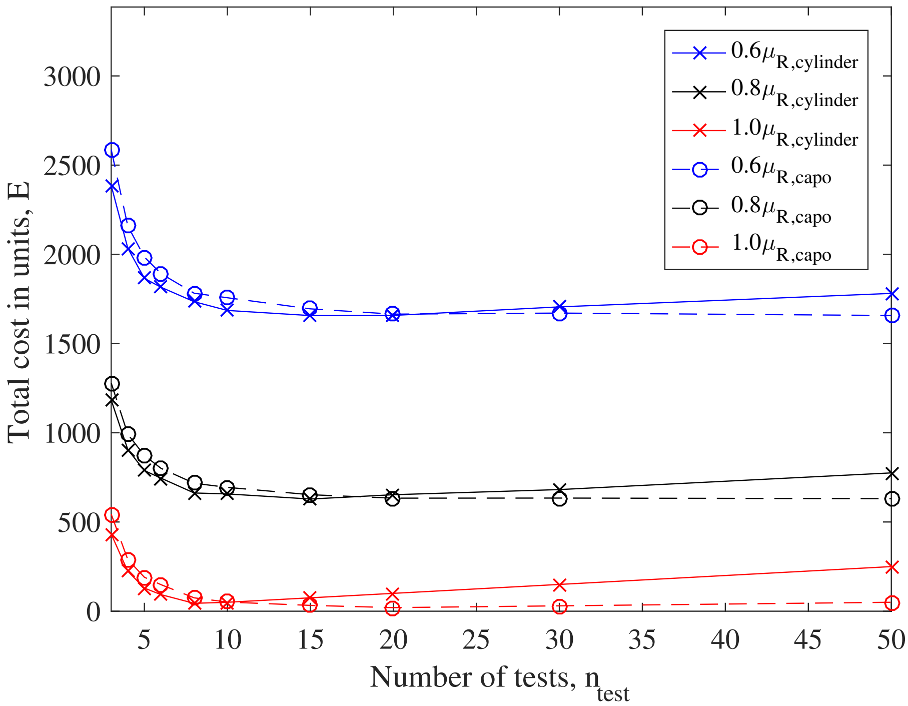

Figure D.5: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

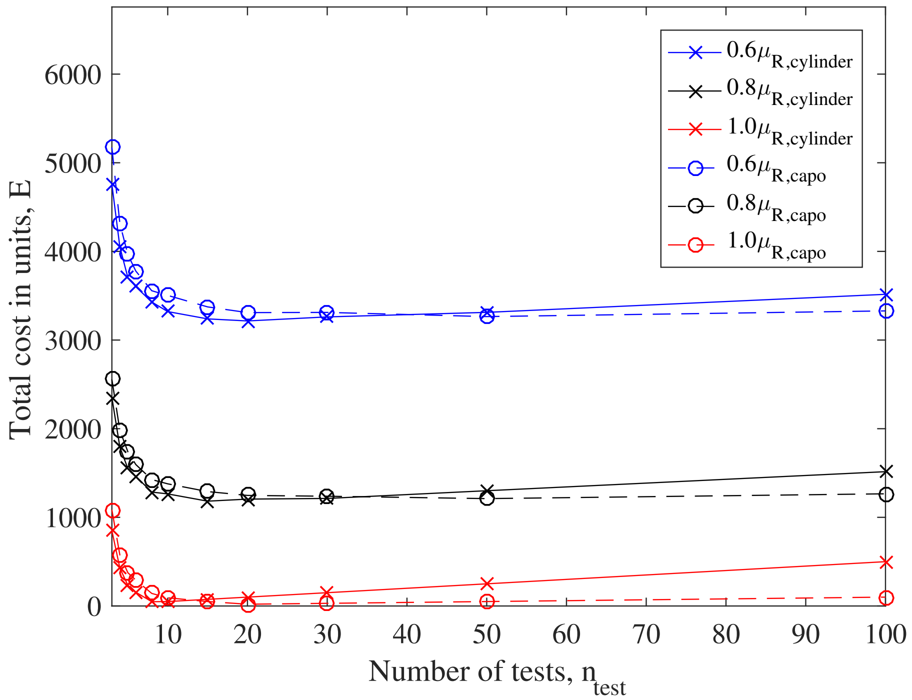

Figure D.6: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

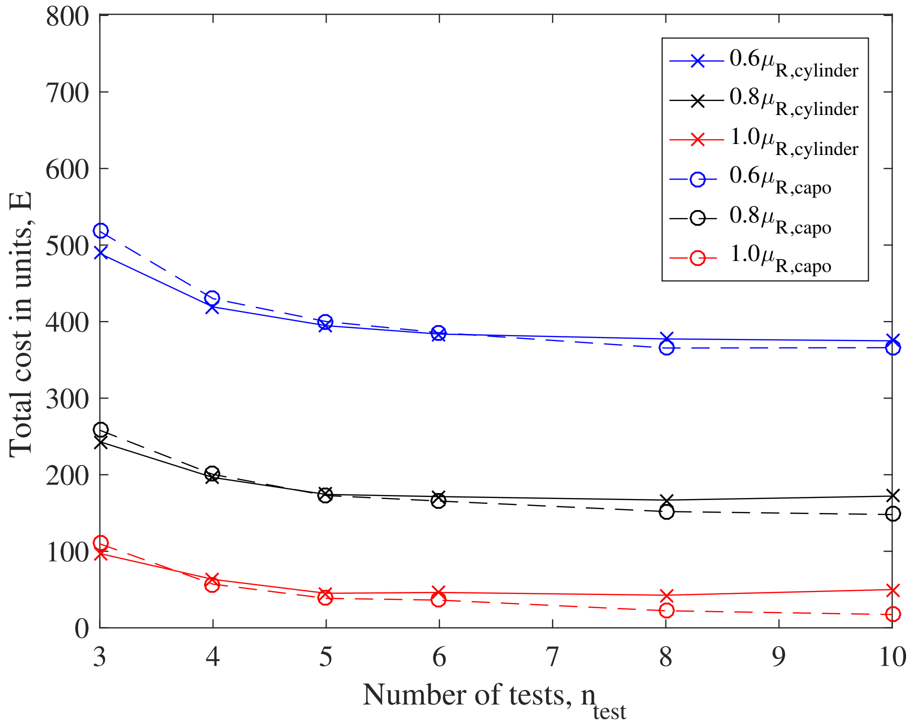

Figure D.7: Cost-benefit analysis of cylinder tests. Total cost for performing the test and reinforcing the component if necessary

