Scalable Denoise-and-Forward

In Bidirectional Relay Networks

Project group 1010
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Abstract:

Interference is considered as the single largest limitation for wireless networks. In this project bidirectional relay networks are considered seeking to exploit the interference in an intelligent fashion, where most existing relay schemes seek to avoid it.
The relay scheme DeNoise and Forward, DNF, is applied as a starting point, as it embraces interference by utilising analog network coding, where the signals are added in the wireless medium. DNF was originally presented using BPSK modulation and a simple relay scenario with two end nodes communicating through a relay node. The main focus in this project is to scale the concept of DNF in order to make it applicable in more realistic relay scenarios.
A scheme based on DNF, Scalable DNF, S-DNF, has been devoloped for networks with more than two end nodes and the functionality of DNF has been extended to also enable other modulation schemes and higher order modulation. Results show good performance potential of the proposed scheme and when different modulation schemes are applied, the performance impact of S-DNF is similar to what is experienced when using the originally proposed BPSK.

Preface

This report documents the thesis project of group 1010, at Aalborg University. The work has been conducted over two semesters; the autumn and spring semester of 2008 and 2009 respectively. From the results of this project the group has composed two papers: *Scalable Denoise-and-Forward in Bidirectional Relay Networks* submitted to Science Direct, and *Physical Layer Network Coding for FSK Systems* submitted to IEEE. A copy of these papers are enclosed with this report.

As a part of this work the project group was invited to a four month research stay at Harvard University, School of Engineering and Applied Sciences, by Senior Fellow, Professor Vahid Tarokh. Therefore a special thanks goes to Prof. Tarokh for making this stay possible and also to postdoc Toshiaki Koike Akino for his very qualified supervision during the stay at Harvard University.

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Chapter 1

Introduction

An important limitation in wireless communication is range. The signal is attenuated during propagation, which limits communication to an area within a certain radius of the transmitter. One way to increase the range of a wireless communication system is to introduce a relay, which receives and forwards the packets to nodes outside the range of the transmitter. This principle has received much attention in the wireless research community lately [Kim et al., 2008]. The relay principle is often used in cooperative schemes with multiple data flows or bidirectional communication with two data flows. An important aspect of the existing work is to make the distribution of data more efficient [Popovski and Yomo, 2006a, Gong et al., 2009].

Another limitation of wireless communication systems is interference, and it must be assumed that any wireless network to some extent experiences interference [Gupta and Kumar, 2000, Bicket et al., 2005]. Interference is especially relevant in relaying scenarios where multiple nodes must share the medium. Existing work treats transmissions not intended for the receiver as pure noise and tries to avoid it by different access methods, e.g. TDMA or FDMA [Akyildiz and Wang, 2005]. In this work the possibilities of exploiting knowledge about the interfering signals in order to coexist with the interference are investigated. An example of a relaying scheme where this is accomplished is DeNoise-and-Forward, DNF, proposed in [Popovski and Yomo, 2006b]. DNF utilises analog network coding, where transmissions to the relay are performed simultaneously and interfere with each other, i.e. the signals are added in the air. This scheme will be the starting point of this project.

In [Popovski and Yomo, 2006b] DNF has been proposed in a simple three node scenario with BPSK modulation. In this work the goal is to scale that concept and make it useful in other scenarios and more sophisticated setups. This means that the DNF scheme should be scaled from the relay network with two end nodes to be applicable in larger and more common networks. The use of BPSK and other coherent modulation schemes require symbol synchronisation and tracking of the phase. Phase tracking is complex and impractical [Proakis and Salehi, 2002, 399], hence it is desired to be able to utilise non-coherent modulation schemes when employing DNF. These modulation schemes only require symbol synchronisation and is therefore more applicable. Moreover it is desired to enable larger constellations in DNF in order to support higher bit rates.

To sum up, this means that the key elements of this project are to scale the concept of DNF with respect to:

- Networks with more than two end nodes
- Non-coherent modulation schemes
- M-ary signal constellations

Report Structure

This report contains ten chapters in addition to this introduction. Their purpose is briefly presented in this section.

2. Background: This chapter holds all background information relevant to this work such as a description of different concepts used in this work.

3. System Description: The system targeted in this work is described in this chapter. In this way the necessary models and assumptions have been introduced for use in later scheme development.

4. Scalable Denoise-and-Forward: In this chapter a scaling of DNF with respect to network size is proposed along with a method for scaling the denoise operation in DNF to M-ary modulation schemes. In addition, possible approaches to error control in the scheme are discussed.

5. Analysis of Decision Regions: This chapter presents an analysis of the decision regions in DNF for two modulation schemes: The simple BPSK scheme used in the original proposal of DNF and BFSK in order to apply a non-coherent modulation scheme.

6. Performance Analysis: The performance analysis of the proposed scheme, when different types of error control are applied, is derived in this chapter.

7. Performance Evaluation: The performance of the proposed scheme is in this chapter compared to relevant existing schemes for bidirectional relaying. This is done through simulation and by using the analysis derived in the previous chapter.

8. Tornado Codes From End Nodes: In order to increase the reliability of the proposed scheme, this chapter investigates the use of Tornado codes as error control in DNF.

9. Scaling of Constellation Size in S-DNF: The binary modulation schemes considered in DNF so far are in this chapter scaled to M-ary constellations. This is done in order to increase the maximum possible bit rate.

10. High Order Constellations and FEC: This chapter investigates a combination of larger modulation schemes and error control using FEC. Hence, the well known trade-off between error resilience and information per symbol is investigated.

11. Conclusion: The final conclusions are drawn and interesting topics for future work are identified in this chapter.

Chapter 2

Background

The area of relay networks and network coding are an essential basis of the problem specified in chapter 1. In order to have the required background knowledge before investigating the problem, this chapter describes these concepts briefly along with recent work within these areas.

2.1 Limitations in Wireless Networks

While wireless networks have a number of clear advantages, such as mobility and low cost of implementation, they also possess disadvantages. Most often nodes in a wireless network have only a single antenna. This means that links in such wireless networks are limited to half duplex communication. Even when multiple antennas are available, both transmitting and receiving at the same time is challenging since the two signals will interfere, unless proper multiplexing is performed, using e.g. spatial multiplexing or frequency division multiplexing.

Another limitation is related to the broadcast nature of wireless networks. Assuming no beamforming is used, a transmitted signal will occupy the entire wireless medium within a certain area, which prevents other communications from taking place therein if interference should be avoided. Interference has been identified as a major limiting factor in wireless communication. [Gupta and Kumar, 2000, Bicket et al., 2005] This limitation is further strengthened by the fact that the interference range is always larger than the communication range.[Beuster et al., 2008] Specifically in the relay scenario, where A and B wish to exchange packets, this means that while A is transmitting to the relay, B can not, and vice versa.

2.2 Digital Modulation Schemes

Modulation is an essential part of wireless communication, and this section briefly describes the elements of modulation, which will be of focus in this project. Modulation refers to the mapping of information represented by digital data bits onto an analog carrier wave [Haykin, 2001, 344]. This is required in order to enable the exchange of information over the analog communication channel.

2.2.1 Signal Constellation

When digital data is modulated the digital information is represented by different characteristics of the carrier wave depending on the utilised modulation scheme, e.g. phase or frequency. This parameter can take different values where each value results in a unique analog signal. The signal constellation refers to the set of unique signals available when modulating the digital bits. Each unique signal can be regarded as a symbol or point in the n-dimensional space, where n depends on the modulation scheme.

As the transmitted signal propagates through the communication channel it is distorted by noise. This means that the received signal point is located around the transmitted signal point. The distance between these two points depends on the noise power, but due to the probabilistic characteristics of channel noise a received signal is most likely to be distributed closely around the transmitted signal point.

In order to demodulate a received signal a node must determine which signal was transmitted given the received distorted signal. Around each signal point in the constellation there exist a region where that signal is most likely to have been transmitted. This region is referred to as a decision region. If a received signal point is located outside the decision region of the transmitted signal, it is interpreted as a wrong signal resulting in an error.

The information that can be represented by each signal in the constellation is given by the cardinality, M, of the constellation. Having M unique signals means that each signal can hold the information of $\log_2(M)$ bits. Hence, if M is increased, the number of bits represented by a single signal increases correspondingly. However, increasing the cardinality of the constellation makes the decision regions smaller, as more regions are sharing the same n-dimensional space. This results in a higher error rate, hence there exists a trade-off between error rate and data rate which should be taken into account when selecting the modulation scheme.

2.2.2 Coherent and Non-coherent Detection

Modulation schemes can be divided into two groups based on which characteristics of the carrier wave they utilise to represent the information. The schemes utilising the phase comprise one group. These schemes require a receiver which tracks the phase of the carrier wave to be able to demodulate. This is referred to as coherent detection. Tracking the phase is complicated and impractical resulting in a fairly complex receiver structure [Proakis and Salehi, 2002, 399]. The remaining modulation schemes comprise the second group. These schemes do not utilise the phase and it is therefore not required to track it at the receiver. This is referred to as non-coherent detection and gives a more simple receiver structure. In the following some relevant examples of coherent and non-coherent modulation schemes are briefly described.

Phase Shift Keying

Phase Shift Keying, PSK, is a coherent modulation scheme, as it utilises the phase to distinguish between the signals in the constellation. In M-ary PSK the constellation consists of a single carrier wave shifted with M unique phase shifts.

Frequency Shift Keying

Frequency Shift Keying, FSK, utilises different frequency bands for the signals in the constellation and is therefore a non-coherent modulation scheme. When using FSK the available bandwidth is divided into M frequency bands.

Amplitude Shift Keying

The final modulation scheme presented is Amplitude Shift Keying, ASK. This scheme is non-coherent as the signals in the constellation are distinguishable by amplitude. Note that for $M = 2$ the signal constellations of PSK and ASK are identical.

2.3 The Relay Network

Recently relay networks have received great interest. In this section the topology of the typical relay network is described. In general a relay network means that nodes communicate trough a relay node instead of communicating with each other directly. This relay can be any node in the network placed between the communicating nodes. As an example consider the Wireless Local Area Network (WLAN), illustrated in Figure 2.1, where two end nodes, A and B, are located on opposite sides of an Access Point (AP).

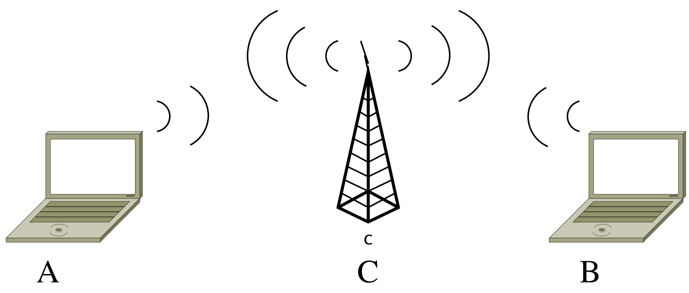


Figure 2.1: A basic relay network with two nodes communicating through the relay node.

The nodes are outside the range of each other but within the range of the AP. In this way the AP can be used as a relay node and forward packets from one node to the other. It is not required that the relay node is an AP or similar network equipment. A third node, C, could also act as a relay if it is located between the communicating nodes.

The work regarding relay networks has so far primarily focused on the scenario with two end nodes and a single relay [Kim et al., 2008], hence this topology is also used as a starting point in this project.

2.4 Related Work

Comprehensive work on relay networks has focused on the operation of the relay and how to achieve good performance. A concept which is promising combined with relay schemes is network coding, hence this section provides a brief introduction to the concept of network coding and different relay schemes.

2.4.1 Network Coding

In general, network coding is the notion for combining data within the network. Combining data is advantageous with respect to data flow, as a single packet can contain information for several nodes. This is particularly effective in wireless networks, where a transmission is received at every node in the proximity. However, when receiving a coded packet, a node can only extract the packet intended for it if all other components in the coded packet are known a priori.

Digitial Network Coding

Digital Network Coding, DNC, is a variant of network coding where the combination of data is performed in the hardware of a node in the network. The combination is done after demodulation, i.e. on digital data, hence the name. A simple example of DNC is to let the relay in Figure 2.1 calculate the XOR of two packets, one from A and one from B. If the relay broadcasts the XOR, both A and B can decode what was intended for them by performing an XOR operation on the received packet and the packet they transmitted themselves. This saves a transmission compared to regular multihop relaying, which is the purpose of network coding. DNC is not restricted to the XOR operation. Any linear combination can be used as long as the coefficients are known to all receivers.

Analog Network Coding

Analog Network Coding (ANC), also called physical network coding, refers to the case where two signals are transmitted concurrently and thereby added in the air. This means that the signals are combined in the physical communication channel and the received signal is the sum of the two signals. As an example consider nodes A and B transmitting to C simultaneously in Figure 2.1. C will receive A+B, which can be broadcast and used for decoding of intended data at both A and B by subtracting their own signal. In this process an additional transmission is saved compared to DNC. However, this is only possible if proper synchronisation of the data is performed in the transmissions to the relay. This is the main challenge in ANC.

2.4.2 Relay Schemes

When two end nodes communicate through a relay the simplest scheme would have the relay operate as a repeater forwarding each transmission from the end nodes. There exist different relay schemes that utilises the relay more efficiently in order to increase the throughput between the end nodes.

Amplify and Forward

The idea of the Amplify and Forward, AF, scheme is that the relay does not attempt to decode the received signal. Instead it just amplifies it and forwards it to the intended receiver. This is a very simple operation, which puts a minimal burden on the relay node. On the other hand, noise is also amplified in the process, which is a potential problem at the intended receiver. [Kim et al., 2008]

Decode and Forward

As the name suggests, Decode and Forward, DF, is a scheme where the relay decodes the signal it receives, before forwarding it to the intended receiver. This is the counterpart of AF, and the advantages and disadvantages are reversed. In this scheme, the relay has to perform a relatively complex operation in the decoding. The advantage in doing so, is that the noise added in the transmission to the relay is removed. [Kim et al., 2008]

Denoise and Forward

The idea of DeNoise and Forward, DNF, is to let A and B transmit their packets to the relay concurrently, i.e. ANC. Due to the broadcast nature of wireless communication, the signals will be added in the air, given proper synchronisation. If BPSK modulation is applied the relay will receive either -2, 0 or $+2$ for each symbol. These three possible symbols are mapped to a binary message indicating either equal (-2 and $+2$) or unequal (0). This compression ensures that a combined packet only needs the same amount of bits as a regular packet. At the same time the mapping removes any noise added during transmission, although decoding is not performed, hence the name. Note that the addition of signals in the air combined with the mapping from -2, 0 and $+2$ to a binary message is effectively an XOR operation. Equal symbols map to one value and unequal symbols map to another. The final denoised packet is referred to as an analog network coded packet. This packet is broadcast to A and B, which can now reconstruct what was intended for them by performing an XOR operation on the received packet and the packet they transmitted themselves. [Popovski and Yomo, 2006a, Popovski and Yomo, 2007, Koike-Akino et al., 2008]

DNF is a way to unite the virtues of AF and DF. The operation is simple since decoding is not required and the noise is removed. Moreover this scheme inherits the decreased number of necessary transmissions provided by ANC, hence the throughput is increased. The drawback is that it also inherits the required synchronisation in ANC. Another drawback is the fact that DNF depends on a priori information, which means an end node must be able to save the transmitted packet in order to extract the desired information from the packet broadcasted by the relay node.

2.4.3 Discussion

The described relay schemes have different advantages, but DNF comes out superior compared to AF and DF. AF and DF have their own strenghts and weaknesses, but DNF is able to combine the strenghts of the two basic schemes, while introducing a few new weaknesses. However, these weaknesses are surmountable, since it is assumed that enough memory is available in order to buffer transmitted data, and that symbol synchronisation between two transmitters is possible.

Chapter 3

System Description

The scenario targeted by this work is described in this chapter. Moreover, assumptions and approximations for the scenario, which are necessary throughout the report, are outlined. Finally, the error model used in both analysis and simulations of the presented schemes are presented.

3.1 The Scenario

The relay network presented in section 2.3, page 7, contains two end nodes. It is desired to expand the topology of the traditional relay network to comprise additional end nodes. Adding additional end nodes to the traditional relay network results in a star topology with end nodes distributed on the circumference of a circle around a single relay node. The end nodes communicate in pairs and two communicating nodes are antipodal, hence they communicate over direct links at a distance of the diameter, d_l, of the circle. As an example it has been chosen to use a network size of eight end nodes. This topology is illustrated in Figure 3.1.

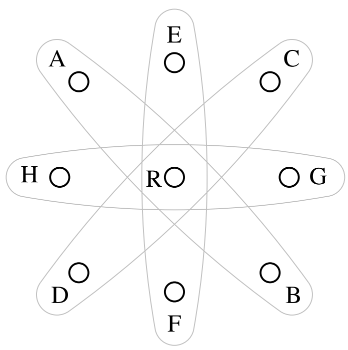


Figure 3.1: Eight end nodes distributed around a relay node.

The eight end nodes comprise four node pairs: $A \Leftrightarrow B$, $C \Leftrightarrow D$, $E \Leftrightarrow F$ and $G \Leftrightarrow H$.

Assumptions and Approximations

The utilised network topology has a number of node pairs and a common relay node services all node pairs. It is therefore necessary to assume an unfair Medium Access Control (MAC) protocol. This means that it is possible to distribute the medium access in favour of the relay and not uniformly between all nodes.

In wireless channels any transmitted signal is distorted by noise and attenuated as the signal propagates through the wireless medium. The error model therefore incorporates a noise component, which is assumed to be Additive White Gaussian Noise (AWGN). The links in the network are assumed to be independent, with an equal noise power level on all links.

When a node pair performs a joint transmission, it is assumed that the two transmitters are synchronised at symbol level. This is a necessary assumption when using analog network coding, in order for the signals to add correctly.

3.2 Error Model

The proposed relaying scheme is designed for the wireless scenario illustrated in Figure 3.1 and in order to analyse the potential of this scheme a model of the wireless channel is required. In this section this model is constructed based on the given assumptions.

When a single node transmits, the received signal only has a single source component, hence the signal from node i received by node j is given by:

$$y_j = h_{ij} \cdot x_i + z \tag{3.1}$$

Where h_{ij} is the channel coefficient and z is the Gaussian noise component, $z \sim \mathcal{N}(0, \sigma^2)$. When two nodes transmit simultaneously the received signal is analog coded with two source components. A signal transmitted jointly from nodes i and j, and received at node k, is therefore given by:

$$y_k = h_{ik} \cdot x_i + h_{jk} \cdot x_j + z \tag{3.2}$$

The model accounts for propagation loss and ergodic phase fading, where the phase, ϕ, is uniformly distributed between 0 and 2π. The channel coefficient, which models the propagation loss in the channel, is given by:

$$h_{ij} = \sqrt{PL(d_{ij})} \tag{3.3}$$

where PL is the path loss factor as a function of the distance, d_{ij}, between the communicating nodes i and j. For simplicity this project utilises the free space loss factor[Appadwedula et al., 1999, Perennou et al., 2005]. This loss factor is given by equation (3.4). [Molish, 2006, 46]

$$PL(d) = \frac{p_{rx}}{p_{tx}} = \left(\frac{4\pi d}{\lambda}\right)^{-\kappa} \tag{3.4}$$

Where λ is the wavelength of the transmitted signal. The path loss exponent, κ, is normally in the range from 2 to 4, and is this project $\kappa = 2$ is assumed. For all nodes the transmitted power, p_{tx}, is assumed to be 1, hence the received power, p_{rx}, equals $PL(d)$.

The diameter d_l of the circular network illustrated in Figure 3.1 is the longest possible distance between two communicating nodes. Hence, the link between two antipodal nodes is the weakest link in the network. The Signal-to-Noise Ratio (SNR) on this link is denoted γ. With the utilised path loss model the power of a propagated signal on a given link is deterministic. Thus, the value of γ dictates the noise power, σ^2, which is given by

$$\sigma^2 = \frac{p_{tx} \cdot PL(d_l)}{\gamma} = \frac{PL(d_l)}{\gamma} \tag{3.5}$$

The scenario and error model, described in this chapter, comprise the system applied in this project. Using this system the scaling of the concept of DNF with respect to larger network sizes, different modulation schemes and larger signal constellation can be investigated.

Chapter 4

Scalable Denoise-and-Forward

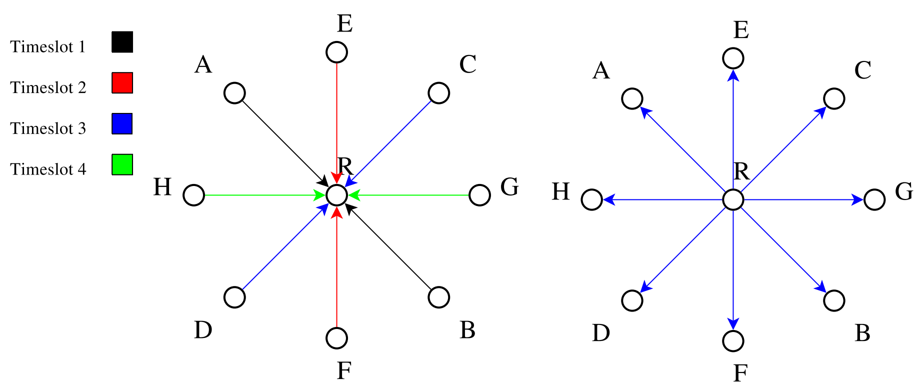
This project seeks to scale the concept of DNF both with respect to network size and constellation size in the modulation. In this chapter, a scheme which scales DNF with respect to network size is described. Moreover, a general concept for scaling the denoise operation in order to enable larger constellations is described. This concept is used in later chapters.

4.1 Introduction

The existing relay schemes, described in section 2.4.2, are not designed for the scenario with several end node pairs. It is therefore desired to design a relay scheme addressing the scenario presented in section 3.1, where multiple pairs of end nodes communicate through a single common relay. In this section a scheme utilising the concept from the DNF scheme is developed. A simple solution would be dividing the scenario into subscenarios of two end nodes and one relay node. In this case the node pairs would comprise the two end nodes and each subscenario would share the same relay node. The access to the relay node is then divided uniformly between the subscenarios. In this way any of the relay schemes described in section 2.3 can be used to exchange packets between the nodes in each of the node pairs. Using regular DNF in this example is referred to as Multiple DNF (M-DNF) and will be used in order to evaluate the performance of the proposed scheme which reuses some of the functionality from regular DNF.

4.2 Scaling With Respect to Network Size

The nodes in the scenario are located in the proximity of each other. This means that it is possible to utilise the broadcast nature of the wireless medium in order to increase the throughput in the network. This section presents a scheme seeking to reduce the number of broadcasts from the relay compared to the case where regular DNF is used on each pair. The proposed scheme is referred to as Scalable DNF, S-DNF. The scheme reuses the first step from regular DNF for each pair, i.e. the nodes transmit in pairs concurrently creating analog network coded packets at the relay. However, due to the broadcast nature of wireless communication each node in the network has a possibility of overhearing the transmissions from the other node pairs. In this way each node may hold a priori information of what was sent to the relay. This information can be used to reduce the number of broadcasts from the relay. As an example consider the case where the four node pairs from Figure 3.1 have transmitted their analog coded packets, $a_i \oplus b_i$ and $c_i \oplus d_i$ etc. to the relay. Each node has overheard the transmissions from the other node pairs and saved the packets, i.e. each end node has three analog coded packets stored and the relay has four. The relay is allowed to combine the analog coded packets from all pairs into one packet using digital network coding. The resulting packet is referred to as a parity packet. In this way it can transmit information of the packet to be decoded to four different end nodes by only broadcasting a single parity packet, $(a_i \oplus b_i) \oplus (c_i \oplus d_i) \oplus (e_i \oplus f_i) \oplus (g_i \oplus h_i)$. Then each node can extract their analog coded packet by XORing the received parity packet with their a priori information, i.e. the overheard packets. Finally the nodes can decode the packet of interest by XORing the analog coded packet with their own packet. This is illustrated in Figure 4.1 as a two-step procedure. In Figure 4.1(a) each node pair in turn broadcasts their packet creating an analog coded packet at the relay and at all end nodes who were able to overhear the transmission. In Figure 4.1(b) the relay broadcasts the combination of the anolog coded packets from which each end node can decode the packet intended for them.



(a) First to fourth timeslot. (b) Fifth timeslot.

Figure 4.1: The operation of S-DNF.

The S-DNF scheme reduces the number of broadcasts from the relay by combining the analog coded packets from the end nodes, which increases the resulting performance of the network. However, the heavier coding used by the relay, i.e. the more packets the relay encode into one packet, the more a priori information is required at the end nodes in order to extract the packet of interest. Hence there is a trade-off between the number of broadcasts from the relay and the probability of the end nodes being able to decode their packet.

The parity packets broadcasted by the relay can be any set of one or more network coded packets with a different number of components. The different possible combinations can be illustrated with a binary tree as in Figure 4.2. The leaf nodes represent the end node packets and each generation in the tree represents a level of network coding. The root node represents the heaviest coding where the relay combines all analog network coded packets into a single packet. This coding level is referred to as Level 0, L0, as illustrated in Figure 4.2. The heavier coding used by the relay the more a priori information is required at the end nodes in order to extract and decode the desired packet. As an example consider the case where an end node receives the root packet from Figure 4.2. In order to decode the packet of interest the node must have overheard the transmissions from all other node pairs in the network.

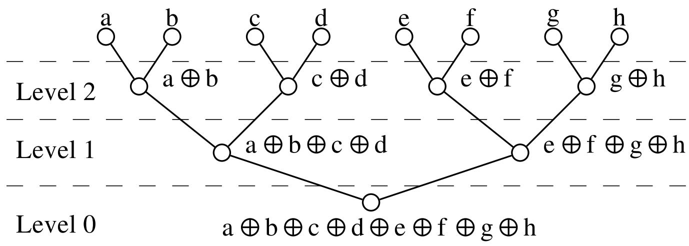


Figure 4.2: A binary tree indicating possible network coding for a network with eight end nodes.

Any combination of analog coded packets is possible and a binary tree only illustrates a subset of the total possible combinations. However, by restricting the possible combinations to those indicated by the binary tree simplifies the description of the details and functionality of the scheme. Note that there is a strong resemblance between the coding used in this scheme and tornado codes. A subset of three nodes in the tree in Figure 4.2, a parent and its two children, corresponds to a simple (3,2) tornado code. The purpose of a tornado code is error control, which is treated in the following.

4.3 Error Control

Due to the erroneous nature of the wireless medium the possibility of packet errors must be taken into account. It is important to determine how, and when, the nodes in the network must check for errors and which actions to take in case an error occurs.

In relay networks nodes communicate over multiple hops. This means that the detection of errors can be performed at the destination node only or for each link, referred to as end-to-end and link-level error detection respectively.

End-to-end detection does not require computation at the intermediate node, which simplifies the relaying protocol. However, when an error is not detected before the end node the proposed scheme becomes vulnerable to errors. As an example consider the case where the relay receives an erroneous packet. The relay does not detect the error but continues its normal operation and combines the packet with one or more of the other analog coded packets. When the parity packet is broadcasted the error propagates to all receiving end nodes. This will render all nodes unable to extract their packet from the parity packet even though they hold the necessary a priori information. This vulnerability becomes more significant if heavier coding is used at the relay since an error then influences more node pairs.

The problem of an increased error vulnerability due to the performed network coding must be addressed. Different approaches to error handling are described next.

4.3.1 Relay Generated Redundancy

If a single level of network coding is chosen from Figure 4.2, it is ensured that all packets are included excactly once in the set of combined packets. In this way, all packets are distributed, and no data is redundant. However, it might be advantageous to introduce redundancy at the relay, in order to address the problem with error vulnerability.

The redundancy is introduced by the relay by extending the set of parity packets to different levels of coding and different combinations of analog coded packets. In this way, if a packet for an end node was included in more than one parity packet, the probability of being able to decode the packet would be higher. This is equivalent to the concept of tornado coding, as mentioned earlier. However, there is a trade-off between the achieved throughput and the number of parity packets broadcast by the relay. Hence at low SNR it may be preferable to use a lighter coding instead of having a large set of redundant parity packets. A significant drawback in the use of relay generated redundancy is the fact that the initial transmissions from the end nodes to the relay are not protected against errors.

4.3.2 Forward Error Correction

Errors in S-DNF can also be handled by using Forward Error Correction (FEC) on the individual data streams in the network. The concept of FEC in communication systems is well known, hence the basics will not be described here, but the purpose of the FEC is to make the end nodes able to decode their intended packets even though errors occur. The advantage of this approach compared to relay generated redundancy is that errors on the joint transmissions to the relay are handled. However, the errors still influence potentially many end nodes, due to the network coding performed by the relay, but if all data streams are coded with FEC, it may by possible to correct them when decoding.

4.3.3 Automatic Repeat Request

Automatic Repeat reQuest (ARQ) will also be considered as the error handling protocol in S-DNF. This protocol is the counterpart of FEC, since it can be characterised as a reactive protocol in contrast to the proactive nature of FEC. ARQ makes use of control packets in order to specify which packets were erroneous and needs to be retransmitted. ARQ protocols exist in different variants with different complexity. The simplest protocol is Stop-and-wait ARQ and more advanced variants are Selective-repeat ARQ and Go-back-N ARQ. In S-DNF the node pairs exchange packets one pair at the time. For simplicity the use of ARQ in S-DNF is limited to the simplest protocol, Stop-and-wait ARQ.

In S-DNF both link-level and end-to-end ARQ are considered. The main difference is that in link-level ARQ the relay should be able to detect errors and ask for retransmissions from the end nodes, while with end-to-end ARQ this is not necessary. Hence, the functionality of the relay when using link-level ARQ is more complex, but it is expected to yield a better throughput.

4.3.4 Performance Potential Over Regular DNF

In order to illustrate the potential of the proposed scheme the performance of S-DNF and M-DNF is compared to Traditional SingleHop, TSH. The comparison is made on delivery of a single packet from each end node to its destination in a network with a topology as in Figure 3.1, on page 11, but with N end nodes. In this description of the potential, perfect links in the network are assumed, hence performance is measured in required time slots. More in depth analysis is performed in chapter 6.

Using TSH a single packet is delivered in one hop. Since N packets are to be transmitted and no packets can be transmitted concurrently, this scheme requires N time slots. M-DNF is able to deliver the two packets transmitted between a pair within two time slots. One slot with joint transmissions from the end nodes to the relay and one slot with a broadcast of the combined packet from the relay node. This operation is carried out for all $\frac{N}{2}$ pairs in the network, hence N time slots are needed with this scheme as when using TSH. This shows that M-DNF has no gain over TSH in the ideal case with perfect links. However, when distances and corresponding link qualities are taken into account, M-DNF will have an advantage since it uses shorter and more reliable links.

