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STUDENT REPORT

Physical aspects of air in pipe systems, including its effect on pipeline flow capacity and surge pressure (water hammer)

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Abstract:

The presence of air in pipe systems may lead to malfunction of pipes where transient flow analysis, not considering the air, might not predict such concern. Therefore, it is essential to understand the effects of air on transient flows.
In this thesis, the basic concepts of transient flow, as well as the transient-flow equations, are first introduced for a better understanding of the phenomenon.
Then, some of the most common sources of air in pipelines, as well as of the effects of air in transient flows, are listed.
The characteristics method, or Method of Characteristics (MOC), is applied for the computation of the main flow variables of pressure head and flow speed in transient flows. A number of computer programs, for the solution of transient-flow problems under various initial and boundary conditions, both with and without the presence of air in them, are presented in MATLAB.
The Volume of Fluid (VOF) model, is, as well, adopted in the computation of the main transient-flow variables. This time, STAR-CCM+, a CFD code, simulation platform, is used.
The results of the MATLAB programs and STAR-CCM+ simulations, are then presented, discussed, and, if available, compared to experimental data.

Acknowledgements

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Above all, I want to thank my supervisor, Torben Larsen, for taking the time to listen to me, for the guidance, and patience.

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Finally, particular thanks to my parents, for always being there for me.

Reading guide

The Harvard citation style is used to incorporate other author's quotes, findings and ideas in order to support and validate conclusions without breaching any intellectual property laws. The referencing system is made up of two main components:

In-text citations including the author's last name and year of publication, shown in brackets. If the source has three or more authors, the first author's surname is used, amid the abbreviation 'et al.' e.g., Bergant, *et al*. (2001). If quoting a particular section of the source, the page number or page range should also be included after the date; e.g. Wylie, E. B. and Streeter, V. L. (1993, p.192)

A reference list outlining all of the sources directly cited; e.g., Bergant, A., et al. (2001) 'Developments in unsteady pipe flow friction modelling', *Journal of Hydraulic Research*, 39(3), pp. 249-257. The list is arranged in alphabetical order by the author's last name. If two, or more sources by the same author are cited, they should be listed in chronological order of year of publication.

Tables and figures are numbered in the order they first appear in the text, each of them displayed with a brief explanatory title and including a caption beneath the table; e.g., Figure 1.1 (a) Sudden closure of the valve; (b) control volume; (Wylie, E. B. and Streeter, V. L. 1993). Appendices are labeled in alphabetical order; e.g., Appendix A.1 Basic Equation of Water Hammer.

Contents

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Nomenclature

A	Cross-sectional area of pipe; Starting point in x-t plane
A_o	Orifice area
a	Wave speed
B	Pipeline characteristic impedance, a/gA; Starting point in x-t plane
B_M, B_P	Known constants in compatibility equations
C^+, C^-	Name of characteristics equations
C_M, C_P	Known constants in compatibility equations
C_d	Orifice discharge coefficient
D	Pipe diameter
F	Force
f	Darcy-Weisbach friction factor
g	Gravitational acceleration
H	Instantaneous pressure head, absolute or relative
$\overline{H}$	Barometric head
H_A	Pressure head at starting point in x-t plane
H_P	Pressure head at unknown computational point in x-t plane of characteristics grid
H_R	Pressure head at reservoir
H_a	Absolute pressure head of the air pocket
H_{max}	Maximum pressure head
H_{min}	Minimum pressure head
H_v	Vapor pressure head
$H_{a,0}$	Steady-state absolute pressure head of the air pocket
H_0	Steady-state pressure head
i	Denotes section number along a pipe
K	Bulk modulus of elasticity
L	Pipe length
m	Mass; polytropic exponent
N	Number of reaches in a pipe
P	Solution point in x-t plane
p	Pressure
Q	Instantaneous flow rate
Q_P	Unknown flow rate at P
Q_0	Steady-state flow rate
R	Pipeline resistance coefficient, $f\Delta x/2gDA^2$
s	Pipe stretch in length
S	Pipeline slope parameter
t	Time; as a subscript denotes partial differentiation
u	Speed of the pipe, x direction
V	Instantaneous flow speed
V_A	Flow speed at starting point in x-t plane
V_P	Unknown flow speed at P
V_0	Steady-state flow speed
$\cancel{V}_a$	Instantaneous volume of the air pocket
$\cancel{V}_c$	Cavity volume
$\cancel{V}_{a,0}$	Steady-state volume of the air pocket
$\cancel{V}_{c,0}$	Steady-state cavity volume
x	Distance along the pipe, from upstream end; as a subscript denotes partial differentiation
Y	Expansion coefficient
z	Elevation of pipe above datum
α	Pipe slope; void fraction
γ	Unit weight of fluid

δ_x	Control volume thickness
ε	Error
λ	Multiplier in characteristics method
ρ	Mass density
τ_0	Shear stress

1 INTRODUCTION

The presence of trapped air in pipelines has a great influence on the behavior of the main flow variables in case of a transient, which can lead to malfunction of the system in situations where transient flow analysis, that does not consider the presence of air, may not foresee such concern. The presence of trapped air, when it accumulates in high points of the system, may, in turn, reduce the effective cross-section of the pipe, increase the friction, and, therefore, increase the head losses through the pipeline. This all leads to a reduced flow capacity, and an increase in energy consumption of the pump; considerable costs.

The overall aim of this thesis is to analyze the above-mentioned effects of air in pipelines. To this end, the characteristics method (MOC) is applied for the computation of the main flow variables of pressure head and flow speed in transient flows. A number of computer programs, for the solution of transient-flow problems under various initial and boundary conditions, both with and without the presence of air in them, are presented in MATLAB. The Volume of Fluid (VOF) model, is, as well, adopted in the computation of the main transient-flow variables. This time, STAR-CCM+, a CFD code, simulation platform, is used. In order to do so, it is first necessary to proper understand the concepts associated to transient flows, as well as the equations behind the characteristics method.

2 TRANSIENT FLOW: CONCEPTS

2.1 Classification of Flow: Terminology

If all flow conditions at any point remain constant with respect to time, the flow is called *steady*. However, if conditions at any point change with time, the flow is known as *unsteady*. The intermediate-stage, in which conditions change from one steady state to another, is called *transient flow*. A transient involving a sudden, great increase or movement, or both, of pressure, is known as *pressure surge* or *surge*. In the past, *water hammer* was used to refer to the term '*pressure surge*', given that the fluid was water. Nonetheless, *hydraulic transient* has become popular since the 1960s (Chaudhry, 2014).

2.2 Basic Equation of Water Hammer

The momentum, and continuity equations, are applied to a control volume, Fig. 2.1; which pictures a section of a pipe. Analysis of the sudden closure of a downstream valve.

The pipe is assumed frictionless, and the fluid, slightly compressible. The flow speed, V, is considered positive in the downstream direction. As for the pipe walls, these are considered as rigid walls; so, the pipe cross-sectional area, A, does not change due to pressure changes; during the transient.

The fluid moves at V_0, and the steady-state pressure head upstream of the reservoir is p (initial conditions). (At) t = 0; the valve is closed, and the fluid nearest to it, brought to rest; V_0 changes to $V_0 + \Delta V$. This change in flow speed; of ΔV, results in an increase in pressure head at the face of the valve, Δp; the fluid is (slightly) compressed; initiates the transient.

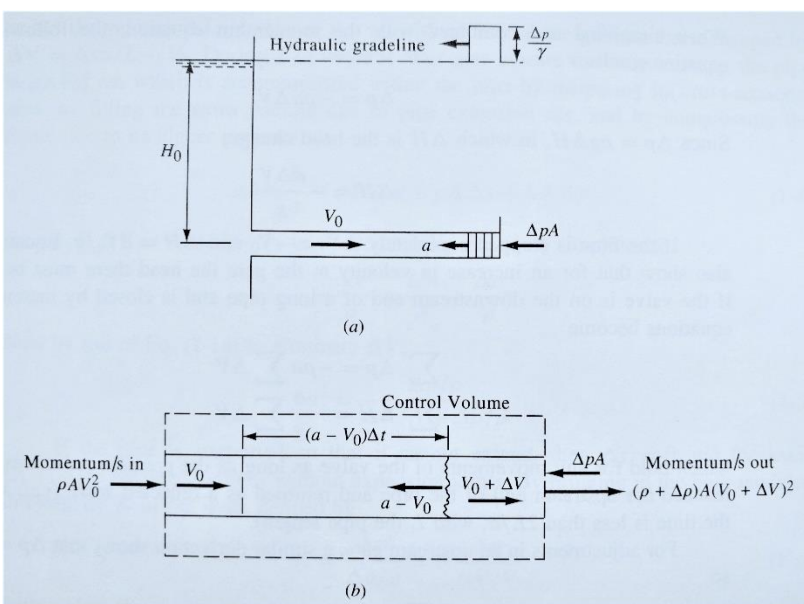


Figure 2.1 (a) Sudden closure of the valve; (b) control volume; (Wylie, E. B. and Streeter, V. L. 1993).

By applying the aforementioned equations (of momentum, and continuity,) to the control volume in Fig. 2.1, the increase in pressure head, Δp, may be determined

$$\sum \Delta \mathrm{p} = \pm \rho \mathrm{a} \sum \Delta \mathrm{V} \tag{2.1}$$

in which ρ is the density of the fluid, and a, the wave speed; Section 2.3. Since Δp = ρgΔH; in which g is the gravitational acceleration, and ΔH, the head change

$$\sum \Delta \mathrm{H} = \pm \frac{a}{g} \sum \Delta \mathrm{V} \tag{2.2}$$

which is the *basic equation of water hammer*; the plus sign is used for waves traveling upstream whereas the minus sign is used for waves traveling downstream.

The complete derivation of the transient flow equation; *basic equation of water hammer*, can be found in Appendix A.1.

2.3 Wave Speed

Let us now consider the pipe walls to be, to some extent, elastic. Therefore, when the valve is closed; (at) t = 0, the pipe may stretch in length, Δs, Fig. 2.2, and its cross-sectional area increase, ΔA, (all) due to the increase in pressure head at the face of the valve, Δp,

based on the foregoing; these assumptions, the *wave speed* equation is

$$a^2 = \frac{K/\rho}{1 + (K/A)(\Delta A/\Delta p)} \tag{2.3}$$

in which K is the bulk modulus of elasticity of the fluid, defined by

$$K = \frac{\Delta p}{\Delta \rho/\rho} \tag{2.4}$$

In case of a very thick-walled pipe, ΔA/Δp is very small (rigid walls), and $a \approx \sqrt{K/\rho}$ is the *wave speed* of a small disturbance in an infinite fluid. Small amounts of entrained gas in the liquid, or gas that has come out of solution, greatly modify the acoustic speed in a pipe, (Wylie, E. B. and Streeter, V. L. 1993).

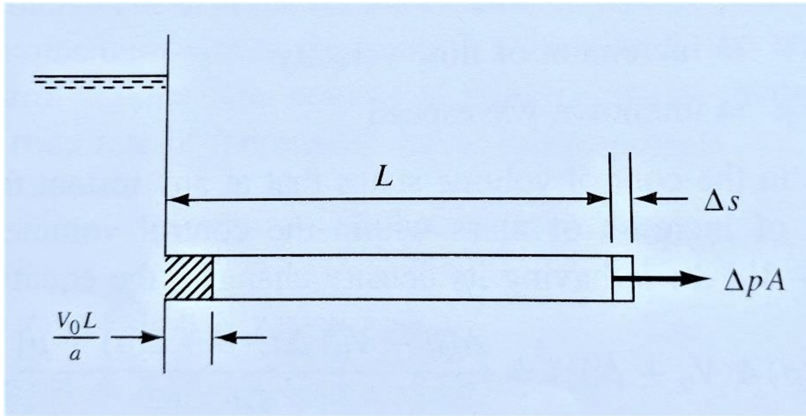


Figure 2.2 Continuity relations in the pipe; (Wylie, E. B. and Streeter, V. L. 1993).

The complete derivation of the *wave speed* equation may be found in Appendix A.2.

2.4 Wave propagation

The same situation, as in Section 2.2, is considered.

a) (At) t = 0; the valve is closed, the fluid nearest to it brought from V_0 to rest, ΔV = -V_0, and the pipe wall stretched, Δs, due to an increase in pressure at the face of the valve (compression of the fluid), ΔH = -(a/g)ΔV. Once the first layer is compressed, the process is repeated for the next layer of fluid. A high-pressure pulse wave is seen as traveling upstream at some wave speed, a, Section 2.2, bringing the fluid to rest, compressing it, and stretching the pipe, as it passes. At t = L/a (seconds), the wave arrives at the upstream (reservoir) end of the pipe, and through its entire length, the pipe is stretched, V = 0, and H = H_0 + ΔH.

b) t = L/a; the high-pressure pulse wave reaches the upstream (reservoir) end of the pipe, and the pressure drops from H_0 + ΔH, in an adjacent (to the reservoir) layer of fluid in the pipe, to H_0, in the reservoir (constant). The fluid begins flowing backwards, brought from rest to -V_0, ΔV = -V_0, and the pipe wall and pressure return to normal; due to the pressure drop. This process is visualized as traveling downstream at a speed a. At t = 2L/a, the wave reaches the valve, and, for the entire length of the pipe, V = -V_0, and H = H.

c) t = 2L/a; the wave reaches the valve, which is yet closed, the fluid adjacent to it is brought from -V_0 to rest, ΔV = V_0, and the pipe wall contracted, -Δs, because of a pressure drop at the face of the valve (compression of the fluid), ΔH = (a/g)ΔV. A low-pressure pulse wave travels upstream at a speed a, and, by the time the wave reaches the upstream end of the pipe, at t = 3L/a, through its entire length, the pipe is contracted, V = 0, and H = H_0 - ΔH.

d) t = 3L/a; the low-pressure pulse wave reaches the upstream reservoir, and the pressure increases from H_0 – ΔH, in an adjacent layer of fluid in the pipe, to H_0, in the reservoir. The fluid is brought to V_0 from rest, and the pipe wall and pressure return to normal; due to the increase in pressure. A high-pressure pulse wave is visualized as traveling downstream at a speed a. At t = 4L/a, the wave arrives at the valve, and, for the entire length of the pipe, $V = V_0$, and $H = H_0$ (initial situation).

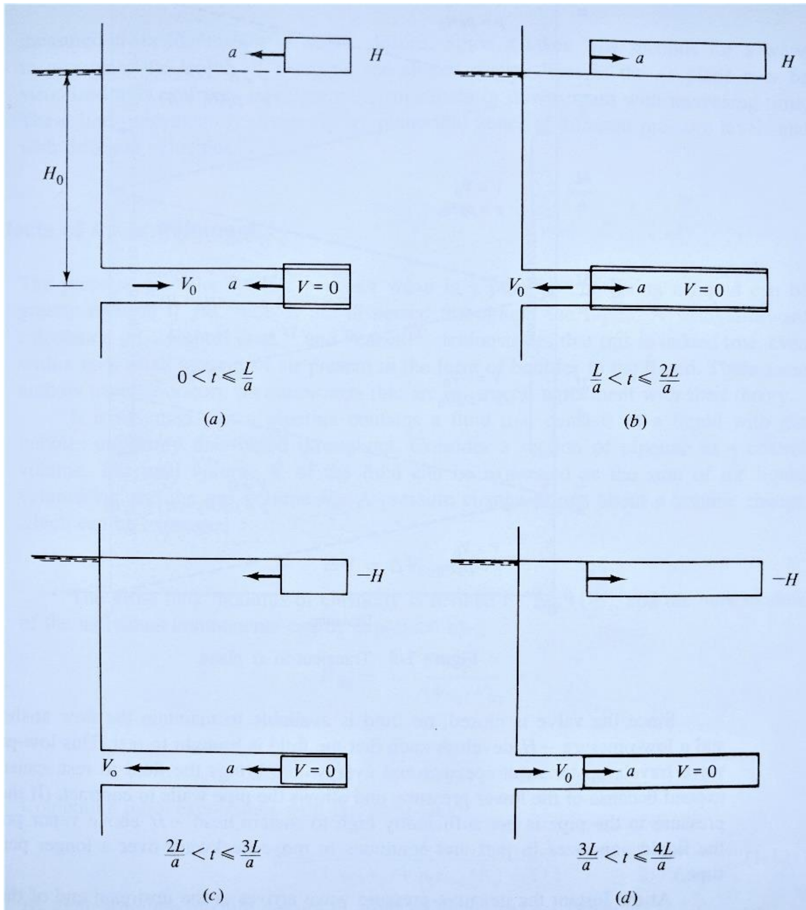


Figure 2.3 Wave propagation; complete cycle after sudden closure of a valve; (Wylie, E. B. and Streeter, V. L. 1993).

2.5 Causes of Transients

As per definition, Section 2.1, a transient-state occurs as long as the steady-state, flow conditions, are being changed; from one steady state to another. The aforementioned changes, in turn, may be due to planned or accidental changes in the settings of the control equipment of a man-made system, or by changes in the inflow or outflow of a natural system.

Some of the main causes of transients in engineering systems could be the opening, or closing of valves in a pipeline, as well as starting or stopping of pumps or compressors, cavitation or column separation, or a sudden increase in a river or sewer inflow; due to a heavy storm. The survey of transients quite often covers situations in which more than one of these causes are present.

3 TRANSIENT-FLOW EQUATIONS

3.1 Equation of Motion

The Newton's second law of motion, $\sum F = ma$, is applied to a control volume (conical tube), Fig. 3.1; full of fluid, of mass density, ρ; average, cross-sectional flow speed, V, and pressure, p, equal to the centerline pressure, converted into hydraulic-grade-line head, H, when necessary, by $p = \rho g(H - z)$; cross-sectional area, A, thickness, δx, and inclined α(°) with respect to the horizontal.

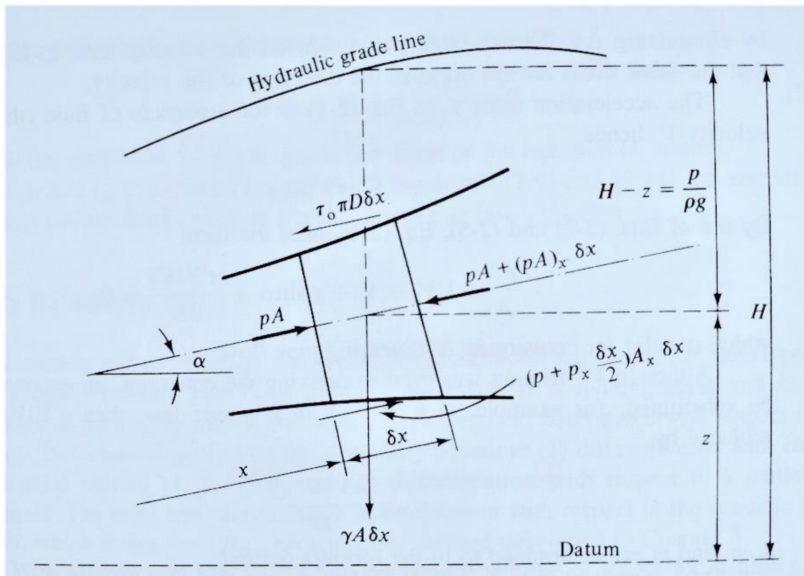


Figure 3.1 Control volume for equation of motion; (Wylie, E. B. and Streeter, V. L. 1993).

τ_0 being the shear stress, D the diameter of the control volume (tube), and γ the unit weight of fluid, thus

$$gH_x + V_t + \frac{fV|V|}{2D} = 0 \tag{3.1}$$

in which f is the Darcy-Weisbach friction factor, and which is the simplified, head form of the *equation of motion*; restricted to less compressible fluid, flowing at low velocities. The subscripts x and t denote partial differentiation, i.e., $p_x = \partial p/\partial x$.

The complete derivation of the *equation of motion* may be found in Appendix B.1.

3.2 Continuity Equation

The continuity equation, applied to a moving control volume, Fig. 3.2; stationary relative to the pipe, it moves or stretches only as the inside surface of the pipe moves and stretches, yields

$$\frac{a^2 V_x}{g} + H_t = 0 \tag{3.2}$$

which is the simplified, head form of the unsteady *continuity equation*; restricted to less compressible fluid, flowing at low velocities; a being the wave speed, Section 2.3.

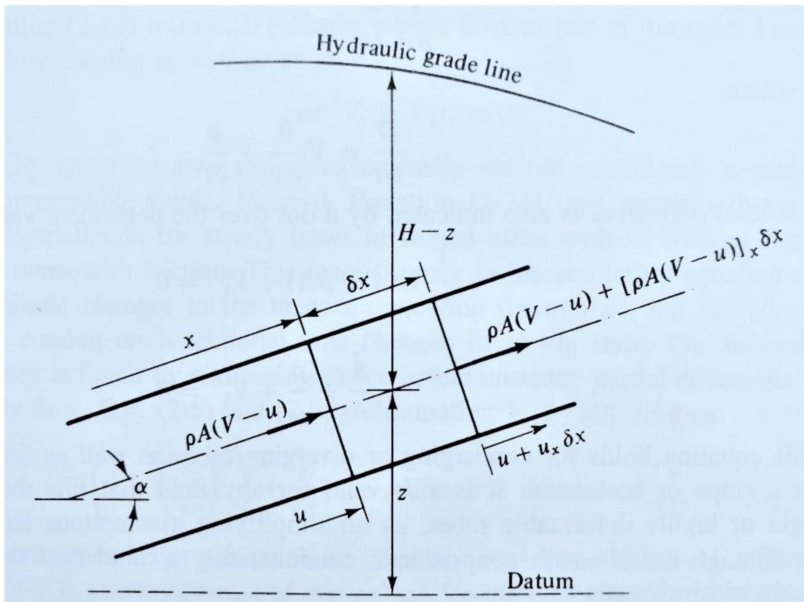


Figure 3.2 Control volume for continuity equation; (Wylie, E. B. and Streeter, V. L. 1993).

u being the speed of the pipe at x. The complete derivation of the *continuity equation* may be found in Appendix B.2.

