System identification and control of centralized refrigeration systems

Aalborg Universitet Esbjerg
Title: System identification and control of centralized refrigeration systems
Due date: 8. January 2009
Student: Kim Christensen
Supervisors Zhenyu Yang & Roozbeh Izadi-Zamanabadi

Titlepage

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Abstract: This project focuses on the investigation of adaptive control strategies for a centralized Danfoss refrigeration system. The interest is specially in adaptive minimum variance control.

A model from Danfoss has been adopted and utilized for data generation and acting as the system which to control.

For a subsystem an estimation model and MV_0 controller has been proposed, it does however have some concerns that needs to be addressed. These concerns has been taken up for discussion, but not realised due to time considerations.

Supervisors: Zhenyu Yang & Roozbeh Izadi-Zamanabadi

Student: Kim Christensen

Preface

This report is written as documentation for my final project at Aalborg University Esbjerg. The project is made in cooperation with Danfoss A/S RA-DC.
The purpose of the report is to give the reader insight into which considerations and processes that was used during the project.

The report is divided into four parts. First the introduction, presenting the project, the system, how the project has been broken down and the controller type that is considered.
The second part presents the work on the system identification and controller design and testing. The third part holds the conclusion. The last part contains the appendix, in which some calculations and model relevant equations and constants has been placed.

In the back of the report a CD is enclosed containing a digital copy of the report along with relevant Matlab and Simulink files.

Danfoss A/S RA-D should have a thanks for the help and hospitality enjoyed during a period of the project. Also a special thanks shall go to Niels Kjoelstad Poulsen for fine literature and advises in stochastic adaptive control.

Aalborg Universitet Esbjerg, fall 2008

Kim Christensen

Contents

I Introduction 1

1 System 2
1.1 Energy transfers . . . 3

2 Project stages 4
2.1 Stage 1 . . . 4
2.2 Stage 2 . . . 4
2.3 Stage 3 . . . 4

3 MVC 5

II Design 6

4 System identification 6
4.1 Noise and disturbance specification . . . 6
4.2 System sampling time, structure and order . . . 7

5 Controller 11
5.1 First controller . . . 11
5.2 Second controller . . . 13
5.3 Third controller . . . 16
5.4 Controller instability issues . . . 18
5.5 Summary . . . 19

6 Actuator control 20
6.1 Pitfalls . . . 20

III Conclusions 22

IV Appendix 23

A System description 23
A.1 Equations . . . 23
A.2 Constants . . . 24

B System type determination 25

Part I
Introduction

In supermarkets refrigeration systems are widely used, and the quality of the goods vastly depend on the performance of these systems. The performance of a model-based algorithm depends highly on the underlying model. Since the dimensions and sizes of industrial refrigeration systems changes depending on the application, the underlying model also changes. However the of the model only the parameters change, leaving the overall dynamic description the same.

The projects goals are therefore (from project proposal):

- Establish dynamic model of an evaporator
- Develop / utilize or adopt an estimation algorithm in order to estimate the model parameters for a given evaporator.
- Develop / adopt a control strategy that can also react to the changes of the model (and the underlying system). In particular a control strategy based on Minimum Variance Control is of interest as it is relatively simple and can be used to achieve an optimal performance.
- Test and verify the algorithms on real data (system)

1 System

The system considered in this report is a display case. These can be found in the local supermarket in varying sizes and temperature ranges.

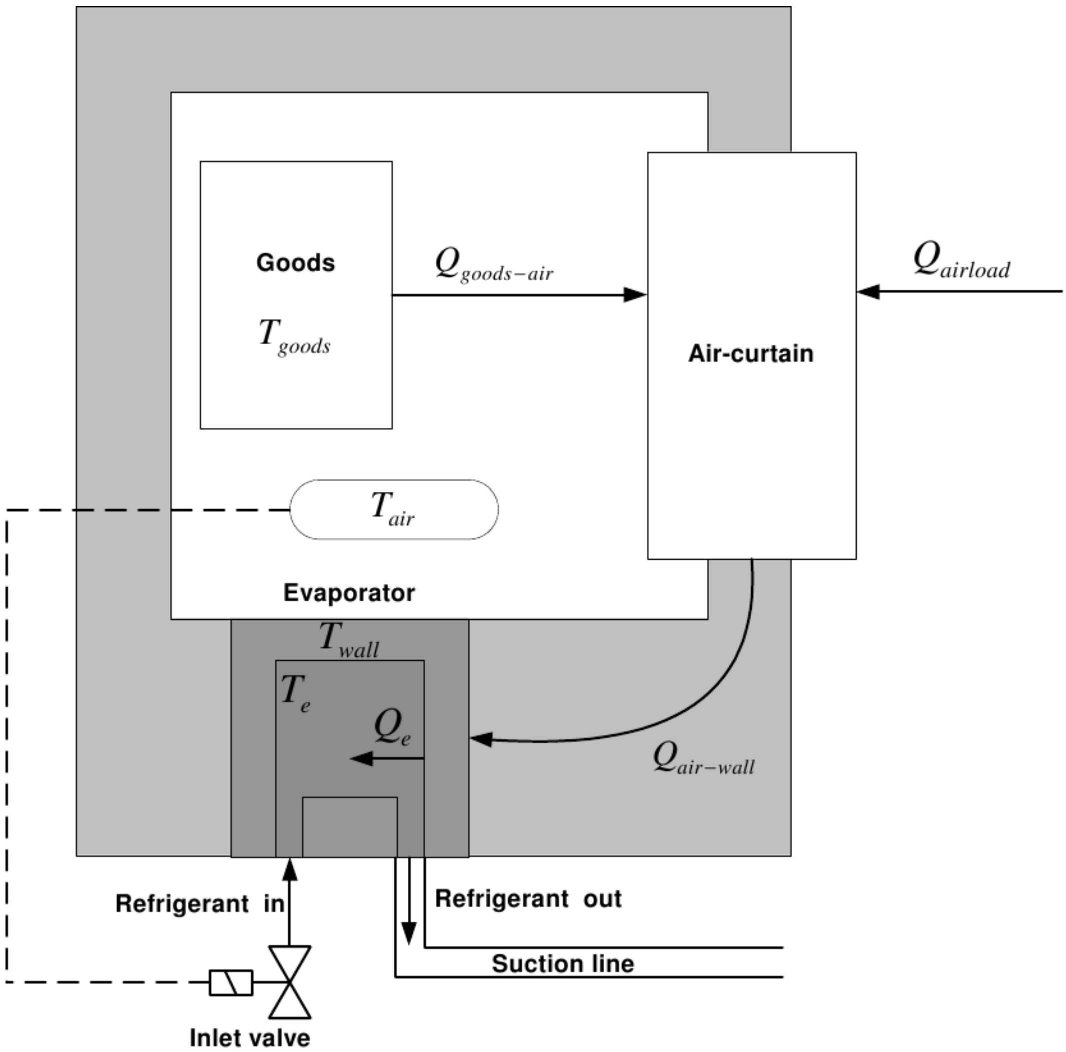

Figure 1: General system

Liquid refrigerant in the evaporator, evaporates absorbing energy from the evaporator wall. Air blown through the evaporator is cooled by delivering energy to the evaporator. The air is then directed as an air-curtain, being heated by the ambient air and the air inside the display case.