The proposed scheme, S-DNF, also utilises joint transmissions in each pair, therefore $\frac{N}{2}$ time slots are needed in order to send all data to the relay node. Moreover a single additional time slot is needed to broadcast the final combined packet, when the heaviest coding is utilised. S-DNF is thus able to deliver all data within $\frac{N}{2} + 1$ time slots. This gives a maximum performance gain of

$$G_{max} = \frac{N}{N/2 + 1} \tag{4.1}$$

Note that G_{max} is 1 for $N = 2$, since in this case only regular DNF is possible, and note that G_{max} approaches 2 as N goes to infinity. In this way S-DNF emulates full duplex communication between the nodes for large N. This maximum gain is only achieved in the ideal case, where all nodes are able to overhear all other transmissions. In the worst case scenario, no nodes can overhear anything, and the relay has to transmit network coded packets containing only two signals, since the end nodes only know their own packet a priori. This is similar to regular DNF, hence the gain is 1. However, for any partial amount of overheard transmissions, a gain between 1 and G_{max} is achievable.

4.3.5 Scaling The Constellation Size

The concept of DNF applied in the proposed scheme uses binary modulation. This simple type of modulation has helped clarifying the functionality of the proposed scheme. In scenarios with poor channel conditions the use of binary modulation may be reasonable. However, when good channel conditions are present higher order modulations can be utilised in order to transmit more data in each symbol. Hence the functionality of the scheme must scale to M-ary modulation schemes.

As described in section 2.2, on page 5, modulation schemes use some characteristic of the carrier wave to represent the digital information. For M-ary modulation schemes this characteristic has M different states, where each state is referred to as a symbol value, S_i, where $i \in \{0; M-1\}$. As an example, this means that the carrier wave in M-FSK can have M different frequencies, as illustrated in Figure 4.3. This means that each symbol in the constellation holds the information of $\log_2(M)$ bits.

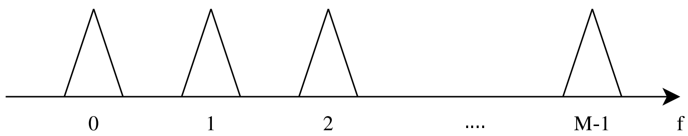


Figure 4.3: The M frequency band utilised by M-ary FSK.

When two nodes, i and j, transmit simultaneously to a third node, k, the received signal is one of 2^M different combinations of the two transmitted symbols. The denoise operation should compress these combinations into M different symbols in order to represent the analog coded packet using the same modulation scheme. This is achieved by adding the number of the two received symbols modulo M. This means that when node i and j transmit symbols S_i and S_j respectively, the denoised analog coded symbol is represented by the symbol S_k, where k is given by

$$k = (i + j) \mod M \tag{4.2}$$

In this way the 2^M different analog coded symbols can be represented using only M symbols, where a priori information is necessary in order to decode one of the source components. To decode the symbol from node j, S_j, node i subtracts the symbol number from the number of the received symbol modulo M. As an example consider the case of quaternary modulation. Node i and j transmit S_3 and S_2 respectively and the relay denoises the received signal. The resulting symbol, S_1 ($(2+3) \mod 4 = 1$), is broadcasted to the node pairs. Node i is then able to extract the symbol from node j, S_2, from the received symbol S_1 using its own symbol, S_3, since $(1-3) \mod 4 = 2$. This method for denoising analog coded packets scales to any M-ary modulation scheme allowing each end node to transmit more data in each transmission.

4.4 Conclusion

In this chapter a scheme which scales DNF to larger network sizes has been presented. Its performance potential has been described and shows a possible gain of a factor 2 in the ideal case. Moreover, a procedure for scaling the concept with respect to constellation size has been presented. The scheme utilises analog and digital network coding to combine packets from different end nodes in order to save transmissions and thus increase the throughput. The resulting packets are XOR combinations of the original data, hence there is a strong resemblance between this coding and tornado codes. The use of tornado codes on the individual data streams from the end nodes will be investigated later. The goal is to increase the error resilience in the system, which is expected to yield a performance improvement at lower SNR.

Chapter 5

Analysis of Decision Regions

In order to analyse the performance of the proposed scheme the error probability must be determined. The error probability in a given network depends on the utilised modulation scheme, hence in this chapter the decision regions for PSK and FSK are derived respectively. Binary modulation is assumed, hence each symbol represents a single bit. The decision regions for single and joint transmissions are derived separately.

5.1 BPSK

In order for BPSK to be useful in practise each transmission must be synchronised with respect to symbols and the phase must be known. PSK has high spectral efficiency, but solutions for detecting and tracking the phase is fairly complex. [Proakis and Salehi, 2002, 399].

5.1.1 Single Transmission

In a transmission from a single node the received symbol is normally distributed around the transmitted symbol, due to the Gaussian noise component. With a single source component the received signal can be demodulated at the receiving nodes according to the utilised modulation scheme. Figure 5.1 illustrates the symbol space with the corresponding Probability Density Functions (PDF), $f(x)$, for BPSK.

With the noise power being uniform on all links the variance of the PDFs will remain the same, regardless of which link is considered. However, due to path loss the amplitude of the received signal, and thereby the mean, μ, of the PDFs, depends on the distance between the communicating nodes. Assuming that each symbol is equiprobable the decision region is bounded by the solid vertical line, Λ_1, placed in zero. Due to symmetry around this bound the probability of error when demodulating a symbol is given by the area denoted α. The Bit Error Rate, BER, is defined as a function of SNR, γ, and for transmissions from node i to node j, the BER, denoted P_b, is given by:

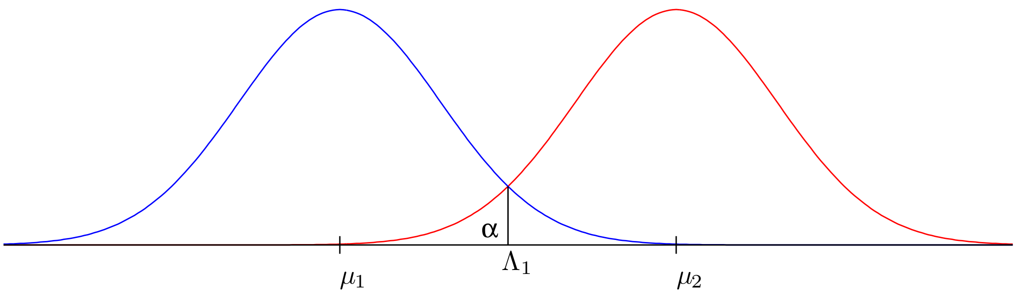


Figure 5.1: The symbol space for a BPSK modulated signal. The bound for the decision region for a single transmission is marked with Λ_1.

$$\begin{aligned} P_b(\gamma_{ij}) &= 2 \cdot \frac{P(X_r < \Lambda_1 | X_t = \mu_2)}{2} \\ &= \int_{-\infty}^{\Lambda_1} f(X_t = \mu_2)\ dx \end{aligned} \tag{5.1}$$

Where the received symbol, X_r, is normally distributed, $X_r \sim \mathcal{N}(\mu_2, \sigma^2)$, and the transmitted symbol $X_t = \mu_2 = \sqrt{\gamma_{ij} \cdot \sigma^2}$. Equation (5.1) is usually written using the complementary error function, erfc(·). [Haykin, 2001, 351]

$$P_b(\gamma_{ij}) = \frac{1}{2} \cdot \mathrm{erfc}\left(\sqrt{\gamma_{ij}}\right) \tag{5.2}$$

With the variance fixed the error probability solely depends on the mean of the PDF, i.e. the magnitude of the received power. This means that with the utilised channel model the distance between the communicating nodes dictates the error probability in the transmission.

5.1.2 Joint Transmission

When nodes i and j perform a joint transmission to node k the sum of the transmitted symbols can take four different values, when the received amplitude of the two signals are unequal. These values are denoted μ_i, where $i \in \{1; 4\}$. Due to AWGN the amplitude of the received signal is normally distributed around μ_i. The symbol space containing these four possible combinations along with their PDFs, $f_{\mu_i}(x)$, are illustrated in Figure 5.2 with dotted lines. Summing the leftmost PDF and the rightmost PDF, denoted $f_{\mu_1}(x)$ and $f_{\mu_4}(x)$ respectively, yields the total PDF for symbols where the two source symbols have equal signs. This is illustrated by the solid blue PDF in Figure 5.2. The two PDFs in the center, $f_{\mu_2}(x)$ and $f_{\mu_3}(x)$, are for combinations of source symbols with opposite signs. The total PDF for symbols with opposite signs is given by the sum of $f_{\mu_2}(x)$ and $f_{\mu_3}(x)$, i.e. the solid green PDF in the center.

The decision region from traditional BPSK cannot be reused for joint transmissions, due to the increased number of possible symbols. It is therefore desired to

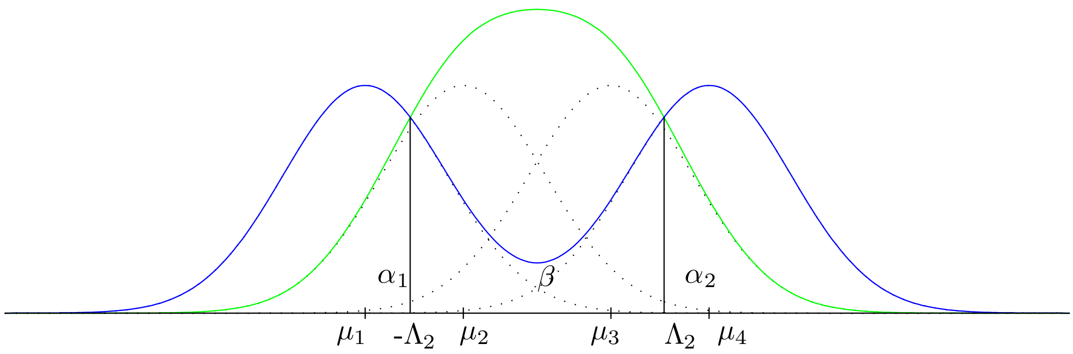


Figure 5.2: The symbol space for the sum of two BPSK modulated signals. The bound for the decision region for a joint transmission is marked by Λ_2.

identify the bound, Λ_2, that yields the lowest error probability, P_b. Given a certain Λ_2 the areas marked $\alpha_{\{1;2\}}$ and β under the PDFs are used when determining the error probability. The area given by the sum of α_1 and α_2 refers to the probability that two different symbols are detected as two equal symbols. Similarly the area β refers to the probability that two equal symbols are interpreted as two different symbols. Hence the total error probability is given by

$$\begin{aligned} P_b =& \frac{1}{4}\left(\int_{-\Lambda_2}^{\Lambda_2} f_{\mu_1}(x)dx + \left(1 - \int_{-\Lambda_2}^{\Lambda_2} f_{\mu_2}(x)dx\right) + \right. \\ & \left.\left(1 - \int_{-\Lambda_2}^{\Lambda_2} f_{\mu_3}(x)dx\right) + \int_{-\Lambda_2}^{\Lambda_2} f_{\mu_4}(x)dx\right) \\ =& \frac{1}{4}\left(F_{\mu_1}(\Lambda_2) - F_{\mu_1}(-\Lambda_2) + (1 - F_{\mu_2}(\Lambda_2) + F_{\mu_2}(-\Lambda_2)) + \right. \\ & \left.(1 - F_{\mu_3}(\Lambda_2) + F_{\mu_3}(-\Lambda_2)) + F_{\mu_4}(\Lambda_2) - F_{\mu_4}(-\Lambda_2)\right) \end{aligned} \tag{5.3}$$

where F_{μ_i} is the primitive function of f_{μ_i}. The Λ_2 yielding the lowest P_b is identified by solving the following equation.

$$0 =(P_b)' \Leftrightarrow \tag{5.4}$$

$$\begin{aligned} 0 =& \frac{1}{4}\left(f_{\mu_1}(\Lambda_2) + f_{\mu_1}(-\Lambda_2) - f_{\mu_2}(\Lambda_2) - f_{\mu_2}(-\Lambda_2) - f_{\mu_3}(\Lambda_2) - f_{\mu_3}(-\Lambda_2) + \right. \\ & \left. f_{\mu_4}(\Lambda_2) + f_{\mu_4}(-\Lambda_2)\right) \end{aligned}$$

Note that differentiating $F_\beta(-\Lambda_2)$ with respect to $+\Lambda_2$ causes a change in sign. Moreover, due to symmetry around zero equation (5.4) can be reduced to:

$$0 = 2 \cdot \left(\frac{f_{\mu_1}(\Lambda_2) - f_{\mu_2}(\Lambda_2) - f_{\mu_3}(\Lambda_2) + f_{\mu_4}(\Lambda_2)}{4}\right) \Leftrightarrow$$

$$f_{\mu_1}(\Lambda_2) + f_{\mu_4}(\Lambda_2) = f_{\mu_2}(\Lambda_2) + f_{\mu_3}(\Lambda_2) \tag{5.5}$$

The right hand side of equation (5.5) is the probability density of the event that two different symbols were transmitted when receiving a signal amplitude equal to Λ_2. Similarly the left hand side is the probability density of the event that two equal symbols were transmitted. From this equation it can be seen that the minimum error probability is achieved when the bound of the decision region is placed where the densities of these two events are equal, i.e. in the intersection of the resulting PDFs, as illustrated in Figure 5.2. The intersection depends on the variance of the PDFs, hence Λ_2 is a function of the noise power, σ^2. The noise power is equal on all links in the network and its magnitude is dictated by the SNR of a given link. This means that for any $\Lambda_2(\sigma^2)$, the BER for an analog coded signal is given by

$$\begin{aligned} P_b(\gamma_{ik}, \gamma_{jk}) = \frac{1}{2} \cdot \big(& P(X_2 > \Lambda_2(\sigma^2) | X_t = \mu_2) + P(X_3 > \Lambda_2(\sigma^2) | X_t = \mu_3) \\ & + P(X_4 < \Lambda_2(\sigma^2) | X_t = \mu_4) - P(X_4 < -\Lambda_2(\sigma^2) | X_t = \mu_4) \big) \end{aligned} \tag{5.6}$$

where
$X_2 \sim \mathcal{N}(\mu_2, \sigma^2)$
$X_3 \sim \mathcal{N}(\mu_3, \sigma^2)$
$X_4 \sim \mathcal{N}(\mu_4, \sigma^2)$

By using the definition of a normal distribution equation (5.5) can be solved for Λ_2 using Matlab. If the signal power received from both node i and j is assumed to be normalised to 1 the function for the decision bound is given by

$$\Lambda_2(\sigma^2) = \frac{1}{2} \cdot \log \left(\exp\left(\frac{2}{\sigma^2}\right)^2 + \sqrt{\exp\left(\frac{2}{\sigma^2}\right)^2 - 1} \right) \tag{5.7}$$

On a given link a function for the BER as a function of SNR for BPSK in DNF can be obtained by substituting equation (5.7) into (5.6). Using this function and equation (5.2) the BER performance of BPSK in DNF and BPSK in a single link transmission is plotted in Figure 5.3.

Figure 5.3 shows a decreasing BER as SNR increases, which was expected. However, the BER for BPSK in DNF is slightly higher than that of BPSK in a single transmission. The functionality of DNF therefore comes at the price of a decrease in BER performance.

5.2 BFSK

In this section the decision regions for the denoise operation in BFSK are determined. When BPSK is applied, it is necessary to assume phase tracking at the relay. However, for non-coherent BFSK it is sufficient to assume symbol synchronisation, since the phase is not utilised in the detection.

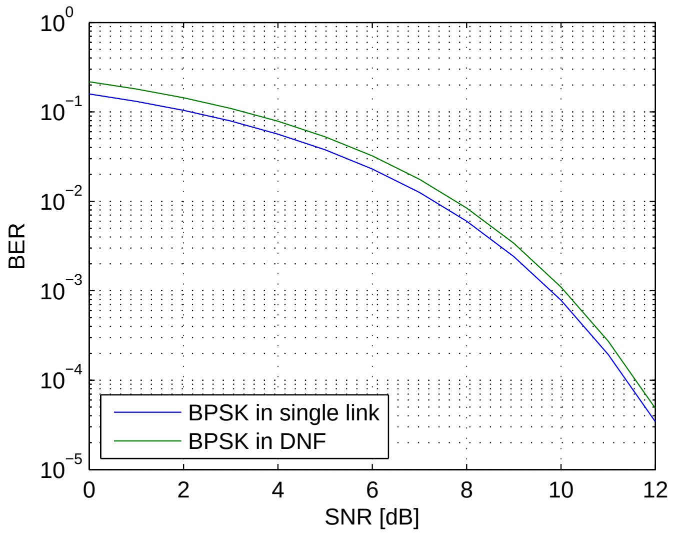


Figure 5.3: BER performance of BPSK in DNF and BPSK in a single link transmission.

5.2.1 Single Transmission

A BFSK receiver samples the energy in two different frequency bands and decide which one the transmitter used for that given symbol. The possible transmitted signals are:

$$s_i(t) = \sqrt{E_s}\cos(2\pi f_i t) \qquad i \in \{1;2\} \tag{5.8}$$

During transmission, a signal will experience a phase shift, ϕ_i, which is unknown to the receiver. Consequently, it must rely on envelope detection, which is realised by a quadrature receiver. In a quadrature receiver the energy of both the in-phase and the quadrature component, with respect to the transmitted signal, are measured. During both of these measurements noise is added. A quadrature receiver is necessary for both frequency bands, which results in a received signal in four dimensions, represented by the four orthonormal functions ψ_1, ψ_2, ψ_3 and ψ_4. The two possible received signals can be represented as:

$$x_1 = \Big(\underbrace{(\sqrt{E_s}\cos\phi_1 + \omega_1)\psi_1, (\sqrt{E_s}\sin\phi_1 + \omega_2)\psi_2}_{\alpha}, \underbrace{\omega_3\psi_3, \omega_4\psi_4}_{\beta}\Big) \tag{5.9}$$

$$x_2 = \Big(\omega_1\psi_1, \omega_2\psi_2, (\sqrt{E_s}\cos\phi_2 + \omega_3)\psi_3, (\sqrt{E_s}\sin\phi_2 + \omega_4)\psi_4\Big) \tag{5.10}$$

From x_i the envelope in both frequency bands can be calculated using Pythagoras theorem on dimensions 1 plus 2 and 3 plus 4, marked by α and β respectively. These envelopes are compared in order to discriminate between s_1 and s_2. If the envelope in α is larger, a decision is made on s_1, and if β is larger, a decision is made on s_2.

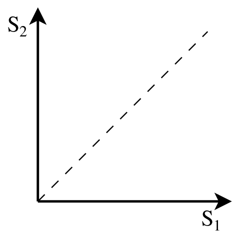


Figure 5.4: Decision regions for a single transmission in BFSK.

The resulting decision regions for a single transmission using BFSK are illustrated in Figure 5.4. Note that assuming AWGN, the envelope in a frequency band containing the signal is Rician distributed, while the envelope in a frequency band containing only noise is Rayleigh distributed. For a transmission from node i to node j this gives a BER, P_b, as a function of SNR given by: [Haykin, 2001, 415]

$$P_b(\gamma_{ij}) = \frac{1}{2} \cdot e^{-\gamma_{ij}} \tag{5.11}$$

5.2.2 Joint Transmission

A significant difference between BPSK and BFSK exists in the utilisation of analog network coding in DNF. In BPSK the transmitted signals are either in phase or in reverse phase, which means that they can be added as scalars. In BFSK, however, an unknown phase difference is present, which means they must be added as vectors. The transmitted signal in a joint transmission by nodes A and B is defined as:

$$s_{ij}(t) = \sqrt{E_{sA}} \cos(2\pi f_i t) + \sqrt{E_{sB}} \cos(2\pi f_j t) \qquad i = 1,2 \qquad j = 1,2 \tag{5.12}$$

Denoting the received signal of a joint transmission x_{ij}, the following possibilities exist.

$$\begin{aligned}
x_{11} = \Big(&(\sqrt{E_{sA}} \cos\phi_{1A} + \sqrt{E_{sB}} \cos\phi_{1B} + \omega_1)\psi_1, \\
&(\sqrt{E_{sA}} \sin\phi_{1A} + \sqrt{E_{sB}} \sin\phi_{1B} + \omega_2)\psi_2, \omega_3\psi_3, \omega_4\psi_4 \Big) \\
x_{12} = \Big(&(\sqrt{E_{sA}} \cos\phi_{1A} + \omega_1)\psi_1, (\sqrt{E_{sA}} \sin\phi_{1A} + \omega_2)\psi_2, (\sqrt{E_{sB}} \cos\phi_{2B} + \omega_3)\psi_3, \\
&(\sqrt{E_{sB}} \sin\phi_{2B} + \omega_4)\psi_4 \Big) \\
x_{21} = \Big(&(\sqrt{E_{sA}} \cos\phi_{1B} + \omega_1)\psi_1, (\sqrt{E_{sA}} \sin\phi_{1B} + \omega_2)\psi_2, (\sqrt{E_{sB}} \cos\phi_{2A} + \omega_3)\psi_3, \\
&(\sqrt{E_{sB}} \sin\phi_{2A} + \omega_4)\psi_4 \Big) \\
x_{22} = \Big(&\omega_1\psi_1, \omega_2\psi_2, (\sqrt{E_{sA}} \cos\phi_{2A} + \sqrt{E_{sB}} \cos\phi_{2B} + \omega_3)\psi_3, \\
&(\sqrt{E_{sA}} \sin\phi_{2A} + \sqrt{E_{sB}} \sin\phi_{2B} + \omega_4)\psi_4 \Big)
\end{aligned}$$

The signals x_{12} and x_{21} are simple to deal with, since the two components do not interfere in these. Both envelopes in x_{12} and x_{21} are Rician distributed. However, when the two transmitters use the same frequency, the signals are added. Figure 5.5 shows a geometrical interpretion of this addition.

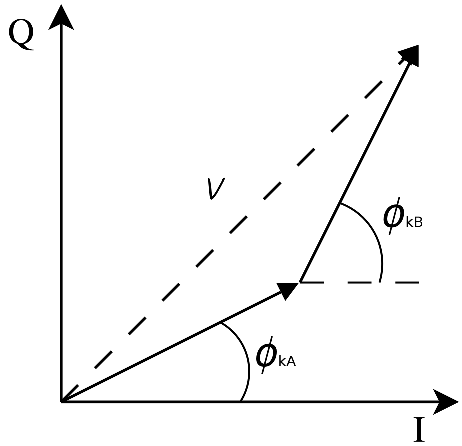


Figure 5.5: The addition of two signals in the same frequency band.

The dashed line, ν, in Figure 5.5 is the total envelope, which is the measure detected by the receiver. Depending on the phase difference between the two components, they will either add as scalars, cancel out or something in between. As a result, ν follows a composite distribution, which can be described as a Rician distribution in which the mean value follows some distribution, which is determined later.

Assuming zero phase difference, the envelope of both x_{11} and x_{22} is $\sqrt{E_{sA}}+\sqrt{E_{sB}}$, which is regarded as the corresponding symbol in the signal-space diagram. The symbols corresponding to x_{12} and x_{21} lies in the first quadrant and are seperated if $\sqrt{E_{sA}} \neq \sqrt{E_{sB}}$, as illustrated in the signal constellation in Figure 5.6. If $\sqrt{E_{sA}} = \sqrt{E_{sB}}$, x_{12} and x_{21} are represented by the same symbol.

Determination of Decision Regions

It has been determined that the four possible symbols in BFSK with analog network coding do not follow the same type of distribution. This means that the optimum decision regions can not be defined by Maximum Likelihood detection. Instead the Maximum A Posteriori Probability, MAP, detection is applied, where the conditional probability density functions of the possible symbols are compared. Note that in DNF it is only necessary to discriminate between the symbols with equal frequency and the symbols with different frequencies. Hence the two dimensional space in Figure 5.6 should be divided into two regions based on the conditional PDFs.

Conditional PDF of Symbols with Different Frequencies: In the case where the received analog coded symbol contains different frequencies the total signal is a

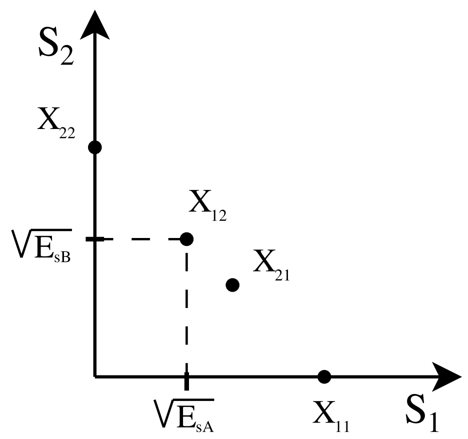


Figure 5.6: Signal-space diagram of a joint transmission using BFSK.

two dimensional vector, whose elements both follow a Rician distribution. A signal vector is defined by the random variable $U = (U_i, U_j)^T$, where U_i and U_j are the envelopes in the two frequency bands respectively. Hence, the joint conditional PDF of U is:

$$\begin{aligned}
f_U(U|s_{12}) =& \frac{U_1}{\sigma^2} \exp\left(\frac{-(U_1^2 + E_{sA})}{2\sigma^2}\right) I_0\left(\frac{U_1\sqrt{E_{sA}}}{\sigma^2}\right) \cdot \\
& \frac{U_2}{\sigma^2} \exp\left(\frac{-(U_2^2 + E_{sB})}{2\sigma^2}\right) I_0\left(\frac{U_2\sqrt{E_{sB}}}{\sigma^2}\right) \\
=& \frac{U_1U_2}{\sigma^4} \exp\left(\frac{-(U_1^2 + E_{sA}) - (U_2^2 + E_{sB})}{2\sigma^2}\right) I_0\left(\frac{U_1\sqrt{E_{sA}}}{\sigma^2}\right) I_0\left(\frac{U_2\sqrt{E_{sB}}}{\sigma^2}\right) \\
f_U(U|s_{21}) =& \frac{U_1U_2}{\sigma^4} \exp\left(\frac{-(U_2^2 + E_{sA}) - (U_1^2 + E_{sB})}{2\sigma^2}\right) I_0\left(\frac{U_2\sqrt{E_{sA}}}{\sigma^2}\right) I_0\left(\frac{U_1\sqrt{E_{sB}}}{\sigma^2}\right)
\end{aligned}$$

Where $I_0(\cdot)$ is the modified zero order Bessel function. Assuming that all symbols are equiprobable, the total joint PDF for symbols with different frequencies is:

$$f_U(U|s_{ij}, i \neq j) = \frac{1}{2}(f_U(U|s_{12}) + f_U(U|s_{21})) \tag{5.13}$$

Figure 5.7 shows a color map of $f_U(U|s_{ij}, i \neq j)$, where $\sqrt{E_{sA}} = 5$, $\sqrt{E_{sB}} = 3$ and $\sigma = 1$ is used as an example.

Conditional PDF of Symbols with Equal Frequencies: When the two transmitters use the same frequency, the remaining frequency band contains only noise. These noise components, ω_i, are orthogonal, hence the resulting envelope is Rayleigh distributed with parameter σ since $\omega_i \sim \mathcal{N}(0, \sigma^2)$. This envelope is caused by pure noise and is referred to as U_k, where $k = 2$ if s_{11} is transmitted and vice versa.

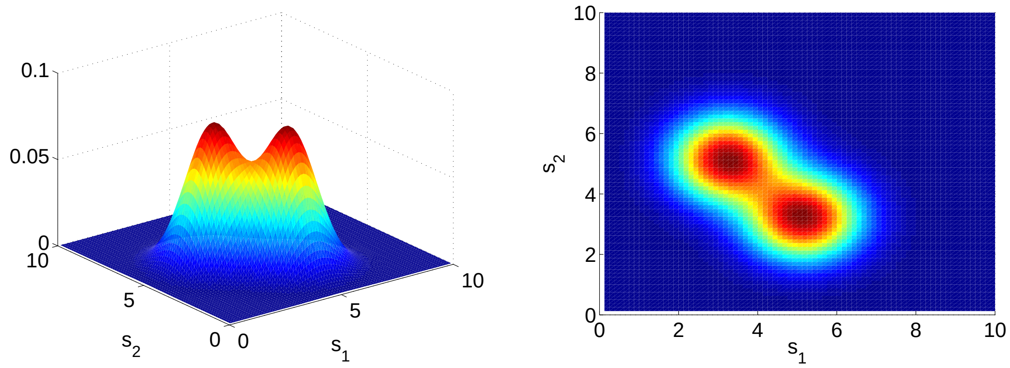


Figure 5.7: The joint PDF for symbols with different frequencies for $\sqrt{E_{sA}} = 5$, $\sqrt{E_{sB}} = 3$ and $\sigma = 1$.

$$f_{U_k}(U_k|s_{ij}, i = j) = \frac{U_k}{\sigma} \exp\left(\frac{-U_k^2}{2\sigma^2}\right) \tag{5.14}$$

The envelope in the used frequency band, U_l, where $l = 1$ if s_{11} is transmitted, follows a composite distribution as stated in section 5.2.2. It is a Rician distribution in which the mean value itself follows a distribution. The composite distribution can thus be expressed as follows.

$$f_{U_l}(U_l|s_{ij}, i = j) = \int_{-\infty}^{\infty} f_\nu(\nu) \cdot \frac{U_l}{\sigma^2} \exp\left(\frac{-(U_l^2 + \nu^2)}{2\sigma^2}\right) I_0\left(\frac{U_l \nu}{\sigma^2}\right) d\nu \tag{5.15}$$

The mean value is the noiseless envelope, ν, which is shown in Figure 5.5. The distribution of ν is a result of the uniform distribution of the phase difference, $\phi_d = \phi_{kB} - \phi_{kA}$. The value of ν only depends on ϕ_d and not on the individual values of ϕ_{kA} and ϕ_{kB}, hence, the reference is changed to ϕ_{kA}. Figure 5.8 illustrates the origin of the distribution of ν.

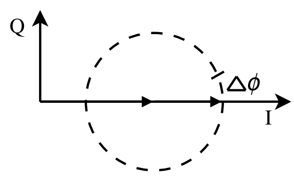


Figure 5.8: The possible total signals for uniformly distributed ϕ_d.

