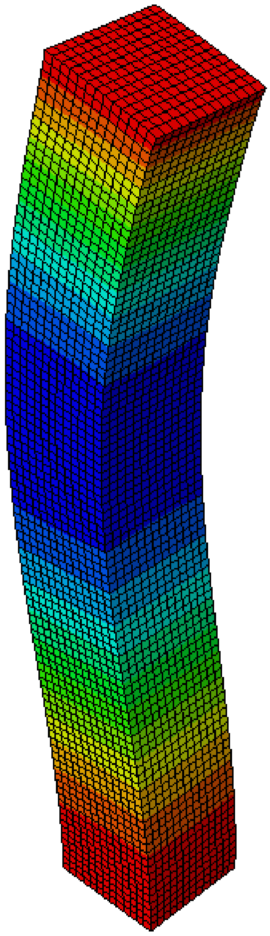
ADVANCED FINITE ELEMENT MODELLING OF REINFORCED CONCRETE



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Advanced finite element modelling of reinforced concrete

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PREFACE

This report presents the Master's thesis written by Nicolás Toro Martinez in agreement with Aalborg University concerning the Msc in structural and civil engineering. This report was written during the period from 1-02-2017 to 16-06-2017. Great gratitude is extended on behalf of the author to my supervisor, Johan Clausen.

All the equations, tables and figures are referenced. Equations are named by the chapter which they are shown, commentary text is written below figures and tables.

TABLE OF CONTENTS

1 INTRODUCTION

In this chapter, the aim of this report will be presented as well as the delimitations established.

1.1 AIM AND MOTIVATION

Nowadays, the use of reinforced concrete is considerably extended for all types of structures. Therefore, it is important to understand the behaviour of this composite material formed of concrete and steel. By doing this, calculations will be more accurate which will help to save high amount and material and thus, reduce the cost of constructions.

As an example, the Three Gorges hydropower Dam constructed in China is shown in the Figure 1 in which millions of tons were needed for its construction.

Figure 1. Three Gorges Dam, China (Wikimedia.org, 2004)

In this report, it will be analysed specifically the mechanical behaviour of concrete and steel when they are bond, i.e., when they are merged together to form reinforced concrete. To fulfil such purpose, 2 different structural elements will be assessed: doubly reinforced concrete beam subjected to bending and reinforced concrete column undergoing axial load.

Modelling of reinforced concrete is complex and the interface between the concrete matrix and steel reinforcement must be accounted for. Non-linear behaviour of reinforce concrete structures will be assessed numerically. Thereafter, the numerical results will be compared to analytical calculations leading to final results which will be presented and discussed.

1.2 OUTLINE OF THE REPORT

The use of shear reinforcement (stirrups) will out of the scope of this report and the yield criteria used will be Von Misses, Mohr Coulomb and Concrete Damage plasticity.

2 MATERIALS

In this chapter, the properties of the composite material to be used (reinforced concrete) will be stated as well as its elastic and plastic behaviour.

2.1 CONCRETE

Concrete or also known as plain concrete stating that is not mixed with any other material, is a highly worldwide used material for construction purposes. Concrete is a composite material itself since it is composed of water, cement and aggregates. Its high use in construction is due to the following advantages presented below:

- It is considered highly **economical** compared to other materials such as steel.
- Its **resistance to fire** which gives considerable safety particularly to buildings.
- For architectural purposes, concrete is nearly always the most suitable option since a wide range of **shapes** can be created.
- The **availability** of the materials to create concrete (aggregates, water and cement) is high at any location.
- Its mechanical properties are well known to withstand loads, particularly its resistance to **compression**.

As all materials, concrete has a main weakness; its low resistance to tension (usually 10% of its compressive resistance). However, this problem is well solved by the addition of steel bars to concrete to form reinforced concrete.

Concrete is considered a brittle material, i.e., it breaks when subjected to specific stress without long plastic deformation. This behaviour is shown in Figure 2

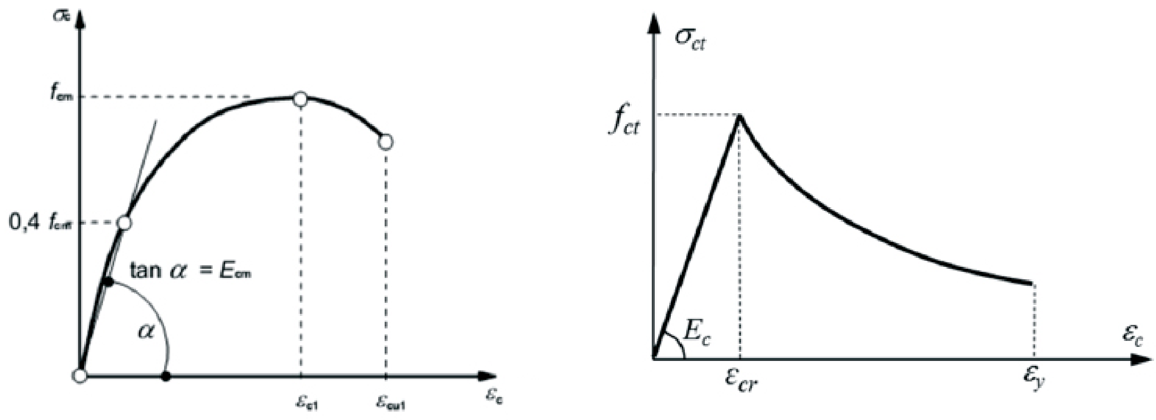

Figure 2. Stress strain curves for concrete in compression and tension (Eurocode, 2004)

Where the left and the right figure represent the compressive and tensile concrete behaviour respectively. In the compression figure, it is observed an initial slope concerning the linear elastic behaviour of concrete which can be defined in terms of the Young's modulus Ecm that ends with the value of the concrete compressive strength fck. Increasing the stress, concrete will enter into the plastic region defined with a curve and will reach the upper point corresponding to the load bearing capacity of the material. After this, concrete undergoes softening (material becomes weaker and continues deforming with less stress due to a reduction of its yield surface) and finally breaks. As for the figure representing tension, elastic region is likewise defines with the Young's modulus (straight line with slope) which ends in the concrete tensile strength fckt. Thereafter, plasticity takes place represented by a curve that ends with the breaking of the material due to tensile stresses.

2.2 STEEL

Steel is composed of the chemical elements iron and carbon. Different to concrete, steel is considered a ductile material (long plastic deformations before breaking) and its advantages are listed below:

- Equal compressive and tensile strength.
- It is a ductile material which gives long deformations before breaking.
- Small cross sectional areas can achieve high resistance against compressive and tensile stresses

Nonetheless, the main disadvantages are its low resistance against fire, corrosion and its high cost. Merging concrete and steel can solve all these drawbacks mentioned before and that is why reinforced concrete is the most suitable solution.

In Figure 3 can be seen the stress strain relationship for steel valid for compression and tension.

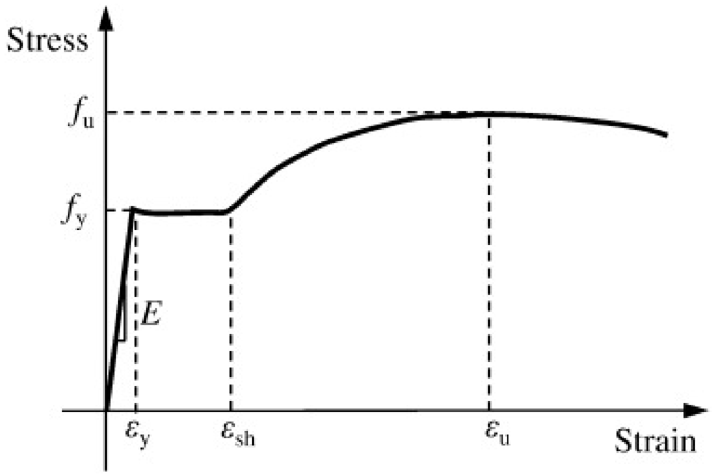

Figure 3. Stress strain curves for steel (Yun & Gardner, 2017)

Similarly to concrete, there is an initial elastic region marks with a slope defining the steel Young's modulus Es. However and contrary to concrete, steel undergoes hardening (material becomes stronger after yielding and thus, the yield surface is increased) and reaches fracture after a long development within the plastic region. The most important steel features apart from hardening is its equal compressive and tensile behaviour.

3 YIELD CRITERIA

In this chapter, the different yield criteria used for the numerical calculations will be described. These are fundamental to represent material plastic behaviour and to verify which criterion fits best with the non linear behaviour of reinforced concrete. Theory of yield criteria can be found in APPENDIX A.

4 RESULTS VERIFICATION

In this chapter, a verification of the accuracy of modelling reinforced concrete will be performed. A reinforced concrete sample will be tested analytical and numerically and the results will be compared to check their reliability.

4.1 ANALLYTICAL CALCULATIONS

A reinforced concrete sample will be tested under axial loading. The supports are assumed to be set in the neutral axis. There will be a pinned support (u_x=0, u_y=0) in one end and a roller support (u_y=0) in the other end. Its geometry and static system are shown in Figure 12 and sample data is shown in Table 1

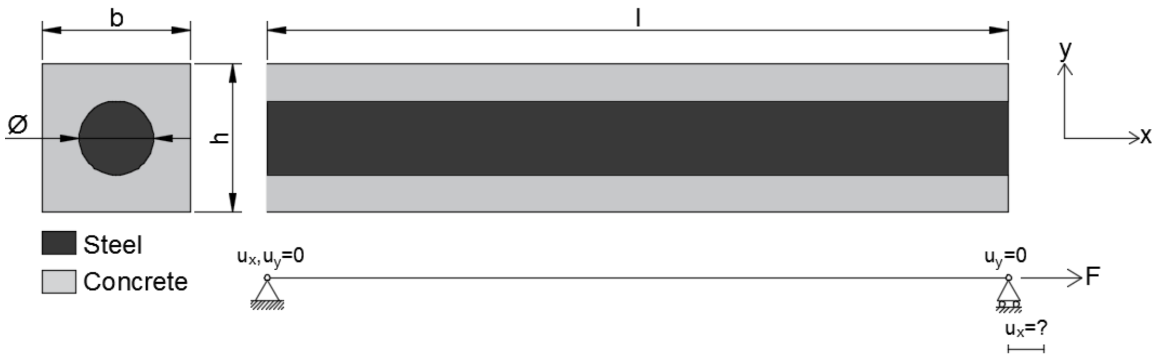

Figure 4. Sample cross section and static system

Length, l[mm]	Width, b [mm]	Height, h [mm]	Diameter, Ø [mm]	Force, F [N]
100	20	20	10	10^4

Table 1. Sample data

As it can be seen on Figure 12, an axial load will be applied to the sample and thus, it will undergo an horizontal displacement u_x. To calculate such displacement, Hook's law must be applied since the materials will be modelled only with elastic response. This is shown in Equation (4.1)

$$F = kx \tag{4.1}$$

Where F is the force, k is the stiffness and x is the displacement. In this case, the notation of x will be u_x since it will be analyzed its horizontal displacement. Hence. The resultant equation will be $F=ku_x$. It is necessary also to consider that the 2 materials of the sample are acting in parallel and not in series Both approaches are shown in Figure 5

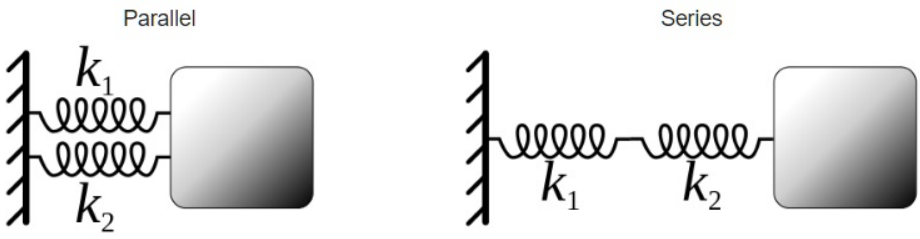

Figure 5. Parallel and series springs based on Hook's law (Wikipedia, 2017)

The approach concerning this sample is the parallel springs. The equivalent axial stiffness of the system k_{eq} can be found in Equation

$$k_{eq} = k_c + k_s \tag{4.2}$$

Where k_c is the concrete axial stiffness

$$k_c = \frac{A_c\, E_c}{l} \tag{4.3}$$

Being A_c the concrete cross sectional area, E_c the concrete Young's modulus and l the length of the sample. And the steel axial stiffness is defined as

$$k_s = \frac{A_s\, E_s}{l} \tag{4.4}$$

Being A_s the steel cross sectional area and E_s the steel Young's modulus.

In Table 2 is presented all the material parameters to take into account for this calculation

Ac [mm2]	As [mm2]	Ec [MPa]	Es [MPa]
321.46	78.54	31500	2.1×10^5

Table 2. Sample material parameters

When using ABAQUS, three types of outcomes are expected to arise when using the option "embedded region". Such outcomes can be calculated analytically and are presented below :

- (1). Concrete and steel are merged so that it is obtained what is represented in Figure 4 as a result.
- (2). The host material (steel) is included without removing the area of the concrete. Therefore, the section will consist of the whole square section filled with concrete plus the circular section of steel.
- (3). Steel is not merged correctly and thus, there will be only concrete in the square section.

Results are presented in Table 3

u_x (1) [mm]	u_x (2) [mm]	u_x (3) [mm]
0.03756	0.03437	0.07936

Table 3. Horizontal displacement for the 3 outcomes calculated analytically

As expected $u_x(3)>u_x(1)>u_x(2)$ since steel is a stiffer material than concrete with a higher Young's modulus E.

4.2 NUMERICAL CALCULATIONS

Calculations in ABAQUS concerning reinforced concrete can be done in different ways. Concrete is normally modelled as a "solid" material whereas steel can be modelled as a solid, wire truss or wire beam:

- (4). Solid: It is necessary to give geometrical dimensions of the cross sectional area as well its material properties.
- (5). Wire truss: It is modelled as a wire (line without area), and thereafter it is assigned a truss section accounting for its cross sectional area and material properties.
- (6). Wire beam: It is modelled as a wire, and thereafter it is assigned a beam section accounting for its section Poisson's ratio and its material properties.