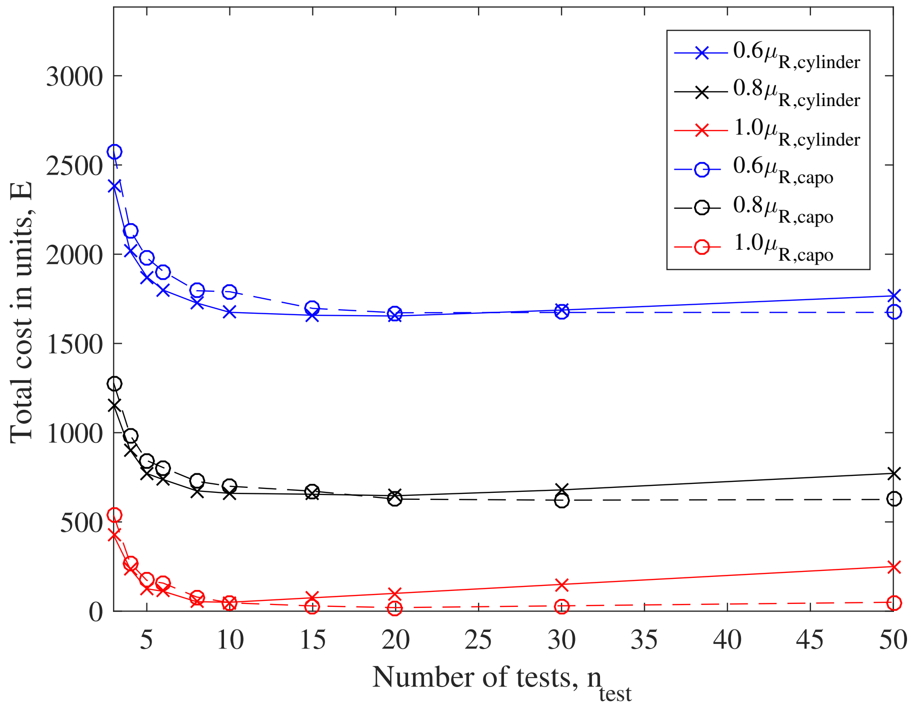

Figure D.8: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

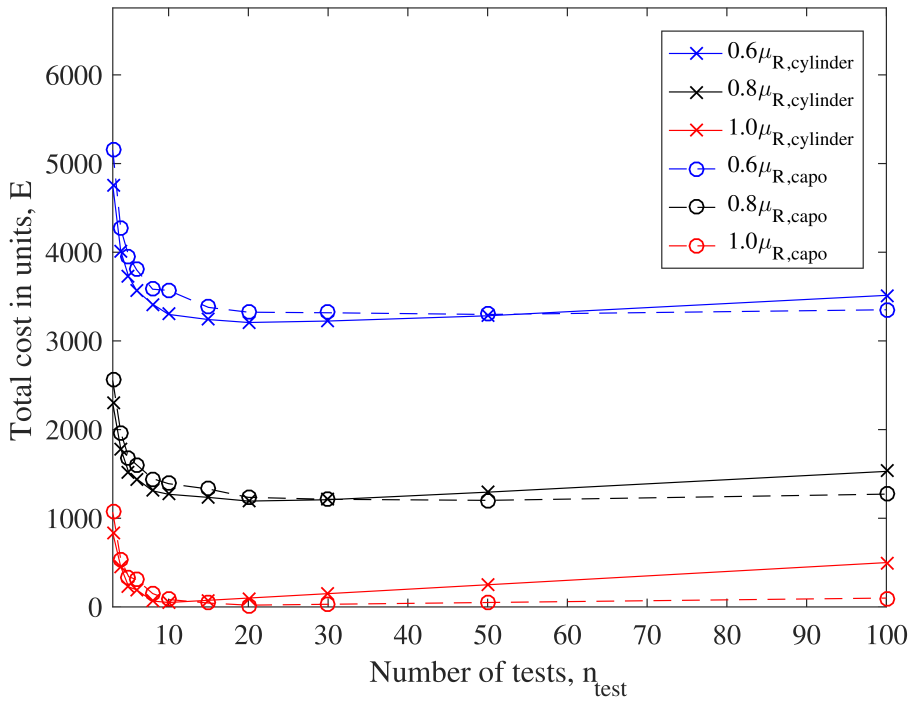

Figure D.9: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

D.0.2 Korrelation = 0.5

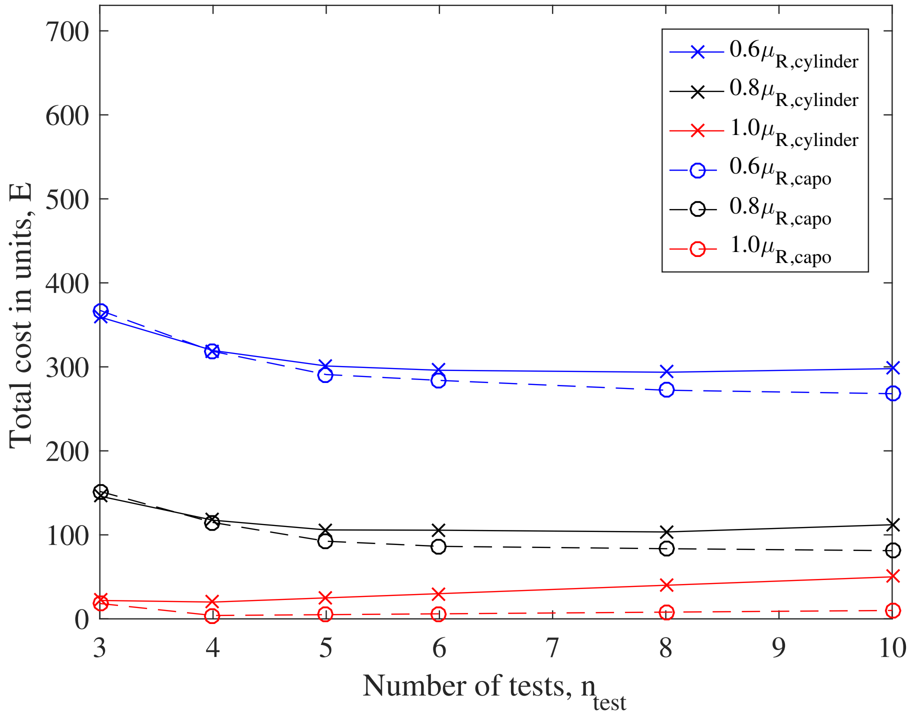

Figure D.10: Cost-benefit analysis of cylinder tests. Total cost for performing the test and reinforcing the component if necessary