3.3 Unsteady Friction

The expression that relates shear stress, τ_0, to average, cross-sectional flow speed, V, in steady, or quasi-steady-state flow; in terms of the Darcy-Weisbach friction factor, f,

$$\tau_0 = \frac{\rho f V|V|}{8} \tag{3.3}$$

is considered to remain valid under unsteady conditions; applied in the derivation of the equation of motion, Section 3.1.

Bergant, A., et al. (2001), tested the quasi-steady friction model, with experimental data; results, obtained by the quasi-steady friction model, give good agreement with the experimental data for the first and second pressure, head rise. Nevertheless, the imbalance between the results increased for later times; imbalance in attenuation of the pressure head; the quasi-steady friction model overestimates the heads, and phase shift; it does not predict the shape of the wave properly. This is not an issue when determining the maximum, or minimum heads.

Bergant, A., et al. (2001), tested, as well, the frequency-dependent friction models of Zielke, W. (1966), and Brunone, B., et al. (1991); same experimental data, substantial upgrade in estimating the attenuation and phase shift of the pressure head traces; to be considered in future studies.

4 AIR IN PIPELINES: EFFECTS ON TRANSIENT FLOW

4.1 Air in Pipelines: Terminology

According to Wisner, P. E., et al. (1975), air may be present in pipelines as *bubbles*, or *pockets*. *Bubbles* are defined as small droplets of air entrapped in water by a turbulent action; e.g. a hydraulic jump, while *pockets* may be defined as air cavities formed as a result of a coalescence of bubbles, or by entrapment of large quantities of air; e.g. during the filling of a pipeline. Vapor, bubble formation and growth in a fluid, due to a pressure drop to vapor pressure, is called *cavitation*. If these bubbles enlarge (merge), filling the entire cross section of the pipe, the phenomenon is referred to as *column separation* (Chaudhry, M. H. 2014). Air, partially bounded by the fluid, is called *entrapped* (or contiguous) air; e.g. an air pocket, while, air, in the form of individual bubbles, separated by relatively thick films of liquid, is referred to as *entrained* air (Zhou, F. 2000).

4.2 Sources

With a view to measure, monitor, and get rid of air that might be found in pipelines, the various means by which air can enter a pipe system are to be understood.

Air coming out of solution; cavitation, or column separation. Water contains about 2% dissolved air under normal conditions of pressure and temperature. The solubility of air in water increases with pressure; and decreases with temperature. Thus, pressurized water, as in pumping systems, is able to withhold more air; than in the case of a gravity-driven flow. The air can come out of the solution as result of a pressure drop to vapor pressure, or an increase in temperature. Once the air is released from the solution, it does not have the ability to return to the solution and will collect in pockets at high points along the pipe.

In addition to air coming out of solution, there are several ways air can be found in pipelines; some of which are listed below (Lauchlan C. S., et al. 2005):

Entrainment at the inflow, or outflow location. Turbulent action, e.g. hydraulic jump. Direct pumping of air into a system; in order to reduce cavitation. There may be insufficient submergence on the pump or vortices may form at the inlet causing air to be entrained into the system. Air transport during filling and emptying of pipelines. Gas formation through biological activity. At sections under negative pressure air can leak in at joints and fittings.

4.3 Effects on Transient Flow

Air tends to become trapped at high points along the system, due to buoyancy; air is lighter than water. The effects of entrapped or entrained air on hydraulic transients can be either beneficial or detrimental, the outcome being entirely dependent on the characteristics of the pipeline affected, and the nature and cause of the transient (also the fraction of air). Some of these effects are listed below (Lauchlan C. S., et al. 2005):

The effective cross section of the pipe is reduced; increased friction, thus increased head losses, leading to a diminished pipe flow capacity, and an increase in energy consumption of the pump. The flow capacity is reduced when the air pocket cannot be transported and removed from the pipe; the flow could even stop completely. When air-mixed water is fed into a turbine, a pressure drop in output occurs, and the efficiency is also reduced. Compression of the air pocket may cause abnormal pressure surges (Wylie, E. B. and

Streeter, V. L. 1993), while expansion of it may lead to sub-atmospheric pressures (Coronado-Hernández, O. E., et al. 2017a); both cases may cause damage to the pipe. According to Wylie, E. B. and Streeter, V. L. (1993), the propagation velocity of a pressure wave in a pipeline containing a liquid can be greatly reduced if gas bubbles are dispersed throughout the liquid; cushioning effect of the air pocket (absorbs energy). The bulk properties of the fluid, such as density and elasticity, are changed; the fluid is now a combination of air and water. Air accumulation in a system may lead to disruption of the flow. This can lead to vibration and structural damage, and cause instabilities of the water surface. In ferrous pipelines the presence of air enhances corrosion by making more oxygen available for the process; hydrogen sulfide in wastewater systems. The presence of air can result in malfunction of measuring devices.

5 CHARACTERISTICS METHOD

A numerical method, for the solution of the transient-flow equations, Section 3, is presented in this chapter. The characteristics method (MOC) transforms the partial, differential equations of motion, and continuity, into ordinary, differential equations. These are then integrated to obtain a finite-difference representation of their variables.

5.1 Characteristic Equations

The motion and continuity; Eqs. (3.1), and (3.2), are a pair of partial differential equations, function of flow speed, V. and pressure head, H, as two, dependent variables, and distance(; through the pipe), x, and time, t, as two, independent variables. The value of the dependent variables depends on the value of the independent variables. These are converted into four ordinary, differential equations by the characteristics method; specified time intervals.

As noted in Sections 3.1, and 3.2, the simplified, pressure-head form of the motion and continuity equations

$$gH_x + V_t + \frac{f}{2D}V|V| = 0 \tag{3.1}$$

$$H_t + \frac{a^2}{g}V_x = 0 \tag{3.2}$$

which, once derived, yield

$$C^+ : \begin{cases} \dfrac{g}{a}\dfrac{dH}{dt} + \dfrac{dV}{dt} + \dfrac{fV|V|}{2D} = 0 & (5.1) \\ \dfrac{dx}{dt} = +a & (5.2) \end{cases}$$

$$C^- : \begin{cases} -\dfrac{g}{a}\dfrac{dH}{dt} + \dfrac{dV}{dt} + \dfrac{fV|V|}{2D} = 0 & (5.3) \\ \dfrac{dx}{dt} = -a & (5.4) \end{cases}$$

Eqs. (5.1) and (5.3) are known as compatibility equations. Eqs. (5.2) and (5.4) plot two straight lines on the x-t plane (if a is constant), Fig. 5.1. These are referred to as the "characteristic" lines; eliminate t, transform the partial differential, into ordinary, differential equations. Nevertheless, Eqs. (3.1) and (3.2) are valid everywhere in the x-t plane, while Eqs. (5.1) and (5.3) are valid only when their respective Eqs. (5.2) and (5.4) are valid. No mathematical approximation is made in this transformation.

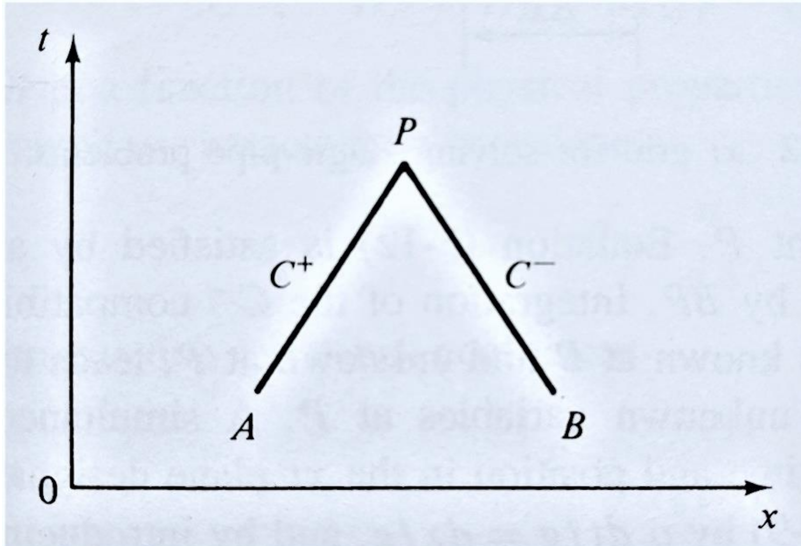


Figure 5.1 Characteristic lines; x-t plane; (Wylie, E. B. and Streeter, V. L. 1993).

The complete transformation of the partial differential, into ordinary, differential equations can be found in Appendix C.1.

5.2 Finite-Difference Equations

A pipeline is divided into an even number of reaches, N, each of Δx, in length, shown in Fig. 5.2; thus, N+1 number of grid intersection points (nodes). A time step of Δt = Δx/a, is defined; even submultiple of the transit time, L/a. Eq. (5.1) is valid along dx/dt = +a, shown by the line AP; C+ line, positive slope. If the velocity, V_A, and pressure head, H_A; dependent variables, are known at A, then Eq. (5.1), may be integrated between A and P (limits), and therefore be written in terms of V_P and H_P; unknown, at point P. The same applies to Eq. (5.3) along C- line. A simultaneous solution (of the two) yields conditions at a particular time and position in the x-t plane; point P.

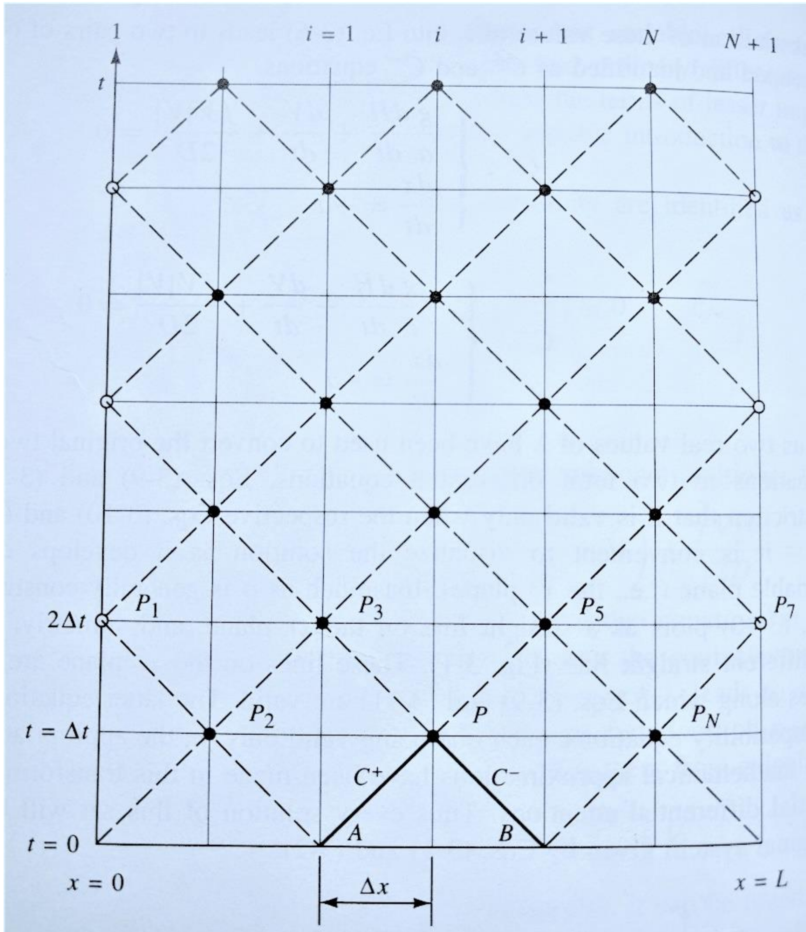


Figure 5.2 x-t grid for solving single-pipe problems; (Wylie, E. B. and Streeter, V. L. 1993).

The study of a hydraulic transient often begins with steady-state conditions at t = 0; initial values of H, and V, are known at each grid intersection point (node), Fig. 5.2. The method consists on finding H, and V, for each grid point along the pipe at t = Δt, then, (same) for t = 2Δt, and so on, until the entire simulation time duration has been covered. By introducing the pipeline area, A, to write the equation in terms of discharge, Q, in place of velocity, V, the integration of Eqs. (5.1) and (5.3), along C+, and C-, respectively, yields

$$C^{+}: H_P = C_P - B_P Q_P \tag{5.5}$$

$$C^{-}: H_P = C_M + B_M Q_P \tag{5.6}$$

At any node, e.g. point P at section i, Fig. 5.2, the two compatibility equations, Eqs. (5.5) and (5.6), are solved simultaneously for the unknowns Q_P and H_P; coefficients C_P, B_P, C_M and B_M are known constants

$$C_P = H_{i-1} + BQ_{i-1} \qquad B_P = B + R|Q_{i-1}| \tag{5.7}$$

$$C_M = H_{i+1} - BQ_{i+1} \qquad B_M = B + R|Q_{i+1}| \tag{5.8}$$

in which B is a function of the physical properties of the fluid and the pipeline, often called the pipeline characteristic impedance

$$B = \frac{a}{gA} \tag{5.9}$$

and R is the pipeline resistance coefficient

$$R = \frac{f\Delta x}{2gDA^2} \tag{5.10}$$

The complete transformation of the partial differential, into *ordinary, differential equations* can be found in Appendix C.2.

5.3 Boundary Conditions

As stated in Section 5.1, Eq. (5.1) is only valid along the C^+ characteristic, AP, while Eq. (5.3), holds along the C^- characteristic, BP. At any node, e.g. point P at section i, Fig. 5.2, the two compatibility equations are solved simultaneously for the unknowns Q_P and H_P. Nevertheless, only one of Eqs. (5.1); downstream end, and Eq. (5.3); upstream, is available at the boundaries, Fig. 5.3; so special, boundary conditions are required. Q_P, and H_P, are determined by solving Eq. (5.1), or (5.3), together with these conditions, imposed at the boundaries. A boundary condition may be, e.g. the end condition of a pipeline; dead-end, presence of a valve, etc.

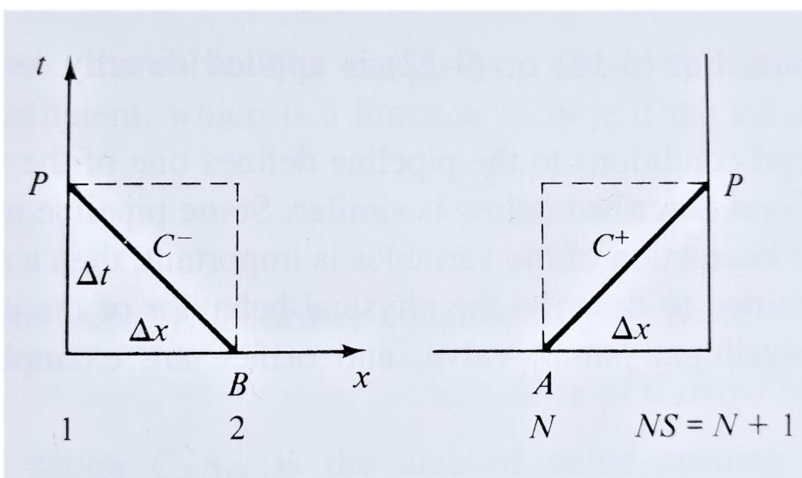


Figure 5.3 Characteristic lines at the boundaries; (Wylie, E. B. and Streeter, V. L. 1993).

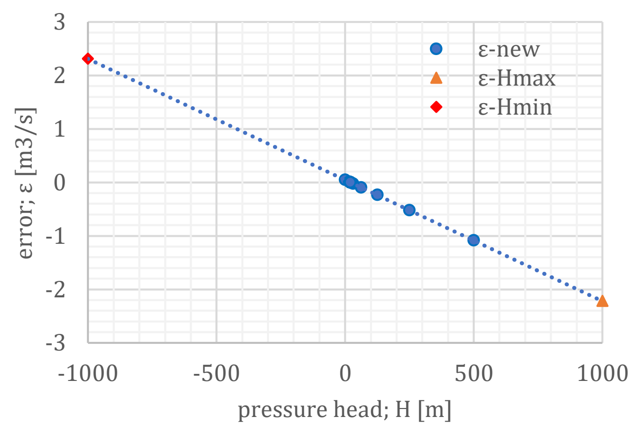
5.4 Error-based Method

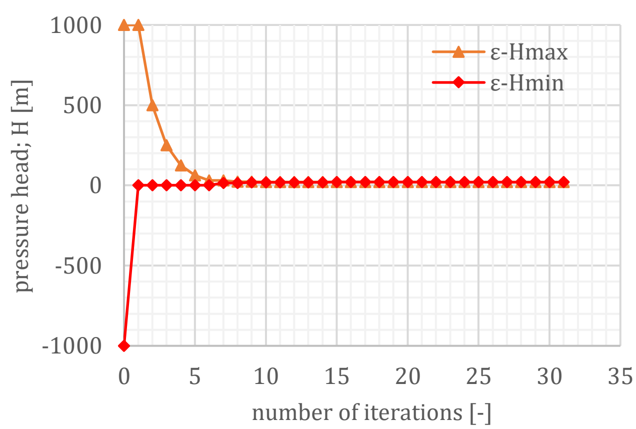
The idea behind this approach is that the flow rate, at point P, Q_P, should be the same when determined by either Eq. (5.1) or (5.3). Thus, the pressure head, H_P, for which Q_P would be the same when calculated by either Eq. (5.1) or (5.3), is to be iteratively solved at each internal node of the pipeline.

Two initial, maximum and minimum values of the pressure head, H_{max} and H_{min}, are assumed as limits of our calculation; the desired H_P has to lie within these limits, so, a wide range is considered. H_P is known, thus Q_P may be determined by only one of Eqs. (5.1) and (5.3); as if it was a boundary. Q_P is determined by both, Eqs. (5.1) and (5.3); it has to be the same, if it is not the same, the difference between the two is calculated. The error, ε, or disturbance, is defined as the deviation of this difference from a "true" value, set equal to 10^{-10} m³/s. Two, maximum and minimum errors, ε-H_{max} and ε-H_{min}, one for each limit, are determined.

A new H_P is then estimated as the arithmetic mean of the initial, H_{max} and H_{min}. Q_P is determined by both Eqs. (5.1) and (5.3), and, again, the new error, ε-new, estimated. According to whether or not ε-new is below, or above the "true" value, the H_P connected to this error is set as new, ε-H_{max} or ε-H_{min}, initial value of the next estimation (iteration).

The process is repeated until the absolute value of ε-new is smaller than that of the "true" value. The desired H_P is the one associated to that latter error. An example of this approach is shown below.





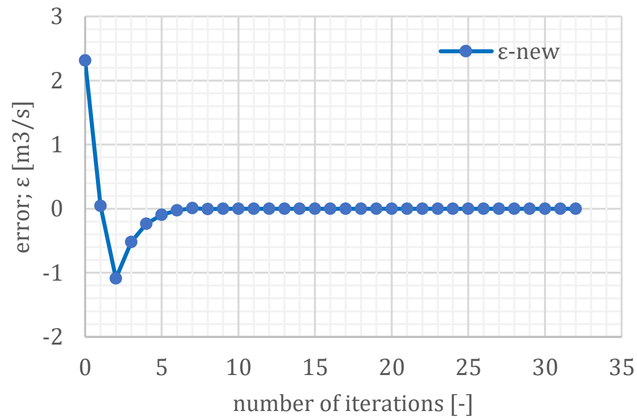


Figure 5.4 Pressure head calculation by means of the error-based method.

The MATLAB code for this approach may take the following form

```
H_high=H_max;
H_low=H_min;
H_M=[H_low H_high];
nn=length(H_M);
Qp_L_M=zeros(1,nn);
Qp_R_M=zeros(1,nn);
e_M=zeros(1,nn);
Cp=H(i,j-1)+B*Q(i,j-1);
Bp=B+R*abs(Q(i,j-1));
Cm=H(i,j+1)-B*Q(i,j+1);
Bm=B+R*abs(Q(i,j+1));
for k=1:nn
    Qp_L_M(1,k)=(Cp-H_M(1,k))/Bp;
    Qp_R_M(1,k)=(H_M(1,k)-Cm)/Bm;
    e_M(1,k)=Qp_L_M(1,k)-Qp_R_M(1,k);
end
e_low=e_M(1,1);
e_high=e_M(1,2);
e_new=1;
while abs(e_new)>=e_lim
    H_new=0.5*(H_high+H_low);
    Qp_L=(Cp-H_new)/Bp;
    Qp_R=(H_new-Cm)/Bm;
    e_new=Qp_L-Qp_R;
    if e_new<-e_lim
        H_high=H_new;
    elseif e_new>e_lim
        H_low=H_new;
    else%if -e_lim<e_new<e_lim
        H_high=H_new;
        H_low=H_new;
    end
end
Hp(i,j)=0.5*(H_high+H_low);
%Qp_L=(Cp-Hp(i,j))/Bp;
Qp_R=(Hp(i,j)-Cm)/Bm;
```

```
Qp(i,j)=Qp_R;
Vp(i,j)=Qp(i,j)/A;
```

in which e_lim is the "true" value of the error.

6 COMPUTER PROGRAMS

MATLAB has been used in order to write four different, characteristics-method based, computer programs, for the solution of the transient-flow equations under various initial and boundary conditions; with, and without the presence of air in them.

For all of them, the fluid is considered to be incompressible, and the pipe walls rigid, so that the pipe cross-sectional area, A, does not change due to pressure changes.

6.1 Pump Stop at Power Failure

The behavior of the main flow variables during a transient, due to pump stop at power failure, is studied by the present program. The results of the program are to be later compared to data collected on the pump station of Hjedsbækvej 198.

