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STUDENT REPORT

AALBORG UNIVERSITY

STRUCTURAL & CIVIL ENGINEERING, FOURTH SEMESTER

MASTER'S THESIS

Reliability-based classification of the load bearing capacity of existing bridges

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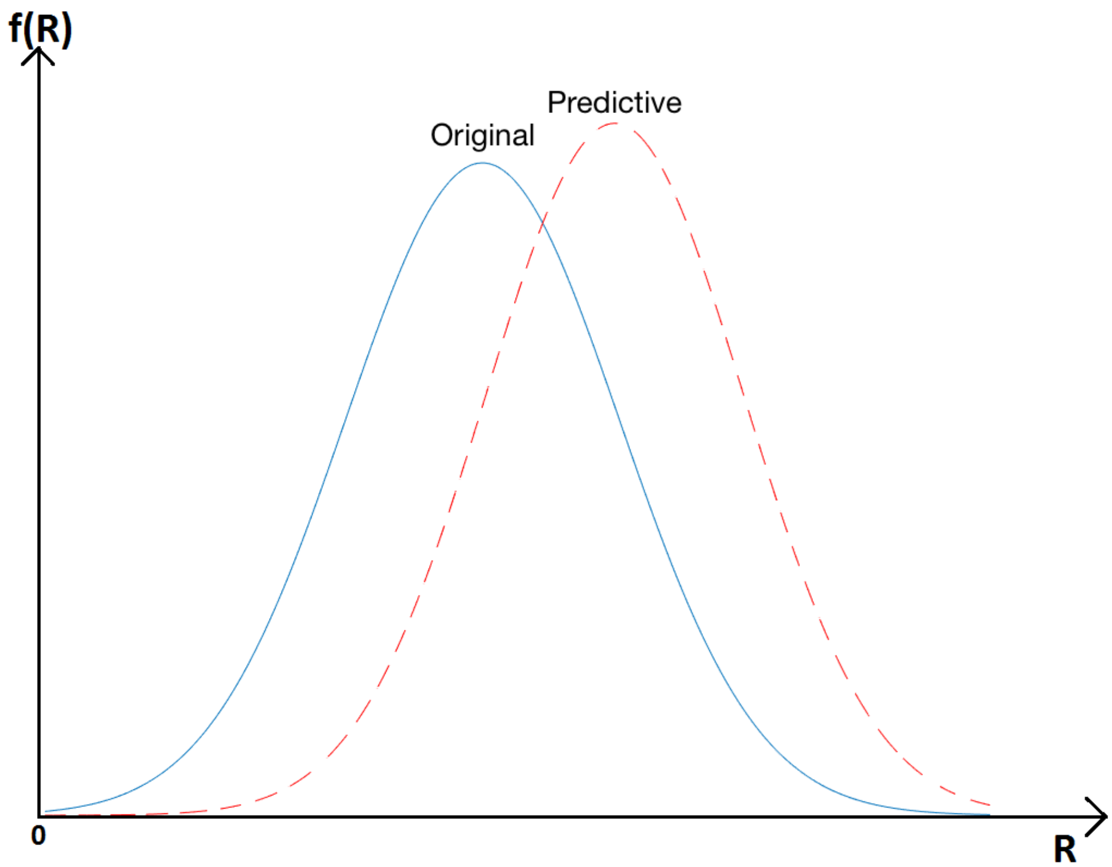

Francisco Manuel Lorente Morales
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Thomas Mans vej 23
9220 Aalborg East

Title:

Reliability-based Classification of the Load Carrying Capacity of Existing Concrete Bridges

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Student:

Francisco Manuel Lorente Morales

Supervisor:

John Dalsgaard Sørensen
Jannie Sønderkær Nielsen

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Abstract:

In this project, a reliability-based classification of the load carrying capacity of existing concrete bridges is carried out. Firstly, a state-of-the-art assessment of existing concrete bridges is presented. Some topics are discussed as for instance the different between assessment and design of bridges, analytic or experimental approach, as well as the procedures and methods of assessment according to European research projects and national codes and guidelines. Then, the stochastic modelling is presented, considering load modelling and material modelling. In the load modelling, the permanent load as well as the variable load, namely traffic load, are modelled. In addition, concrete as well as non-prestressed reinforcement steel are modelled. The model uncertainty is considered also. Once the stochastic modelling is established, a reliability analysis is carried out. Then, these reliability results, measured by the reliability index, β, are improved by different types of updating, which are an updating by proof load tests and updating by test results related to the material parameters, with the aim to avoid costly rehabilitation or demolition of a bridge that in reality fulfills the safety requirements. Finally, an order of implementation of these updating methods is proposed and the recommended future work is mentioned.

Preface

This report is produced by Francisco Manuel Lorente morales, student of the 4th semester of the M.Sc. programme "*Structural & Civil Engineering*" at Aalborg University. The project has been carried out in the period between the 20th of March to the 22st of December 2017, after having received an extension of three months the 2nd of October.

The editor greatly advises the reader to have knowledge about statistics and risk and reliability in engineering.

I would like to thank John Dalsgaard Sørensen and Jannie Sønderkær Nielsen for being the supervisors of this master's thesis. Their guidance, constructive criticism and pleasant meetings were greatly appreciated during the master's thesis period. Additionally, I would like to thank my family and friends, for all the support recieved from them in this period.

Reading Guide

Various references are used throughout the report, by the use of numbers, i.e. the references have the format [1]. The references are collected in a bibliography at the very end of the report, preceding the Appendices.

Labelling of figures and tables are done in accordance to the corresponding chapter for which they are located and a distinction between figures and tables is made. This means that Figure 1.1 and Table 1.1 can be referred to. Equations are labelled by taking the same line of thought, meaning that Eq (1.1) and Eq (2.1) belong to Chapter 1 and 2, respectively.

Contents

CHAPTER 1

Introduction

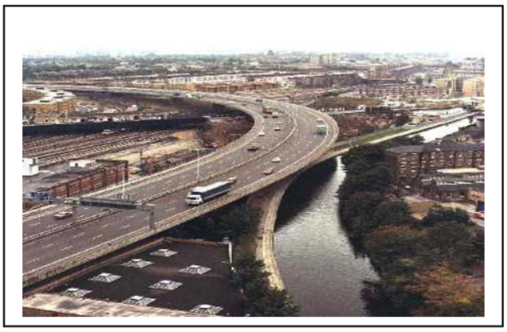
1.1 Background

It is well-known that the transport network is very important for the economy and social development in Europe. It has been important for the growth of the economy and prosperity and it allows that people and goods can move faster, easier and safer. The investment in the road network is very large with bridges being the most costly element. Bridges allow roads to cross valleys, rivers and other obstacles, both human-made and natural. Additionally, they provide access to isolated places [1], see Figure 1.1.

River Soèa Bridges, Slovenia

Estuary Crossing, Normandy, France

Grade separated junction, UK



Elevated motorway, UK

Figure 1.1: Different bridge functions [1].

Bridges are an important part of the roadway and railway transport infrastructure. Since the expansion of the road and railway networks in most of the countries in Europe started in the 20th century, an important part of the bridge stock was constructed more than 100 years ago and several masonry arch bridges were built in the Roman times [2, 3].

In Denmark, within road and railway network there exist several bridges which are more than 50 years old. In addition, a lot of bridges which were built in the last decades with design according to deterministic codes of practice in Denmark from 1949, 1973, 1984 and 1998 [4, 5, 6, 7].

Due to the above mentioned, numerous bridges that are still in service are exposed to traffic loads far higher than those considered in the design. Additionally, because of the insufficient investments in maintenance of bridges, deterioration can be observed in many existing bridges after some years of service [3].

As it was mentioned above, the expansion of the European Community and their continuous economy growth have implied the increase of the traffic loads on European highways and railway in recent years and it is expected to keep increasing in the future. Consequently, robustness of existing bridges will need to be increased as well. Hence, an upgrade of the highway and railway bridge networks is important in order to guarantee that existing bridges are safety enough after increased loads have been applied. This can be achieved in many cases by applying traditional bridge load carrying capacity assessment methods. Nevertheless, current load bearing capacity assessment procedures for existing bridges are commonly implemented from the design standards, which are meant to be for new bridges and might not be suitable for the evaluation of existing bridges [8].

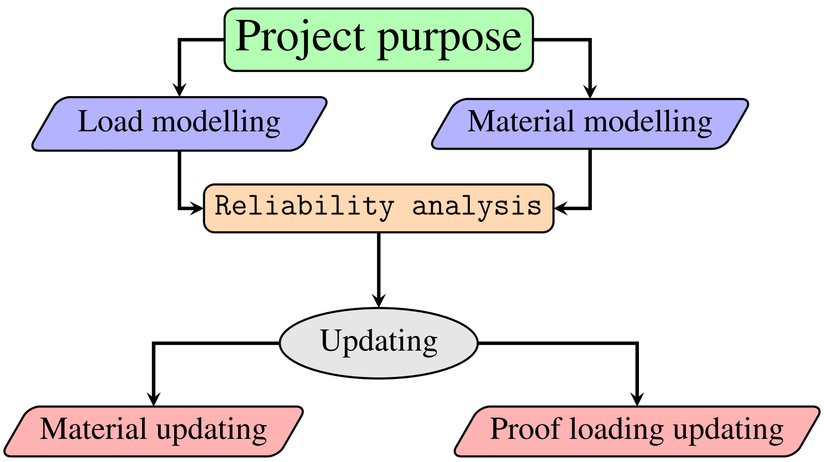
Most methods currently used for bridges safety assessment are based on simple method of analysis as linear elastic analysis and deterministic or semi-probabilistic bearing capacity assessment. In real life, a bridge is a interconnected members system where if one of the members fails, it does not necessarily mean that the whole structure collapses. Thus, the reliability of single members does not represent the whole structure reliability. Moreover, the majority of variables which describes the mechanical properties of materials, structural geometry and loads effects are not deterministic parameters and their characteristic or design values do not always accurately reflect their uncertainties. Because of all the simplifications and conservative assumptions typically made during the design, using the same procedures for existing bridges assessment might lead to consider many bridges unsafe which are in reality sufficient safe. Due to all of this, many researchers have suggested to apply advanced probabilistic analysis methods in order to evaluate the safety of existing structures [1, 2, 8].

It is not generally worthy to carry out a complete structural reliability-based analysis in new bridges design. This is because more advanced analysis would just mean a small decrease in member sizes and in the amount of materials used, being the money saved insignificant in most of the cases. Thus, the important computational effort needed to perform advanced reliability-based analysis is not frequently justified for design. On the other hand, taking into account uncertainty in the estimation of the most important variables can mean a big amount of money saved in the assessment of existing bridges. For instance, it is the case when decisions must be taken with regard to which suitable maintenance actions to carry on such as strengthening, rehabilitation or replacement of bridges that might not satisfy the design equations. Due to the above mentioned, the use of probabilistic methods of assessment for existing bridges are getting more and more wide spread in practical applications. In order to avoid the need to carry out a probability-based analysis for all bridges, some research studies [1, 2, 9, 10, 11, 12] have suggested that structural assessment approaches should be based on different levels with increasing degrees of accuracy and complexity. They suggest to proceed to the next assessment level just if the bridge fails to pass the previous one [8].

This 'step-level' approach has been also proposed in [12], where five levels of assessment are suggested. The most advanced assessment method recommended in this guideline combines stochastic FE-models as a structural analysis with a advanced probability-based analysis. This is the last level checked before costly activities of reparation/strengthening or replacement are carried out.

1.2 Purpose

The purpose of this project is to investigate different methods in order to assess the reliability of existing concrete bridges which do not fulfill the safety requirements before coping with costly rehabilitation/demolition by applying different types of updating, namely proof load updating and material updating. A flowchart with the steps followed is shown below.



1.3 Scope

The following aspects are to be considered:

Stochastic modelling

- The dead load and the permanent load model uncertainty are modelled.

- The traffic load modelled for passage situations, in the specific case of conditional passage in one lane. Therefore, both normal passage, which the case of passage situations, and mixed traffic situations are not considered in this project. Traffic load is the only variable load considered.
- Concerning vehicle modelling, standard vehicles (which are special heavy vehicles) are modelled, not considering the model of ordinary vehicles.
- The dynamics effects of the vehicles are modelled, whereas the transverse and longitudinal positioning of vehicles are not.
- The model uncertainty for traffic load is included.
- The concrete compression strength as well as the concrete strength model uncertainty are modelled.
- The reinforcement strength as well as the reinforcement strength model uncertainty are modelled.

Reliability analysis of a generic existing bridge

- The reliability assessment is performed for ULS (Ultimate limit states), not considering Serviceability limit states (SLS) or accidental limit states (ALS).
- A generic limit state equation is applied in order to carry out the reliability analysis.
- The dead load is normalized with respect to the traffic load.
- Two generic failure modes are considered: one in which the dominant material strength is the concrete compression strength and another one where the reinforcement strength is dominant.
- Two different types of standard vehicles are considered.
- This analysis is performed for existing concrete bridges, where different ratios of the permanent load to variable load are considered.
- A sensitivity analysis is performed.
- The reliability measurement used is the reliability index.

Reliability updating of a generic existing bridge

- An updating by proof load tests is carried out for both concrete compressive strength and reinforced strength as the dominant material strength. In addition, two standard vehicles are considered.
- An updating of the material coefficient of variation is performed.
- An updating of the material strength is carried out.

CHAPTER 2

State-of-the-art assessment of existing concrete bridges

2.1 Introduction

There can be several reasons for performing a reliability analysis. Some of these reasons are mentioned in the following:

- Changes on the bridge due to for instance mechanical damages, deterioration or change of use, see Figure 2.1.
- The bridge was designed based on old design codes and it has to be checked again if the bridge meets the new ones and the new loads requirements.
- The maximum allowed load in a road network is to be increased and there are doubts about if the bridges in this road network will bear the new loads.
- A heavy load that normally is not allowed, it could be for instance the blades of a wind turbine, needs to cross the bridge.

Figure 2.1: Damages in a bridge due to corrosion [13].

In a reliability assessment, the purpose is to check if a bridge is still sufficiently safe. On the other hand, in the design of a new bridge, the aim is to design a bridge which is safe for its entire lifetime. Additionally, there are design uncertainties as for instance loads, mechanical properties of materials, structure geometry, etc. that can be reduced in the reliability assessment as most of them can be measured. Hence, the design codes do not provide the best approach for assessing the safety of existing bridges as they are meant for the design of new bridges and not for reliability assessment, so the procedures are different.

The concepts and procedures for the reliability assessment of existing bridges are introduced in this chapter. Firstly, the differences between design and assessment of bridges are discussed. Secondly, the different steps in order to perform a reliability analysis are mentioned. Afterwards, the concepts of analytic and experimental assessment of existing bridges are presented. Finally, based on national guidelines, research projects reports and recent codes, several procedures for the reliability assessment of existing bridges are introduced. This chapter is based partly on [3].

2.2 Principles of structural assessment

2.2.1 Assessment vs. Design of bridges

When designing a new bridge, the next steps might be followed:

1. Depending on the traffic expected, the geometry of the road is defined.
2. The type of bridge and the lengths of the spans are chosen.
3. The static system and the dimension of the cross-sectional areas of the members are established.
4. Based on models and information from design codes, the loads applied on the bridge are assumed.
5. Load effects in the structural elements are determined and the capacities of the bridge members are calculated. The process might stop when the bearing capacity of all the members of the bridge is greater than the calculated load effects. Otherwise and in order to fulfill the safety requirements, the class of material or the geometry of the cross-sectional area have to be adjusted.

The last three steps consist of an iterative process.

On the other hand, the procedures in the assessment of the load bearing capacity of an existing bridge are different as the situation is quite different. Several uncertainties that appear during the design of the bridge might be reduced in the assessment of existing bridges:

- The geometry is determined and it is possible to measure it as the bridge already exists. However, sometimes there can be some difficulties to measure it, for instance when measuring the reinforcement in concrete bridges.

- By using partial destructive or non-destructive methods, the mechanical properties of the construction materials may be estimated.
- By using hydraulic jacks, the self weight of the entire bridge can be calculated.
- By using a Weigh-in-Motion system, the traffic loads can be estimated.
- Based on load tests and measurements, it is possible to update the bearing capacity of the members of the bridge.

Therefore, it is clear that the amount of available information in reliability assessment of existing bridges is higher than in the design of new bridges. Thus, it is obvious that the uncertainty associated to the reliability assessment is lower than the one related to bridge design. However, in practice not always measurements information is available or it is of low quality. Hence, if this is the case, there is still a relevant uncertainty associated to the structure actual state.

2.2.2 Main stages of assessment

The first step in a load bearing capacity assessment of an existing bridge normally is to evaluate its condition by the examination of existing documents related to the bridge and a preliminary inspection on the site. Then, the information collected can be used to perform a structural assessment.

According to [1], these different stages can be defined in the process of bridge assessment:

1. Examination of documents related to the design and their accuracy.
2. Visual identification of the structure and possible damages on it as a preliminary inspection.
3. Extra investigations may be carried out in order to get more accurate information from the bridge.
4. Assessment of the load bearing capacity and bridge safety.

Different levels of preciseness can be applied in the last two steps of assessment of bridges above mentioned. For some bridges, more accurate information and high-level analysis may be required in which non-linear structural analysis and probability-based assessment can be involved.

2.2.3 Analytic approaches

The traditional way of design and evaluation of safety of existing bridges is the deterministic approach. This approach is based on experience and the safety measures are empirical. The safety measures are quite conservative as many simplifications are assumed [12].

The global safety factors method is the deterministic method most used. This factor of safety is the ratio between the resistance and the load effect. It is advisable not to use as far as possible deterministic verification methods with one global safety factor in the assessment of existing bridges as does not properly reflect reality and contain an important amount of uncertainty. This method is not used in Denmark.

Another approach is the semi-probabilistic approach, which is based on the limit state principle. The Ultimate Limit State (ULS) is introduced, with the aim of ensuring that the failure does not happen either in a member of the structure of in the structure itself. Partial safety factors are applied as a safety measure. In this method, it is possible to take into account the uncertainty on the design parameters. Therefore, the semi-probabilistic approach reflects the reality way better than the deterministic approach. Nevertheless, the semi-probabilistic methods are known to be conservative as well for most of the structures.

As the partial safety factor method has been developed for the design of structures, it is included in most of the design codes.

Finally, another approach to be considered is the probabilistic approach. It is also based on the limit state principle. The safety measure in this method is the probability of failure and the reliability index, which are directly associated. The structure will be considered safe enough when it reaches a required minimum structural reliability, denoted as target reliability. Thus, the safety requirements of the structures are expressed by a maximum accepted probability of failure P_f and its associated minimum accepted reliability index β [12]. Different target reliability values are presented in Table 2.1.

Table 2.1: Required safety index and probability of failure for ultimate limit states, [14].

Failure type	Failure with warning and carrying capacity reserve	Failure with warning but without capacity reserve	Failure without warning
β_t	4.26	4.75	5.20
P_f	10^{-5}	10^{-6}	10^{-7}

The semi-probabilistic approach in combination with the 'Design value format' method is widely applied in design codes and as well in reliability assessment of existing bridges may be accurate enough. Nevertheless, sometimes it could be too conservative and might result in an unnecessary and costly strengthening or bridge replacement. Hence, in those case the application of a probabilistic reliability assessment may be needed.

A scheme of the different analytic approaches is shown in Figure 2.2.

2.2.4 Experimental approach

In addition to the analytic methods, it is possible to apply bridge load tests. Two different types of bridge load tests can be distinguished: Diagnostic tests and proof load tests. While

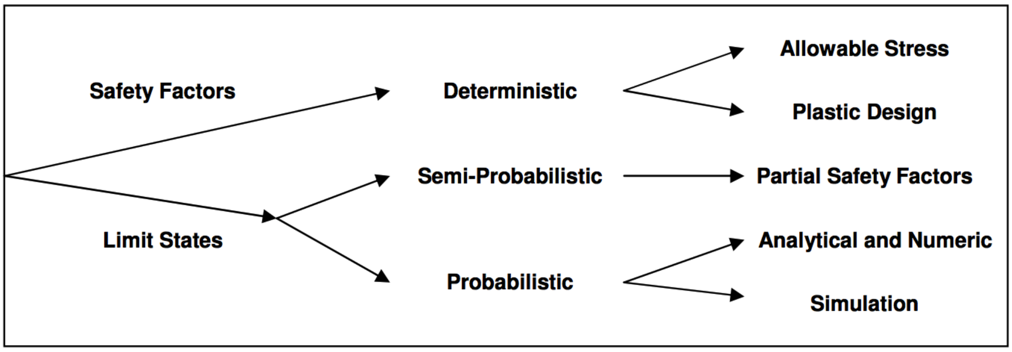


Figure 2.2: Different reliability verification approaches [12].

diagnostic tests are used to check and adjust the predictions of structural models and analytic methods, proof load tests have the advantage of verifying the load bearing capacity directly.

Thanks to the information collected in the proof load test, an updating of the reliability of the bridge can be performed. This information from the proof load test can be added to the reliability assessment in different ways. In one of them, the updating of the probability distribution function (PDF) of the bridge resistance can be performed by truncating the lower tail to the corresponding theoretical distribution at the value reached in the proof load, see Figure 2.3.

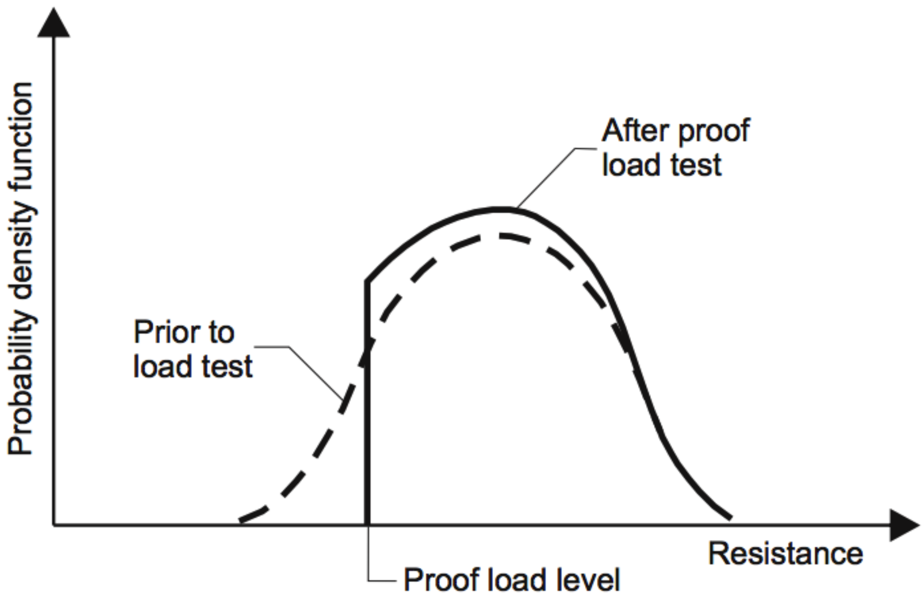


Figure 2.3: PDF truncation of bridge resistance after having applied proof load [3].

The reliability assessment of a bridge is improved by a proof load test as the proof load test updates information related to the load carrying capacity of the bridge. Nevertheless, it is inevitable to run a risk, which is the collapse of the bridge. Hence, it is important to be cautious in order to avoid the collapse of the bridge.

2.3 Procedures and methods of assessment

According to the danish standards [15], the steps for assessment of existing structures are shown in the flowchart presented in Figure 2.4. By following this procedure, the assessment of existing bridges shall be addressed.

In addition, slightly different methods are recommended in other references. For instance, different assessment levels with increasing levels of accuracy and complexity are presented in European research projects as BRIME [1], COST45 [2] and SAMCO [12].

In the following, an overview about the assessment procedures presented in research project reports, guidelines or national codes are presented based mainly on the information collected from [3].

2.3.1 Approaches proposed in European research projects

In European research projects, as for instance BRIME [1], COST345 [2] or SAMCO [12], the existing bridges assessment is proposed to be addressed by using a method with different levels. The different levels included in [12] for the assessment of highway bridges are:

- Level 0, Non-formal qualitative assessment: It is applied mainly for an evaluation of the structure and is based on engineer experience.
- Level 1, Measurement-based determination of load effect: Measured performance values are compared to threshold values which are given in codes in order to perform a serviceability assessment. A structural analysis is not performed at this level.
- Level 2, Partial safety factor method, based on documents review: Information collected from design, construction and inspection documentation is used in order to perform a load bearing capacity and serviceability assessment. Simple structural analysis methods are used and the partial safety factors method is applied.
- Level 3, Partial safety factor method, based on supplementary investigation: Information from non-destructive investigations on the specific site is used in order to carry out a load bearing capacity and serviceability assessment. Refined structural analysis methods are used and the partial safety factors method is applied.
- Level 4, Modified target reliability, partial safety factors modification: Modified partial safety factors are applied to verify the load bearing capacity of the structure.
- Level 5, Full probabilistic assessment: All basic variables with their statistical properties are used for the assessment of the structure. Safety verification is based

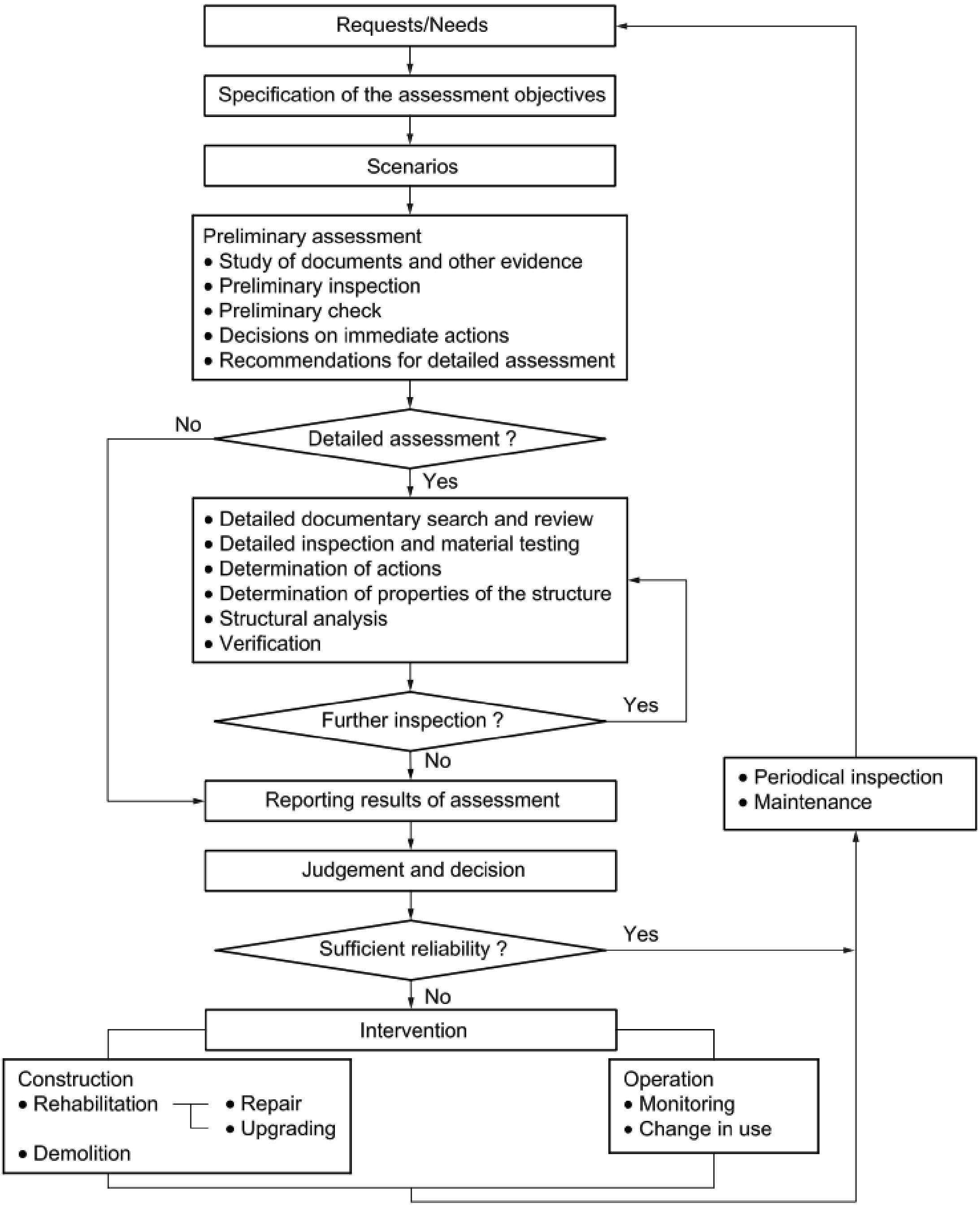


Figure 2.4: Steps for the assessment of existing structures [15].

on a structural reliability analysis instead of the partial safety factors method. In this level the uncertainties are modelled based on a probabilistic approach.

A summary of this methodology based on levels for the assessment of structures can be seen in Figure 2.5.

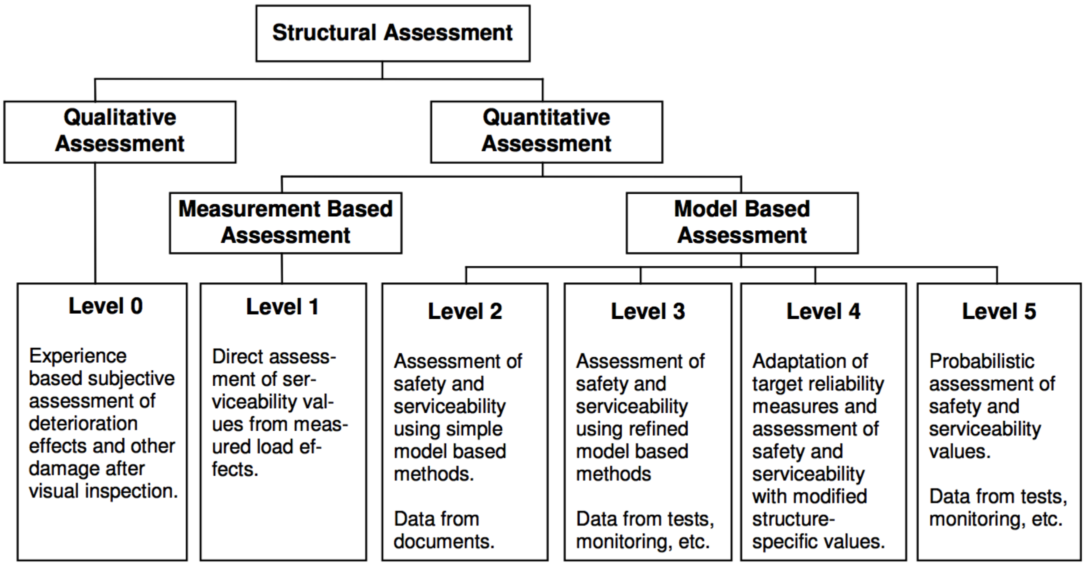


Figure 2.5: Structural assessment levels [12].

2.3.2 Approaches in national codes and guidelines

According to surveys carried out in different European research projects [1] [9], the assessment of existing bridges is still based on design codes. Nevertheless, some countries in Europe as United Kingdom, Denmark or Switzerland have developed national codes or guidelines in order to approach the issue of bridge assessment. Additionally, in Canada and USA different procedures for the assessment of existing bridges have been developed in the last decades and currently they are quite established and being applied in practice. In the following, a short overview about these assessment procedures is presented.

American code (AASHTO LRFR)

The different procedures and strategies in order to carry out an assessment of existing bridges are included in AASHTO LRFR [16]. Different methods can be applied according to this code for extreme traffic loads:

1. Load and Resistance Factor Rating (LRFR): It is the simplest method and is equivalent to the partial safety factors method applied in design codes. The manual recommends the rating to be carried out at the beginning for the design loads. In case the bridge fails the rating at design loads, a rating is performed for legal loads. The traffic load safety factor is calculated in this level taking into account the actual traffic load on the bridge. Regarding the safety factors for permanent loads, they remain as the ones in the design but the wearing surface factor is introduced, that can be modified depending on the direct thickness measurements on the bridge. Nevertheless, the partial safety factor related to resistance has to be determined by the provided formula which takes into account bridge redundancy.

2. Load test methods: Two different load test methods for the assessment of bridges are

distinguished in this manual, which are diagnostics tests and proof load tests. While diagnostic tests are used to check and adjust the predictions of structural models and analytic methods,proof load tests have the advantage of verifying the load bearing capacity directly.

3. Probabilistic methods: In this method, a reliability assessment is performed. Probabilistic models of the basic variables according to [16] in order to apply the first order reliability method (FORM) are suggested. It is recommended to use this assessment method in special situations in which the load effects, material properties, economic impacts or deterioration levels differ greatly with regards to those considered in this manual.

Canadian code (CAN/CSA-S6-06)

One of the sections included in the Canadian code for design of new bridges [17] is related to the assessment of existing bridges. As in [16], three different methods are proposed in the mentioned section:

1. The first method is equivalent to the partial safety factors method applied in design codes. The main difference is that in this case the partial safety factors are calibrated for the assessment of the bridge. Reliability procedures are applied to determine these safety factors.

2. Mean Load Method: This method is used when the bridge fails the assessment in the previously described method. It is a probabilistic method in which the basic variables need to be modelled.