Next, it is set an embedded constraint being steel the embedded material and concrete the host material. In Figure 6 and Figure 7 it can be seen the sample modelled with concrete and steel as a solid

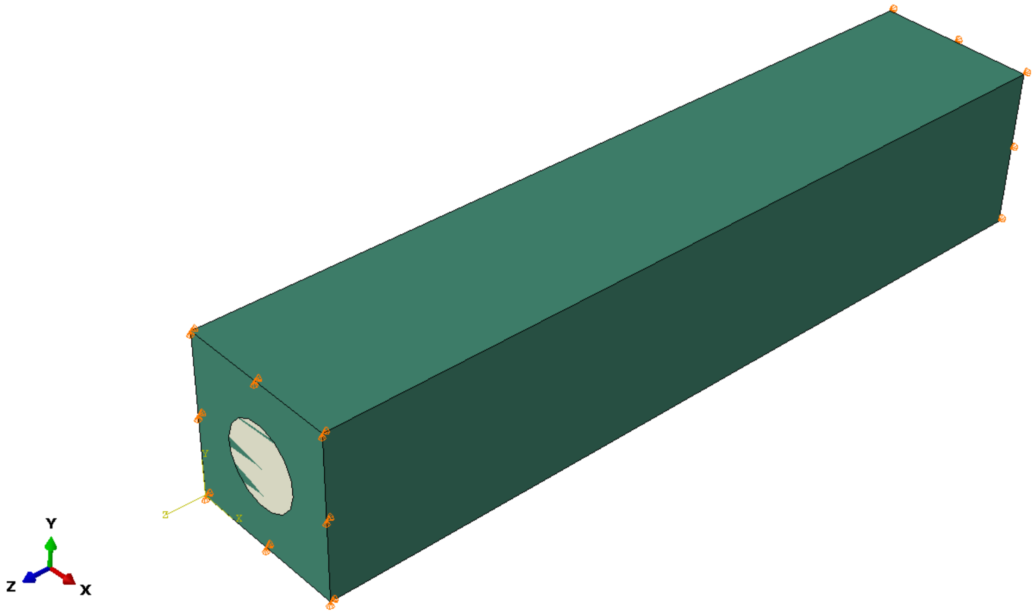

Figure 6. ABAQUS solid reinforced concrete sample isometric view

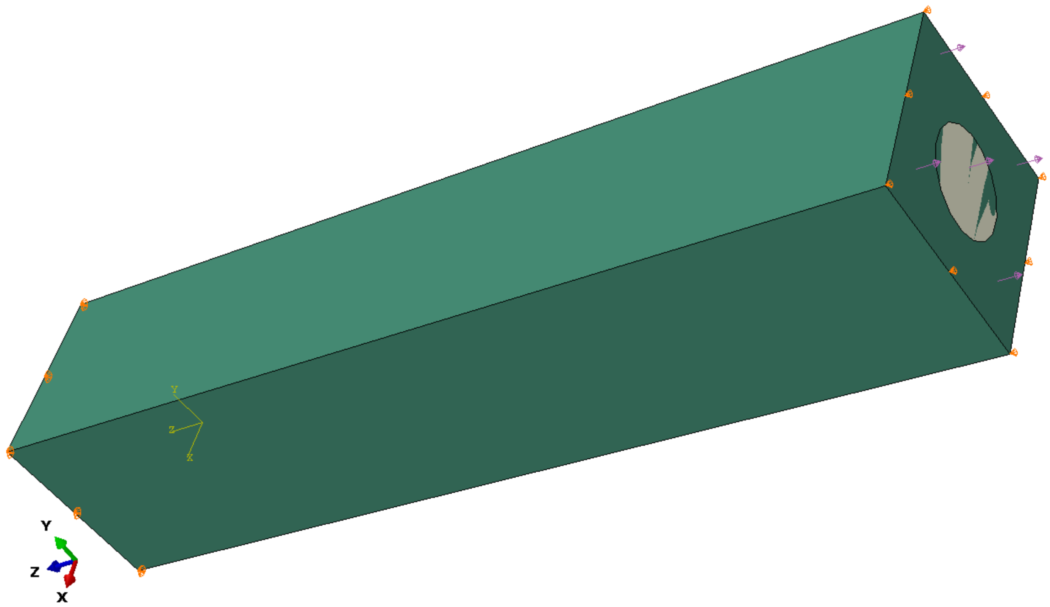

Figure 7. ABAQUS solid reinforced concrete sample rotated view

And in Figure 8 and Figure 9 it is shown the sample modelled with concrete as a solid and steel as a wire truss or beam

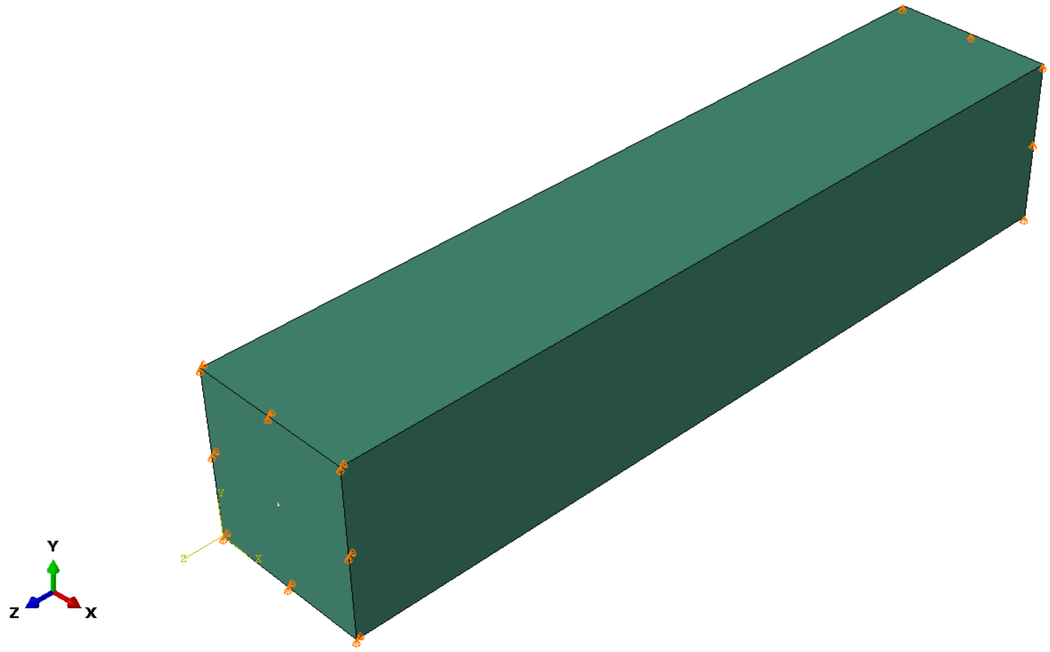

Figure 8. ABAQUS wire truss/beam reinforced concrete sample isometric view

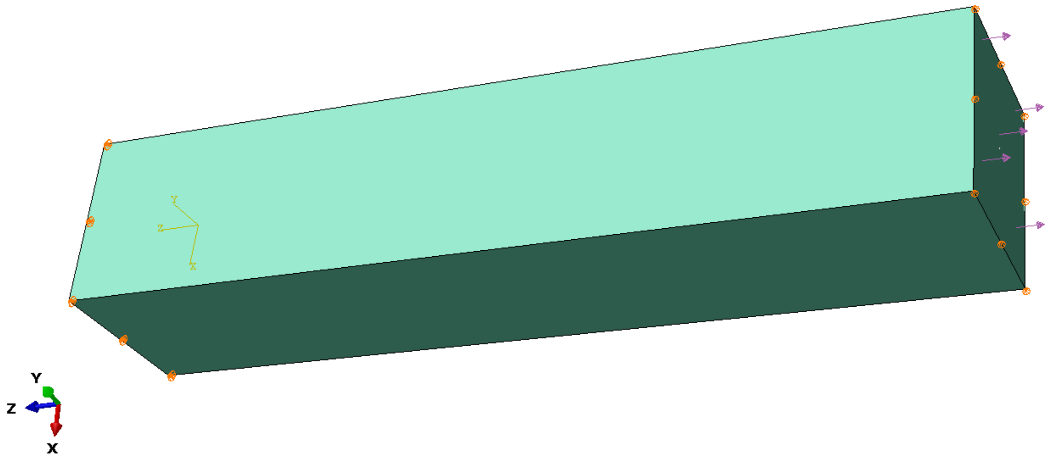

Figure 9. ABAQUS wire truss/beam reinforced concrete sample rotated view

To define the boundary conditions, it is important to account for the principal axis x,y and z. The horizontal displacement u_x calculated analytically will correspond to u_z in numerical calculations since it is seen in Figure 6 that the axis in ABAQUS in z and not x. Therefore, and in order to avoid confusion, $u_x=u_z$ when presenting numerical results.

Since in ABAQUS it is used a 3d model, the axial force used analytically (2d) $f=10^4$ has to be divided by the cross sectional area of the sample (A_{sample}=20x20=400mm2) and consequently, it is obtained the pressure $P=f/A_{sample}=10^4N/400mm^2$=25N/mm2.

The input material parameters introduced in ABAQUS are presented in Table 4 as well as the load.

ν_c [-]	ν_s [-]	Ec [MPa]	Es [MPa]	Pressure, P [MPa]
0.18	0.30	31500	2.1×10^5	25

Table 4. ABAQUS Input parameters for concrete and steel

Where ν_c is the concrete Poisson's ratio and ν_s is the steel Poisson's ratio.

The numerical results are presented in Table 5

u_x (4) [mm]	u_x (5) [mm]	u_x (6) [mm]
0.03905	0.04303	0.04697

Table 5. Horizontal displacement for the 3 outcomes calculated numerically

4.3 COMPARISON

Results obtained analytically and numerically are compared and shown in Table 6 and in Figure 10

u_x (1) [mm]	u_x (2) [mm]	u_x (3) [mm]	u_x (4) [mm]	u_x (5) [mm]	u_x (6) [mm]
0.03756	0.03437	0.07936	0.03905	0.04303	0.04697

Table 6. Displacements for analytical and numerical calculations

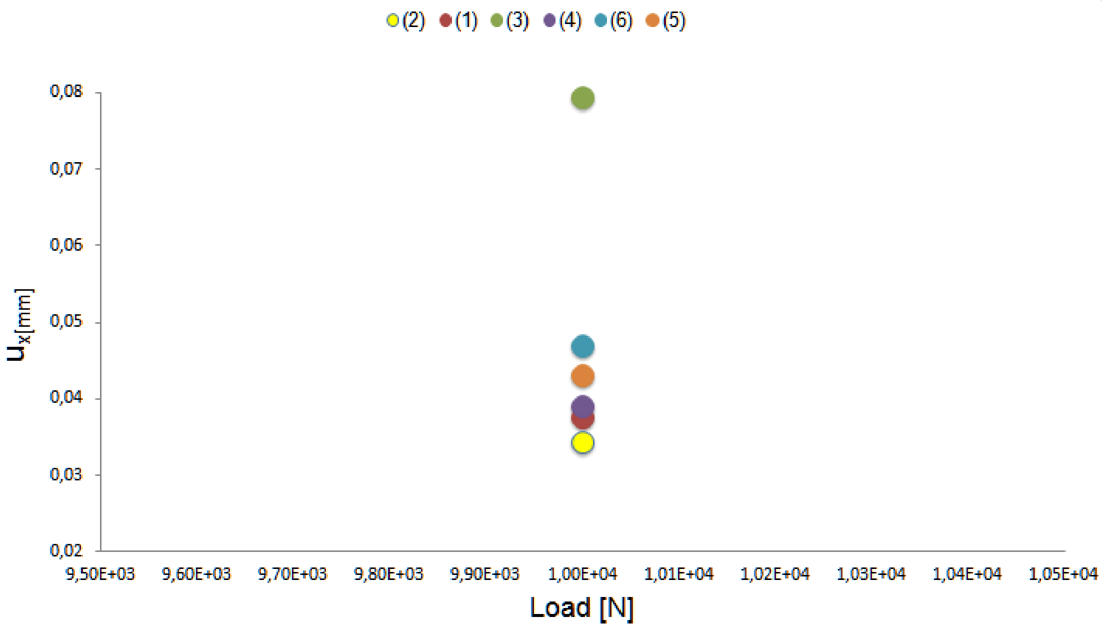

Figure 10. Displacement for analytical and numerical calculations

It can be concluded that, modelling concrete and steel as a solid is the best option (4) since its result is the closest to the real analytical calculation (1). Therefore, for the posterior numerical calculations it will be used the option of modelling steel and concrete as a solid.

PART I

DOUBLY REINFORCED CONCRETE BEAM

1 INTRODUCTION

In this part, a non-linear analysis of a plain and a doubly reinforced concrete beam will performed. Analytical and numerical solutions will be used and conclusions will be drawn.

2 PLAIN CONCRETE BEAM

In this chapter, a plain concrete beam will be tested with analytical and numerical solutions. The beam will be modelled as a elasto-plastic material with the yield criterion von Misses and its non-linear behaviour will be analyzed.

2.1 ANALLYTICAL CALCULATIONS

A standard concrete beam whose characteristics fulfil Eurocode requirements (Eurocode, 2004) subjected to uniform load has been chosen in terms of geometry and elastic and plastic material parameters as shown in Figure 11 and Table 7

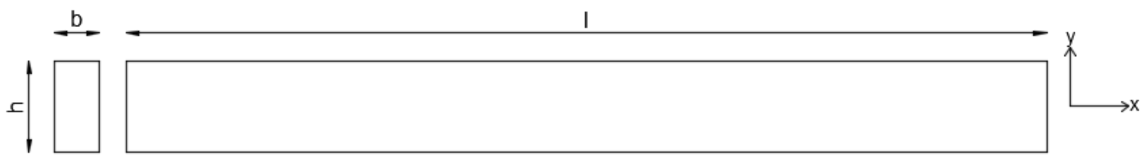

Figure 11. Chosen plain concrete beam geometry

b [mm]	h [mm]	l [mm]	Ec [MPa]	ν [-]	σ_y [MPa]	ε_{pl} [-]
250	500	10^4	31500	0.18	25	0

Table 7. Beam properties

Where σ_y is the C25 concrete yield stress and ε_{pl} is the plastic strain. Non linear analysis will be performed by relating the displacement in the middle of the simply supported beam δ_{mid} in function of an uniform load applied q. The static system is presented in Figure 12

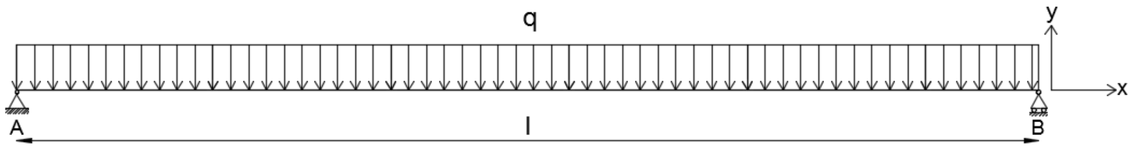

Figure 12. Plain concrete beam static system

In order to calculate δ_{mid}, the Euler-Bernoulli beam theory has to be used. To do so, it is necessary first to establish the equilibrium of forces of the beam. It is made a cut at any point of the beam and it is represented the left side of the beam where is placed the pinned support as shown in Figure 13

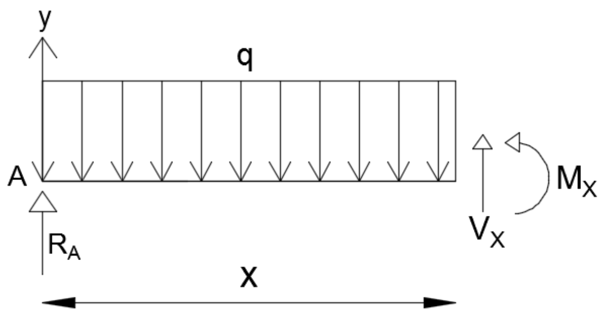

Figure 13. Beam equilibrium

Where R_A is the vertical reaction at point A, X is the distance from the point A to where the cut was made, q the vertical uniform load and M_x is bending moment distribution of the beam. It is taken from where the cut has been made and M_x is obtained

$$M_x = R_A x - \frac{qx^2}{2} \tag{2.1}$$

Based on Euler-Bernoulli beam theory (Wikipedia, 2017), the moment distribution is stated as

$$M_z(x) = -EI\frac{d^2\delta}{dx^2} \tag{2.2}$$

Where I is the second moment of inertia that for a rectangular section is

$$I = \frac{b\,h^3}{12} \tag{2.3}$$

EI is the bending stiffness or flexural rigidity, d the derivative and δ is the deflection of the beam.