The pump station of Hjedsbækvej 198 is located in the municipality of Rebild, in Region Nordjylland. Rebild is enclosed by neighboring municipalities of Aalborg, Vesthimmerlands and Mariagerfjord. Hjedsbækvej 198 collects wastewater from the small town of Suldrup, and the village of Sønderup, both sitting in central Himmerland, and pumps it to the following pump station of Bustedvej 28, on its way to Aalborg Wastewater Treatment Plant West.

6.1.1 Program Setup and Initial Conditions

The model consists of a pump at the upstream end of a pipe, and a 1025-m-long pipe(; L = 1025 m), with a 210-mm nominal diameter(; D = 0.21 m); the pipe profile is shown in Fig. 6.2.

The pump is initially running, at a flow rate, Q_0, of 0.032 m^3/s, i.e. the initial flow speed, V_0, is 0.92 m/s for the entire length of the pipe. The initial pressure head at the pump, H_0, is 56.02 m. The pipe roughness, ε, is 0.05 mm, thus, the Darcy-Weisbach friction factor, f, iteratively solved by means of the Colebrook-White equation, is equal to 0.017. The wave speed, a, is 288 m/s. The transit time, L/a, is (thus) 3.56 s. At t = 0, the power failure occurs, pump stop; the fluid nearest to the pump is gradually brought from V_0 to rest, $V_0 + \Delta V = 0$; thus $\Delta V = -0.92$ m/s. The pressure drop, ΔH, associated to the aforementioned ΔV, may be determined by Eq. (2.2); $\Delta H = -27$ m. The transient is initiated; travels downstream.

Minor losses are neglected along the pipe. The model allows for cavitation, or column separation.

6.1.2 Results

Results are shown at the upstream end of the pipe; at the pump.

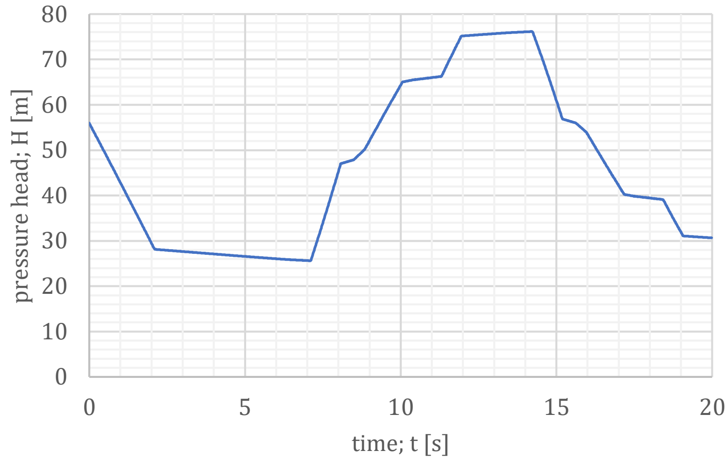


Figure 6.1 Pressure head measurements at the pump.

(i) t = 0; a power failure occurs, pump stop. (ii) $0 < t \leq 2$ s; gradual reduction of the flow speed, $\Delta V = -0.92$ m/s. This ΔV causes pressure to gradually drop at the pump; up to $\Delta H = -27$ m. The transient is initiated; travels downstream. (ii) $2 < t \leq 7$ s; a positive flow still continues away from the pump, due to pressure dropping to vapor pressure, H_v; so $H = H_v$ (fixed), and a new ΔH(, < -27 m), leads to a $\Delta V < -0.92$ m/s. The pressure, H, still decreases, but at a lower rate. In this time, the transient travels back and forth through the pipe. (iii) t = 7 s; the transient is back to the pump, and, for the entire length of the pipe, negative flow; compresses the fluid, increase of H at the pump. (iv) $7 < t \leq 14$ s; the negative flow continues away from the pump; H still increases. The, rather small, fluctuations in H, are caused by cavitation, or column separation (along the pipe). Again, the transient travels back and forth through the pipe during this time. (v) t = 14 s; the transient reaches the pump (again); with a positive flow. H has reached its maximum, and starts decreasing.

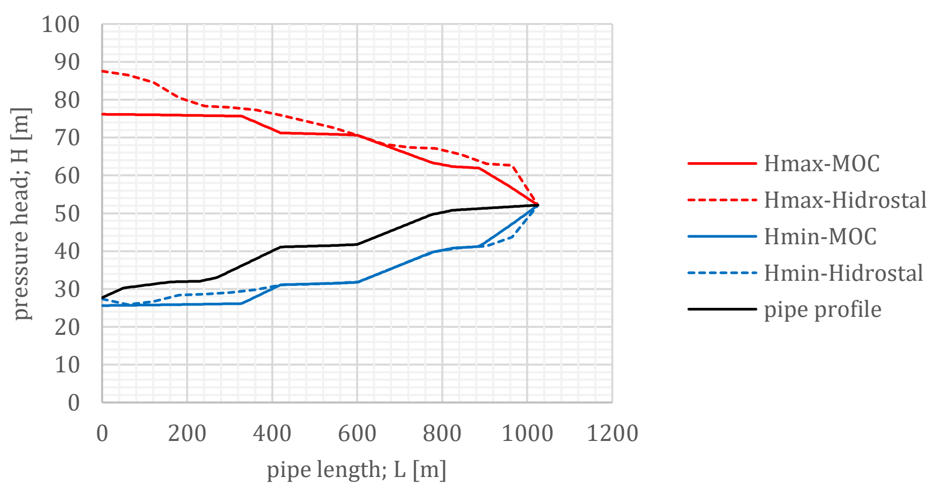


Figure 6.2 Maximum, and minimum pressure head measurements along the pipeline.

The maximum, and minimum values of pressure head, H_{max} and H_{min}, determined by the characteristics method; along the pipe, are compared to data collected on the pump station of Hjedsbækvej 198, in Fig. 6.2.

6.1.3 Conclusions

In spite of its apparent simplicity; constant friction factor assumed, does not consider minor losses, etc., the program is able of truly represent the behavior of the main flow variables during the transient; through the system.

The MATLAB code for this program may be found in Appendix D.1.

6.2 Single Leakage

The influence of a single leakage on the behavior of the main flow variables during a transient, is to be studied by the present program; so that the leakage can be located from this analysis. Under normal, operating conditions, the transient travels along the pipe at some wave speed, and gets reflected at the boundaries. The presence of a leakage (partly,) extra-reflects the pressure signals. The leakage may be located by measuring the time the pressure signal needs to travel from the measuring point to the leakage and vice versa; non-destructive method. The location of leakages in pipelines is a major concern in water distribution systems; due to the economic and social cost associated to water losses.

6.2.1 Program Setup and Initial Conditions

The model consists of a pump at the upstream end of a pipe, a 1025-m-long(, L = 1025 m), horizontal pipe; no leakage in it, with a 210-mm nominal diameter(; D = 0.21 m), and a constant-level reservoir at the downstream end of it.

The pump is initially running, the pump flow rate, Q_0, is 0.032 m^3/s; thus, (flow speed) $V_0 \approx 0.9$ m/s. The initial pressure head, H_0, of the constant-level reservoir is 23.42 m. The pipe roughness, ε, is (assumption) equal to 0.05 mm. The Darcy-Weisbach friction factor, f, is thus (iteratively) solved by means of the Colebrook-White equation; f ≈ 0.018. The wave speed, a, is considered equal to 288 m/s. The area of the orifice, A_o, is considered one-twentieth of the pipe cross-sectional area, A; $A_o = A/20$.

The leakage rate is determined as the difference between the inflow and outflow at the reach containing the leakage. The relationship between the leakage rate, and the pressure head, H(, at the leakage), can be modeled by the orifice equation; for a horizontal pipe

$$\mathrm{Q} = C_d A_o Y \sqrt{\frac{2g\Delta H}{(1-\beta^4)}} \tag{6.1}$$

in which

$$\beta = \frac{D_0}{D_1} \tag{6.2}$$

C_d being the discharge coefficient; equal to 0.61 (sharp edge), D_0 the orifice diameter, and D_1 the pipe diameter. The orifice diameter is considered to be significantly smaller than the pipe diameter; $D_0 << D_1$, then $\beta^4 \approx 0$. Y is the expansion coefficient, equal to 1 for incompressible flow, then

$$Q = C_d A_o \sqrt{2g\Delta H} \tag{6.3}$$

The process may be divided into three steps:

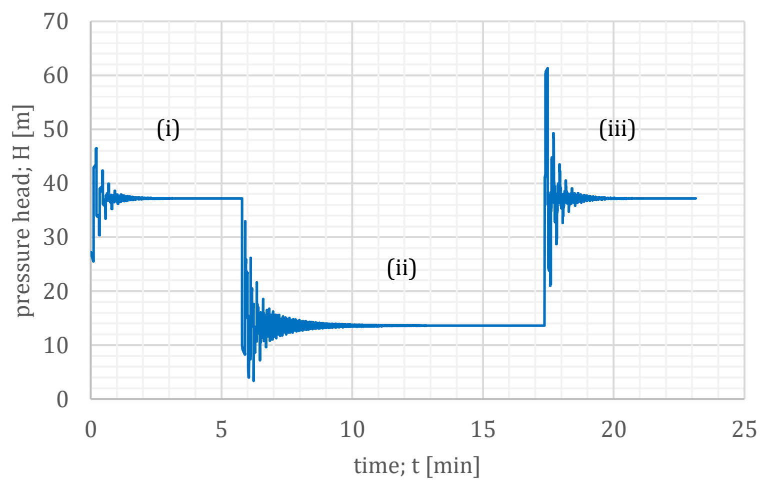


Figure 6.3 Complete process.

(i) t = 0; an orifice is considered at a certain point along the pipe, with a view to generate the initial conditions of pressure head, H, along the pipe, before the stoppage of the pump; which is the starting point of our analysis. A flow is generated, coming out of the pipe through the orifice; equal to the leakage flow rate. This flow is associated to a drop in pressure, which yields a decrease in flow speed at the orifice. A first transient is generated; moving downstream of the orifice. This step is not considered as part of our analysis; does not compare to a real-life situation, e.g. the sudden appearance of an orifice in a pipe may lead to additional pressure fluctuations that cannot be represented by the model.

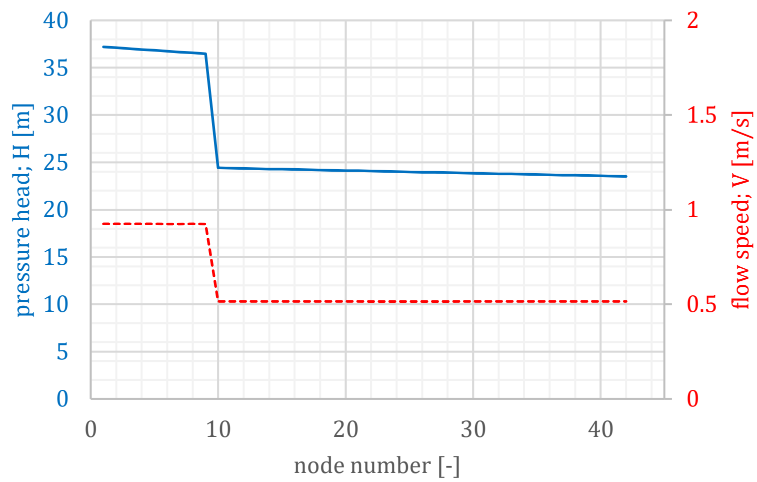


Figure 6.4 Steady-state conditions along the pipe before pump stoppage.

(ii) As soon as the system is back to steady-state conditions, so that the first transient has disappeared completely, and does not affect the new, second transient; sudden stoppage of the pump, the fluid nearest to it is brought to rest, which yields a drop in pressure head at the pump. A new transient is generated; moves downstream of the pump, but, it does not reflect when it reaches the orifice, it continues its way towards the constant-level reservoir, at the downstream end of the pipe. No useful data may be extracted from this step either.

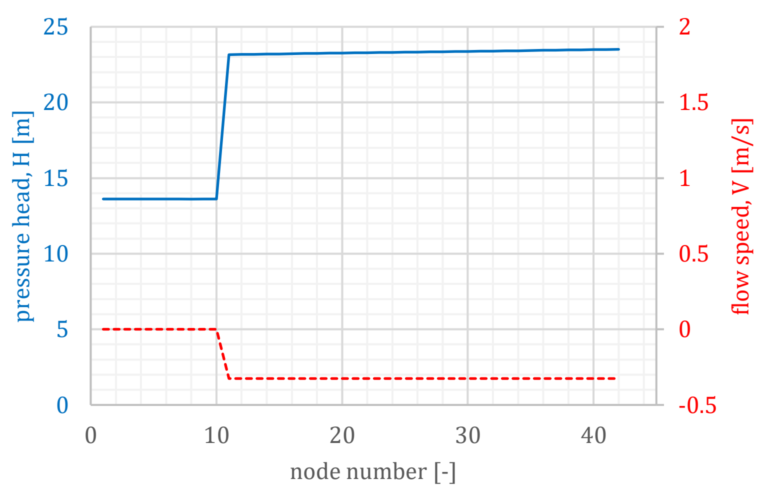


Figure 6.5 Steady-state conditions before pump start.

(iii) The minute the system is, again, back to steady-state conditions (same reasons); sudden start of the pump, increase in flow speed, which yields an increase in pressure (at the pump). A third transient is initiated; (again) moves downstream of the pump and, this time, when it reaches the orifice, reflects, in part, back to the pump, while the transient continues its way towards the constant-level reservoir, at the downstream end of the pipe. The data of this third step is used in the location of the leakage.

6.2.2 Results

Results are shown at the upstream end of the pipe, at the pump.

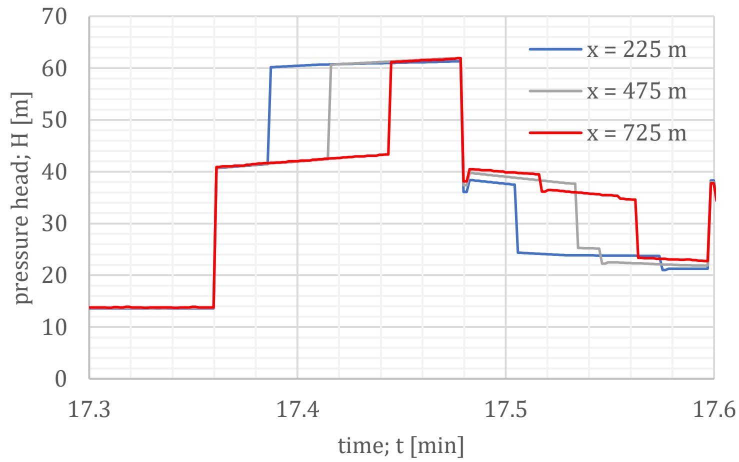


Figure 6.6 Pressure head measurements at the pump for different leakage locations.

Figure 6.6 gives the pressure head, H, at the pump, when the leakage is considered at varying locations, x, along the pipe. The wave, transit time, L/a, is ≈ 3.5 s; thus, the wave needs ≈ 7 s (; ≈ 0.12 min,) to travel back and forth through the pipe. As can be noted in Fig. 6.6, a number of pressure signals reach the pump before that time. As expected, the leakage causes partial reflections of the wave fronts that become small pressure discontinuities in the original pressure trace.

Let us consider the pressure trace associated to the leakage located at x = 225 m. The time it takes for the first pressure signal to reach the pump is ≈ 1.56 s; less than the transit time, suggests the presence of a leakage. The location of the leakage may be determined as x = (at)/2; it is divided by 2 because the pressure signal travels back and forth through the pipe. Therefore, x = (288 m/s · 1.56 s)/2 ≈ 225 m.

6.2.3 Conclusions

The location of the leakage in the pipe can be accurately determined by the analysis of the pressure signals, outcome of the present program.

The MATLAB code for this condition may take the following form

```
e_new=1;
while abs(e_new)>=e_lim
     H_new=0.5*(H_high+H_low);
     Qp_L=(Cp-H_new)/Bp;
     Qp_R=(H_new-Cm)/Bm;
     if j==j_G && H(i,j)>=z_G
         if H(i,j-1)>H(i,j+1)
```

```
                Q_G=Cd*A_G*sqrt(2*g*(H(i,j-1)-z_G));
            else%if H(i,j-1)<H(i,j+1)
                Q_G=Cd*A_G*sqrt(2*g*(H(i,j+1)-z_G));
            end
            e_new=Qp_L-Qp_R-Q_G;
        else%if i<>iLeak
            e_new=Qp_L-Qp_R;
        end
        if e_new<-e_lim
            H_high=H_new;
        elseif e_new>e_lim
            H_low=H_new;
        else%if -eLimit<eNew<eLimit
            H_high=H_new;
            H_low=H_new;
        end
end
Hp(i,j)=0.5*(H_high+H_low);
      Qp_L=(Cp-Hp(i,j))/Bp;
Qp_R=(Hp(i,j)-Cm)/Bm;
if abs(Qp_L)>=abs(Qp_R)
     Qp(i,j)=Qp_R;
else%if Cp-Cm<0
     Qp(i,j)=Qp_L;
end
Vp(i,j)=Qp(i,j)/A;
```

in which j_G, and z_G, are the location; node number, and elevation of the leakage, respectively. The entire MATLAB code can be found in Appendix D.2.

6.3 Isolated Vapor Cavity

The influence of isolated cavitation, or column separation, on the behavior of the main flow variables in the event of a transient, is to be studied by the present program.

6.3.1 Program Setup and Initial Conditions

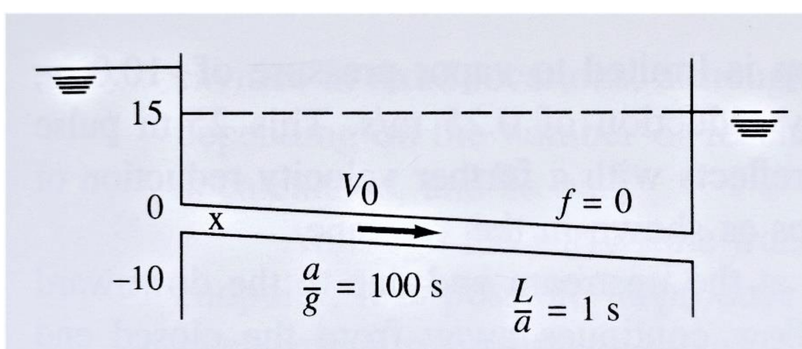


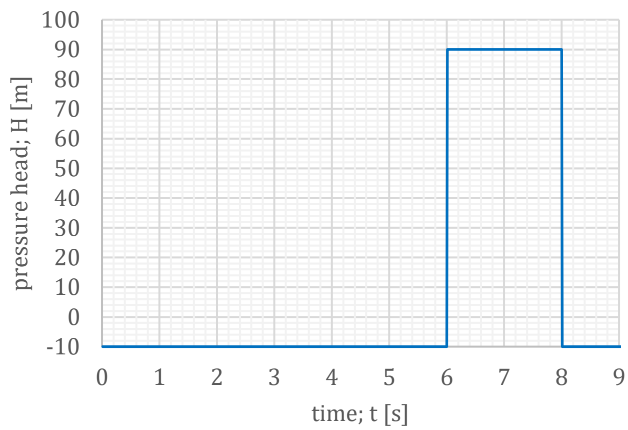
Figure 6.7 Program setup; (Wylie, E. B. and Streeter, V. L. 1993).

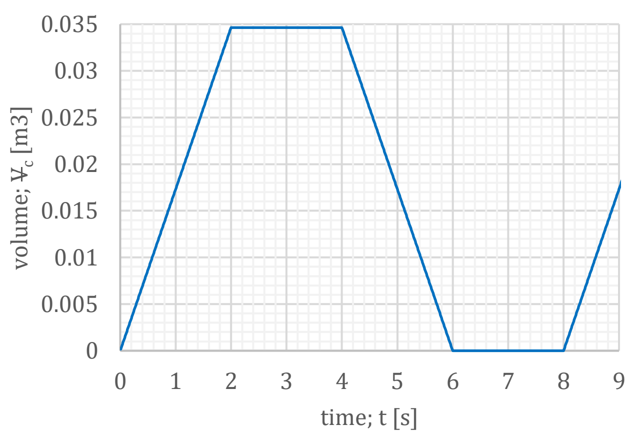
The program is based on one of Wylie, E. B. and Streeter, V. L. (1993, p.192), Fig. 6.7, and consists of a pump at the upstream end of a pipe, a valve (next to it), a 981-m-long pipe(; L = 981 m), with a 210-mm internal diameter(; D = 0.21 m) and a certain negative slope, and a constant-level reservoir at the downstream end of it. The steepness of the slope of the pipe does not affect the process described.

Two cases are analyzed: (i) (initial flow speed) V_0 = 0.75 m/s, and (ii) V_0 = 0.80 m/s. The initial, steady-state pressure head, H_0, for the entire length of the pipe, is 15 m; which is the pressure head, H, of the constant-level reservoir at the end of the pipe (downstream). There is no cavity present at t = 0; $\cancel{V}_{c0}$-= 0. The pipe is considered frictionless; f = 0. The wave speed, a, is 981 m/s. The transit time, L/a, is (thus) 1 s. At t = 0, sudden closure of the valve; H (of the fluid nearest to the valve,) decreases; the drop is limited to vapor pressure, $H_v = H_0 + \Delta H$. The vapor pressure, H_v, is set equal to -10 m; thus ΔH = -25 m. As long as $H \leq H_v$, $H = H_v$ (fixed pressure), and $\cancel{V}_c$ is allowed to grow and collapse in it. The flow speed decrease, ΔV, associated to the aforementioned ΔH, may be determined by Eq. (2.2); ΔV = -0.25 m/s.

