COORDINATED CONTROL OF SEPARATION PROCESSES FOR BETTER PRODUCED WATER TREATMENT

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Abstract:

Two separation processes are used to filtrate the produced water in offshore oil and gas industry. Each process is occurring in specific system. Gravity separator model is defined for the first stage of produced water treatment. Hydro-cyclone model is estimated for the second separation stage. The two systems are influencing each other indicating a relationship between the gravity and hydro-cyclone separator systems. The two defined models are combined forming multi-input multi-output system. Control structure is defined based on the MIMO model in order to investigate coordinated control for better produced water treatment.

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By signing this document, each member of the group confirms participation on equal terms in the process of writing the project. Thus, each member of the group is responsible for the all contents in the project.

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List of variables and parameters for Gravity separator system

Notation	Description	Units
ρ	Water phase density	kg/m^3
ρ_i	Density of the inlet flow for water phase	kg/m^3
$F_{in}(t)$	Inlet flow for water phase	m^3/h
$F_{out}(t)$	Outlet-flow for water phase	m^3/h
$V_w(t)$	Water phase volume	m^3
$A_w(t)$	Water phase cross-section area	m^2
$L1$	Water phase container length	m
R	Radius of gravity separator vessel	m
$h_w(t)$	Water phase level	m
$f(u_w)$	Valve opening function	%
$P_g(t)$	Gravity separator tank pressure	kPa
$P_{ow}(t)$	Water phase outlet pressure	kPa
SG	Specific gravity of water	-
a	System matrix	-
b	Input matrix	-
c	Output matrix	-
Q	State weighting matrix for LQR and LQRI	-
R	Input weighting matrix for LQR and LQRI	-

List of variables and parameters for Hydro-Cyclone separator system

Notation	Description	Units
V_u	Underflow valve opening position	%
V_o	Overflow valve opening position	%
V_u'	Operating point for underflow valve	%
V_o'	Operating point for overflow valve	%
PDR	Pressure difference ratio	-

List of variables and parameters for MIMO system

Notation	Description	Units
x_1 and x_2	PDR state considering underflow input	-
x_3 and x_4	PDR state considering overflow input	-
a_{gsep}	System matrix for water level (scalar)	-
$a_{ii_{hvu}}$	Underflow to PDR system matrix (coefficient)	-
$a_{ii_{hvo}}$	Overflow to PDR system matrix (coefficient)	-
b_{gsep}	Input matrix for water level (scalar)	-
$b_{i_{hvu}}$	Input matrix for underflow to PDR system (coefficient)	-
$b_{i_{hvo}}$	Input matrix for overflow to PDR system (coefficient)	-
c_{gsep}	Output matrix for water level (scalar)	-
$c_{i_{hvu}}$	Output matrix for underflow to PDR system (coefficient)	-
$c_{i_{hvo}}$	Output matrix for overflow to PDR system (coefficient)	-
Q	State weighting matrix for LQR and LQRY	-
R	Input weighting matrix for LQR and LQRY	-

I. Introduction

Top side processing is necessary part for onshore and offshore oil and gas production. In the case of offshore production, a system of valves and pipelines is used to elevate a multi-phase flow to an oil rig processing station. Due to the high complexity of the separation process, control over the processing equipment is required. The focus of this project is evolving around the first and second stage of the separation processes. Moreover, key point of interest is the investigation of control strategy which allows stable operating conditions for the first separation stage while maintaining pressure difference ration (PDR) for the second separation stage in permissible range [1].

The first stage of separating the multi-phase flow to oil, gas and water components is carried by a “Three phase gravity separator system”. Based on the application gravity separators are classified as primary phase separators, test separators, high or low pressure separators etc. Based on the separator tank shape and orientation separators are classified as cylindrical, spherical, horizontal or vertical separator systems. Small scale first stage horizontal three phase gravity separator system with cylindrical body illustrated in Fig I.1 is considered for the scope of this project.

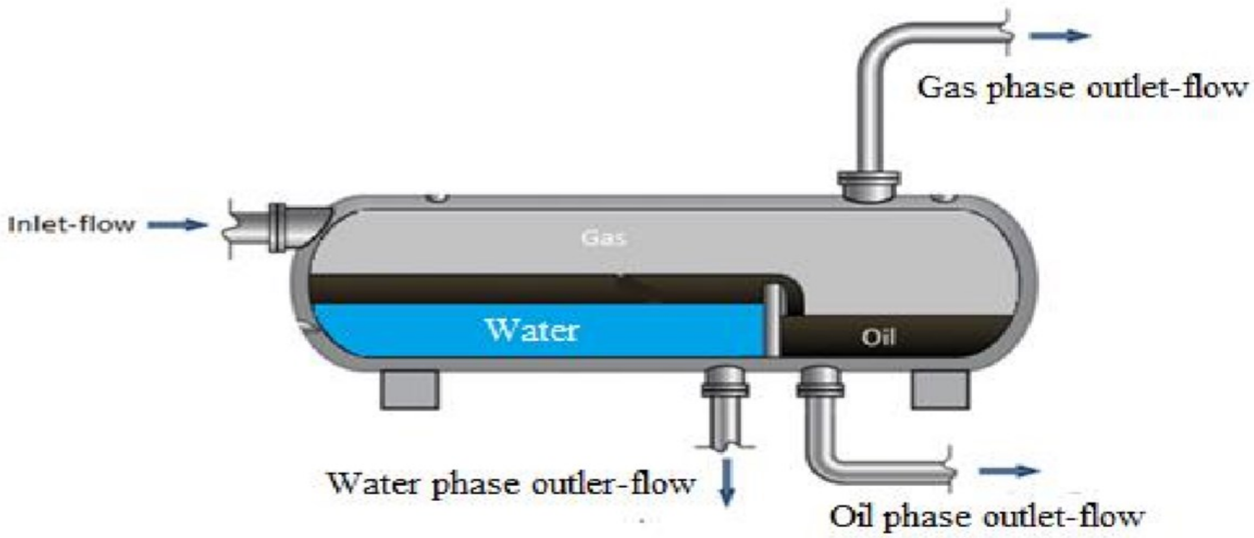


Fig. I.1 Three phase gravity separator system [http://goo.gl/fymPiv]

The multi-phase inlet follow is entering the gravity separator system where the velocity of the stream reduces and splits to liquid and gas phases as in Fig I.1. The liquid phase is entering the first control volume allowing buoyancy to take effect under gravitational influence. Maintaining steady liquid level inside the separator tank results in the further separation of the liquid phases into oil and water products. The formed oil layer is pushed into second control volume by the new amount of liquid entering the separator vessel. The gas and liquid phases are almost fully separated during the first stage of the separation process. However, the water and oil phases cannot be fully separated under the slow dynamics of a gravity separator system. Oil droplets will remain into the water phase outlet flow leaving the separator vessel, requiring further treatment of the produced water.

Secondary hydro-cyclone separator stage is usually implemented to enhance the separation process. The produced water outlet-flow is tangentially injected under pressure to a cyclonic body as illustrated in Fig.I.2.

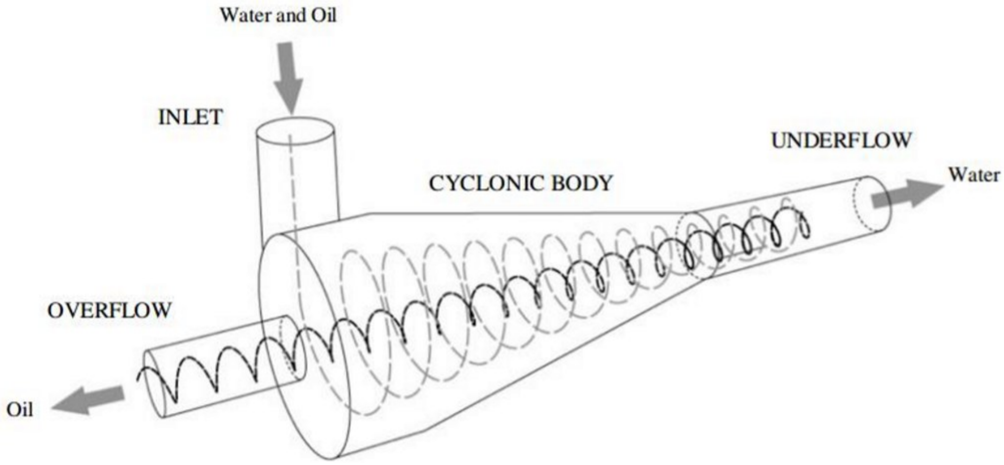


Fig. I.2 Hydro-cyclone separator [1, 2]

The inlet flow entering a hydro-cyclone is developing a vortex system defining two flow patterns. Under the influence of the created centrifugal force the heavier water is pushed through the walls of the hydro-cyclone separator while the lighter oil particles are migrating towards the center. The water is forced to exit the separator body through the underflow while the separated oil exits thought the overflow achieving further filtration of the produced water.

The two systems are cooperating to form a two stage separation process for the produced water treatment. From implementation point of view, the efficiency of the first separation process is not demanding. The gravity separation process is considered efficient, as long as water spills does not occur and only oil enters the second control volume of the system. In contrast the complex second separation process efficiency is heavily constrained and hard to measure. Instead a pressure difference ration (PDR) between the underflow and overflow side of a hydro-cyclone system is used in practice. Maintaining constant PDR in specified range ensures that the proportionality between the underflow and overflow pressures will be maintained. From which follows that the stability of the two flows will be maintained, ensuring that the hydro-cyclone system operates in efficient range.

I.II Project Requirements

The focus of this project is evolving around the first and second stages of the separation process used in offshore oil and gas production. The two systems used are connected in series and dependent on each other in order to maintain stable separation process, introducing a non-conventional problem with two main objectives. The first objective is to maintain steady water level inside the gravity separator vessel. The second objective is to maintain the hydro-cyclone PDR as constant as possible. In order to obtain solution for the stated problem several sub-problems with clear goals are defined as follow.

1. Definition of mathematical model describing three phase gravity separator system.
2. Validation of the nonlinear gravity separator model.
3. Control design and implementation for a gravity separator system
4. Modeling of underflow valve to PDR relationship of a hydro-cyclone separator system.
5. Modeling of overflow valve to PDR relationship of a hydro-cyclone separator system
6. Validation of the two hydro-cyclone models
7. Coupling of derived gravity and hydro-cyclone separator models
8. Validation of the derived multi-input multi-output (MIMO) system
9. Definition of MIMO control strategy.

The defined sub-problems are split into three categories defined as MIMO, gravity and hydro-cyclone separator problems. The first sub-problem is critical requirement to approach the first main objective. Mathematical model representation of a gravity separator system is necessary as first step before obtaining control designed. The second sub-problem is derived from the first. The required model must be compared against empirical data collected from the available small scale gravity separator system located at Aalborg University Esbjerg. After a gravity separator model is obtained and validated a control structure design and implementation is considered as solution to the third requirement, allowing to obtain solution for the gravity separator category of sub-problems.

Two relationships for the hydro-cyclone's PDR are required to approach the secondary main objective. The first relationship considers the necessary quantity of energy required to reach given PDR value. The second relationship associated with the necessary flow split is considered in the fifth sub-problem. The required PDR models must be compared against empirical data collected from the available hydro-cyclone separator system in order to solve sub-problem six, allowing to obtain solution to the hydro-cyclone category of problems.

Proceeding further the three required models must be coupled in order to create a MIMO system suitable for control purposes, implying third validation step before reaching the two main objectives. After solution to sub-problem eight is obtained, a control structure for the MIMO system describing the first and second stages of the produced water separation process is considered, in order to obtain one solution which coordinates the two systems.

I.III Project Specifications

In order to fulfill the stated requirements and solve the main control problem a set of specifications is defined as follow.

1. Derivation of mathematical model describing the water phase level of a gravity separator system.
2. Empirical validation of nonlinear and linear water phase level model.
    a. Basic operating condition for gravity separator system
3. Closed loop design and implementation for water phase level
    a. Proportional Integral (PI) control structure
    b. Internal module control (IMC) structure
    c. Linear quadratic regulator (LQR) control structure
4. Empirical modeling of underflow valve to PDR relationship.
    a. Sufficient operating conditions for gravity and hydro-cyclone separator systems
5. Empirical modeling of overflow valve to PDR relationship
    a. Sufficient operating conditions for gravity and hydro-cyclone separator systems
6. Empirical validation of the identified hydro-cyclone's models
7. Coupling of the hydro-cyclone and gravity separator models
8. Validation of the obtained MIMO system
9. Definition of MIMO LQR solution.

The specifications are assigned under the three required sets of sub-problem categories. Considering that gravity separator systems are complex and have at least three degrees of freedom (3DOF) is decided to focus on one phase only. In order to meet the requirement for stable first stage separation process is decided to model the water phase of a gravity separator system. In depth the control objective of interest is to maintain steady water phase level in order to ensure stable operating conditions for the separation of the two liquid phases.

Proceeding further the derived model must be evaluated under given criteria to determine if the considered model is suitable for control purposes. The validation method chosen is comparison between empirical data and simulated model output. Experiment under basic operating conditions with minimum disturbances acting on the water phase level is considered for the nonlinear model validation procedure. It is decided that the following linearization procedure required to obtain linear time invariant (LTI) system for control purposes must be validated following the criteria applied to the nonlinear model.

Three control structures are considered to finalize the specifications for the gravity separator category of sub-problems. A PI control structure is considered first as the conventional control over gravity separator systems used in oil and gas industry. As second option, an IMC control structure is considered by utilizing the defined gravity separator model. The final option considered is based on optimal control theory and threating the first main project objective as infinite time problem allowing to design theoretical "optimal control" for the water phase level.

The second category of sub-problems requires two models which are considering the flow rate and flow split influence over the hydro-cyclone's PDR. The flow split relationship is manipulated by an overflow valve, while the flow rate relationship is manipulated by an underflow valve. Under consideration of the complexity of hydro-cyclone systems is decided to use empirical modeling approach [3]. Two identification experiments with strict operating conditions are considered for the identification of the required hydro-cyclone model. As part of a system identification procedure the identified models are validated against experiment with similar operating conditions and different operating points for the variables of interest, finalizing the specifications for the hydro-cyclone category of sub-problems.

The final sub-problem category after obtaining control over the water phase level and estimating the two required hydro-cyclone models is to couple the systems. It is decided to validate the new MIMO model based on the experimental data collected during the development of each individual part of the models. The final step to fulfil the project requirements and specification is to derive a MIMO control structure based on combined model. Considering the optimal control theory is decided to use "LQR problem formulation" for the derivation of the required MIMO controller, providing an adequate control design model for the specified problem.

I.III.I Laboratory setup description

The following block diagram is used to illustrate the different lab setup components included in the scope of this project.

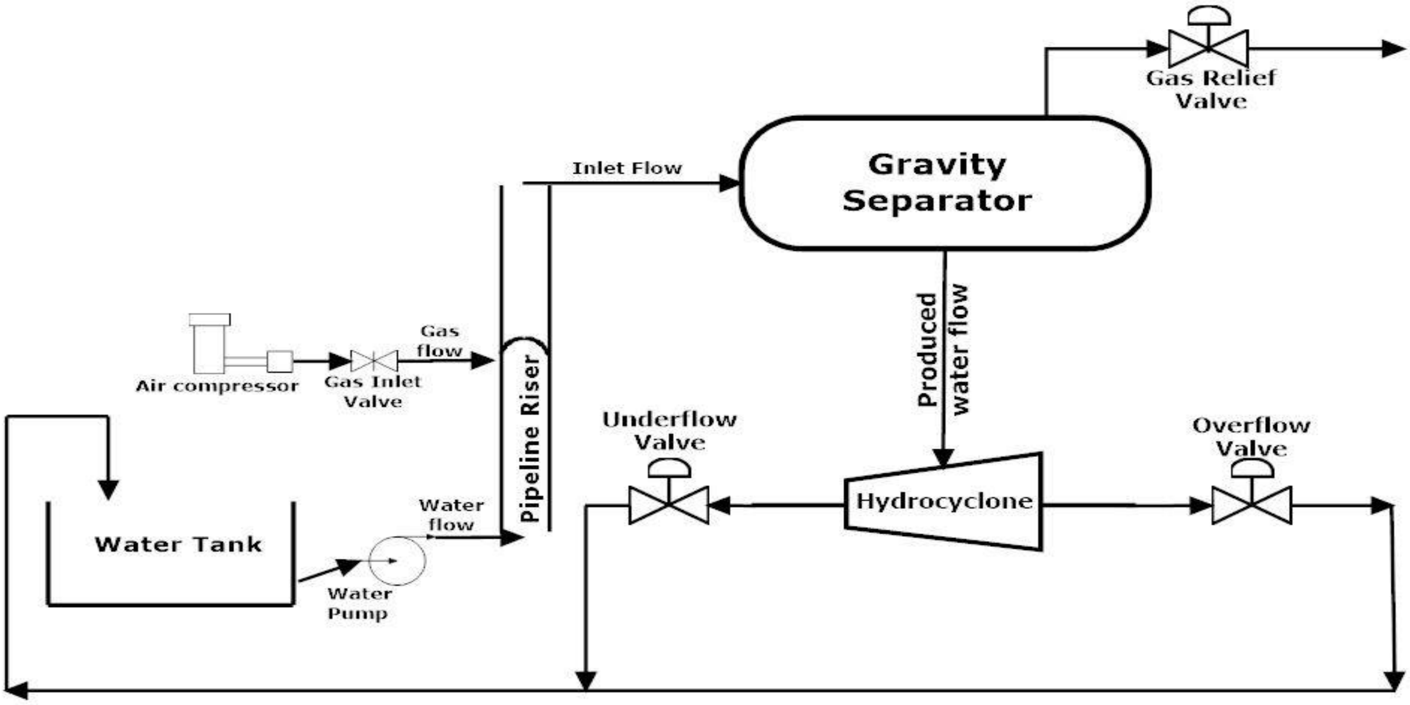


Fig. I.3 Lab setup block diagram

A reservoir tank is used to store sufficient amount of water required to run the lab setup. A centrifugal pump is used to emulate water phase inlet flow. The produced water flow is injected in the bottom of a pipeline riser which elevates the liquid to the level of the gravity separator system, allowing water to enter the separator vessel. A gas inlet valve is used to inject pressurized air, supplied from a compressor, allowing to pressurize the lab setup. A gas relief valve is used to ensure safe operation of the setup under pressurized operating conditions. The emulated gas phase is also injected in the bottom of the riser, forming a two phase flow entering the gravity separator system when water and gas flows are introduced to the setup. The produced water outlet flow leaving the gravity separator vessel is entering the hydro-cyclone separator using system of pipes. The hydro-cyclone's inlet flow is split into underflow and overflow direction. A valve is applied from each side of the hydro-cyclone system, allowing to manipulate the water phase level using an underflow valve, while the PDR is manipulated by an overflow valve. The two flows passing through the underflow and overflow valves are returned to the water storage tank in order to form a closed loop water circulation in the system. The independent components of the lab setup must be controlled in order to form suitable operating conditions for all experiments considered in the scope of this project. In order to speed up the process of lab setup preparation a PDR controller acting on the overflow valve is provided by "Aalborg University Esbjerg". All other controllers are implemented during the experimental process.

II. Material balance model of water phase level

Density is defined as mass per unit volume, allows to represent the total mass of water accumulated in the gravity separator system as the difference between flows entering and leaving the separator vessel. Under the assumption that the density of the water phase accumulated inside the separator vessel does not depend on position (perfect mixing assumption) and considering the fundamental law for conservation of masses defines the material balance equation (1) [4].

$$\rho \frac{dV_w(t)}{dt} = \rho_i F_{in}(t) - \rho F_{out}(t) \quad \textbf{(1)}$$

The rate of change in water phase volume is defined in terms of volumetric flow rates entering and leaving the gravity separator system. By assuming, that the density of the inlet and outlet water-phase streams is equal to the density of the accumulated water phase equation (1) is reduced to (2).

$$\frac{dV_w(t)}{dt} = F_{in}(t) - F_{out}(t) \quad \textbf{(2)}$$

The water phase volume in (3) is expressed as the product between the area of the circular segment at given water phase level and the length of the water phase container.

$$V_w\ (t) = A_w(h_w(t))L_1 = \left(R^2 \operatorname{acos}\left(\frac{R-h_w(t)}{h_w(t)}\right) - \left(R - h_w(t)\right)\sqrt{\left(2Rh_w(t) - h_w^2(t)\right)}\right)L_1 \quad \textbf{(3)}$$

In order to obtain material balance model expressed in terms of the water phase level an adjustment is required. The circular segment area is derivate with respect to the water phase level variable. Following the chain rule, a relationship between the first derivative of water phase volume and level is obtained in (4) see Ref. [AI].

$$\frac{dV_w(t)}{dt} = dA_w L_1 \frac{dh_w(t)}{dt} = \left(\frac{R}{\sqrt{1-\frac{\left(h_w^2(t)-R\right)^2}{h_w^2(t)}}} + \sqrt{2Rh_w(t) - h_w^2(t)} + \frac{(h_w(t)-R)(2h_w(t)-2R)}{\sqrt{4Rh_w(t)-2h_w^2(t)}}\right) L_1 \frac{dh_w(t)}{dt} \quad \textbf{(4)}$$

By rearranging (4) and substituting in (3) the general material balance equation is rewritten in term of the water phase level as follow.

$$\frac{dh_w(t)}{dt} = \frac{1}{dA_w L_1}\left(F_{in}(t) - F_{out}(t)\right) \tag{5}$$

Furthermore, it is decided that the inlet stream entering the separator vessel will be considered disturbed input and the outlet stream is defined as manipulated input. Knowing that the lab setup includes a controllable valve which can manipulate the water phase level inside the separator tank the outlet flow from equation (5) is substituted with the following valve equation in (6) [5].

$$\frac{dh_w(t)}{dt} = \frac{1}{dA_w L_1}\left(F_{in}(t) - \frac{1}{5.8}C_{vw} f(u_w) 500\sqrt{\frac{P_g(t)+\rho g h_w(t) - P_{ow}(t)}{SG}}\right) \tag{6}$$

The outlet stream leaving the separator vessel is influenced by the pressure difference between the total pressure of the water and gas phases inside the separator vessel, the outlet flow pressure, a manipulated input "*u*" representing the valve-opening fraction and a valve coefficient "C_{vw}". Where the "C_{vw}" term in equation (6) is combining the flow area of the valve orifice, the contraction coefficient and head loss coefficient in one. The two constants appearing in the used valve equation are scaling gains, handling the conversion between imperial and SI units with dimensions "*kg/h*" which later is converted to the appropriate dimensions of cubic meter per second

II.I. Empirical experiment for validating nonlinear gravity separator model

In order to validate the model an empirical data is required in agreement with the project specifications. Several operating conditions are taken into consideration. During the experimental period it is discovered that a simple "PI controller" must be applied to the inlet pump in order to ensure constant flow entering the separator vessel. Furthermore, it is discovered that the lab setup is sensitive to operating conditions of water and gas inlet streams entering the system simultaneously leading to the decision of running the first trial of experiments without injecting gas to the water phase inlet flow. The gas and oil relief valves are fully opened during the experiment in order to ensure that the gas phase pressure inside the separator tank is approximately constant and equal to the atmospheric pressure. The desired outcome of the experiment is to observe the transient response of the water phase level between two steady states.

A "PI control structure" is used obtain empirical value for an underflow valve opening which results in steady state for the water phase level. Steady water phase level is achieved with "58.4" percent valve opening. After value for the underflow valve opening position is obtained the validation experiment is conducted. The gravity separator tank is filled with water by keeping the underflow valve fully closed. As time evolves the underflow valve opening position is shifted to the obtained value, including sufficient amount of time to obtain steady state for the water phase level. As time evolves the valve opening fraction is increased with one percent with intention that new steady state must be reached after a period of time. The following figure illustrates the measured water phase level.