Next, Equations (2.1) and (2.2) are connected, integrated 2 times to account for derivative of order two d^2, and finally Equation (2.4) it is obtained

$$EI\delta = \frac{qx^3}{24}(2l - x) + C_1 x + C_2 \tag{2.4}$$

Where C1 and C2 are the integral constants. These can be obtained by setting two boundary conditions:

- 1. Vertical displacement is δ=0 for x=l
- 2. Vertical displacement is δ=0 for x=0

Applying the correspondent boundary conditions and replacing C_1 and C_2 in Equation (2.4) , Equation (2.5) is obtained

$$\delta_{mid} = -\frac{5ql^4}{384EI} \tag{2.5}$$

The next step will be to calculate the uniform load q that brings yielding to the beam, i.e., the load that will cause the beam to pass from the elastic to the plastic regime. To do so, the stress distribution for the elastic regime of the beam due to bending is shown in

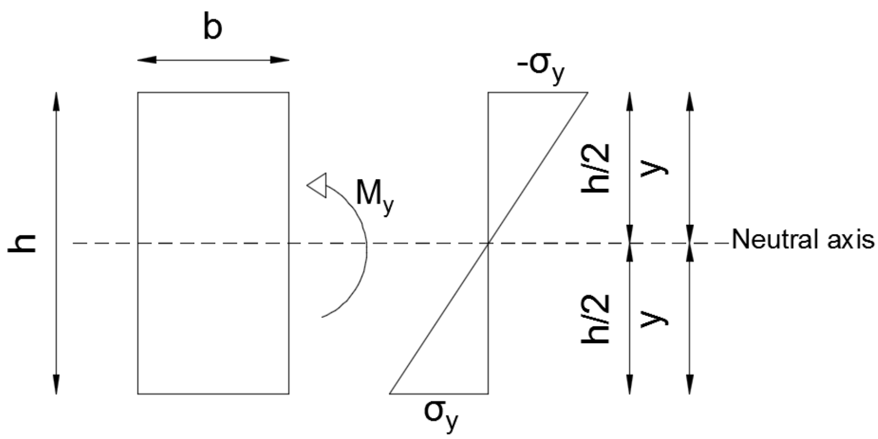

Figure 14. Elastic stress distribution due to yield bending moment

Where M_y is the yielding bending moment, $-\sigma_y$ is the yielding compression stress and σ_y is the yielding tensile stress. In this case σ_y=- σ_y since von Mises criterion is used which states that compressive stresses are equal to tensile stresses. The maximum stress is defined by Navier's formula as shown in Equation (2.6)

$$\sigma_y = \frac{M_y\, y}{I} \tag{2.6}$$

Rearranging the Equation (2.6), M_y can be obtained

$$M_y = \frac{\sigma_y I}{y} \tag{2.7}$$

And the maximum bending moment in a simply supported beam subjected to uniform load is equal to

$$M_{max} = \frac{ql^2}{8} \tag{2.8}$$

Equalizing Equations (2.7) and (2.8), $q_{yielding}$ can be obtained

$$q_y = \frac{8\,\sigma_y I}{y\,l^2} \tag{2.9}$$

Results concerning the elastic behaviour of the plain concrete beam are shown in Table 8

I [mm^4]	M_y [Nmm]	q_y [N/mm]	q_y ABAQUS [N/mm^2]	δ_{mid} [mm]
$2.6x10^9$	$2.6x10^8$	20.83	0.083	33

Table 8. Plain concrete beam elastic regime results

In order to calculate the load bearing capacity of the beam q_p, it has to be accounted for a fully stress plastic distribution due to bending as shown in Figure 15

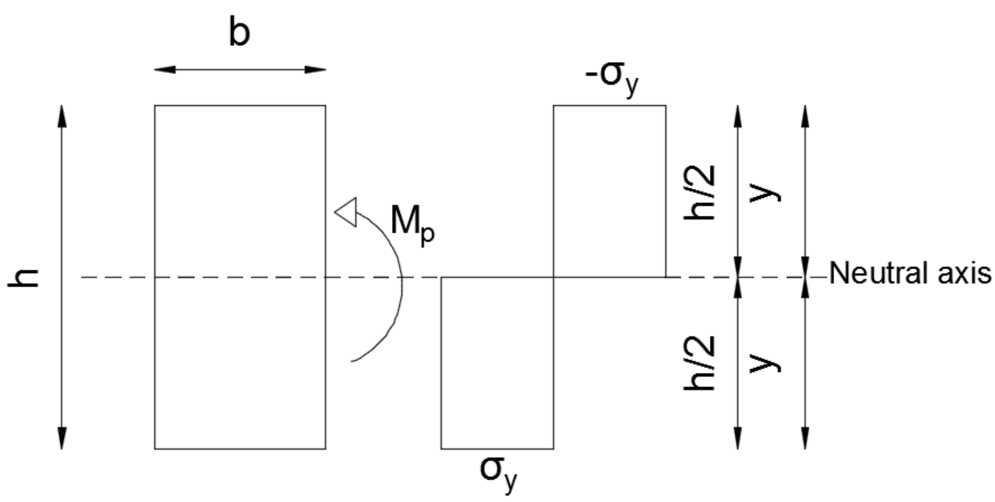

Figure 15. Elastic stress distribution due to plastic bending moment

Where M_p is the plastic bending moment as is given as

$$M_p = \sigma_y \left(\frac{b\,h^2}{4}\right) \tag{2.10}$$

Finally, Equation (2.9) (2.8) is rearranged and it is obtained

$$q_p = \frac{8\,M_P}{l^2} \tag{2.11}$$

Results are presented in

M_p [Nmm]	q_p [N/mm]	qp ABAQUS [N/mm^2]
$3.9\text{x}10^8$	31.25	0.125

Table 9. Plain concrete beam plastic regime results

2.2 NUMERICAL CALCULATIONS

Numerical solutions were performed in the software ABAQUS. Contrary to analytical calculations, the plain concrete beam was modelled in 3d as a solid body. Von Mises criteria was used to define the plasticity characteristics shown in Table 7.

Boundary conditions are similar to the analytical solution. In one end is restricted the horizontal and vertical displacement (u_x=u_y=0) and in the other end it is restricted the vertical displacement (u_y=0). Such boundary conditions are applied at the same height of the neutral axis (h/2). Again, it is important to remember that the beam is modelled along the axis z and not the axis x as in the analytical static system. The plain concrete beam modelled can be visualized in Figure 16 and Figure 17

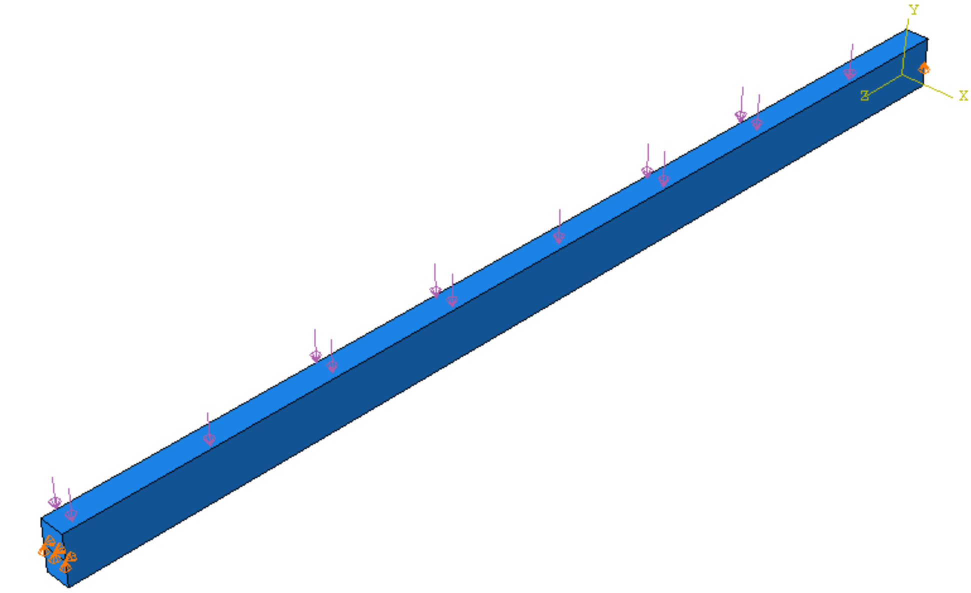

Figure 16. Plain concrete beam isometric view

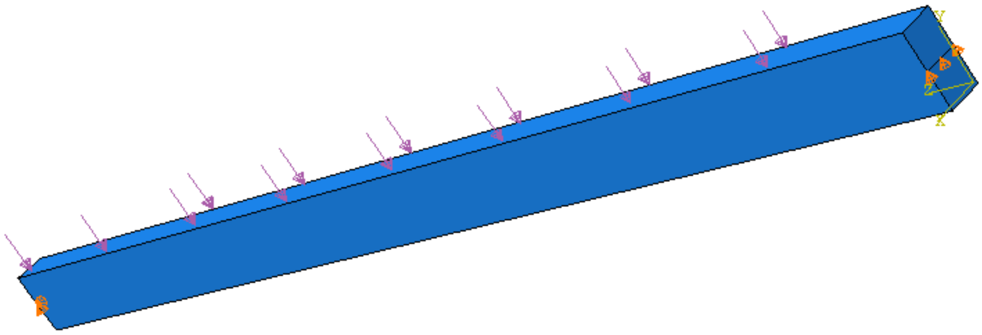

Figure 17. Plain concrete beam rotated view

2.3 COMPARISON

To be able to compared analytical and numerical calculations, analytical results obtained in Table 8 and Table 9 have to be divided by the width of the beam b since analytical is a 2d solution and numerical was done in 3d.

Both approaches are presented and can be compared in Figure 18

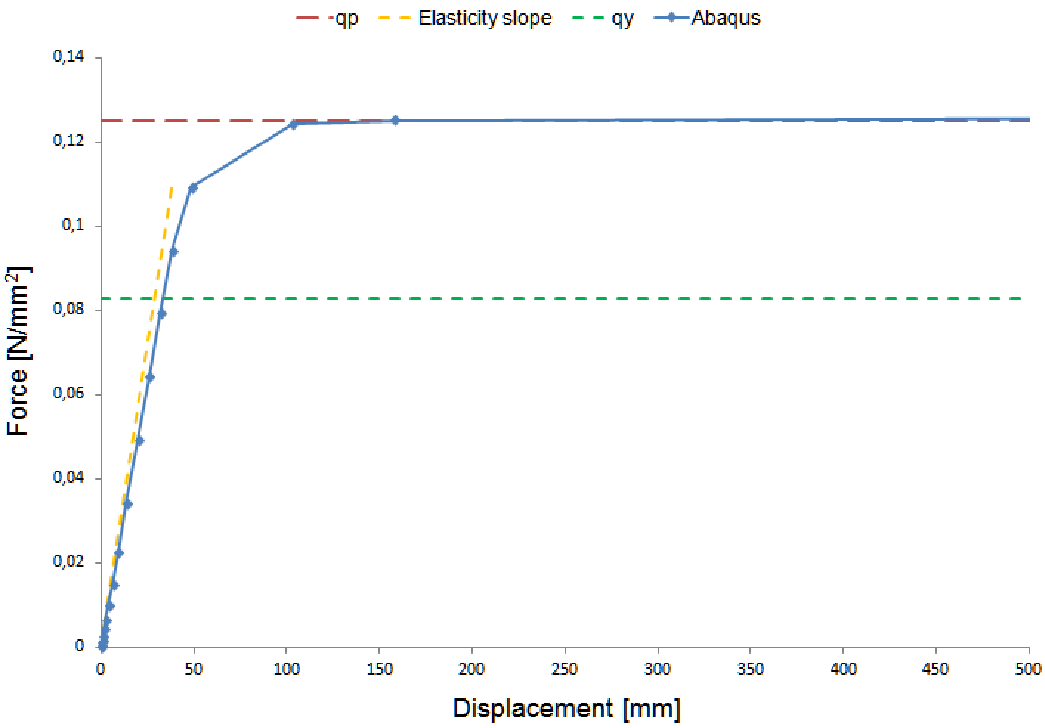

Figure 18. Plain concrete beam comparison results

It can be seen that q_p matches perfectly the numerical solution. q_y is also very accurate although it is hard to know exactly when the elastic part ends and the plastic starts. However, it has been also plotted the analytical elastic slope and it is observed that is nearly similar to the numerical slope. It can be concluded that when modelling plain concrete with von Mises criterion, analytical and numerical calculations are practically similar.

3 DOUBLY REINFORCED CONCRETE BEAM

In this chapter, the presence of reinforcement will be added to obtain a doubly reinforced concrete beam. It will be doubly because it will consist of reinforcement in the compressive and tensile side. The beam will be tested under analytical and numerical calculations and results will be compared. The beam will be modelled with different yield criteria, namely, Von Mises for steel reinforcement and for concrete three different criteria will be used and compared: Von Mises (VM), Mohr coulomb (MC) and Concrete damage plasticity (CDP).

3.1 ANALLYTICAL CALCULATIONS

A standard doubly reinforced concrete beam whose characteristics fulfil Eurocode requirements (Eurocode, 2004) subjected to uniform load has been chosen in terms of geometry and elastic material parameters as shown in Figure 19 and Table 10

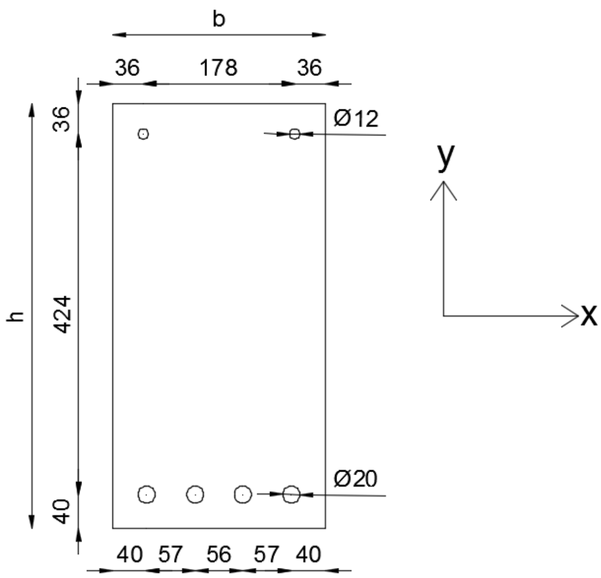

Figure 19. Chosen doubly reinforced concrete beam geometry

b [mm]	h [mm]	l [mm]	ν (steel) [-]	Es [MPa]	Ec [MPa]	ν (concrete) [-]
250	500	10^4	0.30	$2.1x10^5$	31500	0.18

Table 10. Parameters for doubly reinforced concrete beam

The static system considered is similar to the one presented in Figure 12 for the plain concrete beam being simply supported and subjected to an uniform load q.

For SLS calculations, Alternate Design Method (ASD) will be used based on (Jensen, 2011). This is a method based on elastic theory and thus, it is a suitable for linear-elastic calculations.

To apply this method, steel is replaced by an equivalent concrete area determined by their Young's modulus (E_s and E_c) that corresponds to the modular ratio coefficient α

$$\alpha = \frac{E_s}{E_c} \tag{3.1}$$

This can be seen in Figure 20

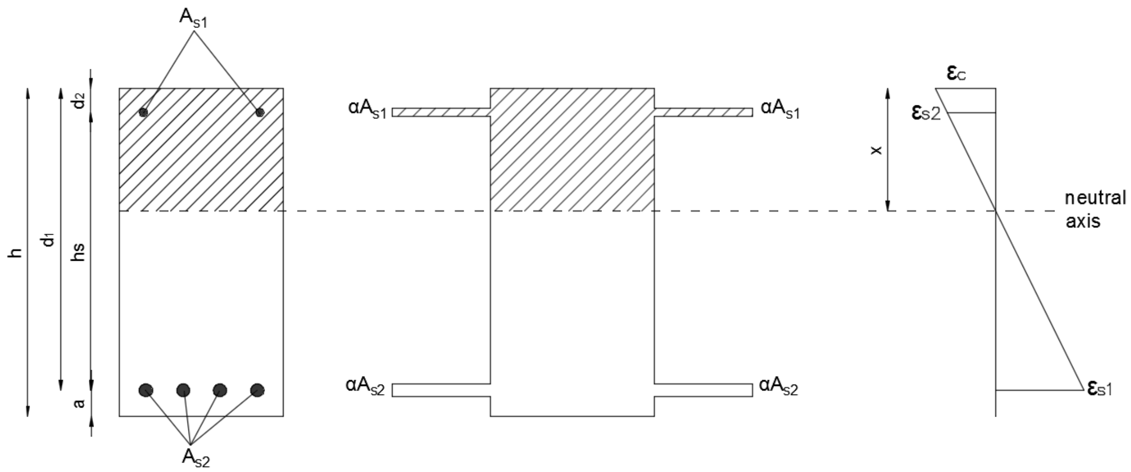

Figure 20. ASD method representation

Where ε_{s1} is the strain of the bars in the tensile zone, ε_{s2} is the strain of the bars in the compressive zone, ε_c is the strain of the concrete and x is the distance from the upper edge to the neutral axis calculated with Equation (3.2). This distance will correspond to the beam height in compression. This is calculated by taking moments of area around the neutral axis.

$$\frac{bx^2}{2} + (\alpha - 1)A_{s2}(x - d_2) - \alpha A_{s1}(d_1 - x) = 0 \tag{3.2}$$

Where A_{s1} and A_{s2} are the compressive and tensile reinforcement area respectively.

From an economical point of view, it is important to check that the compressive reinforcement reaches yielding. If they do not reach yielding, that means the beam is over reinforced. This check is also important to calculate the resisting moment M_r. To do so, three different options can be present depending on the value of x where x_{min} can be defined by taking proportions of the 2 triangles corresponding to the strains in the beam shown in Figure 20

$$\frac{x_{min}}{d_2 - x_{min}} = \frac{\varepsilon_c}{\varepsilon_{s1}} \tag{3.3}$$

Where x_{min} can be simplified and it is obtained

$$x_{min} = d_2 \frac{\varepsilon_c}{\varepsilon_c - \varepsilon_{s1}} \tag{3.4}$$

And x_b is defined

$$x_b = \frac{d_1 \varepsilon_{cu}}{\varepsilon_{cu} - \varepsilon_y} \tag{3.5}$$

Where ε_{cu} and ε_y are the ultimate state strain for concrete and yielding strain for steel respectively and they are given in (Eurocode, 2004).