6.3.2 Results

Results are shown at the upstream end of the pipe; at the cavity. The results appear to be similar for both cases; both V_0. The complete cycle, for V_0 = 0.75 m/s, is detailed below. Any notable difference between the two cases will be commented further on.





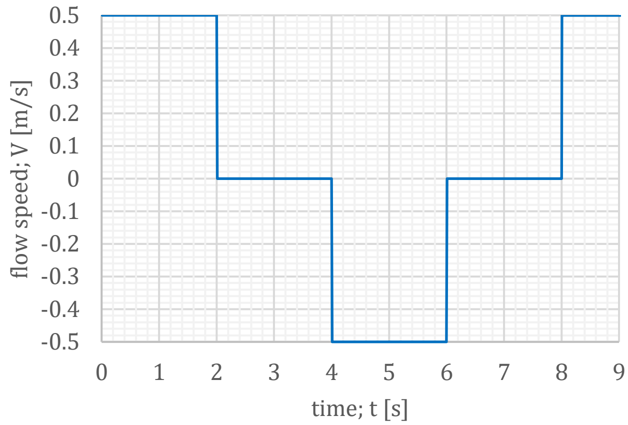
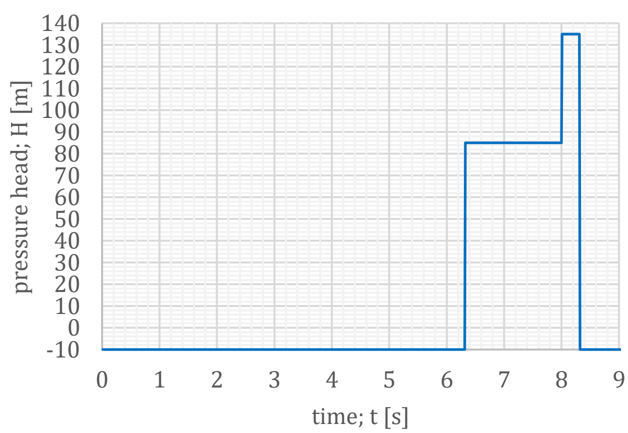


Figure 6.8 Isolated vapor cavity in a single pipeline; V_0 = 0.75 m/s.

Water-column separation occurs only at the upstream end of the pipe due to its negative slope. (i) t = 0 s; the valve is closed, and the pressure drops to H_v = -10 m; ΔH = -25 m. A vapor cavity forms, next to the valve. This ΔH yields a reduction of flow speed; ΔV = -0.25 m/s. The transient is initiated. (ii) 0 < t ≤ 2 s; So long as the positive flow continues away from the closed, upstream end of the pipe, the volume of the vapor cavity, $\cancel{V}_c$, increases. While it is present, the vapor cavity behaves as a constant-pressure boundary; at H = -10 m. (iii) 2 < t ≤ 4 s; The fluid nearest to the valve has been brought to rest, the cavity ceases to grow, and remains at a constant volume. (iv) 4 < t ≤ 6 s; A negative flow, V = -0.5 m/s, returns to the valve; $\cancel{V}_c$ decreases. (v) 6 < t ≤ 8 s; The instant the vapor cavity collapses, the flow is brought to rest, which generates a pressure increase as shown in Fig. 6.8. H rises well above the constant-pressure set as initial condition at the upstream end of the pipe; H_0 = 15 m.



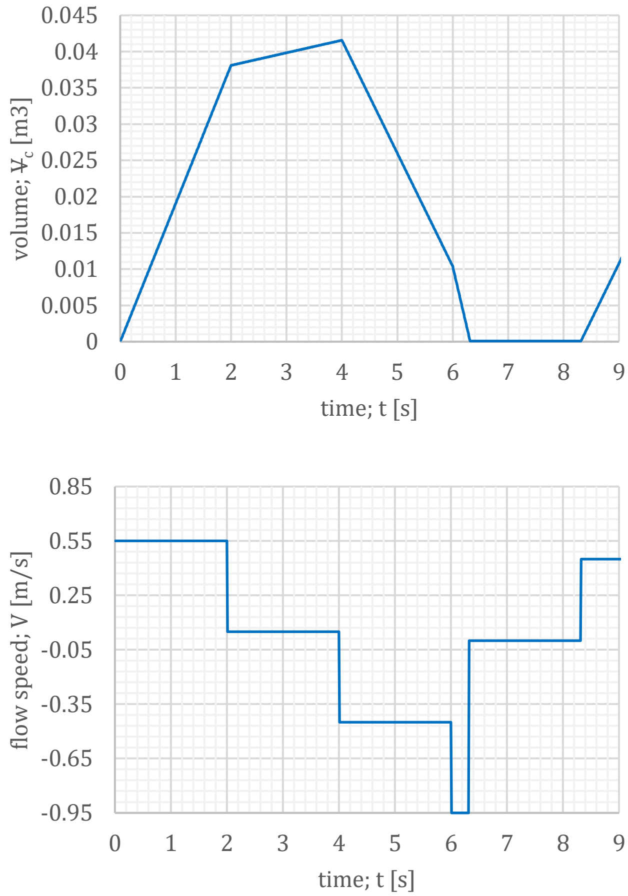


Figure 6.9 Isolated vapor cavity in a single pipeline; V_0 = 0.80 m/s.

(ii) 0 < t ≤ 2 s; The increase rate of the volume of the vapor cavity, $\not{V}_c$, is higher than that of V_0 = 0.75 m/s; higher flow speed, V. (iii) 2 < t ≤ 4 s; $\not{V}_c$ continues to increase; the flow is not at rest, positive flow. Largest $\not{V}_c$ than that of V_0 = 0.75 m/s. (iv) 4 < t ≤ 6 s; The rate of decrease of $\not{V}_c$ is lower than that of V_0 = 0.75 m/s; higher V. Thus, (v) 6 < t ≤ 8 s; the vapor cavity collapses a bit later than for V_0 = 0.75. The maximum H reached is higher than for V_0 = 0.75.

6.3.3 Conclusions

The isolated vapor-cavity program mimics this type of problem with reasonable reliability, at least through the first cavity collapse. This is generally true for cases in which only a discrete cavity is present at a fixed location (Wylie, E. B. and Streeter, V. L. 1993).

The MATLAB code for an internal reach; cavitation, or column separation, calculation, may take the following form

```
if v_C>0
    Hp(i,j)=z_l(i,j)+H_V;
    Qp_R=(Hp(i,j)-Cm)/Bm;
```

```
        Qp_L=Qp(i,j);
        v_C_M(i,c)=v_C+(Qp_R-Qp_L)*dt;
        Qp(i,j)=Qp_R;
        Vp(i,j)=Qp(i,j)/A;
        if v_C_M(i,c)<=0
            while abs(e_new)>=e_lim
                H_new=0.5*(H_high+H_low);
                Qp_R=(H_new-Cm)/Bm;
                e_new=Qp_L-Qp_R;
                if e_new<-e_lim
                    H_high=H_new;
                elseif e_new>e_lim
                    H_low=H_new;
                else%if -e_lim<e_new<e_lim
                    H_high=H_new;
                    H_low=H_new;
                end
            end
            Hp(i,j)=0.5*(H_high+H_low);
            if Hp(i,j)<z_l(i,j)+H_V
                Hp(i,j)=z_l(i,j)+H_V;
            end
            Qp_R=(Hp(i,j)-Cm)/Bm;
            v_C_M(i,c)=v_C+(Qp_R-Qp_L)*dt;
            Qp(i,j)=Qp_R;
            Vp(i,j)=Qp(i,j)/A;
        end
    else%if v_C=0
        while abs(e_new)>=e_lim
            H_new=0.5*(H_high+H_low);
            Qp_L=Qp(i,j);
            Qp_R=(H_new-Cm)/Bm;
            e_new=Qp_L-Qp_R;
            if e_new<-e_lim
                H_high=H_new;
            elseif e_new>e_lim
                H_low=H_new;
            else%if -e_lim<e_new<e_lim
                H_high=H_new;
                H_low=H_new;
            end
        end
        Hp(i,j)=0.5*(H_high+H_low);
        if Hp(i,j)<z_l(i,j)+H_V
            Hp(i,j)=z_l(i,j)+H_V;
        end
        Qp_R=(Hp(i,j)-Cm)/Bm;
        v_C_M(i,c)=v_C+(Qp_R-Qp_L)*dt;
        Qp(i,j)=Qp_R;
        Vp(i,j)=Qp(i,j)/A;
    end
    v_C=v_C_M(i,c);
```

The entire MATLAB code can be found in Appendix D.3.

6.4 Isolated Air Pocket

The influence of isolated air entrapment on the behavior of the main flow variables in the event of a transient, is to be studied by the present program.

6.4.1 Program Setup and Initial Conditions

The program consists of a constant-level reservoir at the upstream end of a pipe, a valve (next to it), and a dead-end, 2000-m-long pipe(; L = 2000 m), with a 210-mm nominal diameter(; D = 0.21 m) and a given positive slope, s, of ≈ 0.005; so that the air is trapped at the downstream, dead end of it (the pipe).

The valve is initially closed. Thus, for the entire length of the pipe, the fluid is at rest; (flow speed) $V_0 = 0$, and the initial, steady-state pressure head, H_0, is 9.95 m. The constant-level(, pressure head) at the reservoir, H_R, is 34.37 m. The initial, trapped air volume, $\forall_{a0}$, is assumed half the volume of a computing reach; thus $\forall_{a0} \approx 0.35\ m^3$. This means that, the same, single computing reach, consists of both fluid and air. The pipe roughness, ε, of PVC and organic glass pipes (assumption), is equal to 0.0015 mm. The Darcy-Weisbach friction factor, f, is then (iteratively) solved by means of the Colebrook-White equation; f ≈ 0.017. The wave speed, a, in a pipe of the (above) mentioned material, is 400 m/s (Zhou, L. 2011); empirical research. The transit time, L/a, is (thus) 5 s. At t = 0, sudden opening of the valve; the pressure head, H, (of the fluid nearest to it) increases from H_0 to H_R (= $H_0 + \Delta H$); thus ΔH = 24.42 m. The increase in flow speed, ΔV, associated to this ΔH may be determined by Eq. (2.2); ΔV ≈ 0.6 m/s.

The gas is considered to follow the reversible polytropic relation

$$H_a \forall_a^{\ m} = H_{a0} \forall_{a0}^{\ m} = C_A \tag{6.4}$$

in which H_a, is the absolute pressure head, $H_a = H_P - z + \overline{H}$, and $\forall_a$, the volume, of the entrapped air at a time t; m, the polytropic exponent, and C_A a constant. Fast transient phenomena are often assumed to be adiabatic processes with m = 1.4 (Zhou, L. 2011a). Therefore, for the relative small air pocket volume and the fast response of the system to the first pressure rise, a polytropic exponent of 1.4 is assumed. $\overline{H}$ is the atmospheric(, or barometric,) pressure, equal to 10.33 m.

By introducing the integrated continuity equation; the minus sign tells that the air volume decreases with positive inflow

$$\frac{d\forall_a}{dt} = -Q \tag{6.5}$$

and, applying the mean value theorem of integrals and the method of the mean in Eq. 6.5, it follows

$$\int_t^{t+\Delta t} d\forall_a = -\int_t^{t+\Delta t} Q(t)dt \tag{6.6}$$

$$\cancel{V}_{a,P} = \cancel{V}_a - \Delta t \frac{(Q_P + Q)}{2} \tag{6.7}$$

thus, Eq. 6.4 can be expressed as

$$(H_P + \bar{H} - z)\left[\cancel{V}_a - \Delta t \frac{(Q_P + Q)}{2}\right]^m = C_A \tag{6.8}$$

which combined with Eq. 5.5, yields

$$F_1 = (C_P - B_P Q_P + \bar{H} - z)\left[\cancel{V}_a - \Delta t \frac{(Q_P + Q)}{2}\right]^m - C_A = 0 \tag{6.9}$$

which is a nonlinear equation in the variable Q_P. Newton’s method is used to (iteratively) solve Eq. 6.9; finds a correction to an estimated value of Q_P by using

$$F_1 + \frac{dF_1}{dQ_P}\Delta Q = 0 \tag{6.10}$$

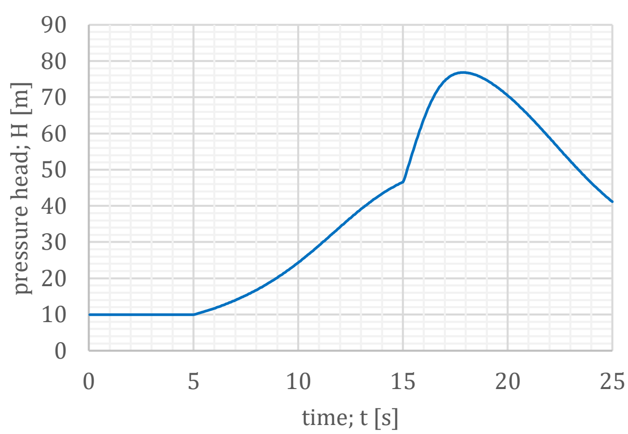
in which, after simplification

$$\frac{dF_1}{dQ_P} = -B_P\left[\cancel{V}_a - \Delta t \frac{(Q_P + Q)}{2}\right]^m - \frac{m \Delta t C_A}{\cancel{V}_a - \Delta t (Q_P + Q)/2} \tag{6.11}$$

ΔQ can, then, be found by isolation in Eq. 6.11.

6.4.2 Results

Results are shown at the downstream, dead end of the pipe; where the trapped air is located.



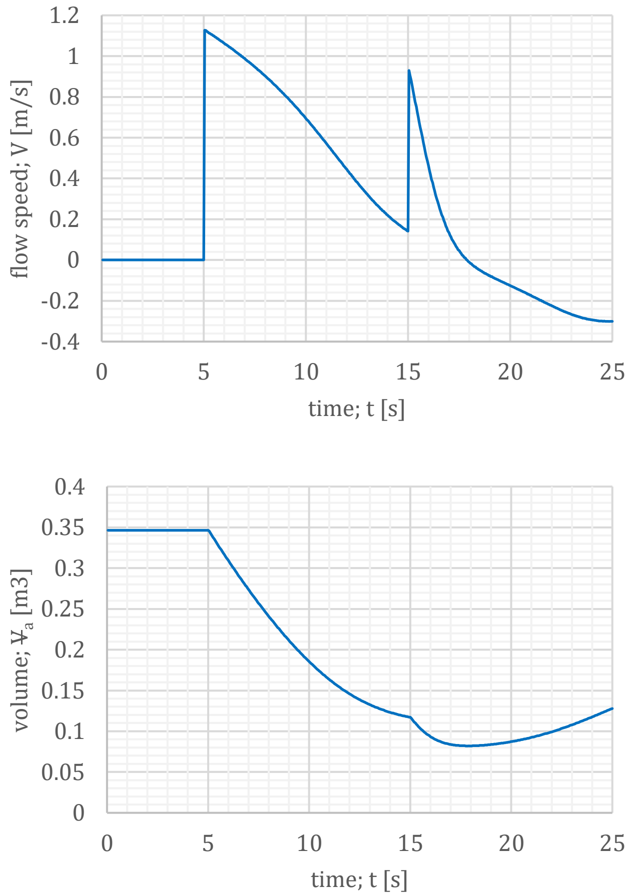


Figure 6.10 Isolated air pocket in a single pipeline.

The air is trapped at the downstream, dead end of the pipe, due to the positive slope, s, of the latter. (i) t = 0 s; sudden opening of the (upstream) valve; increase in pressure head, ΔH, of the fluid nearest to it, which yields an increase of flow speed, ΔV. The transient is initiated; travels downstream. (Still) Initial conditions of pressure, and volume at the air pocket, H_{a0} and $Ꝟ_{a0}$. (ii) 0 < t ≤ 5 s; the positive flow continues away from the valve. At t = 5 s, the transient reaches the trapped air; the pressure head, H, increases, and the volume of the air pocket, $Ꝟ_a$, decreases; increase of flow speed, V (compresses the air pocket). (iii) 5 < t ≤ 15 s; the positive flow continues (compressing the air), but V decreases. H still increases, and $Ꝟ_a$ continues to decrease (but at a lower rate). In that time, the transient has travelled back and forth through the pipe. At t = 15 s the transient reaches the trapped air (again); H increases, and $Ꝟ_a$ decreases; increase of V (further compression of the pocket) (iv) 15 < t ≤ 20 s; V decreases; highest H, and minimum $Ꝟ_a$ when V = 0. The negative flow returns to the valve; H drops, and $Ꝟ_a$ increases (stretched).

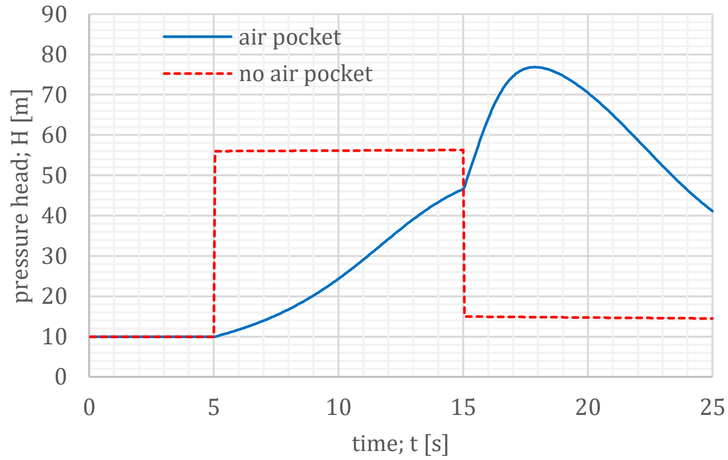


Figure 6.11 Pressure head measurements at the downstream end of the pipe; with and without air pocket.

A comparison of the transient behavior, with and without air pocket, is shown in Fig. 6.11. The maximum H reached is considerable larger in the case where the air pocket is present. The phase shift of the wave is also affected by the presence of the air; longer period.

6.4.3 Conclusions

As mentioned in Section 4.3, a compression of the air pocket may cause abnormal pressure surges (Wylie, E. B. and Streeter, V. L. 1993); relates well to the program results. The presence of the entrapped air as well affects the phase shift of the wave; longer period.

The MATLAB code for this condition may take the following form, beginning with an estimated value of Q_P at the new time step

```
j=no;
Cp=H(i,j-1)+B*Q(i,j-1);
Bp=B+R*abs(Q(i,j-1));
Qp(i,j)=Q(i,j);
u=0;
while u<=KIT
    v_Ap(i,c)=v_A-dt*(Qp(i,j)+Q(i,j))/2;
    if v_Ap(i,c)<v_S
        v_Ap(i,c)<v_S;
    end
    F1=(Cp-Bp*Qp(i,j)-z_l(i,j)+H_bar)*(v_Ap(i,c)^m)-C_A;
    dF1dQp=-m*dt*C_A/v_Ap(i,c)-Bp*v_Ap(i,c)^m;
    dQ=-F1/dF1dQp;
    Qp(i,j)=Qp(i,j)+dQ;
    u=u+1;
end
v_Ap(i,c)=v_A-dt*(Qp(i,j)+Q(i,j))/2;
if v_Ap(i,c)<0
    v_Ap(i,c)=0;
end
```

```
v_A=v_Ap(i,c);
Hp(i,j)=Cp-Bp*Qp(i,j);
Vp(i,j)=Qp(i,j)/A;
```

The constant C_a, C_A, may be defined by Eq. X.X, using H_{a0}, and $\cancel{V}_{a0}$. KIT is the number of iterations in Newton's method, and v_S is a minimum-size air volume, in order to avoid the division by zero. The entire MATLAB code can be found in Appendix D.4.

7 SIMULATIONS

With a view to predict real-life behavior of hydraulic transients in pipes, minimizing the need for, at times costly, time-consuming testing, STAR-CCM+, a CFD code, simulation platform, is used. A simulation offers accurate, less-expensive predictions than experimental testing. Iterative simulation is used to improve the design; no need for repeated testing of physical prototypes (saves time). Besides, it offers a full range of operating, physic conditions; many flows cannot be easily tested in real life.

7.1 (Zhou, L. 2011a)

Zhou, F. (2000), Zhou, F., et al. (2002), and Lee, N. H. (2005) studied the effects of the initial void fraction of entrapped air, α, on the maximum pressure surge of a dead-end, filling horizontal pipe; (rather) large values of α; high inlet pressures. Zhou, F. (2000) and Zhou, F., et al. (2002); 20, 50 and 95.2% (void fractions), and Lee, N. H. (2005); fractions ranging from 5.8 to 44.81%. They found out that the maximum entrapped air pressure increased as air volume decreased; cushioning effect of the entrapment.

The following numerical model is based on an experiment conducted by Zhou, L. (2011a). As in Zhou, F. (2000), Zhou, F., et al. (2002), and Lee, N. H. (2005), the effects of the initial void fraction of entrapped air on the maximum pressure surge of a dead-end, filling (this time) undulated pipe are studied; (in this case) focus on (rather) small values of α, ranging from $\approx$ 0.1 to 8%, under a low inlet pressure. This is of importance because a compression of the entrapped air can result in abnormal pressure surges (Wylie, E. B. and Streeter, V. L. 1993); may cause damage to the pipe when operating unprotected against transient pressures. The numerical model is built in order to analyze the behavior of the main hydraulic variables during the process.

7.1.1 Conceptual model

The conceptual model, Fig. 7.1, consists of a constant-level reservoir at the upstream end of the pipe, a quarter-turn ball valve, BV, a water vent, WV, and a dead-end (closed valve), 4.4445-m-long pipe with a 90-mm internal diameter. The pipe consists of five segments; a 125-cm-long horizontal pipe, a 73-cm-long vertical pipe, a 121.45-cm-long horizontal pipe, a 100-cm-long vertical pipe, and a 25-cm-long horizontal pipe. 20-cm-radius, 90° elbows are assumed as fittings.