Using the energy balances the considered system can be described by the following differential equations.

$$\dot{T}_{wall} = \frac{\dot{Q}_{air-wall} - \dot{Q}_e}{M_{wall} \cdot Cp_{wall}} \quad (1)$$

$$\dot{T}_{goods} = -\frac{\dot{Q}_{goods-air}}{M_{goods} \cdot Cp_{goods}} \quad (2)$$

$$\dot{T}_{air} = \frac{\dot{Q}_{dis} + \dot{Q}_{goods-air} - \dot{Q}_{air-wall}}{M_{air} \cdot Cp_{air}} \quad (3)$$

$$\dot{M}_{ref} = z \cdot \frac{M_{ref,max} - M_{ref}}{\tau_{fill}} - \frac{\dot{Q}_e}{\Delta h_{lg}} \quad (4)$$

The $\dot{Q}_x$'s describes the energy transfer indicated by the subscript. E.g. $\dot{Q}_{air-wall}$ is the transfer of energy from the air to the evaporator wall. The energy transfers are defined in section 1.1.

- z is the opening degree of the expansion valve and the only control input to the system.
- τ_{fill} is the time constant of filling the evaporator.
- Δh_{lg} is the specific latent heat required to evaporate the remaining refrigerant in the evaporator.

The model is used by Danfoss for control purposes and is thus obtained through the department with whom the project is written. The remainder of the relevant modelling can be found in appendix A

1.1 Energy transfers

The energy transfers describes the flow of energy from one part of the system to another. The rate of which the energy flows are governed by a energy transfer coefficient and a temperature difference. The transfer coefficient depends on the material through which the energy flows and the contact area.
In this system the related energy transfers are given by the following equations:

$$\dot{Q}_e = UA_{wall-ref}(M_{ref}) \cdot (T_{wall} - T_e) \quad (5)$$

$$\dot{Q}_{air-wall} = UA_{wall-air} \cdot (T_{air} - T_{wall}) \quad (6)$$

$$\dot{Q}_{goods-air} = UA_{goods-air} \cdot (T_{goods} - T_{air}) \quad (7)$$

2 Project stages

To make the project more manageable, it is divided into stages. The three stages described in the following will generally contain the following

- Stage 1: Use a SISO subsystem of the evaporator and determine its structure for system identification. Implement a identification algorithm to estimate the parameters of the system.
- Stage 2: Examine the identified system from stage 1, and develop an adaptive MV_0 controller for the SISO system.
- Stage 3: Make an controller for expansion valve to control $\dot{Q}_e$.

2.1 Stage 1

The chosen subsystem have T_{air} as output and takes $\dot{Q}_e$ as input. This system is chosen because T_{air} is the quantity which is to be regulated. $\dot{Q}_e$ is not measurable, initially however it is assumed to be known and possible to change directly. In time it will however have to be estimated.

The system can be described by the following set of equations:

$$\dot{T}_{wall} = \frac{\dot{Q}_{air-wall} - \dot{Q}_e}{M_{wall} \cdot Cp_{wall}} \tag{8}$$

$$\dot{T}_{air} = \frac{\dot{Q}_{disturbance} - \dot{Q}_{air-wall}}{M_{air} \cdot Cp_{air}} \tag{9}$$

In equation 9 the heat transfer from the goods and the ambient air surrounding the display case is considered a disturbance, denoted by $\dot{Q}_{disturbance}$. This is due to the temperature of the goods is unknown and will vary with time.

The estimated model is required to have stable zeros of the B and C polynomials, in order of making the minimum variance controller stable.

2.2 Stage 2

Using the model structure and parameters from stage 1, a MV_0 controller is created and tested with a Simulink model. When a stable controller is made, the system identification is added to estimate the parameters along the way, so the adaptation is also used.

2.3 Stage 3

Finally a controller for $\dot{Q}_e$ must be made, using the expansion valve as the actuator. This part of the system is non-linear and would thus need to be linearised. For the resulting linear system a controller and estimator should be made.

3 MVC

Litterature used [1]
The goal of Minimum Variance Control (MV_0C) is to minimize the variance of the system output tracking a certain set-point. The criteria for the control is thus:

$$\bar{J} = E\{(y_{t+k} - \omega_t)^2\} \tag{10}$$

Assuming the following ARMAX model structure:

$$\hat{A}(q^{-1})y_t = q^{-k}\hat{B}(q^{-1})u_t + \hat{C}(q^{-1})e_t + \hat{d} \tag{11}$$

Where d is the mean of the disturbance and

$$\hat{A}(q^{-1}) = 1 + \hat{a}_1 q^{-1} + \cdots + \hat{a}_{na} q^{-na} \tag{12}$$
$$\hat{B}(q^{-1}) = \hat{b}_0 + \hat{b}_1 q^{-1} + \cdots + \hat{b}_{nb} q^{-nb} \tag{13}$$
$$\hat{C}(q^{-1}) = 1 + \hat{c}_1 q^{-1} + \cdots + \hat{c}_{nc} q^{-nc} \tag{14}$$

The MV_0 controller is given by equation 15.

$$\hat{B}(q^{-1})G(q^{-1})u_t = \hat{C}(q^{-1})\omega_t - S(q^{-1})y_t - G(1)\hat{d} \tag{15}$$

Where the polynomials $G(q^{-1})$ and $S(q^{-1})$ is determined by the diophantine equation 18. The order of the polynomials n_g and n_s are given by:

$$n_s = k - 1 \tag{16}$$
$$n_g = max(n_a - 1, n_c - k) \tag{17}$$

$$\hat{C}(q^{-1}) = \hat{A}(q^{-1})G(q^{-1}) + q^{-k}S(q^{-1}) \tag{18}$$

For the MVC controller to be stable, it requires all zeros to be inside the unit circle. To meet that requirement all roots of the B and C polynomials must be inside the unit circle.

Part II
Design

4 System identification

The system identification consists of several parts. First the noise in the system should be analysed as to determine the model structure. Second the system structure, order and sampling time needs to be established. Last an identification method needs to be chosen.

The system considered here is a SISO system, taking $\dot{Q}_e$ as input and T_{air} as output. For testing purposes a test model implemented in Simulink was used.
To choose an appropriate structure a transfer function for the system has been derived, the derivation can be seen in appendix B on page 25. The results are the following discrete transfer function.

$$H(z) = \frac{-8.867 \cdot 10^{-7} z^{-1} - 8.735 \cdot 10^{-7} z^{-2}}{1 - 1.956 z^{-1} + 0.956 z^{-2}} \tag{19}$$

The transfer function suggests a 2nd order system, so this should be the starting premises for the model structure.

4.1 Noise and disturbance specification

The noise can be divided into three major parts.

- Measurement noise from the air temperature sensor
- The load on from the ambient air
- The varying heat transfer from the goods.

Measurement noise

The measurement noise from the sensor should be considered a normally distributed signal $n(\mu, \sigma)$ with $\mu = 0$ and $\sigma = 0.117$.