Finally, Mr is calculated following the criteria of steel yielding before failure in concrete. This is an iterative procedure where Mr will be either the minimum value of M_{rc} and M_{rst}

$$\begin{cases} M_{rc} = \dfrac{bx\sigma_c^{perm}}{2} z + (\alpha - 1)A_{s2}\sigma_c^{perm} h_s \quad for \ \ x_{min} \le x \le xb \\ M_{rst} = A_{s1}\sigma_s^{perm} h_s \end{cases} \tag{3.6}$$

Or the value of M_r will be equal to M_{rst} when $x < x_{min}$

Where M_{rc} is the resisting moment for the concrete area, M_{rst} is the resisting moment for the steel area in tension, σ_c^{perm} is the permissible stress for concrete equal to $0.45f_{ck}$ and σ_s^{perm} is the permissible stress for steel equal to 140 MPa when using a steel class of S350 or lower (in this project it was used a steel class of S345). These values were obtained from standard (ACI318M, 1995)

SLS results are presented in Table 11

M_{rc} [Nmm]	q_{mrc} [N/mm]	δmrc [mm]	εs1 [-]	εs2 [-]	εc [-]	x [mm]
69508917.58	5.56	7.75	0.014	0.0017	0.0035	122.23

Table 11. SLS Results ASD method doubly reinforced concrete beam

Another more recent method used for SLS calculation will be performed. It will be calculated the deflection of the beam in the middle using Equation (2.5). In this case the bending stiffness EI will be different; the cross section is not symmetric and 2 different materials are merged (concrete and steel) to form a composite material. Therefore, it is necessary account for the 2 materials when taking into account its resistance against bending. Neutral axis will not be in the middle since the reinforcement on the top is different from the reinforcement placed on the bottom.

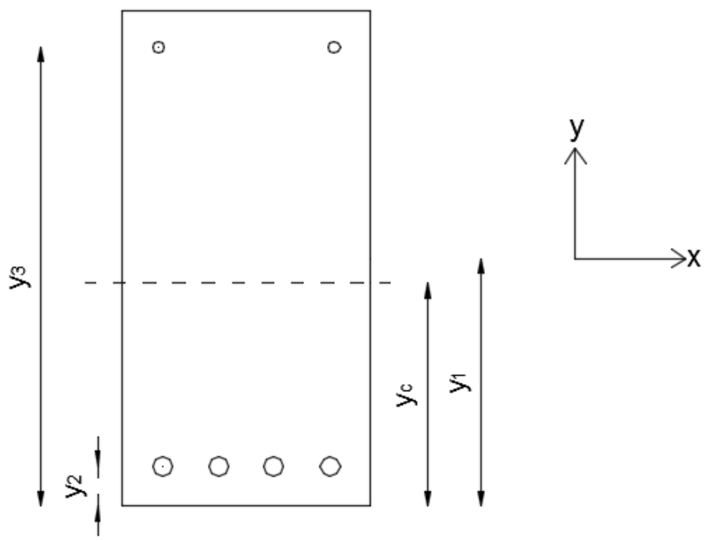

Figure 21. Doubly reinforced concrete beam geometry

Where y1 is the distance from the bottom to the center of the beam, y2 the distance from the bottom of the beam till the center of the tensile reinforcement, y3 the distance from the bottom of the beam till the center of the compressive reinforcement and yc is the distance from the bottom of the beam till the neutral axis.

The neutral axis position yc can be calculated as follows

$$y_c = \frac{A_c\, y_1\, E_c - (A_{s1}\, y_2 + A_{s2}\, y_3)\,(E_c - E_s)}{A_c\, E_c - (A_{s1} + A_{s2})\,(E_c - E_s)} \tag{3.7}$$

Where A_c is the concrete area including the holes and E_c and E_s are the concrete and steel Young's Modulus respectively.

Next it can be obtained the second moment of inertia of the entire section Ics

$$I_{cs} = I_c + A_c(y_1 - y_c)^2 - (n_{h1}(I_{h1} + A_{h1}(y_2 - y_c)^2) + n_{h2}\,(I_{h2} + A_{h2}\,(y_3 - y_c)^2) \tag{3.8}$$

Where nh1 and nh2 are the number of holes corresponding to compressive and tensile reinforcement respectively, A_{h1} and Ah_2 is the singular hole area for compressive and tensile reinforcement respectively, Ih1 and Ih2 are the second moment of inertia of compressive and tensile reinforcement respectively where Ih is equal to

$$I_h = \frac{\pi\left(\frac{\emptyset}{2}\right)^4}{4} \tag{3.9}$$

Finally, the bending stiffness EI is obtained

$$EI = E_c\, I_{cs} + \ n_{h1}E_s(I_{h1} + A_{h1}(y_2 - y_c)^2) + n_{h2}E_s(I_{h2} + A_{h2}(y_3 - y_c)^2) \tag{3.10}$$

Where ∅ is the diameter of the steel bar. The SLS results are presented in

M_y [Nmm]	q_y [N/mm]	δy [mm]
69508917.58	2.08	2.83

Table 12. SLS results

For ULS calculations, it will be calculated the maximum capacity of the beam subjected to bending which corresponds to the maximum bending moment Mp. To do so, it is necessary to account for the stress-strain distribution on the doubly reinforced concrete beam shown in Figure 22

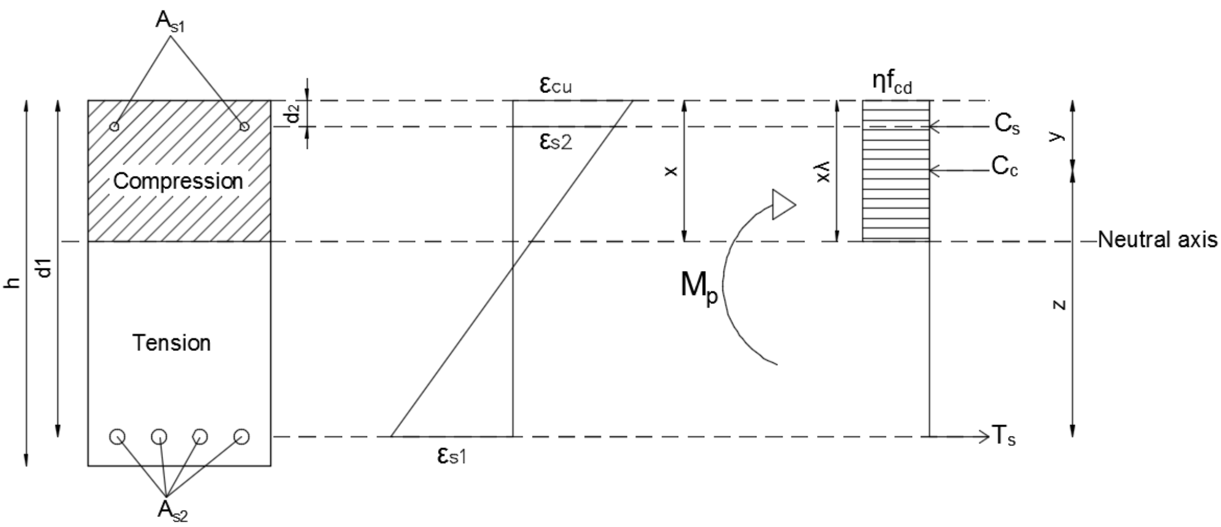

Figure 22. Stress-strain distribution doubly reinforced concrete beam

Where λ is a factor defining the height of the compression zone, z is the forces arm, y is the distance to the resultant compression force Cc, Cs is the resultant force of the reinforcement in compression, Ts is the resultant of the reinforcement in tension, η is a factor defining the effective strength of concrete and Mp is the maximum moment in bending.

If horizontal equilibrium is established it is obtained

$$C_c + C_s = T_s \tag{3.11}$$

Where

$$C_c = f_{ck}\, b\, \lambda x \tag{3.12}$$

$$T_s = f_{s1}\, A_{s1} \tag{3.13}$$

$$C_s = f_{s2}\, A_{s2} \tag{3.14}$$

Being fs1 equal to the steel yielding stress fy=345MPa and fs2 is found by Hook's law

$$f_{s2} = E_s\, \varepsilon_{s2} \tag{3.15}$$

Introducing equations 3.12, 3.13 and 3.14 into the equation 3.11 it is obtained the distance to the neutral axis which divides compression from tension zone

$$x = \frac{A_{s1}\, f_y - A_{s2}\, f_{s2}}{f_{cd} \lambda\, b} \tag{3.16}$$

Finally, it can be determined Mp by taking equilibrium of moments

$$M_p = C_s\,(y - d_2) + T_s\, z \tag{3.17}$$

The results are presented in Table 13

M_p [Nmm]	q_p [N/mm]	δp [mm]
260416666.7	14.92	20.79

Table 13. ULS results Doubly reinforced concrete beam

3.2 NUMERICAL CALCULATIONS

Numerical solutions were performed in the software ABAQUS. Contrary to analytical calculations, the doubly reinforced concrete beam was modelled in 3d as a solid body. Von Mises, Mohr Coulomb, Concrete damage plasticity criteria were used to define the plasticity of concrete whereas Von Misses criterion was used to model steel. The parameters used to define Von Misses for concrete and steel are

σ_y [MPa]	ε_{pl} [-]
25	0

Table 14. Von Misses parameters for concrete

σ_y [MPa]	ε_{pl} [-]
345	0

Table 15. Von Misses parameters for steel

Parameters defining Mohr Coulomb will be chosen according to the recommended values stated in (Abaqus, 2016)

σ_c [MPa]	Φ [º]	k [-]	Ψ [º]	c [-]
25	37	4.02	31/37	6.22

Table 16. Mohr Coulomb parameters concrete

Parameters defining Concrete damage plasticity will be chosen according to the recommended values stated in (Abaqus, 2016)

σ_{t0} [MPa]	$\dot{\varepsilon}_t^{pl}$ [-]	Ψ [º]	e [mm]	$\frac{\sigma_{b0}}{\sigma_{c0}}$ [-]	K_c [-]	μ [-]
2.5	0	31	0.1	1.16	0.667	0

Table 17. Concrete damage plasticity parameters concrete

Hardening parameters will be also used for CDP. There will be used only the hardening compressive parameters since hardening in tensile regime was found unstable and inaccurate.

σ_c [MPa]	$\dot{\varepsilon}_t^{pl}$ [-]
0	0
12	0
13	$2x10^{-5}$
14	3x10-5
15	6x10-5
16	7.5x10-5
17	0.00011
18	0.000145
19	0.00019
20	0.000245
21	0.00032
22	0.000415
23	0.00055
24	0.0008
24.35	0.000983
24.35	0.001083
24	0.0013
23	0.0017
22	0.001975
21	0.00221
20	0.00243

Table 18. Compressive hardening parameters

Boundary conditions are similar to the plain concrete beam except for the boundary conditions concerning the symmetry; that is, it will be modelled a quarter of the beam since it is symmetric in plain axis (axis x and y). Doubly reinforced concrete beam can be seen in Figure 23 and Figure 24

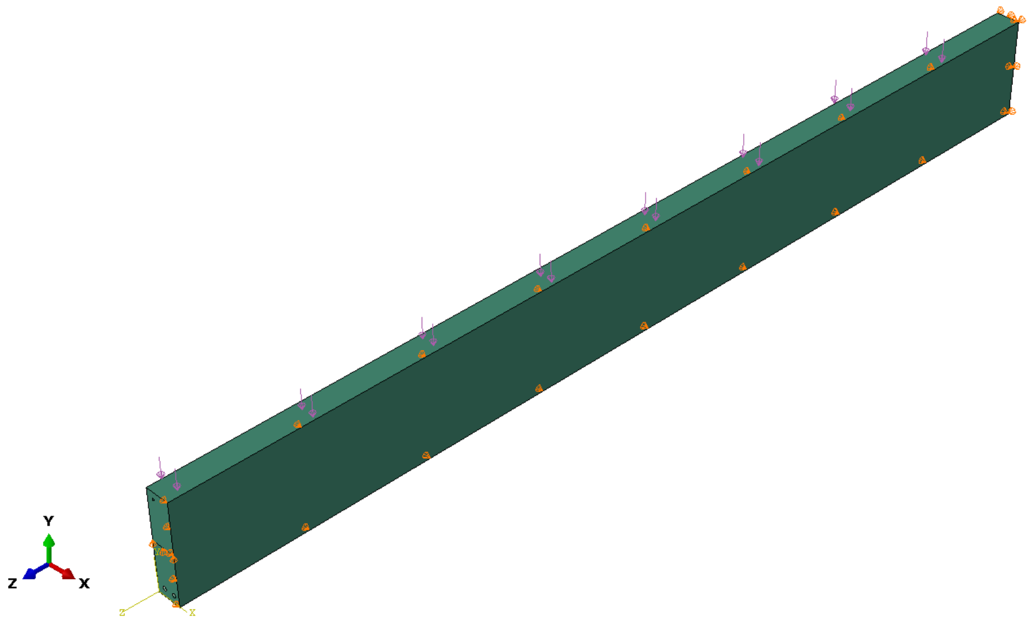

Figure 23. Doubly reinforced concrete beam isometric view

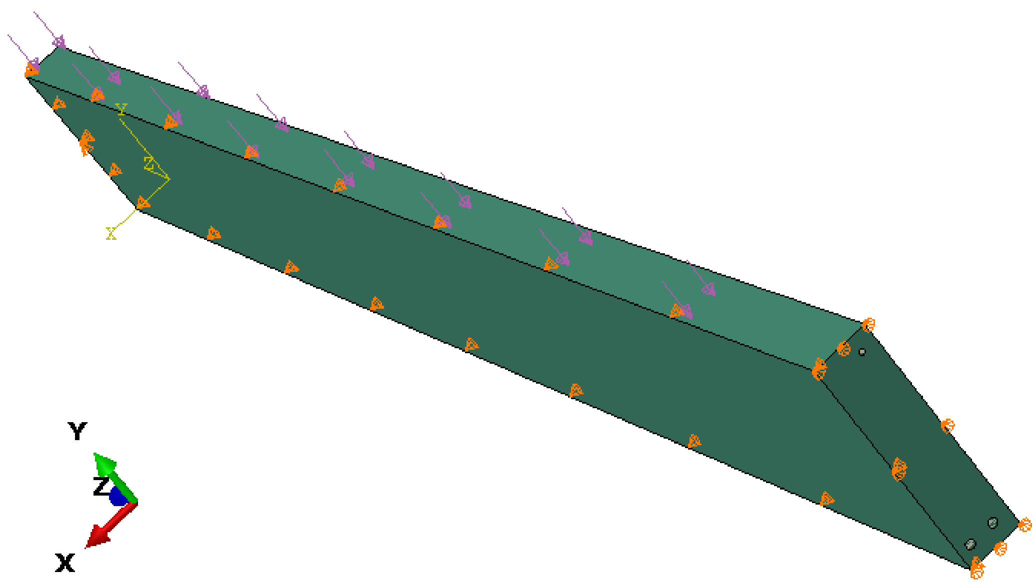

Figure 24. Doubly reinforced concrete bam rotated view

3.3 COMPARISON

When comparing analytical and numerical results, it is important to take into account that the load using in analytical calculations has been divided by the width of the beam in order to compare results properly. This is because analytical calculations are performed in a 2d beam and in numerical is 3d. The results are presented in the next charts:

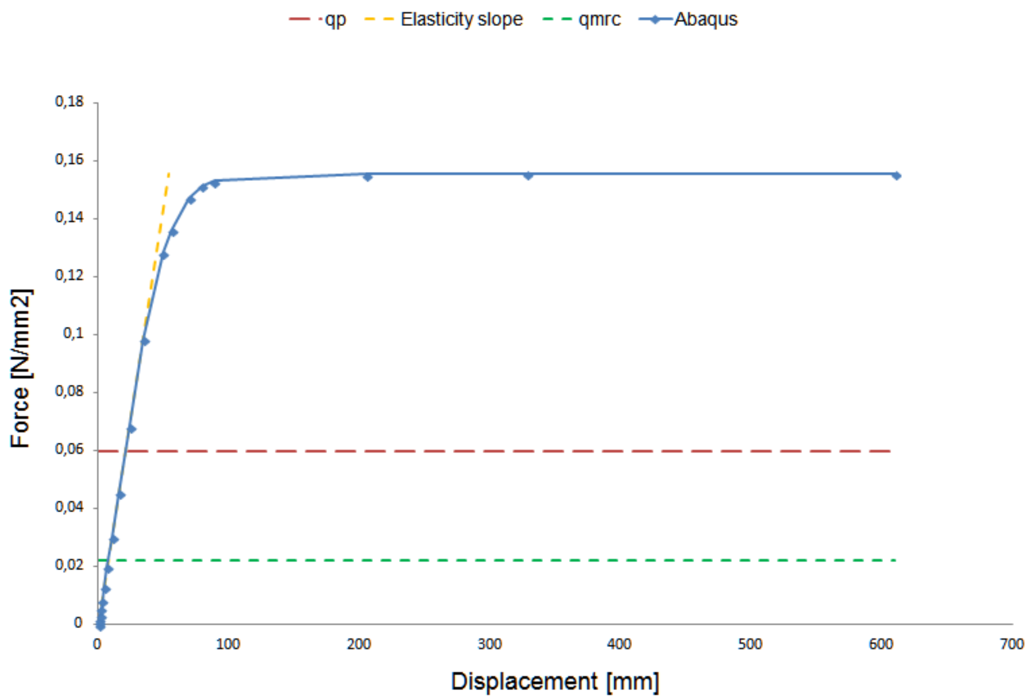

Figure 25. Von Misses concrete modelling

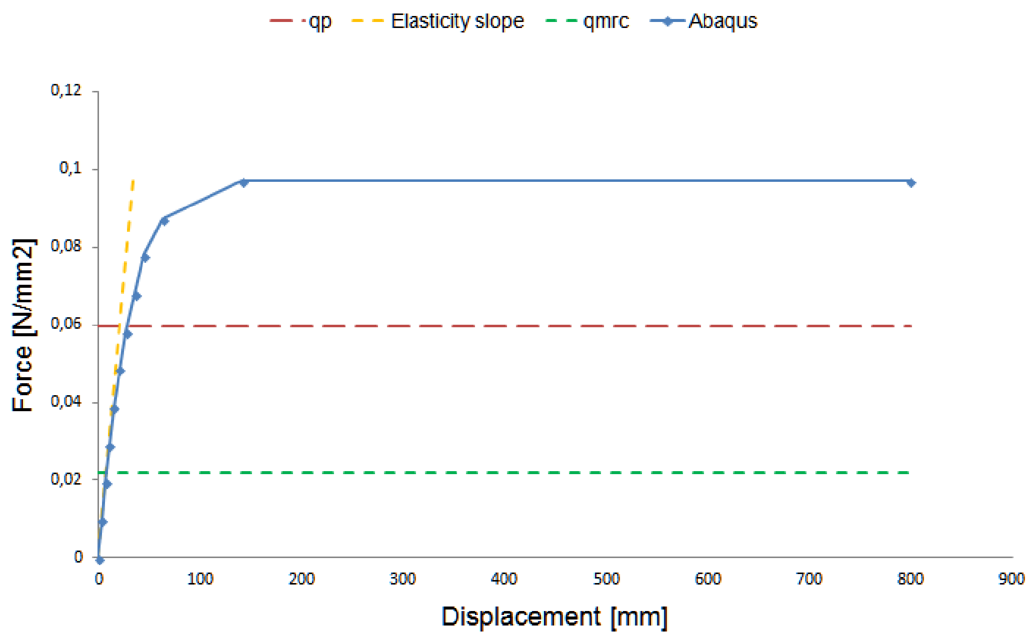

Figure 26. Abaqus Mohr Coulomb modelling

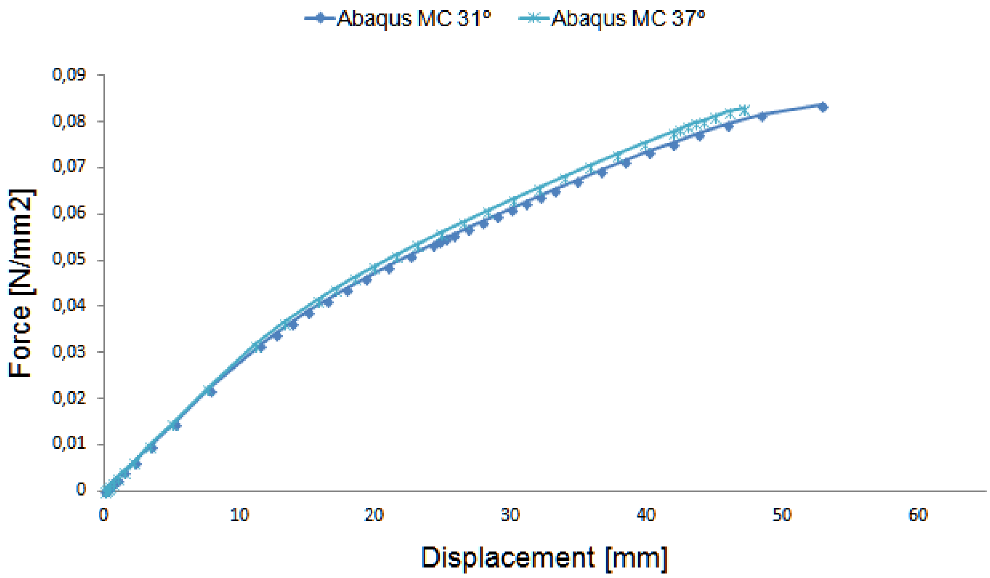

Figure 27. MC modelling dilation angle 37º and 31º

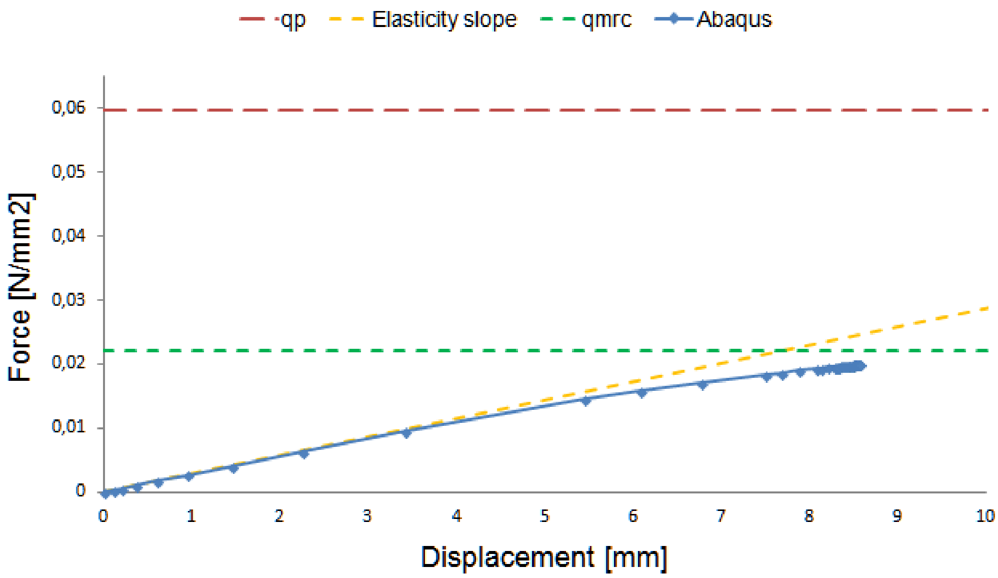

Figure 28. MC Modelling tension cut off 2.5

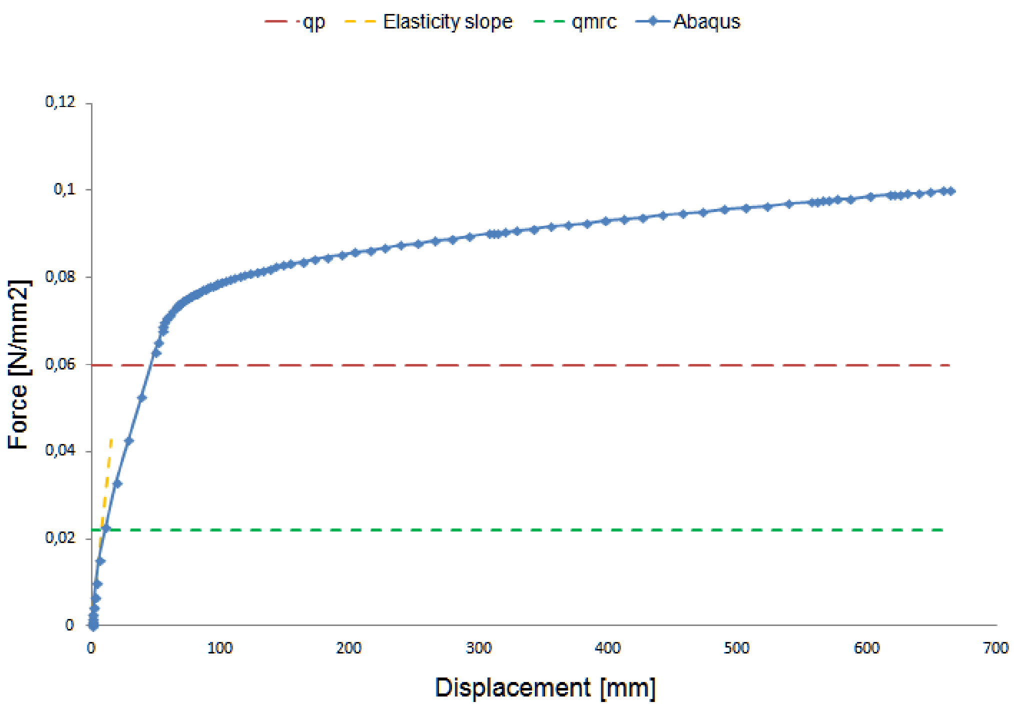

Figure 29. CDP Modelling

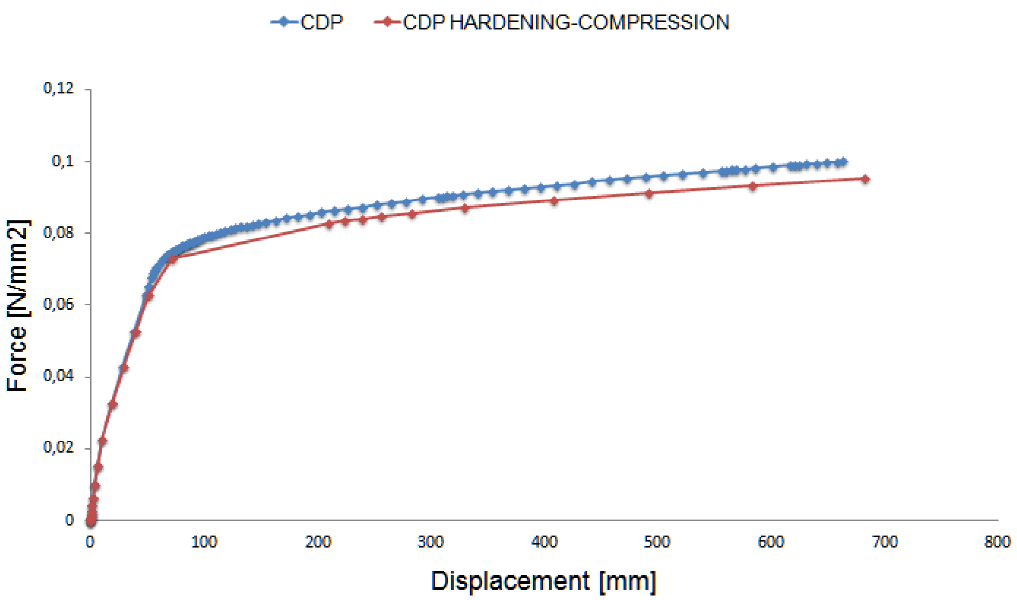

Figure 30. CDP hardening compression

From the above charts, it can be stated and concluded the next points:

- It can be seen that the numerical and analytical results do not match regardless the criterion used, being clearly the analytical solutions more restrictive and thus, more on the safe side.

- By performing the calculations analytically, it is made sure that the structure is more resistant. However, it is also more expensive that it should be.

- When using Mohr Coulomb criterion, it is observed that there is little variation when changing the dilation angle from 37º to 31º.

- The most stable criterion is concrete damage plasticity that shows correctly the elastic, elasto-plastic and plastic regime.

PART II-REINFORCED CONCRETE COLUMN

1 INTRODUCTION

In this part, a non-linear analysis of a plain and a reinforced concrete column will be performed. It will be studied its failure by buckling. Analytical and numerical solutions will be used and conclusions will be drawn.

2 PLAIN CONCRETE COLUMN

In this part, a non-linear analysis of a plain concrete column will be performed. It will be studied its failure by buckling. Analytical and numerical solutions will be used and conclusions will be drawn.

2.1 ANALLYTICAL CALCULATIONS

A standard plain concrete column whose characteristics fulfil Eurocode requirements (Eurocode, 2004) subjected to axial uniform load. Its elastic and plastic parameters are similar to the ones used with the doubly reinforced concrete beam. It is going to be studied the column resistance against buckling. Buckling is a type of deformation as a result of axial compression loads. This leads to have eccentricity in the column (bending). This will occur at lower stress levels than the ultimate normal stress of the column. Buckling depends on many factors like the shape of the column, slenderness, grade of construction imperfection and boundary conditions.

Its geometry and static system of the column that will be analyzed is shown below

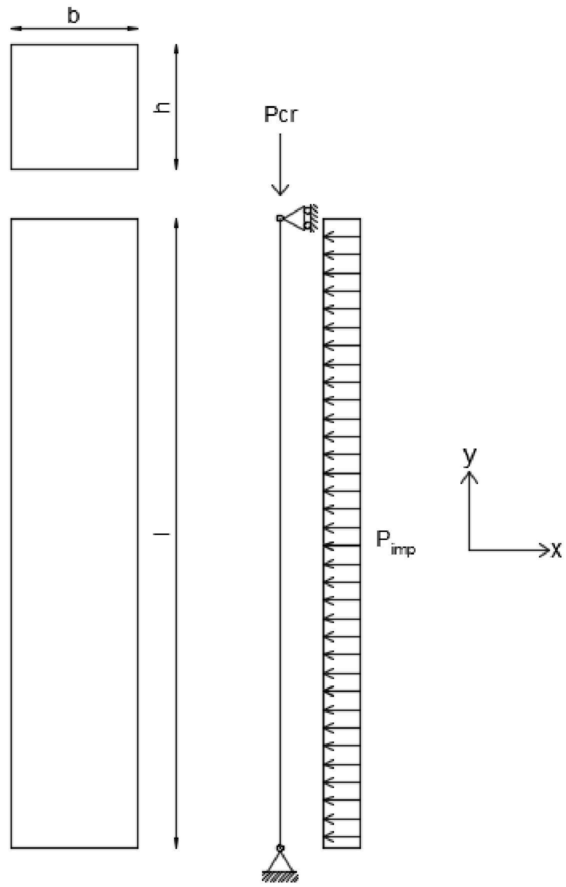

Figure 31. Chosen plain concrete column geometry

b [mm]	h [mm]	l [mm]
400	400	$3x10^3$

Table 19. Plain concrete column geometry and static system

Where Pcr is the critical load corresponding to the allowable load before the column starts to buckle and Pimp is the force that creates the imperfection. This imperfection is necessary for the column to suffer from buckling, otherwise the column would be only subjected to compression and will not develop any bending. In analytical calculations is not necessary to account for such force, however in numerical solutions is necessary.

