Economic Production Optimization of a Power Plant with Constraints

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Abstract:

This project investigates the potential for profit of applying Model Predictive Control (MPC) for production planning in a Combined Heat and Power (CHP) plant which produces heat for District Heating (DH) and electricity. The project is based in the field of energy and conducted as a case study of Horsens Kraftvarmeværk.

The MPC controller generates the production plan by use of an optimization problem which considers plant model, electricity prices, district heating demand and production costs. Horsens Kraftvarmeværk is modeled as a high abstraction level, hybrid state model.

The MPC controller has been tested and evaluated in simulation using empirical data provided by DONG Energy.

Two types of analyses have been conducted in this project, a deterministic and a forecast based analysis. The results showed that the forecast based analysis generated a profit of 99% compared to the deterministic analysis. This is a good result considering the deterministic analysis provides a theoretical upper bound for the Model Predictive Control (MPC) algorithm.

Preface

This project has been conducted as an cooperation between Aalborg University and DONG Energy in the period Feb. 2nd - Jun. 3rd 2015.

The report is divided into three main parts in which a short introduction is given below.

Part 1: provides the preliminary analysis which presents the plant, the environment in which the plant interacts and the available data.

Part 2: describes the Model Predictive Control (MPC) algorithms and the models for the project.

Part 3: depicts the tests, simulations and the associated results of the project.

Acknowledgement

I want to pay a tribute to Kristian Edlund, Senior Energy System Architect at DONG Energy, for the committed support on the project. Kristian Edlund's contributions varies from comments on the report, answering questions about the energy sector and providing the empirical data which is used in the simulations.

Reading Guide

The report is written in a chronological order and it is recommended to read it in that manner. Several topics in the report have references to appendixes which expand on the topic.

The report is split into three parts. The topic of part one is the preliminary analysis. Part two describes the methods used in this reports. Part three handles evaluation and results.

Important Notations

This section provides a short introduction to the most important notation rules in this project. The dummy variable $\mathcal{D}$ is used to illustrate the notations.

Minimum/Maximum Limits

The maximum limit is $\overline{\mathcal{D}}$ and the minimum is denoted as $\underline{\mathcal{D}}$.

Forecasts

Forecasts are denotet with a hat e.g. $\hat{\mathcal{D}}$ is a forecast of $\mathcal{D}$.

Distance from set-point to limit

The distance from set-point to minimum limit is $\underline{\Delta\mathcal{D}}$ and the distance to maximum is $\overline{\Delta\mathcal{D}}$.

Symbol specification

Many symbols in this project have the same unit which means the variables needs to be more specific in the notations. This section provides inside in the structure.

$$(MainSymbol)_{\text{Module,Specifier}}[k] \tag{1}$$

Main Symbol: The main symbol indicates the unit of the variable. All symbols with equal main symbol have the same unit $Q_{\text{WB}i}$ and $Q_{\text{GT,H}}$ are both powers and have unit [MW]. Main symbols are only consisting of one letter.

Module: The first subscript in equation (1) indicates which component/module of the power plant the symbol belongs to. An example is $Q_{\text{GT,E}}$ and $C_{\text{GT,su}}$, where both have GT as first subscript. The subscript GT indicates that the variable is associated to the gas turbine.

Specifier: The specifier is placed after the comma in the symbol, and is the second subscript, seen in (1). The specifier is used to indicate a specific subpart of the variable. An example is when a module has more outputs, $Q_{\text{ST,H}}$ which is the output to district heating and $Q_{\text{ST,E}}$ which is the electricity output.

Contents

Nomenclature

Glossary

Balancing market

A passive market which all players in the energy market are part of. Deviations in energy production vs. day-ahead bid are penalized in the balancing market according to the prices settled in the regulating market.

Elspot

Day-ahead energy market. Bids on production volumes for each hour the following day are bid to Elspot at noon every day and auctioned by the TSO. The Elspot is a sub-market of the Nord Pool Spot.

Nord Pool Spot

Common power trading place for the Nordic and Baltic countries.

Regulating market

The TSO's tool to balance power demand and supply, by activation of regulating bids. Only deterministic producers can bid into the regulating market.

Acronyms

CHP	combined heat and power plant
CHPDH	combined heat and power plant district heating
CO	cooler
GT	gas turbine
HA	heat accumulator

NOIS	Nordic Operator Information System. The NOIS list contains the bids of the regulating market.
PP	Power Plant
ST	steam turbine
TSO	Transmission Service Operator. Energinet.dk is the Danish TSO.
VA	valve
WB	waste burner

Symbols

C_{DN}	down-regulation penalty within a particular hour, defined as $P_S - P_{DN}$	[DKK/MWh]
C_{UP}	up-regulation penalty within a particular hour, defined as $P_{UP} - P_S$	[DKK/MWh]
$C_{\mathrm{CO,su}}$	The start-up cost of the cooler	[DKK]
P_{CO}	Price of consuming 1 MWh energy on the cooler	[DKK/MWh]
$\overline{Q}_{\mathrm{CO}}$	Maximum power consumption from the cooler	[MW]
$\underline{Q}_{\mathrm{CO}}$	Minimum power consumption from the cooler	[MW]
Δ^+	positive part of deviation between bid and actual power production within a particular hour	[MW]
Δ^-	negative part of deviation between bid and actual power production within a particular hour	[MW]
Δ	deviation between bid and actual power production within a particular hour	[MW]
$C_{\mathrm{EP,su}}$	The start-up cost of the external producer	[DKK]
P_{EP}	Price getting 1MWh energy from external producer	[DKK/MWh]
$\overline{Q}_{\mathrm{EP}}$	Maximum power production from the external producer	[MW]
$\underline{Q}_{\mathrm{EP}}$	Minimum power production from the external producer	[MW]
ΔT	The sample time of the discrete system	[hour]
ε	A persentage of the total signal	[·]
Q_{E}	The sum of all electrical power	[MW]
Q_{S}	The sum of all steam power	[MW]
$C_{\mathrm{GT,su}}$	The start-up cost of the gas turbine	[DKK]
$\varepsilon_{\mathrm{GT,E}}$	The percentage of the power output from the gas turbine which is electricity power	[·]

$\varepsilon_{\mathrm{GT,H}}$	The percentage of the power output from the gas turbine which is heat power	[·]
$\varepsilon_{\mathrm{GT,S}}$	The percentage of the power output from the gas turbine which is steam power	[·]
η_{GT}	The power efficiency of the gas turbine	[·]
P_{GT}	Price of producing 1 MWh energy on the gas turbine	[DKK/MWh]
$\overline{Q}_{\mathrm{GT,E}}$	Maximum electric power output from the gas turbine	[MW]
$\underline{Q}_{\mathrm{GT,E}}$	Minimum electric power output from the gas turbine	[MW]
$Q_{\mathrm{GT,E}}$	The power output from the gas turbine in form of electricity	[MW]
$\overline{Q}_{\mathrm{GT,H}}$	Maximum heat power output from the gas turbine	[MW]
$\underline{Q}_{\mathrm{GT,H}}$	Minimum heat power output from the gas turbine	[MW]
$Q_{\mathrm{GT,H}}$	The power output from the gas turbine in form of heat	[MW]
$\overline{Q}_{\mathrm{GT}}$	Maximum power output from the gas turbine	[MW]
$\underline{Q}_{\mathrm{GT}}$	Minimum power output from the gas turbine	[MW]
$\overline{Q}_{\mathrm{GT,S}}$	Maximum steam power output from the gas turbine	[MW]
$\underline{Q}_{\mathrm{GT,S}}$	Minimum steam power output from the gas turbine	[MW]
$Q_{\mathrm{GT,S}}$	The power output from the gas turbine in form of steam	[MW]
Q_{G}	The power input to the steam turbine in form of gas	[MW]
$C_{\mathrm{HA,su}}$	The start-up cost of the heat accumulator	[DKK]
$\overline{E}_{\mathrm{HA}}$	The maximum accumulated energy in the heat accumulator	[MWh]
$\underline{E}_{\mathrm{HA}}$	The minimum accumulated energy in the heat accumulator	[MWh]
E_{HA}	The accumulated energy in the heat accumulator	[MWh]
P_{HA}	Price getting 1MWh energy from heat accumulator	[DKK/MWh]
$\overline{Q}_{\mathrm{HA}}$	Maximum power production from the heat accumulator	[MW]
$\underline{Q}_{\mathrm{HA}}$	Minimum power production from the heat accumulator	[MW]
Q_{HA}	the power output from the heat accumulator	[MW]
Q_{H}	the power input to the heat accumulator from the power plant	[MW] f
P_S	energy spot price within a particular hour, as announced by the TSO the day ahead	[DKK/MWh]
P_{DN}	down-regulation price within a particular hour, determined by bid activations in the regulating market	[DKK/MWh]
P_{UP}	up-regulation price within a particular hour, determined by bid activations in the regulating market	[DKK/MWh]
Q_G	power production sent to the grid within a particular hour	[MW]
$C_{\mathrm{ST,su}}$	The start-up cost of the steam turbine	[DKK]
$\varepsilon_{\mathrm{ST,E}}$	The percentage of the power output from the steam turbine which is electricity power	[·]

$\varepsilon_{\mathrm{ST,H}}$	The percentage of the power output from the steam turbine which is heat power	[·]
η_{ST}	The power efficiency of the steam turbine	[·]
P_{ST}	Price of producing 1 MWh energy on the steam turbine	[DKK/MWh]
$Q_{\mathrm{ST,E}}$	The power output from the steam turbine in form of electricity	[MW]
$Q_{\mathrm{ST,H}}$	The power output from the steam turbine in form of heat	[MW]
$\overline{Q}_{\mathrm{ST}}$	Maximum electric power output from the steam turbine	[MW]
$\underline{Q}_{\mathrm{ST}}$	Minimum power output from steam turbine in operational mode	[MW]
T_S	day-ahead bid of energy production within a particular hour, made by the producer	[MW]
$Q_{\mathrm{W}i}$	Power input to wastburner i in for of waste	[MW]
$\overline{Q}_{\mathrm{VA,H}}$	The maximum power able to by-pass the steam turbine	[MW]
$\underline{Q}_{\mathrm{VA,H}}$	The minimum power able to by-pass the steam turbine	[MW]
$Q_{\mathrm{VA,H}}$	The power which is by-passing the steam turbine and used for heating	[MW]
$\overline{Q}_{\mathrm{VA,S}}$	The maximum input power to the steam turbine in operating state	[MW]
$\underline{Q}_{\mathrm{VA,S}}$	The minimum input power to the steam turbine in operating state	[MW]
$Q_{\mathrm{VA,S}}$	The power which enters the steam turbine	[MW]
$C_{\mathrm{WB,su}}$	The start-up cost of the waste burner	[DKK]
η_{WB}	The power efficiency of waste burner i	[MW]
P_{WB}	Price of producing 1 MWh energy from the waste burner	[DKK/MWh]
$Q_{\mathrm{WB}i}$	Power output from wasteburner i	[MW]
$\overline{Q}_{\mathrm{WB}}$	Maximum power output from waste burner	[MW]
$\underline{Q}_{\mathrm{WB}}$	Minimum power output from waste burner	[MW]
$z_{\mathrm{WB}i}$	The disturbance in output of waste burner i	[·]

Chapter 1

Introduction

This chapter provides an overview of the project. The topics touched upon in the introduction are the field of study, the objective of the research and the problem.

The field of interest in this project is the energy sector. The energy sector is these years going through some major changes. The political goal for the wind energy share in Denmark by 2020 is 50% [Energinet.dk, 2014b]. In 2014, wind power amounted 39.1 % of the total share of produced electricity [Energinet.dk, 2015]. The gradual change from fossil fuel energy to renewable energy is introducing obstacles in which solutions are needed. One major obstacle is the increasing fluctuation in available energy production. The fluctuation is mainly caused by stochastic energy producers as wind turbines and solar cells, which produce energy when wind and sun energy is available, even when consumer demand for energy is low. This causes large variations in the energy prices, and in some cases they become negative [Ingeniøren, 2015], which means the consumer is paid to consume. From a producers point of view negative prices are not of interest because they pay for delivering their product. This calls for the producers, to develop new strategies for how to schedule production. To deal with future challenges, producers need to consider an infrastructure which involves the use of energy storage systems, forecasts on production and consumption and sophisticated control systems.

Taking into account some of the general challenges mentioned above the focus of this project is narrowed down to a single power plant. The type of power plant is a combined heat and power plant (CHP), where the heat is used for district heating. This is sometimes called a combined heat and power plant district heating (CHPDH), but in this project the plant is referred to as a CHP.

The project is conducted as a case study of Horsens Kraftvarmeværk, which is a CHP plant. Horsens Kraftvarmeværk is providing district heating to Horsens area. The Heat is the main product and electricity is the secondary. Horsens Kraftvarmeværk is owned by Fjernvarme Horsens A.m.b.a., which bought it from DONG at a price of 89 million DKK. [EnergiWatch.dk]

A high abstraction level model of the CHP plant, with hybrid states, is build and simulated. The simulation is performed using empirical measurements and forecasts of Horsens Kraftvarmeværk. A set of optimization building blocks, which seek to describe the cost of certain effects in the CHP, have been developed, tested and evaluated.

1.1 Aim of this Research

The main objective of this thesis is to investigate the potential of optimizing the profit of a Combined Heat and Power CHP plant by applying MPC for production planning.

In order to answer the question "does MPC optimize the profit of the power plant?" a comparison with other viable production planning methods is necessary. This is not done in this project, instead this thesis aims to evaluate the viability of MPC for production planning and provide a firm comparison basis for other methods. The research provides estimates of profit and other interesting parameters which can be used in such a comparison.

The results collected from the simulations are used to find interesting behaviors in the interaction between the CHP plant and the MPC algorithm.

1.2 Initial Problem Statement

The initial problem statement is used as a guide line for the decisions made in this thesis.

> How to optimize profit of a Combined Heat and Power CHP plant, using Model Predictive Control (MPC) for production planning, without compromising the operation safeties of the plant.

The following list provides some core questions and how they are attempted answered.

- **Can MPC potentially provide profitable conditions for the CHP?**
  The question can be answered by using deterministic analysis called Value of Perfect Information (VOPI), where ideal conditions for generating profit is created. If the results of the VOPI analysis shows low or even negative profit MPC is considered a non viable method.
- **Can MPC destabilize the CHP?**
  It is very important to answer this question before implementing MPC. This research does not prove stability analytically, instead the stability is heuristically evaluated through simulation.
- **Is the MPC algorithm implementable?**
  Implementability of the MPC algorithm is dependent on several parameters. Examples are stability, solving time etc. These issues is discussed multiple occasions in the thesis.

PART I

PRELIMINARY ANALYSIS

Chapter 2

Preliminary Analysis of the Power Plant

This chapter describes the CHP in general and the environment in which it interacts. The chapter starts with a brief introduction to the environment followed by a description of the CHP.

2.1 District Heating

District Heating (DH) is a concept where one or more centralized production units deliver heat to a district which uses the heat for domestic, industrial and other purposes. Instead of using many small decentralized production units, which produces heat locally, a larger centralized plant delivers the heated water to the costumer through a closed network of pipes. In Denmark, 63 % of the households are heated by district heating [dbdh.dk, 2015].

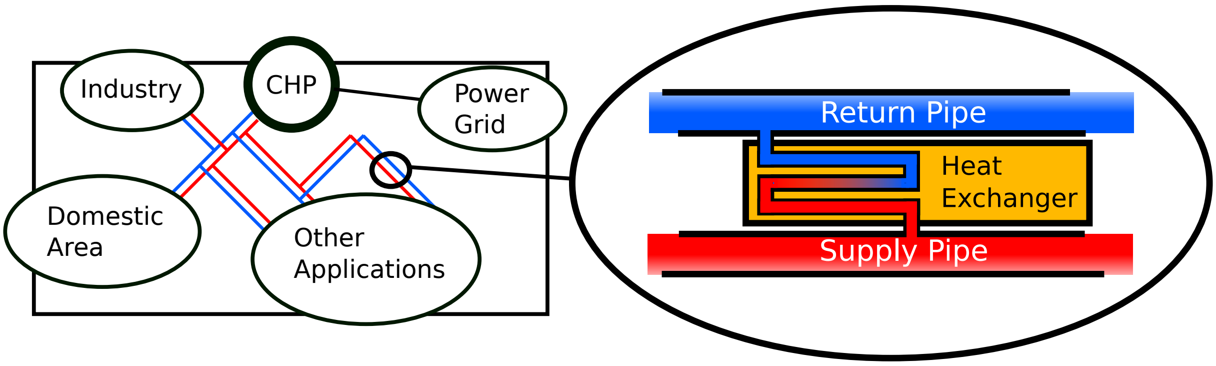

Figure 2.1: Shows a diagram

District heating is often applied as a method to remove excessive heat from a process. Many processes produce heat as byproduct, examples are steam based electricity production and servers parks. District heating is an energy efficient way to deliver usable heat energy. In a steam turbine power plant, the district heating application can raise the efficiency of burned fuel to approximately 85 % [Winther et al., 2012]. In a CHP, a steam turbine is driven by expanding steam on the inlet and condensation of steam at the outlet. This creates a pressure drop over the turbine, which drives the steam trough it. The steam is condensed by removing heat from the system, this heat can then be used as district heating.

District heating is a closed pipe system where heat producers delivers heat to the supply pipe (red pipe in Figure 2.1). The consumers use heat exchangers to take heat out of the system, the water is then send to the return pipe (blue pipe in Figure 2.1). The district heating water leaves the CHP plant at around 80 degrees Celsius through the supply line and return to the plant through the return line at 40 degrees

Celsius.

2.2 The Combined Heat and Power Plant

The CHP plant considered in this project is described in greater detail in this section. The analysis is founded on the case study which is Horsens Kraftvarmeværk. The concepts described are used as guide for the modeling chapter.

2.3 System Diagram of the Power Plant

This section presents the system diagram which shows how the components of the system are linked together. The boxes in Figure 2.2 describes the useful properties of the modules. The modules of the system are further described below.

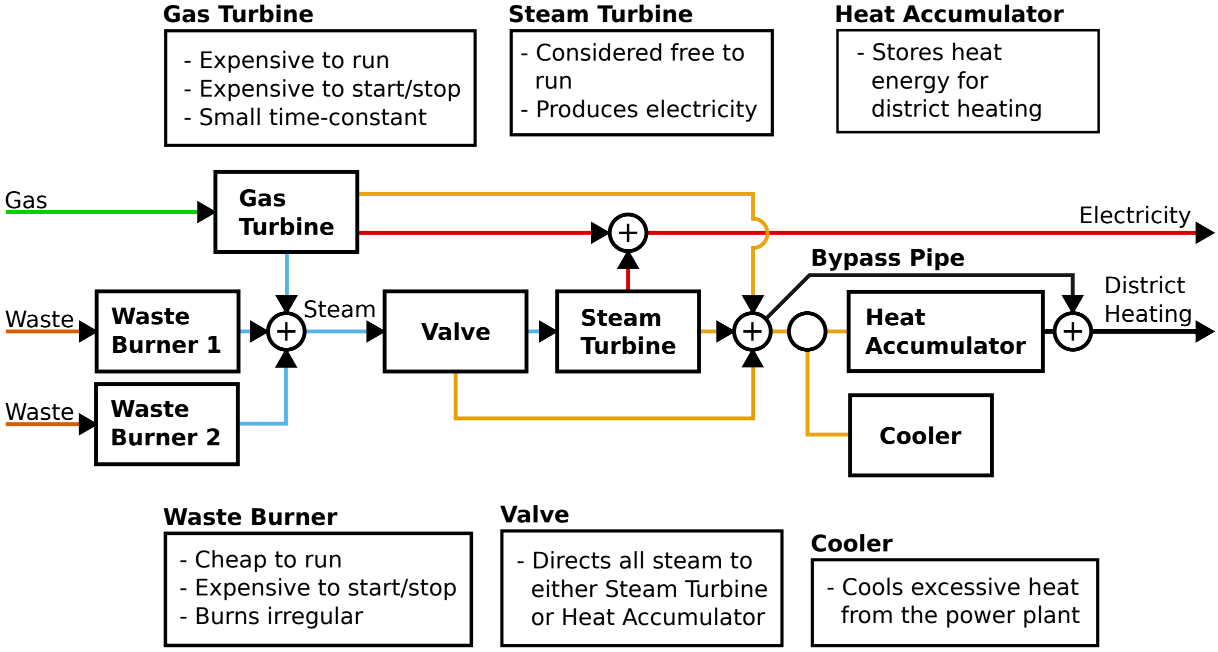

Figure 2.2: Shows the diagram of the power plant. The boxes describes the main properties of the components that makes out the power plant.

Waste burner: The waste burner (WB) delivers the main source of power in the CHP. The CHP is paid to burn the waste. This means the expenses related to burning the waste is almost covered by the payment. This makes the waste burner a cheap source of energy.

Gas Turbine; The gas turbine (GT) is the secondary source of energy. The GT can deliver extra power to the CHP when the waste burner proves insufficient, it is an expensive source of energy. The GT has three outputs, which shows which modules the GT delivers power to. First of all the GT produces electricity directly, but waste heat from the turbine generates superheated steam which can be used in the steam turbine. The heat energy which do not generate superheated steam can be used for district heating.

Steam Turbine: The steam turbine (ST) produces electrical power from the steam generated by the WB and the GT. The waste heat from the steam turbine can be used for district heating.

Valve: The valve (VA) can redirect the steam around the steam turbine. This means the produced steam can be used as energy for district heating instead of producing electricity. This feature is valuable in situations with high district heating demand.

Heat Accumulator: The heat accumulator (HA) is used to store energy. The HA act as buffer which can handle unexpected district heating demand variations.

Cooler: The cooler (CO) is used to consume excessive energy in the power plant. The cooler is used when too much heat is being produced.

2.4 Energy Sources

The power plant produces energy for the power grid and district heating. Due to the law of energy conservation the plant is not producing energy but converting it. The sources for the energy are waste and gas. This section gives an introduction to the advantages and disadvantages of using the energy sources.

2.4.1 Waste as Energy Source

The type of waste considered in this section is the type which goes into the waste burners.

Economical properties: Company is paid to burn waste because it is a product the society needs to get rid of. The income is approximately 150 kr pr. ton waste burned [ingeniøren, 2006].

Technical properties: The waste which goes into the burner is a mix of many different kinds of material. This makes the burning process irregular an the output energy is noisy.

Environmental properties: Burning waste emits less CO_2 than burning carbon based fossil fuels. How much less is uncertain due to the fact that waste is a mix of many different kinds of materials [ingeniøren, 2008].

Society properties: As mentioned above, the act of burning the waste is a way to get rid of the waste that otherwise builds up.

2.4.2 Gas as Energy Source

The type of waste considered in this section is the type which goes into the waste burners.

Economical properties: Comparing gas to waste, it is an expensive source of energy.

Technical properties: The gas is more homogeneous in the composition, than waste, which means it burns more regular than waste. This makes gas a stable energy input.

Environmental properties: Gas is a fossil fuel which means, it reintroduces CO_2 into the atmosphere as an external component.

2.5 Energy Outputs

The Products of the plant are electricity for the power grid and heat for district heating. This section describes how the products are sold in order to gain profit. The electricity and heat are sold in different ways, this is explained below.

2.5.1 Electricity

The electricity is sold on a market called Nord Pool Spot. The income from electricity comes from making a production bid T_{E}, for each hour in following day, on the Day-ahead market. The producer bids by generating a graph which shows the production amount as a function of the electricity price, this is further described in Appendix A.1. The Day-ahead market for the next day closes for bids at 12:00. If the production does not meet the bid T_{E} an imbalance penalty is paid. A more thorough description of the electricity market is found in Appendix A.1.

2.5.2 District Heating

The heat for DH is not sold on a market, instead there exist a deal where the CHP meets the DH demand and it is then paid a fixed price for each delivered MWh.

2.6 Power Plant Control Systems

The plant is considered to have two main control systems. This section gives a brief introduction to the considered controllers of the power plant.

Stabilizing Controller The objective of the stabilizing control system is to supply power according to the district heating demand. The demand changes continuously which means the controller needs a sampling frequency high enough to catch the dynamics of the process. The window which is considered by the controller is short relative to the production schedule controller.

Production Planning Controller: The objective of the production planning controller is to make production decisions based on economical perspectives. The production schedule can consider days ahead in time. The sampling time is slow compared to the stabilizing controller. The production planning controller makes the production decisions based on varies forecasts of demands and prices.

The two controller systems accommodate each other such that the production schedule decides the overall production set-points and the stabilizing-controller corrects deviations between supply and demand in real time, by making small changes from the chosen set-points. The deviations between supply and demand is introduced by forecast errors described.

Chapter 3

Empirical Data

This chapter gives an introduction to the empirical data which is available in this project. The data is provided by DONG Energy, and is gathered from Horsens Kraftvarmeværk. The data is analyzed in order to gain knowledge about behavior, tendencies and probabilities of the data. The data is ordered under two groups, measurements and forecasts.

Description	Symbol	Unit	Time Period	Resolution
Forecast of District Heat Demand	$\hat{r}_{\mathrm{DH}}$	[MW]	9/1-2012 - 31/12-2012	1 hour
Destrict Heat Demand	r_{DH}	[MW]	1/1-2012 - 31/12-2012	1 hour
Elspot Price DK1	P_S	[DKK/MWh]	1/1-2011 - 10/1-2013	1 hour
Up-regulation Penalty	C_{UP}	[DKK/MWh]	1/1-2011 - 10/1-2013	1 hour
Down-regulation Penalty	C_{DN}	[DKK/MWh]	1/1-2011 - 10/1-2013	1 hour
Forecast of Elspot Price DK1	$\hat{P}_S$	[DKK/MWh]	16/12-2012 - 2/2-2013	1 hour
Forecast of Up-regulation Penalty	$\hat{C}_{UP}$	[DKK/MWh]	1/1-2011 - 10/1-2013	1 hour
Forecast of Down-regulation Penalty	$\hat{C}_{DN}$	[DKK/MWh]	1/1-2011 - 10/1-2013	1 hour

Table 3.1: The data for the CHP Plant is delivered by DONG Energy

Table 3.1 presents the available data of interest. The next section analyses the data to search for usable tendencies and properties.

Notation

The notation used for measurements and forecasts are described in this section. Measurements are referred to as any other variable, e.g measurement $\mathcal{D}$. A reference given to a specific sample k of $\mathcal{D}$ is denoted $\mathcal{D}[k]$. A forecast is referred to using a hat, e.g. forecast $\hat{\mathcal{D}}$. References to a specific forecast sample is denoted $\hat{\mathcal{D}}[k|k-n]$, which means $\hat{\mathcal{D}}$ is a forecast for sample k issued in sample $k-n$.

3.1 Analysis of District Heating Demand Data

The District Heating demand is denoted r_{DH}. In order to schedule the production of district heating, forecasts of the demand $\hat{r}_{\mathrm{DH}}$ have been generated. The forecasts predict the demand up to 178 hours into the future. The district heating forecasts are denoted $\hat{r}_{\mathrm{DH}}[k|k-n]$, which means a forecast for hour k is issued in hour $k-n$ given the available information.

Figure 3.1 shows the district heating demand in Horsens area in 2012. The graph in Figure 3.1 is consisting of a low frequency tendency which follows the yearly cycle where the demand for heat is higher in winter than summer. The high frequency tendency follows the daily cycle which is highly effected by human behavior and the daily temperature cycle.

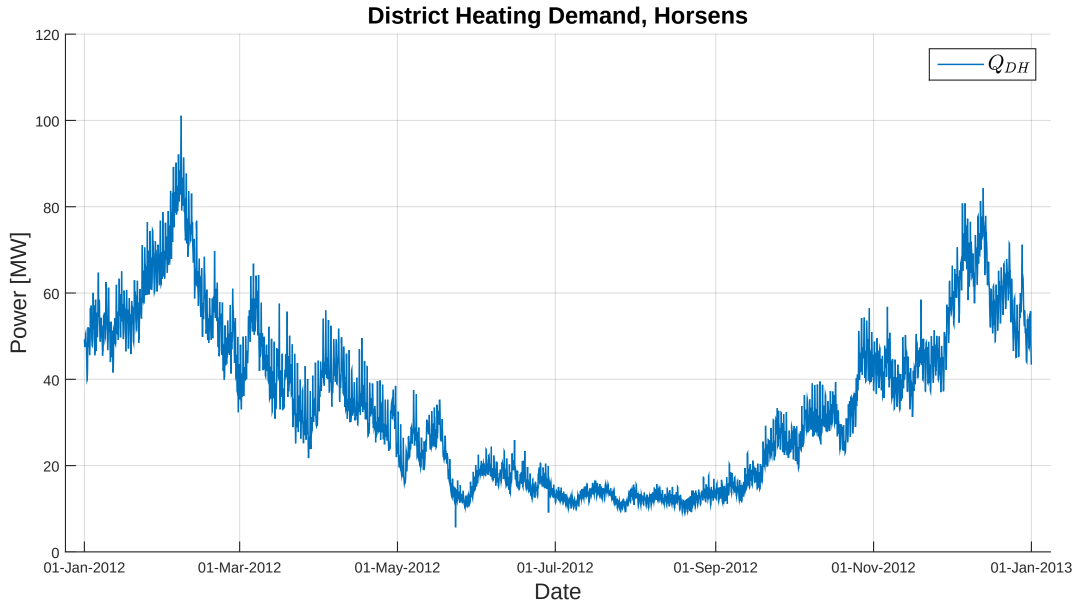

Figure 3.1: Shows the district heating demand in Horsens, 2012. The data is sampled each hour.

The Prediction Error

The prediction error of the district heating demand forecast, introduced in this section, is an important concept for this project an is used throughout the report. Consider the prediction error $e_{\mathrm{DH},n}$ which is defined as

$$e_{\mathrm{DH},n}[k] = \hat{r}_{\mathrm{DH}}[k|k-n] - r_{\mathrm{DH}}[k] \tag{3.1}$$

which describes the error between the forecast and actual district heat demand. The variable n describes how many hours before use, the forecast is issued. In order to investigate the prediction error a series of random variables are defined, $e_{\mathrm{DH},n}$ where $n = \{1, 2, \ldots, 178\}$. Figure 3.3 to 3.4 describes the properties of the random variables $e_{\mathrm{DH},n}[k]$. For each window size between 1 and 178 a histogram is made. Three examples are presented in Figure 3.2. The three figures show the distribution of the prediction error for a forecast made 1, 80 and 160 hours ahead. Taking a look at the x-axis shows that the longer prediction window the larger variance of the error. Two common distributions have been fitted to the histograms. The first is a t-location scale distribution which provides a good fit because it accommodates the heavy tales of the distribution. The second fit is a Normal distribution which is chosen because there are developed many methods to handle normal distributed noise.