In order to derive the distribution of ν, a probability mass function, PMF, is first considered. This is a discrete expression of the distribution of ν, i.e. it expresses the probability of experiencing a ν within a certain $\Delta\nu = [\nu_a; \nu_b]$. A certain $\Delta\nu$ corresponds to a certain $\Delta\phi$, whose relationship is expressed by the difference quotient $\frac{\Delta\phi}{\Delta\nu}$. Note that the probability of experiencing a ν within $\Delta\nu$ can be expressed as

$\frac{\Delta\phi}{\pi}$, because ϕ is uniformly distributed between 0 and 2π and ν is symmetric around π in this interval. See Figure 5.8. The PMF can thus be expressed as $\frac{\Delta\phi}{\pi\Delta\nu}$ and for $\Delta\nu \to 0$ this becomes $\frac{d\phi}{\pi d\nu}$, which expresses the desired PDF. This is derived as follows.

$$\begin{aligned}
\nu &= \sqrt{(\sqrt{E_{sA}} + \sqrt{E_{sB}}\cos\phi)^2 + (\sqrt{E_{sB}}\sin\phi)^2} \\
\nu^2 &= E_{sA} + E_{sB}\cos^2\phi + 2\sqrt{E_{sA}}\sqrt{E_{sB}}\cos\phi + E_{sB}\sin^2\phi \\
\nu^2 &= E_{sA} + 2\sqrt{E_{sA}}\sqrt{E_{sB}}\cos\phi + E_{sB}(\cos^2\phi + \sin^2\phi) \\
\nu^2 &= E_{sA} + 2\sqrt{E_{sA}}\sqrt{E_{sB}}\cos\phi + E_{sB} \\
\cos\phi &= \frac{\nu^2 - E_{sA} - E_{sB}}{2\sqrt{E_{sA}}\sqrt{E_{sB}}} \\
\phi &= \cos^{-1}\left(\frac{\nu^2 - E_{sA} - E_{sB}}{2\sqrt{E_{sA}}\sqrt{E_{sB}}}\right) \\
\frac{d\phi}{d\nu} &= \frac{-\nu}{\sqrt{E_{sA}}\sqrt{E_{sB}}\sqrt{1 - \left(\frac{\nu^2 - E_{sA} - E_{sB}}{2\sqrt{E_{sA}}\sqrt{E_{sB}}}\right)^2}} \\
f_\nu(\nu) = \frac{d\phi}{\pi d\nu} &= \frac{-\nu}{\pi\sqrt{E_{sA}}\sqrt{E_{sB}}\sqrt{1 - \left(\frac{\nu^2 - E_{sA} - E_{sB}}{2\sqrt{E_{sA}}\sqrt{E_{sB}}}\right)^2}} \qquad (5.16)
\end{aligned}$$

By combining equations (5.14), (5.15) and (5.16) the joint conditional PDFs of symbols with equal frequencies can be expressed as:

$$\begin{aligned}
f_U(U|s_{11}) &= f_{U_2}(U_2|s_{11}) \cdot f_{U_1}(U_1|s_{11}) \\
f_U(U|s_{22}) &= f_{U_1}(U_1|s_{22}) \cdot f_{U_2}(U_2|s_{22})
\end{aligned}$$

Hence the total PDF of symbols with equal frequencies is then as follows:

$$f_U(U|s_{ij}, i = j) = \frac{1}{2}(f_U(U|s_{11}) + f_U(U|s_{22}))$$

Figure 5.9 shows a color map of $f_U(U|s_{ij}, i = j)$, where $\sqrt{E_{sA}} = 5$, $\sqrt{E_{sB}} = 3$ and $\sigma = 1$ is used as an example.

Solution to Decision Regions: According to the MAP rule, any point in the two dimensional space in Figure 5.6 should belong to the region represented by the conditional PDF with the highest value in that particular point. This means that the intersection of the two conditional PDFs comprises the decision region bound. Figure 5.10 shows the two PDFs in the same plot and the intersection is also visible.

The intersection is found by solving the following equation.

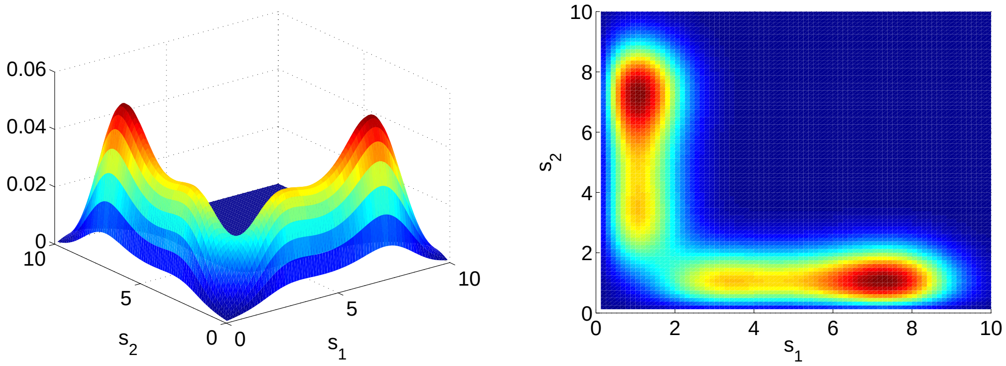


Figure 5.9: The joint PDF for symbols with equal frequencies for $\sqrt{E_{sA}} = 5$, $\sqrt{E_{sB}} = 3$ *and* $\sigma = 1$.

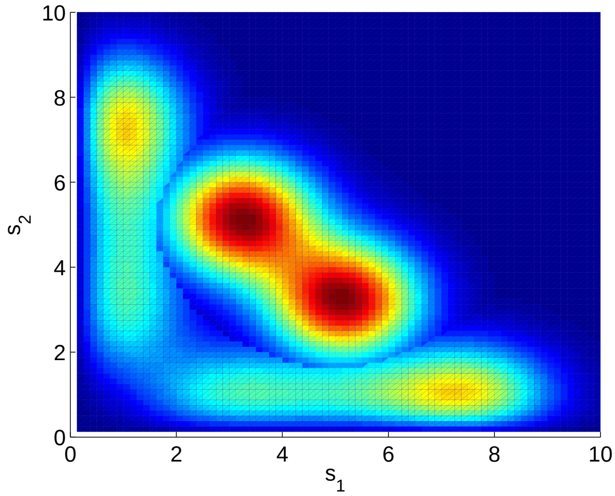


Figure 5.10: Both PDFs for $\sqrt{E_{sA}} = 5$, $\sqrt{E_{sB}} = 3$ *and* $\sigma = 1$.

$$f_U(U|s_{ij}, i \neq j) = f_U(U|s_{ij}, i = j) \tag{5.17}$$

This is a complicated equation, which is not solvable in Matlab. As a consequence a few approximations have been made in order to find a solution. The first approximation is a solution to the integral in equation (5.15). No closed form solution can be found in Matlab, and instead ν is limited to a discrete set of values whose probabilities are found and used in a weighted sum of the corresponding Rician distributions. This approximation can be made arbitrarily accurate by increasing the number of possible values of ν.

The other approximation is necessary to find a solution to the resulting equation in (5.17). It is a function of two variables, U_1 and U_2, and Matlab is not able to find a closed form solution. However, if U_1 is fixed to a discrete set of values, the corresponding equations of the single variable U_2 are solvable. They will either have zero or two solutions depending on U_1. Figure 5.11 shows a plot of a set of solutions found in this way.

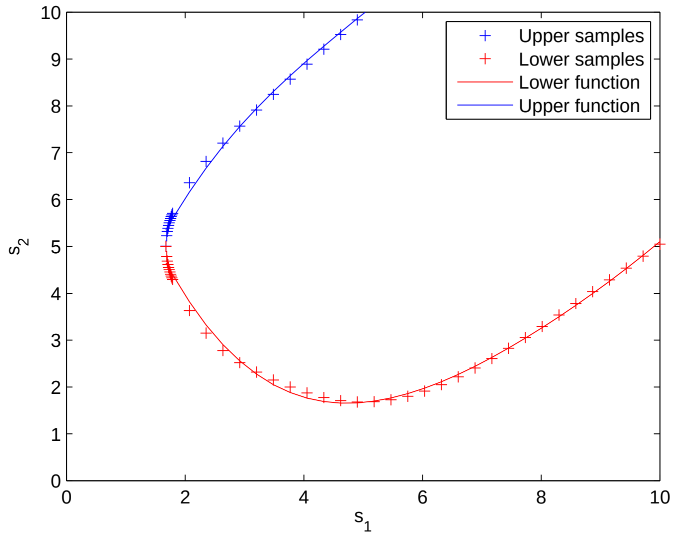


Figure 5.11: Discrete solution to the decision region bound.

The decision region bounds are found by polynomial regression on these points. This gives an upper and a lower function. The decision that different frequencies were used is made if the received signal is above the lower function and below the upper function in the signal space diagram. Using these decision regions the BER performance of BFSK in DNF is determined empirically. In Figure 5.12 this performance is compared to BFSK in a single transmission using equation (5.11).

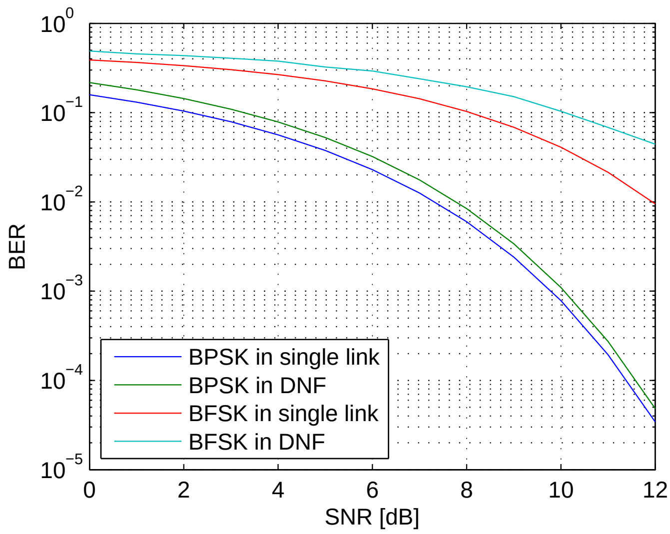


Figure 5.12: BER performance of BFSK in DNF and BFSK in a single transmission plotted together with the corresponding BER performance for BPSK.

Comparing BPSK and BFSK in single transmissions the BER performance is worse for BFSK, as expected. This is due to the decreased distance between the two symbols in the two dimensional symbol space for BFSK compared to the one dimensional symbol space for BPSK. Regarding the BER performance in DNF the denoise operation has a larger impact on BFSK compared to BPSK.

Chapter 6

Performance Analysis

The performance of the proposed scheme, S-DNF, is analysed in this chapter. The purpose of this analysis is to derive equations for evaluating the average throughput in the network using S-DNF. The analysis is carried out for networks incorporating error control using link-level ARQ and end-to-end ARQ respectively. The approach taken in both parts of the analysis is to define a maximum possible throughput for a single end node, t_{max}, based on the number of necessary transmissions in order to deliver a packet. The expected throughput for the node is then defined as t_{max} multiplied by the probability of being able to decode the packet, p_d. Calculating this for every node in the network and averaging gives the desired metric.

$$t = p_d \cdot t_{max} \tag{6.1}$$

6.1 End-to-End ARQ

Using end-to-end error control means that errors are detected by the end nodes only. Hence an end-to-end CRC is assumed and the relay just combines and forwards whatever the code dictates. This means that no sophisticated retransmission scheme is utilised. The network being analysed is the star topology where a number of end nodes communicate in pairs through a common relay.

Given the assumed error control, described in section 3.1, only two outcomes are possible as seen from an end node. After the relay node has broadcasted, an end node achieves either maximum throughput, t_{max}, or zero throughput. Using normalised bandwidth the maximum throughput depends solely on the number of time slots, M, used to distribute the packets, i.e. $t_{max} = 1/M$. The average throughput for a network of n nodes is therefore given by:

$$\bar{t} = \frac{\sum_{i=1}^{n} P_{di} \cdot t_{max}}{n} = \frac{\bar{P}_d}{M} \tag{6.2}$$

where P_{di} is the probability that the $i-$th node is able to decode the packet from the other node in the pair. When deriving the average P_d, it is not possible to use the individual values of P_d for the end nodes to find the mean value, because correlation in the probabilities exist. The P_d of multiple end nodes may depend on the same joint transmission from an end node pair to the relay, because network coding is performed at the relay. It is therefore necessary to condition P_d on the joint transmissions to the relay, in order to decorrelate the individual values of P_d.

In an example of a network with four end nodes, there are two pairs each having a joint transmission to the relay. The outcome of a single joint transmission is binary and a success is denoted (1) and has probability $P_s(\gamma_{iR}, \gamma_{jR})$. A failure is denoted (0) and has probability $P_p(\gamma_{iR}, \gamma_{jR})$. Note that $P_p(\gamma_{iR}, \gamma_{jR}) = 1 - P_s(\gamma_{iR}, \gamma_{jR})$. This means that four different outcomes are possible for two transmissions; (00), (01), (10) and (11). It is then necessary to find the probabilities of these outcomes and to find the corresponding conditional expressions of P_d, $P_d|(xx)$. P_d for a single end node is then given by:

$$P_d = P(00) \cdot P_d|(00) + P(01) \cdot P_d|(01) + P(10) \cdot P_d|(10) + P(11) \cdot P_d|(11) \quad (6.3)$$

When deriving the expression for $P_d|(xx)$, a systematic approach is taken based on the structure of the binary tree. In Figure 6.1 the tree for a network with four end nodes is illustrated. To indicate the network size, $P_d|(xx)$ for this specific network is denoted $P_{d4}|(xx)$.

The analysis will be performed for node A in this tree, but the structure of $P_{d4}|(xx)$ is equal for all end nodes, only indices are changed. For the sake of simplicity, link notation is left out on transmissions from the relay to node A, i.e. $P_s = P_s(\gamma_{RA})$ and $P_p = P_p(\gamma_{RA})$. In the derivation of $P_{d4}|(xx)$ the expression will be constructed step by step, and $P_{d4}|(xx)^*$ denotes an unfinished expression, i.e. a prefix of the final expression.

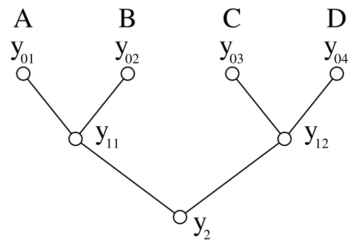


Figure 6.1: The binary tree indicating possible network coding for a network with four end nodes.

Initially the probabilities of the different outcomes in the joint transmissions from the end node pairs to the relay are presented:

$$p(00) = P_p(\gamma_{AR}, \gamma_{BR}) \cdot P_p(\gamma_{CR}, \gamma_{DR})$$
$$p(01) = P_p(\gamma_{AR}, \gamma_{BR}) \cdot P_s(\gamma_{CR}, \gamma_{DR})$$
$$p(10) = P_s(\gamma_{AR}, \gamma_{BR}) \cdot P_p(\gamma_{CR}, \gamma_{DR})$$
$$p(11) = P_s(\gamma_{AR}, \gamma_{BR}) \cdot P_s(\gamma_{CR}, \gamma_{DR})$$

For the derivation of $P_{d4}|(xx)$, note that the easiest way for node A to decode what is intended for it, is by receiving y_{11} from the relay. For this to happen, the XOR of y_{01} and y_{02} must be successfully received at the relay. This is a joint transmission from an end node pair, which is represented by a binary variable depending on (xx). This is denoted J_{AB} and takes the value 1 in the conditions (10) and (11) and zero otherwise. In addition to this, y_{11} must be successfully received by node A. Thus, $P_{d4}|(xx)^*$ equals:

$$P_{d4}|(xx)^* = J_{AB} \cdot P_s \tag{6.4}$$

In Figure 6.1 it is seen that another opportunity for node A to become able to decode exists, namely from receiving y_2. This is not enough, however, since y_{03} and y_{04} are also required at the relay, in addition to y_{01} and y_{02} from earlier, in order to be able to broadcast y_2 in the first place. When adding this to the expression, it must be conditioned on y_{11} not being received. Otherwise, the expression will not be bounded by 1.

$$P_{d4}|(xx)^* = J_{AB}(P_s + P_p \cdot J_{CD} \cdot P_s) \tag{6.5}$$

In this second opportunity to decode, overhearing or the equivalent help from the relay is necessary. This must be included in the expression. In this case the information in y_{12} is necessary. This can be provided by the relay directly or alternatively by overhearing $y_{03} \oplus y_{04}$. The probability of receiving either y_{12} or $y_{03} \oplus y_{04}$ is denoted $G_4^{y_{12}}$.

$$G_4^{y_{12}} = 1 - (1 - P_s(\gamma_{CA}, \gamma_{DA}))P_p \tag{6.6}$$

The final expression for P_{d4} is thus:

$$P_{d4}|(xx) = J_{AB}(P_s + P_p \cdot J_{CD} \cdot G_4^{y_{12}} \cdot P_s) \tag{6.7}$$

Having constructed the expression for $P_{d4}|(xx)$ in this systematic stepwise fashion is very useful when increasing the network size. Note that if it is desired to derive $P_{d8}|(xx)$ for the tree in Figure 6.2, the same elements as already derived for $P_{d4}|(xx)$ would be required. Hence, the expression for $P_{d4}|(xx)$ is a prefix of the expression for $P_{d8}|(xx)$. Deriving $P_{d8}|(xx)$ is thus just a matter of continuing the construction process one step further. This means repeating the process described in equations (6.5) to (6.7). In the next step, the information in y_{22} is necessary. The probability of receiving this information is denoted $G_8^{y_{22}}$ and is a bit more extensive than $G_4^{y_{12}}$, since the necessary help from the relay can come from more combinations. The information in y_{22} can be provided directly by the relay, or in the shape of both y_{13} and y_{14}, which both in turn can be provided directly or by overhearing of either $y_{05} \oplus y_{06}$ or $y_{07} \oplus y_{08}$ respectively. This is a recursive structure, in which the structure

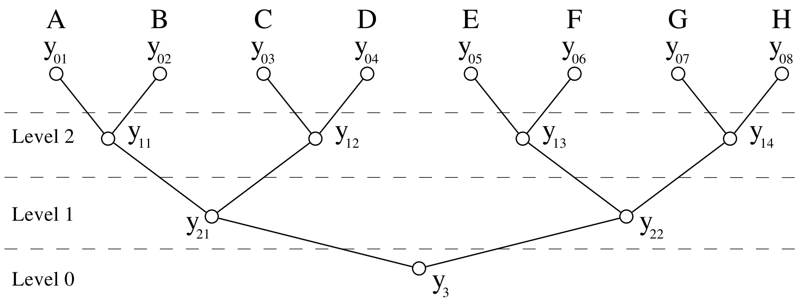


Figure 6.2: The binary tree indicating possible network coding for a network with eight end nodes.

of $G_4^{y_{12}}$ can be reused on y_{13} and y_{14}, as well as on y_{22} for the final expression of $G_8^{y_{22}}$.

$$\begin{aligned} G_4^{y_{13}} &= 1 - (1 - P_s(\gamma_{EA}, \gamma_{FA}))P_p \\ G_4^{y_{14}} &= 1 - (1 - P_s(\gamma_{GA}, \gamma_{HA}))P_p \\ G_8^{y_{22}} &= 1 - (1 - G_4^{y_{13}} \cdot G_4^{y_{14}})P_p \end{aligned} \tag{6.8}$$

$G_8^{y_{22}}$ is the probability that node A holds the necessary information in order to utilise y_3. Note that two additional successful joint transmissions from end node pairs are needed in order to create y_3 at the relay compared to y_{21}. $P_{d8}|(xx)$ can therefore be expressed as:

$$P_{d8}|(xx) = J_{AB} \cdot (P_s + P_p \cdot J_{CD} \cdot G_4^{y_{12}}(P_s + P_p \cdot J_{EF} \cdot J_{GH} \cdot G_8^{y_{22}} \cdot P_s)) \tag{6.9}$$

In these derivations it has been taken into account that there is a probability of receiving every packet included in the binary tree. This is obviously not the case for a specific code unless full redundancy is applied. Thus for a specific code, P_s equals zero on "transmissions" of packets which are not a part of the code. This cancels out the terms including impossible events in the general expression, leaving out an expression that fits the specific code.

6.2 Link Level ARQ

Using link-level error detection means that the relay is able to detect errors and discard erroneous packets. Hence link-level ARQ can be applied, and the simple variant Stop-and-wait ARQ is utilised. In this scenario, this means that if an error occurs in a transmission to the relay, an immediate retransmission is requested, before other nodes are given access to the medium. If errors occur in transmissions from the relay, or an end node is unable to decode due to errors in overheard packets, an immediate retransmission of the necessary ANC packet is requested. In this analysis a topology equal to the one used in section 6.1 is assumed when deriving the necessary expressions.

It is important to note what impact the change in error control has on the starting point used in the analysis in section 6.1. Nodes are no longer experiencing a throughput of either t_{max} or zero, with probabilities P_d and $1 - P_d$ respectively, in a single round. Retransmissions will decrease the throughput in a specific round, making an infinite sequence of throughputs between zero and t_{max} possible, each with a certain probability. These throughputs are denoted t_M, where M is the number of used timeslots in the round in question, $M = 2, 3, 4...\infty$. Depending on the network size and utilised code, M will have a lower bound higher than two. Note that t_∞=0, since.

$$t_M = \frac{1}{M} \tag{6.10}$$

The aim of this analysis is to derive the expected value of t_M.

$$E[t_M] = E\left[\frac{1}{M}\right] = \frac{1}{E[M]} \tag{6.11}$$

In order to find the expected value of t_M, the expected value of M must be determined. M is the sum of three components; the number of necessary transmissions in order to deliver all ANC packets at the relay, M_r, the number of transmissions performed by the relay as dictated by the applied code, M_c, and the number of necessary retransmissions to the end node pairs, in order to correct decoding errors, M_e.

$$M = M_r + M_c + M_e \tag{6.12}$$

Hence, from the linearity of expected value:

$$E[M] = E[M_r] + E[M_c] + E[M_e] \tag{6.13}$$

Where M_c is constant for a deterministic code, and the other two are random variables. First M_r is considered. The probability of k necessary transmissions from a single end node pair, X and Y, to the relay is given by:

$$P(M_{r,XY} = k) = (1 - P_s(\gamma_{XR}, \gamma_{YR}))^{(k-1)} \cdot P_s(\gamma_{XR}, \gamma_{YR}) \tag{6.14}$$

The expected value of M_r in a network with $\frac{n}{2}$ end node pairs is thus.

$$E[M_r] = \frac{n}{2} \sum_{k=1}^{k=\infty} (1 - P_s(\gamma_{XR}, \gamma_{YR}))^{(k-1)} \cdot P_s(\gamma_{XR}, \gamma_{YR}) \cdot k \tag{6.15}$$

For the derivation of $E[M_e]$, an end node pair, X and Y, is again used as an example. M_e is the number of retransmissions by the relay of the ANC packet covering nodes X and Y. Retransmissions are only necessary if the end nodes are unable to decode, hence the expression derived in section 6.1 is reused, though with a small adjustment. The utilised protocol ensures link level reliability, hence the terms corresponding to the probability of having received all necessary data at the relay must be omitted. For a network with four end nodes, P_d equals the following.

$$P_{d4} = P_s + P_p \cdot G_4^{y12} \cdot P_s \tag{6.16}$$

It is assumed that each node has the same conditions for decoding, i.e. the relay must transmit all parity packets for one or more levels of coding. This assumption makes P_d equal for all end nodes.

When an end node pair decodes, it can result in three different states. Either zero, one or two nodes in the pair have decoded its packet. These states are named 0, 1 and 2 respectively. It is now possible to model the retransmission mode with a state model, where the initial state probabilities can be found using P_d. The aim is to calculate the expected number of necessary retransmissions before the system enters state 2. The initial state probabilities are given by:

$$P_{i0} = (1 - P_d)^2 \tag{6.17}$$
$$P_{i1} = 2P_d(1 - P_d) \tag{6.18}$$
$$P_{i2} = P_d^2 \tag{6.19}$$

Figure 6.3 shows the structure of the state model along with the state transition probabilities. Let P_{ij} denote the probability of a transition from state i to state j. As an example P_{01} corresponds to the probability that one of the two nodes in a node pair is able to decode after a retransmission from the relay.

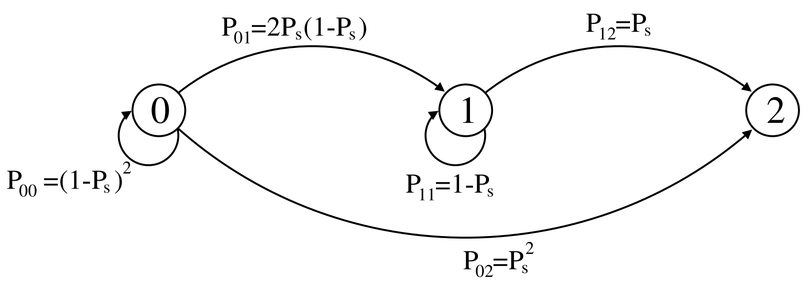


Figure 6.3: A state model of the retransmission mode at the relay.

The approach is to calculate the expected number of retransmissions used in each of the two states 0 and 1, $E[M_{e0}]$ and $E[M_{e1}]$. These are found using the formula for the expected number of trials in a repeated experiment with a given success probability. $E[M_{e0}]$ is thus.

$$E[M_{e0}] = \sum_{k=1}^{\infty} (1 - P_s)^{2(k-1)} \cdot (1 - (1 - P_s)^2) \cdot k \tag{6.20}$$

When the system leaves state 0 it will either enter state 1 or 2. Additional retransmissions are only necessary if state 1 is entered. The probability of entering state 1 rather than state 2 given that state 0 has been left is denoted P_{01c}. It is calculated as follows.

$$P_{01c} = \frac{P_{01}}{P_{01} + P_{02}} = \frac{2P_s(1 - P_s)}{2P_s(1 - P_s) + P_s^2} = \frac{2P_s - 2P_s^2}{2P_s - P_s^2} \tag{6.21}$$

The expected number of retransmissions used in state 1 is.

$$E[M_{e1}] = \sum_{l=1}^{\infty} (1 - P_s)^{(l-1)} \cdot P_s \cdot l \tag{6.22}$$

The total expected number of retransmissions to the end node is a weighted sum of $E[M_{e0}]$ and $E[M_{e1}]$, where the weights are P_{i0} and $P_{i1} + P_{01c}$ respectively, multiplied by $\frac{n}{2}$ which is the number of end node pairs in the network.

$$E[M_e] = \frac{n}{2} \cdot (P_{i0} \cdot E[M_{e0}] + (P_{i1} + P_{01c}) \cdot E[M_{e1}]) \tag{6.23}$$

Note that if a code does not give each node equal conditions for decoding, the expression for $E[M_e]$ would include a sum over the individual end node pairs with corresponding individual P_d and P_s in the expressions for $E[M_{e0}]$ and $E[M_{e1}]$, instead of the multiplication with $\frac{n}{2}$.

Combining equations (6.13), (6.15) and (6.23) and noting that $E[M_c]$ equals M_c, gives:

$$\begin{aligned} E[M] = & M_c + \frac{n}{2}(P_{i0} \cdot E[M_{e0}] + (P_{i1} + P_{01c}) \cdot E[M_{e1}] \\ & + \sum_{k=1}^{k=\infty} (1 - P_s(\gamma_{XR}, \gamma_{YR}))^{(k-1)} \cdot P_s(\gamma_{XR}, \gamma_{YR}) \cdot k) \end{aligned} \tag{6.24}$$

The average throughput in the network can now be calculated by combining equation (6.11) and (6.24).

Chapter 7

Performance Evaluation

S-DNF is evaluated in a number of simulations in this chapter. Initially the core principle of S-DNF is evaluated, where end-to-end ARQ is assumed. Moreover, link level ARQ is evaluated and compared to the results of end-to-end ARQ. Another simulation evaluates a variant of S-DNF where redundancy is generated at the relay. S-DNF where BFSK modulation is utilised is also evaluated and compared to the more simple BPSK, which is used in all other simulations. Finally, a simulation is performed where small scale fading in the channel is taken into account. Each simulation is treated in a seperate section.

7.1 The Scenario

The evaluations are performed for the scenario described in section 3.1. Hence, the network have eight end nodes and a relay node, where the end nodes communicate in pairs through the common relay. Each node generates packets with 128 bytes of random data which are transmitted with unit power using BPSK modulation. Each transmitted symbol is detected using a MAP receiver. It is assumed that the channel has AWGN and the path loss exponent, κ in equation (3.4) on page 13, is set to 2, which corresponds to free space. The noise power on a received signal, SNR^{-1} for unit transmitted power, on a given link is found using equation (3.5) on page 13.

7.2 End-to-End ARQ

The performance of S-DNF is evaluated for two different codes. One where the heaviest possible network coding is used, i.e. only the root packet in the binary tree is transmitted from the relay. This is referred to as Level 0, L0, coding. In the other code the children of the root are transmitted from the relay, which is referred to as Level 1, L1, coding. These two variants of S-DNF are compared to four existing schemes. Traditional Single Hop, TSH, Traditional Multi Hop, TMH, Decode-and-Forward, DF, and a scheme that applies regular DNF multiple times, once for each end node pair. This scheme is referred to as M-DNF. In the simulation the throughput is averaged over the distribution of 30 packets in the network. The results for the scenario with end-to-end ARQ are shown in Figure 7.1 where the throughput is plotted as a function of SNR on the direct link between two communicating nodes, γ. The results of the analysis performed in section 6.1, on page 37, is shown as dashed lines behind the simulation results.

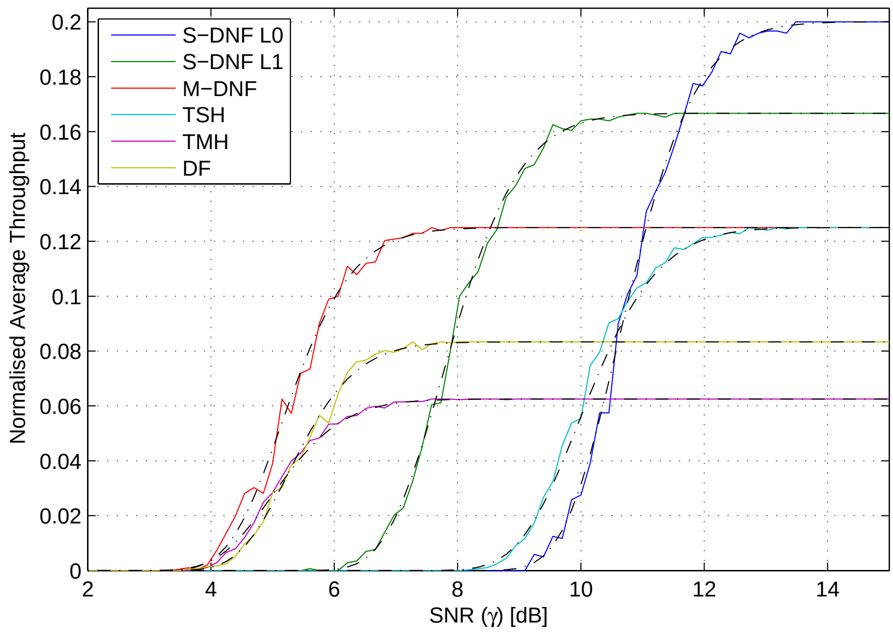


Figure 7.1: The performance of S-DNF, M-DNF, TSH, TMH and DF as a function of γ.