3. Bridge load tests: In [17], this method is considered a complement of the analytic assessment procedure.

Swiss guideline (SIA-269)

The first level of assessment of existing concrete bridges in Switzerland is by application of the one from the actual design code. In case the bridge fails this first assessment, the procedures presented in SIA 462 [18] are to be applied. The method applied is to reduce the partial safety factors by taking other safety measures as for instance periodic or continuous monitoring of the performance of the structure. When these safety measures are applied, regular inspections with a maximum period between them of five years must be carried out. The guideline also allows more accurate analysis and reliability assessment methods to be applied if required.

British guidelines

In the United Kingdom, the assessment codes have been obtained by modifying the design codes. By applying these changes, the strength of bridge materials obtained in new tests can be included in the assessment. Additionally, specific load modelling for the assessment purpose is applied.

In the British guidelines, five assessment levels are suggested with increasing complexity and accuracy. In the first level, a simple analysis is carried out and load and carrying capacity models are quite conservative. In case the bridge fails the first assessment levels, the analysis and models become more advanced up to a fully probabilistic analysis.

The code BD 63/07 [19] might be the most relevant for the assessment of concrete bridges in UK. It is stated in [19] that the experimental approach (bridge load test) should not be used for direct safety assessment as it is in the American [16]. Instead, it may be complementary to the analytic assessment.

Danish guidelines

For danish bridges, a first deterministic assessment proposed by the guideline Vejdirectoratet (1996) [20] has to be carried out. In case the bridge fails this deterministic assessment, the procedure specifies in Vejdirectoratet (2004) [14] shall be followed. In this case, it is a probability-based assessment. As the governing ultimate and serviceability limit states are already known from the deterministic analysis, the probabilistic assessment is performed only for those critical limit states.

It is specified in Vejdirectoratet (2004) [14] the different probabilistic models for load effects as well as for load bearing capacity that may be applied. Additionally, some information is provided in order to perform an updating if for instance some test results from the site are obtained. This project is based on this guideline [14] as the reliability assessment of existing concrete bridges in Denmark is studied.

Chapter 3

Stochastic modelling

3.1 Introduction

The purpose of a safety assessment of a bridge is to verify if the loads applied to the structure exceed or not its carrying capacity or the capacity of one of its members. Nevertheless, it is quite a complicated task to determine the effects of loads applied to the structure and its capacity which involves a high level of uncertainty. Besides, traffic loads are the variables which contain higher uncertainty in addition to be the most significant in the reliability analysis of short-span bridges. However, in the case of concrete bridges, the permanent loads may be taken into account as an important variable as well.

Probabilistic models of bridge permanent loads as well as traffic loads are included in this chapter. Nonetheless, other variable loads as ice load, wind load, temperature load etc. are not included in this report as they might not be that significant in the case of short-span bridges. Besides, accidental loads are also omitted as those cases are considered beyond the scope of this report.

Additionally, the probabilistic models of basic mechanical properties of concrete and steel are analyzed.

This chapter is based on the recommendations provided by the guideline Reliability-based classification of the load carrying capacity of existing bridges [14], which was elaborated by the Road Directorate (Ministry of Transport, Denmark).

3.2 Load modelling

In this section, the load models considered are presented.

Loads on bridges are frequently sorted out based on their variability in magnitude and position in time (permanent or variable, moving or fixed), and based on the kind of structural response (static or dynamic). All above cited load types are stochastic variables, including permanent loads which do not change with time nor create dynamic oscillation. Loads are stochastic because their magnitudes and their effects on the structure are not accurately known [3]. Therefore, in a reliability analysis, loads will be modelled as stochastic variables.

3.2.1 Permanent load

The permanent load is calculated as the weight of the structure and it is usually modelled as a normally distributed variable. Additionally, permanent loads from different sources are assumed to be randomly independent. It is recommended to model the dead load, G, with a coefficient of variation of 5%.

In addition, the permanent load model uncertainty is introduced. The variable Xg is modelled for each permanent load by an independent normally distributed stochastic variable with mean value 0 and a standard deviation of 5% of the mean value of the permanent load associated. Xg is introduced into the computation model by simply adding its value to the relevant basis variables.

3.2.2 Variable loads: Traffic load

Beforehand, it is necessary to introduce that two different kind of vehicles can be classified in Denmark:

1. Ordinary transport, which are cars and trucks with a total weight below 50 tons.
2. Standard vehicles, which are special vehicles classified as class 50-200 for which only a limited number has the permission to cross specific bridges.

Additionally, two situations are considered in [14] for traffic loading:

- Passage situations, which means that the load effect can be calculated considering only one vehicle in each lane. This situation is typical for short influence lengths bridges, which is often less than 60 m. Two different passage situations are considered:
  1. Normal passage: Situation in which standard vehicles cross the bridge without any limitation concerning other traffic.
  2. Conditional passage: Situation in which the only traffic load on the bridge are the standard vehicles. The maximum speed while they cross the bridge is $V = 10$ m/s.
- Mixed traffic situations, defined as situations in which the load effects are calculated by considering standard vehicles combined with mixed traffic in the same lane as the standard vehicle plus in other lanes. This situation is usually for long influence lengths bridges, which is typically more than 60 m.

In the transition situation, i.e situations in which the influence length is around 60 m, it is recommended to study the case as both passage situation and mixed traffic situation.

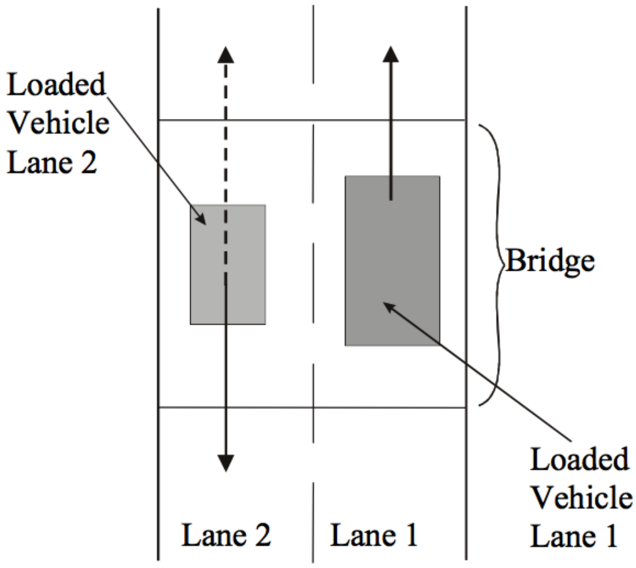


Figure 3.1: Normal passage situation [14].

For the purpose of this report, the conditional passage situation is considered.

The traffic load modelling is based on [21]. Global load effects due to standard vehicles are considered in the following. The maximum annual load effect is determined by:

$$Q = e_Q X_Q (1+\phi)P \tag{3.1}$$

where

Q	Traffic load
e_Q	Coefficient of influence
X_Q	Traffic load model uncertainty
P	Maximum annual vehicle load, modelled by a stochastic variable. Characteristic value: 98%fractile
ϕ	Dynamic factor modelled by a stochastic variable.

Additionally, the maximum annual load distribution function is introduced:

$$F_p(x) = exp(-[1-F_w(x)]N) \tag{3.2}$$

where

$F_w(x)$	Distribution function for load from a single vehicle
N	Number of standard vehicles per year

Vehicle modelling

In[14], both modelling of ordinary transports and standard vehicles are considered. Only standard vehicles are modelled in this report.

For administrative consistency purpose, standard vehicles in Denmark are sorted by classes depending on their weights. The weight of standard vehicles is classified in the range of 50-200 tons and it is assumed to be a normally distributed variable. A table with the different classes is shown in Table 3.1.

Table 3.1: Distribution parameters of the weight standard vehicles, [14].

Standard vehicle	Mean weight [tons]	Standard deviation [tons]
Class 50	53.1	5
Class 60	63.4	5
Class 70	72.2	5
Class 80	82.5	5
Class 90	95.4	5
Class 100	109.2	5
Class 125	131.4	5
Class 150	157.6	5
Class 175	170.2	5
Class 200	201	5

It can be seen that the standard deviation is the same for all the different classes.

It is given as well in [14] an approximation of the amounts of standard vehicles per year depending on the kind of road. The classification can be seen in Table 3.2.

Table 3.2: Number of vehicles per year depending on the road type, [14].

Class / Road type	50	60	70	80	90	100	125	150	175	200
Highways	200	200	200	150	150	100	50	50	50	50
Main roads	100	100	100	80	80	50	20	20	20	20
Other	50	50	50	40	40	20	10	10	10	10

In addition, the dynamic effects produced by the vehicles crossing the bridge need to be taken into account. For normal passage situations, the dynamic effects can be modelled as normally distributed with expected value $\mu = 41.5/W$ and standard deviation $\sigma = 41.5/W$, being W the total vehicle weight in kN. In case of conditional passage situations, the dynamics effects are considered neglected due to the low speed of the vehicles crossing the bridge ($\leq$ 10 km/h).

Finally, the model uncertainty associated to the variable loads can be introduced into the computation model by multiplying the basic parameters by X_Q, being X_Q normally distributed with an $\mu = 1$ and $V = 0.10$ when the uncertainty in the loading model is low, $V = 0.15$ when is medium and $V = 0.20$ when it is high. In case of conditional passage, the loading model uncertainty is low, so $V = 0.10$ is taken for further calculation in this report.

3.3 Material modelling

In this section, the principles for the development of material models are included. Models for the most relevant materials in bridges, in this case concrete and steel for reinforcement are studied.

3.3.1 Material modelling according to the guideline Reliability-Based Classification of the Load Carrying Capacity of Existing Bridges (14)

Concrete

The carrying capacity of a concrete structure is usually model around its most important parameter, which in case of a short column its concrete compressive strength, f_c. In this report, the modelling of the compressive strength of concrete has been based on [6] as recommended in [14]. The concrete compressive strength is given in the form of cylinder strength for structures design on the basis of [6]. The characteristic value of the compressive strength is related to a 5% fractile. The compressive strength is assumed to follow a logarithmic-normal distribution with a mean compressive strength $E[f_c]$ and coefficient of variation V_{fc} given in Table 3.3.

Table 3.3: Concrete compressive strength distribution parameters according to [6].

$f_{ck}[MPa]$	$E[f_c][MPa]$	V_{fc}
5	6.76	0.22
10	12.8	0.18
15	18.9	0.17
20	24.8	0.16
25	30.6	0.15
30	36.2	0.14
35	41.7	0.13
40	47.0	0.12
45	52.8	0.12
50	58.7	0.12

For further calculations in this project, it was chosen to use a concrete with a concrete compressive strength of $f_{ck} = 30MPa$.

Model uncertainty for concrete

By adding the stochastic variable X_m into the computational model, the uncertainty is taken into account. It is introduced into the computational model by multiplying the X_m by the concrete compressive strength in this case. The variable X_m follows a logarithmic-normal distribution with mean value 1 and the variation coefficient V_{X_m} can be calculated as follows:

$$V_{X_m} = \sqrt{V_{X_{m1}}^2 + V_{X_{m2}}^2 + V_{X_{m3}}^2 + 2(\rho_1 V_{X_{m1}} + \rho_2 V_{X_{m2}} + \rho_3 V_{X_{m3}})} \tag{3.3}$$

In the following, the value of all of these parameters are given in Table 3.4, Table 3.5 and Table 3.6.

Computional model accuracy, X_{m1}.

Depending on the accuracy considers for the computation model, different values can be chosen. These are shown in Table 3.4.

Table 3.4: Variation and correlation coefficients concerning the computational model accuracy, [14].

Computational model accuracy	Good	Normal	Poor
$V_{X_{m1}}$	0.04	0.06	0.09
ρ_1	-0.3	0.0	0.3

A normal computational model accuracy was chosen for this project, so $V_{X_{m1}} = 0.06$ and $\rho_1 = 0.0$.

Uncertainty to determine concrete parameters, X_{m2}.

There are different values to define the uncertainty in determining the concrete parameters of the structure. These are given in Table 3.5.
It was chosen to consider the uncertainty in determination of concrete parameters as medium, so $V_{X_{m2}} = 0.06$ and $\rho_2 = 0.0$.

Materials identity, X_{m3}.

The model of the uncertainty of materials identity in bridges is made by the variable X_{m3}, given in Table 3.6.

Table 3.5: Variation and correlation coefficients concerning the uncertainty in determining the concrete parameters, [14].

Uncertainty in determining concrete parameter	Low	Medium	High
$V_{X_{m2}}$	0.04	0.06	0.09
ρ_2	-0.3	0.0	0.3

Table 3.6: Variation and correlation coefficients concerning the uncertainty in materials identity, [14].

Identity of materials	Good	Normal	Poor
$V_{X_{m3}}$	0.04	0.06	0.09
ρ_3	-0.3	0.0	0.3

It was chosen to consider the uncertainty of identity of materials as normal, so $V_{X_{m3}} = 0.06$ and $\rho_3 = 0.0$.

Thus, by introducing all these parameters in Eq. (3.3), a coefficient of variation $V_{X_m} = 0.1$ was obtained. Therefore, the distribution parameters of the bearing capacity model uncertainty are $\mu = 1$ and $V_{X_m} = 0.1$.

Non-prestressed reinforcement

Models for the stochastic modelling of the material parameters for non-prestressed reinforcement are given in the following. The modelling of the reinforcement has been based on [6]. Different kind of bars are considered. The tensile yield stress f_y follows a logarithmic-normal distribution with a standard deviation $\sigma = 25$ MPa without depending on the type of reinforcement. The different type of bars and their respective distribution parameters can be seen in Table 3.7.

For further calculations in this project, it was chosen to use smooth bars with a reinforcement strength of $f_{yk} = 275 MPa$.

Table 3.7: Tensile yield stress for different types of steel non-prestressed reinforcement, [**19**].

Type	Symbol	Diameter [mm]	$f_{yk}[MPa]$	Mean value, μ [MPa]	σ [MPa]
Smooth bars	$Fe360$	≤ 16	235	304	25
Smooth bars	$Fe360$	> 16	225	293	25
Smooth bars	$Fe430$	≤ 16	275	345	25
Smooth bars	$Fe430$	> 16	265	334	25
Smooth bars	$Fe360$	≤ 16	355	426	25
Smooth bars	$Fe360$	> 16	345	416	25
Ribbed bars (Kamstaal)	$Ks410$	-	410	482	25
Ribbed bars (Kamstaal)	$Ks550$	-	550	623	25
Cold-deformed bars (Tensorstaal)	T	-	550	623	25

Model uncertainty for non-prestressed reinforcement

The procedure to determine the model uncertainty for non-prestressed reinforcement is the same as applied in 3.3.1 for concrete and the same material model uncertainty as for concrete has been considered.

3.3.2 Material modelling according to the Danish Standards (22)

According to the Background investigations in relation to the drafting of National Annexes to EN 1990 and EN 1991 [22], both the concrete and the steel for reinforcement can be modelled as followed:

Table 3.8: Coefficient of variation of the strength and model uncertainty variables for concrete and reinforcement.

	$V_{f_c,f_y}[\%]$	$V_{X_m}[\%]$
In situ concrete	14	11
Reinforcement	7	5

The same types of concrete/reinforcement as in Table 3.3 and Table 3.7 are considered, both the variables R and X_m following a logarithmic-normal distribution, and being the mean value of X_m equals to 1. Then, the standard deviation for the concrete model uncertainty $\sigma_{X_mc} = 0.11$ and $\sigma_{X_ms} = 0.05$ while in [14], they are considered to be the same for concrete and reinforcement.

Finally, in the next chapters the models used are the ones related to [14].

Chapter 4

Reliability analysis of a generic existing bridge

4.1 General

This section is partly based on [14].

To ensure the reliability of a structure, different requirements shall be met:

1. The structure has enough safety against failure during its lifetime.
2. The structure works adequately with normal use.
3. The structure has acceptable robustness and durability.

The limit state criteria is used to decide is a structure or a component of its work properly or not. Three different limit states are normally examined:

- Ultimate limit states (ULS).
- Serviceability limit states (SLS).
- Accidental limit states (ALS).

The ultimate limit state is achieved when the failure of a structure or the failure of one of its components occurs. On the other hand, the serviceability limit state is related to failure in normal use. Finally, the accidental limit state corresponds with extreme situation in the structure as accidental loads.

For the scope of this project, only the ultimate limit state (ULS) will be studied.

4.2 Types of failure in ULS

When a load carrying capacity evaluation is carried out, the safety requirement for the ULS depends on the kind of failure predicted [14].

Three different kind of failures can be studied:

1. Failure with warning and with load carrying capacity reserve. The failure is considered to be as ductile failure.

2. Failure with warning but without load carrying capacity reserve. The failure is considered as ductile as well but without extra load carrying capacity.

3. Failure without warning. The failure is considered to be as brittle failure.

To evaluate which is the kind of failure, the given material, component or structure are taken into account.

4.3 Reliability requirements

The reliability index, β, represents the annual probability of failure, i.e with a reference period of one year. It can be defined as follows:

$$\beta = -\Phi^{-1}(P_f) \tag{4.1}$$

where

β	Reliability index
Φ	Standardized normal distribution function
P_f	Probability of failure

The target reliability index, β_t, is related to the type of failure according to [14]. The different values with its respective type of failure are summarized in Table 4.1.

Table 4.1: Required reliability index and probability of failure for ultimate limit states, [14].

Failure type	Failure with warning and carrying capacity reserve	Failure with warning but without capacity reserve	Failure without warning
β_t [-]	4.26	4.75	5.20
P_f [-]	10^{-5}	10^{-6}	10^{-7}

In addition, the target reliability index can be classified also based on economic optimization for different structural classes according to [23]. These target reliability indexes depend on the consequences in case of failure as well as the relative cost of the safety measure, both of them relative to the initial structural construction costs, see Table 4.2. A detailed explanation of each of the consequence classes can be seen in Appendix A.

The target reliability indexes given in Table 4.2 should be considered as indicative for the support of economic optimization and may not be acceptable for what concerns to life safety risks [23].

Finally, according to [21], the target reliability index can be classified in relation to the Consequence Class considered, see Table 4.3.

Table 4.2: Target reliability indexes relative to one year reference period and ultimate limit state, based on economic optimization [23].

Relative cost of safety measures	Consequences of failure Class 2	Class 3	Class 4
Large	$\beta_t = 3.1(P_f \approx 10^{-3})$	$\beta_t = 3.3(P_f \approx 5 \cdot 10^{-4})$	$\beta_t = 3.7(P_f \approx 10^{-4})$
Medium	$\beta_t = 3.7(P_f \approx 10^{-4})$	$\beta_t = 4.2(P_f \approx 10^{-5})$	$\beta_t = 4.4(P_f \approx 5 \cdot 10^{-6})$
Small	$\beta_t = 4.2(P_f \approx 10^{-5})$	$\beta_t = 4.4(P_f \approx 5 \cdot 10^{-6})$	$\beta_t = 4.7(P_f \approx 10^{-6})$

Table 4.3: Target reliability index according to Danish National Annexes for Eurocode EN1990 and EN1991 [21].

Consequence Class	Target reliability index, β_t
CC2	4.8
CC3	5.2

4.4 Failure function and design equations

For the reliability analysis, the following generic ultimate limit state equation has been used [21]:

$$g = zX_mR - ((1-\alpha)(G+Xg) + \alpha X_Q(1+\phi)P) \tag{4.2}$$

where

R	Material strength, modelled as a stochastic variable.
X_m	Material model uncertainty.
X_Q	Traffic load model uncertainty.
X_g	Dead load model uncertainty.
z	Design parameter.
G	Unfavorable permanent load, which is normalized with respect to Q.
α	$\alpha = 1$ means no unfavorable permanent load and $\alpha = 0$ means no variable load.
ϕ	Dynamic factor modelled by a stochastic variable. $\phi_k = 0.25$.

It is noted that in Eq. (4.2) only one variable load is included. When multiple variable loads are considered simultaneously, the recommended load combination factors from EN1990 [24] are used in the design equation but not in the limit state equation, where a stochastic varible is used in order to model the non-dominant load.

In addition, it is important to point out that for some structures with non-linear response, it may be appropriate to make further analysis of the level of safety using the relevant non-linear calculation models [21]. However, this report does not include such studies.

Based on the failure function Eq. (4.2), load combinations (6.10a) and (6.10b) from EN 1990 [24] are applied. The design equations can be written as follows, being z_A and z_B design variables in Eq. (4.3) and Eq. (4.4):

Load combination corresponding to EN 1990: STR / GEO (6.10a) [24]:

$$z_A \frac{R_k}{\gamma_m} - ((1-\alpha)\gamma_{GA,sup} G_k) \geq 0 \tag{4.3}$$

Load combination corresponding to EN 1990: STR / GEO (6.10b) [24]:

$$z_B \frac{R_k}{\gamma_m} - ((1-\alpha)\gamma_{GB,sup} G_k + \alpha\gamma_Q(1+\phi_k)Q_k) \geq 0 \tag{4.4}$$

Before performing a reliability analysis, it is necessary to calculate the design variable, z. For that purpose, the permanent load has to be normalize with regard to the traffic load so z can be calculated. z is calculated from the design equations. According to [24], the safety factors in Table 4.4 shall be applied:

Table 4.4: Safety factors for load combination 6.10a and 6.10b [24].

Safety factors / Load combination	6.10a	6.10b
γ_{mc} [-]	1.45 γ_3	1.45 γ_3
γ_{ms} [-]	1.2 γ_3	1.2 γ_3
γ_G [-]	1.25 K_{FI}	1.0 K_{FI}
γ_Q [-]	0	1.4 K_{FI}

where K_{FI} depends on the consequence class defined in Appendix A and it is $K_{FI} = 1.1$ for Consequence Class CC3 and $K_{FI} = 1.0$ for Consequence Class CC2 since Consequence Class CC1 must not be used for bridges. Additionally, γ_{mc} is the partial safety factor of concrete, γ_{ms} the partial safety factor of the reinforcement and γ_3 depends on the scope of checking, in this case it is considered normal so $\gamma_3 = 1$.

Besides, the bridge is considered to be designed by reinforced concrete with concrete characteristic compressive strength $f_{ck} = 30$ MPa and smooth steel bars as reinforcement with $f_{yk} = 275$ MPa. It is built for vehicles class 100 (Mean weight = 109.2 tons). It is necessary now to calculate the maximum annual traffic load. For that purpose, the maximum annual load distribution is modelled by:

$$F_p(x) = exp(-[1-F_w(x)]N) \tag{4.5}$$

where

$$F_w(x) = \Phi(\frac{x-\mu_w}{\sigma_w}) \quad (4.6)$$

Then, if it is considered $F_p(x) = \Phi(u)$ and by isolating the maximum annual load:

$$P = \mu_w + \sigma_w \Phi^{-1}(1 + \frac{1}{N} ln\Phi(u)) \quad (4.7)$$

where

P	Maximum mean annual load.
μ_w	Mean weight of a vehicle.
σ_w	Standard deviation of the weight of a vehicle.
N	Number of standard vehicles per year for a certain class
u	Standard normal stochastic variable associated with the annual maximum traffic load.

This non-linear function Eq. (4.7) can be solved in MATLAB by a first order approximation.

Considering vehicles class 100, with mean value $\mu = 109.2$ tons and standard deviation $\sigma = 5$ tons, and $N = 100$ standard vehicles per year, a maximum mean annual load of $P = 121.5$ tons and characteristic value $P_k = 126.89$ tons considering 98%fractile.

Two generic failure modes are considered: one in which the dominant material strength is the concrete compression strength (corresponding to a short column) and another one where the reinforced strength is dominant (corresponding to a reinforced concrete beam in bending failure).The design variables z_A and z_B can be calculated from Eq. (4.3) and Eq. (4.4) respectively for the two generic failures modes.

4.4.1 Concrete compression strength as the dominant material strength

By introducing as inputs the parameters in Table 4.5, the design variables z_A and z_B can be calculated from Eq. (4.3) and Eq. (4.4) respectively. The dead load G has been normalized with respect to the traffic load.

Different values of z_A and z_B have been obtained depending of α value. The results can be seen in Table 4.6. It is important for the reader to know that the values of z_A and z_B, which are in m^2, so they have area units, have to be considered as relative and not the dimension of "real bridges", as the permanent load G has been normalized with regard to the annual maximum traffic load.

As it can be seen in Table 4.6, depending on α, either load combination 6.10a Eq. (4.3) or load combination 6.10b Eq. (4.4) can be dominant. For instance, for $\alpha = 0.1$ load

Table 4.5: Value of the different parameters required for the calculation of the design parameter.

Variable	Value
f_{ck} [MPa]	30 (5%fractile)
γ_{mc} [-]	1.45
α [-]	Range of values from 0-1
γ_{GA} [-]	1.375 (CC3)
γ_{GB} [-]	1.1 (CC3)
G_k [tons] $= E[P]$	121.5 (50%fractile)
γ_Q [-]	1.54 (CC3)
Q_k [tons]$= P_k$	126.89 (98%fractile)

Table 4.6: Different values of the design variables depending on α.

α [-]	$z_A[m^2]$	$z_B[m^2]$
0	0.081	0.065
0.1	0.073	0.070
0.2	0.065	0.075
0.3	0.057	0.081
0.4	0.048	0.086
0.5	0.040	0.091
0.6	0.032	0.097
0.7	0.024	0.102
0.8	0.016	0.107
0.9	0.008	0.113
1	0	0.118

combination 6.10a Eq. (4.3) is applied as $z_A > z_B$ whereas for $\alpha = 0.4$ load combination 6.10b Eq. (4.4) is applied as $z_B > z_A$. In the particular case of concrete bridges with traffic load, the range $\alpha = 0.2 - 0.5$ is recommended by [21]. In this range, load combination 6.10b Eq. (4.4) is dominant, so it is the load combination to be used. A reliability analysis will be performed for different values in this range of α.

The reliability analysis is performed by using the software Comrel [25], in which the results are the outputs of the reliability index (β) according to FORM (First order reliability method) and SORM (Second order reliability method) as well as the probability of failure, P_f. The inputs to the software COMREL are shown in Table 4.7.

Different values of the reliability index β_{FORM}, β_{SORM} and probability of failure Pf_{FORM}, Pf_{SORM} have been obtained. They are shown in Table 4.8 and are illustrated in Figure 4.1 . As it can be seen, the values for the reliability index and probability of failure for both

Table 4.7: Inputs to the software COMREL for vehicles class 100.

Variable	Comment	Distribution	μ	σ
f_c [MPa]	Concrete compressive strength	Lognormal	36.2	5.07
G [tons]	Dead load	Constant	121.5	-
X_g [-]	Dead load model uncertainty	Normal	0	6.08
X_Q [-]	Traffic load model uncertainty	Normal	1	0.1
X_{mc} [-]	Concrete strength model uncertainty	Lognormal	1	0.1
α [-]	Parameter	Constant	0.2-0.5	-
U [-]	Standard normal stochastic variable associated with the annual maximum traffic load	Normal	0	1
$z[m^2]$	Design parameter	Constant	0.075, 0.081, 0.086, 0.091	-
ϕ [-]	Dynamic factor	Normal	0.04	0.04
μ_w [tons]	Mean weight of a vehicle	Constant	109.2	-
σ_w [tons]	Standard deviation of the weight of a vehicle	Constant	5	-
N	Number of standard vehicles of a certain class	Constant	100	-

FORM and SORM are quite similar, which indicates that the limit state function is approximately linear around the design point, so a first order reliability method (FORM) in this case is more than enough. FORM and SORM are based on an approximate representation of the limit state function, linear in FORM and quadratic in SORM. In case that the linear state function were highly non-linear, both FORM and SORM might give only a rough approximation of the probability of failure. In such cases, the results can be improved by using a different reliability method, for instance crude Monte Carlo sampling, which consists in simulations. Monte Carlo simulation was applied with 1000000 simulations and the results can be seen in Table 4.8 as β_{MC}, which are quite similar to the ones found by FORM are SORM. It can be observed as well that for higher values of α, the reliability index β is higher as well, due to the design value z_B being higher and because the uncertainty of the different stochastic variables are weighted differently.

Table 4.8: Reliability analysis for different α values and vehicle class 100.

α [-]	z_B $[m^2]$	β_{FORM} [-]	β_{SORM} [-]	β_{MC}	Pf_{FORM} [-]	Pf_{SORM} [-]
0.2	0.075	4.412	4.415	4.43	$5.12 \cdot 10^{-6}$	$5.06 \cdot 10^{-6}$
0.3	0.081	4.807	4.809	4.85	$7.66 \cdot 10^{-7}$	$7.59 \cdot 10^{-7}$
0.4	0.086	5.083	5.084	5.09	$1.87 \cdot 10^{-7}$	$1.85 \cdot 10^{-7}$
0.5	0.091	5.315	5.317	5.37	$5.34 \cdot 10^{-8}$	$5.29 \cdot 10^{-8}$

Therefore, if it is considered the target reliability index β_t depending on the type of failure,

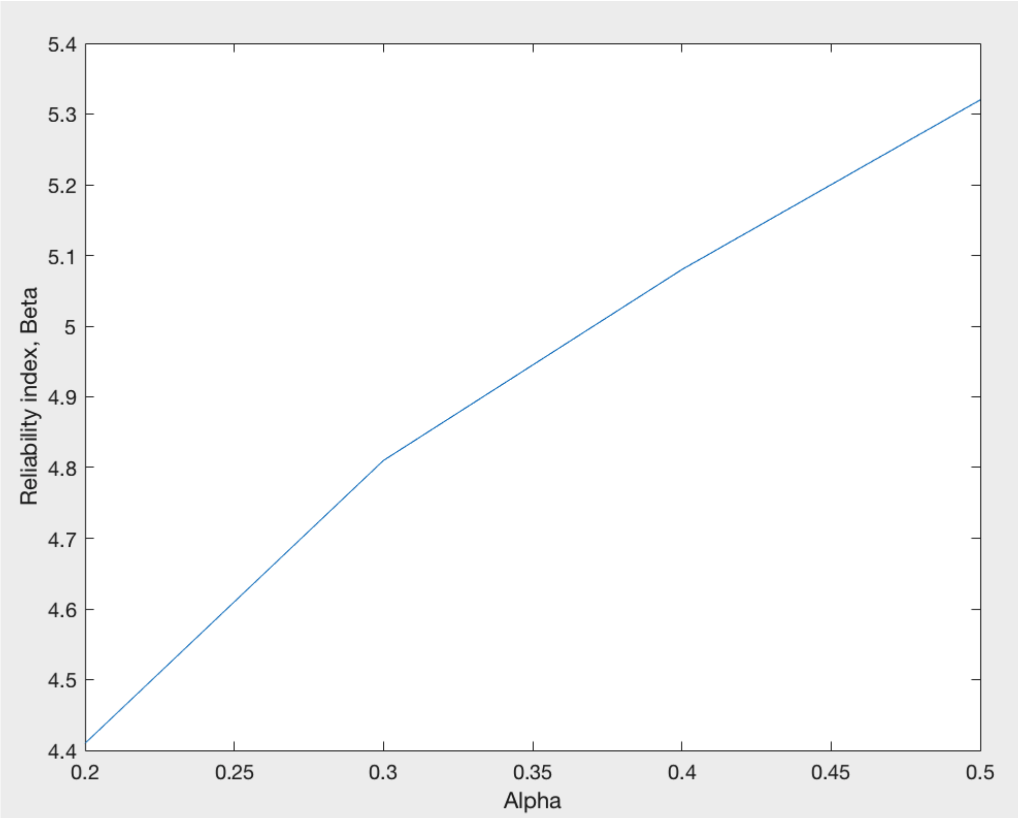


Figure 4.1: α parameter vs. reliability index β .

and the type of failure considered is failure with warning but without capacity reserve, then $\beta_t = 4.75$ as it can be seen in Table 4.1. For $\alpha = 0.3, 0.4, 0.5$, $\beta \geq \beta_t = 4.75$, which means that meet the reliability analysis and the bridge can be considered as safe. On the other hand, For $\alpha = 0.2$, $\beta \leq \beta_t = 4.75$, which means that for those values it does not success the reliability analysis so the bridge in principle "cannot be considered as safe" according to β_t related to the type of failure classification. On the other hand, if β_t is chosen based on the classification depending on economic optimization, the bridge would be considered safe for most of the cases. In any case, an updating in the next chapter will be done, so it is checked if the reliability index can be increased. Another possibility is to calculate an average reliability index β_{av} and compare it to the required reliability index index β_t. The average reliability index was found to be $\beta_{av} = 4.9$, so $\beta_{av} = 4.9 \geq \beta_t = 4.75$, then the reliability would be fulfilled according to this criteria.

Another parameter checked in this reliability assessment is the influence coefficient, which is a sensitivity measure and it is commonly known as the α vector. A plot with the influence coefficient of each of the stochastic variables for the different values of α can be seen in Figure 4.2. The absolute value of the influence coefficient expresses how sensitive the problem is to each of the stochastic variables. If the influence coefficient is positive, the associated random variable is of the capacity type, in this case R and X_m, meaning the reliability increases if the mean of the random variable is increased. If the influence coefficient is negative, in this case X_Q, X_g, U and phi, the variable is of the demand type, consequently the reliability decreases if the mean of the random variable is increased. The square of the influence coefficients sums up to 1. It can be stressed in Figure 4.2 that X_Q, U and phi increase for higher α values, as its influence is higher for higher α values. The opposite happens to X_g, since its influence is lower for higher α values.