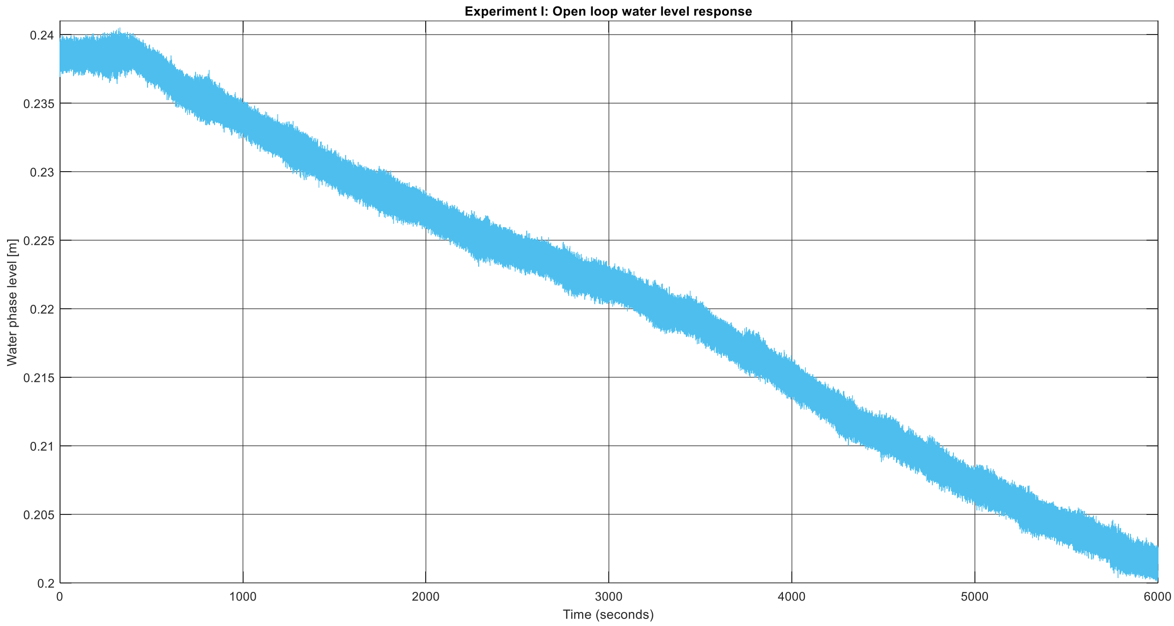


Fig. II.1 Measured water phase level

As expected in the begging of the experiment, a steady state for the water phase level is achieved. The increase in the underflow valve opening position is indicated when the water phase level leaves the steady state observed around four hundred seconds. The transient open loop response of the water phase level is observed in the time range from four hundred to three thousand seconds. The desired second steady state in the water phase level is not reached in the span of the experiment. Based on the specified operating conditions is expected that sufficient amount of time is required to observe second steady state in the water phase level. The measured data indicates that around three thousand five hundred seconds a second steady state is approached. However, measurement noise and unknown fluctuations are influencing the water phase leave response and causing further drain of the separator tank. Concluding that the observed water level dynamics are sufficient under the considered operating conditions and the empirical data will be used for validation the defined material balance model regardless that second steady state for the water phase level is not reached.

II.II. Nonlinear model simulation and validation

After preprocessing the collected empirical data, a set for validation purposes of the derived nonlinear material balance model "*Chapter II equation (6)"* is obtained. Filtered measurements of the inlet and outlet flows, separator tank and water phase outlet flow pressures are used as inputs to the defined water phase level model. The excitation for the modeled relief valve is ideal step change considering one percent increase in the underflow valve opening position. The following results are obtained and compared to the measured water phase leave in Fig. II.3.

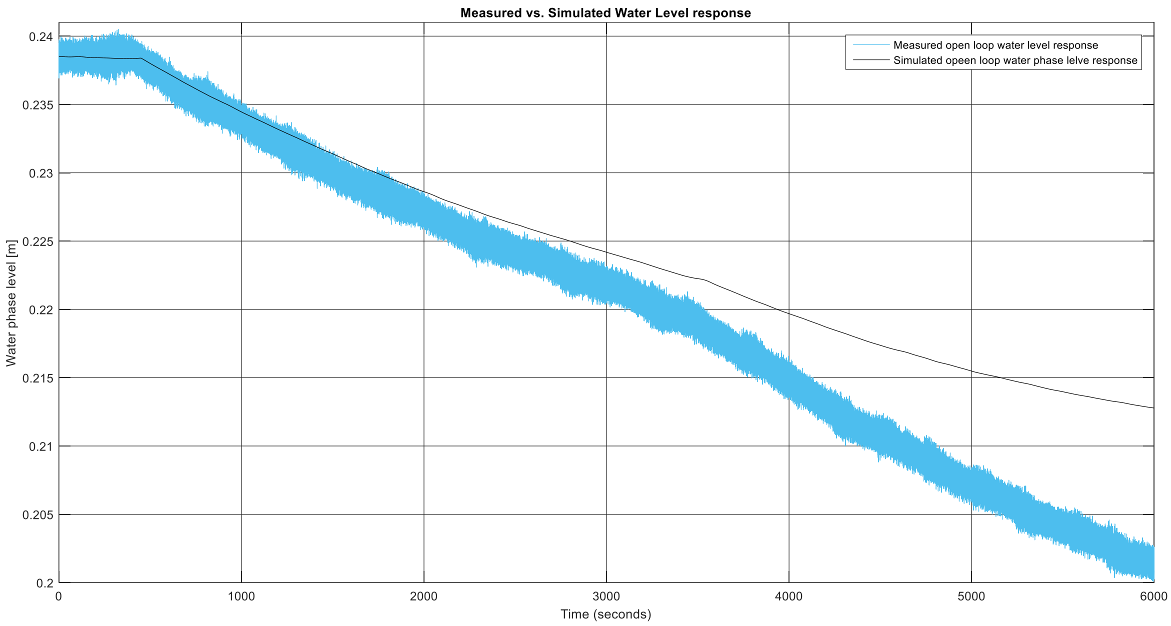


Fig. II.2 Nonlinear model validation for water phase level

The simulated water phase level marked with black curve on Fig. II.3 is following the measured water phase level in the beginning of the simulation. The model is maintaining the first steady state of the water phase level observed in the range between zero and four hundred seconds. At the time when the valve opening of the water relief valve is increased the simulated water phase level starts to decrees in the same manner as the validation data set. It is observed that the curvature of the simulated water phase level response and the obtained data tends to be similar indicating that the model is considering the cylindrical shape of the separator tank. However, several adjustments in the model constants must be taken in consideration in order to obtain the results from Fig. II.3. First the valve constant "C_{vw}" is estimated by rearranging the used valve equation as illustrated in (7) and taking the average value from the produced data set.

$$C_{vw} \approx \frac{1}{N}\sum_{t=1}^{N}\frac{5.8F_{out}(t)}{500\sqrt{\left(P_g(t)+P_w(t)-P_{ow}(t)\right)SG}} \qquad \textbf{(7)}$$

Furthermore, the length of the water storage container inside the gravity separator tank is reduced for the simulations, considering the assumed cylinder shape of the gravity separator tank and excluding the hemisphere geometry of the physical system. Concluding that under the made assumptions and simplifications the simulated level response is approximating the measured data and defined model is representing the modeled gravity separator system up to certain extend.

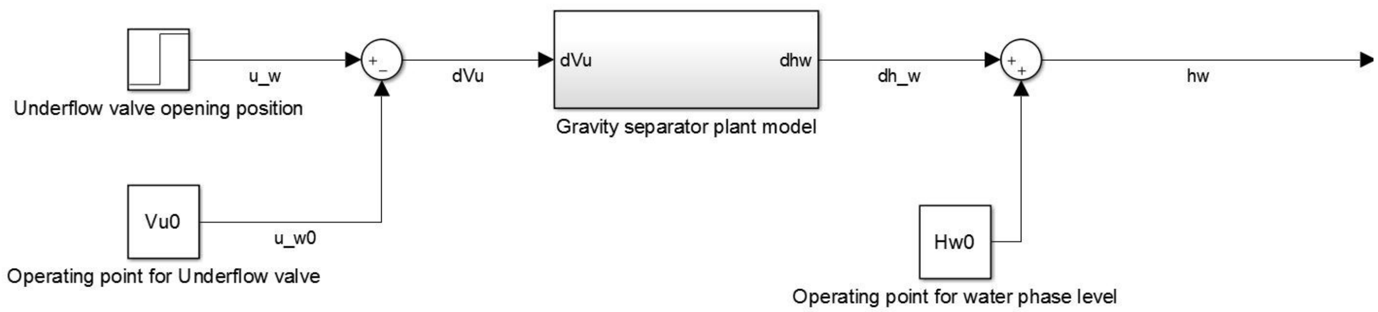
II.III. Linearization of the water phase level model

The defined and validated water phase level model is linearized using Taylor expansion [4]. The chosen operating point is based on calculated average values obtained from the collected empirical data used to validate the nonlinear model. The water phase inlet flow is assumed unknown input disturbance and is fixed as constant during the linearization. Furthermore, is assumed that the pressure inside the separator vessel is also constant, allowing to obtain simple linear model relating the underflow valve opening position to water phase level. The state space representation of the obtained LTI system is defined as follow.

$$\frac{dh_w}{dt} = ah_w(\mathrm{t}) + \mathrm{bu_w}(\mathrm{t}) \tag{8}$$

$$h_w = ch_w \tag{8.1}$$

The water phase level is defined as state variable. The underflow valve opening position is defined as input to the linearized model and the produced model output is defined as water phase level. After values for the three defined coefficients are obtained the state space model is implemented in Simulink environment obtaining the following block diagram.



Step deviation of one percent increase in the underflow valve opening position is considered in order to simulate the LTI system under the scenario of the validation experiment. The input is expressed in deviation terms by subtracting the constant operating point obtained after processing the data. The simulated output level response is summed with the water phase level operating point in order to reconstruct the full system output.

The simulated water level response is compared to the simulated nonlinear model response and the collected empirical data in the following figure.

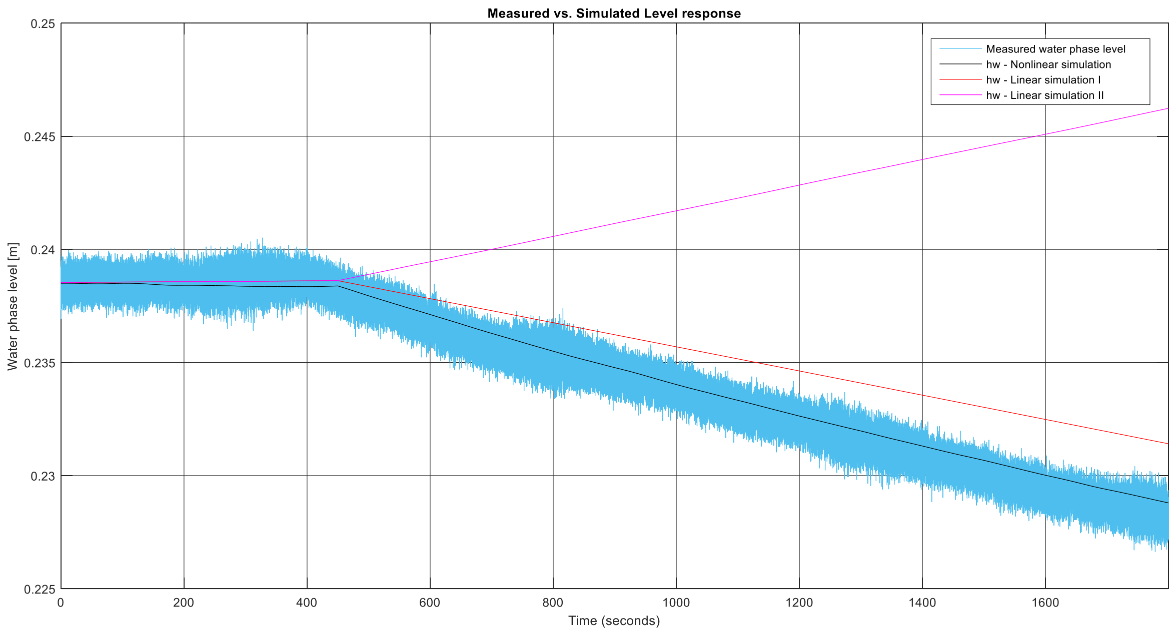


Fig. II.3 Linear model validation

As expected the linear simulation is even further form the original data as it can be seen form Fig. II.4. Two linear model implementations are considered. The curve marked with red color presents the level response under the form scenario of the validation experiment. The curve marked with magenta color presents a linear simulation where the underflow valve opening position is decreased with one percent. Concluding that that the defined LTI system responds for both increase and decrease in the underflow valve opening position. The simulated linear water phase level is becoming less accurate as time evolves and further drift from the defined operating point is observed. Considering that the Nonlinear model require slight modifications and the nonlinear water phase level response is close but not perfect match of the measured water phase level, is decided that the linearization around the given equilibrium point is successful. Leading to the conclusion that the derived in (8) linear model is suitable for control purposes, providing solution to the first half of gravity separator sub-problems.

III. Closed loop Gravity separator system

After the successful linearization of the obtained material balance model three different control structures are considered for the task of controlling the water phase level. As starting point, a “PI control structure” is developed and applied to the lab setup. Furthermore, in order to utilize the produce linear model an “Internal module control structure (IMC)” is obtained providing a model based control over the water phase level. Finally, based on optimal control theory a “Linear quadratic regulator (LQR)” is obtained and applied on the physical system.

III.I. Experiments design

Given the complexity of the lab setup and considering the variety of external and internal disturbances occurring in three phase gravity separator systems, is decided that each controller must be evaluated under set of experiments.

III.I.I. Experiment under atmospheric pressure

The first experiment is idealized scenario in which only one disturbance influencing the system is considered. Constant stream of water is injected into the separator vessel for the duration of the experiment. In order to achieve such flow a “PI controller” is applied to the available pump. Using measurement of the produced flow a feedback loop is obtained and used in order to specify the amount of water injected to the separator vessel. The gas phase and accumulation of pressure inside the separator vessel are neglected by un-pressurizing the separator vessel. This is achieved by keeping the oil and gas relief valves fully opened during the time of the experiment. Finally, in order to incorporate the hydro-cyclone stage into the experiment a “PI controller” for the pressure difference ratio (PDR) is applied to the system.

The experiment is divided into two phases. First an initialization phase is executed for the first two hundred and fifty-five seconds of the experiment. The initialization of the system is required in order to accumulate water inside the separator tank. After approximately twenty-five percent of the total volume of the separator are filled with water the second phase of the experiment is beginning. One thousand tree hundred seconds period of time is given to the system in order to settle down and reach a steady state based on the given reference point. After the system is settled down the same duration of time is used in between all following changes in the reference point for the water phase level. All operating conditions for the first experiment are presented in the following table.

Operating conditions	**Value**	**Units**
$F_{in}(t)$	0.2	L/s
$P_g(t)$	1	Bar
h_{wref}	[0.15, 0.16, 0.17, 0.19]	M

III.I.II. Experiments under seven bar pressure

The second experiment is expanding the first, by increasing the number of disturbances influencing the system during the experiment. By constantly injecting compressed air into the system, a constant gas phase pressure inside the separator vessel is emulated. In order to ensure such condition for the experiment an additional “PI controller” is applied to the available gas phase relief vale. Measurement of the internal pressure in the separator tank is used to form the feedback-loop providing constant gas phase during the experiment. In order to ensure nominal operating conditions for the hydro-cyclone stage and the applied “PDR control” the pressure in the separator tank is fixed to “seven bar”. The water inlet flow is kept constant again. However, the amount of water entering the separator is increased, in order to ensure the accumulation of water given the formed self-regulating pressure loop.

The experiment is divided into three stages. The first stage is initialization, in which the system is prepared for the experiment. The first seven hundred and nighty seconds are used to accumulate the required pressure and twenty-five percent water phase volume. Followed by one approximately twenty minutes settling down period in which a steady state for the water level must be reached. After the system is settled down the second stage of the experiment begins. The reference point for the water level is changed three times within twenty minutes’ interval. The experiment is finalized with a cool down stage. Where the inlet flow of gas and water are cut off, the gas relief valve is opened fully, the accumulated water is slowly led out of the separator tank, ensuring safe shutdown of the lab setup. All operating conditions for the experiment are presented in the following table.

Operating conditions	**Value**	**Units**
$\mathbf{F_{in}(t)}$	0.5	L/s
$\mathbf{P_g(t)}$	7	bar
$\mathbf{h_{wref}}$	[0.15, 0.16, 0.17, 0.19]	m

III.II. PI control of water phase level

The most common controller used in the oil and gas industry to control the water phase level is a "PI structure". As a starting point a "PI controller" is applied to the linearized model as illustrated in the following block diagram.

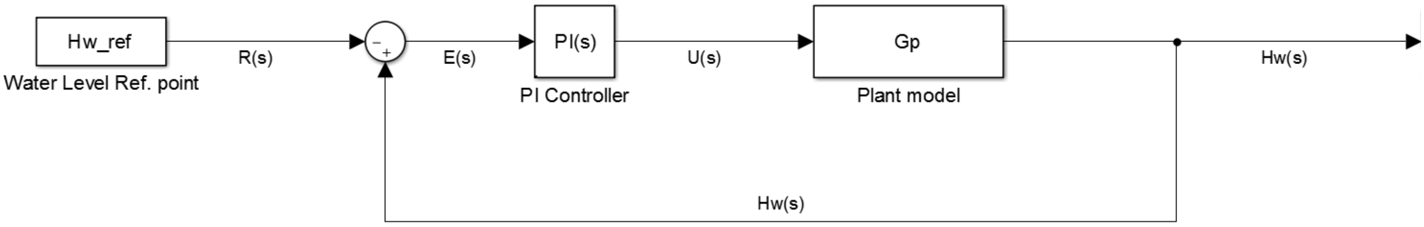


Fig. III.1 Closed loop system with PI control structure

As a standard "PI control structure" a unity feedback of the measured water phase level is used to form the closed loop system. In order to be consistent a transfer function describing the water phase relief valve is obtained from the already developed linear model by utilizing the MATLABS command "ss2tf(A, B, C, D)" see Ref.[AIV]. From observations and literature is noted that a level control problem is classified as revers action problem [6]. While the manipulated variable is increasing the process variable is decreasing. This requires correction in the definition of the error signal as in (1), in order to keep all signs positive.

$$E(s) = Hw(s) - R(s) \tag{1}$$

The developed "PI controller" is in the standard form as in (2). With two tuning parameters "K_p and K_i".

$$PI(s) = K_p + \frac{K_i}{s} \tag{2}$$

After evaluating the obtained close loop transfer function seeRef[AV] [7] is discovered that the system is of type 0 producing positive error constant. Concluding that a steady state error to a step change in the underflow valve opening position is defined as follow

$$e_{ss} = \frac{1}{1 + \mathrm{Kp}} \tag{3}$$

Finally, after the controller is defined and the steady state error is considered the following specifications are defined in order to obtain values for the tunable parameters of the desired "PI controller"

Rise Time	**35 seconds**
Settling time	200 seconds
Overshoot	25%

The specifications are chosen as such in order to ensure connections with the developed experiments. The key point of interest is to obtain a controller which will follow the reference point as good as possible. The process dynamics for the first experiment are relatively slow given the atmospheric pressure inside the separator tank allowing flexible specifications for the designed controller. Given that the integral part of the controller will accumulate error over time, is concluded that integral anti windup must be considered during implementation.

III.II.I. Closed loop system simulation and validation

From the defined specifications and tracking performance defined for the desired “PI controller”, a simulation and implementation of the closed loop system is performed. The simulations are realized in Simulink environment following the block diagram from Fig.III.1. Three different cases of a “PI controller” are considered. The first controller is tuned to be aggressive making the system respond faster on a step change of the reference point. The second controller is tuned to be robust making the system response slower but also removing all possible overshoot. The final version of the controller which is a combination of the first two extreme cases, providing a reasonable tradeoff between aggressive and robust “PI control structure”. The following simulation results are obtained in the following figure.

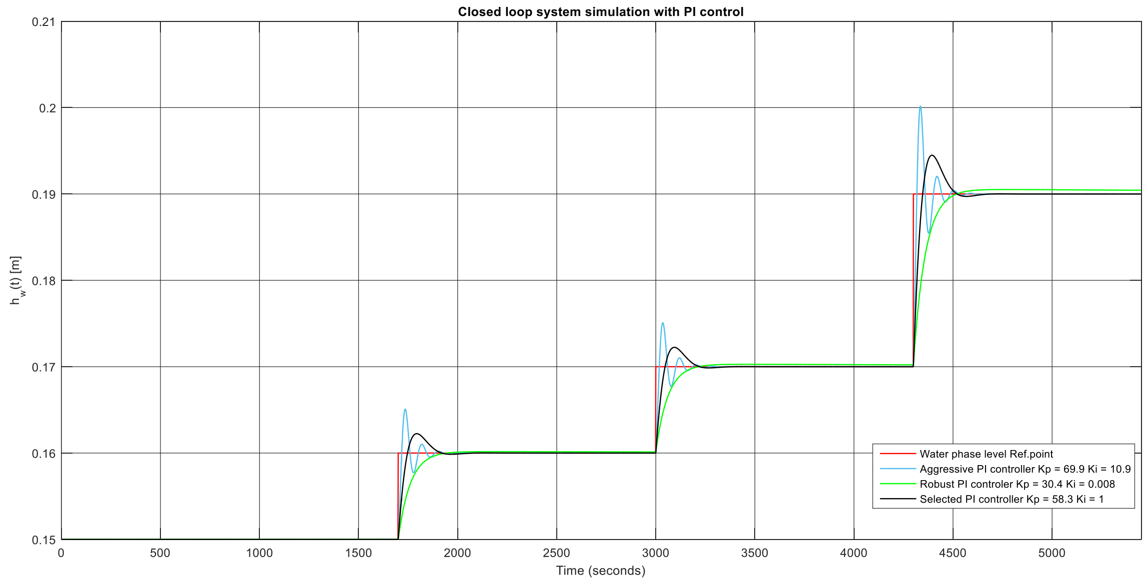


Fig. III.2 Closed loop system with PI control.

It can be seen that the "aggressive PI", marked with blue curve on Fig.III.2, is a fast controller satisfying two of the three specified requirements. The controller fails to pass the overshoot requirement from "Chapter VII Section II", considering the high magnitude of the integral part which leads to oscillations before settling down to the specified reference point value.

The "robust PI controller", indicated with green curve on Fig.III.2, is a total opposite of the aggressive one. The system response is slowed down. Considering that the integral part for this controller is almost negligible it is concluded that the robust controller is tending to the form of proportional control only. This controller satisfies all specifications but fails to track the reference point perfectly. In the time interval from three thousand five hundred seconds until the end the steady state error of the "robust PI controller" is clearly visible. After the range of values for the proportional and integral term is bounded, between the aggressive and robust "PI controller", a trial and error tuning during the simulation procedure is used to obtain values for the two tuning parameters. The selected "PI controller "is then implemented obtaining final values for the two tuning parameters as follow.

$$K_p = 58.370781413401 \tag{4}$$

$$K_i = 1.06656728684162 \tag{4.1}$$

I. PI level control under atmospheric pressure

The obtained and tuned "PI controller" is tested against the operating conditions for the first experiment from "Chapter III Section III.I". One of the main point is to validate the reference point tracking performance of the designed controller. The collected empirical data is then compared to the obtained simulation results in Fig.III.3.

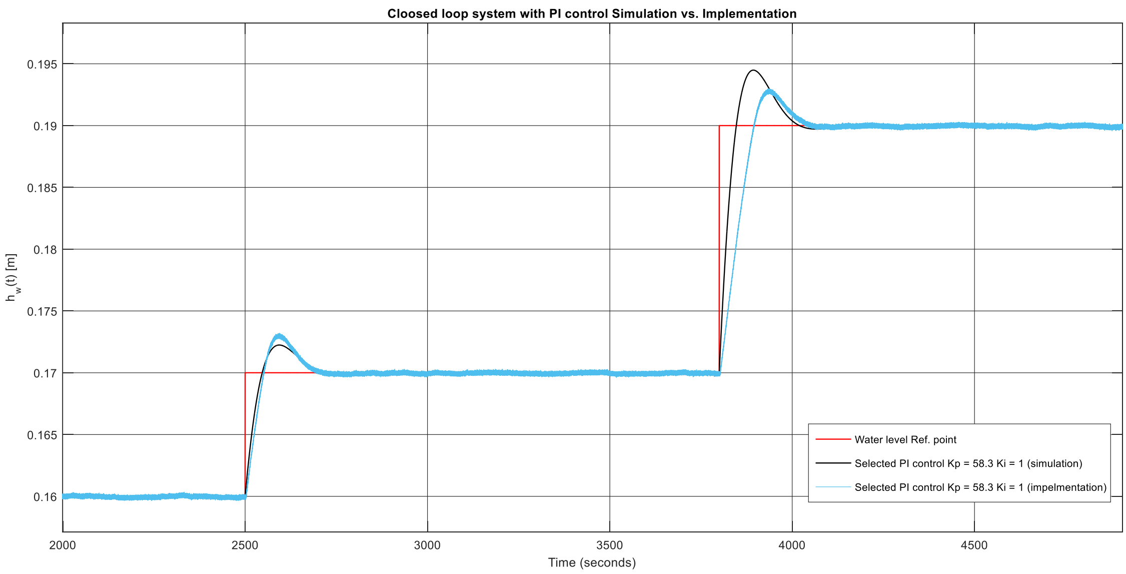


Fig. III.3 Closed loop Simulation vs. Implementation, one to two percent step increase.

. With blue curve on Fig.III.2 is illustrated the raw measurements of the water phase level after the controller implementation. Small overshoot is observed after a new set point is defined due to the integral term of the controller. The simulated closed loop level response is following the same reference point signal used in all experiments. Marked with black curve on Fig.III.3 is the simulated closed loop system response. It is observed that the results between implementation and simulation of the selected "PI controller" are almost identical. For one percent increase in the reference point, observed in the first step on Fig.III.3, the obtained measurements and simulation are almost overlapping. Considering that the rise, settling time and overshoot are almost identical it is reconfirmed that the obtained linear model is accurate within one percent deviation from given equilibrium point. However, a visible difference is observed in the case of two percent increase of the set point. It is observed that for relevantly small changes in the set point the shaped closed loop system is performing grate. However, as the increase in the step from given steady state value is growing, the implementation and simulation start to differ due to the lack of process delay implementation in the designed model. Concluding that the obtained "PI controller" is satisfying the defined specifications achieving good reference tracking performance under the operating conditions of the first experiment.