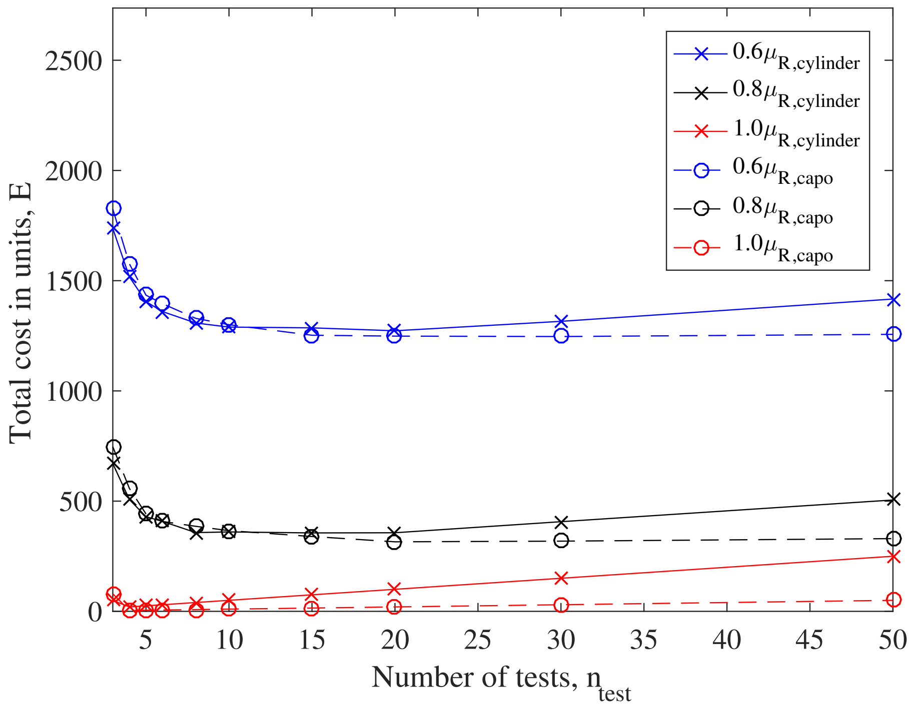

Figure D.11: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

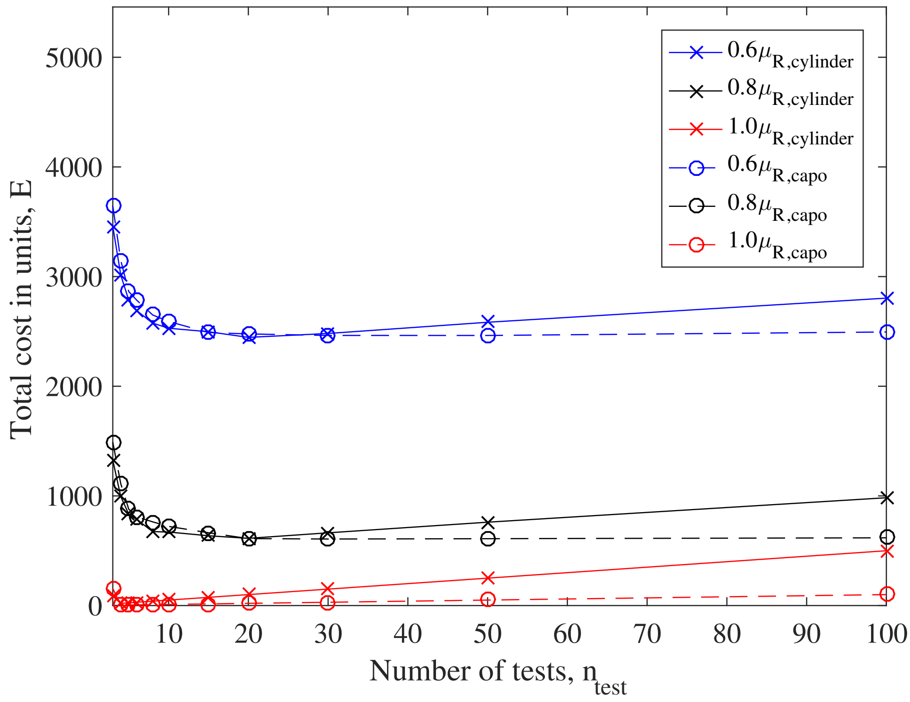

Figure D.12: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

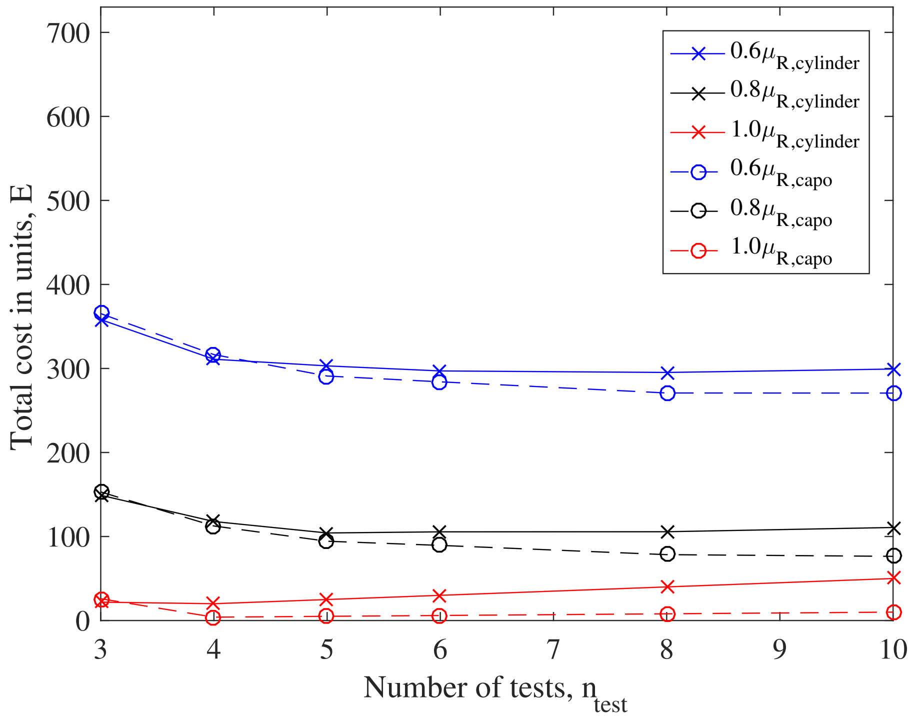

Figure D.13: Cost-benefit analysis of cylinder tests. Total cost for performing the test and reinforcing the component if necessary