Ambient air load

The load of the ambient air is considered constant and of a magnitude of 3000J/s. This is however just a approximation used for simulation purposes. In a real system this load would depend on the temperature difference between the air and the ambient, and other factors such as the contact area.

Heat transfer from goods

The disturbance imposed by the heat transfer from the goods, will start at a value, dependant on initial conditions, and asymptotically go to zero as the temperature difference between air and goods decreases.
This can be modeled as either a slowly changing constant or a first order system.

4.2 System sampling time, structure and order

To determine the sampling time, the rule of 10 samples on a rise time is employed. So to determine the sampling time, a step input was given to the Simulink model and the rise time was measured.

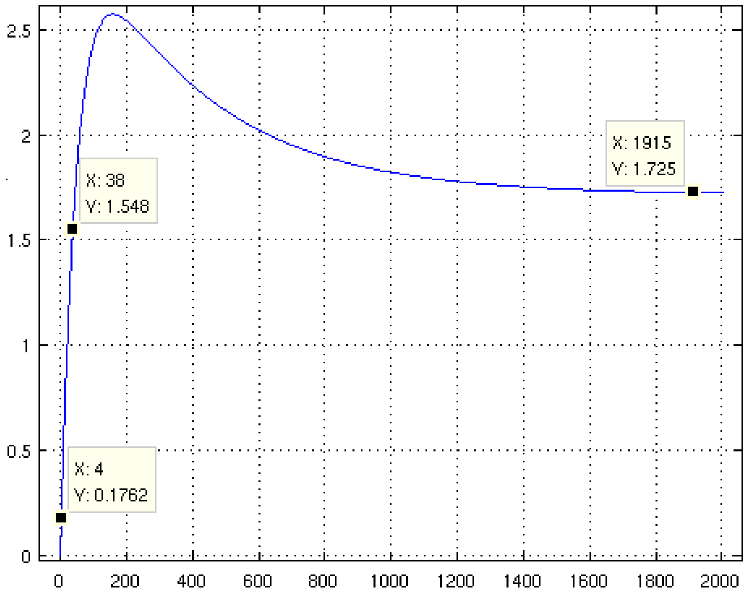

Figure 2: System response to a step input of 3000.

$$T_s = \frac{T_r}{10} = \frac{38s - 4s}{10} = 3.4s \simeq 3s \tag{20}$$

To determine the structure and order of the system the earlier mentioned knowledge regarding the transfer function order and the characteristics of the noise is taken into consideration. This suggests a ARX model of the order 221 with separate parameter for the mean of the noise.
To make a off-line identification Matlab's system identification toolbox is used. This does however not support a model structures with a mean of the noise. This might lead to the models not fitting as they should.
For the test identification, data was generated using the Simulink model. As input

data, a constant with a sampled noise signal overlaid, was used. The input and output can be seen on figures 3 and 4.

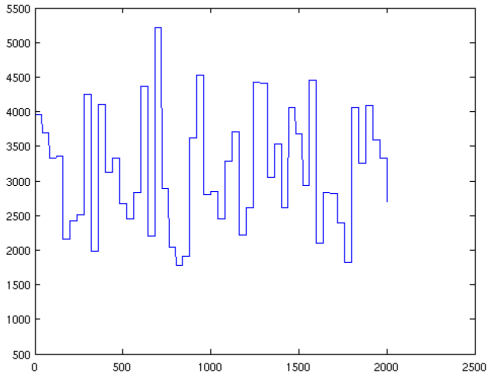

Figure 3: Generated input

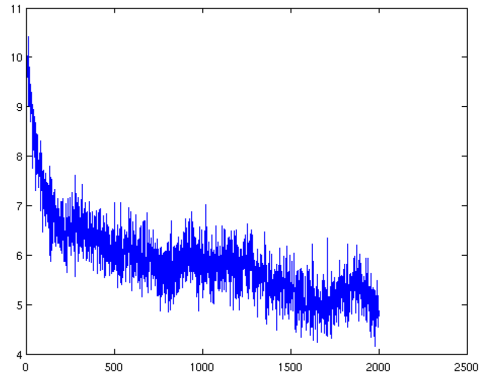

Figure 4: Generated output

Besides the above proposed model, others of ARMAX and ARX structure was tested with varying orders. The models tested can be seen in tables 1 and 2.

Order (ABCk)	Stable zeros	Stable poles	Loss function
1111	yes	marginal	0.132512
2111	yes	marginal	0.132453
2211	no	marginal	0.132414
2121	yes	no	0.124591
2112	yes	marginal	0.132388
2221	no	no	0.12459
2122	yes	no	0.124644
2222	no	marginal	0.13214
3111	yes	marginal	0.13237
3211	no	marginal	0.132335
3121	yes	marginal	0.132257
3112	yes	marginal	0.132365
3221	no	marginal	0.124642
3122	yes	marginal	0.132258
3222	no	marginal	0.132065

Table 1: Table of test identified ARMAX models and some properties

Order (ABk)	Stable zeros	Stable poles	Loss function
111	yes	yes	0.241625
211	yes	yes	0.183005
221	no	yes	0.182633
212	yes	yes	0.182781
222	no	marginal	0.181905
311	no	marginal	0.165182
312	yes	marginal	0.165221
322	no	marginal	0.164762
331	no	marginal	0.164727
332	no	marginal	0.164467
323	no	marginal	0.164723
333	no	marginal	0.164632

Table 2: Table of test identified ARX models and some properties

To choose a model and order, the requirements for the MV_0 controller was taken into account, along with the before mentioned knowledge regarding the model structure and the loss function.

The ARX 221 does however not have stable zeroes, and was on that account excluded for use with the controller type.

An ARX 212 model was also an likely candidate, it was however discarded after unsuccesful attempts to make a stable controller based upon the model. The underlying reason remains uncertain.

The model that was chosen was an ARX 211 with the separate parameter for the noise. This choice is based on the model having stable zeros, and thus will fulfil the requirements for the MV_0 controller.

The model is then described by the following:

$$\hat{A}(q^{-1})y_t = q^{-k}\hat{B}(q^{-1})u_t + \hat{C}(q^{-1})e_t + \hat{d} \tag{21}$$

$$\hat{A}(q^{-1}) = 1 + \hat{a}_1 q^{-1} + \hat{a}_2 q^{-2} \tag{22}$$

$$\hat{B}(q^{-1}) = \hat{b}_0 \tag{23}$$

$$\hat{C}(q^{-1}) = 1 \tag{24}$$

$$k = 1 \tag{25}$$

4.2.1 Choice of identification method

Since the identification should be used on-line with the controller, a recursive method should be used to reduce the amount of computation required for each sample. For an ARX model RLS will be sufficient.

Had the model been of the ARMAX type the usage of either RML or RELS is recommended by the literature. The choice is then dependant on the requirements of convergence and computation load.