Valve saturation and influence of PI level control on the Outlet flow

The range from closed to fully opened position of the water relief valve is mapped between the values of zero and one. Considering that the water phase level must remain in steady state for a given reference point it is expected that all undesired deviations and fluctuations will be transfer to the outlet flow of the system. Fig.III.5 illustrates the water phase outlet flow in comparison with the valve opening of the underflow valve.

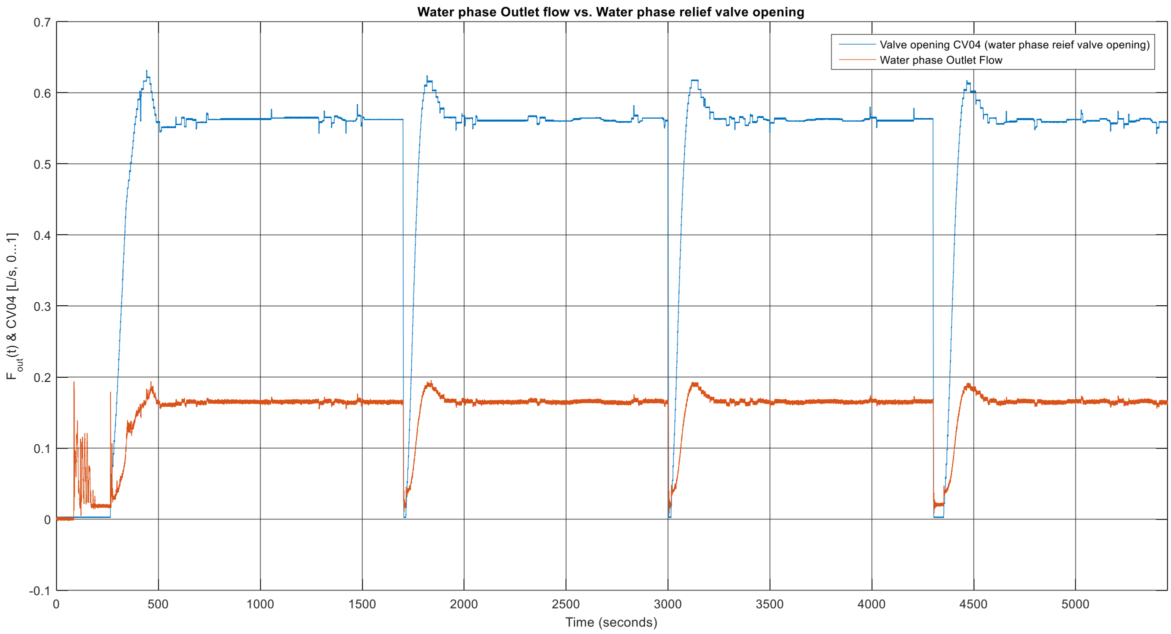


Fig. III.4 Underflow valve opening position vs. Water phase Outlet flow.

As expected when the reference point is tracked accurately the water relief valve is keeping almost constant opening. It must be noted that the observed small deviations in the measured valve opening are micro adjustments of the valve opening in the range of "0.1" percent. After one thousand and seven hundred seconds the initialization phase of the experiment is completed. The following three shift of the reference point for the water phase level are causing the disruptions in the outlet flow and valve opening. It is concluded that with the selected "PI controller" the water phase relief valve is over actuated. For one and two percent increase in the reference point value, the valve is completely closed for short instants of time, followed by quick opening to the point where new steady state for the water phase level is achieved. As a consequence of such action the water phase outlet flow is completely blocked while the transition between two set points is occurring, providing less flow on average leaving the separator vessel.

II. PI level control under 7 bar pressure

After the satisfying results from the first experiment the "PI controller" is tested against the operating conditions of the second experiment. Providing a constant air injection to the system a constant gas phase pressure inside the separator vessel is achieved. In order to satisfy the operating conditions for the hydro cyclone separator the pressure for the second experiment is fixed at seven bar. In order to keep consistency in the experiments the same reference signals used for the first experiment are used for the second one as well. The obtained results regarding the water phase level are presented in Fig.III.6.

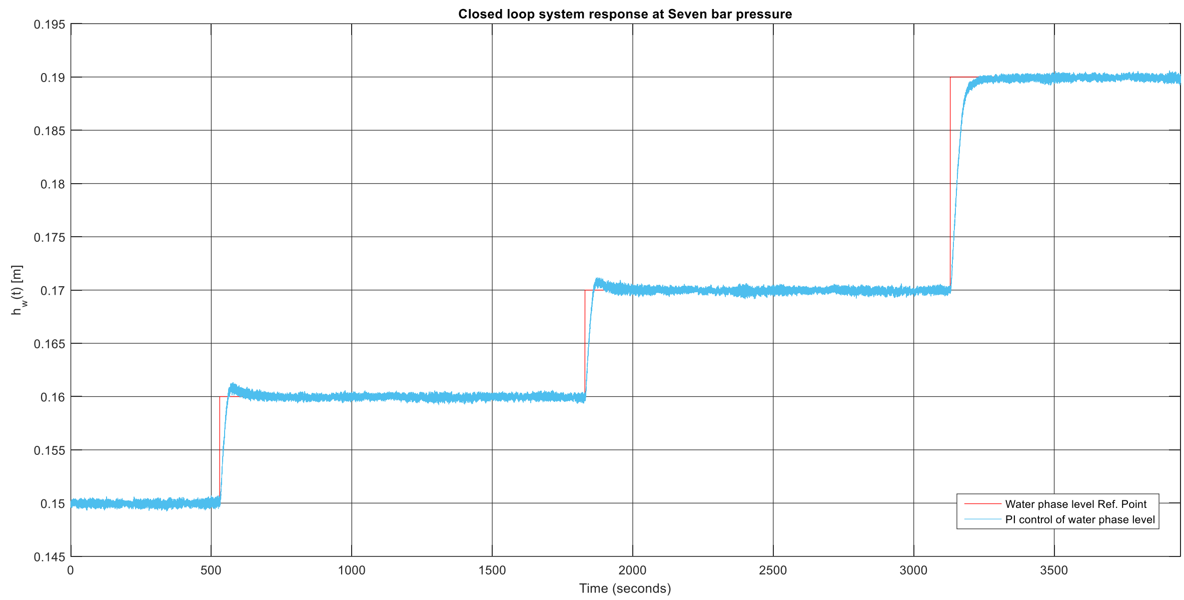


Fig. III.5 Closed loop system response at Seven bar pressure

The designed "PI controller" is following the reference point with minimum error under the operating conditions of the second experiment. The observed overshoot is reduced, considering the gas pressure exerted on the water phase leave and the expected robust response obtained by "PI controllers". With further increase in the reference point step the overshoot is eliminated providing even better results. Concluding that under the operating conditions of the second experiment the defined specifications for the selected “PI controller” are met. Good reference tracking performance is achieved, with stable closed loop level response, under the operating conditions of the second experiment.

Valve saturation and influence of PI level control on the Outlet flow

The second experiment is bringing the system one step closer to real operating conditions of a three phase gravity separator system. The tradeoff between perfect reference tracking and the outlet flow of the separator are presented in Fig.III.8.

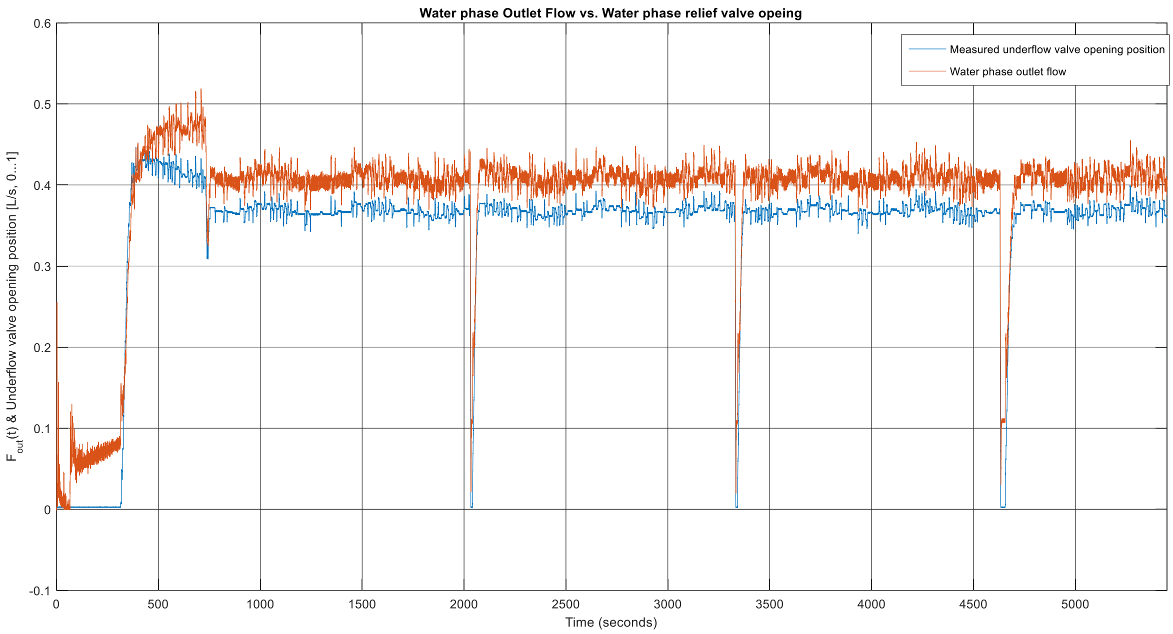


Fig. III.6 Water phase outlet flow vs. Water phase relief valve opening.

The water phase relief valve is again over actuated. The same results obtained from the first experiment are observed in the second experiment also. For one or two percent increase in the reference point the controller is fully closing the valve followed by fast opening in order to compensate and stabilize the water phase level at given steady state value. Concluding that the applied tuning procedure for the “PI controller” provides an aggressive response regarding the water phase leave which can be relaxed in order to reduce the control action exerted on the underflow and ensure robust response in situation of varying water phase inlet flow and gas phase pressure. Indicating that tuning parameters of the “PI controller” must be carefully selected in order to account for all disturbances and provide adequate control over the considered actuator.

III.III Internal module control of water phase level

Different approach to obtain control over the water phase level is to apply model based control to shape the closed loop system. Using the obtained linear model to relate controller settings and calculated model parameters an “Internal model control structure (IMC)” is defined and tested. Block diagram for the closed loop system is presented in Fig.III.9.

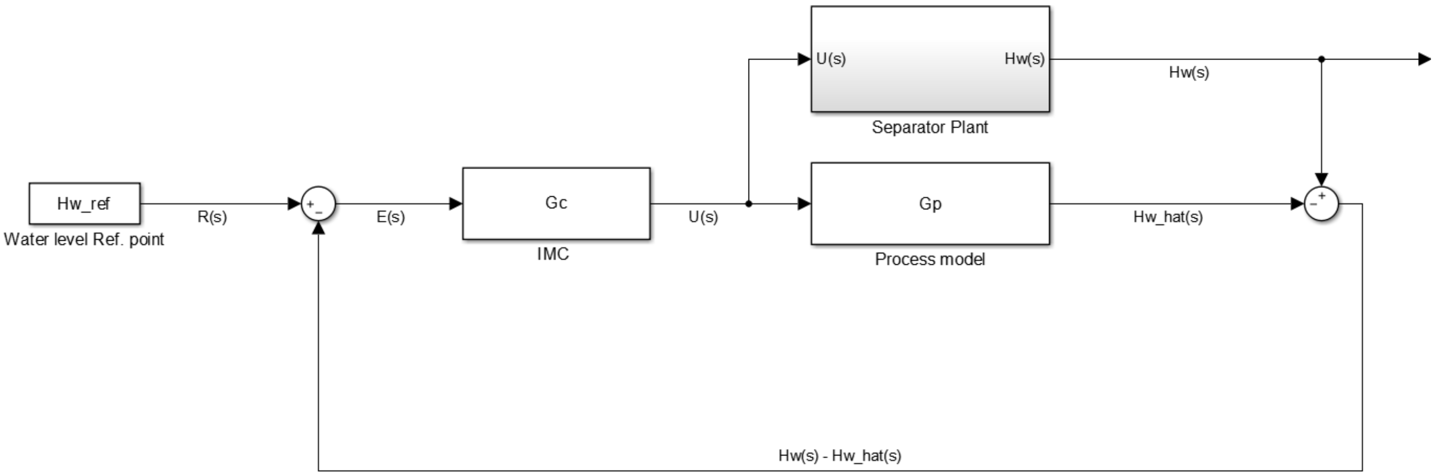


Fig. III.7 Closed loop system with IMC control structure.

The following “IMC design procedure” along with set of conditions is defined in order to obtain the “IMC control structure” [8]

1. Check stability of the process.
    a. Process must be stable
2. Check stability of process model
    a. Process model must be stable
3. Check for right hand plane (RHP) zeros
    a. Process model must not contain RHP zeros
4. Consider process delay
    a. Process model must not include process delay
5. Derive perfect controller
6. Derive implementable controller
    a. Controller must be proper function
7. Check closed loop system stability
    a. Closed loop system must be stable

The water phase level is going to deviate from given set point only under condition that inlet flow is entering the separator vessel, leading to a conclusion that the physical process is stable. The obtained process model “G_p” see Ref[AVI] contains two model parameters, considering that process is modeled by first order transfer function. It is found that the pole of the system is in the left hand side of the complex plane (LHP) ensuring that the process model is also stable. Furthermore, the process model does not contain any zeros in the numerator satisfying the condition regarding RHP zeros and ensuring that the defined "IMC controller" seeRef[AVI] is also stable. The process delay is considered unknown and is excluded from the IMC designed.

After the first three conditions are satisfied the controller transfer function is obtained as the invers of the process model see Ref[AVI]. The derived perfect controller results in improper transfer function, used as a starting point for the implementation of the "IMC controller". An additional transfer function is obtained in order to modify the obtained "IMC controller". Additional tunable parameter for the controller is introduced with the modification. The filter time constant is used to directly tune the speed of the closed loop system response. By multiplying the perfect controller with an "IMC filter", a proper controller transfer function "G_c" is obtained see Ref[AVI].

After the process model and "IMC controller" transfer functions are selected the closed loop system from Fig.III.9 is simulated and implemented. Given that the process, process model and controller are stable it is concluded that the closed loop transfer function see Ref[AVI] is also stable satisfying the last condition of the "IMC design procedure". The feedback signal is constructed as the difference between measured water phase level and process model output. The defined "IMC controller" is subjected to the same specifications and reference point tracking conditions as the "PI controller".

Closed loop simulation and validation

The "IMC control structure" is defined and implemented on the physical lab setup. The same experiments and associated operating conditions are used as basis for validating the model based control structure. In order to compare the different control strategies all specifications and conditions obtained in "Chapter VII Section II" are used for the closed loop system with "IMC control".

IMC level control under atmospheric pressure

The obtained "IMC control structure" is tested against the operating conditions of the first experiment from "Chapter VII Section I". The main goal is to validate the reference tracking performance of the second controller. The simulated and measured closed loop system response is presented in the following figure.

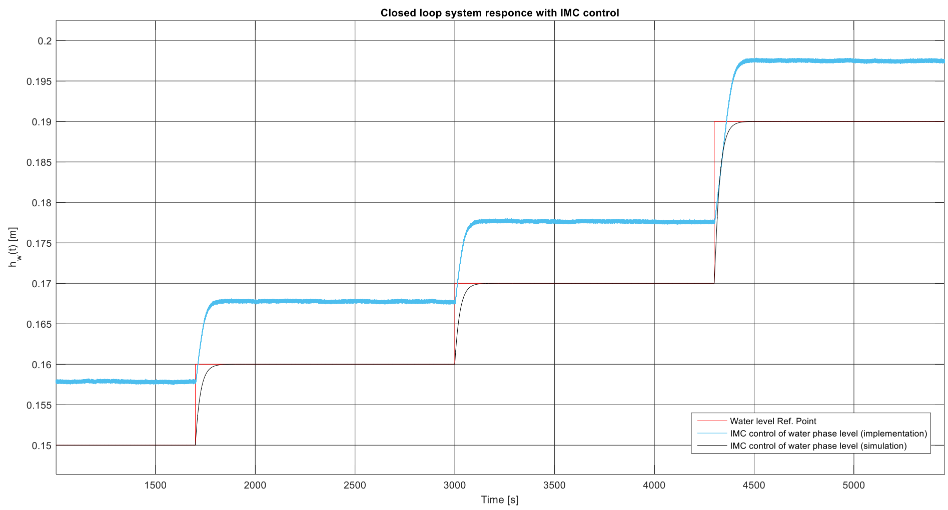


Fig. III.8 Closed loop system response with IMC control.

The simulated closed loop response is perfectly tracking the given reference point, considering that the process model is simplified and the inlet flow entering the system is treated as unknown input disturbance. It must be noted that for one and two percent increase in the reference point the simulated and measured closed loop responses have similar characteristics as observed during the development of the "PI controller". Furthermore, after the implementation of the "IMC controller" it is observed that the closed loop system response has a steady state error. Leading to conclusions that the obtained "IMC control structure" is within the boundaries of the defined specifications, but fails to meet the reference tracking requirement. Indicating that the made model assumptions and simplifications are directly influent the IMC performance and sensitivity to disturbances. Impaling that an addition to the "IMC control structure" must be implemented in order to remove the observed steady state error and meet all specifications and requirement defined in "Chapter VII Section II".

IMC with PI error correction for level control

A modification to the existing "IMC control structure" is made. "PI controller" is cascaded with the "IMC controller" by introducing second feedback loop as in Fig.III.11.

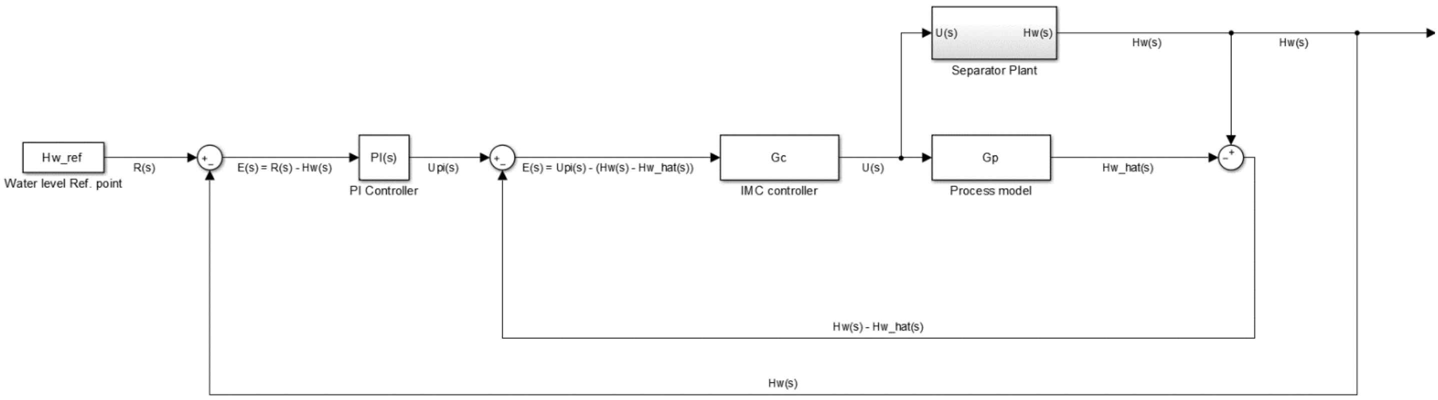


Fig. III.9 IMC/PI control structure

The two feedback loops from Fig.III.11 are used to remove the observed steady state error, produced by the "IMC controller". An outer feedback loop is constructed by using measurement of the water phase level and applying a "PI controller", providing a fine correction of the steady state error regarding the water phase level. Proceeding further an inner feedback loop for the "IMC control structure" is defined as before. The two feedback loops are interconnected by using the "PI controller" output signal "$U_{pi}(s)$" as the new error signal to the "IMC controller".

The designed "PI controller" is tuned first by simulation, to obtain range of values for the two tunable gains. Followed by trial and error tuning method, for the practical implementation of the new "IMC/PI control structure". Three cases of "PI controllers" are applied to the developed "IMC structure, followed by simulation of the closed loop system. The results are presented in the following figure.

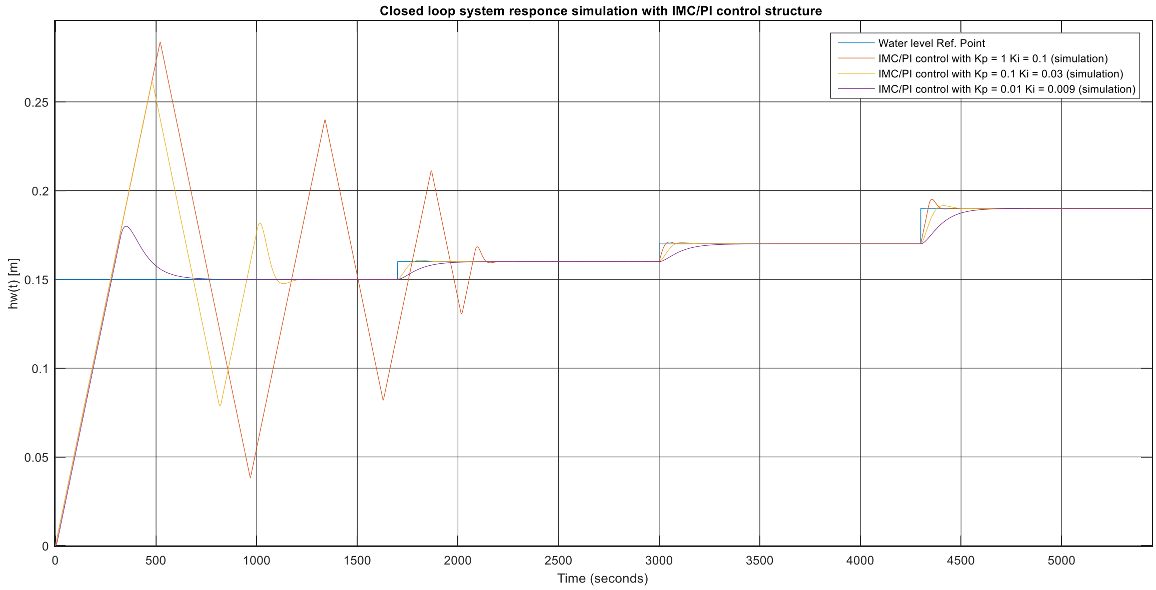


Fig. III.10 Closed loop system simulation with IMC/PI control structure.

As first choice the “IMC control structure” is modified by an “I controller”, marked with the red curve on Fig.III.12. It is observed that by introducing only an integral term the system response is overshooting for more than two thousand seconds failing to reach the first set point for the simulation. The second simulation is executed with the addition of proportional term in order to obtain “PI controller” for the outer feedback loop. The obtained closed loop system response is marked with yellow curve on Fig.III.12. The oscillatory overshooting is reduced by using a “PI controller” indicating that influence of the outer loop must be compensated by small proportional term. Based on the results of the first and second simulation a third “IMC/PI controller” is simulated. The closed loop system response marked with violet curve is considered the best choice as a starting point for the implementation phase. The overshoot is reduced in tradeoff of sluggish closed loop response, with consideration that the closed loop gain is less than the open loop system’s gain [8]. Several iterations after the implementation are required to find final values for the two tunable gains of the considered “PI controller. The values of the proportional and integral term are presented in (7).

$$K_p = 0.01 \tag{7}$$

$$K_i = 0.006 \tag{7.1}$$

It is concluded that the new control structure must satisfy all mentioned specifications and requirements in order to obtain control over the water phase level. Considering that the “IMC/PI control structure” is developed as consequence of the first experiment the modified closed loop system is directly tested against the operating conditions of the second experiment from “Chapter III Section III.I”.

IMC/PI level control under 7 bar pressure

Proceeding further the new “IMC/PI control structure” is tested against the operating conditions of the second experiment from “Chapter III Section III.I”. A comparison between the obtained data and simulated closed loop system response is presented in the following figure.