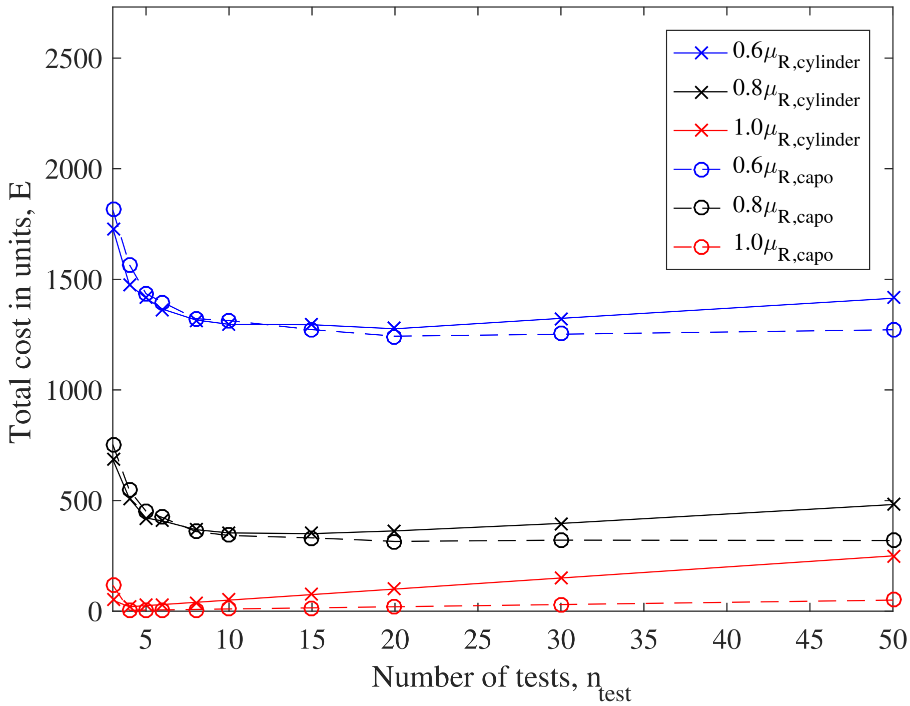

Figure D.14: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

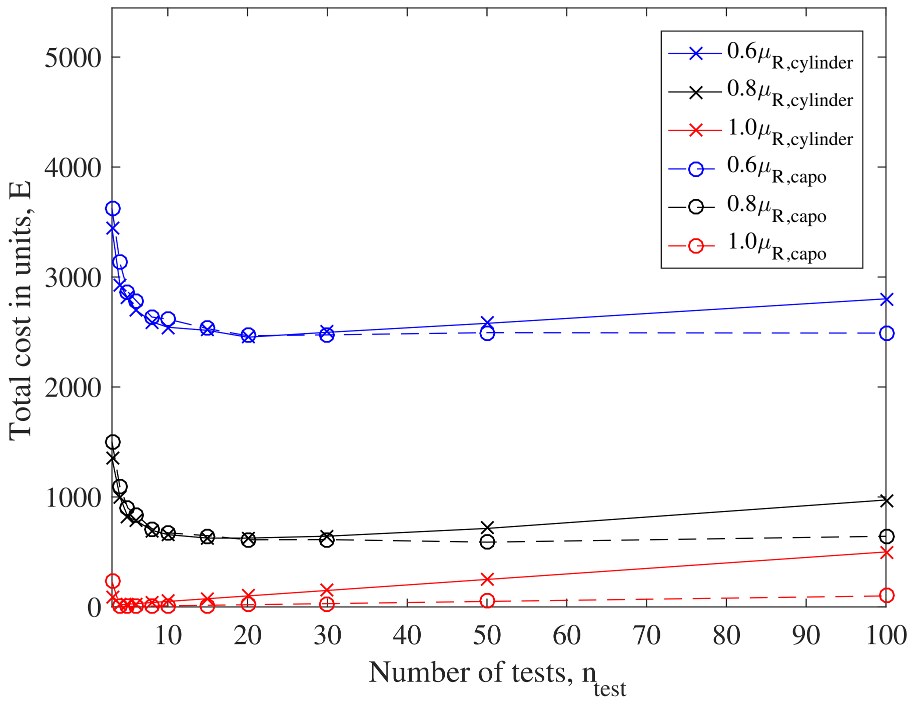

Figure D.15: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

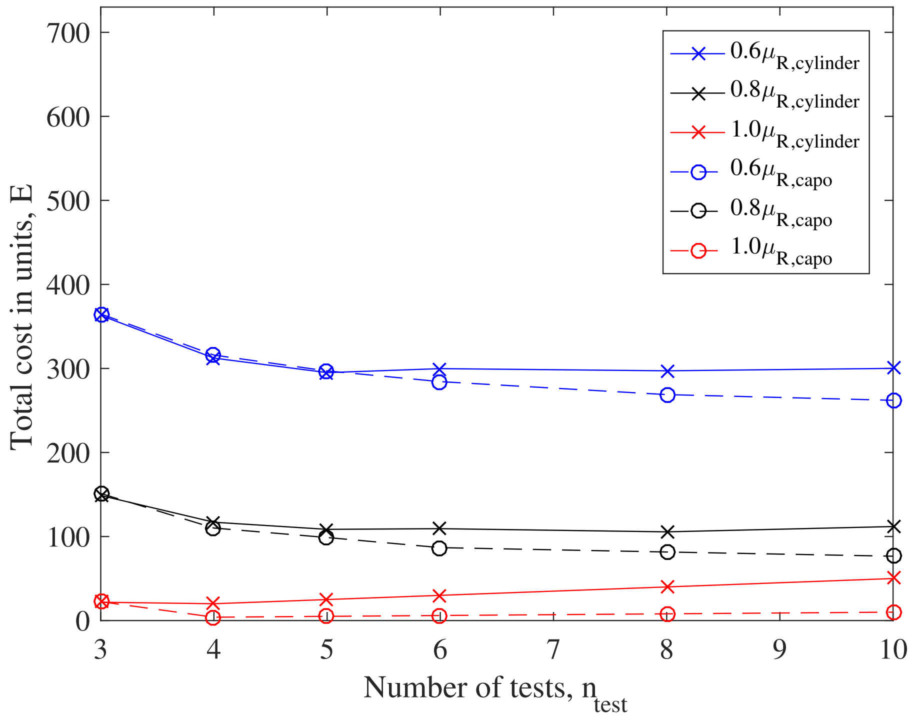

Figure D.16: Cost-benefit analysis of cylinder tests. Total cost for performing the test and reinforcing the component if necessary

