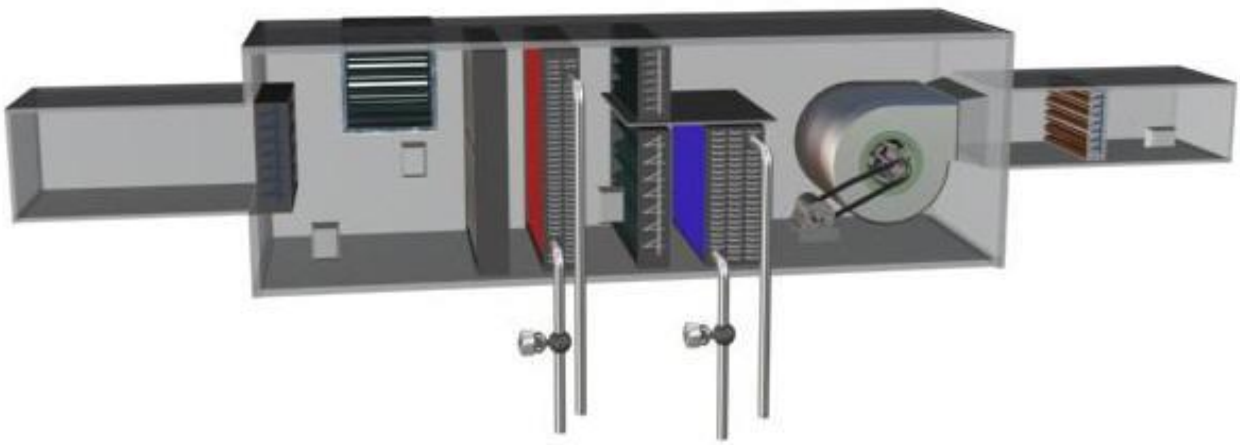
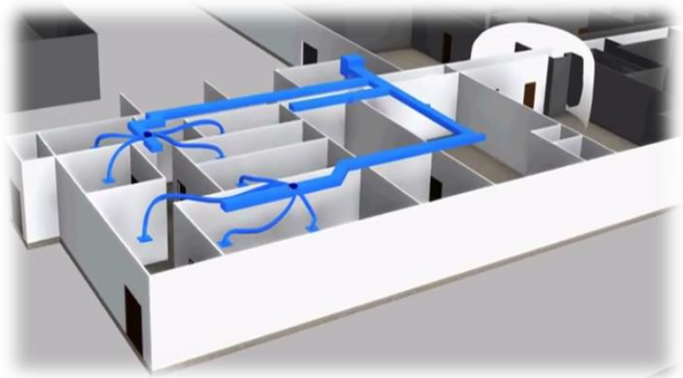
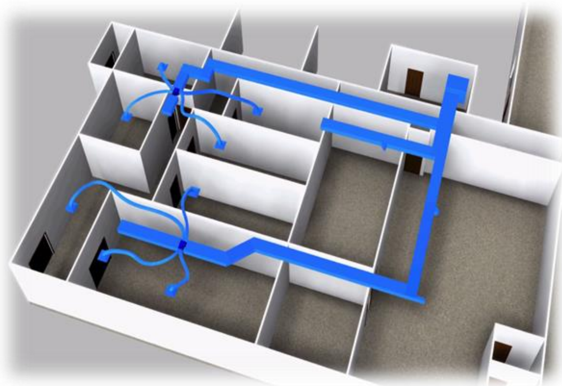
MODELLING AND CONTROL OF A MULTI-ZONE HVAC SYSTEM

For an Unmanned Transformer Platform.







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Preface

This report represents the Master thesis project of the MSc in Sustainable Energy Engineering with specialization in Offshore Energy Systems stated in the energy department at Aalborg University, Esbjerg campus (AAUE).

The authors would like to thank several persons that have contributed and supported this master thesis, both academically and practically. To begin with, we would like to thank our AAU supervisor Dr. Zhenyu Yang for his significant academic advises throughout this thesis. In addition, we would like to thank the head of the HVAC department Mr. Jørgen Steen Hansen and our supervisor and HVAC engineer Mr. Alexander Fateyev, from Semco Maritime A/S, for their valuable help and unparalleled interest in this master thesis throughout the entire process.

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Finally yet importantly, we would like to thank our families and friends for being helpful and supportive during the last two years of studying in Aalborg University.

Title Sheet

Title: Modelling and Control of a multi-zone HVAC-system

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Abstract

Heating ventilation and air conditioning (HVAC) systems are designed to maintain the indoor climate in the desired temperature, humidity, and pressure level for industrial and commercial buildings. Particularly, this master thesis deals with the modelling and control of a multi-zone HVAC system applied in an unmanned offshore platform.

In this scientific work, a pre-investigated single-zone model of a HVAC-system is further developed to a multi-zone model, taking into account the humidity, temperature and the pressurization in each zone. In addition, the interaction between the zones in terms of heat transfer through the walls is considered in this model. Furthermore, the cooling coil, heating coil and humidifier are included in the overall model.

In addition to the above, a set of PI controllers are designed to control the coils, humidifier, the fan, the inlet dampers into the zones and the variable heaters in the zones. Moreover, a Model Predictive Controller (MPC) is designed and compared with the PI controllers after a door opening disturbance is applied.

Finally, PI controllers are implemented to a small ventilation setup in NI LabView, in order to observe and examine their performance. The MPC was not implemented due to a restricted license of the software interface.

Sandra L. Pedersen	**Ioannis Duraj**

Introduction

The HVAC system also known as Heating, Ventilation and Air Conditioning system, is used on offshore platforms to maintain the indoor climate in the different zones, at desired temperatures, appropriate moisture level and to ensure the pressurization. Keeping these three variables within proper ranges is necessary in order to obtain both comfort and safety. The pressurization of the zones is important in order to avoid flammable gasses to enter in the zones, thereby avoiding fire or intoxication. The pressurization is between 25 Pa - 50 Pa higher than the ambient and has to be kept when a door is opened. The moisture level in the air, also referred to as humidity, has to be kept within a range to avoid possible damages in the machines and thereby avoiding malfunctions. The temperature for manned platforms is recommended to be around $21°C$ to $22°C$. This project deals with an offshore unmanned transformer platform where the given specifications were provided by Semco Maritime A/S.

Additionally to the above, the offshore weather conditions can be extreme. Therefore critical weather changes occur, such as from both very hot and humid air in the summer periods to very dry and cold air in winter periods. It is therefore important that the HVAC system can handle these changes.

In the previous project, a model for the heat transfer and pressurization of a single-zone was found and investigated. This model only considered the inflow of air regulated by a fan. The external duct- or ventilation system was not of any consideration.

In addition, a PI-controller was made in order to maintain the inside pressurization by regulating the speed of the fan. The problem with the pre-investigated single zone model was that the reaction of the PI controller was unrealistically fast and the changes in the magnitude of the zone temperature, were significantly small. These statements is illustrated in the figure shown below.

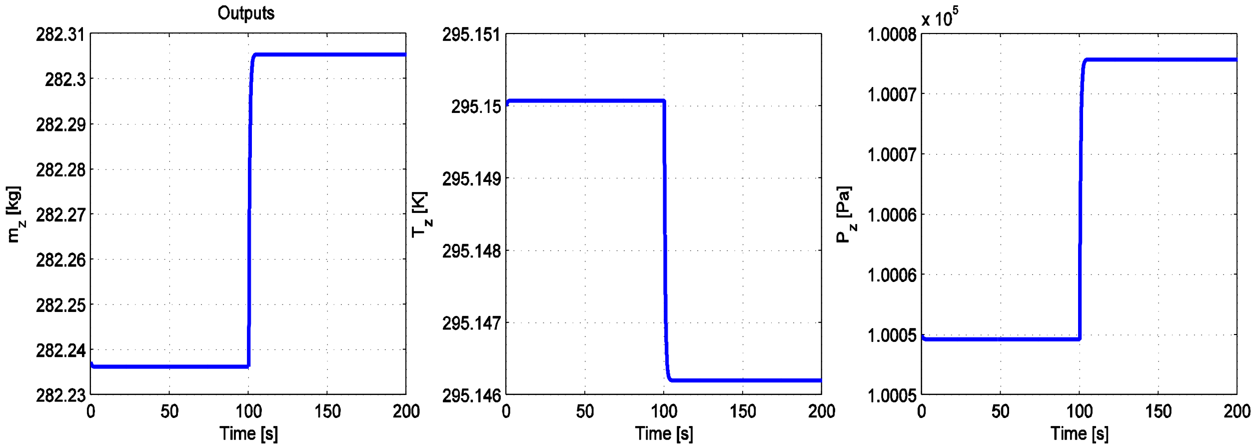

Figure 1 Changes in a) mass of air, b) temperature, c) pressure in the zone. (Pedersen, 2015)

The problem is clearly shown in the figure above. The temperature decreases 0.08 K when the damper is opened. Setting as an inlet temperature lower that the already existing temperature in the zone, it was expected the final zone temperature to decrease. That is due to unrealistically small changes of mass inflow in the zone. A further introduction in the HVAC systems as well as the pre-investigated single zone model are presented in the following chapters.

Problem statement

- How to expand the single zone model to a multi-zone HVAC model?
- How to account for the humidity and heating coil in the model?
- How to improve the existent controller by prediction of future control input?

Problem formulation

The project deals with expanding a pre-investigated model of a HVAC-system. This includes study of how the humidity and the heating of the air can be modelled. In addition, a strategy of regulating the air inlet dampers, which makes sure that the maximum pressure in the zones are not exceeded, will be made. After the overall model is found, the model will be simulated and a controller will be designed in order to obtain a specified temperature, pressure and humidity of the different zones. Taking into account the harsh weather conditions on the offshore situations, a robust control strategy should be implemented and integrated in the controller. The safety regulations for the offshore HVAC applications are very strict and should be taken into consideration due to the danger of the probability of flammable gas insertion in the indoor environment of the platform or in the HVAC system. Considering all the above, the indoor pressurization, temperature and humidity need to be kept within acceptable safety regulation range and are the three main factors to deal with in this project.

Variable	Variable Name	Unit
ΔE	Difference in energy	$[J]$
E	Energy	$[J]$
$\Delta \dot{E}$	Difference in rate of energy	$\left[\frac{J}{s}\right]$
$\dot{E}$	Rate of change in energy	$\left[\frac{J}{s}\right]$
c_p	Specific heat capacity	$\left[\frac{J}{Kg \cdot K}\right]$
$\dot{m}$	Mass flow rate	$\left[\frac{kg}{s}\right]$
m	Mass	$[Kg]$
T	Temperature	$[K]$
Q	Heat transfer	$[J]$
$\dot{Q}$	Heat transfer rate	$\left[\frac{J}{s}\right]$
h	Enthalpy	$\left[\frac{J}{kg}\right]$
ρ	Density	$\left[\frac{Kg}{m^3}\right]$
q	Volumetric mass flow	$\left[\frac{m^3}{s}\right]$
P	Pressure	$Pa]$
V	volume	$[m^3]$
v	Velocity	$\left[\frac{m}{s}\right]$
g	Gravity	$9.82\left[\frac{m}{s^2}\right]$
C_d	Discharge coefficient	$[-]$
C	Heat capacity	$\left[\frac{J}{K}\right]$
C_c	Vena Contracta coefficient	$[-]$
A_0	Dampers opening	$[m^2]$
A_1	Duct diameter	$[m^2]$
A	Area	$[m^2]$
K_{wall}	Thermal conductivity	$\left[\frac{W}{m \cdot K}\right]$
L	Length	$[m]$
U	u-value	$\left[\frac{W}{m^3 K}\right]$
$\bar{h}$	Average convection coefficient	$\left[\frac{W}{m^2 \cdot K}\right]$
v_k	Kinematic viscosity	$\left[\frac{m^2}{s}\right]$
α	Substance constant	$\left[\frac{m^2}{s}\right]$
β	Thermal expansion coefficient	$[K^{-1}]$
T_f		$[K]$
L_e	Entry length	$[m]$
n	Fans sharft speed	$[RPM]$

$\boldsymbol{\omega(t)}$	Fans frequency	$[Hz]$
$\boldsymbol{s}$	Slip	$[-]$
$\boldsymbol{N}$	Gear ratio	$[-]$
$\boldsymbol{p}$	No. of poles	$[-]$
$\boldsymbol{\dot{m}_{wcc,max}}$	Maximum mass flow rate of water inside the cooling coil	$\left[\frac{Kg}{s}\right]$
$\boldsymbol{Opening}$	"Valve opening" – indicates the fraction of the maximum mass flow rate that flows in the heating or cooling coil.	$[-]$
$\boldsymbol{\omega}$	Specific humidity	$\left[\frac{kg\ water\ vapor}{kg\ dry\ air}\right]$
$\boldsymbol{\phi}$	Relative humidity	$[\%]$
$\boldsymbol{\dot{m}_{wcc,out}}$	Condensated water by dehumidification	$\left[\frac{Kg}{s}\right]$
$\boldsymbol{\dot{m}_{w,in}}$	Added water vapor from the humidifier	$\left[\frac{Kg}{s}\right]$
$\boldsymbol{D}$	Diameter of duct	$[m]$
$\boldsymbol{K_L}$	Resistance coefficient	$[-]$
$\boldsymbol{f}$	Friction factor	$[-]$
$\boldsymbol{Re}$	Reynolds number	$[-]$
$\boldsymbol{\epsilon}$	Material roughness	$[m]$
$\boldsymbol{Nu}$	Nusselt number	$[-]$
$\boldsymbol{\mu}$	Dynamic viscosity	$\left[\frac{m^2}{s}\right]$
$\boldsymbol{W_d}$	W is the duct dimension parallel to the blade axis	$[m]$
$\boldsymbol{R_d}$	Perimeter of the duct.	$[m]$
$\boldsymbol{L_d}$	Sum of damper length	$[m]$
$\boldsymbol{n_d}$	Number of damper blades.	$[-]$
$\boldsymbol{\Delta P}$	Pressure drop	$[Pa]$
$\boldsymbol{H_{RH}}$	Height of radiator	$[m]$

Subscriptions	Description
$Wall$	Wall
in	Inflow
out	Outflow
A, B	Point A and Point B
rad	Radiator
$dist$	Disturbance
atm	Ambient - Outside
r	Return
mix	Mixing box
cc	Cooling coil
hc	Heating Coil
h	Humidifier
dyn	Dynamic
$Static$	Static
$zone$	Zone - Room
dp	Dew point
db	Dry-bulb
sat	Saturation
g	
w	Water vapor
a	Dry air
tot	total
d	Duct

Matrices/ coefficients	Description
Least squares	
$\boldsymbol{a_0, a_1, a_2}$	Polynomial coefficients
$\boldsymbol{\Theta_{Ls}}$	Parameter Matrix
$\boldsymbol{Y}$	Output Matrix
$\boldsymbol{\Psi}$	Regression Matrix
State space	
A	Continuous time system matrix
A_d	Discrete time system matrix
B	Continuous time input matrix
B_d	Discrete time input matrix
C	Continuous time output matrix
C_d	Discrete time output matrix
D	Continuous time feedforward matrix
D_d	Discrete time feedforward matrix
λ	Eigenvalue
Ψ	Matrix used when discretizing
T	Sampling time
y	Outputs
x	States

u	Inputs
PI	
K_p	Proportional gain
K_i	Integral gain
K_d	Derivative gain
$e(t)$	Error wrt. time
MPC	
N_p	Prediction horizon
N_u	Control horizon
N_w	Window size
r	Reference/setpoint
Q	Weightings matrix
R	Weightings matrix
Ψ	Prediction matrix
Φ	Prediction matrix
Θ	Prediction matrix
Ψ_y	Prediction matrix – related to the output
Φ_y	Prediction matrix– related to the output
Θ_y	Prediction matrix – related to the output
Y	Outputs matrix
V	Cost function
T	Trajectory matrix
Ω	Overall Weightings matrix
P	Overall Weightings matrix
M	Optimization matrix – used in quadratic programming
H	Optimization matrix – used in quadratic programming
β	Constraint matrix – used in quadratic programming
B	Constraint matrix – used in quadratic programming
E	Constraint matrix
F	Constraint matrix
G	Constraint matrix
W	Constraint matrix
$\mathcal{F}$	Constraint matrix
$\mathcal{G}$	Constraint matrix
f	Constraint vector – constraint the constraints of u
g	Constraint vector - constraint the constraints of y
w	Constraint vector - constraint the constraints of Δu

Abbreviations	**Description**
HVAC	Heating, ventilation and air conditioning
AHU	Air handling unit
PU	Pressurization unit
CAV	Constant air volume
VAV	Variable air volume
PI	Proportional integral
MPC	Model Predictive Control

Table of Contents

1. HVAC system

HVAC as shown in Figure 1 is a fundamental part of any building. To maintain air conditioning in the summer and heating in the winter, heat or cold air is supplied to the different zones of a building and is normally controlled thermostatically. (Duraj, 2014)

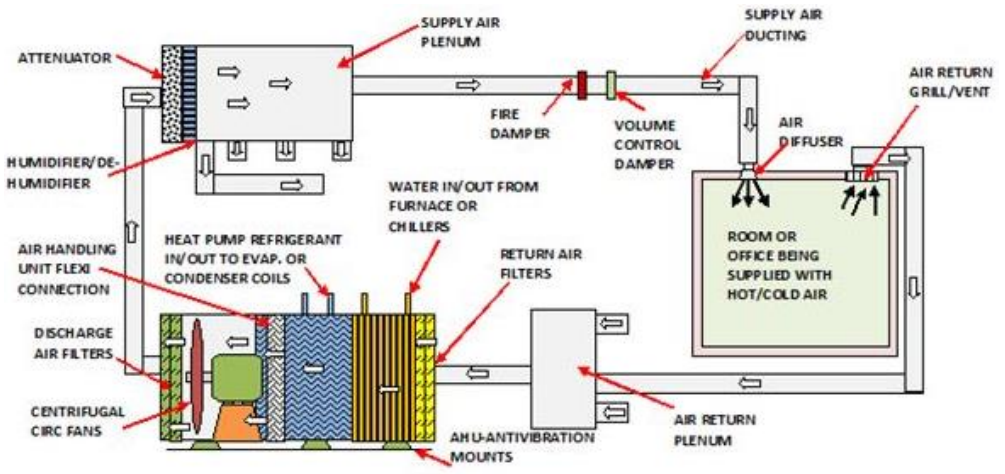

Figure 1 Typical HVAC system showing the ducting distribution and components. (brighthubengineering, n.d.)

The basic components, which they can be described as modern and more efficient units are:

- Air Handling Unit (AHU)
- Filters
- Ducting
- Attenuators
- Volume control and fire/smoke dampers
- Humidifiers and De-humidifiers
- Air return grills
- Air distribution diffusers

The Air handling unit consists of the air circulating fans, the supply and return filters as well as the heating and cooling coils, which supply the hot or chilled water depended on the demand of air hot air or air conditioning respectively. Any other way a combination of an evaporator, condenser and a heat pump can supply the hot and cold air in the unit to the zone.

The fans of the system are the basic device of circulating the heated or cooled air through the supply and return ducting system.

There are suction and discharge filters into the Air Handling Unit (AHU). The suction filters are placed on the return ducting system and their task is to remove any undesirable particles on the air such as dust smoke or fumes before the fan chambers and heating or cooling coils. In addition, the discharge filters function is to abolish any remaining particles into the air before it is circulated again into the supply system ductwork to the man board environment.

The attenuators is a device that is fitted to the ducting system which primary task is to mollify the noise that is created by the circulation of air through the ducting system in order to arrange a less annoying and distracting manpower environment.

Humidifiers function is to use very exquisite spray method in order to drizzle or "spray" water into a dry cold or dry hot air stream. Dehumidifiers on the contrary remove the moisture out of the air stream by driving the wet air into the cooling coils and afterwards deposit the condensed water in a small container, which is placed underneath. Most of the HVAC system building regulations prescribe that the humidity level in the air circulated in the buildings should be maintained in the range of 40% to 60% (Mini-Circuits, 2015). Both devices are of an exceptional importance due to the fact that they maintain healthy environment indoor living conditions and safe operating conditions for the machines. They prevent the growth of molds, mites and mildew into the buildings. Additionally they discard the possibilities of low humidity conditions, which can cause dry skin and throat irritation to the people in the workplace.

The diffusers are located in the zones ceilings and their function is to moderate the discharge velocity of the air stream in order to assure the fair and distribution of the air in the needed direction into the required zones of the building. The louvers of the diffusers are adjusted automatically in order to direct the air into the return ducting system, and they are mounted on the ceilings. The return air stream then is driven into the return ducting system to the discharge plenum, over the return filter and then into the AHU completing in this way the system circuit.

1.1 Variable air volume (VAV) systems

HVAC systems can supply conditioned air into the zone of the buildings by either constant air volume (CAV) or variable air volume (VAV) systems. In the first case, the constant air volume systems are predicated as a high energy consumption systems with not efficient controllability of the indoor zone temperature if compared to the variable air volume systems. Additionally the HVAC systems that include the VAV system are experiencing supplementary passive dehumidification, less noise in the fan and less fatigue in the compressor components.

Variable air volume systems automatically adjust the required indoor zone temperature and pressure in a building. Principally the most important part of a VAV system is the VAV box, which is placed in the terminal of the unit. This particular device-system is connected directly to the indoor zone and its main task is to regulate the required temperature and pressure as well as to counterpoise the fluctuation of the load of these requirements by adjusting the air volume that inserts the zone.

A VAV box is illustrated in Figure 2 p. 2.

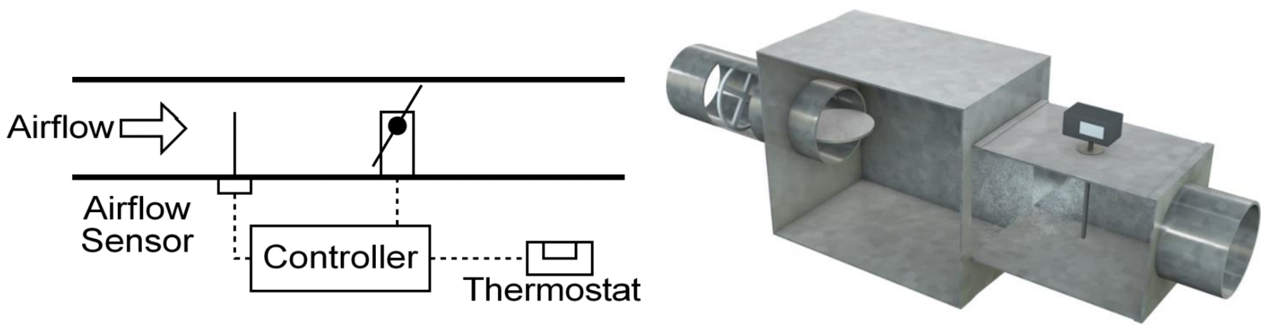

Figure 2 a) VAV box sketch, b) 3D illustration of VAV box. (kmccontrols, n.d.)

The primary air inserts the VAV box after is regulated by the primary air damper which is contained in the terminal unit, and it exists as a secondary airflow into each zone. The control system of the VAV inflects the air damper in various positions of angles between 0° to 90° depending on the volume airflow that is needed into the room. As the angle "drive" the damper to the closure position the volume the pressure difference between the primary and the secondary air flow rises while the volume of the secondary air decrease. Thus, the great advantage of using VAV box terminals is that they provide all the requirements of a comfort and healthy indoor living in a building.

2. Single-Zone modelling

In this chapter, the single-zone model that was found and investigated in the previous project is explained further. In order to carry out the multi-zone model for this project a single zone model has to be firstly introduced.

2.1 Conservation of Energy

In consideration of controlling the pressurization and temperature inside the offshore platform, the energy balance into that zone has to be conserved (Duraj, 2014). In other words, the conservation of energy principle will be used – 1st law of thermodynamic – which states that "energy can be neither created nor destroyed during a process, it can only change forms" (A. Cengel).

The change in the energy content of a system is equal to the difference between the energy input and the energy output, and the conservation of energy principle for any system can be expressed simply as:

$$\Delta E_{zone} = E_{in} - E_{out} \qquad \textit{Eq 2.1}$$

The rate of energy changing is:

$$\Delta \dot{E}_{zone} = \dot{E}_{in} - \dot{E}_{out} \qquad \textit{Eq 2.2}$$

Where,

- $\dot{E}$ is the energy changing rate $\left[\frac{J}{s}\right]$.
- $\Delta\dot{E}$ are the difference in energy changing rate in the zone.

Heat balance in the considered controlled zone depicts that change in energy brings change in temperature:

$$c_p \cdot m_{zone} \cdot \frac{dT_{zone}}{dt} = \dot{Q}_{in} - \dot{Q}_{out} \qquad \textit{Eq 2.3}$$

Where,

- c_p is the heat capacity coefficient of air $\left[\frac{J}{Kg \cdot K}\right]$
- m_{zone} is the mass of the air in the particular space [Kg]
- $\dot{Q}_{in}$ is the heat transfer rate into the space via inlet $\left[\frac{J}{s}\right]$
- $\dot{Q}_{out}$ is the heat transfer rate out of the zone via outlet $\left[\frac{J}{s}\right]$
- $\frac{dT_{zone}}{dt}$ is the change of the temperature T in time dt.

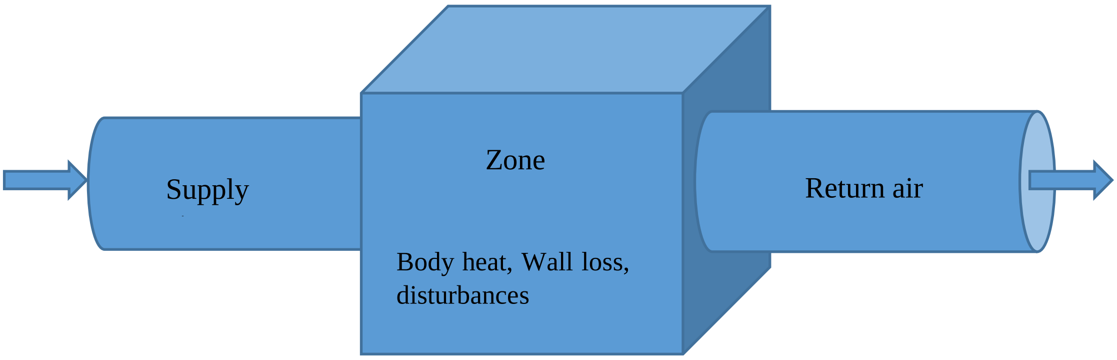

Figure 3 The energy conservation principle. (Pedersen, 2015)

Since the zone will be affected by leakage, radiation from a small amount of sun light, radiators, staff (body heat), computers as well as heat transfer through the walls, the heat transfer equation can be expanded to a more appropriate one as shown in the below equation:

$$c_{air} \cdot m_{zone} \cdot \frac{dT_{zone}}{dt} = \dot{Q}_{in} + \dot{Q}_{out} + \dot{Q}_{rad} + \dot{Q}_{wall} + \dot{Q}_{dist} \qquad \text{Eq 2.4}$$

Where,

- $\dot{Q}_{dist}$ denotes the disturbances such as; body heat, leakage and other appliances $\left[\frac{J}{s}\right]$.
- $\dot{Q}_{rad}$ is the heat transfer from the radiators $\left[\frac{J}{s}\right]$.
- $\dot{Q}_{wall}$ is the heat transfer through the walls $\left[\frac{J}{s}\right]$.

2.2 Inlet heat flow rate

The heat transfer is given by:

$$Q = m \cdot h \qquad \text{Eq 2.5}$$

Where,

- Q is the heat transfer $[J]$
- h is the enthalpy $\left[\frac{J}{kg}\right]$

The change in heat transfer Q can be found form the change in enthalpy between to states. In order to find the equation for the heat transfer rate, the heat entering and leaving a control volume has to be considered.

$$\sum_{in} Q = \sum_{out} Q \qquad \text{Eq 2.6}$$

In general, the overall heat transfer rate is equal to the difference between the sum of heat transfer rate in a zone and the sum of the heat transfer rate that leaves the zone. This statement can be expressed simply by using the below equation:

$$\frac{dQ}{dt} = \sum_{in} \dot{Q} - \sum_{out} \dot{Q} \qquad \text{Eq 2.7}$$

From this, the inlet airflow energy contribution can be determined analytically by the following equation, which was stated above:

$$\dot{Q}_{in} = \dot{m}_{in} \cdot h_{in} - \dot{m}_{in} \cdot h_{zone} \qquad \text{Eq 2.8}$$

The enthalpy can for a ideal gas be given as $h = c_p \cdot T$, therefore the inlet heat transfer rate can be found to be:

$$\dot{Q}_{in} = C_{air} \cdot \dot{m}_{in} \cdot \frac{dT_{zone}}{dt} = C_{air} \cdot \dot{m}_{in} \cdot (T_{in} - T_{zone}) \qquad \text{Eq 2.9}$$

Where,

- $\dot{Q}_{in}$ is the inflow heat transfer rate $\left[\frac{J}{s}\right]$
- c_{air} is the heat capacity coefficient of air $\left[\frac{J}{Kg \cdot K}\right]$
- $\dot{m}_{in}$ is the indoor air mass rate of change $\left[\frac{Kg}{s}\right]$
- T_{in} is the temperature in the duct $[K]$
- T_{zone} is the temperature inside the zone $[K]$.

The indoor air mass rate $\dot{m}_{in}$ can be calculated as:

$$\dot{m}_{in} = \rho_{in} \cdot q_{in} \qquad Eq\ 2.10$$

Where

- ρ_{in} is the density of the air in the duct $\left[\frac{Kg}{m^3}\right]$
- q_{in} is the volumetric flow rate in $\left[\frac{m^3}{s}\right]$ via the inlet duct.

In order to define the volumetric flow rate in the inlet duct Bernoulli's equation is used. This particular equation states that at two points along a streamline in a fluid, all forms of energy are the same (steady flow along a streamline). By using this equation in the generic form (A. Cengel):

$$\int \frac{dp}{\rho} + \left(\frac{1}{2}v^2\right) + g \cdot z = Cst \qquad Eq\ 2.11$$

In order to determine the mechanical energy balance the above principle is re-written:

$$\frac{P}{\rho} + \left(\frac{1}{2}v^2\right) + g \cdot z = Cst \qquad Eq\ 2.12$$

Where,

- P is the static pressure in $[Pa]$
- v is the velocity of the flow $\left[\frac{m}{s}\right]$ along a streamline
- z is the height [m]

Precisely the $\frac{P}{\rho}$ is the dynamic energy, the $\frac{1}{2}V^2$ is the kinetic energy and the $g \cdot z$ is the potential gravitational force which is minimum due to air mass, thereby in this project is neglected:

$$g \cdot z = 0 \qquad Eq\ 2.13$$

From this, *Eq. 2.13* in *Eq.2.12* yields:

$$\frac{P}{\rho} + \left(\frac{1}{2} \cdot v^2\right) = Cst \qquad Eq\ 2.14$$

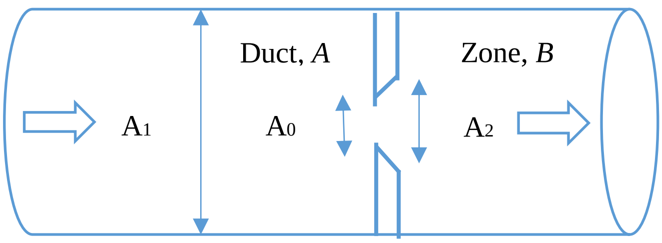

Figure 4 An orifice, damper opening.

If between the damper and the zone there are point A and point B as depicted in Figure 4, taking into account the Bernoulli's principle which states that the inlet flow A and the outlet flow B are equal, results to the following:

$$\frac{P_A}{\rho_A} + \left(\frac{1}{2} \cdot v_A^2\right) = \frac{P_B}{\rho_B} + \left(\frac{1}{2} \cdot v_B^2\right) \qquad Eq\ 2.15$$

Re-arranging the above equation yields in the relationship between the velocities and the pressure differences between the two positions:

$$v_B^2 - v_A^2 = 2 \cdot \left(\frac{P_A}{\rho_A} - \frac{P_B}{\rho_B}\right) \qquad \textit{Eq 2.16}$$

Because point A and B are just before and just after the damper, the density is the same because air has not yet obtained the energy of the zone:

$$\rho = \rho_A = \rho_B \qquad \textit{Eq 2.17}$$

Using *Eq. 2.17* in *Eq. 2.16* results to the following:

$$v_B^2 - v_A^2 = \frac{2}{\rho} \cdot (P_A - P_B) \qquad \textit{Eq 2.18}$$

The velocities in position A and B are unknown in this set of equation thereby they have to be calculated. Taking this into consideration, velocities can be expressed through the equation of the mass flow rate $\dot{m}$ and as a consequence the volumetric flow rate q:

$$\dot{m} = \rho \cdot q \qquad \textit{Eq 2.19}$$

And,

$$q = v \cdot A \qquad \textit{Eq 2.20}$$

As it was stated before the inlet flow in point A before the damper and the outflow in point B just after the damper are equal, which results to:

$$\dot{m}_A = \dot{m}_B \qquad \textit{Eq 2.21}$$

⇔

$$\rho_A \cdot q_A = \rho_B \cdot q_B \qquad \textit{Eq 2.22}$$

⇔

$$v_A \cdot A_A \cdot \rho_A = v_B \cdot A_B \cdot \rho_B \qquad \textit{Eq 2.23}$$

Due to the fact of the changing in the dampers' position, different cross-sectional areas will be created. This fact can be explained by the contraction coefficient C_c which is in other words referred as the vena contacta.

As depicted in the Figure 4, p.5 the different cross-sectional areas A_1 ,A_0 and A_2 can formulate the contraction coefficient as the fraction between the A_0 and A_2:

$$C_c = \frac{A_2}{A_0} \qquad \textit{Eq 2.24}$$

Re-arranged with respect to A_2, due to the fact that it is difficult to actually determine the A_2 area which will be used in following equations:

$$A_2 = C_c \cdot A_0 \qquad \textit{Eq 2.25}$$

Using *Eq. 2.25* in *Eq. 2.23* results in:

$$v_A \cdot A_1 \cdot \rho_A = v_B \cdot (C_c \cdot A_0) \cdot \rho_B \qquad \textit{Eq 2.26}$$

Re-arranging the equation in order to get the flow velocity V_A in the position A:

$$v_A = v_B \cdot \frac{C_c \cdot A_0 \cdot \rho_B}{A_1 \cdot \rho_A} \qquad \textit{Eq 2.27}$$

Using *Eq. 2.27* in *Eq. 2.26* outcomes in finding the velocity of the flow V_B in the point B and as a consequence the flow velocity in the zone, which is very important in order to explain the dynamic of the damper:

$$v_B = \frac{\sqrt{\left[\frac{2}{\rho} \cdot (P_A - P_B)\right]}}{\sqrt{\left[1 - \left(\frac{C_c \cdot A_0 \cdot \rho_B}{A_1 \cdot \rho_A}\right)^2\right]}} \qquad \textit{Eq 2.28}$$

Using Eq.2.28 in Eq. 2.20 results in the following volumetric flow q equation:

$$q = v_{\mathrm{A}} \cdot A_1 = v_{\mathrm{B}} \cdot A_2 = C_c \cdot A_0 \cdot \frac{\sqrt{\left[\frac{2}{\rho} \cdot (P_{\mathrm{A}} - P_{\mathrm{B}})\right]}}{\sqrt{\left[1 - \left(\frac{C_c \cdot A_0 \cdot \rho_B}{A_1 \cdot \rho_A}\right)^2\right]}} \qquad \textit{Eq 2.29}$$

In order to simplify the above equation, the discharge coefficient is introduced as shown below:

$$C_d = \frac{C_c}{\sqrt{\left[1 - \left(\frac{C_c \cdot A_0 \cdot \rho_B}{A_1 \cdot \rho_A}\right)^2\right]}} \qquad \textit{Eq 2.30}$$

Using the discharge coefficient *Eq. 2.30* in *Eq. 2.29* yields:

$$q = A_0 \cdot C_d \cdot \sqrt{\frac{2}{\rho} \cdot (P_{\mathrm{A}} - P_{\mathrm{B}})} \qquad \textit{Eq 2.31}$$

Let $\rho_{\mathrm{A}} = \rho_{in}$, $\rho_{\mathrm{B}} = \rho_{zone}$, $P_{\mathrm{A}} = P_{in}$ and $P_{\mathrm{B}} = P_{zone}$. Substituting these into the above equation results in:

$$q_{in} = A_0 \cdot C_{d,in} \cdot \sqrt{\frac{2}{\rho} \cdot (P_{in} - P_{zone})} \qquad \textit{Eq 2.32}$$

Using the *Eq. 2.32* in *Eq. 2.19* yield:

$$\dot{m}_{in} = \rho_{in} \cdot q_{in} = \rho_{in} \cdot A_0 \cdot C_{d,in} \cdot \sqrt{\frac{2}{\rho} \cdot (P_{in} - P_{zone})} \qquad \textit{Eq 2.33}$$

As a conclusion of all the above, by substituting the *Eq. 2.33* into *Eq. 2.9* results in the following generic equation of the energy contribution equation of the inlet airflow:

$$\dot{Q}_{in} = C_{air} \cdot \rho_{in} \cdot A_0 \cdot C_{d,in} \cdot \sqrt{\frac{2}{\rho} \cdot (P_{in} - P_{zone})} \cdot (T_{in} - T_{zone}) \qquad \textit{Eq 2.34}$$

Precisely the above equation describes the heat transfer rate of the input damper and it can be substituted in the general energy balance equation as it was structured in the *Eq. 2.4*

2.3 Outlet heat flow rate

The outflow air heat transfer rate $\dot{Q}_{out}$ is determined analytically by the following equation, using the same procedure as for $\dot{Q}_{in}$:

$$\dot{Q}_{out} = C_{air} \cdot \dot{m}_{out} \cdot (T_{zone} - T_{out}) \qquad \textit{Eq 2.35}$$

Where,

- $\dot{Q}_{out}$ is the outflow heat transfer rate $\left[\frac{J}{s}\right]$
- $\dot{m}_{out}$ is the outdoor air mass rate $\left[\frac{Kg}{s}\right]$
- T_{out} is the air temperature in the outlet duct $[K]$.

The indoor air mass rate of change $\dot{m}_{out}$ can be calculated as:

$$\dot{m}_{out} = \rho_{zone} \cdot q_{out} \qquad \textit{Eq 2.36}$$

Where, q_{out} is the volumetric flow rate $\left[\frac{m^3}{s}\right]$ and ρ_{out} is the density in the outlet duct $\left[\frac{kg}{m^3}\right]$.

$$q_{in} \geq q_{out}$$

The above restriction is stated as mandatory limitation for the purpose of this project since the pressurization indoor has to be maintained $25\ Pa - 50\ Pa$ over the atmospheric pressure, as the safety regulation states.

Repeating the same procedure as in the air inlet energy model subchapter, in order to find the outflow air heat energy transfer rate $\dot{Q}_{out}$ the volumetric outflow q_{out} has to be calculated:

$$q_{out} = C_{d,out} \cdot A_{0,out} \cdot \sqrt{\frac{2}{\rho} \cdot (P_{zone} - P_{out})} \qquad \textit{Eq 2.37}$$

Where P_{out} is the pressure in the outlet duct $[Pa]$.

For the outlet damper the volumetric outflow is modelled in the same way as for the inlet dampers. The different cross-sectional areas A_1 , A_0 and A_2 can formulate the contraction coefficient as the fraction between the A_0 and A_2:

$$C_c = \frac{A_2}{A_0} \qquad \textit{Eq 2.38}$$

Considering the above, the discharge coefficient C_d is calculated in the same way as for the volumetric inflow in the inlet canal.

$$C_{d,out} = \frac{C_c}{\sqrt{\left[1 - \left(\frac{C_c \cdot A_{0,out} \cdot \rho_{out}}{A_1 \cdot \rho_{zone}}\right)^2\right]}} \qquad \textit{Eq 2.39}$$

Taking into consideration *Eq. 2.39* in *Eq. 2.37* results to:

$$q_{out} = A_{0,out} \cdot \frac{C_c}{\sqrt{\left[1 - \left(\frac{C_c \cdot A_{0,\text{out}} \cdot \rho_{in}}{A_1 \cdot \rho_{zone}}\right)^2\right]}} \cdot \sqrt{\frac{2}{\rho} \cdot (P_{zone} - P_{out})} \qquad \textit{Eq 2.40}$$

Using the above equation to calculate the rate of the outflow air mass $\dot{m}_{out}$:

$$\dot{m}_{out} = \rho_{zone} \cdot C_{d,out} \cdot A_{0,out} \cdot \sqrt{\frac{2}{\rho} \cdot (P_{zone} - P_{out})} \qquad \textit{Eq 2.41}$$

The equation of the energy contribution of the air outflow can be rewritten by substituting *Eq. 2.41* in *Eq. 2.35*, which results in the following generic form:

$$\dot{Q}_{out} = C_{air} \cdot \rho_{zone} \cdot C_{d,out} \cdot A_{0,out} \cdot \sqrt{\frac{2}{\rho} \cdot (P_{zone} - P_{out})} \cdot (T_{zone} - T_{out}) \qquad \textit{Eq 2.42}$$

The above equation describes the heat transfer rate of the outlet pipe and it can be substituted in the general energy balance equation as it was structured in *Eq. 2.4.*

2.4 Wall heat transfer rate

In reality, there will be some heat transfer through the walls due to a temperature difference between the zones and to the ambient condition, depending on the weather conditions. In offshore structures, the walls are extremely well isolated. The conduction can be expressed in terms of Fourier's equation, which is a 3-dimensional PDE with respect to time (A. Cengel). For simplicity the timeless and one-dimensional equation is considered, which is given by:

$$\dot{Q}_{wall} = k_{wall} \cdot A_{wall} \cdot \frac{(T_{atm} - T_{zone})}{L_{wall}} = U_{wall} \cdot A_{wall} \cdot (T_{atm} - T_{zone}) \qquad \textit{Eq 2.43}$$

where,

- k_{wall} is the thermal conductivity of the wall $\left[\frac{W}{m \cdot K}\right]$
- A_{wall} is the surface area of the walls $[m^2]$
- T_{atm} is the outside air temperature $[K]$
- L_{wall} is the wall thickness $[m]$
- U_{wall} is the heat loss coefficient, u-value $\left[\frac{W}{m^3 K}\right]$

The principle of the wall heat transfer is depicted in the figure below, where it is assumed that the wall consists of one material.

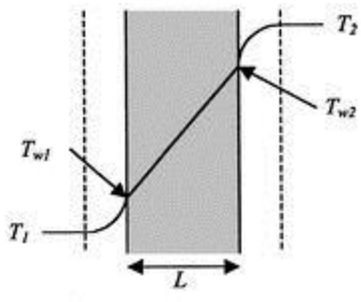

Figure 5 Heat loss through a wall. (MIT, 2015)

2.5 Radiators heat transfer rate

A model for the energy contribution due to the radiator $\dot{Q}_{rad}$ is taken into account and developed. Note that, the notation *rad* does not refer to radiation in this case.

It is assumed that the surface temperature of the radiator is constant and it maintains the required steady state temperature in the zone. Having as a guidance the above limits, the modeling of the heat energy contribution is calculated by using Newton's second law of cooling, which states that (A. Cengel):

$$\dot{Q}_{rad} = \bar{h} \cdot A_{rad} \cdot (T_{rad} - T_{zone}) \qquad \textit{Eq 2.44}$$

Where,

- $\dot{Q}_{rad}$ is the heat energy transfer rate due to the radiator $\left[\frac{J}{S}\right]$
- $\bar{h}$ is the average convection coefficient for the entire surface $\left[\frac{W}{m^2 \cdot K}\right]$
- A_{rad} is the surface area of the radiator $[m^2]$
- T_{rad} is the constant surface temperature $[K]$

To calculate $\bar{h}$ the following equation is used:

$$\bar{h} = Nu \cdot \frac{K_{air}}{H_{RH}} \qquad \textit{Eq 2.45}$$

Where,

- Nu is the Nusselts number [-]
- H_{RH} is the height of the radiator [m]

The Nusselts number is found as:

$$Nu = c \cdot Ra^{n} \qquad \textit{Eq 2.46}$$

The Nusselts number is defined as the ratio of convective to conductive heat transfer across a boundary within a fluid under the same condition, in our case the surface of the radiator. It has to be noticed that in convection term the diffusion as well as the advection are included.

And K_{air} is the heat conductivity of the air, which in our case is:

$$K_{air} = 0.0261 \frac{W}{m \cdot K} \qquad \textit{Eq 2.47}$$

Both c and n are constants which depend on the radiator's surface geometry and the flow regime. Normally the value of c is less than one and the value of n is equal to $\frac{1}{3}$ if it is a turbulent flow, and n is equal to $\frac{1}{4}$ if it is a laminar flow. More analytically, after calculating the Rayleigh number is compared in order to fit in a range of two different cases and substitute the values of c and n, meaning that (A. Cengel):

If $10^4 > Ra > 10^9$ then $Nu = 0.59 \cdot Ra^{\frac{1}{4}}$ so $n = \frac{1}{4}$ and $c = 0.59$

If $10^9 > Ra > 10^{13}$ then $Nu = 0.1 \cdot Ra^{\frac{1}{3}}$ so $n = \frac{1}{3}$ and $c = 0.1$

The Rayleigh number Ra [-] is calculated as it is depicted below:

$$Ra = \frac{g \cdot \beta \cdot H_{RH}^{3} \cdot (T_{rad} - T_{zone})}{v_k \cdot \alpha} \qquad \textit{Eq 2.48}$$

Where,

- v_k is the kinematic viscosity of the fluid $\left[\frac{m^2}{s}\right]$
- α is just a substance constant $\left[\frac{m^2}{s}\right]$
- β is the thermal expansion coefficient $[K^{-1}]$, which is defined as:

$$\beta = \frac{1}{T_f} \qquad \textit{Eq 2.49}$$

Where,

$$T_f = \frac{T_{rad} - T_{zone}}{2} \qquad \textit{Eq 2.50}$$

The kinematic viscosity for air is found as (engineeringtoolbox):

$$v_k = 1.5647 \cdot 10^{-5} \frac{m^2}{s}$$

In addition, the substance constant is:

$$\alpha = 2.2142 \cdot 10^{-5} \frac{m^2}{s}$$

2.6 Overall zone temperature model

The final equation for the change in zone temperature is:

$$\frac{dT_{zone}}{dt} = \frac{\dot{Q}_{in} + \dot{Q}_{out} + \dot{Q}_{rad} + \dot{Q}_{wall}}{c_{air} \cdot m_{zone}} \qquad \textit{Eq 2.51}$$

⇔

$$\begin{aligned}\frac{dT_{zone}}{dt} = &\frac{C_{air} \cdot \rho_{in} \cdot A_0 \cdot C_{d,in} \cdot \sqrt{\frac{2}{\rho} \cdot (P_{in} - P_{zone})} \cdot (T_{in} - T_{zone})}{c_{air} \cdot m_{zone}} \\ &+ \frac{C_{air} \cdot \rho_{zone} \cdot C_{d,out} \cdot A_{0,out} \cdot \sqrt{\frac{2}{\rho} \cdot (P_{zone} - P_{out})} \cdot (T_{zone} - T_{out})}{c_{air} \cdot m_{zone}} \\ &+ \frac{(\bar{h} \cdot A_{rad} \cdot (T_{rad} - T_{zone}) + (U_{wall} \cdot A_{wall} \cdot (T_{atm} - T_{zone}))}{c_{air} \cdot m_{zone}}\end{aligned} \qquad \textit{Eq 2.52}$$

2.7 Conservation of mass

The mass rate balance is equal to the difference of inlet and outlet mass flow rate, and it is given by the following equation:

$$\frac{dm_{zone}}{dt} = \dot{m}_{in} - \dot{m}_{out} \qquad \textit{Eq 2.53}$$

⇔

$$\frac{dm_{zone}}{dt} = \rho_{air} \cdot C_{d,in} \cdot A_{0,in} \sqrt{\frac{2(P_{in} - P_{zone})}{\rho_{in}}} - \rho_{zone} \cdot C_{d,out} \cdot A_{0,out} \sqrt{\frac{2(P_{zone} - P_{out})}{\rho_{zone}}} \qquad \textit{Eq 2.54}$$

2.8 Ideal gas law

The system can be assumed that works under ideal conditions at all times. Thereby the density and pressure can be determined in the zones from the ideal gas law.

$$p_{zone} \cdot v_{zone} = n_{zone} \cdot R_{zone} \cdot T_{zone} \qquad \textit{Eq 2.55}$$

The density in the zone ρ_{zone} is calculated as the correlation between the mass and the volume of the zone.

$$\rho_{zone} = \frac{m_{zone}}{v_{zone}} \qquad \textit{Eq 2.56}$$

⇔

$$\frac{p_{zone}}{\rho_{zone}} = R_{zone} \cdot T_{zone} \qquad \textit{Eq 2.57}$$

⇔

$$P_{zone} = R_{zone} \cdot T_{zone} \cdot \rho_{zone} \qquad \textit{Eq 2.58}$$

From the above, the mass balance becomes:

$$\dot{m}_{zone} = \rho_{air,p} \cdot C_{d,in} \cdot A_{0,in} \sqrt{\frac{2 \cdot (p_{pipe} - p_{zone})}{\rho_{pipe}}} - \left(\frac{m_{zone}}{V_{zone}}\right) \cdot \mathrm{C_{d,o}} A_{0,out} \sqrt{2 \cdot \left(R \cdot T_{zone} - \frac{p_{atm}}{\left(\frac{m_{zone}}{V_{zone}}\right)}\right)} \qquad \textit{Eq 2.59}$$

2.9 Fan

The PU consists of a centrifugal fan, which delivers the needed amount of airflow in order to maintain the indoor air pressure. This is achieved by choosing the operating speed of the fan. In order to find the frequency which relates to the operating speed, the fan curve is used. After the selection of the fan, the fan curve is provided by the manufacture. The fan is chosen based on the systems resistance and required flow rate. In this case, the selected fan is presented in Chapter 4, as the fan has to be able to fulfil the requirement of multiple zones.

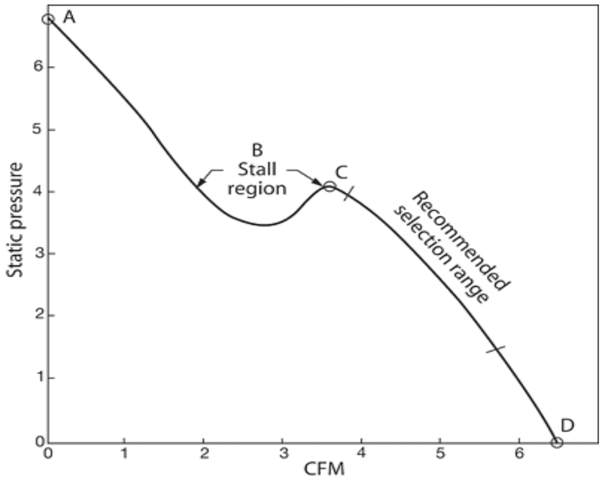

Figure 6 A typical fan curve, relation between static pressure and cubic feet per minute. (esmagazine, n.d.)

2.9.1 Flow development through a centrifugal fan into a duct

The selection of a fan that fulfils the system requirements is of great importance. Subsequently it is important to look at the flow development after the fan, so the duct length for the inlet is long enough. The flow development is illustrated in the Figure 7.

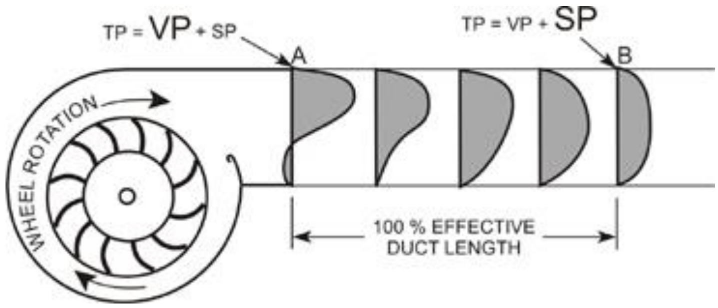

Figure 7 Flow development through a duct. (achrnews, n.d.)

The total pressure ($TP = P_{tot}$) is the sum of the velocity pressure ($VP = P_{dyn}$) and the static pressure ($SP = P_{static}$) as is introduced as (achrnews, n.d.):

$$P_{tot} = P_{dyn} + P_{static} \qquad \text{Eq 2.60}$$

Total pressure is defined as the measure of the energy content of the airstream that is always dropping as the flow proceeds downstream. P_{tot} it can be measured by using a pitot tube pointing directly upstream, connected to a manometer. Additionally, P_{tot} it can take positive and negative values with respect to the atmospheric pressure.

The length between the point at the outlet of the centrifugal fan and downstream the duct where the flow is fully developed is defined as effective duct length (L_e) and is calculated for a turbulent flow by *Eq 2.61* (A. Cengel):

$$L_e = 10 \cdot D \qquad \text{Eq 2.61}$$

The static pressure causes the air in the duct to flow. On the other hand, the velocity pressure results from the air movement and is defined as the pressure that is required to accelerate from zero to some desired velocity. P_{dyn} is proportional to the kinetic energy of the stream of the flow. From the above, it is desirable to have a high P_{static} compared to P_{dyn} develop by the fan.

The process of the conversion the velocity pressure P_{dyn} to static pressure P_{static} in terms of decrease of velocity and total pressure and increase of static pressure is known as static regain.

2.9.2 Fan and Inlet

The movement of the fan wheel pushes the air against the outside of the scroll. Therefore, at the top of the outlet of the fan (point A), as shown in Figure 7 in the velocity profile, there is a “high” velocity whereas at the bottom there is a negative velocity due to the rotating of the air back to the fan at the cut off attempting to re-enter in the fan.

As the air moves towards the duct becomes more uniform across the duct, which means that at point B where the flow is fully developed ,as shown in the figure 1, the static pressure will be higher in value than the velocity pressure compared to the point A.

At the time that the flow is fully developed (point B) the system has gained the static regain. The total pressure is approximately the same from point A to B, only the velocity and static pressure increases or decreases respectively.

In order to achieve the best fan performance is advised as a technical prerequisite that at the outlet of the fan keep the duct as straight as possible and avoid the fittings near the fan outlet in order to eliminate the system effect.

2.9.3 Fan curve

In order to obtain the fan curves for different frequencies from the one provided by the manufacture an approximation can be made. The fan curve can be approximated by a second order polynomial by use of at least three points from the original curve.

The approximation is:

$$\Delta P_t(q_{in}) = a_0 + a_1 \cdot q_{in} + a_2 \cdot q_{in}^2 \qquad \text{Eq 2.62}$$

Where,

- ΔP_t is the differential pressure across the fan $[Pa]$
- The parameters a_1, a_2, a_3

The parameters can be approximated by least squares method. It is an estimation method, which considers the minimum of squared errors (System Identification, Least squares estimation, 2014).

$$\Theta_{Ls} = (\Psi^T \cdot \Psi)^{-1} \cdot \Psi^T \cdot Y \qquad \text{Eq 2.63}$$

Where,

- Y in this case is the point for the pressure difference
- Ψ is the regression vector, containing q_{in} in form of:

$$\Psi = \begin{bmatrix} 1 & q_{in} & q_{in}^2 \end{bmatrix} \qquad \textit{Eq 2.64}$$

2.9.3.1 Affinity laws

After the polynomial for the original fan curve is found, the fan curves for different fan speeds can be found by use of the Affinity laws.

The affinity laws can be expressed in terms of:

$$\text{Volume capacity:} \quad \frac{q_1}{q_2} = \frac{n_1}{n_2} \cdot \frac{d_1}{d_2} \quad \rightarrow \quad \frac{q_1}{q_2} = \frac{n_1}{n_2}$$

$$\text{Head or Pressure:} \quad \frac{dp_{1_1}}{dP_2} = \left(\frac{n_1}{n_2}\right)^2 \cdot \frac{d_1}{d_2} \quad \rightarrow \quad \frac{dp_{1_1}}{dP_2} = \left(\frac{n_1}{n_2}\right)^2 \qquad \textit{Eq 2.65}$$

$$\text{Power:} \quad \frac{P_1}{P_2} = \left(\frac{n_1}{n_2}\right)^3 \cdot \frac{d_1}{d_2} \quad \rightarrow \quad \frac{P_1}{P_2} = \left(\frac{n_1}{n_2}\right)^3$$

Where,

- n is the operating speed of the fan $[RPM]$
- 1 and 2 denotes at the previously operating condition and the current operating condition of the fan, respectively

The diameter change is eliminated in the equation since only the frequency of the fan can be changed.

An asynchronous motor controls the fan speed, where the frequency driver controls the speed of the motor, given by the following equation:

$$\omega(t) = n = 2\pi(1 - s) \cdot \frac{f}{p \cdot N} \qquad \textit{Eq 2.66}$$

Where,

- f is the frequency $[Hz]$.
- p is the number of poles.
- N is the gear ratio.
- s is the slip.

The slip is assumed to be constant. Thereby the following can be found:

$$\text{volume capacity:} \quad q_2 = q_1 \frac{f_2}{f_1}$$

$$\textit{Eq 2.67}$$

$$\text{Pressure:} \quad dP_2 = dP_1 \left(\frac{f_2}{f_1}\right)^2$$

From these equations, the pressure and inflow rate can be determined for different demands of frequency based on the values provided from the fan curve of the manufacture.

2.10 System curve

The system curve is a representation of how much resistance or losses exist throughout the pipe system. It is a relation between the flow rate and differential pressure over the fan.

Using the Bernoulli equation, the sum of all pressure loss between two points along a streamline in a duct can be found to be:

$$P_A + \frac{1}{2} \cdot \rho \cdot v_A^2 + \rho \cdot g \cdot h_A + \Delta P_t = P_B + \frac{1}{2} \cdot \rho \cdot v_B^2 + \rho \cdot g \cdot h_B + f \cdot \rho \cdot g \qquad \text{Eq 2.68}$$

Where,

- ΔP_t is the differential pressure over the fan [Pa]
- f accounts for the friction in the ducting system.

Rearranging for ΔP_t:

$$\Delta P_t = (P_B - P_A) + \frac{v_B^2 - v_A^2}{2} + (h_B - h_A) \cdot \rho \cdot g + f \cdot \rho \cdot g \qquad \text{Eq 2.69}$$

The pressure loss due to friction can be expressed as:

$$f \cdot \rho \cdot g = k \cdot q_{in}^2 \cdot \rho \cdot g \qquad \text{Eq 2.70}$$

Where,

- k is the resistance coefficient [-]

The remaining of the equation can be given as the static pressure that the fan needs to raise:

$$P_{static} = (P_B - P_A) + \frac{v_A^2 - v_B^2}{2} \rho + (h_B - h_A) \cdot \rho_A \cdot g \qquad \text{Eq 2.71}$$

Assuming that the velocity do not have significant changes at the two points, and that $P_A = P_{in}$, $P_B = P_{zone}$ and $\rho_A = \rho_{in}$, the static pressure becomes:

$$P_{static} = (P_{zone} - P_{in}) + (h_2 - h_1) \cdot \rho_{in} \cdot g \qquad \text{Eq 2.72}$$

The differential pressure across the fan then becomes:

$$\Delta P_t = P_{static} + k \cdot q_{in}^2 \cdot \rho_{in} \cdot g \qquad \text{Eq 2.73}$$

Where k is the resistance coefficient, which includes all the minor and major losses in the system such as duct bending, roughness of the ducts material, damper openings etc. This coefficient is introduced further in the chapter for the multiple zone modelling, Chapter 3.4.

2.10.1 Operating point

To achieve the desired pressure or flow rate into the zone a specific operating frequency and differential pressure of the fan is needed. To obtain this, the intersection of the fan and system curves has to be found. This idea is illustrated in Figure 8.

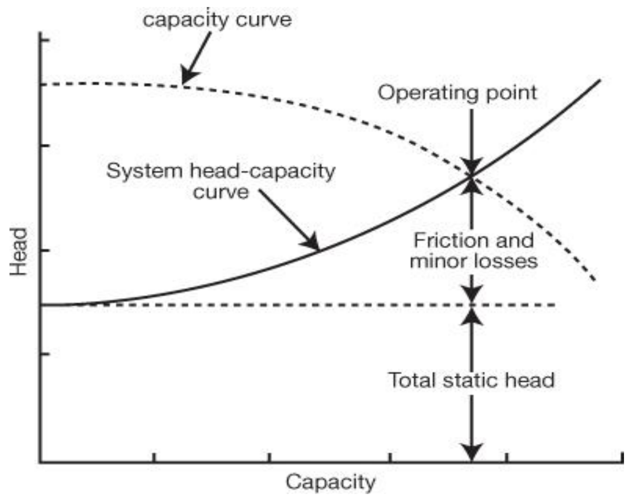

Figure 8 The Operating point. (Controlglobal, u.d.)

As the condition in the system changes, such as damper openings, the resistance and thereby the system curve will change. In order to maintain the required flow rate, the frequency needs to be altered.

3. Model extension

3.1 Heat transfer

In the industrial conditioning the values of temperature and humidity are lying within well-defined limits in order to be carried out the preferred results of the system. In other words the picture of the conditioning of the comfort zone in an industrial or scientific process is quite different if compared with any other appliance of HVAC systems. The choice of the inside conditions is based clearly on what is wanted and not based on statistical survey outcomes. This means that a change of the limits may devastate the preferred work that is already done.

The pre-investigated model will in this section be extended in forms of considering the fraction of return air and the fraction of outside air coming into the mixing box. In addition to this, the moisture or humidity is only considered as a part of the mass balance. The mass balance consider the dry air and water vapor separately for simplicity.

The system is depicted in the block diagram below.

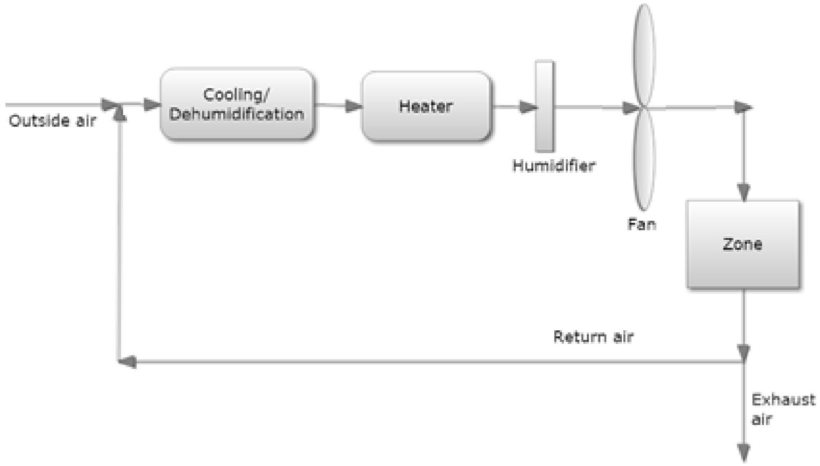

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Figure 9 Block diagram of the whole system.

For the investigation of the humidity changes in the mass balance is only considered when the outside air and return air are mixed, in the cooling coil and in humidifier, when dehumidification and humidification occurs. It is assumed that no extraction of moisture occur in the heating coil. Furthermore, it is assumed that the fan has no influence in the temperature and provides only with changes in the volumetric airflow.

3.1.1 Mixing Box

In the mixing-box the fraction of air that has left the zone and was not exhausted, returns to be mixed with a new amount of outside air. This is done both to provide savings in the heating and to maintain a “fresh-air” quality.

The block diagram of the process is illustrated in Figure 10.

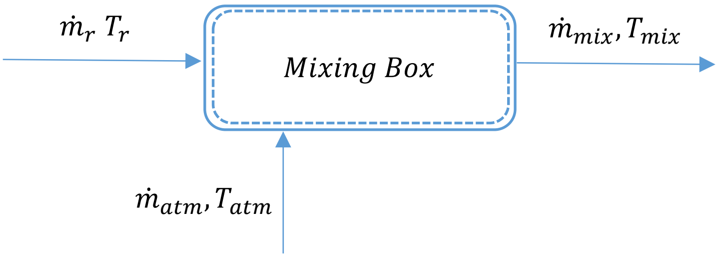

Figure 10 The mixing process.

When the two flow streams are mixed, both heat and moisture is changed due to their previously environment is different in both temperature and humidity.

The return air can be modelled as a fraction of the heat transfer, which leaves the zone:

$$\dot{Q}_r = \beta \cdot \dot{m}_{a,out} \cdot h_{out} \qquad \textit{Eq 3.1}$$

Where,

- β is the fraction of air which is not exhausted [-].
- $\dot{m}_{a,out}$is the outlet mass flow rate $\left[\frac{kg}{s}\right]$ of the zone.
- h_{out} is the enthalpy of the outlet of the zone $\left[\frac{J}{kg}\right]$.
- $\dot{Q}_r$ is the heat transfer rate of the return air $\left[\frac{J}{s}\right]$.

For simplicity in the equations it is assumed to be:

$$\dot{Q}_r = \dot{m}_{a,r} \cdot h_r \qquad \textit{Eq 3.2}$$

Where,

- $\dot{m}_{a,r}$ is a fraction β of $\dot{m}_{a,out}$
- $h_r = h_{out}$.

The amount of mass flow from the outside into the mixing chamber is based on a specific mixing temperature. The heat transfer from outside is expressed as:

$$\dot{Q}_{atm} = \dot{m}_{a,atm} \cdot h_{atm} \qquad \textit{Eq 3.3}$$

Where,

- $\dot{m}_{atm}$ is the mass flow of outside air $\left[\frac{kg}{s}\right]$.
- h_{atm} is the enthalpy of the outside air $\left[\frac{J}{kg}\right]$.
- $\dot{Q}_{atm}$ is the heat transfer rate of the outside air $\left[\frac{J}{s}\right]$.

The heat transfer balance of the mixing box can be found from:

$$\sum_{in} \dot{m}h = \sum_{out} \dot{m}h \qquad \textit{Eq 3.4}$$

Which leads to:

$$\dot{Q}_r + \dot{Q}_{atm} = \dot{Q}_{mix} \quad \rightarrow \dot{m}_{a,r} \cdot h_r + \dot{m}_{a,atm} \cdot h_{atm} = \dot{m}_{a,mix} \cdot h_{mix} \qquad \text{Eq 3.5}$$

⇔

$$\dot{m}_{a,mix} \cdot (h_{atm} - h_r) = \dot{m}_{a,mix} \cdot h_{mix} \qquad \text{Eq 3.6}$$

Substituting for the enthalpy, the expression can be altered to:

$$\dot{Q}_{mix} = \dot{m}_{a,mix} \cdot c_{air} \cdot (T_r - T_{atm}) \qquad \text{Eq 3.7}$$

Where the mass flow rate of dry air is:

$$\dot{m}_{a,mix} = \dot{m}_{a,r} + \dot{m}_{a,atm} \qquad \text{Eq 3.8}$$

And,

- $\dot{m}_{a,mix}$ is the mass flow rate $\left[\frac{Kg}{s}\right]$ in the mixing box.
- T_r is the temperature $[K]$of the return air.
- T_{atm} is the outside temperature $[K]$.

3.1.2 Cooling coil heat transfer

Heat transfer to cooling coil can be described in three stages:

- Airstream transfers heat to the fins and pipes outer surface.
- Then, heat is transferred through the metal of the fins and the wall of the piping.
- Final step, concludes in the main stream of the coolant by passing through the inner walls of the tubes through the surface film of the cooling fluid.

The process of the cooling coil is depicted below.

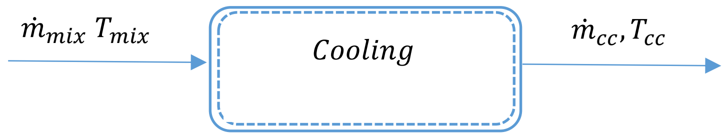

Figure 11 The Cooling process.