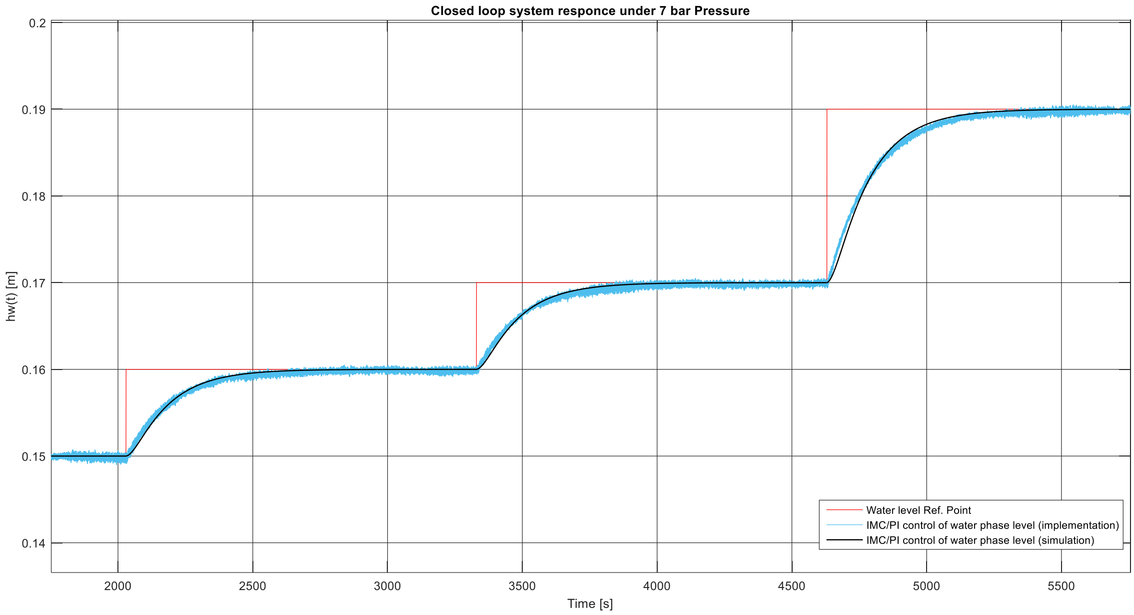


Fig. III.11 Closed loop system response with IMC/PI control structure.

The results are promising. The observed steady state error is corrected. The “IMC/PI control structure” is following the given reference point with minimal error. Furthermore, it is noted that for both one and two percent increase in the reference point the simulated and measured system response are overlapping, indicating that the defined closed loop system with “IMC/PI controller” is achieving good accuracy during simulation in trade off of slow water level response. Observing that an IMC control approach can be obtained based on simplified model in trade off of introducing more tunable parameters to the closed loop system in order to compensate for mismatch between the process and the process model. Furthermore, is observed that the closed loop response with an “IMC/PI controller” is dampened in contrast with the general “IMC control structure” indicating that a tradeoff between the tuning of the two control structures must be considered in order reach the range of controller specifications. Concluding that good results are obtained at higher implementation cost, by assuming that the water inlet flow and gas phase pressure as unknown disturbances acting on the water phase level.

Valve saturation and influence of IMC/PI level control on the Outlet flow

The derived "IMC/PI control structure" is capable of maintaining the water phase level at given reference point, considering some tradeoffs. The influence of the applied controller over the outlet flow leaving the separator vessel, subjected to the signal driving the underflow valve are presented in Fig.III.15.

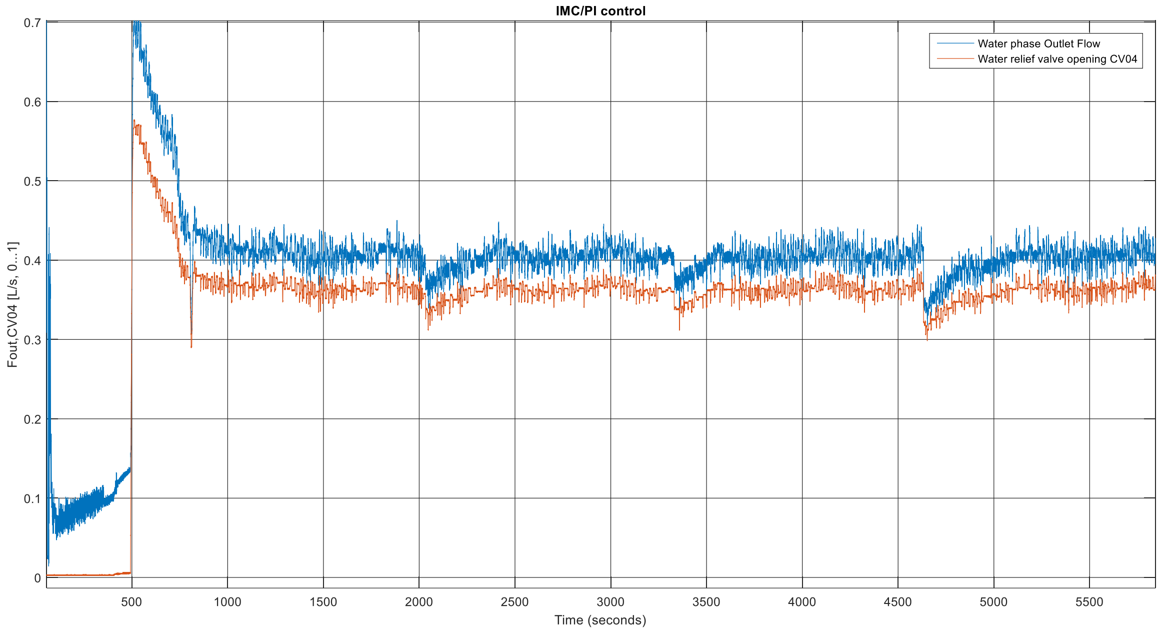


Fig. III.12 Water phase relief valve opening vs. Outlet flow.

The water phase outlet flow from Fig.III.15 is constant on average, considering the operating conditions of the second experiment. The applied control strategy successfully drives the underflow valve with significantly less control effort. The valve is not saturated allowing reference tracking of the water phase level without great intact on the outlet flow leaving the separator vessel. The choice of this particular values for the three tunable parameters of the "IMC/PI control structure", results in slower water phase level response, which directly influences the amount of valve opening required to follow given reference point. The controller is forcing the underflow valve to readjust the valve opening within a range of "2 to 5 percent", manifested as small drop deviations in both valve opening and water phase outlet flow from Fig.III.15. Indicating that the added PI controller is dominant in the considered case directly influencing the obtained closed loop system response. Concluding that a tradeoff between the tuning of the two control structures must be considered in order to modify the closed loop system response with "IMC/PI control structure".

III.IV Linear quadratic regulator (LQR) control of water phase level

Third approach considered to obtain control over the water phase level is to use optimal control technique. By finding an optimal control gain "K_∞" which minimizes the defined in (8) cost function, a "Linear Quadratic Regulator (LQR)" solution to the infinite time problem is obtained [10, 11].

$$J(h_w, u_w) = \int_0^\infty (h_w^T Q h_w + u_w^T R u_w) dt \tag{8}$$

The "Q" parameter is a scalar value considering that the only state of the system is the water phase level. The underflow valve opening position is the defined system input defining a scalar value for the "R" matrix as part of the LQR solution. The values for "Q and R" in (8) are chosen base on "Bryan's rule" see Ref[AVIII] [12].

Both parameters in the cost function are chosen arbitrarily, implying that the chosen design procedure is at best partially optimal, in tradeoff for achieving compromise between control effort and system response speed guaranteeing the stability of the system.

According to the theory [10] the control gain "K" is a time-varying optimal control solution where a portion of the solution produces a constant gain "K_∞". Two methods for computing the desired constant control gain can be considered. The first method is based on solving the "Riccati equation" backwards in time until a steady solution is reached. Where a drawback for this method is the substantial computational power required to obtain the desired solution. The second approach is less demanding and is utilizing the "algebraic Riccatie equation (ARE)" see Ref [AVIII].

After the cost function and values for both weighting matrices are defined a solution for the optimal control gain "K_∞" based on the derived linear model is obtained using Matlab software. The following constrains are satisfied and compared against the returned function output in the table below.

Notation	**Constrain**	**Numerical Value**
$\boldsymbol{Q}$	State weighting matrix must be positive definite $Q > 0$	11.11
$\boldsymbol{R}$	Input weighting matrix must be positive semi definite $R \geq 0$	0.0014
$\boldsymbol{P_\infty}$	Solution to "algebraic Riccati equation" must be positive definite $P_\infty > 0$	225.5855
$\boldsymbol{J(h_w,u_w)}$	The cost function must be positive $J(u_w) > 0$	-

The first three constrains are satisfied. The cost function constrain is not checked numerically. However, the constrain is satisfied implicitly from the first two constrains. The value for the optimal control gain is presented in (9)

$$K_\infty = -90.0225 \tag{9}$$

The final step in order to design practically implementable LQR controller is to introduce reference tracking input with full state feedback. The new control input is defined as the difference between the steady state value of the manipulated input and the steady state error scaled by the obtained state feedback gain "K_{∞}" see Ref[AIII] [13]. The value for reference tracking input is presented in (10)

$$N_r = -90 \tag{10}$$

After a reference input gain is calculated the final block diagram for the closed loop system with LQR control is designed as follow.

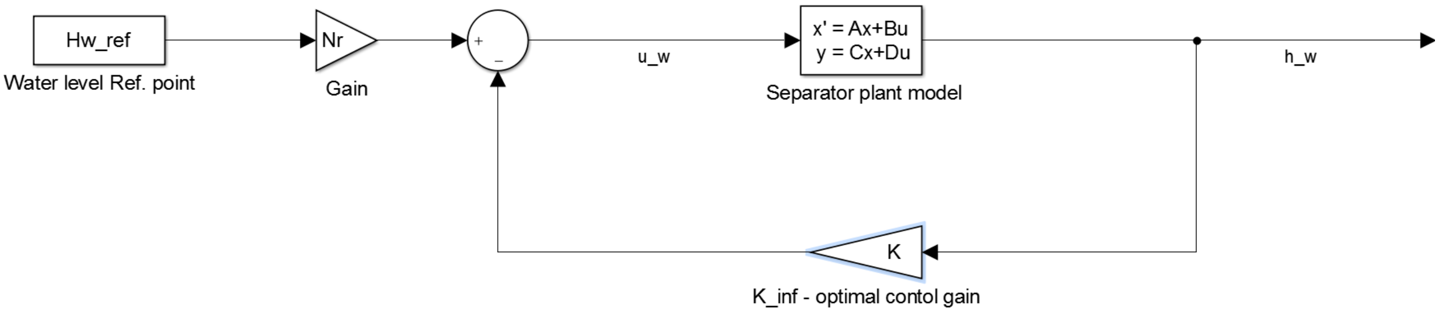


Fig. III.13 LQR control structure.

The closed loop model "*LQRv1.slx*" is described by the block diagram from Fig.III.17. The reference point for the water phase level is scaled by the obtained reference input gain "N_r". The full state feedback gain serves the purpose to bring the system state to zero leading steady state for the water phase level.

Closed loop system simulation and validation

After the derivation and simulation of the obtained LQR control structure, a practical implementation on the physical system is achieved. The new controller is tested against the same set of experiments from "Chapter III Section III.I".

LQR level control under atmospheric pressure

As a starting point the LQR controller is implemented under the defined operating conditions of the first experiment considered. The main goal is to validate the reference tracking performance of the controller, based on simulated and measured closed loop system response presented in Fig.III.18.

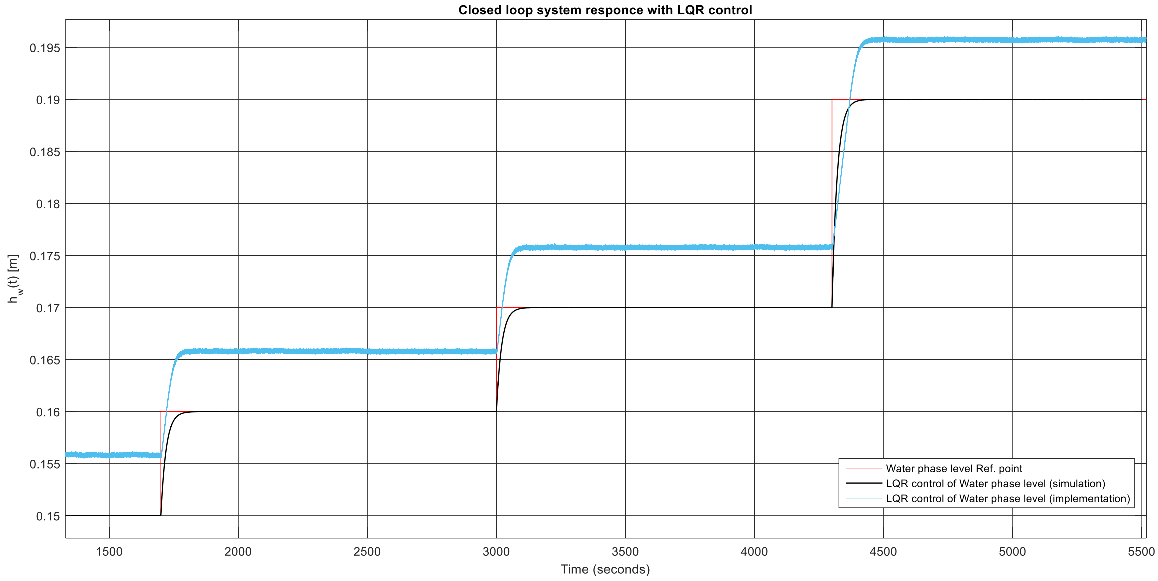


Fig. III.14 Measured vs. Simulated closed loop system response

Three step changes in the reference point are considered. The red line on Fig.III.18 is indicating the water phase level reference point signal with one and two percent increase from given steady state. The simulated closed loop level response, illustrated with black curve, is agreeing with the theory. The simulated LQR is providing perfect control, with consideration that the model does not include any disturbance acting on the system during the simulation. The measured water level response obtained by following the same reference input used in the simulation is indicated with blue curve. The transient responses of the simulated and implemented LQR are almost identical. However, the implemented version of the LQR is producing a steady state error causing an offset from the actual reference, failing to match the required reference tracking performance. Indicating that the obtained theoretical "LQR solution" guarantee that the cost function is minimized, but there is no evidence that the minimization is resulting at zero. Concluding that a modification to the theoretical solution is required to eliminate the observed steady state error.

LQR with Integral control over the water phase level

The choice of "N_r" required to introduce reference tracking input is resulting in zero steady state error given ideal operating conditions. However, in the case of disturbances influencing the system any change in the model parameters causes nonzero error as observed in the LQR implementation. Moreover, considering that the control signal applied to the water relief valve is bigger than zero at any positive reference point regarding the water phase level, it is decided that suitable correction for the LQR control is the application of LQR with integral control [14].

Introducing an integration of the system error and applying extra "integral state" seeRef[AVIII] [14] to the derived linear model, provides the following closed loop system with integral control of the water phase level.

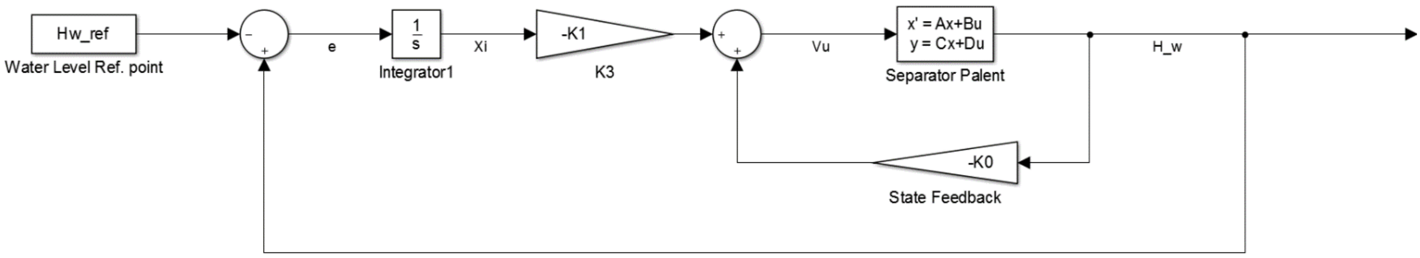


Fig. III.15 Closed loop system with integral control.

The two control gains required to solve for the integral control from Fig.III.19 are calculated based on pole placement procedure SeeRef[AVIII], with pole values defined in (11). The system error is defined as the difference between system output and reference point input. The error is integrated and scaled through the obtained control gain before being summed with the calculated control law.

$$p_{h_w} = -0.01 \tag{11}$$

$$p_i = -0.5 \tag{11.1}$$

Three factors are considered in the choices of vales in (11). It is considered that the three phase gravity separator system operates in low frequencies range, indicating slow output response. The choice of value for the system state pole is considering the pole location of the derived linear model. The root of the transfer function is located near the origin and initially chosen for the pole placement procedure, considering atmospheric pressure in the tank. In case of gas phase pressure disturbance acting on the water phase it is expected that the system dynamic will be speeded up requiring a modification of the original system pole which leads to (11). Finally based on the results obtained through experiments with “PI and IMC/PI level control structures”, is decided that the integration state pole location must be in the range of $-1 \leq p_i < 0$ in order to ensure stable system pole and escape saturation of the underflow valve. The midpoint from the defined range is used as integrator state pole value in (11.1).

After values for the two poles are selected two methods for obtaining values for the two control gains from Fig.III.19 are used and compared. Manual calculation following the described integral control definition [14] is compared to the produced output of predefined software function. The “lqi()” function allows to use the already obtained LQR problem formulation to compute state feedback and integration control gains in agreement with the constructed model. The results from both calculations are presented in the following table.

	Manual calculation	LQI() Function
K_0	-106.3635	-106.7381
K_1	0.887	0.9

Both methods are obtaining similar solution for the required control gains considering the chosen values for the weighting matrix “Q”. It must be noted that with different scaling of the weighting matrices the used lqi() function will produce different results which may lead to better solution. Concluding that the manual calculation is used to understand the procedure of defining integral control and the software calculated gain values will be used for the implementation.

LQRI control under 7 bar gas phase pressure

The derived integral control is simulated and tested against the operating conditions of the second experiment as continuation of the LQR tests. The measure and simulated water phase level response is compared in the following figure.

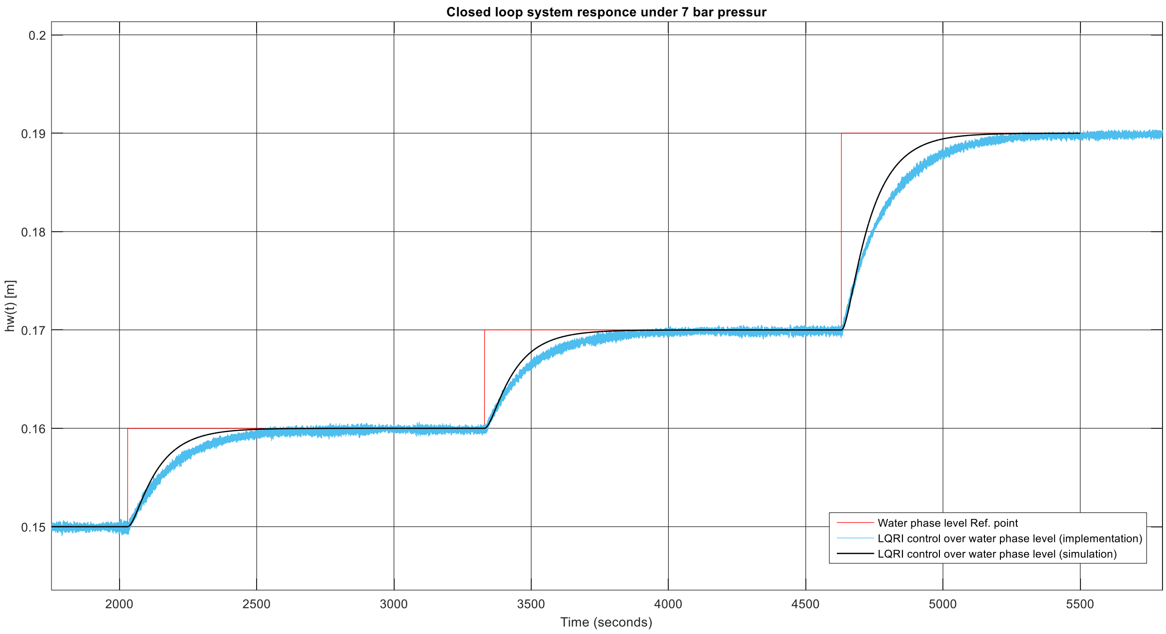


Fig. III.16 Closed loop system response with LQRI control.

The observed steady state error is removed. The measured and simulated closed loop system responses with integral control are almost identical. A difference in the rise time between simulation and implementation is negligible. After the integral error correction is introduced the closed loop system response is dampened, causing increase in the settling time. Observing that the obtained “LQRI controller” provides robust but slow closed loop level response. Indicating that retuning of the considered weighting matrices must be performed in order to ensure faster closed loop system response. Concluding that the for small implementation cost the theoretical “LQR solution” is modified to practical “LQRI solution”, allowing to utilize the optimal control theory to control the water phase level of a gravity separator system.

Valve saturation and influence of LQRI level control on the Outlet flow

The final step in validating that the derived LQRI control structure is to evaluate the control effort exerted on the water relief valve and the influence over the outlet flow leaving the separator vessel. Fig.III.21 is comparing the measured water relief valve opening position and water phase outlet flow.

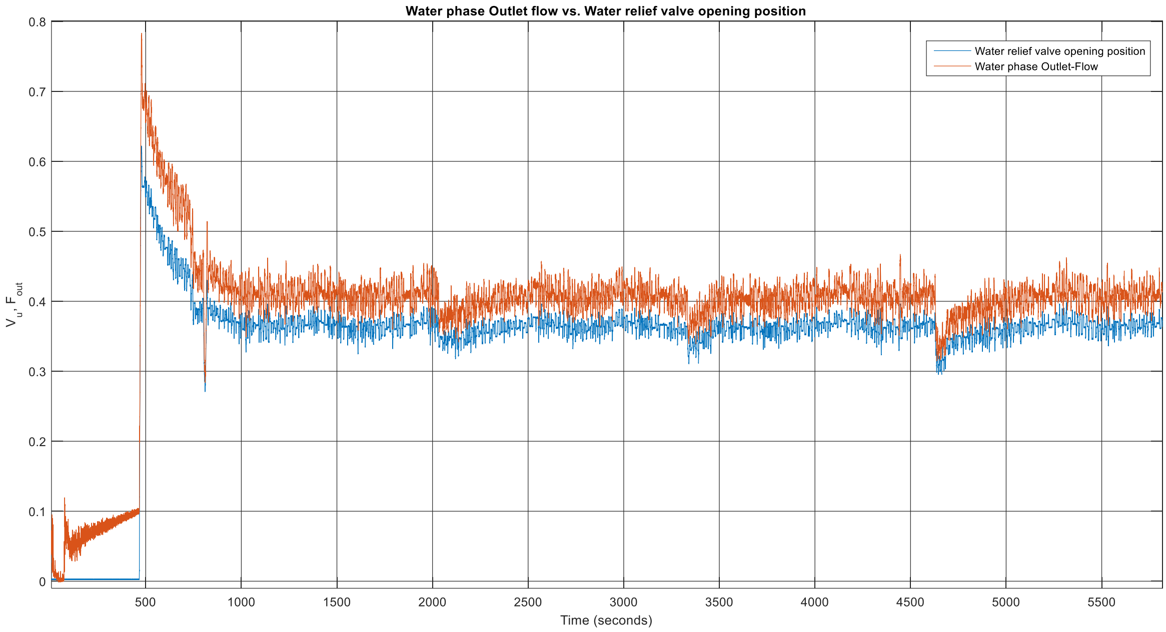


Fig. III.17 Water phase relief valve opening vs. Outlet flow.

The obtained results are similar to the results obtained during the IMC/PI implementation. The water phase outlet flow is constant on average. The control effort required to compensate for step transition in the reference point regarding the water phase level is minimized. For small deviations from given steady state value the underflow valve opening position is readjusted in small range, exerting less influence on the outlet flow leaving the separator vessel. Concluding that the obtained LQRI controller provides suitable control performance under the specified operating conditions.

III.V Comparison between PI, IMCPI, LQR control of water phase level

After the three “Level control structures” are implanted and tested a comparison is made using the data collected from the closed loop experiments with seven bar pressure. The closed loop water level responses to one percent increase in the water level reference point are presented in the following figure.

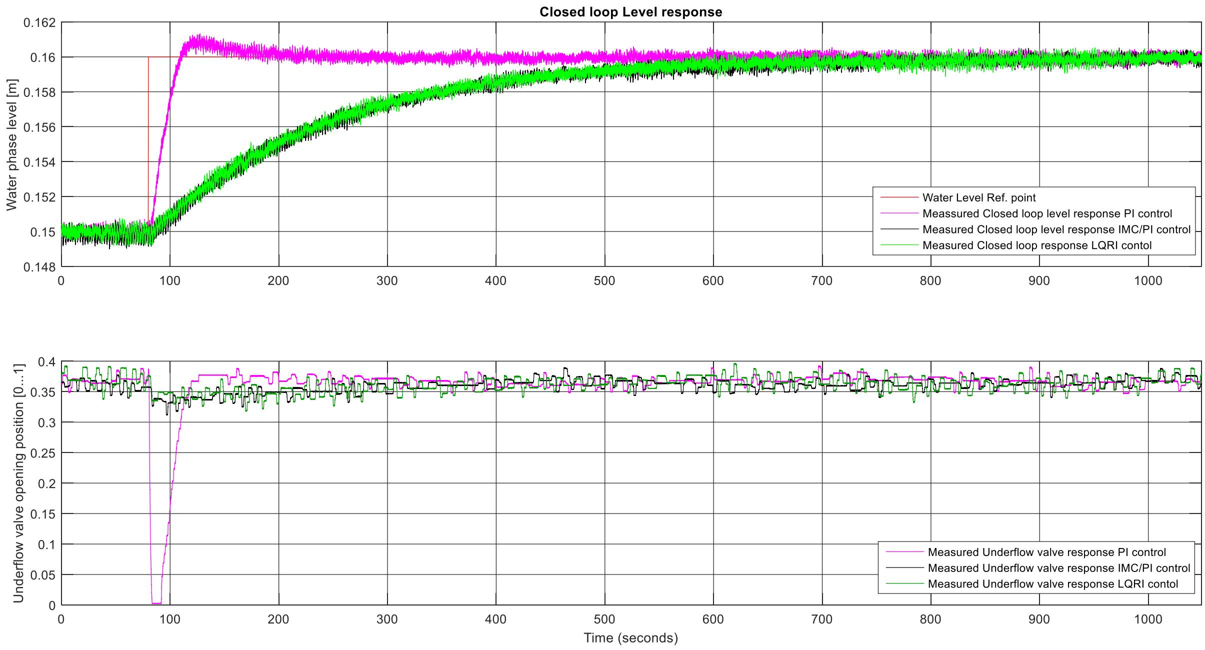


Fig. III.18 Closed Loop controller comparison .