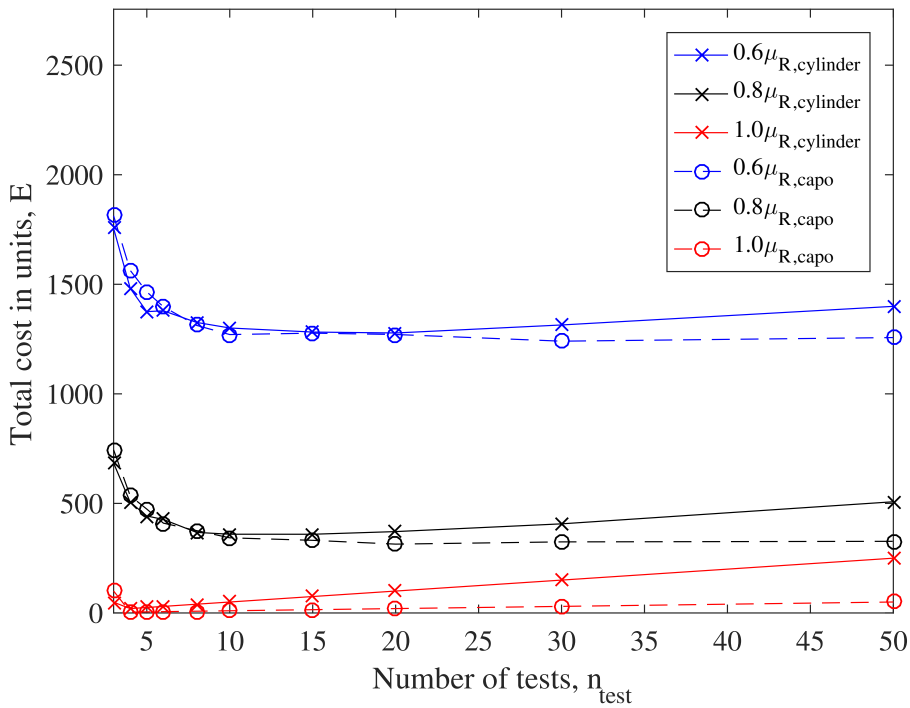

Figure D.17: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

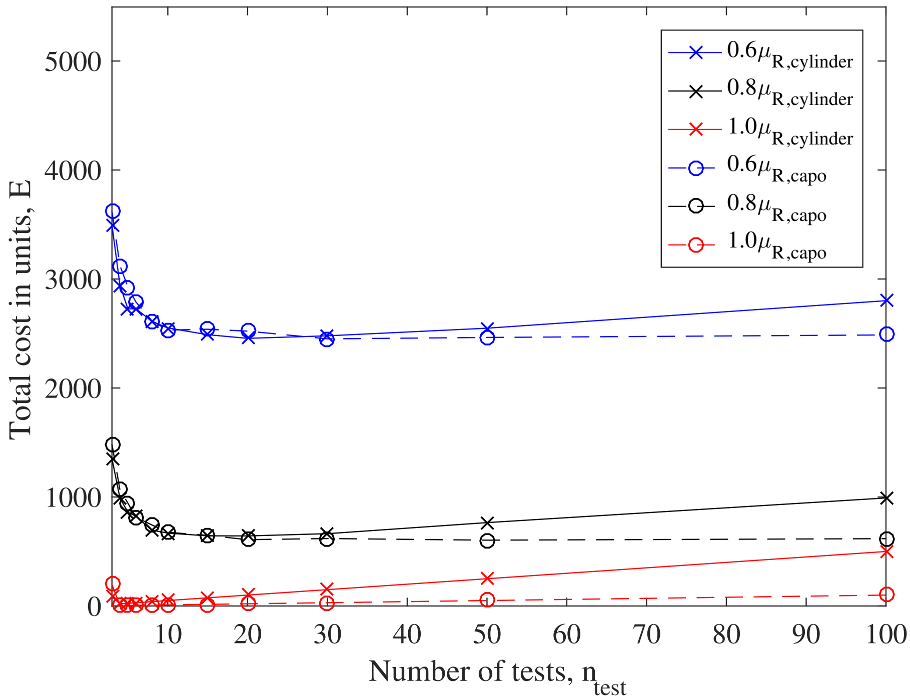

Figure D.18: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

D.0.3 Korrelation = 0.8

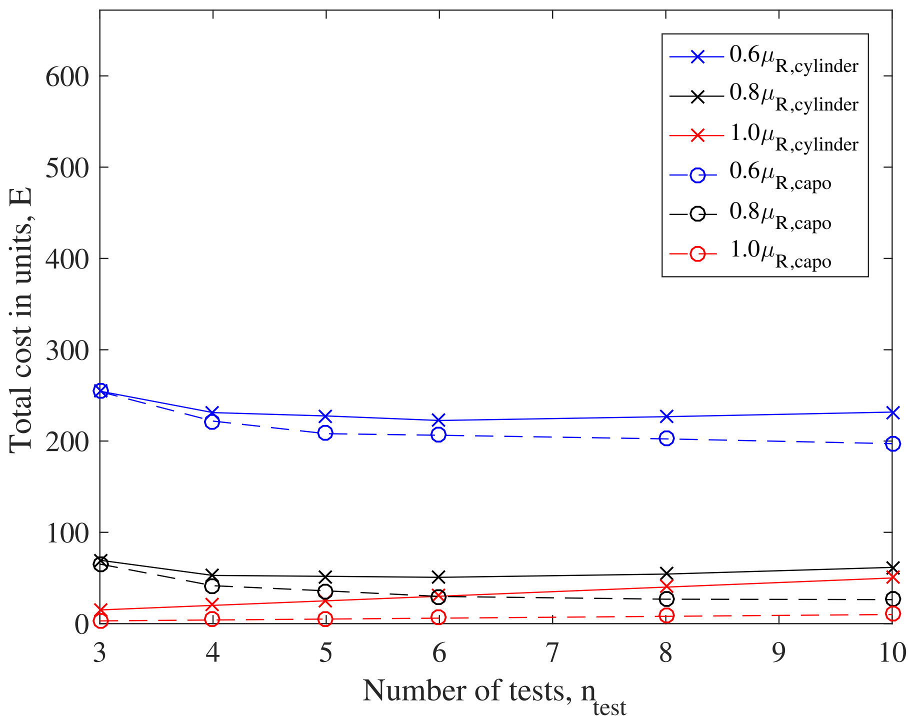

Figure D.19: Cost-benefit analysis of cylinder tests. Total cost for performing the test and reinforcing the component if necessary