In this process, the air is cooled and some heat is extracted from the air, therefore a change in the heat transfer will occur:

$$\sum_{in} \dot{m}h = \dot{Q}_{cc} + \sum_{out} \dot{m}h \qquad \text{Eq 3.9}$$

Where $\dot{Q}_{cc}$ is the heat transfer rate $\left[\frac{J}{s}\right]$, of which the air is cooled down. The heat flow balance for a cooling coil is:

$$\dot{m}_{a,mix} \cdot h_{mix} = \dot{m}_{a,cc} \cdot h_{cc} + \dot{Q}_{cc} \qquad \text{Eq 3.10}$$

The dynamic heat transfer rate of the dry air that is applied in the cooling coil is described by the following fundamental equation (Bourhan, Molhim, & Al-Rousan, 2004):

$$C_{cc} \cdot \frac{dT_{cc}}{dt} = \dot{m}_{mix} \cdot C_{air} \cdot (T_{mix} - T_{cc}) - \dot{m}_{wcc} \cdot C_{pw} \cdot (T_{cc_{w,in}} - T_{cc_{wout}}) \qquad Eq\ 3.11$$

⇔

$$C_{cc} \cdot \frac{dT_{cc}}{dt} = \dot{m}_{mix} \cdot C_{air} \cdot (T_{mix} - T_{cc}) - \dot{m}_{wcc,max} \cdot Opening_{CC} \cdot C_{pw} \cdot (T_{cc_{w,in}} - T_{cc_{wout}}) \qquad Eq\ 3.12$$

Eq 3.12 was used instead of *Eq 3.11*, in order to define the range of the unknown mass flowrate $\dot{m}_{wcc}$ it was therefore used a fixed maximum flow rate. Assuming when the valve is fully open the maximum flow rate is achieved, the $\mathrm{Opening}_{CC}$ indicates how much the valve is open. This approach neglects the orifice equation for the valve and thereby it is assumed a linear relationship between the opening and the mass flow rate.

Where,

$$C_{cc} = m_{cc} \cdot C_{air} \qquad Eq\ 3.13$$

And,

- $m_{cc}$is the mass $[kg]$ of the air inside the cooling coil
- T_{cc} is the temperature$[K]$ of the air inside the cooling coil
- T_{mix} $[K]$ is the temperature of the mixed air inserted to the cooling coil after the mixing box
- $\dot{m}_{wcc}$ is the mass rate $\left[\frac{Kg}{s}\right]$ of the water that flow through the cooling pipe and transfers the temperature of the water into the cooling coil in order to achieve a change (lower) temperature of the air exerted from the cooling coil.
- $T_{cc_{w,in}}$ and $T_{cc_{w,out}}$ are the temperature $[K]$ of the water that come in and out of the cooling coil pipe respectively.
- $Opening_{CC}$ is the opening of the valve in terms of how many percentage flow rate is let through.
- $\dot{m}_{wcc,max}$ is the maximum flow rate $\left[\frac{Kg}{s}\right]$ of the cooling oil

3.1.3 Heating coil heat transfer

The heating process is depicted below.

Figure 12 The heating process.

When the cooled air flow through the heater, no addition in flow rate occurs:

$$\dot{m}_{a,mix} = \dot{m}_{a,hc} \qquad Eq\ 3.14$$

Where $\dot{m}_{a,hc}$ is the mass flow rate out of the heating coil. Through the heater, additional heat is added making the energy balance:

$$\sum_{in} \dot{m}h + \dot{Q}_{hc} = \sum_{out} \dot{m}h \qquad \textit{Eq 3.15}$$

Where $\dot{Q}_{hc}$ is the heat transfer rate $\left[\frac{J}{s}\right]$ of which the air is heated.

The static heat transfer balance therefore becomes:

$$\dot{Q}_{hc} + \dot{m}_{a.mix} \cdot h_{mix} = \dot{m}_{a,hc} \cdot h_{hc} \;\; \rightarrow \;\; \dot{Q}_{hc} = \dot{m}_{a,mix} \cdot (h_{hc} - h_{mix}) \qquad \textit{Eq 3.16}$$

The dynamic heat energy transfer rate by the heating coil is being introduced and calculated by the equation showed below:

$$C_{hc} \cdot \frac{dT_{hc}}{dt} = \dot{m}_{mix} \cdot C_{air} \cdot (T_{hc} - T_{cc}) - \dot{m}_{whc} \cdot C_{pw} \cdot \left(T_{hc_{win}} - T_{hc_{wout}}\right) \qquad \textit{Eq 3.17}$$

⇔

$$C_{hc} \cdot \frac{dT_{hc}}{dt} = \dot{m}_{mix} \cdot C_{air} \cdot (T_{hc} - T_{cc}) - \dot{m}_{whc,max} \cdot Opening_{HC} \cdot C_{pw} \cdot \left(T_{hc_{win}} - T_{hc_{wout}}\right) \qquad \textit{Eq 3.18}$$

Where,

$$C_{hc} = m_{hc} \cdot C_{air} \qquad \textit{Eq 3.19}$$

And,

- m_{hc} is the mass $[kg]$ of the air inside the heating coil.
- $T_{hc}$is the temperature $[K]$ of the air inside the heating coil.
- $T_{cc}$is the temperature [K] after the cooling coil.
- $\dot{m}_{whc}$ is the mass rate $\left[\frac{Kg}{s}\right]$of the water that flow through the heating pipe.
- $\dot{m}_{whc,max}$ is the maximum mass flow rate $\left[\frac{Kg}{s}\right]$ of water in the heating oil
- $Opening_{HC}$ is the opening of the valve of the heating coil

Note that, the minus sign in *Eq 3.18* do not result in heat extracted form the flow. The negative sign is due to a negative coil temperature different, $\left(T_{hc_{win}} - T_{hc_{wout}}\right)$, because the temperature leaving the heater will be less than what is entering. The temperature leaving the heating coil is the inlet temperature, T_{in} which is used in the single zone model.

3.2 Humidity

The humidity is an important factor for the offshore platforms both in case for manned and un-manned. In case of a manned platform, the humidity influences the comfort and health condition of the crew. If the humidity becomes too low, dry skin and irritation can occur, on the other hand if it is too high it can cause less sweet to evaporate and thereby the function to dissipate heat from the body becomes less. In addition, the critical thinking is weakened.

For the unmanned case, the equipment and electrical components are affected. In addition a too high humidity will cause increases in corrosion. Carbon steel is used for some components inside the buildings, therefore low humidity is needed, since a relative humidity above 60% will result in start of corrosion affects . Normally 65% is the required maximum. On the other hand having a too low humidity will cause a large increase in electrostatic charges (ESC). The ESC can result in either overheating of component, thereby causing them not to work proper, or to fail in future operation.

Therefore, the control of the humidity is very important. In this section different perspectives of the humidity is introduced and a proposed model is found. Furthermore, different important terms related to the humidity are explained.

3.2.1 Dry air

Often only the heat transfer is studied when modelling an HVAC-system but when the moisture or also referred to as humidity has to be accounted for, the existence of another gas in the air has to be considered. Per definition (Cengel & Boles), the humidity is a relationship between how much dry air and water vapor there is in an amount of air. This means that instead of considering only one-phase gas, a two-phased gas mixture has to be modelled. In this project the two phases are considered separately. In case of more moisture is added the water vapor mass is increased and vice versa. The water vapor is also referred to saturated water.

Air contains different gases in different amount. Normally air contains some water vapor, this is referred to as atmospheric air, whereas dry air contains no water vapor. It is beneficial to consider the dry air since the composition do almost not change but the vapor changes due to condensation and evaporation throughout the air handling unit.

3.2.2 Saturated Temperature and Pressure

The saturation temperature and pressure is related to when a pure substance changes its phase, e.g. when water starts to boil. The temperature for the boiling point for a constant pressure is called the saturated temperature, T_{sat}. For constant temperature, a saturated pressure can be found in the same manor, P_{sat}. (A. Cengel)

A relation between the temperature, pressure and specific volume is illustrated in the Figure 13, p. 24.

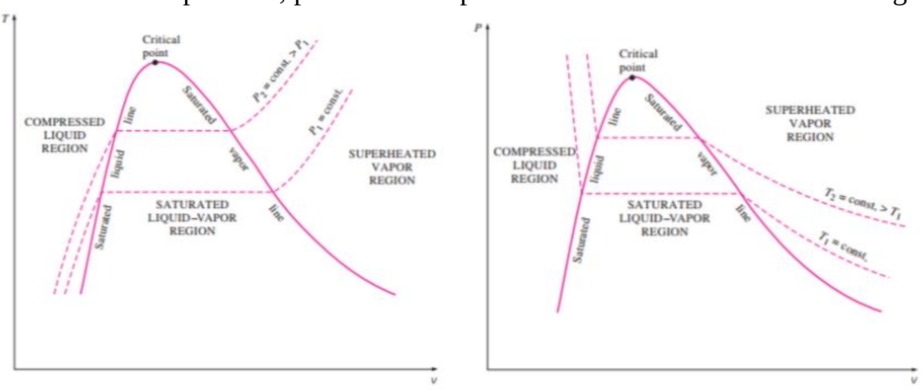

Figure 13. T-v and P-v graphs. (Cengel & Boles)

The large amount of energy that it takes to change the phase is called Latent heat and is the energy absorbed or released during the process.

3.2.3 Dew point

The dew-point temperature T_{dp}, is the temperature at which the water vapor begins to condense in an air-water vapor mixture when it is cooled down at constant pressure. At this point, the moisture capacity of the air equals to the amount hold by the air. A drop in temperature results in condensation.

$$T_{dp} = T_{sat}(P_{sat} = P_{water})$$

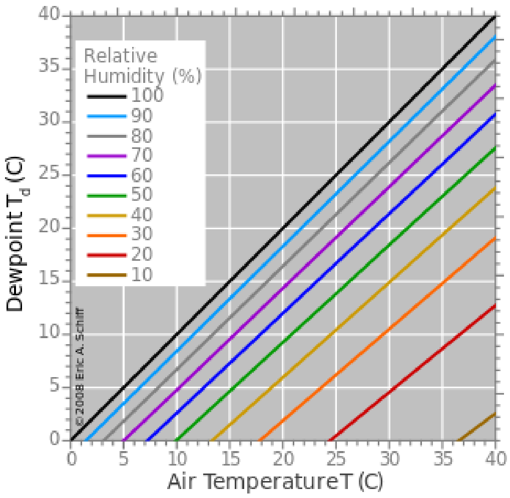

Figure 14 Relationship between the drew point temperature and air temperature (dry bulb) (wikipedia, u.d.)

The dew point temperature can be approximated to be (ajdesigner, 2015):

$$T_{dp} = \left(\frac{\phi}{100}\right)^{\frac{1}{8}} \cdot (112 + 0.9 \cdot T_{db}) + 0.1T_{db} - 112 \qquad \textit{Eq 3.20}$$

Where ϕ is the relative humidity [%] and T_{db} is the dry bulb temperature $[C]$.

Besides the dew point temperature, terms like dry-bulb and wet-bulb temperatures are often used related to moist air. The dry-bulb temperature referrers to the temperature of the air whereas the wet bulb temperature is the temperature of adiabatic saturation (100% RH). This temperature is always lower than the dry bulb temperature (engineeringtoolbox, 2015).

3.2.4 Specific humidity

The specific humidity is also known as the moisture content or humidity ratio and is defined as the mass of water vapor in kg of dry air in an air-water mixture (Cengel & Boles).

The specific humidity ω is defined as:

$$\omega = \frac{m_w}{m_a} \qquad \textit{Eq 3.21}$$

Where m_w and m_a is the mass $[Kg]$ of the water vapor and the dry air respectively.

From the ideal gas law the mass can be found:

$$p \cdot V = m \cdot R \rightarrow m = \frac{p \cdot V}{R \cdot T} \qquad \textit{Eq 3.22}$$

Substituting *Eq. 3.22* into *Eq. 3.21* the following can be obtained assuming same temperature and volume of both gases:

$$\omega = \frac{p_w \cdot V_w \cdot R_a \cdot T_a}{p_a \cdot V_a \cdot R_w \cdot T_w} \rightarrow \quad \omega = \frac{R_a \cdot p_w}{R_w \cdot p_a} \qquad \textit{Eq 3.23}$$

Where $\frac{R_a}{R_w}$ is the relative density of water vapor with respect to dry air. Knowing $R_a = \frac{R}{M_a}$, $R_w = \frac{R}{M_w}$ and that the molar mass of water is $18.02 \frac{g}{mol}$ and for dry air $28.97 \frac{g}{mol}$, the following can be found from a simple substitution:

$$\frac{R_a}{R_w} = \frac{M_w}{M_a} = \frac{18.02}{28.97} = 0.622 \qquad \textit{Eq 3.24}$$

Finally, the specific humidity becomes:

$$\omega = 0.622 \cdot \frac{p_w}{p_a} \qquad \textit{Eq 3.25}$$

The total pressure is the sum of the dry air and water vapor pressure:

$$p_{tot} = p_a + p_w \qquad \textit{Eq 3.26}$$

Combining *Eq 3.25* and *Eq 3.26* the expression for the specific humidity can be found to be:

$$\omega = 0.622 \cdot \frac{p_w}{p_{tot} - p_w} \qquad \textit{Eq 3.27}$$

The water vapor saturated pressure at ambient pressure of 1atm can be found from (engineeringtoolbox, 2015):

$$p_w = 610.78 \times e^{\left(\frac{T_{db}}{T_{db} + 238.3} \times 17.2694\right)} \qquad \textit{Eq 3.28}$$

3.2.5 Relative humidity

The relative humidity, *RH*, is the relationship between the amount of water vapor the air is holding and how much the air can hold. It is defined as the ratio between the pressure of water vapor in moist air at a given temperature, and the pressure of saturated water vapor at the same temperature.

$$\phi = \frac{m_w}{m_g} = \frac{P_w}{P_{sat,w}} \rightarrow \phi = \frac{P_w}{P_{sat,w}} \cdot 100\ \% \qquad \text{Eq 3.29}$$

⇔

$$\phi = \frac{\omega \cdot P_{tot}}{(0.622 + \omega) \cdot P_{sat,w}} \cdot 100\% \qquad \text{Eq 3.30}$$

Where P_g is the pressure of saturated water vapor $[Pa]$.

The relative humidity is related only to the temperature and pressure changes, as the saturation pressure is based on the dry bulb temperature. The relation between the water vapor pressure and its saturation pressure can be expressed as:

$$P_w = \phi \cdot P_{sat,w} \qquad \text{Eq 3.31}$$

The relationship between the dry bulb temperature, specific humidity and relative humidity is depicted in the Figure 15.

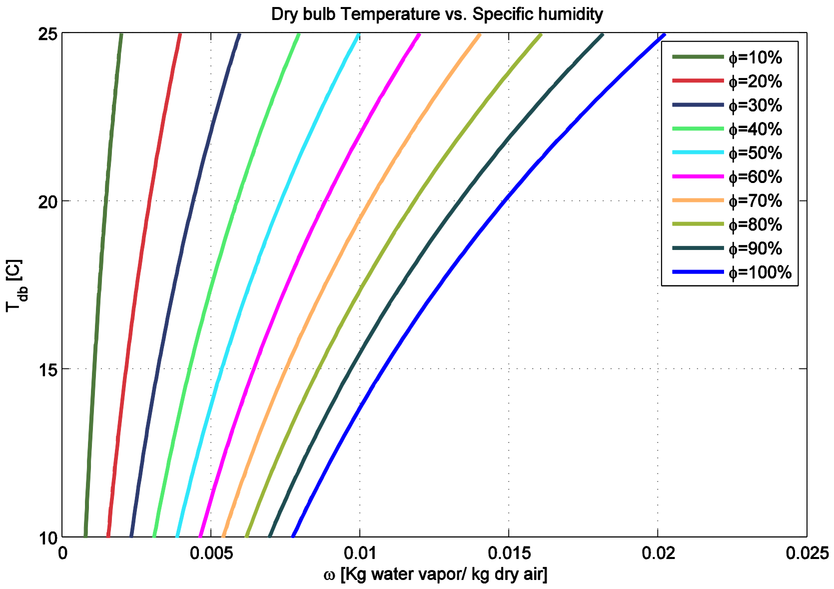

Figure 15 dry-bulb temperature with respect to specific humidity for different RH.

It is possible to see that, the higher relative humidity the larger changes occur in the specific humidity when the temperature increases.

3.2.6 The psychometric chart

The psychometric chart is used to graphically represent psychometric properties of air, such as enthalpy, humidity and dry-bulb temperature. It is often used for processes related to HVAC-system to both understand how the properties changes during the different process throughout the HVAC-system, and to define the comfort zone of which the room is needed to be in. The chart is given for a constant pressure and is normally given for atmospheric pressure, for other pressures some correction has to be made. A general overview of the construction of the chart is depicted in Figure 16.

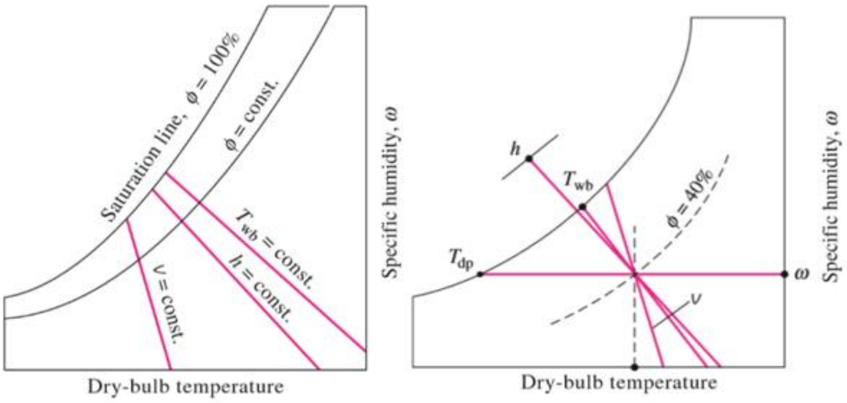

Figure 16 The psychometric chart, indicates constant reference lines. (Cengel & Boles)

Different lines of interest can be found in the charts above; the *Dry-bulb temperature lines* is the vertical dashed line indicated in the chart to the right. Along this line, the temperature is constant. The values of the temperature increases from left to right. The *specific humidity* is the right vertical axis. The *RH* starts from the saturation line, indicated in the left chart, decreasing from 100 % to 0 %. Additionally, the Dew point can be found from drawing a constant line from the specific humidity to the saturation line. Along this line the specific humidity is constant. The process is not necessarily at the dew temperature. If not, the line will intersect the Dry-bulb temperature line. In addition, the enthalpy, specific volume and wet-bulb temperature lines can be found.

3.2.7 Air-conditioning processes

The psychometric-chart can also be used to illustrate the processes, which occur in an air conditioning unit. These processes include simple heating, simple cooling, humidification and dehumidification in order to maintain the desired temperature and humidity in a zone. The processes are depicted in the psychometric chart below. In summer, the process normally is to cool and dehumidify and vice versa.

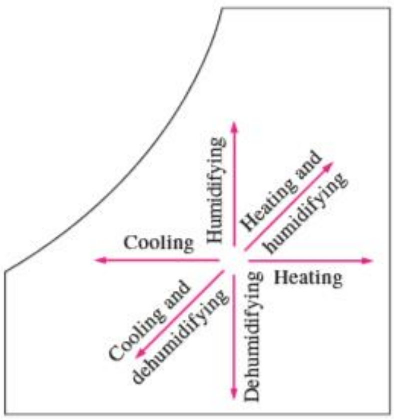

Figure 17 Air conditioning processes.

3.2.7.1 The heating and humidification process

This process is indicated in the left chart in Figure 18. The humidification process is when moisture is added to the air without any changes in temperature (dry bulb). Moisture particles are sprayed into the air where it evaporates and is absorbed. In the chart to the left in the process line between point 2 and point 3. In this project, a humidification process where no temperature is added is considered but in reality, a small change in temperature will occur because the added moisture has a higher temperature.

In the HVAC process heat will be added just before the humidifier. This process is indicated with the line between point 1 and point 2.

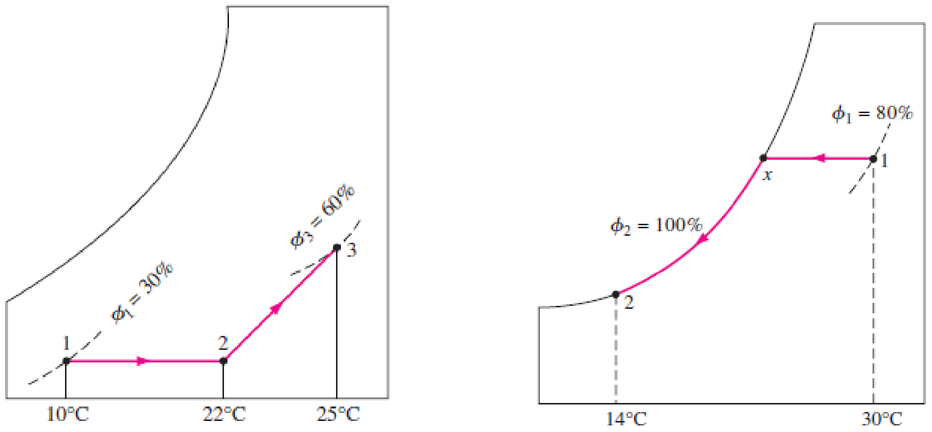

Figure 18 a) Heating and humidification process, b) The dehumidification and cooling process. (Cengel & Boles)

3.2.7.2 The cooling and dehumidification process

The dehumidification process is where moisture is removed by keeping the dry-bulb temperature constant (It will gradually decrease when water is condensed). The process is indicated in the right of Figure 18 by the decreasing line between the point 1's intersection with the saturation line and point 2. As the humidification, the dry-bulb temperature will change by a small amount. This change is not accounted for in this project. In the HVAC-system process the dry air is cooled down by passing a cooling coil in which cold water flows. The air is cooled down to the *Dew* point temperature if dehumidification is needed. At this point, the relative humidity is 100% and the dehumidification process begins. In this process, the drew-point keeps decreasing along the saturation line together with the temperature and specific humidity.

3.2.7.3 Overall HVAC air conditioning process

The overall HVAC-system process is depicted in Figure 19. The chart is used to evaluate which of the processes are needed, for a given zone specification, in order to achieve these specifications. The different sections in the system are denoted by a number:

- Point 1: Mixing box. Process of cooling in order to get to point 2.
- Point 2: After the cooling coil, re-heat in order to reach the point 3 which is the comfort zone.
- Point 3: The comfort zone is characterized by the zone specifications. In other words the comfort zone is the desired specifications for each zone (e.g Temperature, Humidity).

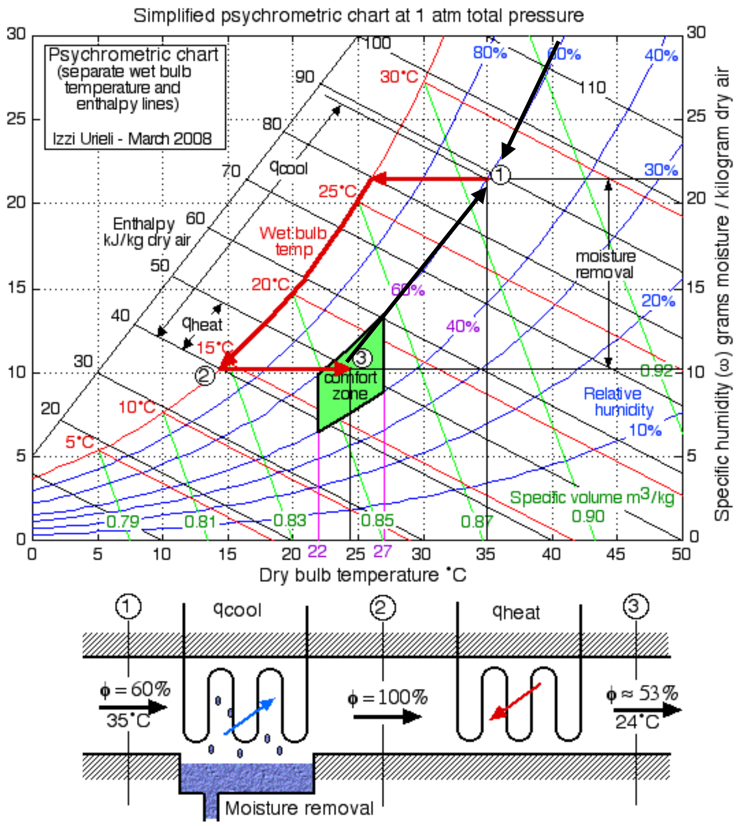

Figure 19 HVAC processes (ohio.edu, n.d.)

In the Figure 19 at state 1, the return air and outside air is mixed and indicated the mixture properties. The black arrow indicates the return and outside air process. Here after the mixing air enters the cooling coil, reducing the dry-bulb temperature and going to the saturation line. The red arrow along this line indicates the condensation process. This cycle continues repeatedly, for given air properties.

As it was already mentioned, the comfort zone is the region specified for the room air properties. This region can be used to see which process are needed for next cycle; e.g. If the humidity of the supply air is to low, only humidification process is needed, depending on the mixing condition.

3.3 Humidity model

The humidity model for this project is based on the assumption that the dry air and water vapor is considered separately. The humidity model is based on the mass balance for each section of the HVAC-model. Starting with the mixing box and ending with the outlet of the zone.

3.3.1 Mixing box

As discuss in chapter 3.2, only a fraction of the outlet air from the zone returns to the mixing box. As the humidity and temperature does not change much when some air is exhausted, it is assumed that the specific- and relative humidity of the outlet is the same for the return air. The flow mass of air from outside into the mixing box is based on having a mixing temperature fixed at $15°C$. The specification of the return water vapor is given as:

$$\dot{m}_{w,r} = \beta \cdot \dot{m}_{w,out}, \qquad \omega_r = \omega_{out}, \qquad \phi_{out} = \phi_r$$

Where,

- $\dot{m}_{w,r}$ and $\dot{m}_{w,out}$ are the mass flow rate of water vapor$\left[\frac{Kg}{s}\right]$of the return and outlet air respectively.
- ω_r and ω_{out} are the specific humidity $\left[\frac{Kg\ moist}{Kg\ dry\ air}\right]$of the return and outlet air respectively.
- ϕ_{out} and ϕ_r are the relative humidity [%]of the return and outlet air respectively.

The process for the mixing box is depicted in the block diagram below:

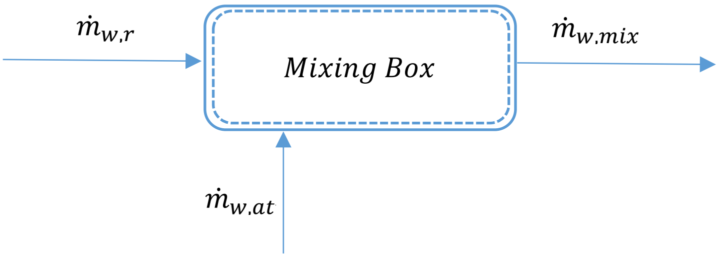

Figure 20 The mixing box.

The mass balance for the control volume of the mixing box is:

$$\dot{m}_{w,r} + \dot{m}_{w,atm} = \dot{m}_{w,mix} \qquad \textit{Eq 3.32}$$

Where $\dot{m}_{w,atm}$ and $\dot{m}_{w,mix}$ are the mass flow rate of water vapor$\left[\frac{Kg}{s}\right]$of the outside and mixing air respectively.

Isolating m_w from *Eq 3.21* results in a relationship between the mass of the dry air and the water vapor:

$$m_w = \omega \cdot m_a \qquad \textit{Eq 3.33}$$

Substituting *Eq 4.31* into *Eq 4.30* results in:

$$\dot{m}_{a,r} \cdot \omega_r + \dot{m}_{a,atm} \cdot \omega_{atm} = \dot{m}_{a,mix} \cdot \omega_{mix} \qquad \textit{Eq 3.34}$$

From this, the static equation for the humidity of the mixture becomes:

$$\omega_{mix} = \frac{\dot{m}_{a,r} \cdot \omega_r + \dot{m}_{a,atm} \cdot \omega_{atm}}{\dot{m}_{a,mix}} \qquad \text{Eq 3.35}$$

Taking the dynamic equation as the:

$$m_{a,mix} \cdot \frac{d\omega_{mix}}{dt} = \dot{m}_r \cdot (\omega_r - \omega_{mix}) + \dot{m}_{atm}(\omega_{atm} - \omega_{mix}) \qquad \text{Eq 3.36}$$

Where $m_{a,mix}$ is the mass $[Kg]$ of dry air in the mixing box.

This is found from a specified volume,V_{mix}, and the density of air. The density of air inside the mixing box is found from the ideal gas law:

$$\rho_{mix} = \frac{P_{mix}}{R_{air} \cdot T_{mix}} \qquad \text{Eq 3.37}$$

The equation for the mass is:

$$m_{a,mix} = \rho_{mix} \cdot V_{mix} \qquad \text{Eq 3.38}$$

3.3.2 Cooling coil and dehumidifier

The mass flow rate of the water vapor into the cooling coil is equal to the mixture of mass flow rate of water vapor. This process is depicted in the block diagram in Figure 21.

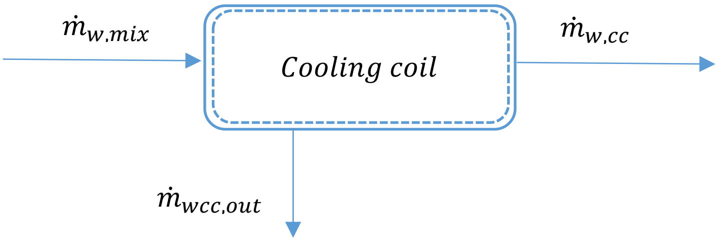

Figure 21 The Cooling coil process.

The water vapor mass balance is:

$$\dot{m}_{w,mix} = \dot{m}_{w,cc} + \dot{m}_{wcc,out} \qquad \text{Eq 3.39}$$

Where,

- $\dot{m}_{wcc,out}$ is the rate of mass condensation of the vapor $\left[\frac{Kg}{s}\right]$.
- $\dot{m}_{w,cc}$ is the mass flow rate of water vapor out of the cooling coil $\left[\frac{Kg}{s}\right]$.

Applying the relationship between the dry air and the water vapor mass, the following equation is obtained:

$$\dot{m}_{a,mix} \cdot \omega_{mix} = \dot{m}_{a,cc} \cdot \omega_{cc} + \dot{m}_{wcc,out} \qquad \text{Eq 3.40}$$

Where ω_{cc} is the specific humidity of the air leaving the cooling coil.

The mass flow rate needs to condense in order to obtain the desired specific humidity of the air leaving the cooling coil. The amount of water vapor needed to dehumidify is calculated as:

$$\dot{m}_{w,out} = \dot{m}_{a,mix} \cdot (\omega_{mix} - \omega_{cc}) \qquad \textit{Eq 3.41}$$

The dynamic of the humidity in the cooling coil can be found to be:

$$m_{a,cc} \cdot \frac{d\omega_{cc}}{dt} = m_{a,mix} \cdot (\omega_{mix} - \omega_{cc}) - \dot{m}_{wcc,out} \qquad \textit{Eq 3.42}$$

Here $m_{a,cc}$ is the mass of the air in the coiling coil and is found accordingly to *Eq.3.38*.

3.3.3 Heater

As the dry air flows through the heating coil and is heated up, no change in state occur therefore specific humidity remains the same whereas the relative humidity will change meaning that $\phi_1 > \phi_2$.

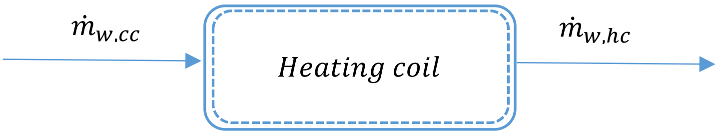

Figure 22 The Heating coil process.

The mass flow rate of the water vapor becomes:

$$\dot{m}_{w,cc} = \dot{m}_{w,hc} \qquad \textit{Eq 3.43}$$

⇔

$$\dot{m}_{a,mix} \cdot \omega_{cc} = \dot{m}_{a,hc} \cdot \omega_{hc} \;\; \rightarrow \cdot\, \omega_{cc} = \omega_{hc} \qquad \textit{Eq. 3.44}$$

Where,

- ω_{hc} is the specific humidity of the air leaving the heating coil.
- $\dot{m}_{w,hc}$ is the mass flow rate of the water vapor leaving the heater $\left[\frac{Kg}{s}\right]$.

3.3.4 Humidifier

Before the air is let into the zone, if needed, some hot water is sprayed into the air to increase the mass of water vapor. The process is depicted in the figure below.

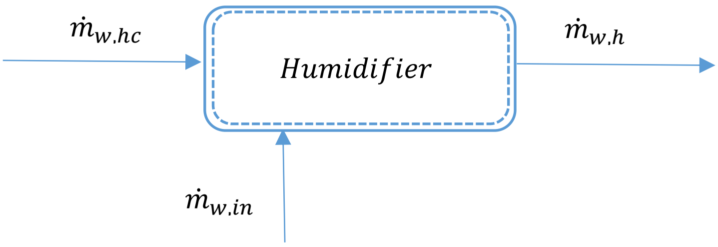

Figure 23 The humidification process.

From applying the mass balance of the water vapor, the following is obtained:

$$\dot{m}_{w,hc} + \dot{m}_{w,in} = \dot{m}_{w,h} \qquad \textit{Eq 3.45}$$

⇔

$$\dot{m}_{a,mix} \cdot \omega_{hc} + \dot{m}_{w,in} = \dot{m}_{a,mix} \cdot \omega_{h} \qquad \textit{Eq 3.46}$$

Where,

- $\dot{m}_{w,in}$ is the added water vapor by the humidifier $\left[\frac{Kg}{s}\right]$.
- ω_h are the specific humidity of the air leaving the humidifier.

The amount of water vapor needed to be added to the air in order to obtain the desired specific humidity in the zone is:

$$\dot{m}_{w,in} = \dot{m}_{a,mix} \cdot (\omega_{hc} - \omega_h) \qquad \text{Eq 3.47}$$

As ω_h is the specific humidity, which has to go into the zone, it should be the set point of the zone but as the outlet of the zone has another relative and specific humidity, ω_h, can be calculates from a steady state condition for the humidity of the zone, which will be further explained in the next section:

$$m_{zone} \cdot \frac{d\omega_{zone}}{dt} = \dot{m}_{in} \cdot (\omega_{in} - \omega_{zone}) - \dot{m}_{out} \cdot (\omega_{zone} - \omega_{out}) \qquad \text{Eq 3.48}$$

Where ω_{zone} is the specific humidity in the zone. In order to determine the amount water vapor needed added, the specific humidity of the inlet air is needed to be found. It is found from solving Eq 3.48 for a steady state condition:

$$0 = \dot{m}_{in} \cdot (\omega_{in} - \omega_z) - \dot{m}_{out} \cdot (\omega_z - \omega_{out}) \qquad \text{Eq 3.49}$$

$\Leftrightarrow$

$$\omega_{in} = 2 \cdot \omega_{z,setpoint} - \omega_{out} \qquad \text{Eq 3.50}$$

Since the inflow of air occurs after the humidification process, the humidity that leaves the humidifier is the one assumed to enter the zone:

$$\omega_h = \omega_{in}$$

The dynamic equation for the humidifier is:

$$m_h \cdot \frac{d\omega_{in}}{dt} = \dot{m}_{a,mix} \cdot (\omega_{cc} - \omega_{in}) + m_{w,in} \qquad \text{Eq 3.51}$$

Where, m_h is the mass of the humidifier.

3.3.5 Zone

No evaporation or condensation in the zone is assumed. The process of the zone is depicted in the figure below:

Figure 24 The process of the zone.

The humidity of the outlet of the zone is assumed to be fixed at all times but a dynamic could be made in order to account for the changes over time. In addition, it should be noticed that after the humidifier the fan changes the pressure in order to obtain a pressurization in the zone. For simplicity, it is assumed that the fan has no influence on the humidity and temperature from this point and into the inlet of the zone. In reality, some changes in temperature due to pressure changes and friction will occur in the duct system.

The water vapor mass balance for the zone is:

$$m_{zone} \cdot \frac{d\omega_{zone}}{dt} = \dot{m}_{a,in} \cdot (\omega_{in} - \omega_{zone}) - \dot{m}_{a,out} \cdot (\omega_{zone} - \omega_{out}) \qquad \textit{Eq 3.52}$$

3.4 Multi zone model

To extend the single-zone model to a multiple zone model, both heat transfer equation and mass balance need to be found for each zone together with the wall heat loss that couples the zone temperatures. In addition, fan type and dampers need to be changed in order to fit the total required inflow rate for all three zones. In addition to the constant radiator, variable radiators are introduced in order to account for the individual zone temperatures.

3.4.1 Heat transfer model

If a difference in the temperatures between neighbor zones exist, then the heat loss between the zones results in a decrease of the temperature in the zone with the highest temperature and in an increase of the other one with the lowest temperature. This temperature change is related to the dynamic of the wall temperatures. To simplify the equations the wall losses to the outside, through the roof and through the floor are combined to one constant value, $\dot{Q}_{wall}$. All losses still are of different values, the are only combined notation wise. The doors U-value is assumed to be the same as the walls, for simplicity.

A sketch over the zones with the heat losses (except for roof and floor) is depicted in the figure below.

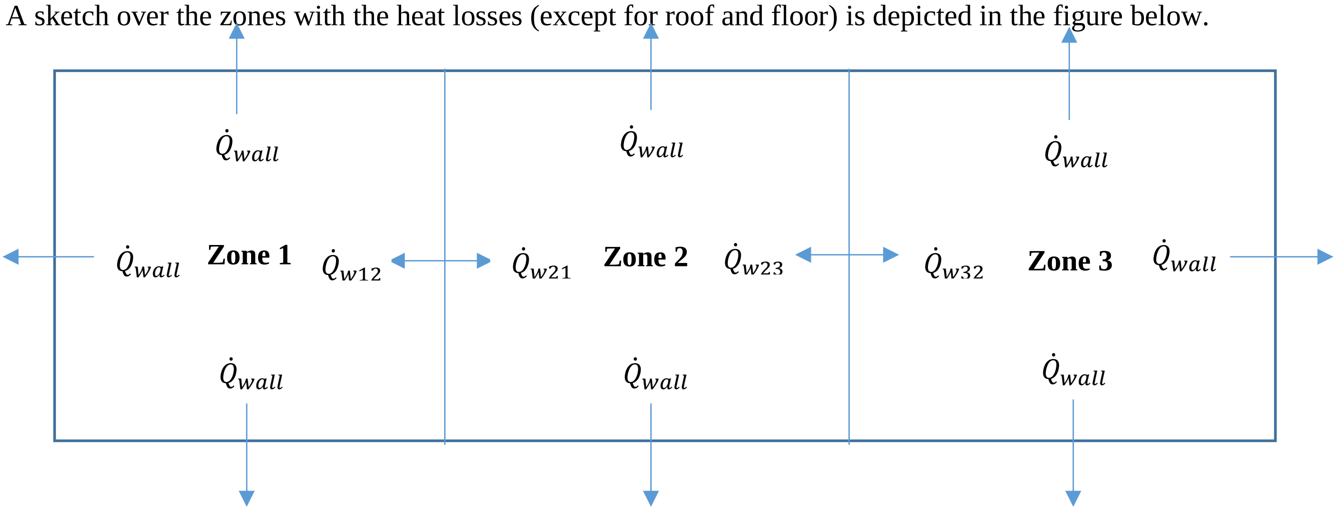

Figure 25 The figure depicts the heat transfer losses notation used in the multi-zone model.

3.4.1.1 Zone 1.

The overall heat transfer balance can for zone 1 is found to be:

$$\Delta\dot{Q}_{z1} = \dot{Q}_{in,z1} + \dot{Q}_{out,z1} + \dot{Q}_{rad,cz1} + 3\cdot\dot{Q}_{wall} + \dot{Q}_{roof} + \dot{Q}_{floor} + \dot{Q}_{w12} + \dot{Q}_{rad,vz1} \qquad \text{Eq 3.53}$$

where,

- $\dot{Q}_{rad,cz1}$ is the heat transfer from the constant radiator.
- $\dot{Q}_{rad,vz1}$ is the heat transfer from the variable radiator.
- $\dot{Q}_{roof}$ is the heat transfer through the roof.
- $\dot{Q}_{floor}$ is the heat transfer through the floor.
- $\dot{Q}_{w12}$ is the heat transfer between zone 1 and 2.

Combining $\dot{Q}_{wall}, \dot{Q}_{roof}$ and $\dot{Q}_{floor}$ to one constant $\dot{Q}_{wall}$, the above equation becomes:

$$\Delta\dot{Q}_{z1} = \dot{Q}_{in} + \dot{Q}_{out} + \dot{Q}_{rad,cz1} + 5\cdot\dot{Q}_{wall} + \dot{Q}_{w12} + \dot{Q}_{rad,vz1} \qquad \text{Eq 3.54}$$

The temperature of zone 1 is calculated as:

$$\frac{dT_{z1}}{dt} = \frac{\dot{Q}_{in} + \dot{Q}_{out} + \dot{Q}_{rad,cz1} + 5 \cdot \dot{Q}_{wall} + \dot{Q}_{w12} + \dot{Q}_{rad,vz1}}{c_p \cdot m_{z1}} \qquad \textit{Eq 3.55}$$

The wall losses was presented in chapter 2.4 to be:

$$\dot{Q}_{wall} = U \cdot A \cdot (T_z - T_{out}) \qquad \textit{Eq 3.56}$$

Regarding the wall loss between the zones, the heat transfer is given by the same equation:

$$\dot{Q}_{w12} = U \cdot A_{12} \cdot (T_z - T_{w12}) \qquad \textit{Eq 3.57}$$

where,

- T_{w12} is the temperature $[K]$ of the wall between zone 1 and zone 2.
- A_{12} is the surface area of the wall $[m^2]$.

For this case, the dynamic of the wall temperature is included to account for both zone temperatures.

The dynamic of the wall temperature can be expressed as:

$$\dot{Q}_{w12} = c_{p,wall} \cdot m_{wall} \cdot \frac{dT_{w12}}{dt} = U_{12} \cdot A_{12} \cdot (T_{z1} - T_{w12}) - U_{21} \cdot A_{21} \cdot (T_{w12} - T_{z2}) \qquad \textit{Eq 3.58}$$

From the above it follows that:

$$U_{12} = U_{21}, \qquad A_{12} = A_{21}$$

The dynamic of the wall temperature becomes:

$$\frac{dT_{w12}}{dt} = \frac{U_{12} \cdot A_{12} \cdot (T_{z1} - T_{w12}) - U_{12} \cdot A_{12} \cdot (T_{w12} - T_{z2})}{c_{p,wall1} \cdot m_{wall1}} \qquad \textit{Eq 3.59}$$

Where m_{wall} is the mass of the wall and $c_{p,wall}$ is the specific heat capacity of the wall.

3.4.1.2 Zone 2.

The overall heat transfer balance for zone 2 can be found as:

$$\Delta\dot{Q}_{z2} = \dot{Q}_{in,z_2} + \dot{Q}_{out,z_2} + \dot{Q}_{rad,cz2} + 2 \cdot \dot{Q}_{wall} + \dot{Q}_{roof} + \dot{Q}_{floor} + \dot{Q}_{w21} + \dot{Q}_{w23} + \dot{Q}_{rad,vz2} \qquad \textit{Eq 3.60}$$

⇔

$$\Delta\dot{Q}_{z1} = \dot{Q}_{in,z_2} + \dot{Q}_{out,z_2} + \dot{Q}_{rad,cz2} + 4 \cdot \dot{Q}_{wall} + \dot{Q}_{w21} + \dot{Q}_{w23} + \dot{Q}_{rad,vz2} \qquad \textit{Eq 3.61}$$

The temperature of zone 1 becomes:

$$\frac{\mathrm{dT}_{z2}}{dt} = \frac{\dot{Q}_{in,z_2} + \dot{Q}_{out,z_2} + \dot{Q}_{rad,cz2} + 4 \cdot \dot{Q}_{wall} + \dot{Q}_{w21} + \dot{Q}_{w23} + \dot{Q}_{rad,vz2}}{c_p \cdot m_{z2}} \qquad Eq\ 3.62$$

The temperature of the wall between zone 1 and zone 2 can be found from *Eq 3.59*.

Regarding the wall loss between the zone 2 and zone 3 is given as the formula between zone 1 and zone 2. The dynamic of the wall temperature can be expressed as:

$$\dot{Q}_{w23} = c_{p,wall} \cdot m_{wall} \cdot \frac{dT_{w22}}{dt} = U_{23} \cdot A_{23} \cdot (T_{z2} - T_{w23}) - U_{23} \cdot A_{23} \cdot (T_{w23} - T_{z3}) \qquad Eq\ 3.63$$

The dynamic of the wall temperature becomes:

$$\frac{dT_{w23}}{dt} = \frac{U_{23} \cdot A_{23} \cdot (T_{z2} - T_{w23}) - U_{23} \cdot A_{23} \cdot (T_{w23} - T_{z3})}{c_{p,wall2} \cdot m_{wall2}} \qquad Eq\ 3.64$$

3.4.1.3 Zone 3.

The overall heat transfer balance can for zone 3 is found as:

$$\Delta\dot{Q}_{z3} = \dot{Q}_{in,z3} + \dot{Q}_{out,z3} + \dot{Q}_{rad,cz3} + 5 \cdot \dot{Q}_{wall} + \dot{Q}_{w23} + \dot{Q}_{rad,vz3} \qquad Eq\ 3.65$$

Where T_{w23} is the temperature $[K]$ for wall 2.

The temperature of zone 1 results as:

$$\frac{\mathrm{dT}_{z3}}{dt} = \frac{\dot{Q}_{in,z_3} + \dot{Q}_{out,z_3} + \dot{Q}_{rad,cz3} + 5 \cdot \dot{Q}_{wall} + \dot{Q}_{w32} + \dot{Q}_{rad,vz3}}{c_p \cdot m_{z_3}} \qquad Eq\ 3.66$$

3.4.2 Temperature equations

By substitution of each heat transfer term, the equation to the expanded temperature equation can be found.

The inlet and outlet heat transfer rate was given by *Eq 2.34* and *Eq 2.42* from this the temperature models result in:

$$\begin{aligned}\frac{dT_{z1}}{dt} = &\frac{\left(c_p \cdot \rho_{in,z_1} \cdot c_{d,in\boldsymbol{z_1}} \cdot A_{in,\boldsymbol{z_1}} \cdot \sqrt{2 \cdot \left(\frac{p_{in,z_1} - p_{z_1}}{\rho_{in,z_1}}\right)} \cdot \left(T_{in} - T_{z_1}\right)\right)}{c_p \cdot m_{z1}} \\ &+ \frac{\left(c_p \cdot \rho_{z_1} \cdot c_{d,out\boldsymbol{z_1}} \cdot A_{out,\boldsymbol{z_1}} \cdot \sqrt{2 \cdot \left(\frac{p_{z,z_1} - p_{out}}{\rho_{z_1}}\right)} \cdot \left(T_{z_1} - T_{atm}\right)\right)}{c_p \cdot m_{z1}} \\ &+ \frac{\dot{Q}_{rad,cz1} + 5 \cdot \dot{Q}_{wall} + U_{12} \cdot A_{12} \cdot (T_{z1} - T_{w12})}{c_p \cdot m_{z1}} + \frac{\bar{h} \cdot A \cdot (T_{radz1} - T_{z_1})}{c_p \cdot m_{z1}}\end{aligned} \qquad Eq\ 3.67$$

$$\frac{dT_{z2}}{dt} = \frac{\left(c_p \cdot \rho_{in,z_2} \cdot c_{d,in\boldsymbol{z_2}} \cdot A_{in,\boldsymbol{z_2}} \cdot \sqrt{2 \cdot \left(\frac{p_{in,z_2} - p_{z_2}}{\rho_{in,z_2}}\right)} \cdot (T_{in} - T_{z_2})\right)}{c_p \cdot m_{z2}} + \frac{\left(c_{air} \cdot \rho_{z_2} \cdot c_{d,out\boldsymbol{z_2}} \cdot A_{out,\boldsymbol{z_2}} \cdot \sqrt{2 \cdot \left(\frac{p_{z,z_2} - p_{out}}{\rho_{z_2}}\right)} \cdot (T_{z_2} - T_{atm})\right)}{c_p \cdot m_{z2}} + \frac{\dot{Q}_{rad,cz2} + 4 \cdot \dot{Q}_{wall} + U_{12} \cdot A_{12} \cdot (T_{z2} - T_{w12}) + U_{23} \cdot A_{23} \cdot (T_{z2} - T_{w23})}{c_p \cdot m_{z2}} + \frac{\bar{h} \cdot A \cdot (T_{radz2} - T_{z_2})}{c_p \cdot m_{z2}} \qquad Eq\ 3.68$$

$$\frac{dT_{z3}}{dt} = \frac{\left(c_p \cdot \rho_{in,z_3} \cdot c_{d,in\boldsymbol{z_3}} \cdot A_{in,\boldsymbol{z_3}} \cdot \sqrt{2 \cdot \left(\frac{p_{in,z_3} - p_{z_3}}{\rho_{in,z_3}}\right)} \cdot (T_{pipe} - T_{z3})\right)}{c_p \cdot m_{z3}} + \frac{\left(c_p \cdot \rho_{z_3} \cdot c_{d,out\boldsymbol{z_3}} \cdot A_{out,\boldsymbol{z_3}} \cdot \sqrt{2 \cdot \left(\frac{p_{z,z_3} - p_{out}}{\rho_{z_3}}\right)} \cdot (T_{z3} - T_{atm})\right)}{c_p \cdot m_{z3}} + \frac{\dot{Q}_{rad,cz3} + 5 \cdot \dot{Q}_{wall} + U_{23} \cdot A_{23} \cdot (T_{z3} - T_{w23})}{c_p \cdot m_{z_3}} + \frac{\bar{h} \cdot A \cdot (T_{radz3} - T_{z3})}{c_p \cdot m_{z3}} \qquad Eq\ 3.69$$

3.4.3 Mass balance

3.4.3.1 Zone 1.

The mass balance for zone 1 is:

$$\Delta \dot{m}_{z1} = \dot{m}_{in} - \dot{m}_{out} \qquad Eq\ 3.70$$

From chapter 2.7, the inflow mass flow rate was found as the product of the volumetric inflow and density of air:

$$\dot{m}_{in} = \rho_{z1} \cdot q_{in,z1} \qquad Eq\ 3.71$$

The volumetric inflow for zone 1 is given as:

$$q_{in,z_1} = c_{d,in\boldsymbol{z_1}} \cdot A_{in,\boldsymbol{z_1}} \cdot \sqrt{2 \cdot \left(\frac{p_{in,z_1} - p_{z_1}}{\rho_{in,z_1}}\right)} \qquad Eq\ 3.72$$

Defining the discharge coefficient by:

$$C_{d,in\boldsymbol{z_1}} = \frac{C_c}{\sqrt{1 - \left(\frac{\rho_{z1} \cdot C_c \cdot A_{in,\boldsymbol{z_1}}}{\rho_{in,z_1} \cdot A_1}\right)^2}} \qquad \textit{Eq 3.73}$$

The mass flow rate leaving zone 1 can be found in the same manner. From this, the equation for the mass flow rate changes in the zone can be found to be:

$$\Delta\dot{m}_{z1} = \rho_{in,z1} \cdot q_{in,z1} - \rho_{z1} \cdot q_{out,z1}$$

⇔

$$\Delta\dot{m}_{z1} = \rho_{in,z1}\left(c_{d,in\boldsymbol{z_1}} \cdot A_{in,\boldsymbol{z_1}} \cdot \sqrt{2 \cdot \left(\frac{p_{in,z_1} - p_{z1}}{\rho_{in,z_1}}\right)}\right) - \rho_{z1}\left(c_{d,out\boldsymbol{z_1}} \cdot A_{out,\boldsymbol{z_1}} \cdot \sqrt{2 \cdot \left(\frac{p_{z1} - p_{out}}{\rho_{z_1}}\right)}\right) \qquad \textit{Eq 3.74}$$

3.4.3.2 Zone 2.

From applying the same equation for zone 2. The mass rate equation becomes:

$$\Delta\dot{m}_{z2} = \rho_{z2} \cdot q_{in,z2} - \rho_{z2} \cdot q_{out,z2}$$

⇔

$$\Delta\dot{m}_{z2} = \rho_{z2}\left(c_{d,in\boldsymbol{z_2}} \cdot A_{in,\boldsymbol{z_2}} \cdot \sqrt{2 \cdot \left(\frac{p_{in,z_2} - p_{z_2}}{\rho_{in,z_2}}\right)}\right) - \rho_{z2}\left(c_{d,out\boldsymbol{z_2}} \cdot A_{out,\boldsymbol{z_2}} \cdot \sqrt{2 \cdot \left(\frac{p_{z2} - p_{out}}{\rho_{z_2}}\right)}\right) \qquad \textit{Eq 3.75}$$

3.4.3.3 Zone 3.

From applying the same equation for zone 3 the mass rate equation becomes:

$$\Delta\dot{m}_{z2} = \rho_{z3} \cdot q_{in,z3} - \rho_{z3} \cdot q_{out,z3}$$

⇔

$$\Delta\dot{m}_{z3} = \rho_{in,z3}\left(c_{d,in\boldsymbol{z_3}} \cdot A_{in,\boldsymbol{z_3}} \cdot \sqrt{2 \cdot \left(\frac{p_{in,z_3} - p_{z_3}}{\rho_{in,z_3}}\right)}\right) - \rho_{z3}\left(c_{d,out\boldsymbol{z_3}} \cdot A_{out,\boldsymbol{z_3}} \cdot \sqrt{2 \cdot \left(\frac{p_{z3} - p_{out}}{\rho_{z_3}}\right)}\right) \qquad \textit{Eq 3.76}$$

3.4.4 Ideal gas law

The pressure equation is needed for each zone. The pressure in the ideal gas can be found to be the following.

3.4.4.1 Zone 1.

The pressure for zone 1 is:

$$P_{z_1} = \frac{m_{z_1}}{V_{z_1}} \cdot R \cdot T_{z_1} \qquad \textit{Eq 3.77}$$

3.4.4.2 Zone 2.

The pressure for zone 2 is:

$$P_{z_2} = \frac{m_{z_2}}{V_{z_2}} \cdot R \cdot T_{z_2} \qquad \textit{Eq 3.78}$$

3.4.4.3 Zone 3.

The pressure for zone 3 is:

$$P_{z_3} = \frac{m_{z_3}}{V_{z_3}} \cdot R \cdot T_{z_3} \qquad \textit{Eq 3.79}$$

3.4.5 Duct system

Expanding the single-zone concept to a multi-zone concept includes furthermore to account for more supply air. It is assumed, that the total required mass flow will equal to the amount need in each zone added together. To look more into this, a duct system is roughly designed wherefrom the system curve of the system is found. In addition, a fan to handle the amount of mass flow has to be found. The fan selection and duct system are given in the next chapter for parameter estimation.

The principle of the multi-zone is depicted below.

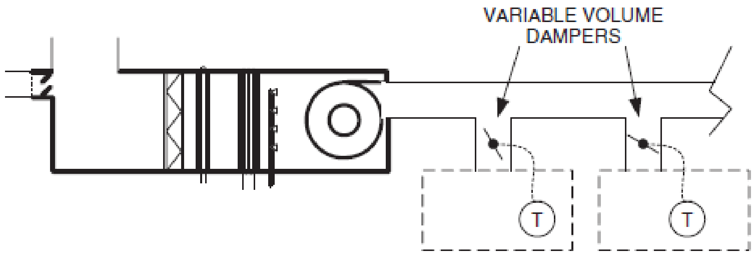

Figure 26 Multi zone concept. (Montgomery & McDowall, 2009)

In this case, the feedback will not be the temperature but the pressure.

3.4.6 Alterations for the system curve

The system curve was presented in chapter 2.10, for this project a slightly different approach is used for the multi-zone.

The pressure loss through the duct system can be found from:

$$P_L = \sum_i f_i \cdot \frac{L_i}{D_i} \cdot \frac{v_i^2}{2} \cdot \rho + \sum_j K_{L,j} \cdot \frac{v_j^2}{2} \cdot \rho \qquad \text{Eq 3.80}$$

Where,

- L_i is the total length of the duct $[m]$.
- D_i is the diameter of the duct $[m]$.
- v_i is the velocity $\left[\frac{m}{s}\right]$.
- $K_{L,j}$ is the resistance coefficient.
- f_i is the friction factor, which can be found from:

$$f\left(Re, \frac{\epsilon}{D}\right): \quad \frac{1}{\sqrt{f}} = -2\log\left(\frac{\frac{\epsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \qquad \text{Eq 3.81}$$

Where ϵ is the roughness $[m]$ of the duct surface.

And,

$$Re = \frac{\rho \cdot v \cdot D}{\mu} \qquad \text{Eq 3.82}$$

This is the same approach as in Chapter 2.10 expressed differently. The difference in this project approach compared to the previous is that the resistance coefficient is found based on the ducts systems duct length, bends and damper openings.

A sketch over different duct shapes and resistance factors are depicted below.

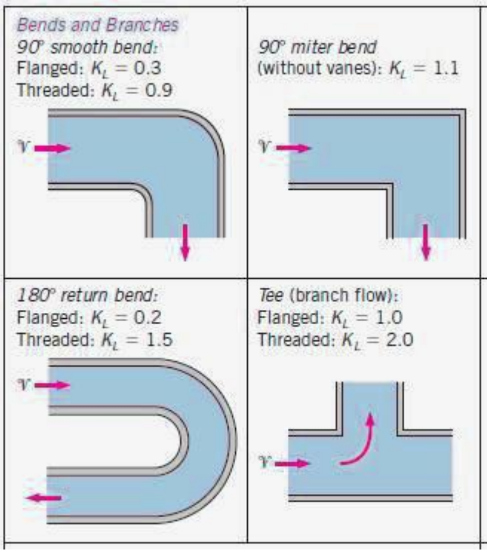

Figure 27 Different shapes and its resistance coefficients. (A. Cengel)

The simple ducts system has three 90°bends, two T-inlets, and three dampers. The resistance coefficient of the dampers can be found from the following equation and table (Nardone):

$$\frac{L_d}{R_d} = \frac{n_d \cdot W_d}{2 \cdot (W_d + H_d)} \qquad \text{Eq 3.83}$$

Where,

- n_d is the number of damper blades.
- W_d is the duct dimension parallel to the blade axis.
- H_d is the duct height $[m]$.
- L_d is the sum of damper length $[m]$.
- R_d is the perimeter of the duct $[m]$.

The resistance coefficient related to different opening angles are given in Table 1 degrees vs. resistance coefficients of dampers.

Damper, Rectangular, Parallel Blades (Brown 1957)

	Co							
L/R	θ, degrees							
	0	10	20	30	40	50	60	70
0.3	0.52	0.79	1.4	2.3	5	9	14	32
0.4	0.52	0.85	1.5	2.4	5	9	16	38
0.5	0.52	0.92	1.5	2.4	5	9	18	45
0.6	0.52	0.92	1.5	2.4	5.4	9	21	45
0.8	0.52	0.92	1.5	2.5	5.4	9	22	55
1	0.52	1	1.6	2.6	5.4	10	24	65
1.5	0.52	1	1.6	2.7	5.4	10	28	102

Table 1 degrees vs. resistance coefficients of dampers. (Nardone)

A function for the opening area can be found and used in the model. This allows the model to account for changing system curve and inlet pressures when the damper openings changes. This function is found in the next chapter.

4. Parameter estimation

In order to simulate the model all the parameters of the model has to be determined. The project considers three zones on an unmanned transformer platform. A schematic of the three zones is provided by Semco Maritime and is depicted in Figure 28. The specifications for each zone is given in Table 2.

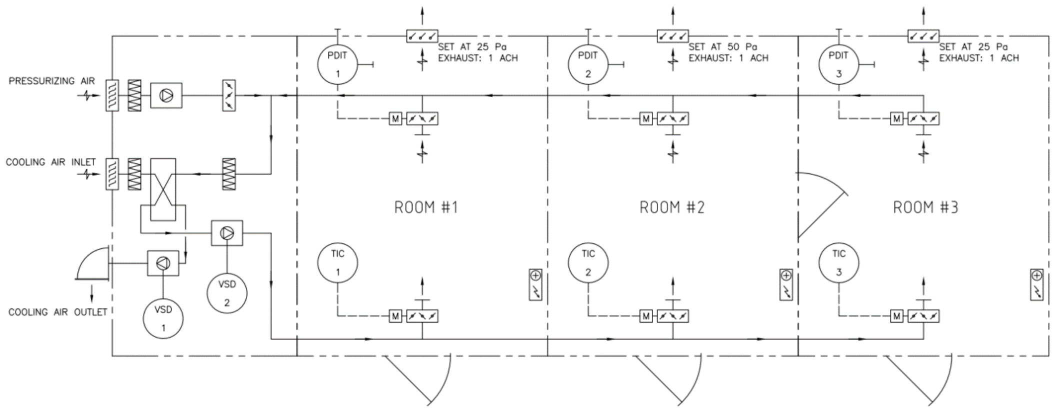

Figure 28 The zone architecture schematic. [Semco Maritime a/s]

PARAMETER	ROOM #1	ROOM #2	ROOM #3
MAX TEMPERATURE	$25°C$	$30°C$	$21°C$
ROOM PRESSURIZATION	$25\ Pa$	$50\ Pa$	$25\ Pa$
ROOM DIMENSIONS [MM]	$12000 \cdot 6500 \cdot 3500$	$10500 \cdot 6500 \cdot 3500$	$4000 \cdot 6500 \cdot 3500$
U-VALUE FOR ROOM	$0.5\frac{W}{m^3K}$	$0.5\frac{W}{m^3K}$	$0.5\frac{W}{m^3K}$

Table 2 indoor design criteria. [Semco Maritime a/s]

4.1 Temperature, pressure and density

The maximum temperatures are given in the table. The temperature in each zones are set to be some degrees below the maximum and at same time, not too far from each other to avoid too much heat losses through the walls between the zones. The temperatures are given as:

$$T_{z1} = 21°C \;\; \rightarrow \;\; T_{z1} = 294.15\ K$$

$$T_{z2} = 22°C \;\; \rightarrow \;\; T_{z2} = 295.15\ K$$

$$T_{zone3} = 19°C \;\; \rightarrow \;\; T_{z3} = 292.15\ K$$

From the Meteorological Institute of Denmark (DMI), it is stated that the annual average ambient temperature for 2014 was $T_{atm} = 8.4°C$, so :

$$T_{atm} = 8.4°C \;\; \rightarrow \;\; T_{atm} = 281.55K$$

Assuming a constant outside pressure at:

$$P_{atm} = 1 \cdot 10^5\ Pa$$

The pressure specification is given in the table. From this, the setpoint for the pressure in each zone is:

$$P_{z1} = 100025\ Pa$$

$$P_{z2} = 100050\ Pa$$

$$P_{z3} = 100025\ Pa$$

From the temperatures and pressures, the density of each zone can be determined from the ideal gas law by a simple substitution.

$$P \cdot V = n \cdot R \cdot T \;\rightarrow\; \rho_{zone} = \frac{P_{zone}}{R_{air} \cdot T_{zone}}$$

R_{air} is the specific gas constant for air, which is $286.9\frac{J}{Kg \cdot K}$ (engineeringtoolbox, n.d.). From this equation, the density of the air in each room is calculated below:

$$\rho_{z1} = \frac{100025\,Pa}{286.9\frac{J}{Kg \cdot K}_{air} \cdot 294.15K} = 1.1812\frac{kg}{m^3}$$

$$\rho_{z2} = \frac{100050\,Pa}{286.9\frac{J}{Kg \cdot K}_{air} \cdot 295.15K} = 1.1815\frac{kg}{m^3}$$

$$\rho_{z3} = \frac{100025\,Pa}{286.9\frac{J}{Kg \cdot K}_{air} \cdot 292.15K} = 1.1934\frac{kg}{m^3}$$

4.2 Supply-air and Supply-duct system

4.2.1 Supply air

The supply air is based on the demand of an air change of 6 times pr. hour. The volume of the three zones are:

$$V_{z1} = 273m^3, \qquad V_{z2} = 238.38\,m^3, \qquad V_{z3} = 91\,m^3$$

This results in a demand of:

$$q_{in,z1} = 0.455\frac{m^3}{s}, \qquad q_{in.z2} = 0.3981\frac{m^3}{s}, \qquad q_{in,z3} = 0.1517\frac{m^3}{s}$$

The total volumetric inflow and thereby the minimum air supplied, which the fan needs to provide is:

$$q_{tot} = q_{z1} + q_{z2} + q_{z3} = 1.0048\frac{m^3}{s}$$

4.2.2 Duct dimensions

To find the system curve, the resistance in the system is needed. The supply duct is considered and the lengths of the ducting system are approximated either by the size of the zones or by the flow regime. The entry length of the duct is the determined by the flow regime. The length has to be long enough for the flow to fully developed. This length is strongly dependent on the flow regime, whether it is laminar or turbulent.

The Reynold number was given by *Eq 3.83*. The velocity can found to be:

$$v = \frac{Q}{A} = \frac{1.004\ \frac{m^3}{s}}{0.2827\,m^2} = 3.54\frac{m}{s}$$

The Reynolds number can be found for a dynamic viscosity of approximately $1.8\frac{kg}{m \cdot s}$.

$$Re = \frac{1.2 \frac{kg}{m^3} \cdot 3.54 \frac{m}{s} \cdot 0.6m}{1.8 \cdot 10^{-5} \frac{kg}{m \cdot s}} = 141470$$

The flow is clearly turbulent. From this, the entry length is defined as (A. Cengel). Letting the diameter to be equal $0.6m$:

$$L_e = 10 \cdot D = 6.00\ m$$

The sketch of the duct lengths are depicted below. These values are chosen, as no specifications exist.

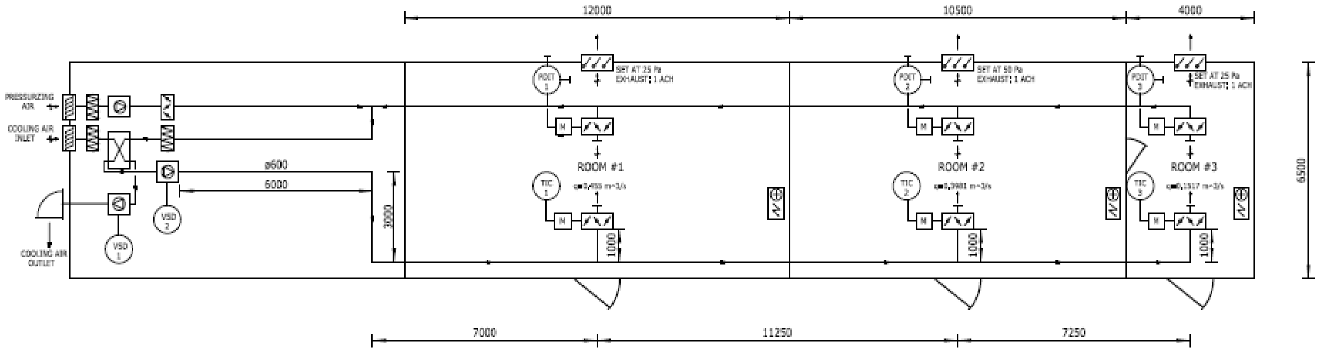

Figure 29 zones dimensions. [Own figure]

4.2.3 System curve

The system curve illustrates how much resistance there are in the given system. The resistance of the major components of the system is part such as bends and inlets. As the shape of these are not specified, approximations are found.

The inlet duct consists of three 90° bends, two T-bends and three dampers. In real applications, balancing dampers are implemented in the inlet duct before the inlet to the zones in order to obtain the same inlet pressure. These dampers are not considered, the inlet pressure is assumed almost constant.

$$3 \cdot 90°\ Bends \rightarrow K_L = 1.1$$

$$2 \cdot\ T\ bends \rightarrow K_L = 1\ to\ 2$$

Regarding the dampers, a resistance functions can be found by interpolating the values given in Table 1 in chapter 3.4.6. The dampers which are used is Halton UTP Balancing damper.