The PI control structure is satisfying the desired specifications. Minimum overshoot before settling down to the specified reference point is observed, in tradeoff of expensive control. The underflow valve is saturated at the point of transition between the two steady states forcing huge action for one percent correction in the water phase level. Concluding that the obtained “PI control” structure is aggressive, providing fast and accurate response, in tradeoff of robustness.

The level response with “IMC/PI controller” is dependent on model parameters, specified filter time constant required to obtain proper controller transfer function and the two tunable parameters of the additional PI correction term. Concluding that the closed loop IMC response is reasonable, based on the slow model dynamics obtained during the modeling phase of the gravity separator system. The control action exerted on the underflow valve is minimized significantly in trade off of slow response in change of the water phase level reference point. Concluding that the “IMC control structure” is implementable but cost inefficient considering the six tuning parameters required to modify the closed loop system response.

Finally, the “LQRI control structure” is considered. The closed loop system response is similar to the response obtained with “IMCPI control structure”. The closed loop level response is over damped in tradeoff of minimum control action required to compensate for one parent increase in the level reference point. Concluding that the “LQRI controller” is a better solution than the “IMC/PI”, considering only the two tunable weighting matrices required to modify the closed loop system response.

Under the formed experimental conditions all controllers are having similar reference tracking performance, implying that all of them are good candidates for "Level control" of the gravity separate system. The significant difference in the responses is caused by the different tuning approaches. However, the results from a worst case scenario experiment in which oscillations in the inlet flow are introduced is considered in order to extend the evaluation of the implemented control structures. The following results are presented in the figure below.

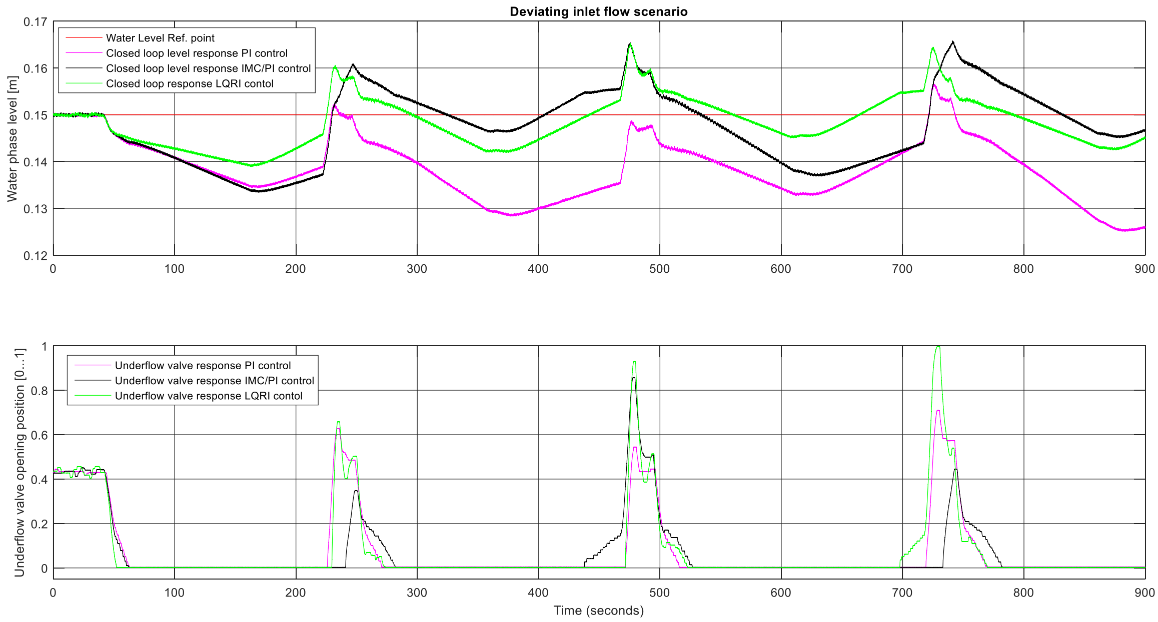


Fig. III.19 Closed loop response under worst case scenario.

The "aggressive PI controller" is heavily influenced by the introduced inlet flow disturbance. The significantly faster closed loop response time of the PI controller, is implying high saturation in the lower limit of the underflow valve causing reaction only when the water phase leave is increased, explaining the observed drift from the specified reference point. The heavy disturbances in addition with errors accumulated in the model caused by the assumed simplifications are clearly influencing the closed loop system response of the "IMC/PI control structure", indicating that the obtained idealized closed loop system response cannot be trusted. Concluding that the derived "IMC/PI controller" requires further tuning and better gravity separator model in order to achieve better results. Only the "LQRI control structure" is maintaining some performance under the considered worst case scenario. Indeed, the reference tracking performance is decreased, but the closed loop system response is oscillating around the specified reference point for the water phase level. Concluding that the derived "LQRI control structure" maintains the observed robustness by considering the infinite gain margin of the "LQR solution".

III.VI Conclusions over closed loop gravity separator system

Two experiments are designed to evaluate the reference tracking performance of the considered control structure. The first experiment considers simple operating conditions under atmospheric pressure in the separator vessel. The second experiment expands the first by pressurizing the system and including second constant disturbance for the duration of the experiment.

Three control structures are designed and implemented in order to obtain control over the water phase level. The first control structure is a “PI controller” used as standard solution to the considered level control problem. The tuning procedure of the “PI controller” is separated into two parts. Using MATLAB’s software for tuning “PID controllers” a range of values for the two tunable parameters is obtained by simulation. The obtained range of values between aggressive and robust PI controller are used for a trial and error method during the implementation phase in order to find suitable gains for the two controller parameters. The obtained results under the conditions of the second experiment considered are satisfying. The “PI controller” follows the specified reference point with minimum error in tradeoff of demanding control action exerted on the underflow valve. Furthermore, considering a worst case scenario in which the water inlet flow is oscillating, concludes that the obtained “PI controller” is too sensitive for realistic operating conditions.

The derived linear representation of the gravity separator system is used to obtain an “IMC control structure” in order to control the water phase level of the gravity separator system. The results obtained under the operating conditions of the first experiment are unsatisfying. A steady state error in the closed loop system response is observed, indicating that all simplification and assumptions considered during the derivation of the linear gravity separator model are directly influencing the closed loop system response. A “PI controller” is introduced to the “IMC control structure” in order to eliminate the steady stat error. A trial and error simulation procedure is executed in order to obtain starting values for the proportional and integral term of the new “IMC/PI control structure”. Further retuning is executed during the implementation of the controller. The observed steady state error is eliminated after the “PI correction term” providing slow level response in tradeoff of reduced control action in comparison with the “PI controller”. Under a worst case scenario, the “IMC/PI control structure” also experience failure, leading to two conclusions. First the simplified model used for the derivation of the IMC structure must be retuned and expanded for more accurate model response before considering an “IMC solution”. Second the implementation cost of the obtained “IMC/PI control structure” is higher than the two other solutions considered. Six tunable parameters are required to obtain closed loop response similar to the response obtained with LQRI control structure.

The final controller considered is a “LQR control structure”. Considering that a “LQR solution” is based on optimal control techniques is expected that the derived controller must provide better results in comparison to the two other implemented control structures. However, the results obtained after the first experiment are associated with steady state error in the water phase level response. Concluding that the considered cost function is minimized, allowing to obtain “optimal control gains”, but it is not guaranteed that the minimization is fixed at zero leading to the observed steady state error. An integration term is introduced to the control structure forming a “LQRI controller”. The new controller is tested against the operating conditions of the second experiment. The steady state error is eliminated after the application of integral state for the error signal providing better results. Furthermore, the considered “LQRI control structure” is providing better response under the conditions of worst case scenario. The reference tracing performance is strongly influenced but oscillating around the defined references point. Concluding that the standard “LQR solution” is good starting point for developing a water phase level controller, but from implementation point of view a “LQRI control structure” must be considered in order to ensure perfect references tracking performance and preserve the “LQR” properties.

IV. Empirical modeling of hydro-cyclone separator system.

The second category of sub-problems requires two models describing a flow rate and flow split to PDR relationships of a hydro-cyclone system. Under the conditions of the available setup, the first relationship is manipulated by adjusting the underflow valve opening position, while the second relationship is manipulated by adjusting the overflow valve opening position, allowing to relate the combined action of the two valves with the associated hydro-cyclone's PDR. Under consideration for the dynamics introduced by the two valves and the complex dynamics occurring in hydro-cyclone systems, it is decided to use "System identification procedure" [in order to obtain a black box model of the hydro-cyclone system as in Fig.IV.1.

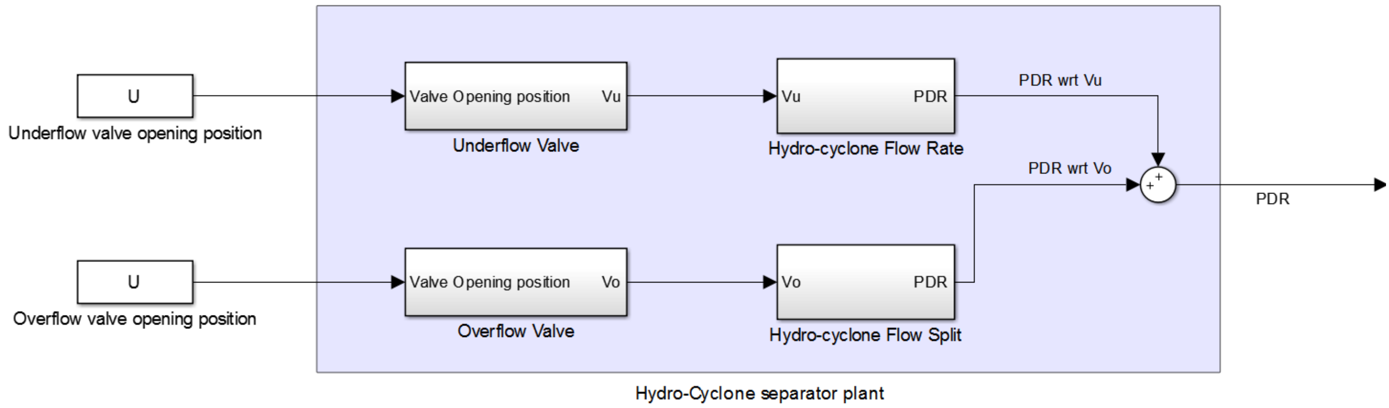


Fig. IV.1 Hydro-Cyclone black box model.

The defined black box hydro-cyclone model is divided into two separate stages. The first stage is describing an input-output (I/O) relationship in terms of underflow valve opening position and PDR. The second stage considers overflow valve opening position to PDR I/O relationship. The combination of outputs from both relationships is forming the total PDR of the hydro-cyclone system, creating the requirement for the identification of a MISO system. However, by treating the two hydro-cyclone stages of the black box model as LTI systems, allows to use the superposition principle reducing the MISO identification requirement to two SISO cases in tradeoff of increasing the amount of identification experiments. Four experiments in total are designed to obtain sufficient amount of data for identification and validation purposes of the two separate stages of the hydro-cyclone model.

IV.I. Identification experiments

The four designed experiments are divided into two groups, defined as identification and validation experiments. The two identification experiments are designed to reach identical operating conditions, under the consideration of the two required SISO identification cases. The first case is considering the underflow valve opening position to PDR relationship. Two sets of data are considered. The input data set is obtained by recording the reference input signal passed to the underflow valve. The output data set is obtained by logging the associated PDR calculation. By manipulating the underflow valve opening position while the overflow valve maintains constant opening, the overflow valve influence to the PDR output is excluded, allowing to obtain informative data set from the first identification experiment.

The second case considers the relationship between overflow valve opening position and the associated PDR. Following the same approach, an I/O data set consisting of recorded reference signal driving the overflow valve and PDR calculation, is considered. In order to obtain the desired data, the operating conditions for the underflow and overflow vales are interchanged. The overflow valve opening position is manipulated, while the underflow valve opening position remains constant, highlighting the desired relationship and providing informative data set from the second identification experiment

The two identification experiments are divided into four stages. The initialization stage defines specific set points for each operating conditions required to ensure stable operation of the gravity and hydro-cyclone separator systems simultaneously.

Initialization phase:

Gravity separator:

Water phase inlet flow - $F_{in}(t)$ $0 \rightarrow 0.6\ L/s$

Water phase level - $h_w(t)$ $0 \rightarrow 0.25\ m$

Tank pressure - $P_g(t)$ $0 \rightarrow 7.5\ bar$

Hydro-cyclone separator:

Pressure difference ratio - $PDR(t)$ $0 \rightarrow 1.5$

Considering that individual operating condition have different time constants, is decided to handle the initialization phase under closed loop configuration. The water phase inlet flow is reaching the defined steady state value first. A “PI feed-back loop” controlling the available pump actuator is implemented, allowing constant flow entering the gravity separator vessel. The second operating condition which reaches the specified set point is the water phase level. The designed and tested “PI Level control structure” is used to maintain steady level, by controlling the underflow valve opening position. After the inlet flow entering the separator vessel is stabilized and the water phase level is fixed at the designed set point, sufficient amount of pressure is accumulated in the separator vessel allowing to reach the defined PDR set point. The provided “PDR controller” is used to maintain steady state for the PDR operating condition, by manipulating the overflow valve opening position. Finally, the gas phase pressure has reached the defined set point, requiring to apply an additional “PI feedback loop” to the available gas relief valve in order to maintain steady state value for the specified gas phase pressure. By providing twenty minutes settling time for all controllers, it is ensured that all four operating conditions are maintaining steady values. This is done while approximately constant valve opening positions for the underflow and overflow valves are achieved, allowing to create the following operating conditions for a “Steady state phase “of the two identification experiments.

Steady state phase:

Gravity separator:

Constant Water phase inlet flow - $0.6\ L/s$

- PI control over inlet flow pump

Constant Separator tank pressure - $7.5\ bar$

- PI control over gravity separator gas relief valve

Constant Water phase level - $0.25\ m$

- "Level control" maintaining constant underflow valve opening position

Hydro-cyclone separator:

Constant PDR - 1.5

- "PDR control" maintaining constant overflow valve opening position

The "Steady state phase" is achieved by forming two categories of closed loop systems. The first category maintains steady water phase inlet flow and separator tank pressure, while the second maintains steady water level and PDR. In order to perform the "Identification phase" for each experiment the second category of closed loop systems is transitioned to open loop. The "Level and PDR controllers" are switched off, allowing to drive the underflow and overflow valves based on pre-calculated equilibrium opening positions. The following two cases of operating conditions are used to collect the required identification data sets.

Identification phases:

Disturbances:

Constant Water phase inlet flow - $0.6\ L/s$

- PI control over inlet flow pump

Constant Separator tank pressure - $7.5\ bar$

- PI control over gravity separator gas relief valve

Operating conditions:

Case I

Underflow valve opening position

- Input signal: $V_u' + \Delta V_u$

Overflow valve opening position

- Input signal: V_o'

Case II

Underflow valve opening position

- Input signal: V_u'

Overflow valve opening position

- Input signal: $V_o' + \Delta V_o$

By keeping the two main disturbances acting on the hydro-cyclone separators constant, while manipulating one of the valves at a time, allows to observe a clear influence over the PDR. A step signal in increments of 1, 2, 3 and 4% up to 10% deviation from the "Steady state" underflow valve opening position is used as excitation for the first identification case. For the second case the point of interest is shifted to the overflow valve. The same input signal is used to record the PDR response, while maintaining constant underflow valve opening.

After the identification phase for each experiment is finished a finalization stage of the two identification experiments is reached. By slowly releasing the accumulated gas and water pressure, the gravity and hydro-cyclone systems are prepared for shutdown.

IV.II. Validation experiments

Two validation experiments are designed in order to proceed with the identification of the hydro-cyclone system. The two validation experiments are executed under identical operating conditions with respect to the underflow or overflow valves. The validation experiments are design using the same approach applied for the identification experiments. Four stages per experiment are considered in order to obtain validation data. The initialization phase for the required validation experiments is designed to reach the following operating points.

Initialization phase:

Gravity separator:

Water phase inlet flow - $F_{in}(t)\ \ 0 \to 0.6\ L/s$

Water phase level - $h_w(t)\ \ 0 \to 0.25\ m$

Tank pressure - $P_g(t)\ \ 0 \to 7.5\ bar$

Hydro-cyclone separator:

Pressure difference ratio - $PDR(t)\ \ 0 \to 1.4$

The three operating points from the gravity separator side are adopted from the identification experiments design. By selecting the same values for water phase inlet flow, level and pressure is ensured that the hydro-cyclone system will be in sufficient operating range. In order to obtain validation data set which is differing from the identification data set, a change in the operating point value for the PDR is made. By selecting different PDR set point, it is ensured that different operating point values for the two required valve opening positions will be achieved. After the initialization phase is executed, the two validation experiments are transitioning to a second stage.

In order to ensure that all operating conditions are settled down, the "Steady state phase" of the two validation experiments is executed under closed loop conditions. The four selected controllers during the identification experiments design are applied to form the "Steady state phase" for the validation experiments. After the gravity and hydro-cyclone systems have settled down an underflow and overflow valve opening operating points are calculated, the experiments are transitioned to the third stage.

In the "Validation phase", the closed loop systems maintaining steady water phase level and PDR are switched off. The constructed deviation signals applied during the identification experiments are reused, allowing to obtain validation data sets which are similar to the identification data but in different operating point. Finally, the two systems are transitioned to finalization stage of the two validation experiments, ensuring the safe shutdown of the two systems.

IV.III. Underflow valve model identification

The results obtained from the first identification experiments are used to form the required I/O identification data set. The identification data is limited to data recorded during the "Identification phase" of the experiment. A low-pass filter is applied to the raw calculation of the PDR. Pass band of 0 to 5 Hz is selected to reduce the associated nonlinearities and measurement noise influence. The first thirty seconds of data is used to calculate average equilibrium point values for the input and output data sets. The calculated values are subtracted from the recorded data, allowing to represent the I/O data set in terms of deviation signals.

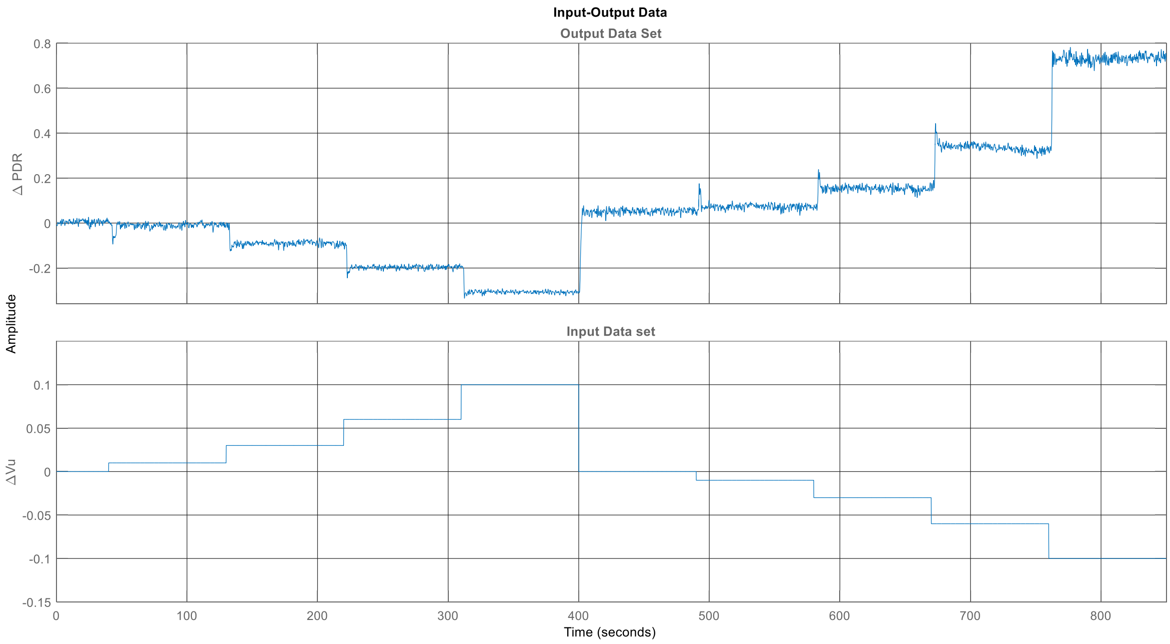


Fig. IV.2 Underflow valve identification data set.

Correlation in the I/O data set is observed under the provided step excitation. The I/O relationship is falling under the revers action problems. By increasing the underflow valve opening position the PDR is decreased and vice versa a decrease in the valve opening position causes increase in the PDR. The prepared identification data set is introduced to "MATLAB's identification toolbox" in order to proceed with the model identification procedure [16].

Different model structures are evaluated during the model identification procedure. Curve fitting validation against the validation data set is used to select the best model candidates. The identification of each model uses the following specifications.

Estimation specifications:

- Model structure: Transfer function model
- Initial condition: zero
- Initialization method: All
- Iteration steps: 20

The initial condition for the selected identification procedure is set to "zero", considering that the identification data set is expressed in terms of deviation signals. By specifying "All" initialization methods, the software compares all predefined methods and chooses the one that yields the smallest value of prediction error norm. It is decided that the default choice of twenty iterations per one identification cycle is sufficient to obtain an estimation for the specified model. Around seventy percent fit is achieved with second and fourth order transfer function models. The transfer functions are evaluated based on step response, pole-zero map and bode diagram as follow.

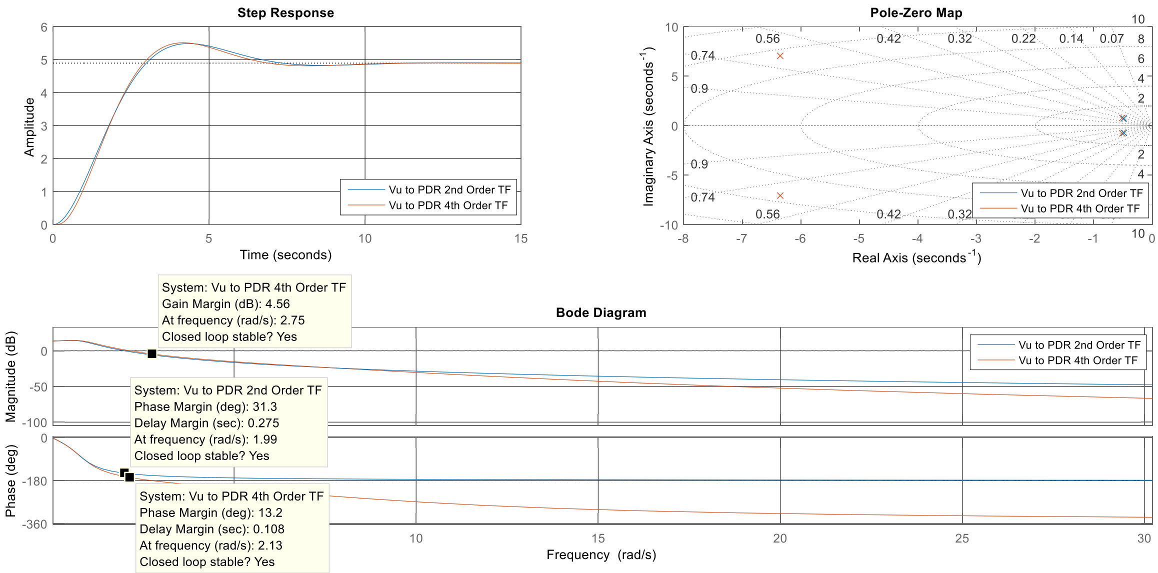


Fig. IV.3 Estimated transfer functions analysis.

The two transfer function models are stable. All poles are located on the left hand side of the complex plane. The obtained step responses have settling time of twelve seconds, in agreement with the fast dynamics of the PDR. Based on the amplitude of the step responses, it is concluded that the estimated "DC gain" for both transfer functions is higher than required and additional tuning must be considered after the first trial of simulations. Finally, the two bode plots are compared. The two transfer functions are closed loop stable. The fourth order model is limited by low gain margin in contrast with the second order model. Based on the obtained results is concluded that the two estimated transfer functions are suitable for simulation purposes.