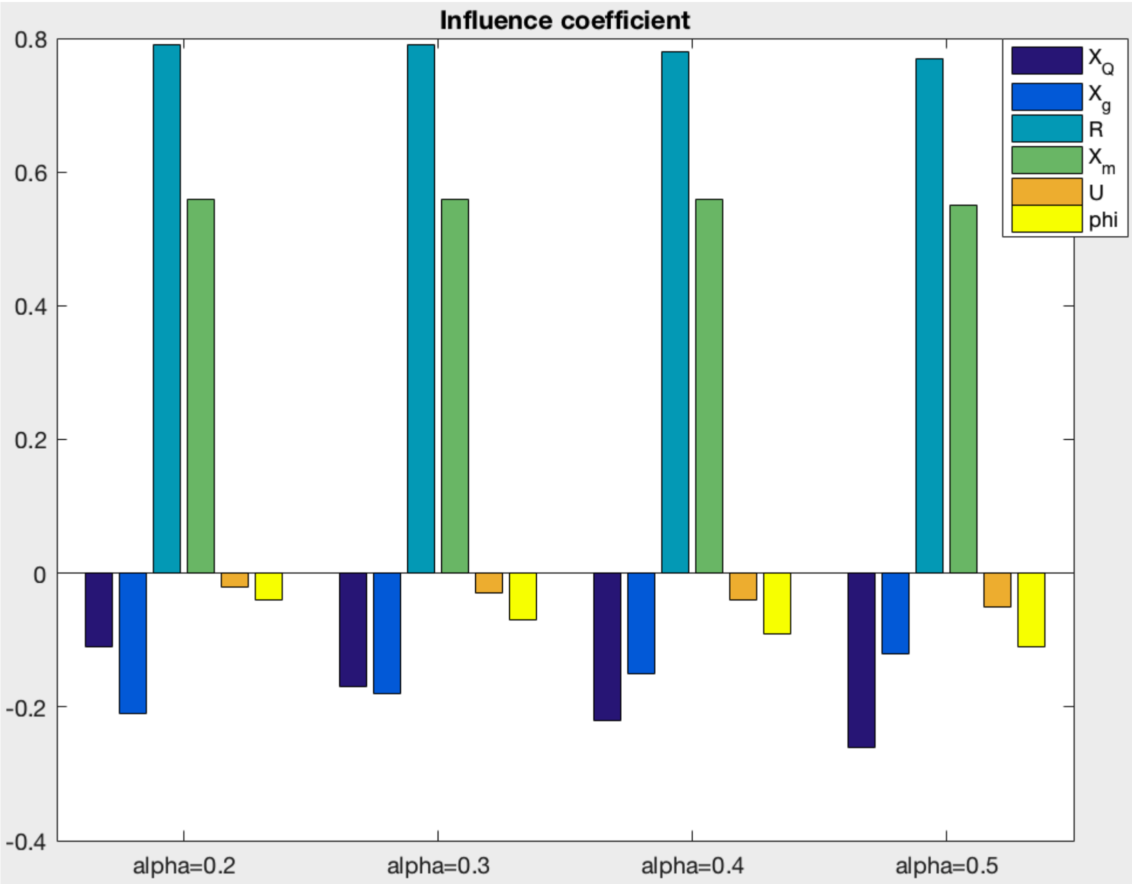


Figure 4.2: Influence coefficients for concrete compression strength as the dominant material strength and standard vehicles class 100.

Then, it is investigated if the bridge would fulfill the reliability analysis if a higher class of standard vehicles are considered, which in this case is class 125, with a mean weight of 131.4 tons, in case an upgrade of the bridge class is proposed. The data necessary for the reliability assessment is shown in Table 4.9 and the result from the reliability assessment in Table 4.10.

An average reliability index $\beta_{av} = 3.98$ was found. Therefore, based on the results obtained from the reliability analysis for standard vehicles class 125, none of them nor the average one would success the reliability analysis. Nevertheless, it must be noted that after an updating of any of the parameters involved, the bridge could success the reliability analysis. In addition, the bridge could success this reliability analysis if the reliability is taken based on economic optimization, see Table 4.2.

Table 4.9: Inputs to the software COMREL for vehicles class 125.

Variable	Comment	Distribution	μ	σ
f_c [MPa]	Concrete compressive strength	Lognormal	36.2	5.07
G [tons]	Dead load	Constant	143.93	-
X_g [-]	Dead load model uncertainty	Normal	0	7.20
X_Q [-]	Traffic load model uncertainty	Normal	1	0.1
X_{mc} [-]	Concrete strength model uncertainty	Lognormal	1	0.1
α [-]	Parameter	Constant	0.2-0.5	-
U [-]	Standard normal stochastic variable associated with the annual maximum traffic load	Normal	0	1
$z[m^2]$	Design parameter	Constant	0.075, 0.081, 0.086, 0.091	-
ϕ [-]	Dynamic factor	Normal	0.04	0.04
μ_w [tons]	Mean weight of a vehicle	Constant	131.4	-
σ_w [tons]	Standard deviation of the weight of a vehicle	Constant	5	-
N	Number of standard vehicles of a certain class	Constant	100	-

Table 4.10: Reliability analysis for different α values and vehicle class 125.

α [-]	$z_B[m]$	β [-]	Pf [-]
0.2	0.075	3.48	$2.5 \cdot 10^{-4}$
0.3	0.081	3.88	$5.32 \cdot 10^{-5}$
0.4	0.086	4.15	$1.63 \cdot 10^{-5}$
0.5	0.091	4.39	$5.57 \cdot 10^{-6}$

Additionally, a convergence analysis was performed to check how much bigger the design parameter should be to fulfill the reliability analysis if the failure type is considered as failure without warning ($\beta_t = 5.20$) for instance for $\alpha = 0.3$. It was found that the design parameter needs to be increased from $z = 0.081m^2$ to $z = 0.087m^2$ to fulfill this ultimate limit state.

4.4.2 Reinforced strength as the dominant material strength

By introducing as inputs the parameters in Table 4.11, the design variables z_A and z_B can be calculated from Eq. (4.3) and Eq. (4.4) respectively for the reinforcement. The results can be seen in Table 4.12.

For the same reason as in Section 4.4.1, load combination 6.10b Eq. (4.4) is domi-

Table 4.11: Value of the different parameters required for the calculation of the design parameter for the reinforcement.

Variable	Value
f_yk [MPa]	275 (5%fractile)
γ_{MS} [-]	1.2
α [-]	Range of values from 0-1
γ_{GA} [-]	1.375 (CC3)
γ_{GB} [-]	1.1 (CC3)
G_k [tons] $= E[P]$	121.5 (50%fractile)
γ_Q [-]	1.54 (CC3)
Q_k [tons]$= P_k$	126.89 (98%fractile)

Table 4.12: Different values of the design variables of the reinforcement depending on α.

α [-]	$z_A[m^2]$	$z_B[m^2]$
0	0.0073	0.0058
0.1	0.0066	0.0063
0.2	0.0058	0.0068
0.3	0.0051	0.0073
0.4	0.0044	0.0078
0.5	0.0036	0.0082
0.6	0.0029	0.0087
0.7	0.0022	0.0092
0.8	0.0015	0.0097
0.9	0.0007	0.0102
1	0	0.0107

nant, so it is the load combination to be used for concrete bridges with traffic load applied. A reliability analysis will be performed for different values of α in the range $\alpha = 0.2 - 0.5$.

As for the last section, the reliability analysis is performed by using the software COMREL [25], in which the result are the outputs of the reliability index (β) according to FORM (First order reliability method). The inputs to the software COMREL are shown in Table 4.13.

Different values of reliability index β_{FORM} and probability of failure Pf_{FORM} have been obtained. They are shown in Table 4.14 and are illustrated in Figure 4.3. It can be observed that for higher values of α, the reliability index β is higher as well, due to the design value z_B being higher and because the uncertainty of the different stochastic variables are weighted differently.

Table 4.13: Inputs to the software COMREL for vehicles class 100.

Variable	Comment	Distribution	μ	σ
f_y [MPa]	Reinforcement strength	Lognormal	345	25
G [tons]	Dead load	Constant	121.5	
X_g [-]	Dead load model uncertainty	Normal	0	6.08
X_Q [-]	Traffic load model uncertainty	Normal	1	0.1
X_{ms} [-]	Reinforcement strength model uncertainty	Lognormal	1	0.1
α [-]	Parameter	Constant	0.2-0.5	
U [-]	Standard normal stochastic variable associated with the annual maximum traffic load	Normal	0	1
$z[m^2]$	Design parameter	Constant	0.0068, 0.0073, 0.0078, 0.0082	
ϕ [-]	Dynamic factor	Normal	0.04	0.04
μ_w [tons]	Mean weight of a vehicle	Constant	109.2	-
σ_w [tons]	Standard deviation of the weight of a vehicle	Constant	5	-
N	Number of standard vehicles of a certain class	Constant	100	-

Table 4.14: Reliability analysis for different α values and vehicle class 100.

α [-]	$z_B[m]$	β [-]	Pf [-]
0.2	0.0068	4.91	$4.60 \cdot 10^{-7}$
0.3	0.0073	5.39	$3.62 \cdot 10^{-8}$
0.4	0.0078	5.78	$3.77 \cdot 10^{-9}$
0.5	0.0082	6.00	$9.59 \cdot 10^{-10}$

Therefore, if it is considered the target reliability index β_t depending on the type of failure, and the type of failure is failure with warning but without capacity reserve, then $\beta_t = 4.75$ as it can be seen in Table 4.1 and for all the different values of α in the range 0.2-0.5 $\beta \geq \beta_t = 4.75$, so the reliability analysis is fulfilled and the bridge can be considered as safe. An average β is calculated for the different values of the α parameter considered, resulting $\beta_{av} = 5.52$, which is way higher than $\beta_t = 4.75$.

In addition, a plot with the influence coefficient, namely the α vector, of each of the stochastic variables for the different values of α can be seen in Figure 4.4.

Additionally, it is investigated if the bridge would fulfill the reliability analysis if a higher class of standard vehicles are considered, which in this case is class 125, with a mean weight of 131.4 tons. The data necessary for the reliability assessment is shown in Ta-

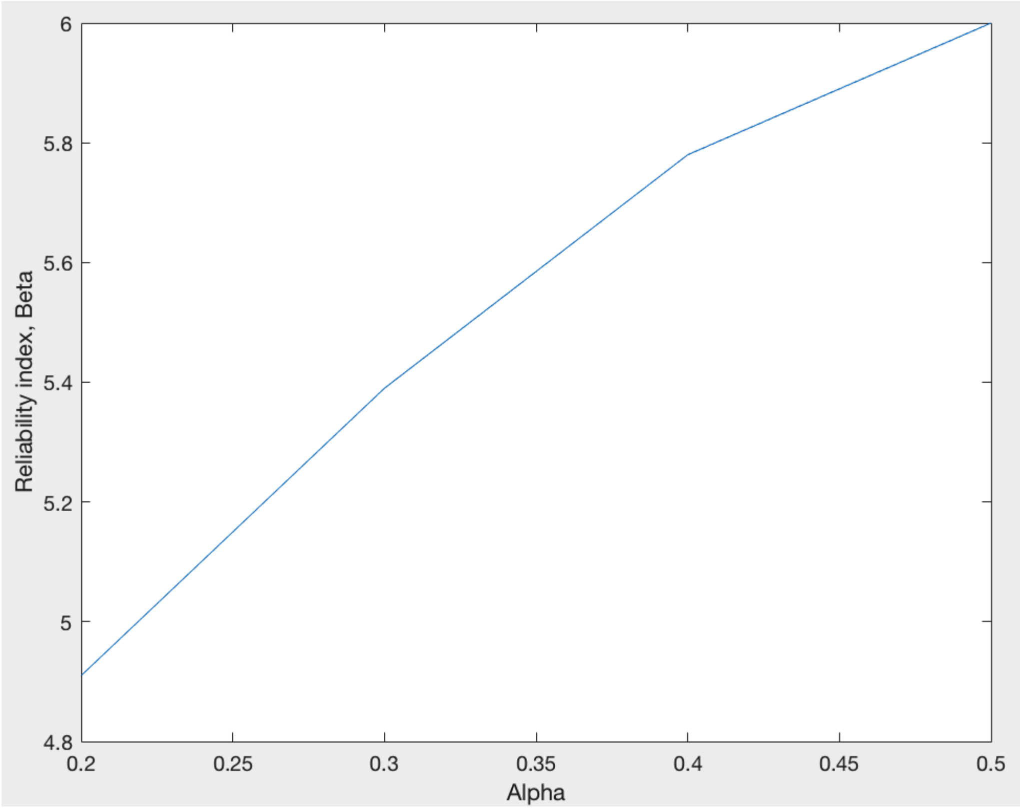


Figure 4.3: α parameter vs. reliability index β .

ble 4.15 and the result from the reliability assessment in Table 4.16.

Then, if an average reliability index is calculated, $\beta_{av} = 4.25$. It does not fulfill the reliability analysis is the target reliability index is $\beta_t = 4.75$. However, it might be possible to reach $\beta_t = 4.75$ after an updating of some parameters involved. That will be checked in the next section.

Finally, it has been checked the difference of the influence coefficient (α vector) for concrete and reinforced. The influence coefficient for the resistance is lower for reinforcement due to the uncertainty related to the resistance for reinforcement is smaller, consequently it can be seen that R and X_m are inverted for concrete and reinforcement, see Figure 4.5.

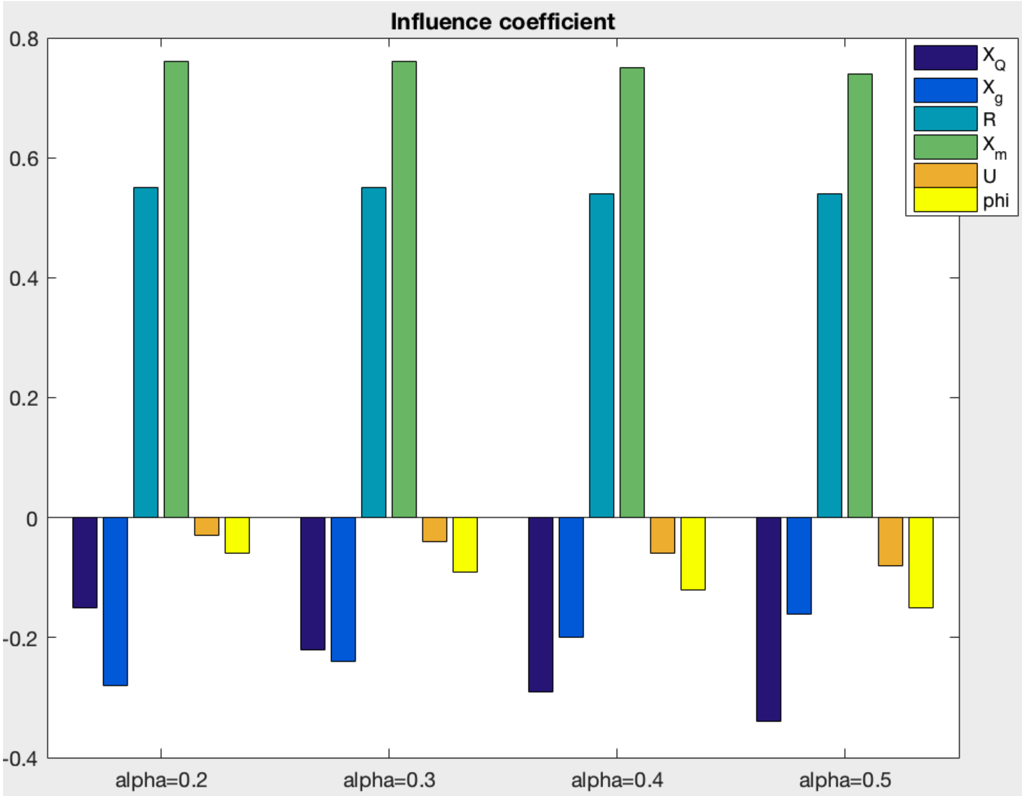


Figure 4.4: Influence coefficients for reinforcement strength as the dominant material strength and standard vehicles class 100.

Table 4.15: Inputs to the software COMREL for vehicles class 125.

Variable	Comment	Distribution	μ	σ
f_y [MPa]	Reinforcement strength	Lognormal	345	25
G [tons]	Dead load	Constant	143.93	
X_g [-]	Dead load model uncertainty	Normal	0	7.20
X_Q [-]	Traffic load model uncertainty	Normal	1	0.1
X_{ms} [-]	Reinforcement strength model uncertainty	Lognormal	1	0.1
α [-]	Parameter	Constant	0.2-0.5	
U [-]	Standard normal stochastic variable associated with the annual maximum traffic load	Normal	0	1
$z[m^2]$	Design parameter	Constant	0.0068, 0.0073, 0.0078, 0.0082	
ϕ [-]	Dynamic factor	Normal	0.04	0.04
μ_w [tons]	Mean weight of a vehicle	Constant	131.4	-
σ_w [tons]	Standard deviation of the weight of a vehicle	Constant	5	-
N	Number of standard vehicles of a certain class	Constant	100	-

Table 4.16: Reliability analysis for different α values and vehicle class 125.

α [-]	$z_B[m]$	β [-]	Pf [-]
0.2	0.0068	3.61	$1.51 \cdot 10^{-4}$
0.3	0.0073	4.10	$2.06 \cdot 10^{-5}$
0.4	0.0078	4.51	$3.20 \cdot 10^{-6}$
0.5	0.0082	4.76	$9.49 \cdot 10^{-7}$

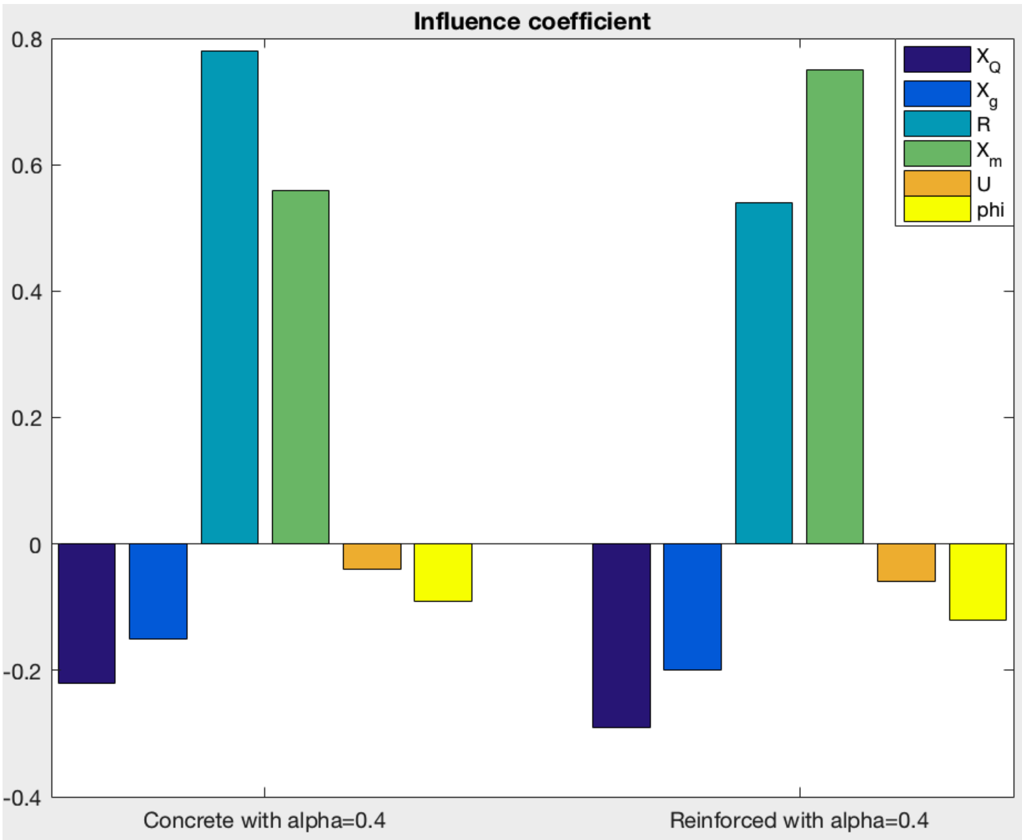


Figure 4.5: Influence coefficient comparison for concrete compression strength and reinforced strength and standard vehicles class 100.

CHAPTER 5

Reliability updating of a generic existing bridge

In order to update the measured quantities in a structure, it is necessary to perform an investigation. Two different inspections can be considered:

- Qualitative inspection: It is a preliminary investigation mainly based on visual inspection and the possible damages on the structure are usually cataloged as "unknown", "severe", "moderate", "minor" etc.
- Quantitative inspection: More accurate investigation which characterizes the current properties of the components of the structure. Updating is based on this kind of inspection.

Different types of test results/observations can be used for the updating of the reliability of an existing bridge:

1. Test results related to material parameters.

2. Proof loading.

3. Loads applied to the structure.

In the following sections, the different updating procedures will be explained and the reliability of the bridge considered in the last chapter will be update based on different data.

5.1 Evaluation of inspection results

Once an investigation have been carried out and some results have been acquired, it is possible to update the properties and reliability of a structure [15]. Two different procedures can be distinguished:

1. Direct updating of the failure probability of the structure.

2. Updating of the individual or multivariate probability distribution of the stochastic variables.

5.1.1 Direct updating of the structural failure probability

By using Bayesian theorem, Eq. (5.2), the structural reliability can be directly updated.

$$P(F \mid I) = \frac{P(F \cap I)}{P(I)} \tag{5.1}$$

where

F	Local or global structural failure.
I	Information gathered by investigation.
$\cap$	Intersection of two element.
$\mid$	Means conditional upon.

If the limit state equation is $g(x)$, where x is the vector of basic variables so the failure F is the failure event that $g(x) < 0$ and the investigation event I is described by the inequality $H > 0$, then the updated failure probability can be written as follows [12]:

$$P(g(x) < 0 \mid H > 0) = \frac{P(g(x) < 0 \cap H > 0)}{P(H > 0)} \tag{5.2}$$

5.1.2 Updating of probability distributions

By equation Eq. (5.4) the procedure to update the individual or multivariate probability distributions is introduced:

$$f_x(X \mid I) = C \cdot P(I \mid x) f_x(X) \tag{5.3}$$

where

$f_x(X \mid I)$	Updated probability density function of the variable X after having updated with information I
X	Basic variable.
C	Normalizing factor.
$P(I \mid x)$	Likelihood function.
$f_x(X)$	Probability density function of X before updating.

Once the updated distribution of the basic variables $f_x(X \mid I)$ have been determined, the updated failure probability $P(F \mid I)$ can be calculated by carrying out a probability-based analysis using structural reliability methods for new structures [12]:

$$P(F \mid I) = \int_{g(x) \leq 0} f_x(X \mid I) dx \tag{5.4}$$

An example of the updating of a probability distribution can be seen in Figure 5.1.

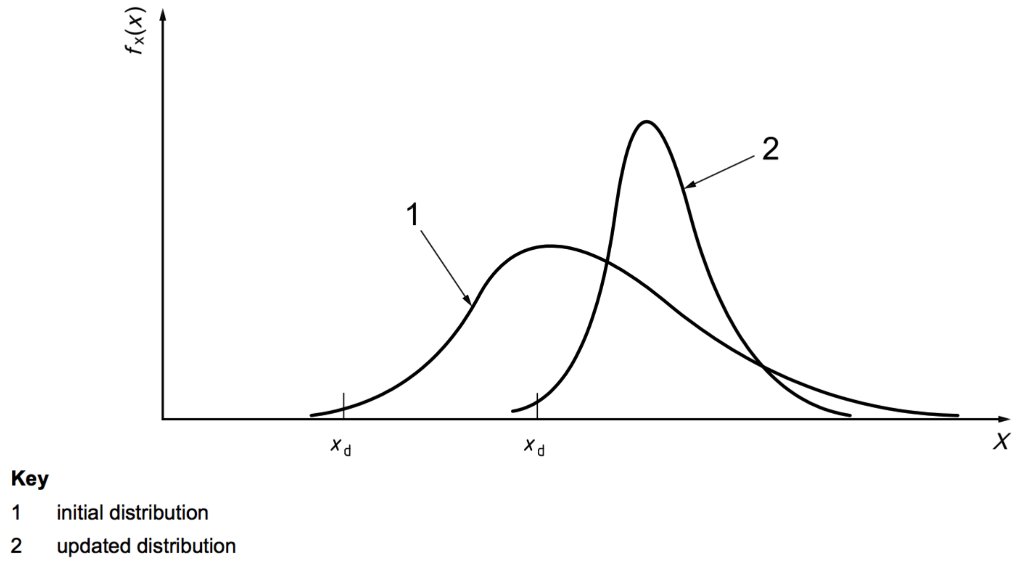


Figure 5.1: Initial and updated probability density function for a variable X [15].

5.2 Direct updating of the structural failure probability

Direct updating of the structural failure probability can be carried out by different types of test results. In this project, the updating will be perform with three different tests results:

1. Proof loading
2. Test results related to material parameters
3. Test results related to load bearing capacity

5.2.1 Updating by proof load tests

When assessing existing bridges with large uncertainties, analytic methods have limitations and tend to be conservative, see Section 2.2.3. Typically, conservative assumptions are made, so it ends up being conservative assessments. In order to reduce these uncertainties, different field testing methods can be applied on the bridge. One of them is the proof load test, in which a load equivalent to the live load is placed on the bridge. After performing the proof load test, if it is proved that the bridge can carry the load applied without sign of damages, the proof load test can be considered as successful and it has been proved that the code requirements are met. Additionally, before performing the proof load test, what is called "the stop criteria" has to be set in order to avoid damages/collapse of the bridge. This stop criteria is a maximum value of the loads applied in the proof loading in order to ensure that damages/collapse does not happen on the bridge.

In this project, and updating of the reliability by proof load tests data will be performed by the software SYSREL [25]. For that purpose, the same generic ultimate state equation as in Chapter 4 Eq. (5.5) has been used and a conditional equation related to the proof load test Eq. (5.6) have been introduced to the software:

$$g1 = zX_mR - ((1-\alpha)(G+Xg) + \alpha X_Q(1+\phi)P) \tag{5.5}$$

$$g2 = ((1-\alpha)(G+Xg) + \alpha P_l) - zX_mR \tag{5.6}$$

where P_l is the load applied in the proof load test.

Then, applying Bayesian theorem:

$$P(g1 < 0 \mid g2 > 0) = \frac{P(g1 < 0 \cap g2 > 0)}{P(g2 > 0)} \tag{5.7}$$

The members of Eq. (5.6) are inverted with regard to Eq. (5.5) because this equation represents the survival of the bridge, so:

$$P_s = 1 - P_f \tag{5.8}$$

where P_s is the probability of survival.

An updating of the proof loading has been made for the two generic failure modes considered in Chapter 4.

Updating of proof loading with concrete compression strength as the dominant material strength

Two cases has been considered for this generic failure mode:

1. Proof loading updating for standard vehicles class 100.
2. Proof loading updating for standard vehicles class 125.

Proof loading updating for standard vehicles class 100

For updating of the proof loading for standard vehicles class 100, the different variables from Table 4.7 has been introduced to SYSREL . In addition, a new variable for the proof loading P_l has been added as a constant and different values for this parameter has been used. The different reliability indexes obtained after performing the updating, β_u, as well as the probability of failure while applying the proof load for updating, $P_{f,proof}$, can be seen in Table 5.1.

Additionally, different plots have been made for each of the different values of α. As an example, it can be seen for $\alpha = 0.2$ in Figure 5.2. For the other values of α, see Appendix C.

Table 5.1: Updated reliability indexes for standard vehicles class 100

$P_l[tons]$ | α	0.2		0.3		0.4		0.5	
	β_u	$P_{f,proof}$	β_u	$P_{f,proof}$	β_u	$P_{f,proof}$	β_u	$P_{f,proof}$
50 $\%P_k$= 63.49	4.42	2.33e-7	4.81	3.32e-9	5.08	4.02e-11	5.31	3.01e-13
60 $\%P_k$=76.19	4.43	4.55e-7	4.81	1.01e-8	5.08	2.49e-10	5.31	4.25e-12
70 $\%P_k$=88.89	4.45	8.34e-6	4.82	2.98e-8	5.08	1.26e-9	5.31	4.29e-11
80 $\%P_k$=101.58	4.49	1.51e-6	4.83	8.03e-8	5.09	5.65e-9	5.32	3.64e-10
90 $\%P_k$=114.28	4.56	2.56e-6	4.87	2.10e-7	5.11	2.38e-8	5.32	2.46e-9
100 $\%P_k$=126.98	4.68	4.50e-6	4.96	5.30e-7	5.16	8.48e-8	5.35	1.43e-8
110 $\%P_k$=139.68	4.89	7.46e-6	5.11	1.24e-6	5.27	2.87e-7	5.44	6.82e-8
120 $\%P_k$=152.38	5.19	1.22e-5	5.35	2.68e-6	5.47	8.76e-7	5.58	2.87e-7
130 $\%P_k$=165.07	5.58	1.98e-5	5.68	5.41e-6	5.75	2.44e-6	5.82	1.07e-6

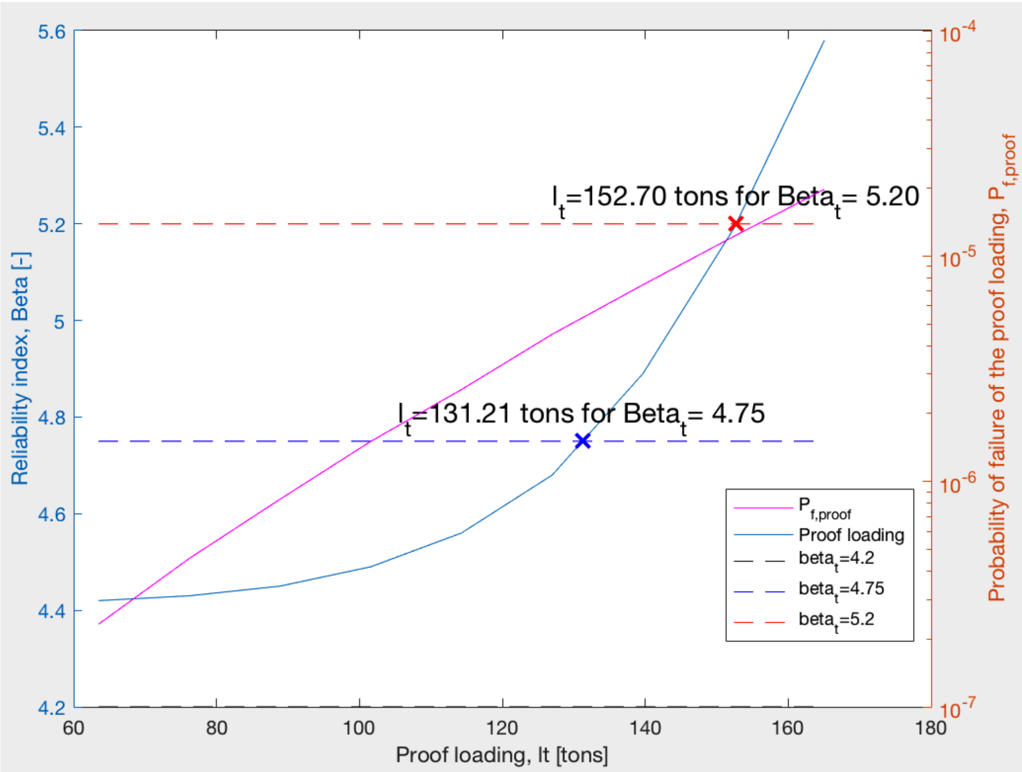


Figure 5.2: Proof loading updating for concrete compression strength, standard vehicles class 100 and $\alpha = 0.2$

As it can be seen in Figure 5.2, the proof loading levels when the possible target reliability indexes are reached are shown by crosses, so in order to obtain higher values of β_u, the load applied on the bridge needs to be higher. In this case, the target reliability indexes considered are $\beta_t = 4.75$ and $\beta_t = 5.20$. $\beta_t = 4.20$ was not considered in this case as β was higher than this value already in the reliability analysis. In addition, the probability of failure while applying the proof loading is represented by the purple line, so you have an idea of how big is the risk of collapse of the bridge when performing a proof load test.

Proof loading updating for standard vehicles class 125

After updating the proof loading for standard vehicles class 125, the different reliability indexes, β_u, in Table 5.2 were obtained.