From Figure 7.1 it is seen that the simulated throughput follows the corresponding expected throughput tightly. This validates the derivation of the analytical expressions and the practical implementation of the simulation, where both are based on the utilised model. Moreover, the results show that the throughput of the different relay schemes starts to rise at different values of γ. This is because the different schemes depend on links of different length. The schemes M-DNF, TMH and DF only depend on the links between relay and end nodes. These links are relatively short compared to the direct link, hence the throughput starts to rise already at $\gamma =\sim 3$ dB.

S-DNF with L0 and L1 coding rely on overheard transmissions on links which are longer than the links between relay and end nodes. Hence, these schemes require a larger SNR before communication is possible. For S-DNF L0, a node requires to overhear all ANC packets in the network and therefore depends on links almost as long as the direct link. This means that using S-DNF L0 communication is not possible until $\gamma =\sim 9$ dB. Note that this is approximately 1 dB more than required by TSH, which may be surprising. However, in TSH a node only requires a single transmission in order to decode the packet. In S-DNF several transmissions are required, which yields a lower probability of decoding compared to TSH.

As the value of γ increases the throughput of the different schemes converge to their maximum throughput. The level of the maximum throughput depends on the number of transmissions required to distribute packets between all node pairs in the network. As expected the existing relaying schemes TMH and DF are the least efficient schemes and therefore achieve the two lowest throughputs. S-DNF L0 is the most efficient scheme, and reaches the highest throughput. When a lighter coding is used the relay broadcasts more parity packets and the maximum throughput decreases. In M-DNF the DNF scheme is used on each node pair, making M-DNF emulate half duplex communication. This is seen from the results, since M-DNF and TSH converges to the same throughput.

Scaling to Larger Networks

The ability of S-DNF to operate at different network sizes is evaluated in this section. The throughput of S-DNF L0, M-DNF and TMH is compared for networks of 4, 8 and 16 end nodes and a single common relay. It is desired to compare the maximum throughput in these scheme, hence $\gamma = 15$ dB is utilised and the resulting throughput is averaged over the distribution of 30 packets.

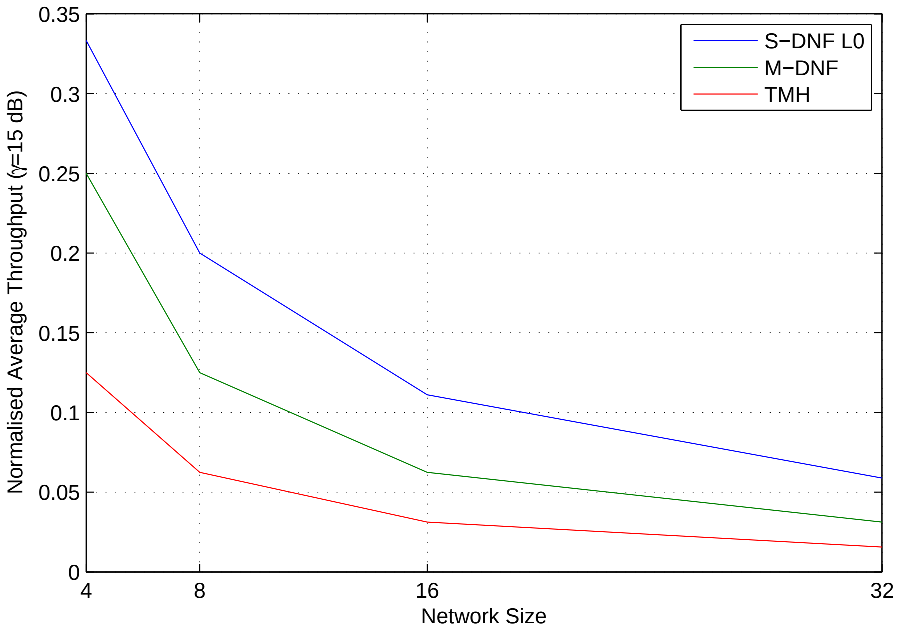


Figure 7.2: The simulated performance of S-DNF, M-DNF and TMH for an increasing network size and $\gamma = 15$ dB.

From Figure 7.2 it can be seen that the performance decreases for all schemes as the network size increases. However, the performance of S-DNF is superior for all network sizes. Moreover, the gain of S-DNF over M-DNF increases as the network size increases. This verifies the claim in section 4.3.4, on page 19, that G_{max} increases for increasing network size.

7.3 Link-Level ARQ

In the evaluation of S-DNF with link-level ARQ M-DNF and TMH are used as reference schemes. The results are shown in Figure 7.3 with the analysis from section 6.2 included as dashed lines. The simulated throughput is averaged over the distribution of 30 packets in the network.

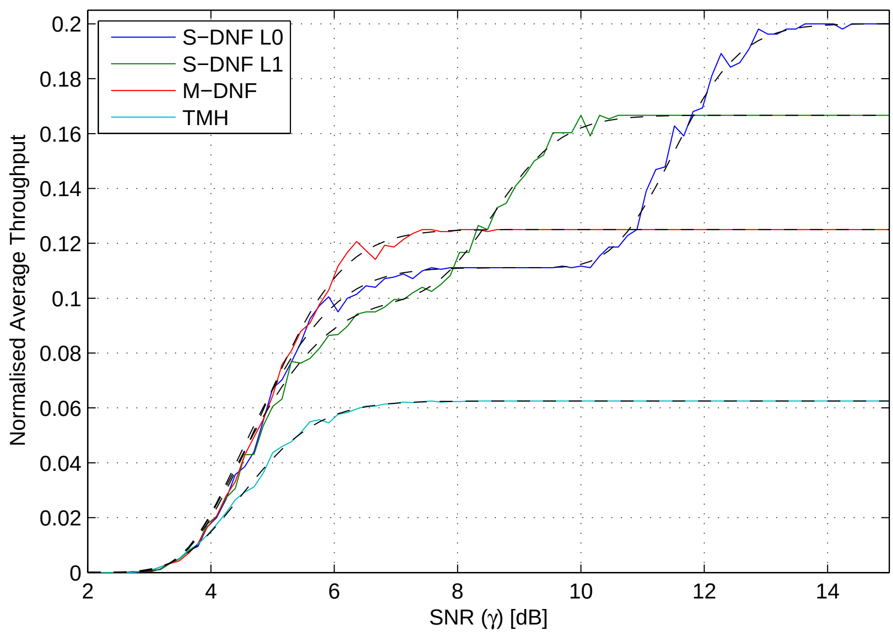


Figure 7.3: The performance of S-DNF, M-DNF and TMH, with link-level ARQ, as a function of γ.

From Figure 7.3 it is seen that all schemes have the initial rise in throughput at the same SNR. This is because all schemes rely on retransmission mode at low SNR, where ANC packets are transmitted from the relay. These packets require no overheard packets from the longer links in the network, hence a non-zero throughput is possible at low SNR even for the variants of S-DNF with heavy network coding at the relay. Another important observation in the figure is the step wise rises in throughput of the S-DNF schemes. This is due to the chosen retransmission mode, where the relay, regardless of the S-DNF code, transmits the needed ANC packets without any additional network coding. This simplifies the retransmission mode, but it also implements an adaptive mechanism, which makes the schemes tend to utilise M-DNF at lower SNR and their specific S-DNF code at higher SNR.

Figure 7.4 shows a comparison of the analysis for end-to-end ARQ and link-level ARQ. An important thing to note is that the common rise in throughput of the schemes using link-level ARQ happens at an SNR lower than the rise for M-DNF with end-to-end ARQ, although this scheme only relies on transmissions on the shortest links in the network. This is attributed to link-level ARQ, where errors do not cause the entire transmission flow to start over, as opposed to end-to-end ARQ.

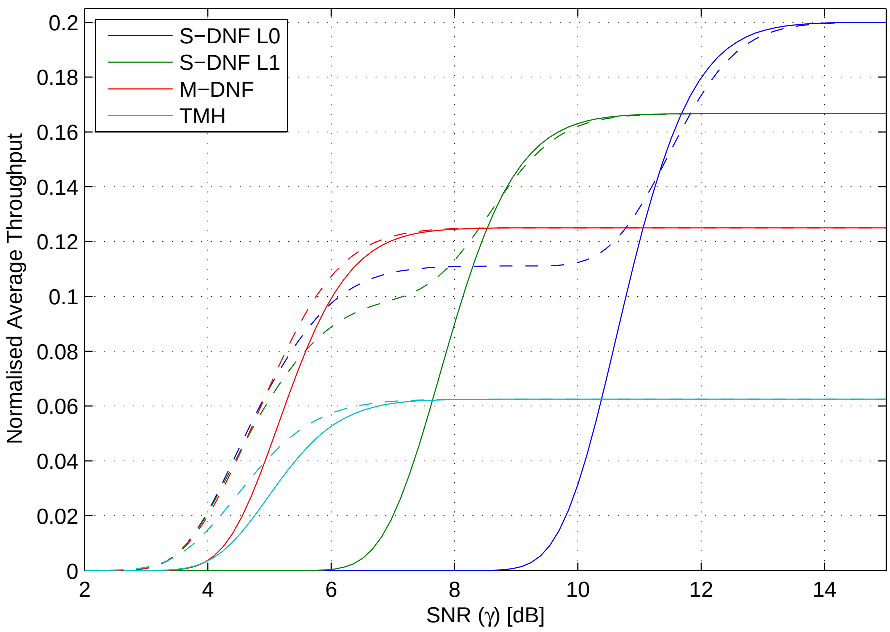


Figure 7.4: The analytical performance of S-DNF, M-DNF and TMH, with both end-to-end and link-level ARQ, as a function of γ.

7.4 Relay Generated Redundancy

In this evaluation redundancy is introduced at the relay as described in section 4.3.1. The evaluated schemes are named according to the utilised levels of network coding from the tree in Figure 6.2. The root packet is referred to as L0, its children L1 and the grand children, which is the ANC packets, is referred to as 'Level 2', L2. Three different variants of relay generated redundancy are evaluated. L0+L1, L0+L2 and L1+L2. They are compared to the basic variants of S-DNF evaluated in section 7.2. Note that S-DNF L2 corresponds to M-DNF in a network of eight end nodes, since M-DNF only broadcasts the ANC packets, which lies in level 2 in an eight node network. In Figure 7.5 both the analytical and simulated throughput is plotted as a function of γ.The expected throughput is plotted as dashed lines along with the corresponding simulated throughput, which is averaged over the distribution of 30 packets.

The results show that the redundancy introduced by the relay has no positive effect on the performance of the S-DNF scheme. When redundancy is added, more packets must be broadcast by the relay, which means the scheme converges to a lower throughput. This is seen if S-DNF L1 and S-DNF L0+L1 are compared. It is natural to assume that this added redundancy will make the scheme able to get information through at a lower SNR, however this is not the case. S-DNF L1 and S-DNF L0+L1 have the rise in throughput at approximately the same SNR. The same is the case if S-DNF L2, S-DNF L0+L2 and S-DNF L1+L2 are compared. Regardless of the redundancy, the rise in throughput is determined by the length of the longest link, which the scheme in question is dependent on. This link is determined by the lowest level of utilised coding. Hence the length of this link is common for schemes with

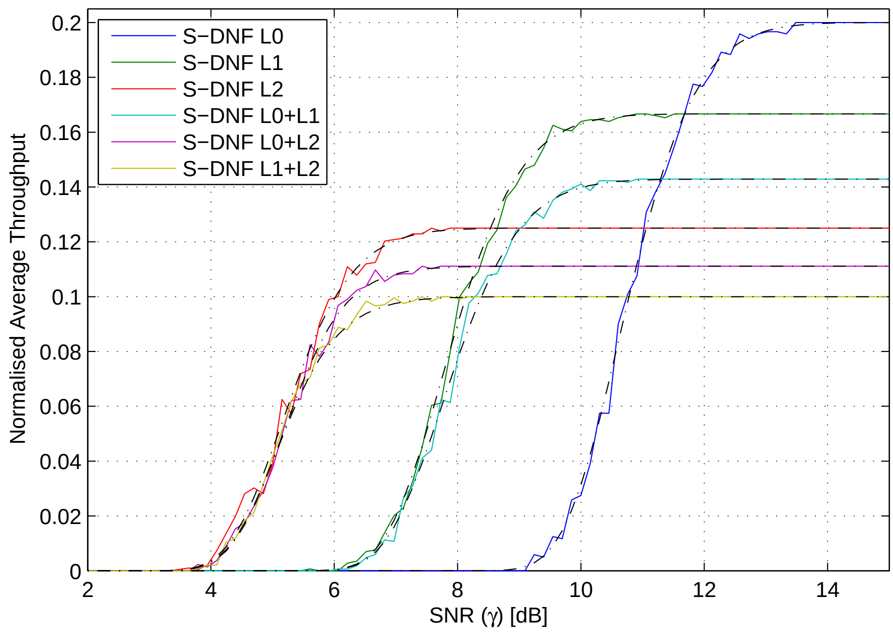


Figure 7.5: The performance of basic variants of S-DNF and variants with redundancy generated at the relay, as a function of γ.

common rise in throughput. It can thus be concluded that generating redundancy at the relay as proposed is not beneficial in the assumed scenario.

7.5 BFSK Modulation

In section 7.2 S-DNF was evaluated using BPSK modulation, which requires tracking of the phase at the receiver. The use of BFSK was investigated in section 5.2.1, on page 27. The BFSK receiver is relatively simple compared to the BPSK receiver, as it relys on envelope detection. However, the simplicity comes at the price of BER performance as illustrated in Figure 5.12, on page 34. In this section the normalised throughput using BFSK and BPSK is compared using the scenario described in section 7.1, where the throughput is averaged over the distribution of 30 packets in the network. In Figure 7.6 the performance of DF and S-DNF with different codes using BFSK is plotted and compared to the same schemes using BPSK.

It is seen that the relative performance of the different relay schemes is similar to the performance using BPSK. However, due to the inferior BER performance of BFSK the schemes require a larger SNR before transmissions are possible, where the effect is most significant in the DNF schemes. This is because DF only uses single link transmissions. The curves for S-DNF with different codes are shifted approximately 6.5 dB to the right for BFSK compared to the curves for BPSK. DF relies on single link transmissions, hence this curve is only shifted approximately 4 dB to the right.

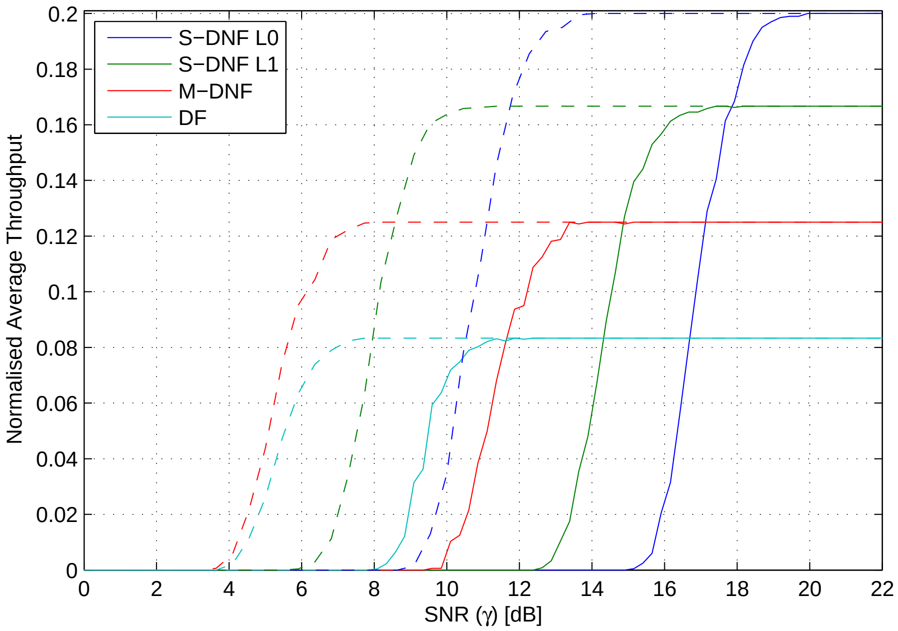


Figure 7.6: The normalised throughput of DF and S-DNF in a scenario using BFSK modulation, plotted with solid lines, is compared to with the same schemes using BPSK modulation, plotted with dashed lines.

7.6 Channel Model With Fading

The performance gain of the proposed relay protocol S-DNF over traditional relay protocols have been evaluated in section 7.2. Those results were based on a fairly simple model of the wireless channel, as the primary focus was the relative performance. In this section the performance of S-DNF is evaluated using a scenario with fading channels. This is done in order to assess the effect of fading in the proposed scheme.

7.6.1 Assumptions

The relay network is assumed to operate in an open environment where each transmission has a Line Of Sight (LOS) component available. This means that the channel experiences small scale fading. The amplitude of the received signal, v_r, is therefore Rician distributed and is implemented as a sum of the LOS component, v_{los}, and a Rayleigh distributed random variable, denoted v_{scat}, representing the total amplitude of the scatter components.

$$\begin{aligned} v_r &= v_{los} + v_{scat} \\ &= \sqrt{p_{los}} + R \cdot e^{j\theta_R} \end{aligned} \tag{7.1}$$

Where p_{los} is the power of the LOS component. R is the Rayleigh distributed amplitude of the scatter components, $R \sim \text{Rayleigh}(\sigma_R)$, and θ_R is the phase of the scatter component, uniformly distributed between 0 and 2π. The standard deviation, σ_R, of a Rayleigh distribution is given by: [Molish, 2006, 75]

$$\sigma_R = \frac{\mu_{scat}}{\sqrt{\frac{\pi}{2}}} \tag{7.2}$$

where μ_{scat} is the mean amplitude of the scatter components given by:

$$\mu_{scat} = \sqrt{\frac{p_{los}}{k}} \tag{7.3}$$

The Rice factor k is the ratio between the power in the LOS component and the total power in the scatter components. Normally k is in the range from 0 to 20. With this model for the fading channel the faded signal is demodulated using v_r in the simulation.

7.6.2 Results

The proposed relay scheme, S-DNF, is evaluated by comparing the normalised throughput of S-DNF and DF. The Rice factor, k, is set to 20 and the throughput is averaged over 50 packets. The results are plotted in Figure 7.7 as a function of the SNR on the direct link, γ, where γ ranges from 0 to 40 dB.

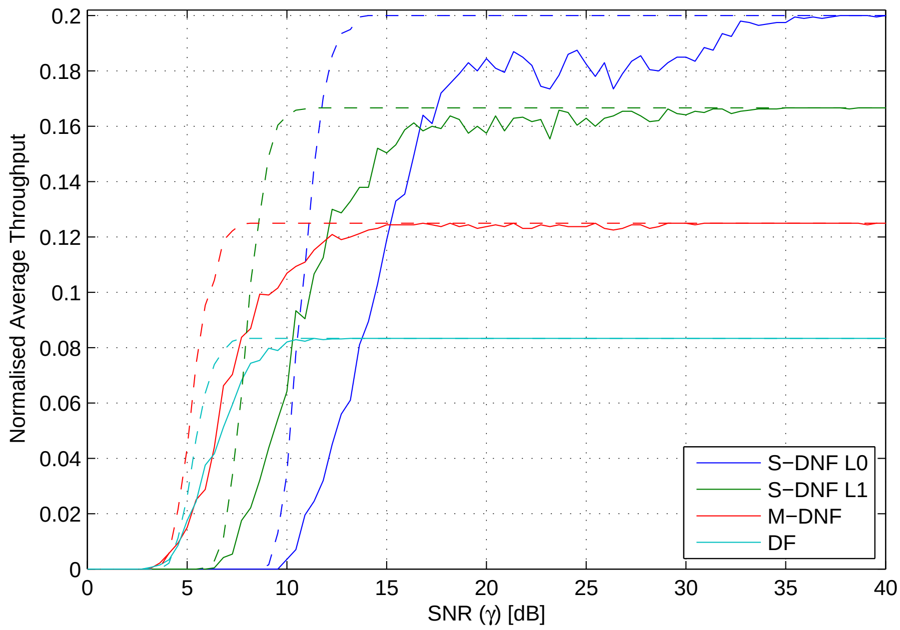


Figure 7.7: Normalised throughput for a scenario with fading channels plotted with solid lines compared to a scenario without fading plotted with dashed lines.

The results show that the throughput of the schemes with and without fading channels starts to rise at the same value of γ. However, the fading channels have a significant impact on the transition before reaching the maximum throughput since the slope is more gentle. This is due to the fact that the fading channels introduce an additional uncertainty to the received signal, hence the task of demodulating becomes more challenging. Moreover, the uncertainty causes the resulting throughput to ripple more compared to the non-fading case even though the throughput is averaged over 50 packets in both cases.

For the schemes where an overheard transmission is required, i.e. S-DNF L0 and S-DNF L1, the fading channels cause the curves to flatten out and not reach their maximum throughput before a γ of approximately 35 dB. When a node overhears a joint transmission the amplitudes of the signal components can be very different, which increases the error probability in the denoise operation. In some unfortunate cases the probabilistic nature of the fading component causes an even larger difference in amplitude, hence the fading increases the error probability of overheard transmissions. This has the largest impact on S-DNF L0, as it requires most overheard transmissions. As the SNR increases the impact of the fading decreases, and at a large γ the schemes reach their maximum throughput. This means that the performance gain when using S-DNF over traditional relay schemes, such as DF, is also present when simulating a more realistic scenario, although a larger SNR is required.

Chapter 8

Tornado Codes From End Nodes

The use of tornado codes on the individial data streams in the network is investigated in this chapter. A tornado code provides an increased error resilience and potentially an increase in throughput at lower SNR. Initially a short description of the principle of tornado coding is given. After this, analysis of the S-DNF scheme when utilising the proposed tornado code is performed, and finally evaluation results are presented.

8.1 The Principle of Tornado Codes

Tornado coding is a systematic erasure code used to increase reliability in a transmission. The idea of tornado coding is to create a number of redundant packets (parity blocks), by XOR combining a number of packets from the source (input blocks). The simplest example of a tornado code is the combination $x_1 \oplus x_2$ of packets x_1 and x_2, i.e. the input blocks are x_1 and x_2 and the parity block is $x_1 \oplus x_2$. The input blocks and parity blocks are referred to as a packet set. When these three packets are transmitted, the receiver will be able to correct a single error with the parity block, since x_1 can be reconstructed from $x_1 \oplus x_2$ if x_2 is succesfully received, and vice versa. This means that both information packets intended for the receiver are decodable if any two out of the three transmitted packets are received succesfully. Hence, this example is a (3,2) Tornado code. Other more reliable codes can be used, where a higher amount of parity blocks are generated. Moreover, a higher amount of input blocks can be supported in a code. In general (n,k) Tornado codes are used, with coding rate $\frac{k}{n}$, all following the same principle.

8.2 Tornado Codes In S-DNF

When utilising tornado codes in S-DNF each end node applies the same code when communicating with the other node in the node pair. This section describes the implementation of tornado codes in S-DNF using the simple (3,2) tornado code as an example. Before describing how this code is implemented, it is desired to analyse the expected number of packets received, $E[n_p]$.

8.2.1 Analysis of A Simple Tornado Code

The simple (3,2) tornado code uses three time slots and in each time slots there exist a probability, P_s, that the packet is received correctly. This gives eight different outcomes which are listed in table 8.1 along with the probability of receiving one, two or no packets, respectively. A '1' and '0' refers to the event that the packet was received correctly or discarded, respectively.

$\|x_1\|$	$\|x_2\|$	$\|x_1 \oplus x_2\|$	n_p	Event Probability
0	0	0	0	$(1-P_s)^2$
0	0	1	0	
0	1	0	1	$2 \cdot (1-P_s)^2 \cdot P_s$
1	0	0	1	
1	0	1	2	$2 \cdot (1-P_s) \cdot P_s^2$
0	1	1	2	
1	1	0	2	P_s^2
1	1	1	2	

Table 8.1: The possible outcomes after transmitting a set of packets using the simple tornadocode.

From table 8.1 the expected number of received packets is given by:

$$\begin{aligned} E[n_p] &= 0 \cdot (1-P_s)^2 + 1 \cdot \left(2 \cdot (1-P_s)^2 \cdot P_s\right) + 2 \cdot \left(2 \cdot (1-P_s) \cdot P_s^2 + P_s^2\right) \\ &= 2P_s \cdot (1 + P_s - P_s^2) \end{aligned} \tag{8.1}$$

In order to evaluate the performance of the utilised tornado code the expected number of received packets is compared to the simplest possible transmission scheme, i.e. where the packets are transmitted unreliably one by one, without any coding. The expected number of received packets in three time slots is found using the same approach as for the tornado code. In table 8.2 the different outcomes are listed.

$\|x_1\|$	$\|x_2\|$	$\|x_3\|$	n_p	Event Probability
0	0	0	0	$(1-P_s)^3$
0	0	1	1	$3 \cdot (1-P_s)^2 \cdot P_s$
0	1	0	1	
1	0	0	1	
1	0	1	2	$3 \cdot (1-P_s) \cdot P_s^2$
0	1	1	2	
1	1	0	2	
1	1	1	3	P_s^3

Table 8.2: The possible outcomes after transmitting a set of packets using the simple unreliable transmission scheme.

The expected number of received packets is then given by:

$$E[n_p] = 0 \cdot (1-P_s)^3 + 1 \cdot \left(3 \cdot (1-P_s)^2 \cdot P_s\right) + 2 \cdot \left(3 \cdot (1-P_s) \cdot P_s^2\right) + 3 \cdot P_s^3$$
$$= 3P_s \tag{8.2}$$

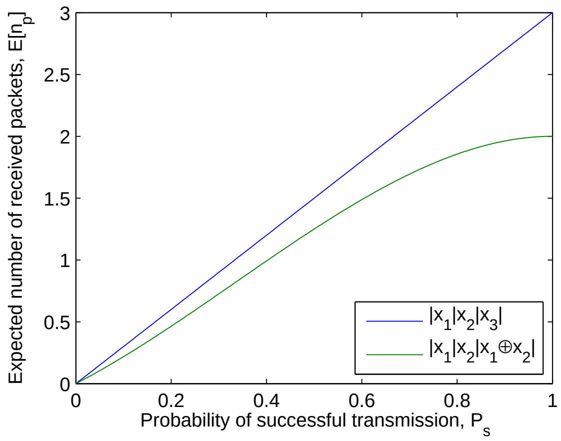


Figure 8.1: The expected number of received packets plotted as a function of P_s for an unreliable retransmission scheme and the simple tornado code.

In Figure 8.1 equation (8.1) and (8.2) are plotted as a function of P_s. This figure shows that there is no value of P_s where the simple tornado code yields an improvement in performance over the simple unreliable transmission scheme, not even for the smallest P_s which may be counter intuitive. The added redundancy decreases the resulting maximum throughput, however, it also increases the reliability. Hence, instead of comparing the performance of the simple tornado code with an unreliable transmission scheme, it should be compared with a reliable scheme, e.g. Stop-and-wait ARQ.

8.2.2 Reliability In S-DNF

In order to make S-DNF a reliable relaying scheme any packet not received correctly must be retransmitted. If the two first packets in the tornado code are received erroneously the entire set is retransmitted. However, if a single packet error occurs, the scheme should keep retransmitting the coded packet only, until both packets can be decoded correctly. In this way the redundancy in the tornado code will reduce the number of required retransmissions compared to Stop-and-wait ARQ without channel coding.

To identify which packets to retransmit the system implements control messages for ACKing or NACKing a transmitted packet. It is assumed that the NACK issued by the receiver holds information about how many of the two information packets have been lost. Since this scenario considers bidirectional relaying these control messages can be piggybacked on the data packets. This means that the only additional time required is for the retransmission of data packets. In the following the reliable scheme is analysed and compared to stop-and-wait ARQ without any source coding.

8.3 Analysis of Tornado Code in Reliable Scheme

Given the retransmission scheme described in section 8.2.2, the transmission can be in a state where none of the input blocks have been received, hence three packets are being transmitted in every attempt. The second possible state is where a single input block has been received, hence the parity block is transmitted in every attempt. The third state is where both packets have been decoded successfully and no retransmissions are requested. This three state model is illustrated in Figure 8.2. The names of the states indicate the number of successfully decoded information packets. Hence, the initial state of a transmission is state 0 and the final state is state 2.

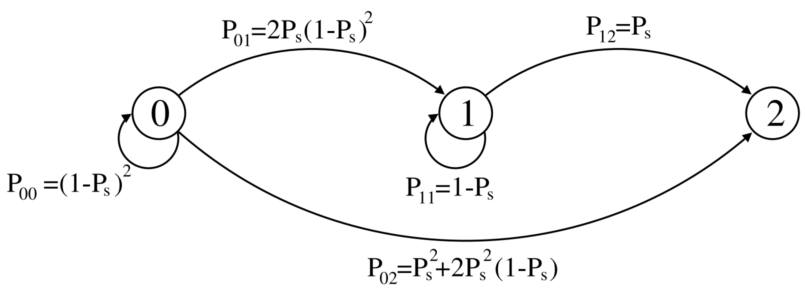


Figure 8.2: A state model of a reliable transmission of a (3,2) Tornado code.

In Figure 8.2 the state transition probabilities are also presented, where P_s is the probability of a successful transmission of a single packet. The state transition probabilities are found in Table 8.1. The aim of the analysis is to derive an expression of the expected number of necessary transmissions to go from state 0 to state 2, $E[M]$. This expression is the sum of two contributions. The expected number of transmissions spent in state 0, $E[M_0]$, and the expected number of transmissions spent in state 1, $E[M_1]$.