Underflow valve model simulation

The estimated transfer function models are validated based on three criteria during the identification procedure. The first criterion of curve fitting the validation and simulation of the PDR system response is used to reduce the number of estimated models. After the model candidates are narrowed down a LTI analysis is performed in order to investigate the open and closed loop stability of the obtained models. Finally, a comparison between the validation and simulation of the PDR responses is considered as third validation criterion. The following block diagram is created to simulate the estimated models based on the recorded validation data set.

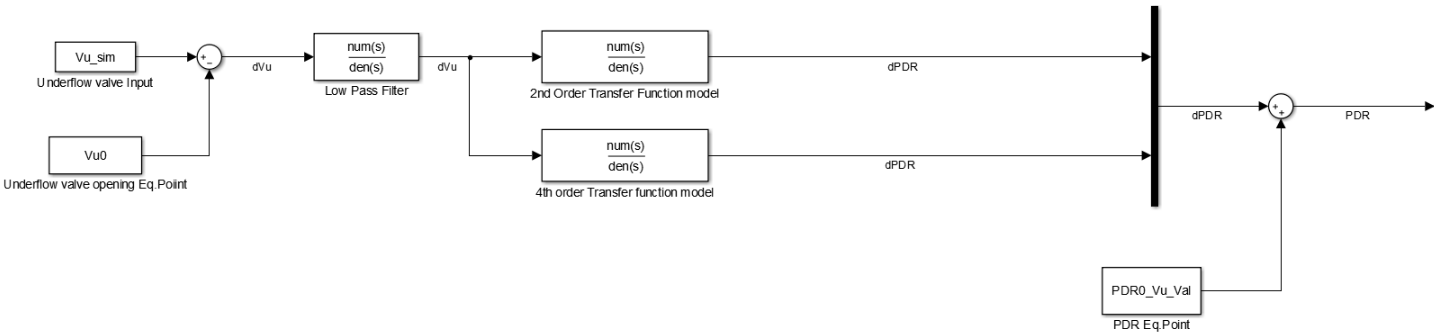


Fig. IV.4 Underflow valve opening position to PDR simulation.

The input signal used for the simulation of the estimated models is formed by subtracting a calculated operating point from the measured valve opening position of the underflow valve. The formed deviation signal is passed through the pre-filtering stage used in the identification procedure. In order to keep the consistency, the low pass filter applied to the identification data set is used in the simulations also. To reconstruct the original output and exclude the filter influence on the simulation, an inverse of the applied filter is required. However, during implementation, it is discovered that the inverse filter transfer function is unstable, directly influencing the simulation results. It is decided to exclude the filter inverse stage in trade off of introducing delay to the simulated output response. Finally, a calculated PDR operating point value is summed with the produced model output, forming the final simulation PDR response. The simulation and validation PDR responses are compared in Fig.IV.5.

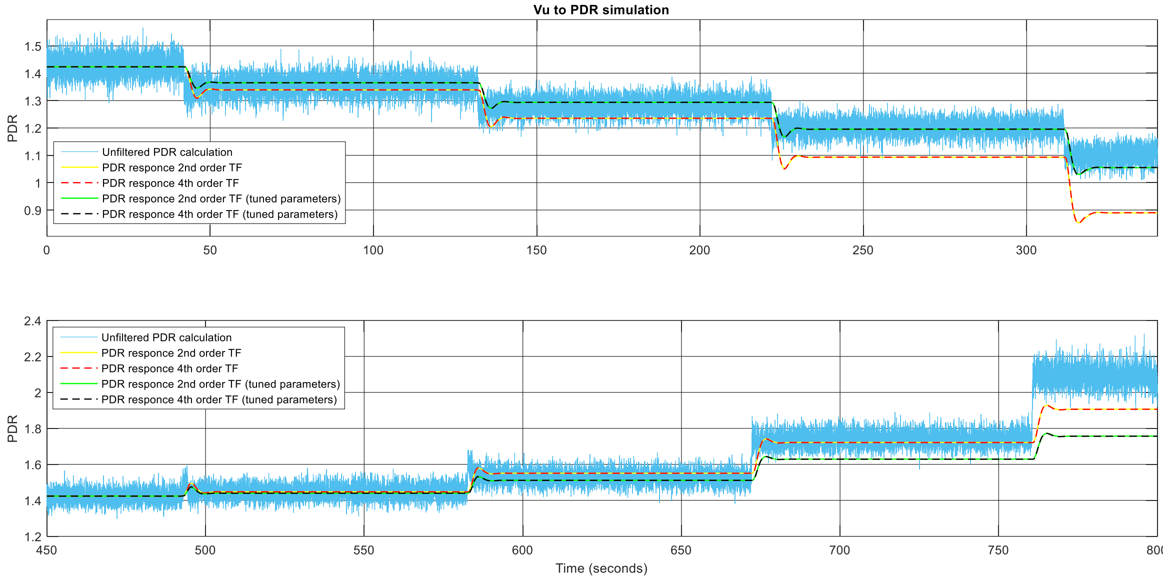


Fig. IV.5 Simulated PDR step response.

The simulation results are split into two parts by dividing the data into two portions for better representation. The first graph from top to bottom in Fig.IV.5 presents the measured and simulated PDR responses under the influence of positive step deviation of the underflow valve opening position. The second graph presents the PDR responses under the influence of negative step deviation of the underflow valve opening position. The pair of solid dash lines of yellow and red colors are reserved for simulation responses of transfer function models with original parameter estimates. The pair of green and black lines is reserved for simulation responses with tuned transfer function parameters.

Both second and fourth order transfer function models are producing similar output responses. The performance of the selected linear models is decreasing as the PDR is shifted further away from the initial state. The best step responses obtained with the original transfer function models are associated with one percent increase or decrees in the underflow valve opening position. In contrast with the results obtained with tuned transfer function parameters. The "dc gain" is reduced by twenty-seven percent for both transfer function, extending the accuracy range of the models up to three percent deviation in the underflow valve opening position. The following detailed Fig.IV.6 illustrates the PDR step response for two present step deviation in the underflow valve opening position.

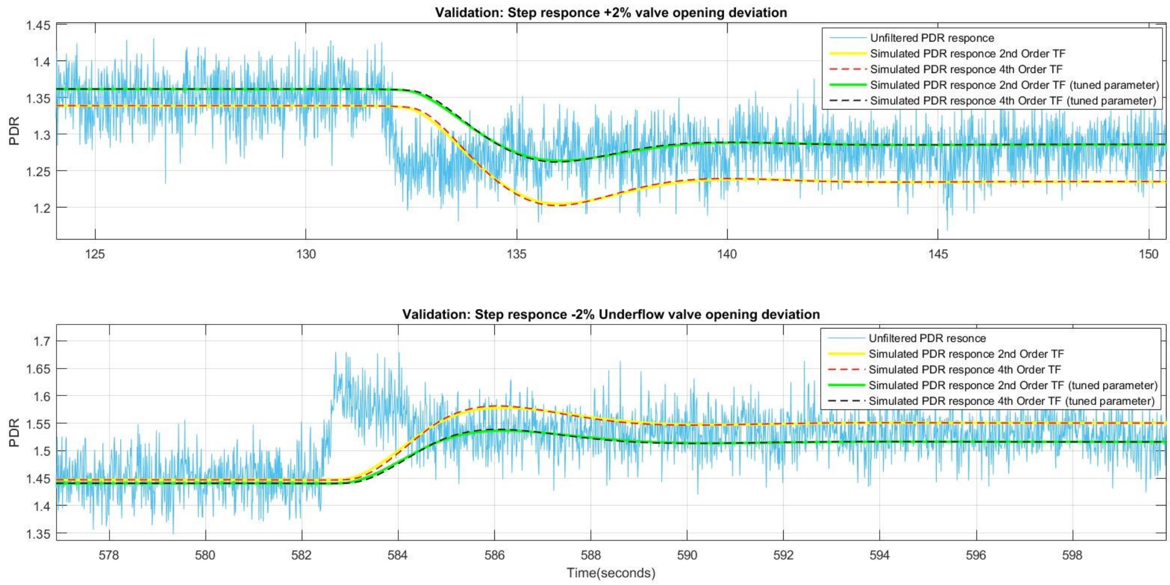


Fig. IV.6 Step response for two percent deviation in valve opening position .

The simulated PDR responses are delayed by the application of the low pass filter. The second and forth order dynamics of the chosen models are sufficient to represent the observed overshoot in the transition between two steady states. The observed steady state error in the simulation is reduced by the tuning of the two transfer function models. Based on the obtained results, it is concluded that the identification of the first relationship of the hydro-cyclone black box model is successful.

IV.IV. Overflow valve model identification

The overflow valve model identification and estimation procedure is executed following the steps defined during the underflow valve identification. The required I/O data set is filtered using the obtained low pass filter. An average operating point is subtracted from both input and output signals forming the deviation signals presented in Fig.IV.7.

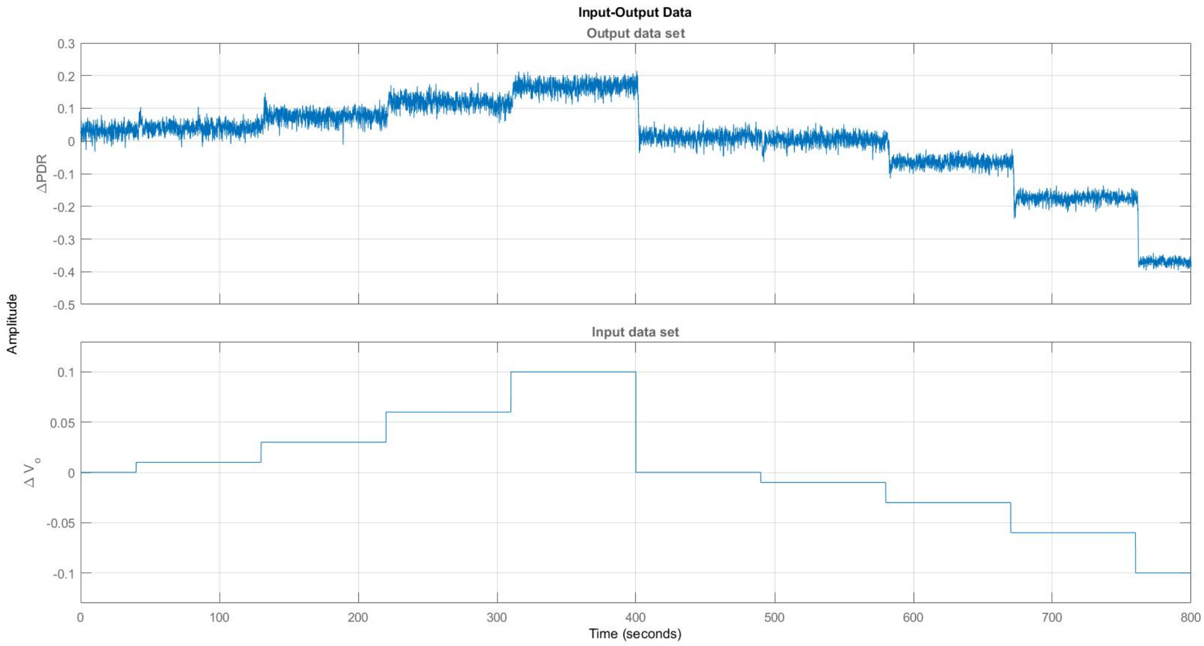


Fig. IV.7 Underflow valve model identification data set.

Relation between the input and output data sets is observed. The second relationship is classified as direct action. Increase in the overflow valve opening position causes increase in the outputted PDR, while decrease in the valve opening cause a decrease in the PDR. The pre-filtered and prepared I/O identification set for identifying the second relationship of the hydro-cyclone models is transferred to the available "Identification toolbox". The stated estimation specifications used for the underflow valve model identification are applied for the second estimation procedure also. Second and fourth order transfer functions are selected after achieving approximately seventy percent curve fitting to the second validation data set. The characteristics of the two estimated transfer functions are evaluated in Fig.IV.8, as second validation stage.

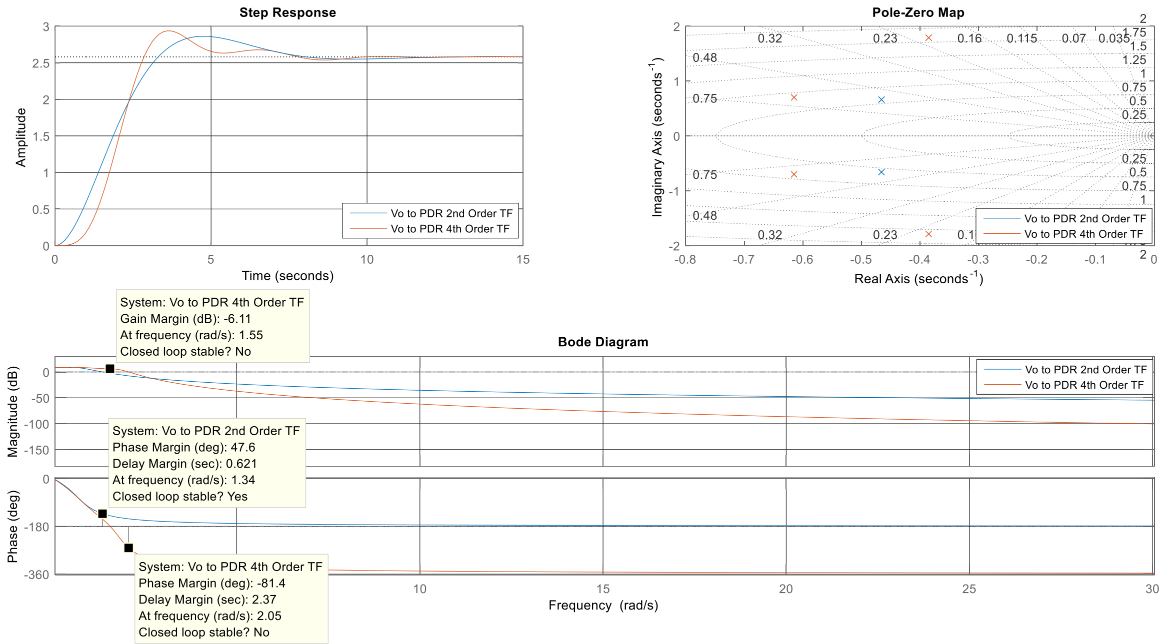


Fig. IV.8 Estimated transfer functions analysis.

The two selected transfer functions are stable. All poles are located on the left hand side of the complex plane. The two step responses are having settling time of eleven to twelve seconds, considered acceptable for representing the PDR dynamics. The second order transfer function is closed loop stable, based on the presented bode diagram. In contrast with the fourth order transfer function. The negative gain margin and more than two seconds delay margin, is showing that the fourth order transfer function can recreate the desired output but is not suitable for closed loop applications. It is decided to discard the fourth order model candidate and execute simulations with the second order transfer function model in order to proceed with the validation procedure.

Overflow valve model simulation

The block diagram from Fig.IV.4 is modified to simulate the obtained second order transfer function model. Two average values for the required operating points are calculated. The operating point for the overflow valve opening is subtracted from the measured opening position. The created deviation signal is filtered and passed to the overflow valve model. The simulated PDR response is summed with the second operating point value forming the final simulation response. The measured and simulated PDR responses are compared in the following figure.

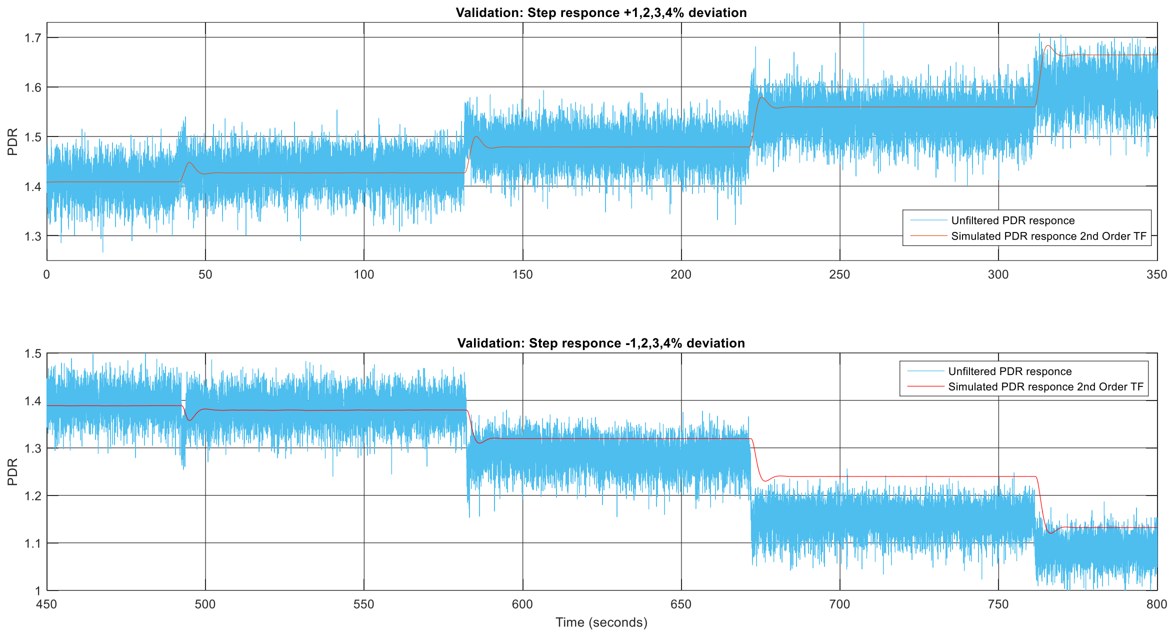


Fig. IV.9 Simulated PDR step response.

The best results are achieved for small deviations from the operating point, without additional modification of the transfer function parameters. The step response for one and two percent increase in the valve opening position, observed in the beginning of the simulation are following the validation data set with minimum error. The second portion of data associated with negative step deviation is less accurate. The best step response is achieved at one percent deviation from the specified operating point, considering the tradeoff between accuracy and simulation over the full duration of the validation data set. The following detailed figure for one percent increase and decrease in the overflow valve opening position is used to evaluate the dynamic performance of the estimated model.

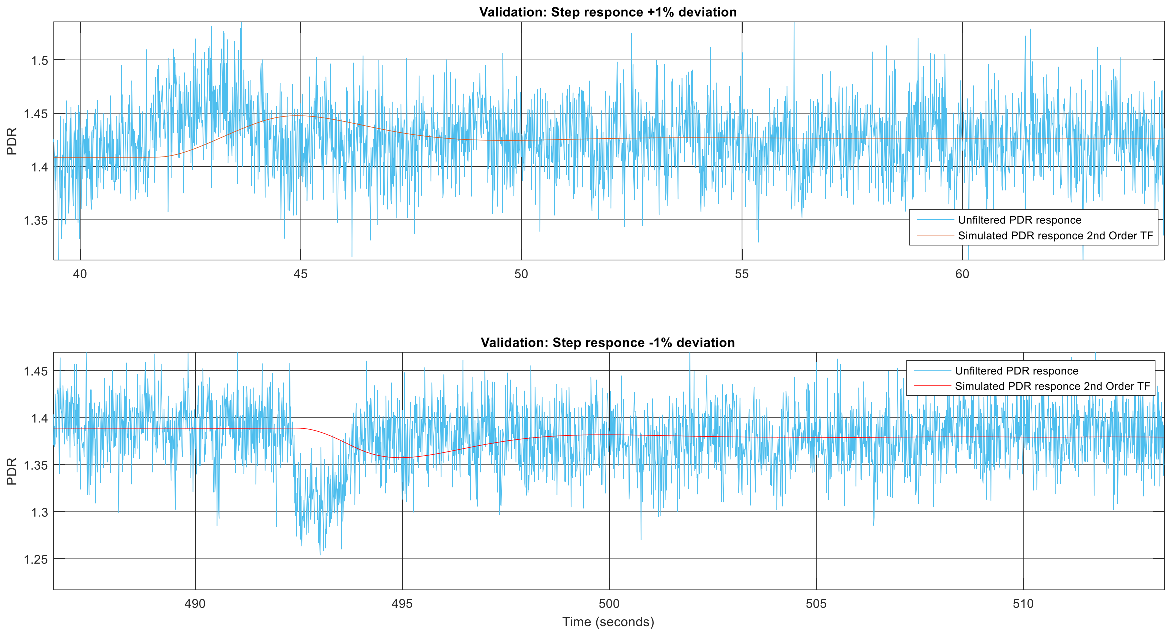


Fig. IV.10 Step response for one percent deviation in valve opening position.

The simulated step response is delayed due to the applied low pass filter. The settling time of five to seven seconds observed in the measured data is extended to eleven second for the simulated PDR response. The overshoot and undershoot dynamics in the transition period between the two steady states is captured by the second order behavior of the estimated transfer function model. Considering the small operating range in which the identified model is validated, it is concluded that the identification procedure for the second relationship of the hydro-cyclone black box model is successful.

IV.V. Conclusions over empirical modeling of hydro-cyclone separator system

Black box system identification is chosen for the empirical modeling of a hydro-cyclone separator system as part of the project requirements and specifications. The black box model is divided into two parts. The first part describes relationship between underflow valve opening position and PDR. The second part describes a relationship between overflow valve opening position and PDR.

Four experiments in total are designed to obtain the informative data sets. Two identification experiments are designed and executed. The identification experiments are designed following similar structure in order to reach suitable operating conditions for the two separator systems. Difference in the “Identification phase” of the experiments is implemented to form the two defined cases, allowing to observe the PDR response based on one valve opening deviation at a time. The collected identification data is stored allowing to proceed with the design of validation experiments. Two validation experiments with operating conditions differing from the identification experiments are executed, allowing to obtain second I/O data set.

After the experimental phase is complete the identification procedure is transitioned to data evaluation and preparation. The I/O data set is reduced by extracting the data recorded during the "Identification and Validation phases" of all experiments. The reduced data sets are filtered, allowing to reduce measurement noise influence over the estimation process. Finally, an estimated operating point values are subtracted from the identification data sets, allowing to obtain I/O relationships in terms of deviation signals.

The prepared identification data sets are transferred to the available "System Identification toolbox", in order to handle the parametric model estimation for the two parts of the hydro-cyclone black box model. Two second and fourth order transfer functions relating underflow and overflow valve opening position to PDR have passed the curve fitting validation. The properties of all model candidates are evaluated as second stage of validation. The two model candidates associated with the underflow valve are open and closed loop stable, passing the second validation stage. While only the second order transfer function associated with the overflow valve have passed the second validation stage.

Comparison between measured and simulated PDR responses is chosen as final stage of validation. Three linear models are created and simulated using the validation data sets. An accurate PDR response simulation is achieved for one percent step deviation in the underflow valve opening position. The accuracy range is extended up to three percent step deviation, by tuning the model parameters. The second and fourth order dynamics of the model candidates are sufficient to represent the general dynamics of the measured PDR response. Under the same simulation conditions, it is proven that the overflow valve model produces accurate simulation results also.

The two validated second order transfer function models are selected to describe the identified hydro-cyclone black box model. The formed hydro cyclone model is both open and closed loop stable, allowing to be used for control purposes.

V. Coupling of gravity and hydro-cyclone separator systems

Three models in total are required to form the MIMO model describing the water phase level and PDR as parts of the gravity and hydro-cyclone separator systems. The derived linear state space model from "Chapter III section I" is used to describe the gravity separator part of the new system. Two choices are available for the underflow part of the hydro-cyclone system. Considering the similarities between the step responses and the difference in the closed loop stability of the two underflow valve models, is decided to use the second order transfer function for the MIMO model design. The last models selected for the MIMO system design is the second order overflow valve model.

After the three models are selected a conversion is required. The two separate parts of the hydro cyclone model are converted into state space form using MATLAB's software. After the conversion all independent parts of the gravity and hydro-cyclone systems are defined in state space form, allowing to augment all matrices and obtain the following state space representation for the MIMO model.

$$\begin{bmatrix} \dot{h}_w \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} a_{g_{sep}} & 0 & 0 & 0 & 0 \\ 0 & a_{11_{h_{vu}}} & a_{12_{h_{vu}}} & 0 & 0 \\ 0 & a_{21_{h_{vu}}} & a_{22_{h_{vu}}} & 0 & 0 \\ 0 & 0 & 0 & a_{11_{h_{vo}}} & a_{12_{h_{vo}}} \\ 0 & 0 & 0 & a_{21_{h_{vo}}} & a_{22_{h_{vo}}} \end{bmatrix} \begin{bmatrix} h_w \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} b_{g_{sep}} & 0 \\ b_{1_{h_{vu}}} & 0 \\ b_{2_{h_{vu}}} & 0 \\ 0 & b_{1_{h_{vo}}} \\ 0 & b_{2_{h_{vo}}} \end{bmatrix} [V_u \; V_o] \tag{1}$$

$$\begin{bmatrix} h_w \\ PDR \end{bmatrix} = \begin{bmatrix} c_{g_{sep}} & 0 & 0 & 0 & 0 \\ 0 & c_{1_{h_{vu}}} & c_{2_{h_{vu}}} & c_{1_{h_{vo}}} & c_{2_{h_{vo}}} \end{bmatrix} \begin{bmatrix} h_w \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{1.1}$$

The new system matrix "A" is with dimensions 5x5 according to the total number of states defined for each system. The input matrix "B" is considering the underflow and overflow valve opening positions in order to manipulate the two control objectives expressed as PDR and water phase level. The underflow valve is acting on both control objectives, while the overflow valve is acting only on the PDR. Finally output matrix "C" is used to obtained the two desired system outputs. The water phase level is obtained based only on the level state, while the PDR output is produced by the combination of PDR outputs associated with each part of the hydro-cyclone black box model. Three simulations are executed to validate the MIMO model output responses.