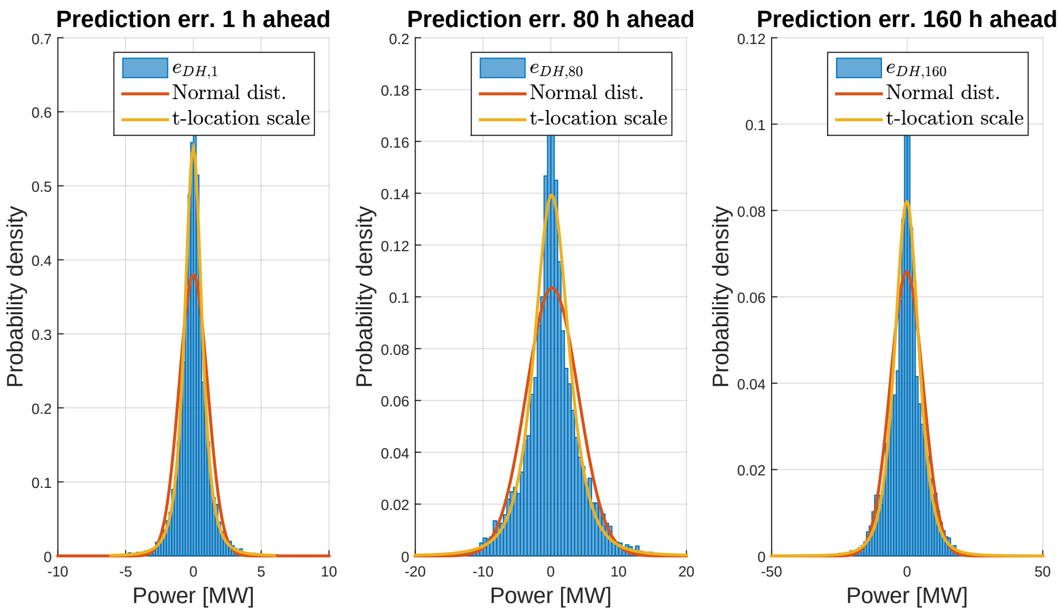

Figure 3.2: Shows the error between the predicted value and the actual result 1, 80 and 160 hours ahead. The longer into the future the prediction is made the higher the variance. Two distributions are fitted in the figure: A normal distribution and a t-location scale distribution.

Figure 3.3 shows the variance and mean value of the prediction error $e_{\mathrm{DH},n}$ for $n = \{1, 2, \ldots 178\}$. Figure 3.3 is closely related to 3.2 because from the variance and mean value the estimated normal distribution can be described. Figure 3.3 presents variance and mean as a function of the prediction window n. As expected the variance is increasing the longer into the future the prediction is maid. The mean value of the prediction errors are zero for the entire window, this indicates that the random variables are unbiased.

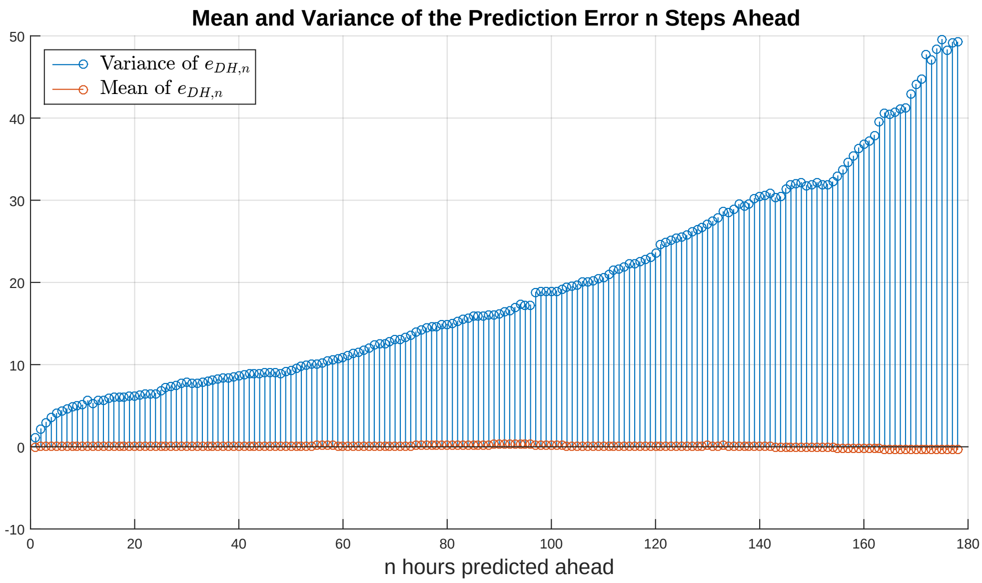

Figure 3.3: Shows how the variance increase with the window size. The mean value of the prediction error is consequently zero.

Figure 3.3 shows how the variance increases with window size the question is "can it be beneficial to consider forecasts with high variance?". Figure 3.4 shows the forecast compared with the resulting demand. The DH forecast $\hat{r}_{\mathrm{DH}}[k|k-1]$, seen in figure 3.4, is significantly more accurate than $\hat{r}_{\mathrm{DH}}[k|k-160]$, but $\hat{r}_{\mathrm{DH}}[k|k-160]$ it is able to catch the slow dynamical trend of the district heating demand, which may become useful.

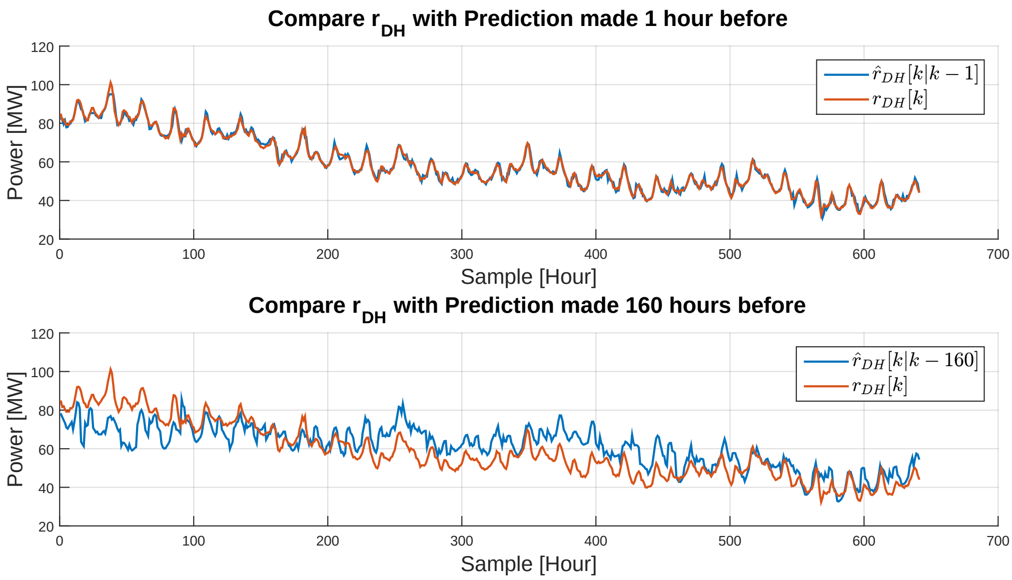

Figure 3.4: The upper graph shows the accuracy of the forecast made 1 hour ahead. The lower graph shows the accuracy of the heat demand forecast made 160 hours ahead.

Figure 3.5 shows the autocorrelation of the prediction error one step ahead in time $e_{\mathrm{DH},1}$. The autocorrelation shows the correlation in time is reasonably low, except with a time lag of 24 and 48 hours, which suggests the prediction algorithm has cyclic behavior.

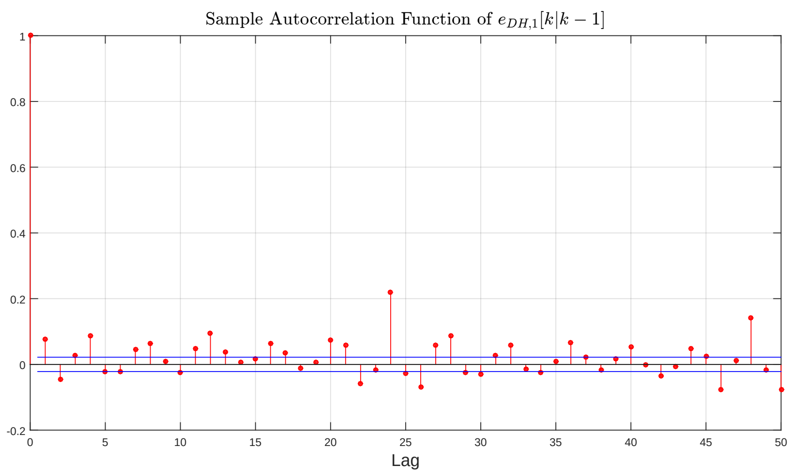

Figure 3.5: The autocorrelation of the prediction error $e_{\mathrm{DH},1}$, which shows that it almost is uncorrelated in time

From the investigation of the district heating data it is concluded that at least $e_{\mathrm{DH},1}$ resembles White Gaussian Noise. This is based on Figure 3.2, which shows $e_{\mathrm{DH},1}$ has zero mean and is normally distributed, and Figure 3.5 which shows low correlation in time.

Chapter 4

Scope of Project

This chapter narrows the scope of project to contain the subjects which are described beneath. Furthermore a brief formulation of subjects which are interesting in relation to the topic but left out of the scope for varies reasons are given.

The power grid is an interplay between many individual consumers and producers of varies sizes. This project focuses on a single CHP plant, and the objectives which are important as an individual power producer.

The CHP plant is described by a high abstract level, hybrid state model. The aspects of controlling the plant on lower abstraction levels are not considered in this project. The plant is assumed stable and set-point controllable.

The aspects of formulating a CHP plant, and the environment in which the plant interacts, as a optimization problem are analyzed.

The facets of bidding on the day-ahead market are considered because the deadline of the day-ahead market is introducing a discontinuous feed of information.

4.1 Subjects Outside the Scope

- Model of the CHP plant which describes the non-linear dynamics. Having a model which describes the non-linear behavior of the plant can prove to be a useful tool to evaluate the performance and limitation of the MPC algorithm.
- Uncertainties in form of parameter deviations are not considered.
- The Regulating market which is the market where the producer can speculate in being in reserve up and down regulation of the power grid.
- Implementation aspects of the MPC algorithm which counts handling infeasible problems in the optimization problem, computation time, proof of stability.

Summary

This concludes Part I which provided an introduction to the CHP plant, the environment in which it interacts and the empirical data. The findings and assumptions made in this part are used in the next to formulate models of the CHP plant and generating optimization building blocks which can be used for the MPC algorithm.

Part II

Model Predictive Control

Chapter 5

Model Predictive Control

This chapter builds the foundation for part II by presenting the general aspects of MPC. The topics discusses in this chapter are later used to formulate the actual controllers.

The MPC algorithm was first developed and used by the industry, which quickly realized the usefulness of the algorithm in varies applications [Maciejowski, 2002, .p xi]. Some of the features of MPC count as the ability to handle multivariate systems, account for limits caused by actuators and safety measures. Today MPC is still a hot topic with new scientific results continuously being published on solving-algorithms and application areas. A couple of examples are [Stephens et al., 2015] which includes game theory perspectives and [Danielson and Borrelli, 2015], which utilizes symmetries in the system. MPC, which is computational heavy, benefits a lot from the continuous increase in calculation power. This means that new application areas open up for MPC all the time. MPC, which started as a method applied to systems with slow dynamics, can today be applied to systems with fast dynamics [Wang and Boyd, 2010].

5.1 The flow of the Model Predictive Control Algorithm

The MPC algorithm is an iterative process which controls a system by solving an optimization problem each time step. The optimization problem considers available information about the system, which is used to decide the control input. The MPC algorithm considers future conditions, of system and environment, by evaluating a window ahead in time. The algorithm is stated in the list below.

1. set $k = k+1$
2. Collect and sort necessary information for the optimization problem. The information consist of plant measurements $y[k]$ and forecasts.
3. Solve the optimization problem over the control window L to find the optimal time sequence of inputs u^*, where

$$u^* = \begin{bmatrix} u^*[k+0] & u^*[k+1] & \cdots & u^*[k+L-1] \end{bmatrix}^T \tag{5.1}$$

4. Apply the first input $u^*[k]$ to the plant, discard the rest.
5. Go to step 1

The implementation of the algorithm is discussed later in the simulation part.

5.2 Timing

This section describes the timing of the MPC algorithm. The MPC algorithm considered in this project is discrete with a sampling time ΔT. The sampling time ΔT is chosen in relation to the sample time of the data, presented in Chapter 3, which is sampled with an one hour interval. This sampling rate is slow in relation to the plant dynamics. The sampling rate lays the foundation for the assumptions on timing of the MPC algorithm presented below.

Assumptions

- The set-points chosen by the MPC algorithm are average values for the hour to come.
- No calculation time.
- The set-points are reached immediately.

The arguments for stating the assumptions are: (1) the sampling time is one hour and it takes approximately a second to calculate the production schedule, (2) the CHP can ramp to the set-point within 5 minutes. Figure 5.1 illustrates the effects of the assumptions. Consider the dashed vertical line in sample k. At that instance the optimization problem is solved, the input is applied and the set-point changed. This is not realistic in reality, but the slow sampling rate makes the assumption reasonable.

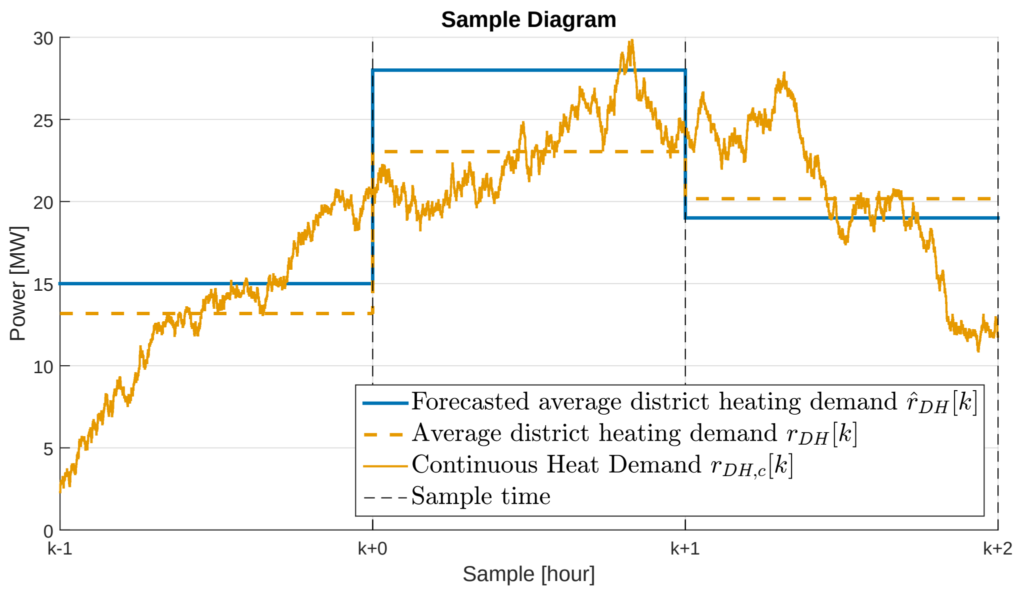

Figure 5.1: Shows how the DH demand and supply signal looks in the simulation compared to reality

In reality the district heating demand changes continuously as seen in Figure 5.1. The value considered for the MPC algorithm is the average heat demand over the hour. The relationship between the continuous heat demand and the average over the hour is seen in

$$r_{\mathrm{DH}}[k] = \frac{1}{\Delta T}\int_{k\Delta T}^{(k+1)\Delta T} r_{\mathrm{DH},c}(t)\mathrm{d}t \tag{5.2}$$

From this point on when referred to heat demand it is the average heat demand which is sampled ones per hour.

In Figure 5.1 the predicted average DH demand $r_{\mathrm{DH}}[k|k-1]$ is presented. The difference between predicted and actual heat demand and the consequences of this difference are discussed later in Section 7.6.

5.3 The Optimization Problem

The backbone of MPC is the optimization problem, which consist of three important components: (1) the plant model, (2) objective function and (3) constraints. The three components are described in this section. A brief introduction to the plant model and the constraints are given before the three components are discussed as a unity in Section 5.3.3.

5.3.1 The Plant Model

In MPC, the model of the plant is an active part of the algorithm. In PID-control, the model is only used to tune the controller. This means (though not recommendable) the controller can be implemented without any knowledge of the model. In MPC the model is an integrated part which means a mathematical formulation of the system is necessary. The model is used to predict the states over the window L, hence the name Model Predictive Control. In the material on MPC the models are described in varies ways. In this project the state space formulation, seen in (5.3), is chosen.

$$x[k+1] = \mathbf{A}x[k] + \mathbf{B}u[k] \tag{5.3a}$$

$$y[k] = \mathbf{C}x[k] + \mathbf{D}u[k] \tag{5.3b}$$

Non-linear models can also be used in MPC. Non-linear models are difficult to handle in MPC because they easily make the optimization problem non-convex. The only non-linear element considered in this project, is the hybrid states of the CHP. This is described in the Section 6.9.

5.3.2 The Constraints

The constraints of the optimization problem can be used in many ways. The first an most obvious is to describe physical plant limitations. These limitations can be decided for varies reasons, examples are production limits, accumulator capacity limit, and security enforced limits. Other constraints are part of optimization models, which are used to generate certain effects in the optimization problem, these models are described in Chapter 7.

5.3.3 The Formulation of the Optimization Problem

The 0-1 Mixed Integer Programming (0-1 MIP), seen in (5.4), represents the general structure for optimization problems for MPC in this thesis. The objective functions and the constraints which are used for the simulations are specified later in the report.

$$\max_{u\in\mathbb{R}^p, b\in\mathbb{B}^h} \sum_{i=0}^{L} \left({c_y}^T y[k+i] + {c_u}^T u[k+i] \right) \tag{5.4a}$$

Subject to

$$i = \{0, 1, \ldots, L-1\}$$

$${c_r}^T y[k+i] = r[k+i] \tag{5.4b}$$

$$x[k+1+i] = \mathbf{A}x[k+i] + \mathbf{B}u[k+i] \tag{5.4c}$$

$$y[k+i] = \mathbf{C}x[k+i] + \mathbf{D}u[k+i] \tag{5.4d}$$

$$\mathbf{0} = \mathbf{C}_z x[k+i] + \mathbf{D}_z u[k+i] \tag{5.4e}$$

$$\mathbf{D}_b u[k+i] + \mathbf{G}_b b[k+i] \leq c_b \tag{5.4f}$$

$$\underline{u} \leq u[k+i] \leq \overline{u} \tag{5.4g}$$

$$\underline{z} \leq z[k+i] \leq \overline{z} \tag{5.4h}$$

where

$y[k+i] \in \mathbb{R}^q$	The outputs of the plant	$[\cdot]$
$u[k+i] \in \mathbb{R}^p$	The inputs to the plant	$[\cdot]$
$x[k+i] \in \mathbb{R}^n$	The states of the plant associated with dynamics	$[\cdot]$
$\mathbf{0}$	A vector containing zeros	$[\cdot]$
$r[k+i] \in \mathbb{R}$	The power demand	[MW]
$\mathbf{c_r}$	A vector that picks out the signals of interest	$[\cdot]$
$\mathbf{b} \in \mathbb{B}^h$	The binary decision variables	$[\cdot]$

The optimization problem (5.4) can be restated as the general 0-1 MIP problem, seen in (5.5), which is described in [Nemhauser and Wolsey, 1988, p. 4]. The general theory on 0-1 MIP problems is further described in Appendix B.

$$\min_{x,b} c^T x + h^T b \tag{5.5a}$$

Subject to

$$d \geq \mathcal{A}x + \mathcal{G}b \tag{5.5b}$$

The 0-1 MIP (5.4) is inspired by the Linear Programming (LP) problem posted in [Hovgaard et al., 2010]. The following sections present some of the main differences between the 0-1 MIP (5.4) and the LP problem posted in [Hovgaard et al., 2010].

0-1 Mixed Integer and Linear Programming

The main difference between (5.4) and the one presented in [Hovgaard et al., 2010] is rooted in the differences between 0-1 MIP and LP. First of all, LP is convex, which is described in Appendix B, and 0-1 MIP is not. 0-1 MIP is not convex due to the nature of integers, which are not continuous. The LP and 0-1 MIP problem looks similar but the computational difference can be very high. The introduction of binary values in the optimization problem can cause combinatorial issues. The combinatorial effect is shown in Example 5.1, where one binary variable, generates two optimization problems.

Example 5.1 (Combinatorics in 0-1 Mixed Integer Programming)
Consider the scaler valued 0-1 MIP problem

$$V = \min_{x\in\mathbb{R},b\in\mathbb{B}} c_0x \;\; \textbf{s.t.} \;\; c_1 \geq x + c_2b \tag{5.6}$$

which can be interpreted as the two linear programming problems

$$V|_{b=0} = \min_{x\in\mathbb{R},b\in\mathbb{B}} c_0x \;\; \textbf{s.t.} \;\; c_1 \geq x \tag{5.7}$$

and

$$V|_{b=1} = \min_{x\in\mathbb{R}} c_0x \;\; \textbf{s.t.} \;\; c_1 \geq x + c_2 \tag{5.8}$$

A first approach, is to solve both (5.7) and (5.8), and choosing b,x according to the set which generates the minimum solution.

$$\min_{x\in\mathbb{R},b\in\mathbb{B}} V = \min\left(V|_{b=0}, V|_{b=1}\right) \tag{5.9}$$

As seen Example 5.1 a problem with one binary variable can be stated as two linear problems. For N binary variables a number of 2^N combinations exists.

The advantage of including binary variables in the optimization problem is that it gives more freedom to design the system. A Mixed Integer Programming (MIP) model can describe hybrid states of a system. Some parts of the system might consist of ON/OFF-states where no middle ground exist. These kinds of binary states can be difficult to model satisfactory with continuous variables. The designer of the optimization problem needs to consider whether the advantages of using binary decision variables outweighs the disadvantages.

Supply and Demand Balance

The second difference between the two optimization problems lies in the supply-demand constraints. In (5.4) the supply-demand constraint is stated as an equality as seen in (5.10), where [Hovgaard et al., 2010] states it as an inequality, as seen in (5.11).

$$c_r{}^T y[k+i] = r[k+i] \tag{5.10}$$

$$c_y{}^T y[k+i] \geq r[k+i] \tag{5.11}$$

An optimization problem which contains (5.10) needs to find a feasible solution where the supply is equal to the demand, where (5.11) demands a solution where the supply is higher than the demand.

Constraints on Input Changes

The third difference between the two optimization problems is the constraint on the input change Δu. In [Hovgaard et al., 2010] the term is included. In this project the assumption is that the CHP do not have physical restrictions on set-point changes. This means constraints on input changes should not be included in the optimization problem. Instead a regularization term which accounts for the ware and tare caused by large set-point changes could be considered.

5.4 Stability

This section gives an introduction to the criteria that ensures stability of the MPC algorithm. Even though, stability of the MPC algorithm applied to the CHP simulation is not proven the aspects of this section can still be taken into account.

Length of Prediction Window

Stability of MPC can be ensured through the length of the prediction window [Maciejowski, 2002, .p 167]. It is important to remember that the optimization problem will not take anything beyond the prediction horizon into account. This means with a too short prediction horizon MPC can put the system into a state where it gets destroyed, because there exist no possible way to recover. A thumb rule is to ensure the window is longer than the slowest time constant of the system. An example is seen in Figure 5.2, where a brake controller needs a prediction window, which can see an obstacle before it enters the minimum brake distance of the car.

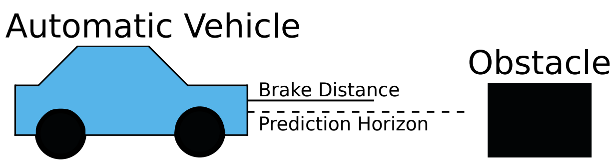

Figure 5.2: Shows how the car is able to brake in time, because the prediction horizon is longer than the brake distance.

Chapter 6
Model of the Combined Heat and Power Plant

This chapter describes the models of the power plant. The objective of the chapter is to formulate the models for use in the MPC algorithm. The constraints are described in Section 6.10 and the hybrid states in Section 6.9.

6.1 High Abstraction Level Hybrid Model of the Power Plant

The model presents a highly simplified version of an otherwise very complex process. The reason is the system is sampled on an hourly basis which hides the dynamics of the system. The model is based on simple input output relations of power. Figure 6.1 describes the signals of the model.

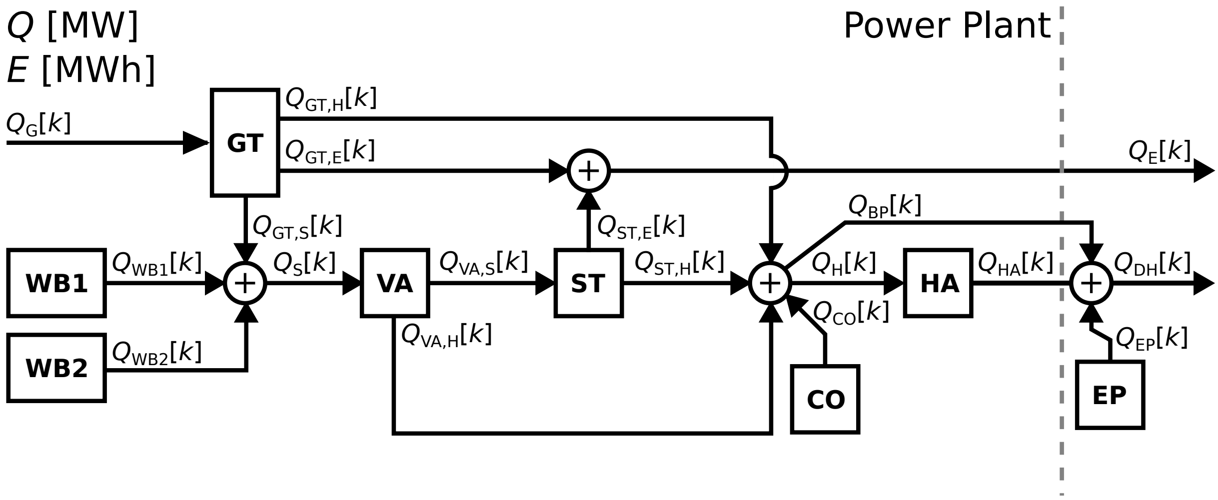

Figure 6.1: Shows the signals of the CHP plant. Q is power and E is Energy. The signals presented in this figure is used from this point forward

To ease the reading of the rest of the thesis it is recommended to learn the acronyms for the plant modules. The gas turbine is GT, The waste burner is WB, the valve is VA, The steam turbine is ST, the cooler is CO, the heat accumulator is HA and the extern production is EP.

Assumptions for the Plant Model

- The power flows of the model are divided into three groups: (1) electrical power (E) which is sold on the power grid, (2) heat power (H) which can be used for DH, and (3) steam power (S) can ether be converted to electricity in a turbine or used for heat.

- The signals, in the model, are described as useful power. This means the power which leaves a block x as output enters block y as input. The losses of energy due to friction, conversion etc. is accounted for within the blocks.
- The different components of the power plant are controlled on lower levels such that the entire system can be set point controlled. This means the dynamics of the system are reduced to proportional gains.
- Negative production is the equivalent to consumption.
- The system is modeled discrete with a sample time ΔT of one hour.
- The slow sampling time means all powers are considered as average values over the timespan ΔT

Common Symbols

This section gives a short introduction to the common symbols used to describe the models. The constant η is used to describe the overall efficiency of the module, which is the ratio between power entering the module and power exiting. An example is shown in (6.1), where MO stands for module.

$$Q_{\mathrm{MO}}[k] = \eta_{\mathrm{MO}} Q_{\mathrm{IN}}[k] \tag{6.1}$$

Some of the modules have multiple power outputs. The total power output is distributed over the different outputs using ε. An example with three power outputs is shown in (6.2).

$$1 = \varepsilon_{\mathrm{MO},H} + \varepsilon_{\mathrm{MO},E} + \varepsilon_{\mathrm{MO},S} \qquad \varepsilon_{\mathrm{MO},H}, \varepsilon_{\mathrm{MO},E}, \varepsilon_{\mathrm{MO},S} > 0 \tag{6.2}$$

Multiplying ε and η gives a map from input power to a specific output. In order to reduce symbols λ is introduced as $\lambda = \varepsilon\eta$. An example is shown in.

$$Q_{\mathrm{MO,S}}[k] = \eta_{\mathrm{MO}} \varepsilon_{\mathrm{MO,S}} \cdot Q_{\mathrm{IN}}[k] = \lambda_{\mathrm{MO,S}} Q_{\mathrm{IN}}[k] \tag{6.3}$$

6.2 Model of the Waste Burners (WB)

This model describes WB i in the power plant. There are two waste burners in the plant. Both waste burners are similar and therefore modeled equally.

$$Q_{\mathrm{WB}i}[k] = \eta_{\mathrm{WB}} Q_{\mathrm{W}i}[k] \qquad 0 \leq \eta_{\mathrm{WB}} \leq 1 \tag{6.4}$$

where

$Q_{\mathrm{WB}i}[k]$	is the power output from waste burner i	[MW]
$Q_{\mathrm{W}i}[k]$	is the chemical power of the waste entering the waste burner i	[MW]
η_{WB}	is power efficiency of the waste burner i	[·]
k	is the time instance	[hour]

Equation (6.4) is describing the output of waste burner i. The waste burner is modelled with an efficiency relation between input and output. The waste boiler has efficiency η_{WB}. The value is found in Table 6.

$$Q_{\mathrm{WB}}[k] = Q_{\mathrm{WB1}}[k] + Q_{\mathrm{WB2}}[k] \tag{6.5}$$

where

$Q_{\mathrm{WB}}[k]$	is the combined power output from the waste burners	[MW]

Equation (6.5) shows the expression for the combined output from the two waste burners.

Important Note on the Waste Burner: Due to slow sampling time, the WB model is a proportional gain, further more it has only one output. In order to save some variables the waste power Q_{W} seen in (6.5) is neglected and the output power Q_{WB} is used directly in the model as input. This can also be seen in Figure 6.1. If the model of the WB were to be described with dynamics Q_{W} should be reintroduced.