Since the model is an ARX 211 the RLS is used. The algorithm can then be expressed by the following

$$\hat{A}(q^{-1})y_t = q^{-k}\hat{B}(q^{-1})u_t + \hat{C}(q^{-1})e_t + \hat{d} \tag{26}$$

$$y_t = \varphi_t^T\hat{\theta}_t + e_t \tag{27}$$

$$\varphi_t^T = \left(-y_{t-1}, -y_{t-2}, u_{t-1}, 1\right) \tag{28}$$

$$\hat{\theta}^T = \left(\hat{a}_1, \hat{a}_2, \hat{b}_0, \hat{d}\right) \tag{29}$$

For each sample the following calculations have to be made:

$$\epsilon_{t+1} = y_{t+1} - \varphi_{t+1}^T\hat{\theta}_t \tag{30}$$

$$P_{t+1}^{-1} = P_t^{-1} + \varphi_{t+1}\varphi_{t+1}^T \tag{31}$$

$$\hat{\theta}_{t+1} = \hat{\theta}_t + P_{t+1}\varphi_{t+1}\epsilon_{t+1} \tag{32}$$

5 Controller

The design of the controller was done through three steps. First the model identified with the Matlab toolbox was used. Next a model identified off-line with the RLS algorithm was used. Finally a adaptive controller using on-line RLS. Here the off-line estimated values is used as initial estimates.

For the off-line estimated systems there has been some mismatch with the DC-gain, causing the controllers to become unstable. This was solved by introducing a gain of -1 after the controller. A discussion of the issue can be found in section 5.4.

In the following T_{air} is substituted for y_t, and $\dot{Q}_e$ for u_t.

5.1 First controller

Following section 4.2 an ARX 211 model was identified. From that model the first controller was derived using the following design procedure.

A note should be taken that the ARX model is considered a special case of the ARMAX model where the C polynomial is equal 1. This is to make the ARX model usable in the MV_0 controller.
Additionally as mentioned in section 4.2 the Matlab toolbox does not support mean of the noise, thus the mean of the noise is equal zero in the first controller.

The system is then given by the following structure

$$\hat{A}(q^{-1})y_t = q^{-k}\hat{B}(q^{-1})u_t + \hat{C}(q^{-1})e_t + \hat{d} \tag{33}$$

Where the polynomials are given by:

$$\hat{A}(q^{-1}) = 1 - 0.4868q^{-1} - 0.4929q^{-2} \tag{34}$$
$$\hat{B}(q^{-1}) = 3.323 \cdot 10^{-5} \tag{35}$$
$$\hat{C}(q^{-1}) = 1 \tag{36}$$
$$\hat{d} = 0 \tag{37}$$
$$k = 1 \tag{38}$$

To determine the controller, equation 18 is used.

$$\hat{C}(q^{-1}) = \hat{A}(q^{-1})G(q^{-1}) + q^{-k}S(q^{-1}) \tag{39}$$
$$1 = (1 + \hat{a}_1 q^{-1} + \hat{a}_2 q^{-2})(g_0) + q^{-1}(s_0 + s_1 q^{-1}) \tag{40}$$

Equating the coefficients of q^n gives the following:

$$1 = g_0 \tag{41}$$
$$0 = \hat{a}_1 + s_0 \Rightarrow s_0 = -\hat{a}_1 \tag{42}$$
$$0 = \hat{a}_2 + s_1 \Rightarrow s_1 = -\hat{a}_2 \tag{43}$$

The polynomials used for the controller is then given by:

$$S(q^{-1}) = s_0 + s_1 q^{-1} = -\hat{a}_1 - \hat{a}_2 q^{-1} = 0.4868 + 0.4929q^{-1} \quad (44)$$

$$G(q^{-1}) = 1 \quad (45)$$

Using equation 15, the difference equation for the controller can be derived:

$$\hat{B}(q^{-1})G(q^{-1})u_t = \hat{C}(q^{-1})\omega_t - S(q^{-1})y_t - G(1)\hat{d} \quad (46)$$

$$(3.323 \cdot 10^{-5})(1)u_t = (1)\omega_t - (0.4868 + 0.4929q^{-1})y_t \quad (47)$$

$$(3.323 \cdot 10^{-5})u_t = \omega_t - 0.4868y_t - 0.4929y_{t-1} \quad (48)$$

$$u_t = \frac{1}{3.323 \cdot 10^{-5}} \cdot (\omega_t - 0.4868y_t - 0.4929y_{t-1}) \quad (49)$$

5.1.1 Test of controller

To test the controller it was put into the Simulink model used to generate data for the system identification.
The output from the simulation can be seen on figure 6.

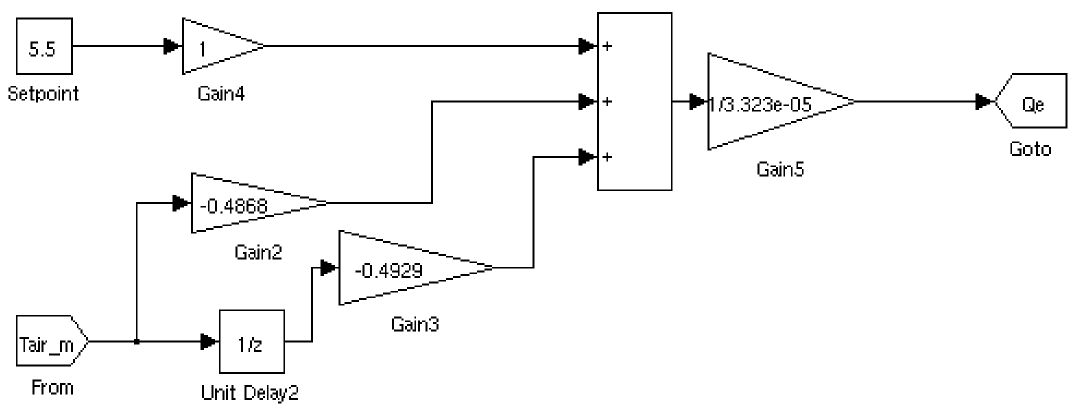

Figure 5: Minimum variance controller in Simulink

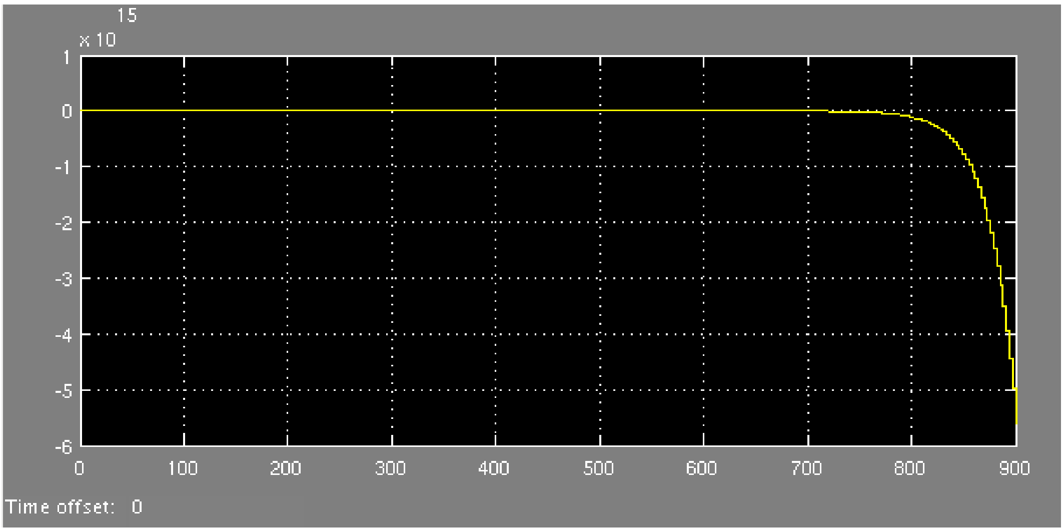

Figure 6: Unstable output from the simulation

As it can be seen the system is not stable with the controller. However by introducing a gain of -1 on the controller output, the system becomes stable as can be seen on figure 7. The instability seems to be caused by the DC-gain of the identified system, this is discussed further in section 5.4.
The controller does however not settle on the desired set-point, which was set to be 5.5.