V.I. Open loop MIMO validation

The open loop validation of the obtained MIMO system model is divided into two parts. The first part consists of comparison between simulated and measured control objectives under the operating conditions formed for the first validation case from “Chapter V Section II”. The second validation part is executed in similar matter under the operating conditions defined for the second validation case. The following block diagram is obtained in order to simulate the open loop validation of the derived MIMO system.

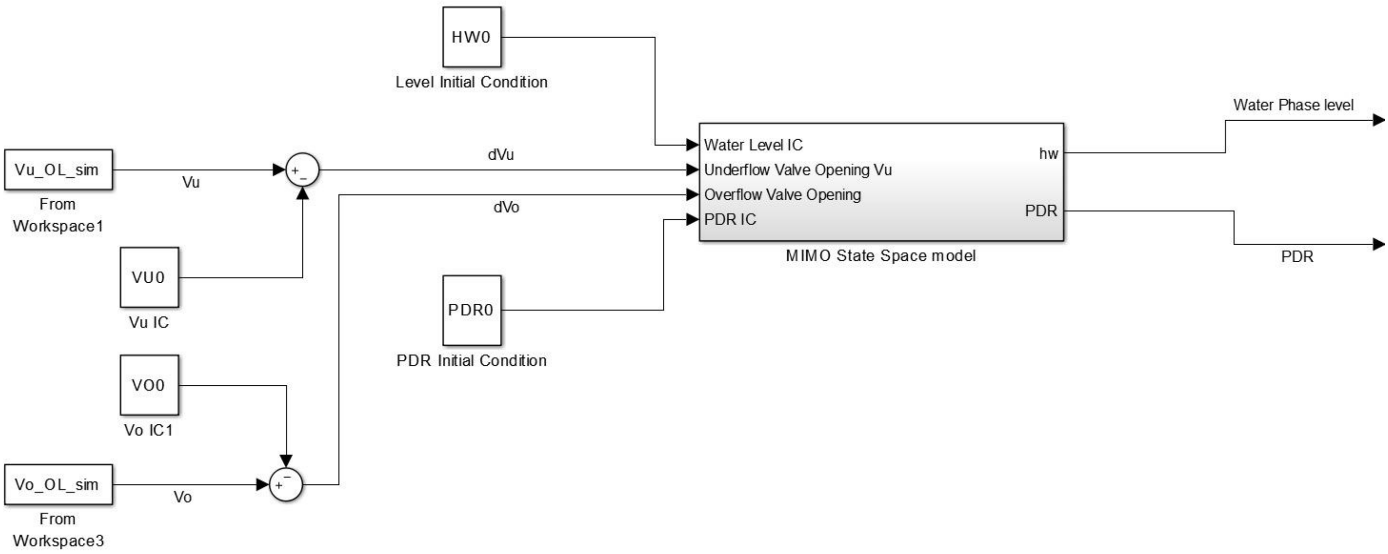


Fig. V.1 MIMO state space simulation model.

Recorded underflow and overflow valve opening positions are used to form the input signals for the simulations. A calculated average operating points are subtracted from both signals in order to express the inputs to the system in deviation terms. The appropriate initial conditions are introduced to the system in order to reconstruct the two system outputs. Under the scenario of the first validation experiment, a step deviation signal is used to drive the underflow valve while the overflow valve maintains constant valve opening position. Proceeding further the driving conditions for the two valves are interchanged. The underflow valve maintains constant opening position while the overflow step deviation is acting on the PDR system output. The following results are obtained for the first part of the open loop MIMO system simulation.

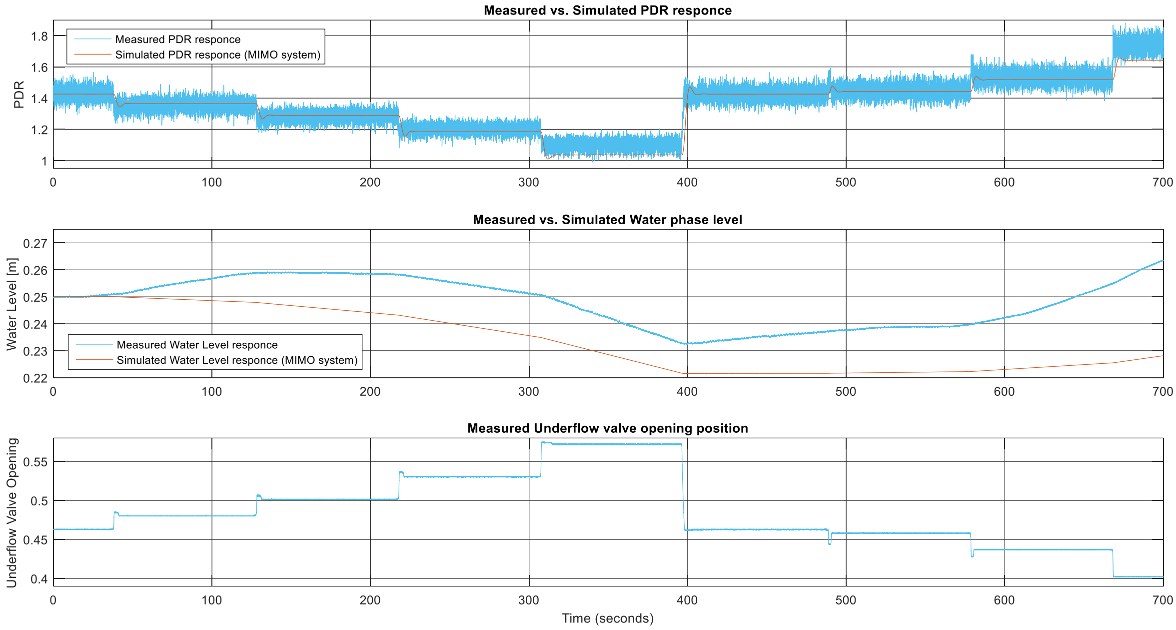


Fig. V.2 MIMO system validation Part I.

The water phase level and PDR are manipulated by the underflow valve opening position as expected. The revers action relationship between the underflow valve and the PDR is preserved. The two second order models describing the hydro-cyclone stage of the MIMO system are responding in the same manner observed during the validation procedure from “Chapter V Section III Subsection I”. Acceptable approximation of the measured PDR is observed for one, two and three percent increase and decrease in the underflow valve opening position.

The simulated and measured water phase level are matching at the steady state in the beginning of the simulation. As time evolve a difference in the two responses is observed. The measured water phase level is illustrating accumulation of water, while the simulation is presenting discharge. The mismatch between simulation and measurement is based on the modeling assumption regarding unknown inlet flow disturbance, which is acting on the water phase level in reality. Based on the obtained results, it is concluded that the simulation model has the general dynamics of the water phase level in trade off of the accuracy achieved using simplifications.

The same model is used to simulate the scenario of the second validation experiment. The underflow valve is kept at constant valve opening position while the overflow valve opening position is manipulated by step deviations signal. The following simulation results are obtained for the second part of the MIMO model validation.

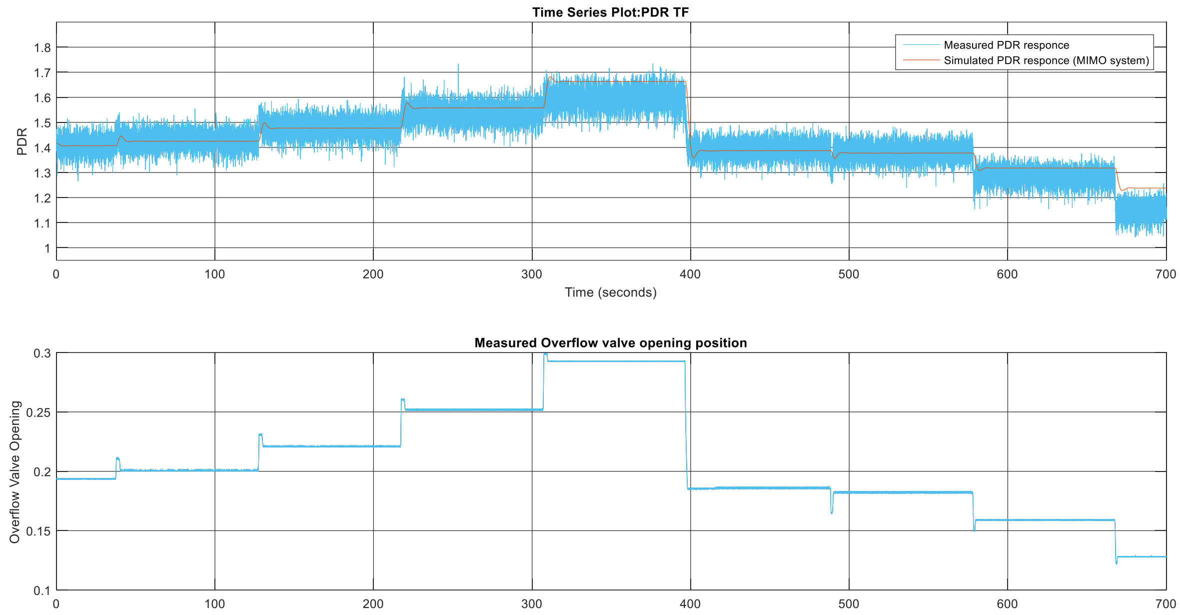


Fig. V.3 MIMO system validation Part II.

The second part of the open loop MIMO system validation is simpler. The overflow valve is manipulating only the PDR considering the identified relationship. The same simulation response achieved in the validation of “Chapter V Section IV Subsection I” is observed in the MIMO model simulation. Good simulation results are obtained for one and two percent step increase in the overflow valve opening position. While only the response associated with one percent decrease in the step deviation is producing satisfying results in the end of the simulation.

The two cases of the considered open loop MIMO system validation are successful. The obtained simulation results regarding the PDR are providing good approximation for small range of step deviations, considering both valve cases. In contrast the simulated water phase level response is less accurate. The simplified model from first principles is following the intention of the measured water phase level in tradeoff of bigger error gap between measured and simulated responses. Based on the obtained results is concluded that the MIMO system is responding in the same manner as each individual part of the model.

V.II. Closed loop validation

In order to validate that the hydro-cyclone part of the MIMO system fully a closed loop evaluation is considered. The following block diagram is used to simulate the MIMO model with two separate control loops.

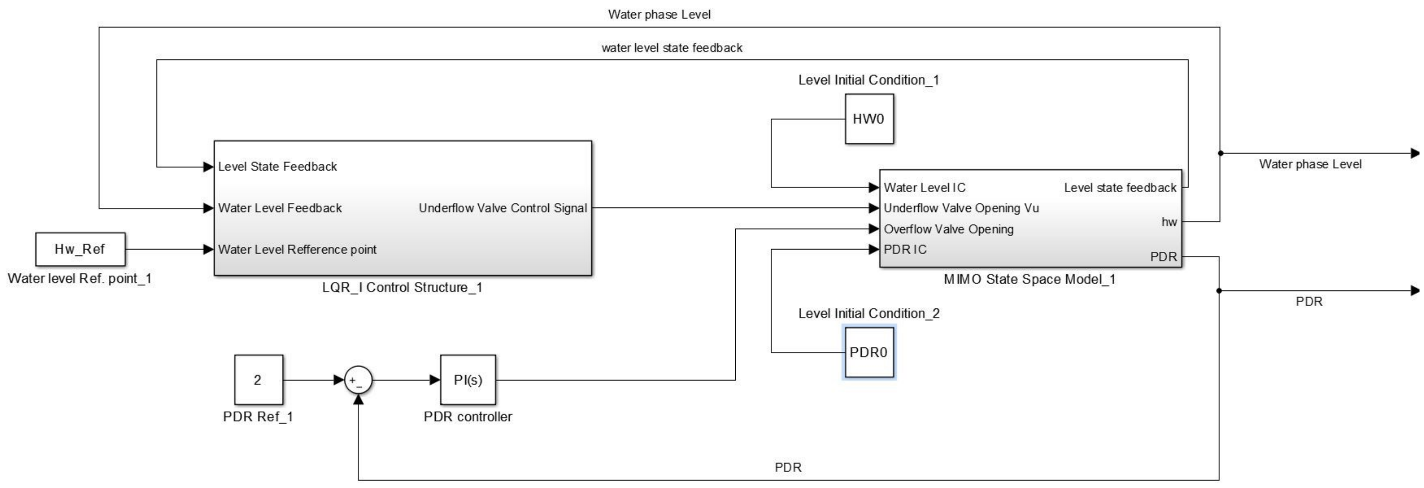


Fig. V.4 Closed loop MIMO system.

The two control objectives are treated separately. The derived "LQRI control structure" is applied to the gravity separator part of the MIMO system. The provided "PDR controller" is applied to the hydro-cyclone separator part of the new model, forming the two closed loop systems. The experiment from "Chapter IV Section I Subsection II" is used for the closed loop MIMO simulation. Recorded data is used to obtain the water phase level reference point. Constant value of two is selected as reference point for the PDR, in agreement with the constant reference point used during the selected experiment. The simulation results are compared to the closed loop measured data in the following figure.

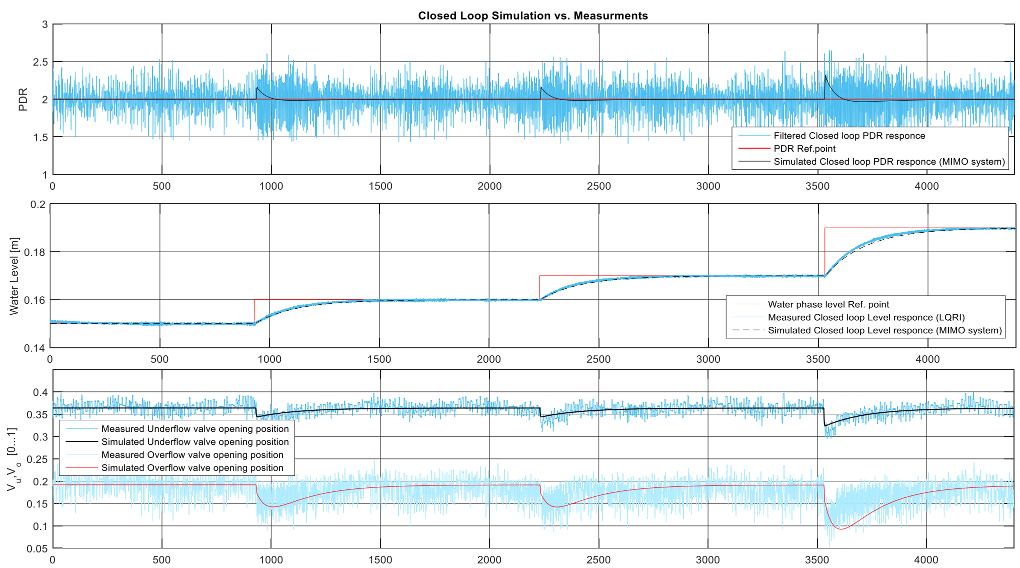


Fig. V.5 Closed loop MIMO validation.

Simple representation of the simulated closed loop system responses is achieved. The simulated closed loop step response of the level is following the measured data, in the second graph from Fig.V.5. The dynamics of the closed loop system are preserved. Concluding that the gravity separator part of the MIMO model is responding identical to the applied controller. The simulated PDR response is following the reference point and the measured data in trade off of dampening the dynamics. An increase in the amplitude of the measured PDR is observed at each step transition of the water level, relating the three peaks observed in the simulated PDR response. Concluding that the hydro-cyclone separator part of the MIMO model is responding in sufficient but simplified manner compared to the measured closed loop data. Finally, the two simulated control signals are summed with the proper operating point values in order to compare measured and simulated valve opening positions. The two simulated inputs to the MIMO model are following the measured data, as sufficient approximation of the measured valve opening positions. Based on the obtained closed loop simulation results is concluded that the MIMO system description have preserved the desired system characteristics under the influence of the applied controllers.

V.III. Conclusions over MIMO model derivation

Three models are selected to represent the gravity and hydro-cyclone separator systems as one MIMO system with water phase leave and PDR control objectives. The derived linear model from “Chapter III Section I” is used to describe the water phase leave of the gravity separator system. The identified and validated hydro-cyclone models required to represent the full black box model are used to form the second part of the MIMO system. After the appropriate conversions are executed the three state space model representations are augmented forming a MIMO model of the new coupled system.

Three steps of validation are considered in order to verify that the performance of the used models is preserved after the combination. The two stage validation used in the identification procedure of the hydro-cyclone models is recreated in order to validate the influence of both valve to the two system output in open loop manner. The underflow valve is influencing both the water phase level and PDR, while the overflow valve is influencing only the PDR output. The obtained results are satisfying. The simulated water phase level response has similar dynamics compared to the measured data, considering the simplification regarding unknown inlet flow disturbance entering the separator vessel. The simulated PDR is identical with the results obtained during the stand alone validation of the underflow part of the hydro-cyclone model. The same procedure is repeated using the second validation data set and obtaining results regarding the influence of the overflow valve to the PDR output of the MIMO system. During the second round of validations is discovered that the assumed minimum influence of the overflow valve to the water phase level is not negligible. Concluding that additional relationship identification can be considered in order to fully describe the overflow valve influence to the water phase level.

The third step in validating the MIMO model is executed under closed loop conditions. Experimental data collected during the development of the level controller is used to form the operating conditions for the closed loop simulation. The “LQRI and PDR controllers” are used to form the two separate closed loop systems controlling the water phase level and the PDR. The obtained simulation results are matching the measured closed loop data. The simulated water phase level response is almost identical with the measured data, while the simulated PDR response is sufficient but simplified in comparison with the measured data.

Based on the simulation results obtained during the three validation steps is concluded that the derived MIMO system model is responding in similar manner as each individual SISO system. Concluding that the derived MIMO system model is suitable for control purposes.

VI. LQR design for the MIMO system

A LQR control structure is selected for the derivation of a MIMO controller with control objectives expressed as PDR and water phase level. The same procedure used during the derivation of “LQR level controller” is reformulated to meet the MIMO requirements. The following cost function is minimized in order to obtain the optimal feedback gains for the “MIMO LQR controller”

$$J(\bar{x},\bar{u}) = \int_{0}^{\infty} (\bar{x}^T Q \bar{x} + \bar{u}^T R \bar{u}) dt \qquad (1)$$

The state weight matrix “Q” is a diagonal positive definite matrix with dimensions 5x5, in agreement with the LQR constrains and number of states considered. The input weighting matrix “R” is also diagonal positive definite matrix with dimension 2x2, considering the two inputs of the system. The “Bryson’s rule” seeRef[AVII] is used as starting point for the selection of weighting values for both matrices. The defined state space and weighting matrices are used to calculate the optimal gains for the MIMO system using MATLAB’s software. The returned outputs of the used software function are evaluated and is confirmed that the obtained gain is “optimal” control gain. Finally reference tracking with full state feedback is introduced to consider none zero reference input for the water phase level and PDR. The scalar case for derivation of reference tracking is extended to meet the MIMO system requirements. The following block diagram is used to simulate the obtained closed loop MIMO system.

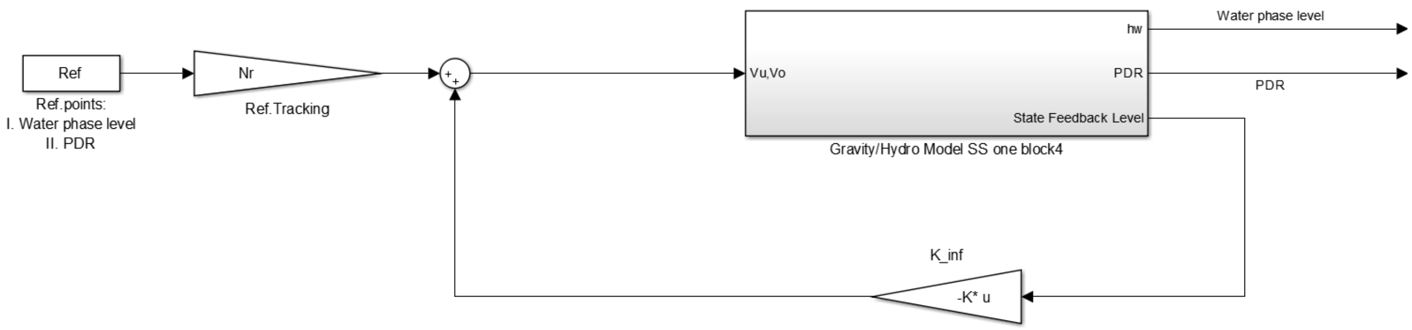


Fig. VI.1 MIMO LQR control structure.

The derived simulation model is tested against scenario in which the reference point for the PDR is constant while step transition in the water phase level reference point is executed. The following simulation results are obtained after trial and error retuning procedure applied for the two weighting matrices.

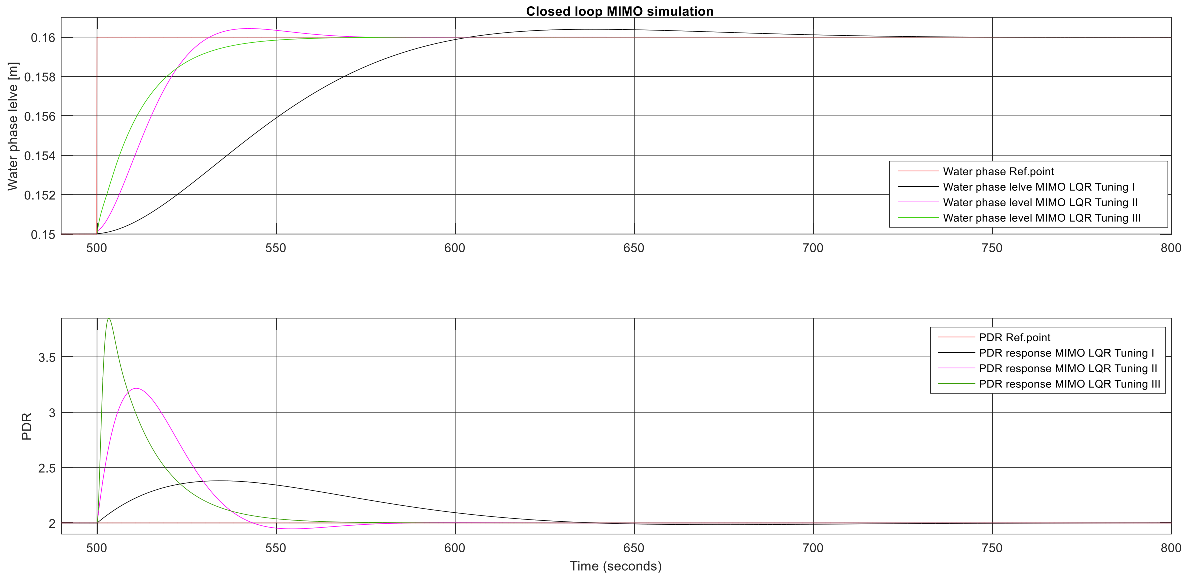


Fig. VI.2 LQR Closed loop system responses.

Three different set of values are chosen to tune the derived MIMO LQR in order to evaluate the controller reference tracking performance for both control objectives. During the tuning procedure is found that the specific range of values can be selected for the input weightings. Furthermore, is observed that the dynamics of the simulated PDR and water phase level are shared. Slower transition to the next reference point regarding the water phase level leads to dampening in the observed PDR overshoot in tradeoff of longer settling time, presented by the black curve from Fig.VI.2. By choosing close values for the weightings of the unknown states associated with the PDR, allows to obtain the closed loop responses marked with magenta color. The settling time for both responses is decreased. The closed loop response of the water phase level is satisfying in contrast with the increase in the overshoot of the PDR response, concluding that a delicate tradeoff between the level and PDR responses must be considered for the implementation of the derived controller. Finally, during the simulations is observed that the overflow valve is causing an overshoot in the underflow valve opening position, due to the simultaneous control over the two valves. Concluding that the appropriate switching time between the two valves must be considered. Based on the obtained results is concluded that the derivation of the “MIMO LQR controller” is successful. However, the weighting of the four unknown state is based on trial and error procedure, indicating that retuning of the controller parameters must be considered in future implementation. Concluding that the cost of implementing of a “MIMO LQR control structure” is increased, considering the amount of tuning parameters required.

VI.I. LQRY design for the MIMO system

In order to reduce the tuning parameters required in the design of “MIMO LQR controller” is decided to reformulate the controller derivation to “MIMO LQRY controller”. The following cost function is considered for the second controller design.

$$J(\bar{y},\bar{u}) = \int_{0}^{\infty} (\bar{y}^{\mathrm{T}} Q \bar{y} + \bar{u}^{T} R \bar{u}) dt \tag{1}$$

Instead of considering the five required model states the selected cost function is redesigned in terms of system inputs and outputs. Considering that the outputs of the “MIMO system” are expressed in terms of the model states scaled trough the output matrix “C” of the obtained “MIMO state space model, allows to define the new “Q” matrix as follow.

$$Q = C^{T} Q' C \tag{2}$$

The design parameter Q' is a diagonal positive definite matrix with dimensions 2x2, in agreement with the “LQR constrains”. The number of tunable weights is decreased to two values related to the water phase level and PDR, allowing to obtain “LQRY controller” following the “LQR design” procedure. After the “optimal” control gains are calculated the returned software function outputs are evaluated, confirming that all constrains are satisfied. The reference tracking with full state feedback is obtained following the same approach taken during the derivation of the “MIMO LQR controller”. The block diagram from Fig.V.I.1 is reused for the simulation of the obtained “MIMO LQRY control structure”. The following simulation results are obtained after several iterations of tuning the two weighting matrixes.