6.3 Model of the Gas Turbine (GT)

The model of the GT is described in this section. The gas turbine burns natural gas to generate power which is used in the power plant.

$$Q_{\mathrm{GT,H}}[k] = \eta_{\mathrm{GT}}\varepsilon_{\mathrm{GT,H}}Q_{\mathrm{G}}[k] = \lambda_{\mathrm{GT,H}}Q_{\mathrm{G}}[k] \tag{6.6}$$

$$Q_{\mathrm{GT,E}}[k] = \eta_{\mathrm{GT}}\varepsilon_{\mathrm{GT,E}}Q_{\mathrm{G}}[k] = \lambda_{\mathrm{GT,E}}Q_{\mathrm{G}}[k] \tag{6.7}$$

$$Q_{\mathrm{GT,S}}[k] = \eta_{\mathrm{GT}}\varepsilon_{\mathrm{GT,S}}Q_{\mathrm{G}}[k] = \lambda_{\mathrm{GT,S}}Q_{\mathrm{G}}[k] \tag{6.8}$$

where

$Q_{\mathrm{GT,H}}[k]$	is the power output from the gas turbine in form of heat	[MW]
$Q_{\mathrm{GT,E}}[k]$	is the power of the from the gas turbine in for of electricity	[MW]
$Q_{\mathrm{GT,S}}[k]$	is the power of the from the gas turbine in for of steam	[MW]
$Q_{\mathrm{G}}[k]$	is the chemical power input in form of burning gas	[MW]
η_{GT}	is power efficiency of the gas turbine	[·]
$\varepsilon_{\mathrm{GT,H}}$	is the percentage of the power output which is heat	[·]
$\varepsilon_{\mathrm{GT,E}}$	is the percentage of the power output which is electricity	[·]
$\varepsilon_{\mathrm{GT,S}}$	is the percentage of the power output which is steam	[·]

The power generated from the gas turbine has three usages: electricity, steam and heat for district heating. Equation (6.6) to (6.8) shows the 3 outputs.

$$1 = \varepsilon_{\mathrm{GT},Q} + \varepsilon_{\mathrm{GT},E} + \varepsilon_{\mathrm{GT},S} \qquad \varepsilon_{\mathrm{GT},Q}, \varepsilon_{\mathrm{GT},E}, \varepsilon_{\mathrm{GT},S} > 0 \tag{6.9}$$

Equation (6.9) shows how ε is used to divide the total produced power upon the three outputs. The use of ε is shown below. Equation (6.10) shows the total power output of the GT.

$$Q_{\mathrm{GT}}[k] = Q_{\mathrm{GT,H}}[k] + Q_{\mathrm{GT,E}}[k] + Q_{\mathrm{GT,S}}[k] \tag{6.10}$$

$$Q_{\mathrm{GT}}[k] = \eta_{\mathrm{GT}}\varepsilon_{\mathrm{GT,H}}Q_{\mathrm{G}}[k] + \eta_{\mathrm{GT}}\varepsilon_{\mathrm{GT,E}}Q_{\mathrm{G}}[k] + \eta_{\mathrm{GT}}\varepsilon_{\mathrm{GT,S}}Q_{\mathrm{G}}[k] \tag{6.11}$$

Substituting (6.6) to (6.8) into (6.10) gives (6.12) .

$$Q_{\mathrm{GT}}[k] = \eta_{\mathrm{GT}}(\varepsilon_{\mathrm{GT,H}} + \varepsilon_{\mathrm{GT,E}} + \varepsilon_{\mathrm{GT,S}})Q_{\mathrm{G}}[k] \tag{6.12}$$

Using the property stated in (6.9) gives (6.13)

$$Q_{\mathrm{GT}}[k] = \eta_{\mathrm{GT}}Q_{\mathrm{G}}[k] \tag{6.13}$$

6.4 Model of the Steam Turbine (ST)

The model of the ST is described in this section. The power generated from the steam turbine is used for two purposes. Electrical power and heat for DH. The input output relation is shown in (6.14) and (6.15).

$$Q_{\mathrm{ST,E}}[k] = \eta_{\mathrm{ST}}\varepsilon_{\mathrm{ST,E}}Q_{\mathrm{VA,S}}[k] = \lambda_{\mathrm{ST,E}}Q_{\mathrm{VA,S}}[k] \tag{6.14}$$

$$Q_{\mathrm{ST,H}}[k] = \eta_{\mathrm{ST}}\varepsilon_{\mathrm{ST,H}}Q_{\mathrm{VA,S}}[k] = \lambda_{\mathrm{ST,H}}Q_{\mathrm{VA,S}}[k] \tag{6.15}$$

where

$Q_{\mathrm{ST,H}}[k]$	is the power output from the gas turbine in form of heat	[MW]
$Q_{\mathrm{ST,E}}[k]$	is the power of the from the gas turbine in for of electricity	[MW]
η_{ST}	is power efficiency of the gas turbine	[$\cdot$]
$\varepsilon_{\mathrm{ST,H}}$	is the percentage of the power output that is heat	[$\cdot$]
$\varepsilon_{\mathrm{ST,E}}$	is the percentage of the power output that is electricity	[$\cdot$]

Similar to the GT, the ST has multiple outputs which are governed by a overall efficiency η_{ST} and ratio constants $\varepsilon_{\mathrm{ST,H}}$ and $\varepsilon_{\mathrm{ST,E}}$.

$$1 = \varepsilon_{\mathrm{GT,H}} + \varepsilon_{\mathrm{GT,E}} \qquad \varepsilon_{\mathrm{GT,H}}, \varepsilon_{\mathrm{GT,E}} > 0 \tag{6.16}$$

6.5 Model of the Heat Accumulator (HA)

The heat accumulator HA is described as a discrete integrator, which accumulates energy through the power input Q_{H} and output power to DH through Q_{HA}.

$$E_{\mathrm{HA}}[k+1] = E_{\mathrm{HA}}[k] + (Q_{\mathrm{H}}[k] - Q_{\mathrm{HA}}[k])\,\Delta T \tag{6.17}$$

where

$E_{\mathrm{HA}}[k]$	is the energy accumulated at time k	[MWh]
$Q_{\mathrm{HA}}[k]$	is the energy leaving the accumulator for district heating	[MW]
$Q_{\mathrm{H}}[k]$	the input power to the heat accumulator from the plant	[MW]
ΔT	is the sampling time	[h]

6.6 Model of the valve (VA)

The objective of the valve is to direct the steam through or bypass the ST. Consider equation

$$Q_{\mathrm{S}}[k] = Q_{\mathrm{VA,S}}[k] + Q_{\mathrm{VA,H}}[k] \tag{6.18}$$

where

$Q_{\mathrm{S}}[k]$	is the power entering the valve	[MW]
$Q_{\mathrm{VA,S}}[k]$	is the power which is directed throught the ST	[MW]
$Q_{\mathrm{VA,H}}[k]$	is the power which bypasses the ST	[MW]

Note: The valve is later used in a discrete state but for now it is just used to divide the steam between ST and the bypass.

6.7 Power Signals

This section describes the general expressions for the signals in the system. The signals of concern are power in form of steam, heat and electricity. Applying Kirchoffs current law in the sums seen in Figure 6.1 gives the following equations.

The steam sum Q_{S} is seen in (6.19a). The steam power variable Q_{S} is describing the total amount of power which is available to run the ST. Substituting the transfer functions for the contributers (6.5) and (6.8) into (6.19a) gives (6.19b).

$$Q_{\mathrm{S}}[k] = Q_{\mathrm{GT,S}}[k] + Q_{\mathrm{WB1}}[k] + Q_{\mathrm{WB2}}[k] \tag{6.19a}$$

$$Q_{\mathrm{S}}[k] = \lambda_{\mathrm{GT,S}} Q_{\mathrm{G}}[k] + Q_{\mathrm{WB1}}[k] + Q_{\mathrm{WB2}}[k] \tag{6.19b}$$

$$Q_{\mathrm{S}}[k] = Q_{\mathrm{VA,S}}[k] + Q_{\mathrm{VA,H}}[k] \tag{6.19c}$$

where
$Q_{\mathrm{S}}[k]$ is the total steam power [MW]

The electrical power output Q_{E} is a sum of the electrical power contributions from the GT and ST. These are the electrical producing components of the CHP.

$$Q_{\mathrm{E}}[k] = Q_{\mathrm{ST,E}}[k] + Q_{\mathrm{GT,E}}[k] = \lambda_{\mathrm{ST,E}}[k] Q_{\mathrm{VA,S}}[k] + \lambda_{\mathrm{GT,E}}[k] Q_{\mathrm{G}}[k] \tag{6.20}$$

Writing the output electrical power as a function of input signal yields (6.21).

$$Q_{\mathrm{E}}[k] = (\lambda_{\mathrm{ST,E}}\lambda_{\mathrm{GT,S}} + \lambda_{\mathrm{GT,E}})\, Q_{\mathrm{G}}[k] + \lambda_{\mathrm{ST,E}} Q_{\mathrm{WB1}}[k] + \lambda_{\mathrm{ST,E}} Q_{\mathrm{WB2}}[k] - \lambda_{\mathrm{ST,E}} Q_{\mathrm{VA,H}}[k] \tag{6.21}$$

where
$Q_{\mathrm{E}}[k]$ is the total electrical power produced from the plant [MW]

The total heat power Q_{H} is a measure of the heat entering the HA for later to be used for district heating.

$$Q_{\mathrm{H}}[k] = Q_{\mathrm{VA,H}}[k] + Q_{\mathrm{ST,H}}[k] + Q_{\mathrm{GT,H}}[k] + Q_{\mathrm{CO,H}}[k] - Q_{\mathrm{BP,H}}[k] \tag{6.22}$$

where
$Q_{\mathrm{H}}[k]$ is the total heat power to the heat accumulator [MW]

The power to district heating Q_{DH} is shown in (6.23).

$$Q_{\mathrm{DH}}[k] = Q_{\mathrm{BP}}[k] + Q_{\mathrm{HA}}[k] + Q_{\mathrm{EP}}[k] \tag{6.23}$$

6.8 The plant model as State Space System

This section presents the equations defined throughout this chapter in matrix form. The formulation of the system is seen in (6.24).

$$x[k+1] = \mathbf{A}x[k] + \mathbf{B}u[k] \tag{6.24a}$$

$$y = \mathbf{D_y}u \tag{6.24b}$$

$$\mathbf{0} = \mathbf{D_z}u \tag{6.24c}$$

$$\underline{u} \le u \le \overline{u} \tag{6.24d}$$

The symbol **0** is a vector containing zeros. The vectors and matrices is showed below.

$$x[k] = E_{\mathrm{HA}}[k] \tag{6.25}$$

$$u[k] = \begin{bmatrix} Q_{\mathrm{G}}[k] & Q_{\mathrm{WB1}}[k] & Q_{\mathrm{WB2}}[k] & Q_{\mathrm{VA,H}}[k] & Q_{\mathrm{VA,S}}[k] & Q_{\mathrm{CO}}[k] & Q_{\mathrm{H}}[k] & Q_{\mathrm{HA}}[k] & Q_{\mathrm{BP}}[k] & Q_{\mathrm{EP}}[k] \end{bmatrix}^T \tag{6.26}$$

$$y[k] = \begin{bmatrix} Q_{\mathrm{E}}[k] & Q_{\mathrm{DH}}[k] \end{bmatrix}^T \tag{6.27}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \Delta T & -\Delta T & 0 \end{bmatrix} \qquad \mathbf{A} = 1 \tag{6.28}$$

The matrix $\mathbf{D_y}$ is made from equation (6.20) and (6.23).

$$\mathbf{D_y} = \begin{bmatrix} \lambda_{\mathrm{GT,E}} & 0 & 0 & 0 & \lambda_{\mathrm{ST,E}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \tag{6.29}$$

The matrix $\mathbf{D_z}$ is generated from the energy balance equations (6.22) and (6.19)

$$\mathbf{D_z} = \begin{bmatrix} \lambda_{\mathrm{GT,S}} & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ \lambda_{\mathrm{GT,H}} & 0 & 0 & 1 & \lambda_{\mathrm{ST,H}} & 1 & -1 & -1 & 0 & 0 \end{bmatrix} \tag{6.30}$$

$$\underline{u} = \begin{bmatrix} \underline{Q}_{\mathrm{G}} & \underline{Q}_{\mathrm{WB1}} & \underline{Q}_{\mathrm{WB2}} & \underline{Q}_{\mathrm{VA,H}} & \underline{Q}_{\mathrm{CO}} & \underline{Q}_{\mathrm{H}} & \underline{Q}_{\mathrm{HA}} & \underline{Q}_{\mathrm{EP}} \end{bmatrix}^T \tag{6.31}$$

$$\overline{u} = \begin{bmatrix} \overline{Q}_{\mathrm{G}} & \overline{Q}_{\mathrm{WB1}} & \overline{Q}_{\mathrm{WB2}} & \overline{Q}_{\mathrm{VA,H}} & \overline{Q}_{\mathrm{CO}} & \overline{Q}_{\mathrm{H}} & \overline{Q}_{\mathrm{HA}} & \overline{Q}_{\mathrm{EP}} \end{bmatrix}^T \tag{6.32}$$

6.9 The Hybrid States of the Power Plant

Several components of the CHP can be in discrete states. This makes the CHP into a hybrid system. The GT can be turned OFF, which means it is not producing any power. The VA can switch discrete between directing all steam through or bypassing the ST, see Figure 6.1. This Section uses the constraints defined in Section 6.10 and the model described in Section 6.1 to outline the states of CHP.

The states of the components GT and VA can be described by binary integers. The interpretations of the binary values seen in (6.33) to (6.34) are formulated as in [Nemhauser and Wolsey, 1988, p. 5]. The state of the GT is denoted by b_{GT}, seen in (6.33).

$$b_{\mathrm{GT}}[k] = \begin{cases} 1 & \text{if GT is ON at sample k} \\ 0 & \text{if GT is OFF at sample k} \end{cases} \tag{6.33}$$

The state of the VA is denoted by b_{VA}, seen in (6.34).

$$b_{\mathrm{VA}}[k] = \begin{cases} 1 & \text{if VA bypasses the ST at sample k} \\ 0 & \text{if VA directs the steam to the ST at sample k} \end{cases} \tag{6.34}$$

The states which are combinations of the component states are shown in Table 6.1. With two components each having two states gives a total number of $2^2 = 4$ states. Table 6.1 gives a short introduction to the states. Each state is further described beneath.

State	$b_{\mathrm{GT}}[k]$	$b_{\mathrm{VA}}[k]$	**Short Description**
v_1	0	0	GT turned off. All steam is directed to ST by VA.
v_2	0	1	GT turned off. The ST is on while by-passed. This means the ST is running in minimum capacity $\underline{Q}_{\mathrm{ST}}$.
v_3	1	0	both the GT and the ST are on. No by-passed heat.
v_4	1	1	GT is turned on. VA by-passes ST. ST running minimum capacity $\underline{Q}_{\mathrm{ST}}$.

Table 6.1: Shows the CHP states as combinations of component states. Only feasible states are named

It is the operational conditions of the power plant that define which state the plant is operating in. The plant can be forced into a state by imposing linear equality and inequality constraints. The following sections describe which constraints that bring the power plant into the different states.

State v_1 ($b_{\mathbf{GT}}[k] = 0$, $b_{\mathbf{VA}}[k] = 0$)

In state v_1 the plant is running without the GT, but the ST are running. All steam produced is directed to the ST. The constraints shown in (6.35) shows the necessary conditions.

$$Q_{\mathrm{VA,S}} = Q_{\mathrm{S}} \qquad Q_{\mathrm{VA,H}} = 0 \qquad Q_{\mathrm{G}} = 0 \tag{6.35}$$

State v_2 ($b_{\mathbf{GT}}[k] = 0$, $b_{\mathbf{VA}}[k] = 1$)

State v_2 describes the plant when the GT is turned OFF and the ST is bypassed while running minimum capacity. This is shown in (6.36) where $Q_{\mathrm{VA,S}}$ is equal to the minimum production value of the ST and $Q_{\mathrm{VA,H}}$ is equal to the remainder.

$$Q_{\mathrm{VA,S}} = \underline{Q}_{\mathrm{VA,S}} \qquad Q_{\mathrm{VA,H}} = Q_{\mathrm{S}} - \underline{Q}_{\mathrm{VA,S}} \qquad Q_{\mathrm{G}} = 0 \tag{6.36}$$

State v_3 ($b_{\mathbf{GT}}[k] = 1$, $b_{\mathbf{VA}}[k] = 0$)

In state v_3 both the GT and ST are running and producing electricity. All produced steam is directed through the ST.

$$Q_{\mathrm{VA,S}} = Q_{\mathrm{S}} \qquad Q_{\mathrm{VA,H}} = 0 \qquad \underline{Q}_{\mathrm{G}} \leq Q_{\mathrm{G}} \leq \overline{Q}_{\mathrm{G}} \tag{6.37}$$

State v_4 ($b_{\mathbf{GT}}[k] = 1$, $b_{\mathbf{VA}}[k] = 1$)

In state v_4, the ST is bypassed but still runs on minimum capacity. The GT is turned ON.

$$Q_{\mathrm{VA,S}} = \underline{Q}_{\mathrm{VA,S}} \qquad Q_{\mathrm{VA,H}} = Q_{\mathrm{S}} - \underline{Q}_{\mathrm{VA,S}} \qquad \underline{Q}_{\mathrm{G}} \leq Q_{\mathrm{G}} \leq \overline{Q}_{\mathrm{G}} \tag{6.38}$$

Generalized Constraints using Binary Variables

This section uses the binary variables, defined from (6.33) to (6.34), to define inequality constraints that cover the constraints described from (6.35) to (6.38). The constraints can be seen in Table 6.2.

Constraint	Variable	Variable = 0	Variable = 1
$b_{\mathrm{GT}}\underline{Q}_{\mathrm{G}} \leq Q_{\mathrm{G}} \leq b_{\mathrm{GT}}\overline{Q}_{\mathrm{G}}$	b_{GT}	$0 \leq Q_{\mathrm{G}} \leq 0$	$\underline{Q}_{\mathrm{G}} \leq Q_{\mathrm{G}} \leq \overline{Q}_{\mathrm{G}}$
$b_{\mathrm{VA}}\underline{Q}_{\mathrm{VA,H}} \leq Q_{\mathrm{VA,H}} \leq b_{\mathrm{VA}}\overline{Q}_{\mathrm{VA,H}}$	b_{VA}	$0 \leq Q_{\mathrm{VA,H}} \leq 0$	$\underline{Q}_{\mathrm{VA,H}} \leq Q_{\mathrm{VA,H}} \leq \overline{Q}_{\mathrm{VA,H}}$
$Q_{\mathrm{VA,S}} \leq b_{\mathrm{VA}}\underline{Q}_{\mathrm{VA,S}} + (1-b_{\mathrm{VA}})\overline{Q}_{\mathrm{VA,S}}$	b_{VA}	$Q_{\mathrm{VA,S}} \leq \overline{Q}_{\mathrm{VA,S}}$	$Q_{\mathrm{VA,S}} \leq \underline{Q}_{\mathrm{VA,S}}$

Table 6.2: Shows the generalized constraints

6.10 The Physical Limits of the Plant and Production Prices

This section describes the constraints on the power plant when it produces energy. The constraints are imposed to keep the plant from running under unrealistic conditions, which might destroy it. All models presented in Chapter 6 have to account for the constraints presented in this section. The values in the tables can be used as validation for the optimization solution. If the result contains one or more values outside the constraints it is considered invalid.

Constraints for Waste Burner 1 and 2

The considered constraints for the waste burner 1 and 2 is shown in table 6.3.

Waste Burner 1, 2	Symbol	Value	Unit
Minimum power production	$\underline{Q}_{\mathrm{WB}}$	10.2	[MW]
Maximum power production	$\overline{Q}_{\mathrm{WB}}$	14.9	[MW]
Energy price	P_{WB}	22	[DKK/MWh]
Start-up cost	$C_{\mathrm{WB,su}}$	-	[DKK]

Table 6.3: The production capacities and costs of the waste burner

Notes: There is a start-up cost for the waste burner, but the burner is only turned off for maintenance or other reasons which is outside the scope of the problem.

Constraints for the Gas Turbine

The considered constraints for the gas turbine are shown in table 6.4.

Gas Turbine	Symbol	Value	Unit
Minimum total power production	$\underline{Q}_{GT}$	32.7	[MW]
Maximum total power production	$\overline{Q}_{GT}$	55.9	[MW]
Minimum electrical power production	$\underline{Q}_{GT,E}$	12	[MW]
Maximum electrical power production	$\overline{Q}_{GT,E}$	20.5	[MW]
Minimum steam power production	$\underline{Q}_{GT,S}$	14.5	[MW]
Maximum electrical steam production	$\overline{Q}_{GT,S}$	24.8	[MW]
Minimum heat power production	$\underline{Q}_{GT,H}$	6.2	[MW]
Maximum heat power production	$\overline{Q}_{GT,H}$	10.6	[MW]
Energy price	P_{GT}	811.7	[DKK/MWh]
Start-up cost	$C_{GT,su}$	10000	[DKK]
Production efficiency	η_{GT}	0.9	[·]
Electricity output per MW gas burned in the GT	$\lambda_{GT,E}$	0.37	[MW]
Heat power output per MW gas burned in the GT	$\lambda_{GT,H}$	0.19	[MW]
Steam power output per MW gas burned in the GT	$\lambda_{GT,S}$	0.44	[MW]

Table 6.4: The production capacities and costs of the waste burner

Notes: The GT can be turned off, which means the power is 0. The cost of starting the GT is 10000 DKK.

Constraints for the Steam Turbine

The considered constraints for the steam turbine are shown in table 6.5.

Steam Turbine	Symbol	Value	Unit
Minimum total power production	$\underline{Q}_{ST}$	12.7	[MW]
Maximum total power production	$\overline{Q}_{ST}$	54.1	[MW]
Minimum electrical power production	$\underline{Q}_{ST,E}$	3	[MW]
Maximum electrical power production	$\overline{Q}_{ST,E}$	12.2	[MW]
Energy price	P_{ST}	0	[DKK/MWh]
Production efficiency	η_{ST}	0.99	[MW]
Electricity output per MW	$\lambda_{ST,E}$	0.23	[MW]
Heat power output per MW	$\lambda_{ST,H}$	0.76	[MW]

Table 6.5: The production capacities and costs of the waste burner

Notes: The Energy price of the ST is considered 0 DKK/MHh because of two reasons: (1) the energy is paid for through the price of the waste burners and the GT, (2) the maintenance is not considered. As the

WB the ST cannot be turned OFF.

Constraints for the Valve

The considered constraints for the valve is shown in table 6.6.

Valve	**Symbol**	**Value**	**Unit**
Minimum power output to ST	$\underline{Q}_{\mathrm{VA,S}}$	12.7	[MW]
Maximum power output to ST	$\overline{Q}_{\mathrm{VA,S}}$	54.1	[MW]
Minimum power by-passing the ST	$\underline{Q}_{\mathrm{VA,H}}$	0	[MW]
Maximum power by-passing the ST	$\overline{Q}_{\mathrm{VA,H}}$	55	[MW]

Table 6.6: The production capacities and costs of the waste burner

Notes: The valve is switching between delivering power to the ST and bypassing the steam turbine in order to deliver more power to district heating.

Constraints for the Heat Accumulator

The considered constraints for the heat accumulator are shown in table 6.7.

Heat Accumulator	**Symbol**	**Value**	**Unit**
Minimum power output	$\underline{Q}_{\mathrm{HA}}$	0	[MW]
Maximum power output	$\overline{Q}_{\mathrm{HA}}$	31	[MW]
Minimum energy capacity	$\underline{E}_{\mathrm{HA}}$	50	[MWh]
Maximum energy capacity	$\overline{E}_{\mathrm{HA}}$	280	[MWh]
Energy price	P_{HA}	-	[DKK/MWh]
Start-up cost	$C_{\mathrm{HA,su}}$	-	[DKK]

Table 6.7: The production capacities and costs of the waste burner

Notes: The heat accumulator has a expended set of borders in the state of energy charge. The heat accumulator must under no circumstances get below 0 MWh and or above 300 MWh. failure to stay within these limits is considered complete failure. A comfort zone is defined to be between 100-250 MWh. The comfort zone is, in this project, only used to compare with the result of the solution.

Constraints for the Cooler

The considered constraints for the cooler are shown in table 6.8.

Cooler	Symbol	Value	Unit
Minimum power production	$\underline{Q}_{\text{CO}}$	0	[MW]
Maximum power production	$\overline{Q}_{\text{CO}}$	-12.25	[MW]
Energy price	P_{CO}	0	[DKK/MWh]
Start-up cost	$C_{\text{CO,su}}$	0	[DKK]

Table 6.8: The production capacities and costs of the waste burner

Notes: The cooler has a negative production which is equivalent to consumption. The cooler has no start-up/running cost, but using this is a waste of energy.

External Producer

The considered constraints for the external producer are shown in table 6.9.

External Producer	Symbol	Value	Unit
Minimum power output	$\underline{Q}_{\text{EP}}$	0	[MW]
Maximum power output	$\overline{Q}_{\text{EP}}$	50	[MW]
Energy price	P_{EP}	1000	[DKK/MWh]
Start-up cost	$C_{\text{EP,su}}$	-	[DKK]

Table 6.9: The production capacities and costs of the waste burner

Notes: The external producer is automatically activated when the heat accumulator reach a certain low point of energy. In this project the external production is included in the optimization problem in order to evaluated whether it can be used strategically in the production.

Chapter 7

Optimization Models

This chapter presents a set of optimization models, that are used as building blocks in the optimization problems, which are part the MPC controllers tested in the simulation part. The building blocks are described individually in this chapter.

7.1 Integrating the Electricity Market into the Objective Function

This section depicts how the mechanics of the energy market are described as an optimization problem. This model enables the MPC algorithm to consider the electricity market when making production decisions. The electricity is valued according to (7.1) [Skajaa, 2013]. For convenience the equations are presented with scalars, but the variables can be interpreted as vectors if the products are substituted with inner products.

$$R_{\mathrm{E}}[k] = P_{\mathrm{E}}[k]Q_{\mathrm{E}}[k] - (C_{\mathrm{DN}}[k]\Delta^{+}[k] + C_{\mathrm{UP}}[k]\Delta^{-}[k]) \tag{7.1}$$

where

P_{E}	The price on electricity	[DKK/MWh]
Q_{E}	The electrical power output from the plant	[MW]
C_{DN}	The down-regulation penalty	[DKK/MWh]
C_{UP}	The up-regulation penalty	[DKK/MWh]

Equation (7.1) describes the income from selling electricity when the penalty, paid for deviating from the day-ahead production bid, is taken into account. The penalty weights are $C_{\mathrm{DN}}, C_{\mathrm{UP}} \geq 0$. Consider equation (7.2)

$$\Delta = Q_{\mathrm{E}} - T_{\mathrm{E}} \tag{7.2}$$

where

Δ	The difference in electricity bid and delivered power	[MW]
Q_{E}	The electrical power output from the plant	[MW]
T_{E}	Electrical production bid	[MW]

which describes the difference between the day-ahead power bid and the electrical power production. The difference Δ can be described by a positive Δ^{+} and negative Δ^{-} component, seen in (7.3). The behavior of Δ^+ and Δ^- are controlled by the following relationship

$$\Delta = \Delta^+ - \Delta^- \tag{7.3}$$

The parts can also be described as $\Delta^+ = \max\{\Delta, 0\}$ and $\Delta^- = \max\{-\Delta, 0\}$, which means $\Delta^+ \geq 0$ and $\Delta^- \geq 0$. The equations (7.1) to (7.3) is implemented as a LP problem, seen in (7.4).

$$\max_{Q_\mathrm{E},\Delta^+,\Delta^-} P_\mathrm{E} Q_\mathrm{E} \Delta T - (C_\mathrm{DN}\Delta^+ + C_\mathrm{UP}\Delta^-)\Delta T - g(Q_\mathrm{E}) \tag{7.4a}$$

Subject to

$$0 \leq Q_\mathrm{E}$$
$$0 \leq \Delta^+$$
$$0 \leq \Delta^-$$
$$0 \leq \Delta^+ - (Q_\mathrm{E} - T_\mathrm{E}) \tag{7.4b}$$
$$0 \leq \Delta^- + Q_\mathrm{E} - T_\mathrm{E} \tag{7.4c}$$

where

$g(Q_\mathrm{E})$ The cost of producing Q_E [DKK]

The mechanism of the optimization problem, seen in (7.4), is controlled by the constraints of the variables. From the formulation of the optimization problem, it can be checked that Δ^- and Δ^+ behave as equation (7.3), except when $C_\mathrm{DN}, C_\mathrm{UP} = 0$, which makes Δ^- and Δ^+ free variables. This is not a problem though, because in that case they are eliminated from the cost-function by multiplying by zero. Given the fact that Δ^- and Δ^+ sometimes becomes free variables means they should not be used for subsequent analysis. The structure seen in (7.4) is build with inspiration from the formulation of the Chebyshev approximation problem seen in [Boyd and Vandenberghe, 2004, p. 6].

Constraint (7.4b) and (7.4c) can be written in the form of linear inequalities, which makes them implementable in linear programming.

$$0 \leq \mathbf{D}_d u_d - d_d \tag{7.5}$$

$$0 \leq u_d \tag{7.6}$$

$$u_d = \begin{bmatrix} Q_\mathrm{E} & \Delta^+ & \Delta^- \end{bmatrix}^T \qquad d_d = \begin{bmatrix} T_\mathrm{E} & -T_\mathrm{E} \end{bmatrix}^T \qquad \mathbf{D}_d = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \tag{7.7}$$

The inequality (7.5) and objective function (7.4a) are used later in the MPC optimization problems. This concludes how the electrical power market is implemented in the optimization problem.

7.2 Event Costs in the Power Plant

The objective of this section is to add a cost which only generates a penalty when a certain event occurs. The event cost is generated using an event indicator. The event indicator ϕ is defined as

$$\phi[i] = \begin{cases} 1 & \text{if the event occurs in sample } i \\ 0 & \text{if the event does not occur in sample } i \end{cases} \qquad i = \{1, 2, \dots, L\} \tag{7.8}$$

The event indicator is the main component of the event cost. When the event is observed a cost can be generated. Consider the objective function seen in (7.9)

$$\max_{b \in \mathbb{B}^L, u \in \mathbb{R}^{nL}} f(b,u) + C^T \phi(b) \tag{7.9}$$

where

L	The length of the considered time window. The vectors are indexed as $i = \{1, 2, \dots, L\}$	[·]
$f(b,u)$	The main cost as a function of continuous and binary values.	[DKK]
b	The vector containing a time sequence of binary valued inputs. $b[i] \in \{0, 1\}$	[·]
$\mathbb{B}^L$	A binary set in dimension L	[·]
u	The vector containing a time sequence of n continuous decision variables.	[·]
$\phi(b)$	An event indicator vector which is a function of the binary inputs	[·]
C	A vector containing the event cost. $C[i] < 0$	[DKK]

As seen in (7.9) the event cost $C^T \phi(b)$ is linear and added to the main cost function $f(b,u)$. The optimization problem is at this point able to account for the cost of triggering the event. The following example describes how the event cost is implemented as a start-up cost to the GT.