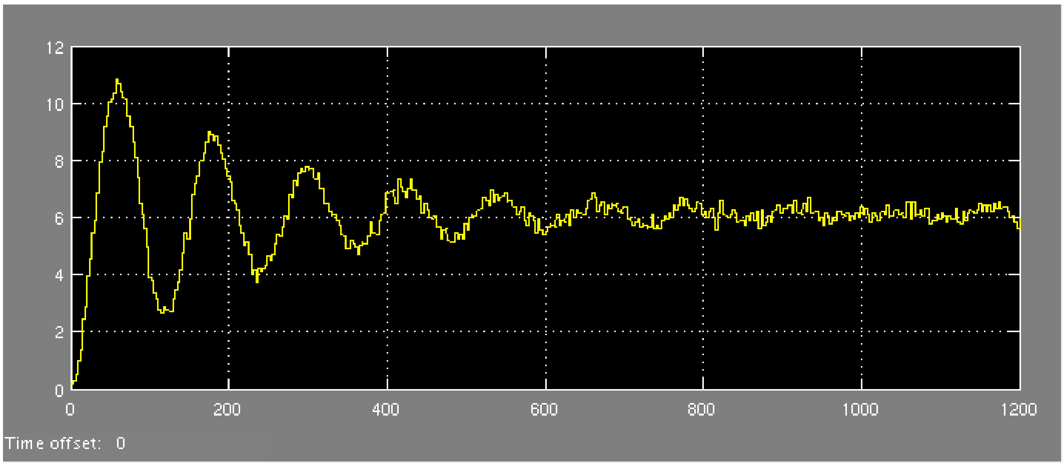

Figure 7: Stable output from the simulation

5.2 Second controller

Since Matlab's system identification toolbox does not support models with a non zero mean of the noise, an recursive system algorithm was run on a similar set of off-line data, and a set of parameters was calculated. The estimated model was then used in the design procedure as described in the first controller.

5.2.1 System identification

To estimate the parameters a similar dataset to that used in the Matlab system identification toolbox were used, along with the algorithm described in section 4.2.1. The dataset was split into two, the first half was used to estimate the parameters, and the second to check the models ability to explain the data.
To gauge how well the estimated model is fitting the data, the square of the difference between the data and the model output was taken and summed up for all samples.

$$error = \sum (y(t) - y_{est}(t))^2 \tag{50}$$

Initially all parameters was set to zero and an estimation was run. On the first estimation the parameters did however not settle on a constant values, figure 8 shows the parameters change in the first estimation. On the figure the x-axis is time in sampels.. The estimates $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_0$ and $\hat{d}$ are the traces with the colours red, blue, black and magenta respectively.
Figure 9 shows the system output (blue) and the estimated output (red), along with the difference between them (black). Like figure 8 the x-axis is time in sampels.

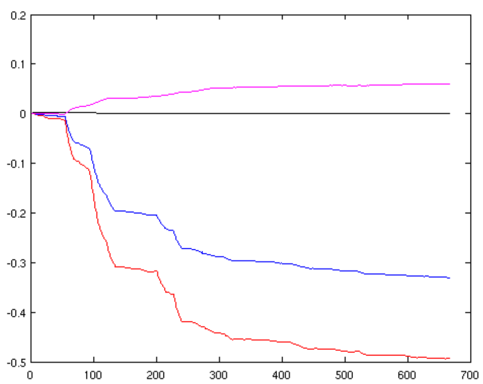

Figure 8: Initial estimation.

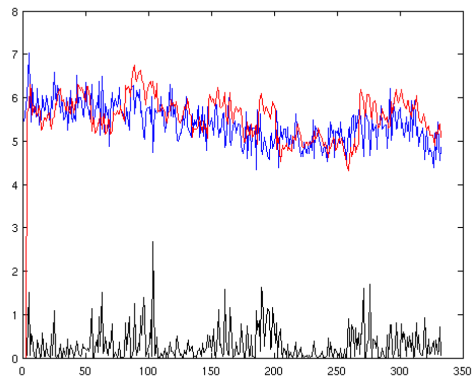

Figure 9: The error from the initial estimation.

After the first estimation the error was summed up to be: $error = 96.5786$.

To get better estimated values, the end estimates was used as initial guesses and the estimation was rerun. This process was done until the parameters became stable and the difference in error was marginal. Figures 10 and 11 shows the estimated values and comparison between system output and the estimated model output respectively. The colour scheme used is the same as on figures 8 and 9.

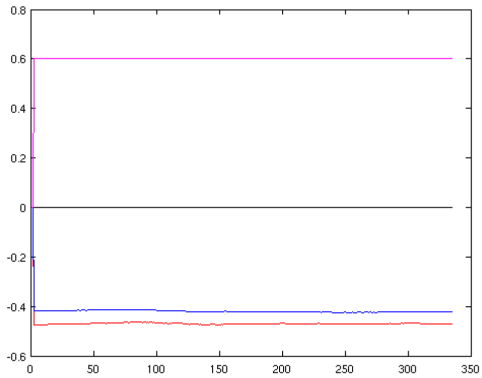

Figure 10: Final estimation.

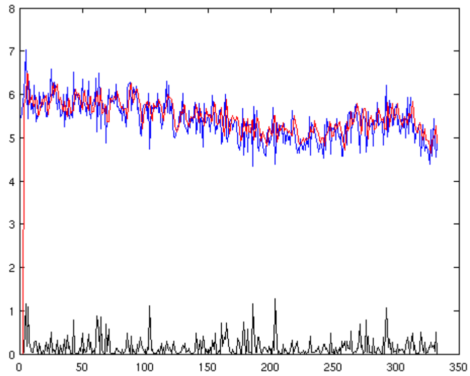

Figure 11: The error from the final estimation.

The error at the last estimation was: $error = 52.8859$.

The off-line estimation did thus end with the following system:

$$\hat{A}(q^{-1})y_t = q^{-k}\hat{B}(q^{-1})u_t + \hat{C}(q^{-1})e_t + \hat{d} \quad (51)$$

$$\hat{A}(q^{-1}) = 1 - 0.5159q^{-1} - 0.3898q^{-2} \quad (52)$$
$$\hat{B}(q^{-1}) = 0.0001 \quad (53)$$
$$\hat{C}(q^{-1}) = 1 \quad (54)$$
$$\hat{d} = 0.599 \quad (55)$$
$$k = 1 \quad (56)$$

5.2.2 Controller and response

The second controller is designed like the first in section 5.1, the only thing changed is the coefficients.
From the design procedure the difference equation for the controller is

$$u_t = \frac{1}{\hat{b}_0} \cdot (\omega_t + \hat{a}_1 y_t + \hat{a}_2 y_{t-1} - \hat{d})$$
$$u_t = \frac{1}{0.0001} \cdot (\omega_t - 0.5159y_t - 0.3898y_{t-1} - 0.599) \quad (57)$$

The controller did however have the same issue as the one identified by the Matlab toolbox. So a gain of -1 was inserted, and the system became stable.
It should be noted that the response of this controller is much faster and the offset has almost been removed (the set-point is still 5.5).