Both $E[M_0]$ and $E[M_1]$ can be expressed using the formula for the expected number of trials in a repeated experiment with a given success probability. In the case of $E[M_1]$, however, it is necessary to weight this contribution with the probability of having to enter state 1 before reaching state 2, i.e. this is the probability of state 1 being the destination state rather than state 2, given that the transmission is leaving state 0. This probability is denoted P_{01c} and is expressed as follows.

$$P_{01c} = \frac{P_{01}}{P_{01} + P_{02}} = \frac{2p(1-p)^2}{p^2 + 2p^2(1-p) + 2p(1-p)^2} = \frac{2(1-p)^2}{2-p} \tag{8.3}$$

Instead of just multiplying with the summation variable as in the traditional formula for the expected number of trials, it is necessary to let the multiplication reflect the number transmissions spent per trial. In the case of $E[M_0]$, this number is 3. For $E[M_1]$ the number is 1. Hence, $E[M]$ can be derived as follows.

$$E[M_0] = \sum_{k=1}^{\infty} (1-p)^{2(k-1)} \cdot (1-(1-p)^2) \cdot 3k$$
$$E[M_1] = P_{01c} \sum_{l=1}^{\infty} (1-p)^{(l-1)} \cdot p \cdot l$$
$$E[M] = E[M_0] + E[M_1] \tag{8.4}$$

8.4 Results

This evaluation uses the setup described in section 7.1, on page 45. It is desired to compare the reliable S-DNF scheme with and without redundancy from the end nodes.

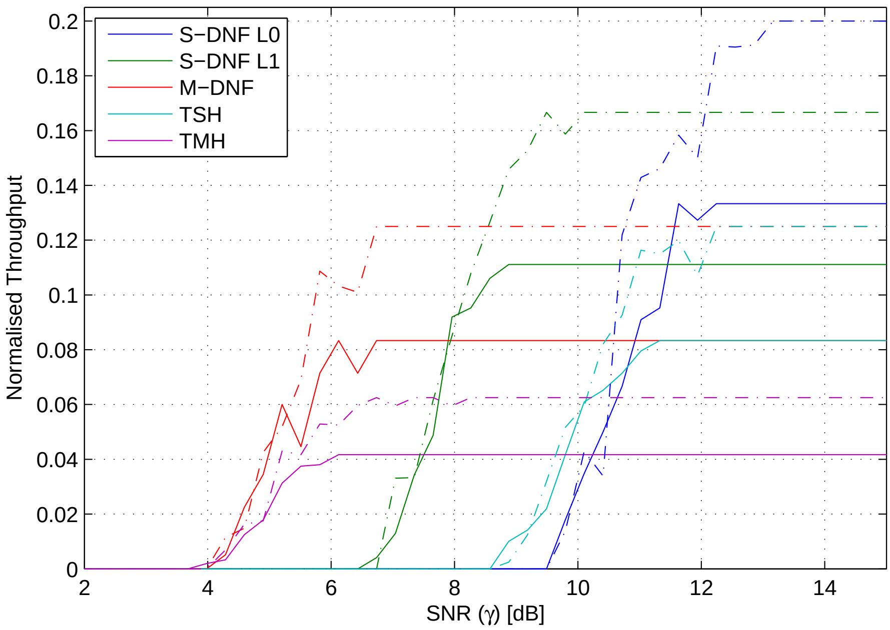


Figure 8.3: The performance with redundancy (solid lines) compared to the performance without redundancy (dashed lines).

Each node is set up to transmit 21 packets, which corresponds to seven complete packet sets using the simple tornado code described in section 8.2. The resulting throughput is then calculated from the number of time slots required to successfully transmit these 21 packets. Let γ denote the SNR on the direct link and thereby the noise power in the network. The normalised throughput is plotted as a function of γ in Figure 8.3, where γ ranges from 0 to 15 dB.

The throughput of the different schemes starts to rise at the same value of γ regardless of the introduced redundancy, but when the tornado code is utilised the throughput of each scheme converges to a 33 % lower value, as every third packet only carries redundant information. This corresponds to the results from the analysis in section 8.2.1, where no trade-off between redundancy and error probability were identified. This means that even at low SNR the increased error resillience is unable to neutralise the impact of transmitting the redundant packets.

8.5 Conclusion

In this chapter the use of tornado codes at the end nodes has been investigated. As an example a simple tornado code was implemented were packet x_1 and x_2 are non-coded data packets, and the third packet is the XOR of x_1 and x_2. Analysis and simulations show that it is not possible to benefit from this type of code, since the added reliability is neutralised by the amount of redundant data.

Chapter 9

Scaling of Constellation Size in S-DNF

In this chapter the scaling of the constellation size for the applied modulation scheme is considered with respect to DNF. So far only binary schemes have been used, but in order to increase throughput M-ary schemes are investigated.

9.1 Introduction

It is well known that increasing the cardinality of a modulation scheme in regular single link transmissions increases the bit rate at the price of a lower BER performance, see section 2.2. What is optimal in a given scenario depends on the SNR of the system. Such a trade-off is also expected to exist in joint transmissions where denoise demodulation is used. This trade-off is investigated for ASK modulation in this chapter. ASK has been chosen because it entails the simplest decision region analysis, where the signal space diagram is one-dimensional. In this investigation focus is on the denoise operation, hence the simple three node relaying scenario, described in section 2.3, is considered. When focusing on the denoise operation it is reasonable to assume that the links between the end nodes and the relay have same channel coefficients. This means the signal components from the two end nodes arrive at the relay with same mean energy per bit.

9.2 The Denoise Operation in M-ASK

As in BPSK, the signals are added as scalars when using M-ASK during joint transmissions. As an example consider 4ASK, where the signal space is illustrated in Figure 9.1. The possible symbols are numbered from 0 to 3 and the bits to symbol mapping is performed using gray coding. The addition of two such signals yields 7 possible output symbols. The resulting signal space is illustrated in Figure 9.2.

The symbols in Figure 9.2 are mapped back to the signal space in Figure 9.1, analogous to the mapping in BPSK, where -2, 0 and $+2$ are mapped to -1 and $+1$. This mapping is performed using the procedure described in section 4.3.5. This

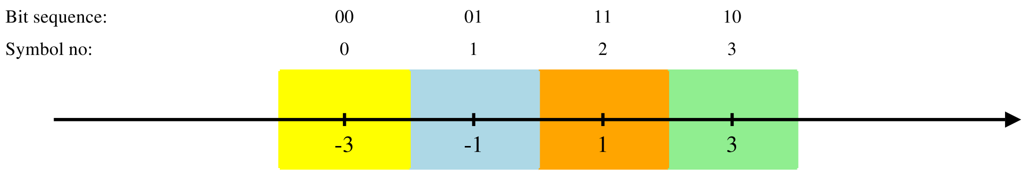


Figure 9.1: An example of a signal space in 4ASK of a single transmitter.

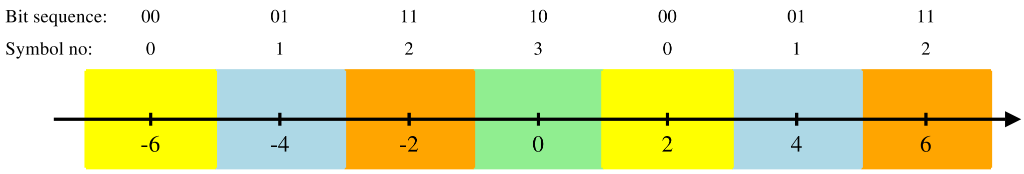


Figure 9.2: The resulting signal space, when two 4ASK signals are added.

procedure states that the sum of the $i-$th and $j-$th symbols should be represented by symbol k, where k is the modulus M sum of i and j and M is the cardinality of the constellation. In the example of 4ASK, this gives a mapping as indicated by the colors in Figures 9.1 and 9.2.

9.3 Decision Regions Bounds

In the analysis of decision regions for the denoise operation in BPSK modulation a solution to the location of a decision region bound between two symbols was found, see section 5.1 on page 23. The solution was found to be the intersecting point between the conditional pdfs of the received signal, conditioned that the symbols in question were transmitted. Here it is important to note that the conditional pdf corresponding to -2 also contributed to the conditional pdf of $+2$ in the solution of the bound between 0 and $+2$ and vice versa, because -2 and $+2$ results in the same decision. The same procedure is used for the analysis of larger constellations in ASK.

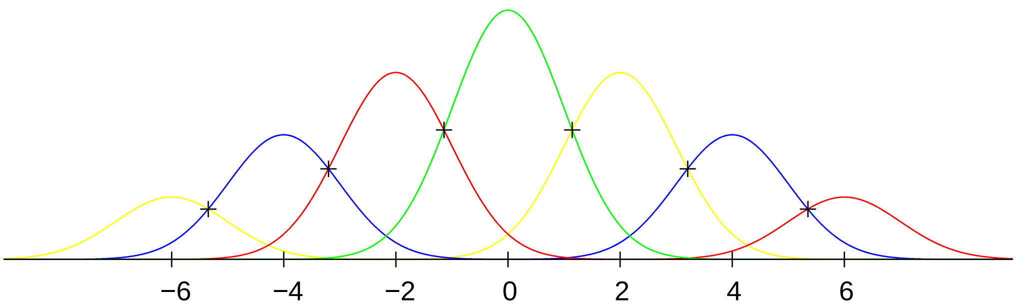


Figure 9.3: The conditional pdfs for the possible symbols in 4ASK.

In Figure 9.3 the conditional pdfs for 4ASK is shown. The difference in height is because the different possible symbols can be created by a different number of combinations. As examples −6 can only be created by (−3, −3) whereas 0 is the result of (−3, +3), (−1, +1), (+1, −1) and (+3, −3). Using the described procedure, the bounds of the decision regions are found. These are indicated by the crosses in Figure 9.3.

9.4 BER Analysis

In this section the BER performance of joint transmissions in M-ASK is analysed and compared to that of regular single link transmissions. In this analysis it is necessary to take into account that one symbol error might contain more than one bit error. As an example consider the symbol −6 in a joint transmission with 4ASK, which is erroneously detected as a −2. This results in two bit errors instead of only one if it had been detected as −4. The severity of an error is indicated by the Hamming weight, which is defined as the ratio between erroneous bits and the total number of bits in a symbol. This means that a −6 detected as a −2 has Hamming weight 1 and a −6 detected as a −4 has Hamming weight $\frac{1}{2}$.

The probability of a symbol with amplitude a being detected as a symbol with amplitude b is denoted $P(X_r = b|X_t = a)$ and the corresponding Hamming weight is denoted ω_{ab}. The lower and upper decision region bounds for symbol b are denoted Λ_{bl} and Λ_{bu} respectively. Note that Λ_{bl} for the symbol with lowest amplitude is $-\infty$ and Λ_{bu} for the symbol with highest amplitude is ∞. $P(X_r = b|X_t = a)$ is thus.

$$P(X_r = b|X_t = a) = \int_{\Lambda_{bl}}^{\Lambda_{bu}} f_a(x)dx \tag{9.1}$$

Where $f_a(x)$ is the conditional pdf of the received signal, given a was transmitted. The BER can be calculated by summing the probabilities of all possible errors, weighted by the corresponding Hamming weights. The BER for joint transmissions with 4ASK is thus.

$$P_{bj,4ASK} = \sum_{a=-6}^{6} \sum_{b=-6}^{6} \omega_{ab} \int_{\Lambda_{bl}}^{\Lambda_{bu}} f_a(x)dx \tag{9.2}$$

Where a and $b \in [-6, -4, -2, 0, 2, 4, 6]$.

In single link transmissions, the same equation applies, only with a different set of possible amplitudes.

$$P_{bs,4ASK} = \sum_{a=-3}^{3} \sum_{b=-3}^{3} \omega_{ab} \int_{\Lambda_{bl}}^{\Lambda_{bu}} f_a(x)dx \tag{9.3}$$

Where a and $b \in [-3, -1, 1, 3]$.

Figure 9.4 shows a comparison of the BER performances of different modulation cardinalities for both denoise demodulation and regular single link demodulation.

This figure shows that the very low performance degradation, when going from single link to DNF, experienced in the binary denoise scheme is maintained at higher cardinalities. This indicates that the proposed procedure for scaling of DNF with respect to constellation size is very efficient. It is possible to scale the concept of DNF to higher cardinalities, at approximately the same expense in BER performance as in single link transmissions.

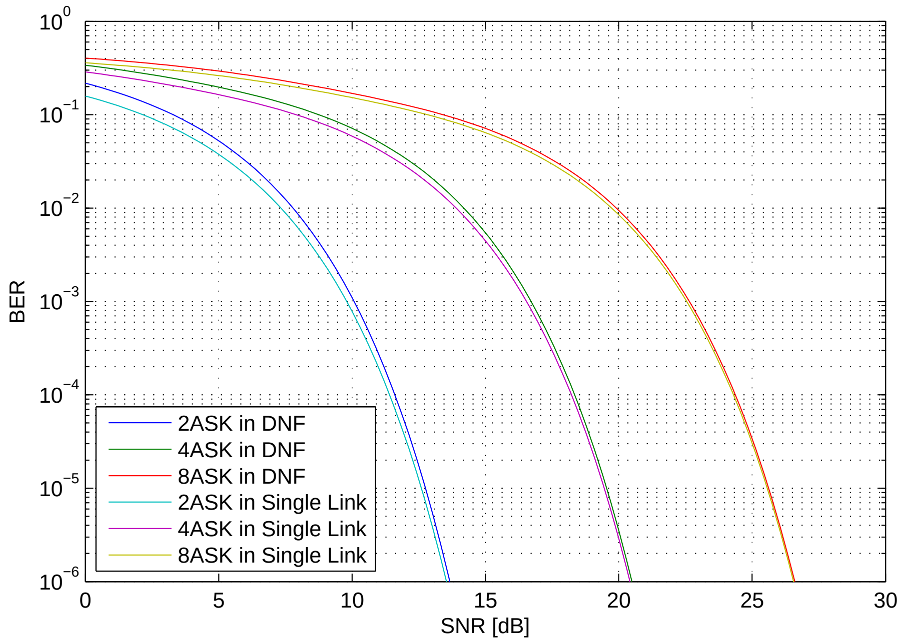


Figure 9.4: The BER performance of different modulation cardinalities in DNF and single link transmissions.

9.5 Results

With the new option to scale DNF to higher modulation cardinalities, it is now interesting to compare different cardinalities with respect to throughput. It is with this performance measure the trade-off between bit rate and reliability becomes evident. The throughput is evaluated in simulations of the three node scenario with 2ASK, 4ASK and 8ASK. The packet length is 1200 bits and 20 packets are transmitted in the simulation. The results of the simulation along with corresponding analysis results are plotted in Figure 9.5 using colored lines and black dotted lines respectively.

The figure shows the expected trade-off between bit rate and error resilience. At low SNR the performance of 2ASK is superior, since it is the only scheme which is able to get any information through the channel. Hence the throughput of both DNF and DF with 2ASK starts to rise at an SNR of $\sim$ 10 dB. At an SNR of approximately 6 dB higher, schemes utilising 4ASK is also able to communicate, and because it has 2 bits per symbol, it converges to an expected throughput which

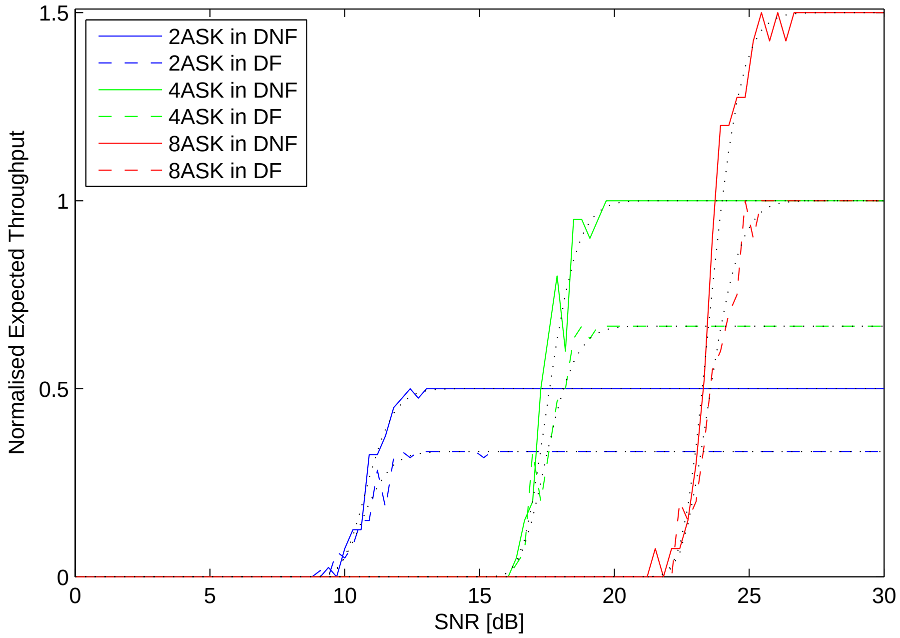


Figure 9.5: Simulation and analysis results of DNF and DF with 2ASK, 4ASK and 8ASK.

is twice as high. At high SNR schemes using 8ASK achieves the highest throughput, since each symbol represents the highest number of bits.

9.6 Conclusion

A procedure for scaling DNF with respect to the cardinality of the modulation scheme has been proposed. Results show that increasing the cardinality using the proposed procedure yields a BER performance decrease similar to the one experienced in DF where single link transmissions are used. This shows that an efficient procedure for scaling DNF to larger modulation schemes has been proposed. Moreover, the expected trade-off between bit rate and reliability has been shown to exist in DNF, as it is the case in single link transmissions.

Chapter 10

High Order Constellations and FEC

In chapter 9 the size of the constellation was increased from binary to M-ary in order to increase the capacity of the relay scheme. However, packing more information into each symbol made the scheme more susceptible to errors. In chapter 8 redundancy was introduced by utilising a tornado code in order to increase the reliability of DNF, but this decreased the maximum throughput of the system as well. In this chapter higher constellations are combined with redundancy in the form of an FEC appended to each packet. In this way the redundancy is introduced at bit level instead of packet level as it is the case when tornado codes are utilised. These two concepts have both advantages and drawbacks regarding the relay scheme and their basis for improvement is very different. At low SNR, increased reliability by introducing redundancy will increase the throughput, where an increased signal constellation will decrease the throughput. At high SNR the opposite applies. The combination of higher constellation sizes and FEC has a well known effect in single link transmissions, and in this chapter the effect in DNF is investigated.

10.1 Introduction

In chapter 8 redundancy was introduced at packet level using a (3,2) tornado code. Results showed that this type of redundancy had no positive effect on the performance with respect to expected throughput. The reason for this is the fact that the redundancy operates at packet level. Even though the applied tornado code has a code rate of $\frac{2}{3}$, which means only $\frac{2}{3}$ of the data is necessary in order to decode, the individual packets are very vulnerable to errors. A single bit error makes an entire packet useless, which means that worst case only two bit errors are enough to make the receiver unable to decode. Redundancy at bit level, however, does not have this high vulnerability. A certain amount of redundancy makes the transmission able to cope with a certain amount of bit errors, no matter how they are distributed in the packet. Therefore, the redundancy considered in this chapter is introduced at bit-level.

In chapter 9, DNF with larger constellations were investigated using M-ASK. This chapter reuses the outcome of this work, hence for details on deriving the decision regions and the general analysis for M-ASK chapter 9, on page 61, should be consulted.

10.2 Assumptions

There exist different codes for generating an FEC. However, when simulating the performance of the scheme with an FEC for each packet the actual FEC is not generated, only its properties are taken into account. It is assumed that the FEC is generated using a perfect (n, k) error correction code, where $k = \frac{2}{3} \cdot n$. This means that a packet contains one third redundant symbols enabling the receiver to recover the packet when up to one sixth of the packet is erroneous. [Haykin, 2001, 654]

As described in section 9.2 gray coding is applied when ordering the symbols in the constellation. This means that the hamming distance between two neighbouring symbols is one. Moreover it is assumed that a symbol error causes a symbol to be interpreted as one of its neighbouring symbols, hence a symbol error only causes one bit error.

In this chapter the three node relay network is utilised. This is the simplest possible relay network, hence any impact on performance can be ascribed to the introduced FEC and M-ary constellations.

10.3 Analysis

For evaluating the expected throughput in DNF with FEC and M-ary constellation, this section derives the necessary expressions. The normalised expected throughput, $E[t]$, experienced by a node in the three node relay network is given by:

$$E[t] = P_f \cdot \frac{k}{n_b} \tag{10.1}$$

where the fraction $\frac{k}{n_b}$ is the ratio between the number of data bits, k, in a packet and the complete packet length in bits, n_b. P_f is the probability that an end node receives the packet from the other end node with no more errors than can be corrected by the FEC.

In order to determine P_f it is necessary to consider the SER for joint and single transmissions denoted P_{yj} and P_{ys} respectively. These probabilities are found using equation (9.2) and (9.3) respectively, on page 63. In these equations the Hamming weight is set to $\omega_{ab} = \frac{1}{log_2(M)}$. This is due to the assumption that all symbol errors contain only one bit error, making the SER equivalent to the BER. For a given node in DNF to receive a symbol correctly a successful joint transmission to the relay followed by a successful single transmission is required. Hence the total probability of successfully receiving a symbol from the other end node is given by:

$$P_y = (1 - P_{yj}) \cdot (1 - P_{ys}) \tag{10.2}$$

The FEC can correct $\frac{n_r}{2}$ bit errors, where n_r is the number of redundant bits, $n_r = n_b - k$. The modulation converts the packet to a sequence of symbols, where the length of this sequence, n_s, depends on the modulation scheme, hence the more bits each symbol represent the shorter sequence of symbols. Assuming that a symbol error at the most causes a single bit error no more than $\frac{n_r}{2}$ symbols out of n_s can be erroneous in order to receive the packet correctly. Let P_i denote the probability of experiencing i symbol errors. Then P_f is the sum of the P_i's where $i \in \left\{0; \frac{n_r}{2}\right\}$:

$$\begin{aligned} P_f &= \sum_{i=0}^{\frac{n_r}{2}} P_i \\ &= \sum_{i=0}^{\frac{n_r}{2}} \binom{n_s}{i} (1 - P_y)^i \cdot P_y^{n_s - i} \end{aligned} \tag{10.3}$$

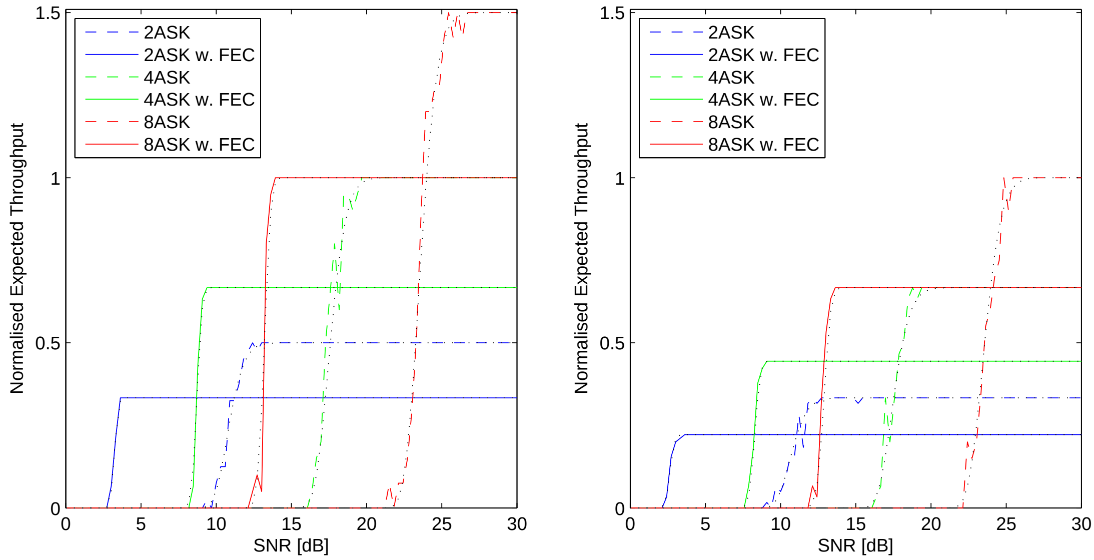
Substituting equation (10.3) into equation (10.1) gives the resulting expression for the expected normalised throughput. This is used for evaluating the performance of DNF when combining bit-level FEC and high order modulation.

10.4 Results

The combination of an M-ary modulation scheme and FEC is evaluated using the expected normalised throughput in equation (10.1) and a simulation of the three node relay network. Three different constellation sizes are used for M-ASK, hence $M \in \{2,\ 4,\ 8\}$. The packets contain random data, but are assumed to be generated using a perfect (1200,800) error correction code, which yields an integer number of symbols for any of the applied constellation sizes. Such a packet contains 400 redundant bits, hence with the assumed relation between bit errors and symbol errors the FEC can correct 200 symbol errors. The simulated throughput is averaged over the exchange of 20 packets. In Figure 10.1 the performance with and without FEC is compared for 2ASK, 4ASK and 8ASK. The normalised throughput is plotted as a function of the SNR on each link. The dotted lines indicate the expected throughput calculated using equation (10.1) and the lines are the throughput from the simulation.

From Figure 10.1 the known trade-off between error resilience and bit rate is evident. For the schemes with FEC the throughput starts to rise at an SNR 7 to 9 dB lower than the non-coded transmissions. Moreover, the maximum throughput is 33 % lower than when no coding is applied. However, it should be noted that by combining FEC and larger constellation sizes the error resilience is improved without compromising the maximum throughput.

Moreover, when comparing Figure 10.1(a) and Figure 10.1(b) it can be seen that coded DNF with M-ASK modulation achieves the same maximum throughput as



(a) DNF (b) DF

Figure 10.1: The performance of relay schemes with FEC (solid lines) compared to relay schemes without FEC (dashed lines) using different constellations.

non-coded DF with M-ASK and offers the same error resilience as coded DF with M-ASK. This is the case for all simulated values of $M = 2, 4, 8$. This means that the error resilience of DNF can be improved by introducing bit-level redundancy at the price of a decrease in maximum achievable throughput. However, DNF will always perform as well as, or better than, DF.

10.5 Conclusion

When increasing the constellation size the throughput increases at high SNR at the price of lower throughput at low SNR. When introducing FEC the opposite applies. The results in this chapter reveals a synergy when combining the two concepts. Introducing FEC to DNF along with a larger constellation size can improve the error resilience without compromising the level of maximum throughput. Compared to DF, coded DNF always performs as well as, or better than, DF.

Chapter 11

Conclusion

With interference as the main limitation in wireless networks this project sought to exploit the interference constructively instead of seeking to avoid it. The relaying scheme DeNoise-and-Forward, DNF, presented in [Popovski and Yomo, 2007], was used as a starting point in the investigations throughout the preceding chapters. This chapter draws the final conclusions on this work and identifies interesting areas for future work.

The concept of DNF was scaled in order to be applied in relay networks with n end nodes, where $n > 2$. This scaling resulted in a new scheme called Scalable DNF (S-DNF) that combines analog network coding with digital network coding at the relay. When the heaviest possible coding is used at the relay each node is required to overhear all transmissions, which makes the proposed scheme more vulnerable to errors. This vulnerability can be decreased by decreasing the coding level at the relay. Through profound analysis and simulations, implementing both end-to-end and link-level error detection, the concept of S-DNF was evaluated. At sufficiently high SNR, S-DNF offers a maximum gain of 2 over traditional DNF as n approaches infinity and the heaviest possible coding is utilised at the relay. Hence, it can be concluded that this project has succeeded in scaling DNF with respect to network size. Moreover, it can be concluded that the developed scheme offers a mechanism for selecting a reasonable trade-off between error resilience and throughput.

In [Popovski and Yomo, 2007] DNF was presented using the binary coherent modulation scheme, BPSK. In order to make the concept of DNF more applicable a denoise operation was developed for the non-coherent modulation scheme, BFSK. Using binary modulation schemes has limitations with respect to achievable throughput, hence in order to increase the bit rate the denoise operation was scaled to operate with an M-ary modulation scheme, where ASK was utilised as an example. Simulations and analysis showed that the proposed procedure for scaling to larger constellations offers the increase in bit rate at approximately the same expense in BER performance as in single link transmissions. It is therefore concluded that a very efficient scaling of DNF with respect to constellation size has been developed.

In order to decrease the vulnerability to errors the use of redundancy was investigated. Redundancy was introduced at two different levels: At packet level by utilising a simple tornado code and at bit level by appending an FEC code to each packet. From analysis it was concluded that only redundancy introduced at bit level will provide the desired trade-off, where the error resilience is improved at the price of a decreased maximum throughput. Moreover, when combining redundancy at bit level with an M-ary modulation scheme it was possible to improve error resilience without compromising the maximum achievable throughput.

This concludes the outcome of this project, where the work on DNF has been taken a step further. The main contributions of this work is the analytical work and simulation of DNF with respect to network size, modulation scheme and constellation size in order to extend the applicability of DNF. This means that DNF is no longer confined to the simple three node scenario and can be applied in more realistic scenarios with respect to network size, error control and modulation scheme. Moreover, the work in this project can be used as a basis for further scaling of DNF. In this way the initial work on exploiting the interference has been scaled to more realistic settings with an option to set various parameters, such as the code at the relay, in order to have the most suitable relay scheme for a given network.

11.1 Future Work

The results achieved in this project is by no means exhaustive for the area of DNF, and the work has revealed areas that would be interesting to look into for future work. These areas are briefly described in this section.

First of all, the focus of this project has been to investigate different possibilities and ideas, hence fairly simple models have been utilised. It would therefore be interesting to utilise more realistic models of the wireless medium in order to validate the results.

Secondly, it has been stressed that the relay should use the same modulation scheme as the end nodes. Hence when M-ary modulation is applied, the denoise operation should compress the possible outcomes, when two symbols are analog coded, into the M symbols in the signal constellation. It would be interesting to investigate the possibility of letting the relay use an M'-ary modulation scheme, where $M' > M$. This makes it possible to consider other mappings from received analog coded symbols to transmitted symbols at the relay. Under some network conditions this might be advantageous, since an other mapping might provide a better trade-off between error resilience and bit rate.

Finally, it would be interesting to combine redundancy in the form of tornado codes and FEC. A receiver should buffer all received packets in a tornado code without performing a CRC. Then by comparing the packets in the tornado code bit wise, soft error detection can be performed. If the XOR of the tornado code does not check on a set of three bits, it is known that either one or three errors have occured. Disregarding the unlikely event of three errors, this gives an error probability of $\frac{1}{3}$ at each of the three bits. On the other hand, if the XOR does check, it is known that either zero or two errors occured. The event of two errors is disregarded, which makes the error probability on all three bits zero. This soft information can be taken into account when decoding to the most probable codeword.

Appendix

Introduction to Worksheets

The appendix enclosed with this work comprises the worksheets documenting the initial work in this project. Hence, in these worksheets the existing work within the area of bidirectional relaying schemes is investigated. Moreover, these worksheet reflect the process of developing and investigating novel ideas concerning bidirectional relaying in order to identify the focus area of this project.

In the following the worksheets are appended as individual chapters, where content is build up and concluded. Hence there is no natural link between the chapters except that they each are a part of the same innovative process.