Example 7.1 (The Cost of Starting the Gas Turbine)
This example describes how the start-up cost of the GT is implemented in the optimization problem. The start-up of the GT is the event of interest. The event cost is formulated as an optimization problem.

$$\max_{b_{\text{GT}} \in \mathbb{B}^L, \delta \in \mathbb{R}^L, u \in U^{nL}} f(b_{\text{GT}}, u) + C_{\text{GT,su}}{}^T \phi_{\text{GT, su}}(b_{\text{GT}}) \tag{7.10}$$

Subject to

$$\phi_{\text{GT},su}[i] = \Delta b_{\text{GT}}[i] + \delta[i]$$
$$\Delta b_{\text{GT}}[i] = b_{\text{GT}}[i] - b_{\text{GT}}[i-1]$$
$$0 \le \delta[i] \le 1$$

where

b_{GT}	A binary valued vector containing the state of the GT	[·]
$C_{\text{GT, su}}$	A vector containing the start-up cost of the GT. $C_{\text{GT, su}}[i] = -10000$	[DKK]
$\phi_{\text{GT, su}}$	A vector function of b_{GT} containing the start-up events of GT	[·]

The objective is to generate an event indicator having properties as seen in (7.11)

$$\phi_{\text{GT, su}}(b_{\text{GT}})[i] = \begin{cases} 1 & \text{if start-up event occurs} \\ 0 & \text{otherwise} \end{cases} \qquad i = \{1, 2, \ldots, L\} \tag{7.11}$$

The following describes the mechamism which gives (7.11) the wanted behavior. Consider the definition of the event vector $\phi_{\text{GT},su}$ seen in (7.12)

$$\phi_{\text{GT},su} \equiv \Delta b_{\text{GT}} + \delta \qquad \phi_{\text{GT},su}[i] \in [0, 1] \tag{7.12}$$

The event vector seen in (7.12) is a sum of two vectors presented in (7.13) and (7.14).

$$\Delta b_{\text{GT}}[i] = b_{\text{GT}}[i] - b_{\text{GT}}[i-1] \qquad i = 1 \ldots L \qquad b_{\text{GT}}[0] = b_{\text{GT},0} \tag{7.13}$$

The difference vector Δb_{GT}, seen in (7.13), holds information about the shifts in the state. The constant $b_{\text{GT},0}$ is the current state of the GT. The value of $\Delta b_{\text{GT}}[i]$ is zero when the state is unchanged, one if the system is being turned on and negative one when the GT is being turned off. Consider δ which is defined as seen in (7.14).

$$\delta \in \mathbb{R}^L \qquad \delta[i] \in [0, 1] \tag{7.14}$$

The vector δ has the behavior seen in (7.15), due to the restrictions made on $\phi_{\text{GT},su}$ in (7.12).

$$\delta[i] = \begin{cases} 1 & \text{if } \Delta b_{\text{GT}}[i] = -1 \\ 0 & \text{otherwise} \end{cases} \tag{7.15}$$

The reason $\delta[i]$ becomes one when $\Delta b_{\text{GT}}[i] = -1$ is to obey the constraints put on $\phi_{\text{GT, su}}$, seen in (7.12). The reason $\delta[i]$ is zero otherwise is because any other value is suboptimal, because $\delta[i]$ is positive semi definite and $C_{\text{GT,su}}[i] < 0$ (strictly negative).

The constraints of the GT start-up vector $\phi_{\text{GT, su}}$ can be written as a linear ineqaulity, seen in (7.16).

$$0 \leq \mathbf{D_e} u_e - d_e \tag{7.16}$$

$$u_e = \begin{bmatrix} b_{\text{GT}}{}^T & \delta_{\text{GT}}{}^T \end{bmatrix}^T \qquad d_e = \begin{bmatrix} b_{\text{GT},0} & 0 & \cdots & 0 \end{bmatrix}^T \tag{7.17}$$

$$\mathbf{D_e} = \left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ldots & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \tag{7.18}$$

The event-cost for the GT is used in all simulations described in the project. The compressed form, seen in (7.16), is used in the optimization problems presented later in the report.

7.3 The Binary Constraints in Standard Form

In this section the inequalities, which contains binary variables, presented in Section 6.9, are stated as constraints of a mixed-integer programming problem described in [Nemhauser and Wolsey, 1988, p. 3]. The Gas Turbine ON/OFF constraint, presented in Table 6.2, is shown in (7.19).

$$b_{\mathrm{GT}}\underline{Q}_{\mathrm{G}} \leq Q_{\mathrm{G}} \leq b_{\mathrm{GT}}\overline{Q}_{\mathrm{G}} \tag{7.19}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \geq \begin{bmatrix} -1 \\ 1 \end{bmatrix} Q_{\mathrm{G}} + \begin{bmatrix} \underline{Q}_{\mathrm{G}} \\ -\overline{Q}_{\mathrm{G}} \end{bmatrix} b_{\mathrm{GT}} \tag{7.20}$$

Consider the Steam Turbine bypass constraint from Table 6.2

$$b_{\mathrm{VA}}\underline{Q}_{\mathrm{VA,H}} \leq Q_{\mathrm{VA,H}} \leq b_{\mathrm{VA}}\overline{Q}_{\mathrm{VA,H}} \tag{7.21}$$

which is formulated as an inequality constraint

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \geq \begin{bmatrix} -1 \\ 1 \end{bmatrix} Q_{\mathrm{VA,H}} + \begin{bmatrix} \underline{Q}_{\mathrm{VA,H}} \\ -\overline{Q}_{\mathrm{VA,H}} \end{bmatrix} b_{\mathrm{VA}} \tag{7.22}$$

The method of using binary values to activate and deactivate constraints, as seen in (7.19) and (7.21), are described in [Bisschop, 1993, p. 75]. The second part of the ST by-pass constraint from Table 6.2

$$b_{\mathrm{VA}}\underline{Q}_{\mathrm{VA,H}} \leq Q_{\mathrm{VA,H}} \leq b_{\mathrm{VA}}\overline{Q}_{\mathrm{VA,H}} \tag{7.23}$$

is reformulated to

$$\overline{Q}_{\mathrm{VA,S}} \geq Q_{\mathrm{VA,S}} + \left(\overline{Q}_{\mathrm{VA,S}} - \underline{Q}_{\mathrm{VA,S}}\right) b_{\mathrm{VA}} \tag{7.24}$$

The matrix inequality, seen in (7.25), is augmented by (7.20) to (7.24).

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \overline{Q}_{\mathrm{VA,S}} \end{bmatrix} \geq \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_{\mathrm{G}} \\ Q_{\mathrm{VA,H}} \\ Q_{\mathrm{VA,S}} \end{bmatrix} + \begin{bmatrix} \underline{Q}_{\mathrm{G}} & 0 \\ -\overline{Q}_{\mathrm{G}} & 0 \\ 0 & \underline{Q}_{\mathrm{VA,H}} \\ 0 & -\overline{Q}_{\mathrm{VA,H}} \\ 0 & \overline{Q}_{\mathrm{VA,S}} - \underline{Q}_{\mathrm{VA,S}} \end{bmatrix} \begin{bmatrix} b_{\mathrm{GT}} \\ b_{\mathrm{VA}} \end{bmatrix} \tag{7.25}$$

The equation (7.25) can be written compactly as

$$d_b \geq \mathbf{D}_b u_b + \mathbf{G}_b b \tag{7.26}$$

The binary inequality shown in (7.25) is on the same form as described in Section B.2. The inequality (7.26) is later used to state the full MPC problems which is used for the test.

7.4 Soft Constraints

MPC being an online real time procedure creates demands on reliability. The constrained optimization problem which is solved on the fly can become infeasible. Many solvers will return an error if an infeasible problem is encountered, this is problematic in an online algorithm where the next input is needed. It can be very difficult to reason whether a problem can be infeasible, which means considerations to avoid such a case are necessary. There exits different ways of solving this issue: (1) repeat the same input as the last, (2) send the next input in the time series calculated in last iteration $u(k+1|k)$, (3) softening constraints to avoid infeasible problems [Maciejowski, 2002, p. 96]. Consider the optimization problem, which contains softening constraints,

$$\min_{u\in\mathbb{R}^n,\omega\in\mathbb{R}^m} f_0(u) + \rho||\omega||^2 \tag{7.27}$$

Subject to

$$b \leq f(u) + \omega$$

$$\underline{u} \leq u \leq \overline{u}$$

where

$f_0(u) : \mathbb{R}^n \to \mathbb{R}$	The original objective function	$[\cdot]$
$f(u) : \mathbb{R}^n \to \mathbb{R}^m$	The constraint function	$[\cdot]$
ω	A non-negative slack variable vector that softens the constraints	$[\cdot]$
ρ	A non-negative large constant	$[\cdot]$
u	The input vector	$[\cdot]$
$\|\|\cdot\|\|$	The 2-norm	$[\cdot]$

The weight ρ needs to be chosen high, relative to the rest of the objective function, such that the constraints are only violated when necessary. The variable ω is zero when the optimization problem is naturally feasible.

7.5 Probabilistic Constraints

This section describes how probabilistic constraints are modeled. The probabilistic constraints are used when random variables are part of the expression. This section is based on [Bisschop, 1993, p. 71]

Consider the following linear constraint

$$\beta_i \leq a_i^T u \qquad u \in \mathbb{R}^n \qquad i = 1, 2 \dots q \tag{7.28}$$

where

β_i A random variable [·]

A solution to a optimization problem, containing the inequality (7.28), is feasible if it satisfies all values of β. If β is defined from $-\infty$ to ∞, the optimization problem is rendered infeasible. To avoid this the demands are weakened such that the constraint is allowed broken with a certain probability α.

$$\textbf{Prob}\left\{\beta_i \leq a_i^T u\right\} \geq 1 - \alpha \tag{7.29}$$

The probabilistic constraint can be turned into a deterministic constraint using the inverse cumulative distribution function $\Phi^{-1}(\cdot)$. The result of β_i has α probability to be over the value of $\Phi^{-1}(1-\alpha)$. This gives the deterministic inequality

$$\Phi^{-1}(1-\alpha) \leq a_i^T u \tag{7.30}$$

The probabilistic constraints are used in the forecast based MPC analysis to provide robustness against prediction errors.

7.6 Model of the Adjustment Capacity of the District Heating Output

The model of the adjustment capacity is used in the forecast based simulations. The notion adjustment capacity is defined as the plants ability to adjust an output using only a specified subpart of the plant. The maximum up-adjustment capacity for district heating is denoted $\overline{\Delta Q}_{DH}$ and maximum down-adjustment capacity is $\underline{\Delta Q}_{DH}$. The concept of power adjustment capacity in the CHP plant is similar to a spinning reserve in the power grid.

The following text provides motivation for designing a power adjustment capacity model for the CHP plant. Recall Figure 5.1, which describes the timing of MPC, it shows the difference between predicted and actual power demand. The forecast based simulation feeds the optimization problem predicted demands $\hat{r}_{\mathrm{DH}}$ which are not equal to the actual demands r_{DH} as seen in (7.31).

$$\hat{r}_{\mathrm{DH}}[k|k-1] \neq r_{\mathrm{DH}}[k] \tag{7.31}$$

The difference between the predicted and actual district heating demand is defined as

$$e_{\mathrm{DH}}[k] = r_{\mathrm{DH}}[k] - \hat{r}_{\mathrm{DH}}[k|k-1] \tag{7.32}$$

if the difference $e_{\mathrm{DH}}[k]$ is positive the demand is higher than predicted and the stability controller needs to up-adjust the power output $\hat{Q}_{\mathrm{DH}}[k]$. Signals are denoted with a hat before adjustment. If $e_{\mathrm{DH}}[k]$ is negative the stability controller needs to down-adjust $\hat{Q}_{\mathrm{DH}}[k]$.

In reality a stabilizing controller continuously tracks the demand in the network using available adjustment capacity. In this project there is no high frequency stabilizing controller under the MPC control loop. The effect of a stabilizing controller is simulated by adjusting the MPC chosen set-point $\hat{Q}_{\mathrm{DH}}[k]$ which is based on the predicted demand $\hat{r}_{\mathrm{DH}}[k|k-1]$ to $Q_{\mathrm{DH}}[k]$ which equals the the actual $r_{\mathrm{DH}}[k]$ demand, which is seen in (7.33).

$$Q_{\mathrm{DH}}[k] = \hat{Q}_{\mathrm{DH}}[k] + \Delta Q_{\mathrm{DH}}[k] \tag{7.33}$$

This set-point change is performed in between two samples and can be interpreted as a disturbance. Figure 7.1 shows the MPC loop with the stabilizing controller as a disturbance.

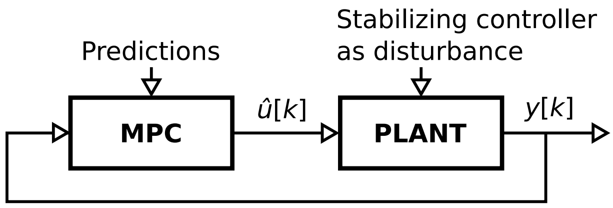

Figure 7.1: Shows how the stabilizing controller, seen from the MPC loop, can be interpreted as a disturbance on the plant

The stabilizing controller can perform the adjustment either by regulating power production units, redirect power flows or a combination of both. The power flows are redirected such that the energy balance equations maintain validity. The set-point change is implemented as an optimization problem which changes the set-point according to the prediction error, this is seen in Appendix F. If the effect of the stabilizing controller is not simulated the production set-point is not moved which means an energy imbalance grows in the system, which in a real plant would give catastrophic results. Appendix C provides a more thorough description of this effect.

7.6.1 Formulating the Models for Adjustment Capacities

The stabilizing controller is assumed, only to be able to control a subpart of the CHP plant. This part is shown in Figure 7.2. The MPC controller needs to allocate enough adjustment capacity for the stabilizing controller to balance supply and demand.

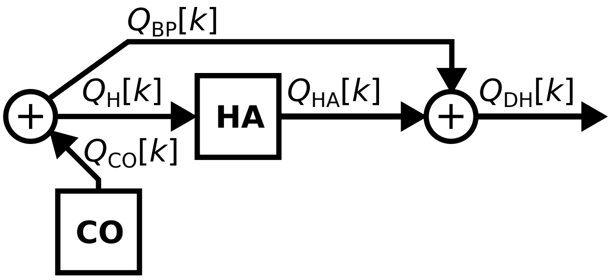

Figure 7.2: Shows the part of the plant, which can be controlled by the stabilizing controller. The objective of the stabilizing controller is to balance supply and demand.

For the stabilizing controller to function the adjustment capacity needs to fulfill following demands

$$\overline{\Delta Q}_{\mathrm{DH}}[k] \geq 0 \tag{7.34a}$$

$$\underline{\Delta Q}_{\mathrm{DH}}[k] \geq 0 \tag{7.34b}$$

$$e_{\mathrm{DH}}[k] < 0 \implies \overline{\Delta Q}_{\mathrm{DH}}[k] > |e_{\mathrm{DH}}[k]| \tag{7.34c}$$

$$e_{\mathrm{DH}}[k] > 0 \implies \underline{\Delta Q}_{\mathrm{DH}}[k] > |e_{\mathrm{DH}}[k]| \tag{7.34d}$$

If the complexity of the plant is high, it can be difficult to accurately describe the adjustment capacities. Instead a lower bound β_{DH} can be established. The lower bound for $\overline{\Delta Q}_{\mathrm{DH}}$ is $\overline{\beta}_{\mathrm{DH}}$ such that $\overline{\Delta Q}_{\mathrm{DH}} \geq \overline{\beta}_{\mathrm{DH}}$ and a lower bound for $\underline{\Delta Q}_{\mathrm{DH}}$ is $\underline{\beta}_{\mathrm{DH}}$ such that $\underline{\Delta Q}_{\mathrm{DH}} \geq \underline{\beta}_{\mathrm{DH}}$.

The estimation for adjustment capacity can be interpreted as a measurement which enables the optimization problem to react on cases with insufficient adjustment capacity. The objective is to formulate the model such it only affects the solution in cases with insufficient adjustment capacity. Formulating the model as a feasibility problem can provide this effect. In that case the model needs to be expressed without an occurrence in the objective function. Before presenting the expressions for adjustment capacities the notion of distance from set-point to operational limits needs to be established. Consider

$$\underline{\Delta Q}[k] = \hat{Q}[k] - \underline{Q} \qquad \underline{\Delta Q}[k] \geq 0 \tag{7.35a}$$

$$\overline{\Delta Q}[k] = \overline{Q} - \hat{Q}[k] \qquad \overline{\Delta Q}[k] \geq 0 \tag{7.35b}$$

where

$\hat{Q}$	The MPC chosen set-point	[MW]
$\underline{Q}$	The lower bound for $\hat{Q}$	[MW]
$\overline{Q}$	The upper bound for $\hat{Q}$	[MW]
$\overline{\Delta Q}$	The up-adjustment capacity for $\hat{Q}$	[MW]
$\underline{\Delta Q}$	The down-adjustment capacity for $\hat{Q}$	[MW]

Figure 7.3 gives examples of distances from set-point to limits in different modules.

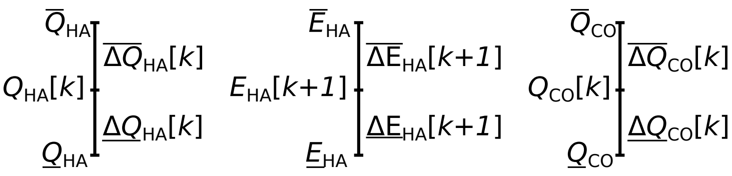

Figure 7.3

The equations (7.35) define the variables which describe the adjustment capacity of an individual module.

The models for up/down-adjustment capacities are seen in the following sections. The models have these particular shapes due to dependencies in the system. The two dependencies which are considered are bottleneck-dependency and shared-dependency. These dependencies are mainly caused by physical limits in the modules, they are further described in Appendix D

Model for Estimating the Up-Adjustment Capacity

Consider the up-adjustment capacity model

$$\overline{\beta}_{\mathrm{DH}}[k] = \min(\underline{\Delta E}_{\mathrm{HA}}[k+1], \overline{\Delta Q}_{\mathrm{HA}}[k]\Delta T + \underline{\Delta Q}_{\mathrm{H}}[k]\Delta T) + \overline{\Delta Q}_{\mathrm{CO}}[k]\Delta T \tag{7.36}$$

where

$\overline{\beta}_{\mathrm{DH}}$	The lower bound for the up-adjustment capacity	[MW]
$\underline{\Delta E}_{\mathrm{HA}}$	The down-adjustment capacity of the heat accumulator state	[MW]
$\overline{\Delta Q}_{\mathrm{HA}}$	The up-adjustment capacity of the heat accumulator output	[MW]
$\underline{\Delta Q}_{\mathrm{H}}$	The down-adjustment capacity of the input to the heat accumulator	[MW]
$\overline{\Delta Q}_{\mathrm{CO}}$	The up-adjustment for the cooler	[MW]

The up-adjustment capacity, seen in (7.36), is limited by four main factors: (1) the amount of energy left in the HA which is described by $\underline{\Delta E}_{\mathrm{HA}}$, (2) the maximum up-adjustment capacity of the slew rate of the HA output $\overline{\Delta Q}_{\mathrm{HA}}$, (3) The amount of power scheduled as input to the HA, that alternatively can be redirected through the by-pass $\hat{Q}_{\mathrm{BP}}$, is described by $\underline{\Delta Q}_{\mathrm{H}}$, (4) The amount of energy scheduled to the CO, which also can be directed through the by-pass to the district heating output, is described by $\overline{\Delta Q}_{\mathrm{CO}}$. The upper limit of the by-pass $\hat{Q}_{\mathrm{BP}}$ is not considered, because the plant cannot reach it.

Model for Estimating the Down-Adjustment Capacity

Consider the down-adjustment capacity model

$$\begin{aligned}\underline{\beta}_{\mathrm{DH}}[k] = \min\Big(&\underline{\Delta Q}_{\mathrm{BP}}[k]\Delta T, \underline{\Delta Q}_{\mathrm{CO}}[k]\Delta T + \max(0, \overline{\Delta E}_{\mathrm{HA}}[k+1] - \underline{\Delta Q}_{\mathrm{HA}}[k]\Delta T)\Big)\\ &+ \min(\overline{\Delta E}_{\mathrm{HA}}[k+1], \underline{\Delta Q}_{\mathrm{HA}}[k]\Delta T)\end{aligned} \tag{7.37}$$

where

$\underline{\beta}_{\mathrm{DH}}$	The lower bound for the up-adjustment capacity	[MW]
$\overline{\Delta E}_{\mathrm{HA}}$	The up-adjustment capacity of the heat accumulator state	[MW]
$\underline{\Delta Q}_{\mathrm{HA}}$	The down-adjustment capacity of the heat accumulator output	[MW]
$\underline{\Delta Q}_{\mathrm{CO}}$	The down-adjustment for the cooler	[MW]
$\underline{\Delta Q}_{\mathrm{BP}}$	The down-adjustment capacity of the power which bypasses the HA	[MW]

The model for down adjustment (7.37) is easiest explained in parts. Consider the model part

$$\min(\overline{\Delta E}_{\mathrm{HA}}[k+1], \underline{\Delta Q}_{\mathrm{HA}}[k]\Delta T) \tag{7.38}$$

which finds the limiting factor between HA charge $\hat{E}_{\mathrm{HA}}$ and slew rate $\hat{Q}_{\mathrm{HA}}$. The expression seen in (7.39) decides the charge capacity of the HA through $\hat{Q}_{\mathrm{H}}$.

$$\max(0, \overline{\Delta E}_{\mathrm{HA}}[k+1] - \underline{\Delta Q}_{\mathrm{HA}}[k]\Delta T) \tag{7.39}$$

The expression (7.39) is used in

$$\min\left(\underline{\Delta Q}_{\mathrm{BP}}[k]\Delta T, \underline{\Delta Q}_{\mathrm{CO}}[k]\Delta T + \max(0, \overline{\Delta E}_{\mathrm{HA}}[k+1] - \underline{\Delta Q}_{\mathrm{HA}}[k]\Delta T)\right) \tag{7.40}$$

which describes the bottleneck between the by-pass and the down adjustment capacity before the by-pass. The power which is taken from the by-pass needs to be directed to eather the HA or the CO. Remember that the by-pass $\hat{Q}_{\mathrm{BP}}$ is not allowed to be negative.

7.6.2 Implementation of the Adjustment Capacity Models

This section describes how the adjustment models are implemented in CVX.

Implementation of the Up-Adjustment Capacity in CVX

The model (7.36) is implemented in CVX, which is seen in (7.41).

$$\overline{\beta}_{\mathrm{DH}}[k] = \min(\underline{\Delta E}_{\mathrm{HA}}[k+1], \overline{\Delta Q}_{\mathrm{HA}}[k]\Delta T + \underline{\Delta Q}_{\mathrm{H}}[k]\Delta T) + \overline{\Delta Q}_{\mathrm{CO}}[k]\Delta T \tag{7.41a}$$

$$\underline{\Delta E}_{\mathrm{HA}}[k+1] = E_{\mathrm{HA}}[k+1] - \underline{E}_{\mathrm{HA}} \tag{7.41b}$$

$$\overline{\Delta Q}_{\mathrm{HA}}[k] = Q_{\mathrm{HA}}[k] - \overline{Q}_{\mathrm{HA}} \tag{7.41c}$$

$$\underline{\Delta Q}_{\mathrm{H}}[k] = Q_{\mathrm{H}}[k] - \underline{Q}_{\mathrm{H}} \tag{7.41d}$$

$$\underline{\Delta Q}_{CO}[k] = Q_{\mathrm{CO}}[k] - \underline{Q}_{\mathrm{CO}} \tag{7.41e}$$

CVX accepts (7.36) in the original form which means it can be implemented directly. The model is not represented in the objective function which means it can be used in a feasibility programming module.

7.6.3 Implementation of the Down-Adjustment Capacity in CVX

The implementation of the model for down-adjustment capacity is more complex than the implementation of the up adjustment capacity because CVX does not accept a direct implementation of (7.37). The implementation of the down-adjustment capacity is a combination of (7.42a) and (7.42b).

$$\underline{\delta}_1^*[k] = \arg(\max_{\underline{\delta}_1 \in \mathbb{R}} -\underline{\alpha}\, \underline{\delta}_1) \tag{7.42a}$$

Subject to

$$0 \leq \underline{\delta}_1$$

$$0 \leq \underline{\delta}_1 - (\overline{\Delta E}_{\text{HA}}[k+1] - \underline{\Delta Q}_{\text{HA}}[k]\Delta T)$$

where

$\underline{\delta}_1^*$	The argument that maximizes the objective function	[MW]
$\underline{\delta}_1$	The continuous desision variable	[MW]
$\underline{\alpha}$	A weight	[·]

$$\underline{\beta}_{\text{DH}}[k] = \min\left(\underline{\Delta Q}_{\text{BP}}[k]\Delta T, \underline{\Delta Q}_{\text{CO}}[k]\Delta T + \underline{\delta}_1^*[k]\right) + \min(\overline{\Delta E}_{\text{HA}}[k+1], \underline{\Delta Q}_{\text{HA}}[k]\Delta T) \tag{7.42b}$$

In the implementation (7.42) is different from (7.37) because the direct implementation of the model violates the CVX ruleset. To make CVX accept the model, the expression

$$\max(0, \overline{\Delta E}_{\text{HA}}[k+1] - \underline{\Delta Q}_{\text{HA}}[k]\Delta T) \tag{7.43}$$

in (7.37) is reformulated to (7.42a) by use of the linear programming implementation of the max-operator, seen in (7.44), which is inspired by [Boyd and Vandenberghe, 2004, p. 6].

$$\max(a,b) = \arg(\max_{\delta \in \mathbb{R}} -\delta) \tag{7.44}$$

Subject to

$$0 \leq \delta - a$$

$$0 \leq \delta - b$$

The result of min-operator can be found by using the solution from (7.44) in (7.45).

$$\min(a,b) = a + b - \max(a,b) \tag{7.45}$$

The implementation of the the down-adjustment capacity has an expression represented in the objective function which means it cannot be part of a feasibility programming problem. **Note:** The implementation seen (7.42) is working when the expression is a stand alone, but in this project it is part of a larger expression which causes it to malfunction. This is further described in Section 11.5.

7.7 Combining the Prediction Error with Adjustment Capacity to form a Back-off Constraint

The formulation of the back-off constraint is inspired by [Hovgaard et al., 2011]. The adjustment capacity for district heating is allocated to accommodate for the district heating prediction error $e_{\text{DH}}[k]$. The adjustment capability should always be larger than the predicted error. Consider the two inequality constraints

$$e_{\text{DH}}^{+} \leq \overline{\beta} \qquad \overline{\beta} \geq 0 \tag{7.46a}$$

$$e_{\text{DH}}^{-} \leq \underline{\beta} \qquad \underline{\beta} \geq 0 \tag{7.46b}$$

where

$\overline{\beta}$	The capacity for up-adjustment	[MW]
$\underline{\beta}$	The capacity for down-adjustment	[MW]
e_{DH}^{+}	The positive part of the prediction error e_{DH}	[MW]
e_{DH}^{-}	The negative part of the prediction error e_{DH}	[MW]

The prediction error e_{DH} is split into the positive and negative part $e_{\text{DH}}^{+} = \max(0, e_{\text{DH}}[k])$ and $e_{\text{DH}}^{-} = \max(0, -e_{\text{DH}}[k])$ where $e_{\text{DH}}^{+}, e_{\text{DH}}^{-} \geq 0$. The equation (7.46a) presents the case where the CHP needs to down-adjust the DH power output and (7.46b) presents the up-adjustment case.

As mentioned in Section 3, $e_{\text{DH}}[k]$ is a random variable which is distributed as white Gaussian noise. This makes the constraint a probability constraint, which means the entire optimization problem becomes infeasible. The two probabilistic constraints (7.46) are reformulated as seen in (7.47).

$$|\Phi_{\text{DH}}^{-1}(1-\alpha)| \leq \overline{\beta}_{\text{DH}} \qquad \overline{\beta}_{\text{DH}} \geq 0 \tag{7.47a}$$

$$|\Phi_{\text{DH}}^{-1}(\alpha)| \leq \underline{\beta}_{\text{DH}} \qquad \underline{\beta}_{\text{DH}} \geq 0 \tag{7.47b}$$

where

$\Phi^{-1}(\cdot)$	The inverse commutative distribution of the prediction error	$[\cdot]$
α	The probability of outcome to be lower than the value given by $\Phi^{-1}(\alpha)$	$[\cdot]$

At this point the distribution of the random variable e_{DH} is used to create the deterministic constraints seen in (7.47). The choice of α decides at which probability the adjustment capacity is allowed to be broken. The equations (7.48) show the combined constraints which link the adjustment capacity and the prediction error of the DH demand.

$$|\Phi_{\text{DH}}^{-1}(\alpha)| \leq \min(\underline{\Delta E}_{\text{HA}}[k+1], \overline{\Delta Q}_{\text{HA}}[k]\Delta T + \underline{\Delta Q}_{\text{H}}[k]\Delta T) + \overline{\Delta Q}_{\text{CO}}[k]\Delta T \tag{7.48a}$$

$$
\begin{aligned}
|\Phi_{\mathrm{DH}}^{-1}(1-\alpha)| \leq \min\Big(&\underline{\Delta Q}_{\mathrm{BP}}[k]\Delta T, \underline{\Delta Q}_{\mathrm{CO}}[k]\Delta T + \max(0, \overline{\Delta E}_{\mathrm{HA}}[k+1] - \underline{\Delta Q}_{\mathrm{HA}}[k]\Delta T)\Big) \\
&+ \min(\overline{\Delta E}_{\mathrm{HA}}[k+1], \underline{\Delta Q}_{\mathrm{HA}}[k]\Delta T)
\end{aligned} \tag{7.48b}
$$

7.8 The End State of the Heat Accumulator

The end state of the HA is implemented in the optimization problems in order to inform the optimization problem, that the energy stored in the HA at the end of the control window still has value beyond the horizon. The expression is stated in

$$
P_{\mathrm{DH}} E_{\mathrm{HA}}[k+L] \tag{7.49}
$$

where

L	The length of the control window	$[\cdot]$
P_{DH}	The price of district heating	[DKK/MWh]
$E_{\mathrm{HA}}[k+L]$	The final state of the heat accumulator	[MWh]

The end state expression is used in the objective functions of the MPC algorithms tested later in the report.