Figure 30 Balancing damper – See Appendix 2 for further specifications.

The blade length is and the blade width is $0.6x0.6m$ and there are 3 blades with the width of a blade of approximately 0.2m. From this the relationship $\frac{L}{R}$ becomes:

$$\frac{L_d}{R_d} = \frac{3 \cdot 0.2m}{2 \cdot (0.2m + 0.6m)} = 0.375$$

Using the dta provided in Table 2. The resistance coefficient with respect to opening area of the dampers are depicted in the figures below.

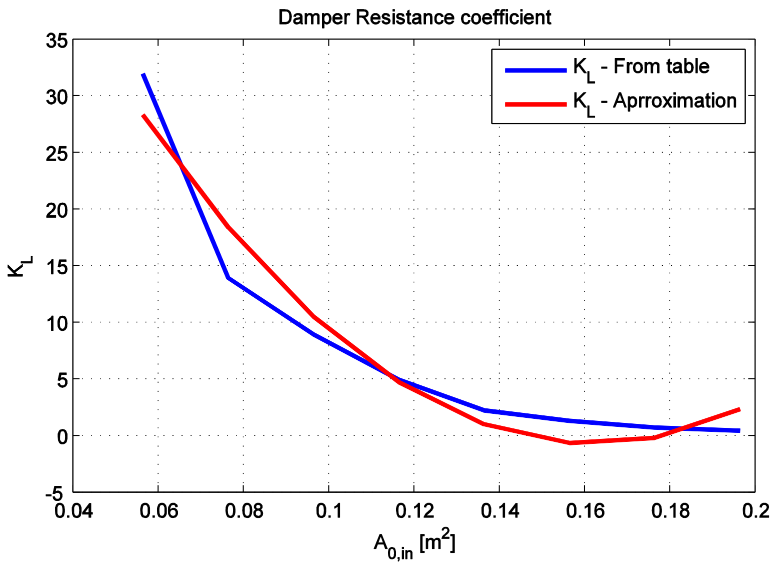

Figure 31 Resistance coefficient with respect to area open

In the figure the values for K_L given by the table is illustrated with the blue points. The red function is the interpolated function for the resistance coefficient.

The function for the opening area are found and used in the model:

$$K_L(A_{0,in}) = 2600.1 \cdot A_{oin}^2 - 841.9 \cdot A_{oin} + 67.5$$

The found equation is depicted as the red function to the right in the figure.

From the function the resistance coefficient of the dampers can be found. Note that, the damper openings are found later but the two parameters are correlated, and iterated until only small changes occur.

$$K_{A0in1}(0.0155\,) = 2600.1 \cdot 0.0155^2 - 841.9 \cdot 0.0155 + 67.5 = 55.07$$
$$K_{A0in2}(0.016\,) = 2600.1 \cdot 0.016^2 - 841.9 \cdot 0.016 + 67.5 = 54.69$$
$$K_{A0in3}(0.0060) = 2600.1 \cdot 0.006^2 - 841.9 \cdot 0.006 + 67.5 = 62.54$$

$$K_{z1} = 55.07, \qquad K_{z2} = 54.69, \qquad K_{z3} = 62.54$$

The total resistance coefficient is:

$$K_{tot} = 177.6$$

The minor losses are the ducts lengths and friction its material. The total length is:

$$L_{tot} = 37.5m$$

For stainless steel the surface roughness is given as:

$$\epsilon = 0.015 \cdot 10^{-3} m \quad \rightarrow \quad \frac{\epsilon}{D} = 2.5 \cdot 10^{-5}$$

The friction can be found from moody charts or from the following equation:

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\frac{\epsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) = -2\log\left(\frac{\frac{\epsilon}{0.6}}{3.7} + \frac{2.51}{141470\sqrt{f}}\right)$$

Solve this results in:

$$f = 0.0164$$

To find he system curve, both the dynamic and static pressure are found. Assuming a height difference of 3.5m:

$$P_{static} = (P_z - P_{atm}) \cdot (h_2 - h_1) \cdot \rho_{in} \cdot g = (100050 - 100000)Pa \cdot (3.5 - 0)m \cdot 1.2\frac{kg}{m^3} \cdot 9.82\frac{m}{s^2}$$
$$= 92.42\ Pa$$

$$P_{dyn} = \left(\frac{\left(f \cdot \frac{L_{tot}}{D}\right) \cdot \left(\frac{\rho_{in} \cdot \left(\frac{q}{A}\right)^2}{2}\right) + K_L \cdot \left(\frac{q}{A}\right)^2}{2 \cdot g}\right) \cdot \rho_{in} \cdot g$$

$$= \left(\frac{\left(0.0164 \cdot \frac{37{,}5m}{0.6m}\right) \cdot \left(\frac{1.2\frac{kg}{m^3} \cdot \left(\frac{1.004\frac{m^3}{s}}{0.2827\ m^2}\right)^2}{2}\right) + 177.6 \cdot \left(\frac{1.004\frac{m^3}{s}}{0.2827 m^2}\right)^2}{2 \cdot 9.82\frac{m}{s^2}}\right) \cdot 1.2\frac{kg}{m^3}$$
$$\cdot 9.82\frac{m}{s^2} P_{dyn} = 1348.7 \cdot q_{tot}^2$$

The total pressure:

$$\Delta P_t = 92.42 + 1060.3 \cdot q_{tot}^2$$

The system curve for different openings are depicted below.

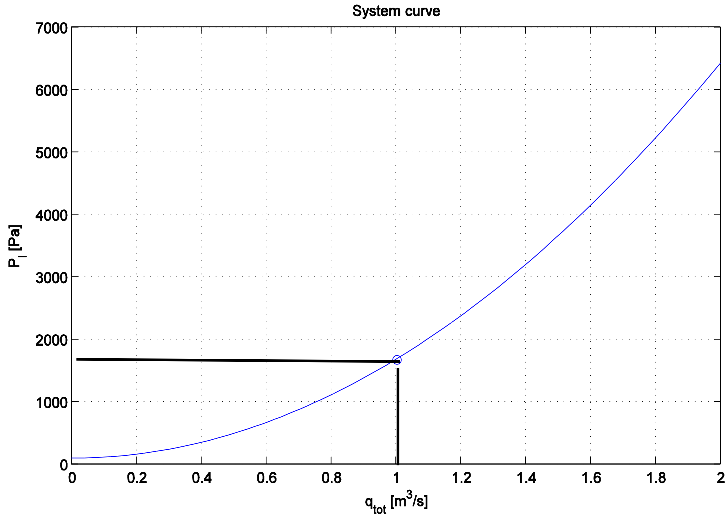

Figure 32 System curve.

To find the fan, the pressure for the required inflow rate has to be found. The pressure needed in order to obtain 1.0048$\frac{m^3}{s}$. The required differential pressure is 1161 Pa. from the specifications, the selected fan is found. The fan is: KBR 355 D2 IE2 thermo-centrifugal fan with max. air flow of 2.09 $\frac{m^3}{s}$.

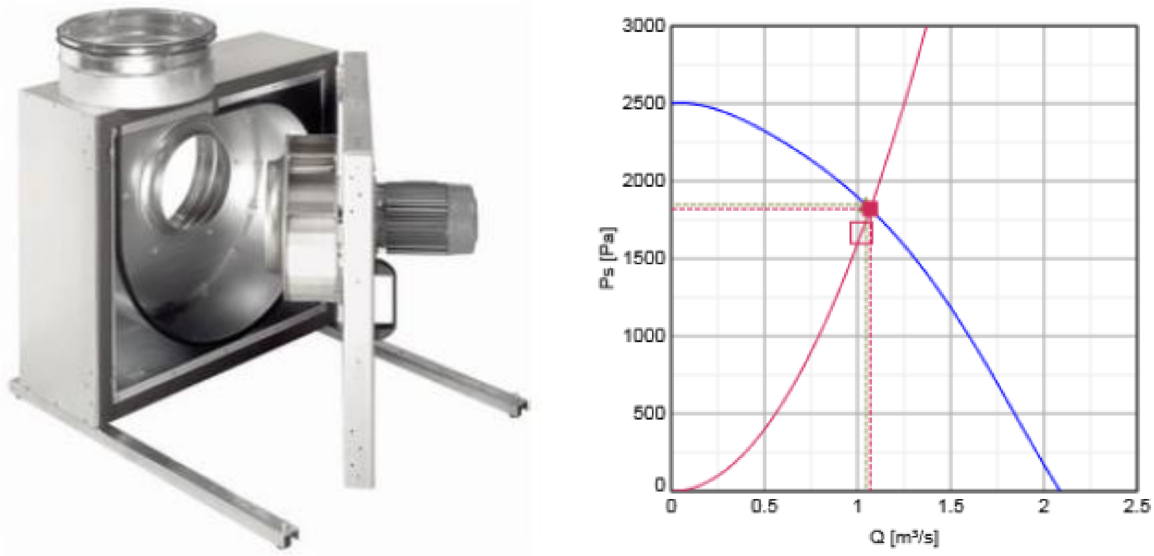

Figure 33 a) Fan illustration, b) Fan curve. See Appendix 1 for further specifications.

4.2.4 Fan curve

From the fan curve presented in section 3.2, three points are found to be:

	POINT 1	POINT 2	POINT 3	POINT 4	POINT 5
$q_{tot@50}\left[\frac{m^3}{s}\right]$	$\mathbf{0.34}$	$\mathbf{0.68}$	$\mathbf{1.24}$	$\mathbf{1.27}$	$\mathbf{1.6}$
$\Delta P_{t@50}\,[Pa]$	2413	2200	1838	1537	989

Table 3 Values from original fan curve.

By using the Least squares method the coefficient of the polynomial can be found to be:

$$\Delta P(q_{tot}) = -843.3 \cdot q_{tot}^2 + 551.9 \cdot q_{tot} + 2289.6$$

From the affinity laws the flow rate and differential pressure across the fan can be found for other frequencies of the fan:

$$q_2 = q_1 \cdot \frac{f_2}{f_1} \qquad \mathrm{d}P_2 = dP_1 \cdot \frac{f_2}{f_1}$$

The new values are given in the table below for different frequencies.

	POINT 1	POINT 2	POINT 3	POINT 4	POINT 5
$q_{tot@45}\left[\frac{m^3}{s}\right]$	0.3060	0.6120	1.1160	1.1430	1.4400
$\Delta P_{t@45}\,[Pa]$	1954.5	1782	1488.8	1245	800.3
$q_{tot@55}\left[\frac{m^3}{s}\right]$	0.3740	0.7480	1.3640	1.3970	1.7600
$\Delta P_{t@55}[Pa]$	2919.70	2662	2224	1859.8	1195.5
$q_{tot@60}\left[\frac{m^3}{s}\right]$	0.4080	0.8160	1.4880	1.5240	1.9200
$\Delta P_{t@60}\,[Pa]$	3474.7	3168	2646.7	2213.3	1422.7

Table 4 Estimated values for other frequencies.

The different polynomials for all the frequencies are depicted together with the system curve in Figure 34.

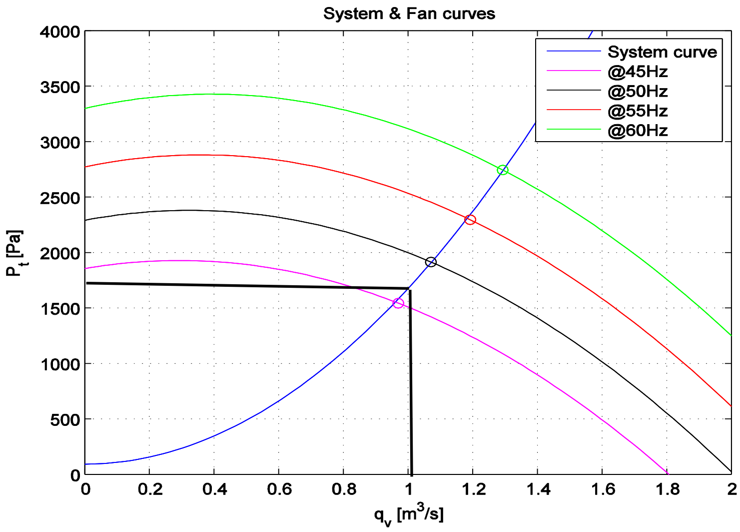

Figure 34 Fan and system curves.

The operating point can be found from the total flow rate that has to be achieved. From this, the fan curve, which intersections with the system curve, at that exact that point is the acquired frequency. The frequency required to obtain the 1.004 $\frac{m^3}{s}$ is 47.5 Hz.

4.3 Damper openings

To calculate the valve openings for the given specifications the dimension of the duct diameter is needed. The duct diameter is set to 0.6m as the resistance in the system with less diameter is too high for given fan selections.

From the given information the area of the pipe, A_1, can be determined:

$$A_{1,in} = \pi \cdot r^2 = \pi \cdot 0.30^2 = 0.2827\, m^2$$

The inflow opening area A_0 can now be found by solving the inflow rate q_{in} to each zone.

The equation for the flow rate into the system is:

$$q_{in_{z1}} = \frac{\sqrt{2\left(\frac{p_{in}}{\rho_{in}} - \frac{p_z}{\rho_z}\right)}}{\sqrt{1-\left(\frac{\rho_z C_{c,in} A_{0,in}}{\rho_{in} A_1}\right)}} \cdot C_c A_{0in} = 0.455\frac{m^3}{s} = \frac{\sqrt{\frac{2}{1.2473\frac{kg}{m^3}}(101300-100025)Pa}}{\sqrt{1-\left(\frac{1.1852\frac{kg}{m^3}\cdot 0.61\cdot A_{0in}}{1.2085\frac{kg}{m^3}}\right)}} 0.61 \cdot A_{0in,z1}$$

$$A_{0,inz1} = 0.016\, m^2$$

$$q_{in_{z2}} = 0.3981\frac{m^3}{s} = \frac{\sqrt{\frac{2}{1.2473\frac{kg}{m^3}}(101080-100050)Pa}}{\sqrt{1-\left(\frac{1.1815\frac{kg}{m^3}\cdot 0.61\cdot A_{0in}}{1.2059\frac{kg}{m^3}}\right)}} 0.61 \cdot A_{0in,z2}$$

$$A_{0,inz2} = 0.0155\, m^2$$

$$q_{in_{z3}} = 0.1517\frac{m^3}{s} = \frac{\sqrt{\frac{2}{1.2473\frac{kg}{m^3}}(101050-100025)Pa}}{\sqrt{1-\left(\frac{1.1815\frac{kg}{m^3}\cdot 0.61\cdot A_{0in}}{1.2059\frac{kg}{m^3}}\right)}} 0.61 \cdot A_{0in,z3}$$

$$A_{0,inz3} = 0.0060\, m^2$$

This will be the minimum position for the damper in order to get the flow changes.

In order to find the opening area of the outlet the mass flow rate is needed. The flow masses of each zone are found to be:

$$\dot{m}_{in,z1} = \rho_{in,z1} \cdot q_{in,z1} = 1.2085\frac{kg}{m^3} \cdot 0.455\frac{m^3}{s} = 0.5499\frac{kg}{s}$$

$$\dot{m}_{in,z2} = \rho_{in,z2} \cdot q_{in,z2} = 1.2059 \frac{kg}{m^3} \cdot 0.3981 \frac{m^3}{s} = 0.4801 \frac{kg}{s}$$

$$\dot{m}_{in,z3} = \rho_{in,z3} \cdot q_{in,z3} = 1.2056 \frac{kg}{m^3} \cdot 0.1517 \frac{m^3}{s} = 0.1829 \frac{kg}{s}$$

Assuming that the zone is in equilibrium the mass balance can be used:

$$\dot{m}_{in} = \dot{m}_{out}$$

From solving the equation of the outlet mass flow with respect to the damper opening, the following can be obtained:

$$A_{0,outz1} = \frac{\dot{m}_{outz1} \cdot V_{z1}}{m_{z1} \cdot \sqrt{2\left(R_{air} \cdot T_z - \frac{P_{out}}{\frac{m_{z1}}{V_{z1}}}\right)}} = \frac{0.5499 \frac{kg}{s} \cdot 273\ m^3}{323.5727\ kg \cdot \sqrt{2\left(286.9 \frac{1}{Kj \cdot K} \cdot 294.15\ K - \frac{1 \cdot 10^5 Pa}{282.24 \frac{kg}{238.88}}\right)}}$$

$$A_{0,outz1} = 0.1134 m^2$$

$$A_{0,outz2} = \frac{0.4801 \frac{kg}{s} \cdot 238.38\ m^3}{281.6525\ kg \cdot \sqrt{2\left(286.9 \frac{1}{Kj \cdot K} \cdot 295.15\ K - \frac{1 \cdot 10^5 Pa}{282.24 \frac{kg}{238.88}}\right)}}$$

$$A_{0,outz2} = 0.0715 m^2$$

$$A_{0,outz3} = \frac{0.1829 \frac{kg}{s} \cdot 91\ m^3}{108.5959\ kg \cdot \sqrt{2\left(286.9 \frac{1}{Kj \cdot K} \cdot 292.15\ K - \frac{1 \cdot 10^5 Pa}{282.24 \frac{kg}{238.88}}\right)}}$$

$$A_{0,outz3} = 0.0387 m^2$$

4.4 Flow rates

4.4.1 Outflow rate

The outflow rate is calculated through the as shown in the following equations.

4.4.1.1 Zone1:

$$q_{outz1} = A_{0,outz1} \cdot \sqrt{\frac{2}{\rho_{zone1}}(P_{zone1} - P_{out})} = 0.1134\ m^2 \cdot \sqrt{\frac{2}{1.1852 \frac{kg}{m^3}}(100025 - 1 \cdot 10^5) Pa}$$

$$q_{outz1} = 0.4639 \frac{m^3}{s}$$

4.4.1.2 Zone2:

$$q_{outz2} = 0.0715 m^2 \cdot \sqrt{\frac{2}{1.1815 \frac{kg}{m^3}} (100025 - 1 \cdot 10^5) Pa}$$

$$q_{zone\#2out} = 0.4063 \frac{m^3}{s}$$

4.4.1.3 Zone3:

$$q_{outz3} = 0.0387 m^2 \cdot \sqrt{\frac{2}{1.1812 \frac{kg}{m^3}} (100025 - 1 \cdot 10^5) Pa}$$

$$q_{outz3} = 0.1533 \frac{m^3}{s}$$

4.4.5 Inflow Heat

To calculate the inflow of heat transfer the specific air capacity is needed. It is for air $c_{air} = 1.012 \frac{J}{Kg \cdot K}$. The output flow heat is:

4.4.5.1 Zone1:

$$\dot{Q}_{in,z1} = c_{air} \cdot m_{in,z1} \cdot T_{in} - \dot{T}_{zone1} = 1.012 \frac{J}{Kg \cdot K} \cdot 0.5499 \frac{kg}{s} \cdot (292.15 - 294.15)$$

$$\dot{Q}_{in} = -1.3056 \frac{J}{s}$$

4.4.5.2 Zone2:

$$\dot{Q}_{in,z2} = 1.012 \frac{J}{Kg \cdot K} \cdot 0.4801 \frac{kg}{s} \cdot (292.15 - 295.15)$$

$$\dot{Q}_{in} = -1.8875 \frac{J}{s}$$

4.4.5.3 Zone3:

$$\dot{Q}_{in,z3} = 1.012 \frac{J}{Kg \cdot K} \cdot 0.1829 \frac{kg}{s} \cdot (292.15 - 292.15)$$

$$\dot{Q}_{in} = 0 \frac{J}{s}$$

4.4.6 Outflow heat

The outlet is determined to be:

4.4.6.1 Zone1:

$$\dot{Q}_{out,z1} = 1.012 \frac{J}{Kg \cdot K} \cdot 0.5499 \frac{kg}{s} \cdot (294.15 - 290.15)$$

$$\dot{Q}_{out,z1} = 2.3450 \frac{J}{s}$$

4.4.6.2 Zone2:

$$\dot{Q}_{out,z2} = 1.012 \frac{J}{Kg \cdot K} \cdot 0.4801 \frac{kg}{s} \cdot (295.15 - 290.15)$$

$$\dot{Q}_{out,z2} = 2.5724 \frac{J}{s}$$

4.4.6.3 Zone3:

$$\dot{Q}_{out,z3} = 1.012 \frac{J}{Kg \cdot K} \cdot 0.1829 \frac{kg}{s} \cdot (292.15 - 290.15)$$

$$\dot{Q}_{out,z3} = 0.3912 \frac{J}{s}$$

4.4.7 Wall losses

The wall losses are divided into two parts; to the outside and to the other zones. The ones to the outside is remains at constant values. The wall temperature to the outside is assumed to have the average between the temperature of the zone and the temperature outside.

$$T_{wall} = \frac{T_z + T_{atm}}{2}$$

This is only assumed as the losses is constant, else the dynamic of the wall temperature could be modelled.

4.4.7.1 Zone1:

The area of the walls interacting with the outside is:

$$A_{wallz1} = (2 \cdot 6.5 \cdot 12 + 2 \cdot 12 \cdot 3.5 + 6.5 \cdot 3.5)m^2 = 262.75\, m^2$$

The U_{wall} is 0.5 $\frac{W}{m^3 K}$. The heat loss through the wall in zone 1 is:

$$\dot{Q}_{wallz1} = U_{wall} \cdot A_{wallz1} \cdot (T_{wallz1} - T_{zone1}) = -827.7\, \frac{J}{s}$$

The area of the walls, which interacts with the different zones are:

$$A_{w12} = A_{w21} = A_{w23} = A_{w32}$$

$$A_{w12} = 6.5 \cdot 3.6 = 23.4\, m^2$$

The heat loss to zone 2

$$\dot{Q}_{wallz1} = U_{wall} \cdot A_{wall12} \cdot (T_{wall12} - T_{zone1}) = 5.85 \frac{J}{s}$$

The initial values of T_{wall12} is found as the average of the two zone temperatures.

4.4.7.2 Zone2:

The heat loss through the walls and the wall area are:

$$A_{wallz2} = (2 \cdot 6.5 \cdot 10.5 + 2 \cdot 10 \cdot 3.6)m^2 = 206.5\, m^2$$

$$\dot{Q}_{wallz2} = -702.1\, \frac{J}{s}$$

The heat loss to zone 1 and zone 3:

$$\dot{Q}_{wallz21} = 0.5 \cdot 23.4 \cdot (294.65 - 295.15) = -5.85 \frac{J}{s}$$

$$\dot{Q}_{wall23} = 0.5 \cdot 23.4 \cdot (293.65 - 295.15) = -17.55 \frac{J}{s}$$

4.4.7.3 Zone3:

The heat loss through the walls and the wall area are:

$$A_{wallz3} = (2 \cdot 6.5 \cdot 4 + 2 \cdot 4 \cdot 3.6 + 6.5 \cdot 3.6)m^2 = 106.75\ m^2$$

$$\dot{Q}_{wallz3} = -282.89\ \frac{J}{s}$$

The heat loss to zone 2:

$$\dot{Q}_{wall32} = 0.5 \cdot 23.4 \cdot (293.65 - 292.15) = 17.55 \frac{J}{s}$$

From the wall losses between the zones, it is possible to see that if the value is negative means that heat leaves and vice versa. The amount lost in zone 2 to zone 1 is $-5.85 \frac{J}{s}$ and the heat achieved in zone 1 by zone 2 is $5.82 \frac{J}{s}$. Only the sign changes.

4.4.8 Radiators heat transfer

The constant radiator is assumed to provide a constant J/s, which accounts for the wall loses to the outside:

$$-\dot{Q}_{wall} = \dot{Q}_{rad,c}$$

Note that a variable radiator is needed to account for the individual heating and will account for the heat losses through the walls between the zones.

4.4.8.1 Zone1:

The Constant radiator has to provide:

$$Q_{rad,cz1} = 827.7\ \frac{J}{s}$$

The Variable radiator has to provide:

$$-(\dot{Q}_{in,z1} - \dot{Q}_{out,z1} + \dot{Q}_{wallz1} + \dot{Q}_{wall12} + \dot{Q}_{rad,cz1}) = Q_{rad,vz1}$$

$$-(-1.3056 - 2345 - 827.7 + 5.85 + 827.7) \frac{J}{s} = 2.1994 \frac{J}{s}$$

The heat the variable radiator has to provide is not much, it could be more beneficial if the constant radiator were less providing less heat and the variable was providing more.

Assuming $A = 10m^2$, if the area should be less due to zone dimensions, the required temperature will increase. The control variable is chosen to be the surface temperature, therefore the other parameters for the radiator has to be found. As most parameters depend on the surface temperature, these parameters is found from $T_{rad} = 295.15$ and the control variable is the one which is directly related to $Q_{rad,v}$. This might result in a small error, but in the control part, this value is not used. The parameters of the radiator are:

$$H_{rh} = 1\,m, \qquad T_{rad} = 295.15\,K, \qquad v = 1.5 \cdot 10^{-5}\frac{m^2}{s}, \qquad \alpha = 2.2142 \cdot 10^{-5}\frac{m^2}{s}$$

The Rayleigh no. is found from:

$$\beta_{z1} = \left(\frac{1}{295.15\,K + 294.15\,K}\right) = 0.0034\,K^{-1}$$

$$Ra_{z1} = \frac{g \cdot beta_{ra1} \cdot (H_{rh}^3) \cdot (T_{rad} - T_{z1})}{v_k \cdot a} = \frac{9.82\frac{m}{s^2} \cdot 0.0034\,K^{-1} \cdot 1^3 m^3 \cdot (295.15\,K - 294.15\,K)}{1.5647 \cdot 10^{-5}\frac{m^2}{s} \cdot 2.2142 \cdot 10^{-5}\,\frac{m^2}{s}}$$
$$= 1.9187 \cdot 10^8$$

Resulting in:

$$c = 0.59, \qquad n = \frac{1}{4}$$

The Nusselt no. is:

$$Nu_{z1} = c \cdot Ra_{z1}^n = 0.59 \cdot (1.9187 \cdot 10^8)^{\frac{1}{4}} = 68.4391$$

And

$$\bar{h}_{z1} = Nu_{z1} \cdot \frac{K_{air}}{H_{rh}} = 68.4391 \cdot \frac{0.0261\frac{W}{m \cdot K}}{1\,m} = 1.8124\frac{W}{m^2 \cdot K}$$

Substituting in and solving for T_{radz1}:

$$\dot{Q}_{rad} = \bar{h}_{z1} \cdot A \cdot (T_{radz1} - T_{zone1})$$

$$-2.1994\frac{J}{s} = 1.8124\frac{W}{m^2 \cdot K} \cdot 10m^2 \cdot (T_{radz1} - 294.15\,K)$$

Solving this equation results in the required surface temperature of the radiator:

$$T_{radz1} = 294.3\,K$$

4.4.8.2 Zone2:

The Constant radiator is:

$$Q_{rad,cz2} = 702.1\,\frac{J}{s}$$

The Variable radiator is:

$$Q_{rad,vz2} = -(-1.8875 - 2.572 - 702.1 - 23.4 + 702.1)\frac{J}{s} = 27.8595\frac{J}{s}$$

The same procedure as in zone 1 is applied to find the different parameters. The list of the parameters are:

$$Ra_{z2} = 9.5773 \cdot 10^8, \qquad Nu_{z2} = 58.3664, \qquad \bar{h}_{z2} = 1.5234\,\frac{W}{m^2 \cdot K}, \qquad T_{radz2} = 296.9375\,K$$

4.4.8.3 Zone3:

The Constant radiator is:

$$Q_{rad,cz3} = 282.89\frac{J}{s}$$

The Variable radiator is:

$$Q_{rad,vz3} = -(0 - 0.3912 - 282.89 + 17.55 + 282.89)\frac{J}{s} = -17.1588\frac{J}{s}$$

As this results in taking heat out of the zone, the constant radiator is dimensioned for less J/s. This could be avoided by dividing the radiators contribution more even.

Subtracting 27 J/s resulting in:

$$Q_{rad,cz3} = 255.89\frac{J}{s}$$

$$Q_{rad,vz3} = 9.842\frac{J}{s}$$

The same procedure as in zone 1 is applied to find the different parameters. The list of the parameters are:

$$Ra_{z3} = 3.8505 \cdot 10^{8}, \qquad Nu_{z3} = 82.6476, \qquad \bar{h}_{z3} = 2.1571\ \frac{W}{m^2 \cdot K}, \qquad T_{radz3} = 292.4106\ K$$

4.5 AHU specifications

4.5.1 Temperature

The temperature in the mixing box is set to 15 $°C$:

$$T_{mix} = 288.15\ °K$$

The temperature of the cooling coil is set to the mixing temperature as the return air is needed only to be heated:

$$T_{cc} = T_{mix}$$

The inlet temperature is set to 19 $°C$:

$$T_{hc} = T_{in} = 292.15\ K$$

4.5.2 Humidity

The desired set point values for the three zones are given in the table shown below.

SPECIFICATION	ZONE 1	ZONE 2	ZONE 3
TEMPERATURE	$T_{z1} = 21°C$	$T_{z2} = 22°C$	$T_{z3} = 19°C$
R/S HUMIDITY	$\omega_{z1} = 0.0074$	$\phi_{z2} = 45\ \%$	$\omega_{z3} = 0.0074$
PRESSURE	25 Pa	50 Pa	25 Pa

Table 5 Specifications for the different zones.

4.5.2.1 Zone 2:

From the specified properties in the saturated pressure of the water vapor and the specific humidity can be found:

$$P_{ws,z2} = 610.78 \cdot e^{\frac{T_{z2}}{T_{z2}+238.3} \cdot 17.2694} = 610.78 \cdot e^{\frac{22°C}{22°C+238.3} \cdot 17.2694} = 2628.9\ Pa$$

$$\omega_{z2} = 0.622 \cdot \frac{P_{ws} \cdot \phi_z}{P_z - P_{ws} \cdot \phi_z} = 0.622 \cdot \frac{2628.9\ Pa \cdot 0.45}{100050\ Pa - 2628.9\ Pa \cdot 0.45} = 0.0074\ \frac{kg\ water\ vapor}{kg\ dryair}$$

4.5.2.2 Zone 1:

Since the pressure and temperature is more specific demands than specific humidity, which needs to be within a range of 35-60%, the same demand for specific humidity in the rooms are required, since deferent demands will result in a change in inflow rate, which influences the pressure and temperature in the zones.

$$P_{ws,z1} = 610.78 \cdot e^{\frac{T_{z1}}{T_{z1}+238.3} \cdot 17.2694} = 610.78 \cdot e^{\frac{21°C}{21°C+238.3} \cdot 17.2694} = 2473.4\ Pa$$

$$\phi_{z1} = \frac{\omega_{z1} \cdot P_{z1}}{(0.622 + \omega_{z2}) \cdot P_{ws,z1}} \cdot 100\ \% = \frac{0.0074 \cdot 100025\ Pa}{(0.622 + 0.0074) \cdot 2473.4\ Pa} \cdot 100\ \% = 47.6\ \%$$

4.5.2.3 Zone 3:

$$P_{ws,z} = 610.78 \cdot e^{\frac{T_{z3}}{T_{z3}+238.3} \cdot 17.2694} = 610.78 \cdot e^{\frac{19°C}{19°C+238.3} \cdot 17.2694} = 2186.3\ Pa$$

$$\phi_{z3} = \frac{\omega_{z3} \cdot P_{z3}}{(0.622 + \omega_{z3}) \cdot P_{ws,z3}} \cdot 100\ \% = \frac{0.0074 \cdot 100025\ Pa}{(0.622 + 0.0074) \cdot 2186.3\ Pa} \cdot 100\ \% = 53.8\ \%$$

It is possible to see that the relative humidity do not exceed the 60%, which is the limit.

4.5.3 Outlet Specification

The outlet air properties of the zones is assumed to be less than in the zone itself. The assumed properties are given as:

$$P_{out} = 1 \cdot 10^5 Pa, \qquad T_{out} = 17°C, \qquad \phi_{out} = 0.45$$

From these values the saturation pressure and specific humidity can be found to be:

$$P_{ws,out} = 610.78 \cdot e^{\frac{17°C}{17°C+238.3} \cdot 17.2694} = 19288\ Pa$$

$$\omega_{out} = 0.622 \cdot \frac{19288\ Pa \cdot 0.45}{100000\ Pa - 19288\ Pa \cdot 0.45} = 0.004833\ \frac{kg\ water\ vapor}{kg\ dryair}$$

4.5.4 Mixing box

For the mixing box, both the outside air properties and the return air has to be determine. Data provided for the relative humidity the offshore site ‘Horns Rev’ is depicted in the figure below.

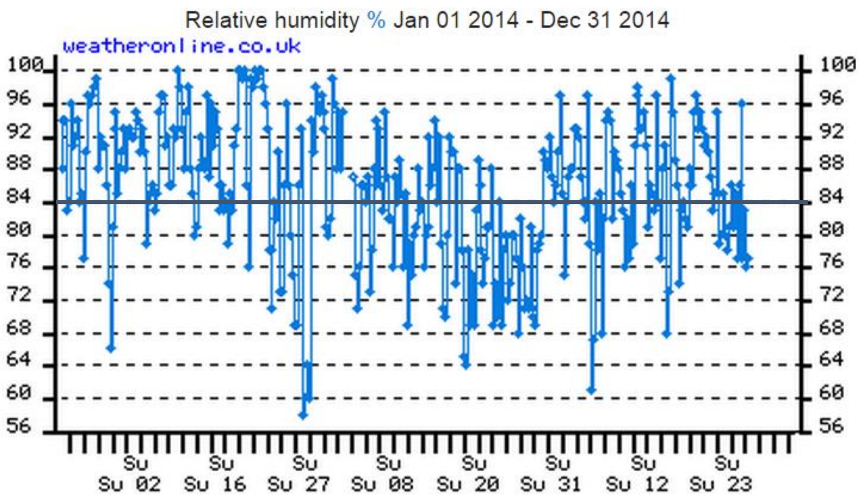

Figure 35 The RH at Horns Rev, Denmark. (Weatheronline)

From the figure above, the relative humidity can be assumed to be 84 %. The offshore average temperature for 2014 could be found to be 8.4°C. The air properties for the outside air are:

$$\phi_{atm} = 84\%, \qquad T_{atm} = 8.4°C, \qquad P_{atm} = P_{mix} = 1 \cdot 10^5 Pa$$

From this:

$$P_{ws,atm} = 610.78 \cdot e^{\frac{8.4°C}{8.4°C+238.3} \cdot 17.2694} = 1099.7\ Pa$$

$$\omega_{atm} = 0.622 \cdot \frac{1099.7\ Pa \cdot 0.84}{100000\ Pa - 1099.7\ Pa \cdot 0.84} = 0.0058 \frac{kg\ water\ vapor}{kg\ dryair}$$

From the heat transfer section, the mass flow rate from outside was determined based on a mixing temperature of approximating 15°C. Finding the saturation vapor pressure for this temperature results in:

$$P_{ws,mix} = 1744.3871\ Pa$$

The humidity of the return air is assumed to be the same as the outlet of the zones as it almost does not change when some air is exhausted.

The equation for the humidity of the mixture was found to be:

$$\omega_{mix} = \frac{\dot{m}_{out} \cdot \beta \cdot \omega_r + \dot{m}_{atm} \cdot \omega_{atm}}{\dot{m}_{out} \cdot \beta + \dot{m}_{atm}}$$

$$= \frac{0.448 \frac{kg}{s} \cdot 0.5 \cdot 0.0048 \frac{kg\ water\ vapor}{kg\ dryair} + 0.0505 \frac{kg}{s} \cdot 0.0058 \frac{kg\ water\ vapor}{kg\ dryair}}{0.448 \frac{kg}{s} \cdot 0.5 + 0.0505 \frac{kg}{s}}$$

$$\omega_{mix} = 0.0058\ \frac{kg\ water\ vapor}{kg\ dryair}$$

$$\phi_{mix} = \frac{P_{mix} \cdot \omega_{mix}}{(0.622 + \omega_{mix})P_{ws,mix}} \cdot 100\% = 63.86\%$$

4.5.5 Cooling coil

The cooling coil is only used for dehumidification if it is needed or if the inlet temperature has to be cooled below the dew point. The relative humidity of the air leaving the cooling coil will be at 100% else no dehumidification will occur. In this case, no dehumidification is needed and all the variables correspond to the mixing variable.

$$\phi_{cc} = \phi_{mix}, \qquad T_{cc} = 15°C, \qquad P_{cc} = 1 \cdot 10^5 Pa, \qquad \omega_{cc} = \omega_{mix}$$

The amount of liquid condensing is given by:

$$\dot{m}_{wcc,out} = (\omega_{mix} - \omega_{cc}) \cdot \dot{m}_{mix} = 0 \frac{kg}{s}$$

If dehumidification is needed, the temperature is set to the dew point temperature for the required specific humidity.

4.5.6 Humidifier

For the humidifier, the specific humidity needed for the inlet is calculated based in a steady state condition for the zone humidity model

$$\omega_{in} = 2 \cdot \omega_{z,setpoint} - \omega_{out} = 2 \cdot 0.0083 \frac{kg\ water\ vapor}{kg\ dryair} - 0.004833\ \frac{kg\ water\ vapor}{kg\ dryair}$$

$$\omega_{in} = 0.0101\ \frac{kg\ water\ vapor}{kg\ dryair}$$

Given the required temperature for the inlet air and the pressure, the following can be found for a mixing temperature of $15°C$, since no cooling in this case are needed.

$$T_{in} = 19°C, \qquad P_h = 1 \cdot 10^5 Pa, \qquad P_{ws,in} = 2186.3\ Pa$$

$$\dot{m}_{win} = (\omega_{mix} - \omega_{cc}) \cdot \dot{m}_{mix} = 0.0017 \frac{kg}{s}$$

5. State space model

For designing a MPC-controller, the state space model has to be found. The linear system is found from steady state condition where the dynamic of the non-linear functions will be zero. The linear point, also known as the operating point, can be given as (Franklin, Powell, & Emami-Naeini, 2010):

$$f(x_0, u_0) = 0 \qquad \textit{Eq 5.1}$$

Where f is the notation of the non-linear equation, x_0 and u_0 is the state and input value in the steady state condition.

The non-linear model can be approximated a linear model by using the Taylor expansion, which can be used to express the non-linear equation in form of the state space model. The linear approximation is only valid in a region close to the steady state point.

An n-order non-linear model can be given as a set of states and input equations for m-inputs and p-outputs (Pedersen, 2015):

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2 .. x_n, u_1, u_2 \dots u_m) \\ &\vdots \\ \dot{x}_n & \end{aligned} \qquad \textit{Eq 5.2}$$

The output function y for the non-linear model is expressed as:

$$\begin{aligned} y_1 &= h_1(x_1, x_2 .. x_n, u_1, u_2 \dots u_m) \\ &\vdots \\ y_p & \end{aligned} \qquad \textit{Eq 5.3}$$

The state space model can be represented by:

$$\begin{aligned} \dot{x} &= A \cdot x + B \cdot u \\ y &= C \cdot x + D \cdot u \end{aligned} \qquad \textit{Eq 5.4}$$

Where,

- A is the system matrix
- B is the input matrix
- C is the output matrix
- D is the feedforward matrix.

The matrices can be defined at the operating point (x_0, u_0) as:

$$\begin{aligned} A &= \left[\frac{\delta f}{\delta x}\right]_{x_0, u_0} \qquad B = \left[\frac{\delta f}{\delta u}\right]_{x_0, u_0} \\ C &= \left[\frac{\delta h}{\delta x}\right]_{x_0, u_0} \qquad D = \left[\frac{\delta h}{\delta u}\right]_{x_0, u_0} \end{aligned} \qquad \textit{Eq 5.5}$$

The matrices will be found by use of the Jacobean matrix:

$$J = \begin{bmatrix} \frac{\delta f_1}{\delta x_1} & \cdots & \frac{\delta f_1}{\delta x_n} \\ \vdots & \ddots & \vdots \\ \frac{\delta f_m}{\delta x_1} & \cdots & \frac{\delta f_m}{\delta x_n} \end{bmatrix} \qquad \text{Eq 5.6}$$

The same operation is required for the output function h.

5.1 Single zone linear model

The non-linear equations for the *temperature* are:

$$f_1 = \frac{dT_{mix}}{dt} = \frac{\dot{m}_{out} \cdot \beta \cdot c_p \cdot (T_{out} - T_{mix}) + \dot{m}_{os} \cdot c_p \cdot (T_{os} - T_{mix})}{m_{a,mix}} \qquad \text{Eq 5.7}$$

$$f_2 = \frac{dT_{cc}}{dt} = \frac{\dot{m}_{mix} \cdot c_p \cdot (T_{mix} - T_{cc}) - \dot{m}_{w,ccMax} \cdot Opening_{cc} \cdot c_{p,w} \cdot \left(T_{in,cc} - T_{out,cc}\right)}{m_{a,cc}} \qquad \text{Eq 5.8}$$

$$f_3 = \frac{dT_{hc}}{dt} = \frac{\dot{m}_{mix} \cdot c_p \cdot (T_{cc} - T_{hc}) - \dot{m}_{w,hcMax} \cdot Opening_{hc} \cdot c_{p,w} \cdot \left(T_{in,hc} - T_{out,hc}\right)}{m_{a,hc}} \qquad \text{Eq 5.9}$$

$$f_4 = \frac{dT_z}{dt} = \frac{\dot{Q}_{in} + \dot{Q}_{out}}{m_z \cdot c_p} + \frac{\dot{Q}_{rad} + \dot{Q}_{wall}}{m_z \cdot c_p} \qquad \text{Eq 5.10}$$

The non-linear equations for the *humidity* are:

$$f_5 = \frac{d\omega_{mix}}{dt} = \frac{\dot{m}_{out} \cdot \beta \cdot (\omega_{out} - \omega_{mix}) + \dot{m}_{os}(\omega_{os} - \omega_{mix})}{m_{a,mix}} \qquad \text{Eq 5.11}$$

$$f_6 = \frac{d\omega_{cc}}{dt} = \frac{m_{a,mix} \cdot (\omega_{mix} - \omega_{cc}) - \dot{m}_{w,out}}{m_{a,cc}} \qquad \text{Eq 5.12}$$

$$f_7 = \frac{d\omega_{in}}{dt} = \frac{\dot{m}_{a,mix} \cdot (\omega_{cc} - \omega_{in}) + m_{w,in}}{m_{a,h}} \qquad \text{Eq 5.13}$$

$$f_8 = \frac{d\omega_z}{dt} = \frac{\dot{m}_{in} \cdot (\omega_{in} - \omega_z) - \dot{m}_{out} \cdot (\omega_z - \omega_{out})}{m_z} \qquad \text{Eq 5.14}$$

The *mass* of the zone is given by:

$$f_9 = \frac{dm_z}{dt}$$

$$= \rho_{air,p} \cdot C_{d,in} \cdot A_{0,in} \sqrt{\frac{2 \cdot (p_{pipe} - p_{zone})}{\rho_{pipe}}} - \left(\frac{m_{zone}}{V_{zone}}\right) \cdot A_{0,out} \sqrt{2 \cdot \left(R \cdot T_{zone} - \frac{p_{atm}}{\left(\frac{m_{zone}}{V_{zone}}\right)}\right)} \qquad \text{Eq 5.15}$$

The states for these non-linear equations are:

Temperature: $T_{mix}, T_{in}, T_{cc}, T_z$

Mass: m_z

Humidity: $\omega_{mix}, \omega_{in}, \omega_{cc}, \omega_z$

The outputs are:

$$h_1 = T_z, \qquad h_2 = \omega_z, \qquad h_3 = P_z = \frac{m_z}{V_z} \cdot R \cdot T_z \qquad \textit{Eq 5.16}$$

Defining the states, input and output in vector form:

$$u = \begin{bmatrix} A_{0,in} \\ P_{in} \\ m_{w,in} \\ m_{w,out} \\ Opening_{hc} \\ Opening_{cc} \end{bmatrix}, \qquad x = \begin{bmatrix} m_z \\ T_{mix} \\ T_z \\ T_{in} \\ T_{cc} \\ \omega_{mix} \\ \omega_z \\ \omega_{cc} \\ \omega_{in} \end{bmatrix}, \qquad y = \begin{bmatrix} T_z \\ \omega_z \\ P_z \end{bmatrix}$$

The state space model for the single zone though linearization becomes:

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{21} & A_{22} & 0 & 0 & A_{25} & 0 & 0 & 0 & 0 \\ A_{31} & A_{32} & A_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{43} & A_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{54} & A_{55} & 0 & 0 & 0 & 0 \\ A_{61} & A_{62} & 0 & 0 & A_{65} & 0 & 0 & 0 & A_{69} \\ A_{71} & A_{72} & 0 & 0 & 0 & 0 & A_{77} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{88} & A_{89} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{98} & A_{99} \end{bmatrix}$$

$$B = \begin{bmatrix} B_{11} & B_{12} & 0 & 0 & 0 & 0 \\ B_{21} & B_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & B_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & B_{54} & 0 & 0 \\ B_{61} & B_{62} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & B_{86} \\ 0 & 0 & 0 & 0 & B_{95} & 0 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ C_{31} & 0 & C_{33} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \mathbf{0}$$

The elements for the multi-zone is given in Appendix, not the single-zone.

5.1.1 Simulation and validation of single zone linear model

The Simulink model for the single zone model is depicted below. To see zone model see Appendix 10.

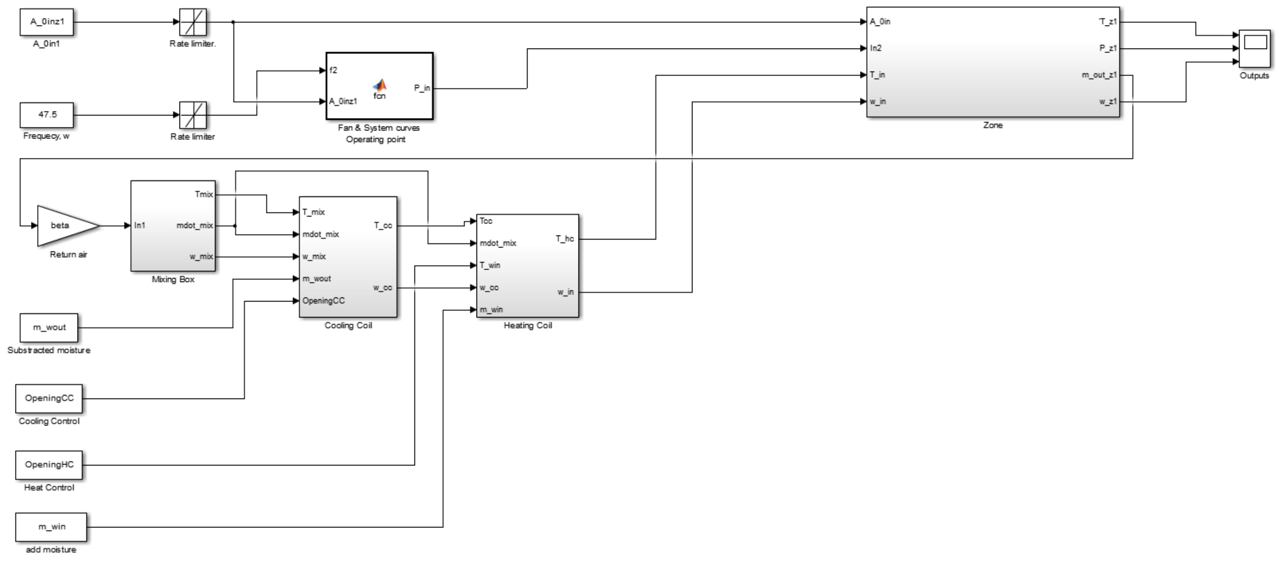

Figure 36 The single zone and the AHU and return air.

The responses of the linear model are validated against the non-linear model. The pressure, temperature and specific humidity in the zone are illustrated in the Figure 37. The non-linear model is depicted with the blue graph and the red graphs depicts the linear model.

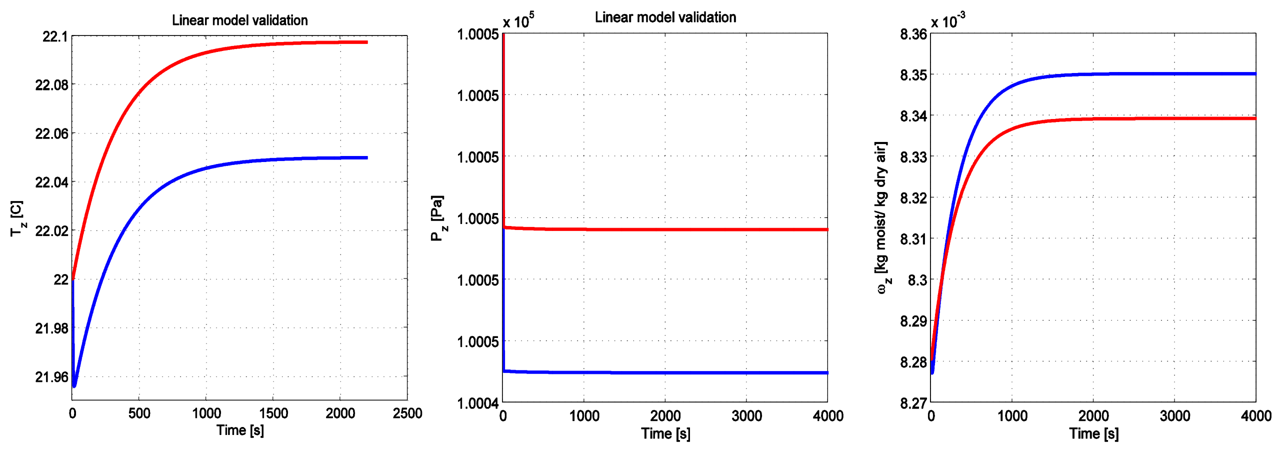

Figure 37 a) Temperature, b) Pressure, c) Specific humidity.

From the figure, it is possible to see that the two models have the same transient response and settles approximately in the same steady state value. The difference between the two models is a very small offset.

To see if the linear model follows the same dynamic for different inputs changes, a step-input in the opening area of the inlet damper is illustrated in Figure 38.The opening area is changed from $0.175m^2$ to $0.22\ m^2$.

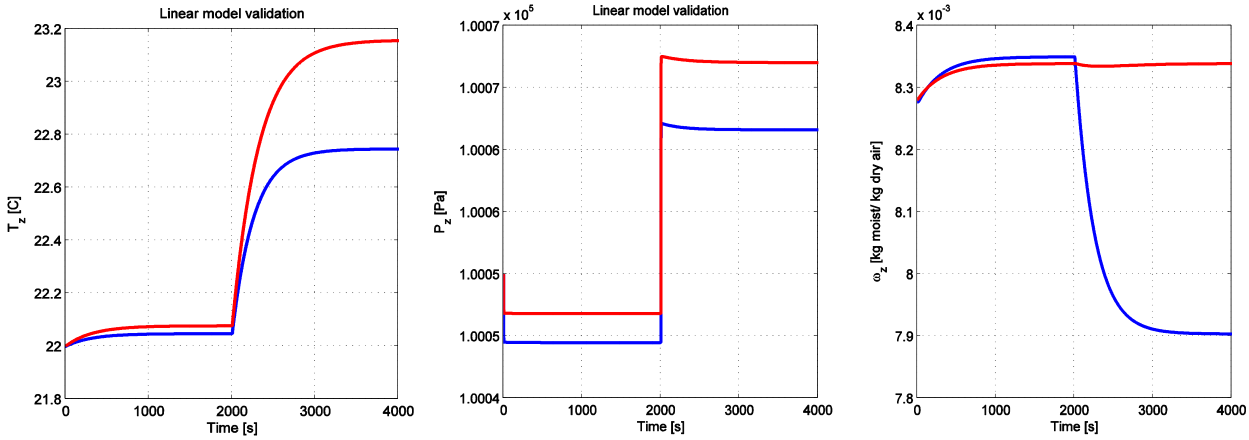

Figure 38 The temperature and mass of air in the zone.

From the figures, it is possible to see that for the input change, the two models have almost the same dynamic and steady state values. The change in the zone temperature has a difference of 0.4°C, which can be acceptable since the responses are very similar.

Regarding the linear humidity response, the change in value for the step input if compared to the non-linear humidity response has a difference of $0.4 \cdot 10^{-3}\,\frac{kg\ moit}{kg\ dry\ air}$. This is a relative large difference. The transient response is the same as the non-linear; the decrease in humidity is just so small it is almost undetectable. The cause of this could be due to that the humidity is highly non-linear and the dynamics is hard to capture when linearizing.

One reason could be the changing return mass flow rate which will cause the specific humidity in the mixing box to change. Without any controller the setpoint cannot be reached and the specific humidity decreases. This is only if the specific humidity in the mixing box decreases as a result of this. This change might be lost when linearizing the model.

5.2 Multi zone linear model

Expanding to multiple zones there will be states relating each zones, as there is heat transfer between the zones.

Adding dynamic of the walls, which will correlate the three zones.

$$f_{16} = \frac{dT_{w12}}{dt} = \frac{U_{12} \cdot A_{12} \cdot (T_{z1} - T_{w12}) - U_{12} \cdot A_{12} \cdot (T_{w12} - T_{z2})}{c_{p,wall1} \cdot m_{wall1}} \qquad \textit{Eq 5.17}$$

$$f_{17} = \frac{dT_{w23}}{dt} = \frac{U_{23} \cdot A_{23} \cdot (T_{z2} - T_{w23}) - U_{23} \cdot A_{23} \cdot (T_{w23} - T_{z3})}{c_{p,wall2} \cdot m_{wall2}} \qquad \textit{Eq 5.18}$$

In addition, the three zone equations are:

$$f_7 = \frac{\mathrm{dT_{z1}}}{dt} = \frac{\dot{Q}_{in,z1} + \dot{Q}_{out,z1} + \dot{Q}_{rad,c} + 5 \cdot \dot{Q}_{wall,z1} + \dot{Q}_{w12} + \dot{Q}_{rad,v}}{c_p \cdot m_{z1}} \qquad \textit{Eq 5.19}$$

$$f_{10} = \frac{\mathrm{dT_{z2}}}{dt} = \frac{\dot{Q}_{in,z2} + \dot{Q}_{out,z2} + \dot{Q}_{rad,c} + 4 \cdot \dot{Q}_{wall} + \dot{Q}_{w21} + \dot{Q}_{w32} + \dot{Q}_{rad,v}}{c_p \cdot m_{z2}} \qquad \textit{Eq 5.20}$$

$$f_{13} = \frac{\mathrm{dT_{z3}}}{dt} = \frac{\dot{Q}_{in,z3} + \dot{Q}_{out,z3} + \dot{Q}_{rad,c} + 5 \cdot \dot{Q}_{wall} + \dot{Q}_{w23} + \dot{Q}_{rad,v}}{c_p \cdot m_{z3}} \qquad \textit{Eq 5.21}$$

One thing to pay attention to is that as only one of the coils can be activated at the time. Therefore there are two state space models, on for a heating and Humidification case, also referred to as winter-case and a cooling and dehumidification case, also referred to as Summer-case. Additionally, the fan will be controlled separately from the MPC, therefore the inlet pressures are excluded in the multi-zone model.

For the multi-zone model, the state, inputs and outputs are:

$$x = \begin{bmatrix} T_{mix} \\ T_{cc} \\ T_{in} \\ \omega_{mix} \\ \omega_{cc} \\ \omega_{in} \\ T_{z_1} \\ m_{z_1} \\ \omega_{z_1} \\ T_{z_2} \\ m_{z_2} \\ \omega_{z_2} \\ T_{z_3} \\ m_{z_3} \\ \omega_{z_3} \\ T_{w12} \\ T_{w23} \end{bmatrix}, \quad y = \begin{bmatrix} T_{z_1} \\ \omega_{z_1} \\ P_{z_1} \\ T_{z_2} \\ \omega_{z_2} \\ P_{z_2} \\ T_{z_3} \\ \omega_{z_3} \\ P_{z_3} \\ T_{in} \end{bmatrix}, \quad u_{winter} = \begin{bmatrix} A_{in,z_1} \\ A_{in,z_2} \\ A_{in,z_3} \\ Opening_{HC} \\ \dot{m}_{w,in} \\ T_{rad,vz1} \\ T_{rad,vz2} \\ T_{rad,vz3} \end{bmatrix}, \quad u_{summer} = \begin{bmatrix} A_{in,z_1} \\ A_{in,z_2} \\ A_{in,z_3} \\ \dot{m}_{wcc,out} \\ Opening_{CC} \\ T_{rad,vz1} \\ T_{rad,vz2} \\ T_{rad,vz3} \end{bmatrix},$$

Next, the state space matrices for the winter case are given.

The elements for the following matrices can be found in the Appendix 11.

$$A = \begin{bmatrix}
A_{1,1} & 0 & 0 & 0 & 0 & 0 & A_{1,7} & A_{1,8} & 0 & A_{1,10} & A_{1,11} & 0 & A_{1,13} & A_{1,14} & 0 & 0 & 0 \\
A_{2,1} & A_{2,2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & A_{3,2} & A_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & A_{4,4} & 0 & 0 & A_{4,7} & A_{4,8} & 0 & A_{4,10} & A_{4,11} & 0 & A_{4,13} & A_{4,14} & 0 & 0 & 0 \\
0 & 0 & 0 & A_{5,4} & A_{5,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & A_{6,5} & A_{6,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & A_{7,7} & A_{8,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & A_{8,3} & 0 & 0 & 0 & A_{8,7} & A_{9,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{9,16} & 0 \\
0 & 0 & 0 & 0 & 0 & A_{9,6} & A_{9,7} & A_{10,8} & A_{9,9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{10,10} & A_{10,11} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & A_{11,3} & 0 & 0 & 0 & 0 & 0 & 0 & A_{11,10} & A_{11,11} & 0 & 0 & 0 & 0 & A_{11,16} & A_{11,17} \\
0 & 0 & 0 & 0 & 0 & A_{12,6} & 0 & 0 & 0 & A_{12,10} & A_{12,11} & A_{12,12} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{13,13} & A_{13,14} & 0 & 0 & 0 \\
0 & 0 & A_{14,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{14,13} & A_{14,14} & 0 & 0 & A_{14,17} \\
0 & 0 & 0 & 0 & 0 & A_{15,6} & 0 & 0 & 0 & 0 & 0 & 0 & A_{15,13} & A_{15,14} & A_{15,15} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{16,8} & 0 & 0 & A_{16,11} & 0 & 0 & 0 & 0 & A_{16,16} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{17,11} & 0 & 0 & A_{17,14} & 0 & 0 & A_{17,17}
\end{bmatrix}$$

$$B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & B_{3,4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & B_{6,5} & 0 & 0 & 0 \\
B_{7,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
B_{8,1} & 0 & 0 & 0 & 0 & B_{8,6} & 0 & 0 \\
B_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & B_{10,2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & B_{11,2} & 0 & 0 & 0 & 0 & B_{11,7} & 0 \\
0 & B_{12,2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & B_{13,3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & B_{14,8} \\
0 & 0 & B_{15,3} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C_{3,7} & C_{3,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{6,10} & C_{6,11} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{9,13} & C_{9,14} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$D = \mathbf{0}$$

5.2.1 Simulation and validation of multiple zone linear model

The non-linear model is illustrated in Figure 39. All the inputs are depicted to the left in the figure. The AHU with heating, cooling, dehumidification and humidification, is depicted in the left bottom half of the figure. Where in the right bottom half of the figure, the relative humidity for both the zones and the AHU are found. At last, the zone models are in the center of the overall model.

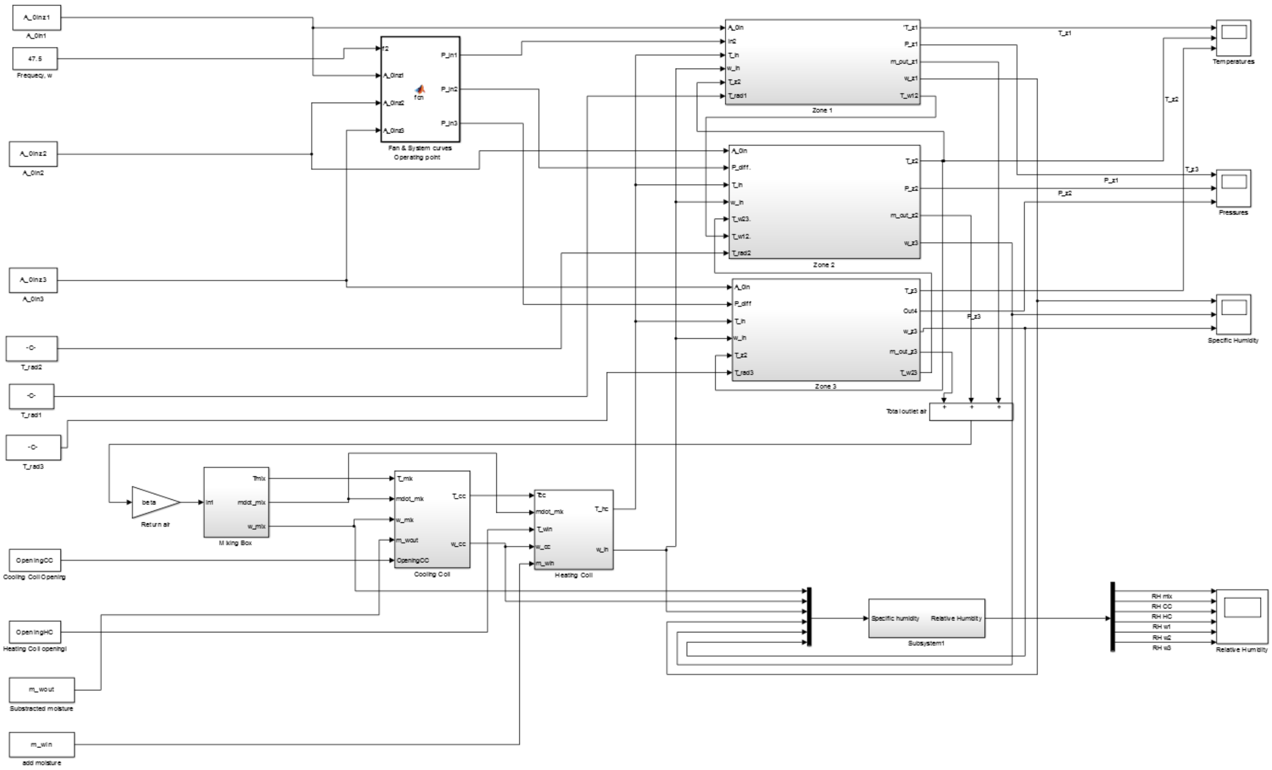

Figure 39 The non-linear model for the multi-zone.

5.2.1.1 Comparison of Linear and Non-linear model

The linear and Non-linear model is validated opposite to each other for different input changes:

- Step input in the dampers opening for zone 2 $A_{0inz2}: 0.0158\ m^2 \rightarrow 0.1\ m^2$
- Step input in the frequency is applied. The change if from $47.5Hz$ to $57.5Hz$.

A change in the damper of zone 2 is illustrated in Figure 40, Figure 41 and Figure 42.

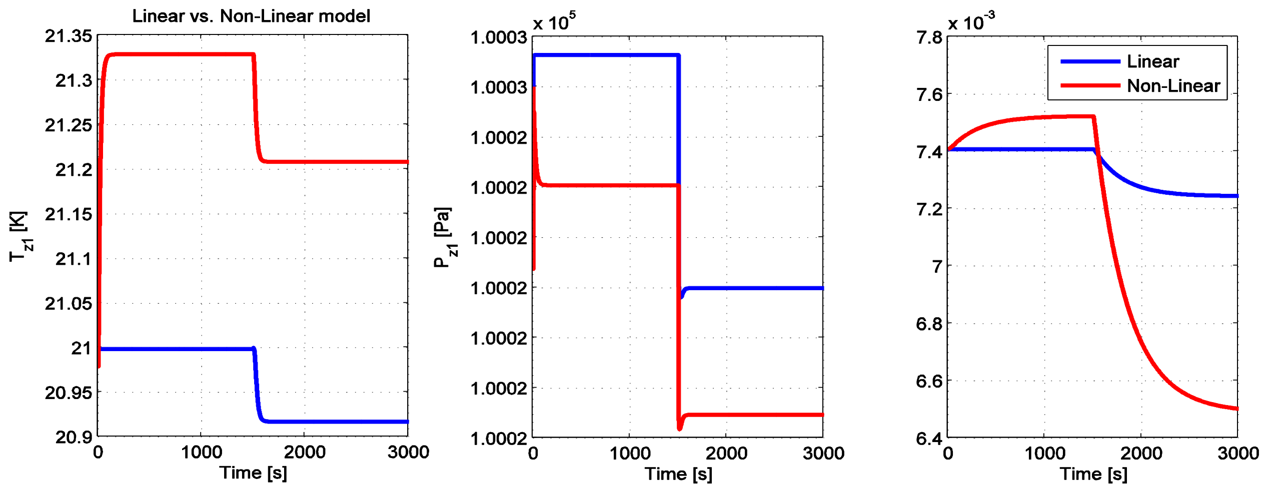

Figure 40 a) Temperature, b) Pressure, c) Specific humidity of zone 1.

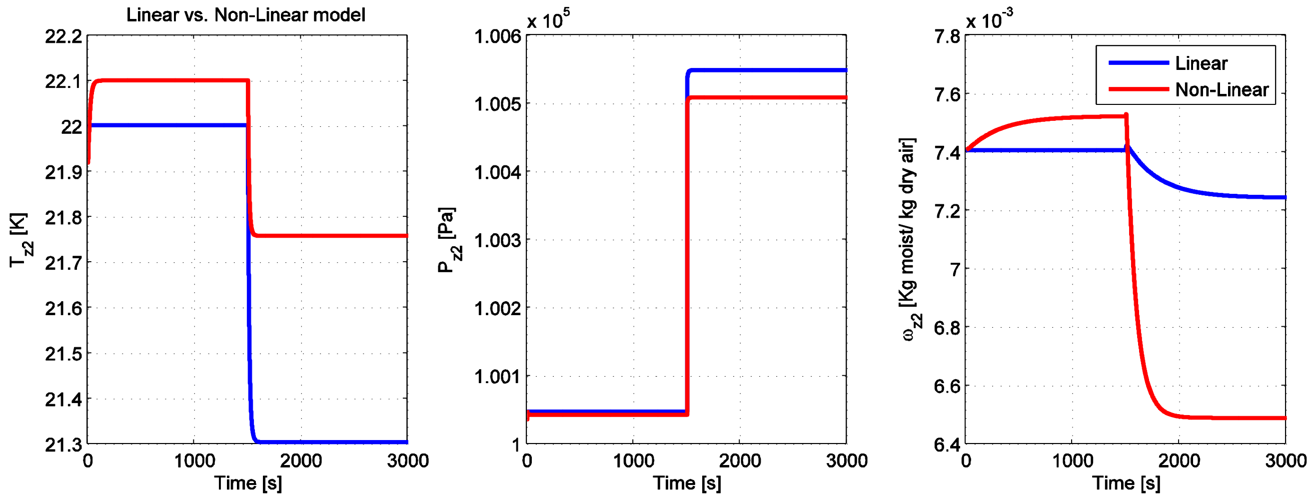

Figure 41 a) Temperature, b) Pressure, c) Specific humidity of zone 2.