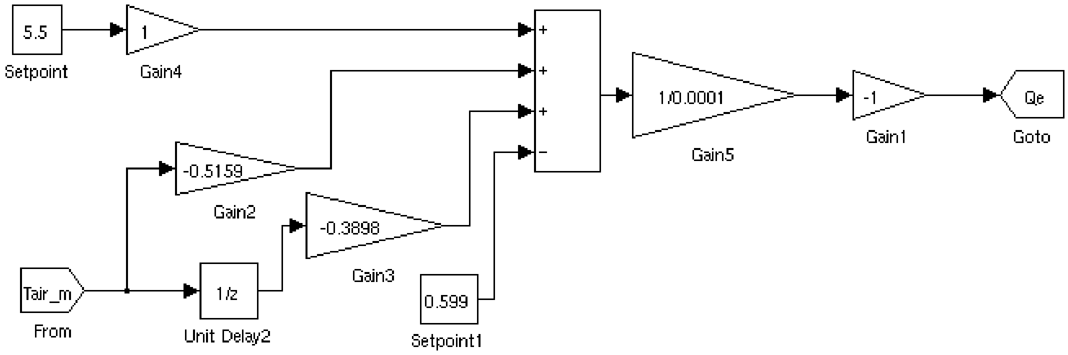

Figure 12: Minimum variance controller in Simulink

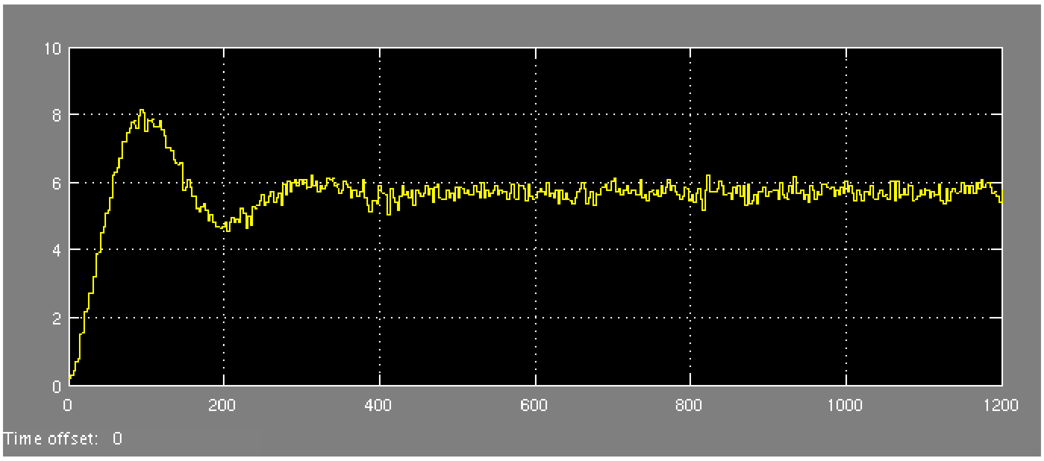

Figure 13: Stable output from the simulation

5.3 Third controller

To add adaptation to the controller, the system identification is run on-line with the controller. The estimation algorithm should however be altered to cope with changing parameters. This can easily be achieved by introducing a forgetting factor to the estimation algorithm. The recursions of RLS with a forgetting factor λ is given by the following:

$$\epsilon_{t+1} = y_{t+1} - \varphi_{t+1}^T \hat{\theta}_t \quad (58)$$

$$P_{t+1}^{-1} = \lambda P_t^{-1} + \varphi_{t+1} \varphi_{t+1}^T \quad (59)$$

$$\hat{\theta}_{t+1} = \hat{\theta}_t + P_{t+1} \varphi_{t+1} \epsilon_{t+1} \quad (60)$$

The forgetting factor should be chosen to be:

$$\lambda \in [0:1] \quad (61)$$

If $\lambda = 1$ the algorithm is standard RLS, and as a rule of thumb, the forgetting factor should be chosen to be:

$$\lambda \in [0.95 : 1[\tag{62}$$

Along with the controller and the system model, the identification algorithm was implemented in Simulink. The choice of forgetting factor was made by testing different values. If the value got less than 0.975 numerical issues would arise with Simulink. The tests showed that varying λ between 0.975 and 0.99 did not impact the controller much, and $\lambda = 0.98$ was chosen.

Making the controller work, posed some issues. Like with the previous controllers, the value of $\hat{b}_0$ caused some troubles. The estimated value of $\hat{b}_0$, is rather unstable, and causes the controller to become unstable if not fixed at a value.
Since the previous controllers required $\hat{b}_0 \simeq -0.0001$, it was fixed at this value, which in turn made the controller stable.
Having $\hat{b}_0$ as a fixed value is an acceptable solution, and this is discussed further in section 5.4. The value does however not seem to be that far from what would be estimated, this is apparent when looking at the estimated value from the simulation. The estimated value of $\hat{b}s_0$ can be seen on figure 14.

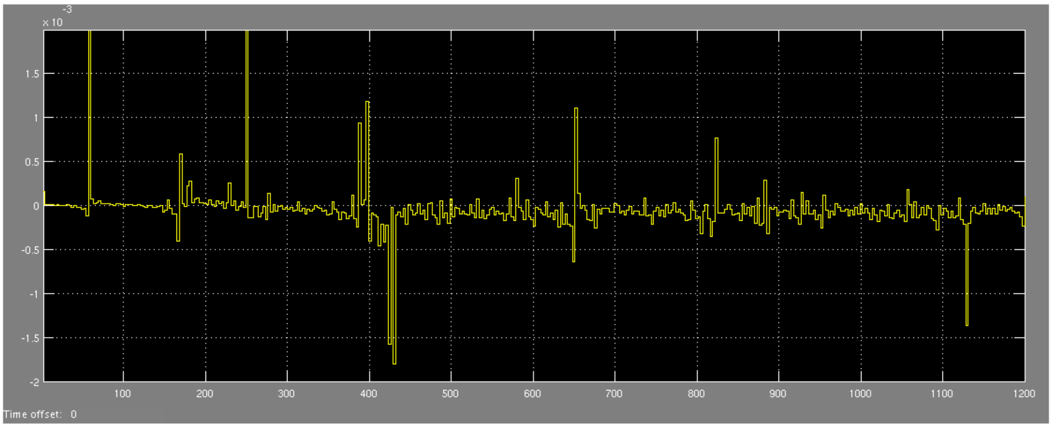

Figure 14: The estimated value of $\hat{b}_0$

The controllers response is shown on figure 15.