1-dimensional Relay Networks

1-dimensional Network

In this section 1-dimensional networks with N (odd) relaying nodes are considered, i.e. chains. An example of such a network is shown in figure 1. Due to the half-duplex nature of wireless nodes, only a maximum of $\frac{N+1}{2}$ nodes can transmit at one time, and this is achieved when every other node is transmitting as depicted in figure 1. Hence, the optimal scheduling of transmissions is to let all even (odd) numbered nodes transmit in even (odd) time slots. How interference cancellation using ANC and DNF can be applied in these networks is investigated in this section.

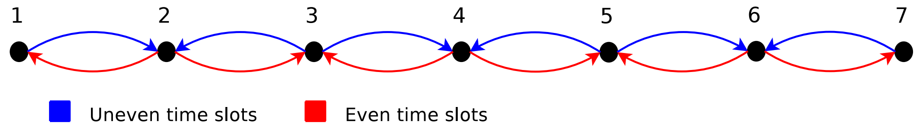


Figure 1: A 1-dimensional network with 5 nodes.

Echo Cancellation

Consider a chain with N=5, that is 3 relaying nodes. Utilization of ANC in this network could be as shown in figure 2. Here it is seen that a steady state is reached where the center node subtracts two times its last transmitted signal $2\gamma_3^{k-1}$. The other relaying nodes just forward the combined signal. The center node is experiencing what is referred to as *echoes* from both its neighbours, which is when its own signal is a part of the signal it receives in the following time slot. This calls for the subtraction of $2\gamma_3^{k-1}$.

Moving on to N=7, the center node still subtracts two times its last transmitted signal, now denoted $2\gamma_4^{k-1}$. However, this time its neighbouring nodes subtract one times their last transmitted signal. The last two relaying nodes subtract nothing. See figure 3. In this network nodes 3 and 5 also experience echoes in steady state, 3 from the left and 5 from the right. Hence the subtractions.

In general in steady state echoes are experienced from the left (right) if a node is in the left (right) side of the chain. The center node experiences echoes from both left and right. The relaying nodes next to the sources/sinks do not experience echoes, because the end nodes always transmit a new signal. Hence, a chain with

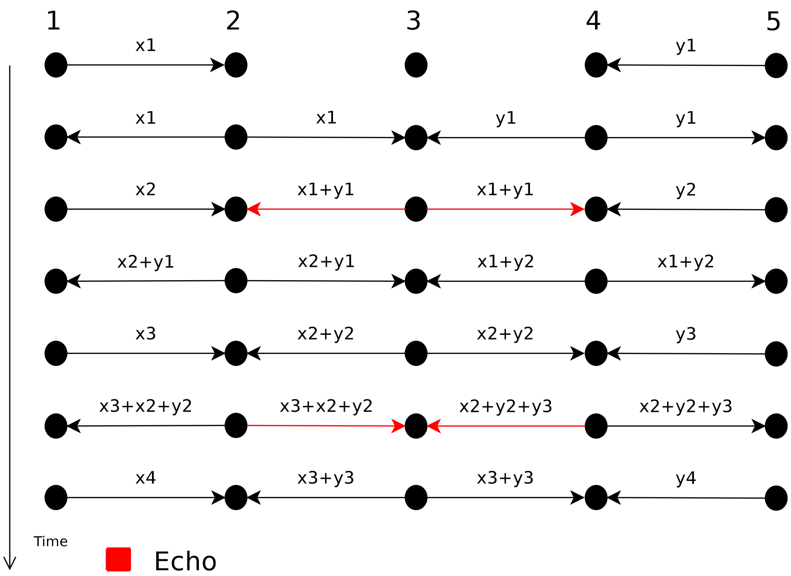


Figure 2: Illustration of the transmissions in each timeslot for a 1-dimensional network with 5 nodes.

arbitrary odd N and center node x, will always reach a steady state where the center node subtracts $2\gamma_x^{k-1}$ and nodes 3 to x-1 and x+1 to N-2 subtract γ_j^{k-1}, where j is the node number. Another important thing to note about the general case is that the largest transmitted signal is a sum of $\frac{N+1}{2}$ signals. This is the signal received by the end nodes. This means that in order to decode the signals, the end nodes must have sufficient buffer to save its last $\frac{N-1}{2}$ transmitted signals. Moreover a valid modulation scheme must be employed, in order to use DNF with signals of this size at the relay nodes.

Discussion

Using the procedure described above, it is quite simple to know what to forward at each node. Once a node knows its position in the chain, it knows whether it should subtract γ_j^{k-1}, $2\gamma_j^{k-1}$ or nothing at all. Once the network reaches a steady state, the subtracted amount is static, given no errors occur or the network topology changes. However, there is a need to find a proper modulation scheme to cope with a sum of $\frac{N+1}{2}$ packets in DNF. The potentially high number of packets in each signal also creates the need of a large buffer in order to recreate transmitted packet. Moreover, a procedure to ensure convergence towards steady state must be developed, since operation in this phase is not equal to the one in steady state.

Subtractions at Relays and Pre-cancellation

The fact that the maximum number of components in a signal is directly proportional to the number of nodes is a major drawback of echo cancellation. In combination with DNF it is difficult to make the mapping of the received signals, when they po-

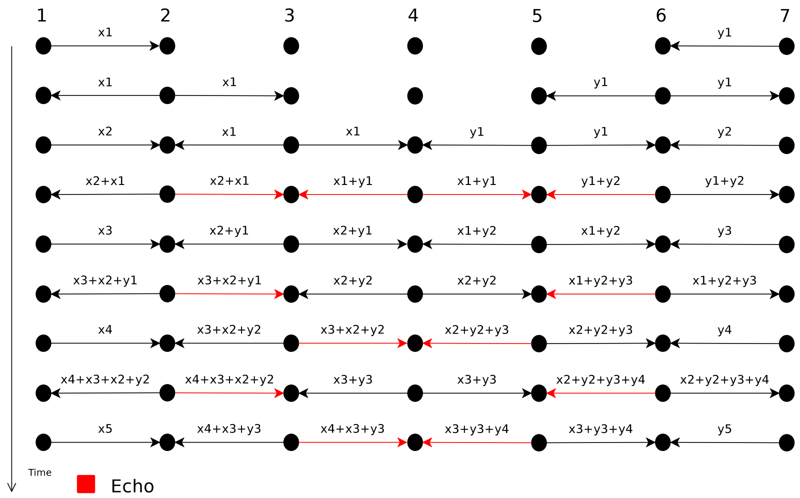


Figure 3: Illustration of the transmissions in each timeslot for a 1-dimensional network with 7 nodes.

tentially consist of $\frac{N+1}{2}$ components. It is desired to keep the number of components as low as possible and preferably independent of N. In the following it is shown how the max number of components can be limited to only two and made independent of N. Having a maximum of two components is key in DNF schemes, since the denoise operation is very simple in this case.

In order to achieve a maximum of only two components, subtractions must be performed at the relays. In the simple bi-directional relaying scenario with three nodes subtractions are performed by the end nodes, utilizing a priori information about the content of the network coded packets. In this scheme, such subtractions are performed at the relays, much like in echo cancellation, however in this case not only echoes are cancelled. When a relay node makes a subtraction, it subtracts its last transmitted signal from the signal sum it just received. That is $\gamma_x^k = \gamma_{x-1}^{k-1} + \gamma_{x+1}^{k-1} - \gamma_x^{k-1}$. In a network with N nodes, the N-4 center nodes perform this subtraction in every transmitting time slot. This means that in the example of a 5 node network, only the center node would subtract.

This is combined with a principle called pre-cancellation, first mentioned in [Kuek et al., 2008]. The idea of this principle is to let the end nodes subtract their last transmitted packet from each new packet they wish to transmit. If we denote the k'th packet from node 1, x_k, this means that the k'th transmitted signal from node 1 would equal $\gamma_1^k = x_k - x_{k-1}$.

If we look at a transmission schedule of this scheme, it would be as shown in figure 4 for a network with 5 nodes. As mentioned above, in this network only the center node uses subtractions. From this figure it is evident that communication between the end nodes through the relays is possible with network coded signals of only two packets. When increasing the number of nodes to seven, the number of nodes using subtraction increases to 3, since the N-4 center nodes should subtract. See figure 5. It is interesting to see that the maximum number of combined packets in the transmitted signals has not increased compared to the smaller five node network.

Increasing the number of nodes even further to nine gives the same result. See figure 6.

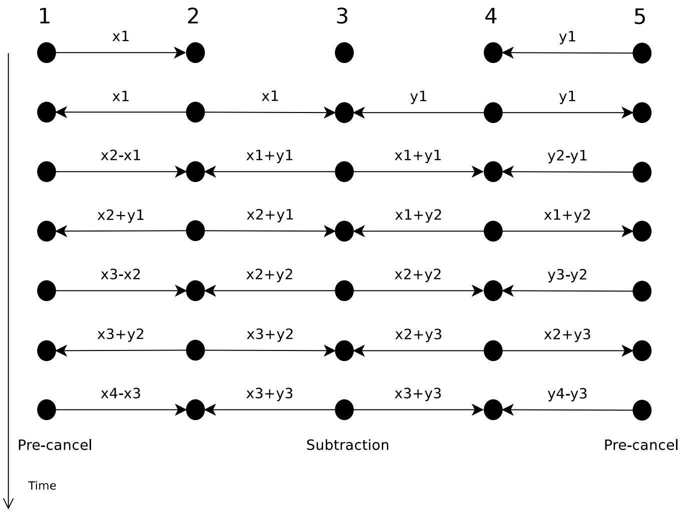


Figure 4: Transmission schedule for the scheme using subtracting relays and pre-cancellation in a 1-dimensional network with 5 nodes.

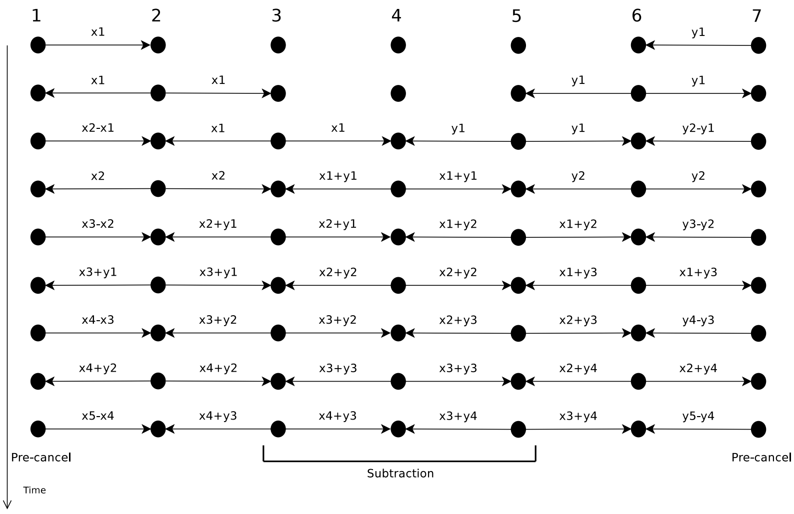


Figure 5: Transmission schedule for the scheme using subtracting relays and pre-cancellation in a 1-dimensional network with 7 nodes.

Discussion

The scheme combining subtractions at the relays and pre-cancellation at the end nodes avoids the significant drawback of echo cancellation, where more than two packets are combined in transmitted signals. This naturally eliminates the drawback of a need for a high buffer at the receiving node in order to reconstruct the transmitted packet. In this scheme the needed buffer is limited to only a single

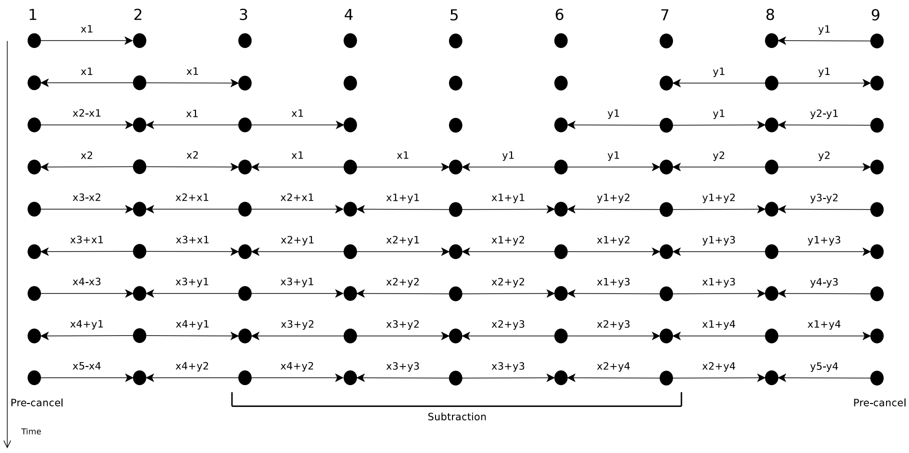


Figure 6: Transmission schedule for the scheme using subtracting relays and pre-cancellation in a 1-dimensional network with 9 nodes.

older packet. In addition, convergence towards steady state is straightforward in this scheme. When a relaying node transmits its second signal, it should go into the subtraction state (if it is not an end node neighbour), where its last transmitted signal is subtracted whenever a new signal is transmitted. An exception exist for the relay nodes neighbouring the center node. They should subtract two times their last transmitted signal, the first time they subtract.

9 Nodes Relay Network

The Scenario

So far research in relaying networks has only focused on scenarios with one dimensional networks of three to four nodes. It is believed that this work can be extended to multidimensional networks of n nodes. Consider the scenario illustrated in figure 7, where nine nodes are placed in a two-dimensional static network.

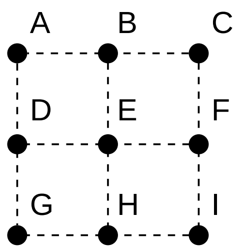


Figure 7: A two-dimensional static network

Constraints

The scenario is investigated under certain constraints in order to simplify some perspectives. A node broadcasts its transmission, but only nodes placed vertically or horizontally to the transmitting node can receive the signal successfully.

Also, these nine nodes should be regarded as the entire network, and not a subsection of a larger two-dimensional network, say a ten-by-ten node grid. In this way the source can only send data to nodes towards the destination and not in the opposite direction.

Two Communicating Nodes

Initially the case where two sources, placed opposite to each other, (node A and I) exchange packets is considered. The rest of the network illustrated in figure 7 constitutes therefore the relay network.

General Considerations

Bi-directional Relay Networks

Following the constraints described above one can imagine how the packets will ripple through the network from source to destination using only the vertical and horizontal hops. There can be several paths through the network. If we isolate one of these paths from the rest of the nodes in the network we can regard this string of nodes as a one dimensional network. In figure 8 two such paths are highlighted with the colours red and green respectively.

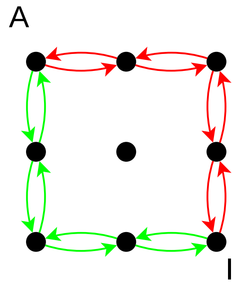


Figure 8: Examples of one-dimensional relay-paths within the two-dimensional network.

By considering a path through the two-dimensional network as a one-dimensional bidirectional relay-network the existing relay schemes can be applied on this particular path.

If each transmission is properly scheduled in the highlighted paths from figure 8, the destination node will simultaneously receive the same signal from two different nodes. In this way the two simultaneous transmissions will interfere constructively and increase the probability of receiving the data successfully.

Scheme Proposal

The main idea of the scheme is to group the nodes between source and destination into supernodes according to which time steps they will receive data. Consider the scenario in figure 9 where node A and I should exchange data through the relay network between them. In the first step (red arrows) nodes A and I transmit their packet to the neighbouring nodes (only the horizontally and vertically neighbouring nodes). This means that in step one, two nodes receive a packet from node A and two other nodes receive a packet from node I, these are grouped into supernodes X and Z respectively. In step two (green arrows), supernodes X and Z transmit their packet to the neighbouring nodes. Three nodes are neighbours to both node X and Z and receives therefore an analog coded signal, these three nodes are grouped into the last supernode, node Y. In step two, node A and I will also receive their own packet again, but it can be subtracted.

In this way the two-dimensional network can be regarded as one-dimensional consisting of supernodes. Some of the nodes in the supernode can make use of channel diversity and thereby increase the probability of decoding the signal correctly.

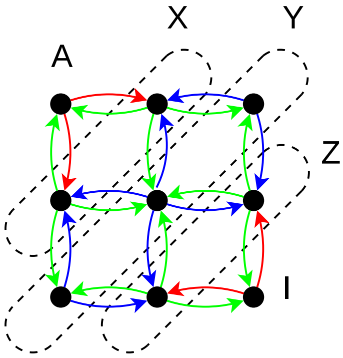


Figure 9: The two dimensional network regarded as a one-dimensional network of five nodes where some of the nodes are supernodes consisting of several nodes.

This can also help to weaken the constraint of only vertical and horizontal transmissions. A diagonal transmission may not be received successfully directly, but if the information is saved in the buffer it can be used as incremental redundancy.

Having a traditional one-dimensional network as illustrated in figure 10 the packets can be relayed as specified in table 1.

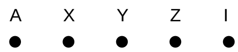


Figure 10: A one-dimensional network of five nodes, where nodes B, C and D are supernodes.

	A		X		Y		Z		I
T_1		$\overset{a_1}{\rightarrow}$	a_1				i_1	$\overset{i_1}{\leftarrow}$	
T_2		$\overset{a_1}{\leftarrow}$		$\overset{a_1}{\rightarrow}$	a_1+i_1	$\overset{i_1}{\leftarrow}$		$\overset{i_1}{\rightarrow}$	
T_3		$\overset{a_2}{\rightarrow}$	a_2+i_1	$\overset{a_1+i_1}{\leftarrow}$		$\overset{a_1+i_1}{\rightarrow}$	a_1+i_2	$\overset{i_1}{\leftarrow}$	
T_4	i_1	$\overset{a_2+i_1}{\leftarrow}$		$\overset{a_2+i_1}{\rightarrow}$	a_2+i_2	$\overset{a_1+i_2}{\leftarrow}$		$\overset{a_1+i_2}{\rightarrow}$	a_1
T_5		$\overset{a_3}{\rightarrow}$	$a_2+a_3+i_2$	$\overset{a_2+i_2}{\leftarrow}$		$\overset{a_2+i_2}{\rightarrow}$	$a_2+i_2+i_3$	$\overset{i_3}{\leftarrow}$	
T_6	i_2^*	$\overset{a_2+a_3+i_2}{\leftarrow}$		$\overset{a_2+a_3+i_2}{\rightarrow}$	$a_3+i_3^{**}$	$\overset{a_2+i_2+i_3}{\leftarrow}$		$\overset{a_2+i_2+i_3}{\rightarrow}$	a_2
T_7		$\overset{a_4}{\rightarrow}$	$a_3+a_4+i_3$	$\overset{a_3+i_3}{\leftarrow}$		$\overset{a_3+i_3}{\rightarrow}$	$a_3+i_3+i_4$	$\overset{i_4}{\leftarrow}$	
T_8	i_3	$\overset{a_3+a_4+i_3}{\leftarrow}$		$\overset{a_3+a_4+i_3}{\rightarrow}$	a_4+i_4	$\overset{a_3+i_3+i_4}{\leftarrow}$		$\overset{a_3+i_3+i_4}{\rightarrow}$	a_3

Table 1: The relaying of packets through eight steps.

- (*): a_2 and i_2 are successfully received as long as the destinations are able to combine their a priori information, e.g. node A must be able to create the signal $a_2 + a_3$ and substract it from $a_2 + a_3 + i_2$ in order to decode i_2.

- (**): Node C must be able to substract two times what it know a priori from T_4 and onwards. The received signal equals $a_2 + a_3 + i_2 + a_2 + i_2 + i_3 = a_3 + i_3 + 2(a_2 + i_2)$.

Which Protocol?

There exist several protocols for relaying networks, but so far, they have not been applied to networks larger than three and four nodes.

- **Decode and Forward (DF):** According to the scheme outlined in table 1 the signal received by the relay nodes is most often coded by ANC in the wireless channel. This will render the relay nodes unable to decode the two packets separately. To use DF the scheme requires more steps to exchange the same number of packets. This would decrease the throughput which is not desired.

- **Amplify and Forward (AF):** This scheme basically just repeats whatever was received, which means that also the channel noise is amplified. The number of noise contributions increase with the number of hops, and the level of amplification of each noise contribution corresponds to the number of times it has been forwarded. This is a significant amount of noise, and may be a deal breaker already with five nodes.

- **De-Noise and Forward (DNF):** This is a novel and more advanced scheme compared to DF and AF. DNF performs a de-noise operation before the ANC coded signal is forwarded. The de-noise operation makes use of a de-noise-map, where the possible ANC coded input signals are mapped onto a set of output signals. These can be decoded by the destination nodes using a priori information. This operation requires the modulation scheme to be chosen very carefully. For networks of no more than three nodes (one relay node) the DNF protocol is somewhat straight forward, but as the number of nodes increases the ANC coded input signals can be ambiguous, and the mapping suddenly becomes complex, if at all possible.

  – The concept of *Pre-Cancellation* may decrease the ambiguity of the input signals. However, the more analog signal operations performed (adding or subtracting signals) the more increases the uncertainty of the resulting signal.

Scalability

If the number of relay nodes between the two nodes exchanging data (node A and I) increases it would still be possible to form supernodes, in which the subnodes receive and transmit the same signals. However, one might imagine a scenario where the two dimensional network spanned by the nodes A and I is a subpart of a larger two-dimensional network. In this way there would be nodes not directly in the path between A and I which could possibly result in a "detour" of the information. This would again result in an unexpected large delay and thereby an unpredictable interference, which is undesired.

In addition to more nodes, it is possible to increase the dimensions of the network, e.g. a three-dimensional network could represent a sensor network. So far a cube network of eight nodes (one in each corner) has been investigated, and the idea of grouping the relay nodes into supernodes also holds for three dimensions. In this case the supernodes become two-dimensional networks.

Four Communicating Nodes

One aspect of scalability is the number of nodes and dimensions as described above. However, the scenario from section 11.1 can also be expanded with respect to sets of sources. In this section the case with four sources is considered. The sources form two pairs, nodes A and I, and C and G, which exchange packets respectively, using the rest of the nodes as relay nodes. The constraints described previously hold for this scenario as well.

Scheme Proposal

In figure 11 the first step is illustrated, where the sources each transmit their first packet to the relay nodes, i.e. the nodes placed vertically or horizontally with respect to each transmitting node.

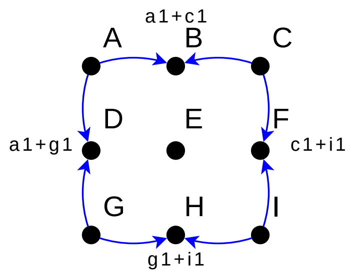


Figure 11: The first step of the exchanging of packets between two sets of two nodes in a two-dimensional network.

Each intermediate relay node receives an ANC coded signal with a component from two of the four sources. Since none of the relay nodes receive the same signal, or combination of signals, it is not possible to reuse the idea of supernodes in this example.

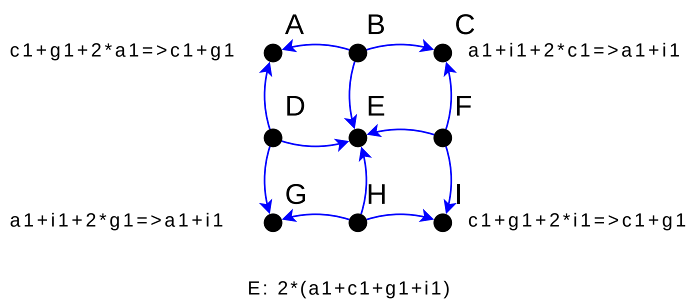


Figure 12: The second step of the communication procedure.

In the second step, illustrated in figure 12, the intermediate relay nodes forwards their ANC coded signal. This results in each source receiving a combination of two source components plus their own initial signal with double amplitude. Each source can remove their own signal from the received signal, but since the remaining part is an ANC coded signal with two unknown components no useful data can be decoded yet. However they should save this signal as it may become useful at a later instant. The signal received by the center relay node is the initial signal from all four sources superimposed with double amplitude.

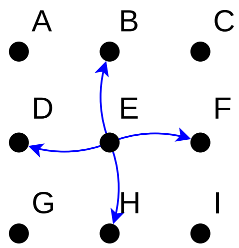


Figure 13: The third step of the communication procedure.

It is chosen to only allow the center node to transmit in the third step, as illustrated in figure 13. The four sources could also have transmitted their second packet, but the resulting ANC coded signal at the intermediate would then contain too many unknown signal components. This would, in the next step, render the sources unable to decode the packet of interest from their opposite node.

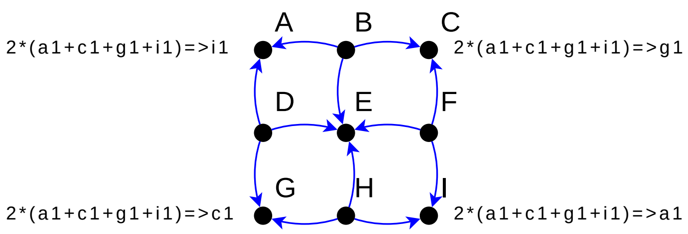


Figure 14: The fourth step of the communication procedure.

The fourth step in figure 14 marks the end of the communication cycle in this scheme. The intermediate nodes forward the ANC coded signal they received from the center node. Using their own initial signal, and the signal saved after step two all four sources are able to decode the initial signal from the opposite node after only four steps. After these four steps the cycle can start over with the sources transmitting their second packet to the intermediate nodes and so on.

Superposition Coding

The Scenario

In a network, like the one in figure 15, with several nodes, the communication between two nodes can be overheard by some of the other nodes.

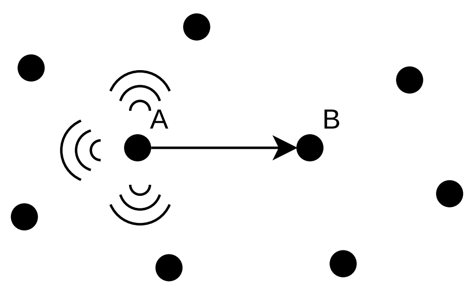


Figure 15: A wireless network with several nodes.

The achievable rate at which a node can receive information from the transmitting node depends on the link quality. Due to the propagation loss the link quality will decrease as the distance increases. In this way a node placed at a larger distance than the destination node will overhear the transmitted information, but the achievable rate will be less than for the destination node. This can be exploited in relay networks. In figure 16 node A transmit packets to node B through the relay node R. Each transmission from A to R is overheard by node B, however due to the low quality link node B is unable to decode the packet.

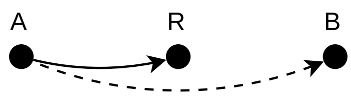


Figure 16: A wireless relay network where the possibility of overhearing is taken into account.

The fact that node B holds 'some' information about the transmitted packet can be used as redundant data in order to increase the reliability of the communication between A and B or to decrease the load on the link between node R and B, since node R does not have to transmit the entire packet.

The Concept of Superposition Coding

Superposition coding is one way to make use of the information overheard by node B from the transmission between node A and R. It is assumed that the links between A and R, and R and B are considerably stronger than the direct link from node A to B.

The packets transmitted from node A consist of two parts, a basic part, x_b, and a superimposed part, x_s. The communication from node A to B can be described as a two step procedure.

1. Node A broadcasts the following signal:

   $$y_0 = \sqrt{\alpha} \cdot x_b + \sqrt{1-\alpha} \cdot x_s$$

   Where α is the superposition power coefficient. Due to the good channel between A and R, both x_b and x_s are decoded successfully at the relay node.

   Node B overhears the transmission from A, and receives the following signal:

   $$y_1 = h_1(\sqrt{\alpha} \cdot x_b + \sqrt{1-\alpha} \cdot x_s) + z_{b1} \tag{1}$$

   Due to the bad channel to A, node B is unable to decode anything and y_1 is therefore saved in the buffer.

2. The relay node recodes the superimposed data, x_s, into the packet x_r and transmits it to node B. The good channel between node R and B allows B to successfully decode x_r to achieve x_s.

   Using available CSI node B can construct the signal $h_1 \cdot \sqrt{1-\alpha} \cdot x_s$ and subtract it from (1):

   $$\begin{aligned} y_2 &= h_1(\sqrt{\alpha} \cdot x_b + \sqrt{1-\alpha} \cdot x_s) + z_{b1} - h_1 \cdot \sqrt{1-\alpha} \cdot x_s \\ &= h_1\sqrt{\alpha} \cdot x_b + z_{b1} \end{aligned} \tag{2}$$

   From (2) node B is able to decode the remaining x_b (*Successive Interference Cancellation, SIC*).

In this way superposition coding allows the destination node to exploit the overheard information in previous transmissions and hereby decrease the load on the relay network. If node A has CSI from the channels in the scenario available, it is possible to optimize the size of the basic and superimposed part of a packet.

Discussion

In multi-dimensional networks relay schemes become fairly complex and employing superposition coding makes no exception. Digital network coding can decrease this effect to some extend. However, the two-dimensional network still suffers from the relatively large number of sources and destinations, which would result in significant delays.

A Relay Network With Three Sources and One Relay

The Scenario

In this worksheet the scenario where three nodes exchanging data using a single relay node is investigated. We will present two different concepts for distributing packets in this network, and point out some of the weaknesses and problems in these distribution procedures.

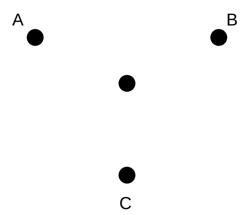


Figure 17: A relay network with three sources and a single relay.

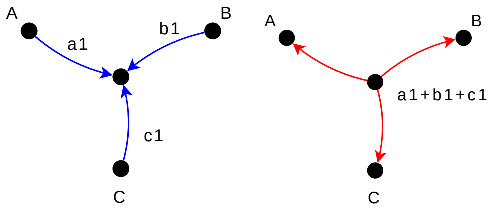
The relay network in question is illustrated in figure 17. The nodes A, B and C have data for each other, however, they can only communicate reliably using the relay node to forward data between them.

Ideas To Exploit The Scenario

The following ideas will describe how data can be exchanged between all sources in the relay network. These ideas will rely heavily on ANC and SIC.

Three Node Broadcast

The first idea is the simplest possible compared to the general case where two nodes exchange data over a relay node. Here an additional source is just added to the network. In the first timeslot all three nodes broadcast their packet. The nodes are unable to listen while they are transmitting so only the relay node receives data. The received signal is an analog coded signal with three components, one from each of the sources. In the second timeslot the relay node broadcast the received information to the sources. This procedure is illustrated in figure 18.