Table 5.2: Updated reliability indexes for standard vehicles class 125

$P_l[tons]$ | α	0.2		0.3		0.4		0.5	
	β_u	$P_{f,proof}$	β_u	$P_{f,proof}$	β_u	$P_{f,proof}$	β_u	$P_{f,proof}$
50 $\%P_k$ = 74.51	3.48	2.25e-5	3.86	6.17e-7	4.14	1.51e-8	4.38	2.05e-10
60 $\%P_k$= 89.41	3.5	3.75e-5	3.86	1.66e-6	4.14	6.82e-8	4.38	2.05e-9
70 $\%P_k$= 104.31	3.53	6.15e-5	3.87	4.10e-6	4.14	2.87e-7	4.39	1.60e-8
80 $\%P_k$= 119.22	3.58	9.96e-5	3.9	9.34e-6	4.15	1.02e-6	4.39	9.44e-8
90 $\%P_k$= 134.12	3.67	1.59e-4	3.96	2.07e-5	4.18	3.24e-6	4.4	5.04e-7
100 $\%P_k$= 149.02	3.83	2.42e-4	4.07	4.25e-5	4.27	9.77e-6	4.45	2.11e-6
110 $\%P_k$= 163.92	4.09	3.76e-4	4.28	8.50e-5	4.42	2.56e-5	4.57	7.80e-6
120 $\%P_k$= 178.82	4.46	5.57e-4	4.58	1.65e-4	4.68	6.41e-5	4.78	2.67e-5
130 $\%P_k$= 193.73	4.91	8.16e-5	4.98	3.02e-4	5.02	1.47e-4	5.08	7.53e-5
140 $\%P_k$= 208.63	5.43	0.0011	5.44	5.19e-4	5.44	3.25e-4	5.46	2.00e-4

The values of the reliability indexes for different proof loading, P_l, for $\alpha = 0.3$ can be seen in Figure 5.3.

As it can be seen in Table 5.2 and Figure 5.3, the increase of β_u for low values of P_l is low, being the curve almost horizontal at the beginning of the proof loading. As long as the proof loading is increased, it starts to increase in a similar way to an exponential curve. In this case, it can be seen that to get a $\beta_u = 4.75$, the probability of failure is not that high, so P_l can be increased at least until $\beta_u = 4.75$.

In addition, a plot which shows the proof loading against α for $\beta_t = 4.75$ and $\beta_t = 5.2$ can be seen in Figure 5.4. As it can be seen in Figure 5.4, the higher the α values are, the lower the proof load P_l has to be in order to reach the target reliability indexes $\beta_t = 4.75$ and $\beta_t = 5.2$.

Updating of proof loading with reinforced strength as the dominant material strength

Proof loading updating for standard vehicles class 100

In this case, it is not necessary to update the proof loading for standard vehicles class 100 for this generic mode. The reason is because they are above $\beta_t = 5.2$ for all α values considered but for $\alpha = 0.2$ (see Table 4.14, which is $\beta = 4.91$, so it is still high and above

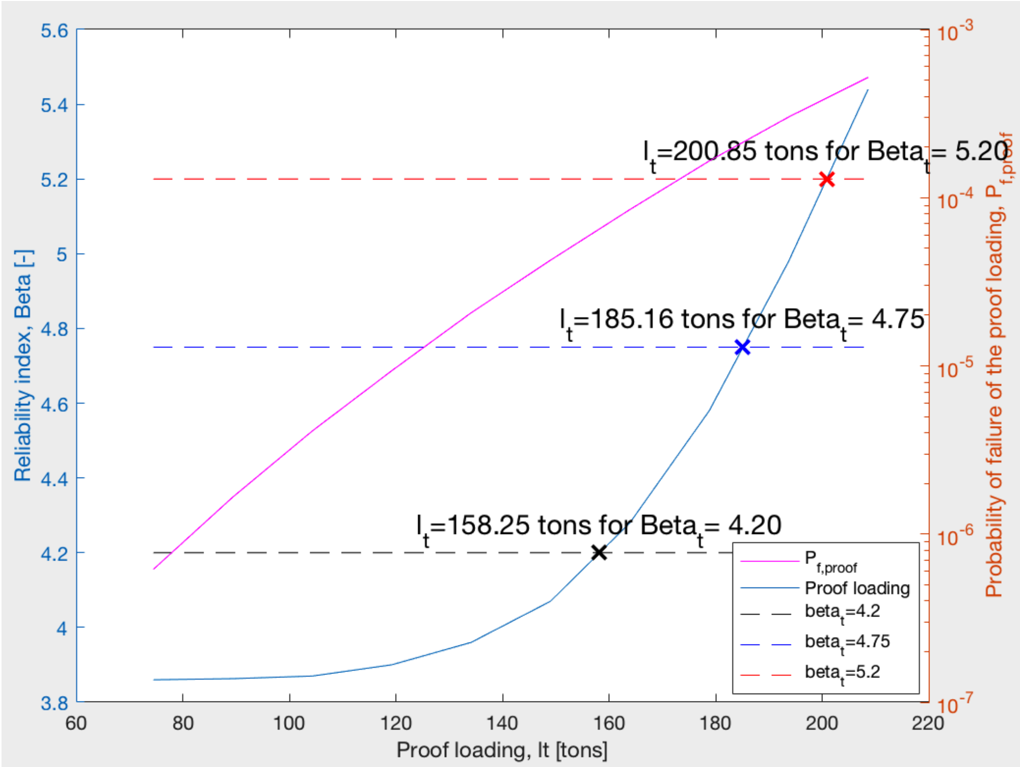


Figure 5.3: Proof loading updating for concrete compression strength, standard vehicles class 125 and $\alpha = 0.3$.

$\beta_t = 4.75$.

Proof loading updating for standard vehicles class 125

For standard vehicles class 125, β values are lower, so an update is required in this case. After performing the proof load updating, the different reliability indexes in Table 5.3 were obtained.

Additionally, the value of the reliability index for different proof loading P_l for $\alpha = 0.3$ can be seen in Figure 5.5.

In addition, a plot which shows the proof loading against α for $\beta_t = 4.75$ and $\beta_t = 5.2$ can be seen in Figure 5.6. As it can be seen in Figure 5.6, the curve for $\beta_t = 4.75$ has no value for $\alpha = 0.5$, the reason for that is that for $\alpha = 0.5$, the reliability index in this case is already higher than $\beta_t = 4.75$, see Table 4.16, so a proof load test is not needed for $\alpha = 0.5$ if the target reliability index is $\beta_t = 4.75$.

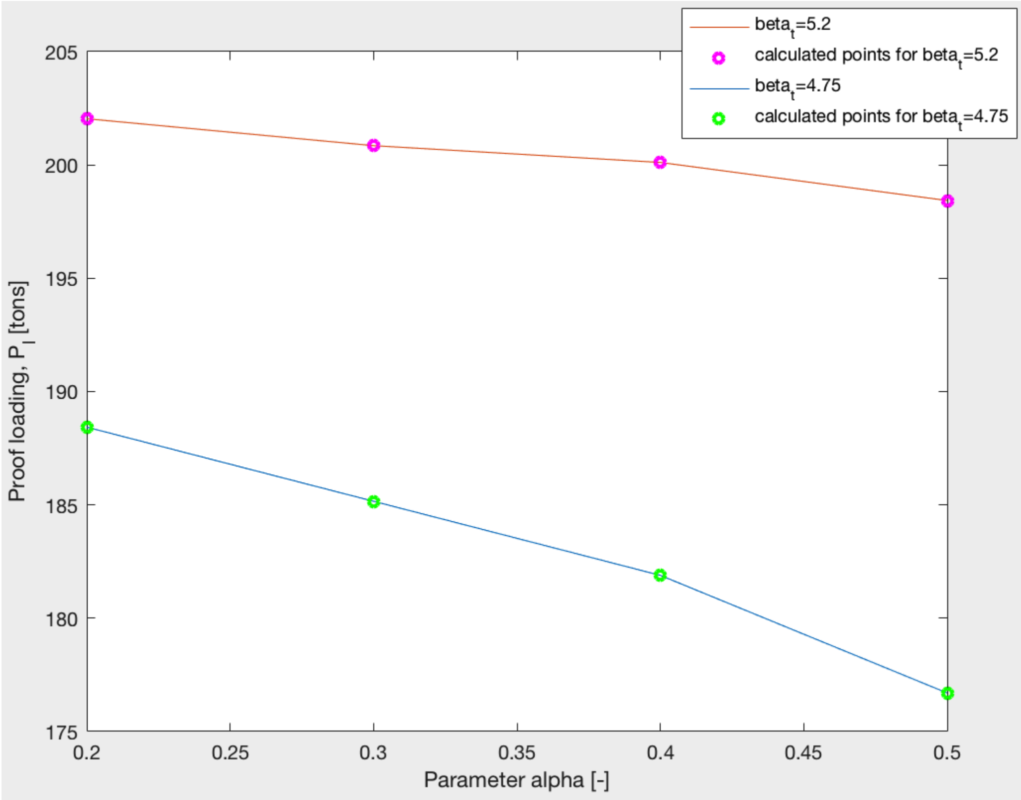


Figure 5.4: Proof loading updating for concrete compression strength, standard vehicles class 125 for different values of alpha considering $\beta_t = 4.75$ and $\beta_t = 5.2$.

Table 5.3: Updated reliability indexes for standard vehicles class 125.

$P_l[tons]$ | α	0.2		0.3		0.4		0.5	
	β_u	$P_{f,proof}$	β_u	$P_{f,proof}$	β_u	$P_{f,proof}$	β_u	$P_{f,proof}$
50 $\%P_k$ = 74.51	3.62	4.10e-6	4.10	2.25e-8	4.51	4.29e-11	4.76	5.86e-14
60 $\%P_k$= 89.41	3.63	8.54e-6	4.1	9.44e-8	4.51	4.68e-10	4.76	2.26e-12
70 $\%P_k$= 104.31	3.65	1.74e-5	4.11	3.52e-7	4.51	4.21e-9	4.76	5.59e-11
80 $\%P_k$= 119.22	3.68	3.45e-5	4.11	1.24e-6	4.51	2.98e-8	4.76	9.27e-10
90 $\%P_k$= 134.12	3.75	6.67e-5	4.15	3.91e-6	4.53	1.79e-7	4.77	1.14e-8
100 $\%P_k$= 149.02	3.88	1.21e-4	4.23	1.12e-5	4.56	9.21e-7	4.78	1.11e-7
110 $\%P_k$= 163.92	4.12	2.16e-4	4.38	3.04e-5	4.66	4.10e-6	4.84	8.34e-7
120 $\%P_k$= 178.82	4.45	3.75e-4	4.64	7.84e-5	4.84	1.52e-5	4.97	4.94e-6
130 $\%P_k$= 193.73	4.88	6.41e-4	4.99	1.85e-4	5.13	5.22e-5	5.20	2.35e-5
140 $\%P_k$= 208.63	5.39	0.001	5.43	4.04e-4	5.5	1.59e-4	5.52	9.57e-5

5.2.2 Updating by test results related to material parameters

Another way of rising the reliability of a bridge, is to update the knowledge of the material parameters by extracting several samples and analysis them in the laboratory. As a preliminary approach, the variation of the reliability index, β_u, is checked by considering the

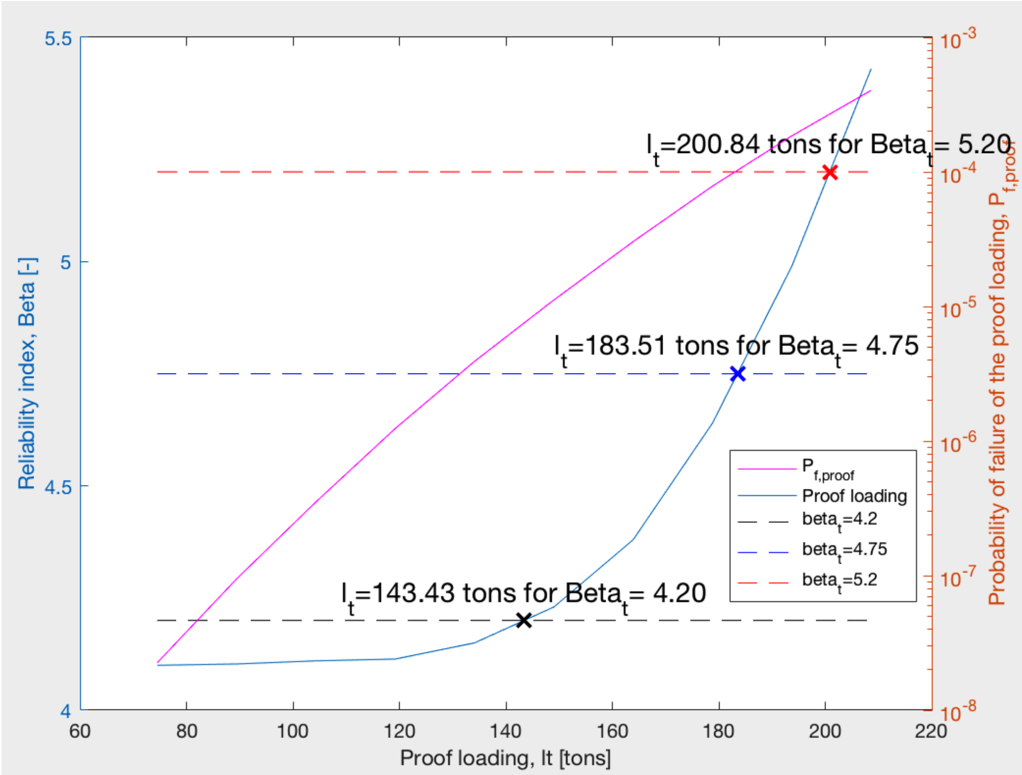


Figure 5.5: Proof loading updating for reinforcement strength, standard vehicles class 125 and $\alpha = 0.3$.

same mean value and different coefficients of variation.

Updating of material coefficient of variation of the concrete compression strength as the dominant material strength

Two cases has been considered, as in the last section, for this generic failure mode:

1. *CV* updating for standard vehicles class 100.
2. *CV* updating for standard vehicles class 125.

Coefficient of variation updating for standard vehicles class 100

Considering the variables in Table 4.7 and by taking different values for the coefficient of variation of the concrete strength stochastic variable, in the range of $CV = [0.12 - 0.22]$, the values of β_u shown in Table 5.4 have been obtained.

The CV considered in the reliability analysis is $CV = 0.14$. Therefore, if the CV is increased, the uncertainty is higher so the reliability index β_u decreases. On the other hand, if it is found in the analyzed test samples that the coefficient of variation of the concrete compression strength decreases, the uncertainty is lower so the reliability index β_u increases, which is the aim of this kind of updating. A plot which illustrates the variation of

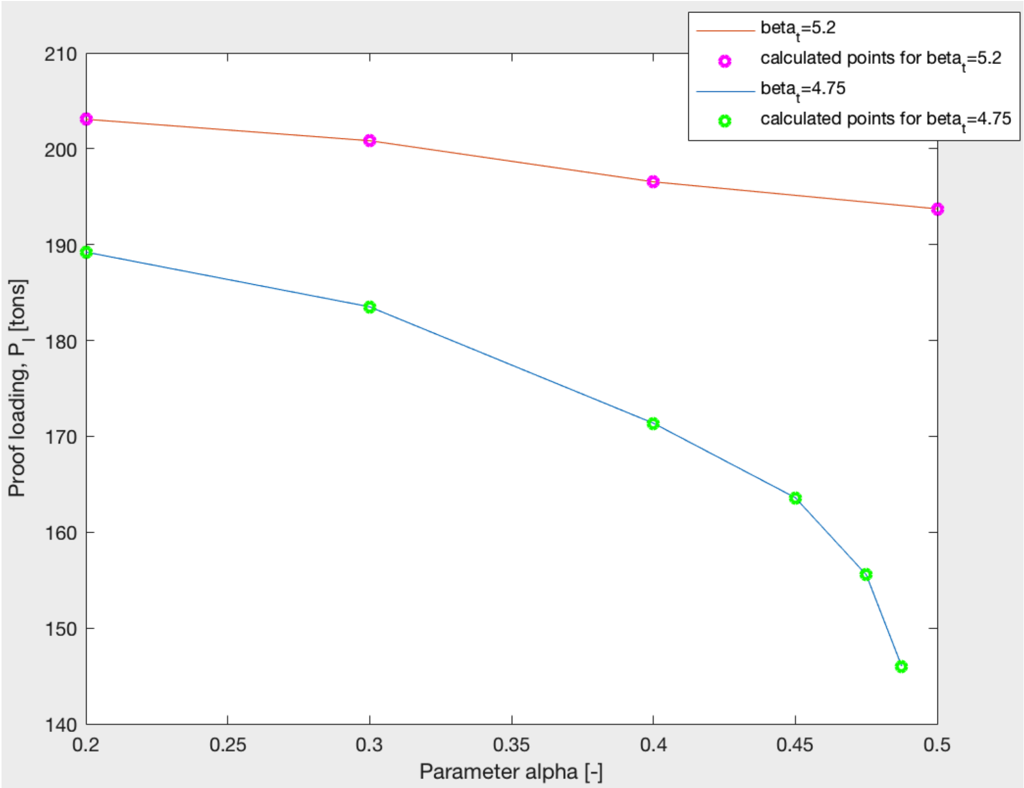


Figure 5.6: Proof loading updating for reinforcement strength, standard vehicles class 125 for different values of alpha considering $\beta_t = 4.75$ and $\beta_t = 5.2$.

Table 5.4: Updated reliability indexes for standard vehicles class 100

$CV \mid \alpha$	0.2	0.3	0.4	0.5
0.12	4.84	5.27	5.56	5.81
0.13	4.62	5.03	5.32	5.55
0.14	4.41	4.81	5.08	5.32
0.15	4.22	4.60	4.86	5.09
0.16	4.04	4.40	4.66	4.88
0.17	3.86	4.21	4.46	4.68
0.18	3.70	4.04	4.28	4.49
0.22	3.15	3.45	3.66	3.85

the reliability index β_u with respect to the coefficient of variation , CV, for $\alpha = 0.4$ can be seen in Figure 5.7. For other α values, see Appendix D.

Coefficient of variation updating for standard vehicles class 125

In this case, for standard vehicles class 125, β values are lower, so an update is required. The different reliability indexes in Table 5.5 were obtained.

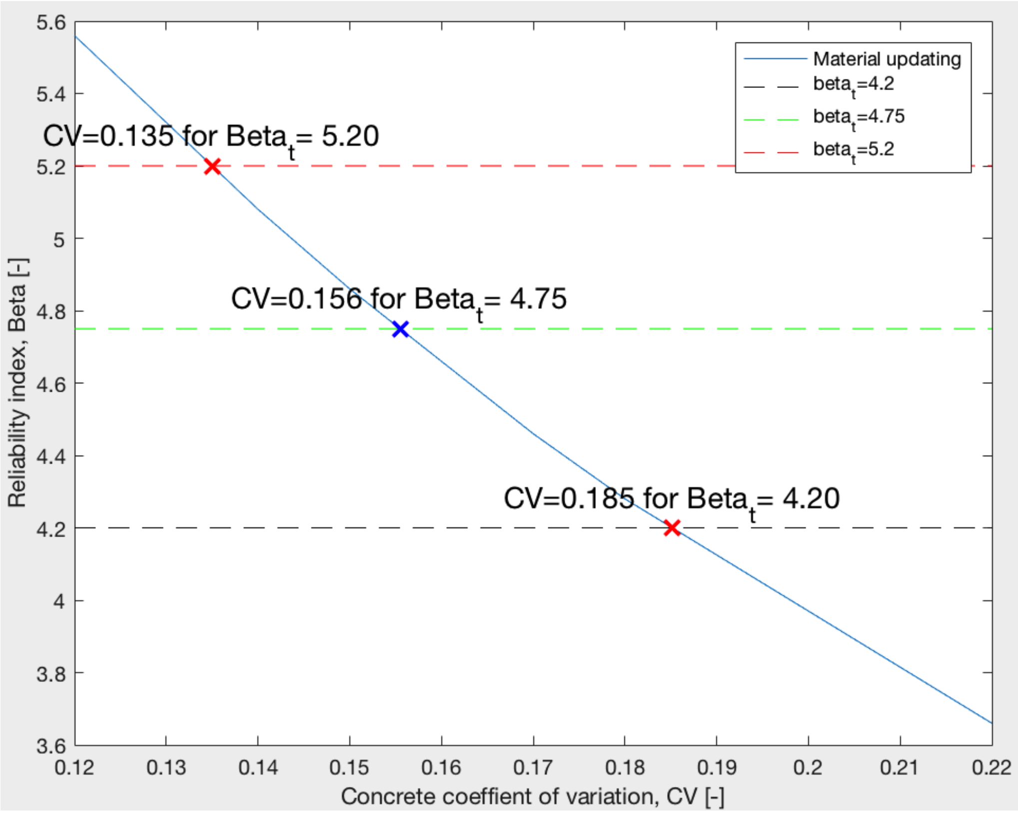


Figure 5.7: Updating of the coefficient of variation for concrete compression strength, standard vehicles class 100 and $\alpha = 0.4$.

Table 5.5: Updated reliability indexes for standard vehicles class 125.

$CV \mid \alpha$	0.2	0.3	0.4	0.5
0.12	3.83	4.26	4.55	4.80
0.13	3.65	4.06	4.35	4.59
0.14	3.48	3.88	4.15	4.39
0.15	3.32	3.70	3.97	4.21
0.16	3.18	3.54	3.80	4.03
0.17	3.04	3.39	3.64	3.86
0.18	2.91	3.25	3.49	3.71
0.22	2.47	2.76	2.98	3.17

A plot with the reliability index, β_u, in function of the coefficient of variation, CV, for $\alpha = 0.3$ can be seen in Figure 5.8. See Appendix D for other values of α.

In the reliability analysis performed in this project, see Chapter 4, the coefficient of variation related to the concrete compression strength chosen was $CV = 0.14$, obtaining $\beta = 4.39$ for $\alpha = 0.5$. If the target reliability index $\beta_t = 4.75$, a coefficient of variation $CV \leq 0.122$ is needed, see Figure 5.8. On the other hand, coefficient of variation $CV \geq 0.151$ would

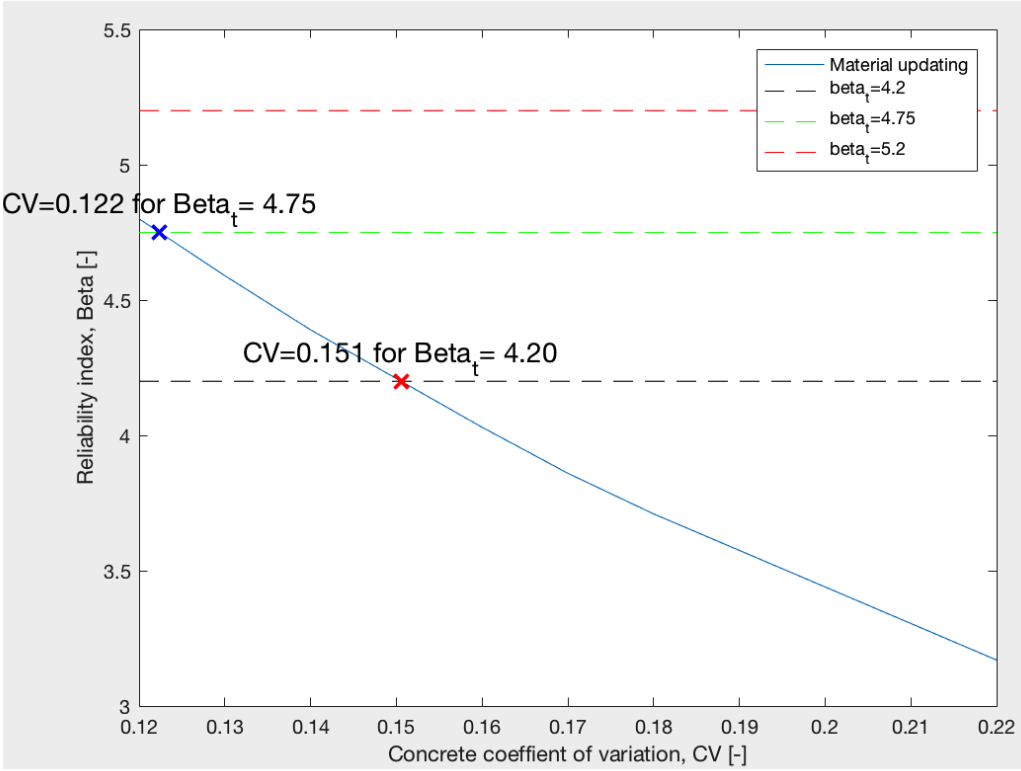


Figure 5.8: Coefficient of variation updating for concrete compression strength, standard vehicles class 125 and $\alpha = 0.5$.

mean the reliability index $\beta_t \leq 4.2$.

Updating of the coefficient of variation of the reinforced strength as the dominant material strength

Proof loading updating for standard vehicles class 100

It has been considered not necessary to update the proof loading for standard vehicles class 100 for this generic mode. The reason is because they are above $\beta_t = 5.2$ for all α values considered but for $\alpha = 0.2$ (see Table 4.14), which is $\beta = 4.91$, so it is high enough and above $\beta_t = 4.75$.

Proof loading updating for standard vehicles class 125

For standard vehicles class 125, β values are lower, so an update is required in this case. Considering several values of the CV, different reliability indexes shown in Table 5.6 were determined.

A plot with the updated reliability index , β_u, in function of the coefficient of variation , CV, for $\alpha = 0.4$ can be seen in Figure 5.9. See Appendix D for other values of α.

In the reliability analysis performed in this project, see Chapter 4, the coefficient of variation related to the reinforced strength chosen was $CV = 0.072$, obtaining $\beta = 4.51$

Table 5.6: Updated reliability indexes for standard vehicles class 125

$CV \mid \alpha$	0.2	0.3	0.4	0.5
0.04	4.09	4.63	5.08	5.34
0.05	3.96	4.48	4.92	5.18
0.06	3.81	4.32	4.75	5.00
0.07	3.65	4.14	4.56	4.81
0.08	3.49	3.97	4.37	4.62
0.09	3.34	3.79	4.18	4.43
0.10	3.18	3.62	4.00	4.24

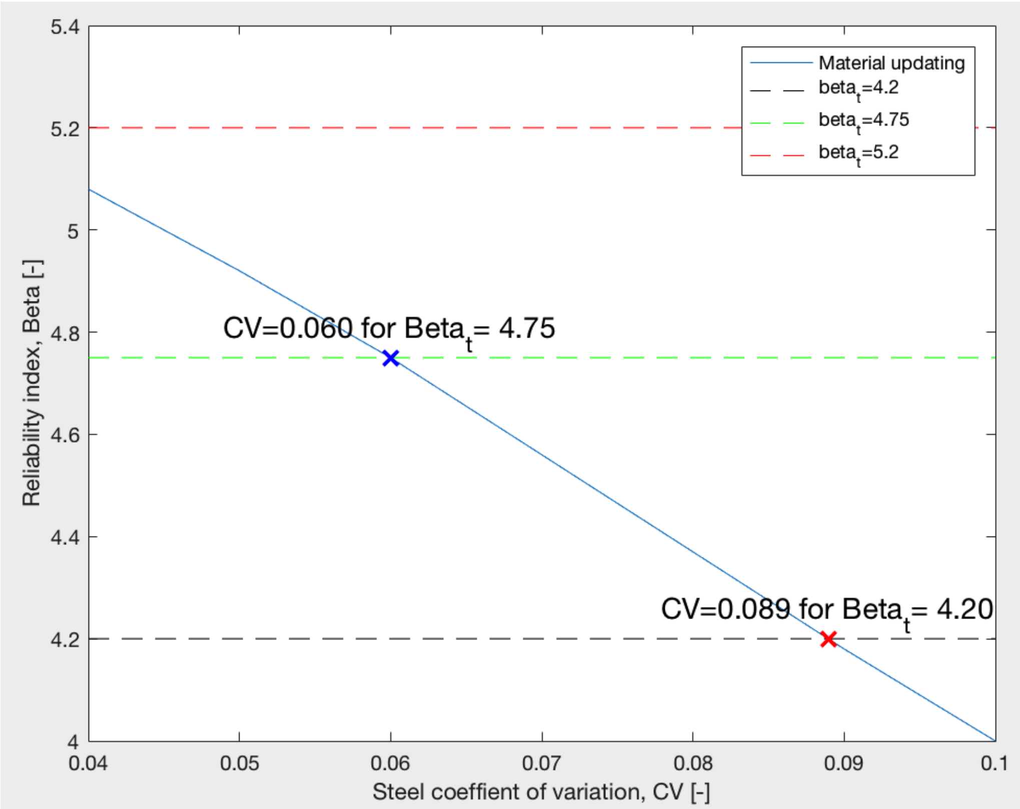


Figure 5.9: Coefficient of variation updating for reinforcement strength, standard vehicles class 125 and $\alpha = 0.4$.

for $\alpha = 0.4$. If the target reliability index $\beta_t = 4.75$, a coefficient of variation $CV \leq 0.06$ is needed, see Figure 5.9. On the other hand, coefficient of variation $CV \geq 0.089$ would mean the reliability index $\beta_t \leq 4.2$.

Updating in material strength testing when considering Normal distribution

When updating the strength of a material, three cases can be considered if the material strength follows a Normal distribution:

- Normal distribution with unknown mean and known standard deviation.
- Normal distribution with known mean and unknown standard deviation.
- Normal distribution with unknown mean and standard deviation

In this section, the concrete compressive strength is meant to follow a normal distribution and the case with unknown mean and known standard deviation will be studied.

In the case of a normally distributed variable with uncertain mean and known standard deviation, the distribution is as followed [26]:

$$f_X(x|\mu,\sigma) = f_N(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} exp(-\frac{1}{2}(\frac{x-\mu}{\sigma})^2) \tag{5.9}$$

where the unknown parameter is μ.

The prior probability density function is assumed Normal and defined as followed:

$$f'_\mu(\mu) = f_N(\mu|\mu',\sigma') = \frac{1}{\sigma'\sqrt{2\pi}} exp(-\frac{1}{2}(\frac{\mu-\mu'}{\sigma'})^2) \tag{5.10}$$

Then, some test results have been collected and represented by $\hat{x} = (\hat{x_1}, \hat{x_2}, ..., \hat{x_n})$ and its average is calculated:

$$\bar{x} = \frac{1}{n}\sum \hat{x_i} \tag{5.11}$$

Therefore, the posterior probability density function becomes Normal:

$$f''_\mu(\mu|\hat{x}) = f_N(\mu|\mu'',\sigma'') \tag{5.12}$$

where:

$$\mu'' = \frac{n\bar{x}\sigma'^2 + \mu'\sigma^2}{n\sigma'^2 + \sigma^2} \tag{5.13}$$

$$\sigma''^2 = \frac{\sigma'^2\sigma^2}{n\sigma'^2 + \sigma^2} \tag{5.14}$$

being n the number of test samples.

Consequently, the predictive probability density function becomes Normal as well:

$$f_X(x|\hat{x}) = f_N(x|\mu'',\sigma''') \tag{5.15}$$

where

$$\sigma''' = \sqrt{(\sigma''^2 + \sigma^2)} \tag{5.16}$$

Therefore, in order to increase the reliability of the concrete bridge by a material updating, different cases have been considered related to the mean value and standard deviation of the concrete compressive strength.

Two main cases have been considered in this analysis:

1. Case when the test average value, $\hat{x}$, is constant and different values of the number of samples are considered ($n = 1, 3, 5, 7, 9, 11$).
2. Case when the number of samples tested, n, is constant and different values of the test average value are considered ($\hat{x} = 30, 33, 36, 39, 42, 45$).

Then, for each of the above cases, there different cases have been considered:

1. Case when $\sigma' = \frac{1}{2}\sigma$ is assumed.
2. Case when $\sigma' = \frac{1}{3}\sigma$ is assumed.
3. Case when $\sigma' = \frac{1}{4}\sigma$ is assumed.

In addition, different α values in the range of $\alpha = 0.2 - 0.5$ are considered for each of the cases.

Test average value constant, $\bar{x} = 39$ MPa with $\sigma' = \frac{1}{2}\sigma$

The first thing to be done is to calibrate the case considering $\sigma'''_{initial}$ equals to the value considered in the reliability analysis, therefore $\sigma'''_{initial} = 5.07$, so σ can be calculated from:

$$\sqrt{\sigma^2 + (\frac{1}{2}\sigma)^2} = 5.07$$

By solving this equation, $\sigma = 4.53$, and consequently $\sigma' = 2.27$.

Having this data, the parameters of the posterior and predictive density functions can be calculated, and as an example, for the case when $n = 1$, is shown below:

$$\mu''(n=1) = \frac{n\bar{x}\sigma'^2 + \mu'\sigma^2}{n\sigma'^2 + \sigma^2} = \frac{1 \cdot 39 \cdot 2.27^2 + 36.2 \cdot 4.53^2}{1 \cdot 2.27^2 + 4.53^2} = 36.79\,\text{MPa}$$

$$\sigma''(n=1) = \sqrt{\frac{\sigma'^2\sigma^2}{n\sigma'^2+\sigma^2}} = \sqrt{\frac{2.27^2\cdot 4.53^2}{1\cdot 2.27^2+4.53^2}} = 2.03\,\text{MPa}$$

where μ' is the mean value of the prior distribution, which is $\mu' = 36.2$ MPa as it was the value used for concrete compressive strength in the reliability analysis. A comparison between the prior and posterior probability density functions of the expected value for concrete compressive strength can be seen in Figure 5.10 for $n = 1$.