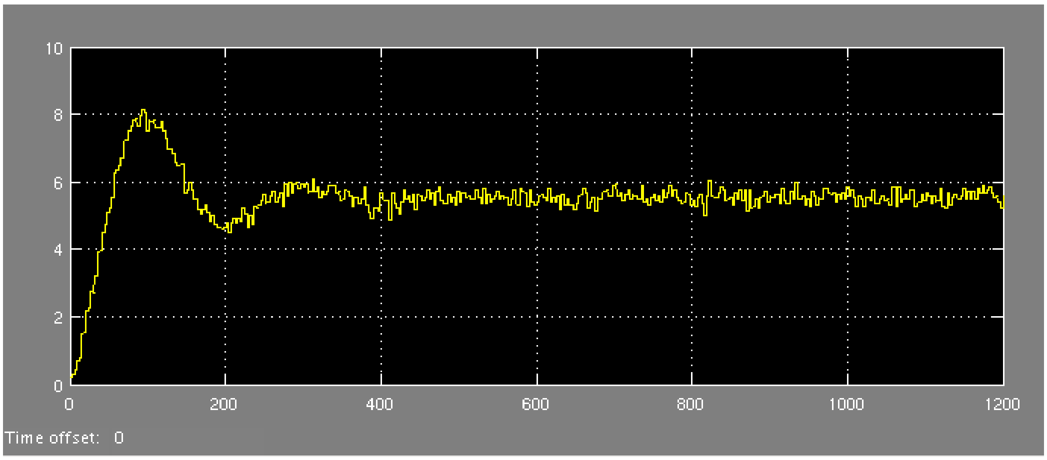

Figure 15: Response of the third controller

5.4 Controller instability issues

To understand the instability with the controllers, the system description which they are build upon must be analysed.
The original system, has an negative infinite gain, thus should the identified system. Common for the first two systems is that their DC-gain is finite and positive. This can explain that the controllers becomes stable when a gain of -1 is introduced.

The underlying reason for the wrong DC-gain can probably be ascribed to the estimate of $\hat{b}_0$. Thus the instability is possibly caused by some issues with the system identification. However the cause for the issue still remains unclear.
Some possible explanations of for the issue can be:

- The data used for the estimation has some issues
- The model structure used for the estimation does not fit the actual system

Estimation data

Considering the model, a positive input should result in a negative output. However due to the nature of the load (viewed as a disturbance) the input needs to have a certain magnitude for this to be true.
This leads to the thought that the data used for the off-line identification does not contain sufficient information or magnitude to accurately describe the system.
When looking at the estimated value of $\hat{b}_0$ on figure 14, the estimate is mostly negative which does give the impression that the above theory might be true.

Model structure

When regarding the model structure and order there are two concerns that should be addressed. First the transfer-function for the model, indicates that the model would fit an ARX 221 and not ARX 211 as chosen. This might be a contributing factor since the information contained in two variables would have to be presented in one.

Then there is the issue about a part of the disturbance. The influence imposed by the energy transfer from the goods, could be modeled as a first order system, and thus the model should have been of the ARMAX structure, however still with a mean of the noise.

5.4.1 Possible solutions

To stabilise the system properly the issues with the identification should be solved. Here there exists some possibilities, non of which was thought of in time to be a part of the project. The obvious possibilities could be:

- Pre-filtering
- Change of model order and/or structure.

Pre-filtering could be used to improve the results of the system identification. This could be achieved by implementing a low-pass filter to filter out frequencies higher than those relevant to the system.

Changing the model could give a better fit and possibly more stable parameters. However following the results from the test identifications in section 4, there exists only a few which would be suitable for use with a MV_0 controller. Thus a change of controller will quite likely accompany a change of model.

5.5 Summary

It is quite apparent that there are some serious issues that needs to be addressed in order of having a proper controller. Most likely a change of model and controller type is needed.
The change in model should be to ARX 221 or ARMAX 2221 depending on how the noise is modeled. Both models should still have a parameter for the mean of the noise. As section 4 implies neither of the models fit for a minimum variance controller. Thus the controller type should be changed to one not affected by the unstable zeroes, which could be a LQG controller.

The issues was however realised too late so it there haven't been time to test it.

6 Actuator control

Due to time limitations this section is a theoretical design of the control of actuator control. A block diagram for the general structure is shown on figure 16.

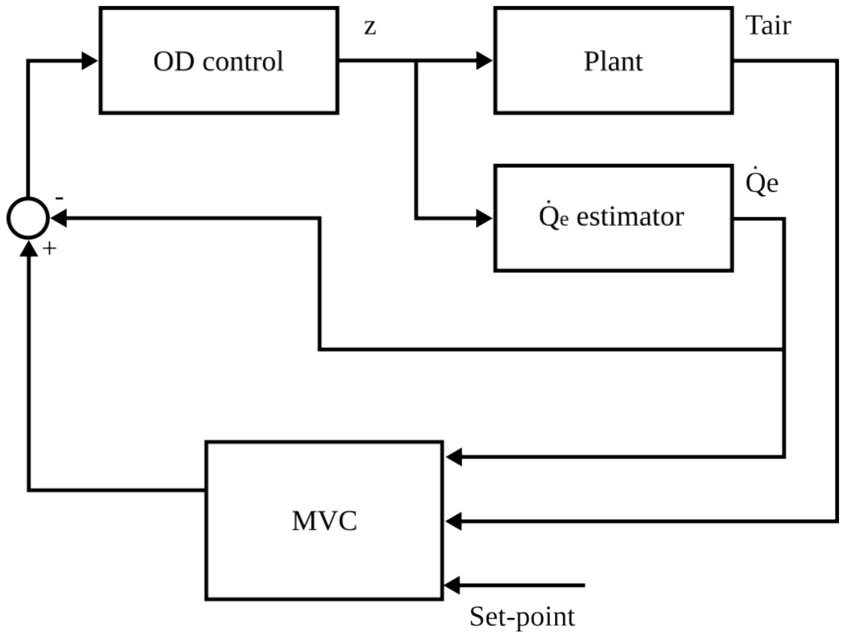

Figure 16: Block diagram of the entire control structure

As previously stated the input ($\dot{Q}_e$) to the MV_0 controller cannot be measured and therefore it has to be estimated. To calculate $\dot{Q}_e$ the mass of refrigerant M_{ref} needs to be estimated. It is possible to make an estimator using the equations found in the system description and appendix A.
The estimated value of $\dot{Q}_e$ can then be used as input for the MV_0 controller and be used as feedback for the linear controller.

The linear controller for the expansion valve controlling $\dot{Q}_e$ should be designed using a linearised version of equation 6. However the size of the working area might be too big for a single equilibrium point, making it necessary to have numerous linearisation and a controllers for each working area.

6.1 Pitfalls

Using the above mentioned control strategy might leave some pitfalls that is worth mention.

The minimum variance controller relies heavily on the system identification, which in turn relies on the accuracy of the measurements taken. Since $\dot{Q}_e$ is an estimate, it's accuracy is paramount to the rest of the control structure.

The actuator control will with this strategy not be adaptive as the rest of the system. This will mean that the algorithm will be targeted for a specific evaporator. This in turn will make limit the algorithm to a subset of industrial refrigeration systems.