Buckling can be calculated analytical accounted for the Euler's equation

$$P_{cr} = \frac{n\,\pi^2 EI}{KL^2} \tag{2.1}$$

Where n is the eigenmode number (in this project it will be studied only the first mode, therefore n=1), EI is the column bending stiffness and K depends on the column boundary conditions shown below

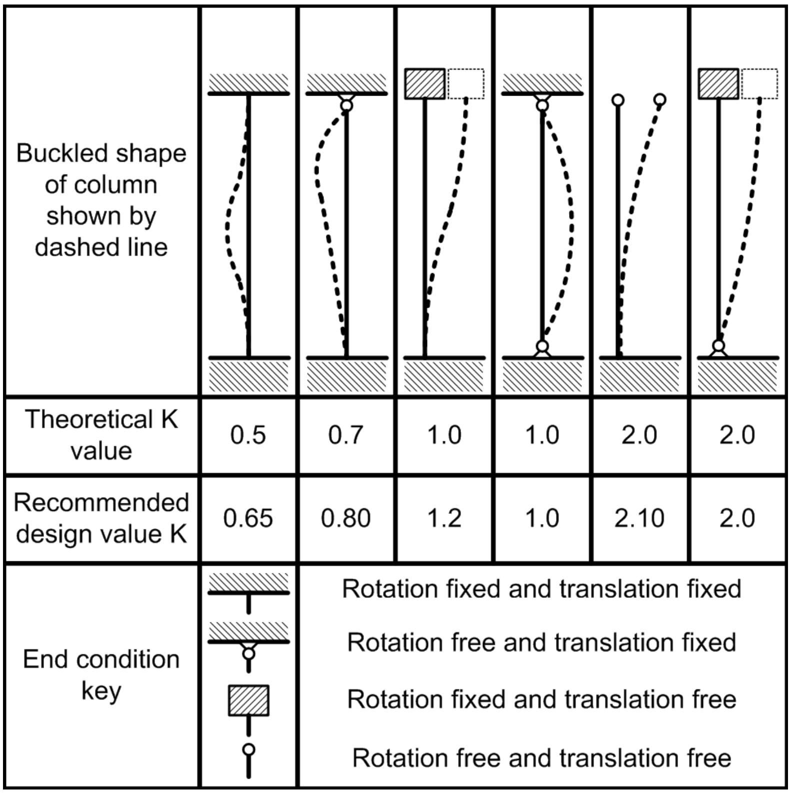

Figure 32. K values buckling (J, 1989)

The theoretical K value for the column will be equal to 0.7 according to the boundary conditions. Later on, it will be verified that the deflection shape corresponds to this one as it will be shown in ABAQUS. The bending stiffness EI will be calculated similarly to the calculation of the doubly reinforced concrete beam. The results for the plain concrete column in elastic regime are shown below

Pcr [N]	EI [Nmm^2]	K [-]
460	$6.72x10^{13}$	0.7

2.2 NUMERICAL CALCULATIONS

The column will be modelled in abaqus with only elastic parameters. It will be applied an axial load equal to the one used in analytical calculations divided by the column cross sectional area in order to obtain a pressure in a 3d model. The upper part of the column will be restraint the x direction (ux=0), in the down part will be restraint the x and y direction (ux=uy=0) and also it will be applied the corresponding boundary conditions to model half of the beam (symmetry). Finally the 2 loads Pcr and Pimp will be applied to the column.

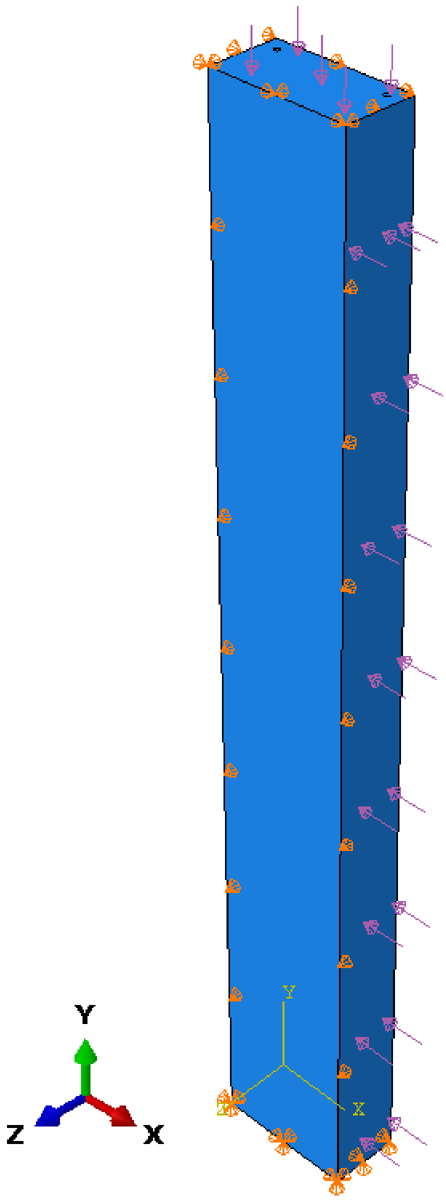

Figure 33. Abaqus plain concrete column

2.3 COMPARISON

Both results, analytical and numerical, can be compared observing the following graph

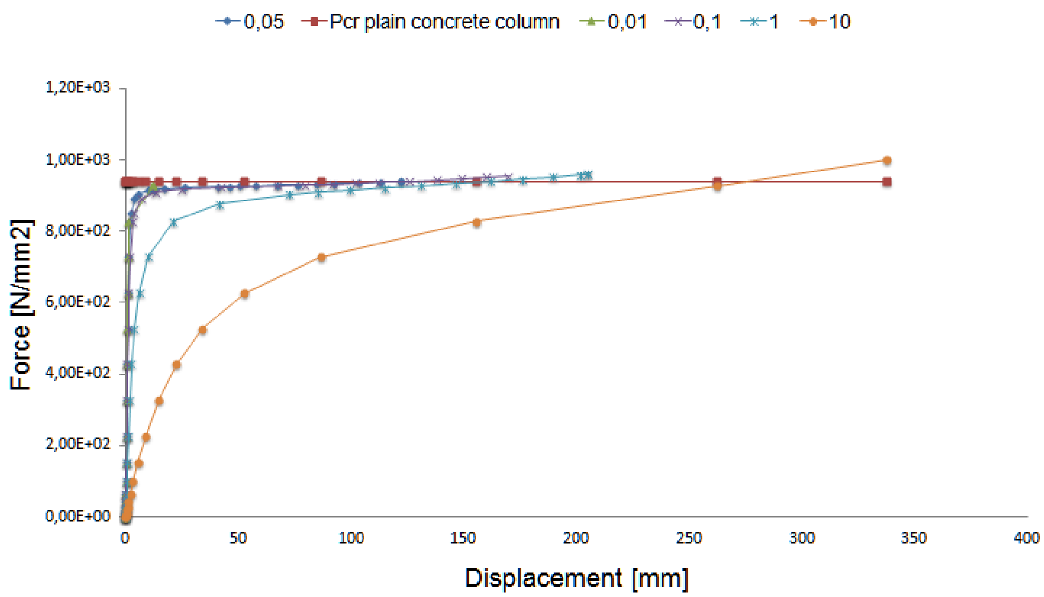

Figure 34. Plain concrete column check

It has been used different values of Pimp in order to see which one matches the best overcome of buckling. It was observed that this is obtained by using a pressure of 0,05N/mm^2.

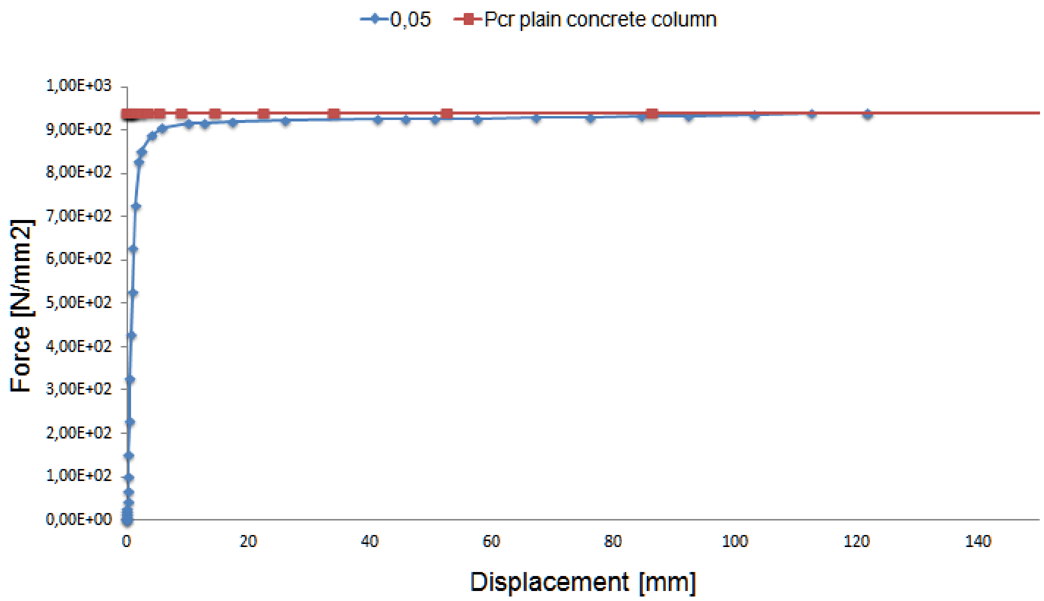

Figure 35. Plain concrete column

It can be concluded that analytical and numerical solutions are practically equal when it comes to elasticity in concrete.

3 REINFORCED CONCRETE COLUMN

In this part, a non-linear analysis of a reinforced concrete column will be performed. It will be studied its failure by buckling. Analytical and numerical solutions will be used and conclusions will be drawn.

3.1 ANALLYTICAL CALCULATIONS

Analytical calculations for a reinforced concrete column within the elastic regime are similar to plain concrete. The only difference is a variation of the bending stiffness EI due to the addition of steel to form a composite material. Bending stiffness will be calculated similarly as the doubly reinforced concrete beam and the cross sectional area geometry in mm can be seen below

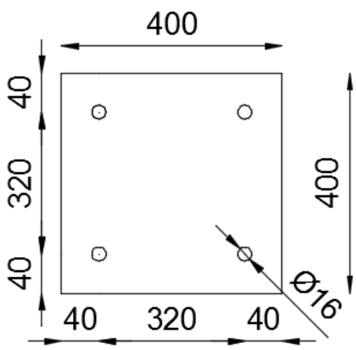

Figure 36. Reinforced concrete column cross sectional area

The results for the elastic reinforced concrete column can be seen in

Pcr [N]	EI [Nmm^2]	K [-]
477	$6.96x10^{13}$	0.7

Table 20. Reinforced concrete column elastic results

Next, it will be studied the buckling of the column when reaching a plastic regime. To do so, it was followed a procedure stated in (Jensen, 2011) which is based on the Engesser's First column theory. The objective is to determine the critical load Pcr that is given as

$$P_{cr} = \sigma_{cr}\, A_c + \sigma_{sc}\, A_s \tag{2.1}$$

Where σ_{cr} is the critical stress of concrete defined as

$$\sigma_{cr} = \frac{\pi^2 E_{\sigma k}}{\left(\frac{l_0}{i}\right)^2} \tag{2.2}$$

Being l_0 the column effective length depending on the boundary conditions (for this column is l_0=0,7l), i is the the minimum radius of gyration

$$i = \sqrt{\frac{I}{A_c}} \tag{2.3}$$

And $E_{\sigma k}$ is the tangential slope for non-linear elastic material and it is related with the initial modulus of elasticity of concrete Ec0k=Ec=31500 MPa.

Being Es the steel modulus of elasticity (210000 MPa).

σ_{sc} is the critical stress of the reinforcement given as

$$\sigma_{cs} = E_s \frac{\sigma_{cr}}{E_c} \tag{2.4}$$

Ac is the concrete area and As is the steel area.

The results are presented below

Pcr [N]	EI [Nmm^2]	K [-]
24.5	6.96x10^{13}	0.7

3.2 NUMERICAL CALCULATIONS

Boundary conditions and geometry are similar to the plain concrete column except for the addition of the reinforcement.

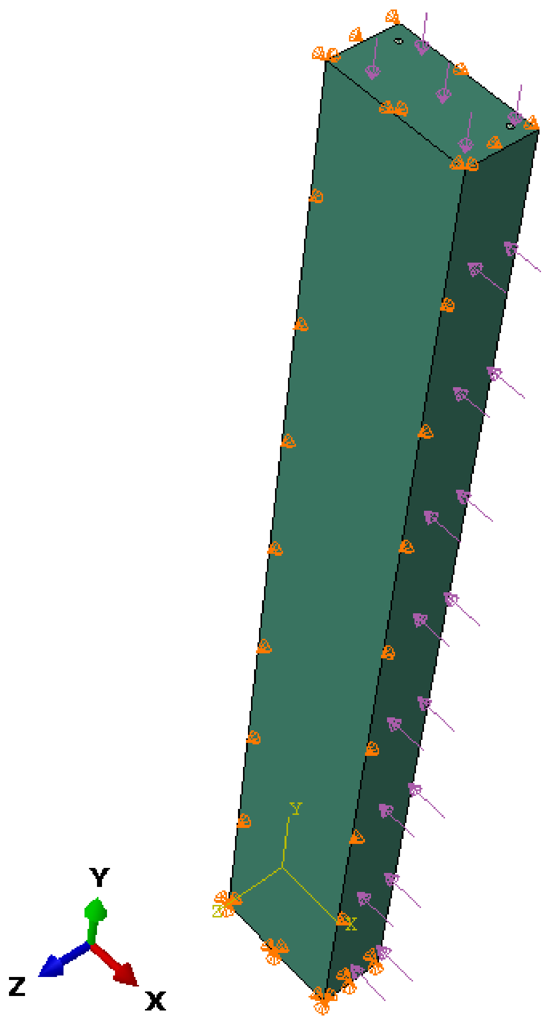

Figure 37. Abaqus reinforced concrete column

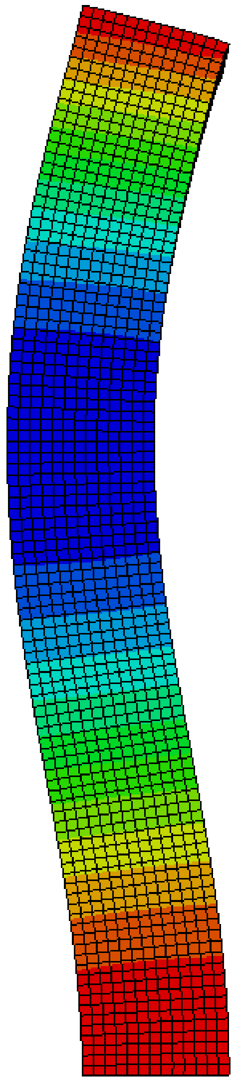

Figure 38. Column buckling deformed shape

3.3 COMPARISON

The differences between analytical and numerical solutions when it comes to the reinforced concrete column in elastic regime are observed in the following charts