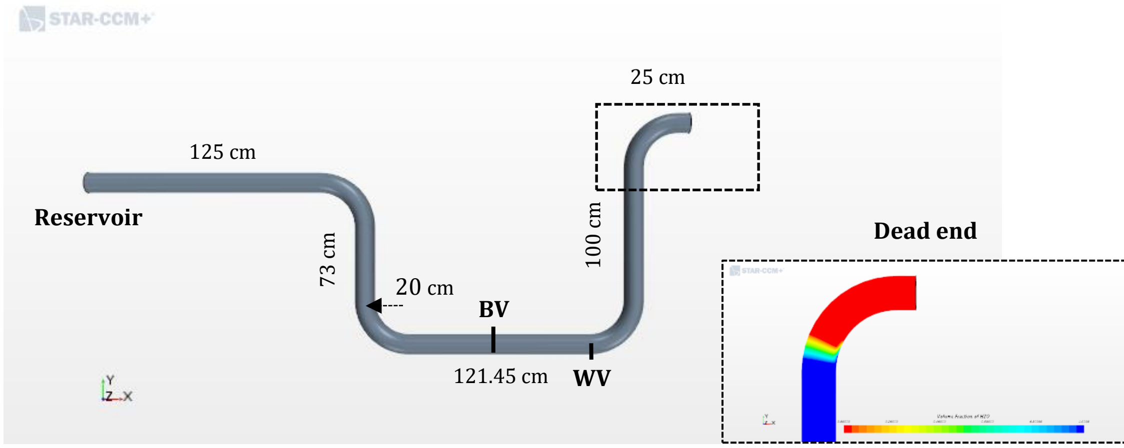


Figure 7.1 Conceptual model; based on (Zhou, L. 2011a).

The datum line, z = 0, is assumed at the centerline of the dead end of the pipe. The air is entrapped at the dead end of the pipe, Fig. 7.1 (in red). The valve located at the dead end of the pipe, and the water vent, are used to regulate the initial fraction of it. The initial elevation of the air-water interface ranges from -0.15 to +0.04 m; α ≈ 8 to 0.1%. The water is at room temperature (20°). Therefore, the water density, ρ, is equal to 998.20 kg/m3, and the dynamic viscosity, ν, to 1.002E-3 Pa-s. The acoustic speed, a, of PVC and organic glass pipes (assumption), filled with water, is 400 m/s (Zhou, L. 2011); empirical research. A Darcy-Weisbach friction factor, f, equal to 0.05 is assumed (Zhou, L. 2011); empirical research.

There are three measuring points (pressure) along the pipe, Fig. 7.2; one immediately upstream of the ball valve, PT1, and two near the dead end of the pipe, PT2 and PT3.

7.1.2 Initial conditions

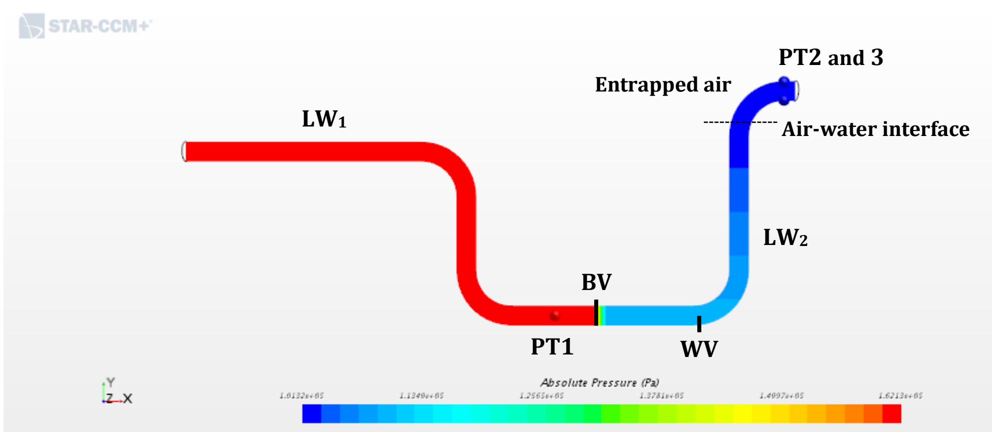


Figure 7.2 Initial pressure distribution; based on (Zhou, L. 2011a).

At t = 0, the ball valve, BV, and the water vent, WV, are closed, which results in two, separated water columns, one before, LW1, and one after, LW2, the ball valve. LW1 is at a pressure, p, of 60801.22 Pa (6.2 m), while LW2 is at hydrostatic pressure, p = ρgh. The entrapped air is at atmospheric (absolute) pressure; 101325 Pa (10.33 m).

7.1.3 Model boundaries

The model consists of three boundaries. The constant-level (upstream) reservoir acts as a pressure-outlet boundary, while the surface, and dead end of the pipe are set as wall boundaries.

A pressure-outlet boundary is a flow-outlet boundary at which the pressure is specified. The pressure-outlet boundary is set as constant-pressure, at a pressure of 60801.22 Pa (6.2 m). The boundary is set so that only water (no air) is allowed to enter the solution domain through this boundary.

The wall boundary represents an impermeable surface that confines fluid or solid regions. The boundary is set as no-slip, meaning that the fluid adheres to the wall, moving with its same velocity; e.g. for a stationary wall, as in our case, the fluid speed is equal to 0 m/s at the wall. This is of importance because, in the case of turbulent flow, near-wall treatments to compute shear are employed. The Darcy-Weisbach friction factor, f, as has been said before, is set equal to 0.05. The pipe roughness, is thus (iteratively) solved by means of the Colebrook-White equation.

7.1.4 Mesh models

The mesh is the discretized representation of the computational domain, which the physics solvers use to provide a numerical solution. The following meshers are selected for generating the mesh:

To improve the overall quality of the existing geometry surface and optimize it for the volume mesh model, the *surface remesher* is used; retriangulates the surface.

The *polyhedral mesher* generates a volume mesh that is composed of arbitrary, polyhedral-shaped cells; suitable for turbulent flow. Long computation time. The *polyhedral mesher* can be used together with the *generalized cylinder mesher*, which generates extruded orthogonal cells along the length cylindrical sections.

The *generalized cylinder mesher* is used to generate an extruded, volume mesh along the length of parts that are considered to be generalized cylinders (our case). It reduces the computation time and improves the rate of convergence. This mesher is best suited to cases where the direction of the fluid is parallel to the vessel wall; such cases can be solved more efficiently by using cells oriented to the direction of the fluid flow.

The *prism layer mesher* is used with a core volume mesh to generate orthogonal prismatic cells next to the wall surfaces or boundaries. This layer of cells is necessary to improve the accuracy of the near-wall flow solution; e.g. resolving the velocity gradients, normal to the wall.

- *Number of prism layers*: The *number of prism layers* parameter controls the number of cell layers that are generated within the prism layer on a boundary. The *number of prism layers* is set equal to 5.
- *Prism layer stretching*: *Prism layer stretching* sets the target growth rate of successive prism layers away from the wall. The *prism layer stretching* parameter is set equal to 1.4.
- *Prism layer total thickness*: The *prism layer thickness* controls the total overall thickness of all the prism layers; (i) relative to base: sets the prism layer total thickness relative to the base size. (ii) Absolute: sets the prism layer total thickness as absolute value with length units. The *prism layer thickness* is set as absolute, and equal to 0.015 m.

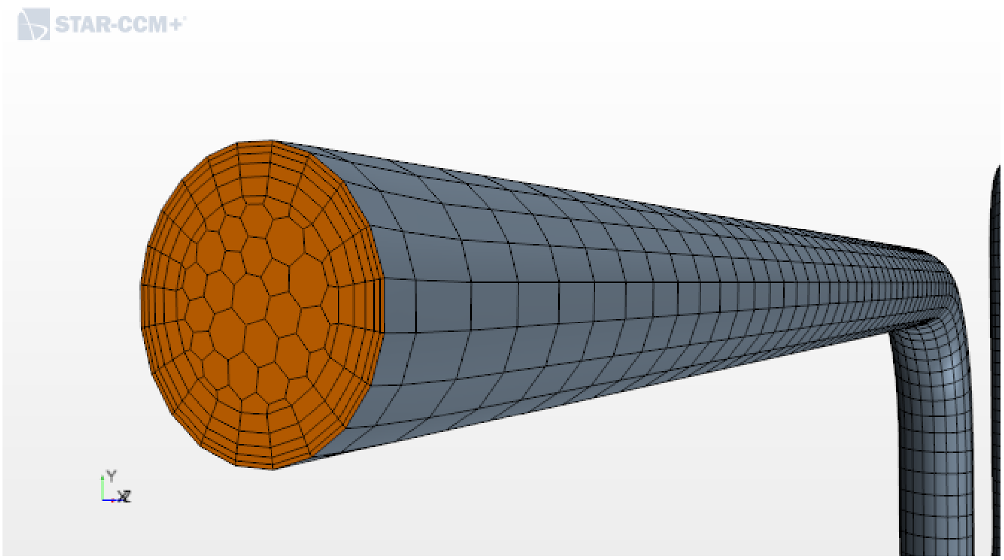


Figure 7.3 Mesh (detail).

The choice of meshers may be questionable. The flow is assumed turbulent, Section 2.1, thus it does not necessarily follow the direction of the vessel wall; one could argue that the generalized cylinder mesher is not the most appropriate mesher to use. At the same time, the extruded volume mesh is generated from a polyhedral mesh; with a large number of cell faces, and suitable for turbulent flows. The size of the cells, and the number of layers are also to be considered. While a small size of the cells, and a large number of layers may lead to a more accurate result, it significantly increases the computation time; a balance between reliability of the results and computation time has been sought.

7.1.5 Physic models

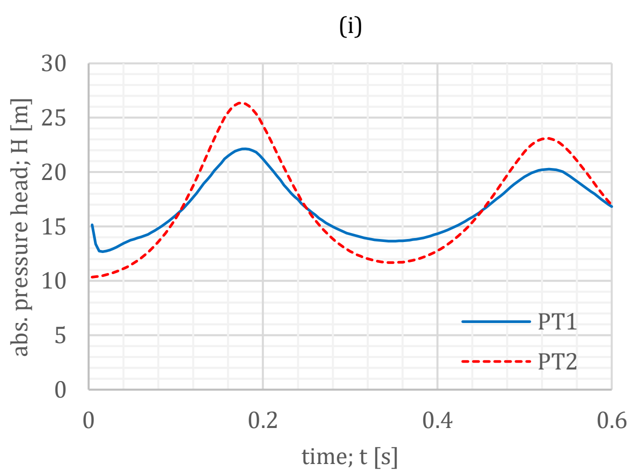
The flow is *three dimensional*. The flow is *turbulent* (assumed); chaotic flow, often considered as the mean flow superposed by eddies causing velocity fluctuations of stochastic nature. The transition from laminar to turbulent flow takes place when the Reynold's number ($Re = VD/\nu$) exceeds a certain value. Experiments show that the transition to turbulent flow takes place at $Re \approx 2300$ (Brorsen, M. 2008); it is possible to verify the assumption. All turbulent flows are, per definition, *unsteady* (assumed); even though one can talk about steady turbulent flow, if the mean flow is steady.

The flow is also *multiphase*; several phases flow in the domain of interest. In modeling terms, a phase is defined as a quantity of matter that has its own physical properties to distinguish it from other phases within a system. A multiphase mixture is a fluid that is composed of multiple phases. The *volume of fluid (VOF) model* is used; this model is suited for systems containing two or more immiscible fluid phases, where each phase constitutes a large structure within the system; on numerical grids capable of resolving the interface between the phases of the mixture. This approach captures movement of the interface between the fluid phases. Two phases are defined, one for the water, and one for the air.

7.1.6 Calibration

The model has to be capable of reproducing real-life situations. Thus, it has to be calibrated against real data (from the experiment). This has not been possible since the initial conditions of the experiment conducted by Zhou, L. (2011a) cannot be met; a valve opening time of 0.1 s cannot be modeled with the data provided in the article (instant opening of the valve assumed).

7.1.8 Results



(ii)

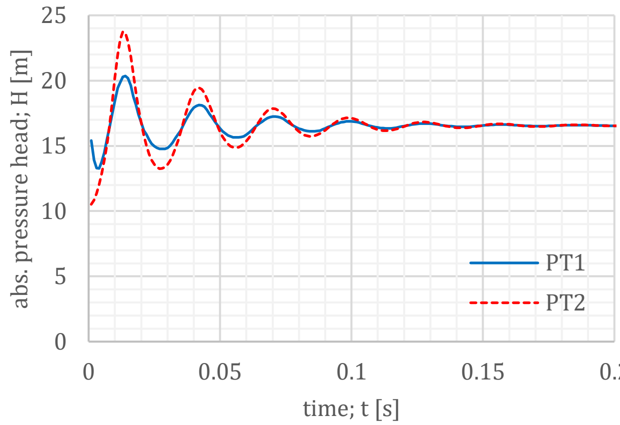
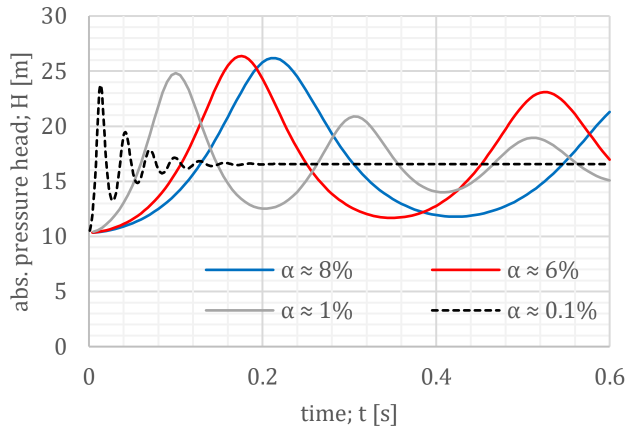


Figure 7.4 Absolute pressure; (i) α ≈ 6%, (ii) α ≈ 0.1%.

(i) t = 0; sudden opening of the ball; increase in pressure, ΔH, of the layer of fluid nearest to it, in LW_2, from p = ρgh to 60801.22 Pa (6.2 m). This ΔH yields an increase of flow speed, ΔV; positive flow. The transient is initiated; travels downstream. (Still) Initial conditions of pressure, and volume at the air pocket, H_{a0} and $\cancel{V}_{a0}$. (ii) The positive flow continues away of the ball valve. The transient reaches the trapped air; the pressure head at the air pocket, $\cancel{H}_a$, increases, and the volume of the air pocket, $\cancel{V}_a$, decreases; it is compressed (cushioning effect). (iii) The fluid in contact with the entrapped air is brought to rest; the entrapped air ceases to decrease. The maximum H has been reached at the entrapped air. (iv) A negative flow returns to the constant-level reservoir; $\cancel{V}_a$ increases (expands), and H decreases.

PT1 is located immediately upstream of the ball valve; higher pressure than downstream of it, that explains the rather small pressure drop in pressure at the very beginning of the process, when the ball valve is opened.

The two measuring points near the dead end of the pipe, where the trapped air is at, yield roughly the same results; thus, only PT2 is shown in Fig. 7.4. The figure shows the absolute pressure head due to a sudden opening of the ball valve; at PT1, and PT2, for (i) α ≈ 6 and (ii) 0.1%. For α ≈ 6%, the air cushioning effect is basically negligible and the water impact force is dominant. Significant difference between the heads at PT2, and at PT1. On the other hand, for α ≈ 0.1%, the maximum head, measured at PT2, decreases and is around the same as in PT1. This is because when α is small, both the air cushioning effect and the water impact force are also small. The wavelength and period is shorter for α ≈ 0.1% than it is for α ≈ 6%, this is because the air cushioning; compressibility, is reduced with α; the pressure surge travels faster through the water; which is incompressible.



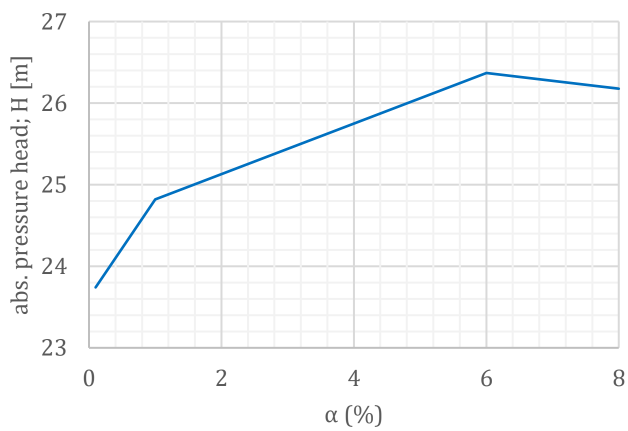


Figure 7.5 Effect of α on the pressure of the air pocket; (i) absolute pressure, recorder by P2, (ii) variation of the maximum pressure of the air pocket.

The highest pressure of the entrapped air increases as α decreases, up to α ≈ 6%; decrease of the air cushioning effect. Thereafter, the highest pressure decreases as α decreases; smaller water impact force due to a lower space for water movement.

7.1.9 Conclusions

The results of our model relate to those of (Zhou, L. 2011a), and are not only coherent with the conclusion of (Zhou, F. 2000), (Zhou, F., et al. 2002), and (Lee, N. H. 2005), but also complimentary for the research in the effects of the initial void fraction of entrapped air on the maximum pressure surge of a dead-end, filling undulated pipe.

7.2 (Coronado-Hernández, O. E., et al. 2017a)

The following, numerical model, is based on an experiment conducted by Coronado-Hernández, O. E., et al. (2017a). The emptying procedure of a water pipeline with an irregular profile and no air valves, (with an air pocket in it) is studied. There is a relative lack of research on the issue (Coronado-Hernández, O. E., et al. 2017a).

This is of importance because: (i) a compression of the entrapped air may cause abnormal pressure surges, (Wylie, E. B. and Streeter, V. L. 1993), (ii) an expansion of the entrapped air may lead to sub-atmospheric pressures (Coronado-Hernández, O. E., et al. 2017a), Section 4.3; both cases may cause damage to the pipe; when operating unprotected against transient pressures. The numerical model is built to analyze the behavior of the main hydraulic variables during the process.

7.2.1 Conceptual model

The conceptual model, Fig. 7.6, is composed of two free-discharge, drain valves, DV_1 and DV_2, located at both ends of the pipe, a water vent, WV, and a 7.3-m-long, PVC pipe with a 51.4-mm internal diameter. The pipe consists of four segments; a 2.25-m-long horizontal pipe, a 1.5-m-long pipe; inclined (+)30°, a 1.5-m-long pipe; inclined (-)30°, and a 2.05-m-long horizontal pipe. No fittings (elbow) are considered.

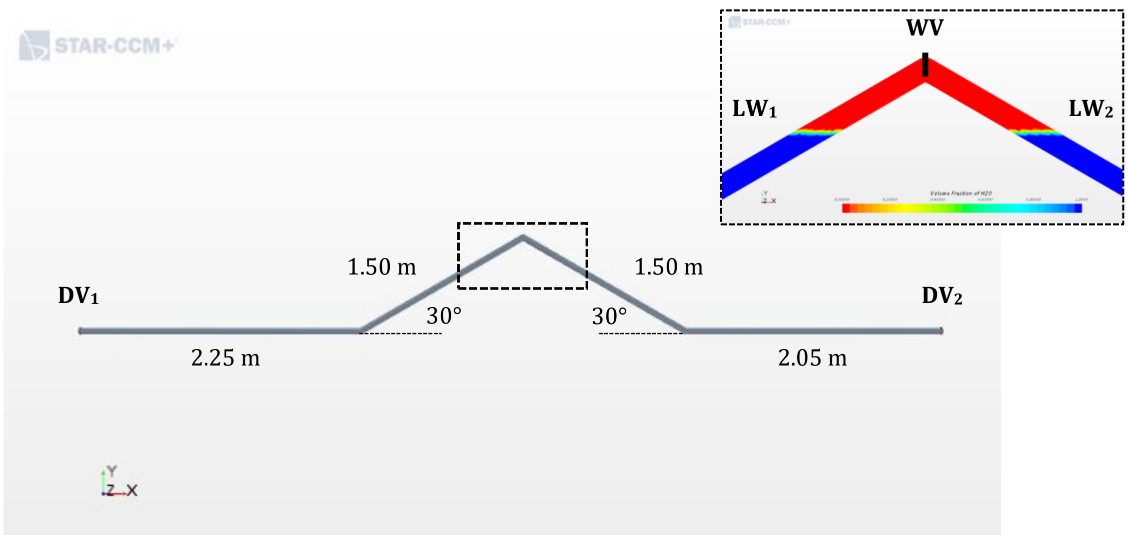


Figure 7.6 Conceptual model; based on (Coronado-Hernández, O. E., et al. 2017a).

The datum line (z = 0 m) is assumed at the centerline of the horizontal pipes. The (only) entrapment, Fig. 7.6 (in red), is symmetrical about WV. DV_1, DV_2, and WV, are used to regulate the initial location (fraction) of it. The initial elevation of the air-water interface is equal to +0.61 m. The water is at room temperature (20°). Therefore, the water density, ρ, is equal to 998.20 kg/m3, and the dynamic viscosity, ν, to 1.002E-3 Pa-s. The acoustic speed, a, of PVC and organic glass pipes, filled with water, is 400 m/s (Zhou, L. 2011); empirical research. A Darcy-Weisbach friction factor, f, equal to 0.05 is assumed (Zhou, L. 2011); empirical research.

There are four measuring points along the pipe, Fig. 7.7; three near the water vent (pressure), PT1, PT2, and PT3, and another one in the 2.05-m-long horizontal pipe, PT4 (flow speed).