Summary

This concludes Part II which presented the following topics: The MPC algorithm in general, The hybrid state model of the CHP plant and the optimization building blocks. These subjects will in the next part be used to formulate the MPC algorithms which are evaluated through simulation.

PART III

SIMULATIONS AND EVALUATIONS

Chapter 8

Introduction to the Evaluation

This chapter presents the tests conducted through simulations. The tests are ordered under two main topics. A deterministic analysis is defined as the case where the information about the future is deterministic. The other test is forecast based which means the MPC algorithm is fed predictions about the future.

8.1 Evaluation Function

This section describes how the tests are evaluated. The evaluation function calculates the profit gained over the test period.

$$R_{\text{EVAL}} = \sum_{k=1}^{M} \left(P[k]^T Q[k]\Delta T + C_{\text{GT,su}}\phi_{\text{GT, su}}[k] - (C_{\text{DN}}[k]\Delta^+[k] + C_{\text{UP}}[k]\Delta^-[k])\Delta T \right) \quad (8.1)$$

$$P[k] = \begin{bmatrix} P_{\text{WB}} & P_{\text{WB}} & P_{\text{G}} & P_{\text{EP}} & P_{\text{E}}[k] & P_{\text{DH}} \end{bmatrix}^T \quad Q[k] = \begin{bmatrix} Q_{\text{WB1}}[k] & Q_{\text{WB2}}[k] & Q_{\text{G}}[k] & Q_{\text{EP}}[k] & Q_{\text{E}}[k] & Q_{\text{DH}}[k] \end{bmatrix}^T \quad (8.2)$$

where

R_{EVAL}	The total profit gained after M hours	[DKK]
P_{E}	Price of electrical energy	[DKK/MWh]
Q_{E}	The power delivered to the electrical grid	[MW]
P_{DH}	The price of energy delivered to district heating	[DKK/MWh]
Q_{DH}	The power delivered to district heating	[MW]
P_{WB}	The price of producing energy on waste burner i	[DKK/MWh]
$Q_{\text{WB}i}$	The power produced from on waste burner i	[MW]
P_{G}	The price on gas energy	[DKK/MWh]
Q_{G}	The used gas power	[MW]
P_{EP}	The price on external production	[DKK/MWh]
C_{UP}	The imbalance penalty for underproduction	[DKK/MWh]
C_{DN}	The imbalance penalty for overproduction	[DKK/MWh]
Δ^+	power imbalance caused by overproduction	[MW]
Δ^-	Power imbalance caused by underproduction	[MW]
Q_{EP}	The external production power	[MW]
$\phi_{\text{GT, su}}$	A vector function of b_{GT} containing the start-up times of GT	[·]
$C_{\text{GT, su}}$	A vector containing the start-up cost of the GT	[DKK]
ΔT	The sample time	[h]

The evaluation function (8.1) is designed to measure profit which is the difference between incomes and expenses. The incomes are the sold district heating energy and electrical energy. The expenses are bought gas, burned waste and start-up costs. Note variables without the sample indicator $[k]$ are constants.

8.2 Test Length

All tests have been evaluated over a course of 8410 hours, which is equivalent to 96 % of a year. The test covers a timespan from January 9^{th} 2012 to December 24^{th} 2012. The sampling time on one hour means a number of 8410 iterations are conducted in each test.

8.3 Deterministic Analysis

In the deterministic analysis the impact of the window length L of the MPC algorithm has been investigated. Even though the MPC is fed deterministic data it cannot evaluate conditions beyond the prediction window. This gives an information advantages to a MPC algorithm with a long window compared to one with a short window. The question is then "how large is the advantage of using a long window and which consequences are attached to using one?".

Evaluations conducted

The tests conducted in deterministic analysis are concerning the window length L of the MPC algorithm. In the simulations, six different window lengths of MPC have been tested and compared. The windows lengths are 3, 6, 12, 24, 48 and 168 hours. The longest window on 168 hours is chosen because it is equivalent to a week. The results are presented in Chapter 10.

Models used to Formulate the Optimization Problem

The tests are conducted using the following models presented in Part 2.

- CHP plant model (Chap. 6)
- Model of the event cost of the GT (Sec. 7.2)
- The binary constraints model (Sec. 7.3).

Common for these models are that they describe costs which are associated with decisions made on deterministic information.

8.4 Forecast Based Analysis

The forecast based analysis investigates how uncertainties affect the results of the MPC algorithm. All data used in the test is forecast which makes the test significantly more realistic compared to the deterministic. The uncertainties between the linear model plant model and actual plant are not investigated though which means the test is still some what idealized. The tests of the forecast based analysis are only conducted with a window of 48 hours. The results from the forecast based test can be compared with the results from the deterministic tests conducted with a 48 hour window.

Evaluations conducted

Transcending from the deterministic analysis to the forecast based introduces new aspects which are interesting to investigate. The tests conducted in the forecast based analysis are mainly concerned with forecast data with different horizon lengths.

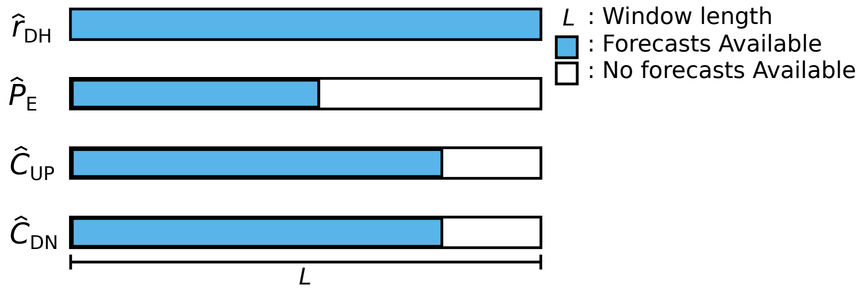

Figure 8.1: Shows the different horizon lengths of the forecast data

Figure 8.1 shows the different lengths of forecast data within the control horizon of the MPC controller. The forecasts of the DH demand $\hat{r}_{DH}$ covers the entire window where the predictions of the electricity price $\hat{P}_{E}$ covers a much smaller part of the window. The approach in this test is to align the lengths of the data vectors by filling blank spaces with pseudo data. Two test are conducted: (1) blank spaces are filled with zeros and (2) the blank spaces are filled with the average predicted value of the specific variable e.g. the blank spaces of $\hat{P}_{E}$ are filled with the average value of $\hat{P}_{E}$.

Models used to Formulate the Optimization Problem

The models which are used in the forecast based tests

- CHP plant model (Chap. 6)
- The binary constraints model (Sec. 7.3)
- The event cost model for the GT (Sec. 7.2)
- Probabilistic constraints (Sec. 7.5)
- Soft constraints (Sec. 7.4)
- Model of the electricity market (Sec. 7.1)

Chapter 9

Simulation Structure

This chapter describes the different phases the simulation encounters when testing the MPC method. The purpose of the simulation is to provide a framework which easily can be configured to test different scenarios in order to reveal mechanisms of the system.

9.1 Flow of the Simulation

Figure 9.1 shows the phases of the simulation. The first two phases are runned ones, the remaining are part of the simulation loop.

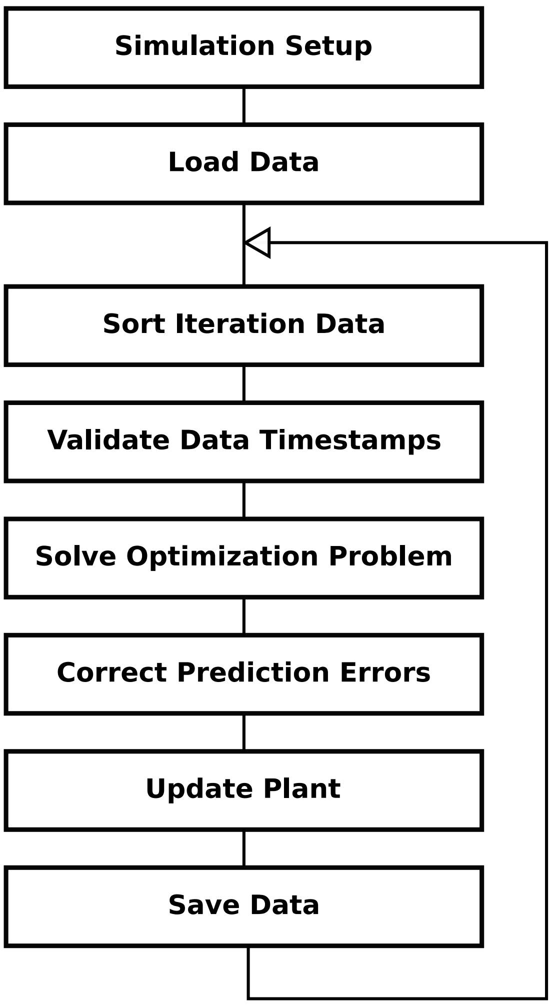

Figure 9.1: Shows the flow chart of the simulation

Simulation Setup

The simulation is initialized in this step. The different parameters of the simulation are assigned according to the chosen setup. The parameters are operation limits and module efficiencies.

Load Data

The empirical data from Horsens Kraftvarmeværk is loaded into the simulation from data files.

Sort Iteration Data

The first step in the simulation loop is sorting the data which is used in the iteration. Most of the data does not need a sorting, but the electricity bid data T_{E} and the predictions for spot price of electricity $\hat{P}_{\text{E}}$ have a special structure. The algorithms for sorting the data are described in Appendix E.

Validate Timestamps

The timestamps of the data used in the iteration are checked for synchronization in time. The forecast data has two timestamps, one that indicates which hour the forecast was issued, and the other indicates the operation hour. Measurement data only has the operational timestamp. All operation timestamps need to fit to sample k.

Solve Optimization Problem

The optimization problem for the MPC algorithm, which is formulated in CVX, is solved using a solver Provided by Gurobi. CVX and Gurobi are introduced in Appendix G.

Correct Prediction Error of District Heating

The gap between supply and demand in district heating which is caused by the prediction error is closed using a optimization problem which redirects power flows in the system using the allocated adjustment capacity. The optimization problem is described in Appendix G.

Save Data

The data is saved in each iteration, which means that data generated before a potential crash is not lost.

9.2 Data structures

This section describes the structures in which the data is ordered. The data is ordered such that the simulation easily can propagate through the data using for-loops.

The forecast data is ordered in matrix structures which is shown in (9.1). The red marking shows the forecasts used for iteration k. Note that the forecasts for iteration k are generated in $k-1$. In iteration $k+1$ the window has moved one row down.

$$\begin{array}{c} \vdots \\ k-1 \\ k \\ k+1 \\ \vdots \end{array} \begin{bmatrix} \vdots & \vdots & \vdots & \cdots & \vdots \\ \hat{\mathcal{F}}[k-1|k-2] & \hat{\mathcal{F}}[k|k-2] & \hat{\mathcal{F}}[k+1|k-2] & \cdots & \hat{\mathcal{F}}[k+L-2|k-2] \\ \hat{\mathcal{F}}[k|k-1] & \hat{\mathcal{F}}[k+1|k-1] & \hat{\mathcal{F}}[k+2|k-1] & \cdots & \hat{\mathcal{F}}[k+L-1|k-1] \\ \hat{\mathcal{F}}[k+1|k] & \hat{\mathcal{F}}[k+2|k] & \hat{\mathcal{F}}[k+3|k] & \cdots & \hat{\mathcal{F}}[k+L|k] \\ \vdots & \vdots & \vdots & \cdots & \vdots \end{bmatrix} \tag{9.1}$$

The measurement data can be ordered in a vector, as seen in (9.2). The data used for iteration k is retrieved by sliding a window of size L through the vector. This is illustrated by the red marking in (9.2).

$$\cdots \quad \mathcal{D}[k-2] \quad \mathcal{D}[k-1] \left[\mathcal{D}[k] \quad \mathcal{D}[k+1] \quad \cdots \quad \mathcal{D}[k+L-1] \right] \mathcal{D}[k+L] \quad \cdots \tag{9.2}$$

9.3 Functionalities for the Simulation Environment

This section describes the added functionalities which are implemented in the simulation in order to test the MPC algorithms properly.

Bidding on the Day-Ahead Market

This functionality is build for the forecast based analysis, which needs production bids for the electricity marked model. The simulation bids in the Day-ahead market according to market rules. As mentioned, in Appendix A, the production bid for the day-ahead market is given before 12:00 am the day before the production is delivered. Consider the Schematic (9.3)

$$\begin{array}{cccccccc} 11:00 & 12:00 & \cdots & 00:00 & 01:00 & \cdots & 23:00 & 00:00 \\ Q_{\mathrm{E}}[k] & \hat{Q}_{\mathrm{E}}[k+1|k] & \cdots & \hat{Q}_{\mathrm{E}}[k+13|k] & \hat{Q}_{\mathrm{E}}[k+14|k] & \cdots & \hat{Q}_{\mathrm{E}}[k+36|k] & \hat{Q}_{\mathrm{E}}[k+37|k] \end{array} \tag{9.3}$$

In the simulation the bidding mechanism bids by evaluating the timestamps of the simulation. If the timestamp yields 11:00 am it is time to generate the bid T_{E}. The bid is generated by using the fresh forecasts of the electricity production $\hat{Q}_{\mathrm{E}}$ provided by the MPC controller. The bid vector T_{E} is assigned the electricity production forecasts for the next day which are marked by red in (9.3).

The algorithm for bidding in the day-ahead market is, together with a schematic containing the daily events, presented in Appendix E.

Sorting the Forecasts of the Electricity Spot Price

The mechanisms of the day-ahead market has an structural impact in the forecast vector of the electricity price $\hat{P}_{\mathbf{E}}$. Due to the announcement of the spot prices P_{E} which happens around 01:00 pm the vector $\hat{P}_{\mathbf{E}}$ becomes a mix of deterministic and predicted values. This mechanism is simulated in the simulation and the algorithm is presented in Appendix E.

Chapter 10

Deterministic Analysis

This chapter analyzes the MPC controller using value of perfect information (VOPI) analysis. The data which is used in a VOPI analysis has no uncertainties. This can also be explained as all stochastic variables are treated as deterministic variables. The optimization problem is fed the actual variable instead of the forecast. This basically demands that the system is able to predict the future which is unrealistic. Having stated that VOPI is an unrealistic approach it still works as a tool to gain knowledge of the bahavior of the system. At the same time it indicates an upper bound to what is achievable with a particular system, because there is always a loss when there are uncertainties in the system.

10.1 The Optimization Problem

This section presents an overview of the optimization problem which is solved each iteration in the deterministic VOPI analysis. The full optimization problem is stated in (10.1), the objection function and constraints are elabortated on in the following sections. The background for the objective function and constraints are described in the referred sections.

$$\max_{u\in\mathbb{U},b\in\mathbb{B}} R_{\text{VOPI}}[k] \qquad \text{(Sec. 10.1.1)} \qquad (10.1a)$$

Subject to

$$i=\{1,2,\ldots,L-1\}$$

$$x[k+1+i]=\mathbf{A}x[k+i]+\mathbf{B}u[k+i] \qquad \text{(Sec. 6.8)} \qquad (10.1b)$$

$$r_{\text{DH}}[k+i]=Q_{\text{DH}}[k+i] \qquad (10.1c)$$

$$y[k+i]=\mathbf{D_y}u[k+i] \qquad \text{(Sec. 6.8)} \qquad (10.1d)$$

$$\mathbf{0}=\mathbf{D_z}u[k+i] \qquad \text{(Sec. 6.8)} \qquad (10.1e)$$

$$\underline{u}\leq u[k+i]\leq\overline{u} \qquad \text{(Sec. 6.8)} \qquad (10.1f)$$

$$0\leq\mathbf{D_e}u_e[k+i]-d_e \qquad \text{(Sec. 7.2)} \qquad (10.1g)$$

$$d_b\geq\mathbf{D_b}u_b[k+i]+\mathbf{G_b}b[k+i] \qquad \text{(Sec. 7.3)} \qquad (10.1h)$$

10.1.1 The Objective Function

The objective function which is used in the deterministic VOPI analysis is shown in (10.2).

$$R_{\text{VOPI}}[k] = \sum_{i=0}^{L-1} \left(P_{\text{y}}[k+i]^T y[k+i]\Delta T + P_{\text{u}}[k+i]^T u[k+i]\Delta T + C_{\text{GT,su}}\phi_{\text{GT, su}}[k+i]) \right) + P_{\text{DH}}E_{\text{HA}}[k+L] \tag{10.2}$$

where

$P_{\text{y}}[k+i]$	The selling prices of electricity and DH at sample $[k+1]$	[DKK/MWh]
$y[k+i]$	The production output of electricity and DH at sample $[k+1]$	[MW]
$P_{\text{u}}[k+i]$	The production prices in the plant	[DKK/MWh]
$u[k+i]$	The vector containing the convex optimization variables	[MW]
ΔT	The sample time	[h]
$C_{\text{GT,su}}$	The cost of starting the gas turbine	[DKK]
$\phi_{\text{GT, su}}[k+i]$	The gas turbine startup signal at sample $[k+i]$	$[\cdot]$

The objective function describes the profit of the plant which is a weight between revenue, material cost and operating cost [Bisschop, 1993, p. 8]. The objective function is chosen linear because revenues and costs can be calculated using actual prices per units. The start-up cost of the GT is represented in the objective function in order to make the optimization problem able to evaluate when it is profitable to start the GT. In Section 2.5.1 the notion of imbalance penalty is introduced. Accounting for imbalance penalties is neglected in the deterministic VOPI analysis because the bid of electricity production is assumed correct, hence no imbalance penalty are paid. The imbalance penalties are used in the forecast based analysis described later. The next section describes the optimization problem where the constraints are included.

10.1.2 The Constraints of the VOPI Optimization Problem

This section provides reasons for choosing the constraints as they are. The state space system shown in (10.1b) describes the dynamics of the system, it is necessary to describe the parts of the system with is state dependent.

- The equality (10.1c) ensures that the supply of district heating Q_{DH} meets the DH demand r_{DH}. The supply and demand needs to be equal to keep the energy balance in the system.
- Equation (10.1d) describes the linear relationship between the input vector u and the production output vector y. The vector y is used in the objective function to state the revenue from the plant.
- The equation (10.1e) describes the nodes where the amount of power entering the node is equal to the power leaving the node, as Kirchoff's current law. These equations connect the modules in the plant.
- The inequalities seen in (10.1f) describe the physical limitations on the inputs.
- The constraint (10.1g) is necessary for the implementation of the event cost.
- The binary inequality (10.1h) is necessary for controlling the binary decisions on the VA and GT.

10.2 Results

This section describes the results from the VOPI analysis. The analysis of the results mainly concerns profit, solving time, window lengths and other interesting phenomenons in the tests.

Profit of Value of Perfect Information

Figure 10.1 presents the resulting profits per hour V_D from the simulations of the deterministic tests.

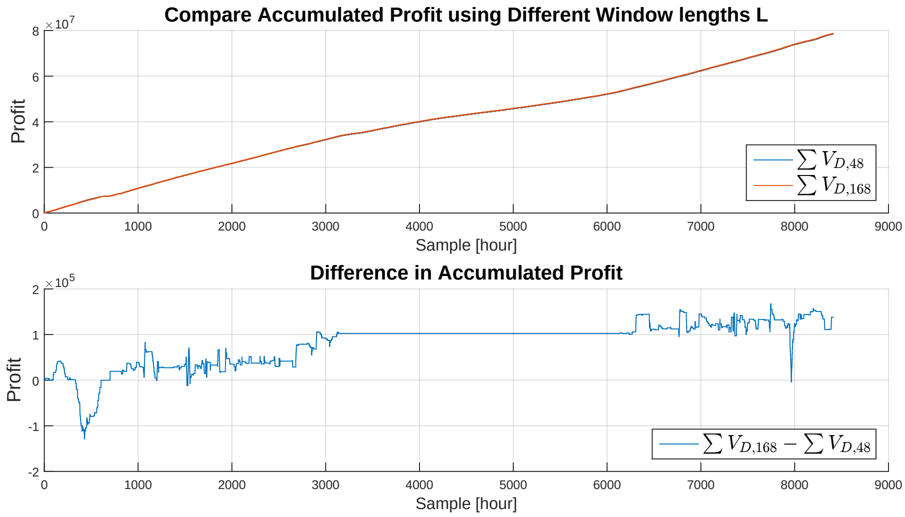

Figure 10.1: (a) Shows the accumulated profit over the full simulation (b) shows the difference in profit created by the different window lengths.

Subplot (a) in Figure 10.1 shows how the profit is steadily increasing over the simulation span. The slope decreases a bit in the middle of the graph but not significantly.

Window Length L [hour]	Profit [mio. DKK]	Average Profit per Hour [DKK]
3	77.542	9220.2
6	77.573	9223.9
12	77.863	9258.3
24	78.235	9302.6
48	78.531	9336.7
168	78.667	9352.9

Table 10.1: The profit gained when testing different window lengths in the simulation of the MPC controlled CHP.

As seen in Figure 10.1 the final difference in profit, between a window length of three compared to 168, is approximate 1.1 mio. The test having a window of three hours made 98.5 % the profit compared to the test with a wondow of 168 hours. The MPC with a window stretching 168 hours demands more computational power which means that it is important to consider the trade off. The behavior of the different window lengths is low therefore is the following analysis conducted on the test with a window of 48 hours.

Start-up Signal to the Gas Turbine

Figure 10.2 shows the timing of the start-up signal for the GT and it is seen that the start-up signal catches the rising flanks of the GT state as intended, which mean that the optimization problem is aware of this event cost. Over the entire simulation the GT was started 60 times. The GT was turned on 40 % of the time.

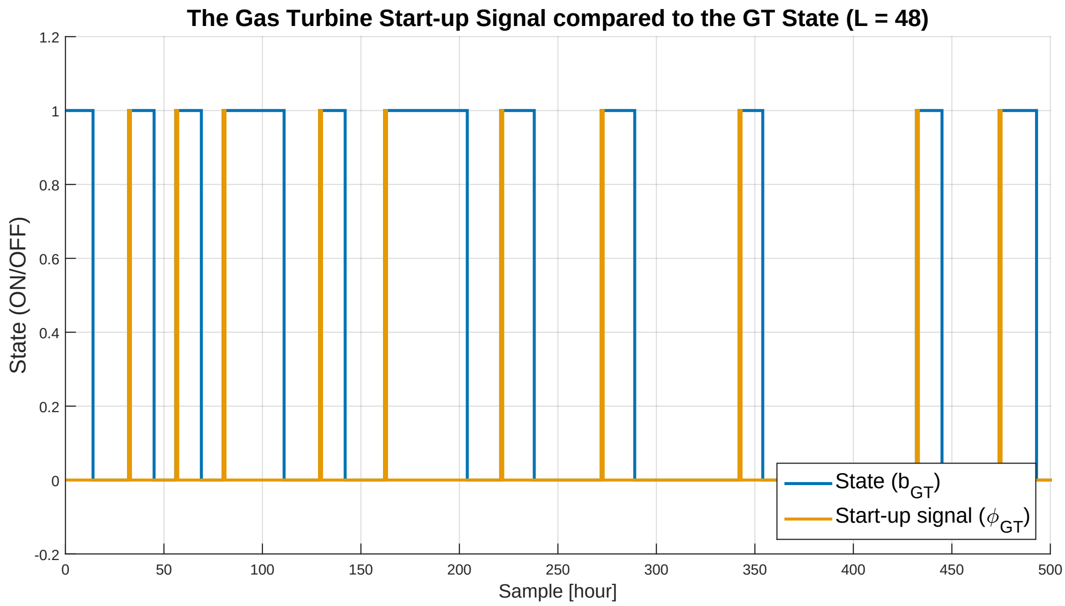

Figure 10.2: Shows the start-up signals fit to the rising flanks of the GT

The Electricity Production

Consider Figure 10.3, which presents the Electricity production Q_E.

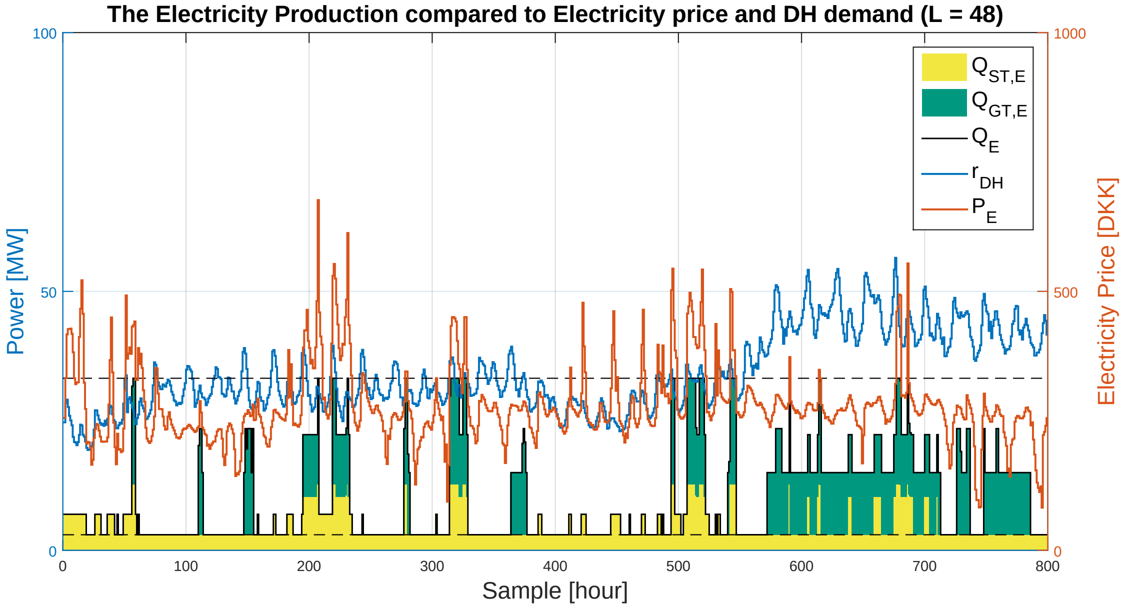

Figure 10.3: Shows the effects of DH demand Q_DH and electricity price P_E on the electricity production Q_E

Figure 10.3 shows how the electricity production Q_E is affected by the DH demand and electricity prices. From sample 1 to approximately 550 the demand for DH is low and the electricity production Q_E only spikes when the electricity prices P_E are favorable. From sample 550 and on the DH demand rises which generate a need for heat production. The electricity and heat production are coupled which means the electricity production also rises.

The input and Output of the Heat Accumulator

This section describes how the input Q_{H} and the output Q_{HA} of the HA are controlled such that the input and output are not active on the same time. Consider Figure 10.4.

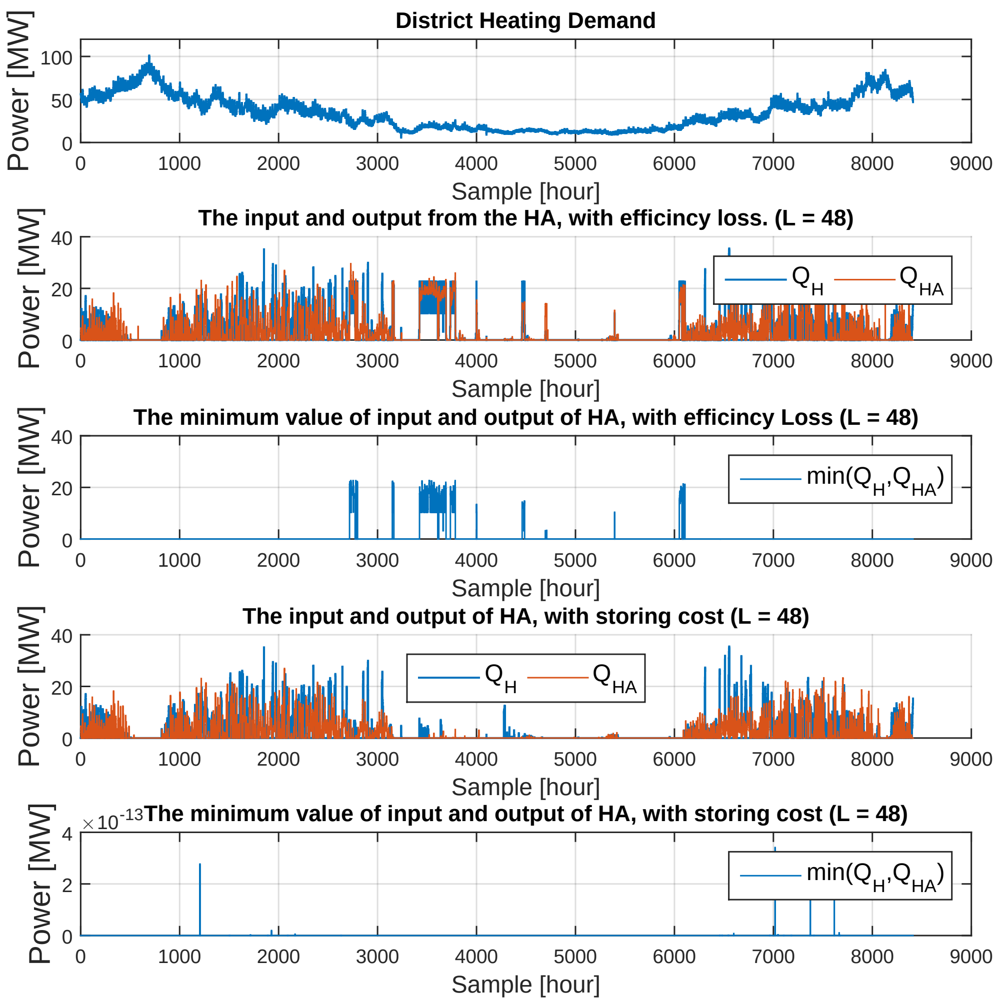

Figure 10.4: (a) Shows the district heating demand. (b) Shows Q_{H} and Q_{HA}, where they are controlled by an efficiency loss in the HA (c) Shows the minimum value of Q_{H} and Q_{HA} from (b). Graph (d) shows Q_{H} and Q_{HA} when they are controlled by a small cost. Graph (e) shows the minimum of Q_{H} and Q_{HA} in (d)

Figure 10.4 describes two approaches taken to control the input and output of the HA. The objective is to avoid an input is present simultaneously with an output, which means that at least one signal always needs to be zero.