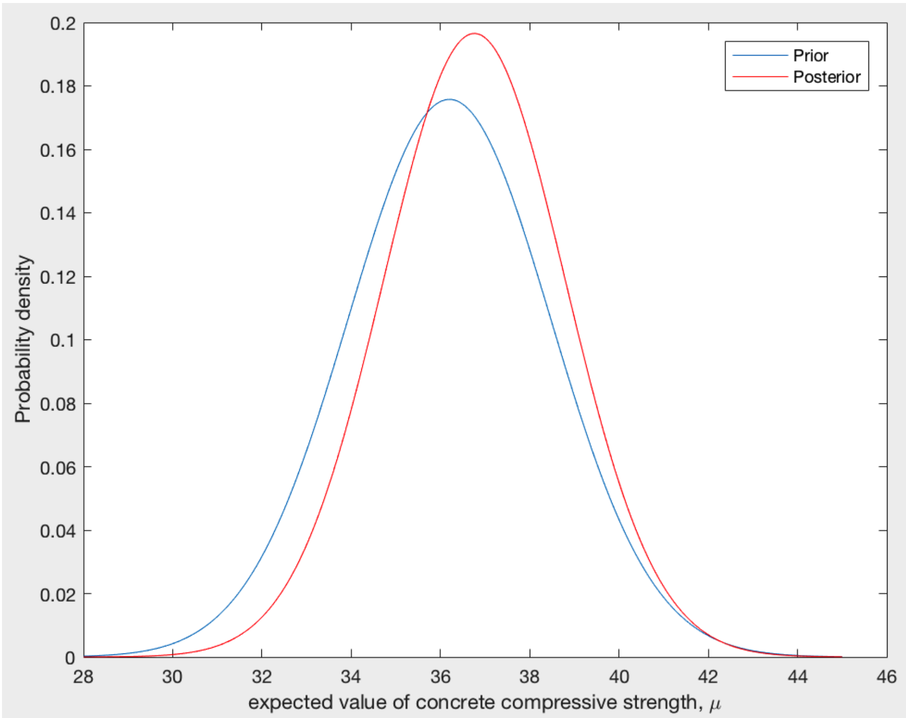


Figure 5.10: *Prior* and *posterior* probability density functions for the expected value of the concrete compressive strength for n=1.

It is interesting to show how is the difference when 11 test samples are taken instead of 1 test sample, so it can be seen in Figure 5.11. It can be seen that the probability density function is modified as the expected value μ'' is higher and the standard deviation σ'' is lower for $n = 11$.

Then, the standard deviation σ''' of the predictive distribution can be calculated:

$$\sigma'''(n=1) = \sqrt{(\sigma''^2+\sigma^2)} = \sqrt{(2.03^2+4.53^2)} = 4.96\,\text{MPa}$$

In Figure 5.12 the original and predictive probability density functions for concrete compressive strength are shown when $n = 1$. In addition, in Figure 5.13 can be seen that the

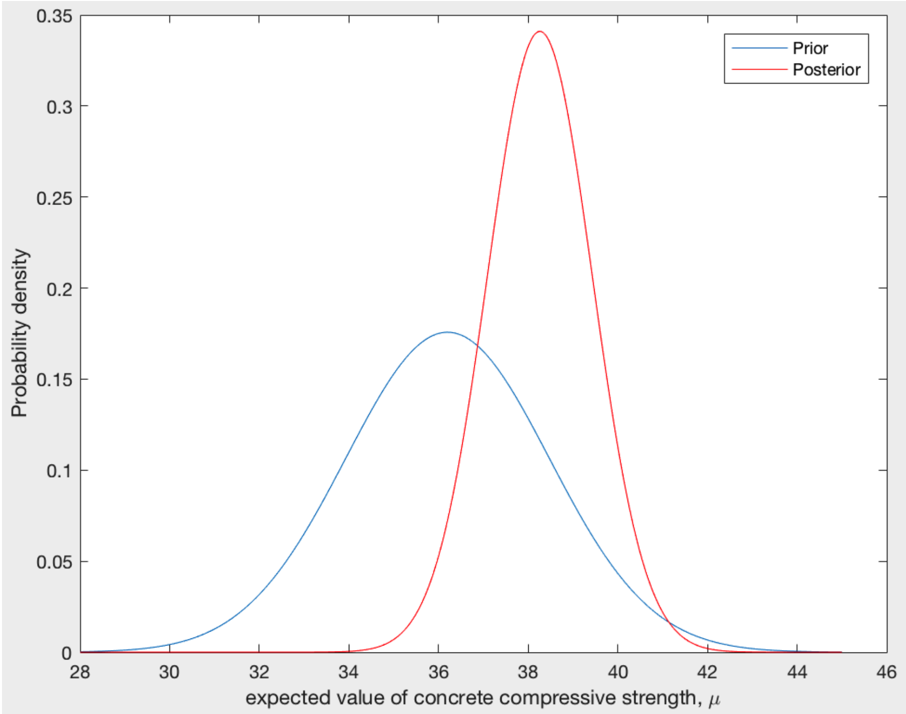


Figure 5.11: *Prior* and *posterior* probability density functions for the expected value of the concrete compressive strength for n=11.

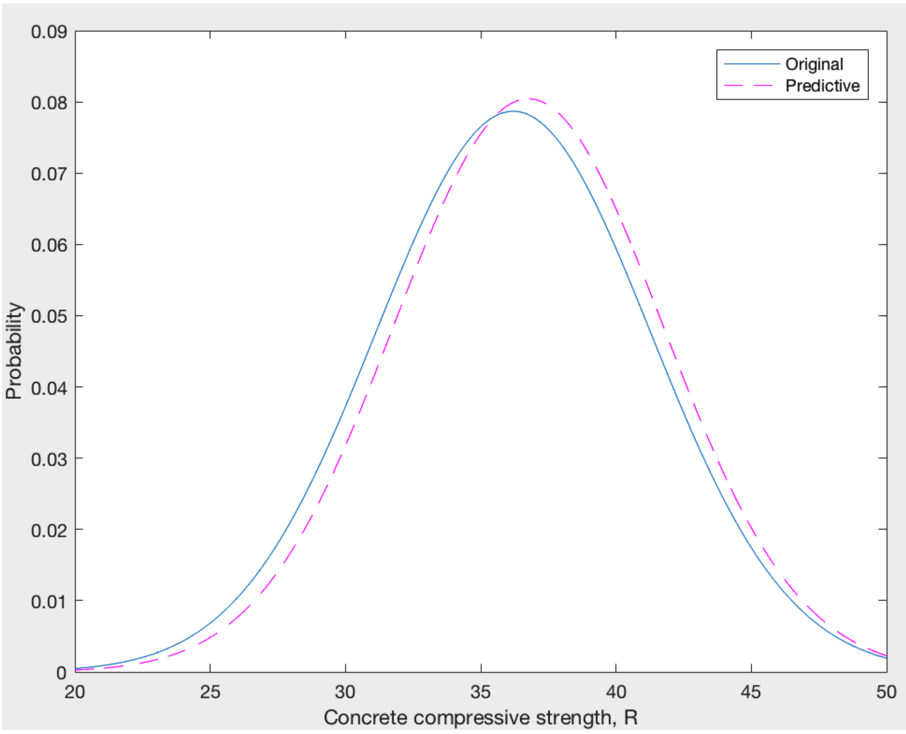


Figure 5.12: *Original* and *predictive* probability density functions for concrete compressive strength for n=1.

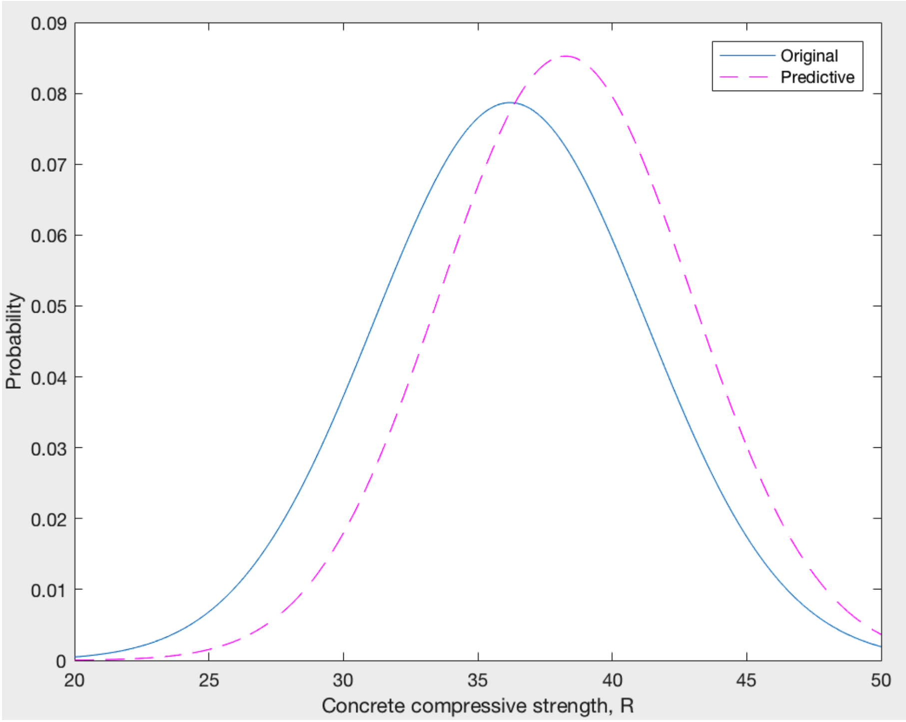


Figure 5.13: *Original* and *predictive* probability density functions for concrete compressive strength for n=11.

predictive density function for $n = 11$ is slightly modified.

A table with the different means and standard deviations obtained for different number of samples, n, are shown in Table 5.7.

Table 5.7: Posterior and predictive expected value and standard deviation.

n	1	3	5	7	9	11
μ''(posterior and predictive)	36.76	37.4	37.76	37.98	38.14	38.26
σ'' (posterior)	2.03	1.71	1.51	1.37	1.26	1.17
σ''' (predictive)	4.96	4.84	4.78	4.73	4.70	4.68

Then, once the predictive distribution has been calculated, it is possible to update the reliability. It is important to point out that the reliability indexes for a Normal distribution are different than the ones found for a Lognormal distribution in last sections, so in this case the values are found to be lower. A table with the reliability indexes values before updating for standard vehicles class 100 can be seen in Table 5.8.

A table with the reliability indexes for the different number of samples, n, and for $\alpha = 0.4$ can be seen in Table 5.9.

Table 5.8: Reliability indexes before updating for a Normal distribution for different α values and vehicle class 100.

α [-]	$z_B[m]$	β [-]
0.2	0.075	3.65
0.3	0.081	3.89
0.4	0.086	4.06
0.5	0.091	4.22

Table 5.9: Reliability indexes for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed

n	1	3	5	7	9	11
$\beta_u(\alpha = 0.4)$	4.24	4.45	4.57	4.66	4.71	4.75

In addition, and in order to show how the reliability changes depending on the number of test samples carried out with test average value constant, Figure 5.14 and Figure 5.15 are shown. In both Figure 5.14 and Figure 5.15 can be seen that the reliability has been increased with respect to the reliability before updating. In addition, it can be seen in Figure 5.14 that the slope of the curve for lower values of n is steeper than for higher values of n, where tends to an horizontal line. A discussion could be to guess the most appropriate number of test samples in material updating, having an economical component as well. In Figure 5.14 can be seen that the number of test samples necessary to reach a target reliability index of $\beta_t = 4.75$ for a constant test average value $\bar{x} = 39$ MPa is $n = 5.5$. Obviously 6 test samples are needed in this case, as it is not possible to perform 5.5 tests.

Test average value constant, $\bar{x} = 39$ MPa with $\sigma' = \frac{1}{3}\sigma$

The same procedure followed in Section 5.2.2 is carried out in this section and the results can be seen in Appendix E.

Test average constant, $\bar{x} = 39$ MPa with $\sigma' = \frac{1}{4}\sigma$

As well, the same procedure followed in Section 5.2.2 is carried out in this section and the results can be seen in Appendix E.

Comparison of the updated reliability indexes depending on the σ and σ' chosen

It is interesting to make a comparison of the reliability depending on the relation between σ and σ'. The results can be seen in Figure 5.16. The conclusion is that for the case when $\sigma' = \frac{1}{2}\sigma$, the reliability as it can be seen in Figure 5.16 is always higher because for this case σ''' is lower and μ'' higher than the other two cases, consequently the reliability is higher.

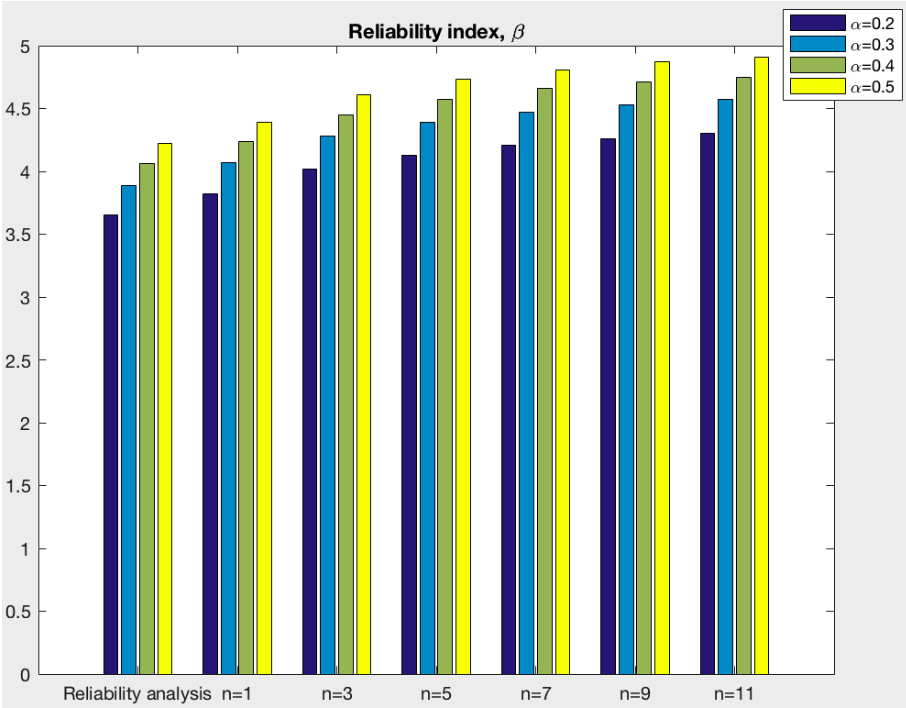


Figure 5.14: Reliability indexes before updating and for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed.

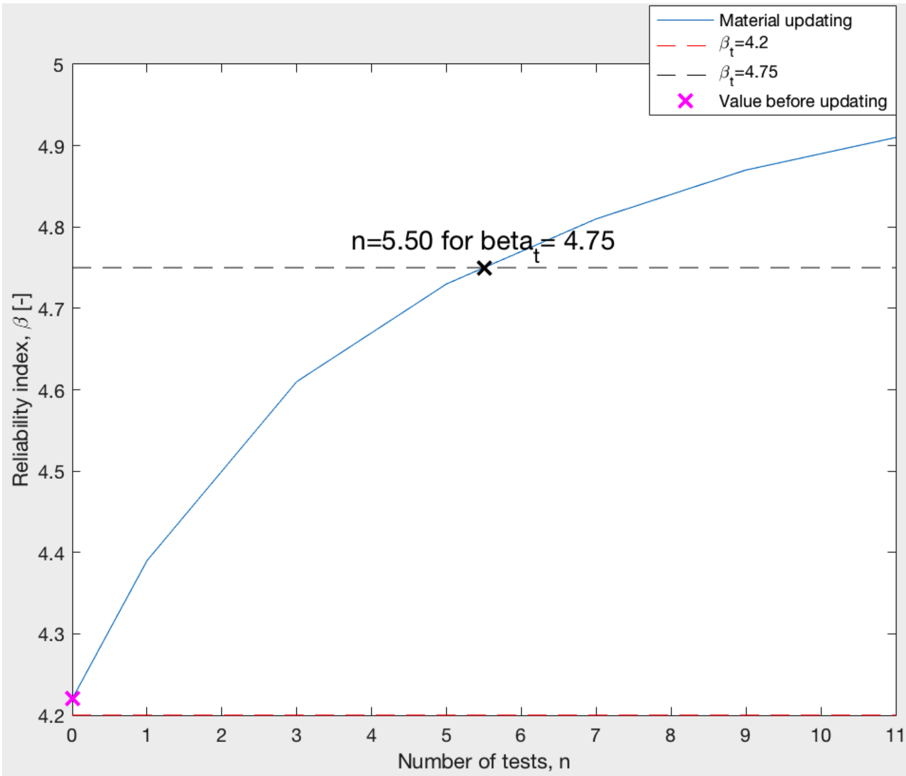


Figure 5.15: Reliability index for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed and $\alpha = 0.5$.

Number of test samples constant, $n = 5$ with $\sigma' = \frac{1}{2}\sigma$

σ and σ' has been already calculated in the last section for the case when $\sigma' = \frac{1}{2}\sigma$, being $\sigma = 4.53$ and $\sigma' = 2.27$.

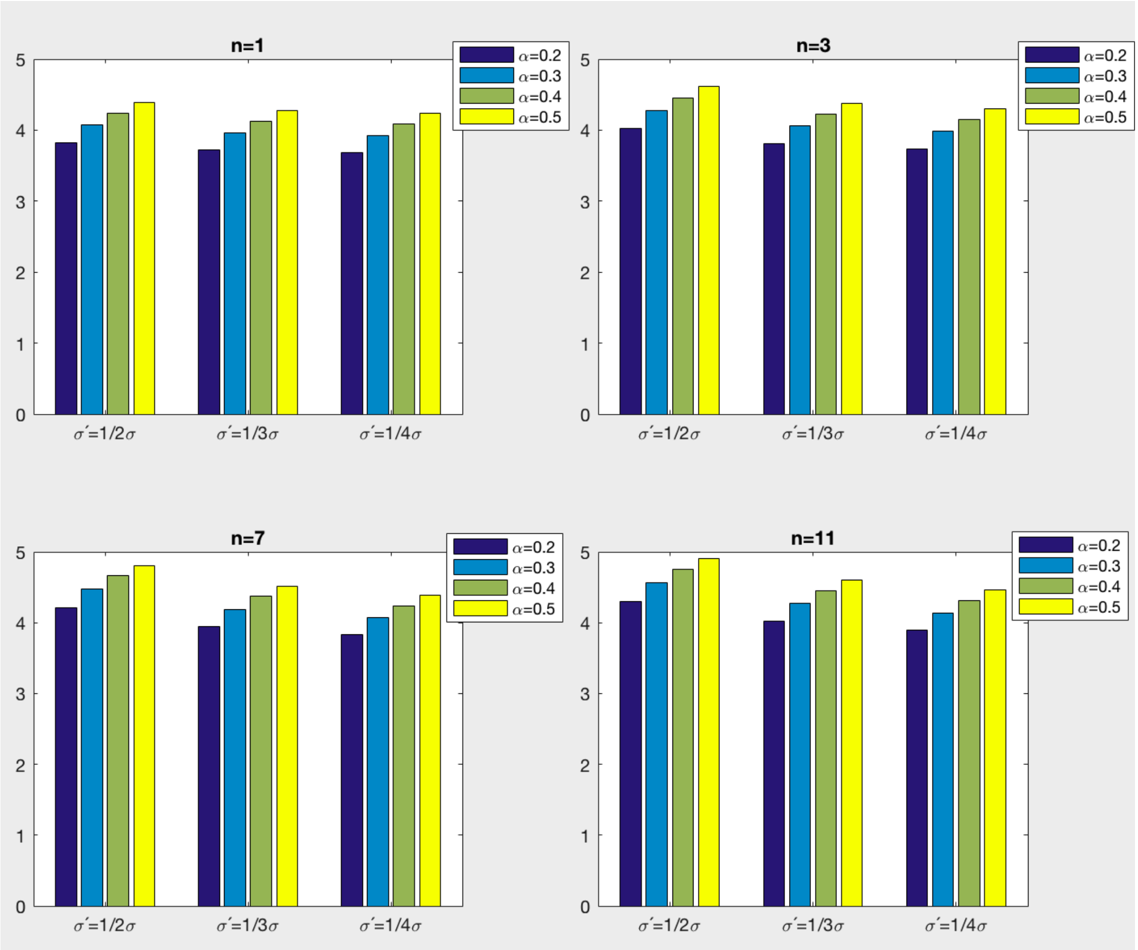


Figure 5.16: Comparison of the reliability depending on the σ and σ' chosen in the calibration

Having this data, the parameters of the posterior and predictive density function can be calculated, and as an example, for the case of $\bar{x} = 30$ MPa is shown below:

$$\mu''(\bar{x} = 30MPa) = \frac{n\bar{x}\sigma'^2 + \mu'\sigma^2}{n\sigma'^2 + \sigma^2} = \frac{5 \cdot 30 \cdot 2.27^2 + 36.2 \cdot 4.53^2}{5 \cdot 2.27^2 + 4.53^2} = 32.75\,\text{MPa}$$

$$\sigma''(\bar{x} = 30MPa) = \sqrt{\frac{\sigma'^2\sigma^2}{n\sigma'^2 + \sigma^2}} = \sqrt{\frac{2.27^2 \cdot 4.53^2}{5 \cdot 2.27^2 + 4.53^2}} = 1.51\,\text{MPa}$$

being μ' the mean value of the prior distribution, which is $\mu' = 36.2$ MPa as it was the value used for concrete compressive strength in the reliability analysis. A comparison between the prior and posterior probability density functions of the expected value of the concrete compressive strength can be seen in Figure 5.17 for $\bar{x} = 30$.

In addition, it is shown when $\bar{x} = 45$ MPa in Figure 5.18. It can be seen that the probability density function for the posterior has the same shape because the standard deviation σ'' is

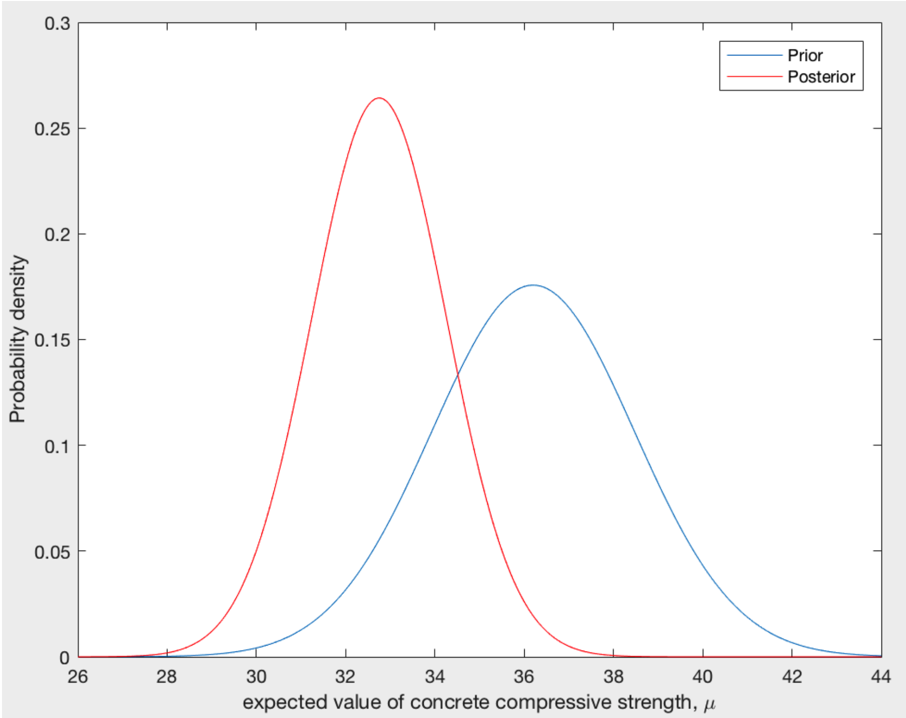


Figure 5.17: *Prior* and *posterior* probability density functions for the expected value of the concrete compressive strength for $\bar{x} = 30$.

the same but displaced forward as the expected value μ'' is obviously higher for $\bar{x} = 45$.

Then, the standard deviation σ''' of the predictive distribution can be calculated:

$$\sigma'''(\bar{x} = 30MPa) = \sqrt{(\sigma''^2 + \sigma^2)} = \sqrt{(1.51^2 + 4.53^2)} = 4.78\,\text{MPa}$$

In Figure 5.19 the original and predictive probability distribution functions for concrete compressive strength are shown for $\bar{x} = 30$.

In addition, the original and predictive probability distribution functions for concrete compressive strength are shown as well for $\bar{x} = 45$, see Figure 5.20. In this case, the predictive probability distribution for $\bar{x} = 45$ is displaced forward with respect to the predictive probability distribution for $\bar{x} = 30$ as the expected value μ'' is obviously higher for $\bar{x} = 45$.

A table with the different expected values obtained for the different test averages values, $\bar{x}$, considered are shown in Table 5.10.

σ'' and σ''' are the same values for all the different $\bar{x}$, as they depend on the number of test samples but not on the test average value $\bar{x}$.

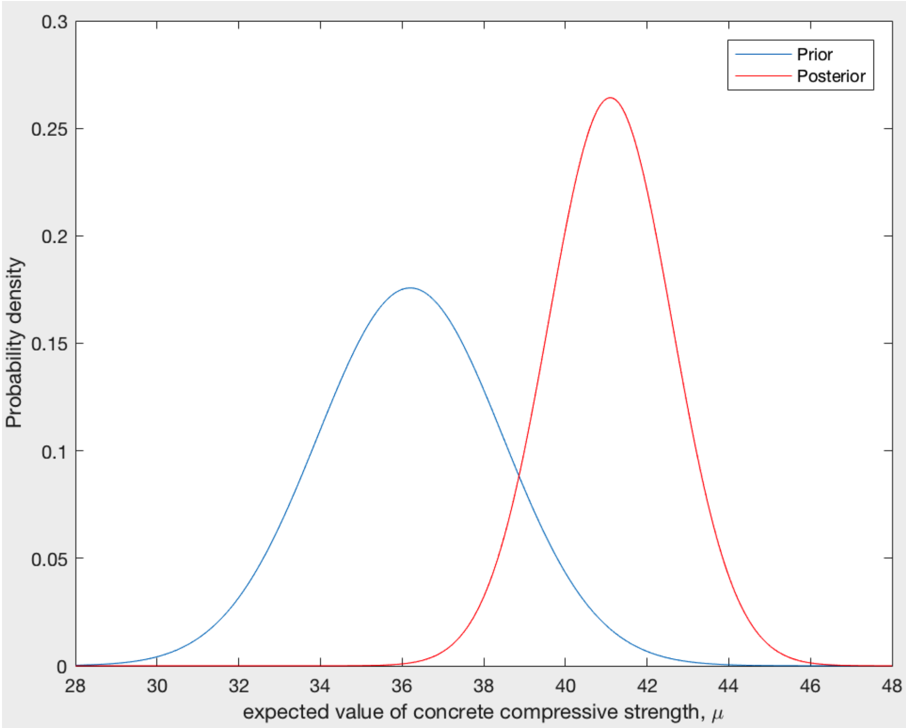


Figure 5.18: *Prior* and *posterior* probability density functions for the expected value of the concrete compressive strength for $\bar{x} = 45$.

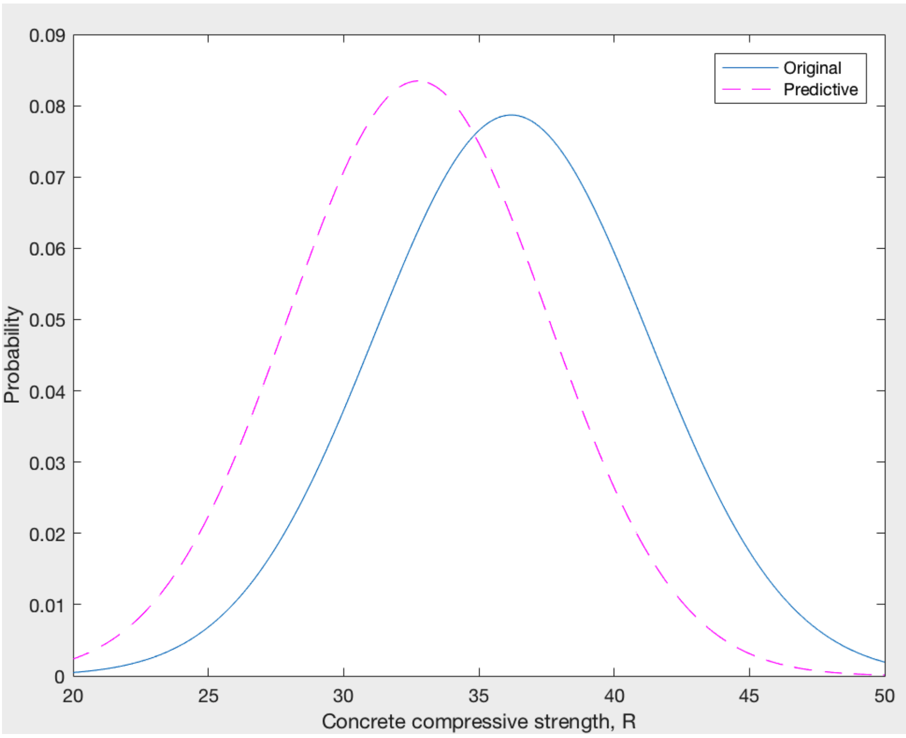


Figure 5.19: *Original* and *predictive* probability distribution functions for concrete compressive strength for $\bar{x} = 30$.

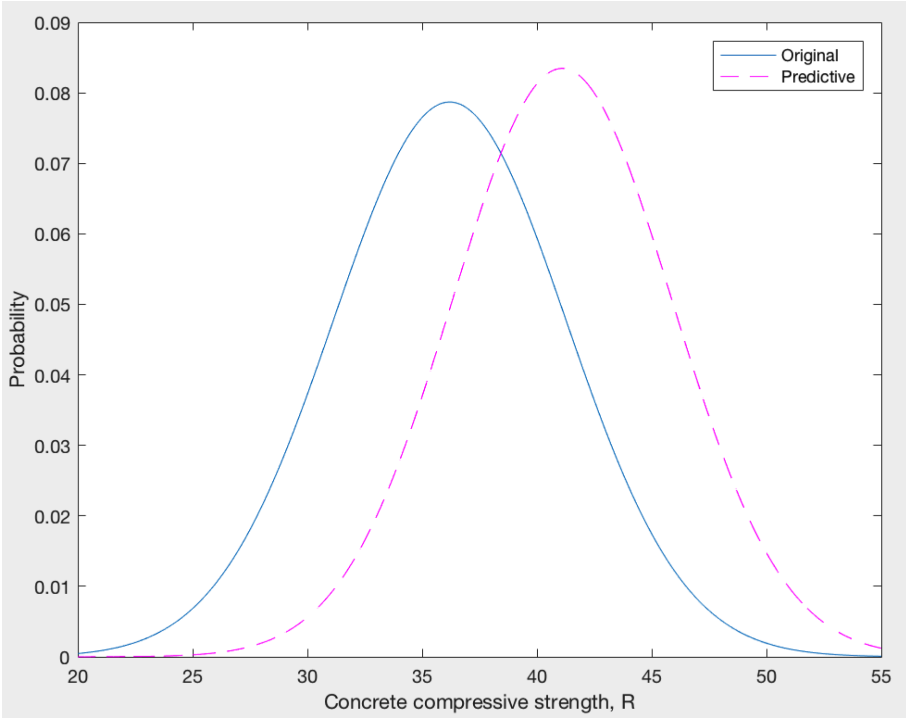


Figure 5.20: *Original* and *predictive* probability distribution functions for concrete compressive strength for $\bar{x} = 45$.

Table 5.10: Posterior and predictive expected value.

$\bar{x}$	30	33	36	39	42	45
μ''(posterior and predictive)	32.75	34.42	36.09	37.76	39.43	41.10

Then, once the predictive distribution has been calculated, it is possible to update the reliability. As it was said before, it is important to have in mind that the reliability indexes for a Normal distribution are different from the ones found for a Lognormal distribution in Chapter 4, so the values were found to be lower. A table with the reliability indexes values before updating for standard vehicles class 100 can be seen in Table 5.8.

A table with the reliability indexes for the different test mean values, $\bar{x}$, and depending on the different α values can be seen in Table 5.11.

In addition, and in order to show how the reliability changes depending on the test average values having a constant number of test samples, Figure 5.21 and Figure 5.22 are shown. In both Figure 5.21 and Figure 5.22 can be seen that the reliability can either be higher or lower than the reliability calculated in Chapter 4. The reason for that is that the average concrete compressive strength tested can be as well lower than the one considered previously. Additionally, it can be seen in Figure 5.22 that the increase of the reliability with respect to the strength average is linear. In Figure 5.22 can be seen that the strength average

Table 5.11: Reliability indexes for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed

$\bar{x}$	30	33	36	39	42	45
$\beta_u(\alpha = 0.2)$	3.17	3.49	3.81	4.13	4.45	4.77
$\beta_u(\alpha = 0.3)$	3.42	3.75	4.07	4.39	4.71	5.03
$\beta_u(\alpha = 0.4)$	3.60	3.92	4.25	4.57	4.89	5.22
$\beta_u(\alpha = 0.5)$	3.75	4.08	4.40	4.73	5.05	5.37

value related to a target reliability index of $\beta_t = 4.2$ for a constant number of samples, $n = 5$, is $\bar{x} = 34.12$ MPa, $\bar{x} = 39.19$ MPa for $\beta_t = 4.75$ and $\bar{x} = 43.41$ MPa for $\beta_t = 5.2$.