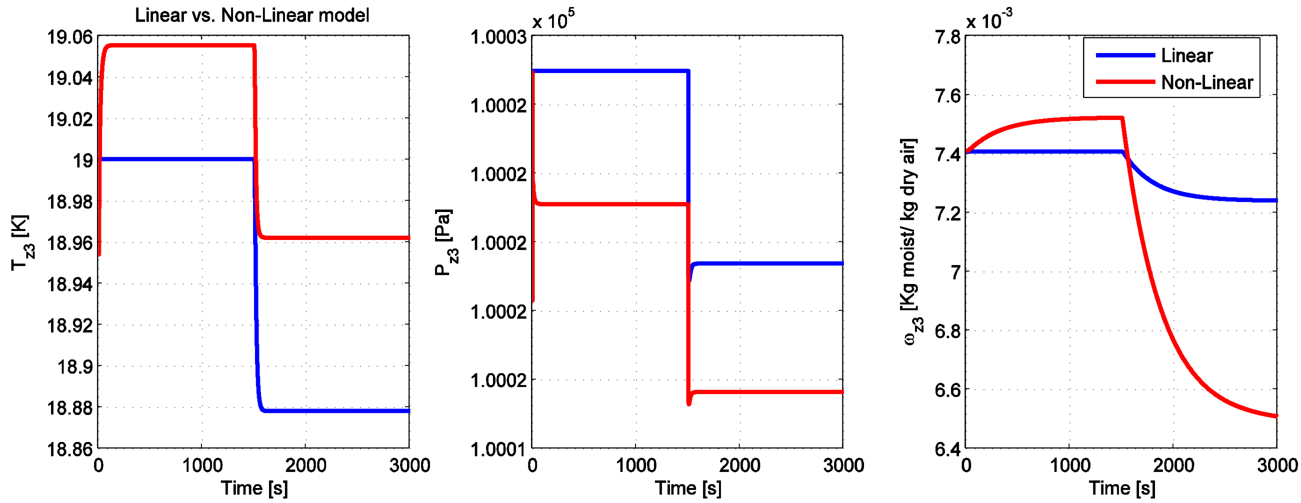

Figure 42 a) Temperature, b) Pressure, c) Specific humidity of zone 3.

From the figures, it is possible to conclude that the temperature and pressure of all the zones are valid. The setpoints of both the linear and non-linear model are the approximately the same, only with small deviations. When applying the change in damper opening, the temperatures and pressures transient response time and magnitude change are the same.

Looking at the specific humidity, there still exist a relative large difference when a change in input occurs. For this case, the difference is $0.8\frac{kg\ moist}{kg\ dry\ air}$.

A change in frequency is shown in Figure 43, Figure 44 and Figure 45.

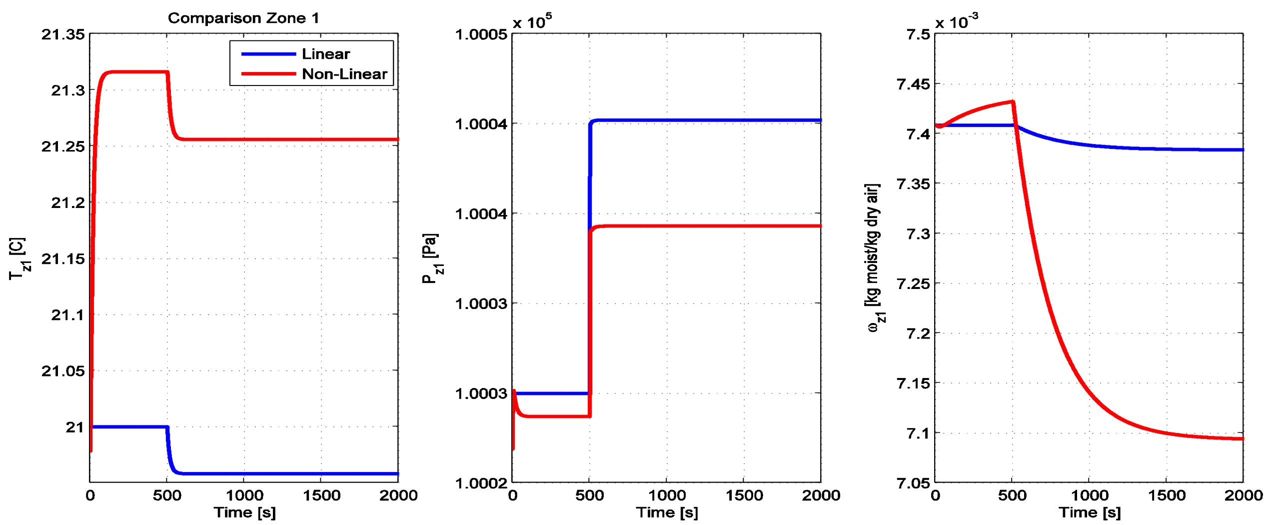

Figure 43 a) Temperature, b) Pressure, c) Specific humidity of zone 1.

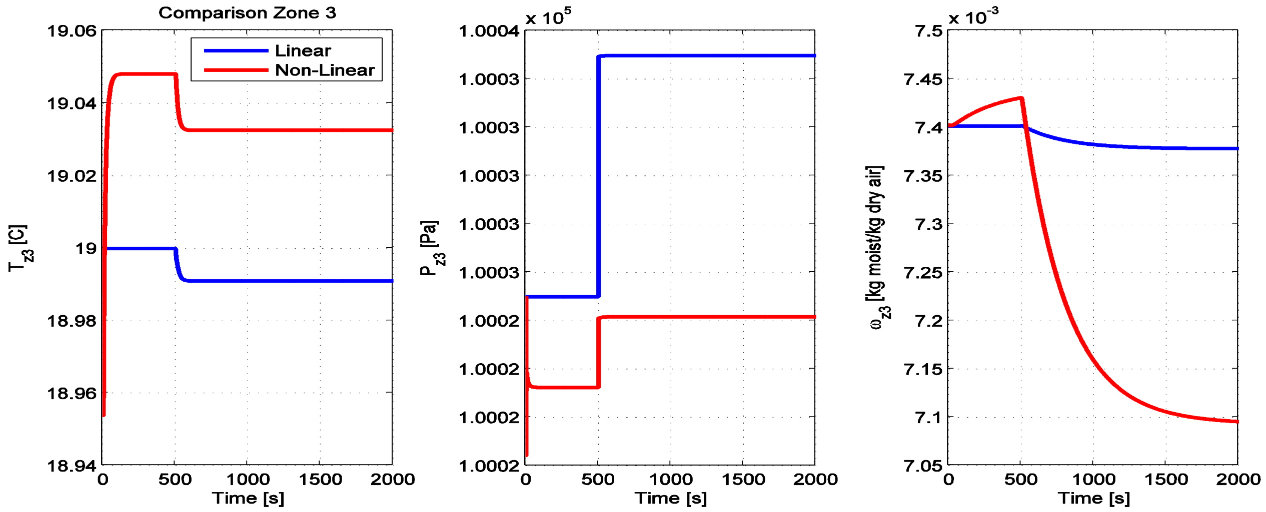

Figure 44 a) Temperature, b) Pressure, c) Specific humidity of zone 2.

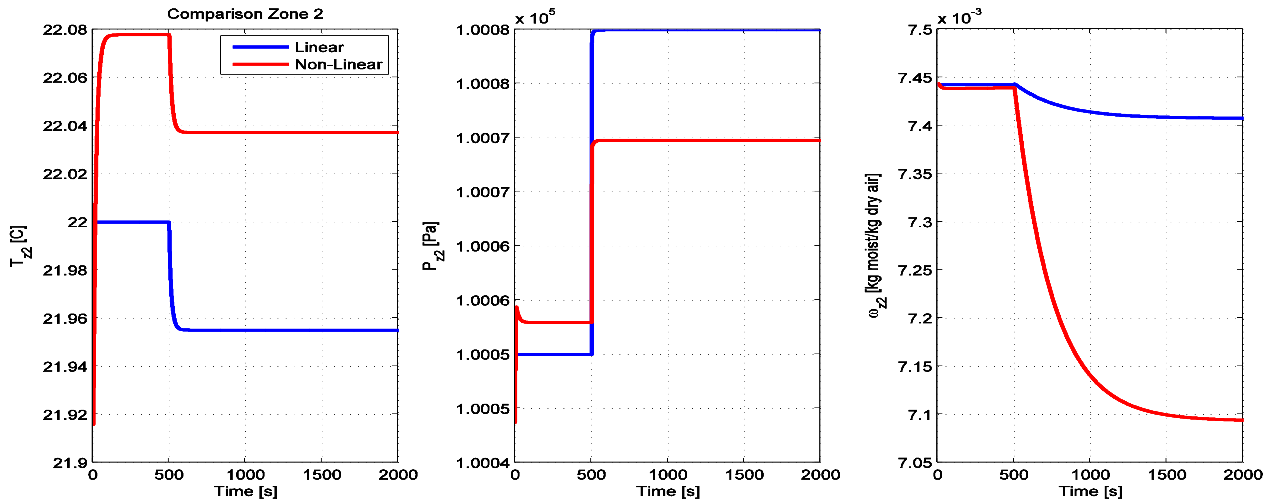

Figure 45 a) Temperature, b) Pressure, c) Specific humidity of zone 3.

From the figures, it is possible to see that the temperatures and pressures of the non-linear and linear models again are very similar. The change in linear pressure is slightly higher than the non-linear, but the difference is in the magnitude of few Pascal.

This time the specific humidity deviates less. This is due to that the specific humidity is strongly depend on the mass flow rate in and out of the zones. Increasing the speed of the fan do not in this case result in a large change in the inlet mass flow rate, whereas in the other case the damper was opened and more air was let in. The reason why decreases instead of increasing, when the damper opens, is because no controller is implemented and the mixing conditions are changing due to the variation of the return air.

5.3 Discrete time state space model

The discrete time state space model is used for designing the MPC. The discrete State space model is represented by (Advanced Motion Control, Difference equations, 2014):

$$(k+1) = A_d \cdot x(k) + B_d \cdot u(k)$$
$$y(k) = C_d \cdot x(k) + D_d \cdot u(k) \qquad \textit{Eq 5.22}$$

Where

$$A_d = e^{AT} = I + A \cdot T + \frac{A^2 \cdot T^2}{2!} \dots = I + A \cdot T \cdot \Psi \qquad \textit{Eq 5.23}$$

$$\Psi = I + \frac{A \cdot T}{2!} + \frac{A^2 \cdot T^2}{3!} \dots \qquad \textit{Eq 5.24}$$

$$B_d = \int_0^t e^{A\lambda} \cdot B \; d\tau = \Psi \cdot T \cdot B \qquad \textit{Eq 5.25}$$

And,

- C_D and D_d are equivalent to the continues time C and D matrices.
- T is the sapling time
- λ is the eigenvalue/s

By choosing a too small sampling time, when converting from the continuous time model to the discrete time can result in instability and the solution of the discrete time model will diverge. For this project, the sampling time is chosen to be 0.01s. This is to make the continuous time non-linear model work together with the discrete signals from the MPC.
The result of for the discrete state space model for zone 1 is depicted in Figure 46. This is for a change in damper opening into zone 2. The change is the same as before; 0.0158 $m^2 \rightarrow 0.1\ m^2$.

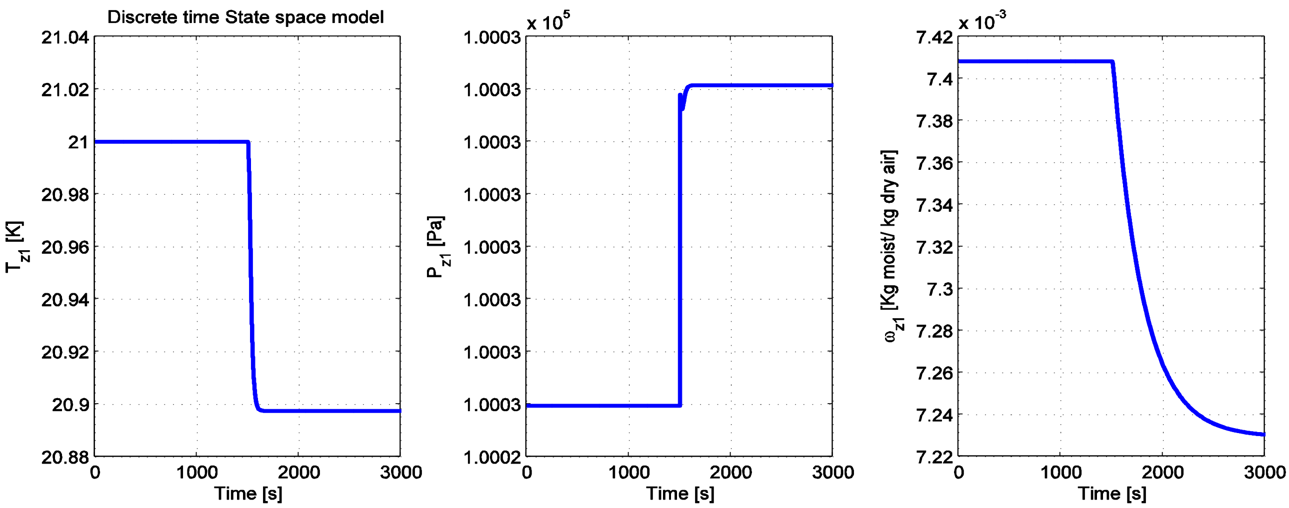

Figure 46 a) Temperature b) Pressure, c) Specific humidity of zone 1.

These responses correspond exactly to the continuous time responses. To see the responses of the other zones see Appendix 7.

6. Control design – PI

In this chapter the PI-controllers for the individual actuators are developed. In general, industrial HVAC units consists of a lot actuators that needs to be controlled. Specifically for this project, the following controllers/actuators are considered:

- Inlet Dampers
- Variable speed fan
- Variable electrical heaters
- Cooling and heating coils

The controllers and the feedback from sensors are illustrated in the 2D sketch below.

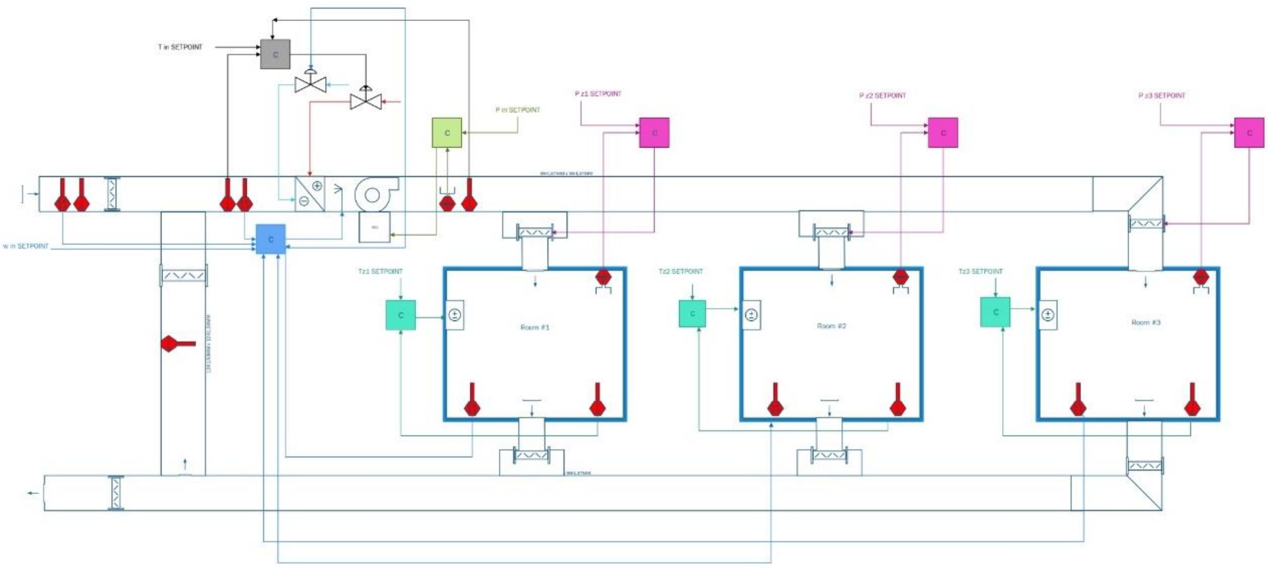

Figure 47 Controllers on the architecture design of the platform.

The following actuators are not controlled but are considered constant:

- Outlet dampers
- Economizer (i.e. inlet damper), exhaust- and return dampers

In the modeling chapter, the mixing temperature was fixed at $15°C$. In this part, it will remain at the same value, but as a part of the design phase, the controllers' performance for different mixing temperature and specific humidity will be investigated.

The PID controller is given by following equation (Franklin, Powell, & Emami-Naeini, 2010):

$$C_{PID} = k_P \cdot e(t) + k_I \cdot \int e(t) + k_D \cdot \frac{de(t)}{dt} \qquad \textit{Eq 6.1}$$

Where,

- $e(t)$ is the error between the setpoint and the actual output value.
- K_p, K_I and K_D is the control gains.
- C_{PID} denotes the controller.

6.1 Sequence of Heating and Cooling Coil

The purpose of the cooling and heating coils are to control the indoor humidity and indoor supply temperature. In order to control these variables feedback from sensors placed in the ducts and in the zones is needed. More in detail, the indoor supply temperature for each zone is been measured by a thermostat in the inlet duct. Moreover, the humidity is measured either in the zones or in the return duct, in this project is measured in the zone. The sensors used to measure the humidity is either a thermostat with a humidity sensor or a hydrometer. The specific humidity is beneficial to be measured at the outlet duct due to it can differ from zone to zone.

6.1.1 Temperature Control

In the VAV-HVAC system, the temperature of the supply air is held constant. On the other hand, the temperature and pressure of the zones is controlled by changing the volume of supplied air. The set point of the supply temperature will be fixed in winter and in summer. Moreover, when dehumidification is needed it will be set to the temperature necessary to dehumidify the air to the required supply specific humidity.

The temperature in both heating and cooling coil is regulated by the opening area of a valve, in the meaning of changing the water mass flow rate in the coils. An important restriction is that only one coil can be controlled at the time. In result of the previews stated only one coil can be controlled at the time, in the meaning of only one coil is active. The coil needed is activated by feedback of the supply temperature which is compared to the setpoint temperature. That means that in the case the temperature goes above the setpoint cooling is needed, otherwise the temperature is below the setpoint heating is needed. This idea is illustrated in the Figure 48.

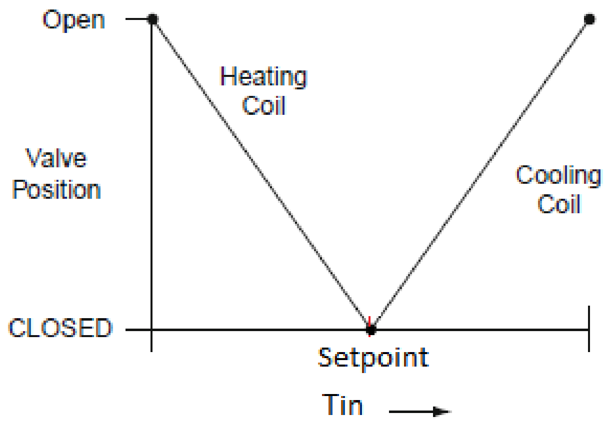

Figure 48 The control sequence of the cooling and heating coils. (Montgomery & McDowall, 2009)

Figure 48 illustrates the sequencing of the valve openings, which depend on the feedback of the supply temperature.

As one can predict, as the temperature vary around the setpoint the control will toggle between the heating- and cooling coil. To avoid this, ASHARE recommend a dead band around the setpoint of 3 degrees (Montgomery & McDowall, 2009).

In addition, to control the supply air by the two coils the economizer damper can be regulate to take in more or less air depending on the outside air in order to reduce energy consumption by cooling or heating less.

6.2 Humidity Control

The specific humidity is controlled by a PI-controller by feeding back the humidity measured in the zones, and compare it to the set point. The specific humidity is controlled by either adding moist from the humidifier or removing moist in the cooling coil, therefore, two controllers are needed. One is depend on the temperature in the cooling coil, whereas the other one is independent of the heating coil, as the specific humidity is considered not to be affected by heating, only the relative humidity.

6.3 Control design

The heating and humidification control block diagram is depicted in Figure 49. The control gain values are given in Table 7.

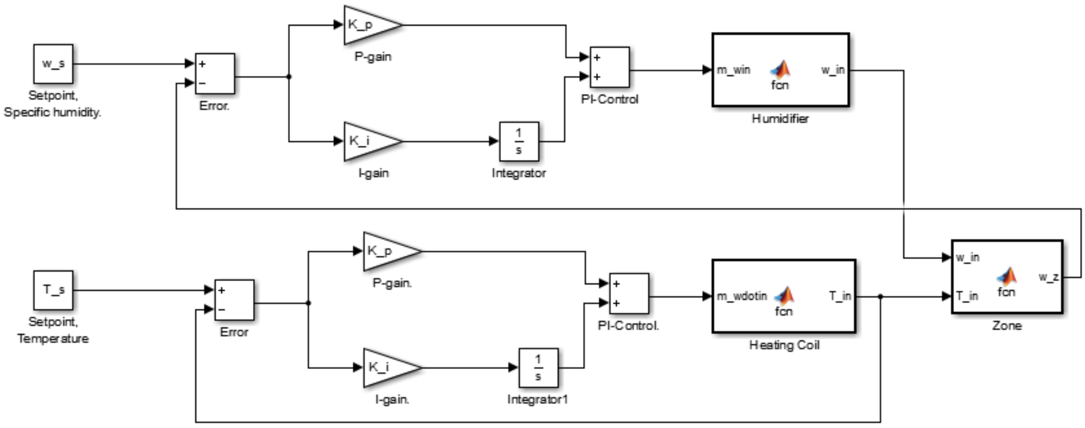

Figure 49 The block diagram of Heating and humidification.

From figure it is clear that the two controllers operate independent. The humidifier is controlled by feedback of specific humidity measurement and it controls by adding more water vapor mass $m_{w,in}$. The temperature is controlled by changing the percentage opening of the valve and thereby causing changes in mass flow in the heating coil.

The Cooling and dehumidification control block diagram is depicted in Figure 50.

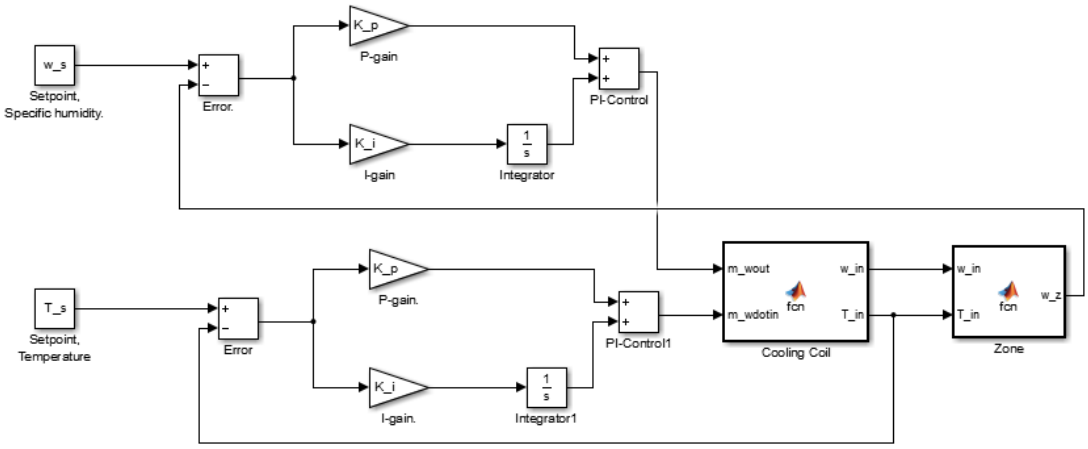

Figure 50 The block diagram of Cooling and dehumidification.

Figure 50 shows that the controller works and which measurements are fed back. The difference from the other block diagram is that the two controllers are depend on each other. They both control the cooling coil and are related by the temperature needs to be at a given set point in order to dehumidify to a given specific humidity. As the relative humidity of the air leaving the coil is 100 % in order to achieve dehumidification, the set point for the supply temperature can be found from the specific humidity set point.

In Simulink, the controllers are all connected to a Mat-file where the different coils and humidifier is activated based on the states of the mixing box. In order to maintain the safety in the HVAC system, the coils are only active when the fan is turned on, otherwise the system could experience damages such as overheating. This can be seen in the a section from the Mat-file below, *LL* denotes lower limit and *HL* is higher limit, *Freq* denotes the frequency of the fan, and thereby makes sure that the fan is operating before any *if*-statements are fulfilled. The lower and higher limits are the dead band of the coils, introduced previously.

```
if Freq ~= 0 && T_mix > T_HL && w_mix > w_HL;          % Cooling and dehumidification

    OpeningHeating=0;                                  % Disable Heating Coil (0% open)
    EnableHumidifier=0;
    EnableDehumidifier=1;

else if Freq ~= 0 && T_mix < T_LL && w_mix < w_LL;    % Heat and humidify

        OpeningCooling=0;
        EnableHumidifier=1;
        EnableDehumidifier=0;

    else if Freq ==0 || T_LL < T_mix && T_mix < T_HL && w_LL < w_mix && w_mix < w_HL;

                % Close all
                OpeningCooling=0;
                OpeningHeating=0;                      % Disable both Coils
                EnableHumidifier=0;                    % Disable both dehum/humidifier
                EnableDehumidifier=0;

        end
    end
end
```

Table 6 Describes the if-statements, which activates the different controllers. See the entire code in Appendix 8.

In the script, there are three cases; *Case 1.* When cooling and dehumidification is needed. Here the mixing condition is above the higher limit, which means it is higher than the supply specifications. *Case 2.* Is the opposite, here the heater and humidifier is enabled. *Case 3.* Is when the mixing conditions are between the lower and higher limit. In this case, everything is turned off. An example could be that the cooling coil is not needed, the *OpeningCooling* is set to zero and vice versa. The same is valid for the humidification- and dehumidification process. If the process is not needed, either *EnableHumidifier* or *EnableDehumidifier* is set to zero. However, it follows that the humidifier cannot be enabled when the cooling coil is on.

The PI-controllers are all manually tuned aiming the system to experience less oscillations and a reasonable settling time. A too fast settling time will not be realistic for this system. As the HVAC system is highly non-linear and the PI-controllers are linear, their performance will differ depending on operating conditions. It is recommended that the controller for the temperature is over damped; this increases settling time and reduces performance, but the overshoot is eliminated.

The responses for the tuned PI-controllers of the specific humidity and supply temperature are illustrated for both a winter and summer case to the controllers' performance.

6.3.1 Winter Case:

The responses for controllers of the heating coil and humidifier are depicted in the below figure.

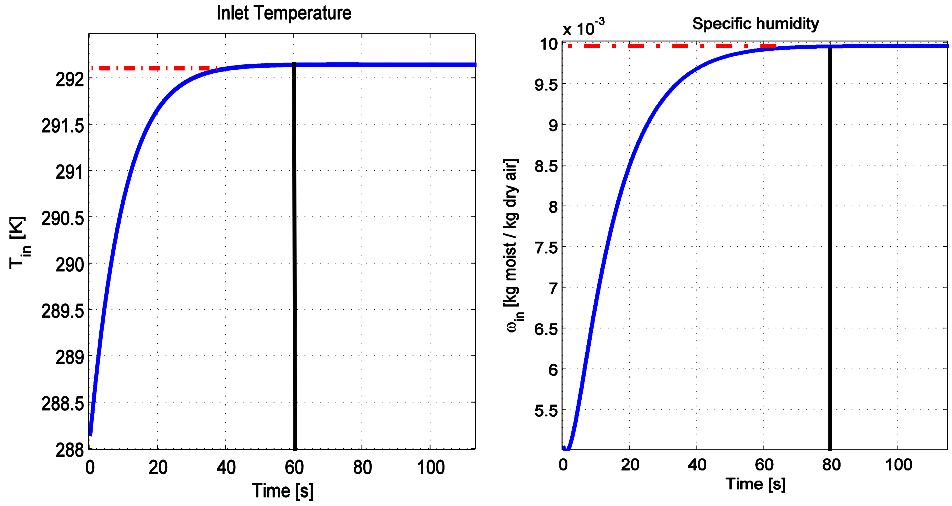

Figure 51 The responses for temperature and specific humidity for heating and humidification

6.3.2 Summer Case:

The responses for controllers of the Cooling coil and dehumidification are depicted in the below figure.

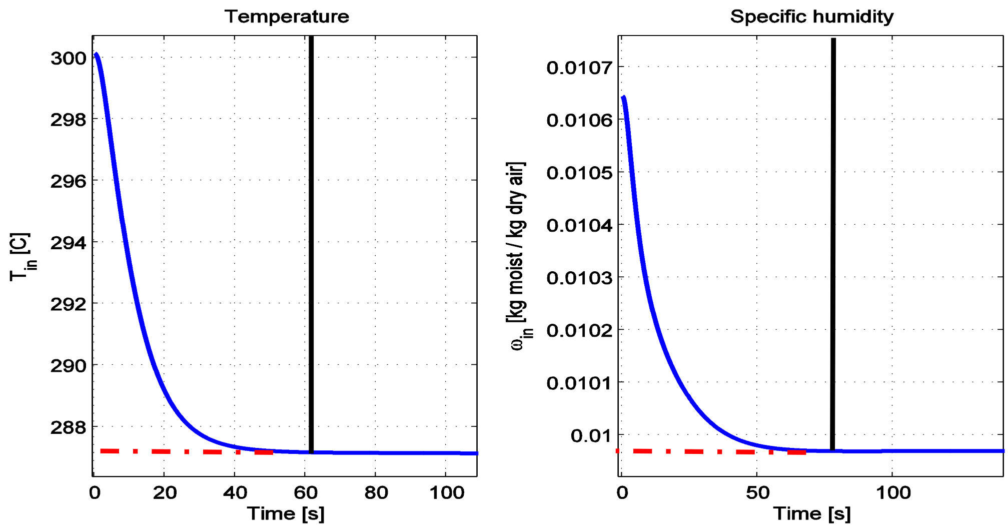

Figure 52 The responses for temperature and specific humidity for cooling and dehumidification.

The settling time can be seen from the above figures. The final control gain values and settling time are:

Unit	K_p	K_i	T_s	Overshoot
Humidifier	30	0.1	$80s$	0%
Dehumidifier	25	0.065	$80s$	0%
Heating Coil	0.01	0.005	$60s$	0%
Cooling Coil	0.01	0.005	$60s$	0%

Table 7 Final control gain values and settling time for each unit.

Note that: As the operating condition changes so do the settling time, as further away the humidity come from the setpoint the longer time it takes to settle. This can be seen in the next section.

6.3.4 Changing mixing conditions - Winter Case

For the winter case three other setpoints are considered:

$$T_{mix} = [283.15, 287.15, 290.15]K$$

$$\omega_{mix} = [0.0037, 0.00505, 0.0058]\frac{kg\ moist}{kg\ dry\ air}$$

The supply temperature, mixing temperature and Valve position of the cooling coil is depicted in Figure 53.

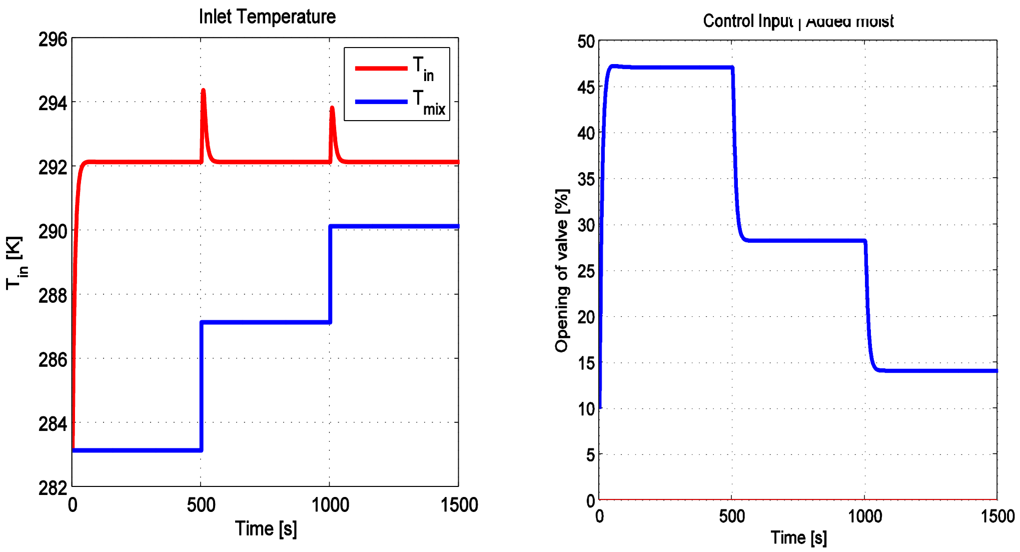

Figure 53 Inlet temperature and Valve opening.

From the figure above it is possible to see that the set point of 292.15 K (19°C) is maintained when the different step inputs are applied.

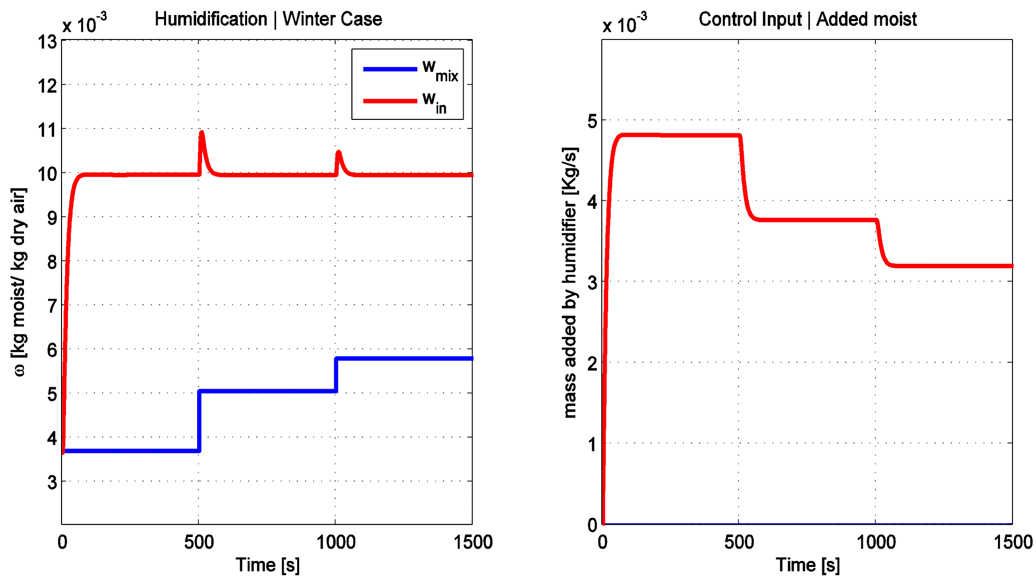

Figure 54 The inlet and mixing specific humidity and added moist.

The humidity is maintained and it is seen that the added moisture to the air decreases with decreasing the difference between the setpoint and the mixing specific humidity.

6.3.3 Changing mixing conditions - Summer Case

If both mixing temperature and humidity are high then cooling and dehumidification is needed. A case of different mixing temperatures and specific humidity are applied to the model to see how the PI-controller works.

The applied steps are:

$$T_{mix} = [295.15, 300.15, 303.15]K$$

$$\omega_{mix} = [0.0105, 0.0125, 0.0168]\frac{kg\ moist}{kg\ dry\ air}$$

The supply temperature, mixing temperature and Valve position of the cooling coil is depicted Figure 55.

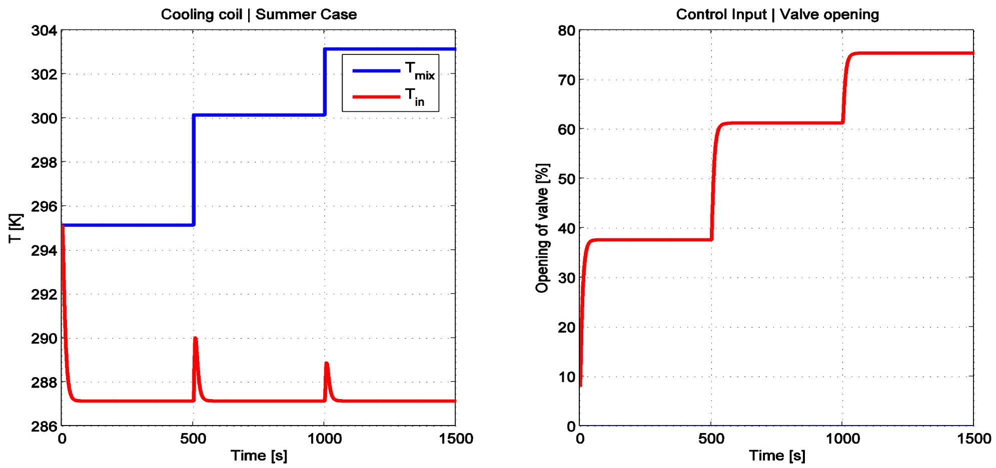

Figure 55 The PI-control of the temperature for different mixing temperatures.

From the figure above is possible to see that the set point of the temperature is maintained (red line) disregarding the increasing mixing temperature (blue line). This is done by opening the valve position more and more (see figure 7-graph to the right). The different set point is smoothly achieved without any overshoot.

Figure 56 illustrates the supply and mixing specific humidity, as well as the removed moist.

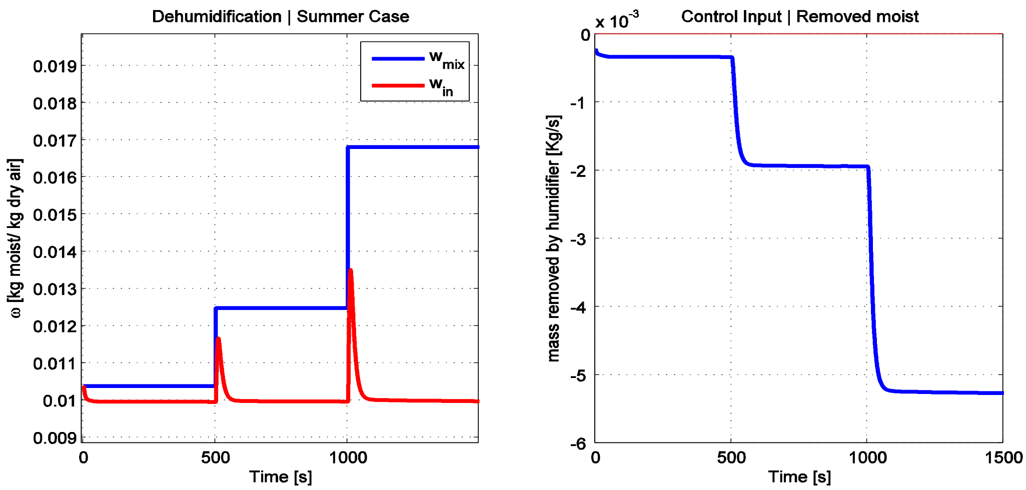

Figure 56The PI-control of the temperature for different mixing specific humidity.

As for the temperature, the humidity is maintained at the set point at all time, except at the transition period.

6.3.4 Zone specific humidity control

In order to observe what happens in the zones specific humidity, a change in the damper opening is applied. Pressure changes lead to an expected change in other zones specific humidity due to the reduction of the mass flow rate in the inlet duct, which is natural because the controller for the fan is not yet implemented. The opening is applied for the first zone and the responses are depicted in the figure below.

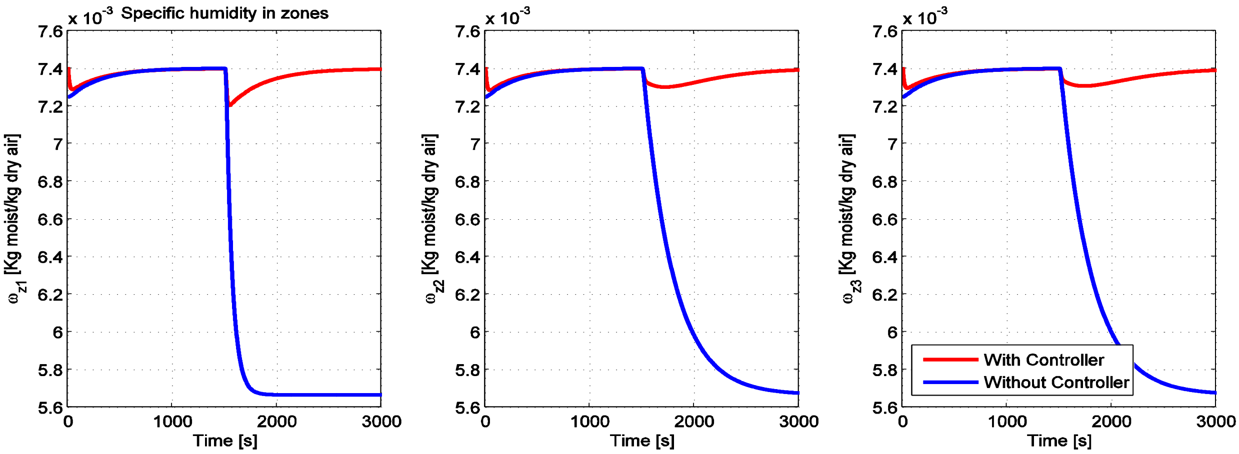

Figure 57 The specific humidity in all the zones with and without humidity control.

Form the above figure, it can be seen that the controller makes sure that the specific humidity only decreases by $0.2\cdot 10^{-3}\ \frac{kg\ moist}{kg\ dry\ air}$ wheras for the uncontrolled model the specific humodot deacreases with $1.7\cdot 10^{-3}\ \frac{kg\ moist}{kg\ dry\ air}$.

The influence of inlet temperature on the zone temperatures are investigated in the next section, where the controllers for the variable radiators are design.

6.4 Remaining controls

In this part of the project, the PI controllers are implemented in the multi-zone model to control and maintain the needed temperature and pressure in each zone. In addition, the PI controller for the fan is implemented in order to control the variable speed of the fan by controlling its frequency. All the PI controllers are tuned manually by inserting different values of the proportional and integral part of the controller.

6.4.1 PI control of zone temperatures

The temperature in the zones is controlled by the variable radiators. In Figure 58 the reaction of each of the PI controller in each zone is shown.

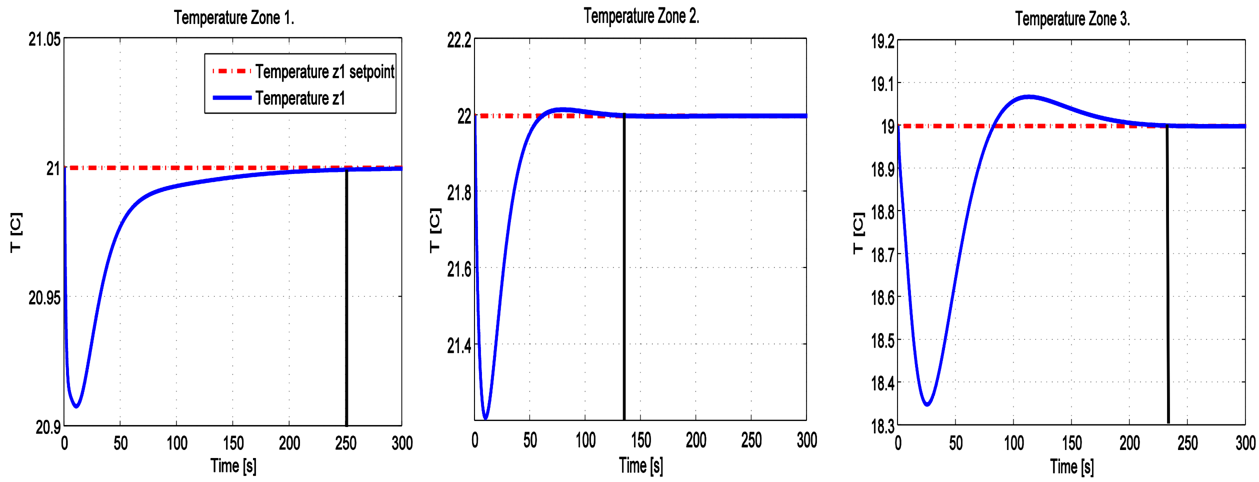

Figure 58 The Temperature of a) zone 1, b) zone 2, c) zone 3.

From the figures, it is possible to see that the temperature settles to the setpoint in the interval of $130s - 250s$. A small overshoot occurs in zone 2 and zone 3, of which the magnitude is in order of $0.02°C$ - $0.07°C$. In the start of all the responses, the transient response starts by decreasing. This is due to a high initial value and changing it to a smaller value will eliminate this “undershoot”.

In the below depicted Table 8 the values of the PI for each zone are depicted.

Zone	K_p	K_i	T_s	O
1	6	0.099	$250s$	0%
2	2.1	0.14	$135s$	$\frac{0.02}{22} = \sim 0\%$
3	0.01	0.009	$230s$	$\frac{0.07}{19} \sim 0\%$

Table 8 The specifications of the variable radiator controllers

6.4.2 Zone pressure

In the Figure 59 the PI controller reaction for each of the 3 zones is illustrated.

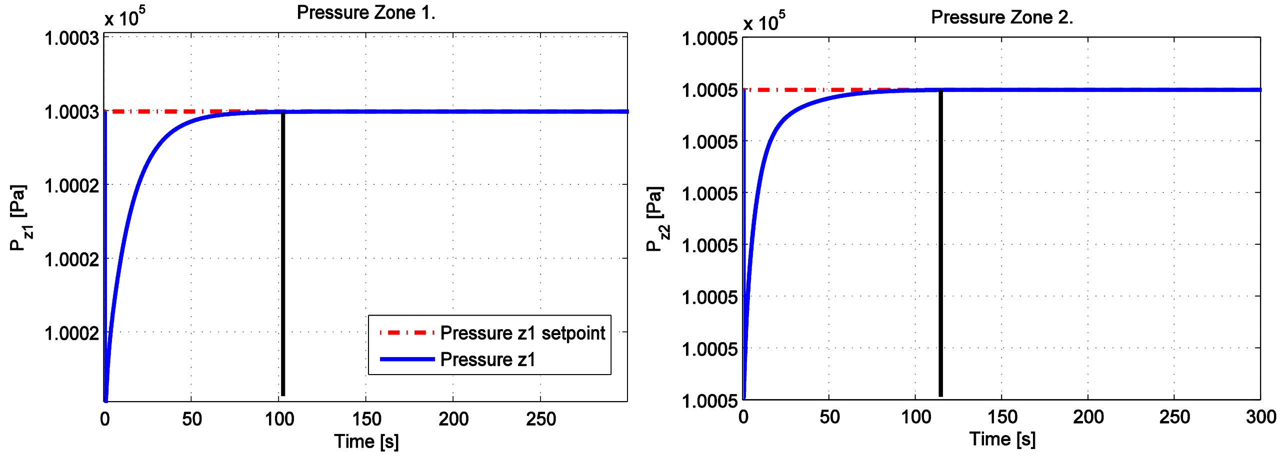

Figure 59 a) The zone 1. Pressure b) The zone 2. Pressure.

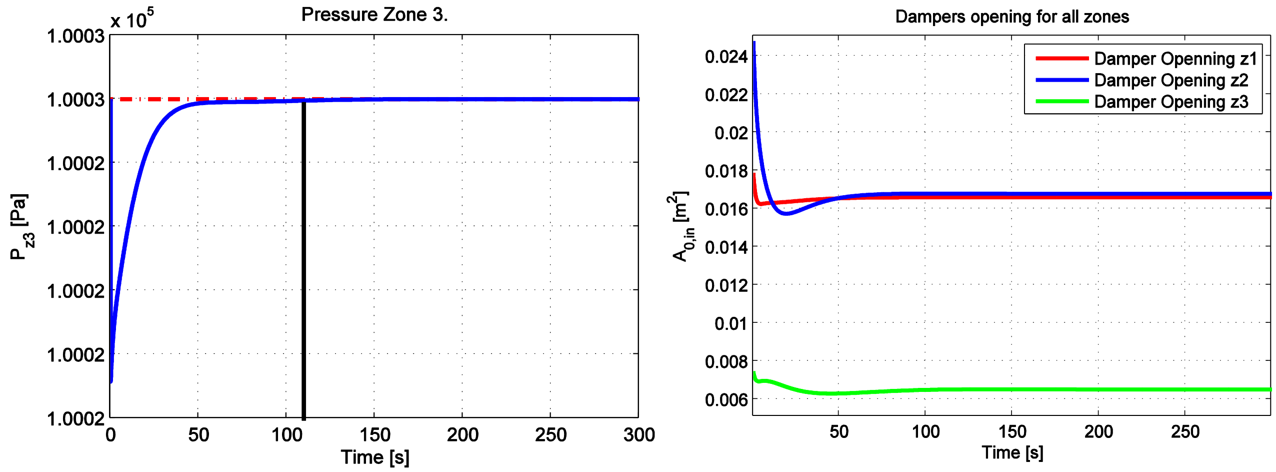

Figure 60 a) The zone 3. Pressure, b) The corresponding damper openings for all three zones.

In the below depicted Table 9 the values of the PI for each zone are depicted.

Zone	K_p	K_i	T_s	O
1	0.0086	0.00060	$105s$	0%
2	0.0086	0.00060	$114s$	0%
3	0.0086	0.00060	$110s$	0%

Table 9 Specifications for the damper controller.

6.4.3 Fan and inlet pressure

In Figure 61, the reaction of the fan's PI controller is depicted.

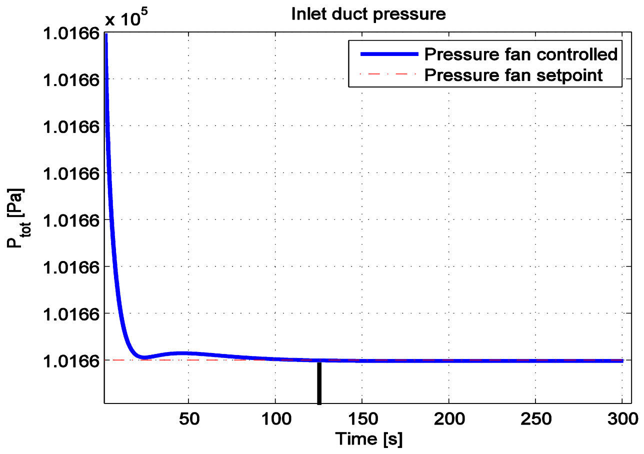

Figure 61 Inlet total pressure provided by the fan

From the figure, it is possible to see that the inlet pressure settles within 125s. Taking into account that the value has to be within 2% of the steady state value, the settling time would be 0s. The deviation from the setpoint is very small, and are in the order of 0-9 Pa, which is basically nothing. The corresponding frequency change is equally small.

In Table 10, the specifications of the controller is given.

K_p	K_i	T_s	O
1.09	0.095	$71s$	0%

Table 10 Controller specification for the fan.

6.4.4 Disturbance – Change in inlet temperature

In this subchapter, changes in the inlet temperature is implemented. In this way, it is able to recognize the effects of the temperature change in all the three zones. In the Figure 62 both the temperature changes and the inlet temperature are depicted.

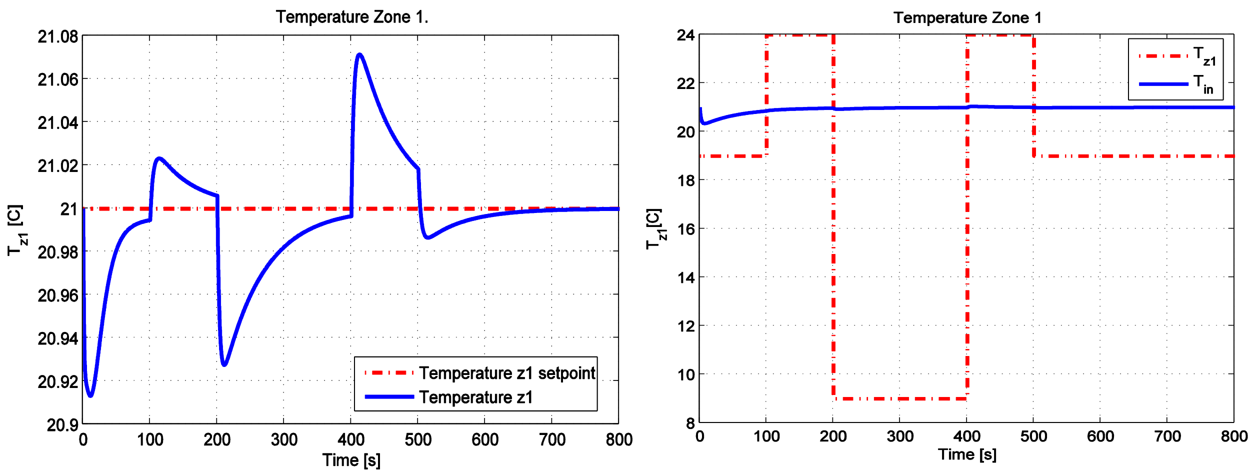

Figure 62 a) The temperature in zone 1. b) The inlet temperature and zone temperature.

In the above figure, the change of the temperature in zone 1 and the PI controller reaction are depicted. Each time the controller is getting another temperature setpoint it always tries to settle down to the temperature setpoint of zone 1, which is $21°C$. The deviations are in order of $0°C - 0.05°C$. This means, no mater the inlet tempereture, the radiator makes sure that the zone temperature is achieved.

The response for zone 2 is depicted in Figure 63.

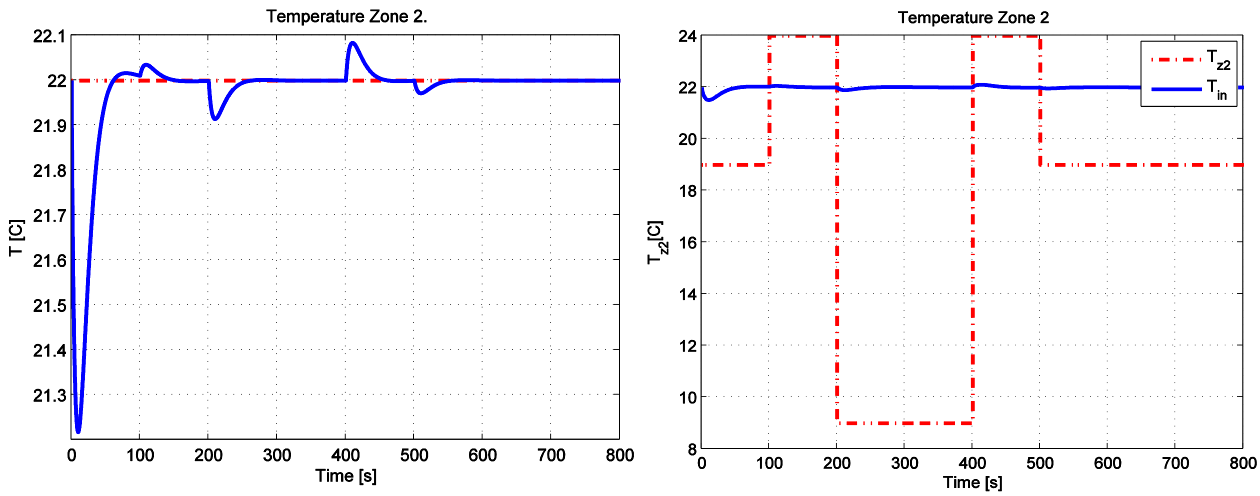

Figure 63 a) The temperature in zone 2. b) The inlet temperature and zone temperature.

In the above figure, the change of the temperature in the zone 2 and the PI controller reaction are depicted. The reaction of the PI controller in the zone 2 seems to manage quite easy the disturbances that are implemented. The deviation is between $0°C$ to $0.8°C$ in the graph to the right, the changes in the zone temperature is plotted togther with the inlet temperature changes. The deviation is very small compared to the already applied changes.

The response for zone 3 is depicted in Figure 64.

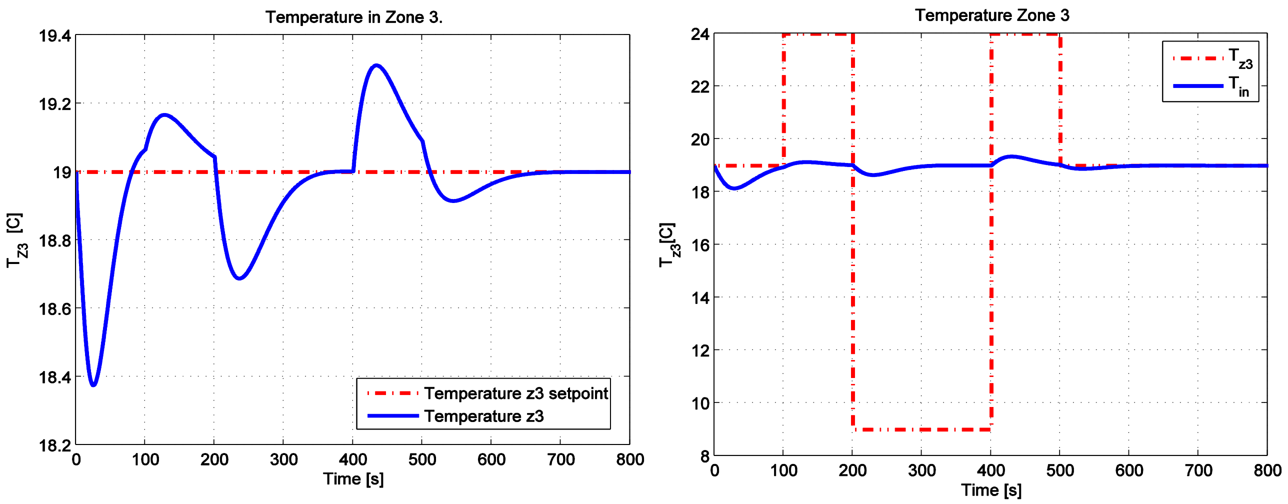

Figure 64 a) Temperature in zone 3 and setpoint, b) Zone and inlet temperature.

From the figure above it can be seen that the temperature tries to settle down each time the inlet temperture changes. The deviations form the setpoint is between $0°C$ to 0.3°.

Disturbance – Change in damper openings

In this sub chapter, it will be implemented different fixed opening area in all inlet dampers at the same time. In this way, the fan's PI reaction will be investigated. The responses of both the dampers and the corresponding pressure changes in the zones are depicted below.

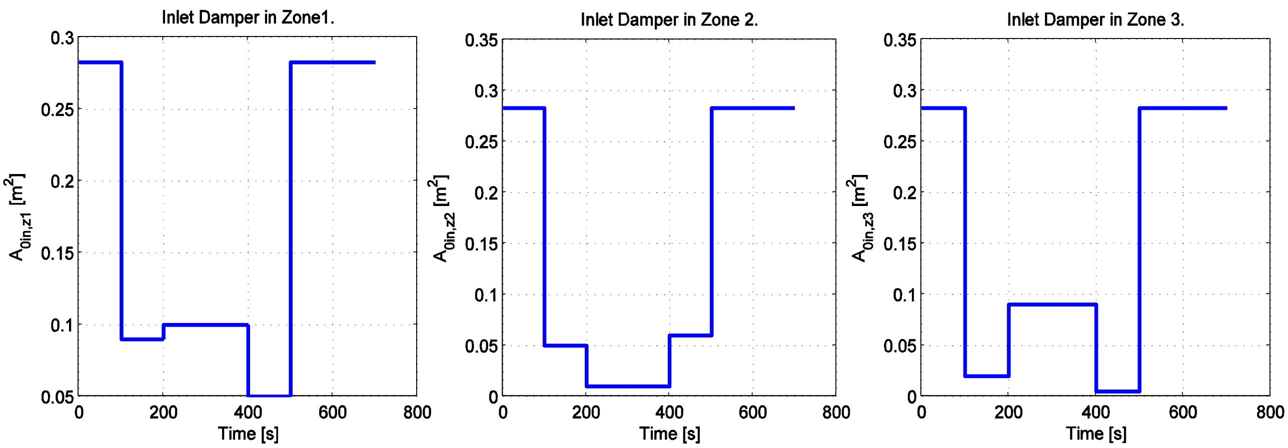

Figure 65 Setpoints for a) Damper zone 1, b) Damper zone 2, c) Damper zone 3.

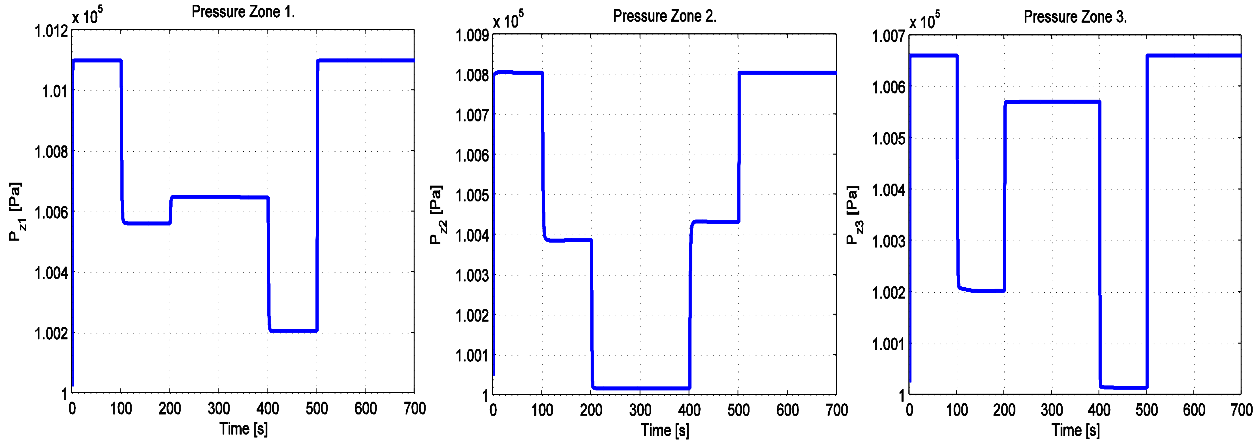

Figure 66 The corresponding pressure changes in a) zone 1, b) zone 2, c) zone 3.

The response of the fan is:

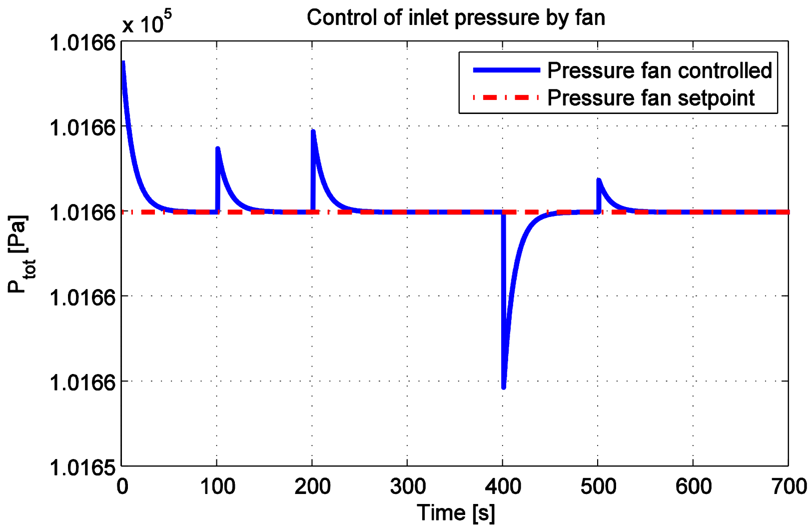

Figure 67 Control of inlet pressure.

Figure 67 illustrates the reaction of the fan in different damper opening areas, which occur in the model as a disturbance in fixed values and times. As it is seemed in the figure, the fan regulates within the 71s to the exact setpoint value, the deviations are almost unnoticeable.

7. Control design – Model Predictive Control

As it was mentioned in the previous chapter, the HVAC system has many actuators that need to be controlled. The PI-control approach handles this by applying individual control to each actuator. In this chapter a new control approach is considered, which controls all the actuators with one single controller. However, the fan is controlled by the PI-controller designed in the previous chapter. Due to that the operating speed is found from the intersection between a fan curve and a systems curve.

This control method is called Model Predictive Control popular as MPC. The MPC adjusts the control input before the changes in output occurs. It solves optimization problems, like minimizing the energy consumption and applies constraints for either actuators or state variables. The standard PI-controller does not take into account the constraints/limits unless limitation and anti-windup is applied.

7.1 MPC

The MPC is a controller that uses dynamic programming[1] to find a sequence of future control inputs, which minimizes a given cost function. The length of this input sequence is given by a control horizon, N_u. In addition to the control horizon the prediction of future state variables is defined as N_p, which indicates how many steps k is the future predicted. It follows that $N_u \leq N_p$. The idea of predicting the future inputs and output is illustrated in Figure 68. If the horizons becomes too small the control becomes closer to the normally used PID strategy, whereas if the horizon becomes too long it could induce oscillation because the MPC control effort becomes more aggressive. Also, a longer horizon could decrease the performance as the time for calculations at each time step increases.

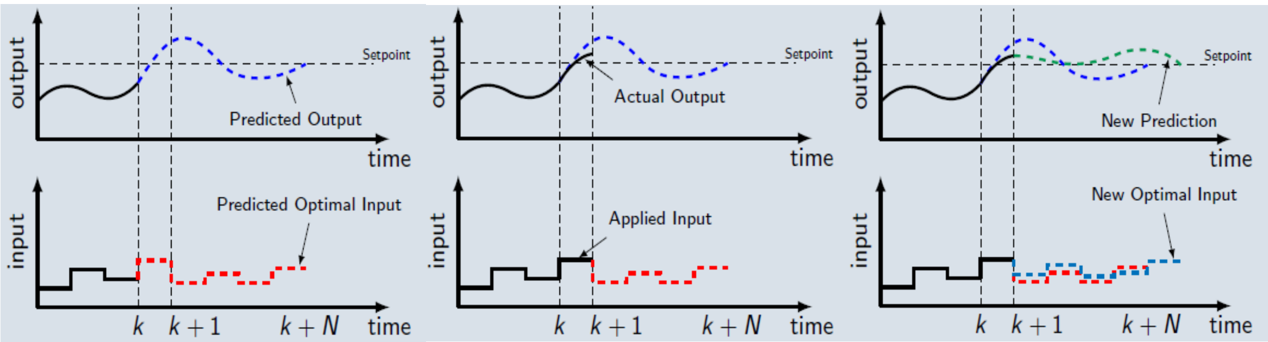

Figure 68 The prediction of input and output sequences and which are applied. [MELB]

From the figure above, it is possible to see that first, the prediction of both inputs and outputs are found, then only the first input in the sequence is applied and then a new prediction is made. This means that at every time step k a new sequence is calculated and only one input is applied. N in the figure is for the case of $N_u = N_p$.

Another thing that the MPC handles is the constraints of the system; such as the actuators operating range or if an output is needed to be within a limited range. In this case, it could be that the temperature inside the zones are needed to be within a range. Normal linear controllers, such as PID's, handles constraint symmetrically, this is depicted in the Figure 69, p.88, for the curves (a) and (b), this means that the input is regulated down far from the constraint, whereas the MPC can run asymmetric and decrease by a lot just before the constraint, see (c) in figure.

[1] Dynamic programming is an optimization method, which simplifies complex optimization problems.

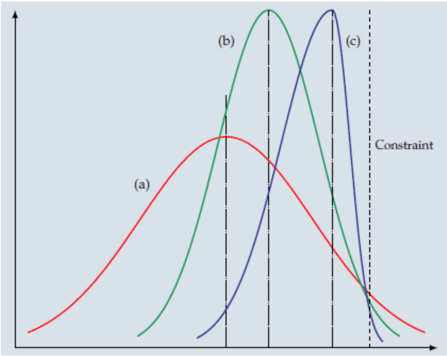

Figure 69 The idea of control close to a constraint. [MELB]

7.1.1 Prediction Model

The MPC is based on the concept of predicting the future inputs and outputs trajectories, so a prediction model has to be found. The prediction model is based on the state space model presented in Chapter 4. The discrete time state space model was given as:

$$x(k+1) = A \cdot x(k) + B \cdot u(k)$$
$$y(k) = C \cdot x(k) + D \cdot u(k) \qquad \textit{Eq 7.1}$$

Where k is the sampling time or step size. From given initial states variables $x_0 = x(k)$ and a previous input $(k-1)$, then a future input sequence $U = \{u_k, u_{k+1}, \dots, u_{k+N_u-1}\}$ can be found. This sequence minimizes a cost function $V(k)$ over the finite horizon N_p. This cost function will be subjected to constraints, in form of upper bounds ub and lower bounds lb.

For the purpose of this project a cost function which handles trajectory tracking or setpoint tracking is considered together with the rate of changes in the control input.