Part III
Conclusions

During the course of the project much time was spend on getting enough understanding of system identification and minimum variance control to be able to use the techniques. Following that most of the remaining time was used on attempting to get controller working against a the simulation model.

For the linear subsystem an identification model was established and estimated. It did however have some issues for which solutions have been proposed.
For the estimated model of the subsystem, a controller have been proposed. The controller does however have some issues related to those of the system identification.

For the last of the project stages, presented in section 2, some thoughts about design have been laid out.

In spite of the issues surrounding the system identification a stable controller was derived. To make it so, an estimate in the controller had to be fixed which is not usable in a real application.

Due to the issues with the system identification and subsequently also the controller it seems that there the model and possibly also controller type needs to be changed. Changing the model will with great likelihood result in a system with unstable zeros forcing the controller to be changed to a type that can handle unstable zeroes, like pole placement or an LQG regulator.

Part IV
Appendix

A System description

This small appendix contains the remaining descriptions for the refrigeration system. This will include the remaining equations describing the system, and constants for the system in consideration.
For the equations that are refrigerant specific approximations for the refrigerant "R134a" is presented.

A.1 Equations

$$UA_{wall-ref}(M_{ref}) = UA_{wall-ref,max}\frac{M_{ref}}{M_{ref,max}} \quad (63)$$

$$T_{SH} = (T_{wall} - T_e)(1 - exp(-UA_{SH} \cdot l_{SH})) \quad (64)$$

$$T_{SH} \equiv T_{out} - T_e \quad (65)$$

$$l_{SH} = \frac{(1-\gamma)\cdot V - \frac{M_{ref}}{\rho_{liq}}}{(1-\gamma)\cdot V} \quad (66)$$

$$V = \frac{M_{ref,max}}{\rho_{liq}\cdot(1-\gamma)} \quad (67)$$

$$P_e = P_{suc} \quad (68)$$

$$T_e = TDewP(P_e) \quad (69)$$
$$\cong -4.3544 \cdot P_e^2 + 29.2240 \cdot P_e - 51.2005 \quad (70)$$

$$\Delta h_{lg} = HDewP(P_e) - HBubP(P_e) \quad (71)$$
$$\cong (0.0217 \cdot P_e^2 - 0.1704 \cdot P_e + 2.2988) \cdot 10^5 \quad (72)$$

$$\rho_{liq} = \frac{1}{V_{BuBP}(P_e)} \quad (73)$$

$$\rho_{suc} \cong 4.6073 \cdot P + 0.3798 \quad (74)$$

$$\frac{d\rho_{suc}}{dP_{suc}} \cong -0.0329 \cdot P^3 + 0.2161 \cdot P^2 - 0.4742 \cdot P + 5.4817 \quad (75)$$

A.2 Constants

Constant	Value	Unit	Description
M_{wall}	200	kg	Mass of the evaporator
Cp_{wall}	1000	$\frac{J}{kg \cdot K}$	Thermal capacity of the evaporator wall
$UA_{wall-ref,max}$	3000	$\frac{J}{s \cdot K}$	Maximum heat transfer coefficient between evaporator wall and refrigerant
$M_{ref,max}$	1.00	kg	Maximal mass of refrigerant
$UA_{air-wall}$	500	$\frac{J}{s \cdot K}$	Heat transfer coefficient between evaporator wall and the air
M_{air}	50	kg	Mass of the air contained in the display case
Cp_{air}	1000	$\frac{J}{kg \cdot K}$	Thermal capacity of the air
M_{goods}	200	kg	Mass of the goods contained in the display case
Cp_{goods}	1000	$\frac{J}{kg \cdot K}$	Thermal capacity of the goods
$UA_{goods-air}$	300	$\frac{J}{s \cdot K}$	Heat transfer coefficient between the goods and the air
τ_{fill}	40	s	Time constant, for filling the evaporator
γ	0.7		Mean void (avg. percentage of gas)
UA_{SH}	1	$\frac{J}{s \cdot K}$	Heat transfer coefficient between the gas and the evaporator wall

Table 3: Constants for the equations

B System type determination

To determine which model structure would be optimal for the system identification, the subsystem in state space form is transformed into a transfer-function.

$$\dot{T}_{wall} = \frac{\dot{Q}_{air-wall} - \dot{Q}_e}{M_{wall} \cdot Cp_{wall}} \quad (76)$$

$$\dot{T}_{air} = \frac{\dot{Q}_{disturbance} - \dot{Q}_{air-wall}}{M_{air} \cdot Cp_{air}} \quad (77)$$

The disturbance have no place in a standard state space model, thus it is neglected for now. To make the matrices more simple the following definitions are used:

$$a = \frac{UA_{wall-air}}{M_{wall} \cdot Cp_{wall}} \quad (78)$$

$$b = \frac{UA_{wall-air}}{M_{air} \cdot Cp_{air}} \quad (79)$$

$$c = \frac{1}{M_{air} \cdot Cp_{air}} \quad (80)$$

The system can then on state space form be expressed by the following

$$\begin{bmatrix} \dot{T}_{wall} \\ \dot{T}_{air} \end{bmatrix} = \begin{bmatrix} -a & a \\ b & -b \end{bmatrix} \cdot \begin{bmatrix} T_{wall} \\ T_{air} \end{bmatrix} + \begin{bmatrix} -c \\ 0 \end{bmatrix} \cdot \dot{Q}_e \quad (81)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} T_{wall} \\ T_{air} \end{bmatrix} \quad (82)$$

$$\dot{x} = Ax + Bu \quad (83)$$

$$y = Cx \quad (84)$$

The transfer-function can be derived by using using the following calculation:

$$\begin{aligned} sx &= Ax + Bu \\ (sI - A)x &= Bu \\ x &= (sI - A)^{-1} \cdot Bu \Rightarrow y = C \cdot (sI - A)^{-1} \cdot Bu \end{aligned} \quad (85)$$

(86)

Solving the equation gives the following input output relationship.

$$\begin{aligned} y &= \frac{-bc}{s^2 + s(a+b)} u \\ T_{air} &= \frac{-bc}{s^2 + s(a+b)} \dot{Q}_e \\ \frac{T_{air}}{\dot{Q}_e} &= \frac{-bc}{s^2 + s(a+b)} = \frac{-7 \cdot 10^{-7}}{s^2 + 0.015s} \end{aligned} \quad (87)$$

As it can be seen the system is of second order.
Applying the final value theorem to equation 87 to get the DC-gain gives:

$$\lim_{s\to 0} \frac{-bc}{s(s+(a+b))} = -\infty \tag{88}$$

The infinite DC-gain is caused by the system having a pure integrator.

To get an idea of what structure the discrete equivalent of the system has, the transfer-function is discreteized using Matlab.

$$H(z) = \frac{-8.867 \cdot 10^{-7} z - 8.735 \cdot 10^{-7}}{z^2 - 1.956z + 0.956} = \frac{-8.867 \cdot 10^{-7} z^{-1} - 8.735 \cdot 10^{-7} z^{-2}}{1 - 1.956z^{-1} + 0.956z^{-2}} \tag{89}$$

References

[1] Niels Kjoelstad Poulsen. Stokastisk adaptiv regulering, January 2007.