Subplots (a) and (b) show the case where the inputs and outputs are controlled by a small artificial efficiency loss on Q_{H} as illustrated in (10.3).

$$E_{\mathrm{HA}}[k+1] = E_{\mathrm{HA}}[k] + (0.9999 Q_{\mathrm{H}}[k] - Q_{\mathrm{HA}}[k])\,\Delta T \tag{10.3}$$

The idea behind the gain loss in (10.3), is to make the optimization problem use Q_{H} when the energy is actually meant to be stored. The behavior of the optimization problem was different than expected. Instead of avoiding the efficiency loss in the HA the optimization problem utilize the functionality to burn power in low DH demand periods. The optimization problem burned power in the HA by having an active input simultaneous with the output. As seen in Subplot 2 the simultaneous input and output come in the low DH demand period where heat power is in excess.

The second approach is to add a small cost to the objective function, as seen in (10.4), which means that the system only uses the HA to store energy and not a way to get rid of excessive heat.

$$\max -0.001 Q_{\mathrm{H}}[k] \tag{10.4}$$

Subplots (c) and (d) in Figure 10.4 show the effect of the small cost, there is no instance where the two signals are greater than zero at the same time.

Solving Time of the Optimization Problem

This section describes the results on solving time of the optimization problem, stated in (10.1), as a function of the control window L .

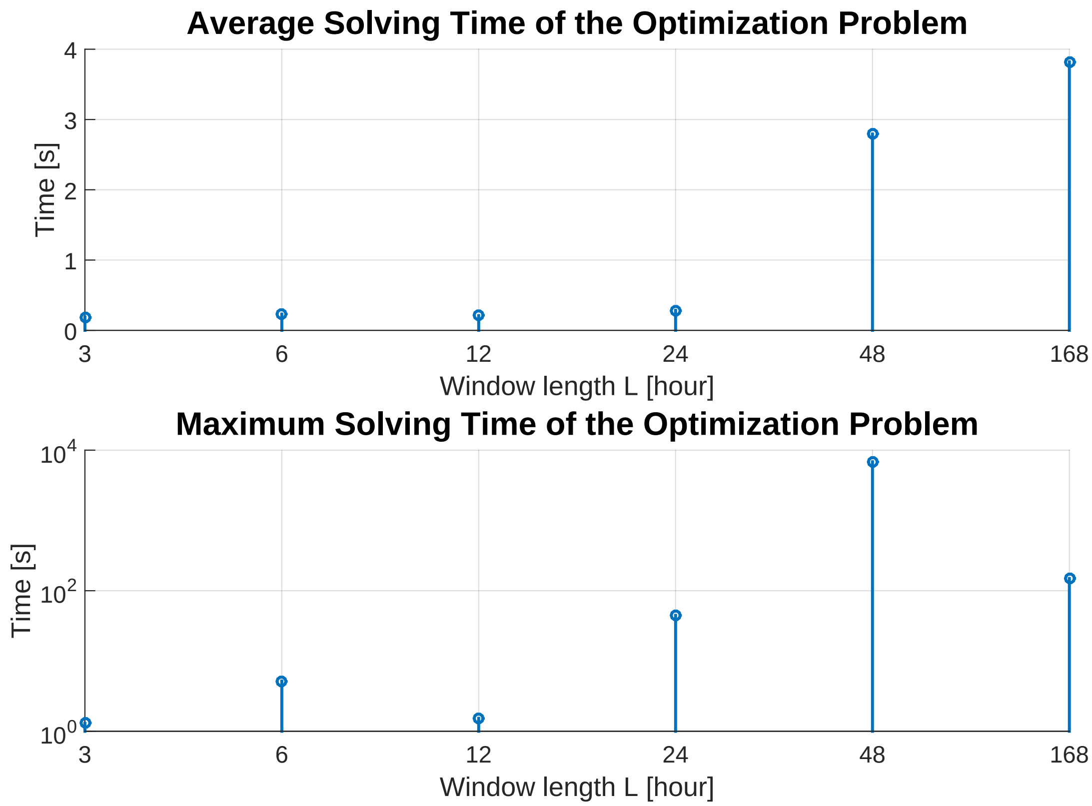

Figure 10.5: Subplot (a) shows the average solving time of the optimization problem. Subplot (b) shows the maximum solving time over the entire test.

The Subplot (a) in Figure 10.5 shows the average solve time. The solve times are low compared to the sampling time of one hour. Subplot (b) shows the maximum solving time during the test. where the average solving times is reasonably low for all tested window lengths, the maximum solving time is high for 24, 48 and 168 hours. Especially 48 hours had a maximum solving time of 114 minutes which is well over the one hour sampling time.

The difference, between the average and maximum solving time, indicates a large variation in solving time. The variation is shown in Figure 10.6 which compares the solving time with a window of 12 and 48 hours.

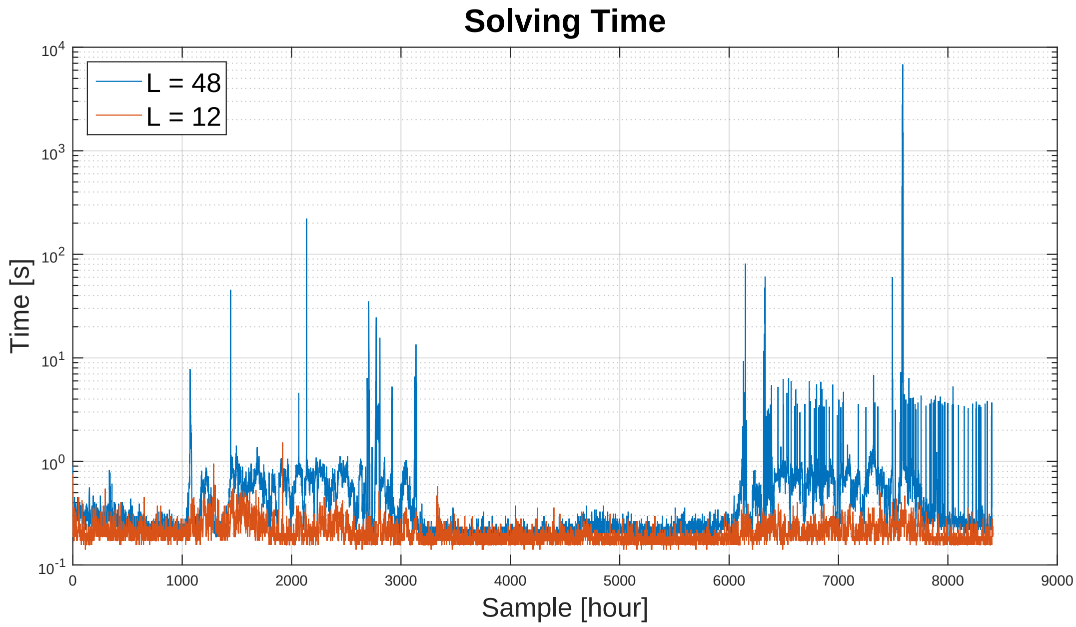

Figure 10.6

The optimization problem is a 0-1 MIP problem which may be the reason to the large variation in solving time. A 12 hour window is chosen for comparison because the solving time is consistency low, where the 48 hour window at some points stalls.

10.3 The Production Units

This section compares the behavior of the production units with the DH demand. Consider Figure 10.7

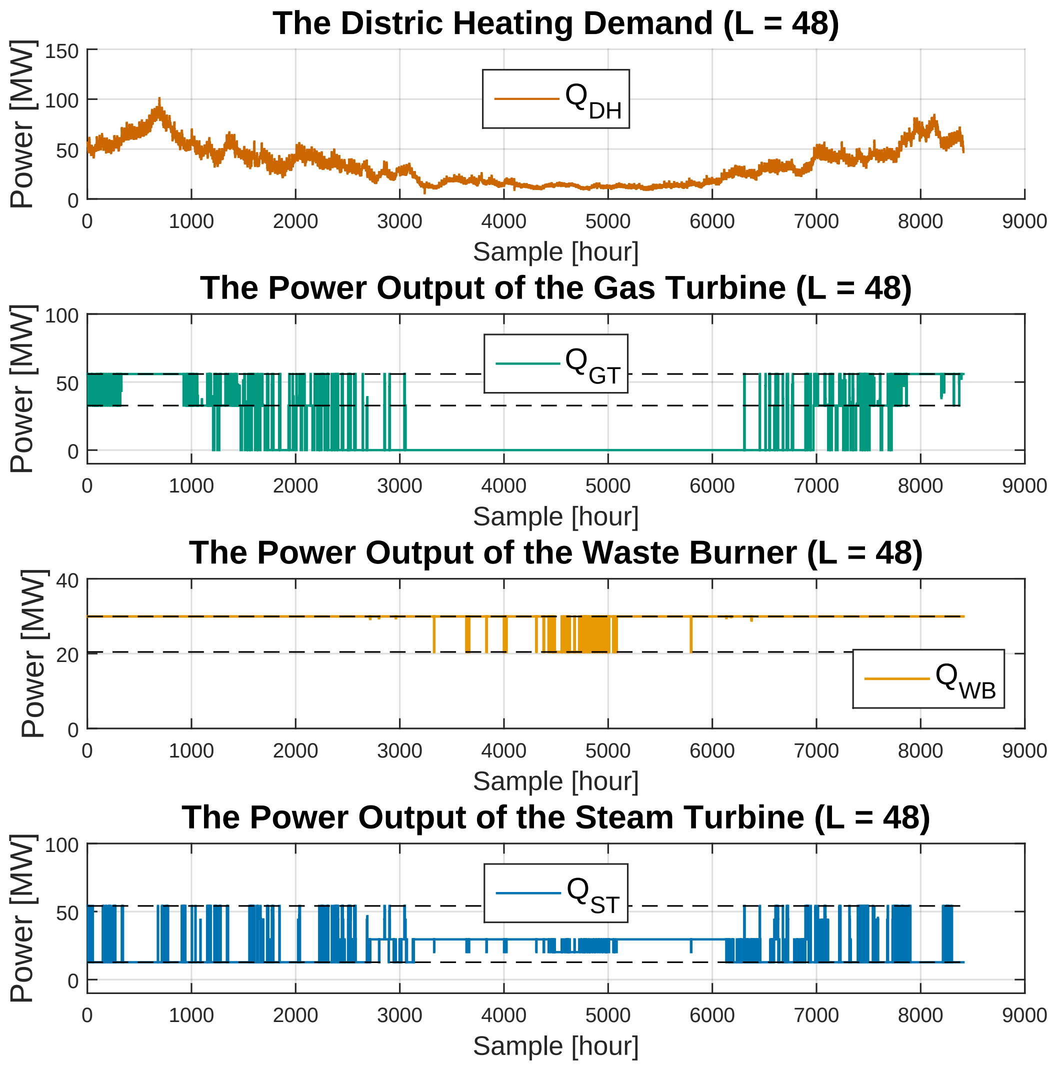

Figure 10.7: (a) shows the DH demand over the test window. (b) Shows the production of the gas turbine. (c) Shows the power production of the waste burner and (d) shows the production from the waste burner.

Figure 10.7 shows that the district heating demand has a large influence on the production in the different units. Furthermore it can be seen that the production units acts differently to the DH demand.

The production of the GT in Subplot (b) shows three different behaviors. When the DH demand is high the GT is producing at full speed, because the waste heat of the GT is needed. In the low demand period the GT is turned OFF. In the transition period, the GT is being turned ON and OFF.

The WB in Subplot (c) is running full capacity throughout the test window, except in very low demand periods. This is due to the very low price of producing from the WB.

Subplot (d) shows how the production times of the ST is close to opposite of the GT. The ST is running frequently when the DH demand is low and is unused for long periods when the demand is high. This effect is explained by the fact that the steam can bypass the ST and be used for heating in high demand periods. In low demand periods the steam turbine is converting some of the steam power to electricity and thereby lowering the heat power.

Chapter 11

Forecast Based Analysis

This chapter analyses the effects of basing the production planning on forecasts. The MPC algorithm is as any other system causal, which means information from the future cannot be used in the MPC algorithm, only predictions. Compared to the VOPI analysis, described in Chapter 10, this chapter only feeds the MPC algorithm forecasts generated before the optimization problem is solved.

11.1 The Optimization Problem for Forecast based Analysis

The optimization problem used in the forecast based analysis is presented in this section, with the objective function presented in Section 11.2. The optimization problem for forecast based analysis contains all the expressions used in the VOPI analysis.

$$\max_{u \in U, b \in B} R_{\text{FORECAST}}[k] \qquad \text{(Sec. 10.1.1)} \qquad (11.1a)$$

Subject to

$$x[k+1+i] = \mathbf{A}x[k+i] + \mathbf{B}u[k+i] \qquad \text{(Sec. 6.8)} \qquad (11.1b)$$

$$\hat{r}_{\text{DH}}[k+i|k-1] = \hat{Q}_{\text{DH}}[k+i] \qquad \text{(Sec. 5.3.3)} \qquad (11.1c)$$

$$y[k+1] = \mathbf{D_y}u[k+i] \qquad \text{(Sec. 6.8)} \qquad (11.1d)$$

$$\mathbf{0} = \mathbf{D_z}u[k+i] \qquad \text{(Sec. 6.8)} \qquad (11.1e)$$

$$\underline{u} \leq u[k+i] \leq \overline{u} \qquad \text{(Sec. 6.8)} \qquad (11.1f)$$

$$0 \leq \mathbf{D_d}u_d[k+i] - d_d \qquad \text{(Sec. 7.1)} \qquad (11.1g)$$

$$0 \leq u_d[k+i] \qquad (11.1h)$$

$$0 \leq \mathbf{D_e}u_e[k+i] - d_e \qquad \text{(Sec. 7.2)} \qquad (11.1i)$$

$$d_b \geq \mathbf{D_b}u_b[k+i] + \mathbf{G_b}b[k+i] \qquad \text{(Sec. 7.3)} \qquad (11.1j)$$

$$|\Phi^{-1}(\alpha)| \leq \overline{\beta}[k] \qquad \text{(Sec. 7.2)} \qquad (11.1k)$$

$$|\Phi^{-1}(1-\alpha)| \leq \underline{\beta}[k] \qquad \text{(Sec. 7.2)} \qquad (11.1l)$$

The constraints, which are added in (11.1), when compared to the VOPI optimization problem, are the probabilistic constraints (11.1k), (11.1l) and the imbalance penalty constraints (11.1g).

11.2 The Objective Function

The objective function used in the forecast based analysis is similar to the one used in the VOPI analysis. The only new expression is the imbalance penalties, shown in (11.4).

$$R_{\mathrm{IO}}[k] = \sum_{i=0}^{L-1} \left({P_u}^T u[k+i]\Delta T + P_{\mathrm{y}}[k+i]^T y[k+i]\Delta T \right) \tag{11.2}$$

$$R_{\mathrm{EVENT}}[k] = \sum_{i=0}^{L-1} \left(C_{\mathrm{GT,su}} \phi_{\mathrm{GT,\,su}}[k+i] \right) \qquad C_{\mathrm{GT,su}} < 0 \tag{11.3}$$

$$R_{\mathrm{IMBALANCE}}[k] = -\sum_{i=0}^{L-1} \left(\hat{C}_{\mathrm{DN}}[k+i]\Delta^+[k+i] + \hat{C}_{\mathrm{UP}}[k+i]\Delta^-[k+i] \right) \tag{11.4}$$

$$R_{\mathrm{END}}[k] = P_{\mathrm{DH}} E_{\mathrm{HA}}[k+L] \tag{11.5}$$

$$R_{\mathrm{FORECAST}}[k] = R_{\mathrm{IO}}[k] + R_{\mathrm{EVENT}}[k] + R_{\mathrm{IMBALANCE}}[k] + R_{\mathrm{END}}[k] \tag{11.6}$$

$$u = \begin{bmatrix} Q_{\mathrm{WB}}[k+i] & Q_{\mathrm{WB}}[k+i] & Q_{\mathrm{G}}[k+i] & \hat{Q}_{\mathrm{EP}}[k+i] \end{bmatrix}^T \tag{11.7}$$

$$y = \begin{bmatrix} Q_{\mathrm{E}}[k+i] & \hat{Q}_{\mathrm{DH}}[k+i] \end{bmatrix}^T \tag{11.8}$$

$$P_{\mathrm{IN}} = \begin{bmatrix} P_{\mathrm{WB}} & P_{\mathrm{WB}} & P_{\mathrm{G}} & P_{\mathrm{EP}} \end{bmatrix}^T \tag{11.9}$$

$$P_y = \begin{bmatrix} \hat{P}_{\mathrm{E}}[k+i]^T & {P_{\mathrm{DH}}}^T \end{bmatrix}^T \tag{11.10}$$

11.3 The Constraints

The forecast based optimization problem contains many repeated constraints from the VOPI analysis. These will not be discussed in this section, but are described in Section 10.1.2. The constraints which are added to the forecast based analysis are described below.

- The Imbalance Penalty inequality, seen in (11.1g), is together with the objective term generating the mechanism which allows the optimization problem to account for Imbalance costs.
- The probabilistic constraints posed as deterministic, seen in (11.1k) and (11.1l) generate the back-off which adjustment capacity to correct prediction errors.

11.4 Results

This section describe the results which are specific for the forecast analysis. The interesting or concerning tendencies of the results are presented in this section.

11.4.1 Profit of the Forecast Based Tests

Figure 11.1 describes the resulting profits $V_{F,0}$, $V_{F,\mathrm{AVE}}$ of the two test variations described in Section 8.4, where missing data (NaN) is substituted with weights of 0 or the average value. Figure 11.1 shows in Subplot (a) and (b) that the income rate is steady for both tests. The profit slope in the middle graph is slightly less steep in the summer period.

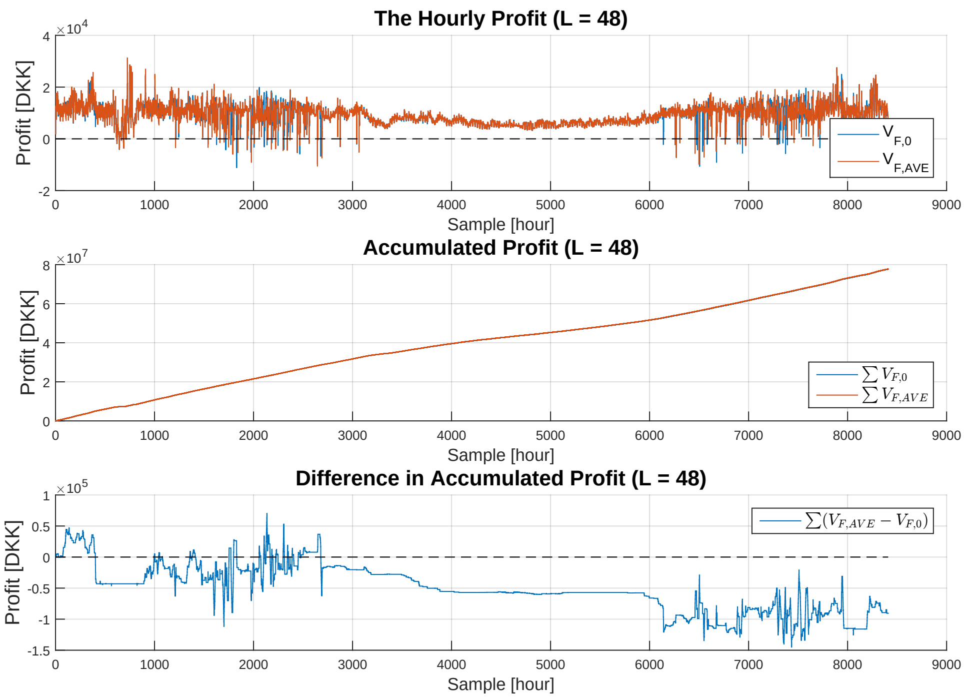

Figure 11.1: shows the comparison between the two tests conducted using forecast based analysis

The difference in profit between the two tests, seen in subplot (c) of Figure 11.1 and Table 11.1, is small compared to the overall profit they both generate. The difference in profit is not big enough to conclude which method is superior. The low difference in profit can indicate that the forecasts which are missing are having very low effect on the results.

Missing forecast	Profit [mio. DKK]	Average Profit per Hour [DKK]
0	77.786	9249.3
AVE	77.696	9238.5

Table 11.1: The production capacities and costs of the waste burner.

11.4.2 Comparison between Heat Accumulator State and Cooler

This section describes an interesting phenomenon between the HA state and the CO. The phenomenon appears in both tests so only one test is presented. Figure 11.2 shows four figures: (a) shows the HA state throughout the test, (b) Shows the Cooler, note that a negative value means it is consuming power, (c) Shows the first entry of the DH demand for each sample (d) Shows the predicted end state of the HA. The graphs depicts the summer period of the test, which has low demand for DH.

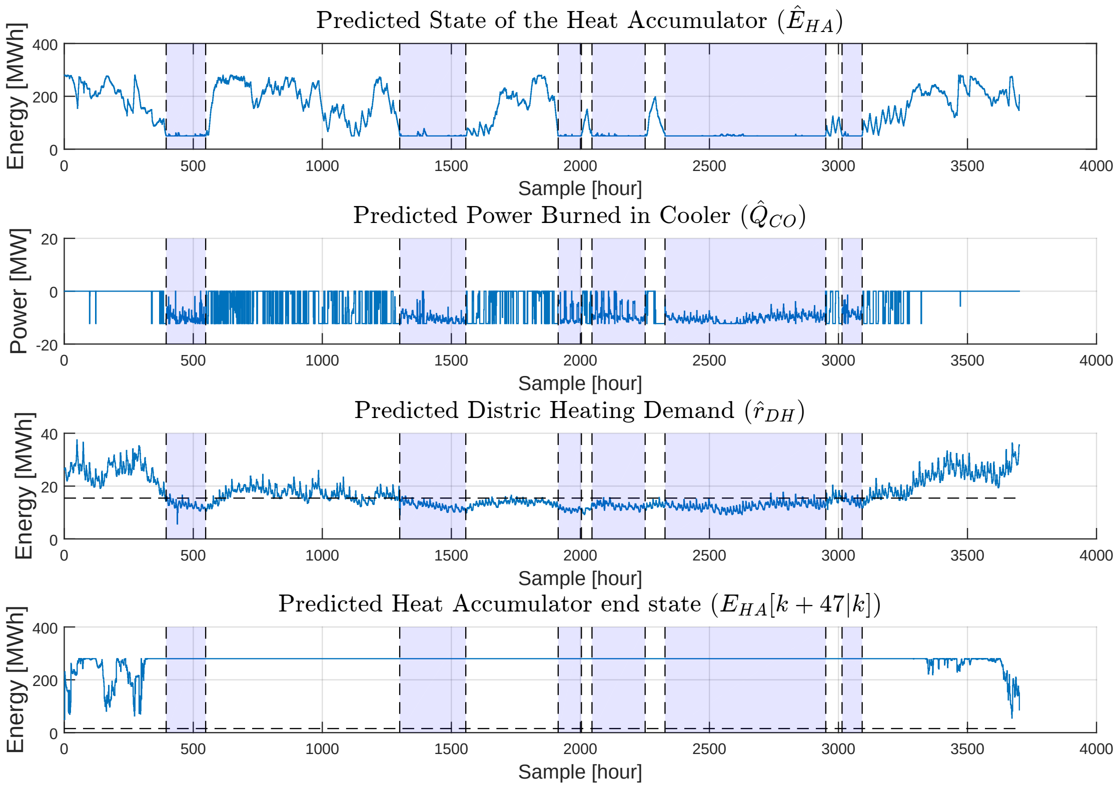

Figure 11.2: Subplot (a) shows the planned state of the HA. (b) shows the power which is burned in the CO. (c) shows the forecast of the DH demand. (d) shows the predicted end state of the HA.

Note the HA state is in minimum charge for extensive periods, which are marked with light blue zones. The second plot shows how the cooler is burning a lot of power in the periods where the HA is empty. The Cooler is even burning more power in these periods than otherwise. The question is "Why does the MPC not assign the excess power to the Heat Accumulator instead of burning it in the Cooler?". It seems suboptimal to waste energy which can be saved for later. A possible answer to the question might be found in graph (c) and (d). Graph (c) shows how the predicted DH demand sometimes falls below the horizontal dotted line which indicates the minimum heat production of the CHP. This means that the CHP is producing more heat power than the district heating can take. The excess heat then needs to ether be saved on the HA or burned in the CO. The predicted DH demand still does not fully explain the lag of energy in the HA, but graph (d) shows the predicted end state which shows a full tank. Only the end state of the HA is valued in the optimization problem of the MPC algorithm, which means the solution seems optimal according to the stated objective function, even though it might be an undesirable effect.

Difference between Bid and Production of Electricity

This section describes the electricity production with respect to the bid given in the day-ahead market. Consider Figure 11.3, which contains information about bid and production.

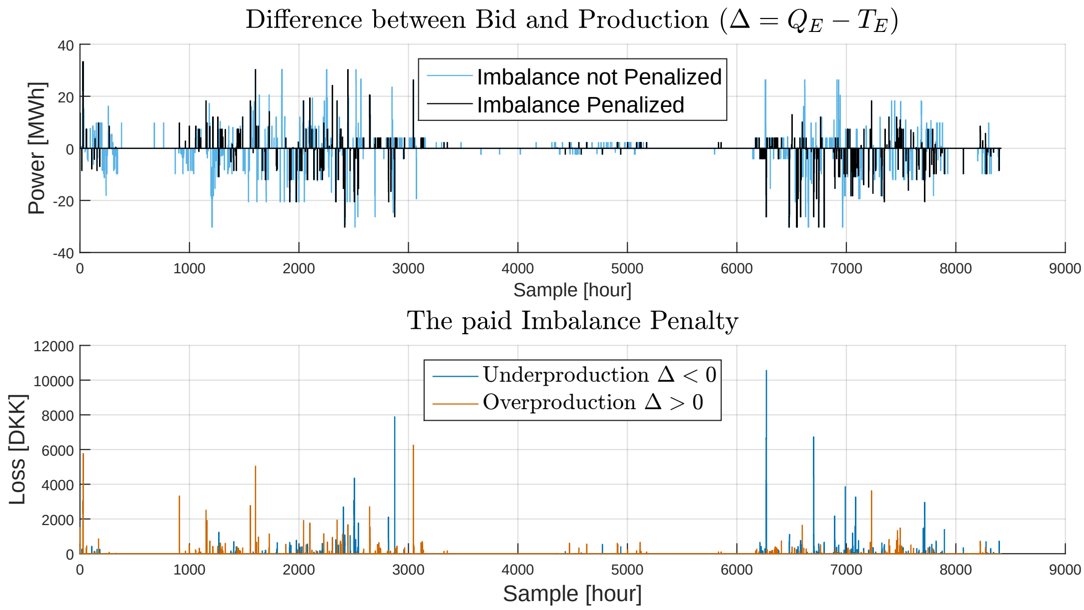

Figure 11.3: (a) Shows the difference of power output in production and bid. The blue bars indicate the situation where the production imbalance helped balancing the overall power grid, thereby no penalty is paid. The black bars present the case where the imbalance contributed to the overall imbalance and thereby an imbalance penalty is paid. (b) Shows the actual imbalances paid in case of overproduction or underproduction.

Figure 11.3 (a) shows the difference between bid and production. The difference Δ reaches errors above 30 MW. In the summer period the production error is small compared to the transition periods spring and fall.

It is important to note that 30 MW production difference is not a large error seen from the grid, but the error almost coincides with the entire production capacity of the CHP plant. This raises the questions, "What happens if the MPC algorithm is applied to a larger power plant? Will the error grow proportionally?", which are not answered in this project, but very important to consider if the algorithm is applied elsewhere.

The production errors occur for several reasons: (1) the imbalance penalty model, described in 7.1, is build such that the optimization problem tries to play the marked. The model is fed forecasts of the imbalance penalties which holds information about whether the marked is in up or down-regulation which can be used to make extra profit. Opposite, when the prediction is wrong the plant looses profit. (2) The second reason is the time delay between making the bid and delivering the power. The bid is based on predictions and the situation is changed when the production is delivered. (3) The third reason is the commitment to deliver District Heating according to the demand. This means the CHP needs to adapt according to the DH demand which means it has to neglect the electricity commitment.

Figure 11.3 (b) shows the penalty paid in each hour, these penalties can be considered pure loses in profit. The accumulated cost of Imbalance Penalties is 294000 DKK during the test.

11.5 An Error in the Implementation of Down-Adjustment Capacity

The results of the forecast based simulation revealed a problem with the estimation of the down-adjustment capacity. The individual test of the optimization building block for down adjustment shows positive results, but combined with the entire optimization problem, the functionality changes.

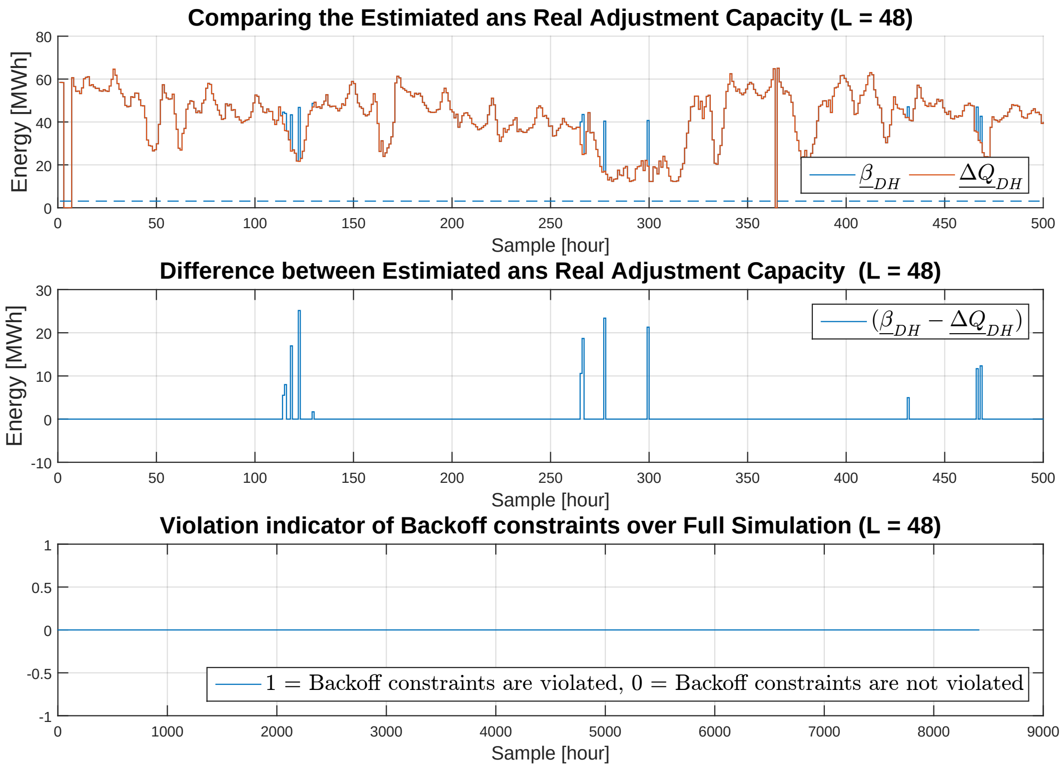

Figure 11.4: Subplot (a) shows the estimated down-adjustment capacity $\underline{\beta}_{\mathrm{DH}}$ sometimes exceeds the real down-adjustment capacity $\underline{\Delta Q}_{\mathrm{DH}}$. Subplot (b) shows the difference between estimated and real down-adjustment capacity. Subplot (c) shows that the estimation error has not caused a violation to the back-off constraint over the entire simulation.