(a) First timeslot. (b) Second timeslot.

Figure 18: A simple procedure for distributing packets between three nodes over a relay node.

The relay node is unable to decode any of the signal components and with three signal components DNF can not be used as forwarding scheme either. Consider the case where the nodes are required to use BPSK, hence either -1 or 1 are transmitted. The resulting signal received by the relay node would therefore take one of the following values: $\{-3, \ -1, \ 1, \ 3\}$. Using DNF the relay would only be able to transmit "all three signals were the same" or "only two out of three were the same" when a binary output from the DNF-scheme is desired. If more information is needed an additional bit can be added, but even then the nodes would experience situations where they are unable to decode the signals from the rest of the relay network. Hence, we consider the use of amplify and forward (AF) in this and the next idea for distribution in the relay network.

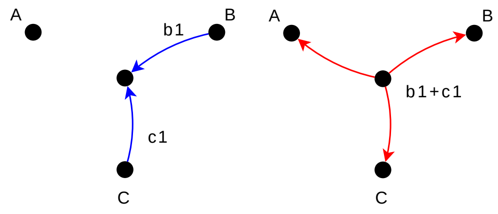
When using AF each source will after the second timeslot have received an ANC coded signal,y, with three signal components. Subtracting their own signal results in a combination of the signals from the remaining nodes, y'. As they have no knowledge a priori of the signal components in y', this signal can not be reduced any further. However, if there is a sufficient difference in the powerlevel of these signal components they can be decoded one by one using SIC.

Additional Thoughts

In some cases it may not be possible to rely directly on SIC as prescribed above. Hence additional information must be transmitted in order to make y' decodable. To determine when this is necessary a simple control system must be designed to ACK and NACK on the decoded packets. If a packets it nacked, i.e. it can not be decoded using SIC, one possibility is to retransmit the entire packet. This would require two entire additional timeslots, since data must be forwarded by the relay node and therefore probably even out any gain obtained from the use of ANC. Instead FEC for one of the signal components could be forwarded to increase the SNR and enable the use of SIC for both signal components to be decoded. This would require less data to be retransmitted and therefore have less impact on performance.

Pairwise Distribution

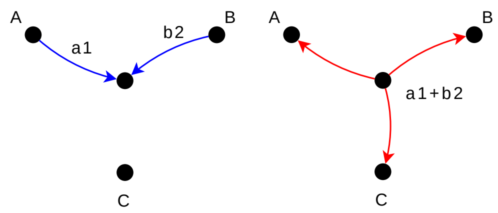
The main difference from this idea and the one just described is that the time of operation is divided between the sources in the network as only two of the source nodes operate simultaneously at a time. The distribution procedure is illustrated in figures 19, 20 and 21.



(a) Timeslot 1. (b) Timeslot 2.

Figure 19: A pairwise procedure for distributing packets between three nodes over a relay node.

In figure 19(a) node B and C are paired, this means that they simultaneously transmit their packets to the relay node. In the subsequent timeslot, see figure 19(b), the relay node broadcast the ANC coded signal. From this nodes B and C can decode packet c_1 and b_1 respectively. Node A also receives the signal from the relay node, and assuming that SIC is applicable both signal components can be decoded.

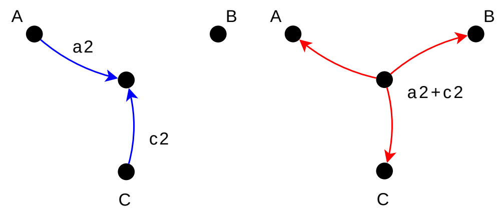


(a) Timeslot 3. (b) Timeslot 4.

Figure 20: A pairwise procedure for distributing packets between three nodes over a relay node.

In figure 20 the pair now consists of nodes A and B. These nodes transmit simultaneously to the relay node (A's first packet and B's second packet). This ANC coded signal is broadcast to all sources in the network, as illustrated in figure 20(b). The remaining node, now node C, must again rely on SIC to decode both signal components.

In the last two illustrated timeslots, see figure 20, node A and C comprise the node pair, and simultaneously transmit their second packet to the relay node. The resulting ANC coded signal is broadcast as usual and the sources decode the data. Nodes A and C decode by subtracting their own signal and B decodes both signal components using SIC.



(a) Timeslot 5. (b) Timeslot 6.

Figure 21: A pairwise procedure for distributing packets between three nodes over a relay node.

Additional Thoughts

For the remaining node, the one not in the pair, this idea relies heavily on SIC, which in some cases may fail to decode both signal components. In this case, as described earlier, one of the signal components could be retransmitted, or FEC could be forwarded. However, any additional data sent to help the remaining node decode the data will decrease the gain obtained by this idea.

It is reasonable to assume that the node not in the pair overhears some of the information transmitted from the pair to the relay node. There may exist a way to exploit this information in combination with the ANC coded signal in order to increase the possibility of decoding both signal components using SIC.

Discussion

The ideas presented above relies on SIC to a large extent. So far we are just familiar with the concept of SIC. To determine the applicability of the presented ideas the concept of SIC will have to be investigated further. We need to know the requirements for SIC to enable the decoding of multiple signal components from an ANC coded signal, e.g. the required weighting between the components. Also, it is necessary to investigate which scenarios would provide these requirements reliably, so retransmissions can be held at a minimum.

Another aspect of the presented ideas is ANC. ANC plays a significant role in almost all ideas presented in previous worksheets. So far it has been assumed that it is possible to perform this simple arithmetic operation on an arbitrary number of signals, but this seems to be a rough assumption. It should therefore be interesting to investigate which limitations we are working under in terms of signal combining. How many signal components can be added and/or subtracted and still sustain the possibility of decoding the remaining components of the ANC coded signal.

Overhearing using FEC

Consider a 1-dimensional network with 5 nodes. See figure 22. Node 1 broadcasts a packet, x_1, in this scheme without using superposition coding. The packet is received and is decodable at node 2. In addition node 3 is able to receive a weak signal, which does not meet the SNR requirement, hence the packet is not decodable. However, node 3 keeps the erroneous packet and is subsequently aided by node 2 in decoding it. The aid is realised by having node 2 generate FEC for x_1 and transmit it to node 3. This is a variant of Incremental Redundancy (IR). The FEC received at node 3 removes the uncertainty about the received packet and it is now decodable. The same procedure is performed from the other side in the network when node 5 transmits y_1. These two-step transmissions to the center node are interleaved during the first four time slots, which makes nodes 2 and 4 able to calculate FEC while the other side transmits. When the center node has received both x_1 and y_1 it combines them using digital network coding and broadcasts the resulting packet. The two-step procedure with incremental redundancy is again used for these transmissions.

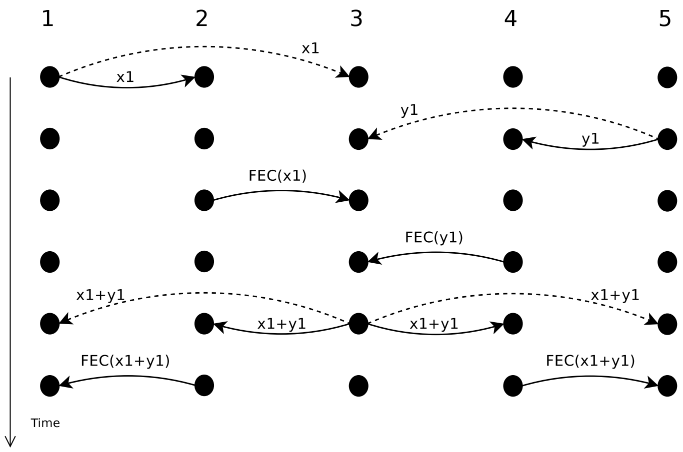


Figure 22: Illustration of a scheme utilizing overhearing in transmissions and incremental redundancy.

Discussion

When assuming the relay nodes have CSI available, the FEC transmissions can be adapted to suit the needs at the receivers. In this case the performance of the above scheme will be closer to the theoretical limit than a scheme with the same transmission schedule but utilizing superposition coding. It should however be noted that the IR-scheme uses six time slots, whereas a scheme ignoring the possibility of overhearing would be able to transmit x_1 and y_1 in only five time slots by also using digitial network coding. See figure 23. However, three of the time slots in the IR-scheme is devoted to transmitting FEC only and these slots are therefore shorter than slots devoted to transmiting the original packet. Hence, the IR-scheme outperforms simple digitial network coding when three FEC slots can be kept shorter than two full packet slots.

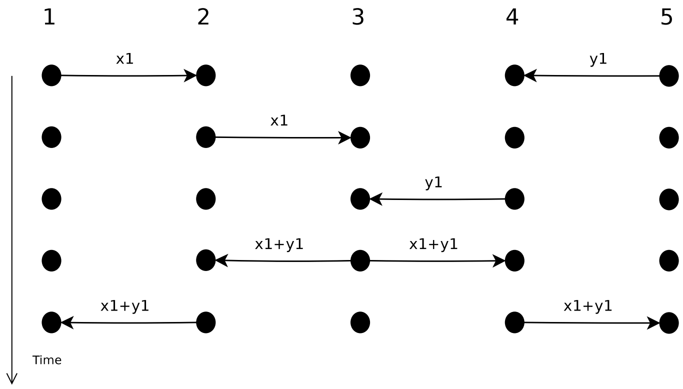


Figure 23: Illustration of a simple digital network coding scheme ignoring overhearing.

Relay Networks With Multiple Orthogonal Channels

What Multiple Channels Can Provide

In wireless networks the interference decrease the overall performance of the network and in previous worksheets we have investigated how the information in this interference could be exploited in order to reduce the impact on network performance. Different approaches have been investigated, all having potential to increase the throughput when the number of communicating nodes are limited to two. If more nodes communicate simultaneously the schemes becomes complex and it is more uncertain that a gain in throughput is achieved with the approaches investigated so far.

A traditional wireless network only use a single channel. This means that all nodes can potentially interfere with each other. However, the IEEE802.11 protocols provide several orthogonal channels. If each set of nodes use a different channel they can communicate even though they are in the range of each other.

In fact, if a node has more than one network interface it can receive and transmit on different channels simultaneously. This possibility gives a new dimension to any communication scenario discussed so far.

Examples

A Single Dimension

Consider the previously described one-dimensional network with two nodes, A and B, communicating over a single relay node, R. In this case each node has two orthogonal channels with identical bandwidth at its disposal instead of just one. Examples of a one-dimensional network utilizing such two orthogonal channels are illustrated in figure 24.

The network reaches a steady state in the second time slot. In this state each node transmits and receives information simultaneously by utilizing the orthogonal channels. Hence in steady state, node A (resp. B) is able to decode a new packet from node B (resp. A) after each time slot.

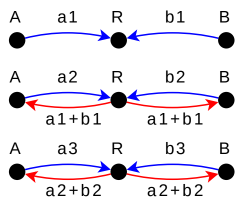
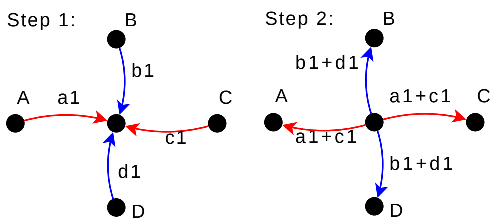


Figure 24: The communication procedure using multiple channels and ANC. One channel is illustrated using red arrows and the other using blue arrows.

Multiple Dimensions

Increasing the number of dimensions also increases complexity, especially if there is more than one set of communicating nodes. In figure 25 a network with two sets of communicating nodes sharing a single relay node is illustrated.



(a) (b)

Figure 25: Two sets of nodes communicating using a single relay node and two orthogonal channels. One channel is illustrated using red arrows and the other using blue arrows.

In this case each set of nodes is assigned one of the two orthogonal channels. The exchange of information in each set becomes therefore identical to one-dimensional ANC relaying with one channel. However, in this case the shared relay node must fully utilize both channels in order to keep up with the rest of the network. If there were more orthogonal channels available, e.g. two channels per set of nodes, the nodes would be able to exchange information as efficient as the one-dimensional ANC network described in section 11.1.

Discussion

Adding a dimension to the scenario in the form of multiple channels may simplify the exchange of information when several sets of nodes are communicating, but nothing comes for free, and simplifying one aspect will most likely add complexity to another. This is also the case when using multiple orthogonal channels in a relay network. In addition to the challenge of precise scheduling, which always exist in relay networks using ANC, the network must also assign which channel to use on the different links, before any packets can be exchanged. In [Kodialam and Nandagopal, 2005] some work have been done investigating two different protocols for assigning channels, a static and a dynamic protocol.

In this worksheet the simplest two-dimensional network topology has been presented, a cross. In this network it is trivial to identify the best path from source to destination. When the topology becomes more complex, e.g. a grid, the scheme will also need proper routing protocols in addition to the channel asigning and relay protocols.

Relay Selection

Relay Selection

In many wireless relay schemes the problem of selecting the optimal relay to assist in communication between two nodes is a key part of the design. The optimal relay should be selected in order to maximize throughput and/or diversity order. In the following a few schemes incorporating analog and digital network coding is described in which relay selection is an important aspect.

Digital Network Coding in Pentagram

Consider a five node network where the nodes form a pentagram. See figure 26. Every node can communicate with all other nodes in the network. Nodes A and B wish to exchange packets, however, the connection between them is weak and errors occur frequently. When an error occurs the receiver piggybacks a NACK to the following packet it transmits. For a single error, conventional rentransmission schemes can be used, e.g. Selective Repeat ARQ. However, if errors occur in succession and on transmissions in both directions, it can be utilised that the other nodes in the network have overheard the transmissions. If one of these nodes has succesfully received both packets intended for A and B, it can combine them with digitial network coding and broadcast the resulting packet. See figure 27. This is effectively a retransmission of both packets in one time slot, which would otherwise require two. Moreover spatial diversity is achieved with this approach. Successive errors in both directions are likely to occur when fading is experienced on the channel, in which case spatial diversity is a desired luxury. An important problem to solve in this scenario is deciding which relay should perform the retransmission, if more than one has the nessecary packets available. It is reasonable to assume that the relays have local CSI available, but a procedure should be designed for selecting the optimal relay.

Analog Network Coding in Pentagram

Another approach in the pentagram network in figure 26 is to utilise network coding in every transmission. If nodes A and B transmit their packets concurrently, all other nodes in the network will receive analog network coded packets. One of these should broadcast this packet, from which nodes A and B can extract their intended

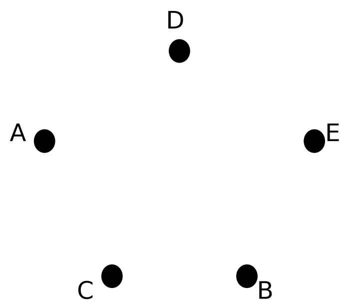


Figure 26: A five node network in which all nodes can communicate with all other nodes.

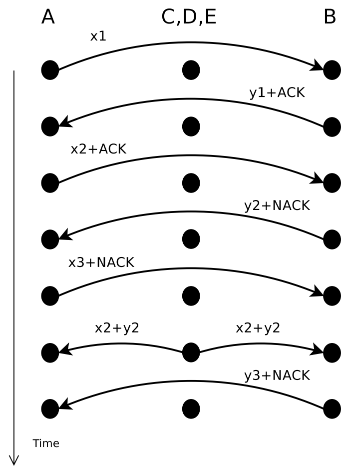


Figure 27: Transmission schedule of a scheme with retransmissions from relay nodes using digital network coding. The center node represents the optimal relay node among C,D and E.

packets using a priori information. In this way every transmission of a single packet from both sides is performed in two steps, as when using direct transmissions. An advantage in this scheme is that the need for retransmission is less likely, because spatial diversity can be utilised in every transmission. In this scheme, as in the DNC scheme above, potentially more than one node would be able to act as the relay node. Hence, again the need for a procedure for selecting the optimal relay is present.

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Papers

Scalable Denoise-and-Forward in Bidirectional Relay Networks

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Abstract

In this paper a novel scalable relaying scheme is proposed based on an existing concept called DeNoise-and-Forward, DNF. We call it Scalable DNF, S-DNF, and it targets the scenario with multiple communication flows through a single common relay. The idea of the scheme is to combine packets at the relay in order to save transmissions. To ensure decodability at the end nodes, a priori information about the content of the combined packets must be available. This is gathered during the initial transmissions to the relay. The trade-off between decodability and number of necessary transmissions is analysed and simulations show, that S-DNF is able to provide a better trade-off than traditional schemes at high SNR.

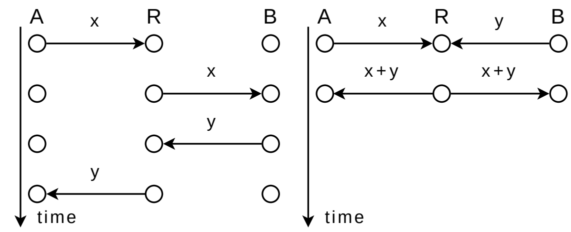
Key words: Analog network coding, Cooperative relaying

1. Introduction

Bidirectional relaying in wireless communication has been the focus of much research recently, see e.g. [4, 6, 5]. In a simple three-node network, the nodes A and B communicate with each other with the help of the relay node R. Fig. 1(a) shows that a Traditional Multi-Hop (TMH) communication protocol would require four time slots for every two packets exchanged between A and B. A large part of the existing work in this area investigates more efficient approaches to the relaying of packets. Examples are Amplify-and-Forward, AF, and Decode-and-Forward, DF, [2], also referred to as Analog Network Coding, ANC [8], and Digital Network Coding, DNC [1], respectively.

Reference [7] presents a promising concept called DeNoise-and-Forward, DNF. The idea is to let A and B transmit their packets to the relay concurrently. This is referred to as a joint transmission. Due to the additive property of the wireless channel, the relay receives a sum of the two transmitted signals. In the idealised case, when both channel gains (from A to R and from B to R) are 1 and BPSK modulation is applied, the relay will receive either -2, 0 or +2 for each symbol. These three possible symbols are mapped to a binary message indicating either equal (-2 and +2) or unequal (0) received symbols. This compression from three to two values enables the relay to use BPSK and ensures that the combined packet can be sent using the same number of symbols. The mapping removes any noise added during transmission, although decoding is not performed, hence the name. This new combined packet is then broadcast to A and B. Both A and B can now reconstruct what was intended for them by performing an XOR operation on the received packet and the packet they transmitted themselves. As shown in figure 1(b), it is possible to exchange packets between A and B in only two time slots using DNF. In [3] the idea of DNF has been extended for higher modulations and fading channels.

In this work we propose a scalable version of DNF, by generalizing it to multiple pairs of end nodes communicating through a single common relay. In the ideal case, and for large networks, the proposed scheme emulates the performance of full duplex communication between end nodes. The idea is to utilize network coding at the relay and allow the end nodes to overhear the joint transmissions from other end node pairs. In this way, each node in the network overhears packets and accumulates a priori information about the data of the other nodes. This allows the relay to send a small amount of information from which each node can extract its target data. The a priori information ensures decodability, while the network coding at the relay saves transmissions and thus increases the throughput. Hence, in this scheme the gain comes from utilizing the packets from other nodes in order to decode the desired information.



(a) Traditional multihop transmissions. (b) DNF.

Figure 1: Existing approaches to bidirectional relaying.

The remainder of this paper is organised as follows. Section 2 introduces the scenario targeted by this work and necessary tools for analysing the system. In section 3 the proposed scheme is described and analysis is performed in section 4. Results are shown in section 5 and finally, conclusions are drawn in section 6.

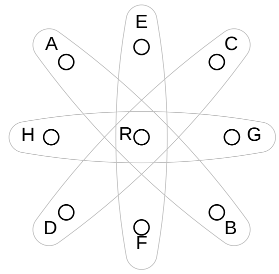


Figure 2: Four pairs of communicating end nodes distributed around a relay node.

2. System Model

Consider the scenario where M node pairs are distributed on the circumference of a circle. We assume that two communicating nodes are antipodal, hence they communicate over direct links at a distance of the diameter, D, of the circle. Thus the channel between two communicating nodes will be weaker than the channel to the nodes in the other pairs. The end nodes are able to overhear the communication in the other node pairs. This is the reference scenario used in this work. In order to aid the communication between the end nodes we place a relay at the center, as illustrated in Fig. 2, where $M = 4$: A↔B, C↔D, E↔F and G↔H. Hence the utilised topology is similar to the wheel topology presented in [1].

In this work we assume end-to-end error detection, i.e. the relay unconditionally forwards the information it receives. Moreover we neglect fading and consider only the Line Of Sight, LOS, component of the signal. For this we use the free space path loss model, where we assume equal antenna gain on all nodes in the network. This model is a far field model and imposes therefore limitations regarding the distance between nodes. However, we assume that the far field assumption holds for all transmissions, hence the loss factor is given by:

$$L(d_{ij}) = \frac{p_{rx}}{p_{tx}} = \left(\frac{4\pi d_{ij}}{\lambda}\right)^{-\kappa} \tag{1}$$

Where d_{ij} is the distance between node i and j, λ is the wavelength of the transmitted signal and κ is the path loss exponent. We assume BPSK modulation with unit power, hence $p_{tx} = 1$. This means that the received power, p_{rx}, equals $L(d_{ij})$. We set the condition that the total amount of transmitted power in the scenarios is equal. Therefore, in the relay case the power is normalised with $\frac{2M}{2M+1}$ for each node.

The SNR on the direct link is fixed to γ. The channel is assumed to be AWGN, where the power density of the noise component, σ_z^2, is uniform throughout the network. With the utilised path loss model the power of a propagated signal on a given link is deterministic. Thus, the value of γ dictates the noise power density, $\sigma_z^2 = \frac{L(D)}{\gamma}$.

Assuming equiprobable symbols, the bound of the decision region, Λ_1, for regular BPSK is placed in the center of the two possible symbols, i.e. $\Lambda_1 = 0$. The BER on transmissions between nodes i and j is denoted $P_b(\gamma_{ij})$, where γ_{ij} is the SNR on the link between node i and j.

When two end nodes perform a joint transmission to a third node we neglect the possibility of extracting one or both packet components using Successive Interference Cancellation (SIC). In fact, SIC is not useful, as it may result in only one packet being decoded correctly, which is insufficient for the final decoding of the desired data, due to the way the relay creates the broadcasted packet. For example, on Fig. 2, when A and B are transmitting, C needs to receive the XOR combination of the two transmitted packets, not the individual packets of A and B. Hence, each receiver, including the relay, makes a decision about the bit that is obtained by XOR of the two transmitted bits. For joint transmissions another decision bound, Λ_2, must be used. This bound is determined in the appendix along with the corresponding BER for a joint transmission from nodes i and j to node k, denoted $P_b(\gamma_{ik}, \gamma_{jk})$.

The Packet Error Rate (PER) for packets of l symbols is given by:

$$P_p(\gamma_{ij}) = 1 - (1 - P_b(\gamma_{ij}))^l \tag{2}$$

Using Eq. (2) we define the probability of a successful transmission between node i and j as $P_s(\gamma_{ij}) = 1 - P_p(\gamma_{ij})$.

3. The Proposed Scheme

The simplest solution for scaling the concept of regular DNF is to use DNF on each of the multiple node pairs and let one pair at the time exchange packets. We refer to this scheme as M-DNF. Succesfull decoding using this scheme only depends on the joint transmission from the end node pair and the broadcast by the relay. However, the gain is limited to the gain offered by regular DNF.

The scheme proposed in this paper is termed Scalable DNF, S-DNF. It reduces the number of broadcasts from the relay compared to M-DNF. In regular DNF each node uses only its own information to decode an analog coded packet from the relay. In S-DNF we allow each node to overhear the other transmissions in the network. In this way each node holds more a priori information compared to regular DNF, which allows the relay to collapse more packets into the broadcast packet. Each end node can then extract the packet intended for them from the relay packet by utilizing their a priori information, gathered by overhearing.

In principle, the packets broadcast by the relay can be any set of the analog network coded packets. The tree on Fig. 3 illustrates the different possible combinations. The leaf nodes represent the end node packets and each generation in the tree represents a level of network coding. The root node represents the heaviest coding (level 0), where the relay collapses all the analog network coded packets into a single packet. In this way the number of broadcasts from the relay is kept at a minimum which maximises the possible throughput. However, the heaviest coding also requires the most a priori information at the end nodes and is thus best suited for networks with good link quality. The two children of the root correspond to another slightly lighter level of coding (level 1). This level of coding

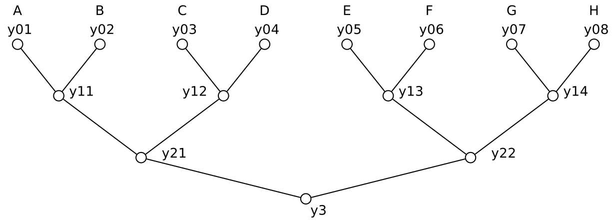


Figure 3: A binary tree indicating possible network coding for a network with eight end nodes.

has a lower maximum throughput but requires less a priori information at the end nodes. Using the lowest level of coding, the relay broadcasts the analog coded packet received from each of the end node pairs without using additional coding. In this case S-DNF operates similarly to M-DNF and the only a priori information required at an end node is its own packet. Hence, there exist a trade-off between decodability and the number of required time slots. The best trade-off between these two depends on the error probabilities in the network. In this way each coding level has different advantages and which level to use is a system design issue.

In a transmission round, an end node goes through the following steps in order to both transmit its own packet and receive its intended packet. 1) Transmit its own packet simultaneously with the other node in the pair. 2) Overhear joint transmissions from other node pairs. 3) Receive the network coded packet broadcast by the relay. 4) Reconstruct the original analog network coded packet from its own pair by XOR'ing with overheard information. 5) Reconstruct its intended packet by XOR'ing with its own packet.

In order to illustrate the potential of the proposed scheme we compare the performance gains of S-DNF and M-DNF over Traditional SingleHop, TSH. The comparison is made on delivery of a single packet from each end node to its destination in a network with a topology as in Fig. 2 but with N end nodes. In this description of the potential, we assume perfect links in the network. Hence performance is measured in required time slots. More in depth analysis is performed in section 4.

Using TSH a single packet is delivered in one hop. Since N packets are to be transmitted and no packets can be transmitted concurrently, this scheme requires N time slots. M-DNF is able to deliver the two packets transmitted between a pair within two time slots. One slot with joint transmissions from the end nodes to the relay and one slot with a broadcast of the combined packet from the relay node. This operation is carried out for all $\frac{N}{2}$ pairs in the network, hence N time slots are needed with this scheme as when using TSH. This shows that M-DNF has no gain over TSH in the ideal case with perfect links. However, when distances and corresponding link qualities are taken into account, M-DNF will have an advantage since it uses shorter and more reliable links.

The proposed scheme, S-DNF, also utilises joint transmissions in each pair, therefore $\frac{N}{2}$ time slots are needed in order to send all data to the relay node. Moreover a single additional time slot is needed to broadcast the final combined packet, when the heaviest coding is utilised. S-DNF is thus able to deliver all data within $\frac{N}{2}+1$ time slots. This gives a performance gain, $G_{max} = \frac{N}{N/2+1}$. Note that G_{max} is 1 for $N = 2$, since in this case only regular DNF is possible, and note that it approaches 2 as N goes to infinity. In this way S-DNF emulates full duplex communication between the nodes for large N. This maximum gain is only achieved in the ideal case, where all nodes are able to overhear all other transmissions. In the worst case scenario, no nodes can overhear anything, and the relay has to transmit network coded packets containing only two signals, since the end nodes only know their own packet a priori. This is just regular DNF, and then the gain is 1. However, for any partial amount of overheard transmissions, a gain between 1 and G_{max} is achievable.

4. Analysis

The proposed scheme, S-DNF, is analysed with the purpose of deriving equations for evaluating the average throughput in a network using S-DNF. The aim is to derive a general expression which is applicable for any given code in S-DNF, i.e. any combination of broadcasts by the relay from the binary tree structure.

The medium access is controlled using round-robin scheduling giving each node pair a chance to exchange packets in each round. Given the assumed error control, see section 2, only two outcomes are possible as seen from an end node. After the relay node has broadcast an end node achieves either maximum throughput, t_{max}, or zero throughput. Using normalised bandwidth the maximum throughput depends solely on the number of time slots, M, used to distribute the packets, i.e. $t_{max} = 1/M$. The average throughput for a network of n nodes is therefore given by:

$$\bar{t} = \frac{\sum_{i=1}^{n} P_{di} \cdot t_{max}}{n} = \frac{\bar{P}_d}{M} \tag{3}$$

where P_{di} is the probability that the i−th node is able to decode the packet from the other node in the pair. When deriving the average P_d, it is not possible to use the individual values of P_d for the end nodes to find the mean value, because correlation in the probabilities exist. The P_d of multiple end nodes may depend on the same joint transmission from an end node pair to the relay, because network coding is performed at the relay. It is therefore necessary to condition P_d on the joint transmissions to the relay, in order to decorrelate the individual values of P_d.

In an example of a network with four end nodes, there are two pairs each having a joint transmission to the relay. The outcome of a single joint transmission is binary and a success is denoted (1) and has probability $P_s(\gamma_{iR}, \gamma_{jR})$. A failure is denoted (0) and has probability $P_p(\gamma_{iR}, \gamma_{jR})$. This means that four different outcomes are possible for two transmissions; (00), (01), (10) and (11). It is then necessary to find the probabilities of these outcomes and to find the corresponding conditional expressions of P_d, $P_d|(xx)$. The average P_d is then given by:

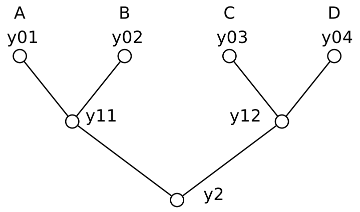


Figure 4: The binary tree indicating possible network coding for a network with four end nodes.

$$\bar{P}_d = P(00) \cdot P_d|(00) + P(01) \cdot P_d|(01) + P(10) \cdot P_d|(10) + P(11) \cdot P_d|(11) \quad (4)$$

When deriving the expression for $P_d|(xx)$, a systematic approach is taken based on the structure of the binary tree. In Fig. 4 the tree for a network with four end nodes is illustrated. To indicate the network size, we denote $P_d|(xx)$ for this specific network $P_{d4}|(xx)$.

The analysis will be performed for node A in this tree, but the structure of $P_{d4}|(xx)$ is equal for all end nodes, only indices are changed. For the sake of simplicity, link notation is left out on transmissions from the relay to node A, i.e. $P_s = P_s(\gamma_{RA})$ and $P_p = P_p(\gamma_{RA})$. In the derivation of $P_{d4}|(xx)$ the expression will be constructed step by step, and $P_{d4}|(xx)^*$ denotes an unfinished expression, i.e. a prefix of the final expression.