The cost function is given as following (M. Maciejowski, 2002):

$$[u_k, u_{k+1}, \dots, u_{k+N_u-1}] = \arg\min V(k):$$

$$\sum_{i=N_w}^{N_p} \big(y(k+i|k) - r(k+i|k)\big)^{\mathrm{T}} Q(k) \big(y(k+i|k) - r(k+i|k)\big) + \sum_{i=0}^{\mathrm{N_u}-1} \Delta\hat{\mathrm{u}}^{\mathrm{T}}(k)\, R(k)\, \Delta\hat{u}(k)$$

$$\text{s.t.} \begin{cases} \mathrm{x}(\mathrm{k}+1) = A_d \mathrm{x}(\mathrm{k}) + B_d \Delta\hat{\mathrm{u}}(\mathrm{k}) \\ y(k+1) = C_d \cdot x(k+1) + D_d \Delta\hat{\mathrm{u}}(\mathrm{k}) \\ \\ x_{lb} \le x(k) \le x_{ub} \\ u_{lb} \le \mathrm{u}(\mathrm{k}) \le u_{ub} \\ \Delta u_{lb} \le \Delta u(k) \le \Delta u_{ub} \end{cases} \qquad \textit{Eq 7.2}$$

Where,

- $y(k+i|k)$ and $r(k+i|k)$ are the output and setpoint predicted at current time.
- $\Delta\hat{u}(k) = \Delta u(k) + u(k-1)$
- N_w is the sampling window.
- Q and R are weighting matrices, which penalizes either the tracking error and rate of changes in the control input. They are given as:

$$Q(k) = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & q_n \end{bmatrix}, \qquad R(k) = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & r_m \end{bmatrix} \qquad \textit{Eq 7.3}$$

Defining a set of future state variables predicted at current time step:

$$x(k+1|k), x(k+2|k), \dots, x(k+N_p|k) \qquad \textit{Eq 7.4}$$

The prediction of these future state variables can be found from the following operation (M. Maciejowski, 2002):

$$\begin{aligned} x(k+1|k) &= A_d x(k) + B_d \Delta\hat{u}(k) \\ x(k+2|k) &= A_d x(k+1|k) + B_d \Delta\hat{u}(k) \\ &= {A_d}^2 x(k) + A_d B_d u(k-1) + B_d \Delta u(k) \\ &\vdots \\ x(k+N_p|k) &= A_{d_p}^{N}\, x(k) + {A_d}^{N_p-1} B_d u(k-1) + {A_d}^{N_p-2} B_d \Delta u(k) \\ &\quad + \cdots + {A_d}^{N_p-N_c} B_d \Delta u(k+N_c) \end{aligned} \qquad \textit{Eq 7.5}$$

From this formulation, the prediction of the future state variables is stated with respect to current state variable and future control input changes. Reformulating the prediction model in forms of matrices yields:

$$x(k+i|k) = \Psi x(k) + \Phi u(k-1) + \Theta \Delta \mathrm{u(k)} \qquad \textit{Eq 7.6}$$

Where,

$$\Psi = \begin{bmatrix} A_d \\ A_d^2 \\ \vdots \\ A_d^{N_p} \end{bmatrix}, \qquad \Phi = \begin{bmatrix} B_d \\ A_d B_d + B_d \\ \vdots \\ \sum_{i=0}^{N_p-1} A_d^i B_d \end{bmatrix}$$

$$\theta = \begin{bmatrix} B_d & 0 & 0 & \dots & 0 \\ A_d B_d + B_d & B_d & 0 & \dots & 0 \\ \vdots & A_d B_d + B_d & B & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \sum_{i=0}^{N_p-1} A_d^i B_d & \sum_{i=0}^{N_p-2} A_d^i B_d & \sum_{i=0}^{N_p-3} A_d^i B_d & \dots & \sum_{i=0}^{N_p-N_u} A_d^i B_d \end{bmatrix} \qquad \textit{Eq 7.7}$$

Substituting this into $y(k+1)$ the prediction of the future outputs can be found from current state variable and control input:

$$Y(k+1) = \Psi_y x(k) + \Phi_y u(k-1) + \Theta_{\mathrm{y}} \Delta u(k) \qquad \textit{Eq 7.8}$$

In addition, a tracking error in form of a matrix $E(k)$ can be defined as:

$$E(k) = \mathrm{T}(k) - \Psi_y x(k) - \Phi_y u(k-1) \qquad \textit{Eq 7.9}$$

Such that the cost function can be written as:

$$V(k) = \left|\left|\Theta_y u(k) - \mathrm{E}(k)\right|\right|_{\Omega}^{2} + \left|\left|\Delta u(k)\right|\right|_{\mathrm{P}}^{2} \qquad \textit{Eq 7.10}$$

Where Ψ_y, Φ_y and Θ_y are corresponds to the me matrices as Ψ, Φ and Θ multiplied with the output matrix C. And,

$$Y(k+1) = \begin{bmatrix} y(k+i|k) \\ \vdots \\ y(k+N_p|k) \end{bmatrix}, \quad T(k) = \begin{bmatrix} r(k+i|k) \\ \vdots \\ r(k+N_p|k) \end{bmatrix}, \quad \Delta u(k) = \begin{bmatrix} \Delta\hat{u}(k|k) \\ \vdots \\ \Delta\hat{u}(k+N_u|k) \end{bmatrix} \qquad \text{Eq 7.11}$$

$$\Omega = \begin{bmatrix} Q(k) & 0 & 0 & 0 \\ 0 & Q(k+1) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & Q(N_p) \end{bmatrix}, \quad \mathrm{P} = \begin{bmatrix} R(k-1) & 0 & 0 & 0 \\ 0 & R((k+1)-1) & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & R(N_u-1) \end{bmatrix} \qquad \text{Eq 7.12}$$

Where Ω and P constains Q and R for each step. These can vary but are considered constant in this case.

The value function can be rewritten to:

$$\begin{aligned} v(k) &= \left|\left|\Theta_y u(k) - \mathrm{E}(k)\right|\right|_{\Omega}^{2} + \left|\left|\Delta u(k)\right|\right|_{\mathrm{P}}^{2} \\ &= \left[\Theta_y u(k) - \mathrm{E}(k)\right]\Omega\left[\Theta_y u(k) - \mathrm{E}(k)\right] + \Delta u(k)\mathrm{P}\Delta u(k) \\ &= \mathrm{E}(k)^T\Omega\mathrm{E}(k) - 2\Delta u(k)^T\Theta_y^T\Omega\mathrm{E}(k) + \Delta u(k)^T\left[\Theta_y^T\Omega\Theta_y\right]\Delta u(k) \\ &= const - \Delta u(k)^T M + \Delta u(k)^T H\Delta u(k) \end{aligned} \qquad \text{Eq 7.13}$$

Where,

$$M = 2\Theta_y\Omega\mathrm{E}(k), \qquad H = \Theta_y\Omega\Theta_y + \mathrm{P} \qquad \text{Eq 7.14}$$

To solve this form of problem the quadratic programming is used. The quadratic programming handles the optimization problem by finding the new control input that will minimize the cost function. The formulation of this problem is defined as:

$$\min_{\Delta u}\frac{1}{2}\Delta u(k)^T H\Delta u(k) + M^T\Delta u(k) \qquad \text{Eq 7.15}$$
$$st.\,\mathrm{B}\Delta u \leq \beta$$

Where B and β is matrices containing the constraints of the given system.

7.1.2 Constraints

The MPC do not only consider the prediction of future state variables and control inputs but also the constraints of the systems actuator and outputs. To use the quadratic programming for solving the optimization problem, the constraint needs to be in form of linear constraint inequalities.

The linear constraint inequalities are (M. Maciejowski, 2002):

$$y_{lb} \leq y(k) \leq y_{ub}$$
$$u_{lb} \leq u(k) \leq u_{ub} \qquad \text{Eq 7.16}$$
$$\Delta u_{lb} \leq \Delta u(k) \leq \Delta u_{ub}$$

Applying the inequalities at each time step results in the following form:

$$E\begin{bmatrix}\Delta u(k)\\1\end{bmatrix}\leq 0$$

$$F\begin{bmatrix}u(k)\\1\end{bmatrix}\leq 0 \qquad \textit{Eq 7.17}$$

$$G\begin{bmatrix}y(k)\\1\end{bmatrix}\leq 0$$

Where E, F and G are matrices consisting of ones and the constraints:

$$E=\left[W_1\dots W_{N_u},w\right],\qquad F=\left[F_1,\dots,F_{N_u},f\right],\qquad G=\left[G_1\dots G_{N_p},g\right] \qquad \textit{Eq 7.18}$$

Where w, f and g contains the values of both the minimum and maximum. $W=[W_1,\dots,W_{N_u}]$, $\mathcal{F}=[F_1,\dots,F_{N_u}]$ and $\mathcal{G}=[G_1,\dots,G_{N_p}]$ have positive ones in the diagonal for maximum constraints and negative ones for minimum constraints.

Re-writing the inequalities with respect to $\Delta u(k)$, the matrices for the quadratic problem can be found. The first inequality can be written as:

$$W\cdot\Delta u(k)+w\leq 0 \qquad \textit{Eq 7.19}$$

⇔

$$W\cdot\Delta u(k)\leq -w \qquad \textit{Eq 7.20}$$

For the second inequality, the following operations can be made:

$$\sum_{i=1}^{H_u}F_i\cdot\hat{u}(k+i-1|k)+f\leq 0 \qquad \textit{Eq 7.21}$$

And,

$$\Delta\hat{u}(k+i-1|k)=u(k-1)+\sum_{j=0}^{i-1}\Delta\hat{u}(k+j|k) \qquad \textit{Eq 7.22}$$

Substituting *Eq 7.22* into *Eq 7.21* the following can be obtained:

$$\sum_{j=1}^{H_u}F_j\cdot\hat{u}(k|k)+\sum_{j=2}^{H_u}F_j\cdot\hat{u}(k+1|k)+\cdots+F_{H_u}\cdot\hat{u}(k+H_u-1|k)+\sum_{j=1}^{H_u}F_j\cdot u(k-1)+f\leq 0 \qquad \textit{Eq 7.23}$$

Making the inequalities with respect to $\Delta u(k)$, the inequality becomes:

$$\mathcal{F}\Delta u(k)\leq -F_1u(k-1)-f \qquad \textit{Eq 7.24}$$

For the third inequality, G was devided into $\mathcal{G}$ and g, thereby the following can be done:

$$Ǵ \cdot \mathrm{Y}(\mathrm{k}+1)+\mathrm{g} \leq 0 \qquad \text{Eq 7.25}$$

⇔

$$Ǵ \cdot \left(\Psi_{\mathrm{y}} x(k)+\Phi_{y} u(k-1)+\Theta_{\mathrm{y}} \Delta u(k)\right) \leq -\mathrm{g} \qquad \text{Eq 7.26}$$

⇔

$$Ǵ \cdot \Theta_{\mathrm{y}} \Delta u(k) \leq Ǵ \cdot \left(\Psi_{\mathrm{y}} x(k)+\Phi_{y} u(k-1)\right)-\mathrm{g} \qquad \text{Eq 7.27}$$

The new inequalities can now be combined to:

$$\begin{bmatrix} \mathcal{F} \\ Ǵ \cdot \Theta_{\mathrm{y}} \\ W \end{bmatrix} \Delta u \leq \begin{bmatrix} -F_1 u(k-1)-f \\ Ǵ \cdot \left(\Psi_{\mathrm{y}} x(k)+\Phi_{y} u(k-1)\right)-\mathrm{g} \\ -w \end{bmatrix} \qquad \text{Eq 7.28}$$

The constraints used are given in Table 11. The dampers are restricted to fully open in 5s, which is $\frac{0.2827 m^2}{5\,s} = 0.0656 \frac{m^2}{s}$. The valves are restricted to fully open in 10s. The addition or subtraction of moisture is unknown but set to $0.001 \frac{kg}{s^2}$. The temperature of the radiators are constraint to have a temperature change of $0.05 \frac{K}{s}$. The radiators are electrical and are expected to respond faster than mass-flow controlled radiators.

Constraints	Minimum Input	Maximum Input	Inputs slope rate
Inputs	u_{min}	u_{max}	Δu_{max}
$\boldsymbol{A_{0inz1}}$	$0\ m^2$	$0.2827\ m^2$	$0.0656\ \frac{m^2}{s}$
$\boldsymbol{A_{0inz1}}$	$0\ m^2$	$0.2827\ m^2$	$0.0656\ \frac{m^2}{s}$
$\boldsymbol{A_{0inz1}}$	$0\ m^2$	$0.2827\ m^2$	$0.0656\ \frac{m^2}{s}$
$\boldsymbol{Opening_{CC}}$	0	1	0.1
$\boldsymbol{Opening_{HC}}$	0	1	0.1
$\boldsymbol{m_{win}}$	$0 \frac{kg}{s}$	$0.05 \frac{kg}{s}$	$0.001 \frac{kg}{s^2}$
$\boldsymbol{m_{wout}}$	$0 \frac{kg}{s}$	$0.05 \frac{kg}{s}$	$0.001 \frac{kg}{s^2}$
$\boldsymbol{T_{rad,vz1}}$	$273.15K$	$310\ K$	$0.05 \frac{K}{s}$
$\boldsymbol{T_{rad,vz2}}$	$273.15K$	$310\ K$	$0.05 \frac{K}{s}$
$\boldsymbol{T_{rad,vz3}}$	$273.15K$	$310\ K$	$0.05 \frac{K}{s}$

Table 11 MPC Constraints.

From these constraint, f and w can be found as:

$$f = \begin{bmatrix} 0 \\ 0.2827 \\ 0 \\ 0.2827 \\ 0 \\ 0.2827 \\ 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0.05 \\ 0 \\ 0.05 \\ 273.15 \\ 310 \\ 273.15 \\ 310 \\ 273.15 \\ 310 \end{bmatrix}, \qquad w = \begin{bmatrix} 0.0656 \\ 0.0656 \\ 0.0656 \\ 0.1 \\ 0.1 \\ 0.001 \\ 0.001 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

The constraints for the output is not applied because they are not violated, but g can be found just by making a reasonable range for the outputs.

7.2 Implementation – Simulink

To test the performance of the MPC, it is first tested on the linear state space model. When the controller is acceptable, the controllers are implemented in the non-linear model. Additionally, there are two cases implemented for the MPC, "winter" and "summer". Therefore, the system is linearized based on either having the cooling coil a dehumidification process or with the heater and humidifier. In the principle many operating points or cases could had been implemented to make the controller even more correct as the non-linear model moves away from the point at which it is linearized about.

7.2.1 Design phase – Tuning

Having a multiple-input-multiple-output system, also known as MIMO system, makes the tuning phase more difficult, as the individual weightings will affect the others relatively. For the input- and prediction horizon, it is important to find a balance between steps into the future and the computation time. When using the MPC as an online control strategy, a long horizon will result in the need for more computation time at each time step. Reducing the horizon will cause less computational effort and reduction of the time needed. In addition, the sampling time could be reduced but in order to run the continuous time non-linear model with the discrete time controller, the sampling time is required to be small.

To design the MPC, the MPC-Toolbox in Matlab is used. The toolbox handles exactly the optimization problem and constraint presented in the last section. The only requirement is a discrete State space model, which was found in Chapter 5.

The MPC toolbox is illustrated below.

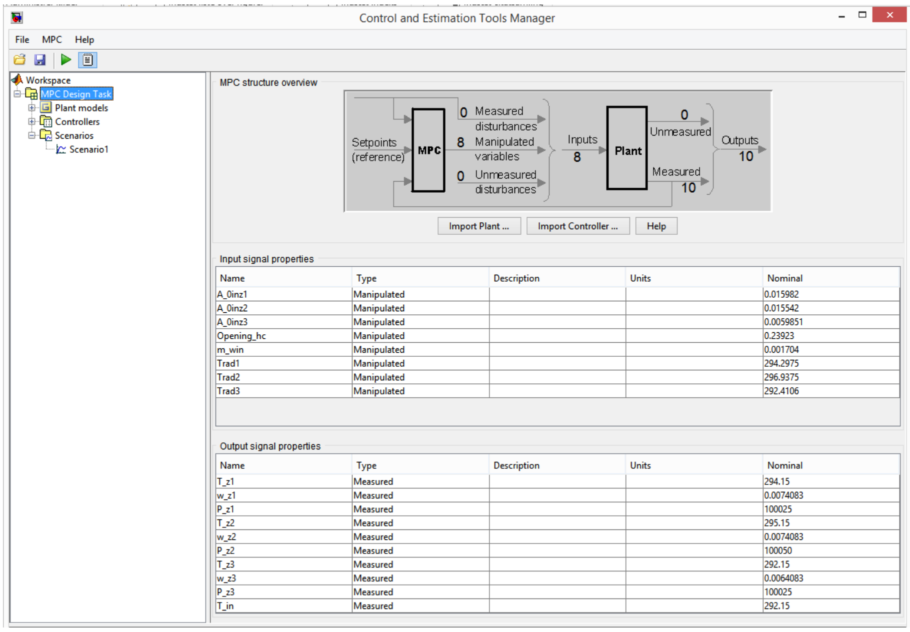

Figure 70 MPC Toolbox

<u>Note that</u>: the steps in the following description can be done arbitrarily.

After importing a discrete time state space model into the toolbox, the horizon for the problem can be chosen for both the control input and state variables, see Figure 71.

The result of choosing a too small horizon is depicted Figure 74, after the introduction to the toolbox.

Horizons

Control interval (time units):	0.01
Prediction horizon (intervals):	60
Control horizon (intervals):	20

Figure 71 Horizons and sample time.

The final prediction horizon is chosen to be 60 intervals whereas the control horizon is 20.

After determine the horizon the constraints are applied as depicted in the figure.

Constraints on manipulated variables

Name	Units	Minimum	Maximum	Max Down Rate	Max Up Rate
A_0inz1		0	0.2827	-0.0656	0.0656
A_0inz2		0	0.2827	-0.0656	0.0656
A_0inz3		0	0.2827	-0.0656	0.0656
Opening_hc		0	1	-0.1	0.1
m_win		0	0.05	-0.001	0.001
Trad1		273.15	310	-0.05	0.05
Trad2		273.15	310	-0.05	0.05
Trad3		273.15	310	-0.05	0.05

Figure 72 Constraints – winter case.

After the constraints are applied, the weightings on both the constraint and the tracking error are enforced. The weightings are increased for the tracking error until it settles to the setpoint. Hereafter both control input and output are weighted to obtain a desired transient response. An underdamped response is desired, but is not always possible, instead small overshoots might occur.

The final weightings are depicted in the below figure.

Input weights

Name	Description	Units	Weight	Rate Weight
A_0inz1			1	1000
A_0inz2			1	10
A_0inz3			1	100
Opening_hc			2	0.3
m_win			2	0.2
Trad1			1	10
Trad2			1	10
Trad3			1	10

Output weights

Name	Description	Units	Weight
T_z1			100
w_z1			1000
P_z1			1
T_z2			100
w_z2			1000
P_z2			1
T_z3			100
w_z3			1000
P_z3			1
T_in			100

Figure 73 Weightings on input and outputs.

With these final values the final controller for a “winter case” is found. The same procedure is implemented for the “summer case”. The linear results for different horizons and weightings are illustrated in the next section.

7.2.1.1 Linear model Simulation

Some of the steps in the tuning phase are illustrated in the below figures. Starting with almost no weighting (Blue graph) and then increasing the weightings to 1 on both the inputs and outputs (Black graph). The next step is to apply individual weightings; 1 on the pressure, 100 for the temperatures and 1000 for the humidity (Purple graph). The last step is to increase the horizon from N_p =5 and N_u = 2 to N_p = 30 and N_u = 10 (Green graph).

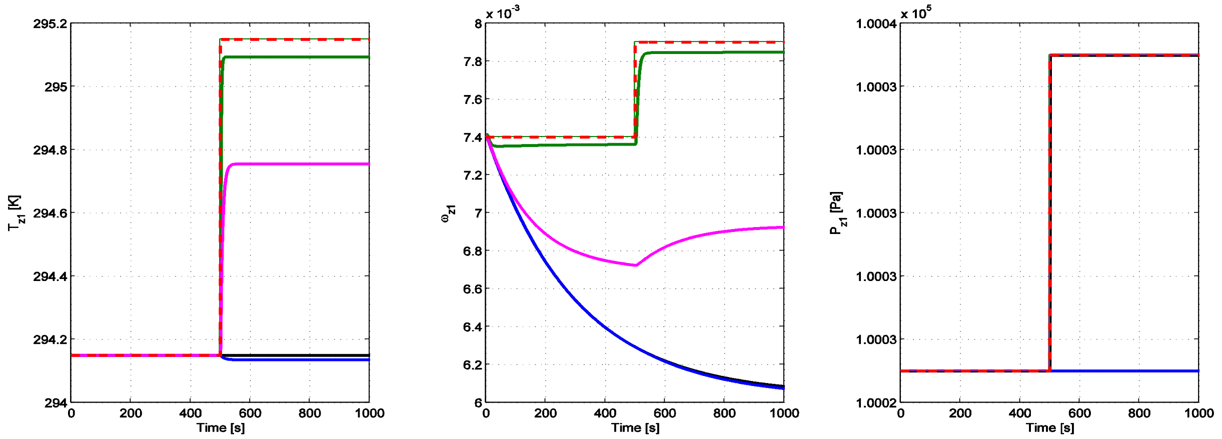

Figure 74 Tuning phase. a) Temperature, b) SH, c) Pressure of zone 1.

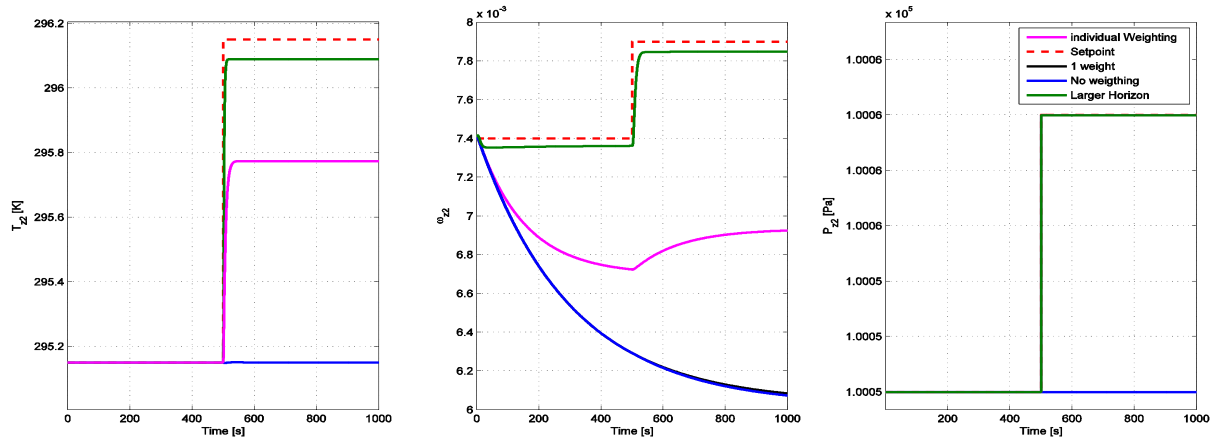

Figure 75 Tuning phase. a) Temperature, b) SH, c) Pressure of zone 2.

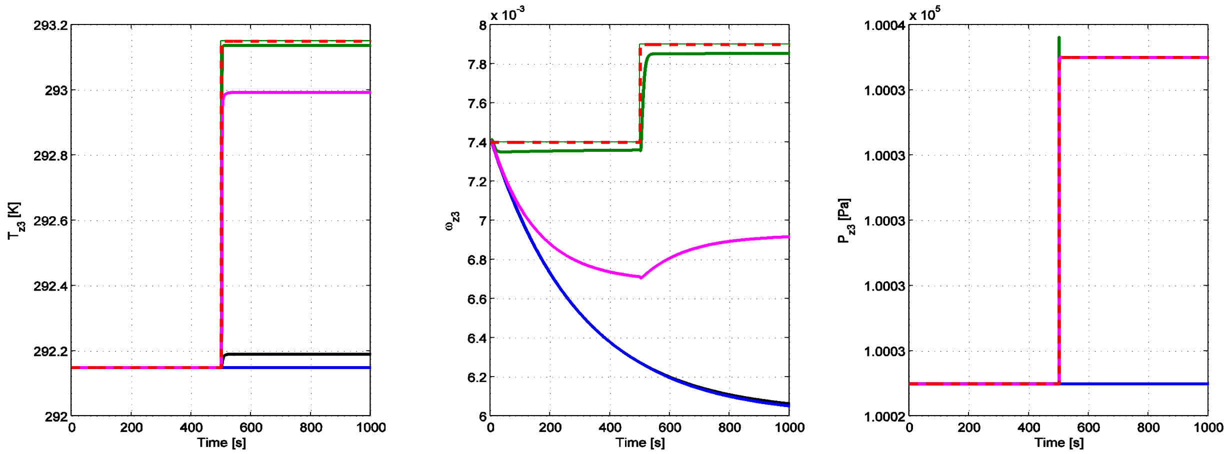

Figure 76 Tuning phase. a) Temperature, b) SH, c) Pressure of zone 3.

From the graphs it is possible to see that, the response of the controller gradually becomes better and better for the higher weightings and longer horizon. Looking at the very last case (Green Line), it is possible to see that the outputs settles very fast. The last part is to weight the rate of the actuators, thereby making the transient response slower, which results in making the control input more realistic. This also results in much better control input without too many discontinuities.

The final controller for the winter case is simulated with the linear model. The output responses are illustrated in the below figures.

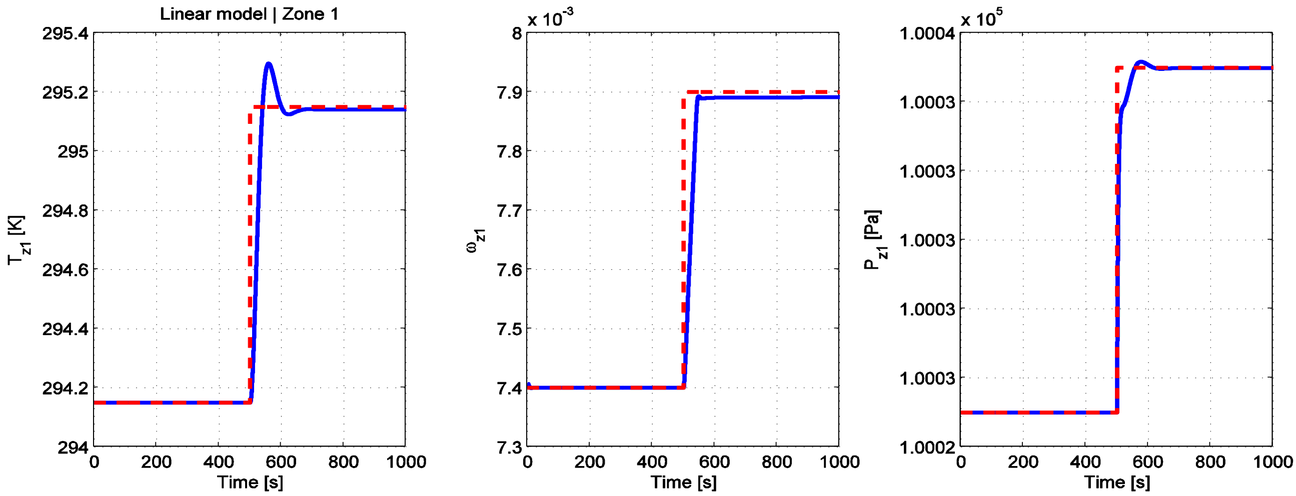

Figure 77 a) Temperature, b) Specific Humidity, c) Pressure of zone 1. Linear model with final MPC.

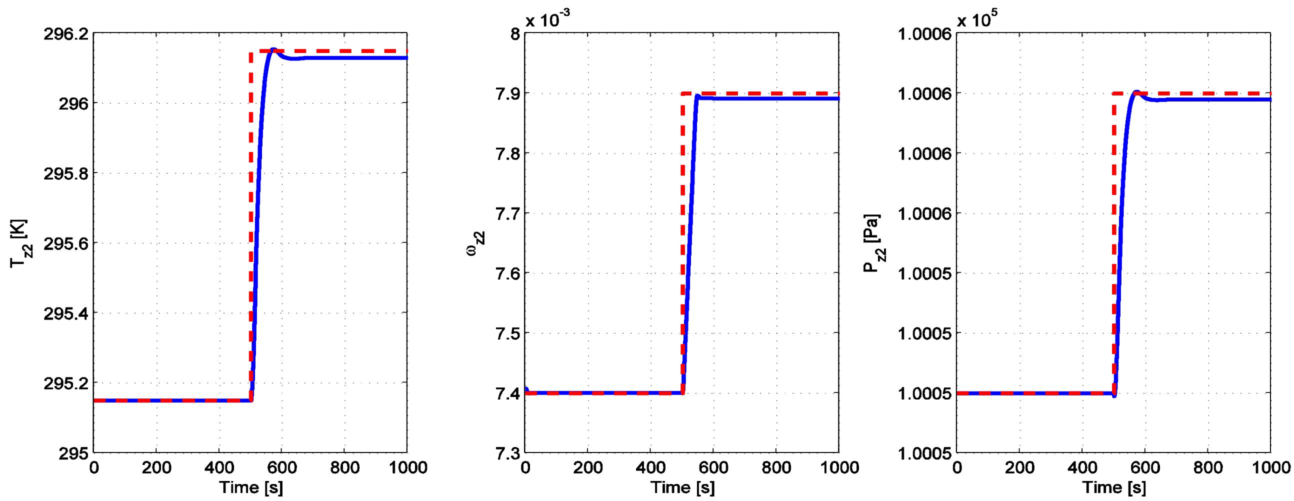

Figure 78 Temperature, b) Specific Humidity, c) Pressure of zone 2. Linear model with final MPC.

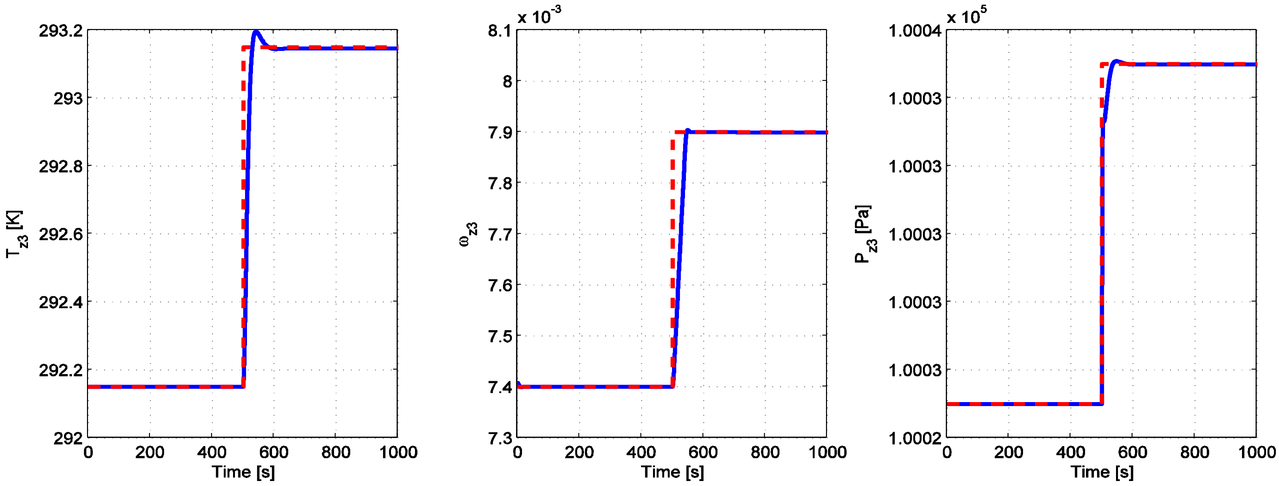

Figure 79 Temperature, b) Specific Humidity, c) Pressure of zone 3. Linear model with final MPC.

From the figures, it is seen that very small deviations occurs in zone 2, else all the outputs settles to the setpoint value, these deviations are acceptable. The performance of the controller will be evaluated after implementing it on the non-linear model.

7.2.1.2 Non-linear model with MPC

From the result is was possible to detect small deviation, this is due to that the controller only works optimal in a region around the linearizing point. Therefore linearizing the model at more operating points or adapting the system at each time step could increase the accuracy, especially for system with large operating ranges, such as the HVAC system.

As mentioned previously, the multiple MPC approach is used as the adaptive is too slow computationally. Linearizing at different operating point, in this case two “points”; one case for the winter and one for the summer.

The idea of this is illustrated in the block diagram below.

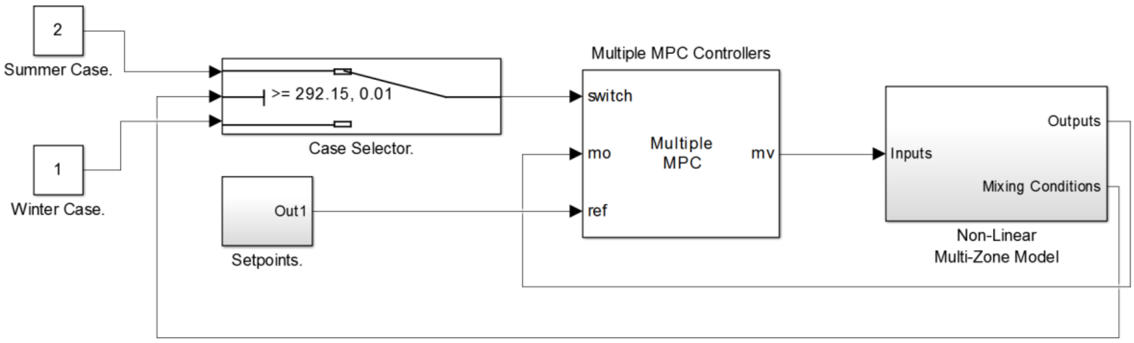

Figure 80 Multiple MPC.

The Multiple MPC block contains the two MPC controllers which each are designed for the summer and winter case. The specific controller is chosen based on the mixing box condition, whether it is higher or lower a value. Both conditions has to be fulfilled this means whenever the temperature and humidity are below this threshold the switch returns 1 else with 2.

7.2.1.2.1 Winter Case

The result is depicted in the figures below, both for the inputs and outputs. The system undergoes a setpoint change from the desired to a higher setpoint to see how the controller tracks the new setpoint.

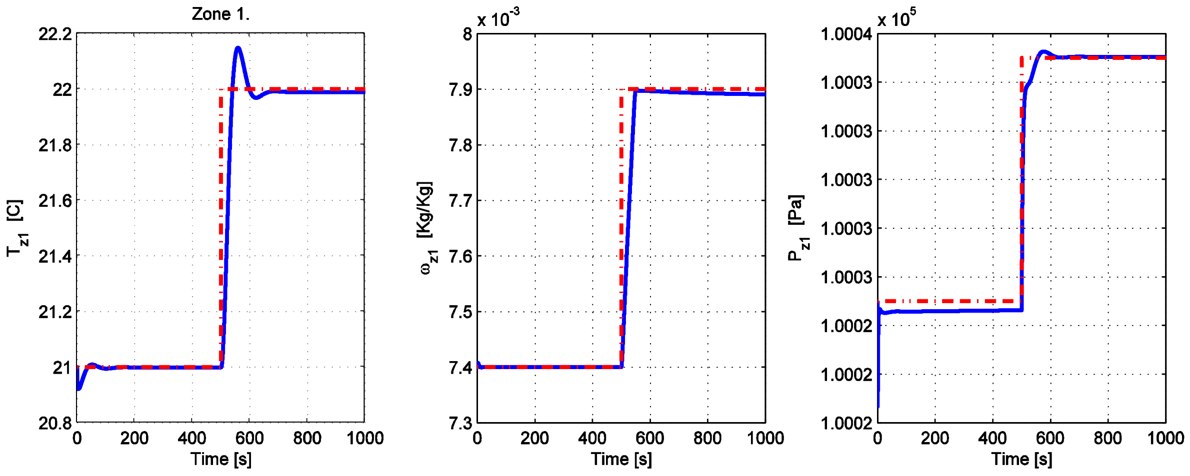

Figure 81 Zone 1. Responses a) Temperature b) Specific humidity c) Pressure.

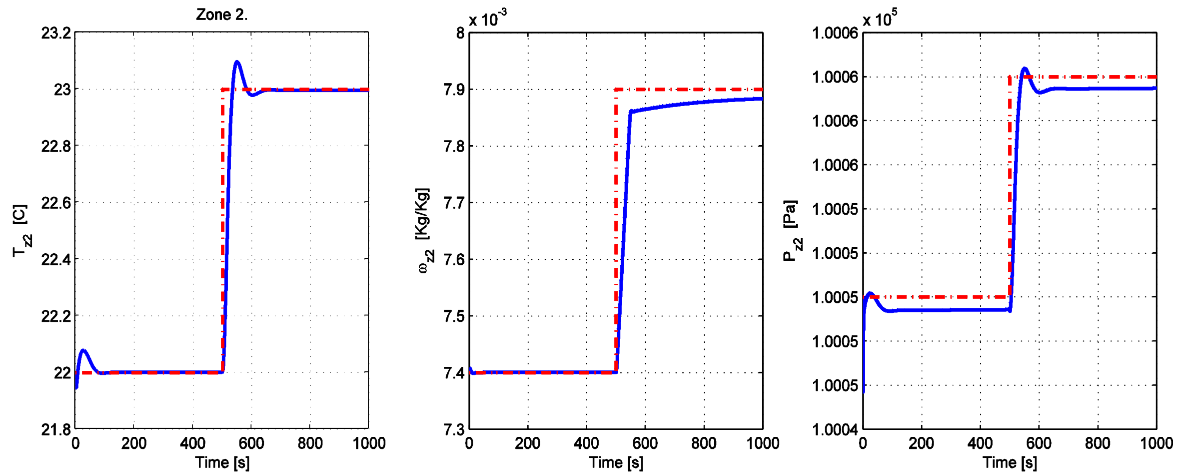

Figure 82 Zone 2. Responses a) Temperature b) Specific humidity c) Pressure.

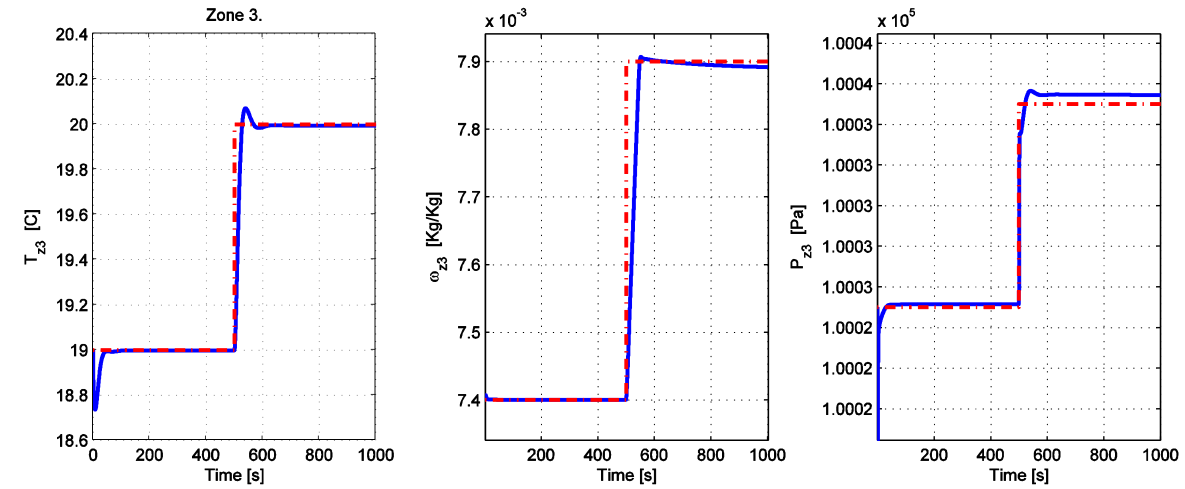

Figure 83 Zone 3. Responses a) Temperature b) Specific humidity c) Pressure.

The corresponding inputs are:

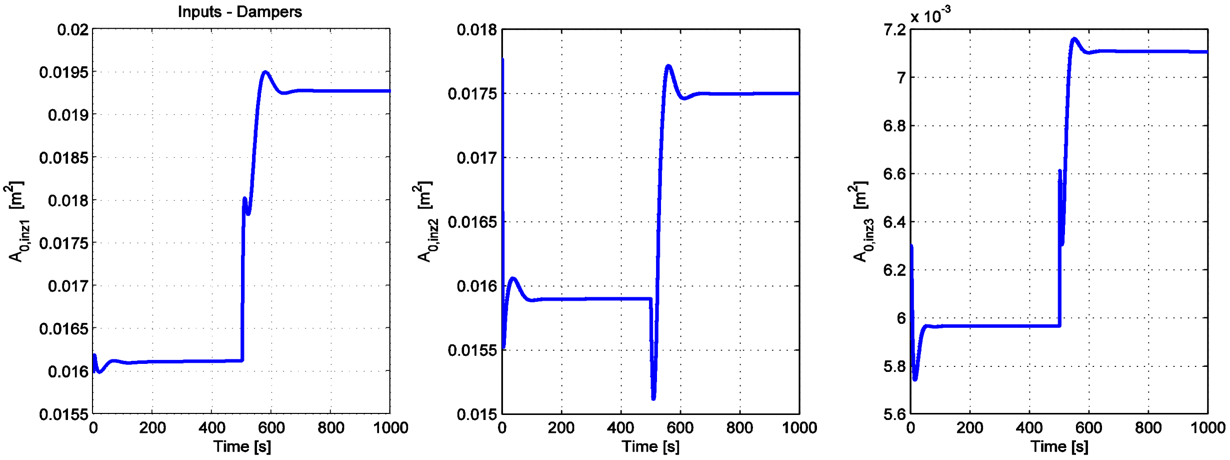

Figure 84 Dampers.

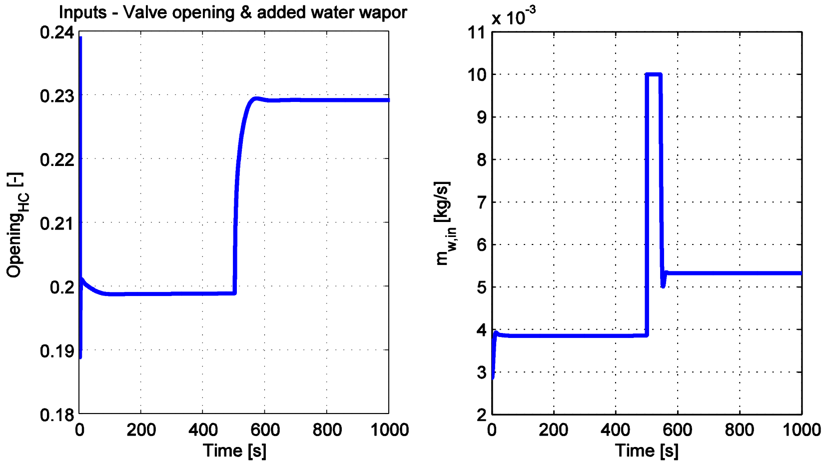

Figure 85 Inputs a) Opening heater valve b) added water vapor.

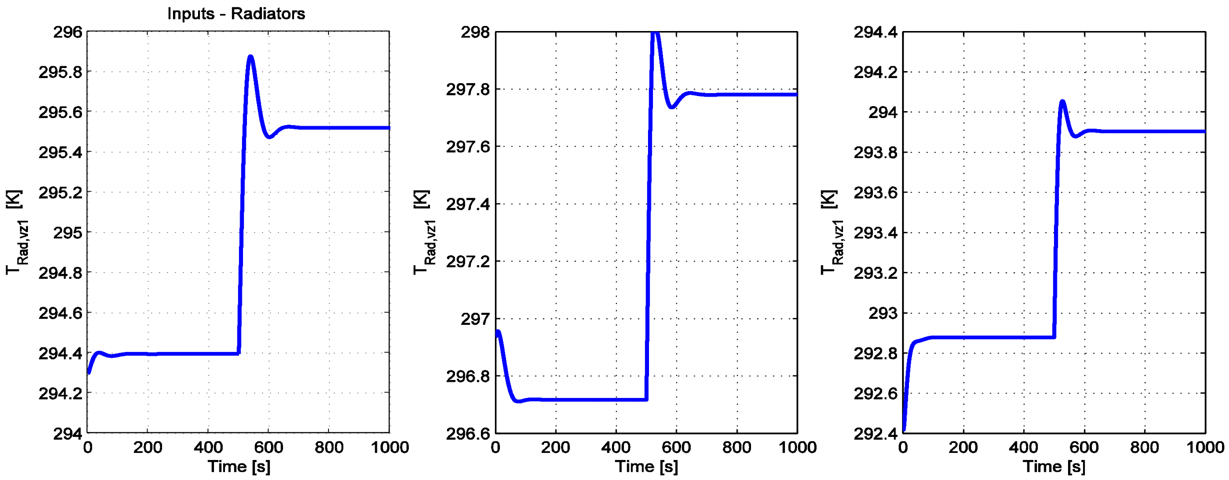

Figure 86 Variable radiators.

It is possible to that the linearizing point is meet perfectly but when a change in setpoint occurs, some small deviations occurs.

The settling times and overshoots are given in Table 12.

	Temperature	Humidity	Pressure
Zone 1.			
Settling time	$130s$	$62s$	$80s$
Overshoot	$0.1\ °C \sim 10\%$	0%	$\sim 0\%$
Zone 2.			
Settling time	$102s$	$100s - 1500s*$	$102s$
Overshoot	$0.1\ °C \sim 10\%$	0%	$\sim 0\%$
Zone 3.			
Settling time	$88s$	$47s$	$92s$
Overshoot	$0.06\ °C \sim 6\%$	0%	$\sim 0\%$

Table 12 Control specifications.

* The humidity do not settle completely. It go within 10% of the setpoint value within 100s where after it slowly converges to the setpoint in 1500s.

Additionally to the winter-case MPC a controller for a summer case is also found. The inputs and outputs responses are illustrated in the figure on the following pages.

7.2.1.2.2 Summer Cass

The summer case where dehumidification is needed, the radiator constant is off. The outputs and control inputs are respectively illustrated in the following figures.

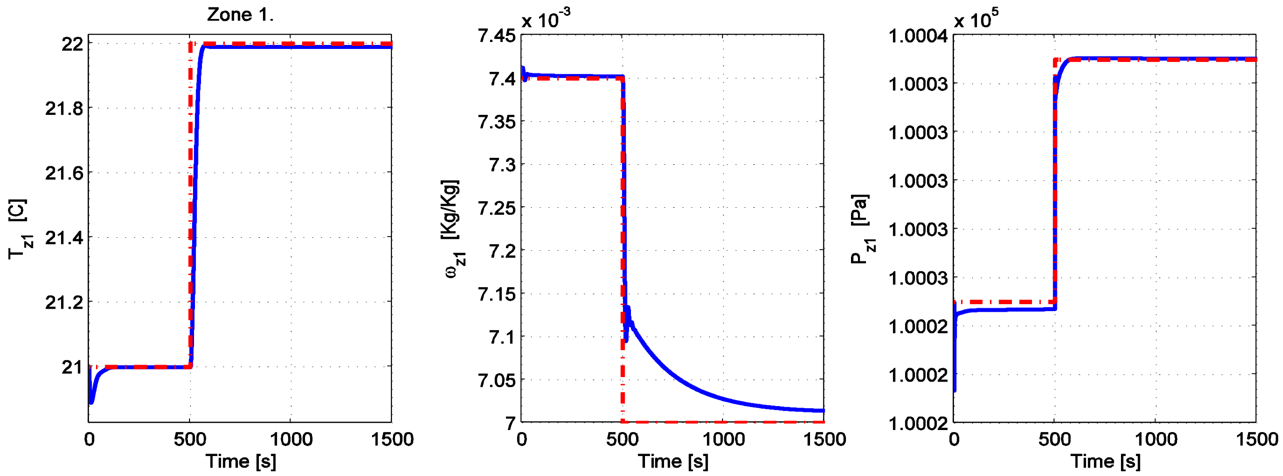

Figure 87 Zone 1 a)Temperature, b)Humidity, c)Pressure.

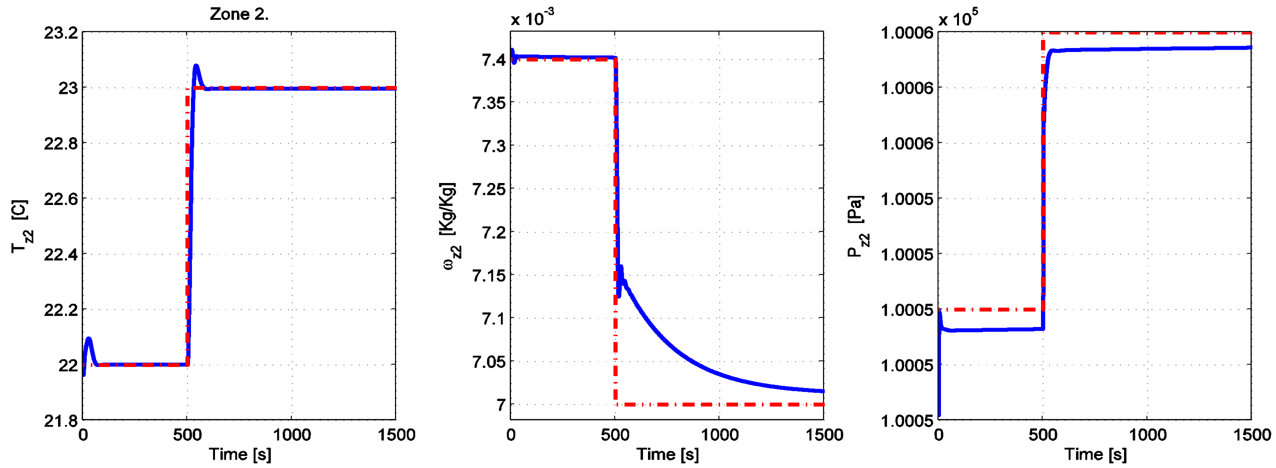

Figure 88 Zone 2 a) Temperature, b) Humidity, c)Pressure.

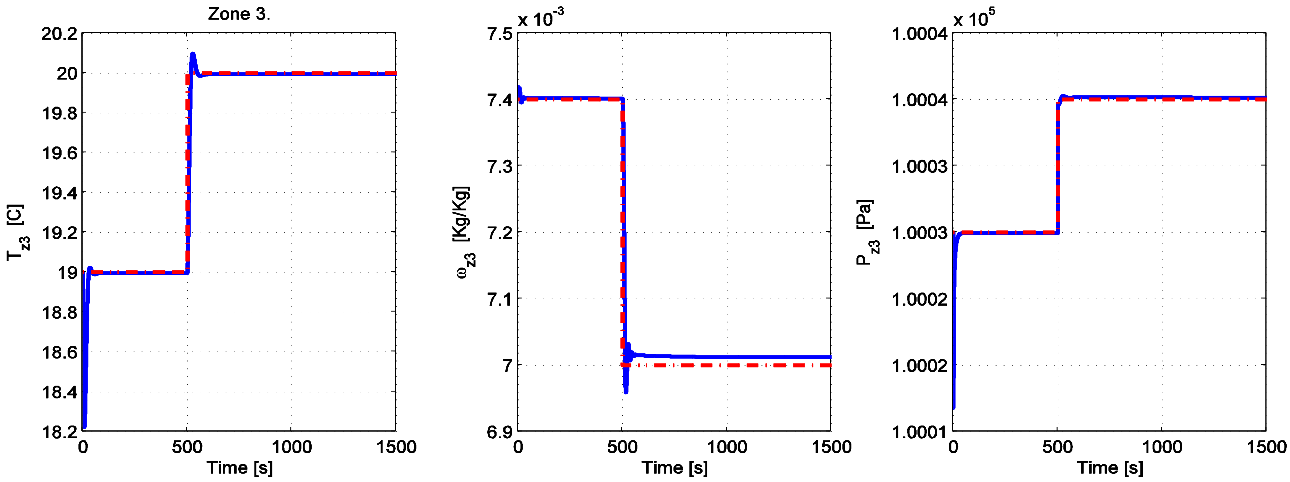

Figure 89 Zone 3 a) Temperature, b) Humidity, c)Pressure.

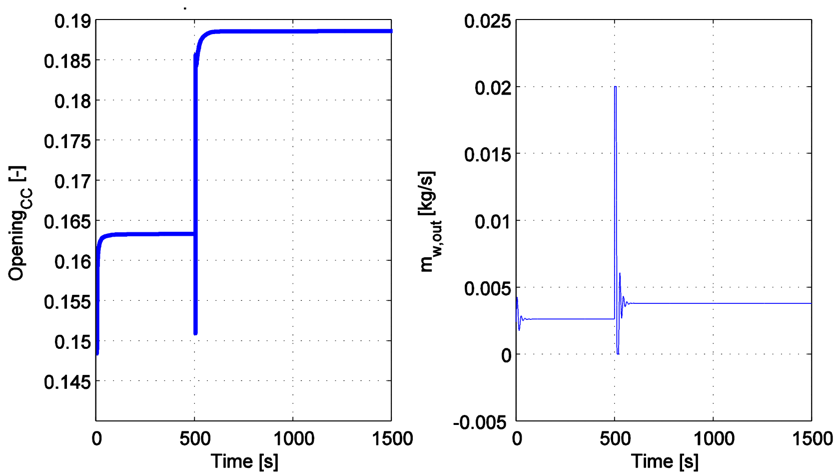

Figure 90 Control input for cooling coil.

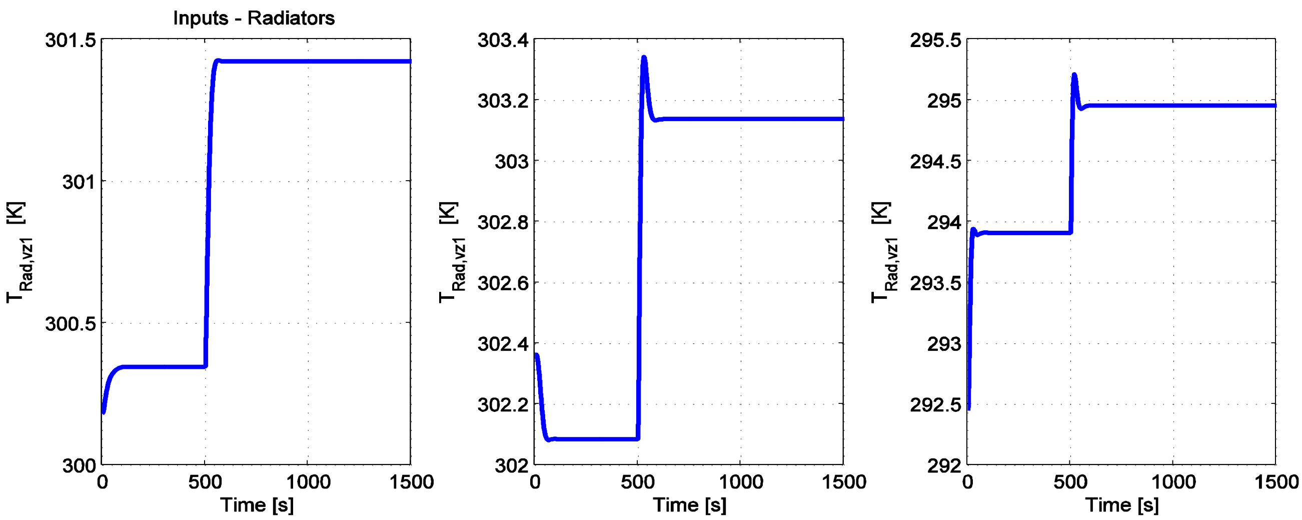

Figure 91 Radiator temperature of a) Zone 1, b) Zone 2, c) Zone 3.

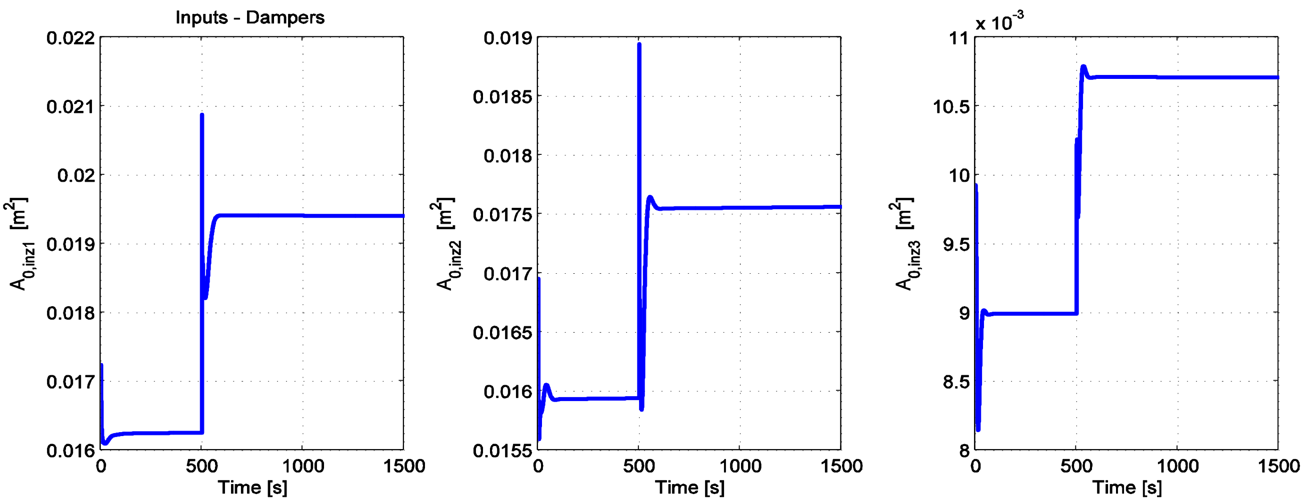

Figure 92 Damper opening of a) Zone 1, b) Zone 2, c) Zone 3.

The response times are approximately the same as for the winter-case. The summer case differs in specific humidity. The controller is not able to reach the setpoint value for zone 1 and zone 2. Applying more weights on the tracking error do not help. The humidity increases at 16s to a value within 37% of the setpoint but hereafter it takes more than 1000s to settle.

The settling time and overshoot for the outputs are given in

	Temperature	**Humidity**	**Pressure**
Zone 1.			
Settling time	$81s$	$1500s$	$84s$
Overshoot	0%	0%	0%
Zone 2.			
Settling time	$103s$	$1500s$	$104s$
Overshoot	$0.1\ °C \sim 10\%$	0%	0%
Zone 3.			
Settling time	$66s$	$46s$	$34s$
Overshoot	$0.1\ °C \sim 10\%$	0%	0%

Table 13 Controller specifications.

From the tables it is possible to see that the MPC controllers are not made much faster than the PI-controllers, except for the specific humidity. However, it is expected that the MPC perform better when a disturbance is applied. This will be tested in the next chapter.

8. Comparison

In this chapter, the two different control methods are compared, to test their performance when not only setpoints are changed. The controllers will be compared for a door opening into the zones.

8.1 Mathematical modelling of disturbance

8.1.1 Mass flow rate

The modelling of the door opening is considered as a natural ventilation technique. More in detail the door opening, will be explained mathematically and implemented in the Non-linear Simulink model.

In order to describe the air flow rate through a door opening, the following semi-empirical model is been used (Heiselberg, 2006):

$$q_{door} = C_d \cdot A \cdot \sqrt{\frac{2 \cdot \Delta p}{\rho_i}} \qquad \text{Eq 8.1}$$

Where,

- q_{door} is the volumetric inflow $\left[\frac{m^3}{s}\right]$ through the door opening,
- A is the opening area $[m^2]$ of the door
- C_d is the discharge coefficient [-]
- Δp $[Pa]$ accounts for the pressure difference across the door opening
- ρ_i refers to density $\left[\frac{kg}{m^3}\right]$ of the air passing the room.

The discharge coefficient C_d is been found, in general cases, in the range of 0,6 – 0,7. The value of C_d is principally depended on the contraction of the opening area and the friction losses which are negligible in the case of window or door opening.

The opening of the door is been accounted as a ventilation through a vertical opening due to the pressure change between indoor and outdoor. The following equation illustrates the correlation between the pressure change and the volumetric flow rate.

$$q_{door} = C_d \cdot A_{door} \cdot \sqrt{\frac{2 \cdot |\Delta p|}{\rho_{zone}}} \cdot \left(\frac{\Delta p}{|\Delta p|}\right) \qquad \text{Eq 8.2}$$

Where the pressure change is:

$$\Delta p = P_{zone} - P_{atm} \qquad \text{Eq 8.3}$$

The term $\frac{\Delta p}{|\Delta p|}$ defines the direction of the flow rate, in the meaning of inserting the building (minus sign) or exerting the building (plus sign)

The mass flow rate leaving the zone can from the above be expressed as:

$$\dot{m}_{door} = \rho_{zone} \cdot q_{door}$$

$$= \frac{m_{zone}}{V_{zone}} \cdot C_d \cdot A_{door} \cdot \sqrt{\frac{2 \cdot (P_{zone} - P_{atm})}{\frac{m_{zone}}{V_{zone}}}} \cdot \left(\frac{P_{zone} - P_{atm}}{|P_{zone} - P_{atm}|}\right) \qquad \text{Eq 8.4}$$

If the pressure is the same, thereby leading to no pressure difference across the door, mass flow rate is based on the temperature or density difference. For this case, there will exist both a mass flow rate in and out of zone but as the zones has to be pressurized, the flow will mainly be going out of the zone, as the pressure outside is less. In addition to this, the wind will contribute with some amount of inflow to the zones when the wind direction is against the door opening. The contribution of the wind is not considered here.

8.1.2 Heat transfer rate

The heat transfer rate can for a door opening with pressure difference can be modelled as convection:

$$\dot{Q}_{door} = \bar{h} \cdot A_{door} \cdot (T_{zone} - T_{atm}) \qquad \textit{Eq 8.5}$$

Where $\bar{h}$ was the heat convection coefficient. $\bar{h}$ can for free convection of air, gasses and dry vapors be found to be in the range of $0.5\frac{W}{m^2K} - 100\frac{W}{m^2K}$ (engineeringtoolbox, n.d.). A value of $0.75\frac{w}{m^2K}$ is used.

The door opening is simulated in the same manner illustrated in the figure below. This takes into account that the door is not fully opened instantly. The time at which the door is open can be seen from the x-axis.

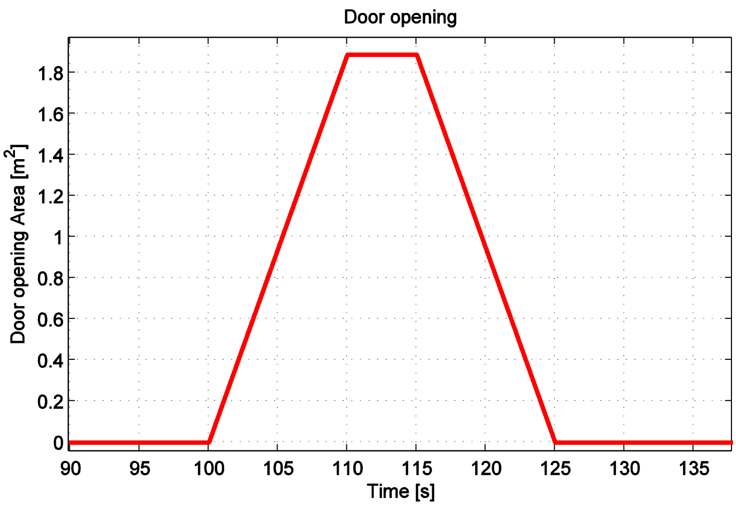

Figure 93 Door opening for simulation.

8.2 Simulation results

The door opening is applied to zone 2 as this zone interact with both zone 1 and zone 3. The results are illustrated in the figures below. Pay attention to that the response for the case with the PI-controllers, the system has already settled, that is why there is no transient period in the start. This is due to different settling times.

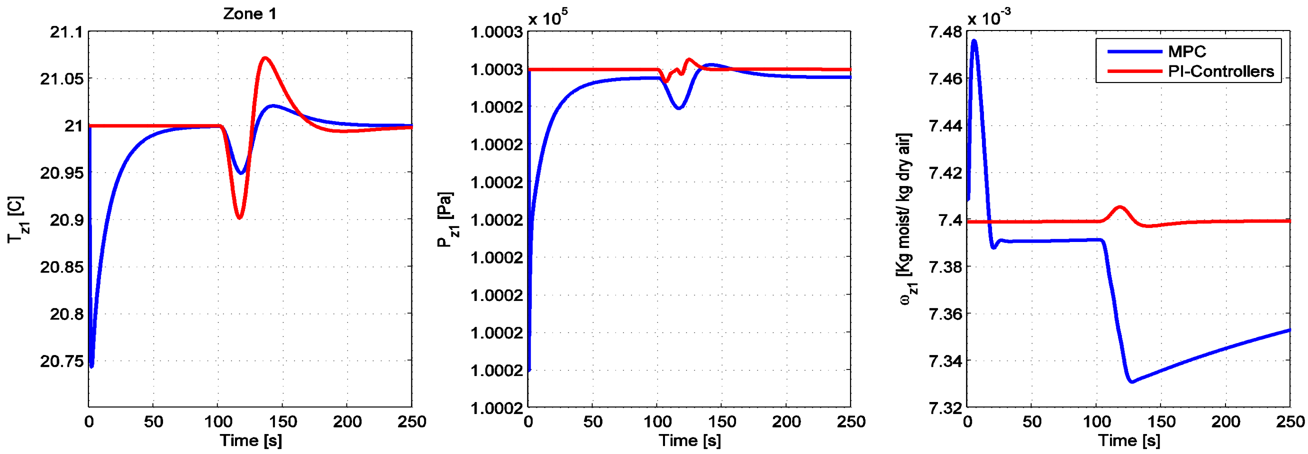

Figure 94 a) Temperature, b) Pressure, c) Specific humidity for zone 1.

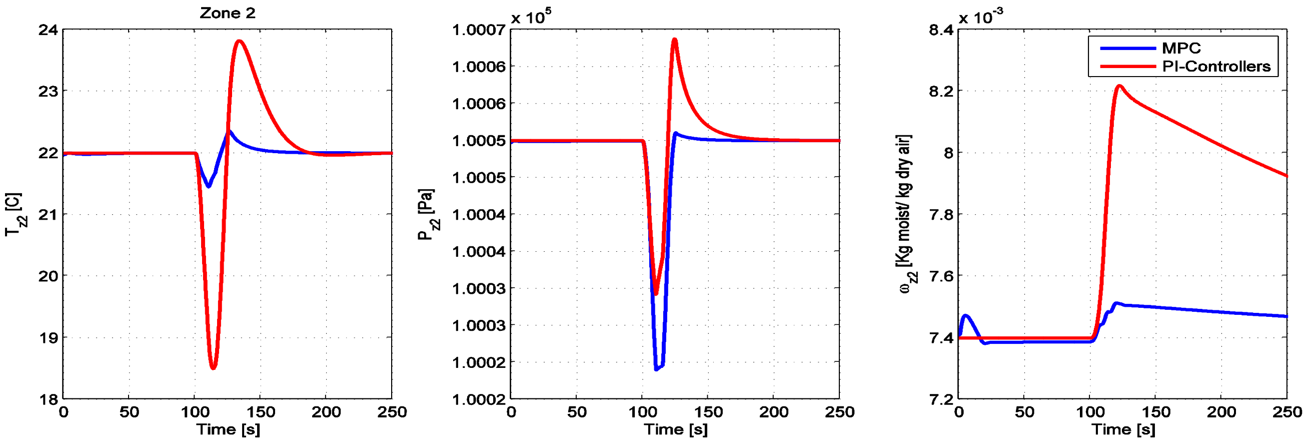

Figure 95 a) Temperature, b) Pressure, c) Specific humidity for zone 2.