7.2.2 Initial conditions

At first, DV_1, DV_2, and WV, are closed. There are two, separated water columns, one before, LW1, and one after, LW2, the entrapped air. Both, LW1 and LW_2, are at hydrostatic pressure, $p = \rho gh$. The entrapped air is at atmospheric (absolute) pressure; 101325 Pa (10.33 m).

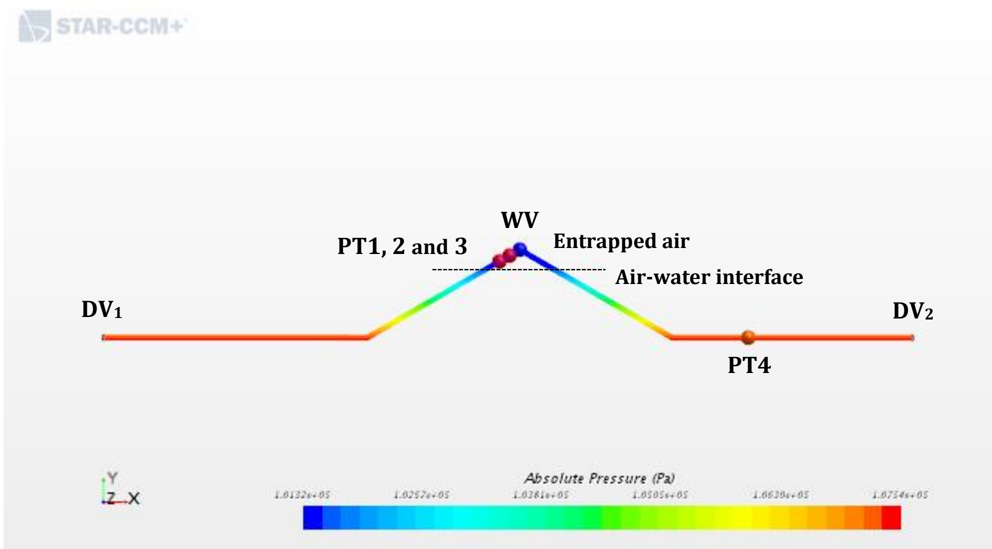


Figure 7.7 Initial pressure distribution; (Coronado-Hernández, O. E., et al. 2017a).

7.2.3 Model Boundaries

The model is composed of three boundaries. DV_1 and DV_2 act both as pressure-outlet boundaries, while the surface of the pipe is set as a wall boundary.

The two pressure-outlet boundaries are set as constant-pressure, at atmospheric (absolute) pressure; 101325 Pa (10.33 m). The boundary is set so that only water (no air) is allowed to enter the solution domain through this boundary.

The boundary is set as no-slip. The Darcy-Weisbach friction factor, f, as has been said before, is set equal to 0.05. The pipe roughness, is thus (iteratively) solved by means of the Colebrook-White equation.

7.2.4 Mesh Models

The same meshers, as in the previous model, Section 7.1, are selected for generating the mesh. This time:

- The *number of prism layers* is set equal to 5.
- The *prism layer stretching* parameter is set equal to 1.2.
- The *prism layer thickness* is set as relative to the base; 30%.

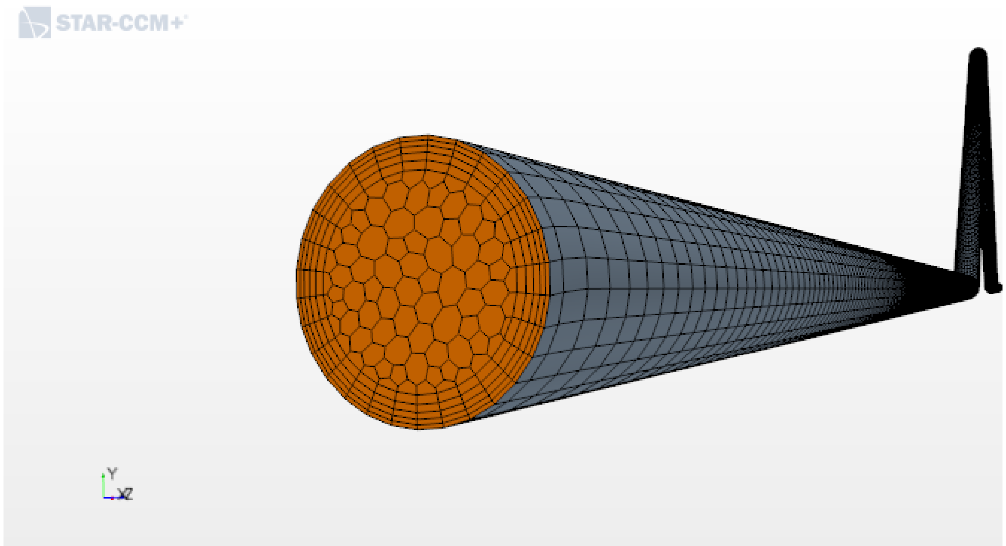


Figure 7.8 Mesh (detail).

7.2.5 Physic models

As in the previous model, Section 7.1, the flow is three dimensional. The flow is assumed turbulent; also unsteady. The flow is multiphase. The *volume of fluid (VOF) model* is thus again used. Two phases are defined, one for the water, and one for the air.

7.2.6 Calibration

The model has to be capable of reproducing real-life situations. Thus, it has to be calibrated against real data (from the experiment). This has not been possible since the initial conditions of the experiment conducted by Coronado-Hernández, O. E., et al. (2017a) cannot be met; a valve opening time of 0.7 s cannot be modeled with the data provided in the article (instant opening of the valve assumed). The air backflow that takes place at the drain valves cannot be properly addressed either; as stated in Section 7.2.3, only water, and no air, is allowed to enter the solution domain through this boundary.

7.2.7 Results

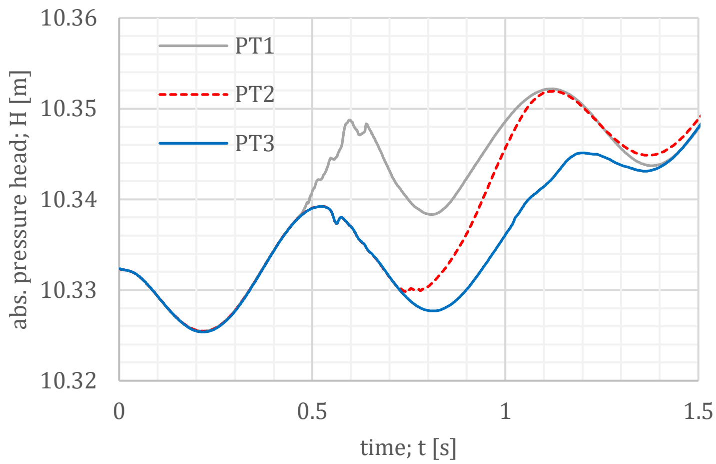


Figure 7.9 Absolute pressure head; at PT1, PT2 and PT3.

The process is (almost) symmetrical; the half-part corresponding to DV_2 is itemized here. (i) The drain valve, DV_2, is opened, and the pressure head, H, at layer of fluid nearest to it (in LW_2) decreases from p = ρgh to 0 Pa (10.33 m); ΔH. This ΔH yields an increase of flow speed at the drain valve, ΔV; positive flow. The transient is initiated; travels upstream. (Still) Initial conditions of pressure, and volume at the air pocket, H_{a0} and $\cancel{V}_{a0}$. (ii) While the positive flow continues, the volume of the entrapped air, $\cancel{V}_a$, increases; it is stretched (expans), and H decreases, sub-atmospheric pressure at the entrapped air. (iii) The fluid in contact with the entrapped air is brought to rest; $\cancel{V}_a$ ceases to increase. The minimum H has been reached at the entrapped air. (iv) As a negative flow travels now through the pipe; water is allowed to enter the solution domain through the boundary, $\cancel{V}_a$ decreases; it is compressed (cushioning effect). (v) The fluid in contact with the entrapped air is brought to rest; $\cancel{V}_a$ ceases to decrease. The maximum H has been reached at the entrapped air; for this cycle. A positive flow is, again, initiated, and the process repeated.

The difference between the results at the various measuring points is due to some of them being located out of the active area of the air pocket.

7.2.8 Conclusions

Even though the model behaves as it was expected, and a sub-atmospheric pressure is reached at the air pocket, the pressure drop is minor and(, in this case,) might not lead to any system failure. The backflow of air through the drain valves and into the air pocket cannot be addressed in here, and may lead to a different behavior than the one displayed in Fig. 7.9.

8 CONCLUSIONS AND FUTURE RESEARCH

A large number of the effects associated to the presence of air in pipelines has been addressed in this thesis. Abnormal pressure surges due to the compression of trapped air, or sub-atmospheric pressures because of the expansion of it, the influence of air on the propagation speed of a pressure wave, or the effects of cavitation, or column separation, have been analyzed and measured. The characteristics-method based computer programs, written in MATLAB, have proven that, in spite of their simplicity, since they do not consider minor losses, the pipe walls are considered rigid, or a constant friction factor is assumed(, etc.), they are able of accurately represent the behavior of the main flow variables in the event of a transient. At the same time, transient-flow analysis has been applied, in one of the programs, for the detection of leakages in pipes, by analyzing the pressure signals in them. This is of importance since, if the location of a leakage along a pipe is known, costs would be reduced; it would only be necessary to dig once, where the leakage has been detected. Single, separate conclusions, can be found in the report for each of the different MATLAB programs. The VOF-based, CFD models provide a good representation of the different situations as well, under various initial and boundary conditions. All this is relevant because both the MATLAB programs, and the CFD models, offer accurate, less-expensive predictions than experimental testing.

Nevertheless, the influence of trapped air on flow capacity, and thus energy consumption, could not be addressed in this thesis. This issue is of major concern due to the considerable costs associated to air valves, chambers, and deepening of trenches in order to provide the minimum pipe slopes, necessary to enable air to leave the system. Ideally, the air would be removed of the system by the flow itself, which is achieved by ensuring a minimum, critical velocity, able to push the trapped along and out of the system. A CFD model of this situation was attempted, not delivering satisfactory results. Therefore, future research should focus on this issue.

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APPENDIX A BASIC CONCEPTS

A.1 Basic Equation of Water Hammer

As stated in Section 2.2; frictionless pipe, and slightly compressible fluid; positive flow speed, V, in the downstream direction, and rigid pipe walls; the cross-sectional area of the control volume, A, does not change due to pressure changes; during the transient.

The fluid moves at V_0, and the steady-state pressure head upstream of the reservoir is p (initial conditions). (At) t = 0; the valve is closed, and the fluid nearest to it, brought to rest; V_0 changes to $V_0 + \Delta V$. This change in flow speed; of ΔV, results in an increase in pressure head at the face of the valve, Δp; the fluid is (slightly) compressed, so the fluid density, ρ_0, changes to $\rho_0 + \Delta\rho$. A high-pressure pulse wave, of magnitude Δp/γ, travels in the upstream direction, at an absolute speed of $a – V_0$; being γ the specific weight of the fluid; γ = ρg, and a the wave speed, Section 2.3. The volume of fluid having its momentum changed is $A(a - V_0)\Delta t$.

The momentum equation states that the resultant force in the x-direction, on a given control volume, is equal to the time rate of increase, plus the net influx of momentum within the already mentioned control volume, both in the x-direction; thus, with the help of Fig. 2.1, the time rate of change of momentum in the positive x-direction is

$$\frac{A(\mathrm{a} - V_0)\Delta t}{\Delta t}[(\rho + \Delta\rho)(V_0 + \Delta \mathrm{V}) - \rho V_0] \tag{A.1}$$

therefore (it follows from the momentum equation that)

$$-\Delta pA = \frac{A(\mathrm{a} - V_0)\Delta t}{\Delta t}[(\rho + \Delta\rho)(V_0 + \Delta \mathrm{V}) - \rho V_0] + (\rho + \Delta\rho)A(V_0 + \Delta \mathrm{V})^2 - \rho A{V_0}^2 \tag{A.2}$$

The continuity equation states that the time rate of increase of mass of a given control volume, is equal to the net mass influx within the aforementioned control volume. As in the previous case, the volume of fluid having its momentum changed is $A(a - V_0)\Delta t$

$$\frac{(\rho + \Delta\rho) - \rho}{\Delta t}A(\mathrm{a} - V_0)\Delta t + \rho A V_0 - (\rho + \Delta\rho)A(V_0 + \Delta \mathrm{V}) = 0 \tag{A.3}$$

simplified and combined with Eq. A.2 (considered that the valve is closed by increments)

$$\sum \Delta \mathrm{p} = \pm\rho\mathrm{a}\sum \Delta \mathrm{V} \tag{A.4}$$

and, since Δp = ρgΔH; in which ΔH is the head change

$$\sum \Delta \mathrm{H} = \pm\frac{a}{g}\sum \Delta \mathrm{V} \tag{A.5}$$

which is the *basic equation of water hammer*; the plus sign is used for waves traveling upstream whereas the minus sign is used for waves traveling downstream.

A.2 Wave Speed

Let us now consider the pipe walls to be, to some extent, elastic. (At) t = 0; the valve is closed, and the pipe may stretch in length, Δs, Fig. 2.2; it is assumed that the valve moves this distance in L/a (seconds; transit time) or, which is the same, has a speed of Δsa/L. In this time period (of L/a), the mass entering the pipe is $\rho A V_0 L/A$. This is possible due to an increase in the cross-sectional area of the pipe, ΔA, the (above mentioned) stretching of the pipe, Δs, and a compression of the fluid, ρ.

$$\rho A V_0 \frac{L}{a} = \rho L \Delta A + \rho A \Delta s + L A \Delta \rho \tag{A.6}$$

The flow speed changes (in L/A) by $\Delta V = \Delta s a/L - V_0$; thus, Eq. A.6 simplifies to

$$-\frac{\Delta V}{a} = \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho} \tag{A.7}$$

which combined with Eq. A.4

$$a^2 = \frac{\Delta p/\rho}{\Delta A/A + \Delta \rho/\rho} \tag{A.8}$$

The bulk modulus of elasticity of a fluid, K, is defined by

$$K = \frac{\Delta p}{\Delta \rho/\rho} \tag{A.9}$$

hence, it follows from Eqs. A.8 and A.9

$$a^2 = \frac{K/\rho}{1 + (K/A)(\Delta A/\Delta p)} \tag{A.10}$$

which is the equation of the *wave speed.*

APPENDIX B BASIC DIFFERENTIAL EQUATIONS FOR TRANSIENT FLOW

B.1 Equation of Motion

The Newton's second law of motion, ∑F = ma, is applied to the control volume in Fig. 3.1. The forces, acting on the control volume, in the x-direction, are the surface normal pressure on both cross-sectional faces, the pressure and shear on the periphery, and the x-component of the gravitational force; thus

$$pA - [pA + (pA)_x \delta x] + \left(p + p_x \frac{\delta x}{2}\right) A_x \delta x - \tau_0 \pi D \delta x - \rho g A \delta x \sin\alpha = \rho A \delta x \dot{V} \tag{B.1}$$

δx^2 is too small; negligible, simplifying, Eq. (B.1) becomes

$$p_x A + \tau_0 \pi D + \rho g A \sin\alpha + \rho A \dot{V} = 0 \tag{B.2}$$

in which τ_0 is the shear stress; Section 3.3

$$\tau_0 = \frac{\rho f V|V|}{8} \tag{B.3}$$

and the dot over the dependent variable V indicates the total derivative with respect to time, i.e., $\dot{V} = VV_x + V_t$; thus Eq. (B.2) can be written as

$$\frac{p_x}{\rho} + VV_x + V_t + g\sin\alpha + \frac{fV|V|}{2D} = 0 \tag{B.4}$$

since VV_x is not considered in steady state, it is only consistent to exclude the term in unsteady flow. This is a common simplification in Eq. (B.4) for low-Mach-number unsteady flows, reducing Eq. (B.4) to

$$\frac{p_x}{\rho} + V_t + g\sin\alpha + \frac{fV|V|}{2D} = 0 \tag{B.5}$$

and, since

$$p_x = \rho g(H_x - z_x) = \rho g(H_x - \sin\alpha) \tag{B.6}$$

Eq. (B.5) becomes

$$gH_x + V_t + \frac{fV|V|}{2D} = 0 \tag{B.7}$$

which is the simplified, head form of the *equation of motion*; restricted to less compressible fluid, flowing at low velocities.

B.2 Continuity Equation

The continuity equation is applied to a moving control volume, Fig. 3.2; stationary relative to the pipe, it moves or stretches only as the inside surface of the pipe moves and stretches

$$-[\rho A(V-u)]_x\,\delta x=\frac{D}{Dt}(\rho A\delta x) \tag{B.8}$$

in which u is the speed of the pipe at x, and D/Dt of, e.g. V, is the same as $\dot{V} = VV_x + V_t$. The time rate of increase od length, δx, of the control volume, is given by

$$\frac{D}{Dt}\,\delta x=u_x\delta x \tag{B.9}$$

Eq. (B.8), thus becomes

$$(\rho AV)_x-(\rho Au)_x+\frac{D'}{Dt}(\rho A)+\rho Au_x=0 \tag{B.10}$$

which, after simplification, may be written as

$$\rho AV_x+V(\rho A)_x+(\rho A)_t=0 \tag{B.11}$$

The last two terms represent the total derivative of ρA with respect to time, then

$$\frac{1}{\rho A}\frac{D}{Dt}(\rho A)+V_x=0 \tag{B.12}$$

which can be as well represented; since D/Dt of, e.g. V, is the same as $\dot{V}$, as

$$\frac{1}{\rho A}\left(\rho\dot{A}+\dot{\rho}A\right)+V_x=0 \tag{B.13}$$

or, which is the same

$$\frac{\dot{A}}{A}+\frac{\dot{\rho}}{\rho}+V_x=0 \tag{B.14}$$

For prismatic tubes, area is a function of pressure only, so, in the first term of Eq. (B.14)

$$\dot{A}=\frac{dA}{dp}\dot{p} \tag{B.15}$$

The second term in Eq. (B.14), considering Eq. (A.9); bulk modulus of elasticity of a fluid, may be written as

$$\frac{\dot{\rho}}{\rho} = \frac{\dot{p}}{K} \tag{B.16}$$

Eq. (B.14), therefore becomes

$$V_x + \frac{\dot{p}}{K}\left(1 + \frac{K}{A}\frac{dA}{dp}\right) = 0 \tag{B.17}$$

which, considering Eq. (A.10), may be written as

$$\rho a^2 V_x + \dot{p} = 0 \tag{B.18}$$

The transport term, Vp_x, in $\dot{p}$ is neglected as being small compared to other terms, Eq. (B18) thus becomes

$$\rho a^2 V_x + p_t = 0 \tag{B.19}$$

and, since

$$p_t = \rho g H_t \tag{B.20}$$

Eq. (B.19) may be written as

$$\frac{a^2 V_x}{g} + H_t = 0 \tag{B.21}$$

which is the simplified, head form of the unsteady *continuity equation*; restricted to less compressible fluid, flowing at low velocities.