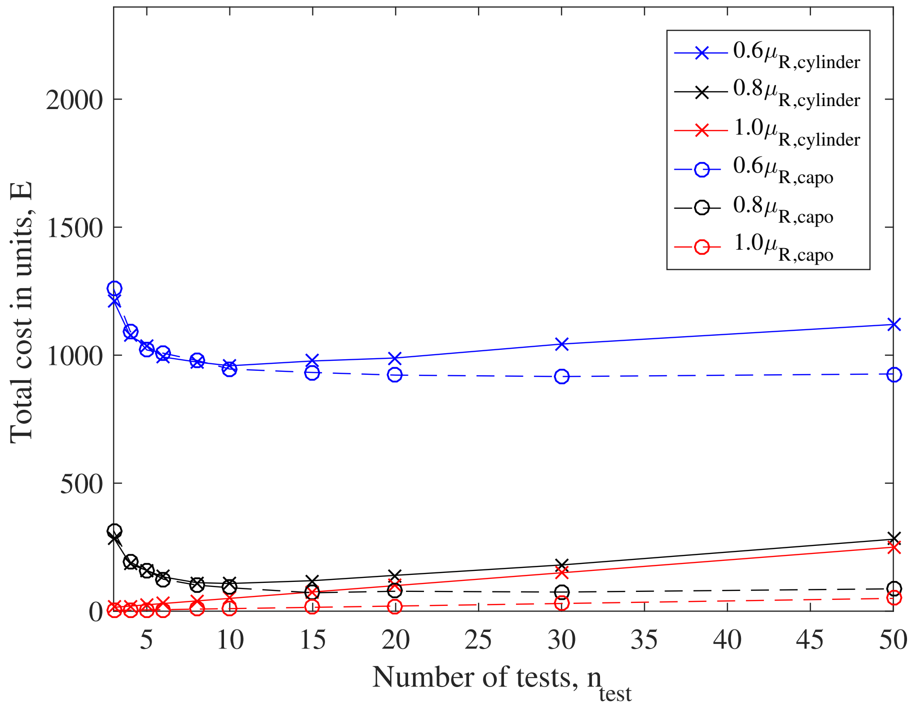

Figure D.20: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

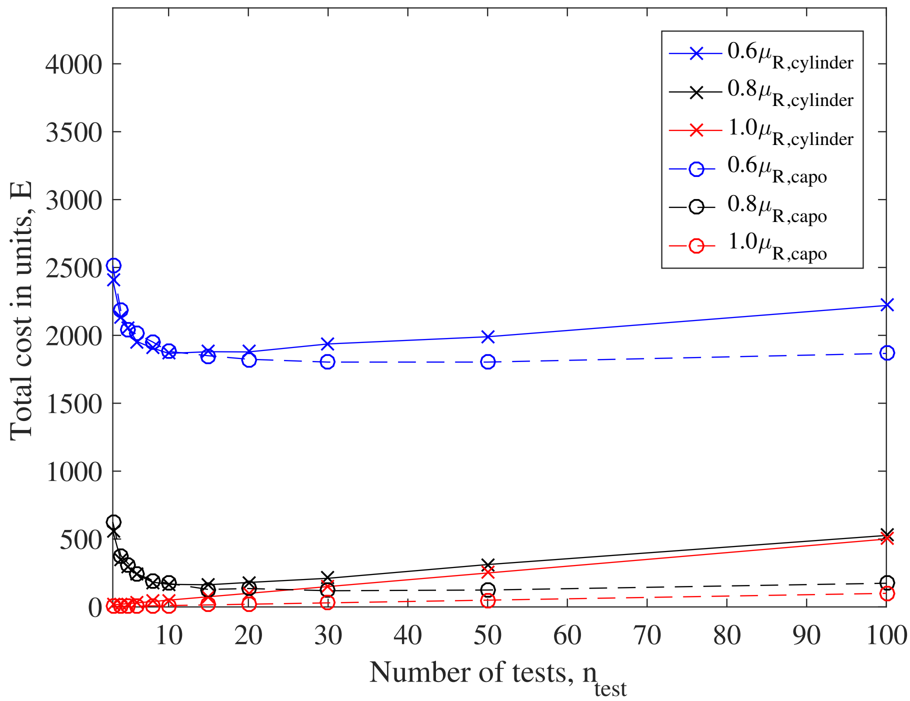

Figure D.21: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

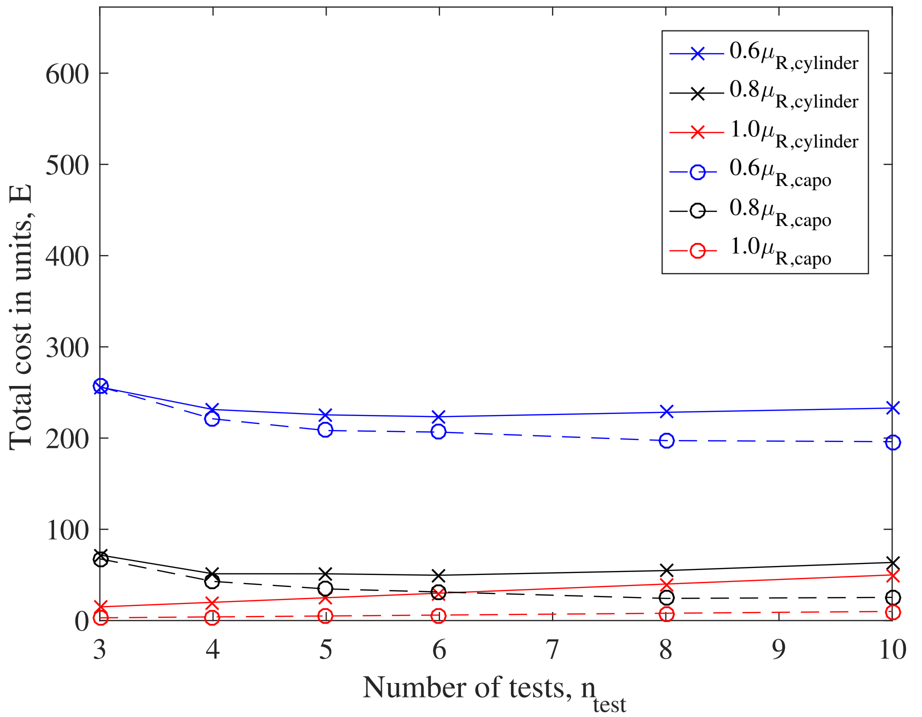

Figure D.22: Cost-benefit analysis of cylinder tests. Total cost for performing the test and reinforcing the component if necessary