Subplot (a) in Figure 11.4 shows how the estimated down-adjustment capacity $\underline{\beta}_{\mathrm{DH}}$ in some cases exceeds the actual down-adjustment capacity $\underline{\Delta Q}_{\mathrm{DH}}$. This violates the rule stated in Section 7.6, which says that $\underline{\Delta Q}_{\mathrm{DH}} \geq \underline{\beta}_{\mathrm{DH}}$. Subplot (b) shows the difference $\underline{\beta}_{\mathrm{DH}} - \underline{\Delta Q}_{\mathrm{DH}}$. The graph shows how much the estimation of down-adjustment capacity exceeds the actual capacity. Subplot (c) shows the consequences of the violation over the entire test. It is seen that no violation of the back off constraints occur in the test. This does not legitimatize the method for estimating the down-adjustment capacity but it legitimatizes the remaining results.

The error is caused by the implementation described in Section 7.6.3. The implementation of

$$\max(0, \overline{\Delta E}_{\mathrm{HA}}[k+1] - \underline{\Delta Q}_{\mathrm{HA}}[k]\Delta T) \tag{11.11}$$

is not imitating correctly. Instead of describing the value of (11.11) and thereby describe the state of the system, the implementation, seen in (7.42a), acts as a cheap (close to free) variable which can be chosen independent of (11.11).

Chapter 12
Compare Deterministic with Forecast Based Analysis

This chapter describes the difference between the deterministic and forecast based tests. The prediction horizons L for both tests are 48 hours.

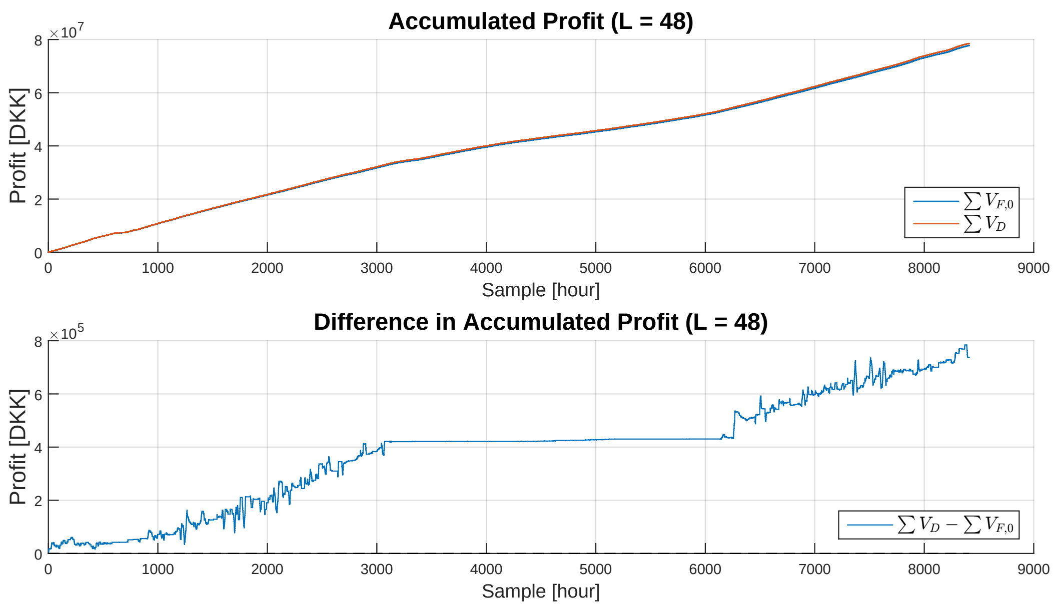

Figure 12.1

Figure 12.1 shows the profit gained from the deterministic and forecast based tests. The hourly profit for the forecast based test is denoted $V_{\mathrm{F},0}$ and the deterministic hourly profit is depicted V_{D}. Subplot (b) shows the difference in accumulated profit. Where the difference grows in the transition periods (Spring and Fall), the forecast based MPC almost performs equally good as the deterministic in the summer period.

The following table shows comparable parameters which gives indicators of the performance of the MPC algorithm.

parameter	Deterministic	Forecast Based	Unit
Profit	78.524	77.786	[mio. DKK]
Number of gas turbine start-ups	60	70	[·]
Energy produced from gas turbine	170.92	171.98	[GWh]
Heat burned in cooler	21.755	21.962	[GWh]
Income per MWh electricity sold	310	305	[DKK/MWh]

Table 12.1: The production capacities and costs of the waste burner.

Profit

The difference in profit between the deterministic and forecast based test is 737 340 DKK, which means the forecast based test makes 99 % of the profit the deterministic test makes. This is an extremely good result considering that the deterministic test is a concept used as an indicator and the forecast based is closer to an implementable version.

Number of Gas Turbine Start-ups

The number of gas turbine starts is an interesting parameter because the start-up is expensive to perform. The forecast based test started the gas turbine 10 times more than the deterministic based test, which is expected to be due to miscalculations caused by uncertainty in the forecasts. The extra gas turbine start-ups account for 100 000 DKK of the difference.

Energy Produced from the Gas Turbine

The forecast based test used the gas turbine 0.62 % more than the deterministic test. The extra production from the gas turbine amounted in an extra cost of 284 860 DKK compared to the deterministic test.

Heat Burned in Cooler

The forecast based test burned 0.96 % more energy in the cooler compared to the deterministic test. Energy burned in the cooler is an efficiency loss.

Income per MWh electricity sold

The parameter, average income per sold MWh electricity is an indicator of how efficient the MPC algorithm is at utilizing the variations the electricity price.

Chapter 13

Conclusion

In this project, the potential of profiting on using MPC for production planning in a CHP plant has been investigated.

A high level, hybrid state model of Horsens Kraftvarmeværk has been formulated. The model describes the couplings and efficiencies of the modules in the plant. The model of Horsen Kraftvarmevæk is specified in Chapter 6

Two 0-1 mixed integer programming optimizations problems, which seek to maximize profit, have been formulated: The first is formulated to the deterministic analysis, seen in Section 11.1 and the second for the forecast based analysis, seen in Section 10.1. Financial influencing topics which are considered counts as:

- The costs associated with producing energy from the different production units.
- The aspects of the electricity market which includes production bid, electricity prices and imbalance penalties.
- The demand for district heating.
- The back-off constraints which are based on a statistical analysis of the district heating forecast data.
- The one time cost of starting the gas turbine.
- The end state of the heat accumulator.

The MPC algorithm, which uses the model and optimization problem to make the production decisions, has been tested in simulation. The simulation is fed empirical data containing measurements and forecasts of the district heating demand, electricity prices and imbalance penalties. Several simulations with different parameters have been conducted and the main results and findings are summed up below.

- The results from the forecast based analysis showed a profit on 99 % of the deterministic analysis. A loss of 1 % in the transition from the deterministic to the forecast based analysis means that very little further profit can be gained by increasing the quality of the forecasts.
- The window length of the MPC control horizon has been investigated. The results from testing the varies window lengths showed that profit increases with window size. The difference is deemed small because the profit of using a window size of three hours amounted in 98.5 % of the profit gained when using a 168 hour window. Besides profit, calculation time of the optimization problem has been evaluated according to the window length. In this case the calculation time increases with window length which is explained by the higher complexity. The results showed that a window of 12 hours had a high profit rate and fast calculation time.

- The issue of having forecasts with different prediction horizons has been investigated in the forecast based analysis. The results from the simulations showed little difference in the results when blank spaces ware filled with different kinds of pseudo data. The conclusion is thereby that if the blank spaces are fare enough from the sample time it is not a problem having forecast data with different prediction horizons.

Chapter 14

Perspective and Discussion

The perspective of this project is to figure out how to generalize the models and methods applied in the project. Many of the concepts used in this report are inspired by methods which are applied to control multiple power plants in a portfolio. The challenge is then to incorporate this specific description of a plant back into the generalized portfolio of controllable plants.

14.1 Future Work

This section discusses topics which are interesting to investigate further.

Reevaluate formulation of Optimization Problem

Not all subjects which have an influence on the profit have been investigated in this project. This section discusses some of the subjects left out.

Investigate the Heat Accumulator

Looking further into the heat accumulator is interesting because it can relieve some of the dependencies between heat and electricity production. Removing some of the dependencies between the heat and electricity output enables the plant to make better production decisions in relation to electricity prices and heat demand. The question is: "how much impact the heat accumulator does when it comes to increasing profit?". The heat accumulator can store energy for eight hours which is fairly low. It is interesting to investigate whether shifting the objective of the heat accumulator from making profit to stability purposes has a large impact on profit.

Efficiency Curves on Production Units

The efficiency of a turbine is dependent on the speed, but no focus has been given to the set-point efficiency in this project. In future work it is interesting to consider the impact of building efficiency curves into the optimization problem. With a linear efficiency the solution to the optimization problem seeks towards bounds. An efficiency curve might let the optimization problem find a more interesting set-point for the combination of production units.

Regularization of Set-Point Changes

Changes in production set-points tares on the system, which means there is a cost associated with this. This cost can be build into the system using a regularization term. It is interesting to explore the effect of implementing a cost which considers set-point changes.

Reevaluate the Plant Model

The plant model is in this project formulated in a high abstraction level. This model is useful when solving the optimization, but a more sophisticated model is needed if a more thorough analysis of the interplay between the CHP and MPC is wanted.

Data

The forecast data proved to be a very efficient substitution for the perfect information. The forecasts provided have zero mean. Investigating the effects of a bias might be future work.

Implementation of the MPC Algorithm

An interesting topic is analyzing which challenges needs to be solved in order to implement. Four major challenges in scope are solving time, stability, algorithm reliability and realistic timing. Solving time and reliability are important for the MPC to run as an online procedure. The stability needs to be guarantied before implemented such the plant does not get destroyed. Realistic timing is important in the implementation because the input cannot be given at same time as it is calculated.

Improving the Production Bids

The implementation of the production bids in this project is very simplified. No consideration on how to do the bidding smart has been performed. This is a topic for future work.

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Appendix A

Principles of the Danish Energy Market

This chapter is cited from a 8. semester report on Aalborg University. Online economic optimization of wind farm operation. 2014 Jakobsen et al. [2014]

In the 1990s the Nordic countries deregulated their power markets, effectively introducing free competition on the market [Nord Pool Spot, 2014a]. In the late 2000s the Baltic countries followed suit, and joined the common power trading place Nord Pool Spot.

The Nord Pool Spot market is divided into several bidding areas, as seen in figure A.1. The division of each country is done by the non-commercial owner of the high voltage backbone grid, known as the transmission service operator (TSO), in order to best avoid transmission bottlenecks. Having a common market place consolidates the overall social welfare by actively reducing the price difference between bidding areas [Nord Pool Spot, 2011b].

Figure A.1: Bidding areas within Nord Pool Spot.

This study is only concerned with the Danish bidding area DK1, which is constituted by Jutland and

Funen. The high voltage backbone grid of the country connects the bidding areas, and is connected to the low voltage distribution net by two regional grids, which are mid-level voltage grids [Energinet.dk, 2014a]. In Denmark the predominant power supply comes from thermal power, but wind power is becoming increasingly important [Nord Pool Spot, 2014a].

A.1 Elspot Day-ahead Market

As much as 99 % of all power trading within Nord Pool Spot is done on the Elspot day-ahead market [Skajaa, 2013]. Elspot is a web-based auction where producers place bids on their expected power production each hour the following day, and buyers similarly place their bids on expected power consumption. The daily routine of the Elspot market is illustrated in table A.1.

Time	Elspot day-ahead daily routine
12:00	Gate closure: Deadline for submitting bids to Elspot. Begin calculations of the Elspot prices.
12:45	Market clearing: Nord Pool Spot publishes the calculated spot prices.

Table A.1: [Ackermann, 2012,pp 531-533], [Nord Pool Spot, 2014c,p 4].

Example A.1 (Elspot volume bids)
The power production bid, T_S, may depend on the settled spot price. Figure A.2 shows an example of a production bid from a producer and a consumption bid from a buyer. The amount of power is bid such that it is dependent on the spot price for that particular hour [Nord Pool Spot, 2014b]. As expected, the producer intends to produce more the higher the price (30 MWh if the price is 60 €, while only 10 MWh if the price is 50 €). Conversely, the buyer intends to buy more the lower the price (50 MWh if the price is 20 €, while only 10 MWh if the price is 40 €).

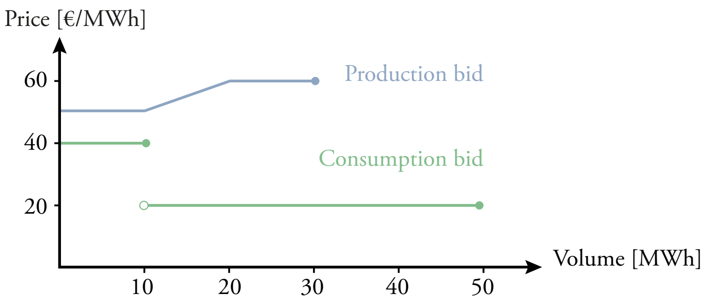

Figure A.2: Production bid from a producer and consumption bid from a buyer. The final volume bid T_S is dependent on the settled Elspot price P_S. [Nord Pool Spot, 2013,p 9]

When the day-ahead bids have been submitted to Nord Pool Spot, the power spot price P_S for each hour is calculated based on the bids of supply and demand, as seen in figure A.3.

The day-ahead expected revenue, R_{Elspot}, in the hour of operation is then given as:

$$R_{\text{Elspot}} = P_S T_S \tag{A.1}$$

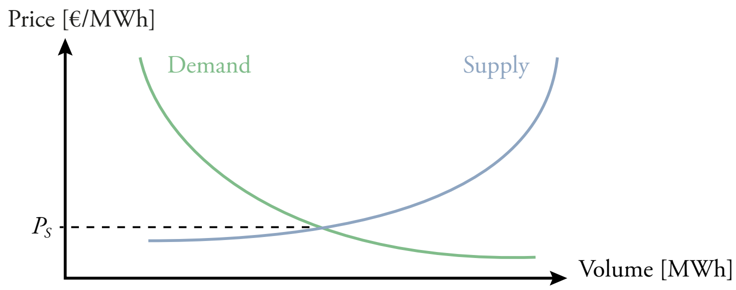

Figure A.3: Demand and supply for the Elspot market depending on price. The spot price P_S is settled as the intersection of the two bidding curves. [Nord Pool Spot, 2013,p 10]

The spot prices in different bidding areas may differ, e.g. if the demand in one area is large compared to the supply and the import capacity cannot accommodate this, the price will be higher in this area than in an area where supply and demand are at the same level or transmission capacity is not a limiting factor.

A.2 Regulating Market

The Regulating market is the TSO's tool to maintain a balance between total production and consumption of power, and obtain a stable frequency in the grid [Nord Pool Spot, 2011a, 2013]. In case the total production exceeds the consumption, the grid frequency will rise above 50 Hz, and conversely if the consumption exceeds the production, the frequency will drop below 50 Hz. As the TSO has no say in the consumers consumption, it must regulate the power production to balance the transmission grid and keep it at a stable frequency.

As power is bought and sold for hourly intervals the day ahead via the Elspot market, the actual production or consumption may vary from the bids previously agreed upon. The TSO will then need to either procure more power from the producers or sell power back to the producers. This is where the regulating market is used.

A.2.1 Bidding in the Regulating Market

A producer can either be committed to submit a bid for every hour of the day for both up- and down-regulation, or the producer may submit a bid if he has the capacity to change the day-ahead bid production. The former type, who is committed to submit bids for every hour, is paid an additional fee for being standby for activation on short notice. The latter type of producer is only paid for the actual production of power, but does not necessarily have to give a bid for up- and/or down-regulation for every hour.

On the regulating market, power is also sold at hourly intervals, but the bidding in the regulating market is open up until 45 minutes prior to the hour of operation. Each producer has bid his expected capacity for regulating production to the Nordic Operator Information System (NOIS) list for the hour of operation. The up- and down-regulation prices are then set in a similar way as for the Elspot day-ahead market. If the TSO wishes to procure a certain amount of power, the bids of the lowest price from the NOIS list are activated until the required amount of power has been reached, and all the producers will be paid the up-regulating price P_{UP}, according to the highest activated bid, for their production.

Example A.2 (Up-regulation by 400 MW)
If the TSO needs to procure an additional 400 MW of power, the up-regulating power is bought by the TSO from all the producers with excess power selling at the lowest price, up until 400 MW is reached, see figure A.4. The last activated bid price will then be the up-regulation price P_{UP} [Nord Pool Spot, 2013].

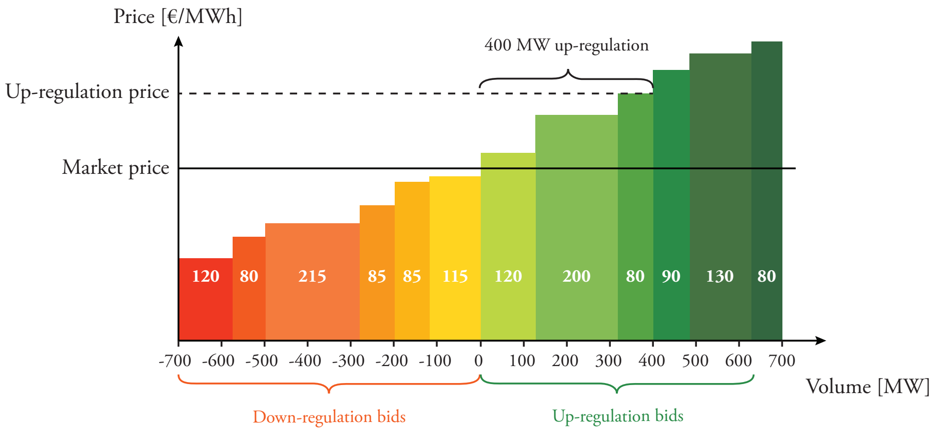

Figure A.4: Activation of bids up until 400 MW from the TSOs NOIS list [Nord Pool Spot, 2013].

Similarly for down-regulation, the producers submit their bids to lower their production compared to the previously settled production bids from the Elspot market. If activated, the producers will then be invoiced the down-regulation price P_{DN} for the production not produced.

During an hour with both up- and down-regulation of the grid, the sum of the regulating power will decide whether the hour has been in up- or down-regulation. Because the spot price is used as the starting price, the up-regulation price is usually higher than the market price (it may also be equal to), and the down-regulation price is usually lower:

$$P_{DN} \leq P_S \leq P_{UP} \tag{A.2}$$

Up- and down-regulation prices can differ between bidding areas. This happens if the interconnection between the bidding areas has reached its maximum capacity. In case the transmission capacity between two adjacent bidding areas has reached its limit and additional power is needed in one of the areas, the TSO might not be able to activate the lowest bids from the NOIS list, as the bidder may be located in the neighboring area to which the transmission capacity is already fully utilized. The TSO will then need to activate another producer from the NOIS list at a higher price, and thus the regulating price in the two areas will be different.

A.3 Balancing Market

The Balancing market is a passive market where the producers cannot place bids. It is a market where the price is settled according to the up- and down-regulation prices set by the regulating market, for the deviations or imbalance from the bid power production, T_S, to the actual power production sent to the grid, Q_G. The production deviation Δ is given as [Skajaa, 2013]:

$$\Delta = Q_G - T_S \tag{A.3}$$

For simplification, this deviation is split up into positive and negative deviations from the contracted production:

$$\Delta = \Delta^+ - \Delta^- \tag{A.4}$$

Both Δ^+ and Δ^- are measured as the absolute deviation in either a positive or negative direction from the contracted value, that is $\Delta^+ = \max\{\Delta, 0\}$ and $\Delta^- = \max\{-\Delta, 0\}$, thus $\Delta^+ \geq 0$ and $\Delta^- \geq 0$. As the Elspot market power production bids are made for each hour, the deviation can only be either positive or negative during any given hour, i.e. for any one producer only Δ^- or Δ^+ is nonzero in a particular hour [Skajaa, 2013]:

$$\Delta^+ \begin{cases} > 0 & \text{iff } Q_G > T_S \\ = 0 & \text{otherwise} \end{cases} \qquad \text{and} \qquad \Delta^- \begin{cases} > 0 & \text{iff } Q_G < T_S \\ = 0 & \text{otherwise} \end{cases} \tag{A.5}$$

The imbalance revenue, compared to the revenue calculated the day ahead in equation A.1, is then calculated as:

$$R_{\text{imbalance}} = P_{DN}\Delta^+ - P_{UP}\Delta^- \tag{A.6}$$

A.3.1 Pricing in the Balancing Market

In case of up-regulation, where the transmission grid is in a power deficit, the producer will be paid the market price for any excess power produced over the previously agreed production. However, if the producer does not meet the bid production, the amount produced less will be invoiced at the up-regulation price, after the producer is paid according to the bid production at market price.

In case of down-regulation, the spot price will be paid for any production less or equal to the bid production, while power production exceeding the Elspot bid will be paid according to the down-regulation price, which is lower than the spot price.

These four cases can be stated mathematically as follows:

1. The market is in **up-regulation**, and the producer is **sending more** to the grid than contracted:

$$Q_G \geq T_S \qquad \Rightarrow \quad \Delta^+ \geq 0 \quad \wedge \quad \Delta^- = 0 \tag{A.7}$$

The producer is producing in favor of the market i.e. producing more than contracted at a time where the market needs more power, hence the producer is paid the spot price for any excess power produced.

$$\left.\begin{aligned} R_{\text{bid}} &= P_S T_S \\ R_{\text{imbalance}} &= P_S \Delta^+ \end{aligned}\right\} \qquad R_{\text{total}} = R_{\text{bid}} + R_{\text{imbalance}} = P_S T_S + P_S \Delta^+ = P_S Q_G \tag{A.8}$$

2. The market is in **down-regulation**, and the producer is **sending more** to the grid than contracted:

$$Q_G \geq T_S \qquad \Rightarrow \quad \Delta^+ \geq 0 \quad \wedge \quad \Delta^- = 0 \tag{A.9}$$

The producer is contributing to the energy surplus of the market i.e. producing more than contracted at a time where the market is in excess of power, hence the producer is penalized by being paid at the down-regulation price for the power produced more than the bid.

$$\left.\begin{aligned} R_{\text{bid}} &= P_S T_S \\ R_{\text{imbalance}} &= P_{DN} \Delta^+ \end{aligned}\right\} \qquad R_{\text{total}} = R_{\text{bid}} + R_{\text{imbalance}} = P_S T_S + P_{DN} \Delta^+ < P_S Q_G \tag{A.10}$$

Example A.3 (The Producer Sent More to the Grid than was Bid in Elspot)
A producer who is contracted 100 MWh for the hour of production, at a spot price of 2 €/MWh does not meet the contract and instead produces 110 MWh. If the market is in up-regulation the producer will be paid full price of 220 € for the produced 110 MWh thus not losing any revenue on the balancing market for not meeting the contract.

However, if the market is in down-regulation, the producer will be paid 200 € for the contracted production, but the remaining 10 MWh are paid at the down-regulation price of 1 €/MWh. This leaves the producer with a total revenue of 210 € which is a potential loss compared to the spot price of 220 € for the produced power.

3. The market is in **up-regulation**, and the producer is **sending less** to the grid than contracted:

$$Q_G \leq T_S \qquad \Rightarrow \quad \Delta^+ = 0 \quad \wedge \quad \Delta^- \geq 0 \tag{A.11}$$

The producer is contributing to the energy deficit of the market i.e. producing less than contracted at a time where the market needs more power, hence the producer is penalized by being invoiced at the up-regulation price for the bid power not produced.

$$\left.\begin{aligned} R_{\text{bid}} &= P_S T_S \\ R_{\text{imbalance}} &= -P_{UP} \Delta^- \end{aligned}\right\} \qquad R_{\text{total}} = R_{\text{bid}} + R_{\text{imbalance}} = P_S T_S - P_{UP} \Delta^- < P_S Q_G \tag{A.12}$$

4. The market is in **down-regulation**, and the producer is **sending less** to the grid than contracted:

$$Q_G \leq T_S \qquad \Rightarrow \quad \Delta^+ = 0 \quad \wedge \quad \Delta^- \geq 0 \tag{A.13}$$

The producer is producing in favor of the market i.e. producing less than contracted at a time where the market is in surplus of power, hence the producer is paid at the spot price for the total production.

$$\left.\begin{aligned} R_{\text{bid}} &= P_S T_S \\ R_{\text{imbalance}} &= -P_S \Delta^- \end{aligned}\right\} \qquad R_{\text{total}} = R_{\text{bid}} + R_{\text{imbalance}} = P_S T_S - P_S \Delta^- = P_S Q_G \tag{A.14}$$

Example A.4 (The Producer Sent Less to the Grid than was Bid in Elspot)
Conversely, the producer who is contracted 100 MWh for the hour of production, at a spot price of 2 €/MWh does not meet the contract and only produces 90 MWh. If the market is in up-regulation, the producer will be paid full price of 200 € for the contracted 100 MWh. For the production deficit of 10 MWh the producer will be invoiced for 10 MWh at the up-regulation price of 3 €/MWh. This leaves the producer with a revenue 170 €, which is a potential loss compared to the spot price of 180 € for the produced power.

However, if the market is in down-regulation, the producer will be paid full price of 200 € for the contracted 100 MWh, and invoiced for the production deficit of 10 MWh at the spot price, leaving the producer with a revenue of 180 € i.e. no loss for the lower actual production.

The up- and down-regulation prices can be handled as penalties, C [Skajaa, 2013,p. 7], defined as:

$$C_{UP} = P_{UP} - P_S \qquad \text{and} \qquad C_{DN} = P_S - P_{DN} \tag{A.15}$$

Only one of the penalties can be nonzero for each hour of operation (within one bidding area). The total revenue can now be expressed in one equation valid for all four cases, as:

$$R_{\text{total}} = P_S Q_G - (C_{DN}\Delta^+ + C_{UP}\Delta^-) \tag{A.16}$$

where

P_S	is the Elspot price	[€/MWh]
Q_G	is the actual production sent to the grid	[MWh]
C_{UP}, C_{DN}	are the up- and down-regulation penalties	[€/MWh]
Δ^+, Δ^-	are the positive/negative part of production deviation, respectively	[MWh]

A.3.2 Summary of Regulation Situations

The market can be in three different regulation situations. In case of up-regulation, the down-regulation penalty is zero, $C_{DN} = 0$; in case of down-regulation, the up-regulation penalty is zero, $C_{UP} = 0$. In case the market is neutral then both $C_{DN} = 0$ and $C_{UP} = 0$.

The producer could also be in two different situations; either over- or under-producing compared to the day-ahead bid. When disregarding the neutral market situation, where penalties are not present, this gives the four different combinations of regulation and production situations.

This is illustrated in table A.2 and figure A.5.

		Market			Market	
	revenue R_{total}	up-regulation $C_{DN}=0$	down-regulation $C_{UP}=0$		up-regulation $C_{DN}=0$	down-regulation $C_{UP}=0$
Producer	over-production $\Delta^- = 0$	$P_S Q_G$	$P_S Q_G - C_{DN}\Delta^+$	=	Case I	Case II
	under-production $\Delta^+ = 0$	$P_S Q_G - C_{UP}\Delta^-$	$P_S Q_G$		Case III	Case IV

Table A.2: Combinations of market and producer situations. These are defined as cases I-IV. The same is illustrated in figure A.5. The colours indicate the balancing prices paid to producers that have produced more than was bid, or invoiced to producers that produced less than was bid [Nord Pool Spot, 2013], with indicating up-regulation prise P_{UP}, indicating market price P_S, and indicating down-regulation price P_{DN}.

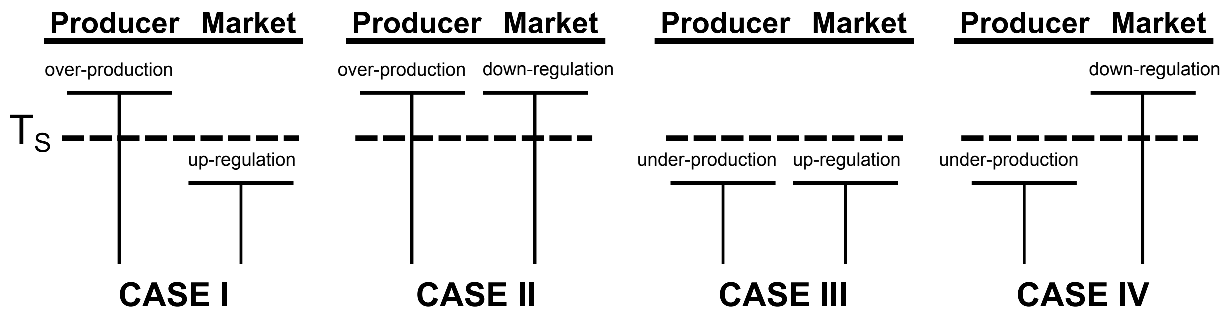

Figure A.5: Combinations of market and producer situations. The dashed line represents the bid (T_S) for the producer, and the market reference (balancing supply and demand). Each ┬ represents the current state of the producer (over- or under-production relative to T_S) or market (up- or down-regulation relative to the 50 Hz reference). The same is illustrated in table A.2.

In figure A.5, the dashed line represents one of two things, depending on the point of view; either the bid T_S, or the market reference of 50 Hz, as seen from the producer or the market, respectively. Each ┬ represents the current state of the producer or the market, i.e. when both the producer and market current state lines are above the dashed line, then the producer is in over-production and the market is in down-regulation. This understanding applies to the remaining cases.

With no control mechanism deciding when the produced energy is sent to the grid, the revenue is strictly dependent on the (stochastic) power production and thereby entirely subject to the market situation (penalties), as seen from equation A.16, which represents all four cases. In the following, a means to control when the produced energy is sent to the grid is presented, thus allowing for revenue optimization.