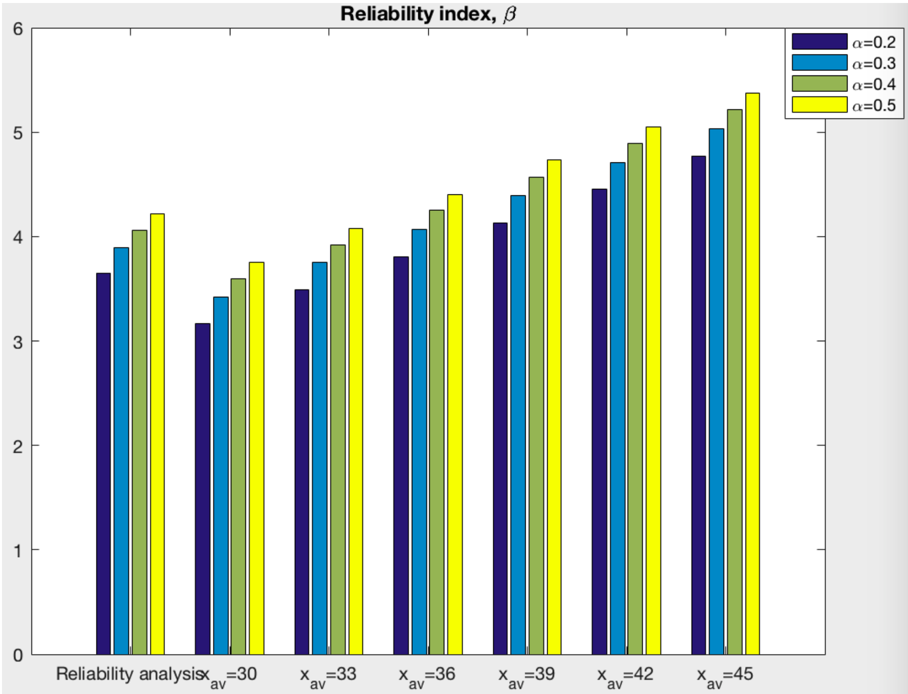


Figure 5.21: Reliability indexes before updating and for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed.

Number of test samples constant, $n = 5$ with $\sigma' = \frac{1}{3}\sigma$

The same procedure followed in Section 5.2.2 is carried out in this section and the results can be seen in Appendix E.

Number of test samples constant, $n = 5$ with $\sigma' = \frac{1}{4}\sigma$

As well, the same procedure followed in Section 5.2.2 is performed in this section and the results can be seen in Appendix E.

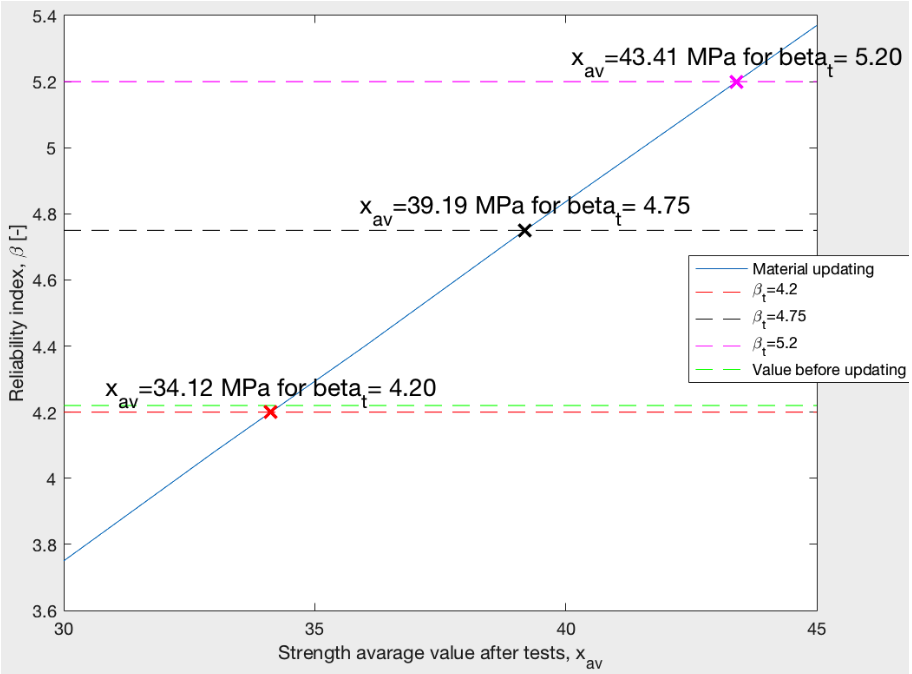


Figure 5.22: Reliability index for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed and $\alpha = 0.5$.

Comparison of the updated reliability indexes depending on the σ and σ' chosen

It is interesting to make a comparison of the reliability indexes calculated depending on the relation between σ and σ'. The results can be seen in Figure 5.23.

In this case, it can be seen that for low values of the test average strength ($\bar{x} = 30$ MPa) the reliability is higher for the case when $\sigma' = \frac{1}{4}\sigma$ than the other two cases. For higher values of the test average strength ($\bar{x} = 39$ MPa or $\bar{x} = 45$ MPa), the reliability index is higher for the case $\sigma' = \frac{1}{2}\sigma$. On the other hand, for values of the average stregth close to ($\bar{x} = 33$ MPa) and as it can be seen in the top right corner of Figure 5.23, the values for the three cases are quite similar. This depends on how the expected value and standard deviation change for each of the cases.

Updating in material strength testing when considering Lognormal distribution

In the last section, the updating in material strength testing when considering Normal distribution was performed. In this section, the updating in material strength testing when considering Lognormal distribution is performed in order to compare the results with the ones when considering normal distribution. Then, in this section the concrete compressive strength and the reinforcement strength are meant to follow a lognormal distribution and the case with unknown mean and known standard deviation will be studied. Standard vehicles class 100 are considered in this section.

In the case with a log-normally distributed variable with uncertain mean and known standard deviation, the density function is as followed:

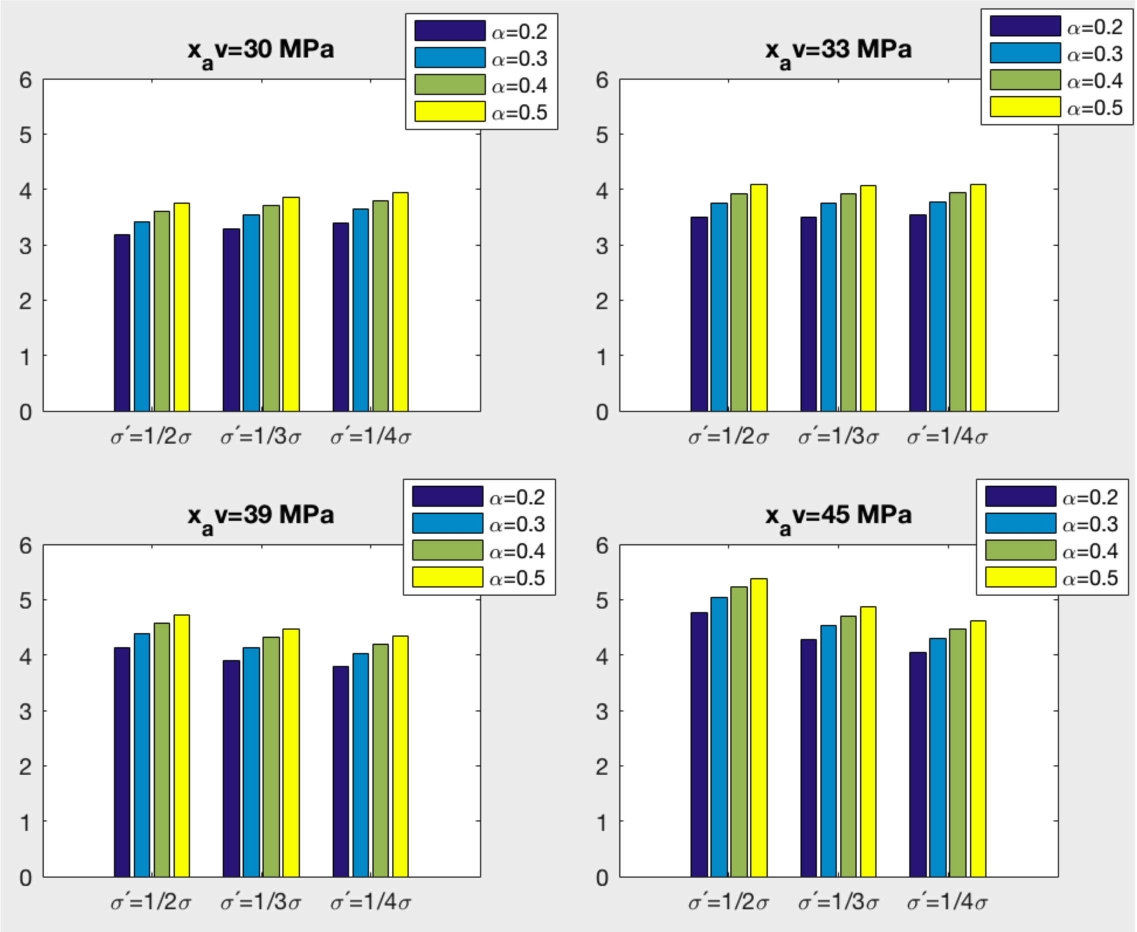


Figure 5.23: Comparison of the reliability depending on the σ and σ' chosen in the calibration.

$$f_X(x|\mu,\sigma) = \frac{1}{x\sigma_Y\sqrt{2\pi}}exp(-\frac{1}{2}(\frac{\ln x - \ln\mu_Y}{\sigma_Y})^2) \tag{5.17}$$

When X is Lognormal distributed, then $Y = \ln X$ becomes Normal distributed with standard deviation:

$$\sigma_Y = \sqrt{\ln(\frac{\sigma^2}{\mu^2}+1)} \approx \frac{\sigma}{\mu} = V \tag{5.18}$$

and expected value:

$$\mu_Y = \ln\mu - \frac{1}{2}\sigma_Y^2 \approx \ln\mu \tag{5.19}$$

where it was checked that they are good approximation of σ_Y and μ_Y as $V < 0.25$. Thus, Eq. (5.18) and Eq. (5.19) were used for the transformation from Normal to Lognomal distribution.

Concrete compressive strength updating when considering Lognormal distribution

The case when $\sigma' = \frac{1}{2}\sigma$ is considered in this section. Here, $\sigma = 4.53$ MPa and $\sigma' = 2.27$ MPa, and $\mu = 36.2$ MPa. Hence, by applying the transformation:

$$\mu_Y = \ln\mu = \ln 36.2 = 3.59$$

and

$$\sigma_Y = \frac{\sigma}{\mu} = V = 0.125$$

Additionally, the prior probability density function is assumed Lognormal and defined as followed:

$$f'_{\mu_Y}(\mu_Y) = f_{Ln}(\mu_Y|\mu'_Y, \sigma'_Y) = \frac{1}{x\sigma'_Y\sqrt{2\pi}}exp(-\frac{1}{2}(\frac{ln\mu_Y - \ln\mu'_Y}{\sigma'_Y})^2) \tag{5.20}$$

Then, the parameters of the Lognormal distribution are $\sigma'_Y = 0.0625$ and $\mu'_Y = 3.59$.

As an example, five test samples are assumed with values $\hat{x} = 37, 38, 39, 40, 41$MPa with $\bar{x} = 39$ MPa. By applying the transformation: $\hat{y} = 3.61, 3.64, 3.66, 3.69, 3.71$. Therefore, the average test sample is calculated to be $\bar{y} = 3.66$.

Therefore, the posterior probability density function becomes Lognormal:

$$f''_{\mu_Y}(\mu_Y|\hat{y}) = f_{Ln}(\mu_Y|\mu''_Y, \sigma''_Y) \tag{5.21}$$

where:

$$\mu''_Y = \frac{n\bar{y}\sigma'^2_Y + \mu'_Y\sigma^2_Y}{n\sigma'^2_Y + \sigma^2_Y} \tag{5.22}$$

$$\sigma''^2_Y = \frac{\sigma'^2_Y\sigma^2_Y}{n\sigma'^2_Y + \sigma^2_Y} \tag{5.23}$$

and n is the number of test samples.

Consequently, the predictive probability density function becomes Lognormal as well:

$$f_X(y|\hat{y}) = f_{Ln}(y|\mu''_Y, \sigma'''_Y) \tag{5.24}$$

where

$$\sigma_Y''' = \sqrt{(\sigma_Y''^2 + \sigma_Y^2)} \tag{5.25}$$

Considering the case where the test average value is constant, a table with the different expected values and standard deviations obtained for different number of samples, n, are shown in Table 5.12.

Table 5.12: Posterior and predictive mean and standard deviation for Lognormal distribution.

n	1	3	5	7	9	11
μ_Y''(posterior and predictive)	3.604	3.621	3.631	3.637	3.641	3.644
σ_Y'' (posterior)	0.056	0.047	0.042	0.038	0.035	0.032
σ_Y''' (predictive)	0.137	0.134	0.132	0.131	0.131	0.130

Then, once the predictive distribution has been calculated, it is possible to update the reliability. The value of the reliability index before updating for the case of $\alpha = 0.5$, which is the one considered in this section for the comparison, is $\beta = 5.315$.

A table with the reliability indexes for the different number of samples, n, can be seen in Table 5.13.

Table 5.13: Reliability indexes for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed and $\alpha = 0.5$.

n	1	3	5	7	9	11
$\beta_u(\alpha = 0.5)$	5.51	5.68	5.78	5.85	5.89	5.93

In Figure 5.24 can be seen the difference concerning the reliability index when the concrete compressive strength is modelled either as Normal or Lognomal distributed for different test samples. The shape of the curves are quite similar but the value of the reliability indexes for the lognormal distribution is considerably higher. Actually, the reliability in this case (when $\alpha = 0.5$) is not necessary to be updated as it was over $\beta_t = 5.2$ already before updating for lognormal distribution. Therefore, the aim of Figure 5.24 is to show the difference when it is modelled as Normal and Lognormal.

On the other hand, if the case where the number of test samples is constant is considered, the different expected values obtained for different number of samples, n, are shown in Table 5.14. In addition, the values $\sigma_Y'' = 0.04$ and $\sigma_Y''' = 0.132$ were calculated.

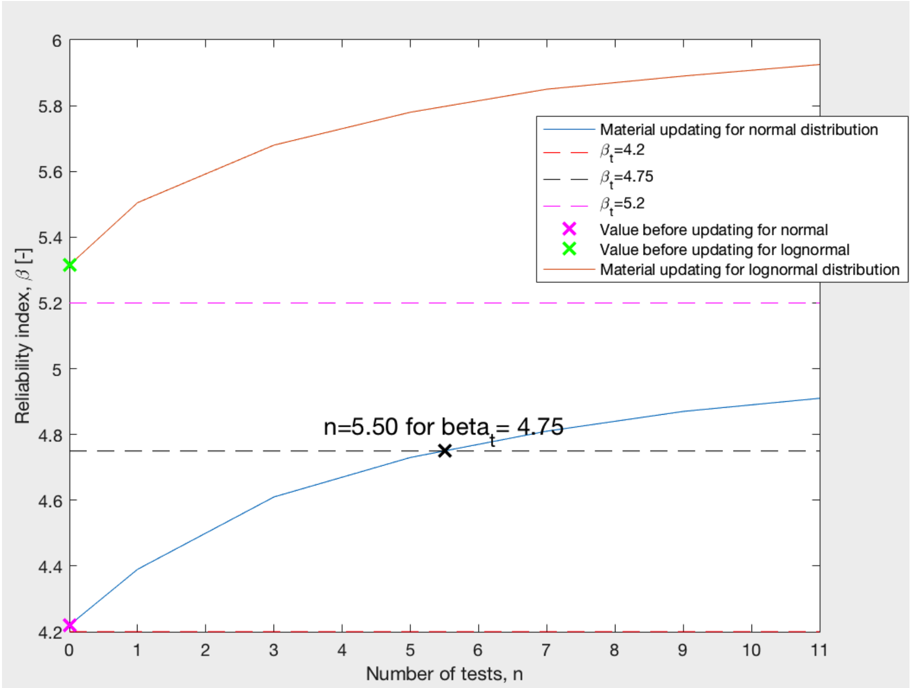


Figure 5.24: Reliability indexes for concrete compression strength, for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed and $\alpha = 0.5$ for normal and lognormal distributions.

Table 5.14: Posterior and predictive mean value.

$\bar{y}$	3.401	3.497	3.584	3.664	3.738	3.807
μ''_Y(posterior and predictive)	3.484	3.538	3.586	3.631	3.672	3.710

Then, once the predictive distribution has been determined, it is possible to update the reliability. The reliability indexes for the different test average values, $\bar{x}$, and for $\alpha = 0.5$ can be seen in Table 5.15.

Table 5.15: Reliability indexes for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed and $\alpha = 0.5$

$\bar{x}$	30	33	36	39	42	45
$\beta_u(\alpha = 0.5)$	4.95	5.25	5.53	5.78	6.02	6.24

In Figure 5.25 can be seen the difference concerning the reliability index when the concrete compressive strength is modelled either as Normal or Lognomal distributed considering different test average values. Again, the values related to the reliability index when it is modelled as Lognormal are considerably higher than for Normal distribution. When it is modelled as a Lognormal distibution, the values of reliability are quite high in this case when $\alpha = 0.5$ was considered.

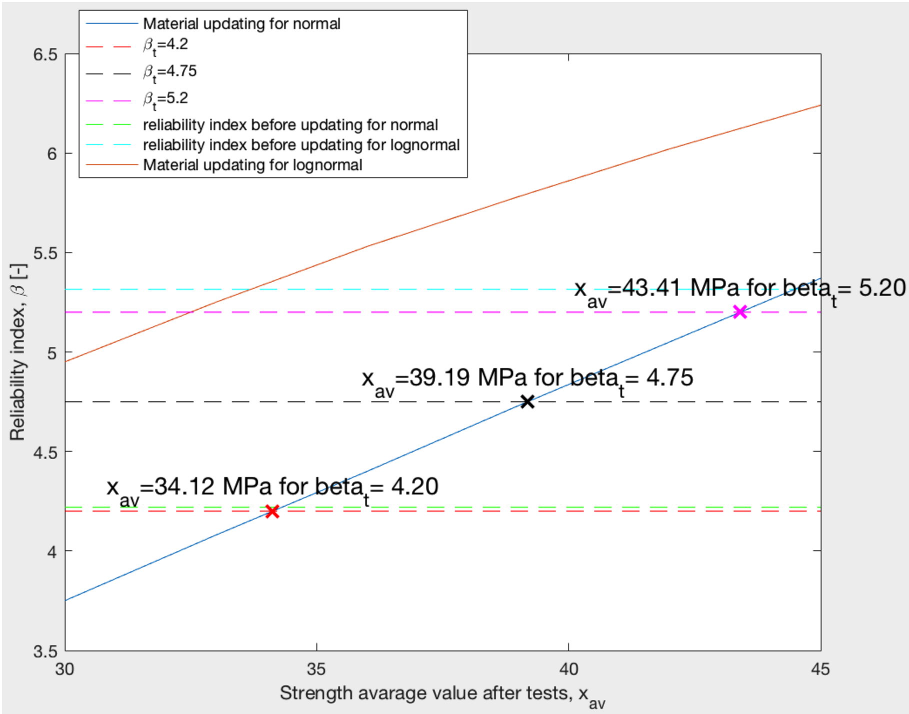


Figure 5.25: Reliability indexes for concrete compression strength, for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed and $\alpha = 0.5$ for normal and lognormal distributions.

Reinforcement strength updating when considering Lognormal distribution

The case when $\sigma' = \frac{1}{2}\sigma$ is considered in this section. Here, $\sigma = 22.36$ MPa and $\sigma' = 11.18$ MPa, and $\mu = 345$ MPa. Hence, by applying the transformation:

$$\mu_Y = \ln \mu = \ln 36.2 = 5.84$$

and

$$\sigma_Y = \frac{\sigma}{\mu} = V = 0.065$$

Then, the same calculations as for concrete compressive strength in the last section has been done. The results can be seen in Figure 5.26 in the case where the test average test value is constant and $\bar{x} = 355$MPa and in Figure 5.27 when the number of test samples is constant and $n = 5$. In Figure 5.26 can be seen the difference concerning the reliability index when the reinforcement strength is modelled either as Normal or Lognormal distributed for different number of test samples. The shape of the curves are quite alike and the value of the reliability indexes for Normal and Lognormal distribution are not that far for reinforcement strength, unlike it is for concrete compressive strength, where the values are quite different, see Figure 5.24. This means that the reinforcement strength is not that sensitive as it is the concrete compressive strength when changing the type of distribution. This result was expected due to the results obtained in the sensitive analysis in Chapter 4.

In addition, either when modelled as a Normal or Lognormal distribution, the values of reliability are quite high in this case where $\alpha = 0.5$ was considered, so it was not needed an updating in this case.

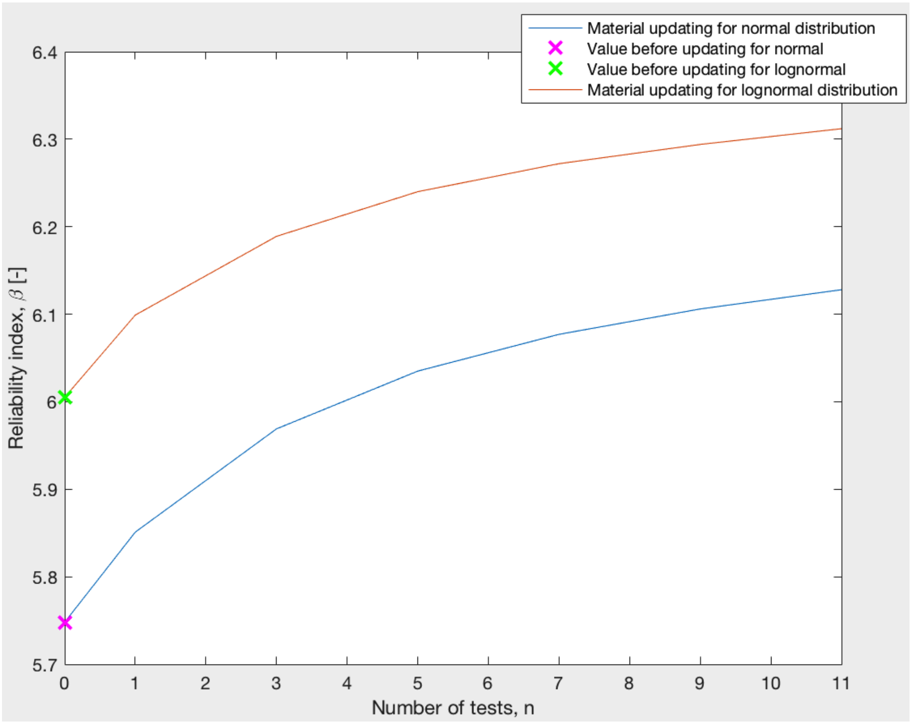


Figure 5.26: Reliability indexes for reinforcement strength, for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed and $\alpha = 0.5$ for normal and Lognormal distributions.

Comparison of concrete vs. reinforcement updating

It was considered interesting to make a comparison between the updating of the concrete compressive strength, f_c, and the reinforcement strength, f_y. For that, a normalization of the reliability index, β, is done in order to compare the results, so this normalized β is the ratio between the reliability index before updating, β_0, and the reliability index in the updating, β_u. The results can be seen in Figure 5.28 and Figure 5.29. It can be seen in both Figure 5.28 and Figure 5.29 that the concrete compressive strength, f_c, is more sensitive to changes than the reinforcement strength, f_y, so the reliability increase (or decrease) faster in the case of the concrete compressive strength. These results were expected as if was proved by a sensitivity analysis with the α vector values in Chapter 4 that the system was quite sensitive to changes in the concrete compressive strength.

Ideal number of test samples, n

The ideal number of test samples depends, of course, on each individual case, as the economical component for instance from one case to another can be quite different. In the case of this report, where the case with unknown mean value and known standard deviation has been considered, and judging based on Figure 5.28, for reinforcement strength seems

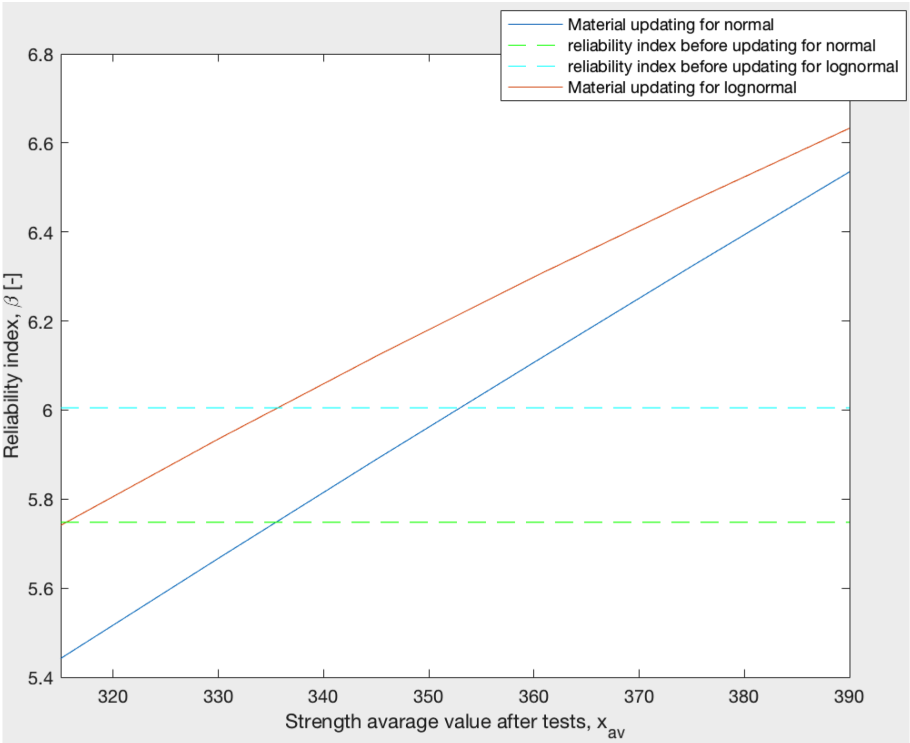


Figure 5.27: Reliability indexes for reinforcement strength, for the case when $\sigma' = \frac{1}{2}\sigma$ is assumed and $\alpha = 0.5$ for normal and lognormal distributions.

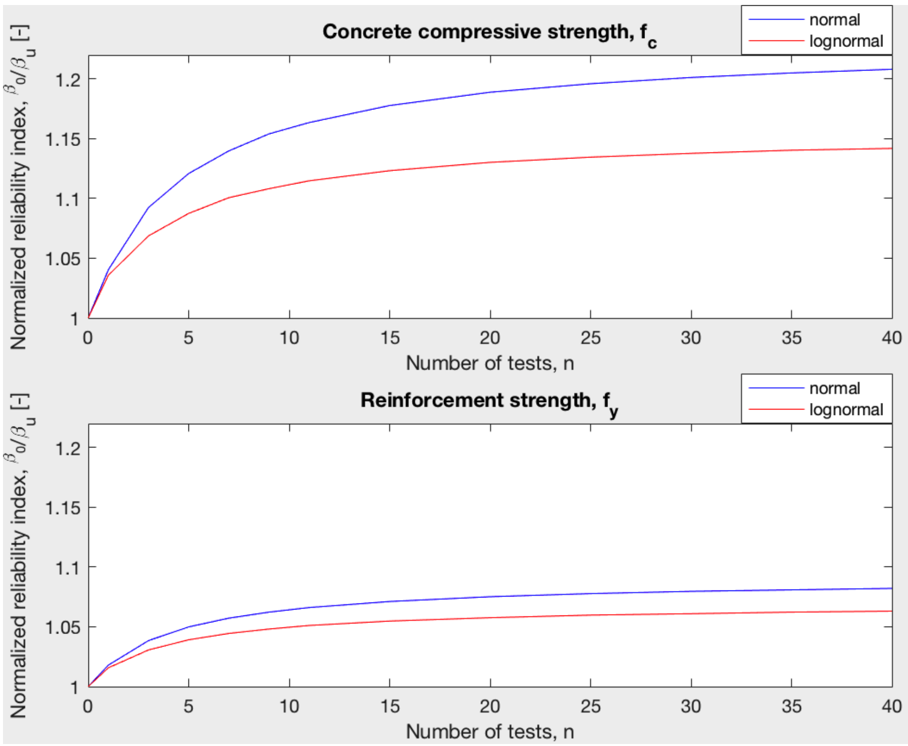


Figure 5.28: Comparison of concrete vs. reinforcement updating when $\bar{x}$ is constant.

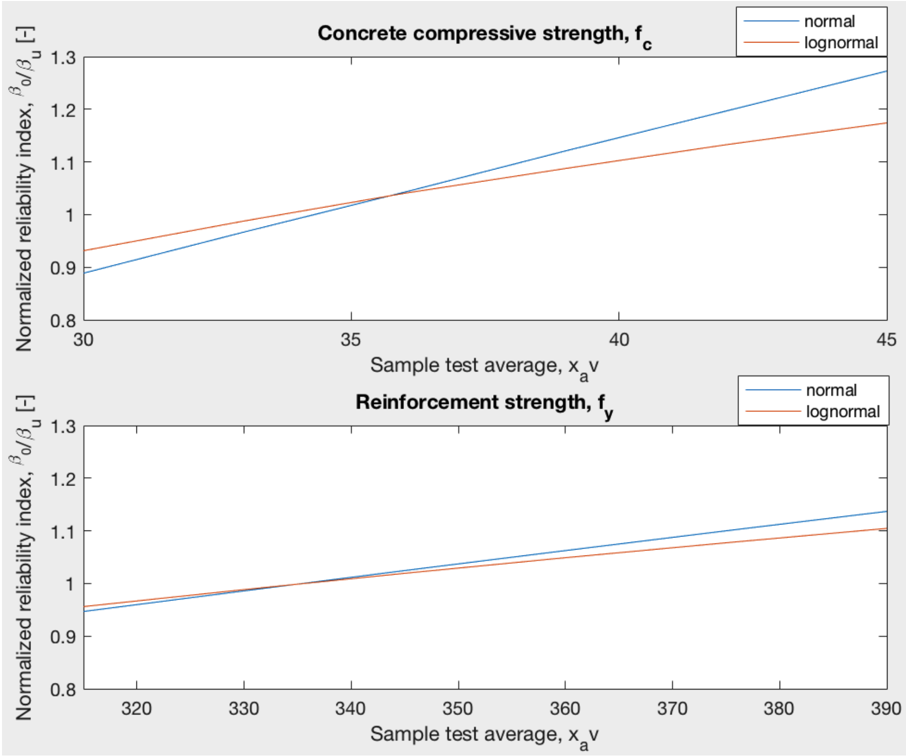


Figure 5.29: Comparison of concrete vs. reinforcement updating when n is constant.

to be quite fine to carry out 5 test samples, as the reliability index does not increase that much after this number of test samples. On the other hand, it seems to be more adequate to perform up to 10 test samples concerning concrete compressive strength. It should be noted that the number of test samples needed highly depends on whether prior information is available or not.

CHAPTER 6

Conclusion

The aim of this project was to investigate different methods in order to assess the reliability of existing concrete bridges which do not fulfill the safety requirements before coping with costly rehabilitation/demolition by applying different types of updating, as proof load updating and material updating. This objective is considered to has been fulfilled and the reached conclusions are exposed in the following.

In the stochastic modelling, both loads and materials were modelled. In load modelling, two different loads were considered: Permanent load, in which the weight of the structure and dead load model uncertainty were modelled and traffic load, in which the standard vehicle weight, the traffic load model uncertainty and the dynamic factor were modelled according to the guideline 'Reliability-Based Classification of the Load Carrying capacity of Existing Bridges'. In material modelling, both concrete and reinforcement were modelled. For both of them, the variables modelled were the material strength and the material model uncertainty, according to the guideline 'Reliability-based classification of the load carrying capacity of existing bridges and according to the Danish standards, applying the modelling according to the first one further in this project. Additionally, a range of different concrete bridges were considered in this project by the parameter α, being α a parameter used to weigh both the permanent and traffic load. Thus, $\alpha = 0$ means no traffic load and $\alpha = 1$ means no unfavorable permanent load considered.

A reliability analysis of a generic existing bridge was performed, with the aim to check if the reliability requirements were fulfilled. Two cases were considered, one when the concrete compression strength is the dominant material strength, and the other one when the reinforcement strength is the dominant material strength. The bridge was designed for standard vehicles class 100. By performing a reliability assessment, it was checked that the reliability requirements established were fulfilled quite good for this class of standard vehicles. Additionally, the case in which the maximum allowed load in a bridge is to be increased was considered (bridge upgrading). Then, the traffic load was increased to class 125 in order to check if the bridge would bear the new loads. After carrying out a reliability analysis, the results showed that the reliability requirements were not fulfilled in this case. In addition, a sensitivity analysis was performed in order to check how was the influence of each of the variables modelled measured by the influence coefficient (α vector). The results showed that the influence of the concrete compressive strength in the case when the concrete compression strength is the dominant material strength is way higher than the influence of the reinforcement strength when the reinforcement strength is the dominant material strength. For the case when the reinforcement strength is the dominant material strength, the reinforcement strength model uncertainty is found to be the most influential variable.