APPENDIX C METHOD OF CHARACTERISTICS

C.1 Characteristic Equations

As noted in Appendix B, the simplified, pressure-head form of the motion and continuity equations is

$$gH_x + V_t + \frac{f}{2D}V|V| = 0 \tag{B.7}$$

$$H_t + \frac{a^2}{g}V_x = 0 \tag{B.21}$$

by multiplying Eq. (B.21) by λ; unknown multiplier, adding to Eq. (B.7), and re-arranging terms (linear combination)

$$\lambda\left(H_x\frac{g}{\lambda} + H_t\right) + \left(V_x\lambda\frac{a^2}{g} + V_t\right) + \frac{f}{2D}V|V| = 0 \tag{C.1}$$

A suitable pair of values of λ allows for the simplification of Eq. (C.1); in which velocity, V, and pressure head, H, are functions of distance(; through the pipe), x, and time, t. If x(, independent variable,) is allowed to be a function of t, then

$$\frac{dV}{dt} = V_x\frac{dx}{dt} + V_t \tag{C.2}$$

$$\frac{dH}{dt} = H_x\frac{dx}{dt} + H_t \tag{C.3}$$

which are the total derivatives of V, and H, respectively. Now, by examination of Eq. (C.1), with Eqs. (C.2) and (C.3) in mind, it can be noted that if

$$\frac{dx}{dt} = \frac{g}{\lambda} = \frac{\lambda a^2}{g} \tag{C.4}$$

Eq. (C.1) becomes

$$\lambda\frac{dH}{dt} + \frac{dV}{dt} + \frac{f}{2D}V|V| = 0 \tag{C.5}$$

which is a finite-difference equation. Two particular values of λ may be found by means of Eq. (C.4)

$$\lambda = \pm\frac{g}{a} \tag{C.6}$$

The relation between x and t is determined by substituting these values of λ back into Eq. (C.4)

$$\frac{dx}{dt} = \pm a \tag{C.7}$$

which represents the variation of position of a wave with respect to time, as a function of the wave speed. When the positive value of λ is used in Eq. (C.4), the positive value of λ must be used in Eq. (C.5); same for the negative λ. The substitution of these values of λ into Eq. (C.5) leads to two pairs of ordinary, differential equations which are grouped and identified as C+ and C-, *characteristic equations*.

$$C^{+}: \begin{cases} \dfrac{g}{a}\dfrac{dH}{dt} + \dfrac{dV}{dt} + \dfrac{fV|V|}{2D} = 0 & \text{(C.8)} \\ \dfrac{dx}{dt} = +a & \text{(C.9)} \end{cases}$$

$$C^{-}: \begin{cases} -\dfrac{g}{a}\dfrac{dH}{dt} + \dfrac{dV}{dt} + \dfrac{fV|V|}{2D} = 0 & \text{(C.10)} \\ \dfrac{dx}{dt} = -a & \text{(C.11)} \end{cases}$$

C.2 Finite-Difference Equations

Let us begin with the compatibility equation along the C+ line, Eq. (C.8)

$$\frac{g}{a}\frac{dH}{dt} + \frac{dV}{dt} + \frac{fV|V|}{2D} = 0 \tag{C.8}$$

By multiplying Eq. (C.8) by a(dt/g) = dx/g, and by introducing the pipeline cross-sectional area, A, to write the equation in terms of discharge, Q, in place of velocity, V, the equation may be placed in a form, suitable for integration along the C+ characteristic

.

$$\int_{H_A}^{H_P} dH + \frac{a}{gA}\int_{Q_A}^{Q_P} dQ + \frac{f}{2gDA^2}\int_{X_A}^{X_P} Q|Q|dx = 0 \qquad \text{C.12)}$$

The integration of Eq. (C.12), and a similar integration, this time along the C-characteristic line between B and P, yields

$$C^{+}: H_P - H_A + \frac{a}{gA}(Q_P - Q_A) + \frac{f\Delta x}{2gDA^2} Q_P|Q_A| = 0 \tag{C.13}$$

$$C^{-}: H_P - H_B - \frac{a}{gA}(Q_P - Q_B) - \frac{f\Delta x}{2gDA^2} Q_P|Q_B| = 0 \tag{C.14}$$

and, solving for H_P, these equations may be written as

$$C^{+}: H_P = H_A - B(Q_P - Q_A) - RQ_P|Q_A| \tag{C.15}$$

$$C^{-}: H_P = H_B + B(Q_P - Q_B) + RQ_P|Q_B| \tag{C.16}$$

in which B is a function of the physical properties of the fluid and the pipeline, often called the pipeline characteristic impedance

$$B = \frac{a}{gA} \tag{C.17}$$

and R is the pipeline resistance coefficient

$$R = \frac{f\Delta x}{2gDA^2} \tag{C.18}$$

Eqs. (C.15) and (C.16), may be written in a simpler form

$$C^+: H_P = C_P - B_P Q_P \tag{C.19}$$

$$C^-: H_P = C_M + B_M Q_P \tag{C.20}$$

coefficients C_P, B_P, C_M and B_M being known constants

$$C_P = H_{i-1} + BQ_{i-1} \qquad B_P = B + R|Q_{i-1}| \tag{C.21}$$

$$C_M = H_{i+1} - BQ_{i+1} \qquad B_M = B + R|Q_{i+1}| \tag{C.22}$$

APPENDIX D MATLAB CODE

D.1 Pump Stop at Power Failure

```
clear all; close all; clc;
%%
%pipe prof. & data
g=9.8105;%local acceleration due to gravity [m/s^{2}]
a=288;%wave speed [m/s] GIVEN
L=1025;%pipe length [m] GIVEN
dx=5;%length incr. [m] GIVEN
no=(L/dx)+1;%no. of calc. points [-]
cp=1:no;%vec.; calc. points
nod=[0 50 160 230 270 420 550 600 775 825 1000 1025];%vec.; stations
[m] GIVEN
z_nod=[27.70 30.25 31.85 32.05 33 41.10 41.50 41.75 49.65 50.80 52
52.12];%vec.; elevs. [m] GIVEN
l=length(nod);%length; n
i=1;
for j=1:l-1%pipe profile
    s=(z_nod(1,j+1)-z_nod(1,j))/(nod(1,j+1)-nod(1,j));
    c=0;
    for i=i:((nod(1,j+1)/dx)+1)
        c=c+1;
        z_l(1,i)=z_nod(1,j)+(c-1)*dx*s;
    end
end
% hold on
% plot_l=plot(cp,z_l,'k');
D=0.21;%pipe diameter [m] GIVEN
%Rh=D/4;%hydraulic radius [m]
A=pi*((D/2).^2);%pipe cross-sec. area [m^{2}]
dt=dx/a;%time incr. [s]
%dt=0.2;%time incr. [s] GIVEN
H_max=100;%max. pressure head; error calc. [m] GIVEN
H_min=-20;%min. pressure head; error calc. [m] GIVEN
%%
%init. cond.
Qi=0.032;%init. flow rate [m^{3}/s] GIVEN
Hi=56.02;%init. pressure head [m] GIVEN
Hf=52.12;%fin. pressure head [m] GIVEN
dh_T=Hi-Hf;%total pressure head incr. [m]
f=dh_T*D*2*g*(A.^2)/(L*(Qi.^2));%Darcy friction factor [-]
%f=0.01741;%Darcy friction factor [-] EXCEL SHEET
dh=f*dx*(Qi.^2)/(D*2*g*(A.^2));%pressure head incr. [m]; Darcy-Weisbach
equation
i=1;
j=1;
H(i,j)=Hi;
for j=2:no
    H(i,j)=H(i,j-1)-dh;
end
for j=1:no
    Q(i,j)=Qi;
    V(i,j)=Q(i,j)/A;
end
```

```
Hp=zeros(1,no);
Vp=zeros(1,no);
fig=figure;
ax1=subplot(2,1,1);
hold on
plot_l=plot(cp,z_l,'k');
plot_H=plot(cp,H,'c--');
plot_Hp=plot(cp,Hp,'b');
hold off
set(ax1,'xLim',[1 no],'yLim',[H_min H_max]);
box on
grid on
grid minor
xlabel(ax1,'Calc. points [-]');
ylabel(ax1,'Pressure head [m]');
title(ax1,'Fluid transient');
ax2=subplot(2,1,2);
hold on
plot_l=plot(cp,z_l,'k');
plot_V=plot(cp,V,'c--');
plot_Vp=plot(cp,Vp,'m');
hold off
set(ax2,'xLim',[1 no],'yLim',[-1 1]);
box on
grid on
grid minor
xlabel(ax2,'Calc. points [-]');
ylabel(ax2,'Velocity [m/s]');
%%
%vid.
movieObj = VideoWriter('Hjedsbaekvej_198_no_cavitation.avi');%create
the movie object
%MovieObj.Framerate = 60;%set the properties if desired (in this case
the frame rate)
open(movieObj);%get the movie object ready for writing
%%
%calc.
t=20;%simulation time [s]
dt_no=t/dt;%no. of time incr. [-]
B=a/(g*A);%pipeline characteristic impedance [s/m^{2}]
R=f*dx/(2*g*D*(A.^2));%pipeline resistance coefficient [s^{2}/m^{5}]
H_V=-10;%vapor pressure head[m]
e_lim=10^-10;%"true" value [m^{3}/s]
H_max_M=H;
H_min_M=H;

i=1;
for c=1:dt_no
    j=1;
    %Qp(i,j)=0;
    Qp(i,j)=-0.0152*(c-1)*dt+0.0319; %EXCEL SHEET
    if Qp(i,j)<0
        Qp(i,j)=0;
    end
    Vp(i,j)=Qp(i,j)/A;
    H_high=H_max;
    H_low=H_min;
```

```
h_M=[H_low H_high];%M as abbrev. for 'matrix'
nn=length(h_M);
Qp_L_M=zeros(1,nn);%L as abbrev. for 'left hand side'
Qp_R_M=zeros(1,nn);%R as abbrev. for 'right hand side'
e_M=zeros(1,nn);
Cm=H(i,j+1)-B*Q(i,j+1);
Bm=B+R*abs(Q(i,j+1));
for k=1:nn
    Qp_L_M(1,k)=Qp(i,j);
    Qp_R_M(1,k)=(h_M(1,k)-Cm)/Bm;
    e_M(1,k)=Qp_L_M(1,k)-Qp_R_M(1,k);
end
e_low=e_M(1,1);
e_high=e_M(1,2);
e_new=1;
while abs(e_new)>e_lim
    H_new=0.5*(H_high+H_low);
    Qp_L=Qp(i,j);
    Qp_R=(H_new-Cm)/Bm;
    e_new=Qp_L-Qp_R;
    if e_new<-e_lim
        H_high=H_new;
    elseif e_new>e_lim
        H_low=H_new;
    else%if -eLimit<eNew<eLimit
        H_high=H_new;
        H_low=H_new;
    end
end
Hp(i,j)=0.5*(H_high+H_low);
if Hp(i,j)<z_l(i,j)+H_V
    Hp(i,j)=z_l(i,j)+H_V;
end
if Hp(i,j)<0
    Hp(i,j)=0;
    Cm=H(i,j+1)-B*Q(i,j+1);
    Bm=B+R*abs(Q(i,j+1));
    Qp(i,j)=(Hp(i,j)-Cm)/Bm;
    Vp(i,j)=Qp(i,j)/A;
end
if Hp(i,j)>H_max_M(i,j)
    H_max_M(i,j)=Hp(i,j);
end
if Hp(i,j)<H_min_M(i,j)
    H_min_M(i,j)=Hp(i,j);
end
G(i,c)=Hp(i,j);%pressure head at the pump [m]
for j=2:(no-1)
    H_high=H_max;
    H_low=H_min;
    Cp=H(i,j-1)+B*Q(i,j-1);
    Bp=B+R*abs(Q(i,j-1));
    Cm=H(i,j+1)-B*Q(i,j+1);
    Bm=B+R*abs(Q(i,j+1));
    e_new=1;
    while abs(e_new)>e_lim
        H_new=0.5*(H_high+H_low);
```

```
                Qp_L=(Cp-H_new)/Bp;
                Qp_R=(H_new-Cm)/Bm;
                e_new=Qp_L-Qp_R;
                if e_new<-e_lim
                    H_high=H_new;
                elseif e_new>e_lim
                    H_low=H_new;
                else%if -eLimit<eNew<eLimit
                    H_high=H_new;
                    H_low=H_new;
                end
            end
            Hp(i,j)=0.5*(H_high+H_low);
            if Hp(i,j)<z_l(i,j)+H_V
                Hp(i,j)=z_l(i,j)+H_V;
            end
            Qp_R=(Hp(i,j)-Cm)/Bm;
            Qp(i,j)=Qp_R;
            Vp(i,j)=Qp(i,j)/A;
            if j==2
                F(i,c)=Vp(i,j);
            end
            if Hp(i,j)>H_max_M(i,j)
                H_max_M(i,j)=Hp(i,j);
            end
            if Hp(i,j)<H_min_M(i,j)
                H_min_M(i,j)=Hp(i,j);
            end
        end
        j=no;
        Hp(i,j)=H(i,no);
        Cp=H(i,j-1)+B*Q(i,j-1);
        Bp=B+R*abs(Q(i,j-1));
        Qp(i,j)=(Cp-Hp(i,j))/Bp;
        Vp(i,j)=Qp(i,j)/A;
        if Hp(i,j)>H_max_M(i,j)
            H_max_M(i,j)=Hp(i,j);
        end
        if Hp(i,j)<H_min_M(i,j)
            H_min_M(i,j)=Hp(i,j);
        end
        for j=1:no
            H(i,j)=Hp(i,j);
            Q(i,j)=Qp(i,j);
        end
        set(plot_Hp,'Ydata',Hp);%updates H
        set(plot_Vp,'Ydata',Vp);%updates V
        drawnow;
        pause(0.0001)
        frame=getframe(fig);%grab the frame
        writeVideo(movieObj,frame);%write the frame to the object
    end
    close(movieObj);
    fig_M=figure;
    hold on
    plot_Max=plot(cp,H_max_M);
    plot_Min=plot(cp,H_min_M);
```

```
hold off
xlim([1 no]);
ylim([H_min H_max]);
box on
grid on
grid minor
xlabel('Calc. points [-]');
fig_G=figure;
t_G=1:dt_no;
plot_G=plot(t_G,G);
xlim([1 dt_no]);
ylim([-20 110]);
box on
grid on
grid minor
xlabel('Time incr. [-]');
ylabel('Pressure head [m]');
ylabel('Pressure head [m]');
```

D.2 Leakage; x = 725 m.

```
clear all; close all; clc;
%%
%pipe prof. & data
L=1025;%pipe length [m]
dx=25;%length incr. [m]
no=(L/dx)+1;%no. of calc. points [-]
cp=1:no;%vec.; calc. points [-]
nod=[0 L];%vec.; nodes [m]
z_nod=[0 0];%vec.; elevs. [m]
l=length(nod);%length; n [-]
i=1;
for j=1:l-1%pipe profile
    s=(z_nod(1,j+1)-z_nod(1,j))/(nod(1,j+1)-nod(1,j));
    c=0;
    for i=i:((nod(1,j+1)/dx)+1)
        c=c+1;
        z_l(1,i)=z_nod(1,j)+(c-1)*dx*s;
    end
end
% hold on
% plot_l=plot(cp,z_l,'k');
D=0.21;%pipe diameter [m]
A=pi*((D/2).^2);%pipe cross-sec. area [m^{2}]
a=288;%wave speed [m/s]
dt=dx/a;%time incr. [s]
%%
%leakage
j_G=30;%orifice node [-]
% z_G=z_l(1,j_G);%orifice elev. [m]
z_G=0;
Cd=0.61;%disch. coeff. [-]; sharp edge
A_G=A/20;%orifice area [m^{2}]
%%
%init. cond.
f=0.01741;%Darcy friction factor [-] EXCEL SHEET
```

```
Qi=0.032;%init. flow [m^{3}/s]
g=9.8105;%local acceleration due to gravity [m/s^{2}]
dh=f*dx*(Qi.^2)/(D*2*g*(A.^2));%pressure head incr. [m]; Darcy-Weisbach
equation
Hf=23.42;%fin. pressure head [m]
i=1;
for j=1:no
    H(i,j)=Hf+(no-(j-1))*dh;
    Q(i,j)=Qi;
    V(i,j)=Q(i,j)/A;
end
%plot_H=plot(cp,H,'c--');
%%
%fig.
Hp=zeros(1,no);
Vp=zeros(1,no);
H_max=100;%max. pressure head; error calc. [m]
H_min=-100;%min. pressure head; error calc. [m]
fig=figure;
ax1=subplot(2,1,1);
hold on
plot_l=plot(cp,z_l,'k');
plot_H=plot(cp,H,'c--');
plot_Hp=plot(cp,Hp,'b');
hold off
set(ax1,'xLim',[1 no],'yLim',[H_min H_max]);
box on
grid on
grid minor
xlabel(ax1,'Calc. points [-]');
ylabel(ax1,'Pressure head [m]');
title(ax1,'Fluid transient');
ax2=subplot(2,1,2);
hold on
plot_l=plot(cp,z_l,'k');
plot_V=plot(cp,V,'c--');
plot_Vp=plot(cp,Vp,'m');
hold off
set(ax2,'xLim',[1 no],'yLim',[-2 2]);
box on
grid on
grid minor
xlabel(ax2,'Calc. points [-]');
ylabel(ax2,'Velocity [m/s]');
%%
%vid.
movieObj = VideoWriter('leakage_transient_node_30.avi');%create the
movie object
%MovieObj.Framerate = 60;%set the properties if desired (in this case
the frame rate)
open(movieObj);%get the movie object ready for writing
%%
%calc.
% t=60;%simulation time [s]
% dt_no=t/dt;%no. of time incr. [-]
dt_no=4000;%no. of time incr. [-]
t=dt_no*dt;%simulation time [s]
```

```
B=a/(g*A);%pipeline characteristic impedance [s/m^{2}]
R=f*dx/(2*g*D*(A.^2));%pipeline resistance coefficient [s^{2}/m^{5}]
e_lim=10.^-10;%"true" value [m^{3}/s]

i=1;
for c=1:dt_no
    j=1;
    Qp(i,j)=Qi;
    H_high=H_max;
    H_low=H_min;
    H_M=[H_low H_high];%M as abbrev. for 'matrix'
    nn=length(H_M);
    Qp_L_M=zeros(1,nn);%L as abbrev. for 'left hand side'
    Qp_R_M=zeros(1,nn);%R as abbrev. for 'right hand side'
    e_M=zeros(1,nn);
    Cm=H(i,j+1)-B*Q(i,j+1);
    Bm=B+R*abs(Q(i,j+1));
    for k=1:nn
        Qp_L_M(1,k)=Qp(i,j);
        Qp_R_M(1,k)=(H_M(1,k)-Cm)/Bm;
        e_M(1,k)=Qp_L_M(1,k)-Qp_R_M(1,k);
    end
    e_low=e_M(1,1);
    e_high=e_M(1,2);
    e_new=1;
    while abs(e_new)>=e_lim
        H_new=0.5*(H_high+H_low);
        Qp_L=Qp(i,j);
        Qp_R=(H_new-Cm)/Bm;
        e_new=Qp_L-Qp_R;
        if e_new<-e_lim
            H_high=H_new;
        elseif e_new>e_lim
            H_low=H_new;
        else%if -eLimit<eNew<eLimit
            H_high=H_new;
            H_low=H_new;
        end
    end
    Hp(i,j)=0.5*(H_high+H_low);
    if Hp(i,j)<0
        Hp(i,j)=0;
        Qp(i,j)=(Hp(i,j)-Cm)/Bm;
    end
    Vp(i,j)=Qp(i,j)/A;
    G(i,c)=Hp(i,j);
    for j=2:(no-1)
        H_high=H_max;
        H_low=H_min;
        Cp=H(i,j-1)+B*Q(i,j-1);
        Bp=B+R*abs(Q(i,j-1));
        Cm=H(i,j+1)-B*Q(i,j+1);
        Bm=B+R*abs(Q(i,j+1));
        e_new=1;
        while abs(e_new)>=e_lim
            H_new=0.5*(H_high+H_low);
            Qp_L=(Cp-H_new)/Bp;
```

```
                Qp_R=(H_new-Cm)/Bm;
                if j==j_G && H(i,j)>=z_G
                    if H(i,j-1)>H(i,j+1)
                        Q_G=Cd*A_G*sqrt(2*g*(H(i,j-1)-z_G));
                    else%if H(i,j-1)<H(i,j+1)
                        Q_G=Cd*A_G*sqrt(2*g*(H(i,j+1)-z_G));
                    end
                    e_new=Qp_L-Qp_R-Q_G;
                else%if i<>iLeak
                    e_new=Qp_L-Qp_R;
                end
                if e_new<-e_lim
                    H_high=H_new;
                elseif e_new>e_lim
                    H_low=H_new;
                else%if -eLimit<eNew<eLimit
                    H_high=H_new;
                    H_low=H_new;
                end
            end
            Hp(i,j)=0.5*(H_high+H_low);
            Qp_L=(Cp-Hp(i,j))/Bp;
            Qp_R=(Hp(i,j)-Cm)/Bm;
            if abs(Qp_L)>=abs(Qp_R)
                Qp(i,j)=Qp_R;
            else%if Cp-Cm<0
                Qp(i,j)=Qp_L;
            end
            Vp(i,j)=Qp(i,j)/A;
        end
        j=no;
        Hp(i,j)=H(i,j);
        Cp=H(i,j-1)+B*Q(i,j-1);
        Bp=B+R*abs(Q(i,j-1));
        Qp(i,j)=(Cp-Hp(i,j))/Bp;
        Vp(i,j)=Qp(i,j)/A;
        for j=1:no
            H(i,j)=Hp(i,j);
            Q(i,j)=Qp(i,j);
        end
        set(plot_Hp,'Ydata',Hp);%updates H
        set(plot_Vp,'Ydata',Vp);%updates V
        drawnow;
        pause(0.001)
        frame=getframe(fig);%grab the frame
        writeVideo(movieObj,frame);%write the frame to the object
    end
    for c=dt_no+1:3*dt_no
        j=1;
        Qp(i,j)=0;
        H_high=H_max;
        H_low=H_min;
        Cm=H(i,j+1)-B*Q(i,j+1);
        Bm=B+R*abs(Q(i,j+1));
        e_new=1;
        while abs(e_new)>=e_lim
            H_new=0.5*(H_high+H_low);
```

```
        Qp_L=Qp(i,j);
        Qp_R=(H_new-Cm)/Bm;
        e_new=Qp_L-Qp_R;
        if e_new<-e_lim
            H_high=H_new;
        elseif e_new>e_lim
            H_low=H_new;
        else%if -eLimit<eNew<eLimit
            H_high=H_new;
            H_low=H_new;
        end
    end
    Hp(i,j)=0.5*(H_high+H_low);
    if Hp(i,j)<0
        Hp(i,j)=0;
        Qp(i,j)=(Hp(i,j)-Cm)/Bm;
    end
    Vp(i,j)=Qp(i,j)/A;
    G(i,c)=Hp(i,j);
    for j=2:(no-1)
        H_high=H_max;
        H_low=H_min;
        Cp=H(i,j-1)+B*Q(i,j-1);
        Bp=B+R*abs(Q(i,j-1));
        Cm=H(i,j+1)-B*Q(i,j+1);
        Bm=B+R*abs(Q(i,j+1));
        e_new=1;
        while abs(e_new)>=e_lim
            H_new=0.5*(H_high+H_low);
            Qp_L=(Cp-H_new)/Bp;
            Qp_R=(H_new-Cm)/Bm;
            if j==j_G && H(i,j)>=z_G
                if H(i,j-1)>H(i,j+1)
                    Q_G=Cd*A_G*sqrt(2*g*(H(i,j-1)-z_G));
                else%if H(i,j-1)<H(i,j+1)
                    Q_G=Cd*A_G*sqrt(2*g*(H(i,j+1)-z_G));
                end
                e_new=Qp_L-Qp_R-Q_G;
            else%if i<>iLeak
                e_new=Qp_L-Qp_R;
            end
            if e_new<-e_lim
                H_high=H_new;
            elseif e_new>e_lim
                H_low=H_new;
            else%if -eLimit<eNew<eLimit
                H_high=H_new;
                H_low=H_new;
            end
        end
        Hp(i,j)=0.5*(H_high+H_low);
        Qp_L=(Cp-Hp(i,j))/Bp;
        Qp_R=(Hp(i,j)-Cm)/Bm;
        if abs(Qp_L)>=abs(Qp_R)
            Qp(i,j)=Qp_R;
        else%if Cp-Cm<0
            Qp(i,j)=Qp_L;
```

```
            end
            Vp(i,j)=Qp(i,j)/A;
        end
        j=no;
        Hp(i,j)=H(i,j);
        Cp=H(i,j-1)+B*Q(i,j-1);
        Bp=B+R*abs(Q(i,j-1));
        Qp(i,j)=(Cp-Hp(i,j))/Bp;
        Vp(i,j)=Qp(i,j)/A;
        for j=1:no
            H(i,j)=Hp(i,j);
            Q(i,j)=Qp(i,j);
        end
        set(plot_Hp,'Ydata',Hp);%updates H
        set(plot_Vp,'Ydata',Vp);%updates V
        drawnow;
        pause(0.001)
        frame=getframe(fig);%grab the frame
        writeVideo(movieObj,frame);%write the frame to the object
    end
    for c=(3*dt_no)+1:4*dt_no
        j=1;
        Qp(i,j)=Qi;
        H_high=H_max;
        H_low=H_min;
        Cm=H(i,j+1)-B*Q(i,j+1);
        Bm=B+R*abs(Q(i,j+1));
        e_new=1;
        while abs(e_new)>=e_lim
            H_new=0.5*(H_high+H_low);
            Qp_L=Qp(i,j);
            Qp_R=(H_new-Cm)/Bm;
            e_new=Qp_L-Qp_R;
            if e_new<-e_lim
                H_high=H_new;
            elseif e_new>e_lim
                H_low=H_new;
            else%if -eLimit<eNew<eLimit
                H_high=H_new;
                H_low=H_new;
            end
        end
        Hp(i,j)=0.5*(H_high+H_low);
        if Hp(i,j)<0
            Hp(i,j)=0;
            Qp(i,j)=(Hp(i,j)-Cm)/Bm;
        end
        Vp(i,j)=Qp(i,j)/A;
        G(i,c)=Hp(i,j);
        for j=2:(no-1)
            H_high=H_max;
            H_low=H_min;
            Cp=H(i,j-1)+B*Q(i,j-1);
            Bp=B+R*abs(Q(i,j-1));
            Cm=H(i,j+1)-B*Q(i,j+1);
            Bm=B+R*abs(Q(i,j+1));
            e_new=1;
```

```
            while abs(e_new)>=e_lim
                H_new=0.5*(H_high+H_low);
                Qp_L=(Cp-H_new)/Bp;
                Qp_R=(H_new-Cm)/Bm;
                if j==j_G && H(i,j)>=z_G
                    if H(i,j-1)>H(i,j+1)
                        Q_G=Cd*A_G*sqrt(2*g*(H(i,j-1)-z_G));
                    else%if H(i,j-1)<H(i,j+1)
                        Q_G=Cd*A_G*sqrt(2*g*(H(i,j+1)-z_G));
                    end
%                     Q_G=Cd*A_G*sqrt(2*g*(H(i,j)-z_G));
                    e_new=Qp_L-Qp_R-Q_G;
                else%if i<>iLeak
                    e_new=Qp_L-Qp_R;
                end
                if e_new<-e_lim
                    H_high=H_new;
                elseif e_new>e_lim
                    H_low=H_new;
                else%if -eLimit<eNew<eLimit
                    H_high=H_new;
                    H_low=H_new;
                end
            end
            Hp(i,j)=0.5*(H_high+H_low);
            Qp_L=(Cp-Hp(i,j))/Bp;
            Qp_R=(Hp(i,j)-Cm)/Bm;
            if abs(Qp_L)>=abs(Qp_R)
                Qp(i,j)=Qp_R;
            else%if Cp-Cm<0
                Qp(i,j)=Qp_L;
            end
            Vp(i,j)=Qp(i,j)/A;
        end
        j=no;
        Hp(i,j)=H(i,j);
        Cp=H(i,j-1)+B*Q(i,j-1);
        Bp=B+R*abs(Q(i,j-1));
        Qp(i,j)=(Cp-Hp(i,j))/Bp;
        Vp(i,j)=Qp(i,j)/A;
        for j=1:no
            H(i,j)=Hp(i,j);
            Q(i,j)=Qp(i,j);
        end
        set(plot_Hp,'Ydata',Hp);%updates H
        set(plot_Vp,'Ydata',Vp);%updates V
        drawnow;
        pause(0.001)
        frame=getframe(fig);%grab the frame
        writeVideo(movieObj,frame);%write the frame to the object
    end
    close(movieObj);
    fig_G=figure;
    t_G=1:4*dt_no;
    plot_G=plot(t_G,G);
```