Appendix B

Mixed Integer and Linear Programs

This appendix provides a brief introduction to the linear and Mixed Integer linear program. Consider the definition of a mathematical optimization problem, stated in [Boyd and Vandenberghe, 2004, p. 1],

$$\min_{x \in \mathbb{R}^n} f_0(x) \tag{B.1a}$$

Subject to

$$i \in \{1, 2, \ldots, m\} \tag{B.1b}$$

$$b_i \geq f_i(x) \tag{B.1c}$$

where

x	The n dimensional vector containing the decision variables	$[\cdot]$
$f_0 : \mathbb{R}^n \to \mathbb{R}$	The objective function of the optimization problem	$[\cdot]$
$f_i : \mathbb{R}^n \to \mathbb{R}$	The i^{th} inequality function in the optimization problem	$[\cdot]$
b_i	The i^{th} bound on the inequalities	$[\cdot]$

The general optimization problem is only stated with inequalities, because an equality can be expressed as two inequalities [Nemhauser and Wolsey, 1988, p. 3]. The optimization problem (B.1) is convex if the objective function and all inequality functions satisfies

$$f_i(\alpha x + \beta y) \leq \alpha f_i(x) + \beta f_i(y) \quad \text{where} \quad i \in \{0, 1, \ldots, m\} \tag{B.2}$$

given

$$\forall x, y \in \mathbb{R}^n \quad \text{and} \quad \forall \alpha, \beta \in \mathbb{R} \quad \text{where} \quad \alpha + \beta = 1 \quad \text{and} \quad \alpha, \beta \geq 0$$

Having stated the general optimization problem and the definition of convexity the linear and linear mixed integer program can be discussed.

B.1 Linear Programming (LP)

This section describes the general properties of linear programming. Consider the general linear program from [Nemhauser and Wolsey, 1988, .p 5]

$$\min_{x \in \mathbb{R}^n} c^T x \tag{B.3}$$

Subject to

$$d \geq Ax \tag{B.4}$$

where

$x \in \mathbb{R}^n$	The decision variables	$[\cdot]$
$c \in \mathbb{R}^n$	The weight vector	$[\cdot]$
$d \in \mathbb{R}^m$	The inequality bounds	$[\cdot]$
$A \in \mathbb{R}^{m \times n}$	The inequality matrix	$[\cdot]$

The inequality constraints are essential for the linear program, because the unconstrained version of linear programming is unbounded, which means the solution goes to infinite.

Linear programming is convex, because the objective function and the inequality functions are linear. Linear functions satisfies the super position principle, which is

$$f_i(\alpha x + \beta y) = \alpha f_i(x) + \beta f_i(y) \quad \text{where} \quad i \in \{0, 1, \ldots, m\} \tag{B.5}$$

which is a special case of (B.2).

Linear programming is a special case of quadratic programming [Boyd and Vandenberghe, 2004, .p 152], which can be seen from

$$\min_{x \in \mathbb{R}^n} \frac{1}{2} x^T Q x + c^T x \tag{B.6}$$

Subject to

$$d \geq Ax \tag{B.7}$$

where

$Q \in \mathbb{R}^{n \times n}$	The weight matrix for the quadratic term	$[\cdot]$

If the weight matrix Q has all entries equal to zero and the constraints are linear the problem is reduced to a linear program, seen in (B.3).

The LP problem can be solved using efficient and widely tested methods as Dantzig's Simplex method and the Interior-Point method [Boyd and Vandenberghe, 2004,6].

B.2 0-1 Mixed Integer Linear Programming (0-1 MILP)

0-1 Mixed Integer Linear Programming (0-1 MILP) is a family of optimization problems where the objective and constraints are linear and some of the decision variables need to take the value zero or one. The family 0-1 Mixed Integer Linear Programming is a sub family to the general Mixed Integer Linear Programming (MILP), where some decision variables takes integer values. Consider the formulation of the general MILP

$$\min_{x\in\mathbb{R}^n, y\in\mathbb{B}^q} c^T x + h^T y \tag{B.8a}$$

Subject to

$$d \geq Ax + Gb \tag{B.8b}$$

where

$x \in \mathbb{R}^n$	The decision variables	$[\cdot]$
$b \in \mathbb{B}^q$	The binary decision variables	$[\cdot]$
$c \in \mathbb{R}^n$	The weight matrix	$[\cdot]$
$d \in \mathbb{R}^m$	The inequality bounds	$[\cdot]$
$A \in \mathbb{R}^{m\times n}$	The inequality matrix for continuous variables	$[\cdot]$
$G \in \mathbb{R}^{m\times q}$	The inequality matrix for the binary variables	$[\cdot]$

Binary decision values can be added to an optimization problem when the use of continuous becomes inadequate. This is the case when the optimization problem has to decide between two possibilities. An example is to decide whether a machine should be ON or OFF. The machine cannot be 30 % ON, as might happen if a continuous variable makes the decision.

It is important to notice that binary decision variables are not used without consequences. The next sections describe some of the downsides of formulating the optimization problem as a 0-1 MILP.

Mixed Integer Programming is Non-Convex

The presence of binary values make the optimization problem non-convex. The reason is that the space of binary values are not convex, because the linear combinations of one and zero leaves the binary set $\mathbb{B}$. This means the traditional methods for solving convex systems cannot be applied directly.

Calculation Heavy

The introduction of binary values in the optimization problem can cause calculation problems e.g. combinatorial growth in number of problems to solve. The 0-1 MILP problem can be formulated as 2^q LP problems, where q is the number binary variables. To avoid solving all 2^q heuristic based solvers have been generated, among these are Branch-and-Bound and Cutting Plane algorithms Nemhauser and Wolsey [1988].

B.2.1 Upper Bound for the Solution

It is possible to find an upper bound for the solution in 0-1 MILP. Solving the 0-1 MILP as a pure LP problem, by treating the binary decision variables as continuous. The 0-1 MILP problem will at best provide an equally good solution as the LP problem, because of the fact that the binary set $\mathbb{B}$ is a subset of the reals $\mathbb{R}$. The pure LP problem simply has a larger feasible space to choose the optimal solution from.

Appendix C

Impact of Forecast Errors in District Heating

In the forecast based analysis, all production set-points for the district heating are chosen based on predictions. The fact that the DH demand for the coming hour is unknown means that the set-point production $\hat{r}_{\mathrm{DH}}[k|k-1]$ deviates from the actual average demand $r_{\mathrm{DH}}[k]$.

$$\hat{r}_{\mathrm{DH}}[k|k-1] \neq r_{\mathrm{DH}}[k] \tag{C.1}$$

The error between the predicted average demand and the actual average demand is defined as, seen in (C.2).

$$e_{\mathrm{DH}}[k] = \hat{r}_{\mathrm{DH}}[k|k-1] - r_{\mathrm{DH}}[k] \tag{C.2}$$

From Section 3 it is known that the error $e_{\mathrm{DH}}[k]$ predicted one hour before can be approximated as standard normal white noise. Consider the accumulated error in the supply $w_{\mathrm{DH}}[k]$.

$$w_{\mathrm{DH}}[k] = w_{\mathrm{DH}}[k-1] + e_{\mathrm{DH}}[k] = \sum_{i=1}^{k} e_{\mathrm{DH}}[i] \qquad e_{\mathrm{DH}}[i] \sim \mathcal{N}(0, \sigma^2) \tag{C.3}$$

Integrating the error, which is white gaussian noise, gives a random walk, where the variance increases linearly at each time step. Take the example between two independent normal distributions.

$$\big(Z \sim \mathcal{N}(\mu_Y + \mu_X, \sigma_Y^2 + \sigma_X^2) \;\mid\; Z = Y + X,\;\; Y \sim \mathcal{N}(\mu_Y, \sigma_Y^2),\;\; X \sim \mathcal{N}(\mu_X, \sigma_X^2)\big) \tag{C.4}$$

Applying the idea of (C.4) to k samples gives the following

$$w_{\mathrm{DH}}[k] \sim \mathcal{N}(0, k\sigma^2) \tag{C.5}$$

The statement (C.5) is illustrated through simulation in Figure C.1. Figure C.1 shows the need for a controller which corrects the error between the forecast based power set-point and the actual demand.

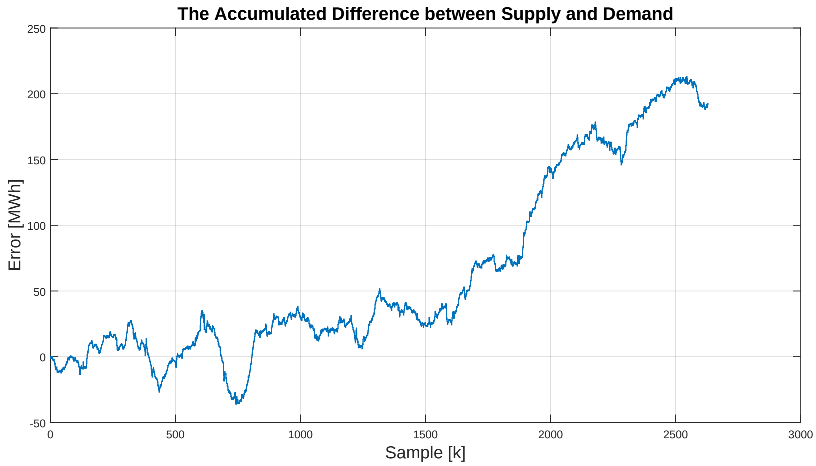

Figure C.1: The accumulated error between supply and demand, when production is based on forecasts

Figure C.1 shows how the energy has accumulated in the DH system.

Appendix D

Dependencies in a System with Limits

The modules the plant are dependent of each other which means that the individual adjustment capacity of a module is not a true indicator of the overall adjustment capacity ΔQ_{DH}. Two kinds of dependencies are considered in this section: (1) Bottleneck-dependency and (2) shared dependency.

D.1 Notation

The equations (7.35) defines the variables which describes the adjustment capacity of an individual module.

$$\underline{\Delta Q}[k] = Q[k] - \underline{Q} \qquad \underline{\Delta Q}[k] \geq 0 \tag{D.1a}$$

$$\overline{\Delta Q}[k] = \overline{Q} - Q[k] \qquad \overline{\Delta Q}[k] \geq 0 \tag{D.1b}$$

The symbol $\underline{\Delta Q}[k]$ is defined as the distance from the set-point to the lower limit and $\overline{\Delta Q}[k]$ is the distance from set-point to upper limit.

D.2 Figure for Adjustment Capacity

The subpart of the plant which can be used for adjustment of the district heating output.

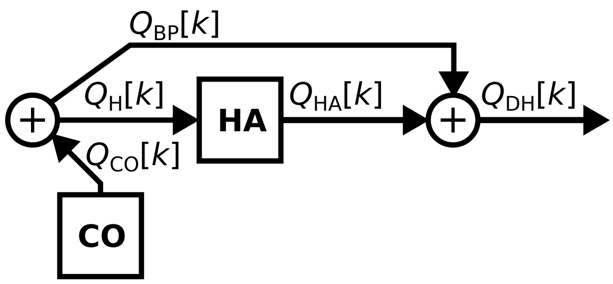

Figure D.1: Reference figure for the dependency examples

D.3 Examples of Dependencies

(1) Bottleneck dependency describes the case where a limit on one variable restraints others. The bottleneck-dependency is definded as the minimum of the bottlenecking signals.

$$\min(a, b) \tag{D.2}$$

This is illustrated in Example D.1.

Example D.1 (Bottleneck-dependency between $\underline{\Delta E}_{\mathrm{HA}}$ and $\overline{\Delta Q}_{\mathrm{HA}}$)
This example describes the up-adjustment capacity of the district heating output by use of the HA output Q_{HA}. Recall the HA difference equation, seen in (D.3), which shows that $Q_{\mathrm{HA}}[k]$ and $E_{\mathrm{HA}}[k+1]$ are dependent on each other.

$$E_{\mathrm{HA}}[k+1] = E_{\mathrm{HA}}[k] + (Q_{\mathrm{H}}[k] - Q_{\mathrm{HA}}[k])\Delta T \tag{D.3}$$

The two following cases describes how the two variables E_{HA} and Q_{HA} can bottlenect each other.

(Case 1) Heat Accumulator Charge Bottleneck: The optimization problem has chosen a set-point for $\hat{Q}_{\mathrm{HA}}$ such that $\overline{\Delta Q}_{\mathrm{HA}}[k] = 10\mathrm{MW}$ and the HA is low on energy e.g. $\underline{\Delta E}_{\mathrm{HA}}[k+1] = 5\mathrm{MWh}$. Only considering the output $Q_{\mathrm{HA}}[k]$ gives the idea that the up-adjustment capacity is $\overline{\Delta Q}_{\mathrm{HA}}[k]\Delta T = 10$ MWh, but given the connection between E_{HA} and Q_{HA}, seen in (D.3), the HA can only deliver 5 MWh extra. The bottleneck can be calculated as $\min(\underline{\Delta E}_{\mathrm{HA}}, \overline{\Delta Q}_{\mathrm{HA}}\Delta T) = 5\mathrm{MWh}$.

(Case 2) Output Slew Rate Bottleneck: In this case the HA is on medium charge which means the output slew rate is the deciding factor. As in case 1 The set-point of Q_{HA} is such that $\overline{\Delta Q}_{\mathrm{HA}}[k] = 10\mathrm{MW}$ and the charge is $\underline{\Delta E}_{\mathrm{HA}}[k+1] = 100\mathrm{MWh}$. The up-adjustment capacity for the HA output is then $\min(\underline{\Delta E}_{\mathrm{HA}}, \overline{\Delta Q}_{\mathrm{HA}}\Delta T)$ = 10MWh.

The second dependency is Shared-dependency. Shared dependency occurs when two variables can drain from the same reservoir. An example of this is again the HA. The MPC chosen state of charge in the HA $\hat{E}_{\mathrm{HA}}[k+1]$ can be lowered in the following ways: (1) down adjusting the input $\hat{Q}_{\mathrm{H}}[k]$, (2) up-adjusting the output $\hat{Q}_{\mathrm{HA}}[k]$, (3) a combination of (1) and (2). The input $\hat{Q}_{\mathrm{H}}[k]$ is interesting to consider because the output to district heating $\hat{Q}_{\mathrm{DH}}[k]$ can be up-adjusted if the power reserved for the HA is redirected through the bypass $\hat{Q}_{\mathrm{BP}}[k]$.

The approach taken to describe the contribution from HA to the up-adjustment capacity of district heating $\overline{\Delta Q}_{\mathrm{DH}}$ when accounting for shared dependency is seen in (D.4).

$$\min(\underline{\Delta E}_{\mathrm{HA}}[k+1], \overline{\Delta Q}_{\mathrm{HA}}[k]\Delta T) + \min(\underline{\Delta Q}_{\mathrm{H}}[k], \max(0, \overline{\Delta E}_{\mathrm{HA}}[k]\Delta T - \overline{\Delta Q}_{\mathrm{HA}}[k]\Delta T)) \tag{D.4}$$

where

$\underline{\Delta E}_{\text{HA}}[k+1]$	The distance from set-point to minimum charge.	[MWh]
$\overline{\Delta Q}_{\text{HA}}[k]$	The maximum up-adjustment capacity of the HA output	[MWh]
$\underline{\Delta Q}_{\text{H}}[k]$	The maximum down adjustment capacity of the input.	[MWh]

The expression for the up-adjustment capacity of district heating provided by the HA is analysed in steps. Step (1) How much additionally can the MPC chosen output $\hat{Q}_{\text{HA}}[k]$ potentially drain from the HA. This is described by expression

$$\min(\underline{\Delta E}_{\text{HA}}[k+1], \overline{\Delta Q}_{\text{HA}}[k]\Delta T) \tag{D.5}$$

which find the limiting factor between charge and output slew rate of the HA. step (2) evaluates whether the HA can be discharged additionally. This is done by calculating a remainder e_{ha}

$$e_{\text{ha}} = \max(0, \overline{\Delta E}_{\text{HA}}[k]\Delta T - \overline{\Delta Q}_{\text{HA}}[k]\Delta T) \tag{D.6}$$

Which describes whether there is energy left in the HA if the output drain full potential. if further is left after step (1) the remainder e_{ha} is positive. If step (2) is positive further energy can indirectly be drained from the HA by redirecting scheduled input power $\hat{Q}_{\text{H}}[k]$ thought the bypass. The potential for down-adjusting the input $\hat{Q}_{\text{H}}[k]$ is calculated as.

$$\min(\underline{\Delta Q}_{\text{H}}[k], e_{\text{ha}}) \tag{D.7}$$

The following gives an example of shared dependency in the HA when the up-adjustment capacity for district heating is estimated.

Example D.2 (Shared-dependency between Q_{H}, Q_{HA} and E_{HA})
The variables Q_{H} and Q_{HA} is chosen such that $\overline{\Delta Q}_{\text{HA}}$ = 20 MW and $\underline{\Delta Q}_{\text{H}}$ = 10 MW. The charge is such that $\underline{\Delta E}_{\text{HA}}$ = 100MWh.

Step (1): Calculating the potential up-adjustment capacity from for the output gives

$$\min(\underline{\Delta E}_{\text{HA}}, \overline{\Delta Q}_{\text{HA}}\Delta T) = \overline{\Delta Q}_{\text{HA}}\Delta T = 20\text{MWh} \tag{D.8}$$

Step (2): Calculate the remainder e_{ha}

$$e_{\text{ha}}[k] = \max(0, \overline{\Delta E}_{\text{HA}}[k]\Delta T - \overline{\Delta Q}_{\text{HA}}[k]\Delta T) = \max(0, 100\text{MWh} - 20\text{MWh}) = 80\text{MWh} \tag{D.9}$$

Step (3): Find potential district heating up-adjustment delivered by input

$$\min(\underline{\Delta Q}_{\text{H}}[k], e_{\text{ha}}) = \underline{\Delta Q}_{\text{H}}[k] = 10\text{MWh} \tag{D.10}$$

The total contribution for the up-adjustment capacity for district heating $\overline{\Delta Q}_{\mathrm{DH}}$ from the HA is

$$\min(\underline{\Delta E}_{\mathrm{HA}}[k+1], \overline{\Delta Q}_{\mathrm{HA}}[k]\Delta T) + \min(\underline{\Delta Q}_{\mathrm{H}}[k], \max(0, \overline{\Delta E}_{\mathrm{HA}}[k]\Delta T - \overline{\Delta Q}_{\mathrm{HA}}[k]\Delta T)) = 30\mathrm{MWh} \tag{D.11}$$

Appendix E

Algorithms used to Sort Data for Simulations

This appendix describes how the data is sorted for the optimization problem solved in the MPC algorithm. The two sorting algorithms are made for the electricity production bid T_E and the electricity price P_E. The hat indicates the value is a forecast and no hat means it is deterministic.

Hour	Clk	Action	P_E	$\hat{P}_E$	T_E	$\hat{T}_E$	$\hat{C}_{DN}$	$\hat{C}_{UP}$
1	00:00	-	24	0	24	$\hat{T}_E$	40	40
2	01:00	-	23	0	23	$\hat{T}_E$	40	40
3	02:00	-	22	0	22	$\hat{T}_E$	40	40
4	03:00	-	21	0	21	$\hat{T}_E$	40	40
5	04:00	-	20	0	20	$\hat{T}_E$	40	40
6	05:00	Spot prize forecast $\hat{P}_E$ for next day has been received	19	24	19	$\hat{T}_E$	40	40
7	06:00	-	18	24	18	$\hat{T}_E$	40	40
8	07:00	-	17	24	17	$\hat{T}_E$	40	40
9	08:00	-	16	24	16	$\hat{T}_E$	40	40
10	09:00	-	15	24	15	$\hat{T}_E$	40	40
11	10:00	-	14	24	14	$\hat{T}_E$	40	40
12	11:00	Generate production bid T_E for next day	13	24	35	$\hat{T}_E$	40	40
13	12:00	-	12	24	36	$\hat{T}_E$	40	40
14	13:00	The Spot price P_E for next day has been published	35	0	35	$\hat{T}_E$	40	40
15	14:00	-	34	0	34	$\hat{T}_E$	40	40
16	15:00	-	33	0	33	$\hat{T}_E$	40	40
17	16:00	-	32	0	32	$\hat{T}_E$	40	40
18	17:00	-	31	0	31	$\hat{T}_E$	40	40
19	18:00	-	30	0	30	$\hat{T}_E$	40	40
20	19:00	-	29	0	29	$\hat{T}_E$	40	40
21	20:00	-	28	0	28	$\hat{T}_E$	40	40
22	21:00	-	27	0	27	$\hat{T}_E$	40	40
23	22:00	-	26	0	26	$\hat{T}_E$	40	40
24	23:00	-	25	0	25	$\hat{T}_E$	40	40

Table E.1: Shows the available data in the different hours of the day. The length of the data vector is the deterministic variables added with the forecasts. An example is: at 06:00 there is $P_E = 18$ and $\hat{P}_E = 24$ data samples which means the data vector is $P_E + \hat{P}_E = 36$

E.1 Notation

This section describes the notation used in the following algorithms. The operator $\ll$ indicates a shift left in the vector e.g $A \ll 1$ means A is shifted one left. Normal brackets () after a vector is a reference to the index of the vector e.g. $A(1)$ is the first entry of the vector. Square brackets after a vector is used when referencing to sample numbers e.g. $A[k+3]$ is sample $k+3$. A colon within the brackets indicate an interval e.g. $A(1:5)$ is the interval 1 to 5 in the index of A. The **NaN** vector is a vector containing empty spaces.

E.2 The Sorting algorithm for Electricity Prices

The electricity price vector used in the MPC algorithm is a concatenation of deterministic prices P_{E} and prediction $\hat{P}_{\text{E}}$. The algorithm is seen in (E.1) is based on the values found in Figure E.1. Assumptions for the algorithm is that the vector $P_{\text{E,MPC}}$ covers 43 or more hours. The **NaN** vector represents empty spaces which can be filled in with different kind of data. This is done in the forecast tests.

Step 1 (E.1)

$k = 0$ // Set the sample number to 0

Initialize $P_{\text{E,MPC}}$ // Make initial data vector

Step 2

Set $k = k+1$ // Increase iteration

$\text{clk} = \text{get\_Time}(k)$ // Get the clock from sample number

$P_{\text{E,MPC}} \ll 1$ // Shift vector left

if $\text{clk} == 05:00$ // Fresh forecast received

$P_{\text{E,MPC}}(20:43) = \hat{P}_{\text{E}}[k+19:k+42|k-1]$ // Update price vector with forecasts

else if $\text{clk} == 13:00$ // The Spot prices are reveiled

$P_{\text{E,MPC}}(1:35) = P_{\text{E}}[k:k+34]$ // Update price vector with spot prices

$P_{\text{E,MPC}}(36:\text{end}) = \mathbf{NaN}$ // Fill NaN into the rest of the vector

end

Go to: Step 2 // Return to step 2

E.3 The Sorting Algorithm for the Production Bid

The sorting algorithm for the production bid T_E differs from the algorithm for the price P_E in the way the forecasts update arrives. The price forecast arrive once a day, where the production forecast is updated once in hour.

Step 1 (E.2)

$k = 0$ // Set the sample number to 0

Initialize $T_{E,MPC}$ // Make initial data vector

Step 2

Set $k = k + 1$ // Increase iteration

$\text{clk} = \text{get\_Time}(k)$ // Get the clock from sample number

$T_{E,MPC} \ll 1$ // Shift vector left

if $\text{clk} \leq 11:00$ // Sending in the production bid

$T_{E,MPC}(24 - \text{clk} : \text{end}) = \hat{Q}_E(24 - \text{clk} : \text{end})$ // Update bid vector with new forecasts

else if $\text{clk} > 11:00$ // The Spot prices are reveiled

$T_{E,MPC}(48 - \text{clk} : \text{end}) = \hat{Q}_E(48 - \text{clk} : \text{end})$ // Update bid vector with estimates

end

Go to: Step 2 // Return to step 2

Appendix F

Correction Key

The correction key, which emulates the stability controller by closing the gab between predicted and actual DH demand, is described in this section. The correction key is substituting a real controller in this simulation and must not be mistaken for an actual controller.

The correction key is posted as an optimization problem which uses the model shown in Figure F.1 to remove the gab caused by the prediction error $e_{\mathrm{DH}}[k]$. The correction key is split up into the two scenarios: (1) the CHP is in underproduction and needs to up-adjust the production, (2) the CHP is in overproduction and needs to down-adjust.

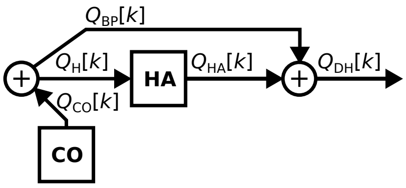

Figure F.1

The optimization problem (F.1) performs the up-adjustment procedure.

$$\min_{\Delta Q_{\mathrm{HA}},\Delta Q_{\mathrm{H}},\Delta Q_{\mathrm{BP}},\Delta Q_{\mathrm{EP}},\Delta Q_{\mathrm{CO}}\in\mathbb{R}_+} \Delta Q_{\mathrm{CO}} + 2\Delta Q_{\mathrm{HA}} + 3\Delta Q_{\mathrm{H}} + 4\Delta Q_{\mathrm{EP}} \tag{F.1a}$$

Subject to

$$Q_{\mathrm{HA}} = \hat{Q}_{\mathrm{HA}}[k] + \Delta Q_{\mathrm{HA}} \tag{F.1b}$$

$$Q_{\mathrm{H}} = \hat{Q}_{\mathrm{H}}[k] - \Delta Q_{\mathrm{H}} \tag{F.1c}$$

$$Q_{\mathrm{EP}} = \hat{Q}_{\mathrm{EP}}[k] + \Delta Q_{\mathrm{EP}} \tag{F.1d}$$

$$Q_{\mathrm{BP}} = \hat{Q}_{\mathrm{BP}}[k] + \Delta Q_{\mathrm{BP}} \tag{F.1e}$$

$$Q_{\mathrm{CO}} = \hat{Q}_{\mathrm{CO}}[k] + \Delta Q_{\mathrm{CO}} \tag{F.1f}$$

$$E_{\mathrm{HA}} = E_{\mathrm{HA}}[k] + (Q_{\mathrm{H}} - Q_{\mathrm{HA}})\Delta T \tag{F.1g}$$

$$e_{\mathrm{DH}} = \Delta Q_{\mathrm{EP}} + \Delta Q_{\mathrm{HA}} + \Delta Q_{\mathrm{BP}} \tag{F.1h}$$

$$\Delta Q_{\mathrm{CO}} = \Delta Q_{\mathrm{H}} + \Delta Q_{\mathrm{BP}} \tag{F.1i}$$

$$\underline{Q} \le Q \le \overline{Q} \tag{F.1j}$$

(F.1k)

The optimization problem (F.1) shows how the a set of variables moves the estimated set-points $\hat{Q}$ to the final set-points Q by solving equation (F.1h). The rest of the constraints ensure the solution still obeys the structure of the system. The weights in the linear cost is chosen from one to four, to exploit the feature of a linear cost, where the problem will use all the capacity of Q_{CO} before moving on to Q_{HA} and etc.

$$\min_{\Delta Q_{\text{HA}},\Delta Q_{\text{H}},\Delta Q_{\text{BP}},\Delta Q_{\text{EP}},\Delta Q_{\text{CO}} \in \mathbb{R}_+} \Delta Q_{\text{CO}} + 2\Delta Q_{\text{HA}} + 3\Delta Q_{\text{H}} + 4\Delta Q_{\text{EP}} \tag{F.2a}$$

Subject to

$$Q_{\text{HA}} = \hat{Q}_{\text{HA}}[k] - \Delta Q_{\text{HA}} \tag{F.2b}$$

$$Q_{\text{H}} = \hat{Q}_{\text{H}}[k] + \Delta Q_{\text{H}} \tag{F.2c}$$

$$Q_{\text{EP}} = \hat{Q}_{\text{EP}}[k] - \Delta Q_{\text{EP}} \tag{F.2d}$$

$$Q_{\text{BP}} = \hat{Q}_{\text{BP}}[k] - \Delta Q_{\text{BP}} \tag{F.2e}$$

$$Q_{\text{CO}} = \hat{Q}_{\text{CO}}[k] - \Delta Q_{\text{CO}} \tag{F.2f}$$

$$E_{\text{HA}}[k+1] = E_{\text{HA}}[k] + (Q_{\text{H}} - Q_{\text{HA}})\Delta T \tag{F.2g}$$

$$e_{\text{DH}} = \Delta Q_{\text{EP}} + \Delta Q_{\text{HA}} + \Delta Q_{\text{BP}} \tag{F.2h}$$

$$\Delta Q_{\text{CO}} = \Delta Q_{\text{H}} + \Delta Q_{\text{BP}} \tag{F.2i}$$

$$\underline{Q} \leq Q \leq \overline{Q} \tag{F.2j}$$

$$Q = \begin{bmatrix} Q_{\text{CO}}[k] & Q_{\text{HA}}[k] & Q_{\text{H}}[k] & Q_{\text{EP}}[k] \end{bmatrix}^T \tag{F.3}$$

$$\overline{Q} = \begin{bmatrix} \overline{Q}_{\text{CO}} & \overline{Q}_{\text{HA}} & \overline{Q}_{\text{H}} & \overline{Q}_{\text{EP}} \end{bmatrix}^T \tag{F.4}$$

$$\underline{Q} = \begin{bmatrix} \underline{Q}_{\text{CO}} & \underline{Q}_{\text{HA}} & \underline{Q}_{\text{H}} & \underline{Q}_{\text{EP}} \end{bmatrix}^T \tag{F.5}$$

The important aspects to note is equation (F.2h) which demands that the change in the variables equals the prediction error.

Appendix G

Software

G.1 MATLAB

MATLAB® is a scripting based high level programming language which is used for numerial based calculation.

MATLAB is in this project used to run the simulation of the MPC controlled CHP plant.

G.2 CVX

CVX is a modeling frame for Disiplined Convex Programming (DCP). CVX is not a solver it is a framework, for MATLAB, which allows the user to post the optimization problem, which obeys the DCP ruleset, in an intuitive way. CVX then translates the posted problem into a form which can be interpreted and solved by a solver.

CVX is in this project used as an interface between MATLAB an the solver Gurobi. The optimization problem is in MATLAB stated similar to the mathematical formulation and by CVX converted into a form which can be interpreted by Gurobi.

G.3 Gurobi

The Gurobi solver can solve varies optimization problem as Linear Programming (LP), Quadratic Programming (QP), Quadratically Constrained Programming (QCP), and MIP Mixed Interger Programming (Mixed-Integer Linear Programming (MILP), Mixed-Integer Quadratic Programming (MIQP), and Mixed-Integer Quadratically Constrained Programming (MIQCP)) problems.

In the project, Gurobi solves the Mixed Integer Linear Program from the MPC algorithm.