In order to increase the reliability, two updating methods were investigated: Updating by proof load tests and updating by test results related to material parameters. In both of them, both the cases when the concrete compression strength is the dominant material strength and when the reinforcement strength is the dominant material strength were studied. In the updating by proof load tests, it was checked how the reliability increases by considering that the bridge can carry a load applied without sign of damages. The probability of failure while applying the proof loading was as well considered, in order to have an idea of how big is the risk of collapse of the bridge when performing a proof load test. When performing a proof load test in a real bridge, it is very important to set a stop criteria, to ensure that the bridge does not collapse while carrying out the proof loading. On the other hand, in the updating by test results related to material parameters, the same expected value and different coefficients of variation for concrete compressive strength and reinforcement strength were considered as a preliminary approach. The results show that the reliability increases as much as the coefficient of variation decreases, since the uncertainty is lower. Then, by using Bayesian statistics, the concrete compressive strength was updated when considering it modelled as a Normal distribution for the case with unknown mean and known standard deviation. Finally, both the concrete compressive strength and the reinforcement strength were updated when considered them modelled as a Lognormal distribution for the case with unknown mean and known standard deviation in order to compare the results when the variables modelled by a Normal distribution. The results showed that the concrete compressive strength is more sensitive to changes so the reliability increases faster as it was expected based on the sensitivity analysis performed in Chapter 4. In addition, it was discussed the ideal number of test samples required in this kind of test. It was found to be 10 test samples for concrete compressive strength and 5 for reinforcement strength based on the results obtained. It probably would not be worthy to perform extra tests as the reliability barely increased and the more tests you perform, the more costly it is.

Thus, the decision-maker should consider in each individual case which updating method is more adequate. In this report, how the reliability varies when applying different updating techniques has been investigated. Nevertheless, the economical part, which is a crucial element in every engineering problem, has been omitted. Hence, the decision will be taken considering which is the method which increases the most the reliability, but being very important as well to consider how expensive is to carry out each of the tests. Additionally, it has to be considered the risk of the bridge collapsing when performing a proof load test.

6.1 Outlook

As a further research, more complex models of traffic loads on road bridges may be considered, as it could be the development of a stochastic modelling for extreme traffic loads (maximum annual load) in bridges with either multiples lanes or congested traffic. It is also relevant better and more realistic (deterministic) models for the load bearing capacity eventually using non-linear numerical models. Another topic which shall be investigated is a stop criteria in proof loading, to ensure that the bridge does not collapse.

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APPENDIX A

Classification of structures in accordance to consequences classes

Structures can be classified depending on the consequence classes according to the document *ISO*2394[23]:

- Class 1: This class is characterized mainly for insignificant material damages. Some structures which are assessed as class 1 are minor wind turbines or small building with few people inside them.
- Class 2: Damages in materials and losses in functionality for owners and operators but little or none losses for the society. Less than 5 fatalities are expected. Some examples of structures are major wind turbines, minor bridges, small off-shore facilities,etc.
- Class 3: Material and functionality affecting a big part of the society. It causes regional disruptions and delays in the mean services of the society for several weeks. Less than 50 fatalities are expected. For instance in this class are included most residential buildings, typical bridges and tunnels, most important offshore facilities,etc.
- Class 4: Disastrous events which cause extreme losses in the services of the society and delays in the whole country for long periods, of the order of months. Less than 500 fatalities are expected. In this class are included for instance most traveled bridges and tunnels, dams, chemical plants, etc.
- Class 5: Catastrophic events which causes losses in the services of the society and disruptions and delays beyond national borders for years. More than 500 fatalities are expected. Some examples of this class are major off-shore plants, major containment of toxic materials, major dams, etc.

According to the National annex to Eurocode: Basis of structural design, the consequences classes are classified in a different way [27], see Table A.1:

Table A.1: Consequence classes according to the National annex to Eurocode: Basis of structural design [27].

Consequences class	Consequences of possible damage	Examples
CC3: High consequences class	High risk of loss of human life, or considerable economic, social or environmental consequences	-Buildings with several storeys where the height to the floor of the uppermost storey is more than 12 m above the ground, if they are often used for accommodating people, e.g. residential or office buildings. -Buildings with large spans, if they are often used by many people, e.g. for concerts, sporting events, theatrical performances, or exhibitions. -Grandstands. -Large road bridges and tunnels. -Large masts and towers. -Large silos near a built-up area. -Dams and similar structures where a failure would result in considerable damage.
CC2: Medium consequences class	Medium risk of loss of human life. Considerable economic, social or environmental consequences.	Buildings or structures not belonging to CC3 or CC1.
CC1: Low consequences class	Low risk of loss of human life, and small or negligible economic, social or environmental consequences.	-1 and 2 storey buildings with moderate spans, which people enter only occasionally, e.g. storage buildings, sheds and small agricultural buildings. -Small masts and towers, including general street masts. -Small silos. -Secondary structural members, e.g. partitions, window and door lintels and cladding.

Appendix B

Logical model in Sysrel

Logical model

The logical model in Sysrel is formed by the logical connection of the different component in a system, which are represented by failure criteria. They are introduced to the Logical Model tab, in which the logical representation of the system is introduced by Cut-Sets (intersection of failure events) [25]. Each of the rows represents a different Cut-Set, being the columns the failure criteria (Components, Constraints) which are introduced in the symbolic expression tab, see Figure B.1. In the last row of the logical model can be introduced a conditional event, i.e. when a conditional failure probability is to be determined. Computation of a conditional probability is in accordance to Eq. (B.1):

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} \tag{B.1}$$

where A are the Cut-Sets and C the conditional event in the last row. The software Sysrel gives the possibility to include the conditional event either as a type Inequality or Equality (which would mean the collapse of the bridge in a proof load test).

The logical model is established with only one Cut-set in this case and a conditional event sets as Inequality. The logical model can be seen in Figure B.1.

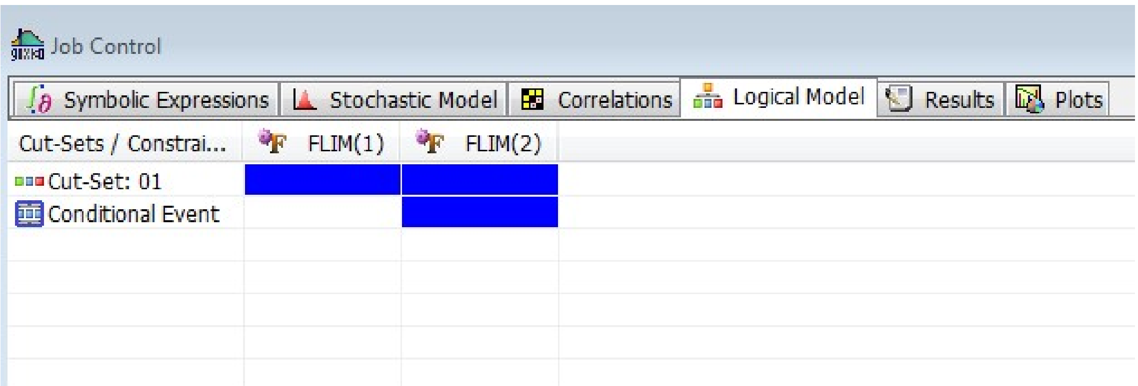

Figure B.1: Logical model in software Sysrel .

APPENDIX C

Updating proof loading plots for the different failure modes and α values

In this chapter, proof loading updating of the reliability index for the different failure modes and taking into account different α values are shown.

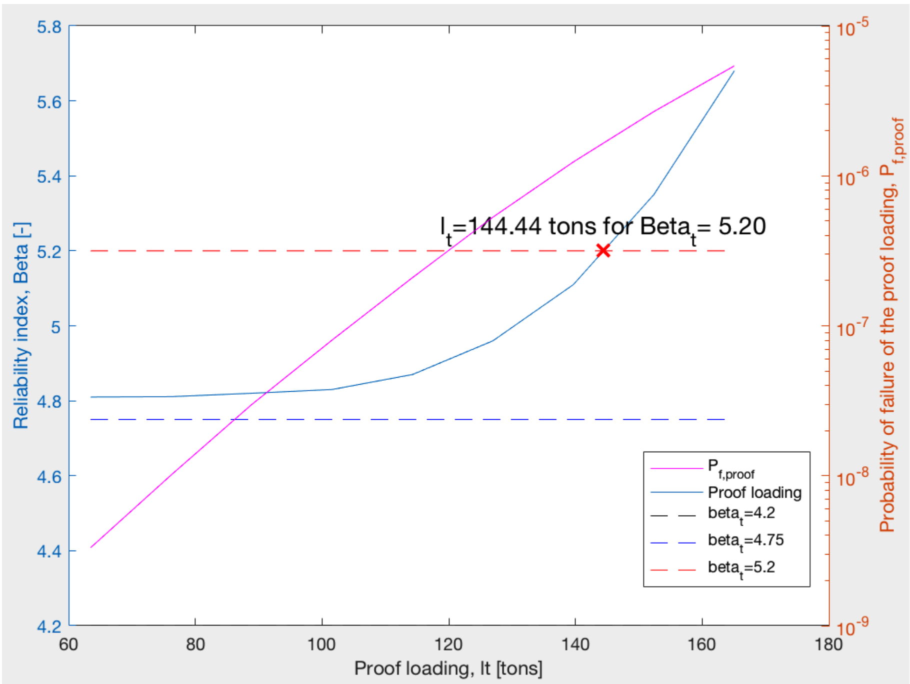

Figure C.1: Proof loading updating for concrete compression strength, standard vehicles class 100 and $\alpha = 0.3$.

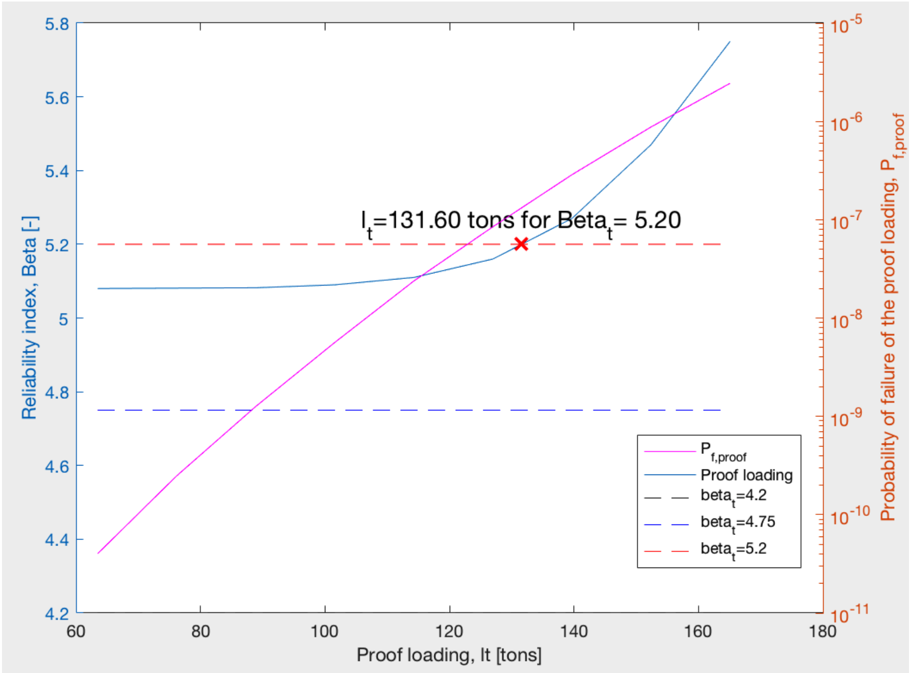

Figure C.2: Proof loading updating for concrete compression strength, standard vehicles class 100 and $\alpha = 0.4$.

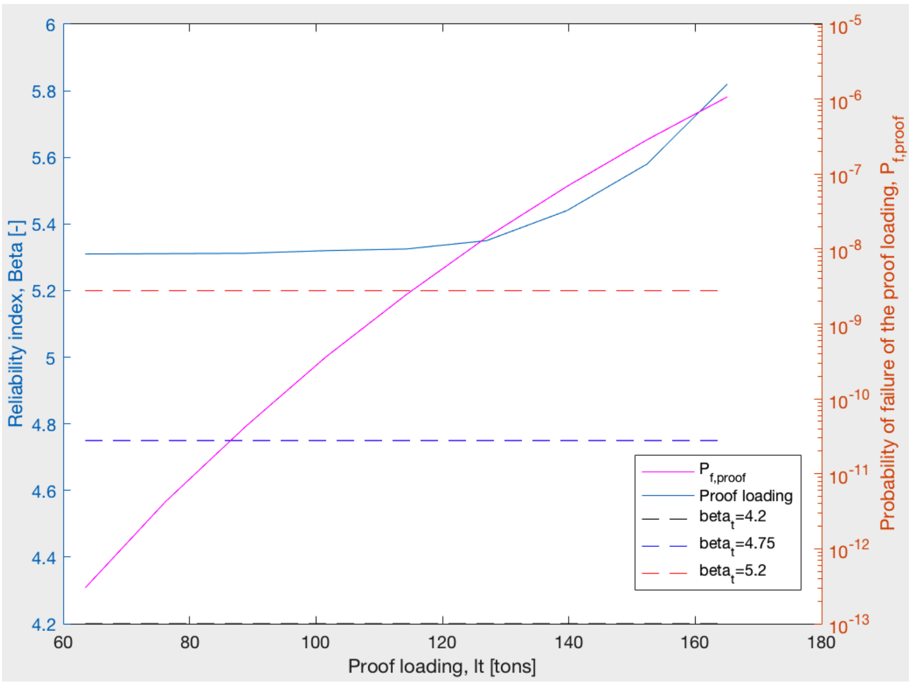

Figure C.3: Proof loading updating for concrete compression strength, standard vehicles class 100 and $\alpha = 0.5$.

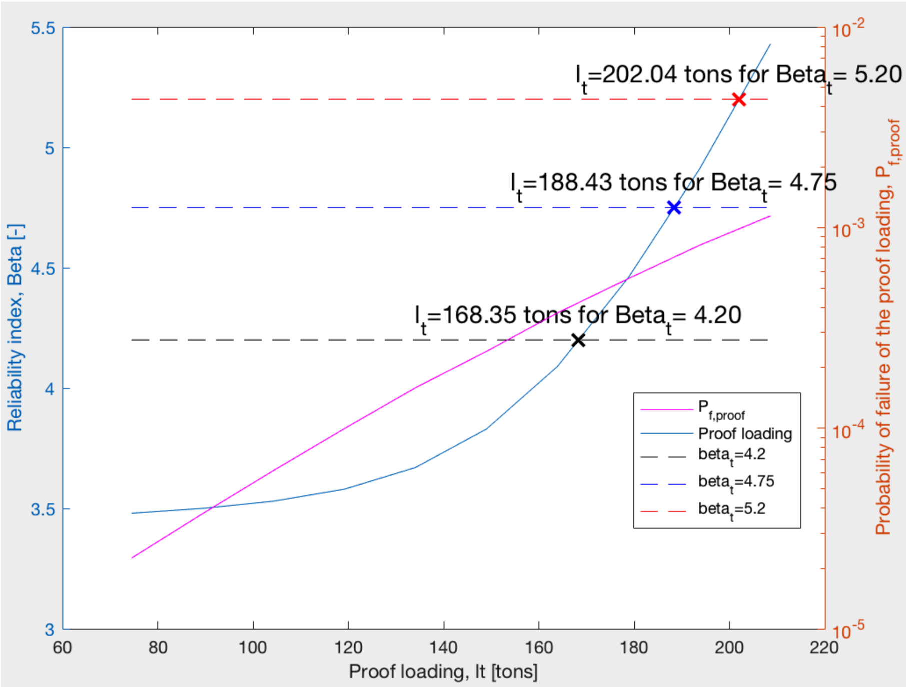

Figure C.4: Proof loading updating for concrete compression strength, standard vehicles class 125 and $\alpha = 0.2$.

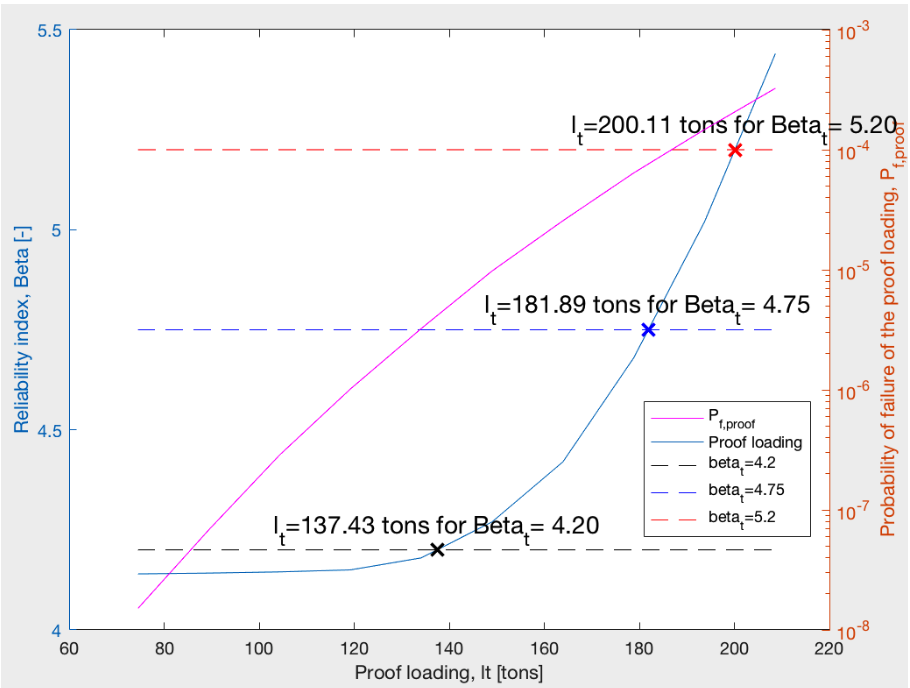

Figure C.5: Proof loading updating for concrete compression strength, standard vehicles class 125 and $\alpha = 0.4$.

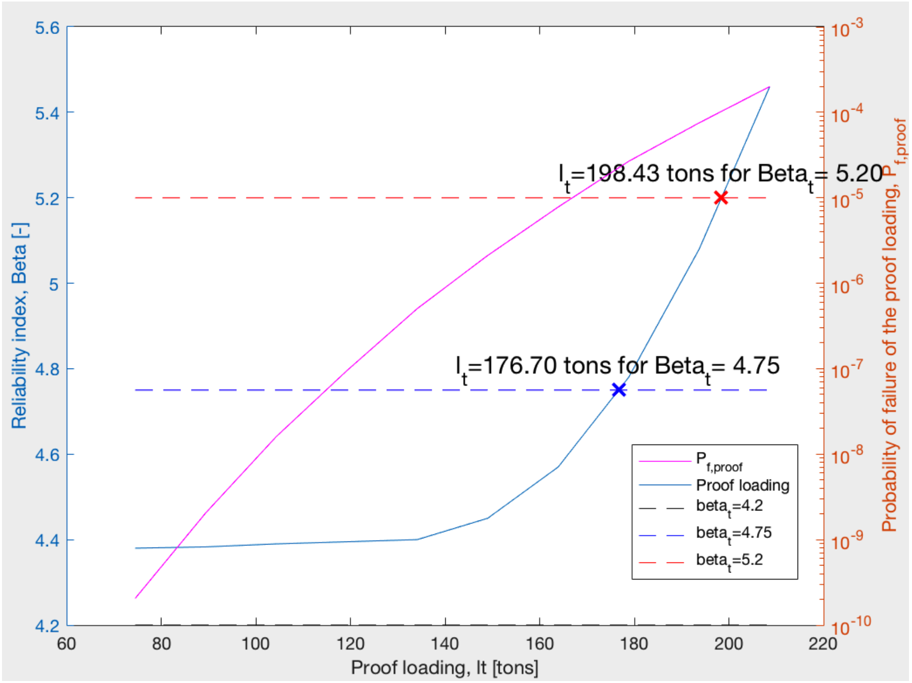

Figure C.6: Proof loading updating for concrete compression strength, standard vehicles class 125 and $\alpha = 0.5$.

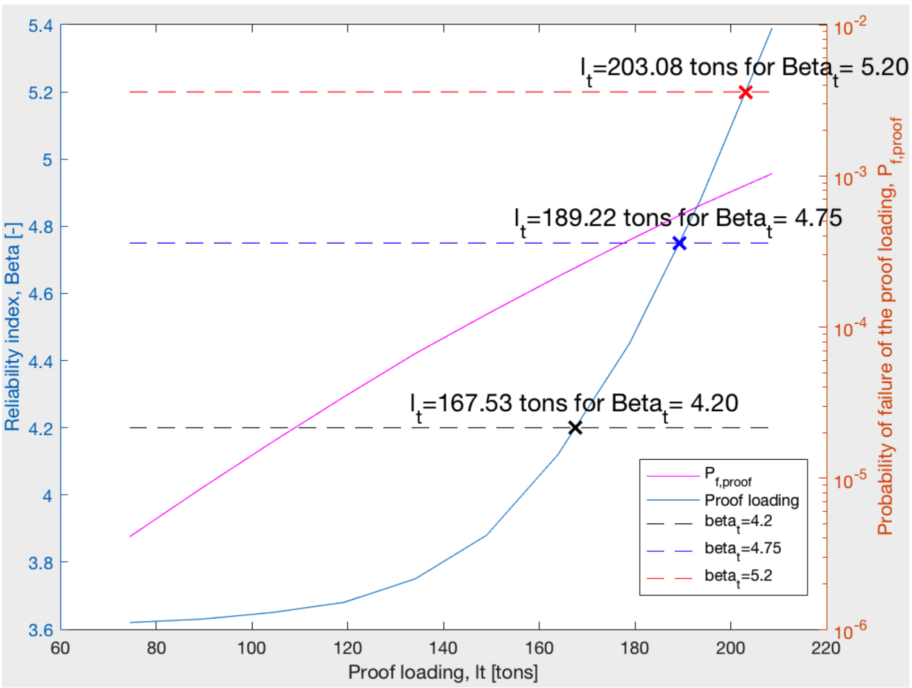

Figure C.7: Proof loading updating for reinforcement strength, standard vehicles class 125 and $\alpha = 0.2$.

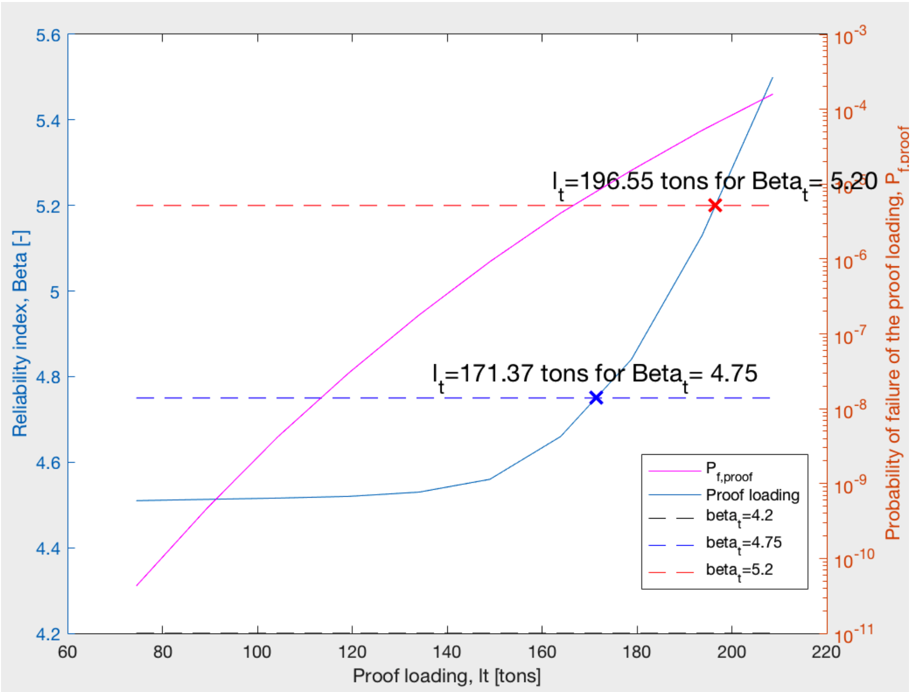

Figure C.8: Proof loading updating for reinforcement strength, standard vehicles class 125 and $\alpha = 0.4$.

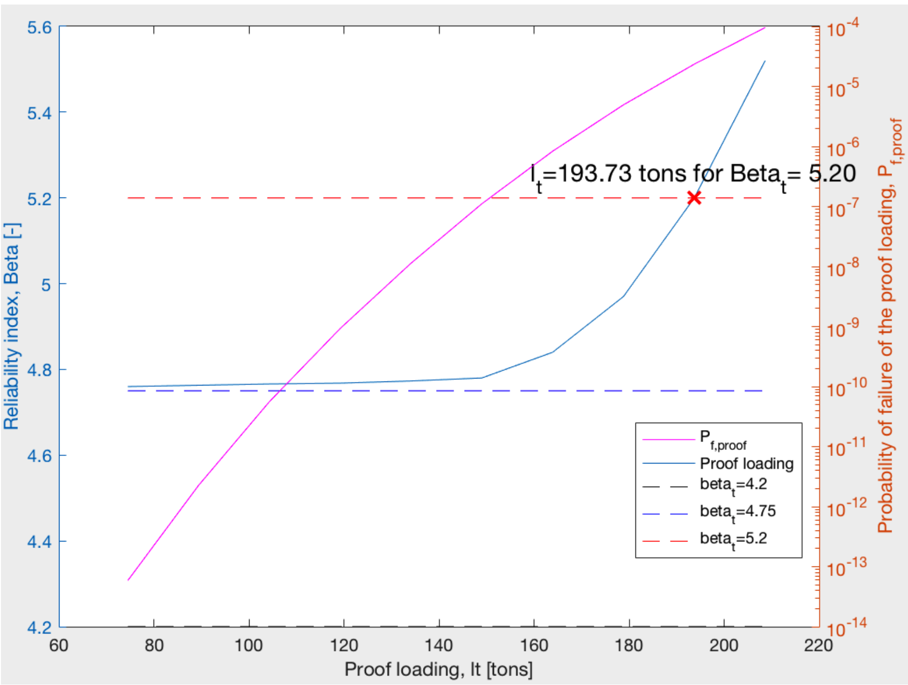

Figure C.9: Proof loading updating for reinforcement strength, standard vehicles class 125 and $\alpha = 0.5$.

Appendix D

Updating coefficient of variation plots for the different failure modes and α values

In this chapter, the updating of the reliability index with regard to the coefficient of variation of the material for the different failure modes and taking into account different α values are shown.

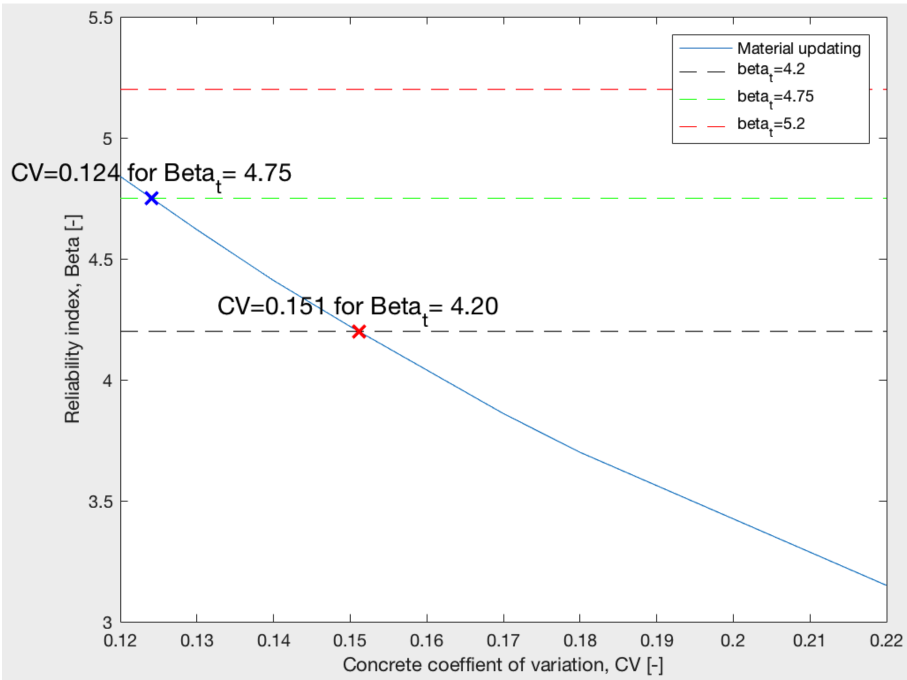

Figure D.1: Coefficient of variation updating for concrete compression strength, standard vehicles class 100 and $\alpha = 0.2$.

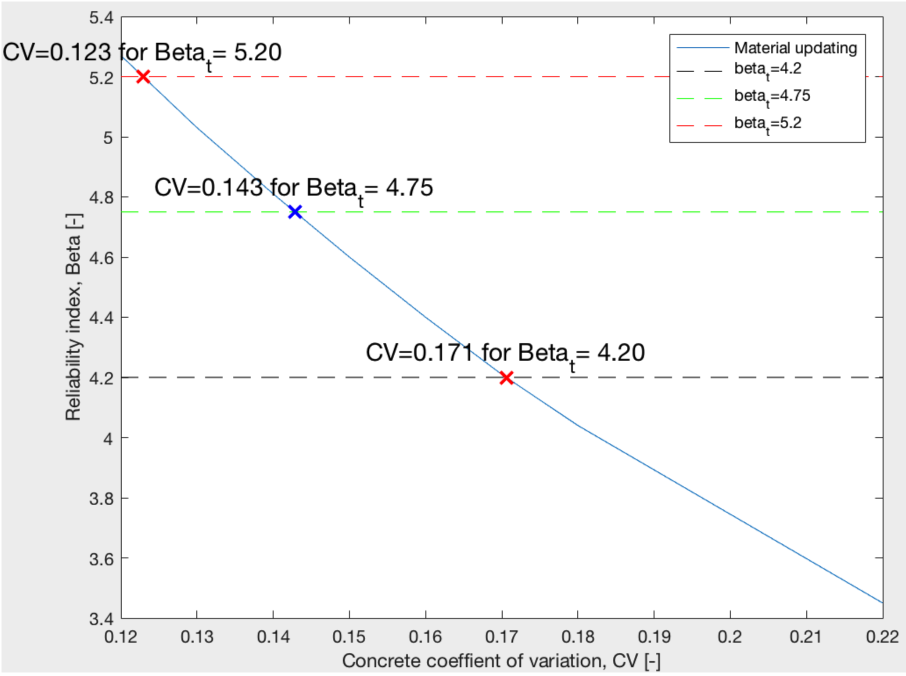

Figure D.2: Coefficient of variation updating for concrete compression strength, standard vehicles class 100 and $\alpha = 0.3$.

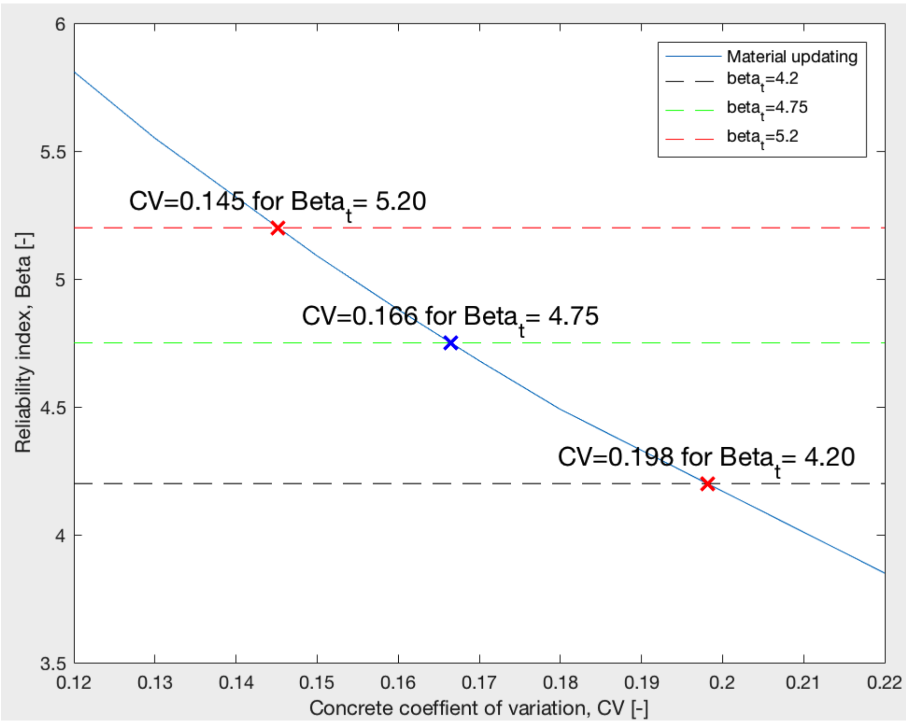

Figure D.3: Coefficient of variation updating for concrete compression strength, standard vehicles class 100 and $\alpha = 0.5$.