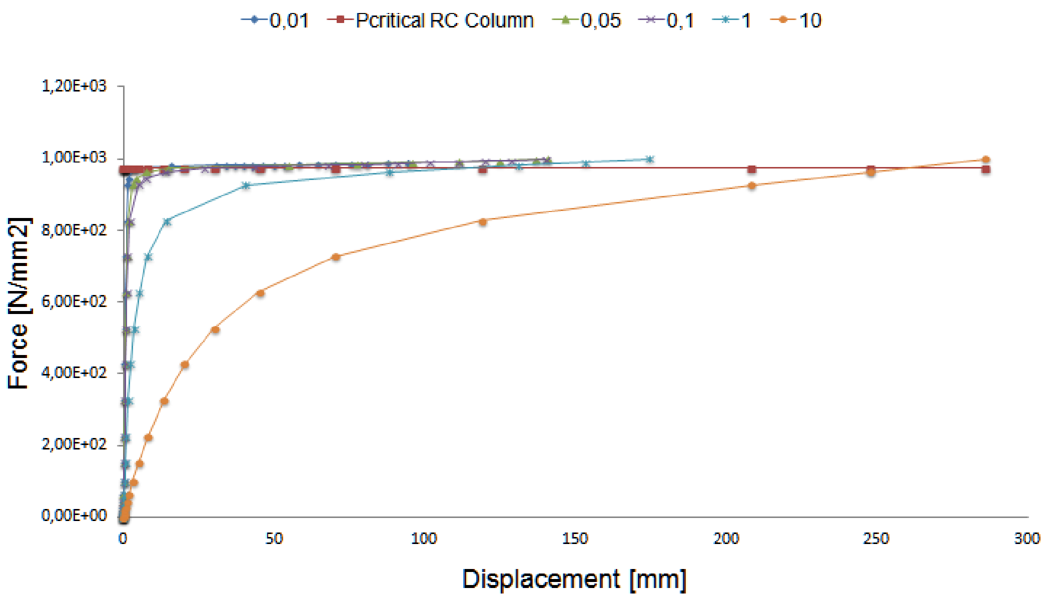

Figure 39. Reinforced concrete column check

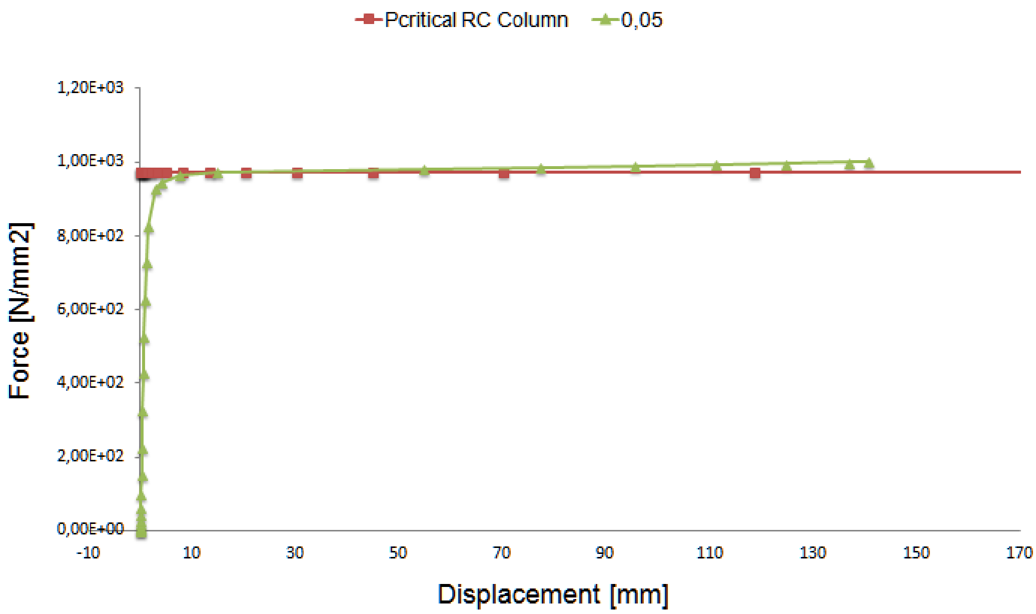

Figure 40. Reinforced concrete column

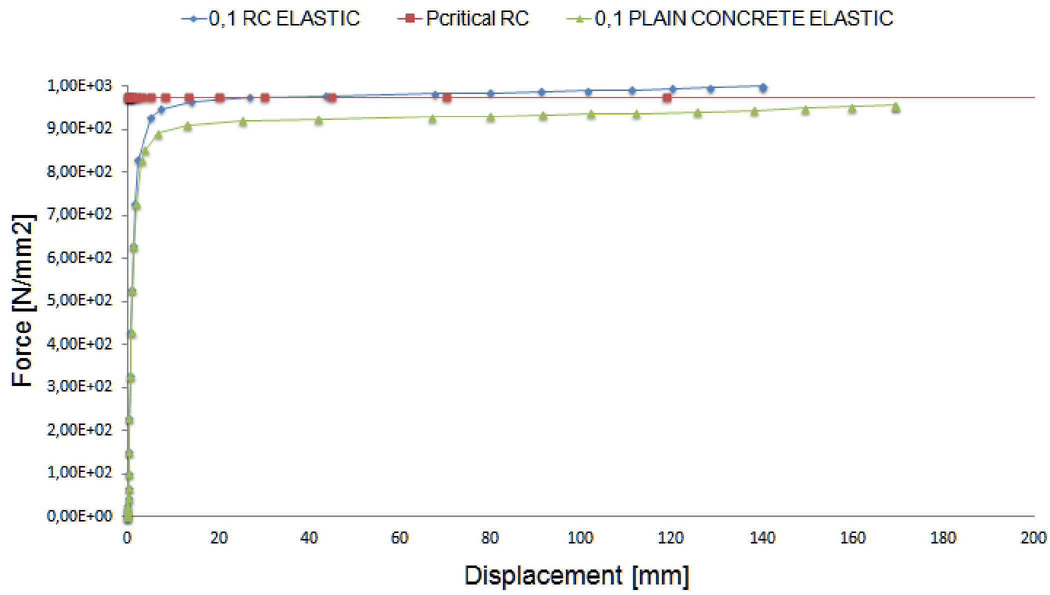

Figure 41. Plain and reinforced concrete column

Once more, it can be seen that when having an elastic behaviour, analytical and numerical solutions are highly similar.

The differences between analytical and numerical solutions when it comes to the reinforced concrete column in elasto-plastic regime are observed in the next charts

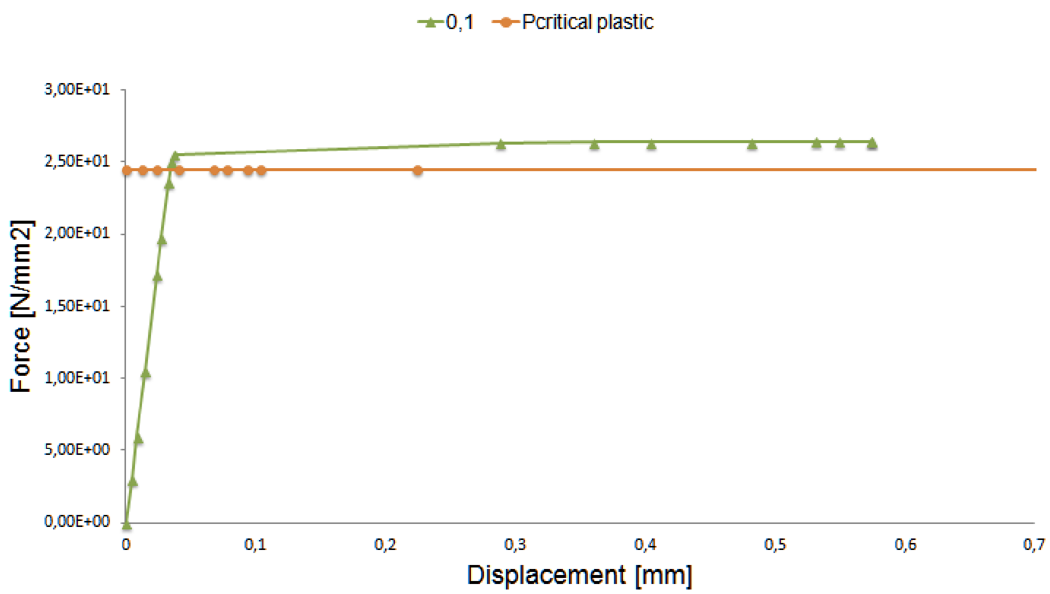

Figure 42. Von Misses reinforced concrete column

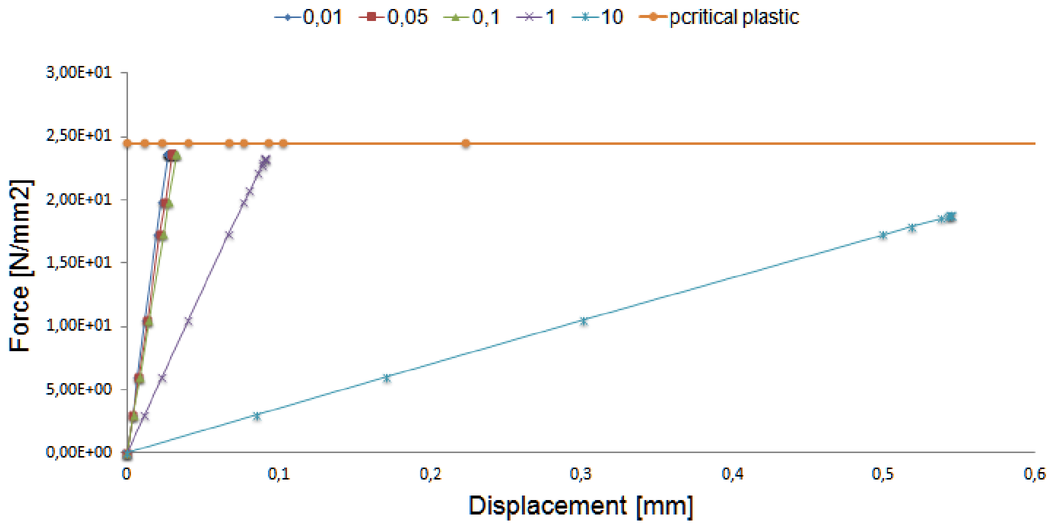

Figure 43. MC reinforced concrete column

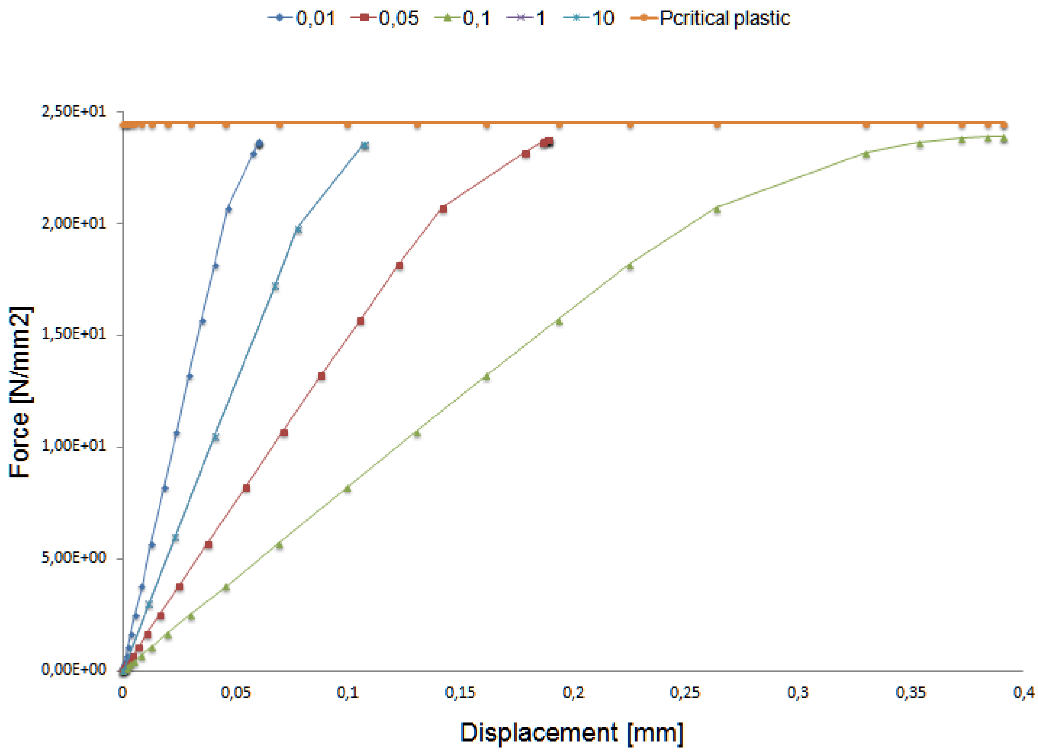

Figure 44. CDP reinforced concrete column

PART III-CONCLUSION

1 CONCLUSIONS

Once checked all the analytical and numerical results, the next conclusions can be drawn:

- It has been observed that numerical solutions for the doubly reinforced concrete beam are less similar than for the study of buckling in the reinforced concrete column. It seems, this is due to the fact that studying bending for a composite material is a more complex matter since there are more assumptions made in the calculations.

- In general, Von Mises and Concrete Damage Plasticity criteria are more stable and accurate than Mohr Coulomb.

- All in all, analytical and numerical solutions appear to be significantly similar for linear elastic and non-linear behaviour.

- Numerical solutions take more time than analytical solutions. From an economical point of view, it can be concluded that it is preferable to use analytical calculations than numerical since they appear to give roughly similar results.

- The use of stirrups might vary the results. This can be important for further research.

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APPENDIX A: YIELD CRITERIA THEORY

1.1 MOHR COULOMB (MC)

MC criterion is defined by the cohesion c and the friction angle φ parameters as seen in Figure 45

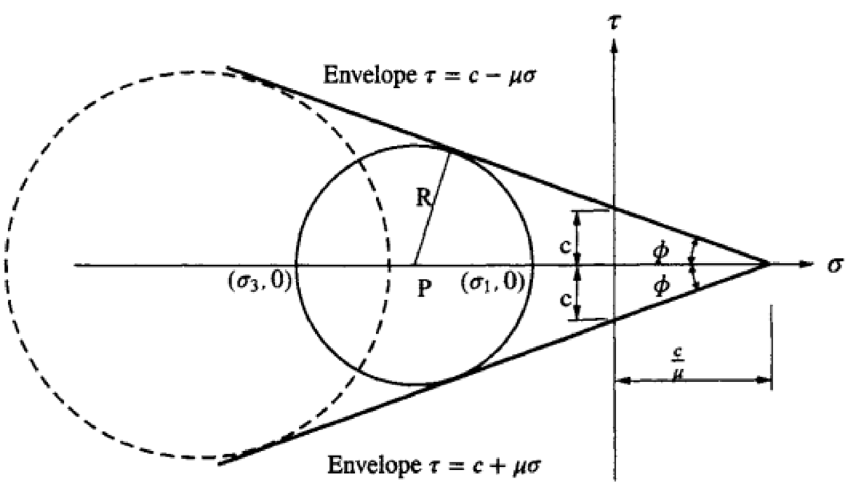

Figure 45. Mohr Coulomb criterion in Mohr diagram (Ottosen, 2005)

This criterion is widely used in numerical calculations since it only requires of few parameters to be defined. Specifically MC is normally used to model concrete and soils since these materials are highly dependent to pressure variations. The base of this criterion is that the material becomes stronger when pressure is increasing.

In the deviatoric plane, MC is defined with the principal stresses σ_1, σ_2 and σ_3 as shown in Figure 46

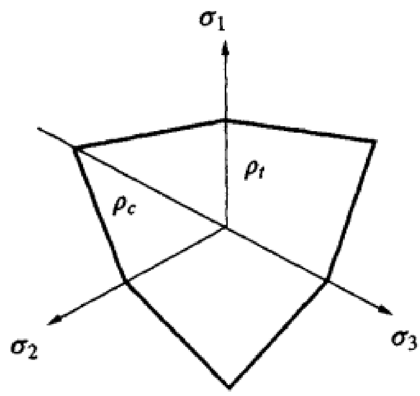

Figure 46. Mohr Coulomb criterion in the deviatoric plane (Ottosen, 2005)

The failure characteristics of concrete based on MC criterion states that $f(\sigma_1,\sigma_2,\sigma_3)=0$ with the convention that $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Assuming that σ_2 is of minor importance it is obtained the relationship between the principal stresses:

$k\sigma_1-\sigma_3-m=0$ where k is the friction parameter and m is a material parameter. Fulfilling the requirements for this expression, it is obtain that $f_{MC}=k\sigma_1-\sigma_3-\sigma_c=0$ where σ_c is the uniaxial concrete compressive strength.

Knowing φ, k can be determined with the Equation (3.1)

$$k = \frac{1 + sin\varphi}{1 - sin\varphi} \tag{3.1}$$

σ_c is determined in function of the cohesion c and k as shown in Equation (3.2)

$$\sigma_c = 2c\sqrt{k} \tag{3.2}$$

And the uniaxial tensile strength σ_t depends on σ_c and k as it can be seen from Equation (3.3)

$$\sigma_t = \frac{\sigma_c}{k} \tag{3.3}$$

It is important to highlight the fact the σ_t can be overestimated since this value is larger than can be observed in reality. Hence, it was added another approach known as Rankine plasticity which decreases this value accounting for what is called "Tension cut-off" and it is defined as $f_{Rankine}=\sigma_1-\sigma_{tm}$ where in this case σ_t is the actual tensile strength or tension cut-off. Applying this, it is obtained the Modified Mohr Coulomb MMC and it is presented in Figure 47

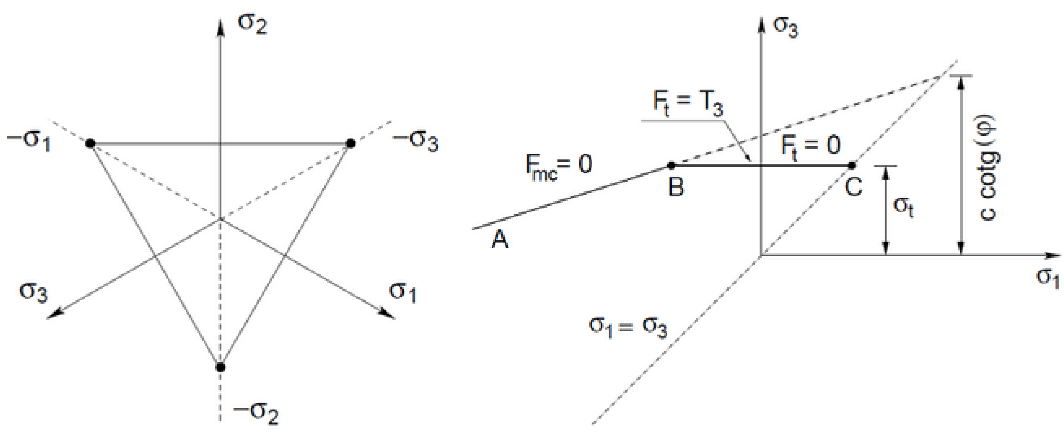

Figure 47. a) Rankine yield condition in the deviatoric plane, b) Modified Mohr coulomb criterion (software, 2013)

Therefore, in the case of ccotgφ>σt, the tensile strength of the material σt is set to the tension cut off.