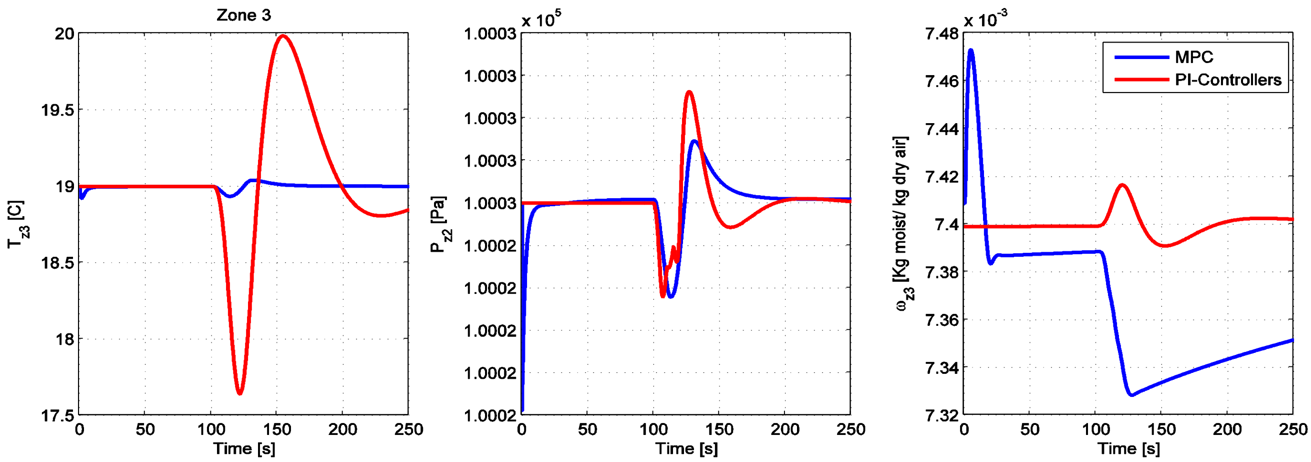

Figure 96 a) Temperature, b) Pressure, c) Specific humidity for zone 3.

The corresponding control inputs are illustrated in the figures below.

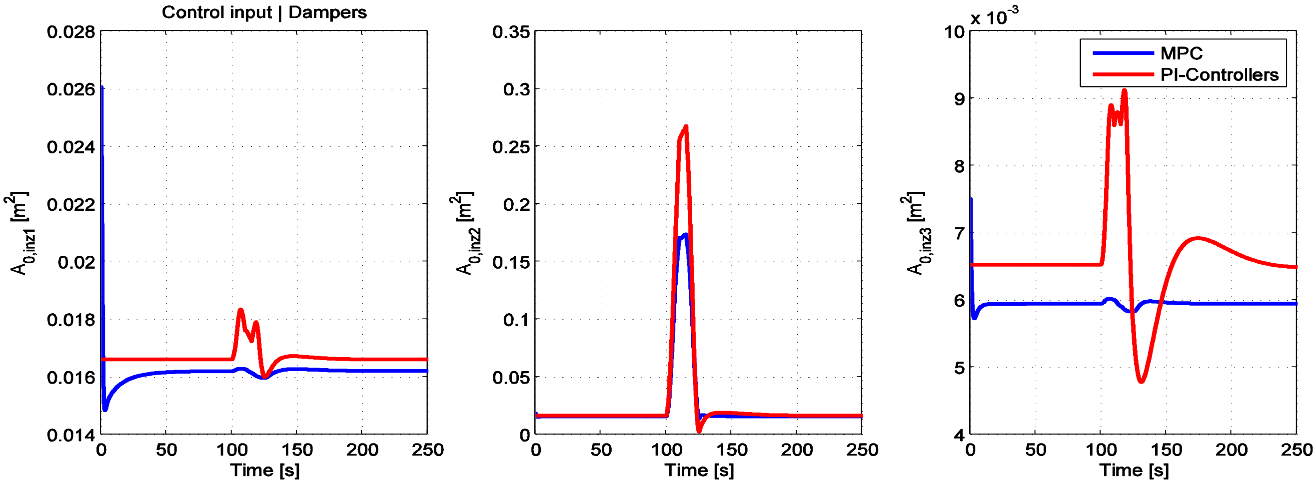

Figure 97 Damper openings of a) Zone 1, b) Zone 2, c) Zone 3.

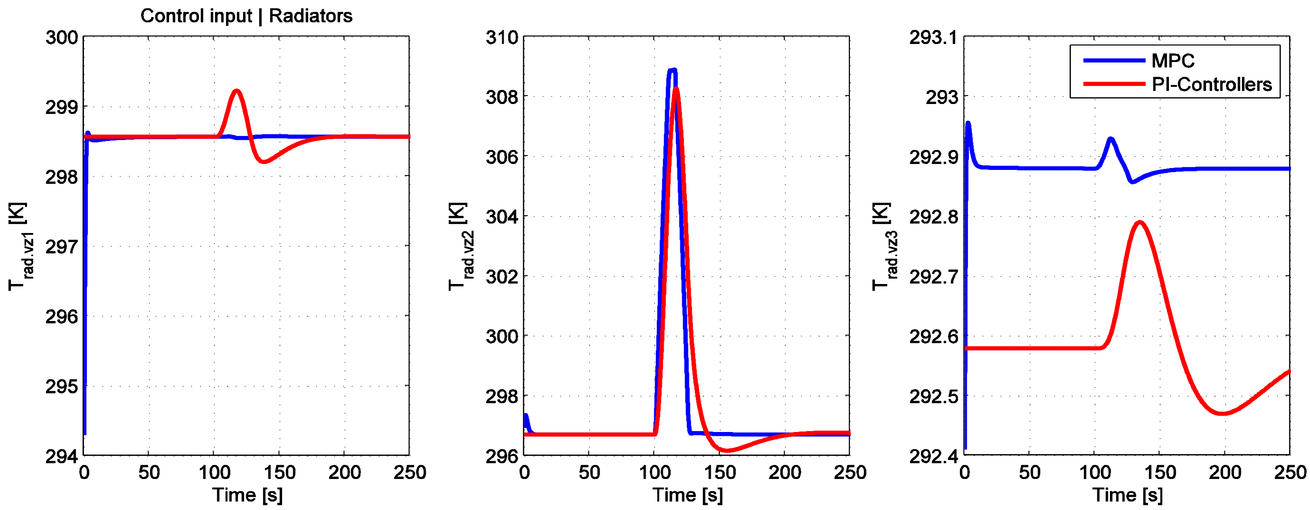

Figure 98 Radiators temperature of a) Zone 1, b) Zone 2, c) Zone 3.

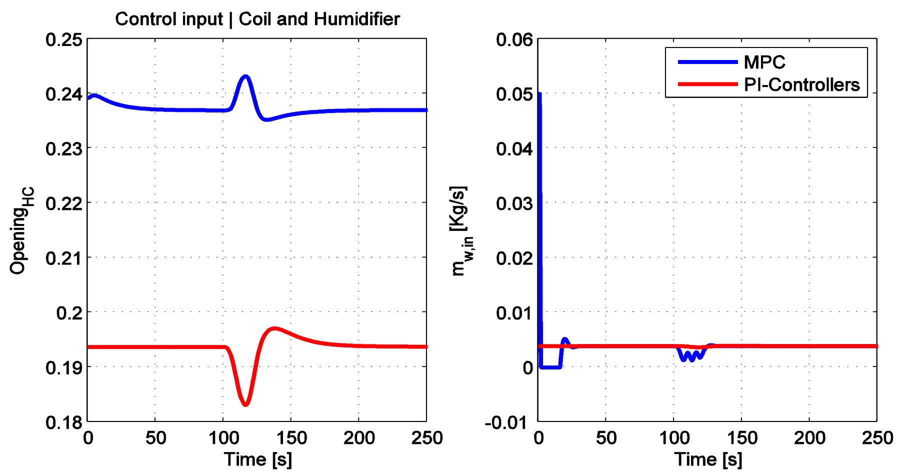

Figure 99 Opening degree for heating coil and added mass from humidifier.

A table over the decreases and increases in the different outputs when the door opening is applied are given .

From comparing the temperature of the zones, it can be seen that for the case with the PI-controllers, the temperatures decreases a lot compared to the case with the MPC. The temperature of zone 1 is practically unaffected. The decrease in temperature for both cases are almost of same magnitudes.

Looking at the temperature in zone 2, the temperature decreases for the PI-case $3.5°C$ while the temperature for the MPC-case decreases with $0.5°C$. Additionally, the MPC-case settles 30s before than the PI-case.

In zone 3, the temperature of PI-case decreases with $1.3°C$ where in the MPC-case it decreases with $0.1°C$ and it takes 150s less to settle.

From comparing the pressures, zone 1 and zone 3 are almost unaffected. In zone 2, the pressure decreases with 30 Pa for the MPC-case and 20 Pa for the PI-case.

Looking at the figure with the damper openings, it is clear that this is a result of the damper in zone 2 is not opened as much for the MPC-case as for the PI-case. A smaller weighting on the dampers might would result in a smaller decrease. Another thing that is important to notice that the overshoot of pressure is avoided by the MPC.

For the specific humidity, the PI-case has a large increase in specific humidity compared to the MPC-case. This is due to the MPC do not open the dampers as much; thereby the PI-case will have much more air inflow with a higher specific humidity compared to what is leaving the zone. The result of not altering the inflow specific humidity in the MPC-case results in a minor decrease, which takes longer time to settle afterwards (Approximately 500s, as no additional water vapor is added).

In general the settling time of the MPC is faster and more robust.

The changes in temperatures are given in the table below:

	PI-Controllers	**MPC**
Temperatures	Decrease/Increase	Decrease/Increase
Zone 1.	$-0.09\,°C$ $+0.07\,°C$	$-0.05\,°C$ $+0.02\,°C$
Zone 2.	$-3.5\,°C$ $+1.82\,°C$	$-0.55\,°C$ $+0.36\,°C$
Zone 3.	$-1.34\,°C$ $+0.98\,°C$	$-0.07\,°C$ $+0.04\,°C$

Table 14 Temperature changes when disturbance is applied.

The changes in pressures are given in the table below:

	PI-Controllers	**MPC**
Pressures	Decrease/Increase	Decrease/Increase
Zone 1.	$-$	$-$
Zone 2.	$-20\ Pa$ $+13\ Pa$	$-30\ Pa$ $+2.0\ Pa$
Zone 3.	$-$	$-$

Table 15 Pressure changes when disturbance is applied.

The changes in specific humidity's are given in the table below:

	PI-Controllers	**MPC**
Specific Humidity	Decrease/Increase	Decrease/Increase
Zone 1.	$-3.0 \cdot 10^{-6}\ \frac{Kg\ Moist}{Kg\ Dry\ air}$ $+9.5 \cdot 10^{-5}\ \frac{Kg\ Moist}{Kg\ Dry\ air}$	$-6.9 \cdot 10^{-5}\ \frac{Kg\ Moist}{Kg\ Dry\ air}$
Zone 2.	$+8.2 \cdot 10^{-4}\ \frac{Kg\ Moist}{Kg\ Dry\ air}$	$+1.2 \cdot 10^{-4}\ \frac{Kg\ Moist}{Kg\ Dry\ air}$
Zone 3.	$-9.0 \cdot 10^{-6}\ \frac{Kg\ Moist}{Kg\ Dry\ air}$ $+1.7 \cdot 10^{-5}\ \frac{Kg\ Moist}{Kg\ Dry\ air}$	$-7.2 \cdot 10^{-5}\ \frac{Kg\ Moist}{Kg\ Dry\ air}$

Table 16 Specific humidity changes when disturbance is applied.

From both the figures and tables, it is possible to conclude that the MPC is better than the PI-controllers. An important fact is that if the radiators rate of change in temperature becomes lower, the decrease in temperature is affected a lot. It would still be possible to weight the constraints on the damper so they can change faster without violating the rate limits of the dampers. However, it would be a cost of a larger deviation in specific humidity when the door is opened.

9. Setup

The setup used for control testing is illustrated in the *Figure 100*.

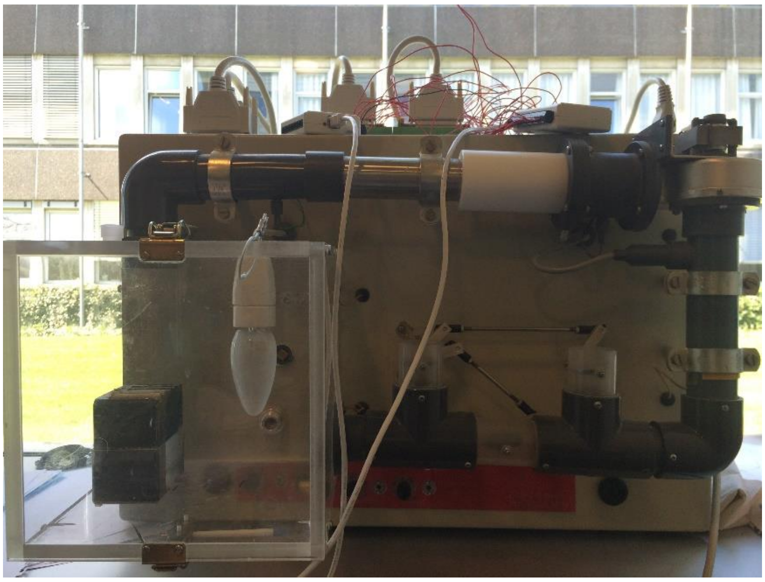

Figure 100 The Setup that was used for validation and control.

9.1 Setup description and interface

A fan provides an airflow which circulates in the plastic pipes. Two dependent dampers, of which the angles are aligned, determine the amount of exhaust and inlet air. The air is sent into a small plastic box, which represents a zone. In the box, there is a lamp, which is considered as internal heat gain or disturbance. The inlet and outlet to the zone is not controlled by any dampers but is fixed at the circulation pipes diameter. A cup of water is placed inside the box in order to add more moisture to the air. The measurements from sensors and actuators are sent to and from two DAQ devices to a computer with an interface in LabVIEW NI.

The ventilation setup consists of:

- A Plastic box (The zone, control volume)
- Cup for water (Add in moisture)
- Inlet and outlet dampers
- A Fan
- A Heater
- A Lamp
- Temperature sensor and Humidity Sensor (Relative Humidity)
- 4. analog switches and controllers
- 2. DAQ Cards, NI USB 6009, depicted in the figure to the right.

Figure 101 The DAQ Card, NI USB 6009, used in this project setup.

The interface is designed in LabVIEW, the pre-existing interface is depicted in the *Figure 102* below.

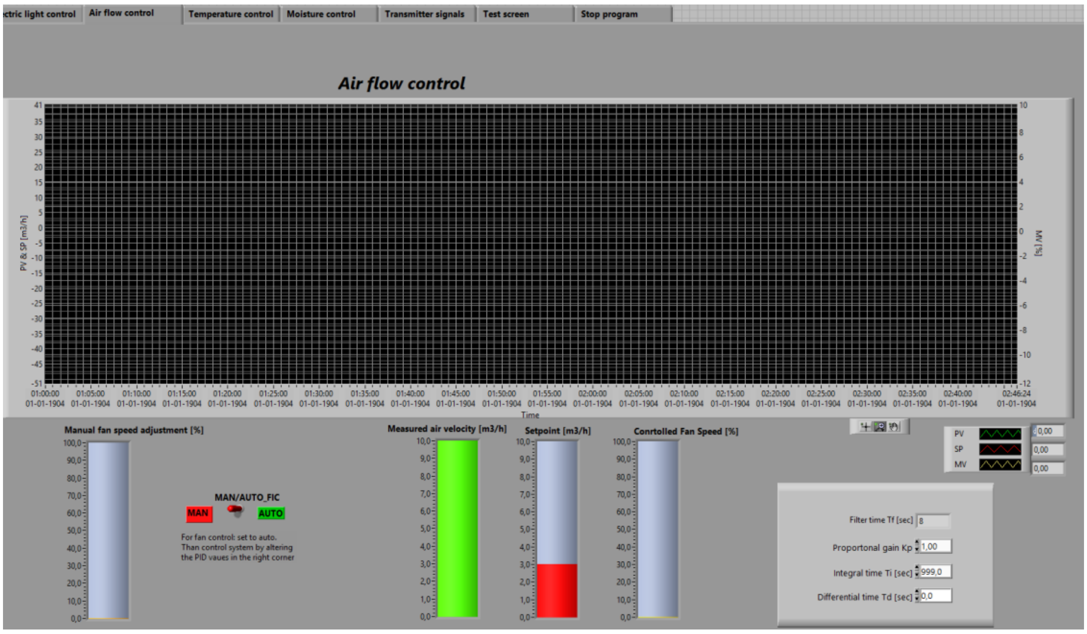

Figure 102 The existing graphical interface.

In the existing interface, controls for both open and closed loop testing are available. Simply by applying the *manual/autonomous* switch depicted in the left-bottom side of the Figure 102. For the purpose of this report, this interface is only used for the open loop testing, due to the fact that the interface is slow and the response between the actual actuation and measurement is delayed a lot due to the filtering time. A lot of filtering is needed which also slows down the measuring and control.

For the appliance of the open loop testing four (4) numeric controls are implemented in the front screen in order to control the actuation precise and at same time.

The data measured, or sent to the setup is handled by two USB 6009 NI data acquisition cards. The cards are limited by its application, and for control purpose, the sampling time needs to be low enough for both DAQ to follow each other.

Each card has 8 analog inputs and 2 analog outputs. It also consists of 12 digital input and output channels but these are not used. The used channels for these testing is given in the tables below.

The two cards input and output are connected as shown in the Table 17.

Card A.	**AI1**	**AI2**	**AI3**	**AI4**	**AO1**	**AO2**
Input (V) (Measurement)	Lamp	Fan	Heater	Damper		
Output (V) (Control input)					Lamp	Fan
Card B.	**AI4**	**AI7**			**AO1**	**AO2**
Input (V) (Measurement)	Temperature (Zone)	RH (Zone)				
Output (V) (Control input)					Heater	Dampers

Table 17 The input and output channels of the DAQ devices.

The voltage of each input ranges between 0-5v. The voltage is converted to the amount of the percentage ON. For temperature (T) and relative humidity (RH) the conversion function is pre-given, by means of relating the voltage into real value of degrees in Celsius and percentage for the temperature and the relative humidity respectively.

For control purpose, a new interface is created and the ranges of the input and output need to be calibrated to the "right" range, as shown in the Table 18.

The ranges of voltage is given below.

	Flow rate $\left[\frac{m^3}{h}\right]$	**Heater [%]**	**Lamp [Lux]**	**Dampers [%]**	**Temp** $[°C]$	**RH [%]**
Actuator/ sensor range	$0\ \frac{m^3}{h} - 6\ \frac{m^3}{h}$	$0\ W - 100\ \%$	$0\ Lux - 1000\ Lux$	$0\ \% - 100\ \%$	$0°C - 100°C$	$0\% - 100\%$
Voltage range	0.988v-3.01v	1.028v-3.5v	3.25v-4.43v	1v to 4.959v	Pre-given relationship	Pre-given relationship
Offset	0.988v~1v	1.028v	3.25v	0.999v~1v		
Conversion function to 100% on	$\frac{6}{2.022} \cdot (V-1)$	$38.88 \cdot (V - 1.028)$	$847.46 \cdot (V - 3.25)$	$25.25 \cdot (V - 1)$	$25 \cdot (V - 1)$	$25 \cdot (V - 1)$

Table 18 Actuator ranges and conversion function

From the above table it is possible to see that there are some pre-given functions to determine the temperature and relative humidity, as it is not possible to measure these values from another sensors and compare with the voltage range.

9.1.1 Filtering

The data logging from the setup is done by a sample rate of 1 kHz and with 1k samples, making the sampling time 1s. The sampling time is very low but it is necessary in order to make the DAQs ports work in sync.

The measurements from the DAQs, due to noise, need to be filtered. A very simple exponential filter is implemented since the cut off frequency is close to 0 and the low pass filter is not suited for every measurement. The measurements of the temperature, before and after the filtering, are depicted in the graphs shown in the Figure 103 below for two different filtering times, 5s and 10 s.

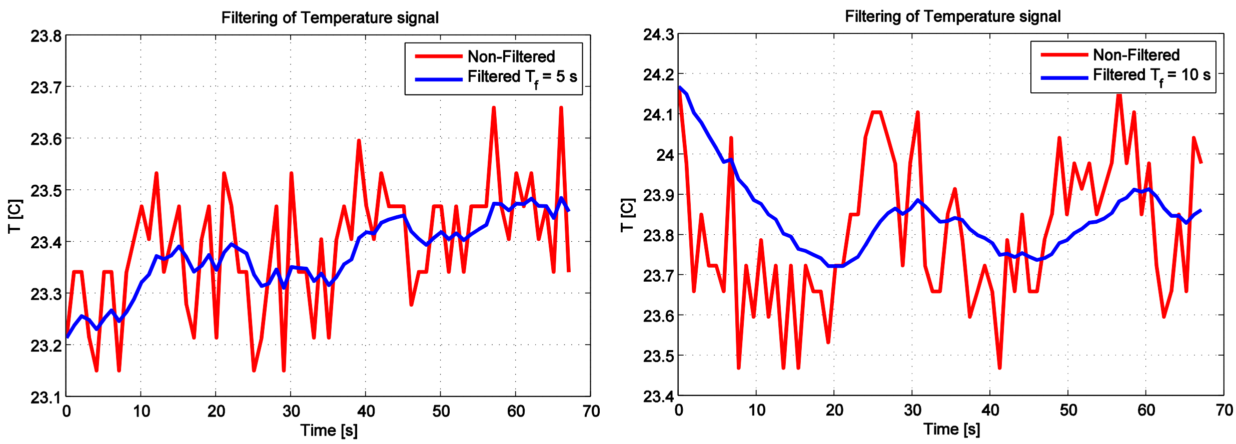

Figure 103 a) Filter time 5s, b) Filter time 10s.

From the graphs above is possible to be seen that choosing the filtering time too large make the response time much slower than when is too small, although in the second choice some noise still exist. Therefore the filter time for the temperature is chosen to be at 8s. The same procedure is applied for the other measurements. Some oscillation still exist but is acceptable, in order to avoid slowing the system even more.

9.1.2 PID-control

In order to run the open loop and closed loop tests, where the PID-controller can be applied for each individual actuator at the time, a “case structure” is used. Switching between open loop, individual control and combined control. The following cases are implemented into the GUI.

1. *Open Loop*
2. *PID-control on all*
3. *PID-control Temp*
4. *PID-control Moist*
5. *PID-Control Air*

Another important factor is that the output voltage range of the DAQ devices is between 0v to 5v therefore the PID-control output must not be negative, since no negative voltage is allowed. The actuation will be controlled by a positive voltage where the magnitude indicates how far is from 0v.

9.1.3 New graphical interface

The new interface is illustrated in Figure 104 below. All the measurements for the actuators are depicted in the four graph in the top, and the temperature and relative humidity is depicted in the two graphs in the bottom part of the figure. For each actuator, except the lamp, the control gains are depicted respectively beneath each graph. Thereby, each actuator can be tuned independently. The reason that the lamp has no controller is due to the fact that is set as a disturbance.

Figure 104 Graphical interface, to see block diagram, see Appendix 9.

In the above figure, the “roll bar controls” are used as a setpoint for either the actuators or for the temperature and RH. The “white roll indicators” illustrate the actuators percentage control input. Furthermore, a “stop button” is implemented in order to terminate the simulation whenever is needed by the user. In addition, a “case selector” is realized, which selects if the setup runs in open loop or one of the closed loop cases.

9.1.4 Limitations and uncertainty

The limitation with this setup is the hardware, which is very old and even though the actuators can run with a high sampling time, the sensors in the system are limited. Therefore, the interface or controllers have a slower response.

Another thing is that the pressurization cannot be maintained due to the fact that the dampers are open to the atmospheric pressure, the control volume is not isolated enough and the dampers' positions are fixed at the same angle.

One of the main uncertainties with the open loop test for the model validation is that the ambient temperature acts as a disturbance, which can vary from day to day with some degrees. The tests are conducted at approximately the same initial temperature, although some starts with some degrees in difference.

9.2 Open loop tests

As a part of the modelling a lot of open loop tests was conducted on the setup, in order to validate the model. Due to some conflicts with the system identification, some round offs and many disturbances in the measurements, the system identification was not completed.

Some of the best measurements are depicted in the below figures. The two figures are for two different setpoints for the heater, 50% and 75% respectively. The tests are conducted in a period of 1h to 2h.

It is possible to see variation in the heaters control input. Beside that the voltage input to the heater differs a lot from time to time, the relationship between the heaters watts is not linear with the voltage provided, the tendency is observed to be more exponential.

The setpoint for the different measurements are given in the legends. Here F denotes the volumetric inflow provided by the fan, D denotes the opening percentage of the dampers and H denotes watts of the heater.

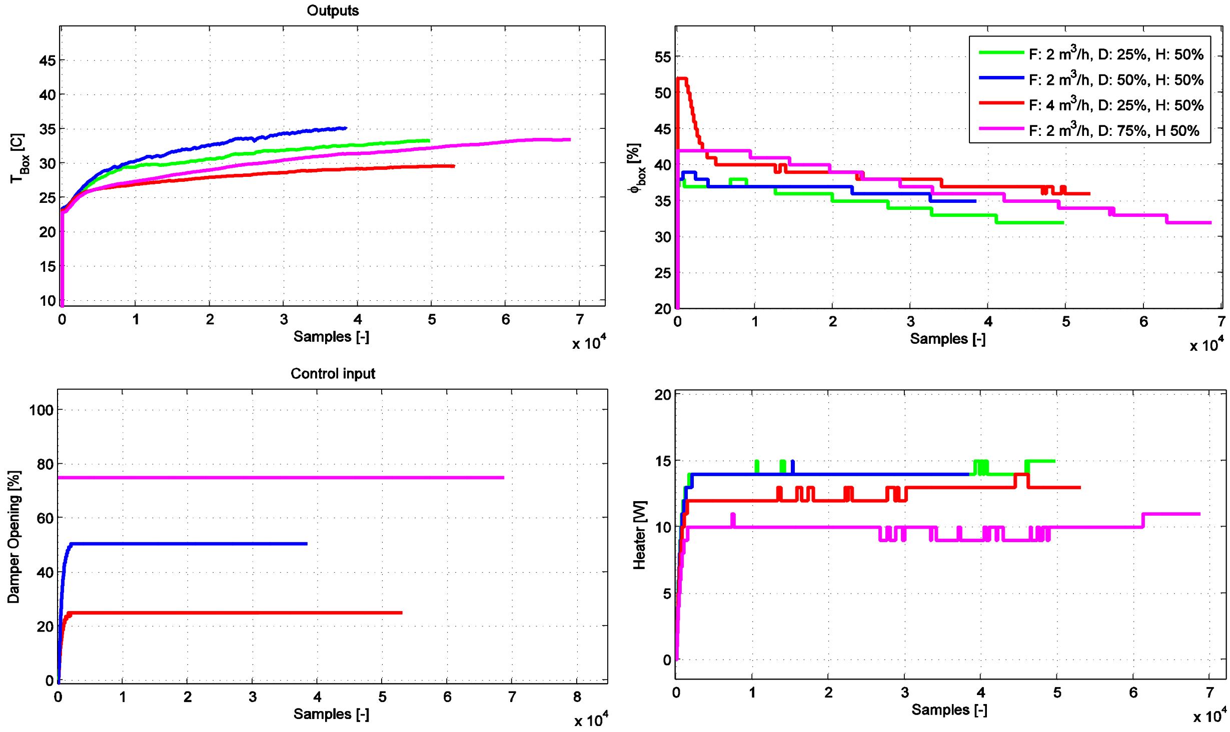

Figure 105 a) Temperature, b) RH, c) Damper opening, d) Heater.

From the above figure, it can be seen that the temperatures has the same increase in the start where after it settles depending on the provided heat and damper opening. From the RH, it can be seen that it gradually decreases with the increasing temperature, which is expected as hotter air can hold more water vapor.

Looking at the measurements for the heater it is possible to see that even though the setpoint is the same, the watts provided still differs by 5 watt.

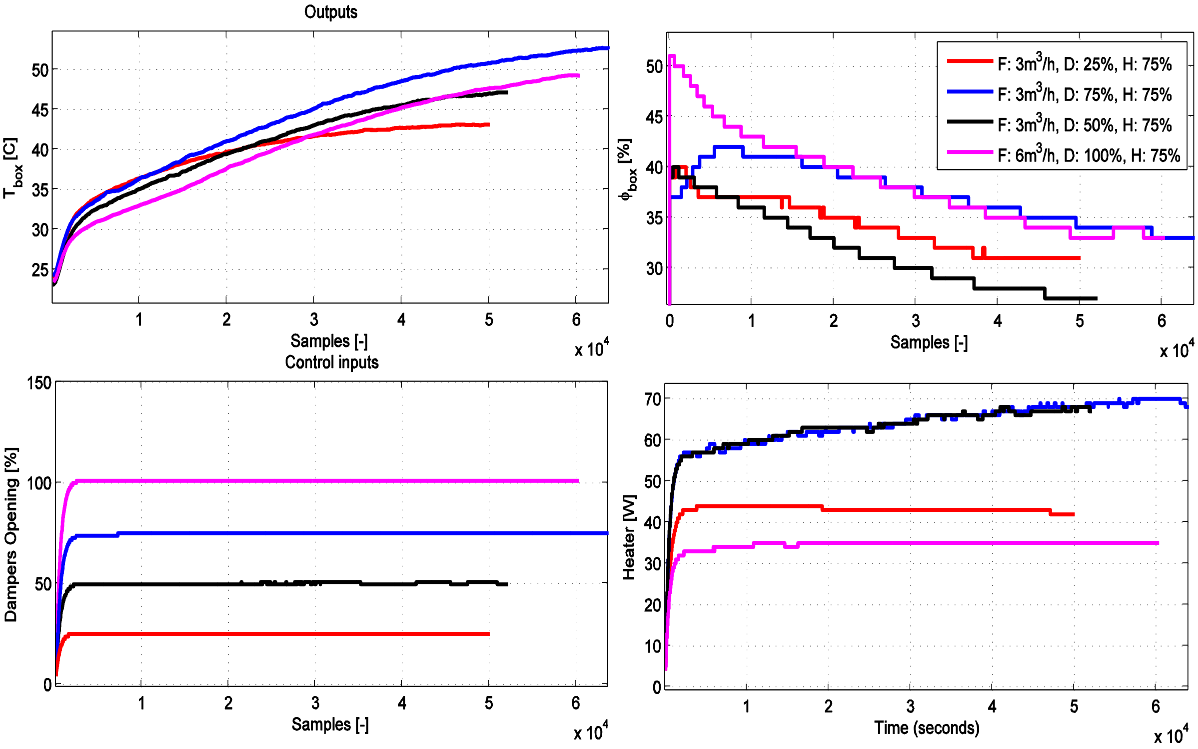

Figure 106 a) Temperature, b) RH, c) Damper opening, d) Heater.

Some improvements for the measurements could had been made by running the tests for a longer period. The temperature has been in steady state for several minutes before the tests are ended. This is almost unnoticeable in the figures as the run time is more than 1 hour.

Another thing which could had improved the system identification process, if the running time was longer, would had been that the RH might have settled down. It seems to need more time than the temperature. At last, more precise measurements for the RH could had been beneficial. Unfortunately some round offs was made when the measurements was saved, thereby making the some of the measurements more discrete.

9.3 Control – PI control

The temperature and relative humidity in the box-zone in the Set up is controlled by using the fan, the dampers and the heater. Compared to the model established in chapter 3, the setup does not have the flexibility to control the specific humidity but only the relative humidity. The specification here is that the RH is achieved by changing the dampers, but can only be controlled within the limits of the temperature within the control volume and the ambient temperature. The temperature is directly controlled by the heating coil but is also influenced by the ambient temperature, as the dampers alter the opening area.

The PI-controller is chosen for all the actuators, as the derivative term can cause unnecessary addition of noise. One of the modified PID approaches could have been used if the derivative term was in need.

The PI-controllers are tuned to be underdamped if possible. As it was mentioned earlier, it is a compromise of a slower response time of the controller, but in this way less oscillation or no overshoot can be achieved. This means that e.g. the heater will not be heating up and cooling down periodically.

Moreover, disturbances are applied to observe the changes of the control input as well as if the setpoint is maintained.

The block diagram for the controller is illustrated in Figure 107 .This is for case 2, where the whole system is controlled at same time.

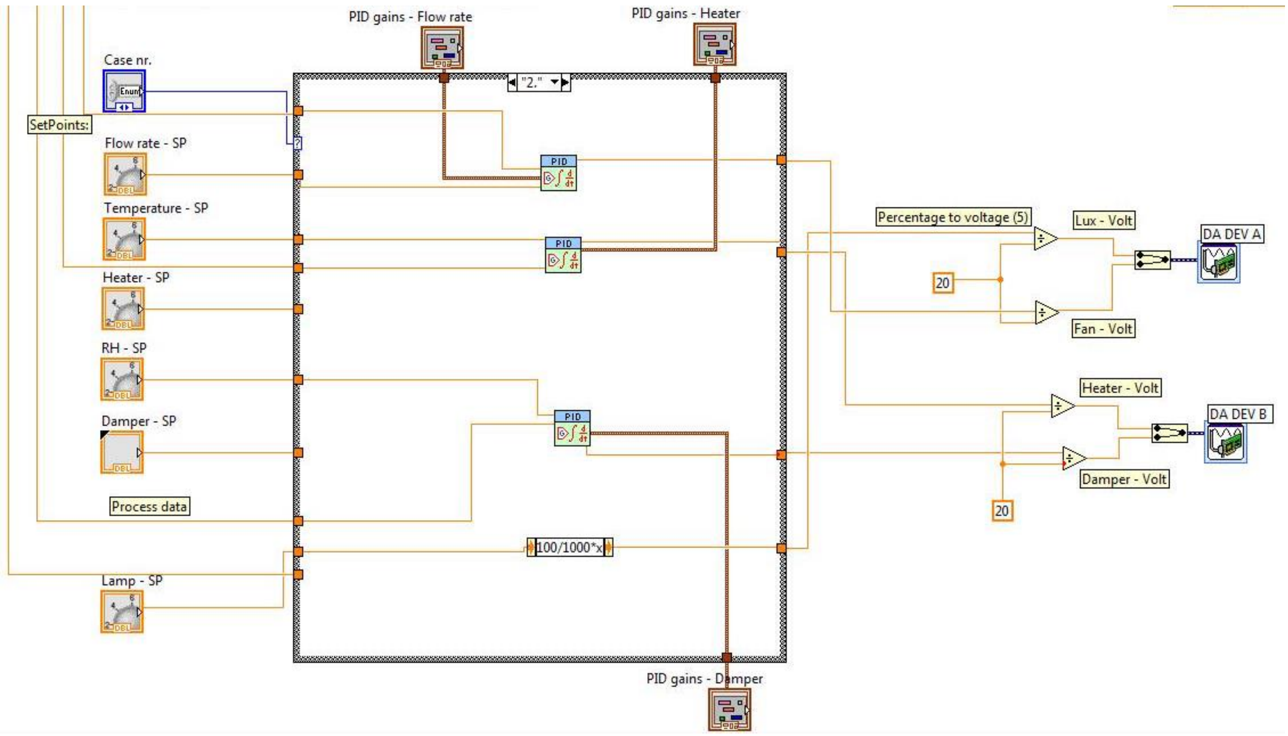

Figure 107 The Case loop for the combined controllers.

In the above figure, the setpoints and process data are depicted in the left side of the block diagram, together with the case control (blue), which dictates which case to run. In the case loop, all PID – controllers are placed with the control gains outside the loop. To the right in the block diagram the output is sent to the DAQs output channels, precisely two outputs for each. Before the control input enters the output port it is divided by a factor of 20 in order to convert it back to voltage, which ranges between 0v-5v.

The case with all the controllers is first used when all the PI-gains for all the actuators are found.

9.3.1 Temperature

As it is already mentioned, no overshoot is desired, therefore the control gains are tuned to avoid them. The PI-gains that were found after the tuning process of the controllers, are given in Table 19. The temperature settles down in around 1.5min to 2.5min depending on the setpoint change. Trying to make it faster will end in continues oscillations in both temperature and control effort.

K_p	K_i	T_s	Overshoot
6	$\frac{1}{T_i} = \frac{1}{0.5} = 2$	Changes with the setpoint changes, here it vary between 100s to 200s	~0%

Table 19 Controller specification

The response of the temperature and control effort of the heater are depicted in the figure below. This is for changing setpoint values.

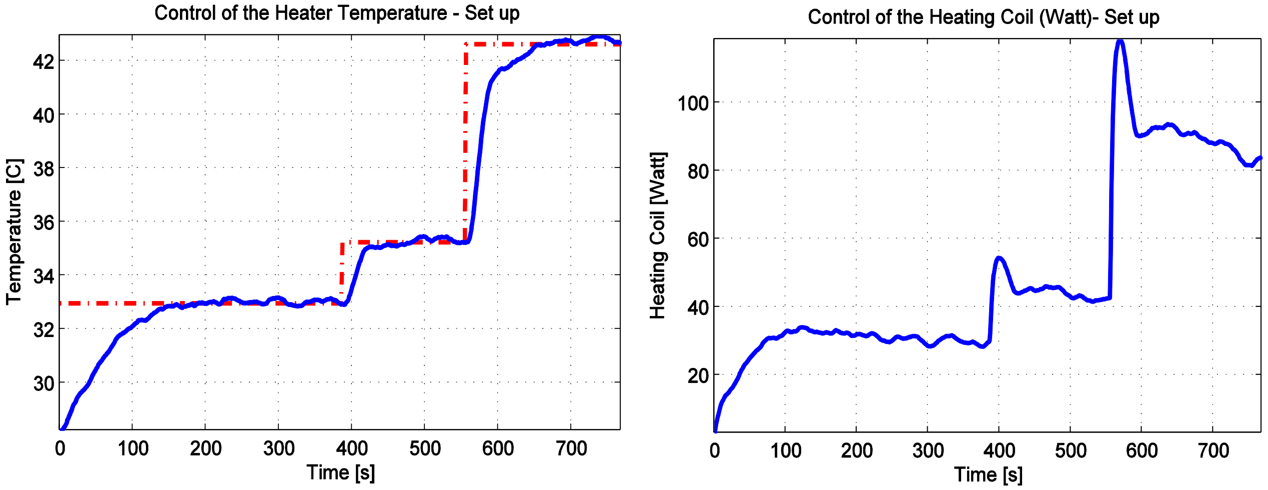

Figure 108 a) The temperature setpoint and actual temperature, b) Control effortof the heater. Lamp off and open damper.

In Figure 108, the temperature is depicted in the left graph and the control effort of the heater in the right graph. It is possible to see that the temperature settles to the setpoint within minutes without much overshoot. It is also possible to see the required percentage [%] that the heater provides in order to keep the temperature at different setpoints.

To test how robust the controller is, various disturbances are applied to the system, such as opening the front side of the control volume, opening a window, adding internal heat gain and different damper openings. The results of this are presented in the next sections.

9.3.1.1 Disturbances – Change in damper openings

Applying different damper openings will cause less or more outside air to enter in the pipe. Thereby, in order to maintain the temperature at the desired setpoint, changes in the amount of heating is a need.

The different applied damper openings are illustrated in the graph to the left side of the Figure 109. The setpoint for the temperature is almost $31°C$. The control effort is depicted to the right side of the figure.

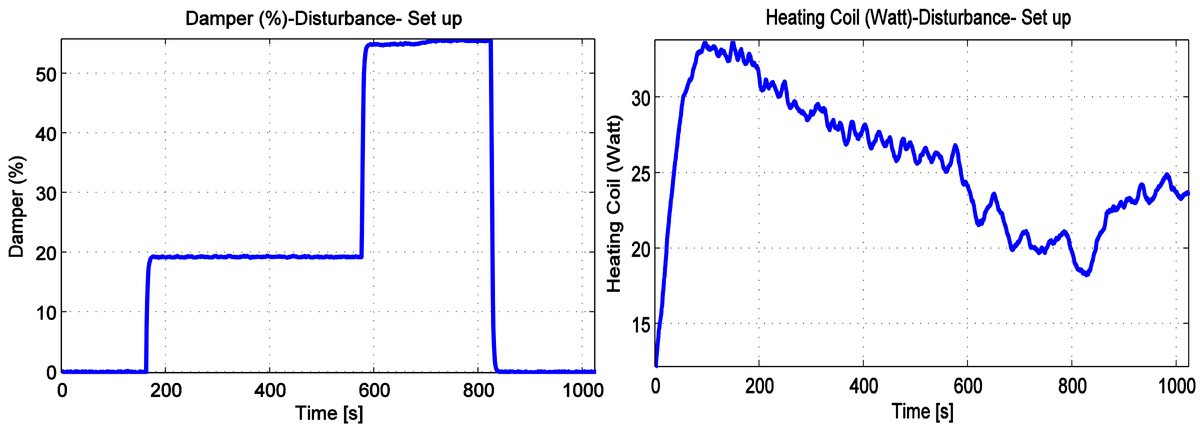

Figure 109 a) Damper opening (0% = fully open), b) Control effort of Heater.

It is possible to see from the above figure, that when the damper closes down, the heater gradually decreases its control effort. This is due to the fact that the ambient temperature is less than the temperature inside the control volume. Therefore, letting less ambient air in and recirculating the inside air will make the return air to the heater warmer and less heating is needed. The effect on the temperature can be seen in Figure 110.

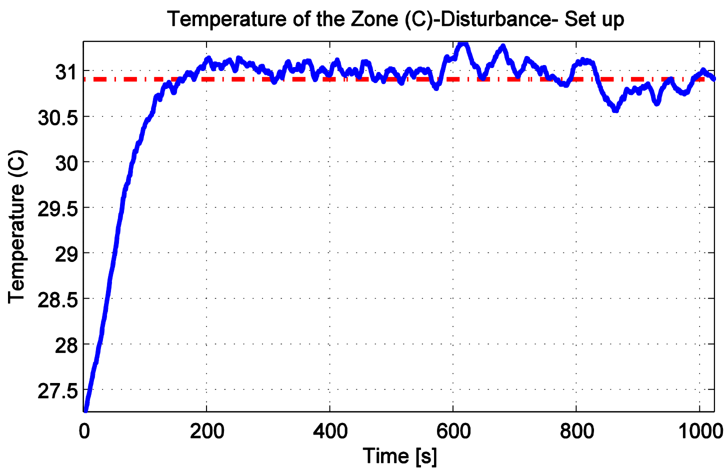

Figure 110 The actual temperature and the setpoint temperature.

One could see that the temperature is approximately at the setpoint and it begins to deviate more when the damper closes. The deviations are in a range of $0°C - 0.5°C$. This means that even though changes in the fraction of air entering and leaving the system occur, the heater makes sure to maintain the temperature within acceptable range. Running the test and disturbances for a longer period could also cause total convergence, without too large fluctuations.

9.3.1.2 Disturbances –Open door

Another disturbance applied to the system is an "Open door" disturbance, which is conducted by opening the front side of the plastic box in the first 30sec and then in 60 sec. Both temperature and control effort of the heater are depicted in the Figure 111 below.

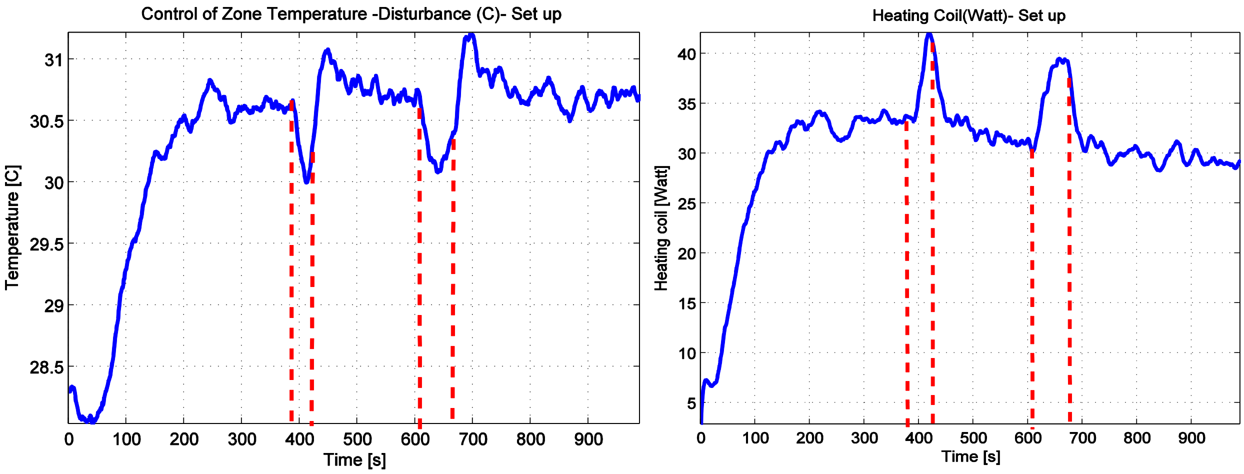

Figure 111 a) Temperature of control volume, b) Control effort heater.

From the above figure it is seen that just before 400s the front side or "door" opens, which results in a decrease of temperature to $0.6°C.$ When this change occurs the control effort of the heater rises almost 7 Watt in order to compensate the change in temperature. At the time that the control efforts reaches the peak value (just after 400s) the temperature begins to increase again. The control effort decreases because the front side is closed again and less heat is needed to compensate for the heat lost by the opening.

The second opening is applied around 600s. The temperature decreases to approximately the same temperature as in the first opening, except that this time it takes a longer time to rise due to the fact that the front side is kept open for a longer time. The control effort is almost of the same magnitude, which is 10Watt. This means, if the front side was to be open even for longer time combined with a lower ambient temperature, the control effort would increase much more. On the other hand leaving the front side open for a longer period, might cause the temperature to settle down but at a cost of large magnitude change in the heater controller.

9.3.1.3 Disturbances –Internal heat gain

The last disturbance applied to the system is the internal heat gain or else the lamp. The lamp setpoint is depicted in the left side of the figure below and the control effort of the heater is depicted in the right side.

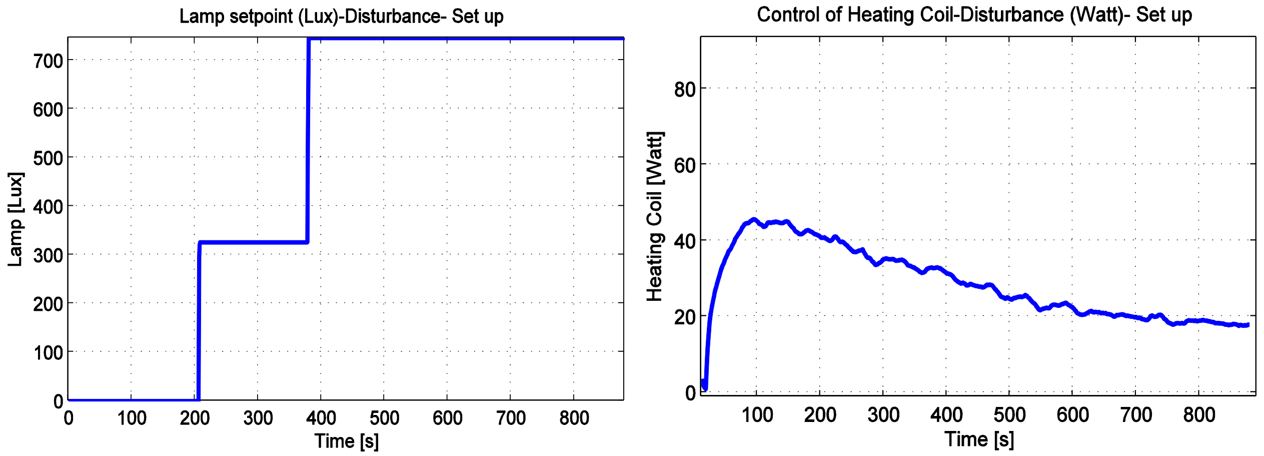

Figure 112 a) Lamp setpoint value, b) Heating coil control effort.

The lamp setpoint varies between 0Lux to 750Lux. When the lamp is turned on, the control effort of the heater starts decreasing as the lamp provides heat to the control volume.

The temperature change throughout time is illustrated in Figure 113.

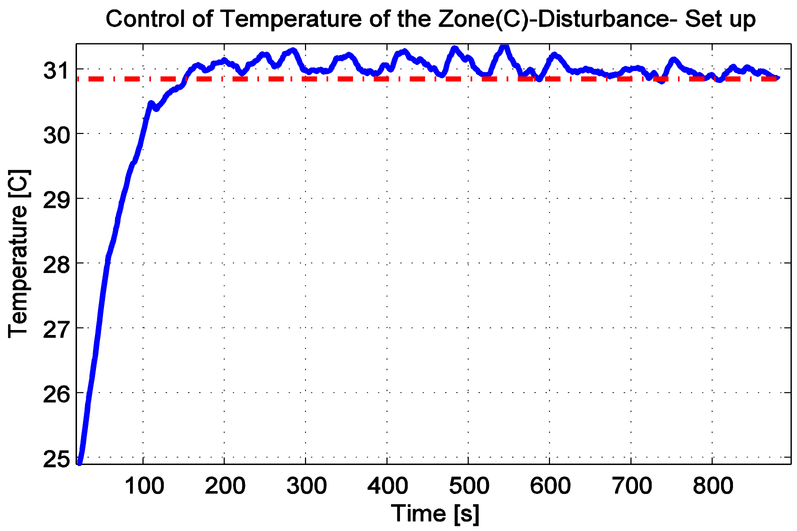

Figure 113 The temperature of the control volume.

It is possible to see that it exists a small error between the setpoint value and the actual temperature, the difference is approximately 0.3$°C$. It can also be seen that the temperature remains almost undisturbed by the increase in Lux-value. In the period of 400s to the 900s, it is possible to see that the temperature settles down. This is due to the test for this Lux-setpoint is conducted for enough time. It takes approximately 500s to settle, which is exactly 8.33 min. This is a relatively long time, but the magnitude of the Lamp is 700 Lux, and without any form of cooling it will take this time to settle down.

9.3.2 Relative humidity

The damper is used in order to control the relative humidity, due to the lack of humidifier or cooling coil. Its operation lies in controlling the opening damper angle in order to either add air, which results in a decrease of the relative humidity *RH*, or fully close damper which results in adding RH.

K_p	K_i	T_s	Overshoot
15	$\frac{1}{T_i} = \frac{1}{0.4} = 2.5$	Changes with the setpoint changes, here it vary between 100s to 200s	~0% Small overshoot for some setpoints.

Table 20 Controller specifications.

The RH and Damper openings for different setpoints, are depicted below. In the graph for the RH, the red dashed line is the real value and the blue is the setpoint.

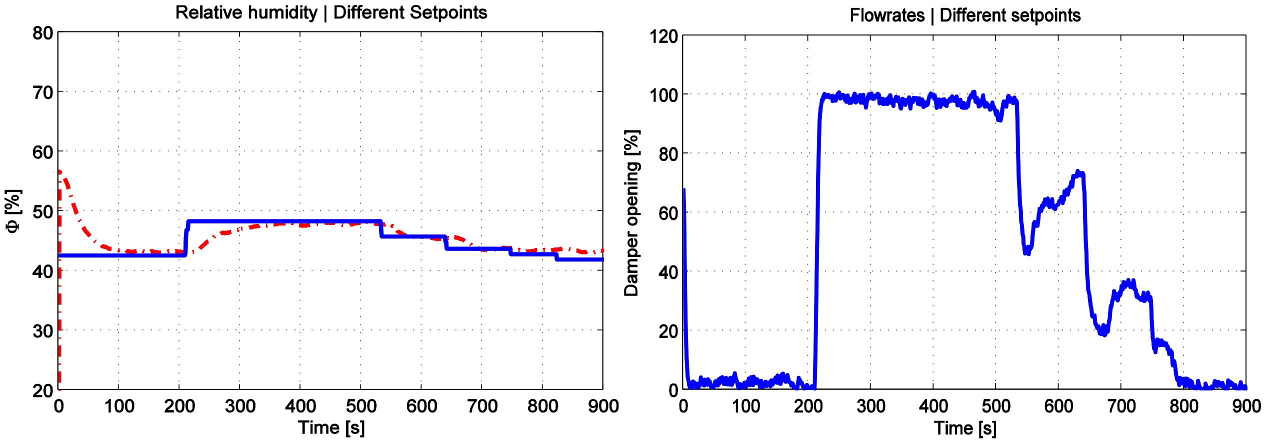

Figure 114 a) RH and different setpoint values, b) Dampers openings.

In the left side of the figure it is possible to see that, the RH settles within 200s but in the end of the test the last setpoint can't be reached as the air specification of the inside and outside air does not allow for a lower RH. At this time, the dampers' position is fully open 0% and cannot let more outside air in, even if it's needed in order for the RH to settle down. This is one of the main limitations of the system, only linking the RH to the dampers position. The way the dampers are controlled is when the RH setpoint is high, the dampers are totally closed. In addition, in the case of the RH setpoint is low, the dampers are fully opened. This is valid disregarding the ambient temperature, specific humidity and RH. It could be that taking in more air would result in a higher RH but the controller does not allow this to happen, as the voltage is non-negative and only goes from 0v-5v. This means no negative values comes from the controller as the lower limit of the output range of the controller is saturated at 0.

In the next sections different disturbances are applied to the system, to see the controller performance.

9.3.2.1 Disturbances – Open door

The door disturbance is applied in both 30s and 60s. As no changes happens in either RH or damper position of the first case, only the case with 60s opening is depicted below.

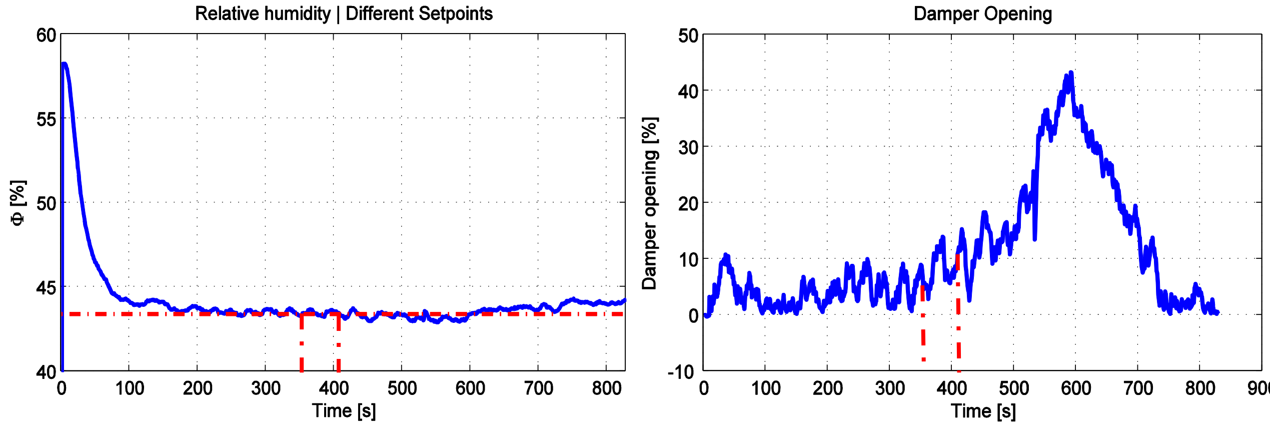

Figure 115 a) RH and setpoint, b) Damper opening.

The opening is applied at 350s. It is possible to see that the dampers position closes down to 40% in order to remain at the setpoint value. As a result of the change of the dampers position, the setpoint is held and only very little deviation occurs. The reason that the setpoint can be maintained results from the fact that the ambient air temperature and RH is very close to the one inside the control volume. In the case of changing the ambient air specifications is tested in the next section.

9.3.2.2 Disturbance – Open window

In this test, the window is opened together with the front plate of the control volume. The results are depicted below.

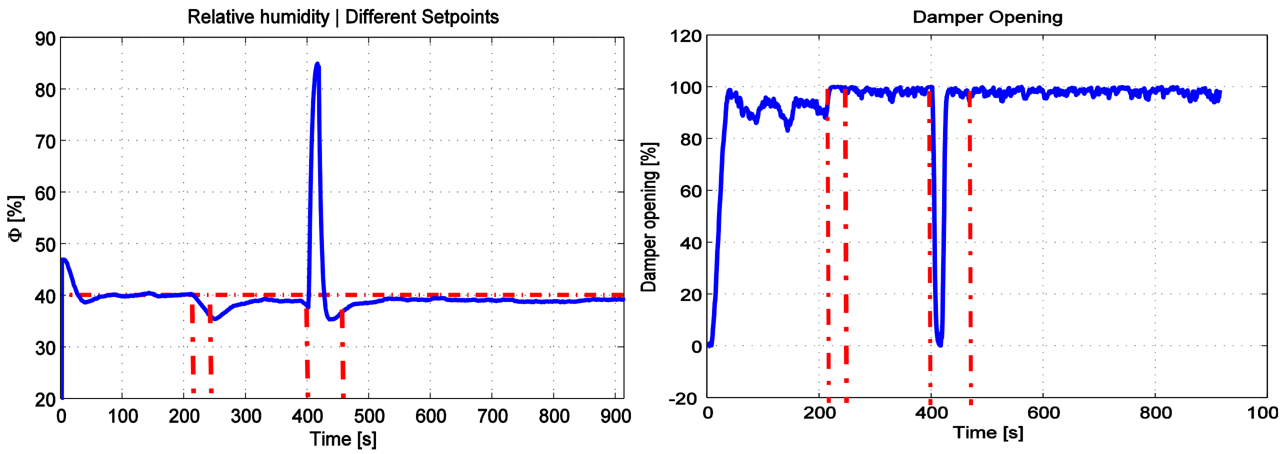

Figure 116 a) RH and setpoint for door opening and a cold environment, b) Dampers opening.

From the graphs it is possible to see that, between 200s-300s the RH decreases as a result of the “door opening”, which is applied at 210s to 240s the dampers close to 100%, and after the front plate is closed again, the RH setpoint is obtained again. The decrease in the RH is approximately 4%. Opening the front plate again, results in a large jump of the RH, which is a measurement error, and then it decreases. The dampers experience the same reactions as before, in order to come back to the setpoint for the RH. This means, the PI-controller cannot prevent the decrease in RH until the front panel is closed. Second time the front plate opens, the RH begins to increase again but not because of damper changes. The dampers are fully closed; therefore it is only due to the mixing of air entering the control volume and the air circulating inside.

9.3.3 Fan

The fan controller is tuned in order to maintain the volumetric flow rate at different damper openings. The pressure is not measured in the set up system. The system is very old and has some limitations. The fan is not strong enough for some damper openings to maintain the maximum flow rate. This will be illustrated in the figures below.

The final PI-gains and specifications are given in the Table 21.

K_p	K_i	T_s	Overshoot
15	$\frac{1}{T_i} = \frac{1}{0.35} = 2.5$	Changes with the setpoint changes, here it vary between 30s to 250s	~0%

Table 21 Controller specifications.

The final controller is tested for different setpoint values and for different damper openings, varying from 0%-100%. Four of the test results are depicted below. These tests are for 20%, 40%, 60% and 80% closed dampers.

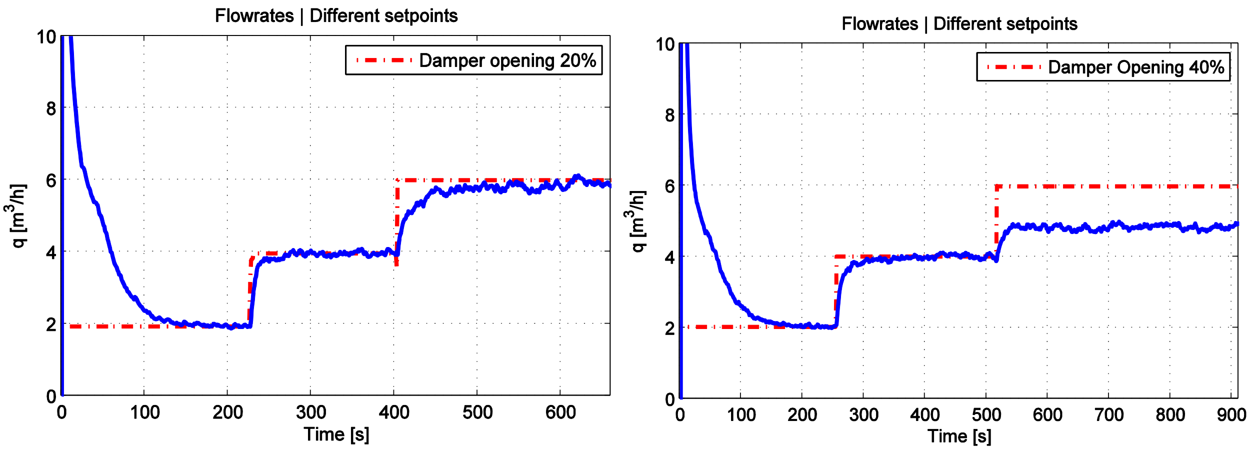

Figure 117 a) Flow rates and setpoints for 20% closed dampers, b) Flow rates and setpoints for 40% closed dampers.

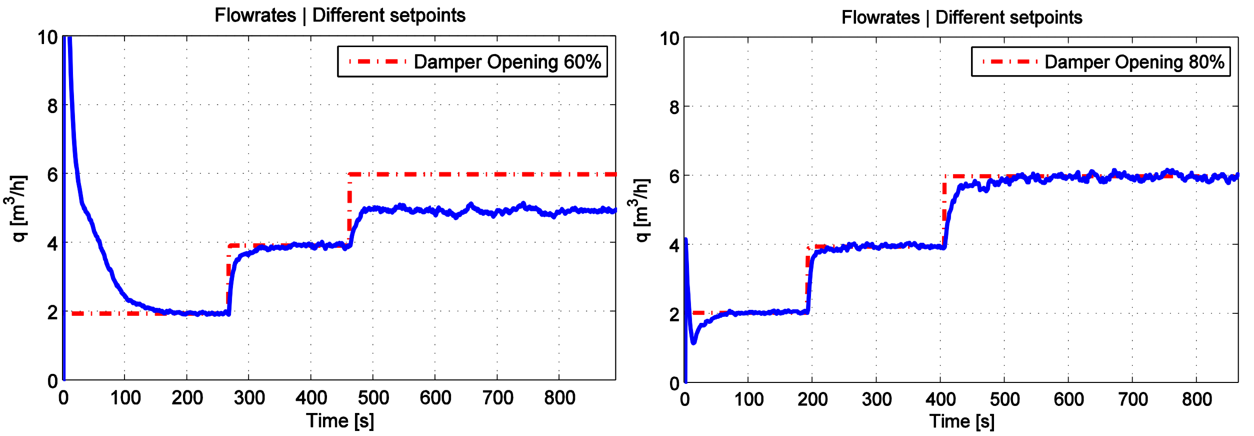

Figure 118 a) Flow rates and setpoints for 60% closed dampers, b) Flow rates and setpoints for 80% closed dampers.

From the graphs it seen that in the case of the dampers are either much closed (80%) or very open (20%) all the flow rates can be achieved. Closing the dampers in the interval of 40% - 60%, a flow rate above $5\frac{m^3}{h}$ cannot be achieved; this means that the performance of the fan is far from optimal when a lot of resistance is induced by the dampers. Based on this observation, the control tests will be running with a flow rate of $5\frac{m^3}{s}$

or below. The settling time for the fan is varying form 30s for small changes to 250s for the upstart setpoint. There is approximately no overshoot, only some fluctuation in the measurements.

9.3.4 Combined Test

The controllers have been found and they are all combined into a combined version. In order to test both the control of temperature and RH, it is important to be aware of the limitations because the two variables are dependent. The relative humidity depends on the specific humidity and the temperature. Therefore there is no point to have a setpoint that violates the range of possible relative humidity for given temperature and specific humidity. In addition, the specific humidity can be found from running a measurement for temperature and relative humidity.

One example is for the first test (Test no. 1), the temperature and RH are measured as:

$$T = 23.5673°C, \qquad RH = 42.93\%$$

From this, the saturation pressure and specific humidity becomes:

$$P_{sat} = 2889.8\ Pa, \qquad \omega = 0.0078\ \frac{kg\ water\ vapor}{kg\ dry\ air}$$

A setpoint for this situation can be: $T_{set} = 25°C$ and $\phi_{set} = 39.35\ \%$

9.3.4.1 Setpoint tracking

Running this test for $4\,\frac{\mathrm{m}^3}{\mathrm{h}}$, the fans capability is shown in the Figure 118 when the dampers is more than 40% to 80 % closed.

The control input, temperature and RH for these specifications are depicted in Figure 119 and Figure 120.

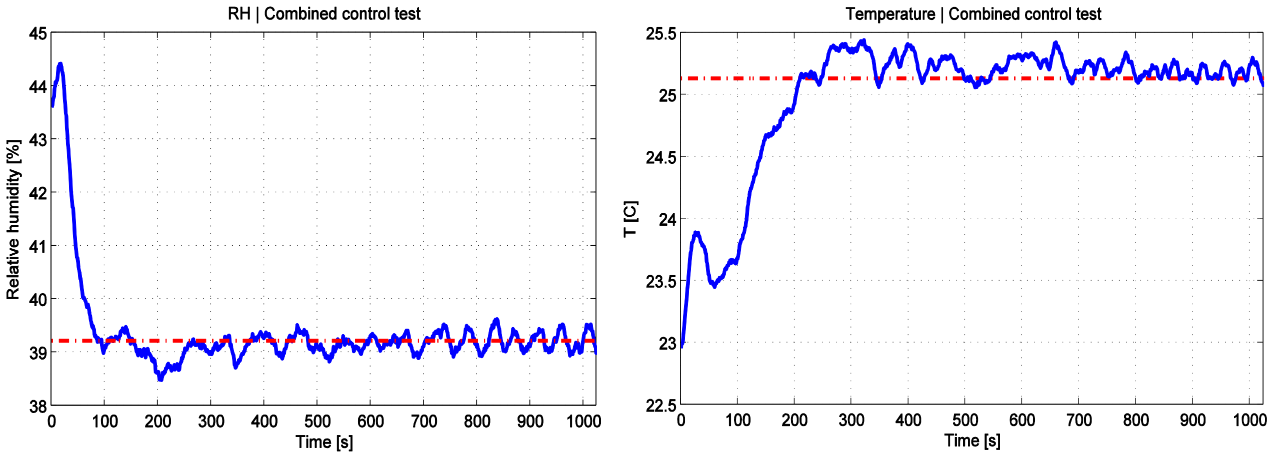

Figure 119 a) The RH, b) The temperature.

From the above figure it is seen that the temperature and RH settles in 100s and 250 sec. In order to maintain the RH the dampers will keep adjusting, allowing the right amount of air in the system. This causes changes in the temperature, and therefore the temperature oscillates within the range of 0.4 C around the setpoint. The RH fluctuates around the setpoint with 0.5%. Some of the fluctuation can be eliminated by having the humidifier, thereby eliminating the need to change the damper position continuously. The damper will be regulated to the position making sure that least heating is needed to achieve the setpoint value for the control volume.

The control inputs are depicted in Figure 120.