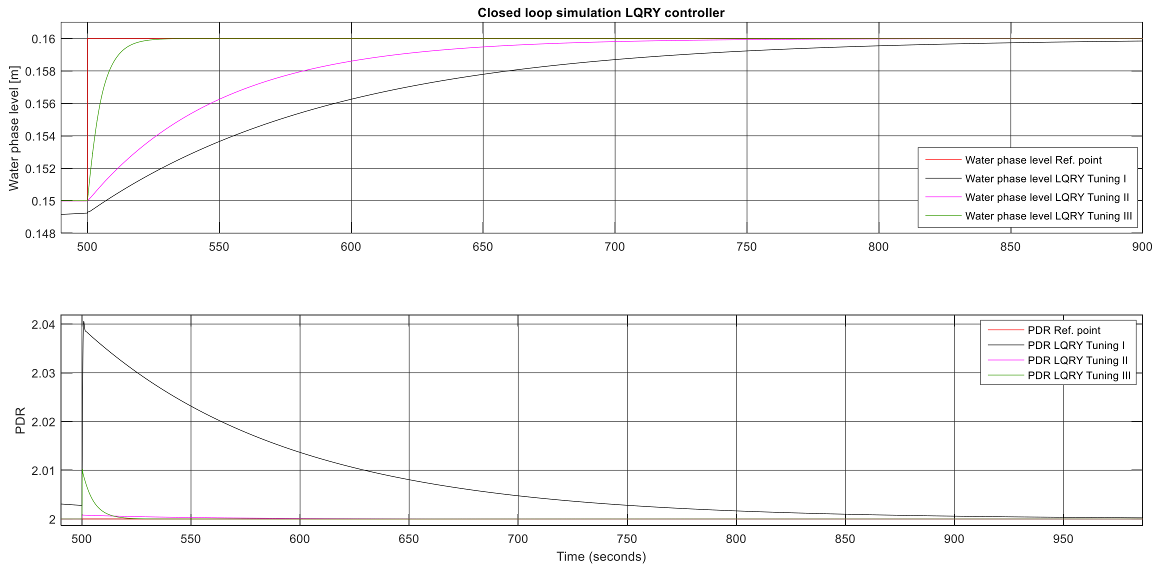
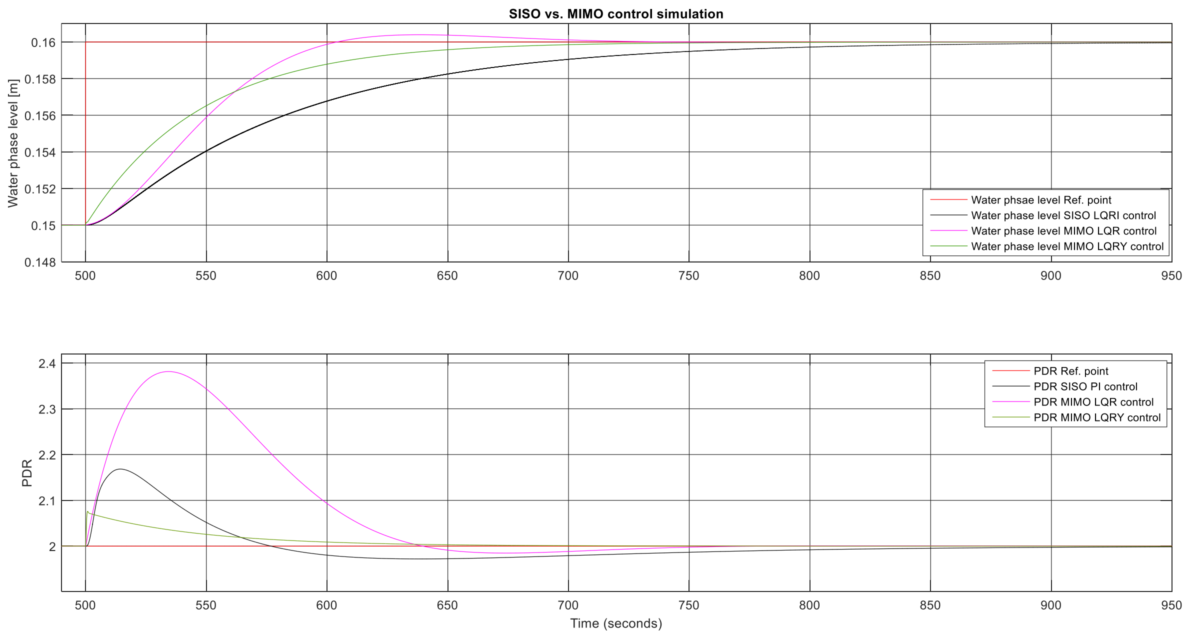


Fig. VI.3 LQRI Closed loop system responses.

The simulation results of the closed loop level responses with “LQRY control structure” are following the introduced reference point signal. A steady state error is observed in the level response marked with black curve, indicating a lower range of values which can be used for the tuning procedure. The magenta and green curves from Fig.VI.4 are obtained by increasing the weights of the two system outputs. The best results regarding the two control objectives are presented by the system responses indicated with magenta color. The rise time regarding the water phase level is in acceptable range ensuring slow but steady transition to the next reference point. In addition, the associated peak observed in the simulated PDR response is reduced indicating good reference tracking performance regarding the PDR. Further increase in the considered weights causes even faster response for both system outputs, in tradeoff of increasing the observed overshot in the simulated PDR response. Based on the obtained results is concluded that the derived “MIMO LQRY control structure” is achieving good simulation results with appropriate tuning of the required weighting matrices. Furthermore, the trial and error tuning procedure is simplified, considering that two weighting parameters associated with the specified inputs and outputs of the MIMO system must be selected. Finally, it is concluded that the obtained “LQRY control structure” is better choice for tuning the controller parameters. However, the implementation cost of the second controller is identical with the “MIMO LQR controller” considering that full state feedback is required and a state estimator must be considered before implementation.

VI.II Comparison between MIMO and SISO control of the gravity and hydro-cyclone separator systems

The final step considered in a simulation comparison between the closed loop responses with two independent control structures acting on the control objectives, and the cooperative “MIMO controllers”. The simulation results are presented in the following figure.



The closed loop simulation results obtained with "MIMO and SISO controllers" are similar regarding the water phase level objective. Different rise and settling time are obtained by considering the different weighting values found for each controller. In contrast the simulated PDR responses are having similar transient behavior but different overshoot amplitudes. The SISO "PDR controller" is producing smaller overshoot in trade off of undershoot before settling down the PDR to the given reference point. The overshoot is amplified after simulation with the "MIMO LQR controller", concluding that the chosen weighting values for each state must be reconsidered in order to obtain better results. The closed loop system responses with "MIMO LQRY control structure" provides better results regarding the simulated PDR response. The observed overshot is decreased allowing to obtain closer to constant results regarding the PDR.

Based on the obtained results is concluded that a tradeoff between treating the two systems independently or as one system is existing. The case of two independent controllers acting on the level and PDR, requires two sets of specifications and stage by stage tuning of the controllers in order to obtain reasonable closed loop performance for each part of the two systems. Furthermore, the exerted control action on the water phase level is acting as unknown disturbance to "PDR controller" and vice versa, indicating that a tradeoff in the tuning of the two controllers is required in order to obtain reasonable solutions. In contrast the "MIMO solution" provides coordinated action for the two manipulated valves, indicating that an account for the influence of one control objective to the other is considered. Finally, the retuning procedure of the "MIMO control structures" is considered faster from implementation point of view, considering the reduction of tuning parameters required.

VI.III. Conclusions of Closed loop MIMO system simulations

Two solutions for the derivation of MIMO control structures are implemented and simulated. First an "MIMO LQR control structure" is derived and simulated. Several decisions and problems are considered in the used trial and error method of finding suitable weighting values for the required matrices. First of all, the three used SISO models which are forming the MIMO model are having different scaling, implying that a rescaling of all model parameters may produce better results. The tuning procedure is extended because four of the model states are unknown, introducing variety of value candidates for the state weighting matrix, indicating higher chance of error and unrealistic closed loop system responses.

The obtained "MIMO LQR solution" is reformulated to "MIMO LQRY control structure" in order to reduce the tuning parameters by applying weights directly to the system outputs instead to the system states. The time for tuning and simulating the "MIMO LQRY control structure" is significantly reduced in comparison with the standard "LQR formulation". Similar simulation results are obtained for the water phase level response. In contrast the closed loop PDR response with "LQRY control structure" provides better results. The observed overshoot in the PDR response is decreased, concluding that a "LQRY solution" provides more adequate tuning options in the case of unknown system states.

The simulated closed loop MIMO system responses are compared to simulated closed loop responses obtained with two independent controllers acting on the water phase level and PDR. The transient behavior of the two system outputs is similar for both control cases. Improvement in the simulated closed loop PDR response is achieved by applying the “MIMO LQRY solution”, impaling that the coordinated control of the two manipulated valves considers the internal relationship between the system responses. In contrast with the “SISO control case” in which the relationship must be considered as unknown disturbance influencing each independent system based on one of the control objectives. Finally, it is noted that the derived “MIMO model and controllers” are building on top of the independent gravity and hydro-cyclone models allowing to obtain the same or better closed loop performance simultaneously and based on fewer tuning parameters.

VII. Project conclusions

Conclusions of gravity separator category of sub-problems

Mathematical model describing the level in the considered gravity separator system is defined. The model is derived from first principles of physics. An experiment with defined operating conditions is conducted to collect empirical data. The collected data is processed and prepared. Open loop simulation of the nonlinear model is executed using the prepared dataset. The simulation results are compared afterwards with the unfiltered water level measurements, being visible that the simulation model follows the same dynamics as the empirical data set.

After linearizing the nonlinear model around the operating point obtained from the empirical data, the first order LTI system is defined and its simulation has been executed considering the operating conditions of the validation experiment. After the comparison of the linear level response, nonlinear model and the measured level responses, it was seen that the linear model follows the behavior of the nonlinear model, concluding that the linearization is successful and the water level model for control purposes is obtained.

The defined LTI system is used to tune a PI controller with the help of simulation. Afterwards the PI controller is implemented and retuned in order to fit the defined specifications.

The linear model is used to define also an IMC structure, which is simulated and implemented. Steady state error is observed after the implementation, concluding that the LTI system is not accurate for an IMC design. By cascading the IMC with a PI controller, the observed steady state error is eliminated, thus enhancing the performance of the designed IMC.

Finally, a LQR is designed and implemented. The steady state error of the water level response is observed in the measurements, but it is not indicated in the simulation made during the LQR design, which indicates that the limitation in the LQR design is reached.

The LQR design is reformulated by introducing an error integration state. After simulating and implementing the new LQRI control structure, it results that the steady state error is eliminated.

The responses of the three control structures are compared under ideal operating, concluding that all three of them are good candidates for the water level control of a gravity separator system. However, during the simulation of the three controllers under worst case scenario operating conditions, it is seen that the PI and IMC control structures are experiencing failure, only the selected LQRI controller being able to achieve marginal stability.

The final simulation under worst case scenario conditions, offers the conclusion that the LQRI control structure through its robustness achieves a better stability without influencing the order of the systems model and also for a small implementation cost, thus providing the solution to the first set of project specifications and requirements.

Conclusions of hydro-cyclone category of sub-problems

Two identification experiments are designed and performed, connecting the required input–output data sets. After processing the identification data, a parametric model estimation is executed and four transfer function models where found that are satisfying the first validation criteria.

After further analysis, two of the four model candidates are discarded, obtaining final model selection. The models are simulated using empirical data from second set of experiments. The selected models have been validated for small range of input-output deviations, concluding that the selected models are suitable for control purposes and obtaining the black box model relating the two hydrocyclone inputs to the associated PDR calculation, also providing the solution to the second set of sub-problems mentioned in the project specifications and requirements.

Conclusions of the MIMO category of sub-problems

The defined gravity and hydrocyclone models are combined forming a MIMO system. The new system is validated on two stages, by comparing the simulation and empirical data, concluding that the defined MIMO model is suitable for control purposes.

Standard LQR solution is defined for the MIMO case and simulated, offering results which have been compared to the simulated results obtained while treating each system separately. This offers the conclusion that appropriate tuning is hard to achieve, considering the unknown states obtained during the hydrocyclone model estimation.

The defined MIMO LQR is redefined in terms of input and output terms, decreasing the tunable parameters number. The new LQRY control structure is simulated and compared using the same approach applied in the MIMO LQR simulation, concluding that the response of the simulated LQRY control structure offers a similar stability as the other control structures functioning independently, although no validation can be done at the moment.

Final conclusions and future work

After comparing the obtained results in the validation experiments and also under a worst case scenario simulation, it can be concluded that the project has achieved a stable control structure for the gravity separator and also a stable simulated control structure of the MIMO system, meeting the project specifications and requirements.

One of the first steps that can be done in any future work of this control system will be the implementation of the MIMO system control structure and validating it against experimental data. Also an important step of the future development, before implementing the control structure in real life, will be the expansion of the working system to be able to handle more degrees of freedom.

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Appendix

AI. Derivation of cross-section area for nonlinear water phase model

Split the original cross section area equation to three separate functions;

$$f\big(h_w(t)\big) = R^2 \operatorname{acos}\left(\frac{R - h_w(t)}{R}\right) \tag{AI}$$

$$g\big(h_w(t)\big) = R - h_w(t) \tag{AI.a}$$

$$k\big(h_w(t)\big) = \sqrt{2Rh_w(t) - h_w^2(t)} \tag{AI.b}$$

Differentiate each function with respect to the water phase level variable:

$$\frac{\partial f\big(h_w(t)\big)}{\partial h_w} = \frac{R}{\sqrt{1 - \frac{(h_w(t) - R)^2}{R^2}}} \tag{AI.c}$$

$$\frac{\partial g\big(h_w(t)\big)}{\partial h_w} = -1 \tag{AI.d}$$

$$\frac{\partial k\big(h_w(t)\big)}{\partial h_w} = -\frac{h_w(t) - R}{\sqrt{(2Rh_w(t) - h_w^2(t)}} \tag{AI.e}$$

Substitute the obtained results into (AI.f) in order to obtain expression in terms of the first derivative of the water phase level

$$dA_w = \left(\frac{\partial f\big(h_w(t)\big)}{\partial h_w} - \left(\frac{\partial g\big(h_w(t)\big)}{\partial h_w}\, k\big(h_w(t)\big) + \frac{\partial k\big(h_w(t)\big)}{\partial h_w}\, g\big(h_w(t)\big)\right)\right)\frac{dh_w(t)}{dt} \tag{AI.f}$$

AIII. Full state feedback and reference tracking input

System:

$$\frac{dh_w}{dt} = ah_w + [b_1\ b_2]\begin{bmatrix} u_w \\ P_g \end{bmatrix} \qquad (AIII)$$

$$h_w = ch_w + dF_{in} \qquad (AIII.a)$$

Control formula:

$$u_w = u_{wss} - K(h_w - h_{wss}) \qquad (AIII.b)$$

From which follows that:

$$h_{wss} = N_x r_{ss} \qquad (AIII.c)$$

$$u_{wss} = N_u r_{ss} \qquad (AIII.d)$$

Considering the steady state system and substituting (AIII.c and d) solution for “Nx and Nu” is obtained as follow.

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} a & b_1 \\ c & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (AIII.e)$$

The combined gain for reference tracking of the state and the control input of the system is obtained in (AIII.f).

$$N_r = N_u + KN_x \qquad (AIII.f)$$

Allowing to rewrite the control law from (AIII.b) for reference tracking purposes as follow.

$$u_w = -Kh_w + N_r r \qquad (AIII.g)$$

AIV. Conversion between state space and transfer function representation

In order to evaluate the system and keep consistency the obtained linear model is converted from state space representation to transfer function representation. By using the command “ss2tf(A, B, C,0)” the model for the water phase level is reduced to SISO case in which a transfer function describing the water phase relief valve is obtained, with restrictions of unknown input disturbance and unknown gas phase pressure. Expressed as equation the conversion is performed as in AIV

$$G_p(s) = C(sI - A)^{-1}B = \frac{b}{s+a} \qquad (AIV)$$

AV. Steady state error of PI controller

Given the defined "PI controller" and the obtained transfer function describing the process model a closed loop transfer function is obtained as follow.

$$G_{cl}(s) = \frac{PI(s)G_p(s)}{1 + PI(s)G_p(s)} = \frac{sbK_p + K_i b}{s^2 + (a + bK_p)s + K_i b} \qquad (AV)$$

The associate transfer function relating the output of the system to the reference point input referred as "Γ(s)" is equivalent of AV and used to defined the error signal as in AV.a.

$$E(s) = H_w(s) - R(s) = \Gamma(s)\mathrm{R}(s) - \mathrm{R}(s) \qquad (AV.a)$$

By rearranging AV.a the reference to error transfer function is obtained in AV.b

$$G_{re}(s) = \frac{E(s)}{R(s)} = \Gamma(s) - 1 \qquad (AV.b)$$

Considering the Final Value theorem and the case of polynomial test input the steady state error of the developed controller is obtained as in AV.c

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{\Gamma(s) - 1}{s^k} = \lim_{s\to 0} \frac{-sK_p b - K_i b}{s^k} \qquad (AV.c)$$

Where the only solution to AV.c is if "k=0" proving that a small steady state error is obtained by solving the limits in AV.c.

AVI. IMC controller derivation

Process model:

$$G_p(s) = \frac{b}{s+a} \qquad (AVI)$$

In order to simplify the "IMC design procedure" several factors are considered. From (AVI) is seen that the process model does not contain right half plane (RHP) zeros, ensuring that the controller is stable and the closed loop system will not lose stability. The existing process delay of the system is considered unknown, allowing to obtain the "IMC controller" transfer function as in (AVI.a).

$$G_c(s) = G_p(s)^{-1} = \frac{s+a}{b} \qquad (AVI.a)$$

The obtained controller from (AVI.a) is an ideal case which agrees with the first two conditions of an "IMC design procedure". However, the controller results in improper transfer function, causing implementation issues and greater sensitivity to changes in the feedback error. If the perfect controller is implemented using classical control configuration the following transfer function is obtained.

$$G_{imc}(s) = \frac{G_c(s)}{1 - G_c(s)G_p(s)} = \frac{G_p(s)^{-1}}{1 - \frac{G_p(s)}{G_p(s)}} = \infty \qquad (AVI.b)$$

Equation (AVI.b) implies that the obtained perfect controller would need infinite control action (infinite gain) to achieve perfect control. Under such conditions it is decided to modify the "IMC controller" by a first order low-pass filter function to solve the improperness issue. The new controller transfer function is obtained in (AVI.b).

$$G_c(s) = f(s)G_p(s)^{-1} = \frac{s+a}{b}\frac{1}{\lambda s + 1} = \frac{s+a}{\lambda b s + b} \qquad (AVI.c)$$

The defined filter time constant λ is a tunable parameter allowing to tune the speed of the closed loop system response. The value of twenty-five is chosen as filter time constant, ensuring operating range in slow frequency band in which the system operates.

Given that the process model does not have perfect knowledge of the process and considering all disturbances to the system are unknown provides the following closed loop transfer function with "IMC control structure".

$$H_w(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)\left(G(s) - G_p(s)\right)} H_{w_{ref}}(s) \qquad (AVI.d)$$

Under the conditions that the process, process model and derived controller are stable all conditions of the "IMC design procedure" are satisfied and the stability of the closed loop system is ensured.

AVII. Solution to LQR problem

"Bryson's rule":

State that the values for "Q and R" weighting matrices can be chosen under the following criterion:

$$Q = \frac{1}{h_w^2} \tag{AVII}$$

$$R = \frac{1}{u_w^2} \tag{AVII.a}$$

In essence the "Q and R" are scaling the variable appearing in the defined cost function so that the maximum acceptable value for each term is one. Providing reasonable choice given that the units of the state variable and input variable are differing numerically from each other. IT is noted that "Bryson's rule" is a starting point to a trial and error method for determining values matching the closed loop system specifications and requirements.

The chosen value for "Q" is based on the maximum allowed water phase level which can be maintained inside the separator vessel. In the case of this system the water phase level must not exceed "0.3" meters, in order to prevent water dripping into the oil phase container. By trial and error, a different value for the R weighting matrix is obtained.

The construction of optimal LQR feedback gain evolves a symmetric positive solution "P" to the "algebraic Riccati equation" stated in (AVII.b).

$$A^T P + PA + Q - (PB)R^{-1}(B^T P) = 0 \tag{AVII.b}$$

for which the closed loop system matrix "$A - BR^{-1}(B^T P)$" is Hurwitz, allowing to rewrite AVII.b in vector form as follow.

$$[P - I]H \begin{bmatrix} I \\ P \end{bmatrix} = 0 \tag{AVII.c}$$

Where "H" is the Hamiltonian matrix expressed in AVII.d

$$H \coloneqq \begin{bmatrix} A - BR^{-1} & -BR^{-1}B^T \\ -Q + R^{-1} & -(A - BR^{-1})^T \end{bmatrix} \epsilon\, \mathcal{R}^{2nx2n} \tag{AVII.d}$$

Given the derived linear model is a single input single output (SISO) system the Hamilton matrix is constant. Providing enough evidence that "H" is in the domain of the Riccati operator. From which follows that the eigenvalue of AVII.d is indeed a root of the optimal, constant gain, closed loop system.

By knowing the optimal roots, a solution for the LQR feedback gain is obtained by using Ackermann's formula presented in AVIII.e.

$$K_{\infty} = R^{-1}(B^{T}P) \qquad (AVII.e)$$

It must be noted that the stated LQR theory is in agreement with the procedure used by the predefined LQR function available in the used computational software, allowing to use the function and validate against the set of constrains associated with solving of the LQR problem.

AVIII. Solution for integral control

The water phase level dynamics are described by the following state space design.

$$\frac{dh_w}{dt} = Ah_w + Bu_w \qquad (AVIII)$$

$$h_w = Ch_w \qquad (AVIII.a)$$

The integral of the system error is defined as in (AVIII.b)

$$e = h_w - r_{h_w} \qquad (AVIII.b)$$

Allowing to use (AVIII.b) as an expression for the extra integral state "x_i" which obeys the following differential equation

$$\dot{x}_i = Ch_w - r_{h_w} \qquad (AVIII.c)$$

Used to define the integral state as in (AVIII.d)

$$x_i = \int_0^t e\, dt \qquad (AVIII.d)$$

The derived state space model describing the dynamics of the water phase level (AVIII) is augmented with the obtained error differential equation (AVIII.c) by obtaining the following state equations.

$$\begin{bmatrix} \dot{x}_i \\ \dot{h}_w \end{bmatrix} = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix} \begin{bmatrix} x_i \\ h_w \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u_w - \begin{bmatrix} 1 \\ 0 \end{bmatrix} r_{h_w} \qquad (AVIII.e)$$

The standard feedback law for the newly defined system (AVIII.e) is defined in (AVIII.f)

$$u_w = -[K_1\ K_0]\begin{bmatrix} x_i \\ h_w \end{bmatrix} \qquad (AVIII.f)$$

The control gain values defined in (AVIII.f) are calculated using pole placement procedure based on the obtained augmented model including the additional integral state. The derived in (AVIII.e) model is second order considering the amount of states, allowing to obtain the following numerical expression of the system based on the selected pole locations.

$$\left(s + p_{x_i}\right)\left(s + p_{h_w}\right) = s^2 + \left(p_{x_i} + p_{h_w}\right)s + \ p_{x_i}\, p_{h_w} \qquad (AVIII.g)$$

The augmented system defined in (AVIII.e) is related to the predefined second order system from (AVIII.g) by evaluating the determinant of the defined system as in (AVIII.h), allowing to obtain expression relating the two control laws required to obtain solution to the integral control problem.

$$\det\left(sI - \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix} \ + \ \begin{bmatrix} 0 \\ B \end{bmatrix}[K_1\ K_0]\right) = s^2 + (A + BK_0)s + BK_1 \qquad (AVIII.h)$$

Providing the second order function obtained by pole placement procedure (AVIII.g) as a solution to the derived second order equation (AVIII.h), describing the closed loop system with integration of the error, allows to obtain isolated expression (AVIII.i) for the unknown control gains based on the specified pole locations.

$$K_0 = \frac{p_{x_i} + p_{h_w} - A}{B} \qquad (AVIII.i)$$

$$K_1 = \frac{p_{x_i}\, p_{h_w}}{B} \qquad (AVIII.ii)$$

The manually calculated feedback gain values from (AVIII.i) are compared to the output of predefined software function for calculating integral control gains based on the derived LQR design by inputting the state and input weighting matrices from the derived LQR control structure.