Initially the probabilities of the different outcomes in the joint transmissions from the end node pairs to the relay are presented:

$$\begin{aligned} p(00) &= P_p(\gamma_{AR}, \gamma_{BR}) \cdot P_p(\gamma_{CR}, \gamma_{DR}) \\ p(01) &= P_p(\gamma_{AR}, \gamma_{BR}) \cdot P_s(\gamma_{CR}, \gamma_{DR}) \\ p(10) &= P_s(\gamma_{AR}, \gamma_{BR}) \cdot P_p(\gamma_{CR}, \gamma_{DR}) \\ p(11) &= P_s(\gamma_{AR}, \gamma_{BR}) \cdot P_s(\gamma_{CR}, \gamma_{DR}) \end{aligned}$$

For the derivation of $P_{d4}|(xx)$, note that the easiest way for node A to decode what is intended for it, is by receiving y_{11} from the relay. For this to happen, the XOR of y_{01} and y_{02} must be successfully received at the relay. This is a joint transmission from an end node pair, which is represented by a binary variable depending on (xx). This is denoted J_{AB} and takes the value 1 in the conditions (10) and (11) and zero otherwise. In addition to this, y_{11} must be successfully received by node A. Thus, $P_{d4}|(xx)^*$ equals:

$$P_{d4}|(xx)^* = J_{AB} \cdot P_s \quad (5)$$

In figure 4 it is seen that another opportunity for node A to become able to decode exists, namely from receiving y_2. This is not enough, however, since we also need y_{03} and y_{04}, in addition to y_{01} and y_{02} from earlier, at the relay in order to be able to broadcast y_2 in the first place. When adding this to the expression, we must condition on y_{11} not being received. Otherwise, the expression will not be bounded by 1.

$$P_{d4}|(xx)^* = J_{AB}(P_s + P_p \cdot J_{CD} \cdot P_s) \quad (6)$$

In this second opportunity to decode, overhearing or the equivalent help from the relay is necessary. This must be included in the expression. In this case the information in y_{12} is necessary. This can be provided by the relay directly or alternatively by overhearing y_{03} and y_{04}. The probability of receiving either y_{12} or y_{03} and y_{04} is denoted $G_4^{y_{12}}$.

$$G_4^{y_{12}} = 1 - (1 - P_s(\gamma_{CA}, \gamma_{DA}))P_p \quad (7)$$

The final expression for P_{d4} is thus:

$$P_{d4}|(xx) = J_{AB}(P_s + P_p \cdot J_{CD} \cdot G_4 \cdot P_s) \quad (8)$$

Having constructed the expression for $P_{d4}|(xx)$ in this systematic stepwise fashion is very useful when increasing the network size. Note that if we wish to derive $P_{d8}|(xx)$ for the tree in Fig. 3, we would need the same elements as already derived for $P_{d4}|(xx)$. Hence, the expression for $P_{d4}|(xx)$ is a prefix of the expression for $P_{d8}|(xx)$. Deriving $P_{d8}|(xx)$ is thus just a matter of continuing the construction process one step further. This means repeating the process described in equations (6) to (8). In this new step, the part provided by G_8 is a bit more extensive, since now the necessary help from the relay can come from more combinations. We note that y_{22} is what is needed. This can be provided directly by the relay, or in the shape of both y_{13} and y_{14}, which both in turn can be provided directly or by overhearing of either y_{05} and y_{06} or y_{07} and y_{08} respectively. This is a recursive structure, in which the structure of G_4 can be reused on y_{13} and y_{14}, as well as on y_{22} for the final expression of G_8.

$$\begin{aligned} G_4^{y_{13}} &= 1 - (1 - P_s(\gamma_{EA}, \gamma_{FA}))P_p \\ G_4^{y_{14}} &= 1 - (1 - P_s(\gamma_{GA}, \gamma_{HA}))P_p \\ G_8 &= 1 - (1 - G_4^{y_{13}} \cdot G_4^{y_{14}})P_p \end{aligned} \quad (9)$$

G_8 is the probability that node A holds the necessary information in order to utilise y_3. Note that two additional successful joint transmissions from end node pairs are needed in order to create y_3 at the relay compared to y_{21}. We can therefore express $P_{d8}|(xx)$ as:

$$P_{d8}|(xx) = J_{AB} \cdot (P_s + P_p \cdot J_{CD} \cdot G_4(P_s + P_p \cdot J_{EF} \cdot J_{GH} \cdot G_8 \cdot P_s)) \quad (10)$$

In these derivations it has been taken into account that there is a probability of receiving every packet included in the binary tree. This is obviously not the case for a specific code unless full redundancy is applied. Thus for a specific code, P_s equals zero on "transmissions" of packets which are not a part of the code. This cancels out the terms including impossible events in the general expression, leaving out an expression that fits the specific code.

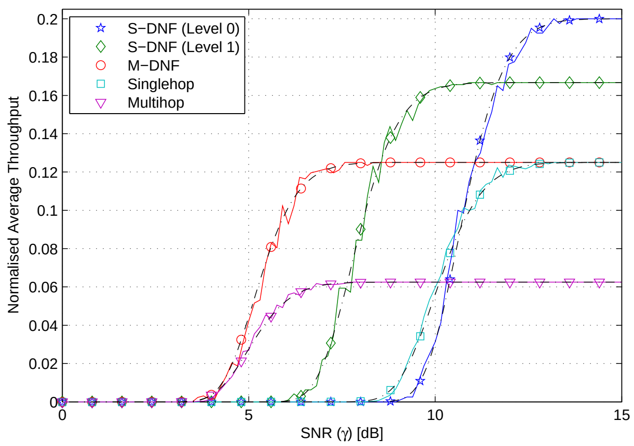


Figure 5: The simulated performance of S-DNF, M-DNF, TSH and TMH plotted on top of their corresponding analytical performance.

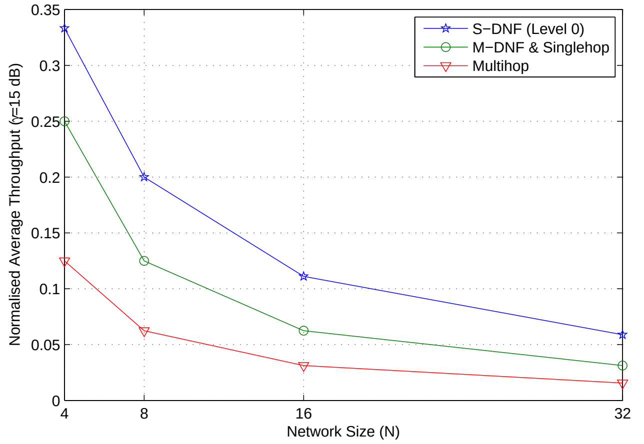


Figure 6: The simulated performance of S-DNF, M-DNF, TSH and TMH for an increasing network size and γ = 15 dB.

5. Results

The proposed scheme is simulated in order to evaluate the performance. The utilised network is similar to the system described in section 2. Hence, we have eight end nodes and a relay node, where the end nodes communicate in pairs through the common relay. Each node generates packets with 128 bytes of random data which are modulated using BPSK. We assume an AWGN channel, where we only account for propagation loss, neglecting fading. The path loss exponent, κ, in Eqn. (1) is set to 2, which corresponds to free space. Each transmitted symbol is detected using a MAP receiver. The performance of the proposed scheme using the heaviest coding is compared with M-DNF, TSH and TMH. Note that S-DNF is comparable to M-DNF when the lowest possible coding level is applied, as described in section 3. In Fig. 5 the normalised throughput of these schemes is plotted as a function of the SNR on the direct link, γ, where each throughput is the average over ten simulations.

The simulated schemes yields the expected throughput as the performance follows the analytic performance closely, plotted as dashed curves in Fig. 5. The probabilistic nature of the AWGN channel is showing as a small ripple on the graphs. At the very low values of γ none of the schemes are able to maintain communication between the end nodes. The link quality increases as γ increases making communication in the network possible. M-DNF and TMH both relies on links between relay and end nodes only. These links have relatively good quality due to the short distance, hence the throughput of M-DNF and TMH becomes non-zero when γ is approximately 3 dB. Since M-DNF uses half the transmissions compared to TMH it converges to a throughput twice as large and the maximum throughput is reached at γ = 8 dB. For S-DNF with level 1 coding the distribution of packets is dependent on an overhearing on slightly longer links, compared to M-DNF. Hence the throughput becomes non-zero when γ is approximately 6 dB and reaches its maximum at γ = 11 dB. At a γ-value around 8 dB the throughput of TSH starts to rise. TSH and S-DNF relies on long links, hence a large γ is required before the throughput starts rising. S-DNF is dependent on several transmissions, hence its throughput does not rise until γ = 9 dB. Both TSH and S-DNF reaches their maximum at γ = 13 dB, where the gain of S-DNF over TSH is 1.625 for a network with eight end nodes. From the graphs in Fig. 5 we see the trade-off between throughput and error resilience. A low coding level will make transmissions possible at low SNR at the cost of maximum throughput. Hence the different levels of coding yields different maximum throughput, where S-DNF with level 0 coding and M-DNF are the upper and lower bounds, respectively.

Fig. 6 shows the same simulation but for a varying network size and a fixed γ. It has been chosen to use γ = 15 dB, since at this value all schemes have converged to their maximum throughput. From the figure it is evident that the performance decreases for all schemes as the network size increases. Moreover, it is seen that S-DNF always has the highest performance and that the gain percentage over TSH increases as the network size increases. This verifies the claim in section 3 that G_{max} increases for increasing network size.

6. Conclusion

A novel scalable relaying scheme based on the existing concept of DeNoise-and-Forward, DNF, has been presented. The proposed scheme, S-DNF, is applicable in a network with an arbitrary number of communicating pairs, all sharing the same relay. This is in contrast to DNF, where only a single node pair at the time exchange packets through the relay. Analysis shows that, by utilising analog and digital network coding, the scheme provides a maximum gain of 2 over direct transmissions, referred to as Traditional Single-Hop (TSH), under the condition that the network size, N, approaches infinity. Moreover, by allowing different levels of applied network coding the proposed scheme offers a trade-off between error resilience and bit rate.

A. Derivation of Decision Region Bound for De-Noise

When nodes i and j perform a joint transmission to node k the sum of the transmitted symbols can take three different values.

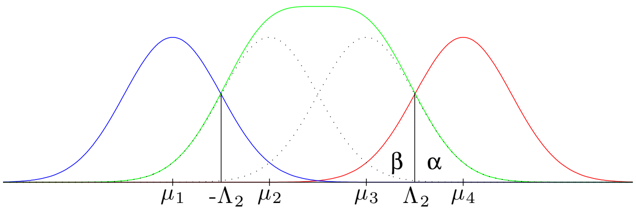


Figure 7: The symbol space for the sum of two BPSK modulated signals. The bound for the decision region for a joint transmission is marked by Λ_2.

However, with the signal power being dependent on the distance the amplitude of the two signals may not be equal. Hence the received analog coded symbol can take four different values, denoted μ_i where $i \in \{1;4\}$. Due to AWGN the amplitude of the received signal is normally distributed around each μ_i. The symbol space containing these four possible combinations along with their PDFs are illustrated in Fig. 7. The leftmost PDF and the rightmost PDF represent symbols where the two source symbols have equal signs and the dotted PDFs are for combinations of source symbols with opposite signs. The solid PDF in the center is the sum of the two dotted PDFs.

The decision region from traditional BPSK cannot be reused for joint transmissions, due to the increased number of possible symbols. It is therefore desired to identify the bound, Λ_2, that yields the lowest error probability, P_b. The solid PDF in the center in Fig. 7 is referred to as f_α. Similarly the rightmost PDF for receiving two symbols with positive amplitude is referred to as f_β. Given a certain Λ_2 the areas marked α and β under these PDFs are used when determining the error probability. The area α refers to the probability that two different symbols are detected as two positive symbols conditioned that two different symbols were transmitted. Similarly the area β refers to the probability that two positive symbols are interpreted as two different symbols. Due to the symmetry around zero we can focus on the right hand side and the total error probability is then given by two times the sum of α and β

$$P_b = 2\cdot\left(\int_{\Lambda_2}^{\infty} f_\alpha(x)dx + \int_{-\Lambda_2}^{\Lambda_2} f_\beta(x)dx\right)$$
$$= 2\cdot\left(F_\alpha(\infty) - F_\alpha(\Lambda_2) + F_\beta(\Lambda_2) - F_\beta(-\Lambda_2)\right) \quad (11)$$

where F_α and F_β are the primitive functions of f_α and f_β respectively. The Λ_2 yielding the lowest P_b is identified by solving the following equation

$$0 = (P_b)'$$
$$= 2\cdot\left(f_\alpha(\infty) - f_\alpha(\Lambda_2) + f_\beta(\Lambda_2) + f_\beta(-\Lambda_2)\right) \quad (12)$$

Note that differentiating $F_\beta(-\Lambda_2)$ with respect to $+\Lambda_2$ causes a change in sign. Moreover, the derivative of $F_\alpha(\infty)$ with respect to $+\Lambda_2$ equals zero, hence

$$0 = 2\cdot\left(-f_\alpha(\Lambda_2) + f_\beta(\Lambda_2) + f_\beta(-\Lambda_2)\right) \Leftrightarrow$$
$$f_\beta(-\Lambda_2) = f_\alpha(\Lambda_2) - f_\beta(\Lambda_2) \quad (13)$$

Usually $f_\beta(-\Lambda_2) \approx 0$, which means the decision bound should be placed in the intersection of f_α and f_β. However, for low SNR $f_\beta(-\Lambda_2)$ is significant and must be taken into account. In this case Λ_2 has a lower value than the value of the intersection.

For any Λ_2), the BER for an analog coded signal is given by

$$P_b(\gamma_{ik}, \gamma_{jk}) = \frac{2\cdot(P(X_{\alpha1} > \Lambda_2) + P(X_{\alpha2} > \Lambda_2))}{4} + \frac{2\cdot(P(X_\beta < \Lambda_2) - P(X_\beta < -\Lambda_2))}{4} \quad (14)$$

where
$X_{\alpha1} \sim \mathcal{N}(\mu_2, \sigma_z^2)$
$X_{\alpha2} \sim \mathcal{N}(\mu_3, \sigma_z^2)$
$X_\beta \sim \mathcal{N}(\mu_4, \sigma_z^2)$

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Physical Layer Network Coding for FSK Systems

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***Abstract*—In this work we extend the existing concept of De-Noise and Forward (DNF) for bidirectional relaying to utilise non-coherent modulation schemes. This is done in order to avoid the requirement of phase tracking in coherent detection. As an example BFSK is considered, and through analysis the decision regions for the denoise operation in DNF are identified. The throughput performance of BFSK in DNF is compared to BPSK.**

I. Introduction

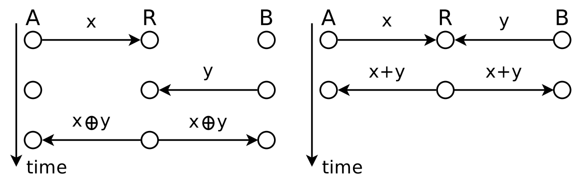
Bidirectional relaying has been the focus of much research within wireless communication recently, [1]–[4]. Traditionally the three node scenario, where nodes A and B communicate with each other through a relaying node R, is considered. Examples of bidirectional relay protocols are Amplify-and-Forward, AF, and Decode-and-Forward, DF, [5], where DF is illustrated in Fig. 1(a). In [6] a concept called DeNoise-and-Forward, DNF, is presented. Here nodes A and B transmit their packets to the relay simultaneously. Assuming proper synchronisation, the signals are added in the air, which is referred to as analog network coding. The relay maps the resulting symbols to a binary message indicating that either equal or different symbols were received. The relay broadcasts this message, which makes an end node able to reconstruct its intended packet by knowing what it transmitted to the relay. Fig. 1(b) shows how packets can be exchanged in only two time slots, when using DNF. The mapping of received symbols to a binary message is effecitvely a remodulation performed in the physical layer, which removes the noise added during transmissions to the relay. This means that the packets are denoised, although decoding is not performed, hence the name.

In [6] BPSK modulation is applied in DNF, hence it is necessary to assume symbol synchronisation and coherent detection. The phase tracking required for coherent detection is impractical, hence non-coherent modulation schemes should be investigated. In this paper we investigate the use of BFSK modulation in DNF. Optimum decision regions are determined through analysis and the expected throughput is presented and compared to that of BPSK in DF and DNF respectively.

II. Analysis of Decision Regions for BFSK

When analysing the decision regions we assume AWGN channels with no interference from other sources. We account for propagation loss and ergodic phase fading, where the phase, ϕ, is uniformly distributed between 0 and 2π. Moreover, symbol synchronisation in joint transmissions is assumed.

FSK systems rely on envelope detection using quadrature receivers. Hence, the received signal is four dimensional and



(a) DF (b) DNF

Fig. 1. Existing approaches to bidirectional relaying.

Gaussian noise components, ω_i, are added to each dimension respectively. The two possible received signals are represented as the following vectors:

$$x_1 = \Big(\underbrace{\sqrt{E_s}\cos\phi_1 + \omega_1, \sqrt{E_s}\sin\phi_1 + \omega_2}_{\alpha}, \underbrace{\omega_3, \omega_4}_{\beta}\Big) \quad (1)$$

$$x_2 = \Big(\omega_1, \omega_2, (\sqrt{E_s}\cos\phi_2 + \omega_3), (\sqrt{E_s}\sin\phi_2 + \omega_4)\Big) \quad (2)$$

The envelope in both frequency bands can be calculated from dimensions 1 plus 2 and 3 plus 4, marked by α and β respectively. Note that assuming AWGN, the envelope in a frequency band containing the signal is Rician distributed, while the envelope in a frequency band containing only noise is Rayleigh distributed.

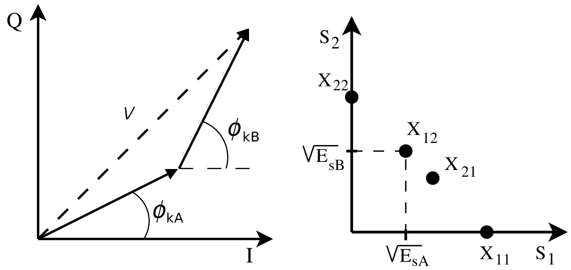
In DNF there exist a significant difference between BPSK and BFSK. For BPSK the transmitted signals are either in phase or in reverse phase, which means that they can be added as scalars. In BFSK, however, they must be added as vectors due to the unknown phase difference. With two possible symbols we have four possible combinations in a joint transmission from nodes A and B. These are denoted x_{ij} where ij refers to the combination of x_1 and x_2 from Eqn. (1) and (2).

$$x_{11} = \Big((\sqrt{E_{sA}}\cos\phi_{1A} + \sqrt{E_{sB}}\cos\phi_{1B} + \omega_1), (\sqrt{E_{sA}}\sin\phi_{1A} + \sqrt{E_{sB}}\sin\phi_{1B} + \omega_2), \omega_3, \omega_4\Big)$$

$$x_{12} = \Big((\sqrt{E_{sA}}\cos\phi_{1A} + \omega_1), (\sqrt{E_{sA}}\sin\phi_{1A} + \omega_2), (\sqrt{E_{sB}}\cos\phi_{2B} + \omega_3), (\sqrt{E_{sB}}\sin\phi_{2B} + \omega_4)\Big)$$

$$x_{21} = \Big((\sqrt{E_{sA}}\cos\phi_{1B} + \omega_1), (\sqrt{E_{sA}}\sin\phi_{1B} + \omega_2), (\sqrt{E_{sB}}\cos\phi_{2A} + \omega_3), (\sqrt{E_{sB}}\sin\phi_{2A} + \omega_4)\Big)$$

$$x_{22} = \Big(\omega_1, \omega_2, (\sqrt{E_{sA}}\cos\phi_{2A} + \sqrt{E_{sB}}\cos\phi_{2B} + \omega_3), (\sqrt{E_{sA}}\sin\phi_{2A} + \sqrt{E_{sB}}\sin\phi_{2B} + \omega_4)\Big)$$



(a) The addition of two signals in the same frequency band.
(b) Signal-space diagram of a joint transmission using BFSK.

Fig. 2. Existing approaches to bidirectional relaying.

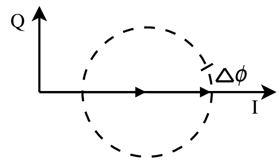


Fig. 3. The possible total signals for uniformly distributed ϕ_d.

The signal components in x_{12} and x_{21} do not interfere, hence the total signal consists of two Rician distributed envelopes. However, when the two nodes transmit the same symbol, the signal components are added. Fig. 2(a) shows a geometrical interpretation of the addition.

The total envelope, ν, detected by the receiver is represented by the dotted line in Fig. 2(a). Depending on the phase difference between the two components, they will either add as scalars, cancel out or something in between. As a result, ν follows a composite distribution, which can be described as a Rician distribution in which the mean value follows some distribution, which is determined later.

Assuming zero phase difference, the envelope of both x_{11} and x_{22} is $\sqrt{E_{sA}} + \sqrt{E_{sB}}$. This is also the coordinate for the symbols in the dimension for the corresponding frequency. The symbols corresponding to x_{12} and x_{21} lie in the first quadrant and if $\sqrt{E_{sA}} = \sqrt{E_{sB}}$ they are represented by the same symbol. If $\sqrt{E_{sA}} \neq \sqrt{E_{sB}}$ however, the symbols are separated resulting in a signal constellation as illustrated in Fig. 2(b), where s_1 and s_2 refers to the dimension of the two frequency bands respectively.

A. Conditional Distributions

The four possible analog coded symbols in BFSK do not follow the same type of distribution, hence the optimum decision regions can not be defined using Maximum Likelihood (ML) detection. Instead Maximum A posteriori Probability (MAP) detection is applied, where the conditional probability density functions of the possible symbols are compared. Note that in DNF we only discriminate between the symbols with equal frequencies and the symbols with different frequencies. Hence the two dimensional space in Fig. 2(b) should be divided into two regions based on the conditional PDFs.

In the case where the received symbol contains different frequencies the total signal is a two dimensional vector, whose elements both follow a Rician distribution. A signal vector is defined by the random variable $U = (U_i, U_j)^T$, where U_i and U_j are the envelopes in the two frequency bands respectively. Hence, the joint conditional PDF of U is:

$$f_U(U|s_{ij}) = \frac{U_i U_j}{\sigma^4} \exp\left(\frac{-(U_i^2 + E_{sA}) - (U_j^2 + E_{sB})}{2\sigma^2}\right) \cdot I_0\left(\frac{U_i\sqrt{E_{sA}}}{\sigma^2}\right) I_0\left(\frac{U_j\sqrt{E_{sB}}}{\sigma^2}\right) \quad (3)$$

Where s_{ij} is the transmitted symbol, and ij is either 12 or 21. I_0 is the modified zero order Bessel function. Assuming that all symbols are equiprobable, the total joint PDF for symbols with different frequencies is:

$$f_U(U|s_{ij}, i \neq j) = \frac{1}{2}(f_U(U|s_{12}) + f_U(U|s_{21})) \quad (4)$$

When the two transmitters use the same frequency, the remaining frequency band contains only noise. These noise components, ω_i, are orthogonal, hence the resulting envelope is Rayleigh distributed with parameter σ since $\omega_i \sim \mathcal{N}(0, \sigma^2)$. This envelope is referred to as U_k, where $k = 2$ if s_{11} is transmitted and vice versa.

$$f_{U_k}(U_k|s_{ij}, i = j) = \frac{U_k}{\sigma^2} \exp\left(\frac{-U_k^2}{2\sigma^2}\right) \quad (5)$$

The envelope in the used frequency band, U_l, where $l = 1$ if s_{11} is transmitted, follows a composite distribution as stated earlier. This distribution is a Rician distribution where the mean value itself follows a distribution. This composite distribution can be expressed as follows.

$$f_{U_l}(U_l|s_{ij}, i = j) = \int_{-\infty}^{\infty} f_\nu(\nu) \cdot \frac{U_l}{\sigma^2} \exp\left(\frac{-(U_l^2 + \nu^2)}{2\sigma^2}\right) I_0\left(\frac{U_l \nu}{\sigma^2}\right) d\nu \quad (6)$$

The mean value is the noiseless envelope, ν, whose distribution is a result of the uniform distribution of the phase difference, $\phi_d = \phi_{kB} - \phi_{kA}$, where k refers to the transmitted frequency. The value of ν depends on ϕ_d and not the individual values of ϕ_{kA} and ϕ_{kB}, hence ϕ_{kA} is used as reference.

In order to derive the distribution of ν, we first consider a probability mass function, PMF. This is a discrete expression of the distribution of ν, i.e. it expresses the probability of experiencing a ν within a certain $\Delta\nu = [\nu_a; \nu_b]$. A certain $\Delta\nu$ corresponds to a certain $\Delta\phi$, whose relationship is expressed by the difference quotient $\frac{\Delta\phi}{\Delta\nu}$. Note that the probability of experiencing a ν within $\Delta\nu$ can be expressed as $\frac{\Delta\phi}{\pi}$, because ϕ is uniformly distributed between 0 and 2π and ν is symmetric around π in this interval, as illustrated in Fig. 3. The PMF can thus be expressed as $\frac{\Delta\phi}{\pi\Delta\nu}$ and for $\Delta\nu \to 0$ this becomes $\frac{d\phi}{\pi d\nu}$, which expresses the PDF we are looking for. This is derived as follows:

$$\nu = \sqrt{(\sqrt{E_{sA}} + \sqrt{E_{sB}}\cos\phi)^2 + (\sqrt{E_{sB}}\sin\phi)^2}$$

$$\phi = \cos^{-1}\left(\frac{\nu^2 - E_{sA} - E_{sB}}{2\sqrt{E_{sA}}\sqrt{E_{sB}}}\right)$$

$$f_\nu(\nu) = \frac{d\phi}{\pi d\nu} = \frac{-\nu}{\pi\sqrt{E_{sA}}\sqrt{E_{sB}}\sqrt{1 - \left(\frac{\nu^2 - E_{sA} - E_{sB}}{2\sqrt{E_{sA}}\sqrt{E_{sB}}}\right)^2}} \quad (7)$$

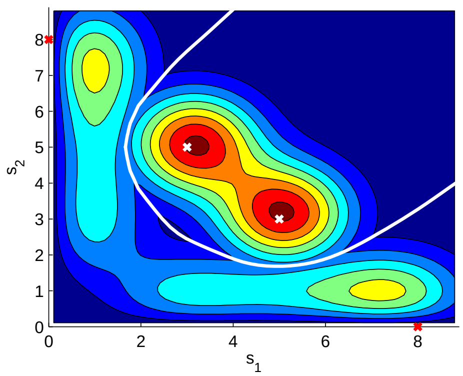


Fig. 4. Both PDFs for $\sqrt{E_{sA}} = 5$, $\sqrt{E_{sB}} = 3$ and $\sigma = 1$.

By combining Eq. (5), (6) and (7) the joint conditional PDFs of symbols x_{ij}, when i and j are equal, can be expressed as $f_U(U|s_{ij}) = f_{U_k}(U_k|s_{ij}) \cdot f_{U_l}(U_l|s_{ij})$. Hence the total PDF of symbols with equal frequencies is then as follows:

$$f_U(U|s_{ij}, i = j) = \frac{1}{2}(f_U(U|s_{11}) + f_U(U|s_{22}))$$

B. The Resulting Decision Regions

According to the MAP detection rule, any point in the two dimensional space in Fig. 2(b) should belong to the region represented by the conditional PDF with the highest density in that particular point. This means that the intersection of the two conditional PDFs comprises the bound of the decision region. In Fig. 4 both PDFs are plotted as a contour plot.

The intersection between the two conditional PDFs is found by solving the following equation:

$$f_U(U|s_{ij}, i \neq j) = f_U(U|s_{ij}, i = j) \quad (8)$$

This is a complex equation, hence in this work it has been solved numerically. This has been done by considering a set of fixed envelopes in s_1 and solving the corresponding equations with only the single variable s_2. A curve indicating the decision region bound can be found by interpolating the solutions. This curve is plotted in Fig. 4 and it agrees with the decision regions indicated by the contour plot.

III. Results

It is known that BPSK outperforms BFSK in regular single link transmissions with respect to BER performance. In this section we compare the performances of the two modulation schemes when applied in DNF and DF respectively.

The BER for BFSK in DNF is determined using a simulation for SNR values between 6 and 23 dB in steps of 1, where $E_{sA} = E_{sB} = 1$. Decision regions for all SNR values are determined as described in section II. As a performance measure we plot the expected throughput, $E[t]$, as a function of SNR, where we assume that the two links have equal SNR ranging from 6 to 23 dB and packets have a length of 128 bytes. In Fig. 5 the expected throughput using DNF and DF is plotted for each modulation scheme respectively. The performance of BPSK in DNF is determined through the analysis in [7]. The results show that using BFSK requires a higher SNR before the throughput converges to its maximal value. DF with BFSK requires ∼4 dB higher SNR, where DNF with BFSK an increase of ∼6 dB. In this way the penalty for using BFSK is more significant in DNF. However, the denoise operation saves a time slot compared to DF, hence the DNF scheme converges to a throughput of 0.5 compared to the 0.33 for DF. If fading was taken into account the relative performance of DNF and DF would be similar, however, a larger SNR would be required before converging to maximum throughput. This is the case for both modulation schemes.

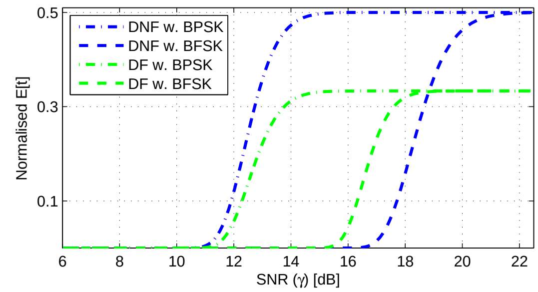


Fig. 5. The expected throughput for DNF and DF using BPSK and BFSK respectively.

IV. Conclusion

The existing concept of De-Noise and Forward (DNF) is based on the coherent modulation scheme BPSK, where the required tracking of phase is impractical. Therefore, this work have extended the concept of DNF to utilise non-coherent modulation schemes, where we have considered BFSK. The decision regions have been identified through analysis. Results shows that BFSK in DNF yields a lower performance compared to BPSK in DNF, as it requires a higher SNR before communication is possible. Hence being independent of the phase requires a larger SNR in order to obtain the same throughput as for BPSK.

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