D.3 Vapor Cavity; V_0 = 0.75 m/s.

```
clear all; close all; clc;
%%
%pipe prof. & data
g=9.8105;%local acc. due to gravity [m/s^{2}]
a=100*g;%wave speed [m/s]
L=a;%pipe length [m]
no=100;%no. of calc. points [-]
dx=L/(no-1);%length incr. [m]
cp=1:no;%vec.; calc. points [-]
nod=[0 L];%vec.; nodes [m]
z_nod=[0 0];%vec.; elev. [m]
l=length(nod);%length; n [-]
i=1;
for j=1:l-1%pipe profile
    s=(z_nod(1,j+1)-z_nod(1,j))/(nod(1,j+1)-nod(1,j));
    c=0;
    for i=i:((nod(1,j+1)/dx)+1)
        c=c+1;
        z_l(1,i)=z_nod(1,j)+(c-1)*dx*s;
    end
end
% hold on
% plot_l=plot(cp,z_l,'k');
D=0.21;%pipe diameter [m]
A=pi*((D/2).^2);%pipe cross-sec. area [m^{2}]
dt=dx/a;%time incr. [s]
%%
%init. cond.
f=0;%Darcy friction factor [-] NO FRICTION; EXCEL SHEET
Vi=0.75;%init. flow speed [m/s] GIVEN
dh=f*dx*(Vi^2)/(D*2*g);%pressure head incr. [m]; Darcy-Weisbach
equation
Hf=15;%fin. pressure head [m]
i=1;
for j=1:no
    H(i,j)=Hf+(no-(j-1))*dh;
    V(i,j)=Vi;
    Q(i,j)=V(i,j)*A;
end
%%
%fig.
Hp=zeros(1,no);
Vp=zeros(1,no);
fig=figure;
ax1=subplot(2,1,1);
hold on
plot_l=plot(cp,z_l,'k');
plot_H=plot(cp,H,'c--');
plot_Hp=plot(cp,Hp,'b');
hold off
set(ax1,'xLim',[1 no],'yLim',[-70 120]);
box on
grid on
grid minor
```

```
xlabel(ax1,'Calc. points [-]');
ylabel(ax1,'Pressure head [m]');
title(ax1,'Fluid transient');
ax2=subplot(2,1,2);
hold on
plot_l=plot(cp,z_l,'k');
plot_V=plot(cp,V,'c--');
plot_Vp=plot(cp,Vp,'m');
hold off
set(ax2,'xLim',[1 no],'yLim',[-1 1]);
box on
grid on
grid minor
xlabel(ax2,'Calc. points [-]');
ylabel(ax2,'Velocity [m/s]');
%%
%vid.
movieObj = VideoWriter('isolated_cavity_no_friction_0.75.avi');%create
the movie object
%MovieObj.Framerate = 60;%set the properties if desired (in this case
the frame rate)
open(movieObj);%get the movie object ready for writing
%%
%calc.
% t=30;%simulation time [s]
% dt_no=t/dt;%no. of time incr. [-]
dt_no=1000;%no. of time incr. [-]
t=dt_no*dt;%simulation time [s]
B=a/(g*A);%pipeline characteristic impedance [s/m^{2}]
R=f*dx/(2*g*D*(A^2));%pipeline resistance coefficient [s^{2}/m^{5}]
H_max=1000;%max. pressure head; error calc. [m]
H_min=-1000;%min. pressure head; error calc. [m] [m]
H_V=-10;%vapor pressure head [m]
v_C=0;%init. cavity volume [m^{3}]
e_lim=10^-10;%"true" value [m^{3}/s]

i=1;
for c=1:dt_no
    j=1;
    Qp(i,j)=0;
    H_high=H_max;
    H_low=H_min;
    H_M=[H_low H_high];%M as abbrev. for 'matrix'
    nn=length(H_M);
    Qp_L_M=zeros(1,nn);%L as abbrev. for 'left hand side'
    Qp_R_M=zeros(1,nn);%R as abbrev. for 'right hand side'
    e_M=zeros(1,nn);
    Cm=H(i,j+1)-B*Q(i,j+1);
    Bm=B+R*abs(Q(i,j+1));
    for k=1:nn
        Qp_L_M(1,k)=Qp(i,j);
        Qp_R_M(1,k)=(H_M(1,k)-Cm)/Bm;
        e_M(1,k)=Qp_L_M(1,k)-Qp_R_M(1,k);
    end
    e_low=e_M(1,1);
    e_high=e_M(1,2);
    e_new=1;
```

```
if v_C>0
    Hp(i,j)=z_l(i,j)+H_V;
    Qp_R=(Hp(i,j)-Cm)/Bm;
    Qp_L=Qp(i,j);
    v_C_M(i,c)=v_C+(Qp_R-Qp_L)*dt;
    Qp(i,j)=Qp_R;
    Vp(i,j)=Qp(i,j)/A;
    if v_C_M(i,c)<=0
        while abs(e_new)>=e_lim
            H_new=0.5*(H_high+H_low);
            Qp_R=(H_new-Cm)/Bm;
            e_new=Qp_L-Qp_R;
            if e_new<-e_lim
                H_high=H_new;
            elseif e_new>e_lim
                H_low=H_new;
            else%if -e_lim<e_new<e_lim
                H_high=H_new;
                H_low=H_new;
            end
        end
        Hp(i,j)=0.5*(H_high+H_low);
        if Hp(i,j)<z_l(i,j)+H_V
            Hp(i,j)=z_l(i,j)+H_V;
        end
        Qp_R=(Hp(i,j)-Cm)/Bm;
        v_C_M(i,c)=v_C+(Qp_R-Qp_L)*dt;
        Qp(i,j)=Qp_R;
        Vp(i,j)=Qp(i,j)/A;
    end
else%if v_C=0
    while abs(e_new)>=e_lim
        H_new=0.5*(H_high+H_low);
        Qp_L=Qp(i,j);
        Qp_R=(H_new-Cm)/Bm;
        e_new=Qp_L-Qp_R;
        if e_new<-e_lim
            H_high=H_new;
        elseif e_new>e_lim
            H_low=H_new;
        else%if -e_lim<e_new<e_lim
            H_high=H_new;
            H_low=H_new;
        end
    end
    Hp(i,j)=0.5*(H_high+H_low);
    if Hp(i,j)<z_l(i,j)+H_V
        Hp(i,j)=z_l(i,j)+H_V;
    end
    Qp_R=(Hp(i,j)-Cm)/Bm;
    v_C_M(i,c)=v_C+(Qp_R-Qp_L)*dt;
    Qp(i,j)=Qp_R;
    Vp(i,j)=Qp(i,j)/A;
end
v_C=v_C_M(i,c);
G(i,c)=Hp(i,j);%pressure head at the pump [m]
S(i,c)=Vp(i,j);%flow speed at the pump [m]
```

```
        T(i,c)=c*dt;%time [s]
        for j=2:(no-1)
            H_high=H_max;
            H_low=H_min;
            Cp=H(i,j-1)+B*Q(i,j-1);
            Bp=B+R*abs(Q(i,j-1));
            Cm=H(i,j+1)-B*Q(i,j+1);
            Bm=B+R*abs(Q(i,j+1));
            e_new=1;
            while abs(e_new)>=e_lim
                H_new=0.5*(H_high+H_low);
                Qp_L=(Cp-H_new)/Bp;
                Qp_R=(H_new-Cm)/Bm;
                e_new=Qp_L-Qp_R;
                if e_new<-e_lim
                    H_high=H_new;
                elseif e_new>e_lim
                    H_low=H_new;
                else%if -e_lim<e_new<e_lim
                    H_high=H_new;
                    H_low=H_new;
                end
            end
            Hp(i,j)=0.5*(H_high+H_low);
            if Hp(i,j)<z_l(i,j)+H_V
                Hp(i,j)=z_l(i,j)+H_V;
            end
            % Qp_L=(Cp-Hp(i,j))/Bp;
            Qp_R=(Hp(i,j)-Cm)/Bm;
            Qp(i,j)=Qp_R;
            Vp(i,j)=Qp(i,j)/A;
        end
        j=no;
        Hp(i,j)=H(i,j);
        Cp=H(i,j-1)+B*Q(i,j-1);
        Bp=B+R*abs(Q(i,j-1));
        Qp(i,j)=(Cp-Hp(i,j))/Bp;
        Vp(i,j)=Qp(i,j)/A;
        for j=1:no
            H(i,j)=Hp(i,j);
            Q(i,j)=Qp(i,j);
        end
        set(plot_Hp,'Ydata',Hp);%updates H
        set(plot_Vp,'Ydata',Vp);%updates V
        drawnow;
        pause(0.001)
        frame=getframe(fig);%grab the frame
        writeVideo(movieObj,frame);%write the frame to the object
    end
    close(movieObj);
    fig_G=figure;
    t_G=1:dt_no;
    plot_G=plot(t_G,G);
    xlim([1 dt_no]);
    ylim([-20 110]);
    box on
    grid on
```

```
    grid minor
    xlabel('Time incr. [-]');
    ylabel('Pressure head [m]');
```

D.4 Air Pocket

```
clear all; close all; clc;
%%
%pipe prof. & data
L=2000;%pipe length [m]
dx=20;%length incr. [m]
no=(L/dx)+1;%no. of calc. points [-]
cp=1:no;%vec.; calc. points [-]
nod=[0 L];%vec.; nodes [m]
z_nod=[0 10];%vec.; elev. [m]
l=length(nod);%length; n [-]
i=1;
for j=1:l-1%pipe profile
    s=(z_nod(1,j+1)-z_nod(1,j))/(nod(1,j+1)-nod(1,j));
    c=0;
    for i=i:((nod(1,j+1)/dx)+1)
        c=c+1;
        z_l(1,i)=z_nod(1,j)+(c-1)*dx*s;
    end
end
% hold on
% plot_l=plot(cp,z_l,'k');
D=0.21;%pipe diameter [m]
A=pi*((D/2).^2);%pipe cross sec. area [m^{2}]
a=400;%speed of pressure pulse [m/s]
dt=dx/a;%time incr. [s]
v_dx=dx*A;
v_Ai=v_dx/2;
%%
%init. cond.
f=0.0172;%Darcy friction factor [-]; can be modeled by the Colebrook-White
equation
Qi=0;%init. flow; before pump stop [m^{3}/s] GIVEN
g=9.8105;%local acceleration due to gravity [m/s^{2}]
dh=f*dx*(Qi^2)/(D*2*g*(A^2));%Darcy-Weisbach equation [m]; per dx
Hi=9.95;%fin. elev.; fin. piez. level [m]
i=1;
for j=1:no
    H(i,j)=Hi+(no-(j-1))*dh;
    Q(i,j)=Qi;
    V(i,j)=Q(i,j)/A;
end
%%
%fig.
Hp=H;
Qp=Q;
Vp=V;
fig=figure;
ax1=subplot(2,1,1);
hold on
plot_l=plot(cp,z_l,'k');
```

```
plot_H=plot(cp,H,'c--');
plot_Hp=plot(cp,Hp,'b');
hold off
set(ax1,'xLim',[1 no],'yLim',[-100 100]);
box on
grid on
grid minor
xlabel(ax1,'Calc. points [-]');
ylabel(ax1,'Pressure head [m]');
title(ax1,'Fluid transient');
ax2=subplot(2,1,2);
hold on
% plot_l=plot(cp,z_l,'k');
plot_V=plot(cp,V,'c--');
plot_Vp=plot(cp,Vp,'m');
hold off
set(ax2,'xLim',[1 no],'yLim',[-2 2]);
box on
grid on
grid minor
xlabel(ax2,'Calc. points [-]');
ylabel(ax2,'Velocity [m/s]');
%%
%vid.
movieObj = VideoWriter('air_pocket.avi');%create the movie object
%MovieObj.Framerate = 60;%set the properties if desired (in this case the
frame rate)
open(movieObj);%get the movie object ready for writing
%%
%air pocket
H_bar=10.33;%barometric head [m]
m=1.4;%polytropic exponent [-]
j=no;
C_A=(Hp(i,j)+H_bar-z_l(1,j))*(v_Ai-dt*(Qp(i,j)+Q(i,j))/2)^m;
v_A=v_Ai;
KIT=100;
v_S=0.0001;
%%
%calc.
% t=30;%simulation time [s]
% dt_no=t/dt;%no. of time increments [-]
dt_no=500;%no. of time increments [-]
t=dt_no*dt;%simulation time [s]
B=a/(g*A);
R=f*dx/(2*g*D*(A^2));
H_max=1000;%max. head; error calc. [m]
H_min=-1000;%min. head; error calc. [m]
e_lim=10^-10;

i=1;
for c=1:dt_no
    j=1;
    Hp(i,j)=34.37;
    Cm=H(i,j+1)-B*Q(i,j+1);
    Bm=B+R*abs(Q(i,j+1));
    Qp(i,j)=(Hp(i,j)-Cm)/Bm;
    Vp(i,j)=Qp(i,j)/A;
```

```
%     Qp(i,j)=0.032;
%     H_high=H_max;
%     H_low=H_min;
%     H_M=[H_low H_high];%M as abbrev. for 'matrix'
%     nn=length(H_M);
%     Qp_L_M=zeros(1,nn);%L as abbrev. for 'left hand side'
%     Qp_R_M=zeros(1,nn);%R as abbrev. for 'right hand side'
%     e_M=zeros(1,nn);
%     Cm=H(i,j+1)-B*Q(i,j+1);
%     Bm=B+R*abs(Q(i,j+1));
%     for k=1:nn
%         Qp_L_M(1,k)=Qp(i,j);
%         Qp_R_M(1,k)=(H_M(1,k)-Cm)/Bm;
%         e_M(1,k)=Qp_L_M(1,k)-Qp_R_M(1,k);
%     end
%     e_low=e_M(1,1);
%     e_high=e_M(1,2);
%     e_new=1;
%     while abs(e_new)>=e_lim
%         H_new=0.5*(H_high+H_low);
%         Qp_L=Qp(i,j);
%         Qp_R=(H_new-Cm)/Bm;
%         e_new=Qp_L-Qp_R;
%         if e_new<-e_lim
%             H_high=H_new;
%         elseif e_new>e_lim
%             H_low=H_new;
%         else%if -e_lim<e_new<e_lim
%             H_high=H_new;
%             H_low=H_new;
%         end
%     end
%     Hp(i,j)=0.5*(H_high+H_low);
%     if Hp(i,j)<0
%         Hp(i,j)=0;
%         Qp(i,j)=(Hp(i,j)-Cm)/Bm;
%     end
%     Vp(i,j)=Qp(i,j)/A;
    for j=2:(no-1)
        H_high=H_max;
        H_low=H_min;
        Cp=H(i,j-1)+B*Q(i,j-1);
        Bp=B+R*abs(Q(i,j-1));
        Cm=H(i,j+1)-B*Q(i,j+1);
        Bm=B+R*abs(Q(i,j+1));
        e_new=1;
        while abs(e_new)>=e_lim
            H_new=0.5*(H_high+H_low);
            Qp_L=(Cp-H_new)/Bp;
            Qp_R=(H_new-Cm)/Bm;
            e_new=Qp_L-Qp_R;
            if e_new<-e_lim
                H_high=H_new;
            elseif e_new>e_lim
                H_low=H_new;
            else%if -e_lim<e_new<e_lim
                H_high=H_new;
```

```
                H_low=H_new;
            end
        end
        Hp(i,j)=0.5*(H_high+H_low);
%          Qp_L=(Cp-Hp(i,j))/Bp;
        Qp_R=(Hp(i,j)-Cm)/Bm;
        Qp(i,j)=Qp_R;
        Vp(i,j)=Qp(i,j)/A;
    end
    j=no;
    Cp=H(i,j-1)+B*Q(i,j-1);
    Bp=B+R*abs(Q(i,j-1));
    Qp(i,j)=Q(i,j);
    u=0;
    while u<=KIT
        v_Ap(i,c)=v_A-dt*(Qp(i,j)+Q(i,j))/2;
        if v_Ap(i,c)<v_S
            v_Ap(i,c)<v_S;
        end
        F1=(Cp-Bp*Qp(i,j)-z_l(i,j)+H_bar)*(v_Ap(i,c)^m)-C_A;
        dF1dQp=-m*dt*C_A/v_Ap(i,c)-Bp*v_Ap(i,c)^m;
        dQ=-F1/dF1dQp;
        Qp(i,j)=Qp(i,j)+dQ;
        u=u+1;
    end
    v_Ap(i,c)=v_A-dt*(Qp(i,j)+Q(i,j))/2;
    if v_Ap(i,c)<0
        v_Ap(i,c)=0;
    end
    Vp(i,j)=Qp(i,j)/A;
    Hp(i,j)=Cp-Bp*Qp(i,j);
    G(i,c)=Hp(i,j);
    F(i,c)=Vp(i,j);
    v_A=v_Ap(i,c);
    for j=1:no
        H(i,j)=Hp(i,j);
        Q(i,j)=Qp(i,j);
    end
    set(plot_Hp,'Ydata',Hp);
    set(plot_Vp,'Ydata',Vp);%updates the the water depths
    drawnow;
    pause(0.001)
    frame=getframe(fig);%grab the frame
    writeVideo(movieObj,frame);%write the frame to the object
end
close(movieObj);
fig_G=figure;
t_G=1:dt_no;
plot_G=plot(t_G,G);
fig_F=figure;
plot_F=plot(t_G,F);
```