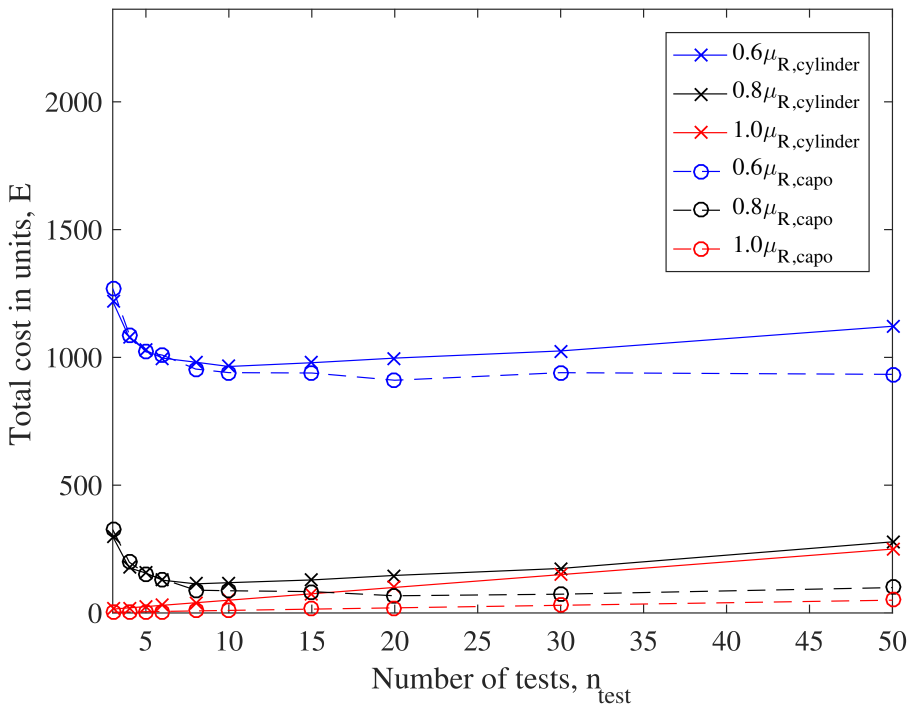

Figure D.23: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

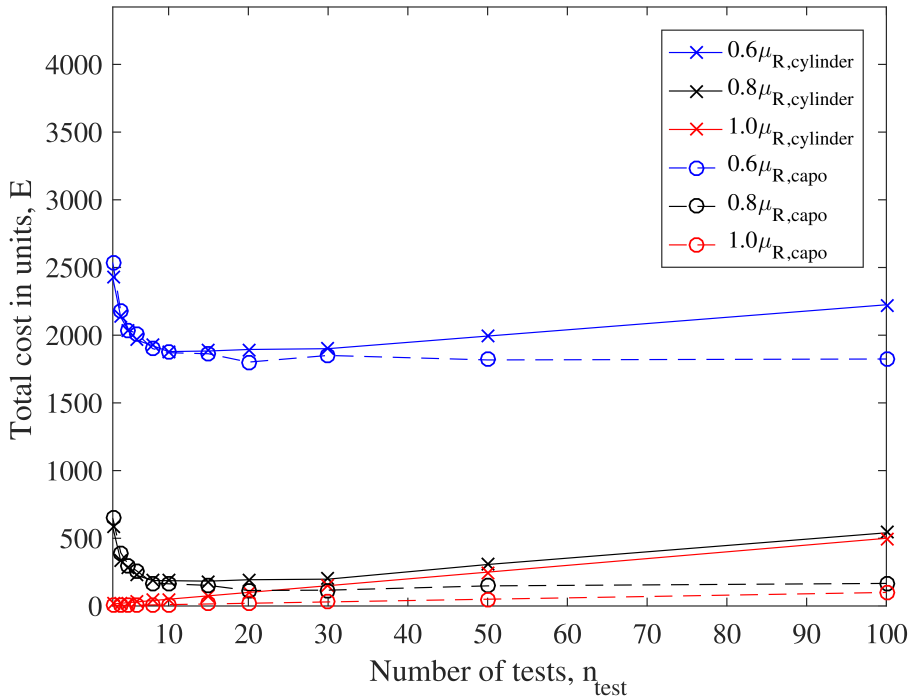

Figure D.24: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

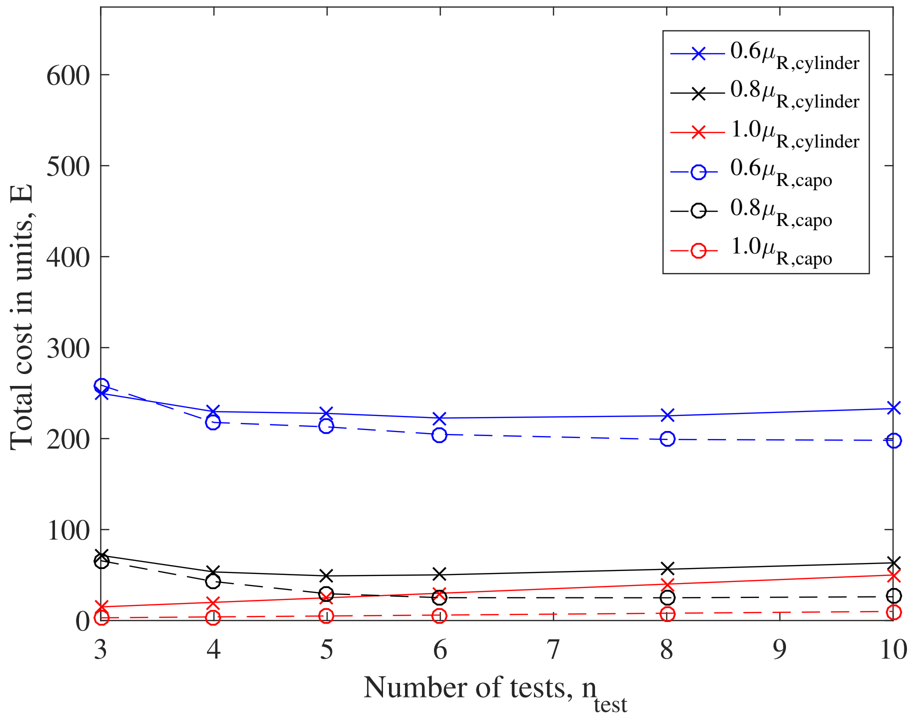

Figure D.25: Cost-benefit analysis of cylinder tests. Total cost for performing the test and reinforcing the component if necessary