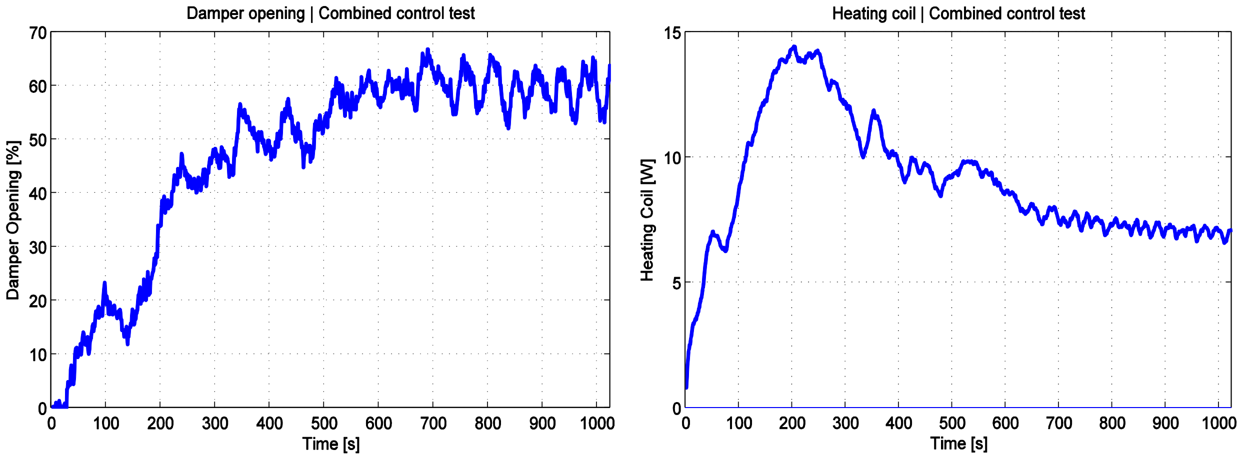

Figure 120 a) The damper opening, b) The heating coil Control effort.

From the above figure it is possible to see that the heater first starts settling down in the end of the runtime because it needs to compensate for the changes in the damper opening. As the damper opening keeps changing, there will also be oscillations in the heaters performance.

The air flow control works disregarding the change in damper opening, see Figure 121.

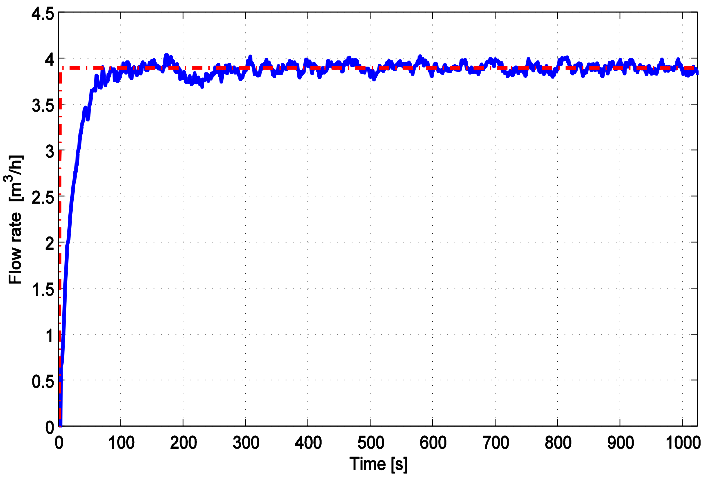

Figure 121 The airflow for Test no. 1.

9.3.4.2 Test no. 2 Different Setpoints tracking

In the next test, different setpoints are applied to the system, to see if the system can follow changing condition. The temperature- and RH setpoints are given in the *Table 22*.

Setpoint	Temperature	RH
1.	$26{,}4°C$	46,4%
2.	$28{,}5°C$	50,6%
3.	$26{,}5°C$	46,7%
4.	$29°C$	48%

Table 22 Setpoints.

The setpoint (Red, dashed line), temperature (Blue line) and RH (Blue line) are depicted in the Figure 122.

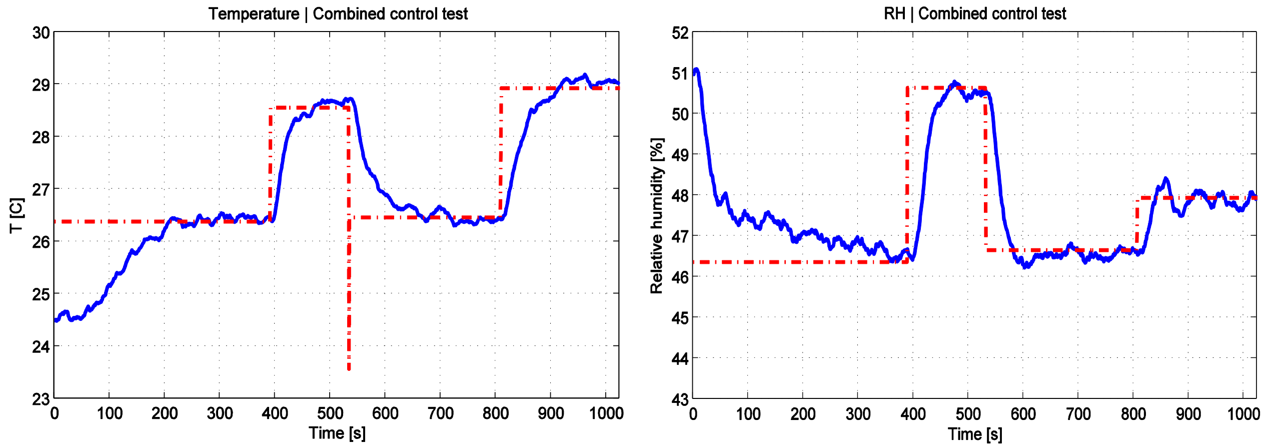

Figure 122 a) Temperature and setpoint values, b) RH and setpoint values.

From the figure, it is possible to see that the temperature settles to the setpoints with changing settling time, depending on the setpoint difference. The fluctuations around the setpoint values are very small. The same follows for the RH, though it can hardly reach the first setpoint, due to that the RH reaches the limit of the range, in which the RH is possible, for the inbox and ambient air specifications. The next setpoint is within this range and is achieved within 100s, which is significant smaller time than the 400s. In general, the RH settles fast, as long as the setpoints are given within the range of possible RH-values. This range varies a lot depends on both temperature and humidity inside and outside the control volume.

The Control effort of the dampers and heater are depicted in Figure 123.

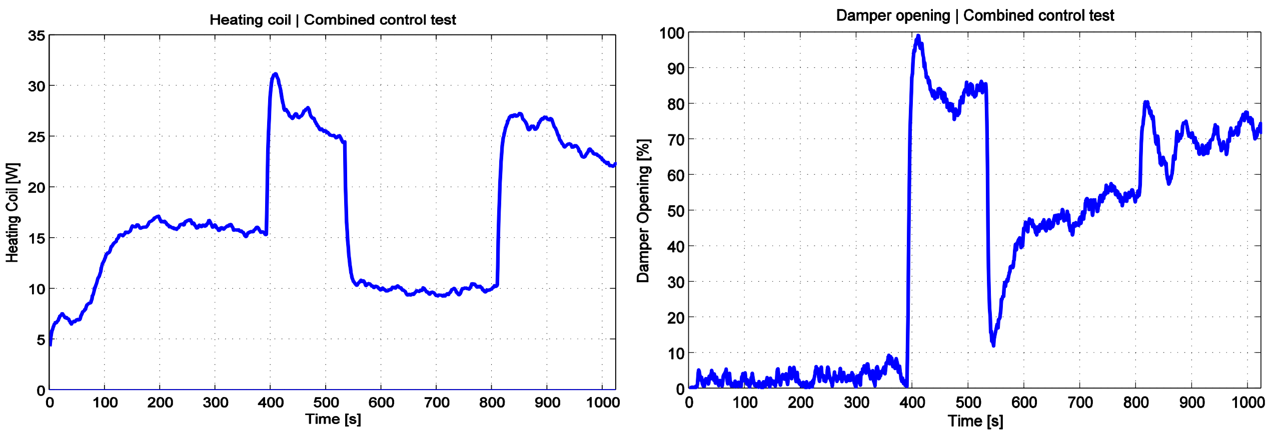

Figure 123 a) The control input from the heating coil, b) The control input from the dampers.

From the figure it is possible to see that at low temperature setpoints the heater is settling fast, as the damper is not affecting the heating much. Increasing the setpoint value to 28.5$°C$-29$°C$, the heaters control effort increases but is needed to be reduced afterwards, as the dampers are closing more than 50%, allowing less air in than what returns from the control volume. This air is already heated.

To see what happens when different disturbances are applied, both a case with the front side open and a case with the window open is applied, letting the cold air in to mix with the ambient air.

9.3.4.3 Disturbances – Door opening

The front side is opened 1/3 of the total area in both 30s, 1min and 2min, in order to simulate what happens when a door in a zone is opened. The temperature different between inside the control volume and the ambient is not large; it will only differentiate by some degrees when the setpoint is relative low (25$°C$ – 27$°C$).

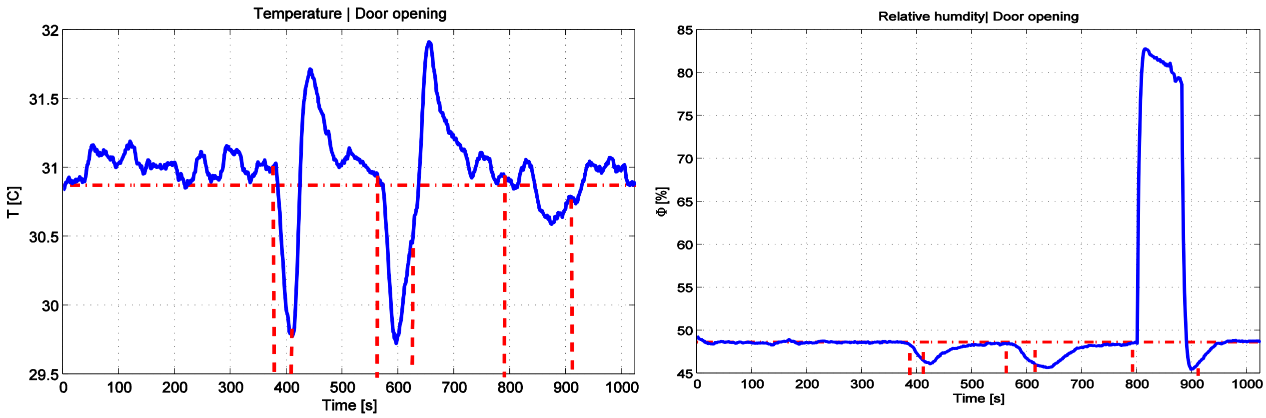

Figure 124 a) The Temperature and Opening intervals, b) RH and opening intervals.

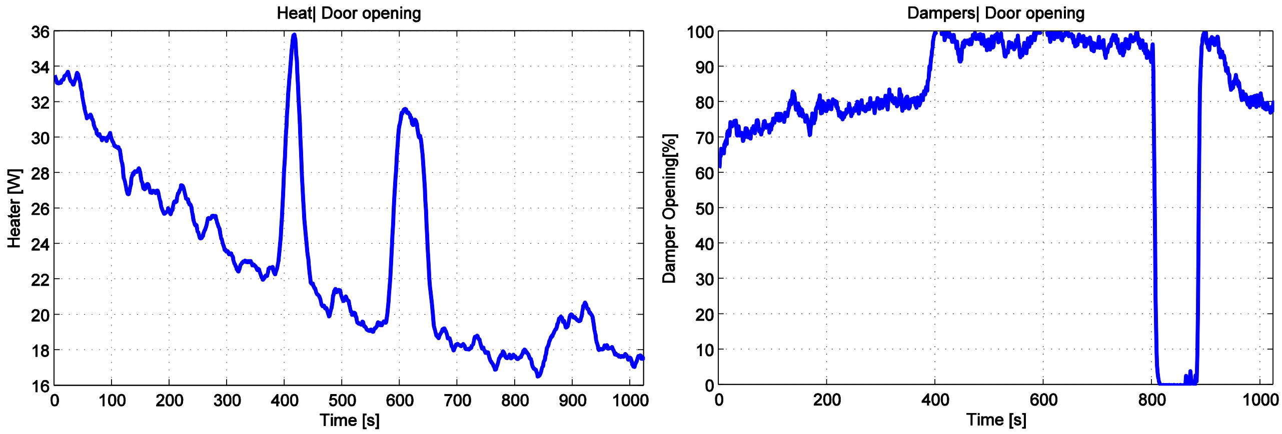

Figure 125 a) The Heaters control input, b) The dampers opening.

In the Temperature and RH graphs given in Figure 124 it is possible to see, both the setpoint - the actual value- and the interval in which the front plate is opened. The control of the heater and dampers corresponding to this are depicted in Figure 125. It is possible to see, that the temperature and RH decreases as the ambient air is let in. The temperature decreases with more than 1$°C$ and the RH with 2%. The heater tries to make the temperature reach the setpoint. This can be seen by looking at the peak at 400s. The same happens in the next interval, but in the third interval of the door opening, which occurs in 2min, the change in RH is the same whereas the temperature decreases only 0.5$°C$ causing only a small rise in the heaters control effort. The large change in RH in the third interval is a disturbance, which disappears after 100s. This is one of the uncertainties in the systems sensor.

9.3.4.4 Disturbance – Ambient air changes

In this test the window in the ambient room is opened, causing the temperature different to grow together with the relative humidity. After the system has settled, the window is opened, together with the front side of the control volume.

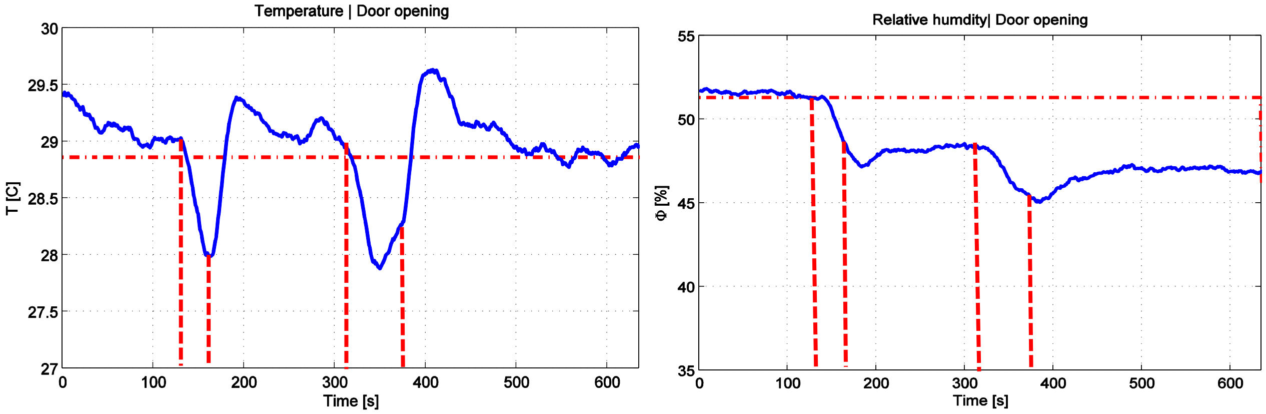

Figure 126 a) The Temperature, b) RH.

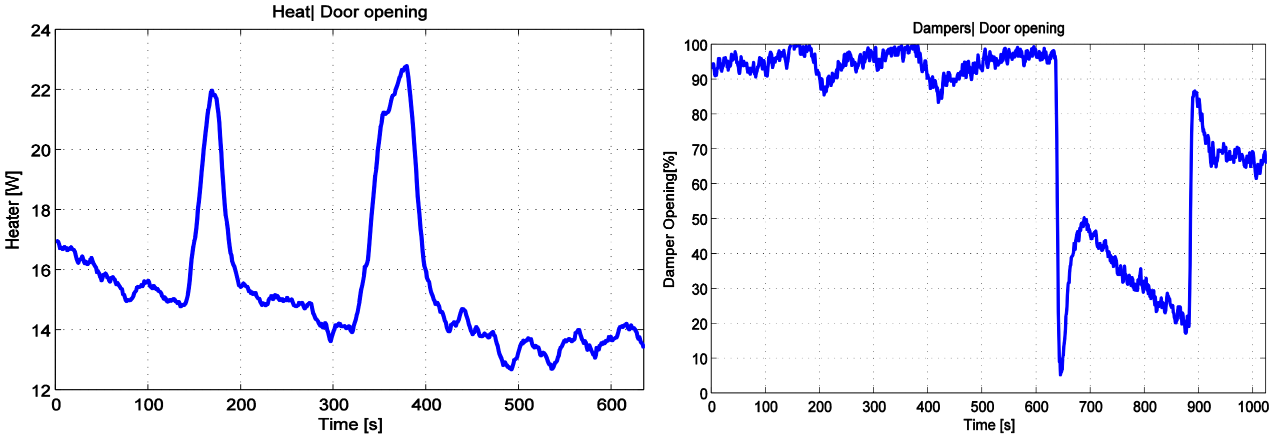

Figure 127 a) Heaters control effort, b) Dampers opening.

The first door opens at 130s here it is possible to see that both temperature and RH decreases. After the front side is closed again the temperature settles, but the RH does not. It is not possible for the RH to settle. The problem with the setup is that it is controlled by a positive voltage and not by both negative and positive voltage range. This means that for some changes, the damper will think it will remain at 0% (0v, fully open) where as it actually should close down and vice versa. Having a voltage range -5v to 5v would make sure to have the ability. One way to see this, is to switch the signs in the feedback loop before the controller, then the dampers work in an opposite way, and will have the same problem just inversed. This is not a problem for the heater. The only problem with the heater is when the air is needed to be cooled down, the only way to do so, is to close the heater completely and wait for the temperature decrease.

9.3.4.5 Different flow rates

Most of the test throughout the testing phase, the mass flow is either set to maximum value ($6\frac{m^3}{h}$) or $4\frac{m^3}{h}$ as this speed is not limited by any damper openings.

To see what happens to the temperature- and RH control at different flow rates, three different flow rates are applied. The applied flow rates are $3\frac{m^3}{h}$, $4\frac{m^3}{h}$ and $5\frac{m^3}{h}$. The flow rates below $3\frac{m^3}{h}$ are not applied when heating is needed, as the heater only turns fully on when the fan provides above $2.5\frac{\mathrm{m}^3}{\mathrm{h}}$ - $3\frac{\mathrm{m}^3}{\mathrm{h}}$.

The test results are depicted in Figure 128.

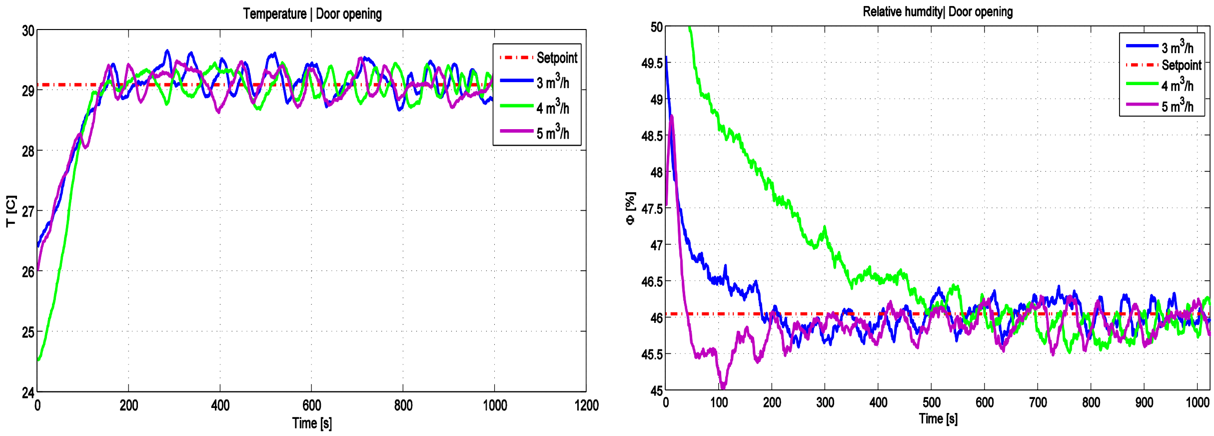

Figure 128 a) The temperature for three different flow rates, b) The RH for three different cases.

From the right graph in the above figure, it is possible to see that for all flow rates the RH settles down in approximately same time. The only one, which do take longer time, is for $4\frac{m^3}{h}$ but this is only due to a different start point, which is further away than the other two start points. If the start point was the same, the settling time can be assumed to approximately the same. The reason why not all three tests are started at exact same point is that the conditions of the room at exact that moment dictates the condition for the initial condition of the air inside the setup. One could try to open window or heat up the air, but again it is very difficult with the conditions provided.

As for the RH, it is possible to see that the temperature settles with at approximately same time, disregarding the flow rates. As the difference in start point makes no difference for the temperature, it means that the RH is more strongly depends on the flow rate.

The goal for the design and testing of these controllers was to compare them with the MPC controllers. Due to limited license this was not possible.

10. Conclusions and further investigation

The single zone model is extended, in terms of a new multi zone concept. The heating coil, cooling coil and humidifier are investigated and implemented in the overall multi zone HVAC model.

Starting with the modelling of the HVAC system and its further implementations, the specification of the fan selection is account in this model, in terms of selecting a realistic fan for this model to provide the necessary flow in the zones. Additionally, inlet dampers are selected for the purpose of this work to achieve a more accurate model of the pressure loss, meaning of adding realistic resistance throughout the duct.

A linear multi zone model is implemented in order to later formulate and apply a model predictive controller. Furthermore, a non-linear model is implemented and the result of linearizing the it had some costs in terms of the specific humidity, as when changes in inputs occurred the changes in the linear model was not as large as in the non-linear.

Several PI controllers were integrated in the non-linear model in order to control various actuators such as dampers, radiators etc. The performance of the PI-controllers was tested against multiple setpoint changes both inside the zones and in the mixing box. Furthermore, the underdamped responses for almost every output of the non linear model was achieved. The problem with the PI-controller is that it do not handle the constraints of the actuators in terms of both operating range and rate. To avoid this a more advanced controller is developed in order to improve the operation of the non-linear model, in terms of both settling time and hen disturbances occur, if possible.

The MPC is implemented both for a winter and a summer case in order investigate the performance at two different operating conditions. Moreover, it is designed based on the linear model and it is obvious when the controller is implemented to the non-linear model some deviation occurs when the non-linear model moves away from the linearizing point, especially the humidity. The deviations that occur in the specific humidity are due to the linear approach is not as accurate for the specific humidity as for the temperature and pressure. Additionally, when moving away from the linearization point the MPC becomes less accurate. Therefore as mentioned an adaptive MPC could had been developed to account for the changes in variable in the non-linear model, but this was too computational heavy and therefore not implemented into the model.

To see further which control strategy is most robust to a disturbance, a simulation of a door opening to the outside is applied. It is obvious for the temperature and specific humidity changes the MPC has the most robust reaction. On the other hand, the pressurization in the zones is maintained above ambient pressure for both controllers.

Finally, the PI control strategy was tested in a small ventilation setup, where an interface was constructed. In addition, several tests for the open loop performance of the set up were conducted in order to validate the new developed model for the humidity. Due to some unfortunate problems with the tests, this was not accomplished.

Keeping in mind that the HVAC systems are vital and crucial set of systems for the industrial section, there will always be a need for improvements in both software and its hardware. In our pointer view further investigation in the HVAC systems could be a more accurate extended model in terms of simulating pressure distribution in the entire ducting system. Another issue to be solved in a further study case could be the affect of the wind direction and speed on the pressure relief dampers in the zones.

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Appendix 1. Fan Specifications

systemair

KBR 355D2 IE2 THERMO FAN

Item no. 33560

Document type: **Product card**
Document date: **2015-04-08**
Generated by: **Systemair Online Catalogue**

Description

- High efficient motor IE2
- Speed-controllable via frequency converter
- Integral cold conductor (PTC)
- Standard motor outside the air stream
- Max temperature of transported air up to 120°C
- Low sound level
- Easy to maintain and reliable

The KBR fans size 355D2 are equipped with high efficient IE2 motors. The KBR fans have impellers manufactured from aluminum with backward-curved blades. The KBR casing is manufactured from double skinned galvanized sheet steel and is insulated with 50 mm mineral wool.

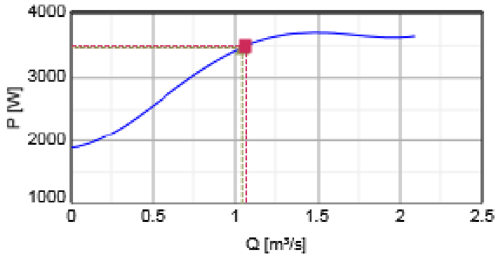
The KBR fans have a swing-out door for easy inspection and service. The direction of the door opening can easily be changed from left to right at site. The fan is isolated from the casing via connectors and anti-vibration dampers are incorporated into the base frame.

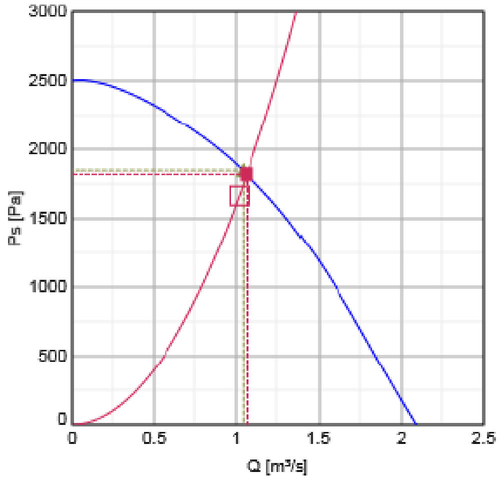
Motor protection is done by cold conductors (PTC), which have to be connected to an external motor protection device.
Please note: Speed controll by voltage, i.e. voltage transformers, is not possible!

In accordance with Commission Regulation (EC) no 640/2009 of the European Parliament - eco-design requirements for electric motors - the new international efficiency classes are binding as of 16 June 2011. These guidelines defined by CEMEP and EPACT are regarded as international standard for energy-saving high-efficiency motors for frequencies of 50 or 60 Hz and make the use of IE2 motors mandatory.
With this new and more efficient technology we offer our customers many advantages such as environmentally friendly operation, reduced energy consumption and hence lower emissions. IE2 motors have a higher efficiency even in part load operation and allow optimum adjustment to the operating point. In addition, the IE2 motors generate less noise and develop less heat, which has a positive influence on the efficiency and the cooling requirement of the motor. Please note: IE2 motors cannot be speed controlled by voltage, i.e. voltage transformers.

Technical parameters

Voltage	400	V
Motor circuit connection	D	
Frequency	50	Hz
Phase	3	~
Input power (P1)	3670	W
Current	6.16	A
Starting current	46.8	A
Max. airflow	2.09	m³/s
Fan impeller speed	2887	r.p.m.
Max. temperature of transported air	120	°C
Sound pressure level at 4 m (free field)	53	dB(A)
Sound pressure level at 10 m (free field)	45	dB(A)
Weight	78	kg
Insulation class, motor	F	
Enclosure class, motor	54	IP





Hydraulic data

	Required point		Working point						
	Q [m³/s]	Ps [Pa]	Q [m³/s]	Ps [Pa]	P [W]	n [r.p.m.]	I [A]	SFP [kW/m³/s]	U [V]
Max efficiency			▲ 1.04	▲ 1848	▲ 3468	2892	5.6	3.32	400
Selection	□ 1.02	□ 1666	■ 1.07	■ 1821	■ 3494	2892	5.64	3.27	400

Acoustics

		Mid-frequency band, Hz							
355D2		**Tot**	**125**	**250**	**500**	**1k**	**2k**	**4k**	**8k**
LwA Inlet	dB(A)	94	92	91	86	84	80	74	71
LwA Outlet	dB(A)	96	94	93	88	86	82	76	73
LwA Surrounding	dB(A)	76	74	73	68	66	62	56	53

Measuring point: qv = 1,04 m3/s, Ps = 1848Pa

Appendix 2. Damper Specifications

Halton UTP dampers are used to balance airflow rates in high pressure ductwork. Dampers meet international standards for rectangular and round ducts. In the open position, the blades face the direction of flow and do not cause a significant pressure loss. The UTP is used as a balancing damper in applications where reliability is important.

- For balancing of air intake and exhaust
- Available as ATEX approved. Shock tested.
- Leakage class of a closed damper according to EN 1751 class 1. Tested size 1000x1000 mm.
- Outer frame of galvanized, painted or stainless steel. Blades of galvanized or stainless steel with double sheet construction. Maintenance-free stainless steel bearings and shafts.
- Electrical, pneumatic or manual operation system available
- UTP dampers can be supplied with connection pieces for round duct
- Maximum duct pressure for damper construction 5000 Pa and maximum air velocity 15 m/s. In case of high duct pressure, contact Halton Marine for finding the most suitable solution.

PART	MATERIAL	FINISHING
Frame	Carbon steel	Painted or galvanised
Frame	Stainless steel EN 1.4301 (AISI304), EN 1.4404 (AISI316L), EN 1.4432 (AISI316L)	-
Blades	Steel	Galvanized
Blades	Stainless steel EN 1.4301 (AISI304), EN 1.4404 (AISI316L), EN 1.4432 (AISI316L)	-
Maintenance-free bearings	Stainless steel EN 1.4404 (AISI316L) / Option: bronze bearings available	-
Shafts	Stainless steel EN 1.4404 (AISI316L)	-

UTP - Balancing Damper

Halton MARINE

UTP DIMENSIONS AND MATERIAL THICKNESS

UTP balancing dampers are manufactured to international standards for both rectangular (width B 100-1200 mm and height H 100-1600 mm, 1 mm division) and circular ducts (Ø100-1250 mm). Non-standard dimensions available on request.

Standard flange width 27 mm. Flanges and drilling also available according to ISO 15138 standards.

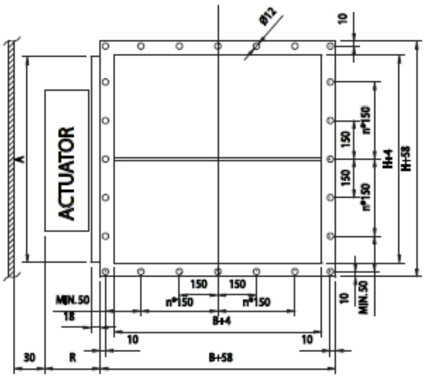
Modular construction sizes available up to 2400x3200 mm. Frame thicknesses from 3 mm to 10 mm. Standard frame thickness is 3 mm.

Actuator effect on dimensions

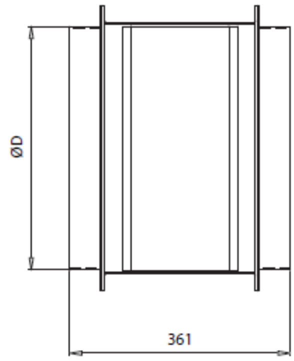
ACTUATOR		DIMENSIONS	
		R	A
Manual	Handle	95	H
Electrical	GGA 326.1E	100	H≤300 = 300 H>300 = H
Pneumatic PNL	Linear actuator 245/300 N	170	H≤500 = 500 H>500 = H
Pneumatic PNR	Pneumatic rotating actuator AT100	170	H≤300 = 300 H>300 = H

The above table contains only some examples of actuators and their effect on dimensions.

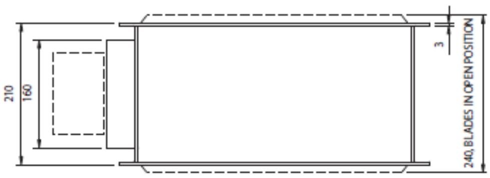
GENERAL UTP DRAWING



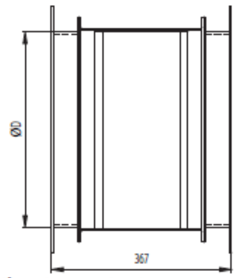
UTP CIRCULAR CONNECTIONS



GENERAL UTP DRAWING, TOP



UTP CIRCULAR, WITH CONNECTION FLANGES



WEIGHTS OF STANDARD HALTON MARINE UTP DAMPERS (KG) without an actuator

WEIGHTS OF STANDARD HALTON MARINE UTP DAMPERS (KG) without an actuator

H / Height	B / Width (mm)											
mm	100	200	300	400	500	600	700	800	900	1000	1100	1200
100	4	6	7	9	10	12	13	15	16	17	19	20
200	6	8	9	11	13	14	16	17	19	21	22	24
300	8	10	12	14	15	17	19	21	22	24	26	28
400	10	12	14	16	18	20	22	23	25	27	29	31
500	13	15	17	19	21	23	25	28	30	32	34	36
600	15	17	19	21	24	26	28	30	33	35	37	39
700	17	20	22	25	27	29	32	34	37	39	42	44
800	19	22	24	27	29	32	35	37	40	43	45	48
900	21	24	27	30	33	36	38	41	44	47	50	53
1000	23	26	29	32	35	38	41	44	47	50	53	56
1100	26	29	32	35	38	42	45	48	51	55	58	61
1200	27	31	34	37	41	44	48	51	54	58	61	64
1300	30	33	37	41	44	48	51	55	58	62	66	69
1400	32	35	39	43	47	50	54	58	61	65	69	73
1500	34	38	42	46	50	54	58	62	66	70	74	77
1600	36	40	44	48	52	56	60	65	69	73	77	81

D2 øD	Weight
mm	kg
100	7
125	8
160	11
200	12
250	17
315	19
400	26
500	34
630	44
800	59
1000	80
1250	110

Examples of actuator weights: **UTP-EL** GGA 326.1E 2,3 kg, GNA 326.1E 1,3 kg, BF230 +3,2 kg, BLF230 +1,7 kg, ExMax/Redmax +3,5 kg, CSQP +3 kg, **UTP-PNR** AT100 +2,1 kg, AT100 as AISI316 4,4 kg, AT50 1,2 kg, **UTP-PNL** Roder +4 kg, **UTP-MAN** +1 kg. **Control enclosure** +4 kg.

Appendix 3. Parameter Estimation

```
%% 2015 Scriptf : PARAMETER ESTIMATION - Multizone
R=286.9;
C_air=1.02;
T_in=292.15;

Cc=0.61;
T_z1=294.15;
T_z2=295.15;
T_z3=292.15;
T_atm=281.55;
T_out=290.15;

P_z1=100025;
P_z2=100050;
P_z3=100025;
P_atm=1e5;

\rho_out=P_atm/(R·T_out);

q_inz1=0.455;
q_inz2=0.3981;
q_inz3=0.1517;
q_inztot=q_inz1+q_inz2+q_inz3;

L_1=15;
L_2=26.25;
L_3=L_2+7.25;
D=0.6;
A_1in=(pi·D^2)/4;
f=0.0164;

A_0inz1=0.0149;
A_0inz2=0.0134;
A_0inz3= 0.0051;

K_A0in1=2600.1·A_0inz1^2-841.9·A_0inz1+67.5;
K_A0in2=2600.1·A_0inz2^2-841.9·A_0inz2+67.5;
K_A0in3=2600.1·A_0inz3^2-841.9·A_0inz3+67.5;
Dp_l1=((f·L_1/D)·(q_inztot/A_1in)^2/2)+((1.6+K_A0in1)·(q_inztot/A_1in)^2)/2;
Dp_l2=((f·L_2/D)·((q_inztot-
q_inz1)/A_1in)^2/2)+(((1.6+K_A0in1+1)+K_A0in2)·((q_inztot-q_inz1)/A_1in)^2)/2;
Dp_l3=((f·L_3/D)·((q_inztot-q_inz1-
q_inz2)/A_1in)^2/2)+(((1.6+K_A0in1+1)+K_A0in2+K_A0in3)·((q_inztot-q_inz1-
q_inz2)/A_1in)^2)/2;
P_in=1660+1e5;

P_in1=P_in-Dp_l1;
P_in2=P_in-Dp_l1-Dp_l2;
P_in3=P_in-Dp_l1-Dp_l2-Dp_l3;

\rho_inz1=P_in1/(R·T_in);
\rho_inz2=P_in2/(R·T_in);
\rho_inz3=P_in3/(R·T_in);
```

```
\rho_z1=P_z1/(R·T_z1);
\rho_z2=P_z2/(R·T_z2);
\rho_z3=P_z3/(R·T_z3);

V_z1=273;
V_z2=238.38;
V_z3=91;

syms A_0inz1 A_0inz2 A_0inz3 A_0utz1 A_0utz2 A_0utz3
A_0inz1=double(solve(q_inz1 == A_0inz1·Cc·(sqrt(2/\rho_inz1·(P_in1-
P_z1)))/(sqrt(1-(\rho_z1·Cc·A_0inz1/(\rho_inz1·A_1in)))),A_0inz1));
A_0inz2=double(solve(q_inz2 == A_0inz2·Cc·(sqrt(2/\rho_inz2·(P_in2-
P_z2)))/(sqrt(1-(\rho_z2·Cc·A_0inz2/(\rho_inz2·A_1in)))),A_0inz2));
A_0inz3=double(solve(q_inz3 == A_0inz3·Cc·(sqrt(2/\rho_inz3·(P_in3-
P_z3)))/(sqrt(1-(\rho_z3·Cc·A_0inz3/(\rho_inz3·A_1in)))),A_0inz3));

m_dotInz1=\rho_inz1·q_inz1;
m_dotInz2=\rho_inz2·q_inz2;
m_dotInz3=\rho_inz3·q_inz3;

m_dotOutz1=m_dotInz1;
m_dotOutz2=m_dotInz2;
m_dotOutz3=m_dotInz3;

m_z1=\rho_z1·V_z1;
m_z2=\rho_z2·V_z2;
m_z3=\rho_z3·V_z3;

\rho_atm=P_atm/(R·T_atm);

A_0utz1=double(solve(m_dotOutz1==m_z1/V_z1·A_0utz1·Cc·(sqrt(2·(R·T_z1-
P_atm/(m_z1/V_z1))))/(sqrt(1-
(\rho_out·Cc·A_0utz1/(m_z1/V_z1·A_1in))^2)),A_0utz1));
A_0utz2=double(solve(m_dotOutz2==m_z2/V_z2·A_0utz2·Cc·(sqrt(2·(R·T_z2-
P_atm/(m_z2/V_z2))))/(sqrt(1-
(\rho_out·Cc·A_0utz2/(m_z2/V_z2·A_1in))^2)),A_0utz2));
A_0utz3=double(solve(m_dotOutz3==m_z3/V_z3·A_0utz3·Cc·(sqrt(2·(R·T_z3-
P_atm/(m_z3/V_z3))))/(sqrt(1-
(\rho_out·Cc·A_0utz3/(m_z3/V_z3·A_1in))^2)),A_0utz3));

q_0utz1=m_dotOutz1/(m_z1/V_z1);
q_0utz2=m_dotOutz2/(m_z2/V_z2);
q_0utz3=m_dotOutz3/(m_z3/V_z3);

Cp=1.06;

Cd_Inz1=Cc/sqrt(1-((\rho_z1·Cc·A_0inz1)/\rho_inz1));
Cd_Inz2=Cc/sqrt(1-((\rho_z2·Cc·A_0inz2)/\rho_inz2));
Cd_Inz3=Cc/sqrt(1-((\rho_z3·Cc·A_0inz3)/\rho_inz3));

Cd_Outz1=Cc/sqrt(1-((\rho_atm·Cc·A_0utz1)/\rho_z1));
Cd_Outz2=Cc/sqrt(1-((\rho_atm·Cc·A_0utz2)/\rho_z2));
Cd_Outz3=Cc/sqrt(1-((\rho_atm·Cc·A_0utz3)/\rho_z3));

Q_dotInz1=Cp·\rho_inz1·Cd_Inz1·A_0inz1·sqrt(2·((P_in-P_z1)/\rho_inz1))·(T_in-
T_z1);
```

```
Q_dotInz2=Cp·\rho_inz2·Cd_Inz2·A_0inz2·sqrt(2·((P_in-P_z2)/\rho_inz2))·(T_in-
T_z2);
Q_dotInz3=Cp·\rho_inz3·Cd_Inz3·A_0inz3·sqrt(2·((P_in-P_z3)/\rho_inz3))·(T_in-
T_z3);

Q_dotOutz1=Cp·\rho_z1·Cd_Outz1·A_0utz1·sqrt(2·((P_z1-P_atm)/\rho_z1))·(T_z1-
T_out);
Q_dotOutz2=Cp·\rho_z2·Cd_Outz2·A_0utz2·sqrt(2·((P_z2-P_atm)/\rho_z2))·(T_z2-
T_out);
Q_dotOutz3=Cp·\rho_z3·Cd_Outz3·A_0utz3·sqrt(2·((P_z3-P_atm)/\rho_z3))·(T_z3-
T_out);

U=0.5;
\rho_Steel=7.8;
V_wall=0.4·3.5·6.5;
m_wall=V_wall·\rho_Steel;
C_pSteel=0.452;

A_wallz1=(12.5·3.5·2+6.5·3.5+12.5·6.5);
A_wallz2=(10.5·3.5·2+10.5·6.5);
A_wallz3=(4·3.5·2+6.5·3.5+4·6.5);
A_12=6.5·3.5;

Q_wallz1=U·A_wallz1·((T_atm+T_z1)/2-T_z1);
Q_wallz2=U·A_wallz2·((T_atm+T_z2)/2-T_z2);
Q_wallz3=U·A_wallz3·((T_atm+T_z3)/2-T_z3);
Q_w12=U·A_12·(T_z2-(T_z2+T_z1)/2);
Q_w23=U·A_12·(T_z2-(T_z2+T_z3)/2);

H_rh=1;
g=9.81;
T_rad=295.15;
c=0.59;
n=1/4;
v_k=1.5647e-5;
a=2.2142e-5;
T_f=(T_rad+T_z1)/2;
beta_ra=1/T_f;
K_air=0.0261;
Ra=(g·beta_ra·(H_rh^3)·(T_rad-T_z1))/(v_k·a);

Nu=c·Ra^n;
h_dash=Nu·K_air/H_rh;
syms A_radz1 A_radz2 A_radz3
Q_radz1=abs(Q_wallz1);
Q_radz2=abs(Q_wallz2);
Q_radz3=abs(Q_wallz3);

A_radz1=double(solve(Q_radz1==h_dash·A_radz1·(T_rad-T_z1),A_radz1));
A_radz2=double(solve(Q_radz2==h_dash·A_radz2·(T_rad-T_z2),A_radz2));
A_radz3=double(solve(Q_radz3==h_dash·A_radz3·(T_rad-T_z3),A_radz3));

syms T_rad1 T_rad2 T_rad3
T_rad1=solve(-(Q_dotInz1-Q_dotOutz1+(Q_radz1-
(Q_w12+2))+Q_wallz1+Q_w12)==h_dash·10·(T_rad1-T_z1),T_rad1);
T_rad2=solve(-(Q_dotInz2-Q_dotOutz2+(Q_radz2-
(Q_w12+Q_w23+5))+Q_wallz2+Q_w12+Q_w23)==h_dash·10·(T_rad2-T_z2),T_rad2);
```

```
T_rad3=solve(-(Q_dotInz3-Q_dotOutz3+(Q_radz3-
(Q_w23+2))+Q_wallz3+Q_w23)==h_dash·10·(T_rad3-T_z3),T_rad3);

beta=0.5;
m_dotr=beta·(m_dotOutz1+m_dotOutz2+m_dotOutz3);
Tmix=15+273.15;

syms m_dotos
m_dotos=double(solve(Tmix==(m_dotr·T_out+m_dotos·T_atm)/(m_dotr+m_dotos),m_dot
os));

m_dotmix=m_dotr+m_dotos;
Tcc=Tmix;
m_dotw=8.02e-5·998;
Tw_out=283.15;
Cpw=4.18;
syms Tw_in
Tw_in=double(solve(T_in==Tcc+m_dotw·Cpw·(Tw_in-Tw_out),Tw_in));

%% Humidity

% Zone specifications:
Phi_z1=0.476;
Phi_z2=0.45;
Phi_z3=0.538;

P_wsz1=610.78·exp((((T_z1-273.15)/((T_z1-273.15)+238.3)·17.2694)));
P_wsz2=610.78·exp((((T_z2-273.15)/((T_z2-273.15)+238.3)·17.2694)));
P_wsz3=610.78·exp((((T_z3-273.15)/((T_z3-273.15)+238.3)·17.2694)));

w_z1=0.622.·Phi_z1.·P_wsz1./(P_z1-Phi_z1.·P_wsz1);
w_z2=0.622.·Phi_z1.·P_wsz1./(P_z1-Phi_z1.·P_wsz1);
w_z3=0.622.·Phi_z1.·P_wsz1./(P_z1-Phi_z1.·P_wsz1);

% Outlet/Return Parameters:
Phi_out=0.40;
P_wsOUT=610.78·exp((((T_out-273.15)/((T_out-273.15)+238.3)·17.2694)));
w_out=0.622.·Phi_out.·P_wsOUT./(P_atm-Phi_out.·P_wsOUT);

% Mixing Box:
w_mix_INITIAL=0.004;
Phi_os=0.84;
P_wsOS=610.78·exp((((T_atm-273.15)/((T_atm-273.15)+238.3)·17.2694)));
w_os=0.622.·Phi_os.·P_wsOS./(P_atm-Phi_os.·P_wsOS);
w_r=w_out;
T_r=T_out;
P_wsMIX=610.78·exp((((Tmix-273.15)/((Tmix-273.15)+238.3)·17.2694)));
w_mix=(m_dotr·w_r+m_dotos·w_os)/(m_dotr+m_dotos);
Phi_mix=w_mix·P_atm/((0.622+w_mix)·P_wsMIX);

% In this case no dehumidification and cooling
Phi_cc=Phi_mix;
P_wscc=610.78·exp((((Tcc-273.15)/((Tcc-273.15)+238.3)·17.2694)));
w_cc=0.622.·Phi_cc.·P_wscc./(P_atm-Phi_cc.·P_wscc);
m_wout=(w_mix-w_cc)·m_dotmix;

Tw_incc=Tmix;
```

```
Tw_outcc=Tw_incc;

w_in=2·w_z1-w_out;                                  % Inlet humidity, might need to be
higher
                                                    % temperature dependent.
P_wsh=610.78·exp((((T_in-273.15)/((T_in-273.15)+238.3)·17.2694)));
m_win=(w_in-w_cc)·m_dotmix;                         % What we want to control
Phi_h=w_in·P_atm/((0.622+w_in)·P_wsh);



% masses
\rho_mix=P_atm/(R·(Tmix));
\rho_cc=P_atm/(R·(Tcc));
\rho_h=P_atm/(R·(T_in));

V_h=1.5;
V_cc=1.5;
V_mix=1.5;

m_h=\rho_h·V_h;
m_cc=\rho_h·V_cc;
m_mix=\rho_mix·V_mix;
```

Appendix 4. System curve

```
% clear all; close all; clc;
%% New System curve

% Zone data:
V1=273;
V2=238.38;
V3=91;

q1=0.455;
q2=0.3981;
q3=0.1517;
qtot=q1+q2+q3;
D=0.6;
epsilon=0.015e-3;
r=D/2;
A=pi·r^2;
v=qtot/A;
mu=1.8e-5;
\rho_in=1.2;
Re=\rho_in·v·D/mu;

K_L=177.2;
syms f
f=double(solve(1/sqrt(f)==-2·log10(((epsilon/D)/3.7)+(2.51)/(Re·sqrt(f))),f));
L_tot=37.5;
g=9.82;

H_1=0;
H_2=3.6;
P_z=100050;
P_atm=1e5;

syms  q
P_dyn=((f·L_tot/D)·((\rho_in·(q./A).^2)/2)+K_L·(q./A).^2/(2·g))·\rho_in·g;
P_static=(P_z-P_atm)+(H_2-H_1)·\rho_in·g;

P_L=P_static+P_dyn;

%% Plot
q=linspace(0,2);
% A=0.0707·0.5;
plot(q,subs((P_L(:))))
hold on
legend('60% Open','70% Open','80% Open','90% Open','100% Open')
grid on
ylabel('P_l [Pa]')
xlabel('q_{tot} [m^3/s]')
title('System curve')
```

Appendix 5. Fan curve

```
% clear all; clc; close all;
%% Least Squares for 50 Hz.
f1=50;

Pt1=[2413 2200 1838 1537 988]';
qv1=[0.34 0.68 1.04 1.27 1.60]';

Y=Pt1;
Psi=[qv1.^2 qv1 [1;1;1;1;1]];

Theta1=inv(Psi'·Psi)·Psi'·Y;

%% Least Squares for 55 Hz.
% Affinity law, same Diameter, differen frequency.
f2=55;
qv2=qv1·f2/f1;
Pt2=Pt1·(f2/f1)^2;

Y2=Pt2;
Psi2=[qv2.^2 qv2 [1;1;1;1;1]];

Theta2=inv(Psi2'·Psi2)·Psi2'·Y2;
%%  Least Squares for 60 Hz.
% Affinity law, same Diameter, differen frequency.
f3=60;
qv3=qv2·f3/f2;
Pt3=Pt2·(f3/f2)^2;

Y3=Pt3;
Psi3=[qv3.^2 qv3 [1;1;1;1;1]];

Theta3=inv(Psi3'·Psi3)·Psi3'·Y3;
%%  Least Squares for 45 Hz.
% Affinity law, same Diameter, differen frequency.
f4=45;
qv4=qv1·f4/f1;
Pt4=Pt1·(f4/f1)^2;

Y4=Pt4;
Psi4=[qv4.^2 qv4 [1;1;1;1;1]];

Theta4=inv(Psi4'·Psi4)·Psi4'·Y4;
%% System curve
run('PArameterEStimation.m')
\rho_in=1.244;
g=9.81;
P_hyd=P_z-P_atm+2·\rho_in·g;
% Make looop
Pt=750;
```

```
%% Plots

% For different Damper openings:
% A_0in=[0.00947 0.0165 0.02525 0.0353 0.05239 0.07069]';
run('Multi_Zone_systemcurve.m')
% figure(2)
% subplot(1,2,1)
qv=linspace(0,2);
Pt1=Theta1(1)·qv.^2+Theta1(2)·qv+Theta1(3);          % Plot different fan
curves
Pt2=Theta2(1)·qv.^2+Theta2(2)·qv+Theta2(3);
Pt3=Theta3(1)·qv.^2+Theta3(2)·qv+Theta3(3);
Pt4=Theta4(1)·qv.^2+Theta4(2)·qv+Theta4(3);
PtSys=P_hyd+K_L·qv.^2·\rho_in·g;                          % Plots the system
curve

hold on
plot(qv,Pt4,'m',qv,Pt1,'k',qv,Pt2,'r',qv,Pt3,'g')%,q,subs((P_L(:))))%,qv,doubl
e(subs(PtSys(1,:))),'k')
legend('60% Open','70% Open','80% Open','90% Open','100% Open')
grid on
title('System & Fan curves')
axis([0 2 0 4000])
ylabel('P_t [Pa]')
xlabel('q_v [m^3/s]')
legend('System curve','@45Hz','@50Hz','@55Hz','@60Hz')
hold on
plot(0.9495,1455,'mo',1.071,1825,'bo',1.172,2167,'ro',1.293,2619,'go')
```

Appendix 6. State space – Linearizer

```
%% symbols
% clear all
syms P_in \rho_inz1 \rho_inz2 \rho_inz3 A_0inz1 A_0inz2 A_0inz3 P_in1 P_in2
P_in3
    P_z1 P_z2 P_z3...
    m_z1 T_z1 A_0utz1 V_z1 m_z2 T_z2 A_0utz2 R V_z2 m_z3 T_z3 A_0utz3 V_z3
    P_atm Q_radz1 Q_radz2 Q_radz3...
    Cc A_1in h A_rad T_rad T_atm Q_wallz1 Q_wallz2 Q_wallz3 C_air T_in Cp visc
    beta mu k g D a Cd_Outz1 Cd_Outz2 Cd_Outz3 A_12 ...
    k_wall A_wall L_wall m_wout beta Cp T_out Tmix m_dotos Cp U A T_os Tmix
    m_mix m_cc m_h C_pSteel m_wall...
    T_outcc T_incc Cpw m_win Tcc m_dotmix T_hc T_inhc T_outhc T_w12 T_w23 A_r
    h_dash1 h_dash2 h_dash3 T_rad1 T_rad2 T_rad3...
    w_out w_mix m_dotos w_os w_mix w_cc m_dotwcc m_dotOUTz1 m_dotOUTz2
    m_dotOUTz3 w_in m_h m_win w_z1 w_z2 w_z3 m_dotout m_z...
    T_f3 T_rad beta_ra3 Ra_3 H_rh v_k a c n h_dash3 Nu_3 K_air...
    T_f2 beta_ra2 Ra_2 Nu_2 h_dash2 Nu_2 T_z1 T_z2 T_z3 ...
    T_f1 beta_ra1 Ra_1 Nu_1 h_dash1 \rho_z1 \rho_z2 \rho_z3 \rho_out
\rho_atm...
    Phi_z1 P_wsz1 P_z1 Phi_z2 Phi_z2 P_wsz2 P_z2 Phi_z3  Phi_z3 P_wsz3 P_z3
    Phi_z3 w_z1 w_z2 w_z3 m_dotwMAX OpeningCC OpeningHC dT_hc dT_cc;

%% Ideal gas law
P_z1=(m_z1/V_z1)·R·T_z1;          % Zone 1.
P_z2=(m_z2/V_z2)·R·T_z2;          % Zone 2.
P_z3=(m_z3/V_z3)·R·T_z3;          % Zone 3.

%% Mass balance

% Zone 1.
Cd_Outz1=Cc/sqrt(1-((\rho_atm·Cc·A_0utz1)/((m_z1/V_z1)·A_1in))^2);
Cd_Outz2=Cc/sqrt(1-((\rho_atm·Cc·A_0utz2)/((m_z2/V_z2)·A_1in))^2);
Cd_Outz3=Cc/sqrt(1-((\rho_atm·Cc·A_0utz3)/((m_z3/V_z3)·A_1in))^2);

C_dz1=Cc/sqrt(1-(((m_z1/V_z1)·Cc·A_0inz1)/(\rho_inz1·A_1in))^2);
m_dotOUTz1=m_z1/V_z1·A_0utz1·Cc·(sqrt(2·(R·T_z1-P_atm/(m_z1/V_z1))))/(sqrt(1-
(\rho_out·Cc·A_0utz1/(m_z1/V_z1·A_1in))^2));
m_dotINz1=\rho_inz1·A_0inz1·Cc·(sqrt(2/\rho_inz1·(P_in1-P_z1)))/(sqrt(1-
(\rho_z1·Cc·A_0inz1/(\rho_inz1·A_1in))^2));
m_dotz1= m_dotINz1-m_dotOUTz1;

% Zone 1.
C_dz2=Cc/sqrt(1-(((m_z2/V_z2)·Cc·A_0inz2)/(\rho_inz2·A_1in))^2);
m_dotOUTz2=m_z2/V_z2·A_0utz2·Cc·(sqrt(2·(R·T_z2-P_atm/(m_z2/V_z2))))/(sqrt(1-
(\rho_out·Cc·A_0utz2/(m_z2/V_z2·A_1in))^2));
m_dotINz2=\rho_inz2· A_0inz2·Cc·(sqrt(2/\rho_inz2·(P_in2-P_z2)))/(sqrt(1-
(\rho_z2·Cc·A_0inz2/(\rho_inz2·A_1in))^2));
m_dotz2= m_dotINz2-m_dotOUTz2;

% Zone 1.
C_dz3=Cc/sqrt(1-(((m_z3/V_z3)·Cc·A_0inz3)/(\rho_inz3·A_1in))^2);
m_dotOUTz3=m_z3/V_z3·A_0utz3·Cc·(sqrt(2·(R·T_z3-P_atm/(m_z3/V_z3))))/(sqrt(1-
(\rho_out·Cc·A_0utz3/(m_z3/V_z3·A_1in))^2));
```

```
m_dotINz3=\rho_inz3·A_0inz3·Cc·(sqrt(2/\rho_inz3·(P_in3-P_z3)))/(sqrt(1-
(\rho_z3·Cc·A_0inz3/(\rho_inz3·A_1in)^2)));
m_dotz3= m_dotINz3-m_dotOUTz3;
%% Zone wall losses
Q_wall12=A_12·U·(T_w12-T_z1);
Q_wall21=A_12·U·(T_w12-T_z2);
Q_wall23=A_12·U·(T_w23-T_z2);
Q_wall32=A_12·U·(T_w23-T_z3);

%%
Q_radv1=10·h_dash1·(T_rad1+0.5-T_z1);
Q_radv2=10·h_dash2·(T_rad2-T_z2);
Q_radv3=10·h_dash3·(T_rad3+0.3-T_z3);

%% Temperature
T_dotz1=((C_air·\rho_inz1·C_dz1·A_0inz1·sqrt(2·(P_in1-P_z1)/\rho_inz1)·(T_in-
T_z1))-(C_air·(m_z1/V_z1)·A_0utz1·Cd_Outz1·sqrt(2·(R·T_z1-
P_atm/(m_z1/V_z1)))·(T_z1-T_out))+Q_wall12+Q_radv1)/(C_air·m_z1)+((Q_radz1-
5)+Q_wallz1)/(Cp·m_z1);
T_dotz2=((C_air·\rho_inz2·C_dz2·A_0inz2·sqrt(2·(P_in2-P_z2)/\rho_inz2)·(T_in-
T_z2))-(C_air·(m_z2/V_z2)·A_0utz2·Cd_Outz2·sqrt(2·(R·T_z2-
P_atm/(m_z2/V_z2)))·(T_z2-
T_out))+Q_wall21+Q_wall23+Q_radv2)/(C_air·m_z2)+((Q_radz2-
22.77)+Q_wallz2)/(Cp·m_z2);
T_dotz3=((C_air·\rho_inz3·C_dz3·A_0inz3·sqrt(2·(P_in3-P_z3)/\rho_inz3)·(T_in-
T_z3))-(C_air·(m_z3/V_z3)·A_0utz3·Cd_Outz3·sqrt(2·(R·T_z3-
P_atm/(m_z3/V_z3)))·(T_z3-T_out))+Q_wall32+Q_radv3)/(C_air·m_z3)+(((Q_radz3-
27))+Q_wallz3)/(Cp·m_z3);

%% Extended Energy model
m_dotOUTtot=m_dotOUTz1+m_dotOUTz2+m_dotOUTz3;

T_dotMIX=(m_dotOUTtot·beta·Cp·(T_out-Tmix)+m_dotos·Cp·(T_os-Tmix))/(m_mix);
T_dotCC=(m_dotmix·Cp·(Tmix-Tcc)-m_dotwMAX·OpeningCC·Cpw·dT_cc)/(m_cc);
T_dotIN=(m_dotmix·Cp·(Tcc-T_in)-m_dotwMAX·OpeningHC·Cpw·dT_hc)/(m_h);

%% Extended Humidity model

w_dotz1=(m_dotINz1·(w_in-w_z1)-m_dotOUTz1·(w_z1-w_out))/m_z1;    % Zone 1.
w_dotz2=(m_dotINz2·(w_in-w_z2)-m_dotOUTz2·(w_z2-w_out))/m_z2;    % Zone 2.
w_dotz3=(m_dotINz3·(w_in-w_z3)-m_dotOUTz3·(w_z3-w_out))/m_z3;    % Zone 3.

w_dotmix=(m_dotOUTtot·beta·(w_out-w_mix)+m_dotos·(w_os-w_mix))/(m_mix);
w_dotcc=(m_dotmix·(w_mix-w_cc)-m_wout)/(m_cc);
w_dotin=(m_dotmix·(w_cc-w_in)+m_win)/(m_h); %wm-wmis

%% Wall
T_dotw12=(U·A_12·(T_z1-T_w12)-U·A_12·(T_w12-T_z2))/(C_pSteel·m_wall);
T_dotw23=(U·A_12·(T_z3-T_w23)-U·A_12·(T_w23-T_z2))/(C_pSteel·m_wall);

%% Output function
h1=(m_z1/V_z1)·R·T_z1;
h2=(m_z2/V_z2)·R·T_z2;
h3=(m_z3/V_z3)·R·T_z3;
```

```
%% Linearization
Inputs=[A_0inz1 A_0inz2 A_0inz3 OpeningHC m_win T_rad1 T_rad2 T_rad3];
States=[Tmix, Tcc, T_in, w_mix, w_cc, w_in, m_z1, T_z1, w_z1, m_z2, T_z2,
w_z2, m_z3, T_z3, w_z3 T_w12 T_w23];
Outputs=[T_z1 w_z1 P_z1 T_z2 w_z2 P_z2 T_z3 w_z3 P_z3 T_in];

x1=States(1); x2=States(2); x3=States(3); x4=States(4); x5=States(5);
x6=States(6); x7=States(7); x8=States(8); x9=States(9);
x10=States(10); x11=States(11); x12=States(12); x13=States(13);
x14=States(14); x15=States(15); x16=States(16); x17=States(17);

u1=Inputs(1); u2=Inputs(2); u3=Inputs(3); u4=Inputs(4); u5=Inputs(5);
u6=Inputs(6); u7=Inputs(7); u8=Inputs(8);
%% State space

Alin=[ diff(T_dotMIX,x1) diff(T_dotMIX,x2) diff(T_dotMIX,x3)
diff(T_dotMIX,x4)  diff(T_dotMIX,x5)   diff(T_dotMIX,x6)  diff(T_dotMIX,x7)
diff(T_dotMIX,x8)   diff(T_dotMIX,x9) diff(T_dotMIX,x10) diff(T_dotMIX,x11)
diff(T_dotMIX,x12)  diff(T_dotMIX,x13)  diff(T_dotMIX,x14)  diff(T_dotMIX,x15)
diff(T_dotMIX,x16)  diff(T_dotMIX,x17)
    diff(T_dotCC,x1) diff(T_dotCC,x2) diff(T_dotCC,x3)  diff(T_dotCC,x4)
diff(T_dotCC,x5)  diff(T_dotCC,x6)  diff(T_dotCC,x7)  diff(T_dotCC,x8)
diff(T_dotCC,x9) diff(T_dotCC,x10) diff(T_dotCC,x11) diff(T_dotCC,x12)
diff(T_dotCC,x13)  diff(T_dotCC,x14)  diff(T_dotCC,x15)  diff(T_dotCC,x16)
diff(T_dotCC,x17)
    diff(T_dotIN,x1) diff(T_dotIN,x2) diff(T_dotIN,x3)  diff(T_dotIN,x4)
diff(T_dotIN,x5)  diff(T_dotIN,x6)  diff(T_dotIN,x7)  diff(T_dotIN,x8)
diff(T_dotIN,x9) diff(T_dotIN,x10) diff(T_dotIN,x11) diff(T_dotIN,x12)
diff(T_dotIN,x13)  diff(T_dotIN,x14)  diff(T_dotIN,x15)  diff(T_dotIN,x16)
diff(T_dotIN,x17)
    diff(w_dotmix,x1) diff(w_dotmix,x2) diff(w_dotmix,x3)  diff(w_dotmix,x4)
diff(w_dotmix,x5)  diff(w_dotmix,x6)  diff(w_dotmix,x7)  diff(w_dotmix,x8)
diff(w_dotmix,x9)   diff(w_dotmix,x10) diff(w_dotmix,x11) diff(w_dotmix,x12)
diff(w_dotmix,x13)  diff(w_dotmix,x14)  diff(w_dotmix,x15)  diff(w_dotmix,x16)
diff(w_dotmix,x17)
    diff(w_dotcc,x1) diff(w_dotcc,x2) diff(w_dotcc,x3)  diff(w_dotcc,x4)
diff(w_dotcc,x5)  diff(w_dotcc,x6)  diff(w_dotcc,x7)  diff(w_dotcc,x8)
diff(w_dotcc,x9) diff(w_dotcc,x10) diff(w_dotcc,x11) diff(w_dotcc,x12)
diff(w_dotcc,x13)  diff(w_dotcc,x14)  diff(w_dotcc,x15)  diff(w_dotcc,x16)
diff(w_dotcc,x17)
    diff(w_dotin,x1) diff(w_dotin,x2) diff(w_dotin,x3)  diff(w_dotin,x4)
diff(w_dotin,x5)  diff(w_dotin,x6)  diff(w_dotin,x7)  diff(w_dotin,x8)
diff(w_dotin,x9) diff(w_dotin,x10) diff(w_dotin,x11) diff(w_dotin,x12)
diff(w_dotin,x13)  diff(w_dotin,x14)  diff(w_dotin,x15)  diff(w_dotin,x16)
diff(w_dotin,x17)
    diff(m_dotz1,x1) diff(m_dotz1,x2) diff(m_dotz1,x3)  diff(m_dotz1,x4)
diff(m_dotz1,x5)  diff(m_dotz1,x6)  diff(m_dotz1,x7)  diff(m_dotz1,x8)
diff(m_dotz1,x9) diff(m_dotz1,x10) diff(m_dotz1,x11) diff(m_dotz1,x12)
diff(m_dotz1,x13)  diff(m_dotz1,x14)  diff(m_dotz1,x15)  diff(m_dotz1,x16)
diff(m_dotz1,x17)
    diff(T_dotz1,x1) diff(T_dotz1,x2) diff(T_dotz1,x3)  diff(T_dotz1,x4)
diff(T_dotz1,x5)  diff(T_dotz1,x6)  diff(T_dotz1,x7)  diff(T_dotz1,x8)
diff(T_dotz1,x9) diff(T_dotz1,x10) diff(T_dotz1,x11) diff(T_dotz1,x12)
diff(T_dotz1,x13)  diff(T_dotz1,x14)  diff(T_dotz1,x15)  diff(T_dotz1,x16)
diff(T_dotz1,x17)
    diff(w_dotz1,x1) diff(w_dotz1,x2) diff(w_dotz1,x3)  diff(w_dotz1,x4)
diff(w_dotz1,x5)  diff(w_dotz1,x6)  diff(w_dotz1,x7)  diff(w_dotz1,x8)
```

```
diff(w_dotz1,x9) diff(w_dotz1,x10) diff(w_dotz1,x11) diff(w_dotz1,x12)
diff(w_dotz1,x13)  diff(w_dotz1,x14)  diff(w_dotz1,x15)  diff(w_dotz1,x16)
diff(w_dotz1,x17)
    diff(m_dotz2,x1) diff(m_dotz2,x2) diff(m_dotz2,x3)  diff(m_dotz2,x4)
diff(m_dotz2,x5)  diff(m_dotz2,x6)  diff(m_dotz2,x7)  diff(m_dotz2,x8)
diff(m_dotz2,x9) diff(m_dotz2,x10) diff(m_dotz2,x11) diff(m_dotz2,x12)
diff(m_dotz2,x13)  diff(m_dotz2,x14)  diff(m_dotz2,x15)  diff(m_dotz2,x16)
diff(m_dotz2,x17)
    diff(T_dotz2,x1) diff(T_dotz2,x2) diff(T_dotz2,x3)  diff(T_dotz2,x4)
diff(T_dotz2,x5)  diff(T_dotz2,x6)  diff(T_dotz2,x7)  diff(T_dotz2,x8)
diff(T_dotz2,x9) diff(T_dotz2,x10) diff(T_dotz2,x11) diff(T_dotz2,x12)
diff(T_dotz2,x13)  diff(T_dotz2,x14)  diff(T_dotz2,x15)  diff(T_dotz2,x16)
diff(T_dotz2,x17)
    diff(w_dotz2,x1) diff(w_dotz2,x2) diff(w_dotz2,x3)  diff(w_dotz2,x4)
diff(w_dotz2,x5)  diff(w_dotz2,x6)  diff(w_dotz2,x7)  diff(w_dotz2,x8)
diff(w_dotz2,x9) diff(w_dotz2,x10) diff(w_dotz2,x11) diff(w_dotz2,x12)
diff(w_dotz2,x13)  diff(w_dotz2,x14)  diff(w_dotz2,x15)  diff(w_dotz2,x16)
diff(w_dotz2,x17)
    diff(m_dotz3,x1) diff(m_dotz3,x2) diff(m_dotz3,x3)  diff(m_dotz3,x4)
diff(m_dotz3,x5)  diff(m_dotz3,x6)  diff(m_dotz3,x7)  diff(m_dotz3,x8)
diff(m_dotz3,x9) diff(m_dotz3,x10) diff(m_dotz3,x11) diff(m_dotz3,x12)
diff(m_dotz3,x13)  diff(m_dotz3,x14)  diff(m_dotz3,x15)  diff(m_dotz3,x16)
diff(m_dotz3,x17)
    diff(T_dotz3,x1) diff(T_dotz3,x2) diff(T_dotz3,x3)  diff(T_dotz3,x4)
diff(T_dotz3,x5)  diff(T_dotz3,x6)  diff(T_dotz3,x7)  diff(T_dotz3,x8)
diff(T_dotz3,x9) diff(T_dotz3,x10) diff(T_dotz3,x11) diff(T_dotz3,x12)
diff(T_dotz3,x13)  diff(T_dotz3,x14)  diff(T_dotz3,x15)  diff(T_dotz3,x16)
diff(T_dotz3,x17)
    diff(w_dotz3,x1) diff(w_dotz3,x2) diff(w_dotz3,x3)  diff(w_dotz3,x4)
diff(w_dotz3,x5)  diff(w_dotz3,x6)  diff(w_dotz3,x7)  diff(w_dotz3,x8)
diff(w_dotz3,x9) diff(w_dotz3,x10) diff(w_dotz3,x11) diff(w_dotz3,x12)
diff(w_dotz3,x13)  diff(w_dotz3,x14)  diff(w_dotz3,x15)  diff(w_dotz3,x16)
diff(w_dotz3,x17)
    diff(T_dotw12,x1) diff(T_dotw12,x2) diff(T_dotw12,x3)  diff(T_dotw12,x4)
diff(T_dotw12,x5)  diff(T_dotw12,x6)  diff(T_dotw12,x7)  diff(T_dotw12,x8)
diff(T_dotw12,x9) diff(T_dotw12,x10) diff(T_dotw12,x11) diff(T_dotw12,x12)
diff(T_dotw12,x13)  diff(T_dotw12,x14)  diff(T_dotw12,x15)  diff(T_dotw12,x16)
diff(T_dotw12,x17)
    diff(T_dotw23,x1) diff(T_dotw23,x2) diff(T_dotw23,x3)  diff(T_dotw23,x4)
diff(T_dotw23,x5)  diff(T_dotw23,x6)  diff(T_dotw23,x7)  diff(T_dotw23,x8)
diff(T_dotw23,x9) diff(T_dotw23,x10) diff(T_dotw23,x11) diff(T_dotw23,x12)
diff(T_dotw23,x13)  diff(T_dotw23,x14)  diff(T_dotw23,x15)  diff(T_dotw23,x16)
diff(T_dotw23,x17)
    ];

Blin=[diff(T_dotMIX,u1) diff(T_dotMIX,u2) diff(T_dotMIX,u3) diff(T_dotMIX,u4)
diff(T_dotMIX,u5) diff(T_dotMIX,u6)  diff(T_dotMIX,u7) diff(T_dotMIX,u8);
diff(T_dotCC,u1) diff(T_dotCC,u2) diff(T_dotCC,u3) diff(T_dotCC,u4)
diff(T_dotCC,u5) diff(T_dotCC,u6) diff(T_dotCC,u7) diff(T_dotCC,u8);
diff(T_dotIN,u1) diff(T_dotIN,u2) diff(T_dotIN,u3) diff(T_dotIN,u4)
diff(T_dotIN,u5) diff(T_dotIN,u6) diff(T_dotIN,u7) diff(T_dotIN,u8);
diff(w_dotmix,u1) diff(w_dotmix,u2) diff(w_dotmix,u3) diff(w_dotmix,u4)
diff(w_dotmix,u5) diff(w_dotmix,u6)  diff(w_dotmix,u7) diff(w_dotmix,u8);
diff(w_dotcc,u1) diff(w_dotcc,u2) diff(w_dotcc,u3) diff(w_dotcc,u4)
diff(w_dotcc,u5) diff(w_dotcc,u6)  diff(w_dotcc,u7) diff(w_dotcc,u8);
diff(w_dotin,u1) diff(w_dotin,u2) diff(w_dotin,u3) diff(w_dotin,u4)
diff(w_dotin,u5) diff(w_dotin,u6) diff(w_dotin,u7) diff(w_dotin,u8);
diff(m_dotz1,u1) diff(m_dotz1,u2)  diff(m_dotz1,u3)  diff(m_dotz1,u4)
diff(m_dotz1,u5)  diff(m_dotz1,u6)  diff(m_dotz1,u7)  diff(m_dotz1,u8);
```

```
diff(T_dotz1,u1) diff(T_dotz1,u2) diff(T_dotz1,u3) diff(T_dotz1,u4)
diff(T_dotz1,u5) diff(T_dotz1,u6) diff(T_dotz1,u7) diff(T_dotz1,u8);
diff(w_dotz1,u1) diff(w_dotz1,u2) diff(w_dotz1,u3) diff(w_dotz1,u4)
diff(w_dotz1,u5) diff(w_dotz1,u6) diff(w_dotz1,u7) diff(w_dotz1,u8);
diff(m_dotz2,u1) diff(m_dotz2,u2)  diff(m_dotz2,u3)  diff(m_dotz2,u4)
diff(m_dotz2,u5)  diff(m_dotz2,u6)  diff(m_dotz2,u7)  diff(m_dotz2,u8);
diff(T_dotz2,u1) diff(T_dotz2,u2) diff(T_dotz2,u3) diff(T_dotz2,u4)
diff(T_dotz2,u5) diff(T_dotz2,u6) diff(T_dotz2,u7) diff(T_dotz2,u8);
diff(w_dotz2,u1) diff(w_dotz2,u2) diff(w_dotz2,u3) diff(w_dotz2,u4)
diff(w_dotz2,u5) diff(w_dotz2,u6) diff(w_dotz2,u7) diff(w_dotz2,u8);
diff(m_dotz3,u1) diff(m_dotz3,u2)  diff(m_dotz3,u3)  diff(m_dotz3,u4)
diff(m_dotz3,u5)  diff(m_dotz3,u6)  diff(m_dotz3,u7)  diff(m_dotz3,u8);
diff(T_dotz3,u1) diff(T_dotz3,u2) diff(T_dotz3,u3) diff(T_dotz3,u4)
diff(T_dotz3,u5) diff(T_dotz3,u6) diff(T_dotz3,u7) diff(T_dotz3,u8);
diff(w_dotz3,u1) diff(w_dotz3,u2) diff(w_dotz3,u3) diff(w_dotz3,u4)
diff(w_dotz3,u5) diff(w_dotz3,u6) diff(w_dotz3,u7) diff(w_dotz3,u8);
diff(T_dotw12,u1) diff(T_dotw12,u2) diff(T_dotw12,u3) diff(T_dotw12,u4)
diff(T_dotw12,u5) diff(T_dotw12,u6)  diff(T_dotw12,u7) diff(T_dotw12,u8);
diff(T_dotw23,u1) diff(T_dotw23,u2) diff(T_dotw23,u3) diff(T_dotw23,u4)
diff(T_dotw23,u5) diff(T_dotw23,u6) diff(T_dotw23,u7) diff(T_dotw23,u8)];

Clin=[0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0;
     0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0;
     diff(h1,x1) diff(h1,x2) diff(h1,x3) diff(h1,x4) diff(h1,x5) diff(h1,x6)
diff(h1,x7) diff(h1,x8) diff(h1,x9) diff(h1,x10) diff(h1,x11) diff(h1,x12)
diff(h1,x13) diff(h1,x14) diff(h1,x15) diff(h1,x16) diff(h1,x17);
     0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0;
     0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0;
     diff(h2,x1) diff(h2,x2) diff(h2,x3) diff(h2,x4) diff(h2,x5) diff(h2,x6)
diff(h2,x7) diff(h2,x8) diff(h2,x9) diff(h2,x10) diff(h2,x11) diff(h2,x12)
diff(h2,x13) diff(h2,x14) diff(h2,x15) diff(h2,x16) diff(h2,x17);
     0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0;
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0;
     diff(h3,x1) diff(h3,x2) diff(h3,x3) diff(h3,x4) diff(h3,x5) diff(h3,x6)
diff(h3,x7) diff(h3,x8) diff(h3,x9) diff(h3,x10) diff(h3,x11) diff(h3,x12)
diff(h3,x13) diff(h3,x14) diff(h3,x15) diff(h3,x16) diff(h3,x17)
     0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0];

Ts=0.01;
Sys=ss(double(subs(Alin)),double(subs(Blin)),double(subs(Clin)),0);
SysD=c2d(Sys,Ts);
[Ad, Bd, Cd, Dd]=ssdata(SysD);
[Ac, Bc, Cc, Dc]=ssdata(Sys);
```

Appendix 7. Discrete time State space responses

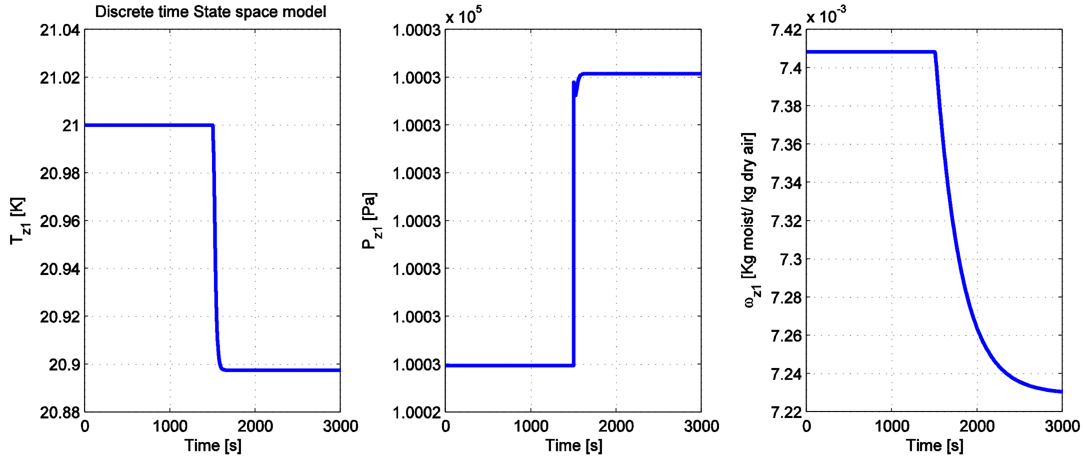


Figure 1 a) Temperature, b) Pressure, c) Specific humidity of zone 1.