Another parameter that is needed to define this criterion is the dilation angle ψ, which is related to the volumetric strain of the material ε_v as shown in Figure 48

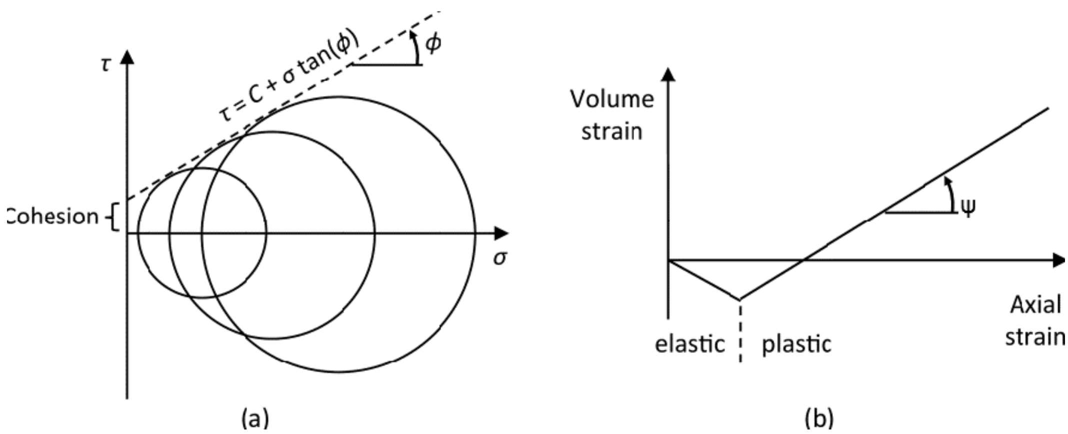

Figure 48. a) friction angle representation, b) dilation angle representation (Iverson, et al., 2014)

1.2 CONCRETE DAMAGE PLASTICITY (CDP)

This constitutive model is found in software ABAQUS and it is indicated mainly for solving concrete structures and others quasi-brittle materials (Abaqus, 2016). CDP yield surface is shown in Figure 49

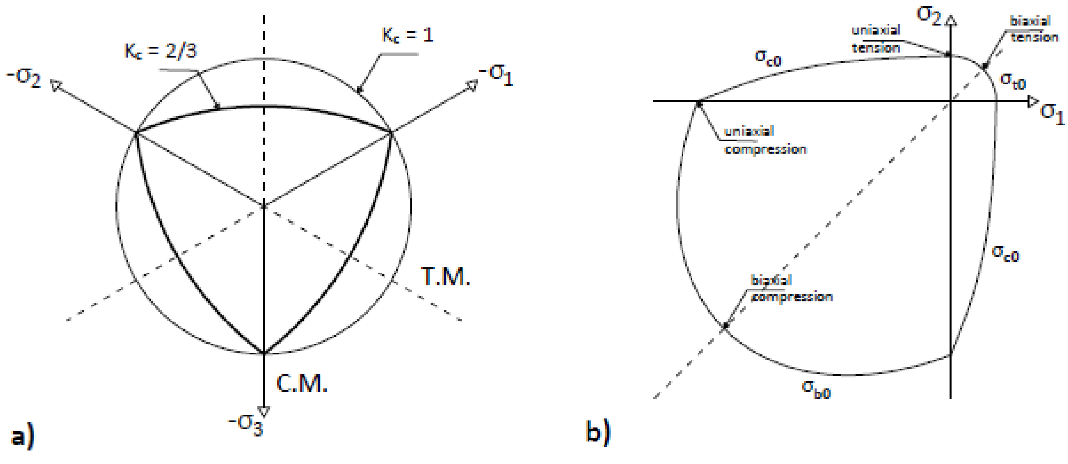

Figure 49. CDP in a) Deviatoric plane b) Plane stress plane (Abaqus, 2016)

CDP can be understood as a modification of the Drucker-Prager criterion by changing the circular failure surface by a shape defined with the parameter k_c. Its yield function was stated by (J, 1989) and posteriously, some corrections were made by (Lee, 1998). The yield function is provided in (3.4)

$$f = \frac{1}{1-\alpha}\left(q - 3\alpha p + \beta(\dot{\varepsilon}^{pl})\sigma_{max} - \gamma(-\sigma_{max})\right) - \sigma_c(\dot{\varepsilon}_c^{pl}) = 0 \qquad (3.4)$$

Where

$$\alpha = \frac{\left(\frac{\sigma_{b0}}{\sigma_{c0}}\right) - 1}{2\left(\frac{\sigma_{b0}}{\sigma_{c0}}\right) - 1} \quad , \quad 0 \leq \alpha \leq 0.5 \qquad (3.5)$$

and

$$\beta = \frac{\sigma_c\left(\dot{\varepsilon}_c^{pl}\right)}{\sigma_t\left(\dot{\varepsilon}_t^{pl}\right)}(1-\alpha) - (1+\alpha) \qquad (3.6)$$

Being γ

$$\gamma = \frac{3(1-K_c)}{2K_c - 1} \qquad (3.7)$$

Where σ_{max} is the maximum principal effective stress, σ_{b0}/σ_{c0} is the ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress, K_c is the ratio of the second stress invariant on the tensile meridian q(TM) to the compressive meridian q(CM) at yield for any pressure p, $\sigma_t\left(\dot{\varepsilon}_t^{pl}\right)$ is the effective tensile cohesion stress and $\sigma_c\left(\dot{\varepsilon}_c^{pl}\right)$ is the effective compressive cohesion stress.

The main purpose of the CDP model is to deal with stiffness recovery when subjected to dynamic loading (Abaqus, 2016). The stress strain curve based on CDP is shown in Figure 50

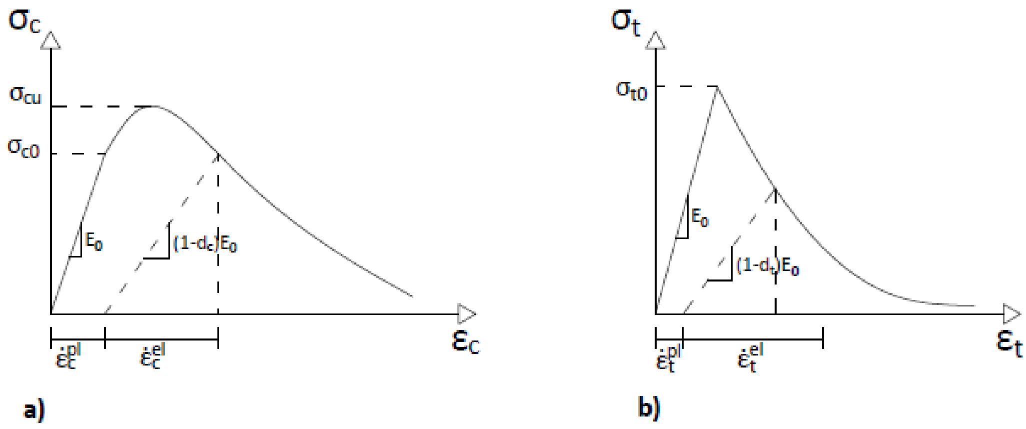

Figure 50. a) Uniaxial loading in compression , b) Uniaxial loading in tension (Abaqus, 2016)

In a) it can be observed that there is an initial elastic regime marked with a straight line corresponding to the Youngs modulus E, thereafter the plastic regime starts to be developed from the yield compressive stress σ_{c0} until it reaches the ultimate compressive stress σ_{cu} and finally, concrete undergoes softening.

In b) it can be appreciated a similar behaviour to a) except for the fact that the yield stress is equal to the ultimate tensile stress σ_{t0}.

1.3 VON MISES (VM)

Von misses yield criterion is usually used for ductile materials like metals. The von Misses stress is calculated and then it is compared with the yield stress to the material to check if the yield limit has been exceeded. The VM yield surface in principal stress coordinates correspond to a circular cylinder as can be observed in Figure 51

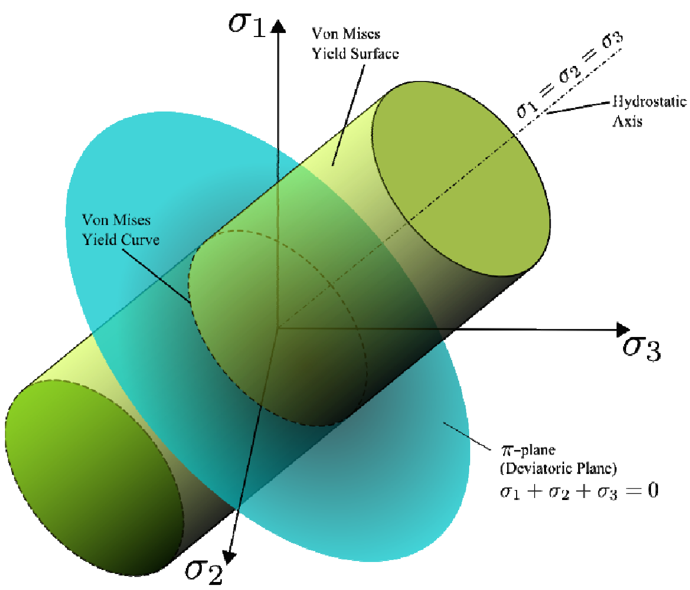

Figure 51. Von Misses yield surface in principal stress coordinates (wikipedia, 2017)

The yield surface of the resultant cylinder defines the VM yield criterion and it is given in Equation (3.8)

$$\sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]} = \sigma_{y0} \qquad (3.8)$$

Where σ_1, σ_2, σ_3 are the principal stresses and σ_{y0} is the yield stress. If VM yield criterion is represented in the deviatoric plane, it can be seen that the limits for compression, tension and shear and coincident since it has a circular shape. This means that if a material, for instance steel, is modelled with this criterion, its resistances to shear, compression and tension are the same. This property is very common in brittle materials. This can be observed in figure

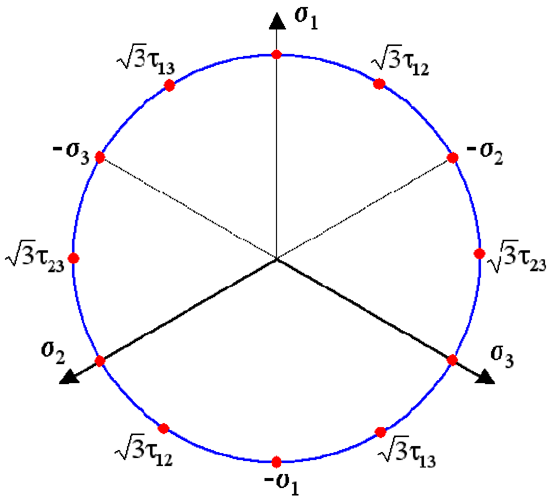

Figure 52. VM criteria in the deviatoric plane (Continuummechanics, 2011)

Therefore, to define VM yield criteria it is only needed the yield stress σ_y and the absolute plastic strain ε_p.

APPENDIX B: RESULTS VERIFICATION

CONCRETE-STEEL SAMPLE

$As=\pi * r^2=78.54mm^2$

$Ac=b*h-\pi * r^2=321.46mm^2$

$u_x=F/k_c+k_s=0.03756mm$

CONCRETE-STEEL-CONCRETE SAMPLE

$As=\pi * r^2=78.54mm^2$

$Ac=b*h=400mm^2$

$u_x=F/k_c+k_s=0.03437mm$

CONCRETE SAMPLE

$As=0$

$Ac=b*h=400mm^2$

$u_x=F/(k_c+k_s)=0.07936mm$

APPENDIX C: PLAIN CONCRETE BEAM

ELASTIC CALCULATIONS

$I=b*h^3/12=2604166666.67\ mm^4$

$EI=82031250000105\ Nmm^2$

$My=\sigma_y*I/y=260416666.67\ Nmm$

$q_y= 8*\sigma_y*I/y*l^2=20.83N/mm$

$\delta mid=5*qy*l4/384*EI=33mm$

PLASTIC CALCULATIONS

$M_p= \sigma_y*(b\ h^2/4)=390625000\ Nmm$

$q_p=8*Mp/l^2= 31.25N/mm$

APPENDIX D: REINFORCED CONCRETE BEAM

CALCULATION OF X

A_{s1}=1256.63 mm^2

A_{s2}=226.20 mm^2

$\alpha = 6.67$

x=122.23 mm

x_{min}=66.32 mm

x_b=247.06mm

STRAINS CALCULATIONS

εcu=0.035

εy=0.0016

εs1=0.014

εs2=0.0017

MOMENTS CALCULATIONS

Mrc=69508917.58 Nmm

Mrst=74593200,64 Nmm

BEAM DISTANCES

Y_c=240.85mm

Y1=250mm

Y2=40mm

Y3=464mm

Ics=2604166667mm^4

Ic=2552650899mm^4

Ih1=7853.98mm4

Ih2=1017.87mm4

SLS RESULTS

EI=9.34X10^{13} MPa

qmrc=5.56 N/mm

δmrc=7.75 mm

My=69508917.58 Nmm

qy=2.08 N/mm

δy=20.79mm

ULS RESULTS

Cs=78037.21 N

Ts=433535.28 N

x=71.09 mm

Mp=186506578.9 Nmm

qp=14.92 N/mm

δp=20.79mm

η=1

λ=0.8

NUMERICAL CALCULATIONS

VON MISES

σy=25 MPa

εpl=0

σy=345 MPa

εpl=0

MOHR COULOMB

σc=25 MPa

Φ=37º

K=4.02

Ψ=31º/37º

C=6.22

CONCRETE DAMAGE PLASTICITY

$\sigma t0$=2.5

ε_t^{pl} =0

ψ=31º

e=0.1mm

σ_{b0}/σ_{c0}=1.16

Kc=0.667

μ=0

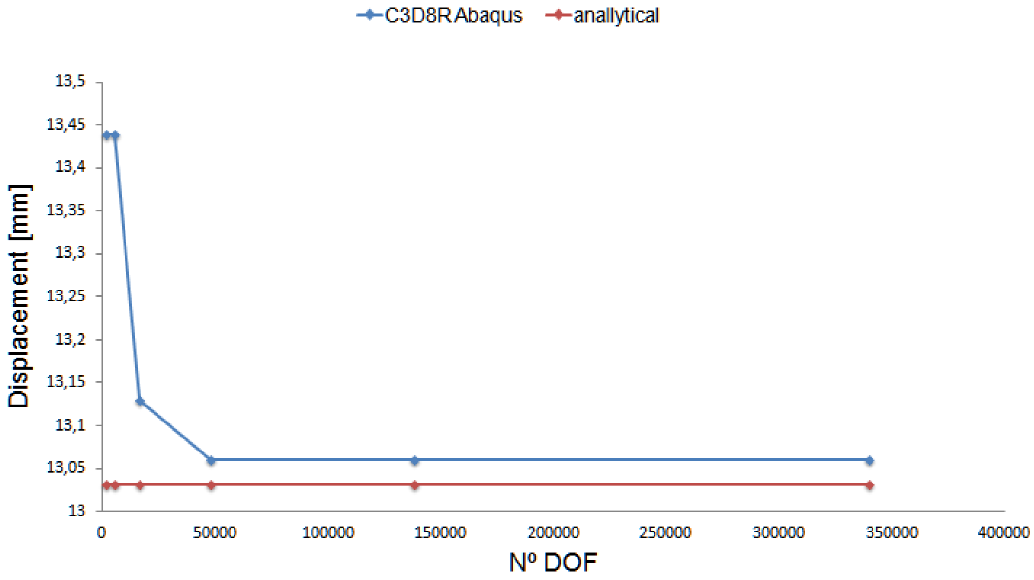

Figure 53. Convergence analysis 8 nodes element

APPENDIX E: REINFORCED CONCRETE COLUMN

PLAIN CONCRETE COLUMN (ELASTIC)

EI=$6.72X10^{13}$ Nmm^2

Pcr=460 N

K=0.7

REINFORCED CONCRETE COLUMN (ELASTIC)

Pcr=477 N

EI=$6.96x10^{13}$ Nmm^2

K=0.7

REINFORCED CONCRETE COLUMN (PLASTIC)

Pcr=24.5 N

EI=$6.96X10^{13}$ Nmm^2

K=0.7

σcr=24.39 MPa

σsc=162.62 MPa

i=115,47 mm