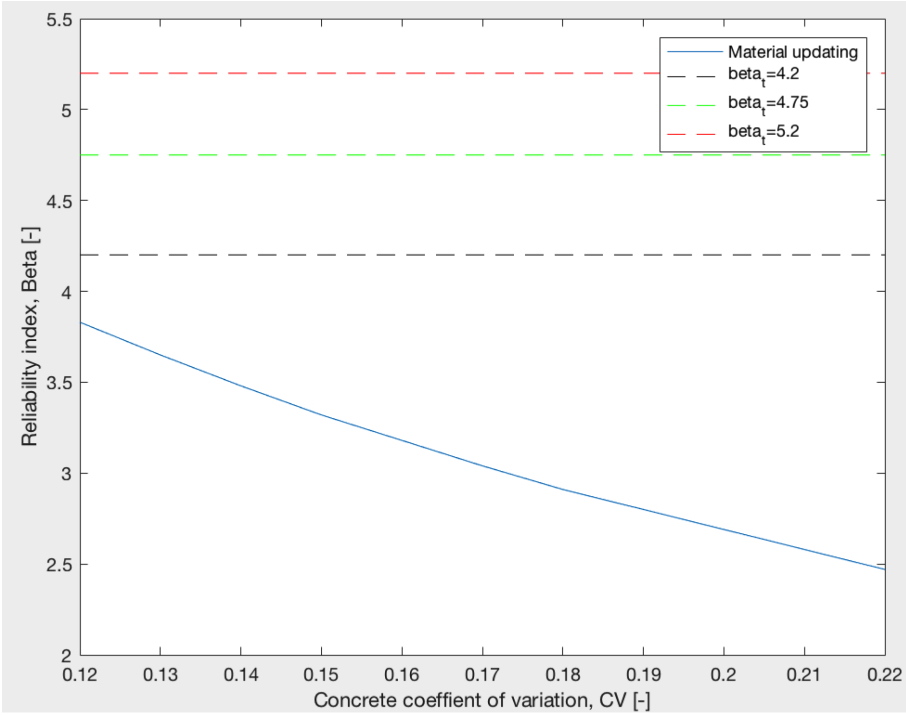

Figure D.4: Coefficient of variation updating for concrete compression strength, standard vehicles class 125 and $\alpha = 0.2$.

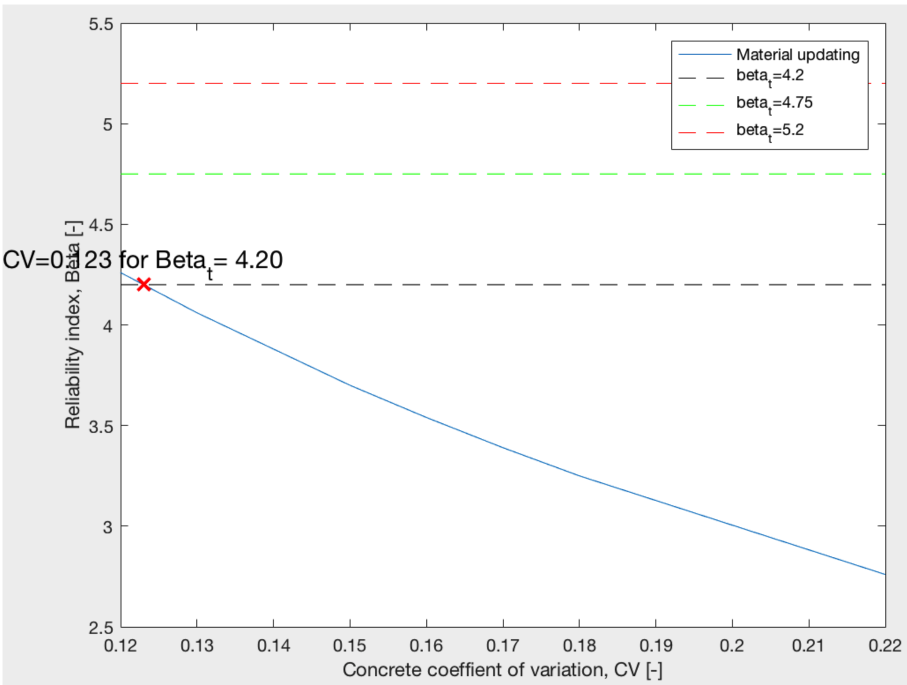

Figure D.5: Coefficient of variation updating for concrete compression strength, standard vehicles class 125 and $\alpha = 0.3$.

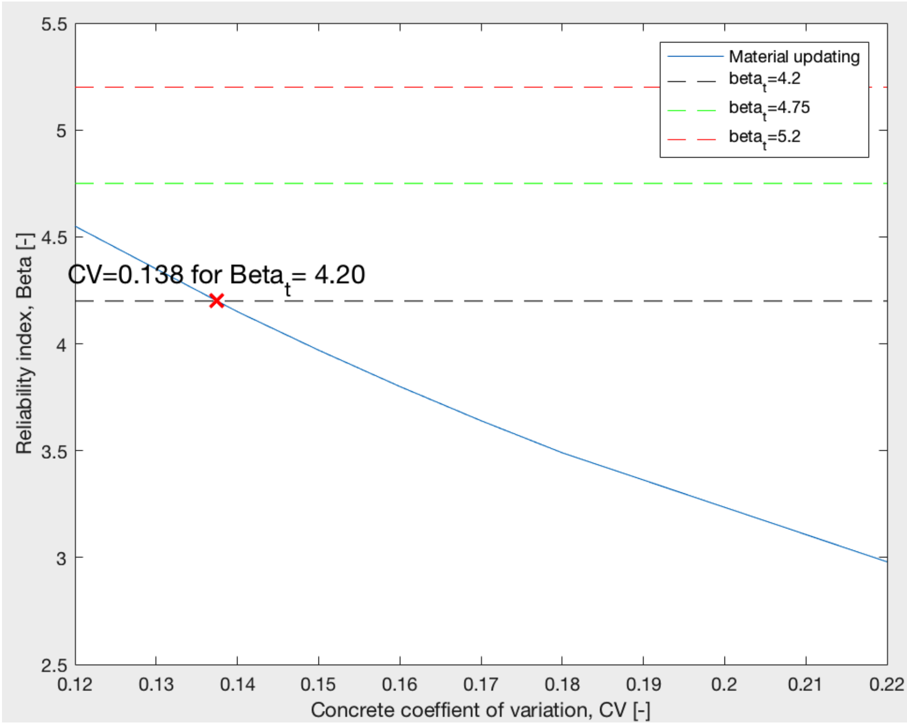

Figure D.6: Coefficient of variation updating for concrete compression strength, standard vehicles class 125 and $\alpha = 0.4$.

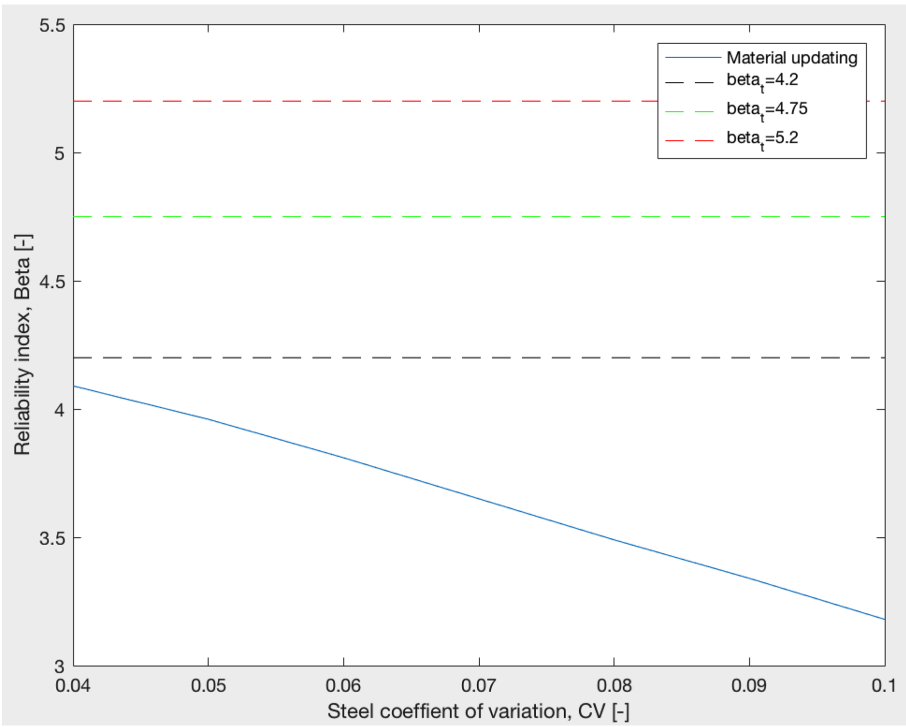

Figure D.7: Coefficient of variation updating for reinforcement strength, standard vehicles class 125 and $\alpha = 0.2$.

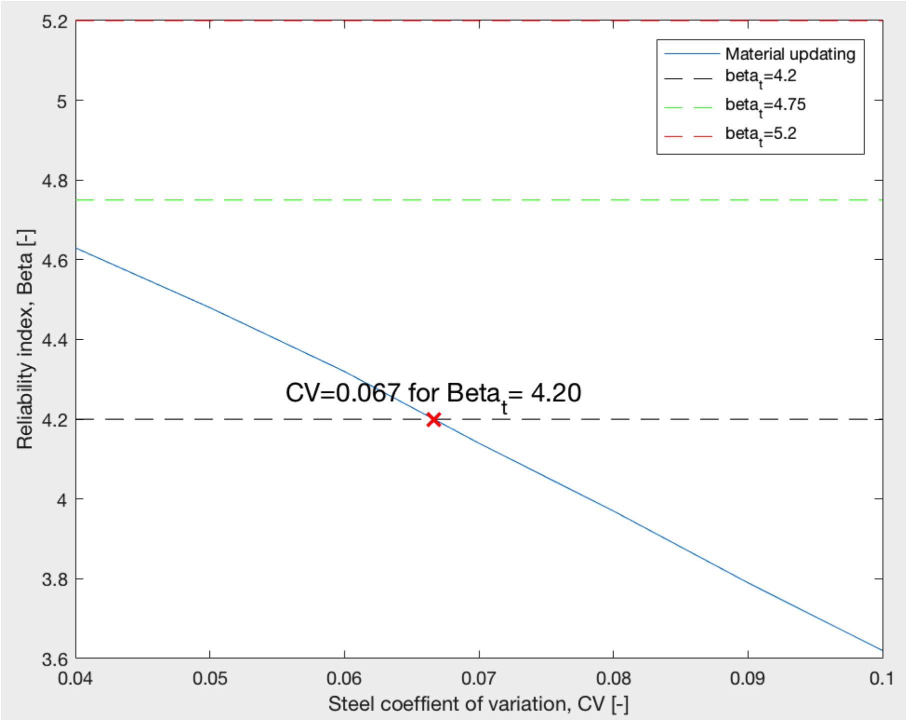

Figure D.8: Coefficient of variation updating for reinforcement strength, standard vehicles class 125 and $\alpha = 0.3$.

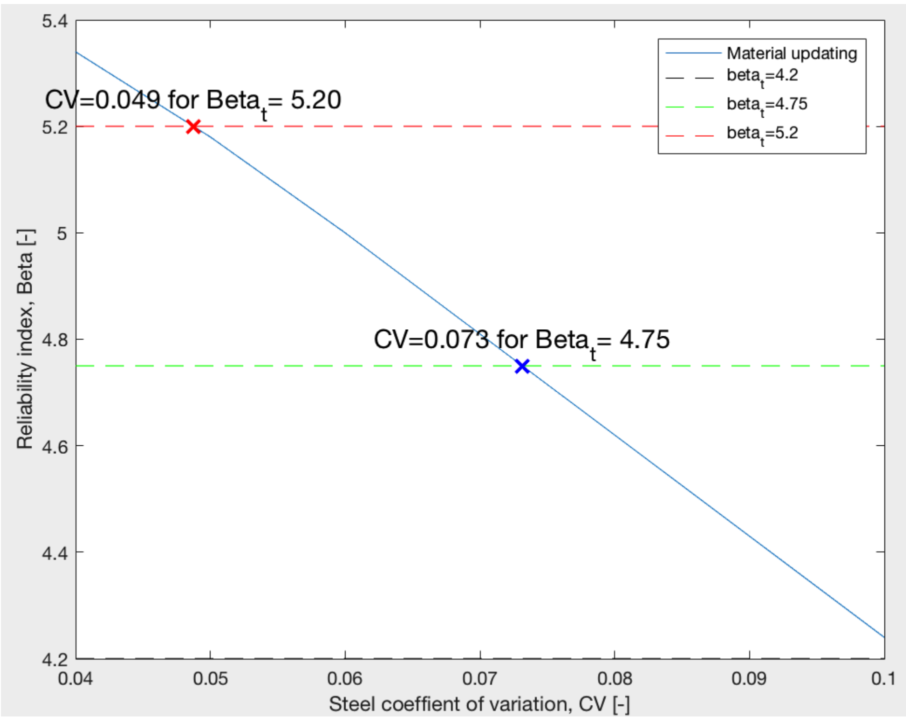

Figure D.9: Coefficient of variation updating for reinforcement strength, standard vehicles class 125 and $\alpha = 0.5$.

Appendix E

Updating concrete compressive strength as a normally distributed variable

In this chapter, material updating of the reliability index for concrete compressive strength, considering different values of σ and σ' and taking into account different values of the number of test samples n and test average $\bar{x}$ are shown.

Test average constant, $\bar{x} = 39$ MPa with $\sigma' = \frac{1}{3}\sigma$; $\sigma = 4.81, \sigma = 1.60$

Table E.1: Posterior and predictive mean and standard deviation when $\sigma' = \frac{1}{3}\sigma$ is assumed.

	n					
	1	3	5	7	9	11
μ''(posterior and predictive)	36.48	36.90	37.20	37.42	37.60	37.74
σ'' (posterior)	1.52	1.39	1.28	1.20	1.13	1.07
σ''' (predictive)	5.04	5.01	4.98	4.96	4.94	4.93

Table E.2: Reliability indexes for the case when $\sigma' = \frac{1}{3}\sigma$ is assumed.

	n					
	1	3	5	7	9	11
$\beta_u(\alpha = 0.2)$	3.72	3.81	3.89	3.94	3.99	4.02
$\beta_u(\alpha = 0.3)$	3.96	4.06	4.14	4.19	4.24	4.27
$\beta_u(\alpha = 0.4)$	4.13	4.23	4.31	4.37	4.41	4.45
$\beta_u(\alpha = 0.5)$	4.28	4.38	4.46	4.51	4.56	4.60

Figure E.1: Reliability index for the case when $\sigma' = \frac{1}{3}\sigma$ is assumed and $\alpha = 0.5$.

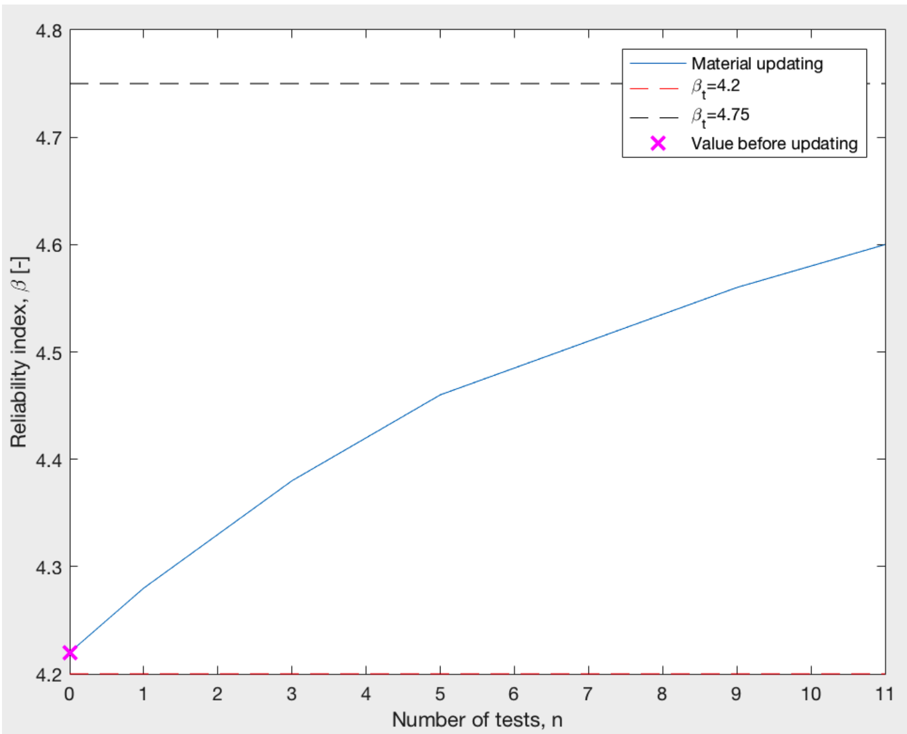

Figure E.2: Reliability index for the case when $\sigma' = \frac{1}{3}\sigma$ is assumed and $\alpha = 0.5$.

Test average constant, $\bar{x} = 39$ MPa with $\sigma' = \frac{1}{4}\sigma$; $\sigma = 4.92, \sigma = 1.23$

Table E.3: Posterior and predictive mean and standard deviation when $\sigma' = \frac{1}{4}\sigma$ is assumed.

	n					
	1	3	5	7	9	11
μ''(posterior and predictive)	36.36	36.64	36.87	37.05	37.21	37.34
σ'' (posterior)	1.19	1.13	1.07	1.03	0.98	0.95
σ''' (predictive)	5.06	5.05	5.04	5.03	5.02	5.01

Table E.4: Reliability indexes for the case when $\sigma' = \frac{1}{4}\sigma$ is assumed.

	n					
	1	3	5	7	9	11
$\beta_u(\alpha = 0.2)$	3.68	3.74	3.79	3.83	3.86	3.89
$\beta_u(\alpha = 0.3)$	3.92	3.98	4.03	4.07	4.11	4.14
$\beta_u(\alpha = 0.4)$	4.09	4.15	4.20	4.24	4.28	4.31
$\beta_u(\alpha = 0.5)$	4.24	4.30	4.35	4.39	4.43	4.46

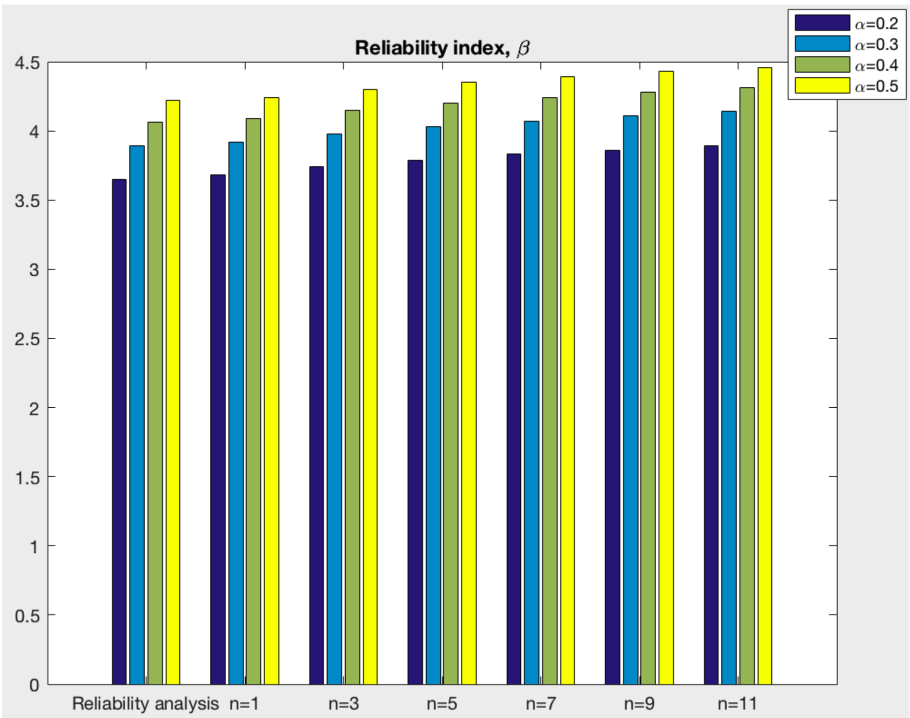

Figure E.3: Reliability index for the case when $\sigma' = \frac{1}{4}\sigma$ is assumed and $\alpha = 0.5$.

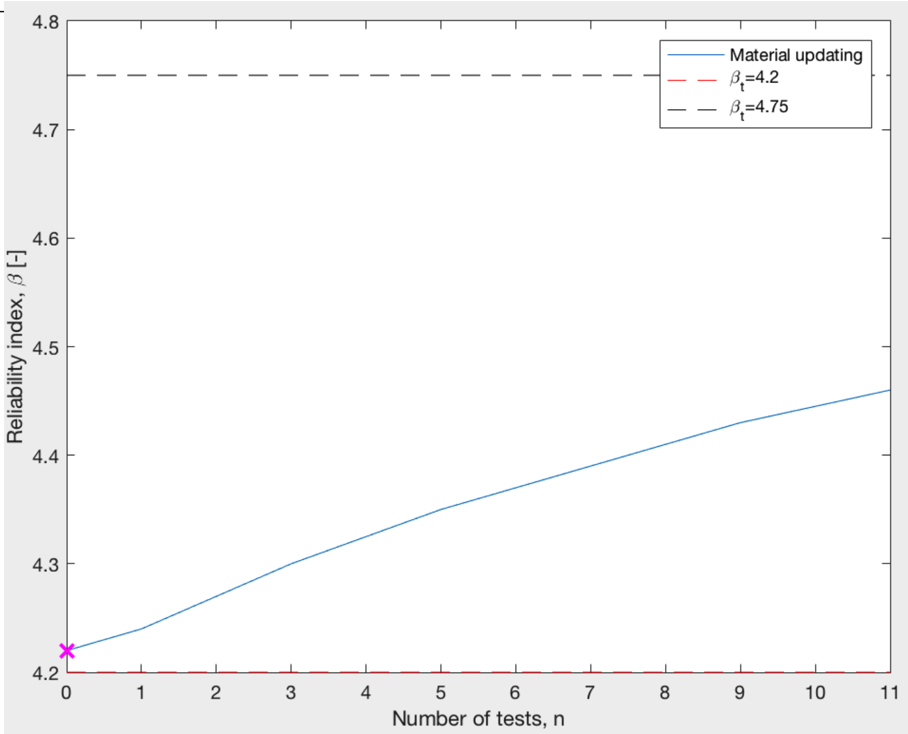

Figure E.4: Reliability index for the case when $\sigma' = \frac{1}{4}\sigma$ is assumed and $\alpha = 0.5$.

Number of test samples constant, $n = 5$ with $\sigma' = \frac{1}{3}\sigma$; $\sigma = 4.81, \sigma' = 1.6, \sigma'' = 1.28, \sigma''' = 4.98$

Table E.5: Posterior and predictive mean value when $\sigma' = \frac{1}{3}\sigma$ is assumed.

	$\bar{x}$					
	30	33	36	39	42	45
μ''(posterior and predictive)	33.99	35.06	36.13	37.20	38.27	39.33

Table E.6: Reliability indexes for the case when $\sigma' = \frac{1}{3}\sigma$ is assumed

	n					
	30	33	36	39	42	45
$\beta_u(\alpha = 0.2)$	3.29	3.49	3.69	3.89	4.09	4.28
$\beta_u(\alpha = 0.3)$	3.54	3.74	3.94	4.14	4.34	4.53
$\beta_u(\alpha = 0.4)$	3.71	3.91	4.11	4.31	4.51	4.71
$\beta_u(\alpha = 0.5)$	3.85	4.06	4.26	4.46	4.66	4.86

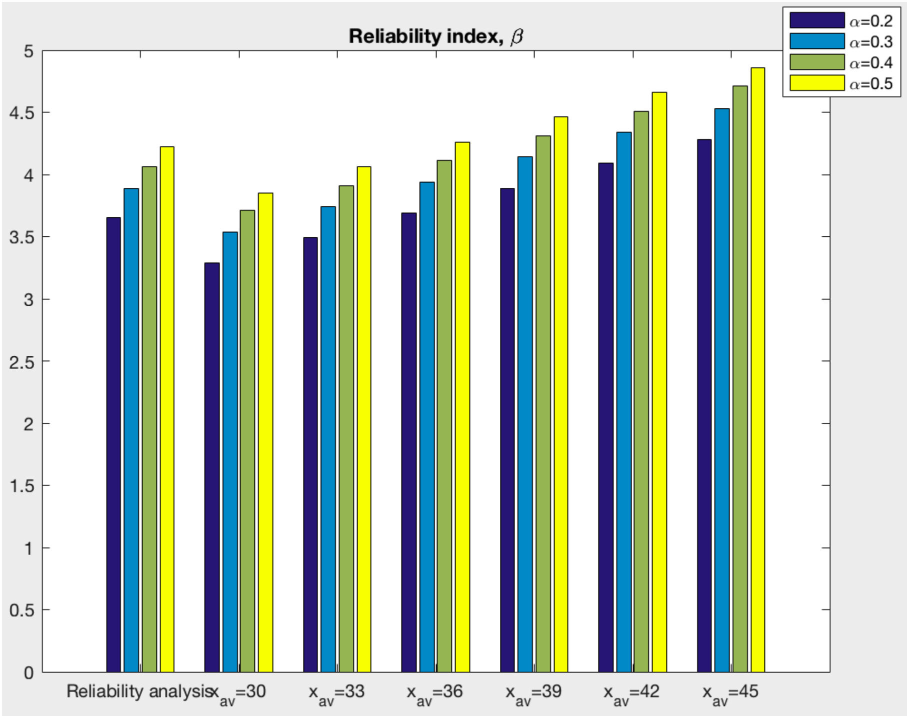

Figure E.5: Reliability index for the case when $\sigma' = \frac{1}{3}\sigma$ is assumed and $\alpha = 0.5$.

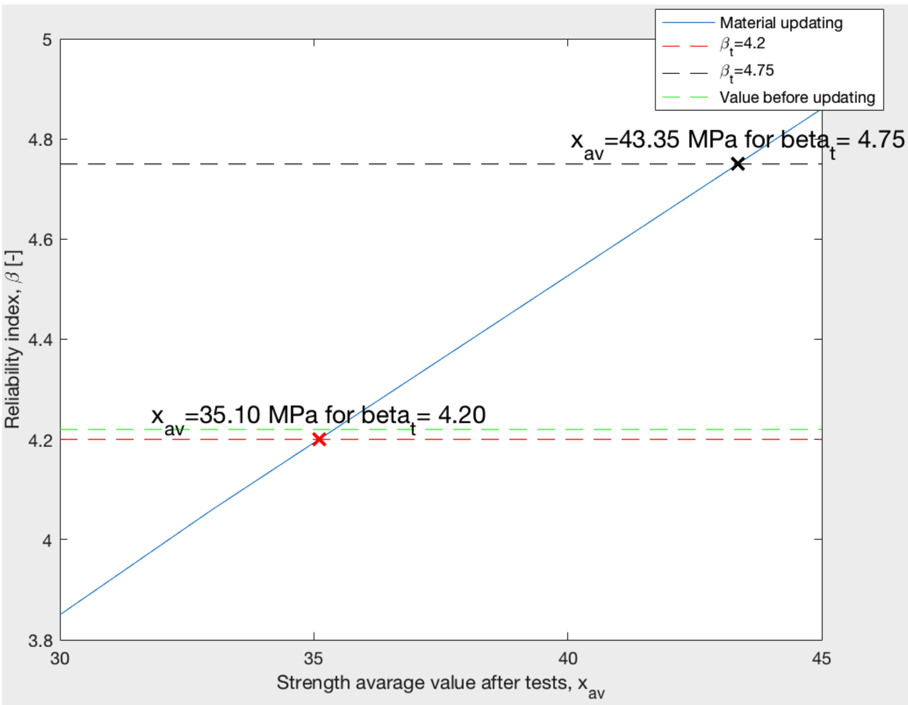

Figure E.6: Reliability index for the case when $\sigma' = \frac{1}{3}\sigma$ is assumed and $\alpha = 0.5$.

Number of test samples constant, $n = 5$ with $\sigma' = \frac{1}{4}\sigma$; $\sigma = 4.92, \sigma' = 1.23, \sigma'' = 1.07, \sigma''' = 5.04$

Table E.7: Posterior and predictive mean value when $\sigma' = \frac{1}{4}\sigma$ is assumed.

	$\bar{x}$					
	30	33	36	39	42	45
μ''(posterior and predictive)	34.72	35.44	36.15	36.87	37.58	38.29

Table E.8: Reliability indexes for the case when $\sigma' = \frac{1}{4}\sigma$ is assumed

	n					
	30	33	36	39	42	45
$\beta_u(\alpha = 0.2)$	3.39	3.53	3.66	3.79	3.92	4.05
$\beta_u(\alpha = 0.3)$	3.64	3.77	3.90	4.03	4.16	4.29
$\beta_u(\alpha = 0.4)$	3.80	3.94	4.07	4.20	4.33	4.47
$\beta_u(\alpha = 0.5)$	3.95	4.08	4.22	4.35	4.48	4.61

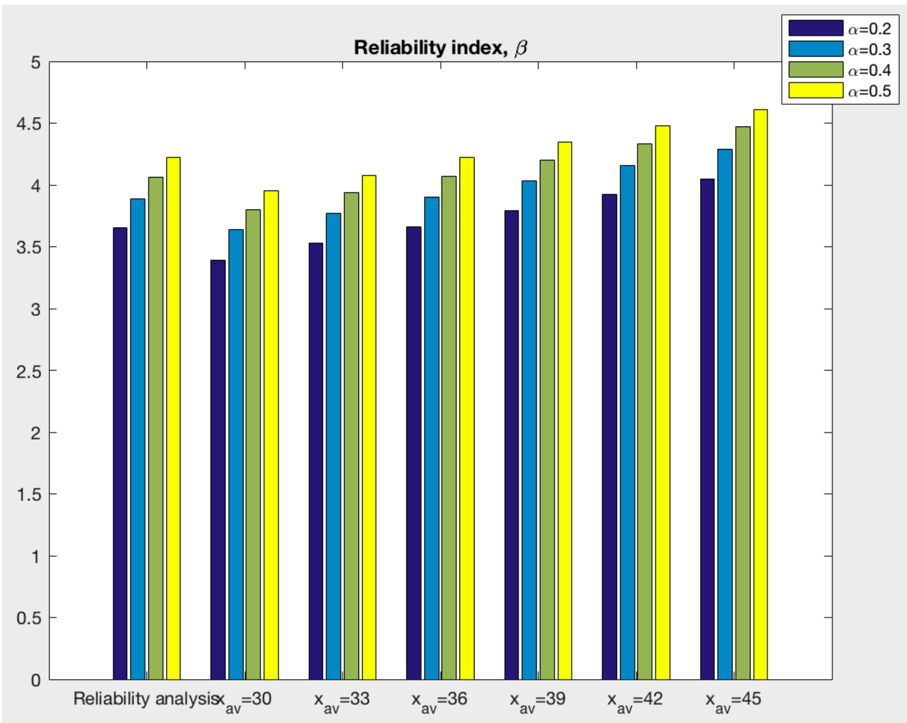

Figure E.7: Reliability index for the case when $\sigma' = \frac{1}{4}\sigma$ is assumed and $\alpha = 0.5$.

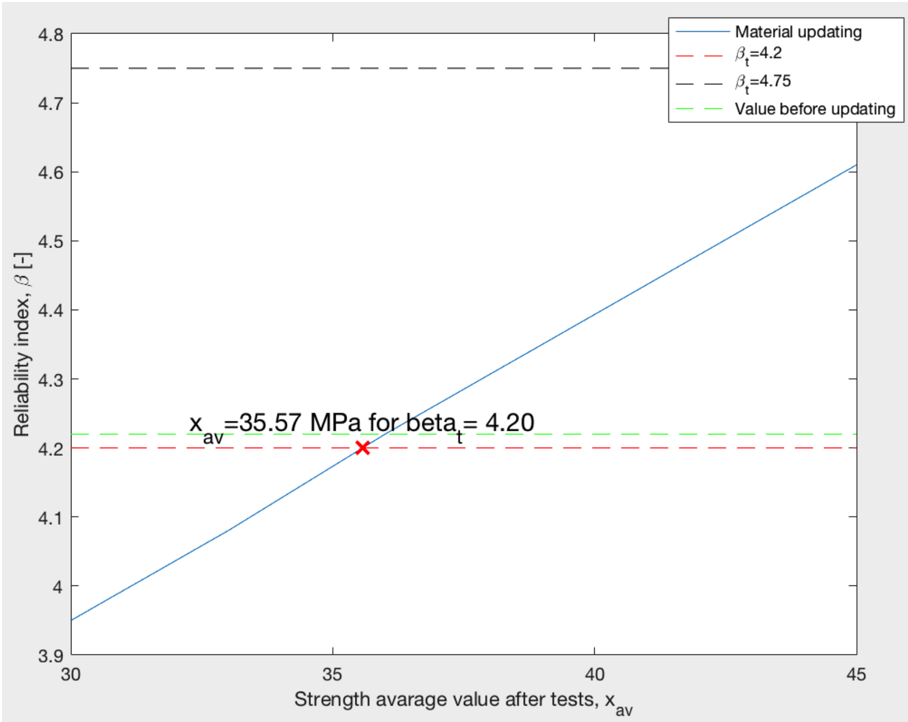

Figure E.8: Reliability index for the case when $\sigma' = \frac{1}{4}\sigma$ is assumed and $\alpha = 0.5$.