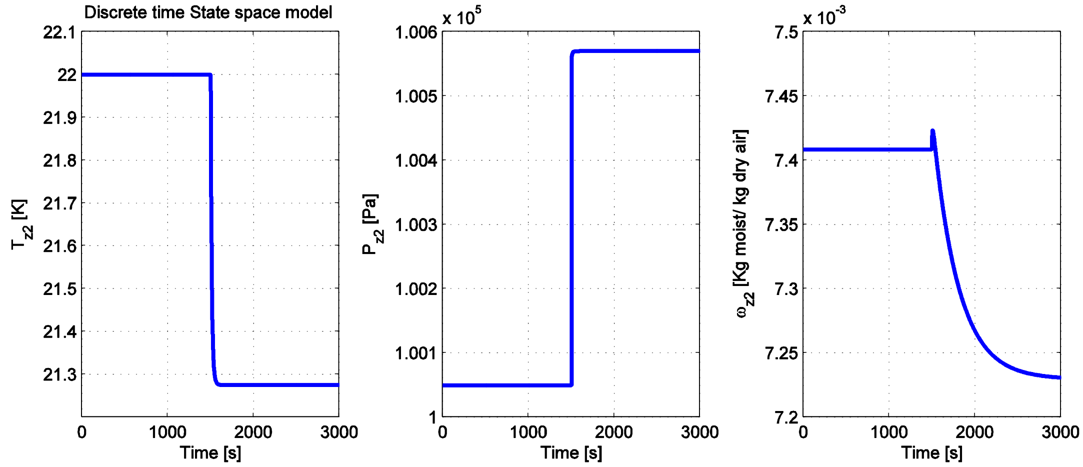


Figure 2 a) Temperature, b) Pressure, c) Specific humidity of zone 2.

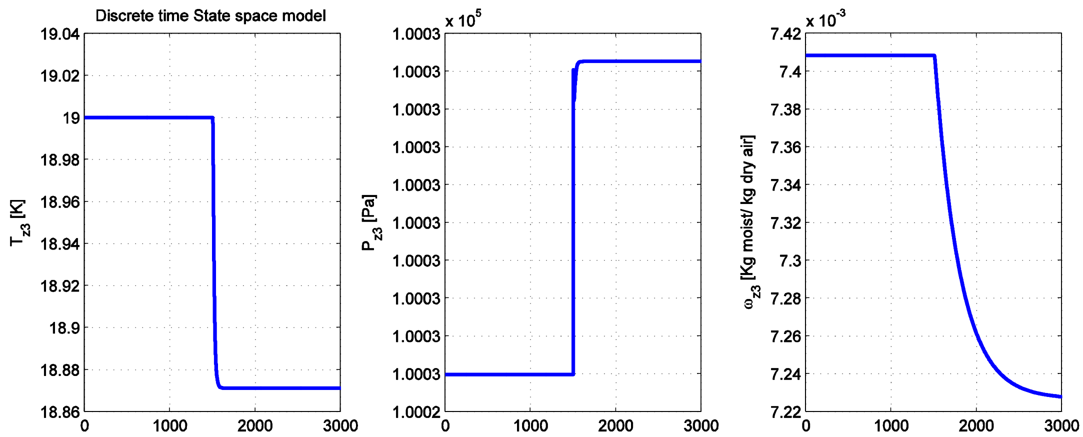


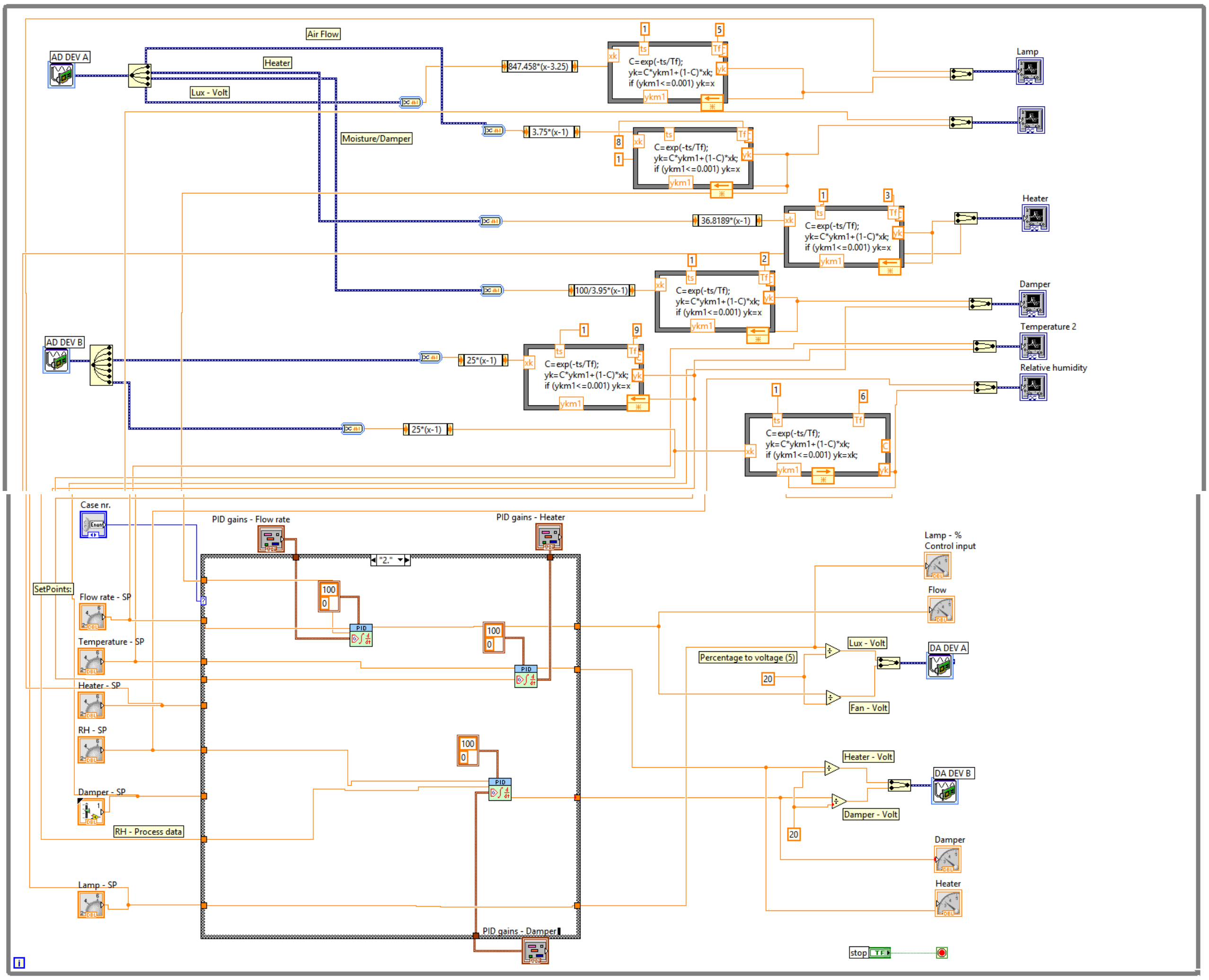
Figure 3 a) Temperature, b) Pressure, c) Specific humidity of zone 3.

Appendix 8. AHU control script

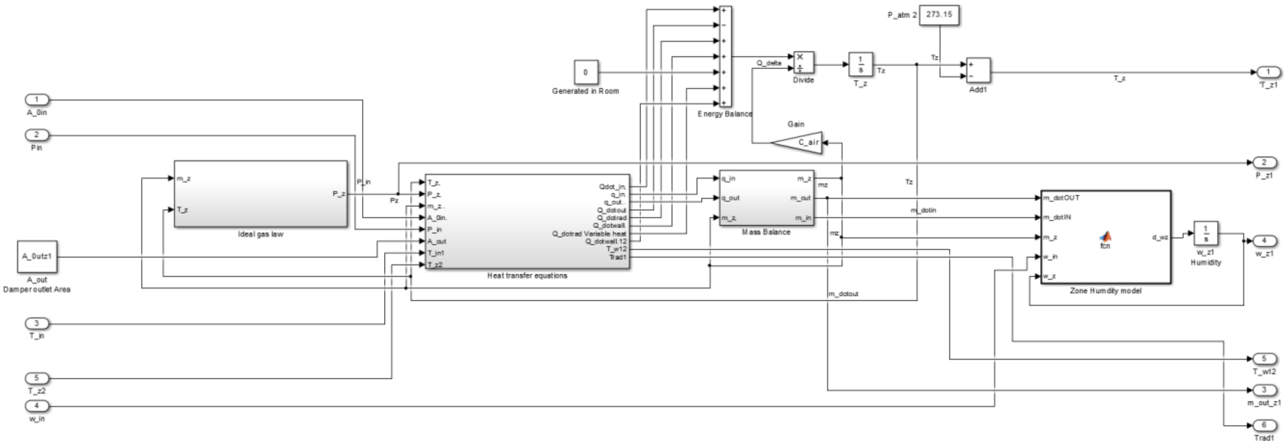
```
function [dT_cc, dT_hc, dw_cc, dw_h] = fcn(T_mix, T_in, m_dotmix, Freq, T_cc,
T_hc, w_mix, w_cc, w_h, w_zsetpoint, RH_cc ,m_wdotout ,m_wdotin,
OpeningCooling,OpeningHeating)

Cp=1.02;
Cw=4.16;
m_dotMAX=0.4;
EnableDehumidifier=0;
EnableHumidifier=0;
w_out=0.0048;
w_s=2·w_zsetpoint-w_out;

T_HL=T_in+1.5;                                                             %
3 Degrees deadband
T_LL=T_in-1.5;

dThc=-10;
dTcc=10;
m_cc=1.7896;
m_h=1.7896;

w_HL=w_s+0.00004;
w_LL=w_s-0.00004;

if Freq ~= 0 && T_mix > T_HL && w_mix > w_HL;
    OpeningHeating=0;
    EnableHumidifier=0;
    EnableDehumidifier=1;

else if Freq ~= 0 && T_mix < T_LL && w_mix < w_LL;
% If Fan is ON AND Tmix more than Tin.
        % Heat and humidify

        OpeningCooling=0;           % Disable Cooling Coil
        EnableHumidifier=1;
        EnableDehumidifier=0;
 else if Freq ==0 || T_LL < T_mix && T_mix < T_HL && w_LL < w_mix && w_mix <
w_HL;                       % If Fan is not ON.
                % Close!
                OpeningCooling=0;
                % Disable both
                OpeningHeating=0;
                EnableHumidifier=0;
                EnableDehumidifier=0;
            end
        end
    end
end
```

```
% OpeningCooling and OpeningHeater is the control input sent from the
controllers, if it is not set to zero. The same follows for m_wdotout and
m_dotwin.

if OpeningCooling==0 && OpeningHeating>0 && EnableHumidifier==1 &&
EnableDehumidifier==0;

    m_dotwcc=OpeningCooling·m_dotMAX;
    m_dotwhc=OpeningHeating·m_dotMAX;

    w_s=2·w_zsetpoint-w_out;                                           %
    m_wdotout=0;

else if OpeningCooling==0 && OpeningHeating==0 && EnableHumidifier==1 &&
EnableDehumidifier==0;

    m_dotwcc=OpeningCooling·m_dotMAX;
    m_dotwhc=OpeningHeating·m_dotMAX;

    w_s=2·w_zsetpoint-w_out;                                           %
    m_wdotout=0;

else if OpeningHeating==0 && OpeningCooling>0&& EnableHumidifier==0 &&
EnableDehumidifier==1;

        m_dotwcc=OpeningCooling·m_dotMAX;
        m_dotwhc=OpeningHeating·m_dotMAX;

        m_wdotin=0;
        w_s=2·w_zsetpoint-w_out;

    else if OpeningHeating==0 && OpeningCooling==0 && EnableHumidifier==0 &&
EnableDehumidifier==0;

            m_dotwcc=OpeningCooling·m_dotMAX;
            m_dotwhc=OpeningHeating·m_dotMAX;
            m_wdotin=0;
            m_wdotout=0;

       else % do nothing
            m_dotwcc=0;
            m_dotwhc=0;
            m_wdotin=0;
            m_wdotout=0;
       end
    end
  end
end

dT_cc=((T_mix-T_cc)·Cp·m_dotmix-m_dotwcc·Cw·dTcc)/(m_cc·Cp);
dw_cc=(m_dotmix·(w_mix-w_cc)+m_wdotout)/m_cc;
dT_hc=((T_cc-T_hc)·Cp·m_dotmix-m_dotwhc·Cw·dThc)/(m_cc·Cp);
dw_h=(m_dotmix·(w_cc-w_h)+m_wdotin)/m_h;

end
```

Appendix 9. NI Labview interface



Appendix 10. Zone model



Appendix 11. Multiple Zone State Space elements

$$A_{1,1} = -\frac{Cp \cdot m_{dotos} + Cp \cdot \beta \cdot \left(\frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot x_8 \cdot \left(R \cdot x_7 - \frac{P_{atm} \cdot V_{z1}}{x_8}\right)^{\frac{1}{2}}}{V_{z1} \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2}\right)^{\frac{1}{2}}} + \frac{2^{\frac{1}{2}} \cdot A_{0utz2} \cdot Cc \cdot x_{11} \cdot \left(R \cdot x_{10} - \frac{P_{atm} \cdot V_{z2}}{x_{11}}\right)^{\frac{1}{2}}}{V_{z2} \cdot \left(1 - \frac{A_{0utz2}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot x_{11}^2}\right)^{\frac{1}{2}}} + \frac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot x_{14} \cdot \left(R \cdot x_{13} - \frac{P_{atm} \cdot V_{z3}}{x_{14}}\right)^{\frac{1}{2}}}{V_{z3} \cdot \left(1 - \frac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot x_{14}^2}\right)^{\frac{1}{2}}} \right)}{m_{mix}}$$

$$A_{2,1} = \frac{Cp \cdot m_{dotmix}}{m_{cc}}$$

$$A_{2,2} = -\frac{Cp \cdot m_{dotmix}}{m_{cc}}$$

$$A_{3,2} = \frac{Cp \cdot m_{dotmix}}{m_h}$$

$$A_{3,3} = -\frac{Cp \cdot m_{dotmix}}{m_h}$$

$$A_{8,3} = \frac{2^{\frac{1}{2}} \cdot u_1 \cdot Cc \cdot \rho_{inz1} \cdot \left(\frac{2 \cdot P_{in1} - \frac{2 \cdot R \cdot x_7 \cdot x_8}{V_{z1}}}{2 \cdot \rho_{inz1}} \right)^{\frac{1}{2}}}{x_8 \cdot (1 - u_1^2)^{\frac{1}{2}}}$$

$$A_{11,3} = \frac{2^{\frac{1}{2}} \cdot u_2^2 \cdot Cc \cdot \rho_{inz2} \cdot \left(\frac{2 \cdot P_{in2} - \frac{2 \cdot R \cdot x_{10} \cdot x_{11}}{V_{z2}}}{2 \cdot \rho_{inz2}} \right)^{\frac{1}{2}}}{x_{11} \cdot \left(1 - \frac{A_{0inz2}^2 \cdot Cc^2 \cdot x_{11}^2}{A_{1in}^2 \cdot \rho_{inz2}^2 \cdot V_{z2}^2}\right)^{\frac{1}{2}}}$$

$$A_{14,3} = \frac{2^{\frac{1}{2}} \cdot u_3 \cdot Cc \cdot \rho_{inz3} \cdot \left(\frac{2 \cdot P_{in3} - \frac{2 \cdot R \cdot x_{13} \cdot x_{14}}{V_{z3}}}{2 \cdot \rho_{inz3}} \right)^{\frac{1}{2}}}{x_{14} \cdot \left(1 - \frac{u_3^2 \cdot Cc^2 \cdot x_{14}^2}{A_{1in}^2 \cdot \rho_{inz3}^2 \cdot V_{z3}^2}\right)^{\frac{1}{2}}}$$

$$A_{4,4} = -\frac{m_{dotos} + beta \cdot \left(\frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot x_8 \cdot \left(R \cdot x_7 - \frac{P_{atm} \cdot V_{z1}}{x_8}\right)^{\frac{1}{2}}}{V_{z1} \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2}\right)^{\frac{1}{2}}} + \frac{2^{\frac{1}{2}} \cdot A_{0utz2} \cdot Cc \cdot x_{11} \cdot \left(R \cdot x_{10} - \frac{P_{atm} \cdot V_{z2}}{x_{11}}\right)^{\frac{1}{2}}}{V_{z2} \cdot \left(1 - \frac{A_{0utz2}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot x_{11}^2}\right)^{\frac{1}{2}}} + \frac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot x_{14} \cdot \left(R \cdot x_{13} - \frac{P_{atm} \cdot V_{z3}}{x_{14}}\right)^{\frac{1}{2}}}{V_{z3} \cdot \left(1 - \frac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot x_{14}^2}\right)^{\frac{1}{2}}} \right)}{m_{mix}}$$

$$A_{5,4} = \frac{m_{dotmix}}{m_{cc}}$$

$$A_{5,5} = -\frac{m_{dotmix}}{m_{cc}}$$

$$A_{6,5} = \frac{m_{dotmix}}{m_h}$$

$$A_{6,6} = -\frac{m_{dotmix}}{m_h}$$

Appendix

$$A_{9,6} = \frac{2^{\frac{1}{2}} \cdot u_1 \cdot Cc \cdot \rho_{inz1} \cdot \left(\frac{P_{in1} - \frac{R \cdot x_7 \cdot x_8}{V_{z1}}}{\rho_{inz1}} \right)^{\frac{1}{2}}}{x_8 \cdot \left(1 - \frac{u_1^2 \cdot Cc^2 \cdot \rho_{inz1}^2}{A_{1in}^2 \cdot \rho_{inz1}^2} \right)^{\frac{1}{2}}}$$

$$A_{12,6} = \frac{2^{\frac{1}{2}} \cdot u_2 \cdot Cc \cdot \rho_{inz2} \cdot \left(\frac{P_{in2} - \frac{R \cdot x_{10} \cdot x_{11}}{V_{z2}}}{\rho_{inz2}} \right)^{\frac{1}{2}}}{m_{z2} \cdot \left(1 - \frac{u_2^2 \cdot Cc^2 \cdot \rho_{inz2}^2}{A_{1in}^2 \cdot \rho_{inz2}^2} \right)^{\frac{1}{2}}}$$

$$A_{15,6} = \frac{2^{\frac{1}{2}} \cdot u_3 \cdot Cc \cdot \rho_{inz3} \cdot \left(\frac{P_{in3} - \frac{R \cdot x_{13} \cdot x_{14}}{V_{z3}}}{\rho_{inz3}^2} \right)^{\frac{1}{2}}}{m_{z3} \cdot \left(1 - \frac{u_3 \cdot Cc \cdot \rho_{inz3}^2}{A_{1in}^2 \cdot \rho_{inz3}^2} \right)^{\frac{1}{2}}}$$

$$A_{4,7} = \frac{\beta \cdot (x_4 - w_{out}) \cdot \left(\frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot \left(R \cdot x_7 - \frac{P_{atm} \cdot V_{z1}}{x_8} \right)^{\frac{1}{2}}}{V_{z1} \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2} \right)^{\frac{1}{2}}} + \frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot P_{atm}}{2 \cdot x_8 \cdot \left(R \cdot X_7 - \frac{P_{atm} \cdot V_{z1}}{x_8} \right)^{\frac{1}{2}} \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2} \right)^{\frac{1}{2}}} - \frac{2^{\frac{1}{2}} \cdot A_{0utz1}^3 \cdot Cc^3 \cdot \rho_{out}^2 \cdot V_{z1} \cdot \left(R \cdot x_7 - \frac{P_{atm} \cdot V_{z1}}{x_8^2} \right)^{\frac{1}{2}}}{A_{1in}^2 \cdot X_8^2 \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2} \right)^{\frac{3}{2}}} \right)}{m_{mix}}$$

$$A_{7,7} = \frac{2^{\frac{1}{2}} \cdot A_{0utz1}^3 \cdot Cc^3 \cdot \rho_{out}^2 \cdot V_{z1} \cdot \left(R \cdot x_7 - \frac{P_{atm} \cdot V_{z1}}{x_8} \right)^{\frac{1}{2}}}{A_{1in}^2 \cdot x_8^2 \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2} \right)^{\frac{3}{2}}} - \frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot P_{atm}}{2 \cdot x_8 \cdot \left(R \cdot X_7 - \frac{P_{atm} \cdot V_{z1}}{x_8} \right)^{\frac{1}{2}} \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2} \right)^{\frac{1}{2}}} - \frac{2^{\frac{1}{2}} \cdot U_1 \cdot Cc \cdot R \cdot x_7}{2 \cdot V_{z1} \cdot \left(1 - \frac{U_1^2 \cdot Cc^2 \cdot \rho_{z1}^2}{A_{1in}^2 \cdot \rho_{inz1}^2} \right)^{\frac{1}{2}} \cdot \left(\frac{P_{in1} - \frac{R \cdot X_7 \cdot X_8}{V_{z1}}}{\rho_{inz1}^2} \right)^{\frac{1}{2}}} - \frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot \left(R \cdot x_7 - \frac{P_{atm} \cdot V_{z1}}{x_8} \right)^{\frac{1}{2}}}{V_{z1} \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2} \right)^{\frac{1}{2}}}$$

$$A_{1,7} = \frac{Cp \cdot \beta \cdot (T_{out} - x_1) \cdot \left(\frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot \left(R \cdot x_7 - \frac{P_{atm} \cdot V_{z1}}{x_8} \right)^{\frac{1}{2}}}{V_{z1} \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2} \right)^{\frac{1}{2}}} + \frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot P_{atm}}{2 \cdot x_8 \cdot \left(R \cdot X_7 - \frac{P_{atm} \cdot V_{z1}}{x_8} \right)^{\frac{1}{2}} \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2} \right)^{\frac{1}{2}}} - \frac{2^{\frac{1}{2}} \cdot A_{0utz1}^3 \cdot Cc^3 \cdot \rho_{out}^2 \cdot V_{z1} \cdot \left(R \cdot x_7 - \frac{P_{atm} \cdot V_{z1}}{x_8} \right)^{\frac{1}{2}}}{A_{1in}^2 \cdot x_8^2 \cdot \left(1 - \frac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot x_8^2} \right)^{\frac{3}{2}}} \right)}{m_{mix}}$$

Appendix

$$A_{8,7} = \frac{\dfrac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}} \cdot (w_{out} - X_9)}{V_{z1} \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot P_{atm} \cdot (w_{out} - X_9)}{2 \cdot X_8 \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot U_1 \cdot Cc \cdot R \cdot X_7 \cdot (X_6 - X_9)}{2 \cdot V_{z1} \cdot \left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot \rho_{inz1}^2}{A_{1in}^2 \cdot \rho_{inz1}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in1} - \dfrac{R \cdot X_7 \cdot X_8}{V_{z1}}}{\rho_{inz1}}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot A_{0utz1}^3 \cdot Cc^3 \cdot \backslash rho_{out}^2 \cdot V_{z1} \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}} \cdot (w_{out} - X_9)}{A_{1in}^2 \cdot X_8^2 \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{3}{2}}}}{X_8}$$

$$- \frac{\dfrac{2^{\frac{1}{2}} \cdot U_1 \cdot Cc \cdot \rho_{inz1} \cdot (X_6 - X_9) \cdot \left(\dfrac{P_{in1} - \dfrac{R \cdot X_7 \cdot X_8}{V_{z1}}}{\rho_{inz1}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot \rho_{inz1}^2}{A_{1in}^2 \cdot \rho_{inz1}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot X_8 \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}} \cdot (w_{out} - X_9)}{V_{z1} \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{1}{2}}}}{X_8^2}$$

$$A_{1,8} = \frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot Cp \cdot R \cdot beta \cdot X_8 \cdot (T_{out} - X_1)}{2 \cdot V_{z1} \cdot m_{mix} \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{1}{2}}}$$

$$A_{4,8} = -\frac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot R \cdot \beta \cdot X_8 \cdot (X_4 - w_{out})}{2 \cdot V_{z1} \cdot m_{mix} \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{1}{2}}}$$

$$A_{8,8} = -\frac{10 \cdot h_{dash1} + A_{12} \cdot U + \dfrac{2^{\frac{1}{2}} \cdot U_1 \cdot C_{air} \cdot Cc \cdot \rho_{inz1} \cdot \left(\dfrac{2 \cdot P_{in1} - \dfrac{2 \cdot R \cdot X_7 \cdot X_8}{V_{z1}}}{2 \cdot \rho_{inz1}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot X_8^2}{A_{1in}^2 \cdot \rho_{inz1}^2 \cdot V_{z1}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot C_{air} \cdot Cc \cdot X_8 \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}}}{V_{z1} \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{atm}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{1}{2}}}}{C_{air} \cdot X_8} + \cdots$$

$$\cdots \frac{- \dfrac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot C_{air} \cdot Cc \cdot R \cdot X_8 \cdot (T_{out} - X_7)}{2 \cdot V_{z1} \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{atm}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot U_1 \cdot C_{air} \cdot Cc \cdot R \cdot X_8 \cdot (X_3 - X_7)}{2 \cdot V_{z1} \cdot \left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot X_8^2}{A_{1in}^2 \cdot \rho_{inz1}^2 \cdot V_{z1}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{2 \cdot P_{in1} - \dfrac{2 \cdot R \cdot X_7 \cdot X_8}{V_{z1}}}{2 \cdot \rho_{inz1}}\right)^{\frac{1}{2}}}}{C_{air} \cdot X_8}$$

$$A_{9,8} = -\frac{\dfrac{2^{\frac{1}{2}} \cdot U_1 \cdot Cc \cdot R \cdot X_8 \cdot (X_6 - X_9)}{2 \cdot V_{z1} \cdot \left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot \rho_{inz1}^2}{A_{1in}^2 \cdot \rho_{inz1}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in1} - \dfrac{R \cdot X_7 \cdot X_8}{V_{z1}}}{\rho_{inz1}}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot R \cdot X_8 \cdot (w_{out} - X_9)}{2 \cdot V_{z1} \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{1}{2}}}}{X_8}$$

$$A_{9,9} = -\frac{\dfrac{2^{\frac{1}{2}} \cdot U_1 \cdot Cc \cdot \rho_{inz1} \cdot \left(\dfrac{P_{in1} - \dfrac{R \cdot X_7 \cdot X_8}{V_{z1}}}{\rho_{inz1}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot \rho_{inz1}^2}{A_{1in}^2 \cdot \rho_{inz1}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz1} \cdot Cc \cdot X_8 \cdot \left(R \cdot X_7 - \dfrac{P_{atm} \cdot V_{z1}}{X_8}\right)^{\frac{1}{2}}}{V_{z1} \cdot \left(1 - \dfrac{A_{0utz1}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z1}^2}{A_{1in}^2 \cdot X_8^2}\right)^{\frac{1}{2}}}}{X_8}$$

Appendix

$$A_{4,10} = -\frac{\beta\cdot(X_4 - w_{out})\cdot\left(\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot Cc\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}}{V_{z2}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}+\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot Cc\cdot P_{atm}}{2\cdot X_{11}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}-\frac{2^{\frac{1}{2}}\cdot A_{0utz2}^3\cdot Cc^3\cdot\rho_{out}^2\cdot V_{z2}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}}{A_{1in}^2\cdot X_{11}^2\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{3}{2}}}\right)}{m_{mix}}$$

$$A_{1,10} = \frac{Cp\cdot\beta\cdot(T_{out} - X_1)\cdot\left(\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot Cc\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}}{V_{z2}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}+\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot Cc\cdot P_{atm}}{2\cdot X_{11}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}-\frac{2^{\frac{1}{2}}\cdot A_{0utz2}^3\cdot Cc^3\cdot\rho_{out}^2\cdot V_{z2}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}}{A_{1in}^2\cdot X_{11}^2\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{3}{2}}}\right)}{m_{mix}}$$

$$\begin{aligned}A_{10,10} = {} & \frac{2^{\frac{1}{2}}\cdot A_{0utz2}^3\cdot Cc^3\cdot\rho_{out}^2\cdot V_{z2}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}}{A_{1in}^2\cdot X_{11}^2\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{3}{2}}}-\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot Cc\cdot P_{atm}}{2\cdot X_{11}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}} \\ & -\frac{2^{\frac{1}{2}}\cdot U_2\cdot Cc\cdot R\cdot X_{10}}{2\cdot V_{z2}\cdot\left(1-\frac{U_2^2\cdot Cc^2\cdot\rho_{z2}^2}{A_{1in}^2\cdot\rho_{inz2}^2}\right)^{\frac{1}{2}}\cdot\left(\frac{P_{in2}-\frac{R\cdot X_{10}\cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}}-\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot Cc\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}}{V_{z2}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}\end{aligned}$$

$$\begin{aligned}A_{11,10} \\ = {} & \frac{\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot C_{air}\cdot Cc\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}\cdot(T_{out}-X_{10})}{V_{z2}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{atm}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}+\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot C_{air}\cdot Cc\cdot P_{atm}\cdot(T_{out}-X_{10})}{2\cdot X_{11}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{atm}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}-\frac{2^{\frac{1}{2}}\cdot U_2\cdot C_{air}\cdot Cc\cdot R\cdot X_{10}\cdot(X_3-X_{10})}{2\cdot V_{z2}\cdot\left(1-\frac{U_2^2\cdot Cc^2\cdot X_{11}^2}{A_{1in}^2\cdot\rho_{inz2}^2\cdot V_{z2}^2}\right)^{\frac{1}{2}}\cdot\left(\frac{2\cdot P_{in2}-\frac{2\cdot R\cdot X_{10}\cdot X_{11}}{V_{z2}}}{2\cdot\rho_{inz2}}\right)^{\frac{1}{2}}}}{C_{air}\cdot X_{11}}+.. \\ & \frac{\frac{2^{\frac{1}{2}}\cdot U_2^3\cdot C_{air}\cdot Cc^3\cdot X_{11}\cdot(X_3-X_{10})\cdot\left(\frac{2\cdot P_{in2}-\frac{2\cdot R\cdot X_{10}\cdot X_{11}}{V_{z2}}}{2\cdot\rho_{inz2}}\right)^{\frac{1}{2}}}{A_{1in}^2\cdot\backslash rho_{inz2}\cdot V_{z2}^2\cdot\left(1-\frac{U_2^2\cdot Cc^2\cdot X_{11}^2}{A_{1in}^2\cdot\rho_{inz2}^2\cdot V_{z2}^2}\right)^{\frac{3}{2}}}-\frac{2^{\frac{1}{2}}\cdot A_{0utz2}^3\cdot C_{air}\cdot Cc^3\cdot\rho_{atm}^2\cdot V_{z2}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}\cdot(T_{out}-X_{10})}{A_{1in}^2\cdot X_{11}^2\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{atm}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{3}{2}}}-10\cdot h_{dash2}\cdot(U_7-X_{10})+A_{12}\cdot U\cdot(X_{16}-X_{10})+A_{12}\cdot U\cdot(X_{17}-X_{10})}{C_{air}\cdot X_{11}} \\ & +\frac{\frac{2^{\frac{1}{2}}\cdot U_2\cdot C_{air}\cdot Cc\cdot\backslash rho_{inz2}\cdot(X_3-X_{10})\cdot\left(\frac{2\cdot P_{in2}-\frac{2\cdot R\cdot X_{10}\cdot X_{11}}{V_{z2}}}{2\cdot\rho_{inz2}}\right)^{\frac{1}{2}}}{\left(1-\frac{U_2^2\cdot Cc^2\cdot X_{11}^2}{A_{1in}^2\cdot\rho_{inz2}^2\cdot V_{z2}^2}\right)^{\frac{1}{2}}}+\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot C_{air}\cdot Cc\cdot X_{11}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}\cdot(T_{out}-X_{10})}{V_{z2}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{atm}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}}{C_{air}\cdot X_{11}^2}-\frac{Q_{radz2}+Q_{wallz2}-\frac{2277}{100}}{Cp\cdot X_{11}^2}\end{aligned}$$

$$A_{12,10} = \frac{\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot Cc\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}\cdot(w_{out}-X_{12})}{V_{z2}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}+\frac{2^{\frac{1}{2}}\cdot A_{0utz2}\cdot Cc\cdot P_{atm}\cdot(w_{out}-X_{12})}{2\cdot X_{11}\cdot\left(R\cdot X_{10}-\frac{P_{atm}\cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}\cdot\left(1-\frac{A_{0utz2}^2\cdot Cc^2\cdot\rho_{out}^2\cdot V_{z2}^2}{A_{1in}^2\cdot X_{11}^2}\right)^{\frac{1}{2}}}}{X_{11}}-\cdots$$

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$$\frac{\dfrac{2^{\frac{1}{2}} \cdot U_2 \cdot Cc \cdot R \cdot X_{10} \cdot (X_6 - X_{12})}{2 \cdot V_{z2} \cdot \left(1 - \dfrac{U_2^2 \cdot Cc^2 \cdot \rho_{z2}^2}{A_{1in}^2 \cdot \rho_{inz2}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in2} - \dfrac{R \cdot X_{10} \cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot A_{0utz2}^3 \cdot Cc^3 \cdot \rho_{out}^2 \cdot V_{z2} \cdot \left(R \cdot X_{10} - \dfrac{P_{atm} \cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}} \cdot (w_{out} - X_{12})}{A_{1in}^2 \cdot X_{11}^2 \cdot \left(1 - \dfrac{A_{0utz2}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot X_{11}^2}\right)^{\frac{3}{2}}}}{X_{11}} - \cdots$$

$$\frac{\dfrac{2^{\frac{1}{2}} \cdot U_2 \cdot Cc \cdot \rho_{inz2} \cdot (X_6 - X_{12}) \cdot \left(\dfrac{P_{in2} - \dfrac{R \cdot X_{10} \cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_2^2 \cdot Cc^2 \cdot \rho_{z2}^2}{A_{1in}^2 \cdot \rho_{inz2}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz2} \cdot Cc \cdot X_{11} \cdot \left(R \cdot X_{10} - \dfrac{P_{atm} \cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}} \cdot (w_{out} - X_{12})}{V_{z2} \cdot \left(1 - \dfrac{A_{0utz2}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot X_{11}^2}\right)^{\frac{1}{2}}}}{X_{11}^2}$$

$$A_{1,11} = \frac{2^{\frac{1}{2}} \cdot A_{0utz2} \cdot Cc \cdot Cp \cdot R \cdot \beta \cdot X_{11} \cdot (T_{out} - X_1)}{2 \cdot V_{z2} \cdot m_{mix} \cdot \left(R \cdot X_{10} - \dfrac{P_{atm} \cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz2}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot X_{11}^2}\right)^{\frac{1}{2}}}$$

$$A_{4,11} = -\frac{2^{\frac{1}{2}} \cdot A_{0utz2} \cdot Cc \cdot R \cdot \beta \cdot X_{11} \cdot (X_4 - w_{out})}{2 \cdot V_{z2} \cdot m_{mix} \cdot \left(R \cdot X_{10} - \dfrac{P_{atm} \cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz2}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot X_{11}^2}\right)^{\frac{1}{2}}}$$

$$A_{10,11} = -\frac{2^{\frac{1}{2}} \cdot U_2 \cdot Cc \cdot R \cdot X_{11}}{2 \cdot V_{z2} \cdot \left(1 - \dfrac{U_2^2 \cdot Cc^2 \cdot \rho_{z2}^2}{A_{1in}^2 \cdot \rho_{inz2}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in2} - \dfrac{R \cdot X_{10} \cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}} - \frac{2^{\frac{1}{2}} \cdot A_{0utz2} \cdot Cc \cdot R \cdot X_{11}}{2 \cdot V_{z2} \cdot \left(R \cdot X_{10} - \dfrac{P_{atm} \cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz2}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot X_{11}^2}\right)^{\frac{1}{2}}}$$

$$A_{11,11} = -\frac{2^{\frac{1}{2}} \cdot U_2 \cdot Cc \cdot R \cdot X_{11}}{2 \cdot V_{z2} \cdot \left(1 - \dfrac{U_2^2 \cdot Cc^2 \cdot \rho_{z2}^2}{A_{1in}^2 \cdot \rho_{inz2}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in2} - \dfrac{R \cdot X_{10} \cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}} - \frac{2^{\frac{1}{2}} \cdot A_{0utz2} \cdot Cc \cdot R \cdot X_{11}}{2 \cdot V_{z2} \cdot \left(R \cdot X_{10} - \dfrac{P_{atm} \cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz2}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot X_{11}^2}\right)^{\frac{1}{2}}}$$

$$A_{12,11} = -\frac{\dfrac{2^{\frac{1}{2}} \cdot U_2 \cdot Cc \cdot R \cdot X_{11} \cdot (X_6 - X_{12})}{2 \cdot V_{z2} \cdot \left(1 - \dfrac{U_2^2 \cdot Cc^2 \cdot \rho_{z2}^2}{A_{1in}^2 \cdot \rho_{inz2}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in2} - \dfrac{R \cdot X_{10} \cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot A_{0utz2} \cdot Cc \cdot R \cdot X_{11} \cdot (w_{out} - X_{12})}{2 \cdot V_{z2} \cdot \left(R \cdot X_{10} - \dfrac{P_{atm} \cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz2}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot X_{11}^2}\right)^{\frac{1}{2}}}}{X_{11}}$$

$$A_{16,11} = \frac{A_{12} \cdot U}{C_{pSteel} \cdot m_{wall}}$$

$$A_{4,13} = -\frac{\beta \cdot (X_4 - w_{out}) \cdot \left(\dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}}}{V_{z3} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot P_{atm}}{2 \cdot X_{14} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3}^3 \cdot Cc^3 \cdot \rho_{out}^2 \cdot V_{z3} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}}}{A_{1in}^2 \cdot X_{14}^2 \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{3}{2}}}\right)}{m_{mix}}$$

$$A_{17,11} = \frac{A_{12} \cdot U}{C_{pSteel} \cdot m_{wall}}$$

Appendix

$$A_{12,12} = -\frac{\dfrac{2^{\frac{1}{2}} \cdot U_2 \cdot Cc \cdot \rho_{inz2} \cdot \left(\dfrac{P_{in2} - \dfrac{R \cdot X_{10} \cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_2^2 \cdot Cc^2 \cdot\cdot \rho_{z2}^2}{A_{1in}^2 \cdot\cdot \rho_{inz2}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz2} \cdot Cc \cdot X_{11} \cdot \left(R \cdot X_{10} - \dfrac{P_{atm} \cdot V_{z2}}{X_{11}}\right)^{\frac{1}{2}}}{V_{z2} \cdot \left(1 - \dfrac{A_{0utz2}^2 \cdot Cc^2 \cdot\cdot \rho_{out}^2 \cdot V_{z2}^2}{A_{1in}^2 \cdot X_{11}^2}\right)^{\frac{1}{2}}}}{X_{11}}$$

$$A_{1,13} = \frac{Cp \cdot \beta \cdot (T_{out} - X_1) \cdot \left(\dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}}}{V_{z3} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot P_{atm}}{2 \cdot X_{14} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3}^3 \cdot Cc^3 \cdot \rho_{out}^2 \cdot V_{z3} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}}}{A_{1in}^2 \cdot X_{14}^2 \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{3}{2}}}\right)}{m_{mix}}$$

$$A_{13,13} = \frac{2^{\frac{1}{2}} \cdot A_{0utz3}^3 \cdot Cc^3 \cdot \rho_{out}^2 \cdot V_{z3} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}}}{A_{1in}^2 \cdot X_{14}^2 \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{3}{2}}} - \frac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot P_{atm}}{2 \cdot X_{14} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}$$

$$- \frac{2^{\frac{1}{2}} \cdot U_3 \cdot Cc \cdot R \cdot X_{13}}{2 \cdot V_{z3} \cdot \left(1 - \dfrac{U_3 \cdot Cc \cdot \rho_{z3}}{A_{1in}^2 \cdot \rho_{inz3}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in3} - \dfrac{R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}} - \frac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}}}{V_{z3} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}$$

$A_{14,13}$

$$= \frac{\dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot C_{air} \cdot Cc \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot (T_{out} - X_{13})}{V_{z3} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{atm}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot C_{air} \cdot Cc \cdot P_{atm} \cdot (T_{out} - X_{13})}{2 \cdot X_{14} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{atm}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot U_3 \cdot C_{air} \cdot Cc \cdot R \cdot X_{13} \cdot (X_3 - X_{13})}{2 \cdot V_{z3} \cdot \left(1 - \dfrac{U_3^2 \cdot Cc^2 \cdot X_{14}^2}{A_{1in}^2 \cdot \rho_{inz3}^2 \cdot V_{z3}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{2 \cdot P_{in3} - \dfrac{2 \cdot R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{2 \cdot \rho_{inz3}}\right)^{\frac{1}{2}}}}{C_{air} \cdot X_{14}} \ldots$$

$$+ \frac{\dfrac{2^{\frac{1}{2}} \cdot U_3^3 \cdot C_{air} \cdot Cc^3 \cdot X_{14} \cdot (X_3 - X_{13}) \cdot \left(\dfrac{2 \cdot P_{in3} - \dfrac{2 \cdot R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{2 \cdot \rho_{inz3}}\right)^{\frac{1}{2}}}{A_{1in}^2 \cdot \rho_{inz3} \cdot V_{z3}^2 \cdot \left(1 - \dfrac{U_3^2 \cdot Cc^2 \cdot X_{14}^2}{A_{1in}^2 \cdot \rho_{inz3}^2 \cdot V_{z3}^2}\right)^{\frac{3}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3}^3 \cdot C_{air} \cdot Cc^3 \cdot \rho_{atm}^2 \cdot V_{z3} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot (T_{out} - X_{13})}{A_{1in}^2 \cdot X_{14}^2 \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{atm}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{3}{2}}}}{C_{air} \cdot X_{14}} \ldots$$

$$- \frac{10 \cdot h_{dash3} \cdot \left(U_8 - X_{13} + \dfrac{3}{10}\right) + A_{12} \cdot U \cdot (X_{17} - X_{13}) + \dfrac{2^{\frac{1}{2}} \cdot U_3 \cdot C_{air} \cdot Cc \cdot \rho_{inz3} \cdot (X_3 - X_{13}) \cdot \left(\dfrac{2 \cdot P_{in3} - \dfrac{2 \cdot R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{2 \cdot \rho_{inz3}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_3^2 \cdot Cc^2 \cdot X_{14}^2}{A_{1in}^2 \cdot \rho_{inz3}^2 \cdot V_{z3}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot C_{air} \cdot Cc \cdot X_{14} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot (T_{out} - X_{13})}{V_{z3} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{atm}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}}{C_{air} \cdot X_{14}^2}$$

$$- \frac{Q_{radz3} + Q_{wallz3} - 27}{Cp \cdot X_{14}^2}$$

Appendix

$$A_{15,13} = \frac{\dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot (w_{out} - X_{15})}{V_{z3} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot P_{atm} \cdot (w_{out} - X_{15})}{2 \cdot X_{14} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}}{X_{14}} \ldots$$

$$- \frac{\dfrac{2^{\frac{1}{2}} \cdot U_3 \cdot Cc \cdot R \cdot X_{13} \cdot (X_6 - X_{15})}{2 \cdot V_{z3} \cdot \left(1 - \dfrac{U_3 \cdot Cc \cdot \rho_{z3}}{A_{1in}^2 \cdot \rho_{inz3}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in3} - \dfrac{R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3}^3 \cdot Cc^3 \cdot \rho_{out}^2 \cdot V_{z3} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot (w_{out} - X_{15})}{A_{1in}^2 \cdot X_{14}^2 \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{3}{2}}}}{X_{14}} \ldots$$

$$- \frac{\dfrac{2^{\frac{1}{2}} \cdot U_3 \cdot Cc \cdot \rho_{inz3} \cdot (X_6 - X_{15}) \cdot \left(\dfrac{P_{in3} - \dfrac{R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_3 \cdot Cc \cdot \rho_{z3}}{A_{1in}^2 \cdot \rho_{inz3}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot X_{14} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot (w_{out} - X_{15})}{V_{z3} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}}{X_{14}^2}$$

$$A_{1,14} = \frac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot Cp \cdot R \cdot \beta \cdot X_{14} \cdot (T_{out} - X_1)}{2 \cdot V_{z3} \cdot m_{mix} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}$$

$$A_{4,14} = -\frac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot R \cdot \beta \cdot X_{14} \cdot (X_4 - w_{out})}{2 \cdot V_{z3} \cdot m_{mix} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}$$

$$A_{13,14} = -\frac{2^{\frac{1}{2}} \cdot U_3 \cdot Cc \cdot R \cdot X_{14}}{2 \cdot V_{z3} \cdot \left(1 - \dfrac{U_3 \cdot Cc \cdot \rho_{z3}}{A_{1in}^2 \cdot \rho_{inz3}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in3} - \dfrac{R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}} - \frac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot R \cdot X_{14}}{2 \cdot V_{z3} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}$$

$$A_{14,14} = -\frac{10 \cdot h_{dash3} + A_{12} \cdot U + \dfrac{2^{\frac{1}{2}} \cdot U_3 \cdot C_{air} \cdot Cc \cdot \backslash rho_{inz3} \cdot \left(\dfrac{2 \cdot P_{in3} - \dfrac{2 \cdot R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{2 \cdot \rho_{inz3}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_3^2 \cdot Cc^2 \cdot X_{14}^2}{A_{1in}^2 \cdot \rho_{inz3}^2 \cdot V_{z3}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot C_{air} \cdot Cc \cdot X_{14} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}}}{V_{z3} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{atm}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}}{C_{air} \cdot X_{14}} \ldots$$

$$- \frac{\dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot C_{air} \cdot Cc \cdot R \cdot X_{14} \cdot (T_{out} - X_{13})}{2 \cdot V_{z3} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{atm}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot U_3 \cdot C_{air} \cdot Cc \cdot R \cdot X_{14} \cdot (X_3 - X_{13})}{2 \cdot V_{z3} \cdot \left(1 - \dfrac{U_3^2 \cdot Cc^2 \cdot X_{14}^2}{A_{1in}^2 \cdot \rho_{inz3}^2 \cdot V_{z3}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{2 \cdot P_{in3} - \dfrac{2 \cdot R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{2 \cdot \rho_{inz3}}\right)^{\frac{1}{2}}}}{C_{air} \cdot X_{14}}$$

$$A_{15,14} = \frac{\dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot R \cdot X_{14} \cdot (w_{out} - X_{15})}{2 \cdot V_{z3} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}} - \dfrac{2^{\frac{1}{2}} \cdot U_3 \cdot Cc \cdot R \cdot X_{14} \cdot (X_6 - X_{15})}{2 \cdot V_{z3} \cdot \left(1 - \dfrac{U_3 \cdot Cc \cdot \backslash rho_{z3}}{A_{1in}^2 \cdot \rho_{inz3}^2}\right)^{\frac{1}{2}} \cdot \left(\dfrac{P_{in3} - \dfrac{R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}}}{X_{14}}$$

Appendix

$$A_{17,14} = \frac{A_{12} \cdot U}{C_{pSteel} \cdot m_{wall}}$$

$$A_{15,15} = -\frac{\dfrac{2^{\frac{1}{2}} \cdot U_3 \cdot Cc \cdot \rho_{inz3} \cdot \left(\dfrac{P_{in3} - \dfrac{R \cdot X_{13} \cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_3 \cdot Cc \cdot \rho_{z3}}{A_{1in}^2 \cdot \rho_{inz3}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot A_{0utz3} \cdot Cc \cdot X_{14} \cdot \left(R \cdot X_{13} - \dfrac{P_{atm} \cdot V_{z3}}{X_{14}}\right)^{\frac{1}{2}}}{V_{z3} \cdot \left(1 - \dfrac{A_{0utz3}^2 \cdot Cc^2 \cdot \rho_{out}^2 \cdot V_{z3}^2}{A_{1in}^2 \cdot X_{14}^2}\right)^{\frac{1}{2}}}}{X_{14}}$$

$$A_{11,16} = \frac{A_{12} \cdot U}{C_{air} \cdot X_{11}}$$

$$A_{16,16} = -\frac{2 \cdot A_{12} \cdot U}{C_{pSteel} \cdot m_{wall}}$$

$$A_{11,17} = \frac{A_{12} \cdot U}{C_{air} \cdot X_{11}}$$

$$A_{14,17} = \frac{A_{12} \cdot U}{C_{air} \cdot X_{14}}$$

$$A_{17,17} = -\frac{2 \cdot A_{12} \cdot U}{C_{pSteel} \cdot m_{wall}}$$

$$B_{7,1} = \frac{2^{\frac{1}{2}} \cdot Cc \cdot \rho_{inz1} \cdot \left(\dfrac{P_{in1} - \dfrac{R \cdot X_7 \cdot X_8}{V_{z1}}}{\rho_{inz1}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot \rho_{z1}}{A_{1in}^2 \cdot \rho_{inz1}^2}\right)^{\frac{1}{2}}} + \frac{2^{\frac{1}{2}} \cdot U_1^2 \cdot Cc^3 \cdot \backslash rho_{z1}^2 \cdot \left(\dfrac{P_{in1} - \dfrac{R \cdot X_7 \cdot X_8}{V_{z1}}}{\rho_{inz1}}\right)^{\frac{1}{2}}}{A_{1in}^2 \cdot \backslash rho_{inz1} \cdot \left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot \rho_{z1}^2}{A_{1in}^2 \cdot \rho_{inz1}^2}\right)^{\frac{3}{2}}}$$

$$B_{8,1} = \frac{\dfrac{2^{\frac{1}{2}} \cdot C_{air} \cdot Cc \cdot \rho_{inz1} \cdot (X_3 - X_7) \cdot \left(\dfrac{2 \cdot P_{in1} - \dfrac{2 \cdot R \cdot X_7 \cdot X_8}{V_{z1}}}{2 \cdot \rho_{inz1}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot X_8^2}{A_{1in}^2 \cdot \rho_{inz1}^2 \cdot V_{z1}^2}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot U_1^2 \cdot C_{air} \cdot Cc^3 \cdot X_8^2 \cdot (X_3 - X_7) \cdot \left(\dfrac{2 \cdot P_{in1} - \dfrac{2 \cdot R \cdot X_7 \cdot X_8}{V_{z1}}}{2 \cdot \rho_{inz1}}\right)^{\frac{1}{2}}}{A_{1in}^2 \cdot \rho_{inz1} \cdot V_{z1}^2 \cdot \left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot X_8^2}{A_{1in}^2 \cdot \rho_{inz1}^2 \cdot V_{z1}^2}\right)^{\frac{3}{2}}}}{C_{air} \cdot X_8}$$

$$B_{9,1} = \frac{\dfrac{2^{\frac{1}{2}} \cdot Cc \cdot \rho_{inz1} \cdot (X_6 - X_9) \cdot \left(\dfrac{P_{in1} - \dfrac{R \cdot X_7 \cdot X_8}{V_{z1}}}{\rho_{inz1}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot \backslash rho_{z1}^2}{A_{1in}^2 \cdot \rho_{inz1}{}^{\wedge}\text{"}}\right)^{\frac{1}{2}}} + \dfrac{2^{\frac{1}{2}} \cdot U_1^2 \cdot Cc^3 \cdot \rho_{z1}^2 \cdot (X_6 - X_9) \cdot \left(\dfrac{P_{in1} - \dfrac{R \cdot X_7 \cdot X_8}{V_{z1}}}{\rho_{inz1}}\right)^{\frac{1}{2}}}{A_{1in}^2 \cdot \rho_{inz1} \cdot \left(1 - \dfrac{U_1^2 \cdot Cc^2 \cdot \rho_{z1}^2}{A_{1in}^2 \cdot \rho_{inz1}^2}\right)^{\frac{3}{2}}}}{X_8}$$

$$B_{10,2} = \frac{2^{\frac{1}{2}} \cdot Cc \cdot \rho_{inz2} \cdot \left(\dfrac{P_{in2} - \dfrac{R \cdot X_{10} \cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}}{\left(1 - \dfrac{U_2^2 \cdot Cc^2 \cdot \rho_{z2}^2}{A_{1in}^2 \cdot \rho_{inz2}^2}\right)^{\frac{1}{2}}} + \frac{2^{\frac{1}{2}} \cdot U_2^2 \cdot Cc^3 \cdot \rho_{z2}^2 \cdot \left(\dfrac{P_{in2} - \dfrac{R \cdot X_{10} \cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}}{A_{1in}^2 \cdot \rho_{inz2} \cdot \left(1 - \dfrac{U_2^2 \cdot Cc^2 \cdot \rho_{z2}^2}{A_{1in}^2 \cdot \rho_{inz2}^2}\right)^{\frac{3}{2}}}$$

Appendix

$$B_{11,2}=\frac{\dfrac{2^{\frac{1}{2}}\cdot C_{air}\cdot Cc\cdot\rho_{inz2}\cdot(X_3-\ X_{10})\cdot\left(\dfrac{2\cdot P_{in2}-\dfrac{2\cdot R\cdot X_{10}\cdot X_{11}}{V_{z2}}}{2\cdot\rho_{inz2}}\right)^{\frac{1}{2}}}{\left(1\ -\dfrac{U_2^2\cdot Cc^2\cdot X_{11}^2}{A_{1in}^2\cdot\rho_{inz2}^2\cdot V_{z2}^2}\right)^{\frac{1}{2}}}+\dfrac{2^{\frac{1}{2}}\cdot U_2^2\cdot C_{air}\cdot Cc^3\cdot X_{11}^2\cdot(X_3-\ X_{10})\cdot\left(\dfrac{2\cdot P_{in2}-\dfrac{2\cdot R\cdot X_{10}\cdot X_{11}}{V_{z2}}}{2\cdot\rho_{inz2}}\right)^{\frac{1}{2}}}{A_{1in}^2\cdot\rho_{inz2}\cdot V_{z2}^2\cdot\left(1\ -\dfrac{U_2^2\cdot Cc^2\cdot X_{11}^2}{A_{1in}^2\cdot\rho_{inz2}^2\cdot V_{z2}^2}\right)^{\frac{3}{2}}}}{C_{air}\cdot X_{11}}$$

$$B_{12,2}=\frac{\dfrac{2^{\frac{1}{2}}\cdot Cc\cdot\rho_{inz2}\cdot(X_6-\ X_{12})\cdot\left(\dfrac{P_{in2}-\dfrac{R\cdot X_{10}\cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}}{\left(1\ -\dfrac{U_2^2\cdot Cc^2\cdot\rho_{z2}^2}{A_{1in}^2\cdot\rho_{inz2}^2}\right)^{\frac{1}{2}}}+\dfrac{2^{\frac{1}{2}}\cdot U_2^2\cdot Cc^3\cdot\rho_{z2}^2\cdot(X_6-\ X_{12})\cdot\left(\dfrac{P_{in2}-\dfrac{R\cdot X_{10}\cdot X_{11}}{V_{z2}}}{\rho_{inz2}}\right)^{\frac{1}{2}}}{A_{1in}^2\cdot\rho_{inz2}\cdot\left(1\ -\dfrac{U_2^2\cdot Cc^2\cdot\rho_{z2}^2}{A_{1in}^2\cdot\rho_{inz2}^2}\right)^{\frac{3}{2}}}}{X_{11}}$$

$$B_{13,3}=\frac{2^{\frac{1}{2}}\cdot Cc\cdot\rho_{inz3}\cdot\left(\dfrac{P_{in3}-\dfrac{R\cdot X_{13}\cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}}{\left(1\ -\dfrac{U_3\cdot Cc\cdot\rho_{z3}}{A_{1in}^2\cdot\rho_{inz3}^2}\right)^{\frac{1}{2}}}+\frac{2^{\frac{1}{2}}\cdot U_3\cdot Cc^2\cdot\rho_{z3}\cdot\left(\dfrac{P_{in3}-\dfrac{R\cdot X_{13}\cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}}{2\cdot A_{1in}^2\cdot\rho_{inz3}\cdot\left(1\ -\dfrac{U_3\cdot Cc\cdot\rho_{z3}}{A_{1in}^2\cdot\rho_{inz3}^2}\right)^{\frac{3}{2}}}$$

$$B_{15,3}=\frac{\dfrac{2^{\frac{1}{2}}\cdot Cc\cdot\rho_{inz3}\cdot(X_6-\ X_{15})\cdot\left(\dfrac{P_{in3}-\dfrac{R\cdot X_{13}\cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}}{\left(1\ -\dfrac{U_3\cdot Cc\cdot\rho_{z3}}{A_{1in}^2\cdot\rho_{inz3}^2}\right)^{\frac{1}{2}}}+\dfrac{2^{\frac{1}{2}}\cdot U_3\cdot Cc^2\cdot\rho_{z3}\cdot(X_6-\ X_{15})\cdot\left(\dfrac{P_{in3}-\dfrac{R\cdot X_{13}\cdot X_{14}}{V_{z3}}}{\rho_{inz3}}\right)^{\frac{1}{2}}}{2\cdot A_{1in}^2\cdot\rho_{inz3}\cdot\left(1\ -\dfrac{U_3\cdot Cc\cdot\rho_{z3}}{A_{1in}^2\cdot\rho_{inz3}^2}\right)^{\frac{3}{2}}}}{X_{14}}$$

$$B_{3,4}=-\frac{Cpw\cdot(T_{win,hc}-T_{wouthc})\cdot m_{dotwMAX}}{m_h}$$

$$B_{6,5}=\frac{1}{m_h}$$

$$B_{8,6}=\frac{10\cdot h_{dash1}}{C_{air}\cdot X_8}$$

$$B_{11,7}=\frac{10\cdot h_{dash2}}{C_{air}\cdot X_{11}}$$

$$B_{14,8}=\frac{10\cdot h_{dash3}}{C_{air}\cdot X_{14}}$$

$$C_{3,7}=\frac{R\cdot X_7}{V_{z1}}$$

$$C_{3,8}=\frac{R\cdot X_8}{V_{z1}}$$

$$C_{6,10}=\frac{R\cdot X_{10}}{V_{z2}}$$

$$C_{6,10}=\frac{R\cdot X_{11}}{V_{z2}}$$

$$C_{12,13} = \frac{R \cdot X_{13}}{V_{z3}}$$

$$C_{13,14} = \frac{R \cdot X_{14}}{V_{z3}}$$