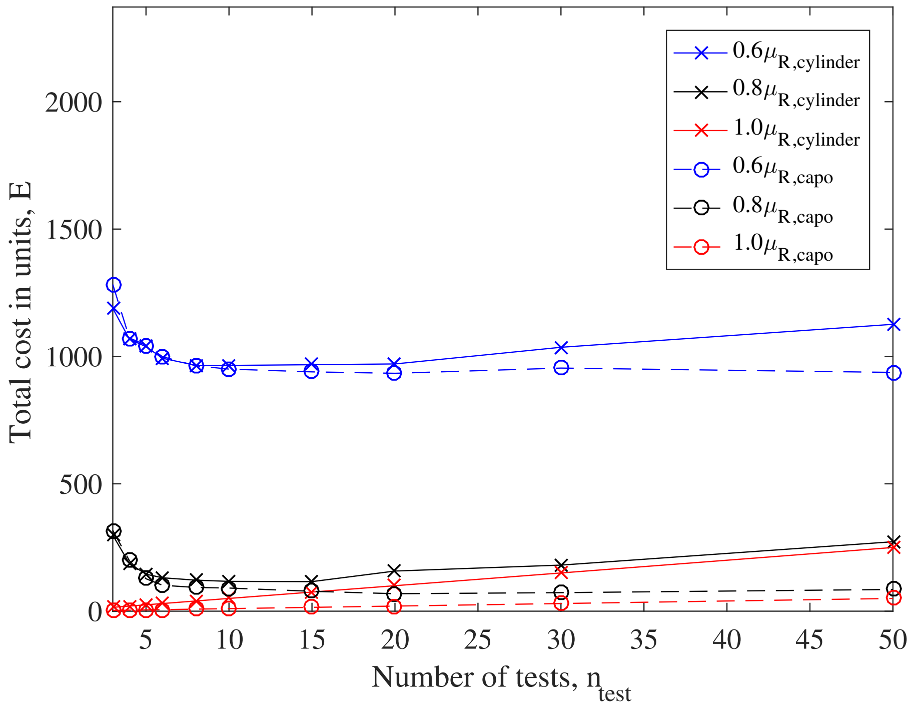

Figure D.26: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

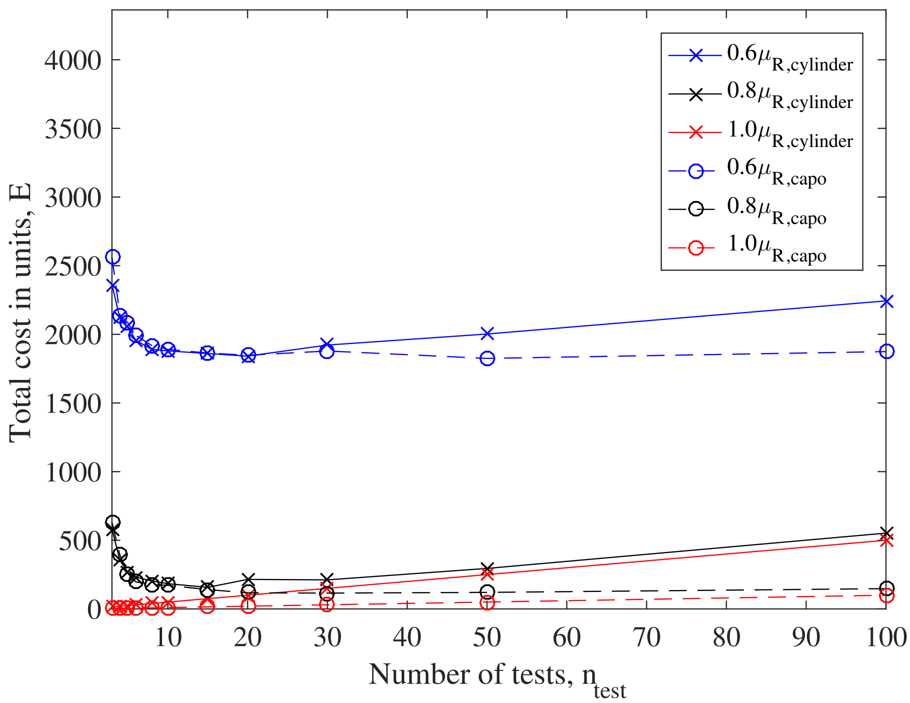

Figure D.27: Cost-benefit analysis of CAPO tests. Total cost for performing the test and reinforcing the component if necessary

Appendix

E Data by Krenchel and Bickley

150 x 300 mm cylinder test			LOK-TEST			CAPO-TEST			Serie	Max.aggr.	Aggr.type
MPa	n	V (%)	kN	n	V (%)	kN	n	V (%)		mm	
33.0	6	3.1	30.0	12	5.6	29.1	12	5.6	A-18	8 mm	Sea gravel
37.1	2	4.0	33.8	4	7.4	35.4	4	4.7	A-17	8 mm	Sea gravel
24.9	4	3.6	24.6	8	9.8	22.9	8	4.9	A-12-B	8 mm	Sea gravel
44.2	6	2.7	38.5	6	4.3	37.9	6	3.4	A-30	8 mm	Sea gravel
30.5	4	5.5	31.3	8	8.3	31.1	8	10.7	A-31	8 mm	Sea gravel
16.3	4	6.0	15.8	8	7.1	17.1	8	5.9	A-28-C	8 mm	Sea gravel
42.8	3	4,7	38.8	6	5.9	39.8	6	6.4	A-22	8 mm	Sea gravel
3.3	6	2.6	**4.7**	12	16.6	**3.4**	8	22.7	B-10-1	**16 mm**	Sea gravel
7.6	6	3.0	**8.7**	12	11.9	**9.0**	12	9.0	B-7-A	**16 mm**	Sea gravel
14.3	6	5.0	**14.9**	12	7.7	**13.0**	12	9.4	B-7-B	**16 mm**	Sea gravel
72.0	4	5.5	**62.7**	8	7.1	**63.4**	8	6.8	B-31-X	**16 mm**	Granite
76.1	4	4.3	**66.8**	8	8.5	**66.9**	8	6.7	B-32-X2	**16 mm**	Granite
75.4	4	3.7	**64.8**	8	7.0	**65.9**	8	6.4	B-32-X3	**16 mm**	Granite
77.0	4	4.8	**65.8**	8	5.9	**66.2**	8	6.5	B-32-X4	**16 mm**	Granite
74.0	9	3.5	**61.6**	24	6.7	**60.6**	24	5.7	B-32	**16 mm**	Granite
28.2	4	3.1	**28.7**	8	5.4	**29.1**	8	6.4	B-13	**16 mm**	Sea gravel
29.5	3	3.5	**33.1**	6	9.1	**32.3**	6	8.6	B-16	**16 mm**	Sea gravel
44.2	6	2.7	**39.2**	6	6.7	**39.7**	6	4.6	B-3	**16 mm**	Sea gravel
39.1	4	4.0	**35.3**	8	5.4	**35.1**	8	5.9	B-2-1	**16 mm**	Sea gravel
25.4	6	5.0	**22.7**	12	7.3	**21.1**	12	6.1	B-4	**16 mm**	Sea gravel
30.8	4	3.3	**30.6**	8	6.9	**30.9**	8	8.2	B-21	**16 mm**	Sea gravel
29.4	3	6.6	**33.5**	6	6.7	**32.7**	6	7.8	B-5	**16 mm**	Sea gravel
42.6	6	3.4	**39.1**	12	6.1	**40.6**	12	5.9	B-32-2	**16 mm**	Sea gravel
39.3	4	4.3	**35.8**	8	5.7	**37.7**	8	6.9	B-20-1	**16 mm**	Sea gravel
38.6	3	3.8	**34.2**	6	7.1	**35.8**	6	5.4	B-29-A	**16 mm**	Sea gravel
40.0	4	4.0	**36.1**	8	5.7	**35.5**	6	6.9	B-14-2	**16 mm**	Sea gravel
37.4	2	2.3	**33.7**	4	6.5	**33.0**	4	2.6	B-11	**16 mm**	Sea gravel
24.7	6	3.6	**21.8**	12	6.0	**21.2**	12	12.9	B-23	**16 mm**	Sea gravel
31.0	4	5.0	30.9	8	7.2	32.5	8	6.4	C-26-1	32 mm	Sea gravel
34.2	4	3.7	31.8	8	8.8	29.9	8	7.4	C-18-1	32 mm	Sea gravel
41.8	6	4.0	39.3	12	7.6	40.5	12	6.4	C-25-1	32 mm	Sea gravel
40.2	4	3.3	37.3	8	5.8	36.9	8	7.2	C-9-1	32 mm	Sea gravel
38.1	3	3.8	35.1	6	7.0	35.8	6	5.2	C-15-1	32 mm	Sea gravel
28.8	4	4.5	26.4	8	9.9	25.8	8	10.1	C-19-1	32 mm	Sea gravel
26.3	4	4.1	22.7	8	12.0	24.2	8	7.1	C-6-A	32 mm	Sea gravel
38.8	4	5.4	38.1	8	7.4	37.4	8	6.9	C-8-1	32 mm	Sea gravel
42.6	6	3.4	38.8	6	3.2	39.8	6	3.4	C-25-2	32 mm	Sea gravel

Department of Structural Engineering, DTU, Denmark, original data with supplementing data
February, 1987

Figure E.1: Data by Krenchel and Bickley from 1987 for cylinder and CAPO-tests.